Chapter 15

Wednesday, July 13, 2016 2:02 PM

$\boxed{15.1}$ Principle of linear impulse & momentum.

→ Integration of EOM ⇒ principle of impulse & Momentum.

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$\vec{a}, \vec{v}$ wrt an inertial frame of reference.

@ $t_1 \rightarrow v_1$

@ $t_2 \rightarrow v_2$

$$\sum \int_{t_1}^{t_2} \vec{F}\,dt = \int_{v_1}^{v_2} m\,d\vec{v}$$

$$\sum \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$$ (Principle of linear impulse & momentum)

* Find v_2 when v_1 and all forces are known.

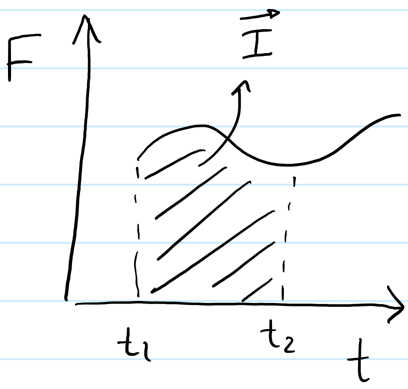
* Linear momentum: $\vec{L} = m\vec{v}$ [same direction as $\vec{v}$]
[kg.m/s]

* Linear impulse: a measure of the effect of a force during the time the force acts

$\vec{I} = \int \vec{F}\,dt$ [same direction as $\vec{F}$] [N.s]

→ Graphically Linear impulse is the area under the

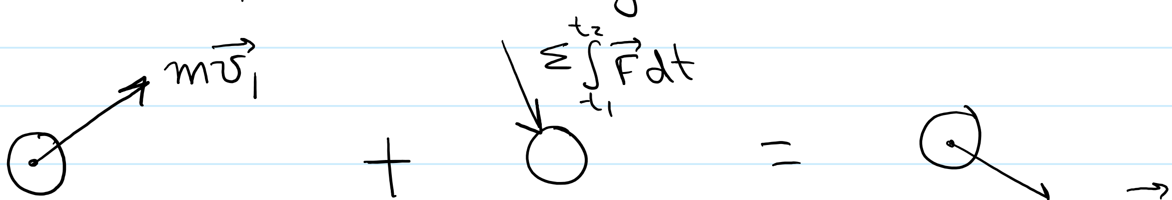
F – t curve



For problem solving : $m\vec{v}_1 + \sum \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2$

the initial momentum of a particle at time t_1 plus the sum of all impulses applied to the particle from t_1 to t_2 is equivalent to the final momentum of the particle at t_2.

Impulse and Momentum Diagrams :



Initial momentum Diagram

Impulse diagram

Final momentum Diagram.

In x, y, z coordinates

$$m v_{x_1} + \int_{t_1}^{t_2} F_x \, dt = m v_{x_2}$$

$$m v_{y_1} + \int_{t_1}^{t_2} F_y \, dt = m v_{y_2}$$

$$m v_{z_1} + \int_{t_1}^{t_2} F_z \, dt = m v_{z_2}$$

$\boxed{15.2}$ Principle of linear impulse and momentum for a system of particles

$$\sum m_i (v_i)_1 + \sum \int_{t_1}^{t_2} \vec{F_i} \, dt = \sum m_i (v_i)_2$$

in terms of the mass center. G of the system:

$$m(v_G)_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = m(v_G)_2$$

$\boxed{15.3}$ Conservation of linear momentum for a system of particles:

$$\text{if } \sum \int_{t_1}^{t_2} \vec{F} dt = 0 \Rightarrow \sum m_i (v_i)_1 = \sum m_i (v_i)_2$$

$$m(v_G)_1 = m(v_G)_2$$

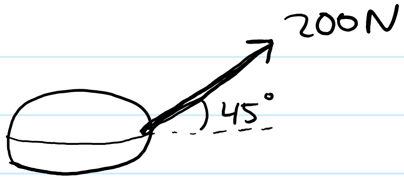
$$(v_G)_1 = (v_G)_2$$

* Velocity of the mass center for the system does not change when no external impulses are applied to the system.

