Chapter 19

Monday, August 8, 2016 1:16 PM

$\boxed{19.1}$ Linear + Angular Momentum.

Force, velocity, time $\Rightarrow$ impulse momentum methods

* Linear Momentum

$$\vec{L} = m\vec{v}_G \qquad (kg \cdot m/s) \quad (slug \cdot ft/s)$$

* Angular Momentum

$$\vec{H}_G = I_G \vec{\omega} \qquad (kg \cdot m^2/s) \quad (slug \cdot ft^2/s)$$

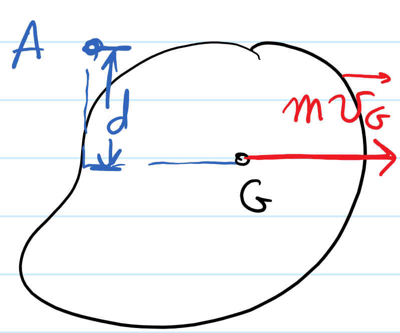
$$\vec{H}_P = I_G\vec{\omega} + m\vec{v}_{G_x}(x) + m\vec{v}_{G_y}(y)$$

x, y represent the distance between P and G.

Consider 3 cases

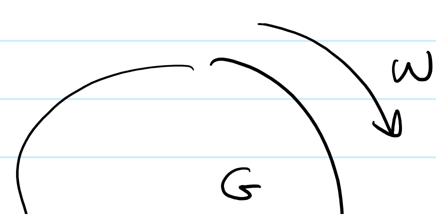
1) Translation:

$$\vec{L} = m\vec{v}_G$$
$$H_G = 0$$
$$H_P = m v_G d.$$



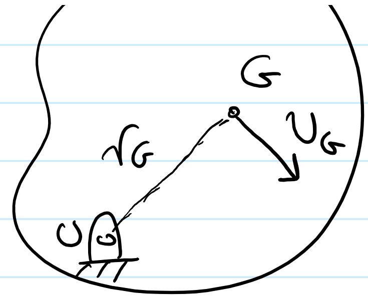
2) Rotation about a fixed axis

$$\vec{L} = m\vec{v}_G$$



$$\vec{L} = m\vec{v}_G$$

$$\vec{H}_G = I_G\vec{\omega}$$



$$H_o = I_G\vec{\omega} + m v_G r_G$$

since for rotation $v_G = r_G \omega$

$$\vec{H}_o = I_G\vec{\omega} + m r_G^2 \vec{\omega}$$

$$= \vec{\omega}\left(I_G + m r_G^2\right)$$

$$\vec{H}_o = I_o\vec{\omega}.$$

3) General Planar Motion

$$\vec{L} = m\vec{v}_G$$

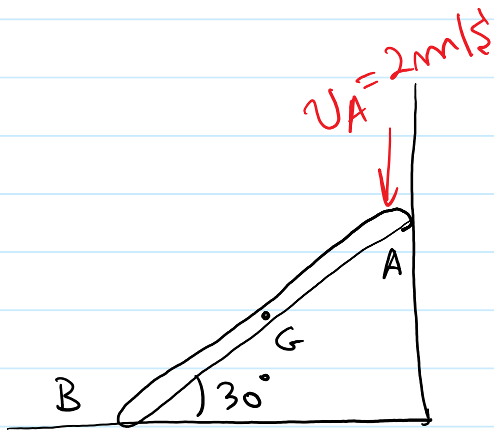
$$H_G = I_G\vec{\omega}$$

$$H_A = I_G\vec{\omega} + m v_G (d)$$

d: perp. distance from G to A

$$H_{IC} = I_{IC}\,\omega$$

<u>example</u>

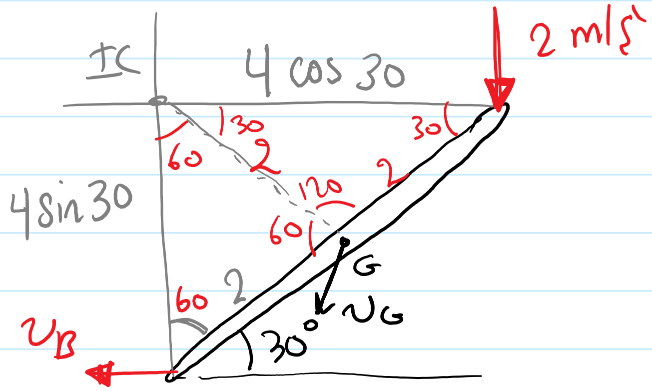


$L = 4\,m$
$m = 5\,kg.$

Find H_G
H_{IC}

IC | 4 cos 30 | 2 m/s

$r_{G/IC} = 2\,m.$



$r_{G/IC} = 2 \text{ m}.$

$$\omega = \frac{v_A}{r_{A/IC}} = \frac{2}{4\cos 30} = 0.577 \text{ rad/s}$$

$$v_G = \omega r_{G/IC} = 0.577(2)$$

$$v_G = 1.155 \text{ m/s}$$

$$\circlearrowright \; H_G = I_G\omega = \frac{1}{12} m L^2 \omega = 3.85 \text{ kgm/s}$$

$$\circlearrowright \; H_{IC} = I_G\omega + m v_G (r_{G/IC})$$

$$\text{or} \quad H_{IC} = I_{IC}\omega = 15.4 \text{ kg.m/s}$$

$\boxed{19.2}$ Principle of Impulse & Momentum

$$m\vec{v}_{G_1} + \int_{t_1}^{t_2} \Sigma\vec{F}\,dt = m\vec{v}_{G_2} \quad \text{(linear impulse \& Momentum)}$$

$$I_G\vec{\omega}_1 + \int_{t_1}^{t_2} \Sigma\vec{M}_G\,dt = I_G\vec{\omega}_2 \quad \text{(angular impulse \& Momentum)}$$

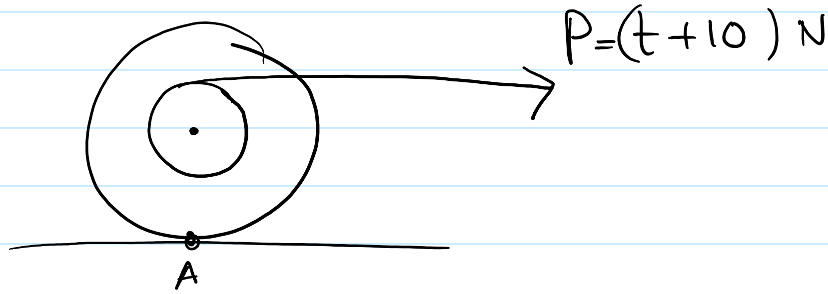
$$m v_{Gx_1} + \int_{t_1}^{t_2} \Sigma F_x\,dt = m v_{Gx_2}$$

$$m v_{Gy_1} + \int_{t_1}^{t_2} \Sigma F_y\,dt = m v_{Gy_2}$$

$$I_G\omega_1 + \int_{t_1}^{t_2} \Sigma M_G\,dt = I_G\omega_2$$

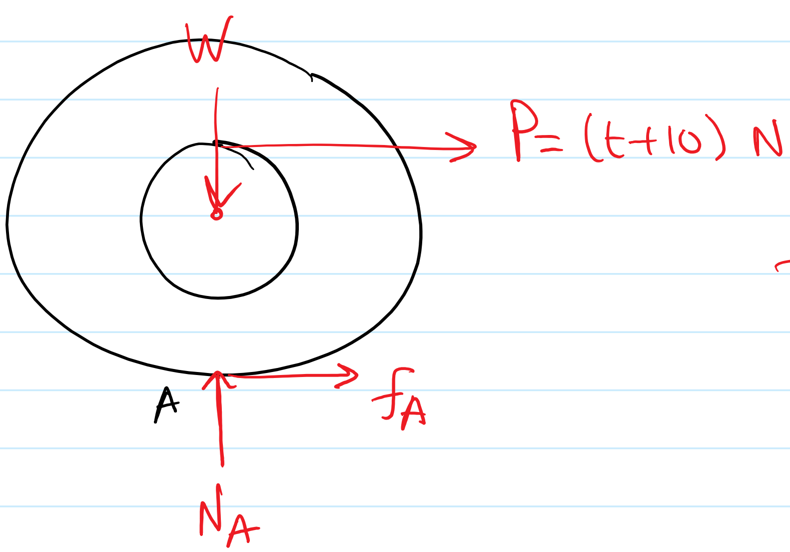
$$I_G\omega_1 + \int_{t_1}^{t_2} \Sigma M_G\,dt = I_G\omega_2$$

