Chapter 17

Thursday, July 28, 2016 1:10 PM

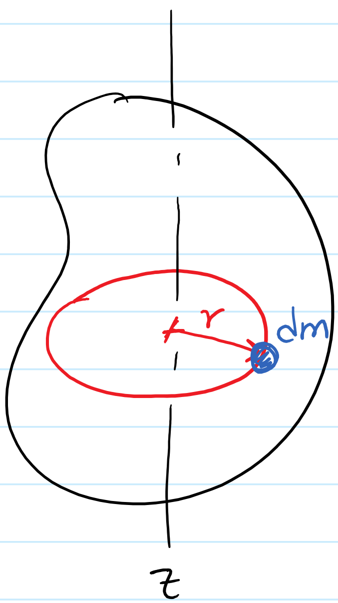
Planar kinetics of Rigid Bodies: Force & Acceleration

$\boxed{17.1}$ Mass moment of Inertia

Force $\rightarrow$ $\underbrace{\text{translation}}$ + Rotation

$\vec{F} = m\vec{a}$ $\qquad$ $\vec{M} = I\vec{\alpha}$

force $\quad$ mass $\quad$ acceleration $\qquad$ moment $\quad$ Inertia $\quad$ Angular acc.



$$I = \int_m r^2 \, dm$$

r is the perpendicular distance from the axis z to dm.

Change the axis $\rightarrow$ r is changed $\rightarrow$ I is changed.

* Choose the axis that passes through G: gravitation center and perpendicular to the plane. (I_G)

I_G +ve always
[$kg\,m^2$] [$slug \cdot ft^2$]

$$I_G = \int_m r^2\, dm$$

$$I_G = \int_V r^2 \rho\, dV \qquad \rho \Rightarrow \text{density}$$

if ρ is constant $\Rightarrow$ $I_G = \rho \int_V r^2 dV$

* Parallel axis theorem: If the moment of inertia of the body about an axis passing through G is known then the moment of inertia about any parallel axis is

$$I = I_G + md^2$$

where d is the perpendicular distance between 2 axes

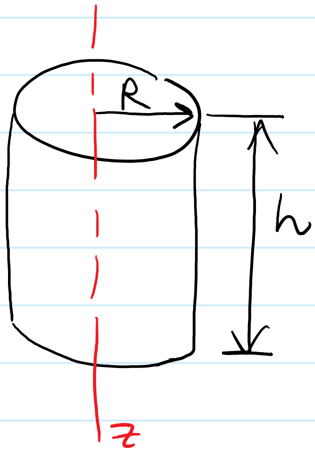
* Radius of Gyration $k = \sqrt{\dfrac{I}{m}}$

$I = mk^2$

* Composite Bodies

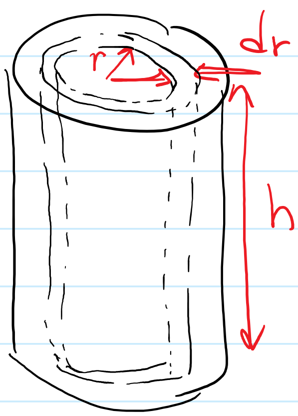
moment of inertia for several parts can be added or subtracted as long as they are computed about the same axis

example



$$I_z = \int_m r^2\,dm$$

$$= \int_V r^2 \rho\, dV =$$



$$dV = (2\pi r) h\, dr$$

$$I_z = \int_0^R r^3 \rho\, 2\pi h\, dr$$

$$= \rho\, 2\pi h \int_0^R r^3\, dr$$

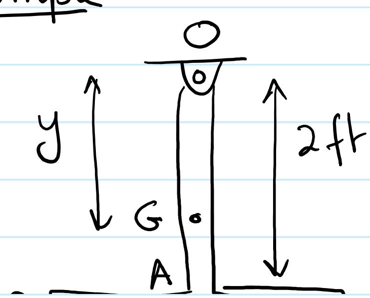
$$= 2\pi \rho h \left.\frac{r^4}{4}\right|_0^R$$