<u>Note:</u> i) Impulsive forces: Large forces acting for

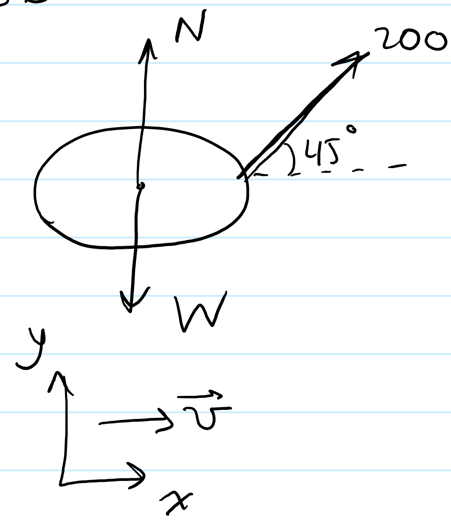
a very short period of time (strike, explosion)
2) Non-impulsive forces: small forces compared to impulsive forces (weight, light spring ...)

Example 15.1



$m = 100$ kg.
originallay at rest.
force acts for 10 seconds
Find v_2
N

FBD



$(\xrightarrow{+})$

$$m_1 v_{x_1} + \sum \int_{t_1}^{t_2} F_x \, dt = m v_{x_2}$$

$$0 + \int_0^{10} (200 \cos 45) \, dt = m v_{x_2}$$

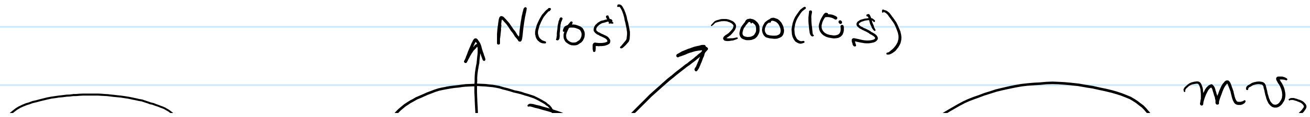
$$200 \cos 45 \, (10 - 0) = 100 \, v_{x_2}$$