<u>example</u>



$r = 0.4\,m$
$R = 0.75\,m$
$m = 100\,kg$
$k_G = 0.35\,m$

Initially at rest, find ω in 5 seconds
rolling without slipping @ A



f_A is also variable

Since f_A is unknown, ω is the same for all points

Apply impulse & Momentum @ point A

$$I_A\omega_1 + \int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

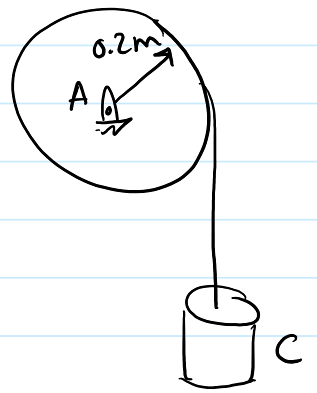
5

t_1

$$\int_0^5 (10+t)(0.75+0.4)\,dt = (\underbrace{I_G}_{mk_G^2} + mR^2)\,\omega_2$$

$$\boxed{\omega_2 = 1.05 \text{ rad/s}}$$

example



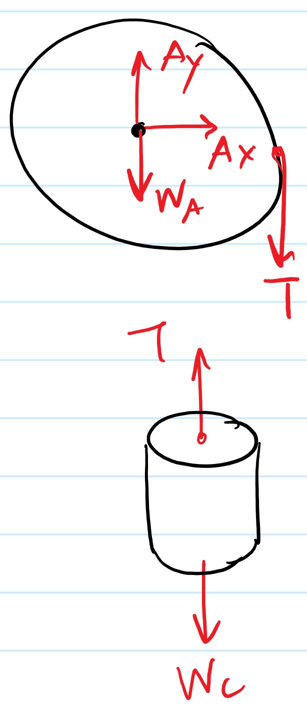
$m_C = 6$ kg

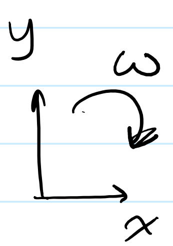
$m_A = 20$ kg

$I_A = 0.4$ kg m^2

$(v_C)_1 = 2$ m/s ↓

find v_C after 3 s





Angular Impulse + Momentum about point A

$$I_A\omega_1 + \int_0^3 M_A\,dt = I_A\omega_2$$

$$\boxed{0.4\omega_1 + \int_0^3 Tr\,dt = 0.4\omega_2}$$

1 $\quad \omega_1 = \frac{v_{B1}}{r} = \frac{2}{0.2} = 10$ rad/s

Linear impulse & Momentum for C

$$m(v_c)_1 + \int_0^3 \Sigma F\, dt = m(v_c)_2$$

$$6(-2) + \int_0^3 (T - W_c)\, dt = 6(v_{c_2})$$

$$\boxed{-12 + \int_0^3 T - (6 \times 9.81)\, dt = 6(v_{c_2})} \quad \text{— (2)}$$

$$\omega_2 = \frac{(v_{c_2})}{0.2}$$

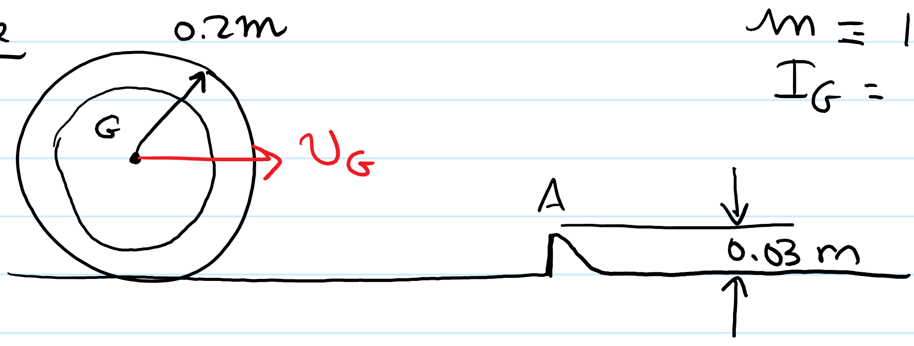
$$\boxed{\text{Solving for } (v_c)_2 = 13 \text{ m/s} \downarrow}$$

