ملخص مادة..

ديناميكا

لجنة الميكانيك

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بسم الله الرحمن الرحيم

الحمد لله والصلاة والسلام على سيدنا محمد وعلى اله وصحبه أجمعين . الحمد لله حمدا كثيرا طيبا مباركا يليق بجلال وجهه وعظيم سلطانه , الحمد لله الذي جعل لنا من العلم نورا نهتدي به والحمد لله الذي من علينا بأتمام هذا الملخص لمادة " الديناميكا ".

نتقدم نحن " لجنة الميكانيك " بتلخيصنا هذا الى زملائنا الطلاب والى كل من يجمعنا بهم رباط العلم سائلين المولى أن يتقبله منا وأن ينال اعجابكم , وأن لا نكون قد قصرنا أو أهملنا فيه .

نحيطكم علما بأن هذا التلخيص لا يغني عن شرح المدرس في المحاضرة والرجوع الى الكتاب .
حيث تتطلب المادة قراءة جميع الأمثلة وحل العديد من الأسئلة الموجودة في نهاية الكتاب.

المرجع المعتمد لكاتب التلخيص هو :
ENGINEERING MECHANICS DYNAMICS 12ed
by HIBBLER

نسأل الله لكم التوفيق ودوام النجاح والتفوق .

لجنة الميكانيك

Chapter (12):

Statics
1. at rest
2. moves with constant velocity.

Dynamics
- Kinematics (علم الحركة)
- Kinetics "The forces that cause the motion".

- (12.2) Rectilinear kinematics ; continuous motion:

(a) Position: - distance (S)
- displacement ($\vec{r}$) or (ΔS)

(b) Velocity: - Instantanuous velocity → $v = \frac{ds}{dt}$ or $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$
- Average (velocity\speed)

Velocity

$v_{avg} = \frac{\Delta s}{\Delta t}$

or $v_{avg} = \frac{\Delta \vec{r}}{\Delta t}$

Speed

$(v_{sp})_{avg} = \frac{S_T}{\Delta t}$

(c) Acceleration: - instantanuous "a" → $a = \frac{dv}{dt}$ or $a = \frac{d^2s}{dt^2}$

or $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

- Average acceleration → $a_{avg} = \frac{\Delta v}{\Delta t}$

أهم ثلاث معادلات في ال kinematics →

1) $v = \frac{ds}{dt}$

2) $a = \frac{dv}{dt}$

3) $\boxed{a\,ds = v\,dv}$

معادلة مهمة جداً.

* Constant Acceleration : فقط عندما يكون التسارع ثابت

↳ $a = a_c$

equations of kinematics :

1. $a_c = \frac{dv}{dt}$

2. $v = \frac{ds}{dt}$

3. $a_c ds = v dv$

[1] Velocity as a function of time :

$$a_c = \frac{dv}{dt} \rightarrow \int_{v_0}^{v} dv = \int_{0}^{t} a \, dt$$

$$\boxed{v = v_0 + a_c t}$$

[2] Position as a function of time :

$$v = \frac{ds}{dt} \rightarrow ds = v \, dt$$

تكامل $\int_{s_0}^{s} ds = \int_{0}^{t} (v_0 + a_c t) \, dt$

$$\boxed{s = s_0 + v_0 t + \frac{1}{2} a_c t^2}$$

[3] Velocity as a function of position:

$$v \, dv = a_c ds \rightarrow \int_{v_0}^{v} v \, dv = \int_{s_0}^{s} a_c ds$$

$$\boxed{v^2 = v_0^2 + 2a_c (s - s_0)}$$

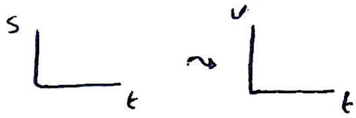
• Note : We can't use constant acceleration when the acceleration is a function of time.

مهم

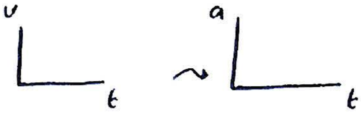
- Rectilinear Kinematics, Erratic motion :
غير منتظم

• The s-t, v-t, and a-t Graphs

s-t ↓ v-t : slope of s-t graph = $\boxed{\frac{ds}{dt} = v}$ ⇒



v-t ↓ a-t : slope of v-t graph = $\boxed{\frac{dv}{dt} = a}$ ⇒

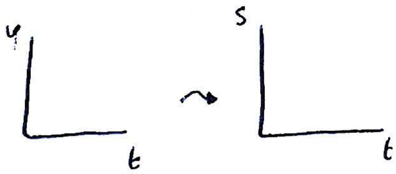


$a = \frac{dv}{dt}$

a-t ↓ v-t : $\int a\,dt = \Delta v$
area under a-t graph ⇒

$v = \frac{ds}{dt}$

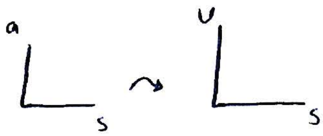
v-t ↓ s-t : $\int v\,dt = \Delta s$
area under v-t graph ⇒



• The v-s and a-s Graphs

$v\,dv = a\,ds$

a-s ↓ v-s : $\frac{1}{2}(v^2 - v_0^2) = \underbrace{\int_{s_0}^{s} a\,ds}_{\text{area under a-s graphs}}$ ⇒

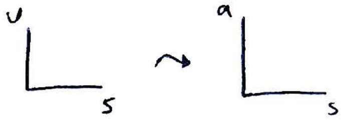


$$a\,ds = v\,dv$$

v-s ↓ a-s

$$a = v\left(\frac{dv}{ds}\right)$$

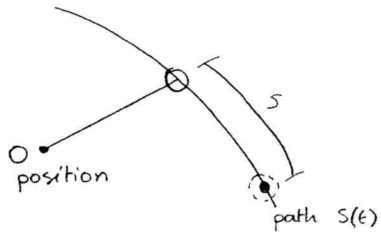
velocity times slope of v-s graph

$\Rightarrow$ 

- (12-4) : General Curvilinear Motion :

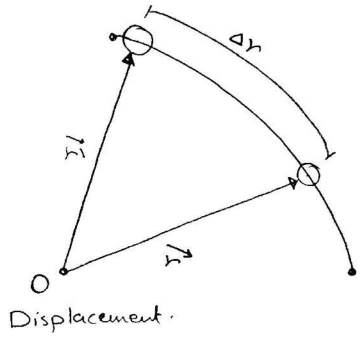
. If the path function is $s(t)$

1) Position



2) Displacement

$$r' = r + \Delta r$$
$$\Delta r = r' - r$$



3) Velocity

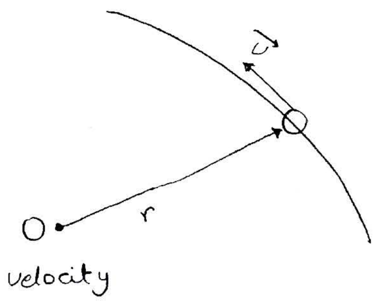
$$v_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

instantanuous → velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

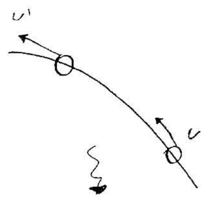
→ speed

$$v = \frac{ds}{dt}$$



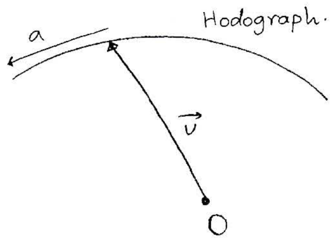
4) Acceleration

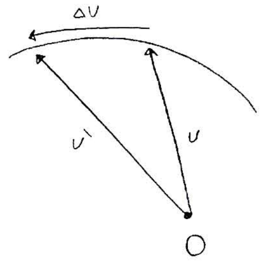
$$a_{avg} = \frac{\Delta v}{\Delta t}$$



instantanuous

$$\hookrightarrow a = \frac{d\vec{v}}{dt} = \frac{d^2 r}{dt^2}$$





$\underline{\underline{v(t)}}$

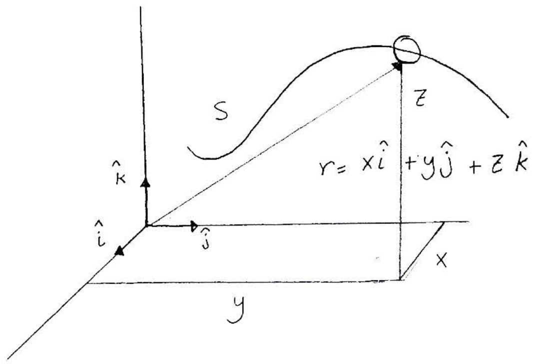
- (12.5) : Curvilinear Motion : Rectangular Components :

1) Position

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

$$|r| = \sqrt{x^2 + y^2 + z^2}$$

$$\vec{u_r} = \frac{\vec{r}}{|r|} \quad \text{"unit vector"}$$



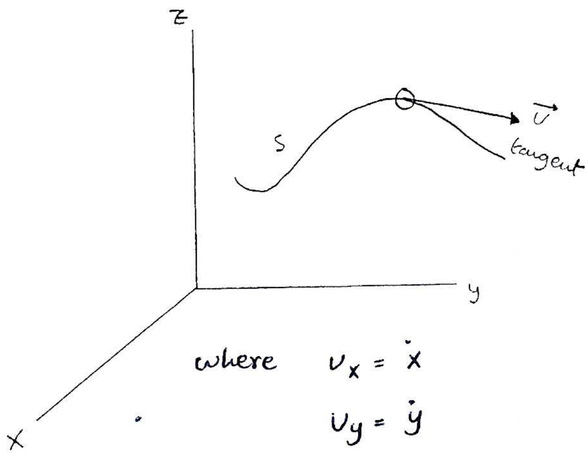
2) Velocity

$$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}}$$

$$|v| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\vec{u}_v = \frac{\vec{v}}{|v|}$$

$\hookrightarrow$ always tangent to the path $s(t)$.



where $v_x = \dot{x}$

$v_y = \dot{y}$

$v_z = \dot{z}$

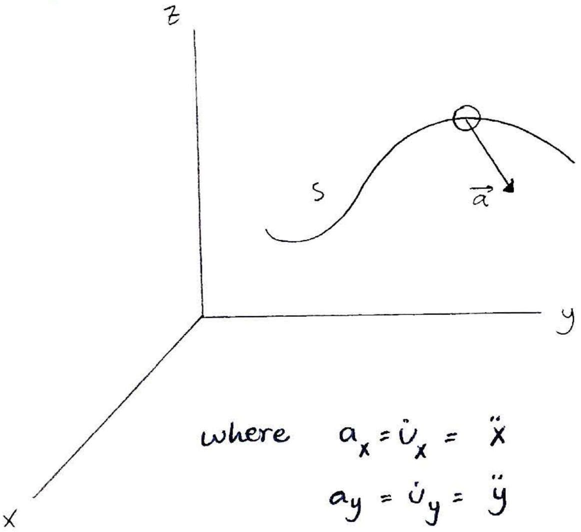
3) Acceleration

$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\vec{u}_a = \frac{\vec{a}}{|a|}$$ "unit vector"

→ not tangent to the path $s(t)$.



where $a_x = \dot{v}_x = \ddot{x}$

$a_y = \dot{v}_y = \ddot{y}$

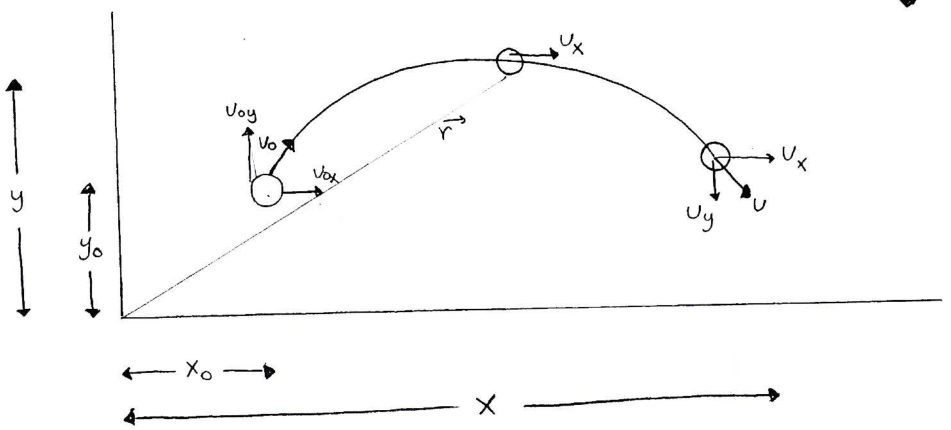
$a_z = \dot{v}_z = \ddot{z}$

- (12.6): Motion of a Projectile:

$$a_c = g = 9.81 \quad m/s^2$$

$$g = 32.2 \quad ft/s^2$$

$a = g$



- Horizontal: $x = x_o + (v_o)_x t$

مع العلم أن v_x ثابتة

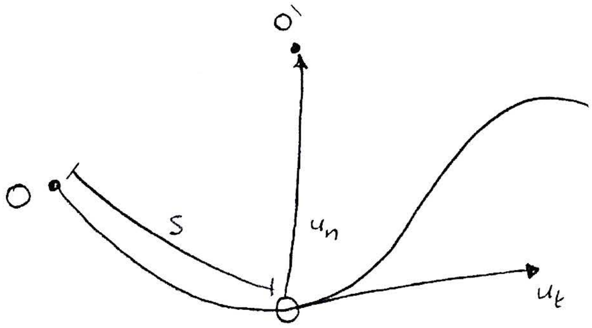
- Vertical:
  → $v_y = v_{oy} + at$
  → $y = y_o + v_{oy}t + \frac{1}{2}at^2$
  → $v_y^2 = v_{oy}^2 + 2a(y - y_o)$

you can use just two equations

-(12.7): Curvilinear Motion: Normal and tangentional components:

ρ (rho) : radious of curvature

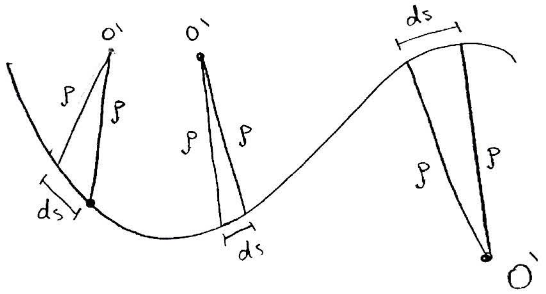
O\` : center of curvature



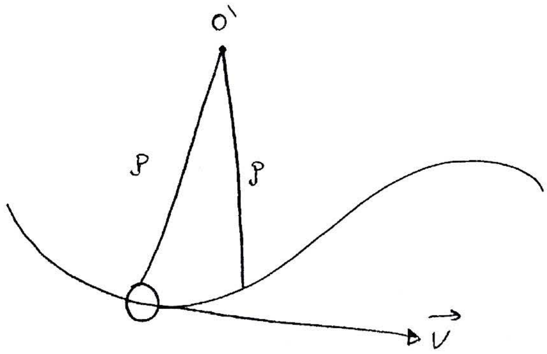
position

$$\boxed{\vec{v} = v\,\vec{u}_t}$$ اتجاه

$$\boxed{v = \dot{s}}$$ مقدار



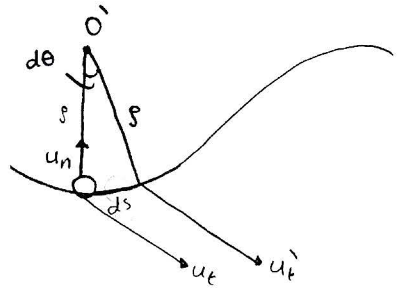
radious of curvature



velocity

* Acceleration:

$$\vec{v} = v\,\vec{u}_t$$



$$\boxed{\vec{a} = \dot{v} = \dot{v}\,\vec{u}_t + v\,\dot{\vec{u}}_t}$$

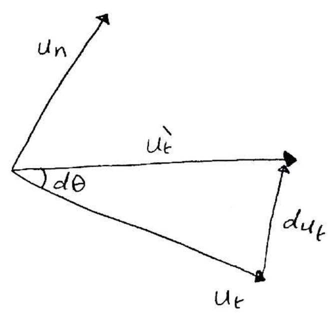
To find $\dot{u}_t$

$\vec{u}_t$: is an unit vector in direction of t-axis and its magnitude is $\boxed{1}$

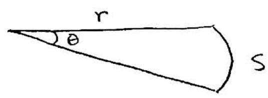
du_t : is an arc

$\boxed{du_t = (1)\,d\theta}$ magnitude of du_t

$\boxed{d\vec{u}_t = d\theta\,\vec{u}_n}$ direction of du_t



. The direction of $d\vec{u}_t$ is in the direction of $\vec{u}_n$



$s = r\theta$

$$\dot{\vec{u}}_t = \dot{\theta}\,\vec{u}_n \qquad ds = \rho\,d\theta$$

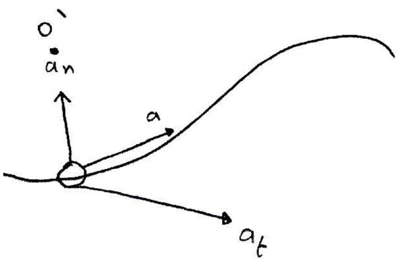
$$\dot{\vec{u}}_t = \frac{v}{\rho}\,\vec{u}_n \qquad d\theta = \frac{ds}{\rho} = \frac{v}{\rho}$$

neo $\boxed{\vec{a} = a_t\,\vec{u}_t + a_n\,\vec{u}_n}$ where: $a_t = \dot{v}$

$a_t\,ds = v\,dv$

$a = \sqrt{a_t^2 + a_n^2}$

and $a_n = \dfrac{v^2}{\rho}$

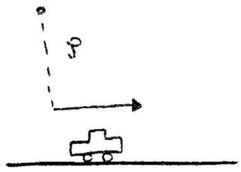


• Cases :

1. The Particle moves along straight line:

results : $\rho \longrightarrow \infty$

$$a_n = \frac{v^2}{\infty} = 0$$



then, $a = a_t$

$\vec{a} = a_t \vec{U_t}$ ... The time rate of change in the magnitude of the velocity.

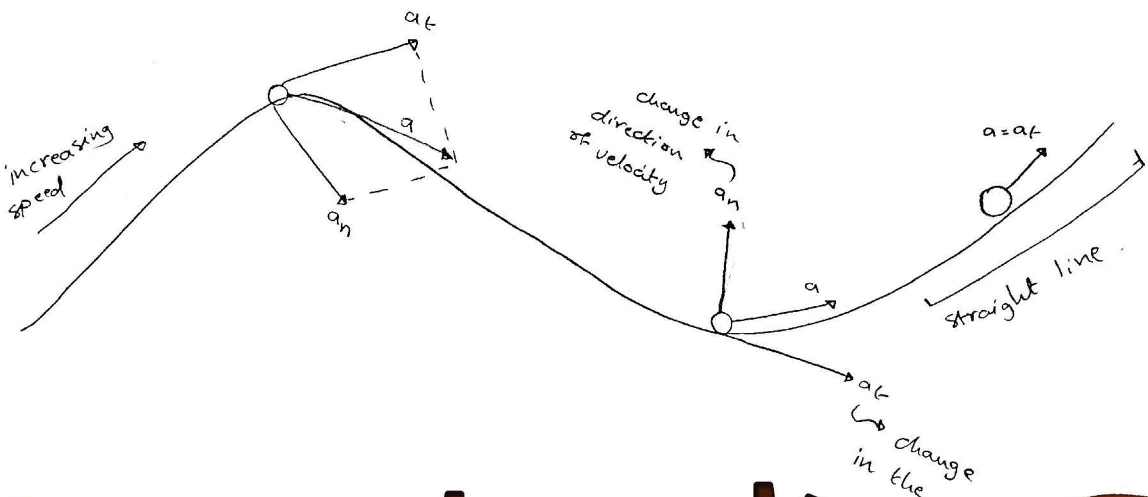
2. The particle moves along a curve with a constant speed:

results: $v \longrightarrow$ constant

$$a_t = \dot{v} = 0$$

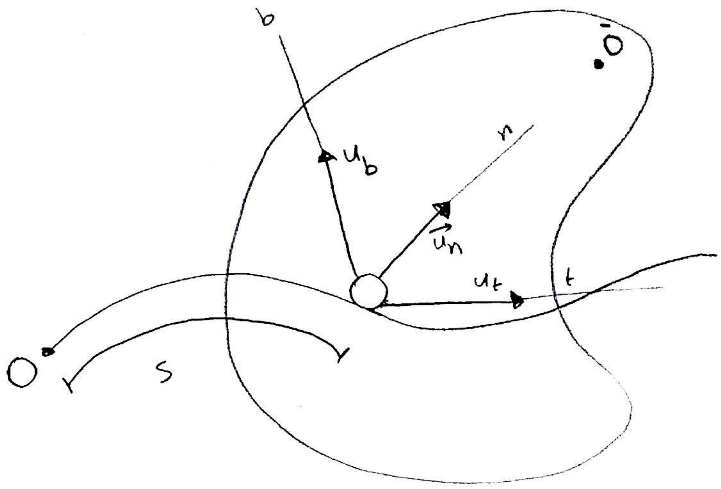
then, $a = a_n$

$\vec{a} = a_n \vec{U_n}$ ... The time rate of change in the direction of the velocity.



* Three - Dimensional Motion:

u_b : binormal.



$\vec{u}_b = \vec{u}_t \times \vec{u}_n$
$\vec{u}_n = \vec{u}_b \times \vec{u}_t$
$\vec{u}_t = \vec{u}_n \times \vec{u}_b$

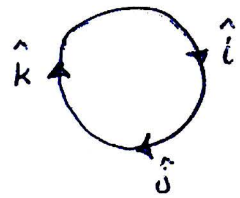
Remember ^^ ..

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{k} = \hat{i}$

$\Leftarrow$



- The +ve t-axis acts in the direction of motion
- The +ve n-axis is directed toward the path's center of curvature.

* Velocity:
  - Direction → is always tangent to the path
  - Magnitude → is $v = \dot{s}$

* Tangentional Acceleration : (a_t).

is the result of the time rate of change in the magnitude of Velocity.

a_t acts → in +ve "s" direction if the particle's speed is ↑
→ in −ve "s" direction if the particle's speed is ↓

• The relations between a_t, v, t and s are the same as for rectilinear motion, namely

$$a_t = \dot{v} \quad , \quad a_t\, ds = v\, dv$$

⟹ If a_t is constant , $a_t = (a_t)_c$

→
1. $s = s_0 + v_0 t + \frac{1}{2}(a_t)_c\, t^2$
2. $v = v_0 + (a_t)_c\, t$
3. $v^2 = v_0^2 + 2(a_t)_c\,(s - s_0)$.

* Normal Acceleration : (a_n).

is the result of the time rate of change in the direction of the velocity.

Direction → a_n is always directed toward the center of curvature of the path (+ve n-axis)

Magnitude → $a_n = \dfrac{v^2}{\rho}$

If the path is expressed as $Y = f(x)$ the radious of curvature ρ at any point on the path is determine from:

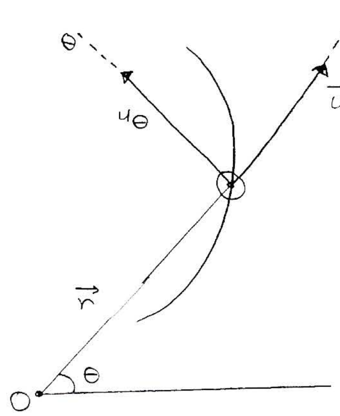
$$\boxed{\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left| d^2y / dx^2 \right|}}$$

-(12.8) Curvilinear Motion: Cylindrical Components:

- Polar coordinates

  r: radial coordinates

  θ: transverse coordinate



Position

- θ: between fixed refference line and and r-axis

→ Position:

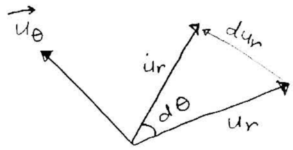
$$\vec{r} = r\vec{u}_r$$

مشتقة

→ Velocity:

$$\vec{v} = \dot{r}\vec{u}_r + r\dot{\vec{u}}_r$$

To find $\dot{\vec{u}}_r$:



$$du_r = d\theta$$

$$\dot{\vec{u}}_r = \dot{\theta}\,\vec{u}_\theta$$

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{rad}{s}$$

↳ angular velocity

$$v = \dot{r}\vec{u}_r + r\dot{\vec{u}}_r$$

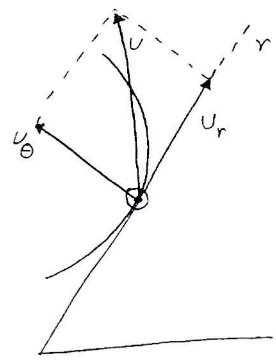
$$\vec{v} = v_r\vec{u}_r + v_\theta\vec{u}_\theta \qquad \begin{bmatrix} v_r = \dot{r} \\ v_\theta = r\dot{\theta} \end{bmatrix}$$

- magnitude:

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

- direction:

  is tangent to the path.



velocity

Acceleration

$$v = \dot{r}u_r + r\dot{\theta}\,u_\theta$$

اشتقاق

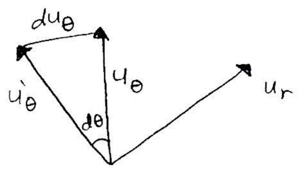
$$\vec{a} = \ddot{r}\vec{u}_r + \dot{r}\dot{\vec{u}}_r + \dot{r}\dot{\theta}\vec{u}_\theta + r\ddot{\theta}\vec{u}_\theta + r\dot{\theta}\dot{\vec{u}}_\theta$$

$$\dot{\vec{u}}_r = \dot{\theta}\vec{u}_\theta$$

· To find $\dot{\vec{u}}_\theta$

$$\dot{u} = \dot{\theta}$$

$$\dot{\vec{u}} = -\dot{\theta}\vec{u}_r$$



$$a = \ddot{r}\vec{u}_r + \dot{r}\dot{\theta}\vec{u}_\theta + \dot{r}\dot{\theta}\vec{u}_\theta + r\ddot{\theta}\vec{u}_\theta - r\dot{\theta}^2\vec{u}_r$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$$

$$\vec{a} = a_r\vec{u}_r + a_\theta\vec{u}_\theta \qquad \left[\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{aligned}\right]$$

where $\ddot{\theta} = \dfrac{d^2\theta}{dt^2} = \dfrac{rad}{s^2}$

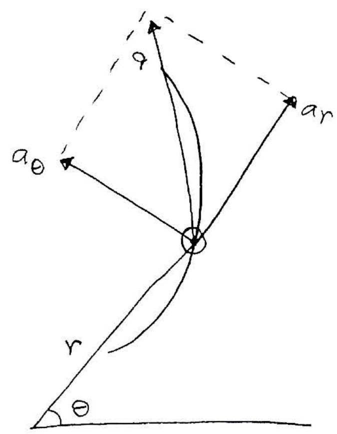
↳ angular acceleration.

