Chapter 18

Friday, August 5, 2016 7:40 PM

18.1 Kinetic energy

If a rigid body has a velocity $\vec{v}_G$, an angular velocity ω, and a mass m, a mass moment of inertia I_G then its kinetic energy is calculated by:

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

- Translation $\omega = 0$

$$T = \frac{1}{2} m v_G^2$$

- Rotation about a fixed axis $v_G = \omega r_G$

$$T = \frac{1}{2} m (\omega r_G)^2 + \frac{1}{2} \cdot I_G \omega^2$$

$$= \frac{1}{2}\left(m r_G^2 + I_G\right)\omega^2$$

$T = \frac{1}{2} I_O \omega^2 \rightarrow O$ is a point on the axis of rotation.

Note $T = \frac{1}{2} I_{IC} \omega^2$

18.2 Work of a Force

Revision:
$$U = \int \vec{F} \cdot d\vec{r}$$

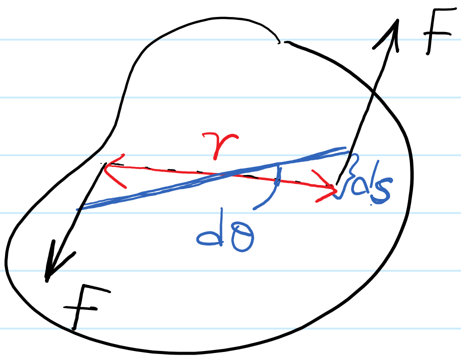
constant force $\Rightarrow$ $U = F\cos\theta \, r$

Weight $\Rightarrow$ $U_W = -W \, \Delta y$

Spring force $\Rightarrow$ $U_s = -\frac{1}{2}k\left(S_2^2 - S_1^2\right)$

Friction force
- $\rightarrow$ No slipping $\rightarrow$ No work
- $\rightarrow$ Slipping $\rightarrow$ it does work.

$\boxed{18.3}$ Work of a couple moment



$$ds = \frac{r}{2}d\theta \quad , \quad M = Fr$$

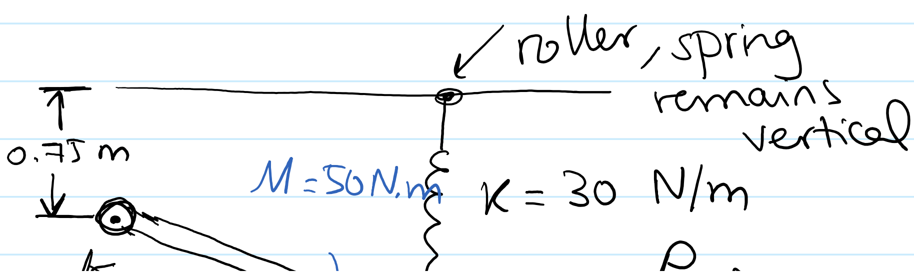
$$dU_M = F\frac{r}{2}d\theta + F\frac{r}{2}d\theta$$

$$\int dU_M = \int Fr\,d\theta$$

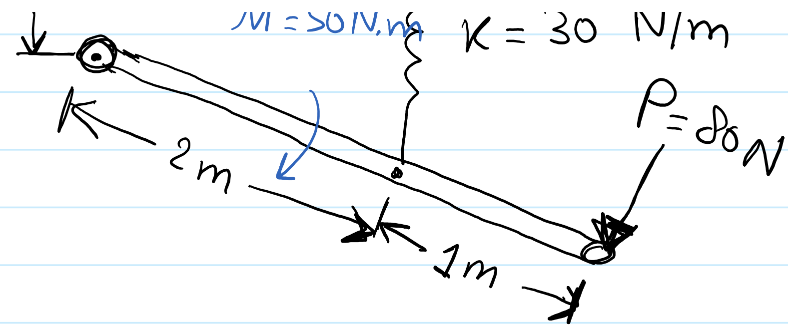
$$U_M = \int_{\theta_1}^{\theta_2} Fr\,d\theta$$