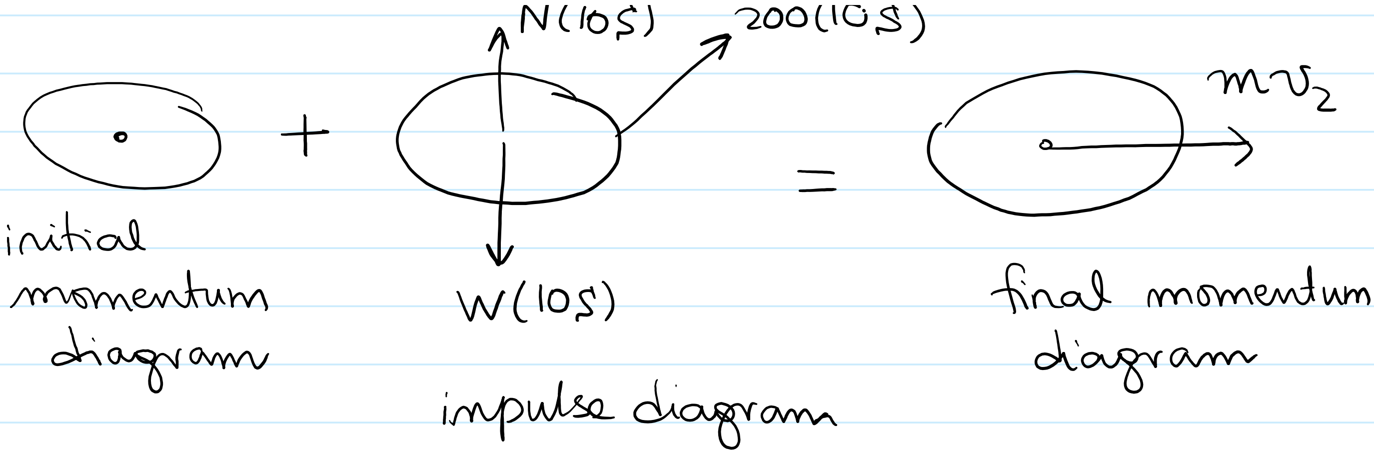
$$v_{x_2} = 14.1 \text{ m/s}$$

$(\overset{+}{\uparrow})$ $$m v_{y_1} + \sum \int_{t_1}^{t_2} F_y \, dt = m v_{y_2}$$

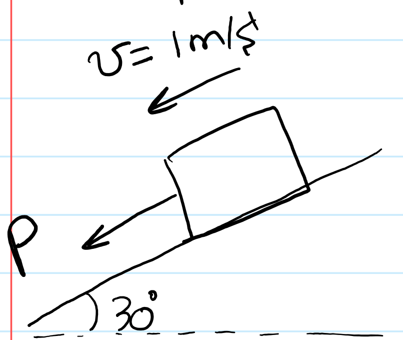
$$0 + \int_{t_1}^{t_2} (N_c + 200 \sin 45 - W) \, dt = 0$$

$$N_c = 840 \text{ N}.$$





Example 15.2



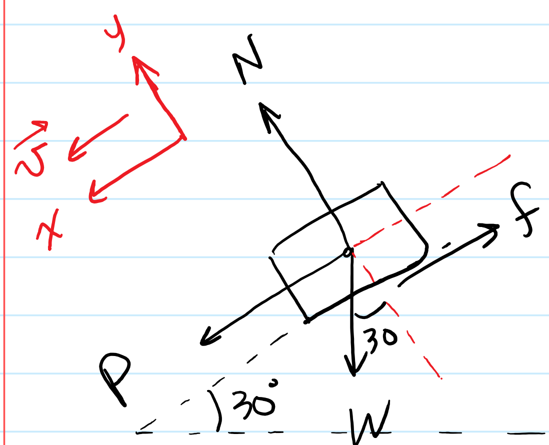
$W = 250$ N

$P = 100t$ N

$t = 2$ s

$\mu_k = 0.3$

find v_2



$(x \swarrow +)$

$$m v_{x_1} + \sum \int_{t_1}^{t_2} F_x \, dt = m v_{x_2}$$

$$\frac{250}{9.81}(1) + \int_0^2 (100t + W\sin 30 - 0.3N)\,dt = \frac{250}{9.81} v_2$$

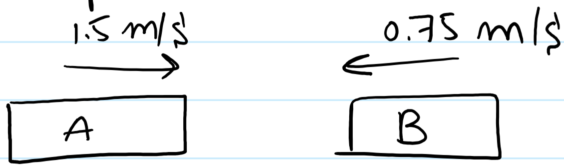
$$25.48 + \left.\frac{100t^2}{2}\right|_0^2 + W\sin 30(2) - 0.3N(2) = 25.48\, v_2$$

