Clutches and Brakes

Clutches and brakes depend on friction in order to function.

Design objectives of brake and clutch design:
- Maximize friction
- Keep it uniform over a wide range of operation
- Minimize wear.

Clutch:
The function of a *clutch* is to permits smooth, gradual connection and disconnection of two members with common axis of rotation.

Brake:
Acts the same as the *clutch* but one member is fixed.

Types of clutches and brakes:

1- Friction type
2- Magnetic
3- Eddy current
4- Hydrodynamic (fluid coupling)

In this course, we will consider only friction type brakes and clutch

Friction Type Clutches and Brakes:

These types of brakes must satisfy the following requirement
1- The friction torque must be produced by acceptable force.
2- The energy produced must be dissipated without producing high temperature.
3- Reduce friction wear to give acceptable life.

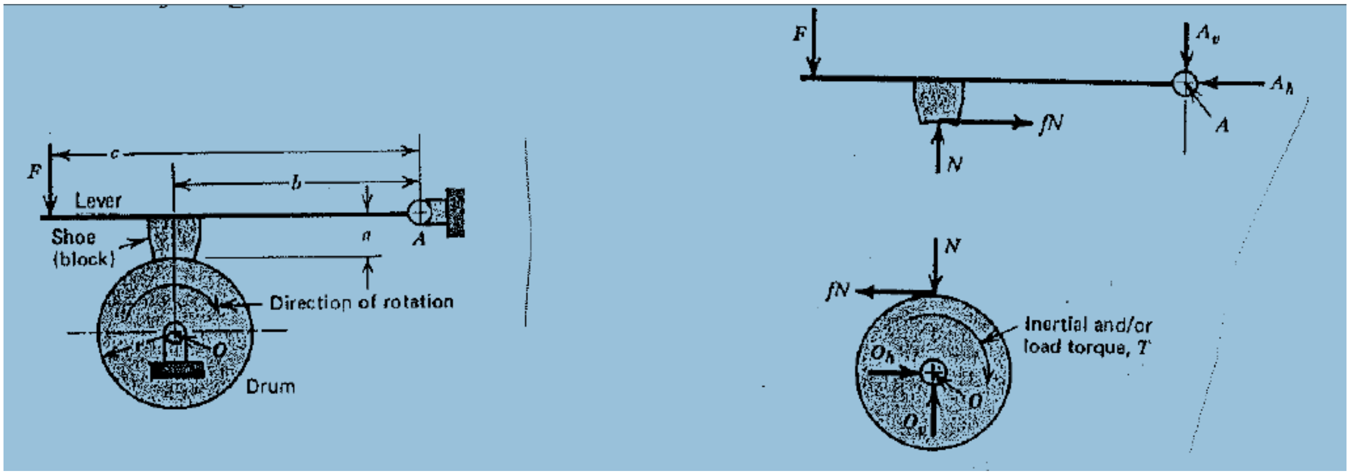
Drum Brakes

Drum brakes are of two types:
1- *External Shoe* that contracts to bear against an outer drum surface.
2- *Internal Shoe* that expands to contact the inner drum surface.

Short- Shoe Drum Brake:

The shoe contact only a small segment of drum periphery
➔ *Short-Shoe* Analysis: assume that the friction and normal force act at the center of contact.

A Free- Body diagram for clockwise drum rotation:



Taking moment about pivot A for the shoe and lever:

$$\sum M_A = 0 \qquad \Rightarrow \qquad Fc + fNa - bN = 0 \qquad (1)$$

Taking moment about O of the drum:

$$\sum M_O = 0 \qquad \Rightarrow \qquad T = fNr \qquad (2)$$

Solving (1) for N and substituting in (2):

$$\Rightarrow \quad N = \frac{Fc}{(b - fa)}$$

$$\Rightarrow \quad T = \frac{fFcr}{(b - fa)}$$

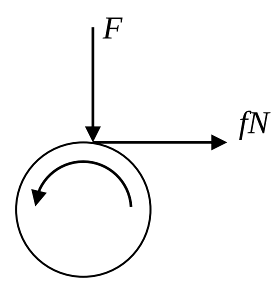
T = friction torque[Inertial or load torque required for equilibrium]

Self-Energizing and Self-de-Energizing Brakes:

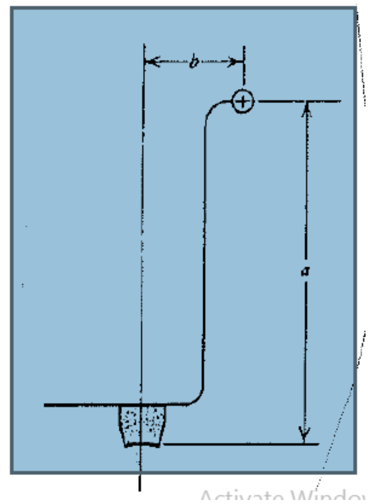
A brake is considered *self- energizing* if the moment friction force (fNa) assists the applied force (F) in applying the brake (reduce the force F).
Self-energizing is produced for *cw* rotation of the drum.
Self-de-energizing brake: if the moment of friction force opposes the application of the brake (increase the force F required for brakeing).
Self-de-energizing is produced for *ccw* rotation of the drum.

Friction force opposes direction of rotation

$$T = \frac{fFcr}{(b + fa)}$$



ccw



Note:

T (self-energizing) > *T* (self-de-energizing)

Self Locking Brake:

For cw rotation: $$T = \frac{fFcr}{(b - fa)}$$

If : $(b\text{-}fa) \leq 0$ ➔ no torque required to produce the brake (self-locking)

Self -Locking brake ➔ $b \leq fa$

For example: For cw rotation if f= 0.3
Self locking ➔ $b \leq 0.3a$
➔ Self locking: $F = 0$ (no force is required to produce the brake)

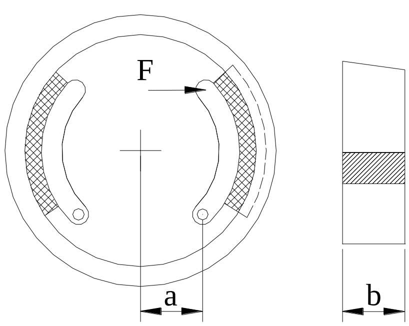
Long Shoe Drum Brake:

Long –Shoe: contact arc $\theta \geq 45°$.

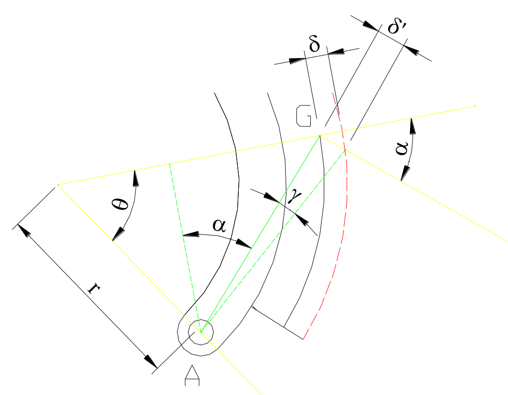
- Internal Shoe
- External Shoe

Internal Shoe Drum Brake:

It is normally used for automotive rear wheel .
Function:
Both shoes pivot about pin and are forced against the inner surface of the drum by a hydraulic piston at the ends. The spring is used to retract the shoes to the original position



(a)



(b)

To determine the pressure *p* of lining on the drum caused by applying the force *F* on the left shoe which is deformed (as shown by dashed line)
δ = Radial deflection

