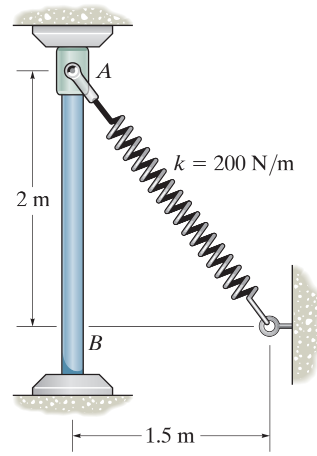
14–78.

The spring has a stiffness $k = 200\ \text{N/m}$ and an unstretched length of 0.5 m. If it is attached to the 3-kg smooth collar and the collar is released from rest at A, determine the speed of the collar when it reaches B. Neglect the size of the collar.



SOLUTION

Potential Energy. With reference to the datum set through B, the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 3(9.81)(2) = 58.86\ \text{J}$$
$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{1.5^2 + 2^2} - 0.5 = 2.00\ \text{m}$ and $x_B = 1.5 - 0.5 = 1.00\ \text{m}$. Thus, the elastic potential energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(200)\left(2.00^2\right) = 400\ \text{J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(200)\left(1.00^2\right) = 100\ \text{J}$$

Conservation of Energy. Since the collar is released from rest at A, $T_A = 0$.

$$T_A + V_A = T_B + V_B$$

$$0 + 58.86 + 400 = \frac{1}{2}(3)v_B^2 + 0 + 100$$

$$v_B = 15.47\ \text{m/s} = 15.5\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 15.5\ \text{m/s}$