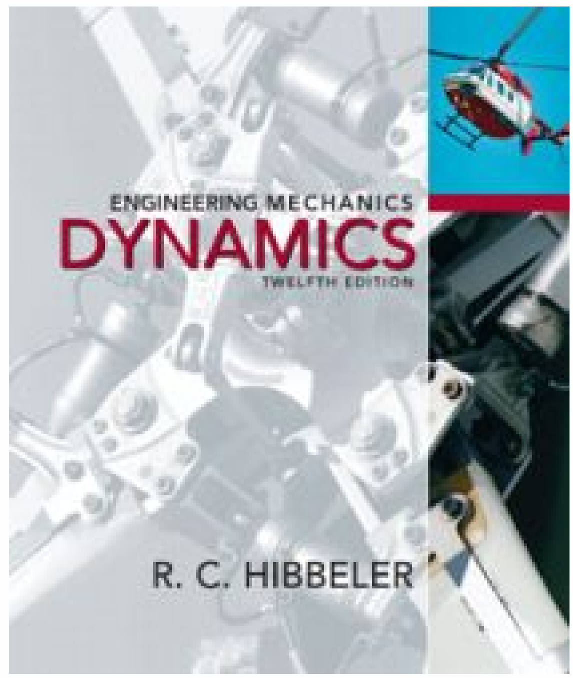
ENGINEERING MECHANICS

DYNAMICS

TWELFTH EDITION

R. C. HIBBELER



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Upper Saddle River, NJ 07458

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•12–1. A car starts from rest and with constant acceleration achieves a velocity of 15 m/s when it travels a distance of 200 m. Determine the acceleration of the car and the time required.

Kinematics:

$v_0 = 0, v = 15 \text{ m/s}, s_0 = 0, \text{and } s = 200 \text{ m}.$

$$(\overset{+}{\rightarrow}) \qquad v^2 = {v_0}^2 + 2a_c(s - s_0)$$

$$15^2 = 0^2 + 2a_c(200 - 0)$$

$$a_c = 0.5625 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$15 = 0 + 0.5625t$$

$$t = 26.7 \text{ s} \qquad \textbf{Ans.}$$

12–2. A train starts from rest at a station and travels with a constant acceleration of 1 m/s^2. Determine the velocity of the train when $t = 30$ s and the distance traveled during this time.

Kinematics:

$a_c = 1 \text{ m/s}^2, v_0 = 0, s_0 = 0, \text{and } t = 30 \text{ s}.$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$= 0 + 1(30) = 30 \text{ m/s} \qquad \textbf{Ans.}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$= 0 + 0 + \frac{1}{2}(1)\left(30^2\right)$$

$$= 450 \text{ m} \qquad \textbf{Ans.}$$

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12–3. An elevator descends from rest with an acceleration of $5\,\text{ft/s}^2$ until it achieves a velocity of $15\,\text{ft/s}$. Determine the time required and the distance traveled.

Kinematics:

$a_c = 5\text{ ft/s}^2$, $v_0 = 0$, $v = 15\text{ ft/s}$, and $s_0 = 0$.

$(+\downarrow)\qquad v = v_0 + a_c t$

$15 = 0 + 5t$

$t = 3\text{s}$ **Ans.**

$(+\downarrow)\qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$15^2 = 0^2 + 2(5)(s - 0)$

$s = 22.5\text{ ft}$ **Ans.**

***12–4.** A car is traveling at $15\,\text{m/s}$, when the traffic light 50 m ahead turns yellow. Determine the required constant deceleration of the car and the time needed to stop the car at the light.

Kinematics:

$v_0 = 0$, $s_0 = 0$, $s = 50\text{ m}$ and $v_0 = 15\text{ m/s}$.