· magnitude :

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$$

· direction:

is determined from the vector addition of its two components



acceleration

* Cylindrical Coordinates:

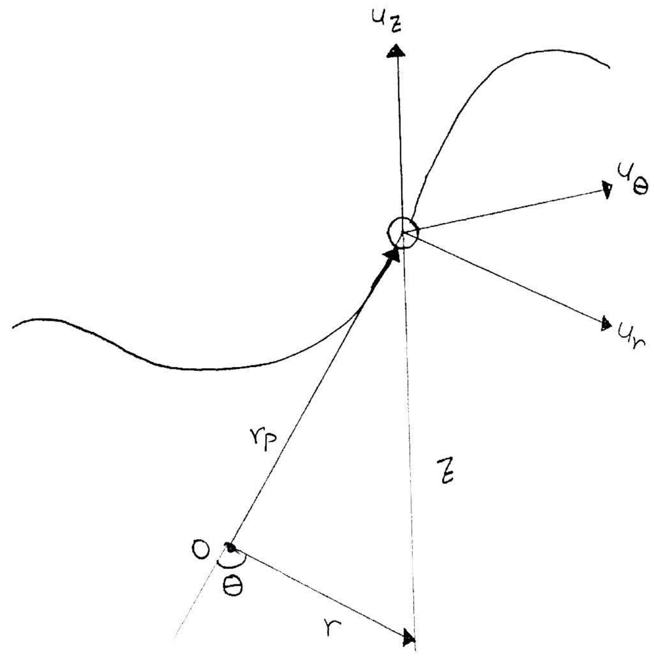
$r \,,\, \theta \,,\, z$

direction $\vec{u}_z$

magnitude is constant

$u_z = \text{const.}$

$\dot{u}_z = 0$ (zero)



1) $\vec{r}_p = r\vec{u}_r + z\,\vec{u}_z$

2) $\vec{v} = \dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta + \dot{z}\,\vec{u}_z$

3) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\vec{u}_\theta + \ddot{z}\,\vec{u}_z$

* Two types of problems generally occur:

① If the polar coordinates are specified as time parameteric equation

$r = r(t)$
and $\theta = \theta(t)$ $\Rightarrow$ Then, the time derivatives can be found directly.

② If the time-parametric equations are not given, then the path

$r = f(\theta)$ $\Rightarrow$ must be known

to solve it, → use chain rule \ to find relation between $\dot{r}$ and $\dot{\theta}$

$\ddot{r}$ and $\ddot{\theta}$

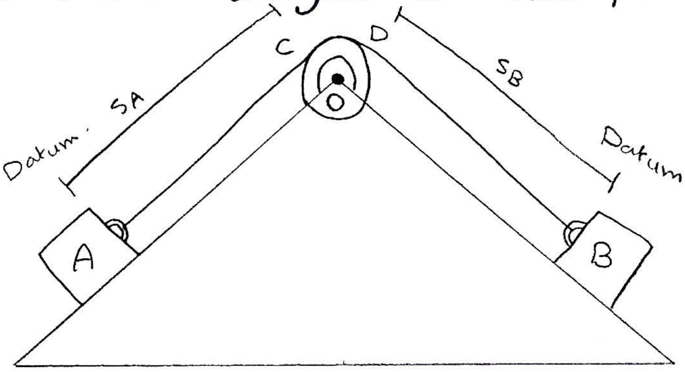
* To Solve Problems → evaluate : $r, \dot{r}, \ddot{r}, \dot{\theta}, \ddot{\theta}$

r : $\vec{r} = r \vec{u}_r$

v : $v_r = \dot{r}$ $\quad v_\theta = r\dot{\theta}$

a : $a_r = \ddot{r} - r\dot{\theta}^2$ $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

- (12.9): Absolute Dependent motion analysis of Two particles:



• Specific the location of the block using position coordinates s_A, s_B.

1) Is measured from a fixed point "O" or fixed datum line.

2) Is measured along each inclined plane in the direction of motion of each block

3) has a positive sense from C to A and D to B

in fig. → $s_A + l_{CD} + s_B = l_{total}$

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0$$

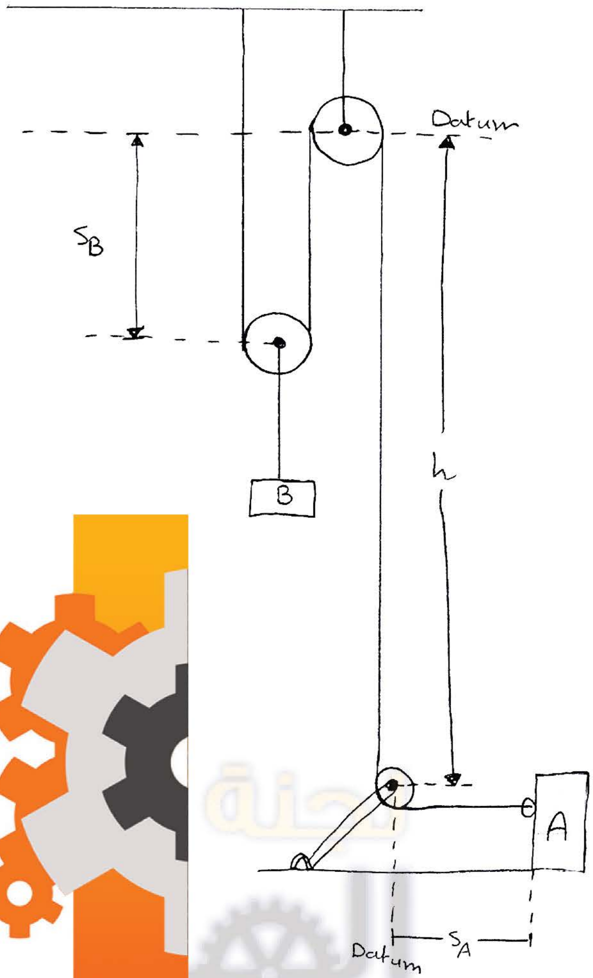
$$v_B = -v_A$$

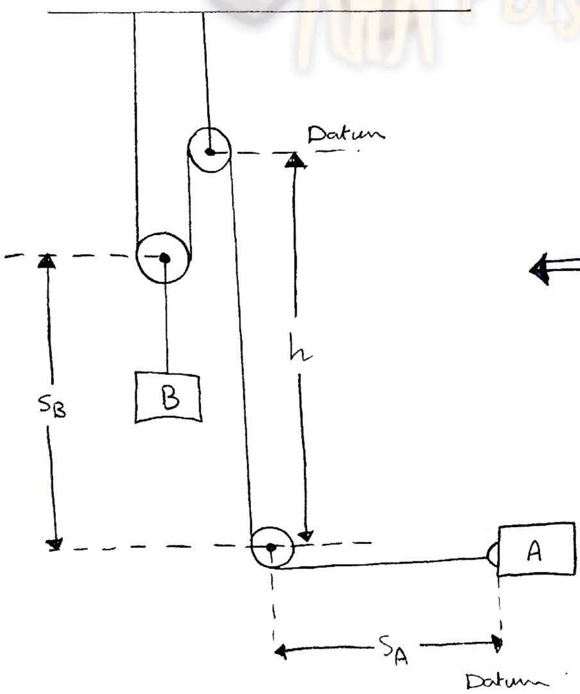
$$a_B = -a_A$$

$$2S_B + h + S_A = L_T$$

$$2v_B = -v_A$$

$$2a_B = -a_A$$





$$2(h - S_B) + h + S_A = L$$

$$2v_B = v_A$$

$$2a_B = a_A$$

* Chapter (13): Kinetics of a particle :

Force and Acceleration ..

- (13.1) : Newton's Second Law of Motion :

Dynamics

→ kinematics

→ kinetics

deals with the relationship between:

- The change in motion of body

and - The forces that cause this change

When

an unbalanced force acts on a particle → The particle will accelerate in the direction of the force with a magnitude that is proportional to the force.

- The force and acceleration are Directly proportional

$$F \propto a$$

$$F = ma \rightarrow m = \frac{F}{a}$$

$$a = \frac{dv}{dt}$$

$$\Rightarrow F = \frac{d(mv)}{dt}$$

momentum ← $P = mv$

$$\boxed{F = \frac{dP}{dt}}$$

→ The unbalanced force acting on the particle is proportional to the time rate of change of the particle's linear momentum.

* Time is not an absolute quantity as assumed by Newton equation of motion $F = ma$

1. fails to predict the exact behaviour of a particle when the particle's speed approaches the speed of light ($0.3\ G\ m/s$)

2. Its conclusion invalid when particles are the size of an atom and move close to one another.

• **Newton's Law of Gravitational Attraction:**

- Define: A law governing the mutual attraction between any two particles.

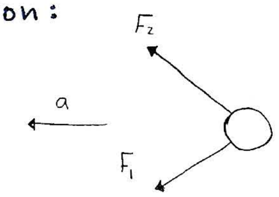
$$F = G\frac{m_1 m_2}{r^2} \quad ; \quad G = 66.73 \times 10^{-12}\ m^3/kg\cdot s^2$$

- If the particle located at\near earth only gravitational force between earth and particle

- For most engineering calculations, (g) is a point on the surface of the earth at sea level, and a latitude of 45°, which is considered the "Standard Location".

- (13.2): The equation of motion:

$\Sigma F = ma$



if $F_R = \Sigma F = 0$

$\therefore\ a = 0$

↳ so, the particle will either

- remain at rest

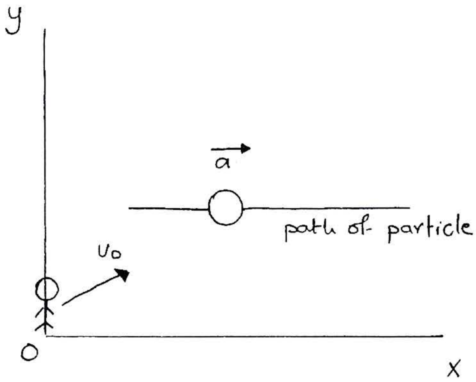
or - move along straight-line path with constant velocity as static equilibrium.

- Inertial Refference Frame (Newtonian refference frame):

- The acceleration of particle must be measured w.r.t a refference frame that is either:

1) Fixed

or 2) translates with a constant velocity.



_ex₁: The motions of rockets of sattelites → I.R.F (stars)

_ex₂: The motions of particles on earth → I.R.F (earth)

F.B.D

represents the particle to be free of its surrounding supports\ forces.

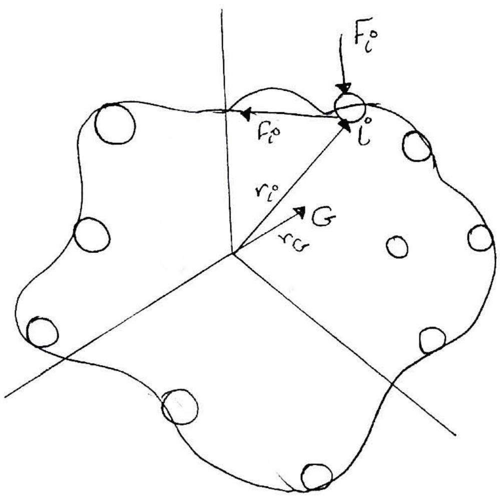
K.D (kinetics)

Pertains to the particle's motion as caused by the forces.

-(13.3): Equation of motion for asystem of particles:

F_i : external force

f_i : internal force.



$$F_i + f_i = m_i a_i$$

$$\sum F_i + \sum f_i = \sum m_i a_i$$

If aparticle carried out $\sum f_i = 0$

$$\sum F_i = \sum m_i a_i$$

G: Center of mass.

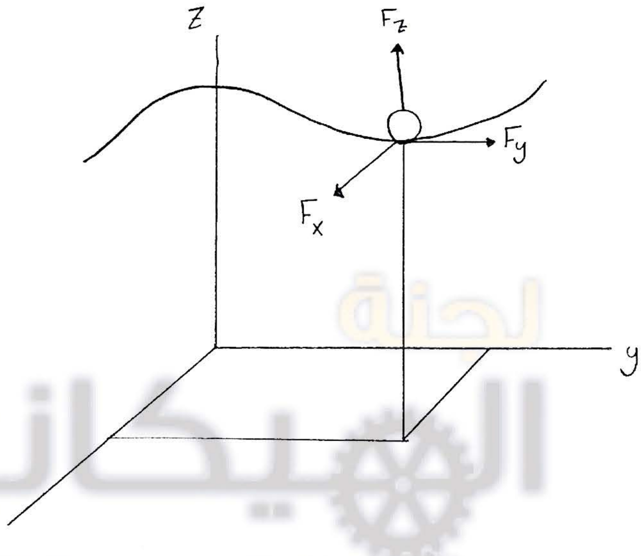
- By definition of the center of mass

$$\boxed{mr_G = \sum m_i r_i} \rightsquigarrow \underline{ma_G = \sum m_i a_i}$$

where $m = \sum m_i$ is the total mass of all particles

$$\sum F = ma_G$$

- (13.4) : Equations of motion : Rectangular Coordinates :



$$\sum F = ma$$

$$\sum F_x i + \sum F_y j + \sum F_z k = m(a_x i + a_y j + a_z k)$$

$\therefore$

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

→ If we have rough surface

its necessary to use the frictional equation:

$$F_f = \mu_k N$$

If the particle is on the verge (على وشك) of relative motion

$$F_s = \mu_s N$$

If we have spring (an elastic spring having negligible mass)

$$F_s = KS \quad ; \quad S = L - L_o$$

the spring force.

L : the deformed length

L_o : the undeformed length

- Kinematics:

- If acceleration is a function of time

$\rightarrow$ use $a = \frac{dv}{dt}$, $v = \frac{ds}{dt}$

which, when integrated $\xrightarrow{\text{yield}}$ The particle's $\underline{v}$ and $\underline{s}$

- If acceleration is a function of displacement

$\xrightarrow{\text{integrate}}$ $a\,ds = v\,dv$

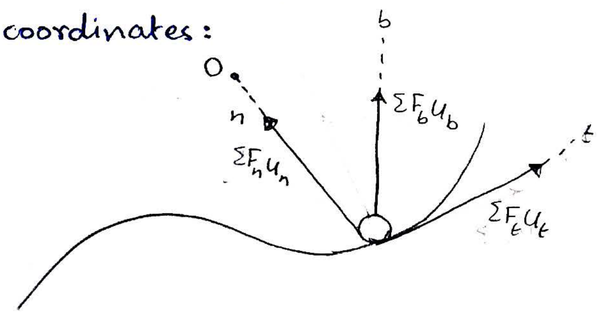
to obtain $\longrightarrow$ v as a function of position.