$(y \nwarrow +)$ $\quad N - W\cos 30 = 0$

$$\boxed{N = 216.5\,N}$$

$$\boxed{v_2 = 13.56\ m/s}$$

Example 15.4



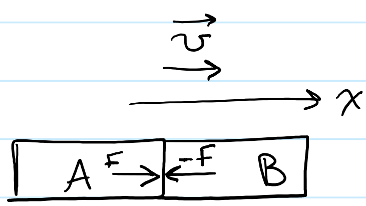
$m_A = 15\ Mg = 15000\ kg$

$m_B = 12\ Mg = 12000\ kg.$

Collide + Couple

Find speed of both cars right after collision

Favg in 0.8 s



$$m_A v_{A_1} + m_B v_{B_1} = (m_A + m_B) v_2$$

$$15000(1.5) + (12000)(-0.75) = (15000 + 12000) v_2$$

$$\boxed{v_2 = 0.5\ m/s \rightarrow}$$

$$m_A v_{A_1} + \int_{t_1}^{t_2} F\,dt = m_A v_{A_2}$$

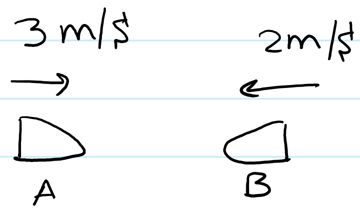
$$15000(1.5) + \int_0^{0.8} F_{avg}\,dt = 15000(0.5)$$

$$15000(1.5) + F_{avg}(0.8 - 0) = 15000(0.5)$$

$$\boxed{F_{avg} = 18.8\ kN}$$

$$\boxed{F_{avg} = 18.8 \text{ kN}}$$

Example 15.6



$m_A = m_B = 150$ kg

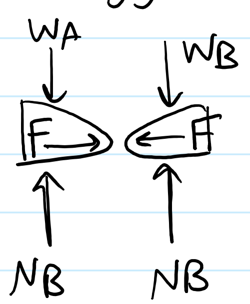
collide head-on, no energy is lost
find v_{A_2} v_{B_2}

$$m_A v_{A_1} + m_B v_{B_1} = m_A v_{A_2} + m_B v_{B_2}$$

$$150(3) + 150(-2) = 150 v_{A_2} + 150 v_{B_2}$$

$$\boxed{v_{A_2} = 1 - v_{B_2}}$$

No energy is lost $\rightarrow$ Conservation of energy



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m_A v_{A_1}^2 + \frac{1}{2} m_B v_{B_1}^2 = \frac{1}{2} m_A v_{A_2}^2 + \frac{1}{2} m_B v_{B_2}^2$$

$$\boxed{v_{A_2}^2 + v_{B_2}^2 = 13}$$

$$v_{B_2}^2 - v_{B_2} - 6 = 0$$

$$\boxed{v_{B_2} = 3 \text{ m/s} \rightarrow} \qquad v_{B_2} = -2 \text{ m/s} \leftarrow$$

$$\boxed{v_{B_2} = 3\ m/s \rightarrow} \qquad v_{B_2} = -2\ m/s \leftarrow$$

$$\boxed{v_{A_2} = -2\ m/s \leftarrow}$$

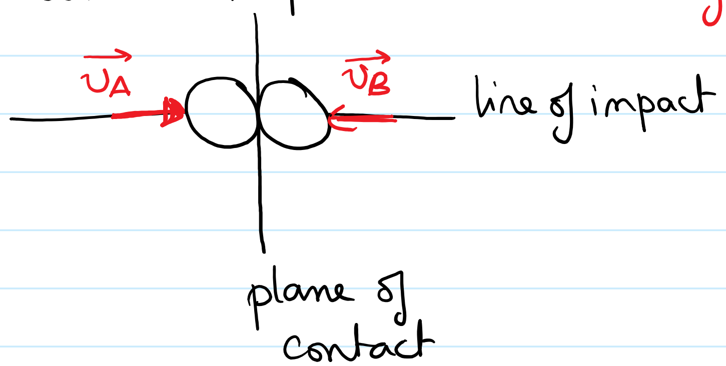
$\boxed{15.4}$ Impact (تصادم)

Impact → 2 bodies collide → very short period of time → large impulsive forces.