$$U_M = \int_{\theta_1}^{\theta_2} M\,d\theta$$

<u>example</u>

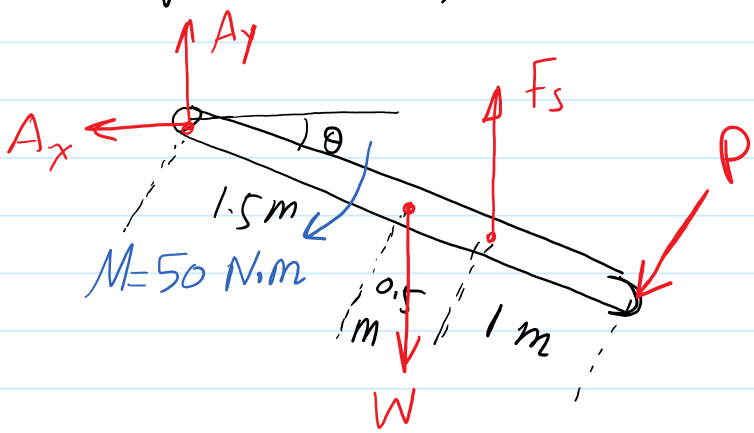


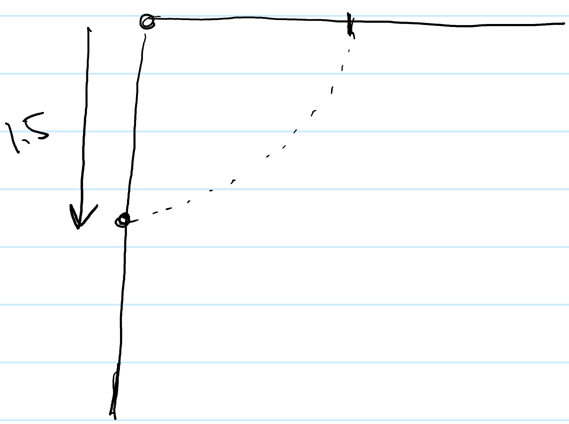
$m = 10\ kg$



$S_{unst} = 0.5\ m$

Find work done from $\theta = 0°$, $\theta = 90°$





$$U_W = 98.1\,(1.5)$$

$$\boxed{U_W = 147.2\ J}$$

$$U_M = M(\theta_2 - \theta_1)$$

$$= 50\left(\frac{\pi}{2} - 0\right)$$

$$\boxed{U_M = 78.5\ J}$$

@ $\theta = 0°$ $\quad S_1 = 0.75 - 0.5 = 0.25\ m$

@ $\theta = 90°$ $\quad S_2 = 2 + 0.75 - 0.5 = 2.25\ m$

$$U_s = -\frac{1}{2}\,30\left(2.25^2 - 0.25^2\right) = -75\ J$$

P moves the bar in a curve

$$S = \frac{\pi}{2} 3 = 4.712 \text{ m}$$

$$U_P = 80(4.712) = 377 \text{ J}$$

Pin reactions A_x, A_y do no work

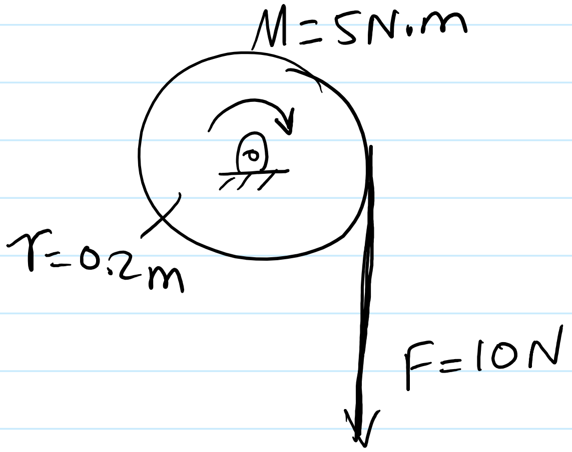
Total Work $= U_T = 147.2 + 78.5 - 75 + 377$

$$\boxed{U_T = 528 \text{ J}}$$

18.4 Principle of Work & Energy

$$T_2 = T_1 + \Sigma U_{1\to 2}$$

example



$m = 30$ kg

Find # of rev to get to

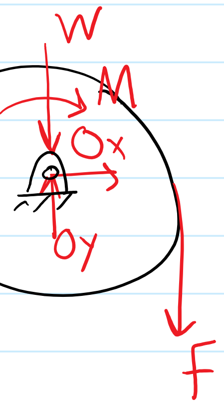
$\omega = 20$ rad/s

starting from rest

$$T_2 = T_1 + U_{1\rightarrow 2}$$

$$T_2 = \frac{1}{2} I_o \omega_2^2 = \frac{1}{2}\left(\frac{1}{2} m r^2\right)(20)^2 = 120\,J$$