- If acceleration is constant, use $\rightarrow$ $v = v_0 + a_c t$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$v^2 = v_0^2 + 2a_c(s - s_0).$$

- (13·5): Equations of motion: Normal and Tangentional coordinates:



Inertial Coordinates System

when a particle moves along curved path which is known

the equation of motion for the particle may be written in the tangentional, normal, and binormal directions.

$$\Sigma F = ma$$

$$\Sigma F_t u_t + \Sigma F_n u_n + \Sigma F_b u_b = m a_t + m a_n$$

$$\boxed{a_t = \frac{dv}{dt}} \quad \text{or} \quad \boxed{a_t = v\frac{dv}{ds}}$$

$$\boxed{a_n = \frac{v^2}{\rho}}$$

$$\boxed{\begin{aligned} \Sigma F_t &= m a_t \\ \Sigma F_n &= m a_n \\ \Sigma F_b &= 0 \end{aligned}}$$

→ If

ΣF_t is in the direction of motion (+ve)
ΣF_t is in the opposite direction of motion (−ve)

→ If

ΣF_n is toward the path's center of curvature (+ve)
ΣF_n is in opposite direction of path's center of curvature (−ve)

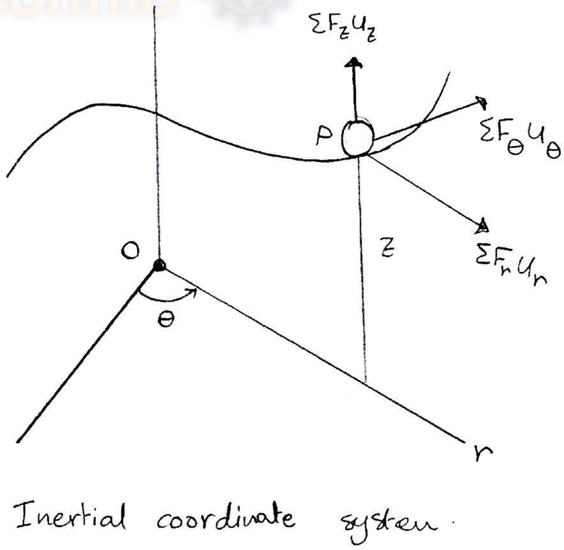
- If the path is defined as $y = f(x)$, then the radius of curvature at the point where the particle is located can be obtained from

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left| d^2y/dx^2 \right|}$$

−(13.6) : Equation of motion : Cylindrical coordinates :

$$\Sigma F = ma$$

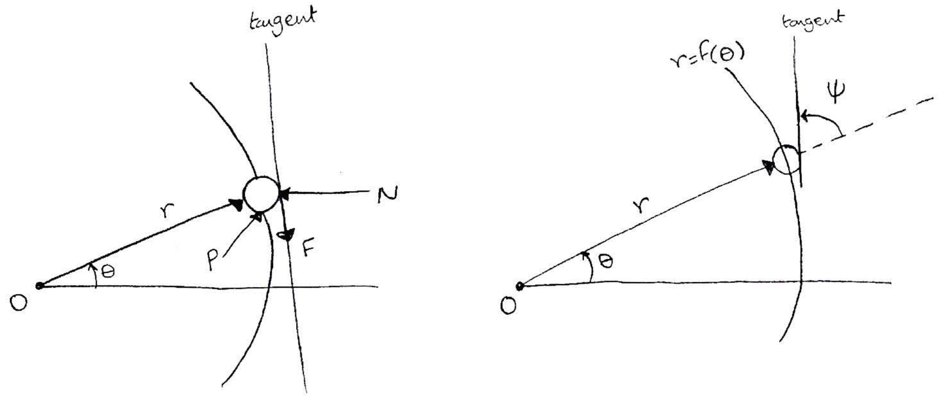
$$\boxed{\Sigma F_r u_r + \Sigma F_\theta u_\theta + \Sigma F_z u_z = ma_r u_r + ma_\theta u_\theta + ma_z u_z}$$



Inertial coordinate system.

$$\Sigma F_r = ma_r$$
$$\Sigma F_\theta = ma_\theta$$
$$\Sigma F_z = ma_z$$

* Tangentional and Normal Forces :



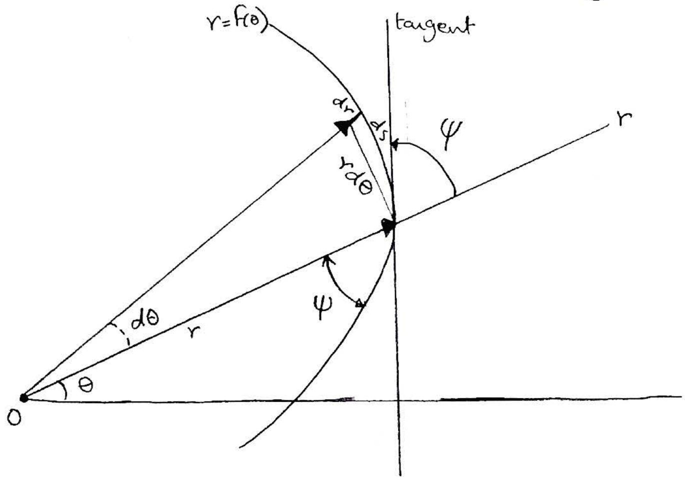
P : the force causes the particle to move along a path $\boxed{r = f(\theta)}$

N : the normal force which the path exerts on the particle (always perpendicular to the tangent)

F : the frictional force, always acts along the tangent in the opposite direction of motion.

The directions of N and F can be specified relative to the radial coordinate (ψ)

ψ between the extended radial line and the tangent to the curve.



- ds : the displaced distance
- dr : the component of displacement in the radial direction.
- $rd\theta$: the component of displacement in the transverse direction

$$\boxed{\tan \psi = \frac{r}{dr/d\theta}}$$

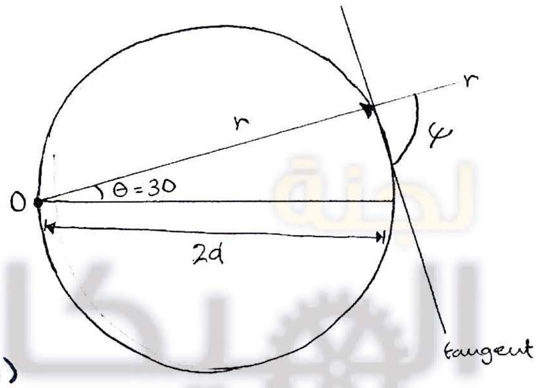
If

ψ → $(+ve)$: so it is measured from the extended radial line to the tangent in C.C.W sense <u>or</u> in the positive direction of θ.

→ $(-ve)$: it is measured in the opposite direction to positive θ.

- example :

$$r = a(1 + \cos\theta)$$

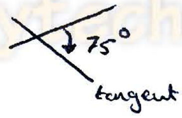


⇒ solution :

$$\dot{r} = -a\sin\theta$$

$$\tan\psi = \frac{r}{\dot{r}} = \frac{a(1+\cos\theta)}{-a\sin\theta}$$

$$= \ominus 75^\circ$$



C.W.

· remember :

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$a_z = \ddot{z}$$

$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$$

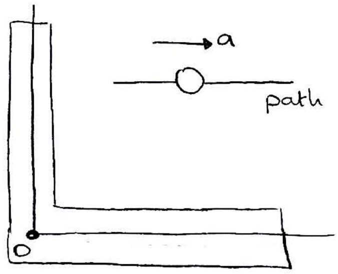
$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\Sigma F_z = m\ddot{z}$$

- Chapter Review :-

- Kinetics : is the study of the relation between forces and acceleration they cause
  ($\Sigma F = ma$) Newton's 2nd law of motion.

- Inertial coordinate systems :



fixed or translate with a constant v.

- To describe rectilinear motion :

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$
$$\Sigma F_z = ma_z .$$

- Normal and Tangentional (n,t) Axes :

$$\Sigma F_t = ma_t \;,\; \Sigma F_n = ma_n \;,\; \Sigma F_b = \text{zero}$$

$$a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$

<u>or</u> $$a_t = v\frac{dv}{ds} \qquad \rho = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{|d^2y \backslash dx^2|}$$

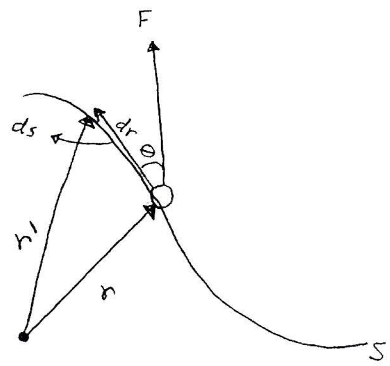
- Cylindrical Coordinates :

$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$$
$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$
$$\Sigma F_z = m\ddot{z}$$

* Chapter (14): Kinetics of a particle: Work and Energy.

- (14.1): The work of a force:

• define: a force F will do work on a particle only when the particle undergoes a displacement in the direction of Force.



The work done by F is a scalar quantity

$$dU = F\,ds\,\cos\theta$$

$$dU = F \cdot dr$$

This result may be interpreted in one of two ways:

1) The product of (F) and The component of displacement ($ds\cos\theta$) in direction of Force.

2) The product of (ds) and the component of Force ($F\cos\theta$) in direction of displacement.

→ If $0° \leq \theta < 90°$ → The force component of displacement have the same sense.
→ The work is (+ve)

→ If $90° < \theta \leq 180°$ → The force component of displacement will have opposite sense.
→ The work is (−ve)

→ If $\theta = 90°$ → $dU = 0$ or if the force is applied at a fixed point (No displacement)

• $dU = F\,ds\,\cos\theta$ → ($1\,J = 1\,N \cdot m$)

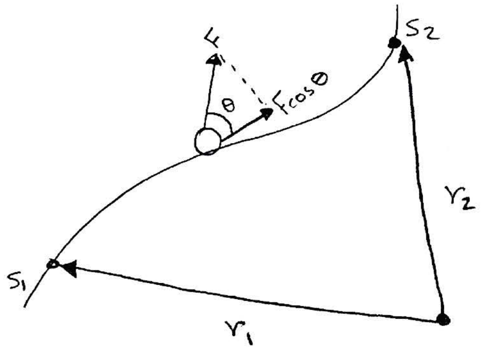
$dU = F\,ds\,\cos\theta \rightarrow (1\,J = 1\,N.m)$

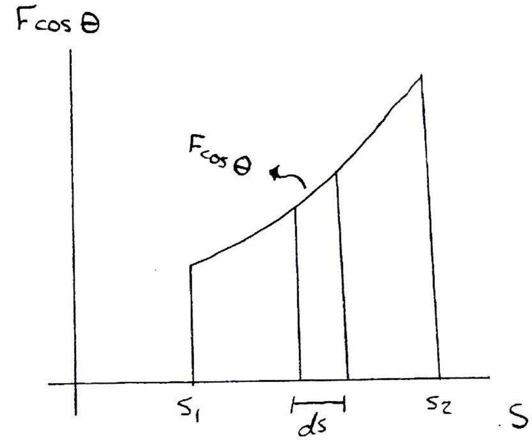
- Moment has the same unit (N.m) with work
- Moment is a vector quantity
- Work is a scalar quantity

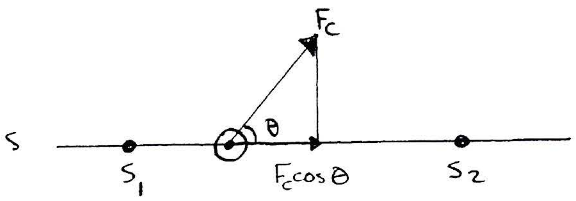
Whereas

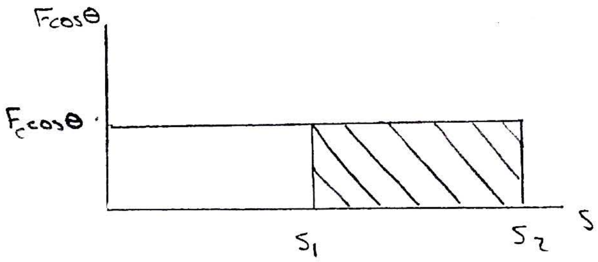
$$\boxed{U_{1-2} = \int_{r_1}^{r_2} F.dr = \int_{s_1}^{s_2} F\cos\theta\,ds}$$

work of a variable force from point to another.









$$U_{1-2} = F_c\cos\theta \int_{s_1}^{s_2} ds$$

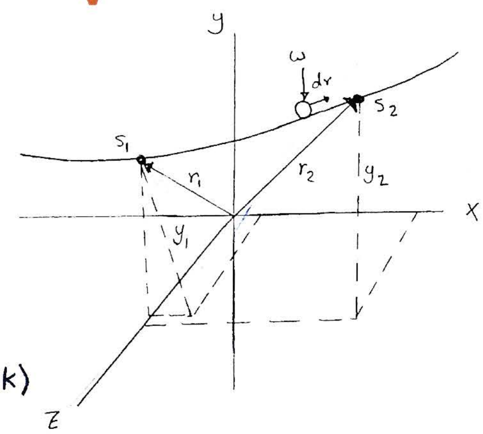
or

$$\boxed{U_{1-2} = F_c\cos\theta\,(s_2 - s_1)}$$

work of a constant force moving along straight line.

Here, the work of (F) represents the area of the rectangular.