Two types of impact:

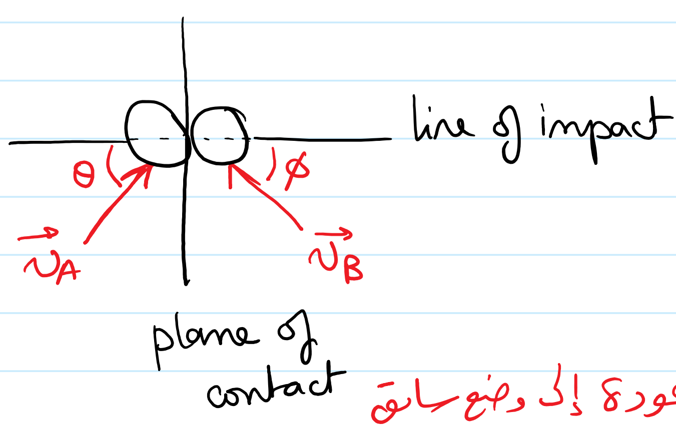
1) Central impact

Direction of motion is along line of impact.



2) Oblique Impact:

Direction of motion of one or both particles makes an angle with the line of impact.



عودة إلى وضع سابق

* Coefficient of Restitution (e): ratio of the relative velocity of the particles' separation just after

impact to the relative velocity of the particles' approach just before impact

$$e = \frac{v_{B_2} - v_{A_2}}{v_{A_1} - v_{B_1}}$$ } depends on velocities + body sizes and shapes

$0 \leq e \leq 1$ by experiment.

1) If $e = 1 \Rightarrow$ elastic impact (perfectly elastic)
Deformation impact = restitution impact
* Does not happen in reality.

2) If $e = 0 \Rightarrow$ plastic impact (inelastic)
No restitution impact → particles stick together and move with a common velocity.

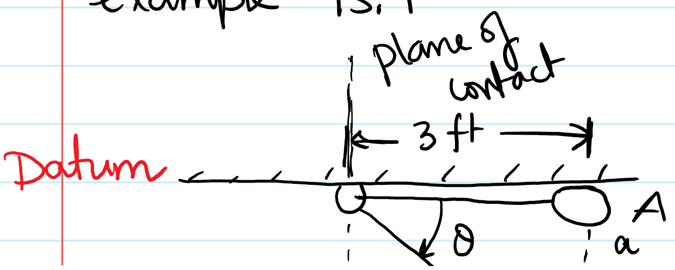
Note: The principle of work and energy cannot be used for impact problems → we don't know internal forces or how they change.

To calculate energy loss in impact (kinetic, heat) sound, deformation

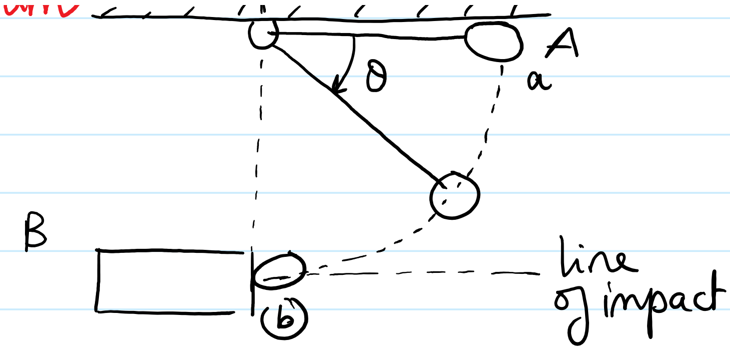
$$\sum U_{1\to 2} = \sum T_2 - \sum T_1$$

For elastic impacts → No energy loss
For plastic impacts → Max energy loss.

example 15.9



$W_A = 6\text{ lb}$
bag A is released from rest @ $\theta = 0°$
after falling $\theta = 90°$
strikes box $W_B = 18\text{-lb}$.



strikes box $W_B = 18$-lb.

