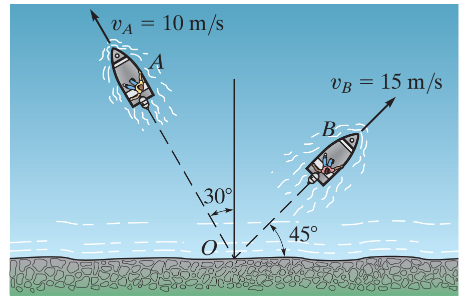
12–223.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 10\text{ m/s}$ and $v_B = 15\text{ m/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 600 m apart?



SOLUTION

Relative Velocity. The velocity triangle shown in Fig. a is drawn based on the relative velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$. Using the cosine law,

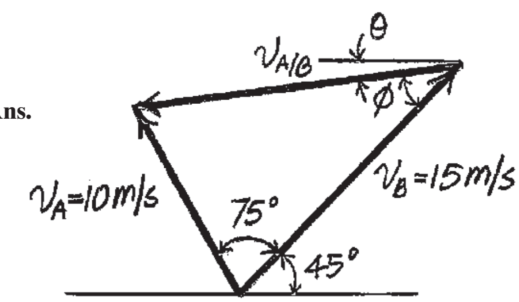
$$v_{A/B} = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 75^\circ} = 15.73\text{ m/s} = 15.7\text{ m/s} \qquad \textbf{Ans.}$$

Then, the sine law gives

$$\frac{\sin\phi}{10} = \frac{\sin 75^\circ}{15.73} \qquad \phi = 37.89^\circ$$

The direction of $\mathbf{v}_{A/B}$ is defined by

$$\theta = 45^\circ - \phi = 45^\circ - 37.89^\circ = 7.11^\circ \; ⦣ \qquad \textbf{Ans.}$$



(a)

Alternatively, we can express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form

$$\mathbf{v}_A = \{-10\sin 30^\circ\mathbf{i} + 10\cos 30^\circ\mathbf{j}\}\text{ m/s} = \{-5.00\mathbf{i} + 5\sqrt{3}\mathbf{j}\}\text{ m/s}$$

$$\mathbf{v}_B = \{15\cos 45^\circ\mathbf{i} + 15\sin 45^\circ\mathbf{j}\}\text{ m/s} = \{7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j}\}\text{ m/s}.$$

Applying the relative velocity equation

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-500\mathbf{i} + 5\sqrt{3}\mathbf{j} = 7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j} + \mathbf{v}_{A/B}$$

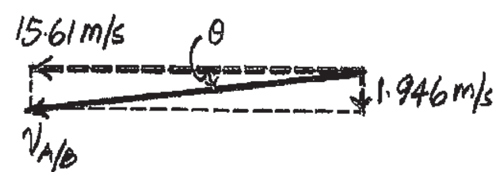
$$\mathbf{v}_{A/B} = \{-15.61\mathbf{i} - 1.946\mathbf{j}\}\text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-15.61)^2 + (-1.946)^2} = 15.73\text{ m/s} = 15.7\text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. b,

$$\theta = \tan^{-1}\left(\frac{1.946}{15.61}\right) = 7.1088^\circ = 7.11^\circ \; ⦣ \qquad \textbf{Ans.}$$



(b)

Here $s_{A/B} = 600$ m. Thus

$$t = \frac{s_{A/B}}{v_{A/B}} = \frac{600}{15.73} = 38.15\text{ s} = 38.1\text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_{A/B} = 15.7$ m/s
$\theta = 7.11^\circ$ ⦣
$t = 38.1$ s