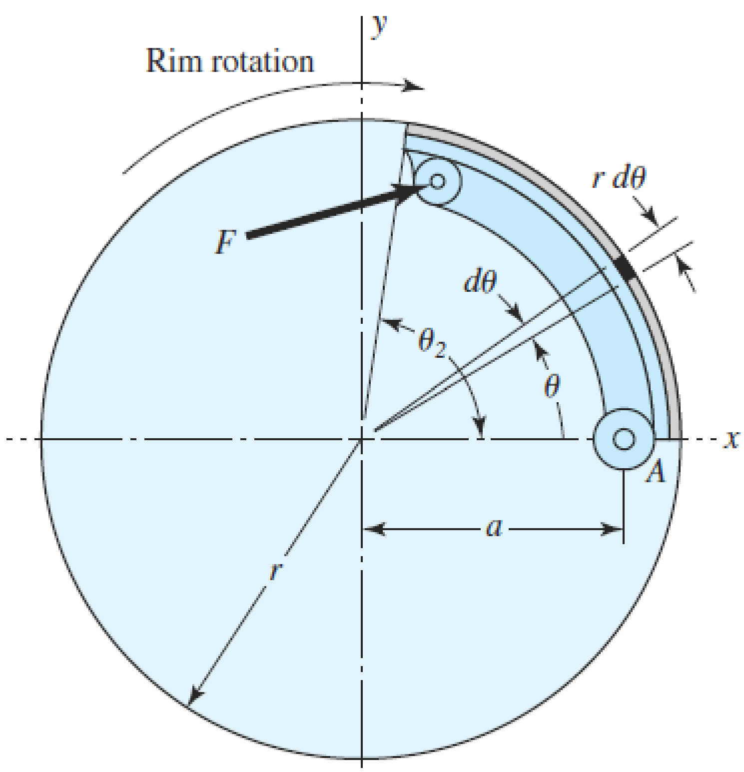
$\delta = \delta' \cos\theta = \gamma AG \cos\alpha$ γ= small angle of shoe rotation.
$\delta = r\sin\theta = \gamma r \sin\theta$ $AG\cos\alpha = AN = r\sin\theta$
$p \propto \delta \propto \sin\theta$ $p = k\delta$, p= pressure on the drum, k= stiffness constant of lining.

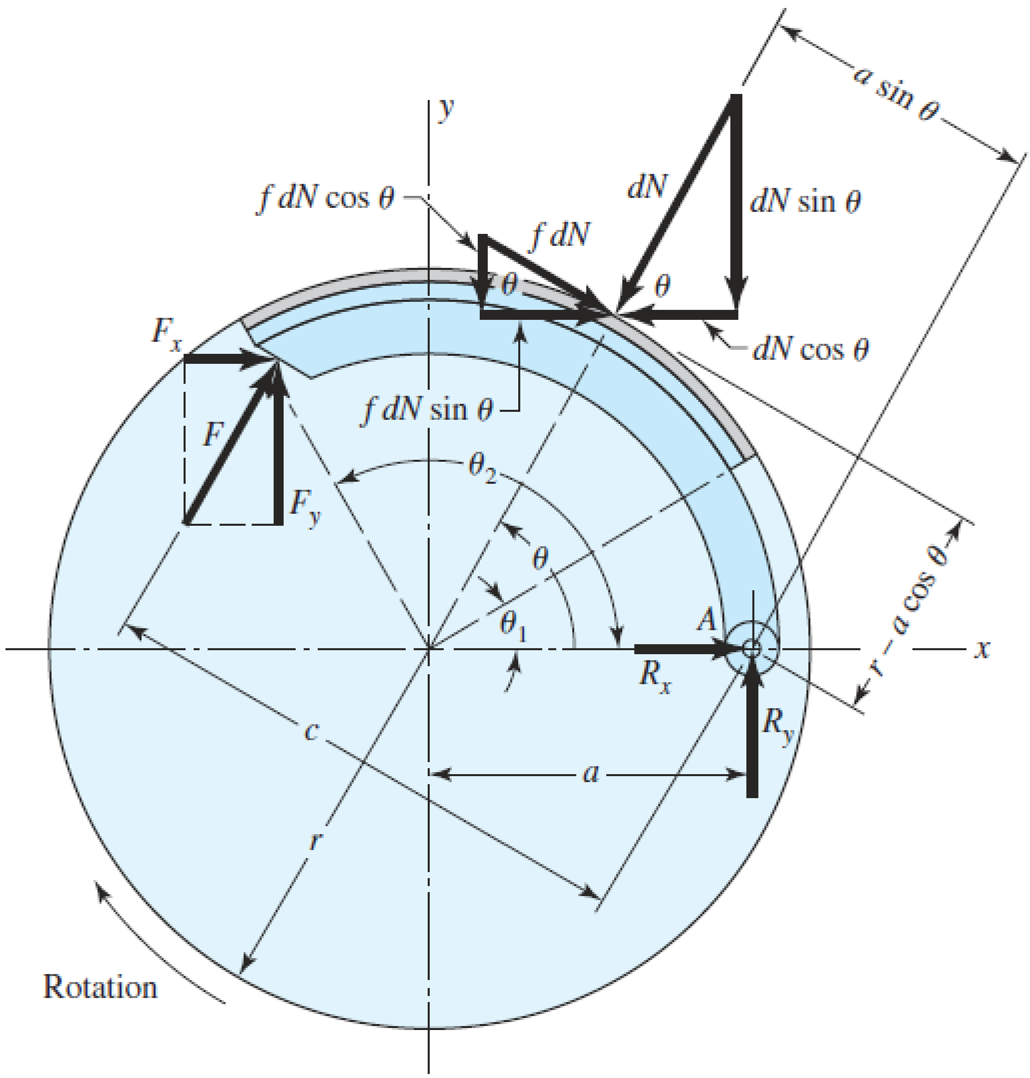
p_{max} ➔ $\theta = \theta_{max}$

$$\Rightarrow \quad \frac{p}{\sin\theta} = \frac{p_{max}}{\sin\theta_{max}} \qquad (1)$$

$(\sin\theta)_{max} = 1.0$ when $\theta_{max} = 90^\circ$ $\Rightarrow$ at $(\theta = 90)$ $p = p_{max} \Rightarrow$ max wear

➔ Good design: max thickness of friction material at point of max. pressure.





It is required to find the friction torque and the reactions at the hinge of the shoe.

Drum rotating *cw*:

Taking an infinitesimal area (*dA*)then integrating over the whole contact area

$dA = brd\theta$ where: b = face width of friction material

$dN = pbrd\theta$ (2)

From (1):

$$p = \frac{p_{\max}}{\sin\theta_{\max}}\sin\theta$$

(3)

Substituting (3) in (1):

$$\Rightarrow \qquad dN = p_{\max} \frac{br\sin\theta \ d\theta}{\sin\theta_{\max}}$$

<u>To find the actuating force *F*:</u>

1- Frictional moment about A:

$$M_f = \int fdN(r - a\cos\theta) = \frac{fbrp_{\max}}{\sin\theta_{\max}} \int_{\theta 1}^{\theta 2} \sin\theta(r - a\cos\theta)d\theta$$

2- Normal force Moment about A:

$$M_N = \int dN(a\sin\theta) = \frac{brap_{\max}}{\sin\theta_{\max}} \int_{\theta 1}^{\theta 2} (\sin\theta)^2 d\theta$$

3- Summing moment about A

$$\sum M_A = Fc - M_N + M_f = 0 \qquad \Rightarrow \qquad F = \frac{M_N - M_f}{c}$$ ➔ (*self-energizing*)

If $M_N = M_f \ \Rightarrow \ \mathrm{F} = 0$ ➔ *(Self-locking)* No actuating force is required .

In Automotive application, brake must be strongly self – energizing while staying away from self-locking.

$$\Rightarrow \qquad M_N \geq M_f \qquad \text{recommended value}: M_N = (1.25 - 1.5) \quad M_f$$

<u>Frictional Torque:</u>

$$T = \int dF_f r = \int frdN = \frac{fbr^2 p_{\max}}{\sin\theta_{\max}} \int_{\theta 1}^{\theta 2} \sin\theta \, d\theta = \frac{fbr^2 p_{\max}(\cos\theta_1 - \cos\theta_2)}{\sin\theta_{\max}}$$

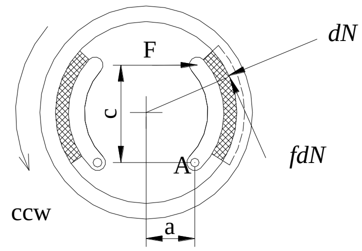
<u>Reaction at hinge A:</u>

$$\sum F_h = 0 \quad \Rightarrow \quad R_x = \frac{brp_{\max}}{\sin\theta_{\max}}(A - fB) - F_x$$

$$\sum F_v = 0 \quad \Rightarrow \quad R_y = \frac{brp_{\max}}{\sin\theta_{\max}}(B + fA) - F_y$$

<u>**Drum rotating *ccw*:**</u>

The friction force is opposite to the direction of rotation



$$\sum M_A = Fc - M_N - M_f = 0 \qquad \Rightarrow \qquad F = \frac{M_N + M_f}{c}$$ ➔ (*Self-de-energizing*)

$$R_x = \frac{brp_{\max}}{\sin\theta_{\max}}(A + fB) - F_x$$

$$R_y = \frac{brp_{\max}}{\sin\theta_{\max}}(B - fA) - F_y$$

where:

$$A = \int_{\theta 1}^{\theta 2} \sin\theta \cos\theta \, d\theta = \left(\frac{1}{2}\sin\theta\right)_{\theta 1}^{\theta 2}$$

$$B = \int_{\theta 1}^{\theta 2} \sin^2\theta \, d\theta = \left(\frac{\theta}{2} - \frac{1}{4}\sin\theta\right)_{\theta 1}^{\theta 2}$$

Note: the reference is always in the center of the drum. The positive x-axis is taken through the hinge. The positive y-axis is always in the direction of the shoe.