$$T_1 = 0$$

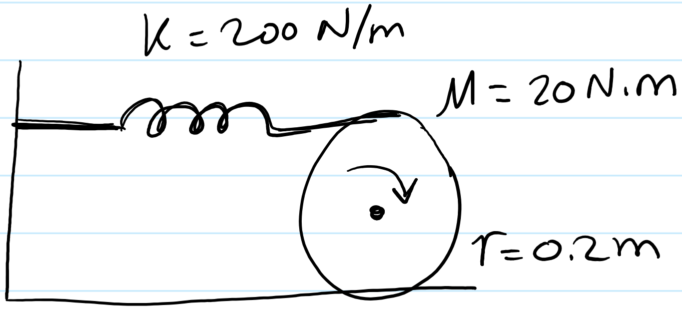


$$U_{1-2} = U_M + U_F$$
$$= M\theta + F s'$$
$$= 5\theta + 10(r\theta)$$

$$120 = 0 + 5\theta + 2\theta \Rightarrow \theta = 17.14 \text{ rad}$$

$$\boxed{\theta = 2.73 \text{ rev}}$$

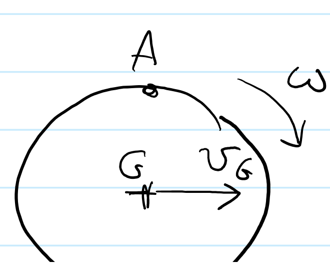
example



$W = 200\,N$
$k_G = 0.15$
rolls from rest
without slipping

Find ω after G moves 0.1 m
spring is initially unstretched.

$$T_1 = 0$$



$$T_2 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega_2^2$$

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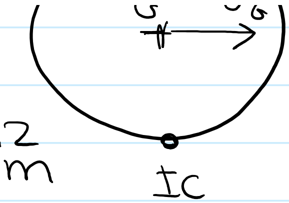
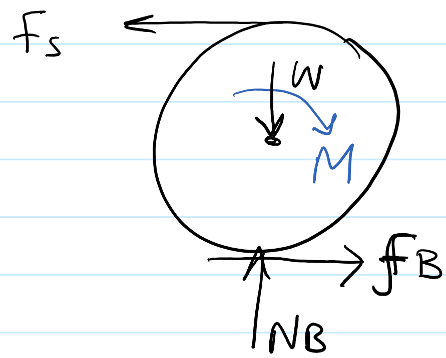


$$T_2 = \frac{1}{2} I_{IC} \omega_2^2$$

$$= \frac{1}{2}\left(I_G + md^2\right)\omega_2^2$$

$$= \frac{1}{2}\left(\frac{200}{9.81}(0.15)^2 + \frac{200}{9.81}(0.2)^2\right)\omega_2^2 = 0.6371\,\omega_2^2$$



$U_W = 0$, $\Delta y = 0$

$U_{fB} = 0$, No slipping

$U_{NB} = 0$, Reaction force

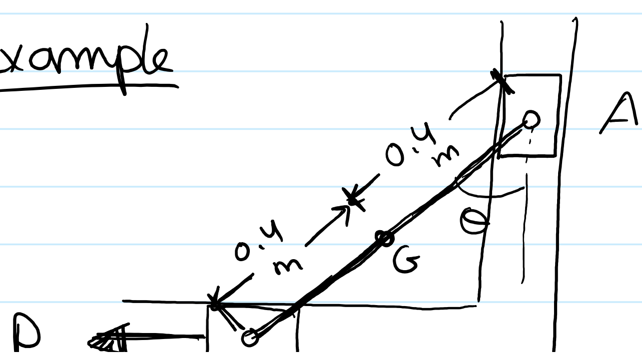
$U_M = M\theta \qquad \theta = s_G / r_{G/IC} = 0.1/0.2 = 0.5 \text{ rad}$