• Work of a weight :



$dr = dx\,i + dy\,j + dz\,k$

$\omega = -\omega j$

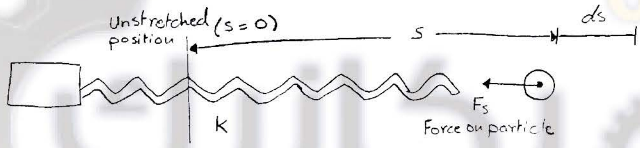
$U_{1-2} = \int F \cdot dr$

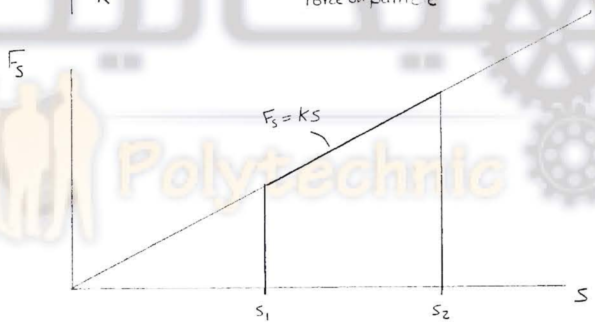
$= \int_{r_1}^{r_2} (-\omega j) \cdot (dx i + dy j + dz k)$

$= \int_{y_1}^{y_2} -\omega\, dy = -\omega(y_2 - y_1)$

$\boxed{U_{1-2} = -\omega \Delta y}$ if Δy is downward $(-\Delta y)$
Them work of the weight is $(+ve)$

• Work of a spring force :





If an elastic spring is elongated a distance $\underline{ds}$ "1st figure"
Then :

$dU = -F_s\,ds = -ks\,ds$

$U_{1-2} = \int_{s_1}^{s_2} F_s\,ds = \int_{s_1}^{s_2} -ks\,ds$

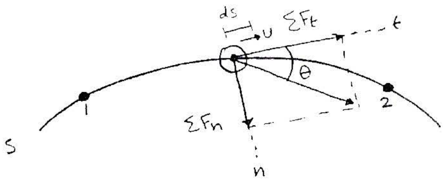
The work is (-ve) since F_s acts in opposite since to ds.

$$\boxed{U_{1-2} = -\frac{1}{2}K(s_2^2 - s_1^2)}$$

The work represents the trapezoidal area under the line $F_s = ks$
(Fig. "2") ..

- (14-2) : Principle of Work and Energy :

$$\Sigma F_t = m a_t$$



$$a_t = \upsilon \frac{d\upsilon}{ds}$$

$$\Sigma F_t = m\upsilon \frac{d\upsilon}{ds}$$

$$\Sigma \int_{s_1}^{s_2} F_t \, ds = \int_{\upsilon_1}^{\upsilon_2} m \upsilon \, d\upsilon$$

$$\boxed{\Sigma \int_{s_1}^{s_2} F_t \, ds = \frac{1}{2} m \left(\upsilon_2^2 - \upsilon_1^2\right)} \qquad \Sigma F_t = \Sigma F \cos\theta$$

or $$\boxed{\Sigma U_{1-2} = \frac{1}{2} m \left(\upsilon_2^2 - \upsilon_1^2\right)} \qquad T = \frac{1}{2} m \upsilon^2$$

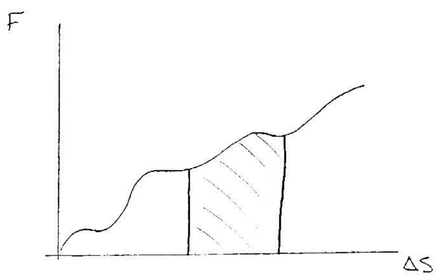
or $$\boxed{T_1 + \Sigma U_{1-2} = T_2}$$

- T (kinetic energy) is always (+ve)

  $$T = \frac{1}{2} m \upsilon^2$$

- Work is (+ve) if $\underline{F}$ and $\underline{\Delta r}$ are in the same sence
- Forces that are fuctions of displacement must be integrated to obtain the work.

- Graphically :- The work is equal to the area under The force - displacement curve.



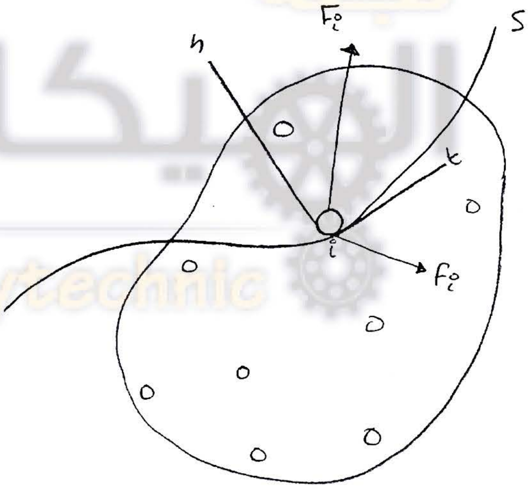
* The work of a weight is the product of the weight magnitude and the vertical displacement.

  $U_w = \mp W Y$ it's positive when the weight moves downwards.

* The work of a spring is

  $U_s = \frac{1}{2} K s^2$ where k: is the spring stiffness and s: is the stretch\compression of the spring.

- (14.3): Priciple of Work and Energy for a system of particles:



F_i :- a resultant external force.

f_i :- a resultant internal force which all the other particles exert on the i^{th} particle.

- (14.4) : Power and Efficiency :

- **Power :**

$$\boxed{P = \frac{dU}{dt}}$$

scalar, scalar, scalar

if $dU = F \cdot dr$

$$P = F \frac{dr}{dt} \Rightarrow \boxed{P = \vec{F} \cdot \vec{U}}$$

vector, vector

$$P = \frac{dU}{dt} = F \cdot U \Rightarrow [1\ W] = 1\left[J/s\right] = 1\left[N \cdot m/s\right]$$

In SI system.

- **Efficiency :** is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine

$$\boxed{\epsilon = \frac{\text{Power Output}}{\text{Power input}}}$$

if energy supplied to the machine occurs during the same time interval , Then :

$$\boxed{\epsilon = \frac{\text{energy output}}{\text{energy input}}}$$

- If the body is accelerating $\rightarrow \Sigma F = ma$ (To Find F)
- If you have F and U $\rightarrow P = \vec{F} \cdot \vec{U}$ or $P = F U \cos\theta$
- If you have work and time $\rightarrow P_{avg.} = \frac{\Delta U}{\Delta t}$

- (14-5) : Conservative forces and Potential Energy :

. conservative force : is independent of the path and depends only on the force's initial and final position.

- ex: 1. The weight of a particle depends only on the vertical dicplacement
2. The Force by a spring depends only on the spring's elongation\compression.

. Unconservative force : is defined as the capacity for doing work.

If a particle is originally at rest , then

$$\boxed{\Sigma U_{1-2} = T_2}$$

The kinetic energy is equal to the wok that must be done on the particle to pring it from a state of rest to a speed v.

- When energy comes from the position of the particle, measured from a fixed datum or reference plane it is called potential energy .

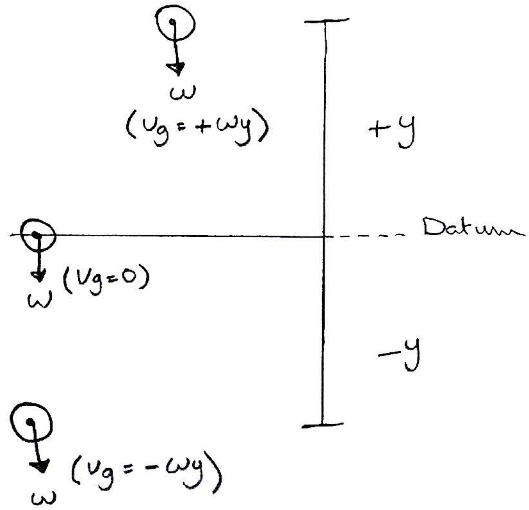
In mechanics, the potential energy created by gravity (weight) or an elastic spring is important ..

* Gravitational Potential Energy :

$$\boxed{U_g = Wy}$$

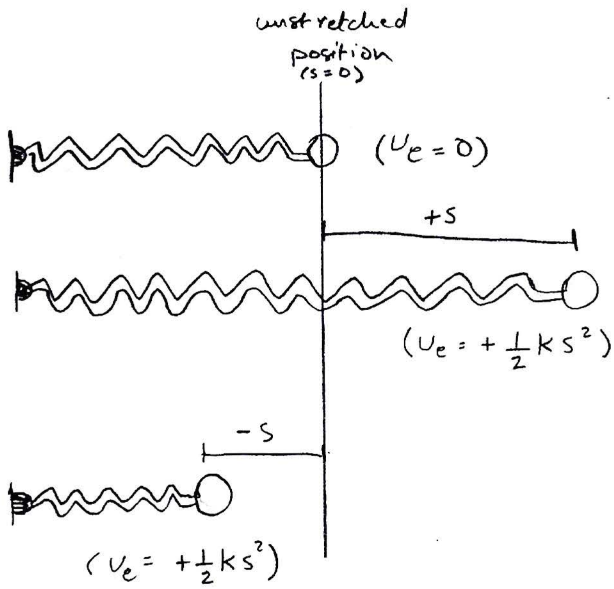
if y is $(+ve) \rightarrow U_g$ is $(+ve)$

if y is $(-ve) \rightarrow U_g$ is $(-ve)$



* Elastic Potential Energy :

$$\boxed{U_e = \frac{1}{2} k s^2}$$

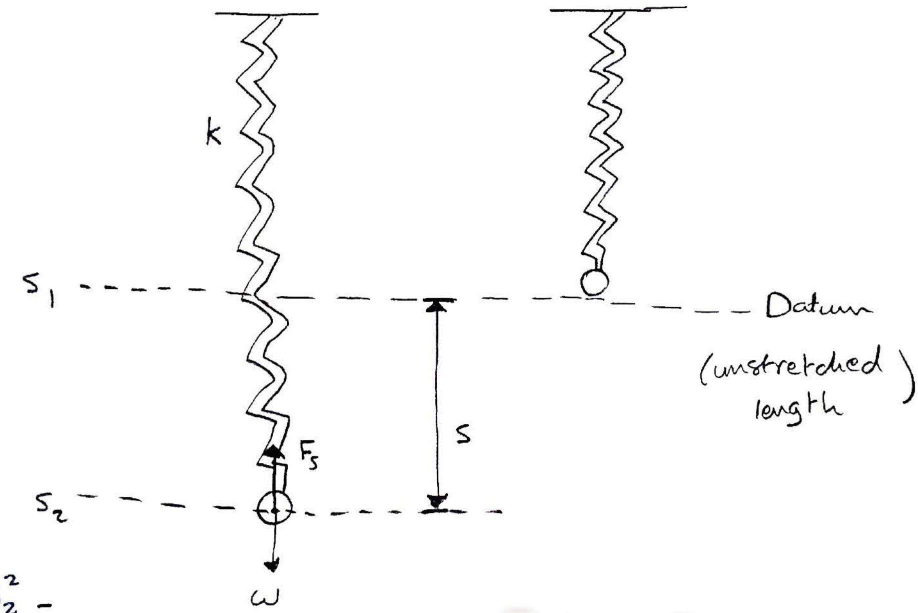


* Potential Function :

If a particle is subjected to both gravitational and elastic forces

$$\boxed{V = U_g + U_e} \qquad \boxed{U_{1-2} = U_1 - U_2}$$

- Example:



$$U = U_g + U_e$$
$$= -\omega s + \frac{1}{2}ks^2$$

$$U_{1-2} = U_1 - U_2$$
$$= -\omega s_1 + \frac{1}{2}ks_1 - (-\omega s_2 + \frac{1}{2}ks_2^2)$$
$$= \omega(s_2 - s_1) - (\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$$

- $\boxed{dU = -dU_{(x,y,z)}}$

- $\boxed{dU = F \cdot dr = F_x dx + F_y dy + F_z dz}$

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

i.e. $\boxed{\vec{F} = -\Delta U}$

$U_g = \omega y$ --- when y is (+ve)

$\rightarrow F_y = -\frac{\partial U}{\partial y} \rightarrow F_y = -\frac{\partial}{\partial y}(\omega y)$

$$F_y = -\omega$$

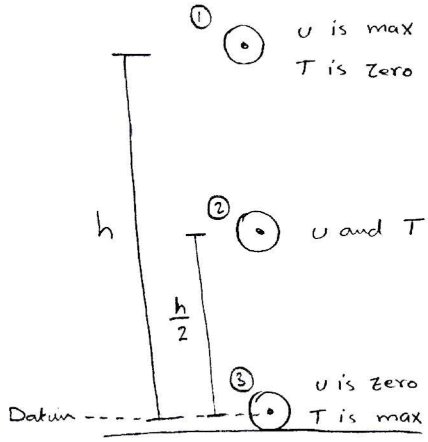
- (14-6): Conservation of Energy:

$$\boxed{(\Sigma U_{1-2})_{cons.} = U_1 - U_2}$$

$$\boxed{T_1 + U_1 + (\Sigma U_{1-2})_{uncon.} = T_2 + U_2}$$

$$\boxed{T_1 + U_1 = T_2 + U_2}$$ if only conservative forces do work.

$$\boxed{E_i = T_i + U_i}$$ The total mechanical energy..



① $E = T_1 + U_1$

$= 0 + wh = \underline{wh}$

② $E = T_2 + U_2$

$= \frac{1}{2}mv^2 + w(\frac{h}{2})$

$= \frac{1}{2}\underbrace{mg}_{w}h + w(\frac{h}{2}) = \underline{wh}$

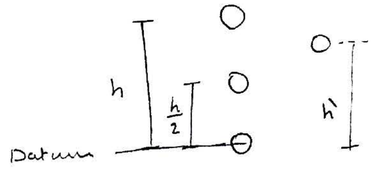
③ $E = T_3 + U_3$

$v = \sqrt{2gh}$

$= \frac{1}{2}m(2gh) + 0 = \underline{wh}$

$E_L = w(h - h')$

an energy losses.



when it came back

* System of Particles:

If a system of paticles is subjected only to conservative forces , Then

$$\boxed{\Sigma T_1 + \Sigma U_1 = \Sigma T_2 + \Sigma U_2}$$

$$\Sigma T + \Sigma U = \text{constant.}$$

* Procedure for analysis:

. The conservation of energy equation is easier to apply than the principle of work and energy

⇒ conserv. of energy eqn ⤳ $\Sigma T_1 + \Sigma U_1 = \Sigma T_2 + \Sigma U_2$

⇒ principle of work and energy ⤳ $T_1 + \Sigma U_{1-2} = T_2$

Beause

The cons. eqn. requires specifying the particle's kinetics and potential energies at only two points a long the path.