19.3 Conservation of Momentum

If $\Sigma F = 0 \rightarrow$ linear momentum is conserved
$\Sigma M = 0 \rightarrow$ angular " " "

(forces are non-impulsive)

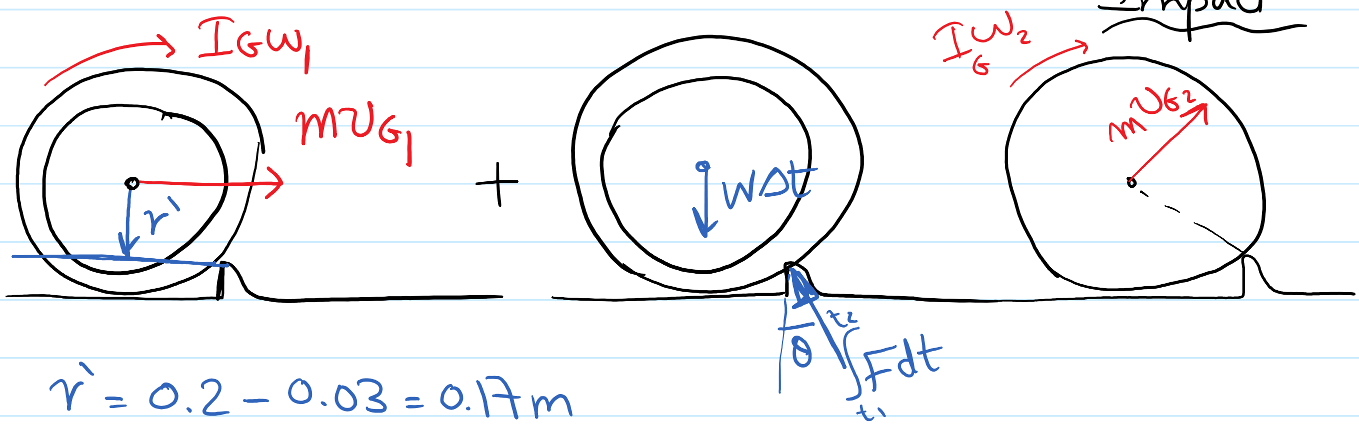
<u>example</u>



$m = 10$ kg
$I_G = 0.156$ kg.m^2

No slip, No rebound $\rightarrow$ find $(v_G)_{min}$ for the wheel to roll over A

Just before impact | Impact | Just after Impact



$r' = 0.2 - 0.03 = 0.17\,m$

Angular Momentum about A is conserved

$$(H_A)_1 = (H_A)_2$$

$$H_{G_1} + m v_{G_1} r' = H_{G_2} + m v_{G_2} r$$

$$I_G \omega_1 + m v_{G_1} r' = I_G \omega_2 + m v_{G_2} r \quad \text{——— (1)}$$

No slipping $\Rightarrow \omega = \frac{v_G}{r} \Rightarrow \omega_1 = 5 v_{G_1}$