$e = 0.5$

1) Velocities after impact

2) Loss of energy

$x \leftarrow$

From a → b only weight acts ⇒ conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2} m_A v_{A_1}^2 + m_A g(-3)$$

$$\frac{1}{2}\frac{6}{32.2} v_{A_1}^2 - 18 = 0 \Rightarrow \boxed{v_{A_1} = 13.90 \text{ ft/s}}$$

Conservation of momentum.

$$m_A v_{A_1} + \underbrace{\cancel{m_B v_{B_1}}}_{0} = m_A v_{A_2} + m_B v_{B_2}$$

$$\boxed{v_{A_2} = 13.9 - 3 v_{B_2}}$$

$$e = \frac{v_{B_2} - v_{A_2}}{v_{A_1} - v_{B_1}} \Rightarrow 0.5 = \frac{v_{B_2} - v_{A_2}}{13.9 - 0}$$

$v_{A_2} = -1.74$ ft/s $\rightarrow$

$v_{B_2} = 5.21$ ft/s $\leftarrow$

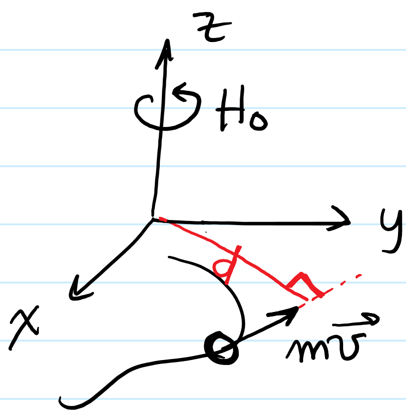
Loss of energy $\sum U_{1\to 2} = T_2 - T_1$

$= -10.1 \text{ lb.ft}$

Note: Impact with ceiling or ground
→ mass is unknown → assume velocity before and after impact to be zero.

15.5 Angular momentum

Angular momentum: Moment of the particle's momentum about O (H_o)



Scalar Representation
$(H_o)_z = mv(d)$

d: moment arm
(perpendicular distance from O to the line of action of $m\vec{v}$.

$[kg\, m^2/s]$ Direction: RHR

Vector Representation:

$\vec{H_o} = \vec{r} \times m\vec{v}$

$\vec{r}$: position vector from O to the particle.

$$\vec{H_o} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

15.6 Relation between Moment of Force and Angular Momentum.

$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{F} = m\dot{\vec{v}}$$

Moment of $\vec{F}$ $\qquad \sum M_o = \vec{r} \times \sum \vec{F}$

$$= \vec{r} \times m\dot{\vec{v}}$$

$$\vec{H}_o = \vec{r} \times m\vec{v}$$

$$\frac{d}{dt}(\vec{H}_o) = \frac{d}{dt}(\vec{r} \times m\vec{v})$$

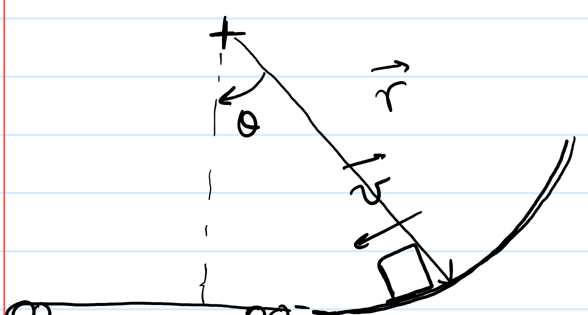
$$= \vec{r} \times m\dot{\vec{v}} + \dot{\vec{r}} \times m\vec{v}$$

$$\dot{\vec{H}}_o = \sum M_o + \underbrace{\dot{\vec{r}} \times m\dot{\vec{r}}}_{0}$$

$$\boxed{\sum M_o = \dot{\vec{H}}_o}$$

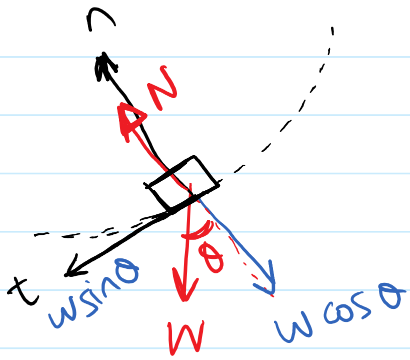
This is similar to: $\sum \vec{F} = \dot{L}$