$$I_z = \rho h \frac{\pi}{2} R^4$$

$$m = \rho R^2 \pi h$$

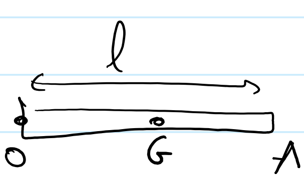
$$\boxed{I_z = \frac{1}{2} m R^2}$$

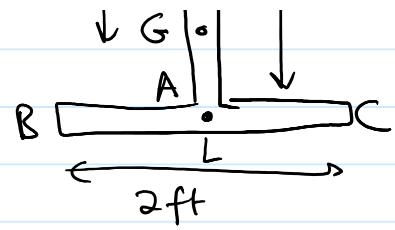
example



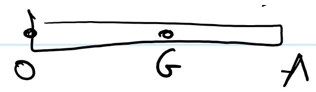
find I_O, I_G $\quad W_{OA} = W_{BC} = 10\text{ lb}$

$$I_O = \frac{1}{3} m \ell^2$$





$I_O = \frac{1}{3} m l^2$



$I_G = \frac{1}{12} m l^2$

$$(I_{OA})_O = \frac{1}{3} m l^2 = \frac{1}{3}\left(\frac{10}{32.2}\right)(2)^2 = 0.414 \text{ slug}\cdot\text{ft}^2$$

$$(I_{BC})_O = (I_{BC})_L + md^2$$

$$= \frac{1}{12} m l^2 + m(2)^2 = 1.346 \text{ slug}\cdot\text{ft}^2$$

$$I_O = 0.414 + 1.346 = 1.76 \text{ slug}\cdot\text{ft}^2$$

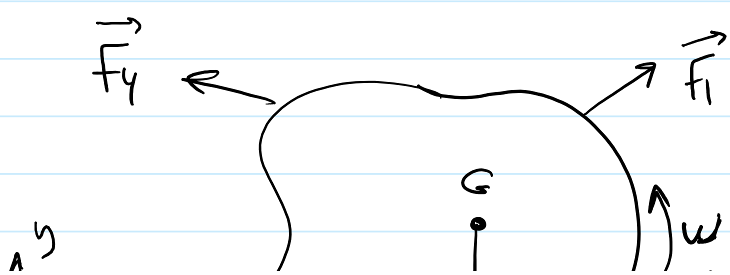
$$y = \frac{\Sigma my}{\Sigma m} = \frac{\left(\frac{10}{32.2} \times 1\right) + \left(\frac{10}{32.2} \times 2\right)}{\frac{20}{32.2}} = \frac{3}{2} = 1.5 \text{ ft}$$

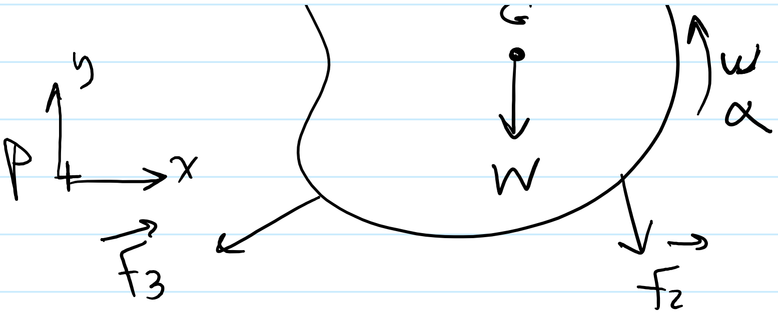
$$I_O = I_G + md^2$$

$$1.76 = I_G + \frac{20}{32.2}(1.5)^2$$

$$\boxed{I_G = 0.362 \text{ slug}\cdot\text{ft}^2}$$

17.2 Planar kinetic equations of motion





Translation EOM:

$\Sigma \vec{F} = m \vec{a}_G$ or $\Sigma F_x = m(a_G)_x$
$\Sigma F_y = m(a_G)_y$.

Rotation EOM:

$$\Sigma M_P = \Sigma (\mathcal{M}_K)_P$$

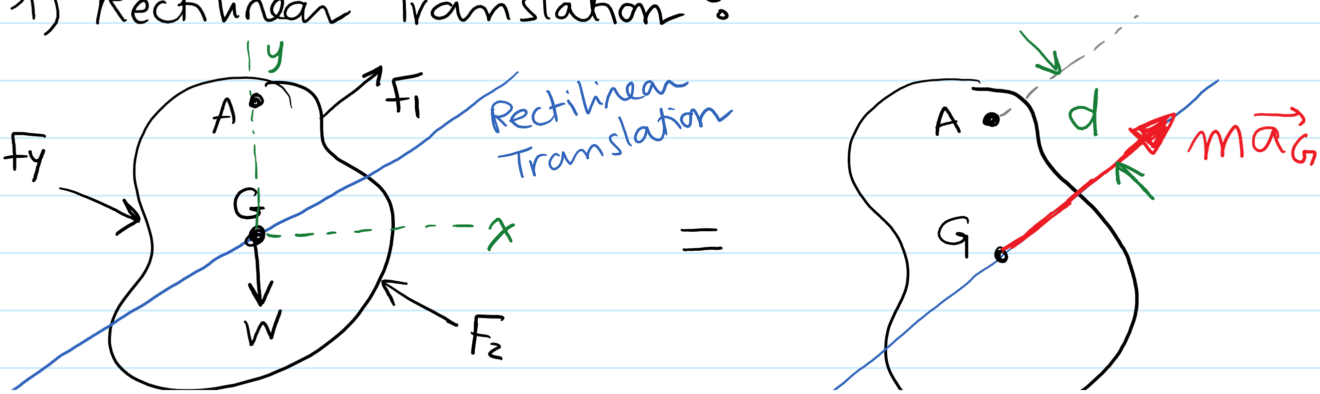
$\mathcal{M}_K$: kinetic moments : moments of $I_G \vec{\alpha}$, $m\vec{a}_G$ about point P

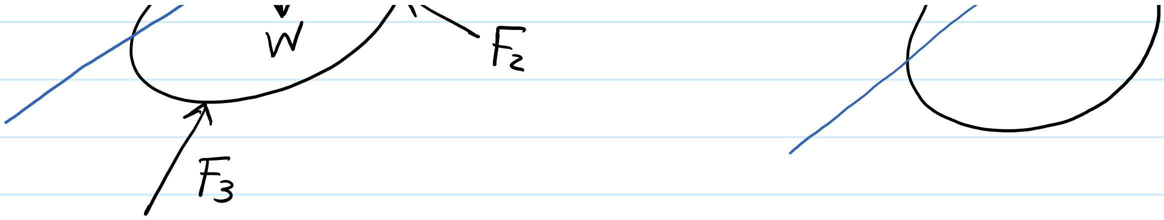
if $P = G$

then $\Sigma M_G = I_G \alpha$.

$\boxed{17.3}$ EOM: Translation: All points have same $\vec{a}$
$\alpha = 0$

1) Rectilinear Translation:



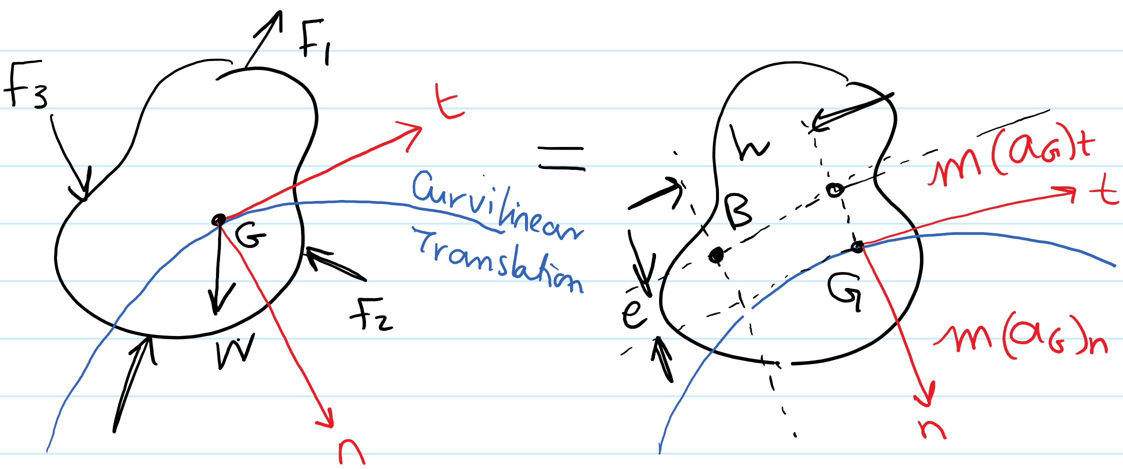


$$\sum F_x = m(a_G)_x$$
$$\sum F_y = m(a_G)_y$$

$$\sum M_G = I_G \alpha = 0$$

$$\sum M_A = (m a_G) d$$

2) Curvilinear Translation:



$$\sum F_n = m(a_G)_n$$
$$\sum F_t = m(a_G)_t$$

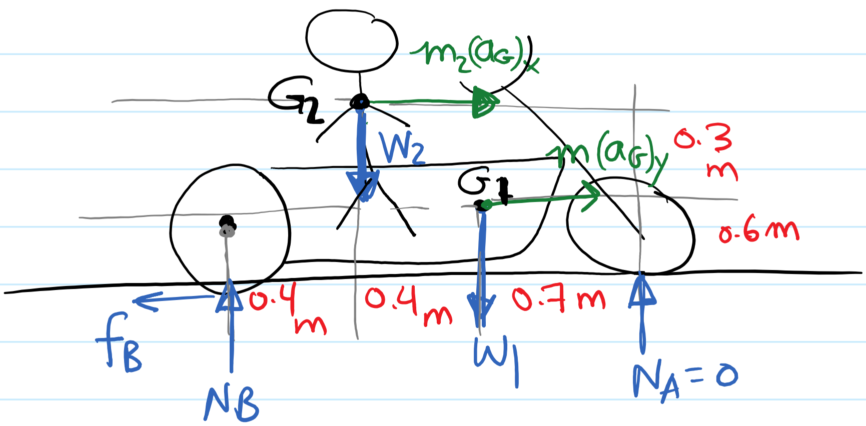
$$\sum M_G = 0$$

$$\sum M_B = e\left[m(a_G)_t\right] - h\left[m(a_G)_n\right]$$

<u>example</u> 17.6

$m_1 = 125\ kg$

example 17.6



$m_1 = 125$ kg
$m_2 = 75$ kg

find $\vec{a}$, $\mu_{s\,min}$

for a wheely.

front wheel free is to roll

$\overset{+}{\rightarrow}$ $\Sigma F_x = -f_B = (m_1 + m_2)(a_G)_x$

$$\boxed{-f_B = 200\, a_{G_x}}$$

$\uparrow +$ $\Sigma F_y = N_B - W_1 - W_2 = (m_1 + m_2)(0)$

$$\boxed{N_B = W_1 + W_2 = 1962\text{ N}}$$

$(+\circlearrowleft)$ $\Sigma M_B = \Sigma (M_k)_B$

$-0.4\, W_2 - 0.8\, W_1 = -m_2(a_G)_x(0.9) - m_1(a_{Gx})(0.6)$

$(a_G)_x = 8.95 \text{ m/s}^2 \rightarrow$

$$\boxed{f_B = -1790\text{ N}}$$

$$\boxed{(\mu_s)_{min} = \frac{f_B}{N_B} = 0.912}$$

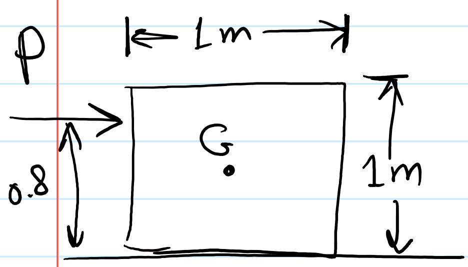
$f_B < \mu_s N_B$

$\rightarrow$ slipping

$\mu \;\; \searrow \;\; f_B$

$$\mu_s > \frac{f_B}{N_B}$$

example 17.7



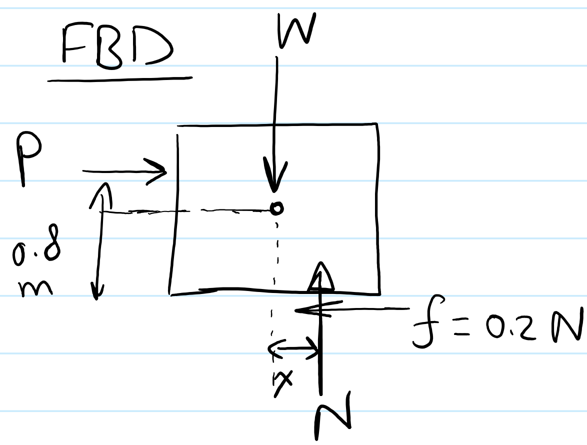
$m = 50\ kg$

$\mu_k = 0.2$

$P = 600\ N$

find $\vec{a}_G$

FBD



for sliding find
$0 \leq x \leq 0.5\,m$

* if $x > 0.5\,m$
$\rightarrow$ tipping will occur about point A

$f \leq 0.2N$

$$\xrightarrow{+} \Sigma F_x = P - 0.2N = m(a_G)_x$$

$$+\uparrow \Sigma F_y = N - W = 0$$

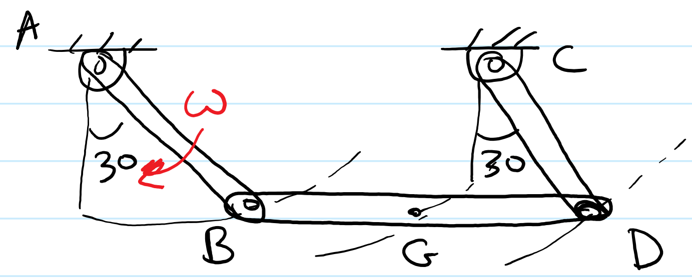
$$\circlearrowright + \; M_G = 0 = -600(0.3) + N_C\,x - f(0.5) = 0$$