<u>Internal Shoe Drum Brake:</u>

- Normally used in automotive rear wheel
- Energizing and de-energizing characteristics are determined by:
    1- Noting the direction of friction force acting upon the shoe surface
    2- Determining if this force tends to bring the shoe into or out of contact of the drum

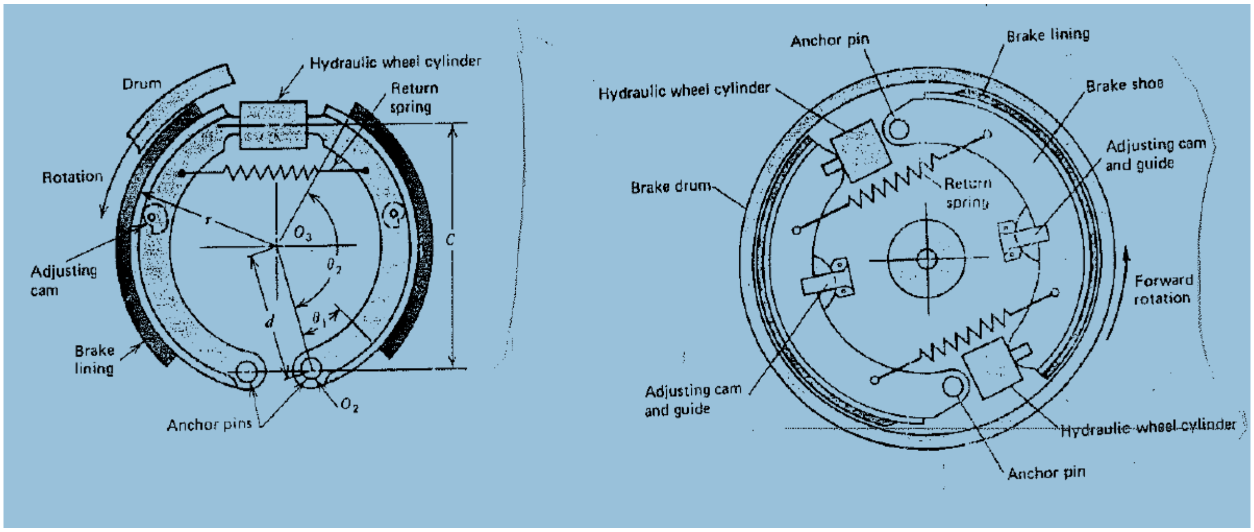
For the left figure shown below:

Left shoe: self – energizing
Right shoe: self-de-energizing

In automotive brakes ➔ self-energizing is desired ➔ decrease pedal force
But self-locking must be avoided

<u>Improved Design:</u>

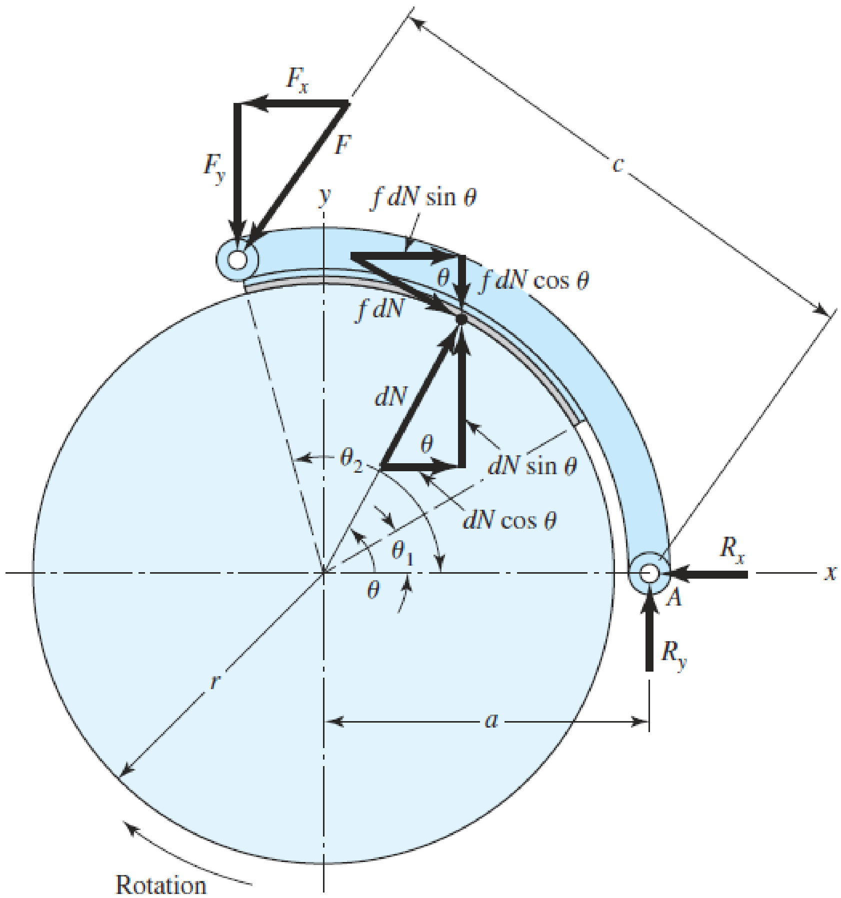
The shoes are inverted and each one is actuated by a separate cylinder.
This arrangement will increase the self- energizing action
➔ Both shoes are self-energizing in forward car motion.
➔ This arrangement is used in automotive front wheels.



<u>Long Shoe Drum Brake with External Shoe:</u>

Consider the external brake shown in the figure. We have proved in previous sections that:

$$p = \frac{p_{\max}}{\sin\theta_{\max}} \sin\theta$$



1- <u>Drum rotating *cw*:</u>

Summing moment about A:

$$\sum M_A = Fc - M_N - M_f = 0$$

$$\Rightarrow \quad F = \frac{M_N + M_f}{c} \qquad \text{(Self-de-energizing)}$$

Large force is required to balance these moments (friction moment does not assist the force to produce the brake).

Where M_N is the moment of normal force:

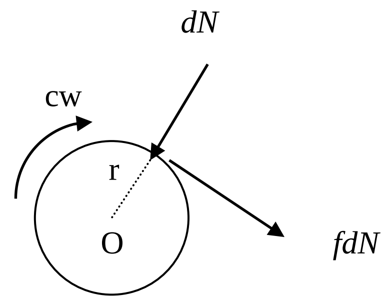
$$M_N = \int dN(a\sin\theta) = \frac{brap_{\max}}{\sin\theta_{\max}} \int_{\theta 1}^{\theta 2} (\sin\theta)^2\, d\theta = \frac{brap_{\max}}{4\sin\theta_{\max}} \left[2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1\right]$$

Moment of friction force:

$$M_f = \int f dN(r - a\cos\theta)d\theta = \frac{fbrp_{\max}}{\sin\theta_{\max}} \int_{\theta 1}^{\theta 2} \sin\theta(r - a\cos\theta)\, d\theta$$

Substituting the relation: $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$$M_f = \frac{fbrp_{\max}}{\sin\theta_{\max}}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$



Friction Torque:

$$T = \int dF_f r = \int frdN = \frac{fbr^2 p_{\max}}{\sin\theta_{\max}} \int_{\theta 1}^{\theta 2} \sin\theta\, d\theta = \frac{fbr^2 p_{\max}(\cos\theta_1 - \cos\theta_2)}{\sin\theta_{\max}}$$

Reaction forces at hinge A:

Reaction forces is obtained summing forces in horizontal and vertical direction:

$$\sum F_h = 0 \quad \Rightarrow \quad R_x = \frac{brp_{\max}}{\sin\theta_{\max}}(A + fB) - F_x$$

$$\sum F_v = 0 \quad \Rightarrow \quad R_y = \frac{brp_{\max}}{\sin\theta_{\max}}(fA - B) + F_y$$

where:

$$A = \int_{\theta 1}^{\theta 2} \sin\theta\cos\theta\, d\theta = \left(\frac{1}{2}\sin\theta\right)_{\theta 1}^{\theta 2}$$

$$B = \int_{\theta 1}^{\theta 2} \sin^2\theta\, d\theta = \left(\frac{\theta}{2} - \frac{1}{4}\sin\theta\right)_{\theta 1}^{\theta 2}$$

2. Drum Rotating ccw:

Summing moment about A:

$$\sum M_A = Fc - M_N + M_f = 0$$

$$\Rightarrow \quad F = \frac{M_N - M_f}{c} \qquad \text{(Self-energizing)}$$

For *ccw* less force is desired to obtain the brake, the friction moment assists the force to produce the brake.