$(\overset{+}{\rightarrow})\qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$0 = 15^2 + 2a_c(50 - 0)$

$a_c = -2.25\text{ m/s}^2 = 2.25\text{ m/s}^2 \leftarrow$ **Ans.**

$(\overset{+}{\rightarrow})\qquad v = v_0 + a_c t$

$0 = 15 + (-2.25)t$

$t = 6.67\text{ s}$ **Ans.**

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•12–5. A particle is moving along a straight line with the acceleration $a = (12t - 3t^{1/2})\ \text{ft/s}^2$, where t is in seconds. Determine the velocity and the position of the particle as a function of time. When $t = 0$, $v = 0$ and $s = 15\ \text{ft}$.

Velocity:

$(\overset{+}{\rightarrow})$ $$dv = a\,dt$$

$$\int_0^v dv = \int_0^t \left(12t - 3t^{1/2}\right)dt$$

$$v\Big|_0^v = \left(6t^2 - 2t^{3/2}\right)\Big|_0^t$$

$$v = \left(6t^2 - 2t^{3/2}\right)\text{ft/s} \qquad \textbf{Ans.}$$

Position: Using this result and the initial condition $s = 15$ ft at $t = 0$ s,

$(\overset{+}{\rightarrow})$ $$ds = v\,dt$$

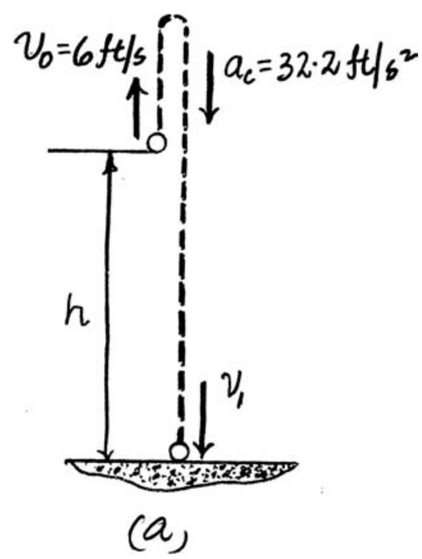
$$\int_{15\text{ ft}}^s ds = \int_0^t \left(6t^2 - 2t^{3/2}\right)dt$$

$$s\Big|_{15\text{ ft}}^s = \left(2t^3 - \frac{4}{5}t^{5/2}\right)\Big|_0^t$$

$$s = \left(2t^3 - \frac{4}{5}t^{5/2} + 15\right)\text{ft} \qquad \textbf{Ans.}$$

12–6. A ball is released from the bottom of an elevator which is traveling upward with a velocity of 6 ft/s. If the ball strikes the bottom of the elevator shaft in 3 s, determine the height of the elevator from the bottom of the shaft at the instant the ball is released. Also, find the velocity of the ball when it strikes the bottom of the shaft.

Kinematics: When the ball is released, its velocity will be the same as the elevator at the instant of release. Thus, $v_0 = 6$ ft/s. Also, $t = 3$ s, $s_0 = 0$, $s = -h$, and $a_c = -32.2\ \text{ft/s}^2$.



$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-h = 0 + 6(3) + \frac{1}{2}(-32.2)\left(3^2\right)$$

$$h = 127\text{ ft} \qquad \textbf{Ans.}$$

$(+\uparrow)$ $$v = v_0 + a_c t$$

$$v = 6 + (-32.2)(3)$$

$$= -90.6\text{ ft/s} = 90.6\text{ ft/s}\ \downarrow \qquad \textbf{Ans.}$$

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12–7. A car has an initial speed of 25 m/s and a constant deceleration of 3 m/s^2. Determine the velocity of the car when $t = 4$ s. What is the displacement of the car during the 4-s time interval? How much time is needed to stop the car?

$$v = v_0 + a_c t$$

$$v = 25 + (-3)(4) = 13 \text{ m/s}$$ **Ans.**

$$\Delta s = s - s_0 = v_0 t + \frac{1}{2} a_c t^2$$

$$\Delta s = s - 0 = 25(4) + \frac{1}{2}(-3)(4)^2 = 76 \text{ m}$$ **Ans.**

$$v = v_0 + a_c t$$

$$0 = 25 + (-3)(t)$$

$$t = 8.33 \text{ s}$$ **Ans.**

***12–8.** If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2$ ft/s^2 to the left.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20 \text{ ft}$$ **Ans.**

•12–9. The acceleration of a particle traveling along a straight line is $a = k/v$, where k is a constant. If $s = 0$, $v = v_0$ when $t = 0$, determine the velocity of the particle as a function of time t.