$U_s = \frac{1}{2} k s^2$, $s_A = r_{A/IC}\theta = (0.4)(0.5) = 0.2 \text{ m}$

$$T_1 + U_s + U_M = T_2$$

$$\boxed{\omega = 3.07 \text{ rad/s}}$$

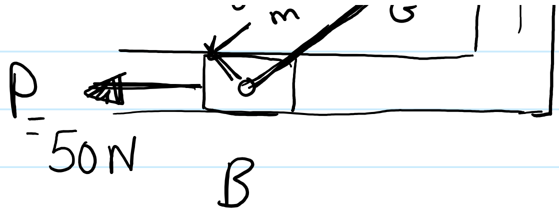
example



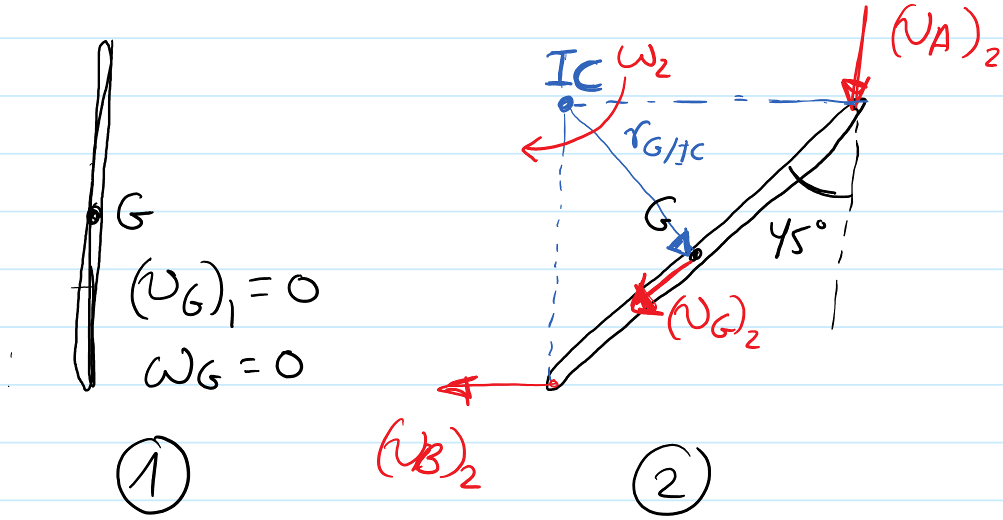
m = 10 kg

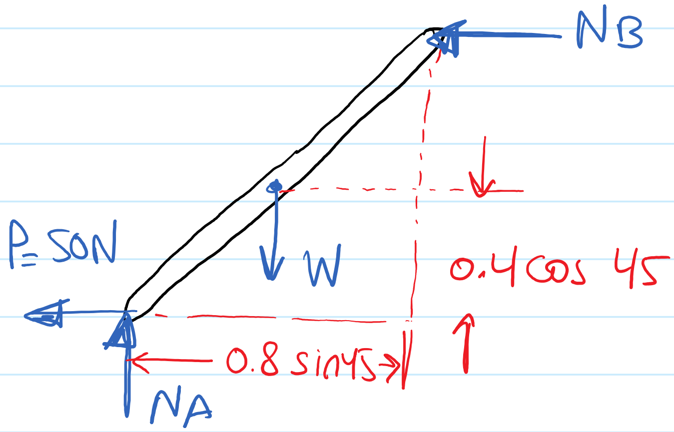
rod is at rest @ $\theta = 0°$

find ω @ $\theta = 45°$



find ω @ $\theta = 45°$





$$r_{G/IC} = 0.4 \tan 45$$

$$(v_G)_2 = r_{G/IC}\,\omega_2$$

$$(v_G)_2 = 0.4\,\omega_2$$

$$T_2 = T_1 + U_{1\to 2}$$

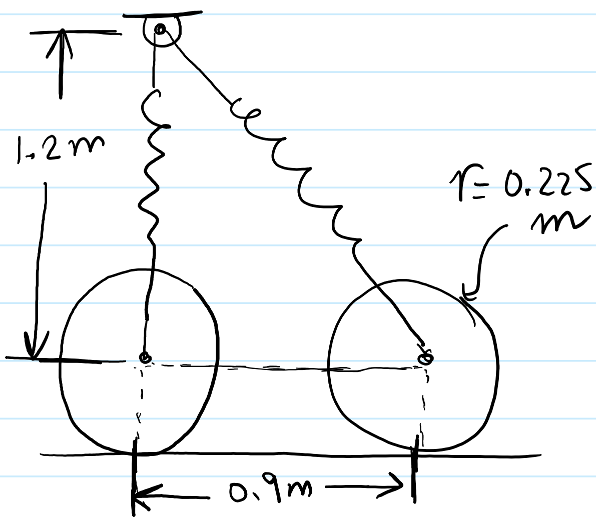
$$\frac{1}{2} m (v_G^2)_2 + \frac{1}{2} I_G \omega_2^2 = 0 + U_W + U_P$$

$$\frac{1}{2} m (0.4\omega_2)^2 + \frac{1}{2} \times \frac{1}{12} m\,\omega_2^2 = W(0.4 - 0.4\cos 45) + P(0.8 \sin 45)$$

$$\boxed{\omega_2 = 6.11 \text{ rad/s} \circlearrowright}$$

$\boxed{18.5}$ Conservation of Energy

If all forces are conservative $\rightarrow T_1 + V_1 = T_2 + V_2$



$m = 15$ kg

$k_G = 0.18$ m

$k = 30$ N/m

unstretched length = 0.3 m

released from rest.
find ω when G moves to the left 0.9 m

$$V_1 = V_e = \frac{1}{2} k s_1^2 = \frac{1}{2}(30)\left(\sqrt{1.2^2 + 0.9^2} - 0.3\right)^2 = 21.6\text{J}$$

$$V_2 = V_e = \frac{1}{2} k s_2^2 = \frac{1}{2}(30)(1.2 - 0.3)^2 = 12.15\text{J}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} I_{IC} \omega_2^2 = \frac{1}{2}\left(I_G + mr^2\right)\omega_2^2$$

$$\underline{\text{or}} \quad \frac{1}{2} m \underbrace{v_G^2}_{(\omega r)^2} + \frac{1}{2} I_G \omega_2^2$$

$$T_1 + V_1 = T_2 + V_2 \Rightarrow \omega = 3.9 \text{ rad/s} \circlearrowleft$$