- Draw 2 diagrams showing the particle located at its initial and final points along the path
- If the particle is subjected to vertical displacement establish fixed Horiz. datum to find U_g.
- $U_g = wy \rightarrow y$ is (+ve) upward from the drum and (−ve) downward from the datum.

  $U_e = \frac{1}{2} K s^2$
- $T_1 + U_1 = T_2 + U_2$
- when you determine $T = \frac{1}{2} m v^2$ remember that v must be measured from an internal reference frame.

* Chapter (15): Kinetics of a particle :

Impulse and Momentum :

$$\sum F = ma = m\frac{dv}{dt}$$

$$\sum\int_{t_1}^{t_2} F\,dt = m\int_{v_1}^{v_2} dv = mv_2 - mv_1$$

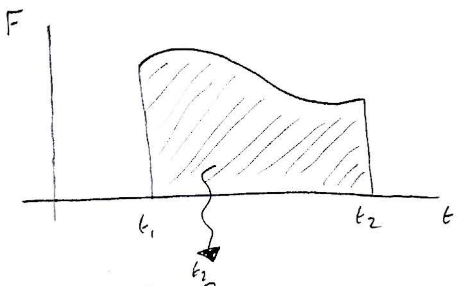
if $v_2 = ?$

1. $\sum F = m a$
2. integrate $a = \frac{dv}{dt} \rightarrow v_2$

- **Linear Momentum** $L = mv$
  - The linear momentum vector has the same direction as v
  - Its units of mass-velocity (kg·m/s).

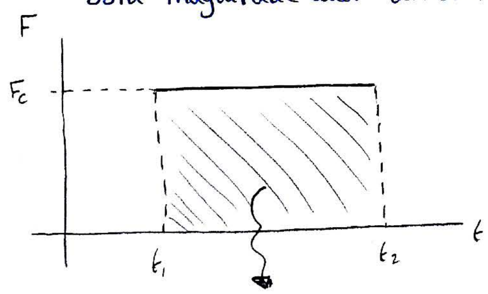
- **Linear Impulse** $I = \int F\,dt$
  - The linear impulse acts in the same direction of F
  - Its units of Force-time (N·s).

[1] If force is expressed as a function of time



$$I = \int_{t_1}^{t_2} F(t)\,dt$$

[2] If force is constant in both magnitude and direction



$$I = F_c\,(t_2 - t_1).$$

- Although the units for impulse and momentum are defined differently, it can be shown that eq. $\sum\int_{t_1}^{t_2} F\,dt = mv_2 - mv_1$ is dimensionally <u>Homogeneous</u>.

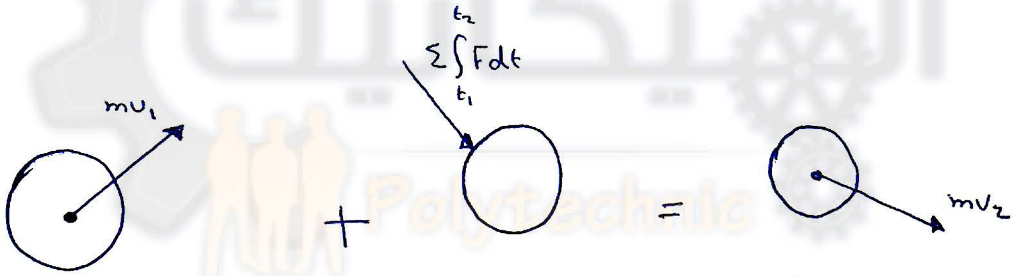
* Principle of linear Impulse and Momentum:

$$\boxed{m v_1 + \sum \int_{t_1}^{t_2} F \, dt = m v_2}$$

→ The sum of all impulses applied to the particle from t_1 to t_2

→ If we resolved into its x, y, z components:

$$\left\{ \begin{aligned} m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt &= m(v_x)_2 \\ m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt &= m(v_y)_2 \\ m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt &= m(v_z)_2 \end{aligned} \right.$$



initial momentum diagram

impulse diagram

final momentum diagram.

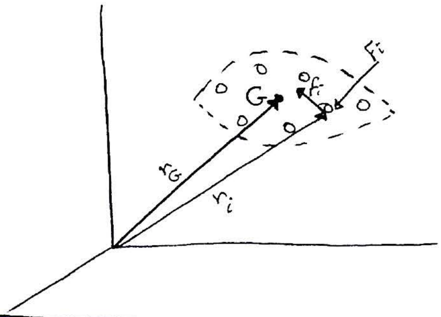
- The principle of linear impulse and momentum is used to solve problems involving Force, time, and velocity (F, t, v)

** كل قوة تصنع تأثير .. حتى لو لم تصنع شغل **

impulse — no work

-(15-2): Priciple of Linear Impulse and Momentum For a System of particles:

$\sum f_i$: internal forces by Newton's 3rd law they occur in equal but opposite collinear pairs.



$$\sum F_i = \sum m_i \frac{dv_i}{dt}$$

$$\boxed{\sum m_i (v_i)_1 + \sum \int_{t_1}^{t_2} F_i \, dt = \sum m_i (v_i)_2}$$

$$m r_G = \sum m_i r_i \longrightarrow m = \sum m_i$$

$$\boxed{m v_G = \sum m_i v_i} \longrightarrow \boxed{m(v_G)_1 + \sum \int_{t_1}^{t_2} F_i \, dt = m(v_G)_2}$$

-(15-3): Conservation of linear Momentum for a system of particles:

when $\sum F_i = 0 \rightarrow$ $\boxed{\sum m_i (v_i)_1 = \sum m_i (v_i)_2}$ The conservation of linear momentum.

The total linear momentum for a system of partcles remains <u>constant</u> during the time period t_1 to t_2

$$m v_G = \sum m_i v_i$$

$$\rightarrow (v_G)_1 = (v_G)_2 .$$

- The velocity (U_G) of the mass center for the system of particles does not change if no external impulses are applied to the system.

- The conservation of linear momentum is often applied when particles collide (تصادم) or interact (تفاعل)

* If the time period over which the motion is studied is very short

[1] some of the external impulses may be neglected or considered approximately equal zero
The forces causing these negligible impulses are called "nonimpulsive forces".

[2] Forces which are very large act for a very short period of time produce a significant change in momentum and are called "impulsive forces"
They of course, cannot be neglected.

- Impulsive forces Normally occure due to
  1. an explosion
  or
  2. the striking of one body against another.

- Nonimpulsive forces may include
  1. The weight of a body
  2. The force imparted (منقولة) by a slightly deformed spring having a relatively small stiffness.
  3. any force that is very small compared with other larger.

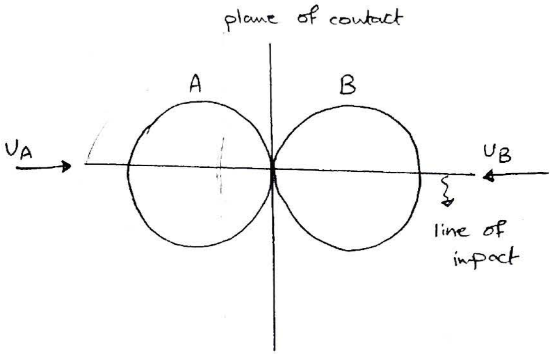
* The effect of striking a tennis ball with a racket During the very short time of interaction,
  - The force of the racket on the ball is impulsive since it changes to ball's momentum drastically. بشكل كبير
  - By comparison, the ball's weight will have a negligible effect on the change in momentum, and therefore it is nonimpulsive.

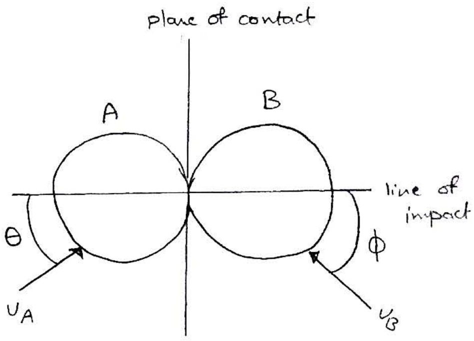
* If an impulse-momentum analysis is considered during the much longer time of flight after the racket-ball interaction, then the impulse of the ball's weight is important since it, along with air resistance, causes the change in the momentum of the ball.

- (15-4): Impact :-
  - Impact occurs when 2 bodies collide with each other during a very short period of time, causing relatively large (impulsive) forces to be exerted between the bodies.

→ ex: [1] The striking of a hammer on a nail
[2] The striking of a golf club on a ball.

* There are 2 Types of impact:





[1] Central impact

occurs

when direction of motion of mass centers of two colliding particles is along a line (line of impact) passing through the mass centers of the particles.

[2] Oblique impact

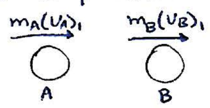
occurs

when the motion of one/both of the particles make an angle with the line of impact.

• Central Impact:

To illustrate the method for analyzing the mechanics of impact

(a)

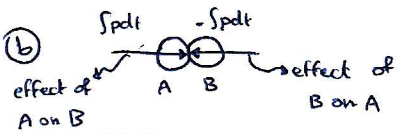


$(v_A)_1 > (v_B)_1$

Before impact

initial momenta

(b)



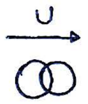
Deformation impulse

During the collision

"The particles must be though as deformable/non rigid"

They exert an equal but opposite deformation impulse $\int P dt$ on each other.

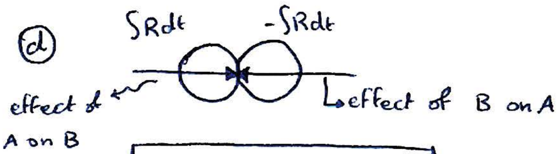
(c)



Maximum deformation

<u>only</u> at the instant of max-deformation will both particles move with a common velocity "U", since their relative motion is zero.

(d)



Restitution impulse

تعويض / العودة إلى الوضع السابق

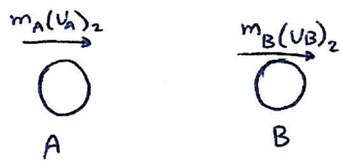
either → return to their original shape
or → remain permenantly deformed

The equal but opposite restitution impulse $\int R\,dt$ pushes the particles apart from one another

in reality $\int P\,dt > \int R\,dt$

(deformation inpulse) > (Restitution impulse)

(e)



Final momenta

$(U_B)_2 > (U_A)_2$

After impact

* Since during collision "The internal impulses" of deformation and restitution <u>cancel</u> <u>momentum</u> for the system of particles is conserved

$$\boxed{m_A(U_A)_1 + m_B(U_B)_1 = m_A(U_A)_2 + m_B(U_B)_2} \quad \text{----} \; \boxed{1}$$

⇒ we need another equation to determine $(V_A)_2$ and $(V_B)_2$:

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{U-(V_A)_2}{(V_A)_1 - U}$$

For A during deformation phase a, b, c: $m_A (V_A)_1 - \int P\,dt = m_A U$

For the restitution phas c, d, e: $m_A U - \int R\,dt = m_A (V_A)_2$

For B during deformation phase a, b, c: $m_B (V_B)_1 + \int P\,dt = m_B U$

For the restitution phase c, d, e: $m_B U + \int R\,dt = m_B (V_B)_2$

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{(V_B)_2 - U}{U - (V_B)_1}$$

$$\Rightarrow \boxed{e = \frac{(V_B)_2 - (V_A)_2}{(V_A)_1 - (V_B)_1}}$$

where e: the coefficient of restitution (The ratio of restitution to deformation).

* It is important to carefully establish a sign convention for deffining the positive direction for both V_A and V_B.

* If a negative value results from the solution of either $(V_A)_2$ or $(V_B)_2$; it indicates motion is to the left if you assume that right is the (+ve) direction

* Coefficient of Restitution:

$$e = \frac{(V_B)_2 - (V_A)_2}{(V_A)_1 - (V_B)_1}$$, "e" varies appreciably with (شكل ملحوظ)

impact velocity — size and shape.

↝ The values of (e) between zero and One

$0 < e < 1$

* Elastic Impact $(e=1)$:

- If the collision is Perfectly elastic
  $\rightarrow \int P\,dt = \int R\,dt$ (equal and opposite)

- Although in reality this can never be acheived, $e=1$ for an elastic collision.

* Plastic Impact $(e=0)$: another name inelastic
- In this case there is no restitution impulse
  $\int R\,dt = 0$

so that, after collision both particles couple\stick together and move with a common velocity.

* Notes :

1. The priciple of work and energy cannot be used for the analysis of impact problems since it is not possible to know how the internal forces of deformation and restitution vary or displace during the collision.

2. The energy loss during collision is $\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

3. The energy loss occurs because some of the initial kinetic energy of the particle is transformed into:
   A) Thermal energy
   as well as B) creating sound
   as well as C) localized deformation of the material.