<u>example 15.12</u>



m

find $\vec{H}_o$, $\vec{a}_t$



$\vec{v}$ tangent to the path

$\vec{r} \perp \vec{v}$

$$|\vec{H}_o| = mvr$$

$$\sum \vec{M}_o = \dot{\vec{H}}_o$$

$$W \sin\theta \, r = m \dot{v} r$$

$$\vec{a}_t = \dot{v} = \frac{W \sin\theta}{m} = g \sin\theta$$

Another Method : EOM in tangential direction.

$$\sum \vec{F}_t = m\vec{a}_t = W\sin\theta = m\vec{a}_t$$

$$\boxed{\vec{a}_t = g\sin\theta}$$

15.7 Principle of Angular impulse & Momentum

$$\sum \vec{M}_o = \sum \dot{\vec{H}}_o$$

$$\sum \vec{M}_o = \frac{d\vec{H}_o}{dt}$$

$$\int_{t_1}^{t_2} \sum \vec{M}_o \, dt = \int_{(H_o)_1}^{(H_o)_2} d\vec{H}_o$$

$$\int_{t_1}^{t_2} \sum \vec{M}_o \, dt = \vec{H}_{o_2} - \vec{H}_{o_1} .$$

$$\vec{H}_{o_1} + \underbrace{\int_{t_1}^{t_2} \sum M_o \, dt} = \vec{H}_{o_2}$$

Angular impulse $= \int_{t_1}^{t_2} (\vec{r} \times \sum \vec{F}) \, dt$

* Conservation of Angular Momentum:

if $\int_{t_1}^{t_2} \sum M_o \, dt = 0 \rightarrow$ No angular impulse

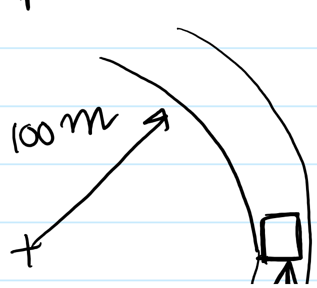
$$(\vec{H}_o)_1 = (\vec{H}_o)_2$$

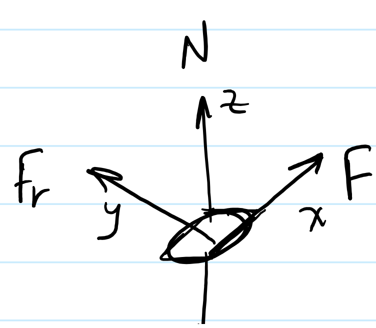
<u>Note</u>: if No external impulse is applied

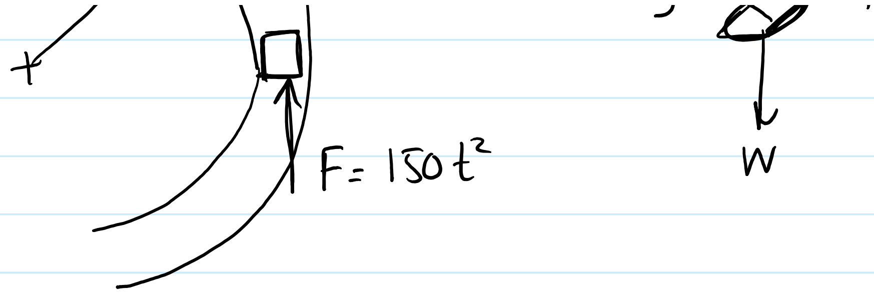
linear & Angular momentums are conserved

In the case of central forces → No moment
→ Angular is conserved but linear is not.