$$\boxed{N = 490.5\ N}$$

$$\boxed{x = 0.467\ m}$$

$$\boxed{a_G = 10\ m/s^2}$$

Example 17.8

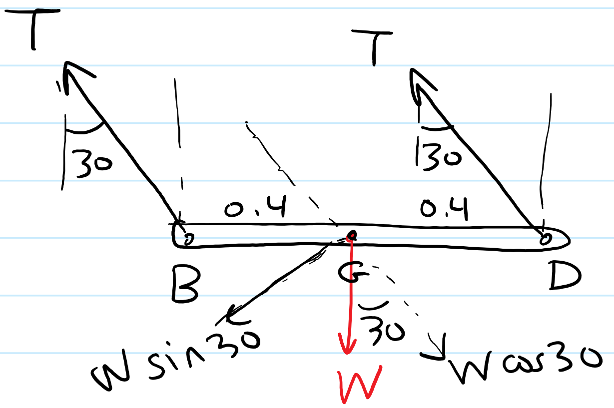


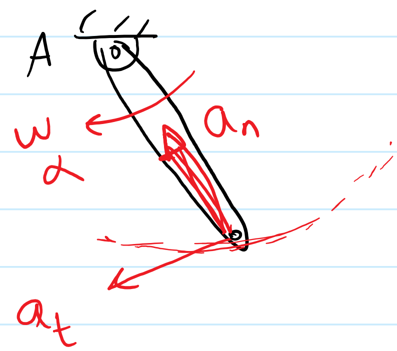
AB = CD = 0.5 m

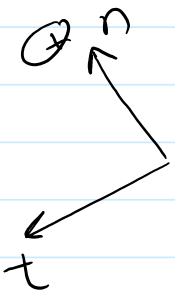
BD = 0.8 m

$\omega = 6$ rad/s

Find forces in each rod







$$\Sigma \vec{F}_n : \quad T_B + T_D - W\cos 30 = m(a_G)_n$$

$$\Sigma \vec{F}_t = \quad W\sin 30 = m(a_G)_t$$

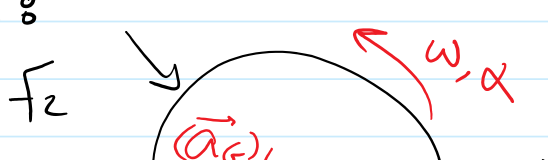
$$\circlearrowleft^{+} \Sigma M_G = \; -T_B\cos 30(0.4) + T_D\cos 30(0.4) = 0$$

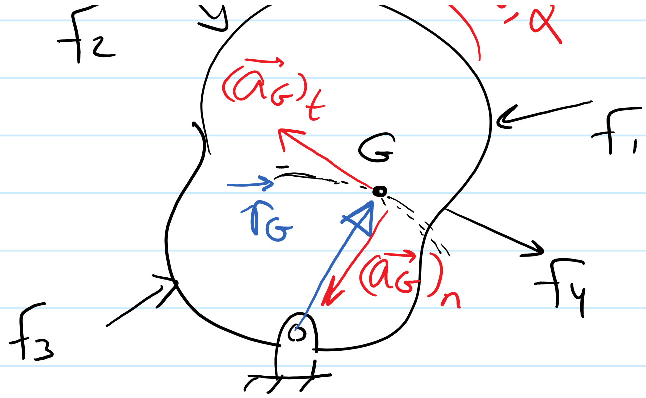
$$(a_G)_n = \omega^2 r = 18\,\text{m/s}^2$$