4. If the impact is perfectly elastic $\rightsquigarrow$ no energy is lost whereas if the collision is plastic $\rightsquigarrow$ the energy lost is max

* Procedure for analysis (central impact):

provided معطى
1. The coefficient of restitution (e)
2. The mass of each particle.
3. each particle's initial velocity.

requierments
$(v_A)_2$ and $(v_B)_2$

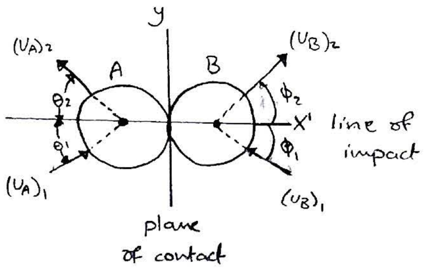
$$\sum m v_1 = \sum m v_2 \quad \ldots (1)$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \quad \ldots (2)$$

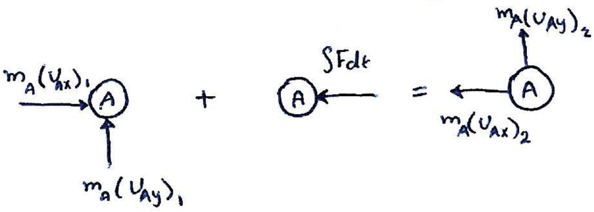
• Oblique Impact:

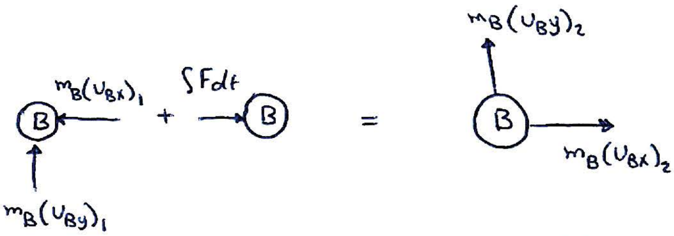
- when oblique impact occurs
→ the particle's move away from each other with velocities having
1. unknown directions
2. unknown magnitudes

provided
The initial velocities



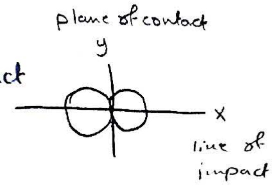
requirments
Four unknowns
↝ represented either as
$(v_A)_2$, $(v_B)_2$, θ_2, ϕ_2
or as the x and y componets of the final velocities.





× Procdure for analysis (Oblique Impact):

- y-axis is established within the plane of contact
  x-axis is established along the line of impact



- The impulsive forces of deformation and restitution acts only in the x direction.

- requierments : $(U_{Ax})_2$, $(U_{Ay})_2$, $(U_{Bx})_2$, and $(U_{By})_2$

  (1) momentum of system is conserved along x-axis (the line of impact)

$$\sum m(U_x)_1 = \sum m(U_x)_2 \quad \text{----} \quad (1)$$

  (2) The coeffiecient of restitution relates the relative velocity components of the particles along x-axis (the line of impact)

$$e = \frac{(U_{Bx})_2 - (U_{Ax})_2}{(U_{Ax})_1 - (U_{Bx})_1} \quad \text{----} \quad (2)$$

From (1) and (2) Find $(U_{Ax})_2$ and $(U_{Bx})_2$.

  (3) momentum of particle A is conserved along (y-axis) since no impulse acts on particle A in this direction

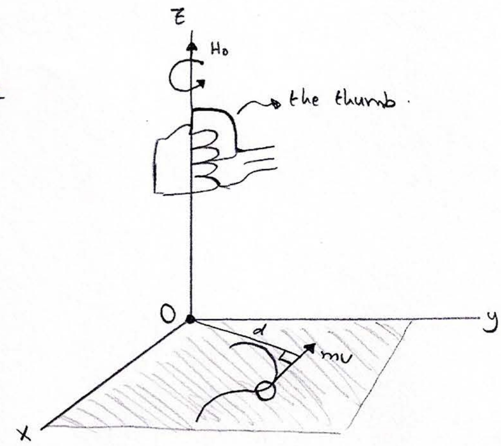
$$m_A(U_{Ay})_1 = m_A(U_{Ay})_2 \quad \text{or} \quad (U_{Ay})_1 = (U_{Ay})_2 \quad \text{---}(3)$$

  (4) momentum of particle B is conserved along (y-axis) since no impulse acts on particle B in this direction

$$(U_{By})_1 = (U_{By})_2 \quad \text{---} \quad (4)$$

From (3) and (4) Find $(U_{Ay})_2$ and $(U_{By})_2$

- (15.5): Angular Momentum:-



- The angular momentum of a particle about point O is defined as the "moment" of the particle's linear momentum about O

- Since this concept is analogous (مشابه) to finding the moment of a force about a point, The angular momentum, H_o, is sometimes reffered to as the moment of momentum

- Scalar Formulation:

If a particle moves along a curve lying in the x-y plane, the angular momentum at any instant about point O (actually the z-axis) by using a scalar Forumulation

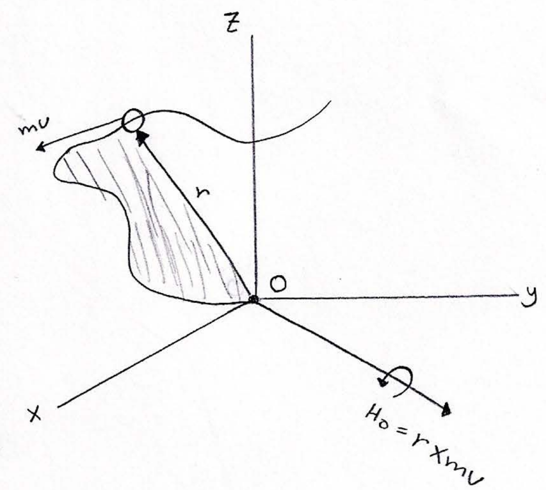
$$(H_o)_z = (d)(mv) \quad \text{in } [kg \cdot m^2 \backslash s]$$

⇒ The direction of H_o is defined by the right-hand rule (from mv to O).

- Vector Formulation:

If the particle moves along a space curve,

The vector cross product can be used to determine the angular momentum about O



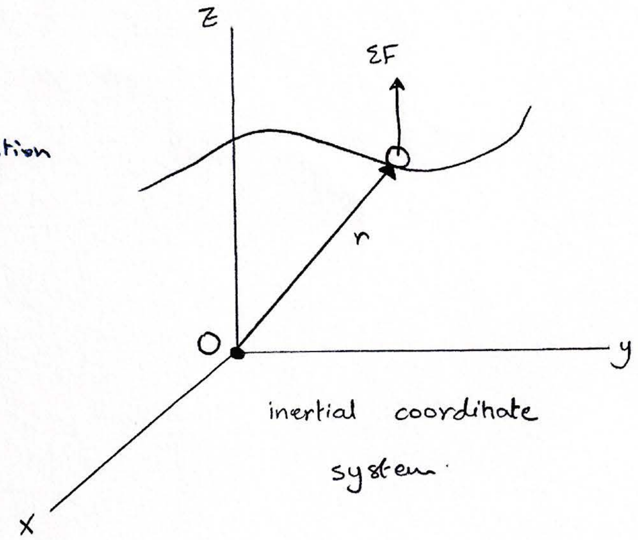
$$H_o = \vec{r} \times m\vec{v}$$

r: from "O" to particle

remember cross product

$$H_o = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

- (15.6): Relation Between Moment of a Force and Angular Momentum:



$\Sigma F = m\dot{v}$ ... the equation of motion

$\Sigma M_o = r \times \Sigma F$

$\Sigma M_o = r \times m\dot{v}$

$H_o = r \times mv$

$\rightarrow \dot{H}_o = \frac{d}{dt}(r \times mv)$

$\dot{H}_o = \underbrace{\dot{r} \times mv}_{} + r \times m\dot{v}$

$\dot{r} \times m\dot{r} = m(\dot{r} \times \dot{r}) = 0$ since the cross product of a vector with itself equal zero

$$\boxed{\Sigma M_o = \dot{H}_o}$$

$\Sigma M_o = r \times \Sigma F$

$= r \times m\dot{v}$

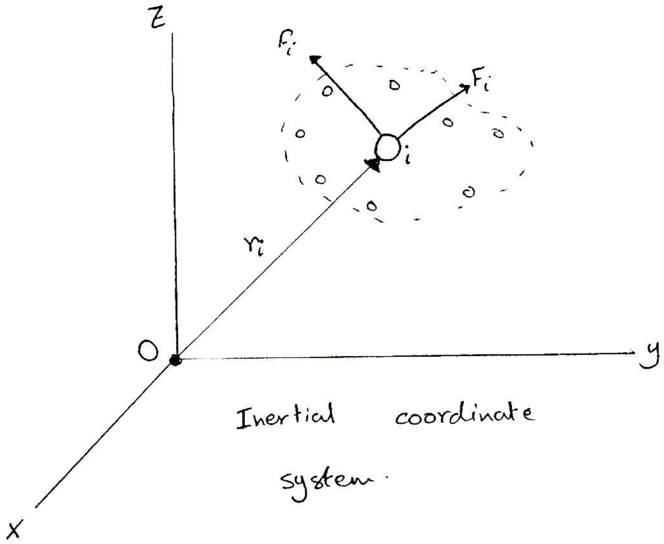
The time rate of change of the particle's angular momentum about point O

If $L = mv$ where L: The particle's linear momentum

Then $\boxed{\Sigma F = \dot{L}}$ $\dot{L} = m\dot{v}$

* System of Particles:

$$\Sigma M_o = \dot{H}_o$$



Inertial coordinate system.

The forces acting on the arbitrary i^{th} particle of the system consist of a resultant external force F_i and a resultant internal force f_i

$$(r_i \times F_i) + (r_i \times f_i) = (\dot{H}_i)_o$$

$$\Sigma(r_i \times F_i) + \underbrace{\Sigma(r_i \times f_i)}_{\text{is zero}} = \Sigma(\dot{H}_i)_o$$

↳ since the internal forces occur in equal but opposite collinear pairs

Hence the moment of each pair about point O is zero.

$$\Sigma M_o = \dot{H}_o$$

the sum of the moments about point O of all the external forces acting on a system of particles.

Although O has been choosen here as the origin of coordinate, it actually can represent any fixed point in the inertial frame of reference.

- (15.7) : Principle of angular Impulse and Momentums.

$$\Sigma M_o = \dot{H}_o$$

$$\Sigma M_o = \frac{d}{dt}(H_o)$$

$$\int_{t_1}^{t_2} \Sigma M_o dt = \int_{H_{o1}}^{H_{o2}} d(H_o)$$

$$\Sigma \int_{t_1}^{t_2} M_o dt = (H_o)_2 - (H_o)_1$$

$$\boxed{(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o\, dt = (H_o)_2}$$

Priciple of angular impulse and momentum.

$(H_o)_1$: The initial angular momenta

$(H_o)_2$: The final angular momenta.

are defined as the moment of the linear momentum of the particle ($H_o = r \times mU$) at the instants t_1 and t_2 respectively.

$\Sigma \int M_o dt$: The angular impulse

It is determined by integrating - w.r.t time - the moments of all the forces acting on the particle over the time period t_1 to t_2.

* Since the moment of a force about point O is $M_o = r \times F$, The angular impulse may be expressed in vector form as

$$\boxed{\text{angular impulse} = \int_{t_1}^{t_2} M_o\, dt = \int_{t_1}^{t_2} (r \times F)\, dt}$$

The principle of angular impulse and momentum for a system of particles

$$\boxed{\Sigma (H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o dt = \Sigma (H_o)_2}$$

* These impulses are created only by the moments of the external forces acting on the system

where, for the i^{th} particle

$$\boxed{M_o = r_i \times F_i}$$

* Vector Formulation:

Using impulse and momentum principle, it is therefore possible to write two equations which define the particle's motion, namely,

linear impulse and momentum

$$m v_1 + \sum \int_{t_1}^{t_2} F\,dt = m v_2$$

angular impulse and momentum

$$(H_o)_1 + \sum \int_{t_1}^{t_2} M_o\,dt = (H_o)_2$$

* Scalar Formulation:

The above equations can be expressed in x, y, z component from yielding a total of six scalar equations.

If the particle is confined to move in the x–y plane, three scalar equations can be written to express the motion

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$(H_o)_1 + \sum \int_{t_1}^{t_2} M_o\,dt = (H_o)_2$$

about z-axis.

* Coseuation of angular momentum :-

• When the $\boxed{\text{angular impulses}}$ acting on aparticle are all zero during the time t_1 to t_2.

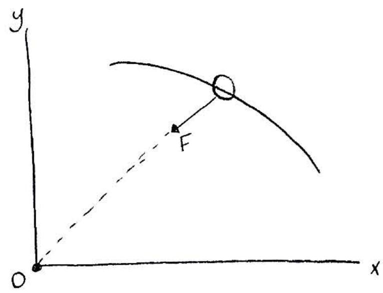


Fig (15.23).

$$\boxed{(H_o)_1 = (H_o)_2}$$ conservation of angular momentum.

• If no external impulse is applied to the particle

1. $mU_1 = mU_2$
2. $(H_o)_1 = (H_o)_2$

} Both linear and angular momentum will be conserved.

• When the particle is subjected only to $\boxed{\text{a central force}}$ as shown in (Fig 15.23),
The impulsive central force F is always directed toward point O. Hence, the angular impulse (moment) created by F about the z-axis is always zero

So

$$(H_o)_1 = (H_o)_2 \quad \text{--- about z-axis.}$$

* Procedure for analysis :-

• To obtain conservation of angular momentum the moments of all forces (impulses) must either

be parallel or pass through the axi's

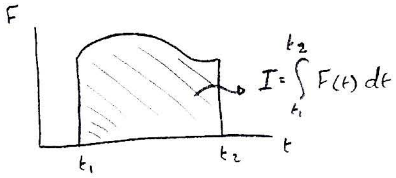
so as to create zero moment throughout the time period t_1 to t_2

• You may draw the impulse and momentum diagrams for the particle instead of Free-body diagram.