<u>example 15.13</u>







$v_1 = 5\ m/s$

Find v is $5\ s$

$$(H_z)_1 + \sum \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

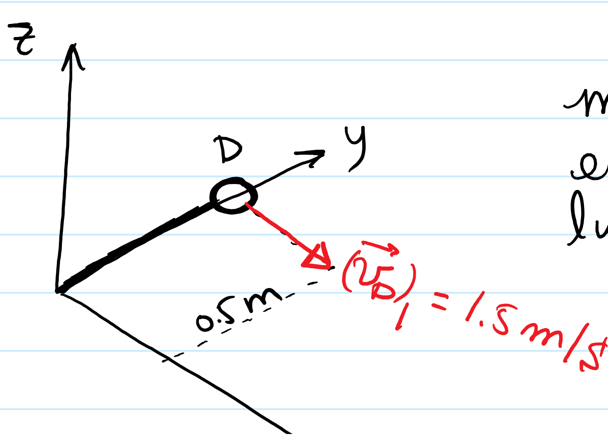
$$r m v_1 + \int_0^5 r F dt = r m v_2$$

$$r m v_1 + \int_0^5 r\, 150 t^2\, dt = r m v_2$$

$$r m v_1 + r\, 150 \frac{t^3}{3} \Big|_0^5 = r m v_2$$

$$\boxed{v_2 = 9.17\ m/s}$$

example 15.15



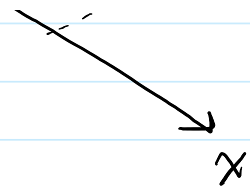
$m_D = 2\ kg$

elastic chord $K_c = 20\ N/m$

$l_{unstretched} = 0.5\ m$

Find:

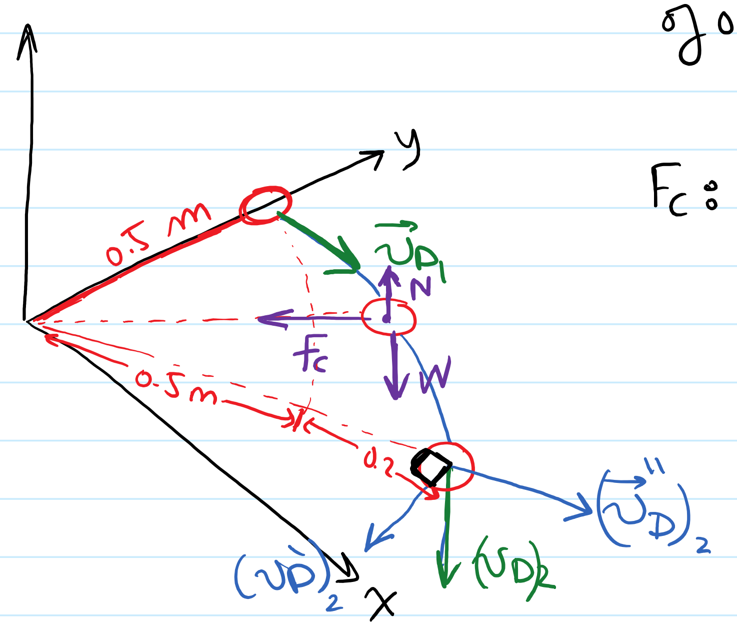
1) the rate at which the chord



m/s

1) the rate at which the chord is being stretched,

2) $\vec{v}_D$ @ a stretch of 0.2 m



F_c : Chord force

W, N have no moment about o, parallel

F_c has " " " o, intersection

$\Rightarrow$ Conservation of Angular momentum.

$$(H_o)_1 = (H_o)_2$$

$$r_1 m_D (v_{D_1}) = r_2 m_D (v'_{D_2})$$

perpendicular to r_2

$$0.5 \times 2 \times 1.5 = (0.5+0.2)(2)(v'_{D_2})$$

$$\boxed{(v'_D)_2 = 1.071 \text{ m/s}}$$

All forces are conservative $\rightarrow$ Conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m_D (v_D)_1^2 + \frac{1}{2} k (0)^2 = \frac{1}{2} m (v_D)_2^2 + \frac{1}{2}(20)(0.2)^2$$

$$\boxed{v_{D_2} = 1.36 \text{ m/s}}$$

$$\boxed{(v_D'')_2 = \sqrt{v_{D_2}^2 - (v_D')_2^2} = 0.838 \text{ m/s}}$$