$$(a_G)_t = 4.905 \;\; \text{m/s}^2$$

$$\boxed{T = 1.32\ \text{kN}}$$

17.4 Equation of Motion: Rotation about a fixed axis:



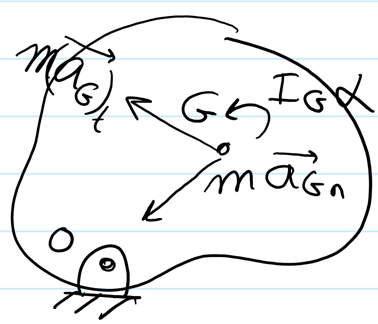


$$|(\vec{a}_G)_t| = \alpha r_G \qquad \Sigma F_n = m\omega r_G^2$$

$$|(\vec{a}_G)_n| = \omega r_G^2 \qquad \Sigma F_t = m\alpha r_G$$

$$\Sigma M_G = I_G \alpha$$

$$\Sigma M_O = \Sigma (M_k)_O$$



$$\Sigma M_O = m(a_G)_t r_G + I_G \alpha$$

$$= m\alpha r_G^2 + I_G \alpha$$

$$= \alpha(I_G + m r_G^2)$$

$$\boxed{\Sigma M_O = I_O \alpha}$$

Read examples 17.9, 17.10, 17.11, 17.12

$\boxed{17.5}$ Equations of Motion: General Planar Motion

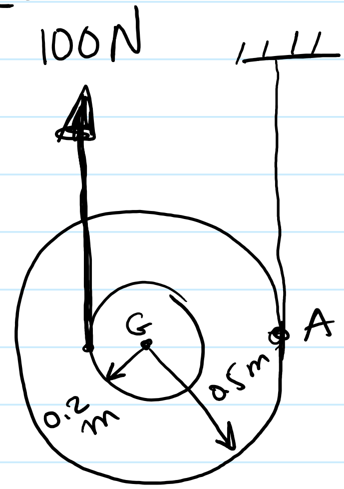
$$\Sigma F_x = m a_{Gx}$$
$$\Sigma F_y = m a_{Gy}$$
$$\Sigma M_G = I_G \alpha$$

In some problems it is more convenient to sum moments about another point $\underline{P}$ rather than G

$$\Sigma F_x = m a_{Gx}$$
$$\Sigma F_y = m a_{Gy}$$
$$\Sigma M_P = \Sigma (\mathcal{M}_k)_P$$

$$\Sigma M_{IC} = I_{IC}\,\alpha$$

<u>example:</u>

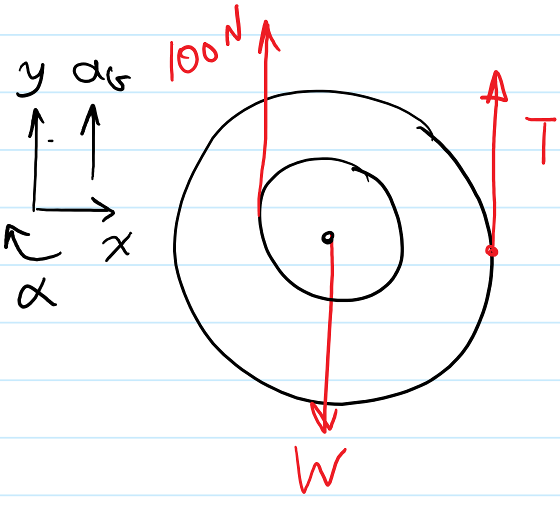


$m = 8$ kg

$k_G = 0.35$ m

find $\underline{\alpha}$

$$I_G = m k_G^2 = 8(0.35)^2$$
$$= 0.98 \text{ kg}\cdot\text{m}^2$$



$$\Sigma F_y = \underline{T} - 100 - W = m \underline{a_{Gy}}$$

$$\Sigma M_G = I_G\,\alpha$$

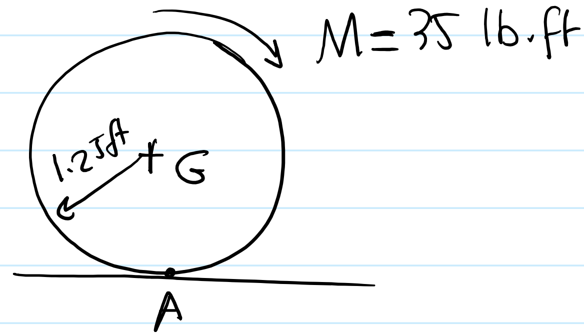
$$100(0.2) - T(0.5) = I_G\,\underline{\alpha}$$