* Chapter Review:-

- Impulse:

1. Is defined as the product of force and time
2. Graphically it represents the area under F-t diagram.



- Principle of Impulse and Momentum:

1. When the equation of motion $\Sigma F = ma$ and the kinematic equation $a = dv/dt$ are combined we obtain the principle of impulse and momentum

$$\boxed{mv_1 + \Sigma \int_{t_1}^{t_2} F\,dt = mv_2}$$

2. This equation is used to solve problems that involve Force, velocity, Time (F, v, T).

- Conservation of linear Momentum:

1. If the principle of impulse and momentum is applied to a system of particles.

$$mv_1 + \Sigma \int_{t_1}^{t_2} F\,dt = mv_2$$

Then

a) The collisions between particles produce internal impulses that are equal, opposite, and collinear, and therefore cancel from equation $(\Sigma f_i = 0)$

b) If an external impulse is small, that is, the force is small and the time is short, then the impulse can be classified as nonimpulsive and can be neglected

$$\boxed{\Sigma m_i (v_i)_1 = \Sigma m_i (v_i)_2} \text{ conserved.}$$

- Impact:

1. when two particles A and B have a direct impact, the internal impulse between them is equal, opposite, and collinear, then

$$\boxed{m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2}$$

2. If the final velocities are unknown, a second equation is needed for solution

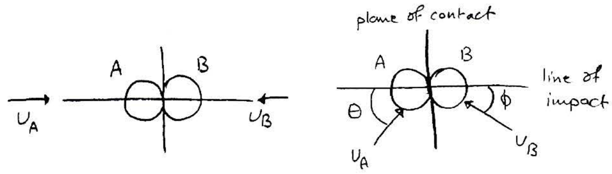
coefficient of restitution. ←
$$\boxed{e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}}$$

a) if the collision is elastic, no energy is lost and $\boxed{e=1}$

b) for a plastic (inelastic) collision $\boxed{e=0}$

3. If the impact is oblique, then the conservation of momentum for the system and (e) apply along the line of impact

Types of impact

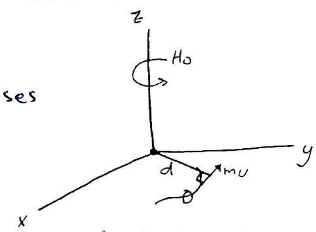


central   oblique ..

4. Conservation of momentum for each particle applies along the plane of contact because no impulse acts on it.

- Principle of angular impulse and momentum:

1. The moment of the linear momentum about Z-axis is called "The angular momentum".



$$(H_o)_z = (d)(mv)$$

2. This equation is used to determine unkown impulses

$M_{\text{about axis}} = 0$ if → F/mv is parallel to axis
→ F/mv passes through axis

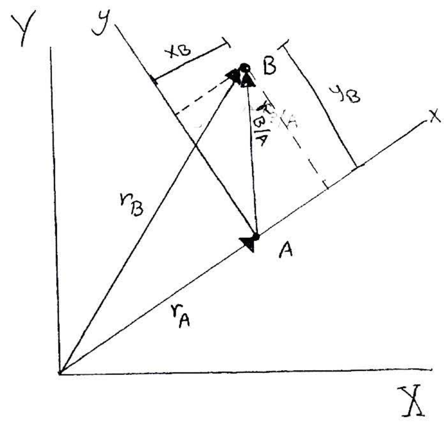
$$\boxed{(H_o)_1 + \Sigma\int_{t_1}^{t_2} M_o\,dt = (H_o)_2}$$

- (16.8): Relative Motion Analysis using Rotating Axes:

- This type of analysis is useful for determining the motion of points located on same rigid body
or The motion of points located on several Pin-connected bodies.

- In following analysis two equations will be devoloped which relate the velocity and acceleration of two points.

- One of two points (The base point A) is the origin of a moving axes.



* Position:

- A, B → their location is specified by r_A, r_B → which are measured w.r.t X, Y, Z
- A (the base point) → represent the origin of x, y, z which is assumed to be both translating and rotating w.r.t X, Y, Z
- $r_{B/A}$ → Position of B w.r.t A
- $r_{B/A}$ → may be expressed either → in terms of unit vectors along X, Y axes i.e. I and J
  or → by unit vectors along x, y axes i.e. i and j

- For the development → $r_{B/A}$ will be measured w.r.t x, y axes

$$r_{B/A} = x_B \hat{i} + y_B \hat{j}$$

- The angular (velocity and acceleration) of X, Y axes are ω and α respectively.
- but the angular (velocity and acceleration) of x, y axes are Ω (omega) and $\dot{\Omega} = d\Omega/dt$ respectively.

- Don't read below Now.. be patient!!!

part (1)

$$U_B = U_A + \Omega \times r_{B\backslash A} + (U_{B\backslash A})_{xyz} \quad \cdots\cdots (1)$$

$$U_B = U_A + \omega \times r_{B\backslash A} \quad \cdots\cdots\cdots (2)$$

If you compare (1) with (2) → eqn.(2) is valid for a translating axes, and the only difference is $(U_{B|A})_{xyz}$
where Ω : angular velocity caused by rotation of x, y, z

part (2)

$$a_B = a_A + \dot{\Omega} \times r_{B|A} + \Omega \times (\Omega \times r_{B|A}) + 2\Omega \times (U_{B|A})_{xyz} + (a_{B|A})_{xyz} \cdots\cdots (1)$$

$$a_B = a_A + \alpha \times r_{B|A} - \omega^2 r_{B|A} \quad \cdots\cdots (2)$$

If you compare (1) with (2) → eqn.(2) is valid for a translating axes and the diffence is $2\Omega \times (U_{B|A})_{xyz}$ and $(a_{B|A})_{xyz}$

$(2\Omega \times (U_{B|A})_{xyz})$: is called The cariolis acceleration

- it represents the difference in the acceleration of B as measured from nonrotating and rotating x, y, z axes.
- Cariolis acc. will always be perpindicular to both Ω and $(U_{B|A})_{xyz}$
- We use cariolic acc. when studying the accelerations and forces which act on rockets, long-range projectiles, or other bodies having motions whose measurements are significantly affected by the rotation of earth.. 8D ! طالبانا

* Velocity :-

- velocity of point B is determined by taking the time derivative of $r_B = r_A + r_{B|A} \longrightarrow U_B = U_A + \underbrace{U_{B|A}}_{} \rightarrow \frac{dr_{B|A}}{dt}$

$$\rightarrow \frac{dr_{B|A}}{dt} = \frac{d}{dt}(x_B i + y_B j) = \frac{dx_B}{dt} i + x_B \frac{di}{dt} + \frac{dy_B}{dt} j + y_B \frac{dj}{dt}$$

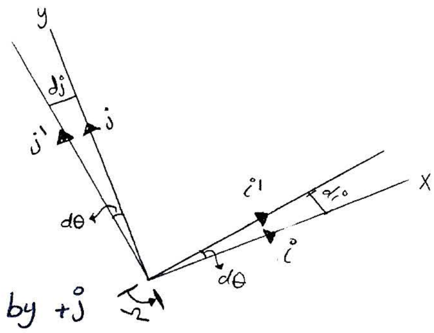
$$= \left(\frac{dx_B}{dt} i + \frac{dy_B}{dt} j\right) + \left(x_B \frac{di}{dt} + y_B \frac{dj}{dt}\right)$$

- The two terms in the first set represent the components of velocity of B measured by x, y, z

These terms will be denoted by vector $(U_{B|A})_{xyz}$

- In the second set, represent the instantaneous time rate of change of unit vectors i, j measured by X, Y, Z

- These changes (di and dj) are due to the rotation $d\theta$ of the x, y, z axes



causing $i \xrightarrow{\text{to become}} i' = i + di$

$j \xrightarrow{\text{to become}} j' = j + dj$

- The magnitude of both di and dj are equal $1 d\theta$
- The direction of di is defined by $+j$
- " " " dj " " " $-i$

$i \simeq i' = 1$

$j \simeq j' = 1$

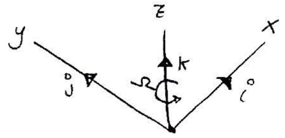
- $\dfrac{di}{dt} = \dfrac{d\theta}{dt}(j) = \Omega j$ $\qquad \dfrac{dj}{dt} = \dfrac{d\theta}{dt}(-i) = -\Omega i$

We can express the above derivatives in terms of the cross product as :

$$\frac{di}{dt} = \vec{\Omega} \times i \qquad \frac{dj}{dt} = \vec{\Omega} \times j$$

- **Note:** $\Omega \hat{k} \times 1 i = \Omega j$, $\Omega \hat{k} \times 1 j = -\Omega i$

where $\vec{\Omega} = \Omega \hat{k}$



- Back to $dr_{B|A} \backslash dt$

$$dr_{B|A} \backslash dt = \left(\frac{dx_B}{dt} i + \frac{dy_B}{dt} j\right) + \left(x_B \frac{di}{dt} + y_B \frac{dj}{dt}\right)$$

$$= U_{B|A} + (x_B \Omega \times i + y_B \Omega \times j)$$

$$= U_{B|A} + \Omega \times r_{B|A}$$

⇛ Back Back!!

$$U_B = U_A + dr_{B|A} / dt$$

$$\boxed{U_B = U_A + \Omega \times r_{B|A} + (U_{B|A})_{xyz}}$$

- U_B : absolute velocity of B measured from X, Y, Z
- U_A : " " " origin of x, y, z measured from X, Y, Z
- Ω : angular velocity of x, y, z measured from X, Y, Z
- $r_{B|A}$: position of B w.r.t A
- $(U_{B|A})_{xyz}$: velocity of B w.r.t A measured by x, y, z

• Now! Back two Pages and look at part (1)! ^^

* Acceleration :-

- acceleration of point B is determined by taking the time derivative of $U_B = U_A + \Omega \times r_{B|A} + (U_{B|A})_{xyz}$

$$a_B = a_A + \dot{\Omega} \times r_{B|A} + \underbrace{\Omega \times \frac{dr_{B|A}}{dt}}_{1} + \underbrace{\frac{d(U_{B|A})_{xyz}}{dt}}_{2}$$

$$\frac{dr_{B|A}}{dt} = (U_{B|A})_{xyz} + \Omega \times r_{B|A}$$

$$\Omega \times \frac{dr_{B|A}}{dt} = \Omega \times (U_{B|A})_{xyz} + \Omega \times (\Omega \times r_{B|A}) \quad \text{..... sub. in } 1$$

$$(U_{B|A})_{xyz} = (U_{B|A})_x \hat{i} + (U_{B|A})_y \hat{j}$$

$$\frac{d(U_{B|A})_{xyz}}{dt} = \left[\frac{d(U_{B|A})_x}{dt}\hat{i} + \frac{d(U_{B|A})_y}{dt}\hat{j}\right] + \left[(U_{B|A})_x \frac{d\hat{i}}{dt} + (U_{B|A})_y \frac{d\hat{j}}{dt}\right]$$

$$= (a_{B|A})_{xyz} + \left[(U_{B|A})_x \Omega \times \hat{i} + (U_{B|A})_y \Omega \times \hat{j}\right]$$

$$= (a_{B|A})_{xyz} + \Omega \times \left[(v_{B|A})_x \, i + (v_{B|A})_y \, j \right]$$

$$\frac{d(v_{B|A})_{xyz}}{dt} = (a_{B|A})_{xyz} + \Omega \times (v_{B|A})_{xyz} \quad \text{.... sub. in (2)}$$

The final form is:

$$\boxed{a_B = a_A + \dot{\Omega} \times r_{B|A} + \Omega \times (\Omega \times r_{B|A}) + 2\Omega \times (v_{B|A})_{xyz} + (a_{B|A})_{xyz}}$$

Now! Back three Pages and look at Part (2)! ^_^

again

$$a_B = a_A + \dot{\Omega} \times r_{B|A} + \Omega \times (\Omega \times r_{B|A}) + 2\Omega \times (v_{B|A})_{xyz} + (a_{B|a})_{xyz}$$

- a_B : acceleration of B measured from X, Y, Z
- a_A : " " the origin (A) of x, y, z measured from X, Y, Z
- $(a_{B|A})_{xyz}$, $(v_{B|A})_{xyz}$: acceleration and velocity of B w.r.t A measured by rotating x, y, z.
- $\dot{\Omega}, \Omega$: angular acceleration and angular velocity of x, y, z measured from X, Y, Z
- $r_{B|A}$: position of B w.r.t A.

again

$$\boxed{v_B = v_A + \Omega \times r_{B|A} + (v_{B|A})_{xyz}}$$

where

$\Omega \times r_{B|A}$: angular velocity effect caused by rotating of x, y, z

$$a_B = a_A + \dot{\Omega} \times r_{B|A} + \Omega \times (\Omega \times r_{B|A}) + 2\Omega \times (v_{B|A})_{xyz} + (a_{B|A})_{xyz}$$

where

- $\dot{\Omega} \times r_{B|A}$: angular acceleration effect caused by rotation of x, y, z
- $\Omega \times (\Omega \times r_{B|A})$: angular velocity effect caused by ratation of x, y, z
- $2\Omega \times (v_{B|A})_{xyz}$: combined effect of B moving relative to x, y, z and rotation of x, y, z (interacting motion).

* Procedure for Analysis :-

- Equations of velocity and acceleration which we taked in this section can be applied to solve problems invoIving the planar motion of particles or rigid bodies using the following procedure ..

- Choose an appropriate location for the origin and proper orientation of X, Y, Z and x, y, z.

- Most often solutions are easily obtained if :
  1. The origins are coincident
  2. The corresponding axes are collinear
  3. The correponding axes are parallel.

- The moving frame (axes) should be selected fixed to the body or device a long which the relative motion occurs.

- After defining the origin A of the moving reference and specifying the moving point B
  These equations should be written :

$$v_B = v_A + \Omega \times r_{B|A} + (v_{B|A})_{xyz}$$

$$a_B = a_A + \dot{\Omega} \times r_{B|A} + \Omega \times (\Omega \times r_{B|A}) + 2\Omega \times (v_{B|A})_{xyz} + (a_{B|A})_{xyz}.$$

- The cartesian components of all these vectors may be expressed along either X, Y, Z or x, y, z

The choise is arbitary provided a consistent set of unit vectors is used
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- Motion of the moving reference is expressed by

$$v_A , a_A , \Omega , \dot{\Omega}$$

and Motion of B with respect to the moving axis is expressed by

$$r_{B|A} , (v_{B|A})_{xyz} , (a_{B|A})_{xyz} .$$