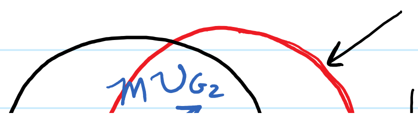
$$\omega_2 = 5 v_{G_2}$$

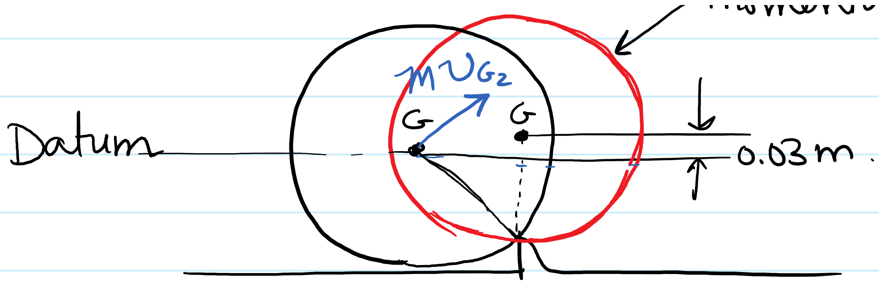
$\therefore$ equ (1) becomes

$$v_{G_2} = 0.8921\, v_{G_1}$$

* Conservation of Energy after impact

momentarily stops



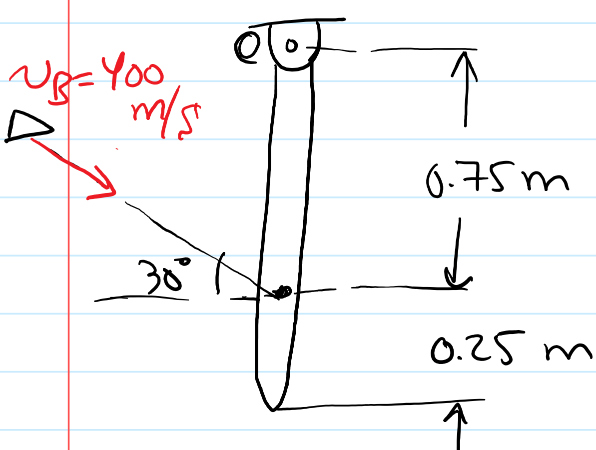


$$T_2 = V_3 \qquad\qquad T_3 = 0$$
$$V_2 = 0$$

$$\frac{1}{2} m v_{G_2}^2 + \frac{1}{2} I_G \omega_2^2 = mg(0.03)$$

$$(v_G)_1 = 0.729 \ m/s \longrightarrow$$

<u>example</u>

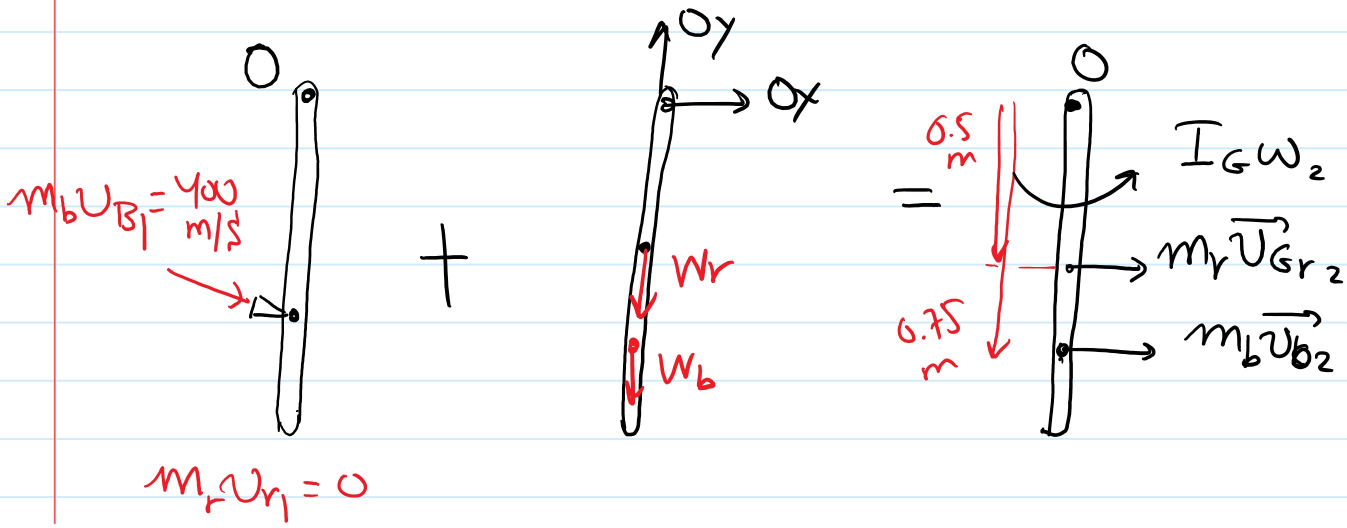


$m_r = 5$ kg

$v_1 = \omega_1 = 0$

$m_b = 4g = 0.004$ kg

find ω_2, bullet is embedded.