If $M_f \geq M_N$ ➔ self-energizing brake is self-locking

It is often desired to make brake self-energizing, while staying away from self-locking.

➔ In brake design: $f \geq (25-50\%)$ true value, $\Rightarrow M_f \approx M_N$

Hinge Reactions:

$$\sum F_h = 0 \quad \Rightarrow \quad R_x = \frac{brp_{\max}}{\sin\theta_{\max}}(A - fB) - F_x$$

$$\sum F_v = 0 \quad \Rightarrow \quad R_y = \frac{brp_{\max}}{\sin\theta_{\max}}(-fA - B) + F_y$$

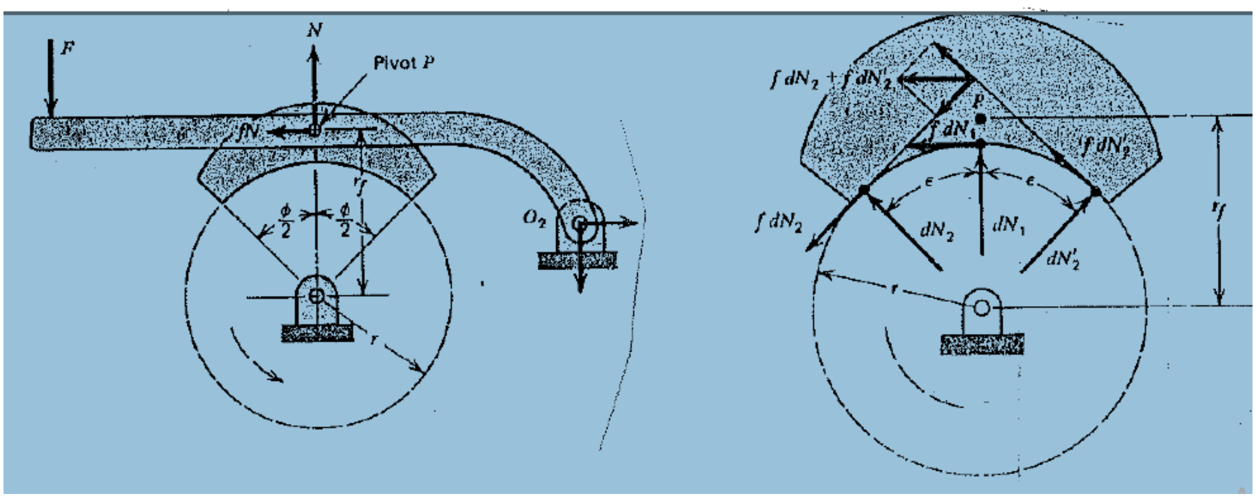
Symmetrical External Shoe Drum Brake:

If the pivot P is located at the intersection of the resultant normal force (*N*) and frictional force (*fN*) ➔ the shoe will not rotate around **P**.

This is desirable to equalize wear.

In practice pivot **P** will move closer to the drum as wear occurs

➔ increase the tendency of the shoe to pivot around **P**.



Integrating over the symmetrical shoe surface, the intersection point of the normal and friction forces r_f:

$$\Rightarrow \quad \left(\int M_f\right)_{rf} = 0$$

$$r_f = \frac{4r\sin\left(\phi/2\right)}{\phi + \sin\phi}$$

where:

r_f : is measured from the drum center. Note that $r_f > r$

ϕ = total shoe angle.

With the pivot placed at this point, the moment about P is zero. The reactions at P are:

$R_x = fN$ →

$R_y = N$ ↓

Friction Torque:

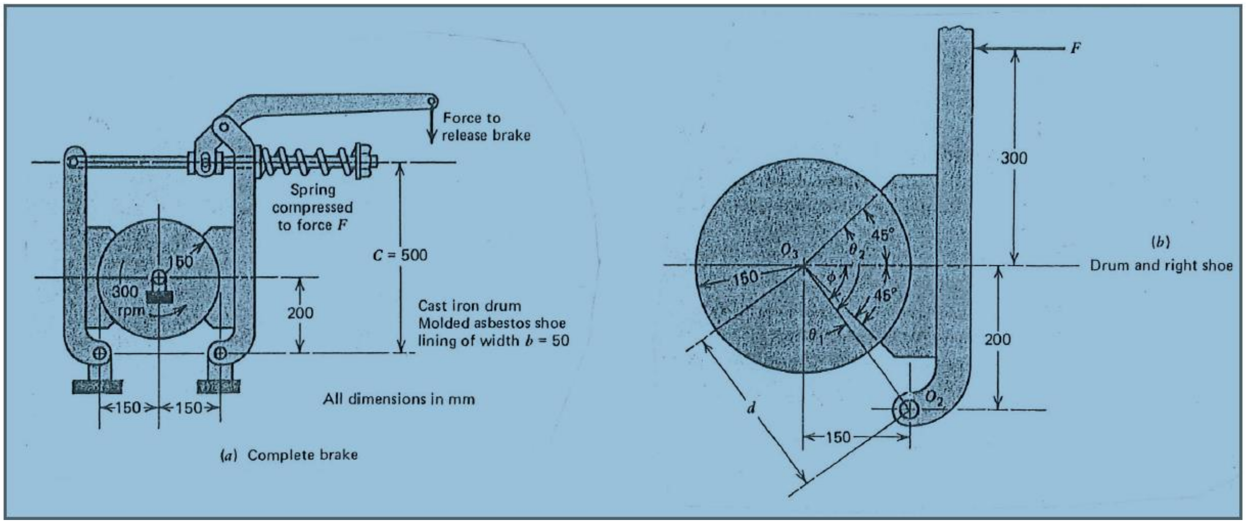
The friction torque, which is equal to the brakeing torque is:

$T = fNr_f$

where: $r_f > r$, because the resulting frictional force acting on any symmetrical shoe is located above the drum surface.

Example:

Double shoe drum brake with spring force F applied at distance c= 500 mm to both shoes. The brake is released by a solenoid. If f = 0.3 and P_{max} = 446 kPa, b=50 mm. Determine an appropriate value of spring force, and the resulting brake torque and power absorption for 300 rpm drum rotation in either directions.



Solution:

1- Right shoe:

$\phi = \tan^{-1}(200/150) = 53.13°$ ➔ $\theta_1 = 8.13°$ and $\theta_2 = 98.13°$.

2- $a = \sqrt{200^2 + 150^2} = 250$ mm.