$$a_G = \alpha r = 0.5\alpha$$

$$\Rightarrow \boxed{\alpha = 10.3 \text{ rad/s}^2}$$

$$\Rightarrow \boxed{\alpha = 10.3 \ rad/s^2}$$

example 17.14

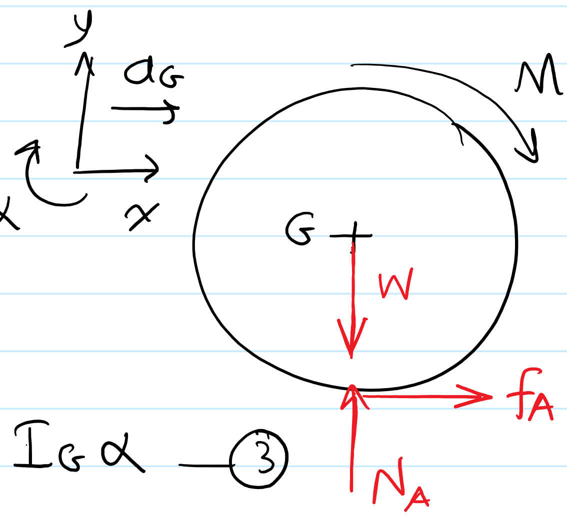


$W = 50 \ lb$

$k_G = 0.7 \ ft$

$\mu_s = 0.3$

$\mu_k = 0.25$ Find $\vec{a}_G$



$\xrightarrow{+} \ \Sigma F_x = f_A = m a_{G_x}$ — (1)

$+\uparrow \ \Sigma F_y = W - N_A = 0$ — (2)

$+\circlearrowright \ \Sigma M_G = M - f_A(1.25) = I_G \alpha$ — (3)

$$I_G = m k_G^2 = \frac{50}{32.2}(0.7)^2 = 0.761 \ slug \cdot ft^2$$

Assume No slipping $\rightarrow f_A \leq \mu_s N_A$

$\vec{a}_G = 1.25 \alpha$ — (4)

Solving $\Rightarrow$ $N_A = 50 \ lb$

$f_A = 21.3 \ lb$

$\alpha = 11 \ rad/s^2$

$\vec{a}_G = 13.7 \ ft/s^2$

Check assumption $f_A \leq \mu_s N_A$

$21.3 \text{ lb} \leq 0.3(50)$

$21.3 \overset{?}{\leq} 15 \quad X$

assumption is wrong $\Rightarrow f_A = \mu_k N_A$ — (4)

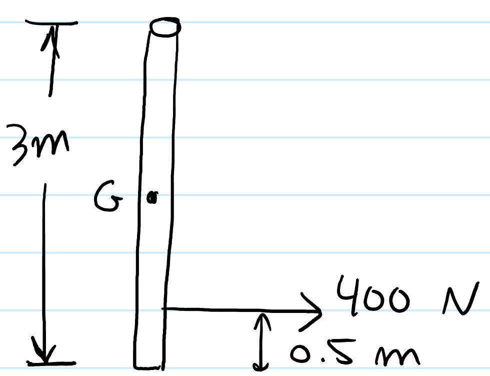
$f_A = 12.5 \text{ lb}$

$N_A = 50 \text{ lb}$

$\alpha = 25.5 \text{ rad/s}^2$

$a_G = 8.05 \text{ ft/s}^2 \rightarrow$

<u>example 17.15</u>

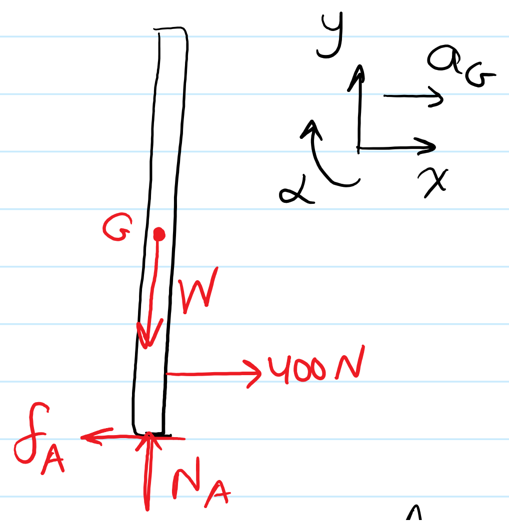


$m = 100 \text{ kg}$

$\mu_s = 0.3 \quad \mu_k = 0.25$

originally @ rest

find α

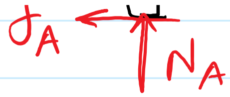


$I_G = \frac{1}{12} m l^2$

$\Sigma F_x = 400 - f_A = 100 a_G$ (1)

$\Sigma F_y = N_A - W = 0$ (2)

$\Sigma M_G = f_A(1.5) - 400(1) = I_G \alpha$ (3)



(3)

Assume No slipping: $a_G = \alpha r$ (4)

point A is a pivot.