Velocity:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{k/v}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{1}{k} v\,dv$$

$$t\Big|_0^t = \frac{1}{2k} v^2 \Big|_{v_0}^{v}$$

$$t = \frac{1}{2k}\left(v^2 - v_0^{\,2}\right)$$

$$v = \sqrt{2kt + v_0^{\,2}}$$ **Ans.**

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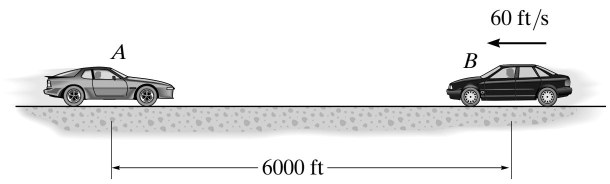
12–10. Car A starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 80 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, car B located 6000 ft down the road is traveling towards A at a constant speed of 60 ft/s. Determine the distance traveled by car A when they pass each other.



Distance Traveled: Time for car A to achives $v = 80$ ft/s can be obtained by applying Eq. 12–4.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$80 = 0 + 6t$$

$$t = 13.33 \text{ s}$$

The distance car A travels for this part of motion can be determined by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$80^2 = 0 + 2(6)(s_1 - 0)$$

$$s_1 = 533.33 \text{ ft}$$

For the second part of motion, car A travels with a constant velocity of $v = 80$ ft/s and the distance traveled in $t' = (t_1 - 13.33)$ s (t_1 is the total time) is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt' = 80(t_1 - 13.33)$$

Car B travels in the opposite direction with a constant velocity of $v = 60$ ft/s and the distance traveled in t_1 is

$$(\overset{+}{\rightarrow}) \qquad s_3 = vt_1 = 60t_1$$

It is required that

$$s_1 + s_2 + s_3 = 6000$$

$$533.33 + 80(t_1 - 13.33) + 60t_1 = 6000$$

$$t_1 = 46.67 \text{ s}$$

The distance traveled by car A is

$$s_A = s_1 + s_2 = 533.33 + 80(46.67 - 13.33) = 3200 \text{ ft} \qquad \textbf{Ans.}$$

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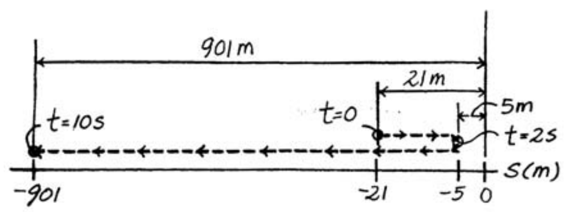
12–11. A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

$v = 12 - 3t^2$ **(1)**

$a = \dfrac{dv}{dt} = -6t\Big|_{t=4} = -24\ \text{m/s}^2$ **Ans.**

$\displaystyle\int_{-10}^{s} ds = \int_{1}^{t} v\,dt = \int_{1}^{t} \left(12 - 3t^2\right)dt$



$s + 10 = 12t - t^3 - 11$

$s = 12t - t^3 - 21$

$s|_{t=0} = -21$

$s|_{t=10} = -901$

$\Delta s = -901 - (-21) = -880$ m **Ans.**

From Eq. (1):

$v = 0$ when $t = 2s$

$s|_{t=2} = 12(2) - (2)^3 - 21 = -5$

$s_T = (21 - 5) + (901 - 5) = 912$ m **Ans.**

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***12–12.** A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{[1]}$$

At $v = 0$, from Eq.[1]

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{[2]}$$

At $t = 3.00$ s, from Eq. [2]

$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m} \qquad \textbf{Ans.}$$

•12–13. A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.