3- $\theta_2 > 90$ deg., $\sin\theta_{max} = 1$

4- $M_N = \dfrac{brap_{max}}{4\sin\theta_{max}}\left[2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1\right]$

$$M_N = \frac{(50)(150)(250)p_{max}}{4\times 1}\left[2\left(\frac{\pi}{2}\right) - \sin 196.26^\circ + \sin 16.26^\circ\right] = -1735\times 10^3\, p_{max},$$

P_{max} in MPa

5- $M_f = \dfrac{fbrp_{max}}{\sin\theta_{max}}\left[r(\cos\theta_1 - \cos\theta_2) + \dfrac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$

$$M_f = \frac{fp_{max}(50)(150)}{1}\left[150(\cos 8.13^\circ - \cos 98.13^\circ) + \frac{250}{4}(\cos 196.26^\circ - \cos 16.26^\circ)\right] = 373\times 10^3\, fp_{max}$$

6- Brake Torque

$$T = \frac{fbr^2 p_{max}(\cos\theta_1 - \cos\theta_2)}{\sin\theta_{max}} = \frac{(50)(150)^2\, fp_{max}(\cos 8.13^\circ - \cos 98.13^\circ)}{1} = 1273\times 10^3\, fp_{max}$$

$$\sum M_{o1} = Fc + M_f - M_N = 0 \quad \Rightarrow \quad F = \frac{M_N - M_f}{c} \text{ (self-energizing)}$$

➔ Fs= spring force = 1448 N

7- For left shoe, the spring force will produce the brake pressure. Since, left shoe is self-de-energizing, P_{max} will be less than that for right shoe.

$$\sum M_{o2} = Fc - M_f - M_N = 0 \qquad \Rightarrow \quad 1448(500) - 1735\times10^3 p_{max} - 373\times10^3(0.3)p_{max} = 0$$

p_{max}= 0.392 MPa

8- Total braking torque

$$T = T_{RS} + T_{LS} = 1273\times10^3(0.3)(0.446 + 0.392) = 320 \text{ N.m.}$$

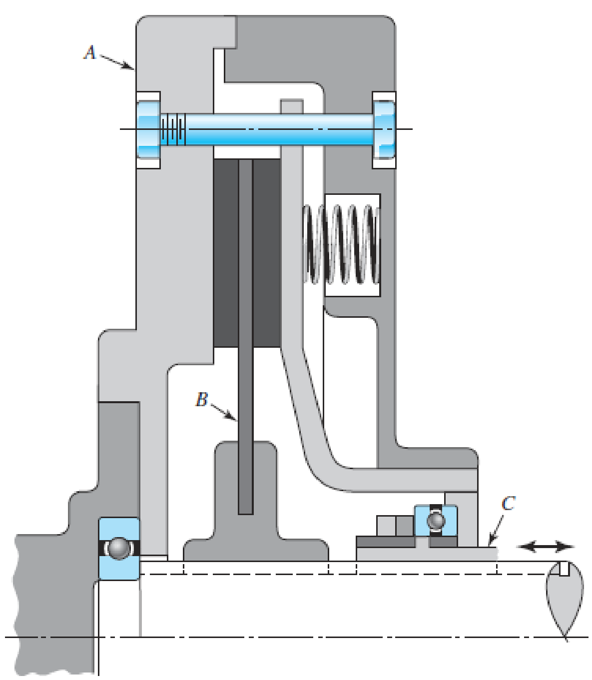
9- Power = T×ω

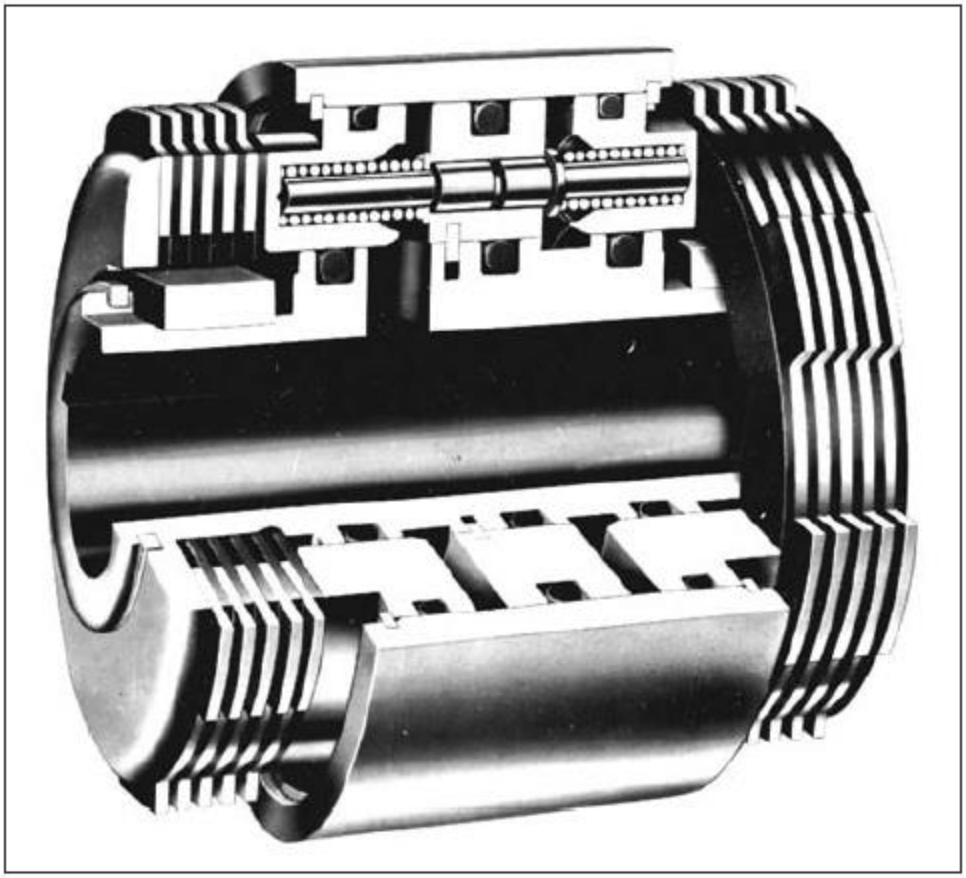
$$Power = T\times\omega = 320\times300\left(\frac{2\pi}{60}\right) = 9549 \text{ watt (applies for both directions of rotation)}$$

Disk Clutch

Disk clutch operates dry or wet

1- Dry clutch: Automotive clutch with manual transmission
2- Wet clutch: multi disk clutches used in automotive automatic transmission. The oil serves as a coolant, multiple disks compensate for reduced coefficient of friction.

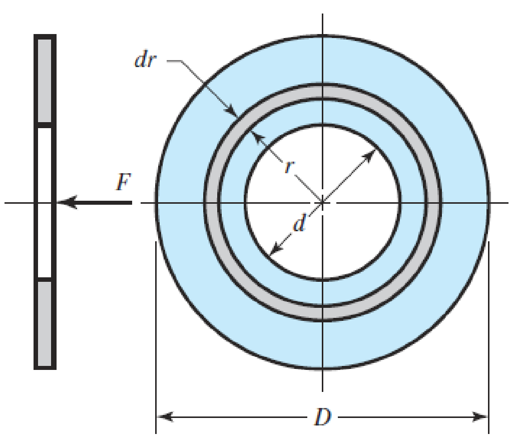




Estimating the axial clamping force and torque capacity is based on the two basic assumptions:

(a) Uniform pressure:

This assumption is valid for new, accurately manufactured clutches, with rigid outer disks.



The normal force acting on differential ring of radius *r* is

$$dF = (2\pi r dr)p$$

Axial Clamping Force:

$$\Rightarrow \quad F = \int_{ri}^{ro} 2\pi\, prdr = \pi\, p\left(r_o^2 - r_i^2\right) \qquad (1)$$

Friction Torque:

$dT = fdFr = (2\pi rdr)pfr$ $\qquad$ f : is assumed to be constant

Total torque:

$$T = \int_{ri}^{ro} 2\pi\, pfr^2 dr = \tfrac{2}{3}\pi\, pf\left(r_o^3 - r_i^3\right) \qquad \text{(for one driving interface)}$$

Multiple Clutch interfaces:

For N friction interfaces transmitting torque in parallel ➔ Total torque capacity:

$$T = \tfrac{2}{3}\pi\, pf\left(r_o^3 - r_i^3\right)N \qquad (2)$$

From eq. (1):

$$p = \frac{F}{\pi\left(r_o^2 - r_i^2\right)} \qquad (4)$$

Substituting (4) in (2):

$$T = \frac{2fF\left(r_o^3 - r_i^3\right)}{3\left(r_o^2 - r_i^2\right)}N \qquad (5)$$

(b) Uniform Wear:

Wear rate $\propto$ Friction Work rate
Friction Work = Friction Force × Rubbing velocity
For constant f ➔ Wear $\propto$ pressure × Sliding velocity ➔ Wear $\propto p \times v_e$
For disk clutch:
➔ $v_e \propto r$ ➔ Wear $\propto p \times r$

➔ Uniform Wear results from uniform rate of friction work ➔ $pv_e = C$
➔ $pr = C$

➔ p_{max} at r_{min}

For disk clutch with: r_o = outside radius, r_i = inside radius

➔ p_{max} at $r = r_i$

$pr = C = p_{max} r_i$

$$\Rightarrow \quad F = \int_{ri}^{ro} 2\pi \; p_{max} r_i dr = 2\pi \; p_{max} r_i (r_o - r_i)$$

$$T = \int_{ri}^{ro} 2\pi \; p_{max} r_i \, f r dr N = \pi \; p_{max} r_i \, f \left(r_o^2 - r_i^2\right) N \qquad (6)$$

$$\Rightarrow \quad T = fF\left(\frac{r_o + r_i}{2}\right)N$$

Assumption of uniform wear gives lower clutch capacity than uniform pressure.