Solving 1 → 4 ⟹
$N_A = 981\ N$
$f_A = 300\ N$
$a_G = 1\ m/s^2$
$\alpha = 0.667\ rad/s^2$

Check assumption: $f_A \overset{?}{\leq} \mu_s N_A$

$300 \overset{?}{\leq} 0.3(981)$

$300N > 294\ N$ X

point A slips ⟹ $f_A = \mu_k N_A$ — (4)'

Solving (1) → (4)' ⟹
$f_A = 245$
$N_A = 981\ N$

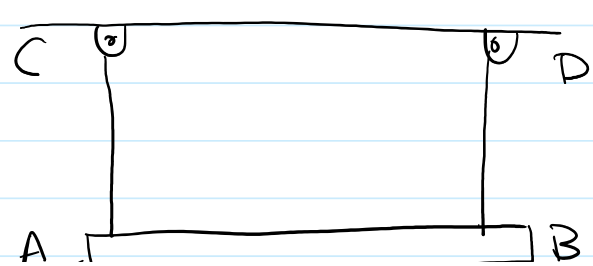
$a_G = 1.55\ m/s^2$

$\alpha = -0.428\ rad/s^2$

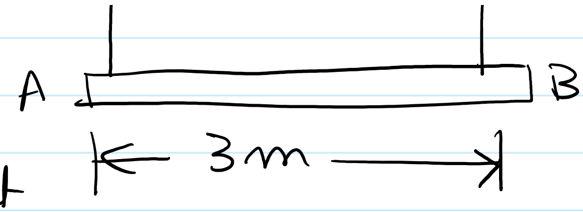
$= 0.428\ rad/s^2$ ↺

example 17.16

$m = 50\ kg$

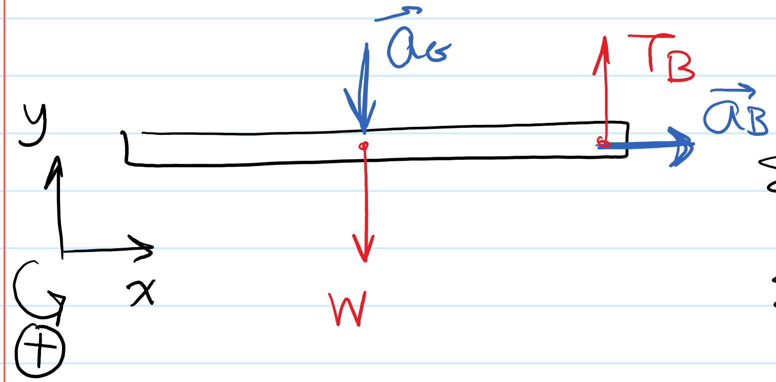


$m = 50$ kg



Find T_{BD}

α after CA is cut



$\Sigma F_x = 0 = m a_{Gx}$

$\Sigma F_y = T_B - W = m a_{Gy}$ —— ①

$\Sigma M_G = I_G \alpha$

$T_B \left(\frac{3}{2}\right) = \frac{1}{12} m L^2 \alpha$ —— ②

Since released from rest $v_B = 0$

$(a_B)_n = 0$

$(a_B)_t = a_B \hat{i}$

$\vec{a}_G = \vec{a}_B + \vec{\alpha} \times \vec{r}_{G/B} - \omega^2 \vec{r}_{G/B}$

$-a_G \hat{j} = a_B \hat{i} + (\alpha \hat{k} \times -1.5 \hat{i}) - 0$

$a_B = 0$

$(a_G)_y = 1.5 \alpha$ —— ③

Solving ① → ③ $\Rightarrow$ $\alpha = 4.905 \text{ rad/s}^2$

$T_B = 123$ N

$(a_G)_y = 7.36 \ m/s^2$

Note: When there is friction, μ_s, μ_k are given

1) Find f from EOM
2) Assume No slipping (use $a = \alpha r$)
3) check assumption

$$f \leq \mu_s N$$

if true: Solution is correct
if false: set $f = \mu_k N$

$a = \alpha r$ does not work anymore
Resolve equations while $f = \mu_k N$.