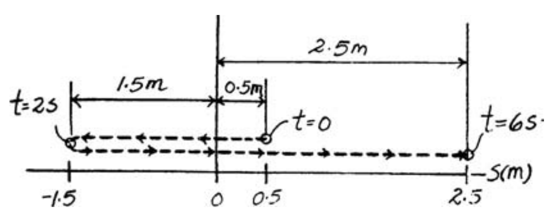
$\Delta s = (s_C - s_A) = 2$ m

$s_T = (0.5 + 1.5 + 1.5 + 2.5) = 6$ m

$t = (2 + 4) = 6$ s

$$v_{avg} = \frac{\Delta s}{t} = \frac{2}{6} = 0.333\ \text{m/s} \qquad \textbf{Ans.}$$

$$(v_{sp})_{avg} = \frac{s_T}{t} = \frac{6}{6} = 1\ \text{m/s} \qquad \textbf{Ans.}$$



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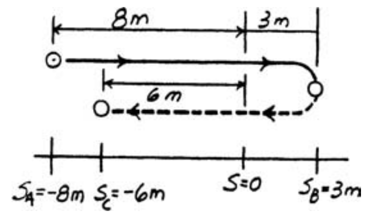
12–14. A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

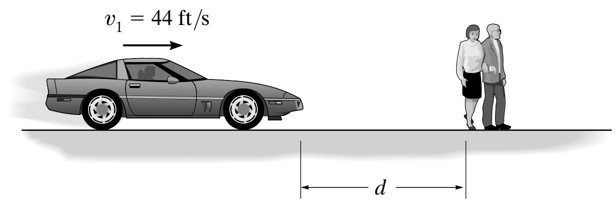
$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \to B} = 8 + 3 = 11.0$ m and $s_{B \to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{Tot} = s_{A \to B} + s_{B \to C} = 11.0 - 9.00 = 20.0$ m.

$$\left(v_{sp}\right)_{avg} = \frac{s_{Tot}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$



12–15. Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 132)$$

$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

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***12–16.** As a train accelerates uniformly it passes successive kilometer marks while traveling at velocities of 2 m/s and then 10 m/s. Determine the train's velocity when it passes the next kilometer mark and the time it takes to travel the 2-km distance.

Kinematics: For the first kilometer of the journey, $v_0 = 2$ m/s, $v = 10$ m/s, $s_0 = 0$, and $s = 1000$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$10^2 = 2^2 + 2a_c(1000 - 0)$$

$$a_c = 0.048 \text{ m/s}^2$$

For the second kilometer, $v_0 = 10$ m/s, $s_0 = 1000$ m, $s = 2000$ m, and 0.048 m/s^2. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 10^2 + 2(0.048)(2000 - 1000)$$

$$v = 14 \text{ m/s} \qquad \textbf{Ans.}$$

For the whole journey, $v_0 = 2$ m/s, $v = 14$ m/s, and 0.048 m/s^2. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$14 = 2 + 0.048t$$

$$t = 250 \text{ s} \qquad \textbf{Ans.}$$

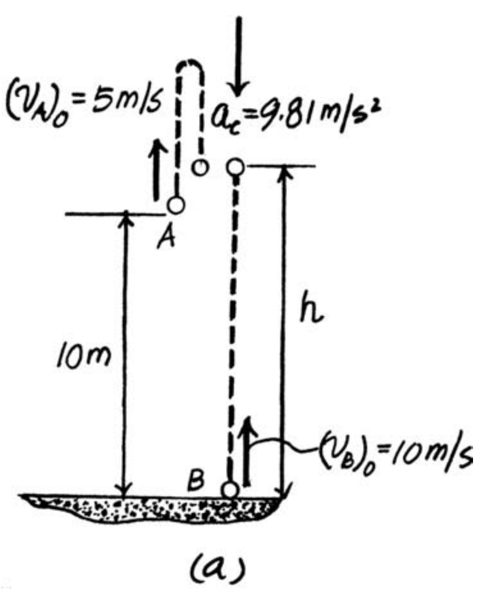
•12–17. A ball is thrown with an upward velocity of 5 m/s from the top of a 10-m high building. One second later another ball is thrown vertically from the ground with a velocity of 10 m/s. Determine the height from the ground where the two balls pass each other.