<u>Clutch Design</u>: Clutches are designed for uniform wear.

From eq. (6): Max. torque for a given outside radius is:

$r_i = \sqrt{1/3}\, r_o = 0.58 r_o$

Recommended Range: $r_i = \; 0.45\, r_o \; - \; 0.8\, r_o$

<u>Disk Brake:</u>

A brake is similar to clutch except that one of the shafts is replaced by a fixed member.
This type of brakes is unsatisfactory because of inadequate cooling.
For this reason, *caliper disk brakes* are commonly used.
Drum brake can be designed for self-energizing this has an advantage and disadvantage:
Advantage: reduced braking effort
Disadvantage: slight change of friction coefficient (as a result of temperature rise or moisture) will cause a large change in pedal force required for braking.
Disk brakes has no self-energization, hence is not sensitive to changes in friction coefficient.

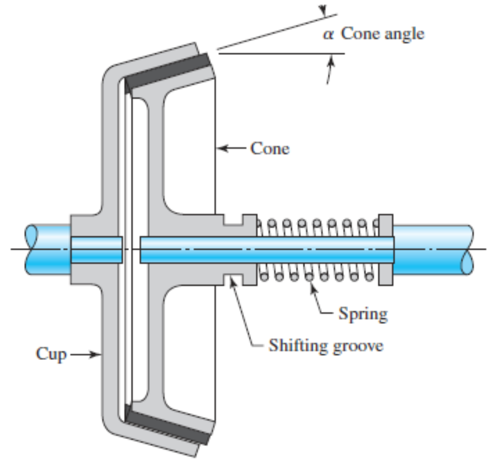
Caliper disk brake have replaced front drum brakes due to:
- Greater cooling capacity
- High resistance to fade
- Drum brake tends to distort more with heat and large forces than disk brakes.

<u>Cone Clutch and Brakes:</u>

Cone clutch and brake can be considered as the general case of which disk brake is a special case, the cone angle $\alpha = 90°$.
Cone clutch normally has one friction face ➔ N=1
The effect of the cone angle is to reduce the clamping force.

For the cone clutch shown on the figure:

$$dA = 2\pi r dr / \sin\alpha$$



Normal force on *dA*:

$$dN = p dA = (2\pi r dr) p / \sin\alpha$$

Axial Clamping force:

$$dF = dN \sin\alpha = (2\pi\, r dr) p$$

Friction Torque:

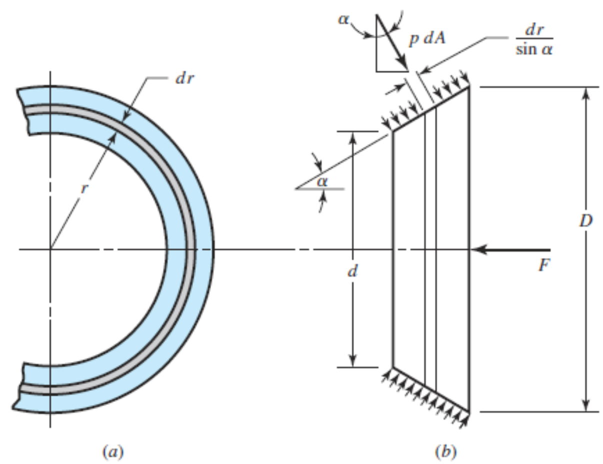
$$dT = dN f r = 2\pi p f\, r^2 dr / \sin\alpha$$

(a) Uniform pressure:

$$F = \int_{ri}^{ro} 2\pi\ p r dr = \pi\ p\left(r_o^2 - r_i^2\right)$$

$$T = \tfrac{2}{3}\pi\ p f\left(r_o^3 - r_i^3\right) \Big/ \sin\alpha$$

$$T = \frac{2 f F\left(r_o^3 - r_i^3\right)}{3\left(r_o^2 - r_i^2\right)} \frac{1}{\sin\alpha}$$



(b) uniform Wear:

$$F = 2\pi\ p_{\max} r_i \left(r_o - r_i\right)$$

$$T = \frac{\pi\ p_{\max} r_i f\left(r_o^2 - r_i^2\right)}{\sin\alpha}$$

$$T = fF\left(\frac{r_o + r_i}{2}\right) \Big/ \sin\alpha$$

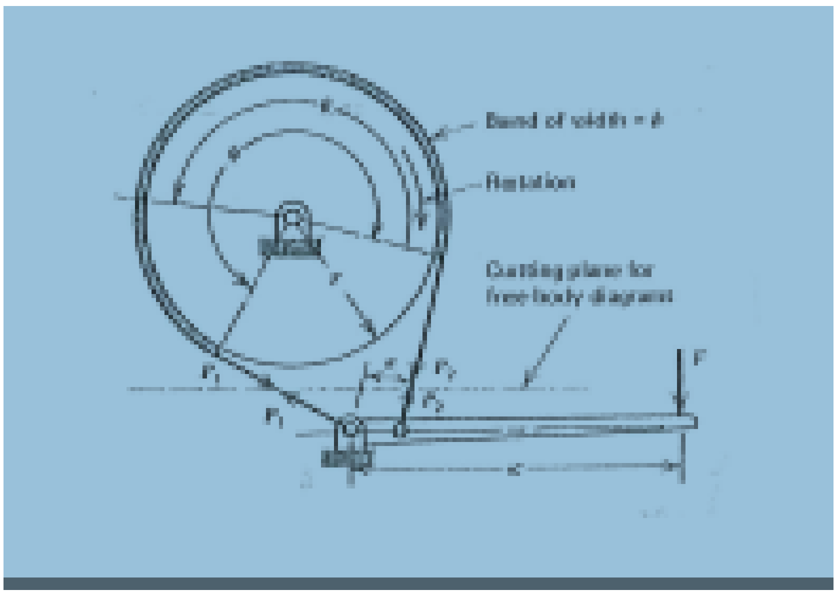
Note:
The smaller α ➔ the lower the clamping force ➔ the higher the friction torque.
If α < 8° ➔ the clutch tend to grab and become difficult to disengage.

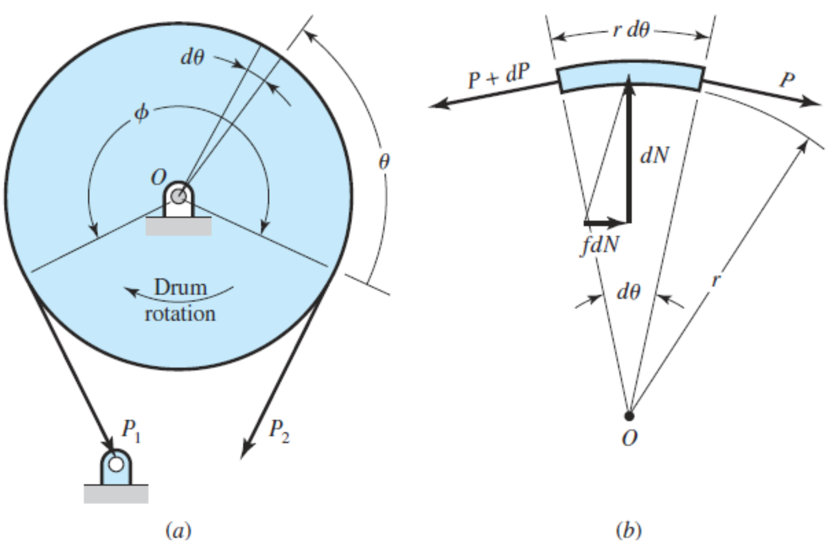
➔ 8°<α< 15°

Optimum value: α=12°.