Conservation of Angular momentum.

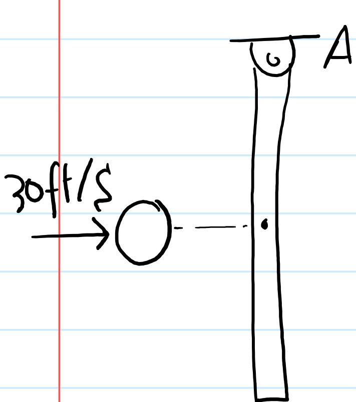
$$\Sigma (H_O)_1 = \Sigma (H_O)_2$$

$$m_B v_{B_1} (\cos 30)(0.75) = I_G \omega_2^2 + m_R v_{G_2}(0.5) + m_b v_{b_2}(0.75)$$

$$\boxed{1.039 = 0.003\, v_{B_2} + 2.5\, v_{G_2} + 0.4167\, \omega_2} \quad (1)$$

$$\left.\begin{aligned} v_{G_2} &= \omega_2 (0.5) \\ v_{B_2} &= \omega_2 (0.75) \end{aligned}\right\} \Rightarrow \omega_2 = 0.623 \text{ rad/s}$$

$\boxed{19.4}$ Eccentric Impact



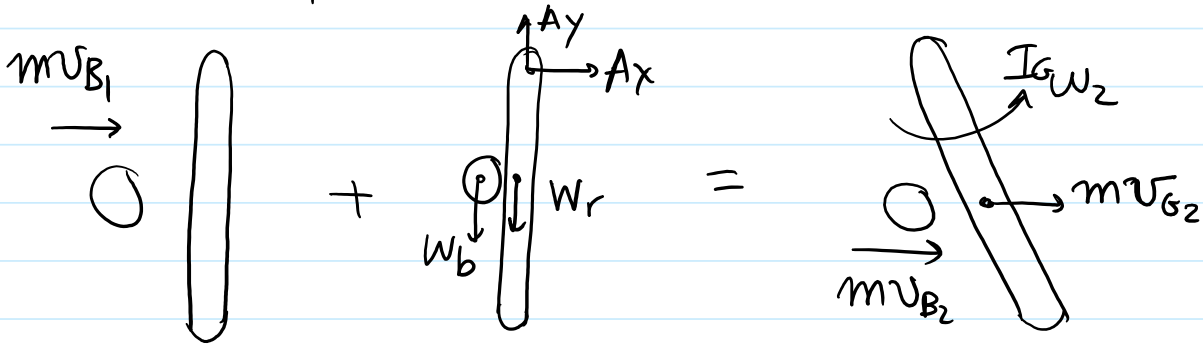
L = 3 ft
W_r = 10 lb
W_b = 2 lb

ball strikes center.

find $(\omega_r)_2$ $\quad e = 0.4$

$$(H_A)_1 = (H_A)_2$$

$(H_A)_1 = (H_A)_2$



$$m_B (v_B)_1 (1.5) + 0 + 0 = m_B (v_B)_2 (1.5) + m_r v_{G_2} (1.5) + I_G \omega_2$$

$$v_{G_2} = 1.5\,\omega_2$$

$$2.795 = 0.09317\, v_{B_2} + 0.9317\,\omega_2$$

$\overset{+}{\rightarrow}$

$$e = \frac{v_{G_2} - v_{B_2}}{v_{B_1} - v_{G_1}} = 0.4$$

$$\frac{1.4\,\omega_2 - v_{B_2}}{30 - 0} = 0.4$$

$$\Rightarrow 12 = 1.5\,\omega_2 - (v_B)_2$$

$$\boxed{\begin{aligned} v_{B_2} &= -6.52 \text{ ft/s} \quad \leftarrow \\ \omega_2 &= 3.65 \text{ rad/s} \quad \circlearrowleft \end{aligned}}$$