Band Brakes:

The band is made of steel, lined with a woven friction material





For *cw* drum rotation:

Friction force increases P_1 and decrease P_2 , ➔ $P_1 > P_2$

Free-body diagram of Drum:

$$T = (P_1 - P_2)r$$

Free-body diagram of Lever:

$$F = \frac{aP_2}{c}$$

Forces acing on an element of the band:

For small angle $d\theta$

$$\sum F_h = 0$$

➔ $dP = fdN$ (1)

$$\sum F_h = 0$$

➔ $dN = 2\left(p\frac{d\theta}{2}\right) = Pd\theta$ (2)

Substituting (2) in (1):

➔ $dP = f\ Pd\theta$

➔ $\frac{dP}{P} = f\ d\theta$

Band force varies [P_1-P_2] between band contact angle θ =[0 - ϕ]

$\int_{P2}^{P1}\frac{dP}{P} = f\int_0^{\phi} d\theta$ ➔ $\ln\frac{P_1}{P_2} = f\phi$ ➔ $\frac{P_1}{P_2} = e^{f\phi}$

But:

$dN = pdA = pbrd\theta$, where: p = pressure between drum and band.

$$dN = Pd\theta$$

➔ $P = pbr$

➔ $P \propto p$

p_{max} occurs at $P_{max} = P_1$

at $\theta = \phi$ $P = P_{max} = P_1$

→ at θ = ϕ $P_1 = p_{max} b r$

For *cw* rotation: Self-energizing

$F = \frac{aP_2}{c}$, where $P_2 < P_1$ [friction force increases P_1 and decreases P_2]

For *ccw* rotation: Self-de-energizing

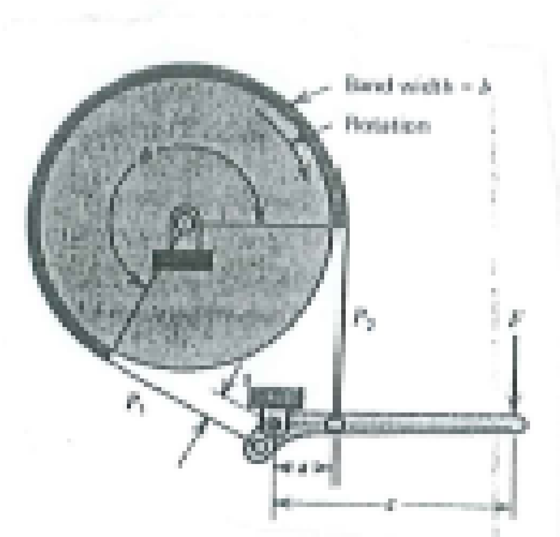
$F = \frac{aP_2}{c}$, where $P_1 < P_2$ [friction force increases P_2 and decreases P_1]

Differential Band Brake:

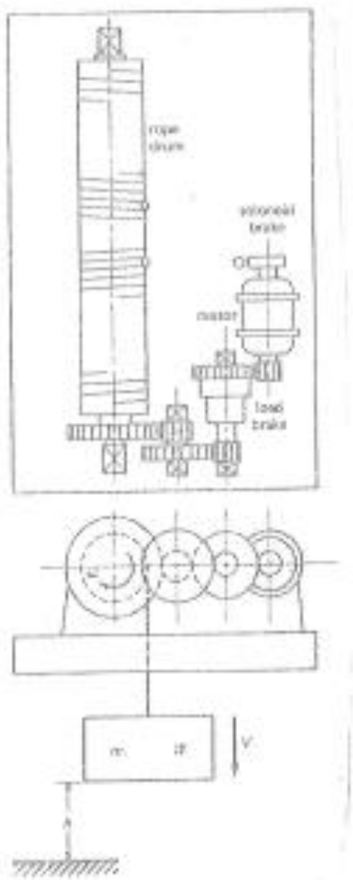
To increase the self-energizing action the differential

$F = \frac{(P_2 a - P_1 s)}{c}$, where $P_1 < P_2$ due to friction force

For self energizing: $s < a$ (to insure that the lever will tighten the band at distance *a* more than at distance *s*).



Energy Consideration in Brakes and Clutches:



The hoist shown in the figure is required to lower a mass (m)

Assume that the brake is applied at time = t_1

<u>At t_1</u>

$\omega = \omega_1$, $V = V_1$, $h = h_1$

<u>At t_2</u> the values are reduced

$\omega = \omega_2$, $V = V_2$, $h = h_2$

during time $(t_2 - t_1)$

Work done by brake = wk_B
Rolling friction, Bearing friction, air resistance = wk_R
Motor work = wk_M
wk_M = Work done by the machine in driving energy consuming device.
If the motor drives the machine during braking period ➔ wk_M = -ve

Total work done = Change in Energy

$$wk_B + wk_R + wk_M = \sum \frac{m}{2}(V^2{}_1 - V^2{}_2) + \sum \frac{I}{2}\left(\omega_1^2 - \omega_2^2\right) + \sum W\left(h_1 - h_2\right)$$

the summation is made for different masses at their corresponding speeds and positions.

wk_B = mechanical energy transferred into heat at the brake .

$$wk_B = \int_{\psi_1}^{\psi_2} T d\psi = T(\psi_2 - \psi_1)$$

ψ = angular displacement of the drum

For constant torque:

If T= const. ➔ linear deceleration = a= const.
➔ Angular deceleration = α = const.

<u>Velocity displacement relation:</u>

$$V_2 = V_1 - a(t_2 - t_1)$$
$$S_2 = S_1 + V_1(t_2 - t_1) - \frac{a}{2}(t_2 - t_1)^2 = S_1 + \frac{V_1 + V_2}{2}(t_2 - t_1)$$

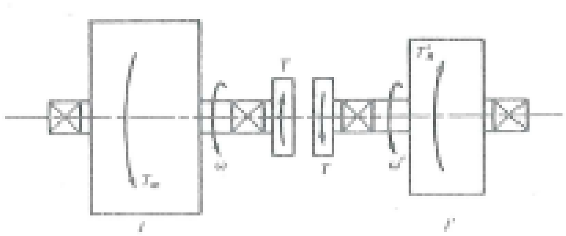
and angular velocity and displacement:

$$\omega_2 = \omega_1 - \alpha(t_2 - t_1)$$

$$\psi_2 = \psi_1 + \omega_1(t_2 - t_1) - \frac{\alpha}{2}(t_2 - t_1)^2 = \psi_1 + \frac{\omega_1 + \omega_2}{2}(t_2 - t_1)$$

<u>Clutch Work Energy Relationship:</u>

In the clutching of two masses initially rotated in the same direction with different speeds, the higher speed mass supplies energy to lower speed mass.
In general: the work done by friction equal the net change in energy of the system.
Consider two masses with *I* and *I′*, rotating in the same direction with initial velocities ω> ω′



Assuming that clutch torque , T= const. during engagement

At time, t_1=0
$\omega > \omega'$
Left Shaft:
$$T = -I\frac{d\omega}{dt}$$
Right Shaft
$$T = I'\frac{d\omega'}{dt}$$
Integrating both equations:
$$\omega = \omega_1 - \frac{T}{I}t$$
$$\omega' = \omega_1' + \frac{T}{I'}t \qquad (1)$$
A t=t_2 , relative motion ends (both shaft rotate at the same speed)

➔ $\omega_2 = \omega'_2$
$$\omega_1 - \frac{T}{I}t_2 = \omega_1' + \frac{T}{I'}t_2$$
$$t_2 = \frac{\omega_1 - \omega_1'}{T}\frac{II'}{I + I'}$$
➔ the time required for engagement is proportional to velocity difference

t_2 = time required for clutch engagement
Subst. t_2 in eq. (1)

➔ Common velocity:
$$\omega_2 = \omega_2' = \frac{\omega_1 I + \omega_1' I'}{I + I'}$$
Displacement after time t_2:
$$\psi = \int_0^t \omega\, dt = \int_0^t \left(\omega_1 - \frac{T}{I}t\right)dt = \omega_1 t - \frac{Tt^2}{2I}$$
$$\psi' = \int_0^t \omega'\, dt = \int_0^t \left(\omega_1' + \frac{T}{I}t\right)dt = \omega_1' t + \frac{Tt^2}{2I'}$$
The work done by the clutch after time t:

Work = Clutch Torque× Relative displacement
$$w_K = T(\psi - \psi') = T(\omega_1 - \omega_1')t - \frac{T^2(I + I')}{2II'}t^2$$
Clutch work at completion of engagement, $t = t_2$

$$w_{KC} = \frac{II'}{2(I+I')}(\omega_1 - \omega_1')^2$$

w_{kc} = energy dissipated by the clutch or brake.

➔ energy dissipated is proportional to velocity difference squared.

S. I. units:

wkc = *E* = Joule

U. S. units:

wkc = *E* = lb.in., wkc = E = (lb.in./9336)= Btu.

Energy Absorption and Cooling:

Heat transfer equation of clutch or brake:

$$H = CA\Delta T = CA(T_s - T_a)$$

H = time rate of heat dissipation (Watt or hp)

C = overall heat transfer coefficient per unit area (W/m^2 °C, hp/in.^{2} °F)

A = exposed heat –dissipating area (m^2 , in^2)

T_s = average temperature of heat-dissipating surfaces (°C, °F)

T_a = Air temperature

Or

$$E = mC\Delta T = mC(T_s - T_a)$$

E = heat dissipation (Joule or lb.in)

C = specific heat (J/m^2 °C, lb.in/in.^{2} °F)

m = mass of brake or clutch (lb_m, Kg), (Btu = 9336 lb.in)

For steel and Cast Iron: C = 0.12 Btu/(lb_m. °F) = 500 J/(Kg. °C)

The rate of heat generation per unit area of friction interface is equal to the product of Clamping pressure, coefficient of friction and rubbing velocity

$H = f\,pV$

An emperical values for *pV* is obtained by brake manufacturer and is given below:

	PV	
Operating Conditions	(psi) (ft/min)	(KPa)(m/s)
Continuous poor heat dissp.	30,000	1050
Occasional, poor heat dissp.	60,000	2100
Continuous good heat dissp.	85,000	3000

The ability of brake and clutch to absorb energy without reaching destructive temperatures can be increased by:

a- Increasing exposed surface area

b- Increasing air flow by minimizing air flow restrictions, and maximizing air pumping action of rotating parts.

c- Increasing the mass and specific heat of contact parts.

Example:
An inertial load is to bring to rest in 10 sec. by a braking torque 2800 lb.in. The initial speed is 1600 rpm, and the heat dissipating surface weight 40 lb. Find the temperature rise.

Solution:

$$\omega_1 = \frac{2\pi n}{60} = \frac{2\pi 1600}{60} = 168 \text{ rad/s}$$

$\omega_2 = 0$

but: $t_2 = \frac{\omega_1 - \omega_1'}{T}\frac{II'}{I+I'}$ = 10 sec

$$\frac{II'}{I+I'} = \frac{Tt_2}{\omega_1 - \omega_1'} = \frac{2800(10)}{168} = 166.7 \text{ lb.s}^2\text{. in.}$$

$$w_{KC} = \frac{II'}{2(I+I')}(\omega_1 - \omega_1')^2 = \frac{166.7(168)^2}{2} = 2.35(10^6) \text{ lb.in.}$$

$$H = mC\Delta T$$

H= $2.35(10^6)/9336$ = 252 Btu

ΔT= 252/[0.12(40)]=52.5 °F

Friction Material:

One of the mating material is normally metal (cast iron or steel) it must have:
1- Good friction characteristics (which is relatively stable over temp. range)
2- Good thermal conductivity
3- Good resistance to wear and thermal fatigue
4- mooth surface finish to minimize wear of the mating friction material.

The friction material must have the following requirements:
1- High dynamic coefficient of friction that is relatively stable over a usable temp range and not sensitive to moisture, dirt and oil
2- Static coefficient of friction exceeds the dynamic value by as little as possible to avoid slip-stick chatter and noise.
3- High resistance to abrasive and adhesive wear
4- Good thermal conductivity
5- Good temperature stability or resistance to fade (reduction in friction coefficient at elevated temp.)

Types of friction material:
1- Molded: most common and least costly. Used for heavy duty application and drum brakes. Molded asbestos contains asbestos fiber and friction modifiers
2- Woven: more flexible (suitable for band brake)

Asbestos lining is used for heavy machinery, perform better when contaminants as mud, grease, and dirt presents.

3- Sintered: made of mixture of copper and iron particles with friction modifiers. Used in clutch and brakes for heavy duty applications.

Tables (16-1,2) list properties of typical friction materials

Example:

The figure shows a band brake used with a punch press. The brake is to be engaged when the crank is 120° past bottom dead center and bring the crank to rest at top dead center. The crank assembly has I= 12 N.m s^2 and rotating at 40 rpm when the brake is engaged. Max. allowed pressure p= 0.18 MPa, f = 0.35.

(a) Determine the required band width.
(b) Determine the required force, F
(c) Would any combination of direction of rotation or coefficient of friction make brake self-locking.

Solution:

(a) Work done by the brake = Change in K.E. of the system

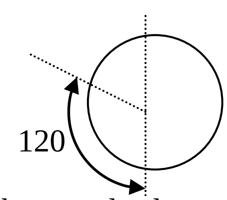
$$\Delta K.E. = \tfrac{1}{2} I\left(\omega_1^2 - \omega_2^2\right) = \tfrac{1}{2}(12)\left(\frac{40\times 2\pi}{60}\right)^2 = 105.275 \text{ J}$$

Brake done by brake = Torque ×angular displacement

$$w_{kB} = T(\psi_2 - \psi_1)$$

$$(\psi_2 - \psi_1) = 180 - 120 = 60^\circ = 1.047 \text{ rad}$$

Top dead center



bottom dead center

$$\Delta K.E. = T(\psi_2 - \psi_1)$$

$$T = \frac{\Delta K.E}{(\psi_2 - \psi_1)} = \frac{105.275}{1.047} = 100.56 \text{ N.m}$$

Free body of the drum:

$$T = (P_1 - P_2)r$$

$$\Rightarrow \quad (P_1 - P_2) = \frac{T}{r} = \frac{100.56}{0.12} = 837.92 \text{ N} \qquad (1)$$

But:

$$\frac{P_1}{P_2} = e^{f\phi}$$

$$\phi = 258^\circ,\quad f = 0.35 \quad \Rightarrow \quad \frac{P_1}{P_2} = e^{\left(0.35\times\left(\frac{258\times\pi}{180}\right)\right)} = 4.835 \qquad (2)$$

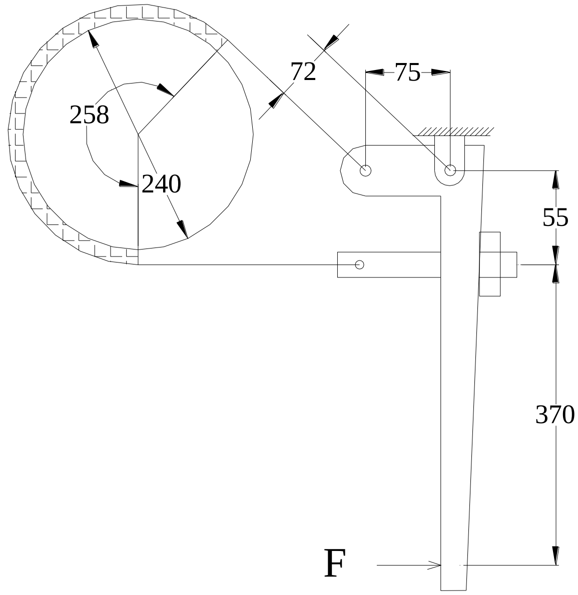
From (1) and (2):

P_1 = 218.41 N, P_2 = 1056.16 N

$$P_1 = p_{\max} b r \qquad b = \frac{P_1}{p_{\max} r} = \frac{1056.16}{\left(0.18\times 10^6\right)0.12} = 49 \text{ mm}$$

(b) Free body of the lever

$$\sum M_o = F(370+55) - P_2(72) - P_1(55) = 0 \quad \Rightarrow \quad F = 173.7 \text{ N}$$


72
75
258
240
55
370
F