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = 5$ m/s, $(s_A)_0 = 0$, $s_A = (h - 10)$ m, $t_A = t'$, and $a_c = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2} a_c t_A^2$$

$$h - 10 = 0 + 5t' + \frac{1}{2}(-9.81)(t')^2$$

$$h = 5t' - 4.905(t')^2 + 10 \qquad \textbf{(1)}$$



Motion of ball B is with $(v_B)_0 = 10$ m/s, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - 1$ and $a_c = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2} a_c t_B^2$$

$$h = 0 + 10(t' - 1) + \frac{1}{2}(-9.81)(t' - 1)^2$$

$$h = 19.81t' - 4.905(t')^2 - 14.905 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$h = 4.54 \text{ m} \qquad \textbf{Ans.}$$

$$t' = 1.68 \text{ m}$$

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12–18. A car starts from rest and moves with a constant acceleration of 1.5 m/s^2 until it achieves a velocity of 25 m/s. It then travels with constant velocity for 60 seconds. Determine the average speed and the total distance traveled.

Kinematics: For stage (1) of the motion, $v_0 = 0$, $s_0 = 0$, $v = 25$ m/s, and $a_c = 1.5$ m/s^2.

$(\overset{+}{\rightarrow})$ $\quad v = v_0 + a_c t$

$$25 = 0 + 1.5t_1$$

$$t_1 = 16.67 \text{ s}$$

$(\overset{+}{\rightarrow})$ $\quad v^2 = {v_0}^2 + 2a_c(s - s_0)$

$$25^2 = 0 + 2(1.5)(s_1 - 0)$$

$$s_1 = 208.33 \text{ m}$$

For stage (2) of the motion, $s_0 = 108.22$ ft, $v_0 = 25$ ft/s, $t = 60$ s, and $a_c = 0$. Thus,

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s = 208.33 + 25(60) + 0$$

$$= 1708.33\text{ft} = 1708 \text{ m}$$ **Ans.**

The average speed of the car is then

$$v_{avg} = \frac{s}{t_1 + t_2} = \frac{1708.33}{16.67 + 60} = 22.3 \text{ m/s}$$ **Ans.**

12–19. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s^2, decelerate at 0.3 ft/s^2, and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.

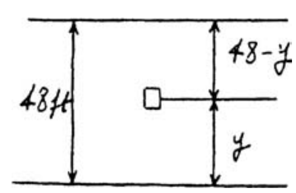
$+\uparrow \quad v^2 = v_0^2 + 2\, a_c\, (s - s_0)$

$v_{max}^2 = 0 + 2(0.6)(y - 0)$

$0 = v_{max}^2 + 2(-0.3)(48 - \text{y})$

$0 = 1.2\, y - 0.6(48 - y)$

$y = 16.0$ ft, $\qquad v_{max} = 4.382$ ft/s < 8 ft/s



$+\uparrow \quad v = v_0 + a_c\, t$

$4.382 = 0 + 0.6\, t_1$

$t_1 = 7.303$ s

$0 = 4.382 - 0.3\, t_2$

$t_2 = 14.61$ s

$t = t_1 + t_2 = 21.9$ s **Ans.**

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***12–20.** A particle is moving along a straight line such that its speed is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\,ds = \int_0^t -4\,dt$$

$$-s^{-1}\Big|_2^s = -4t\Big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4\left(\frac{2}{8t + 1}\right)^2 = \left(-\frac{16}{(8t + 1)^2}\right)\text{ m/s}$$ **Ans.**

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \left(\frac{256}{(8t + 1)^3}\right)\text{ m/s}^2$$ **Ans.**

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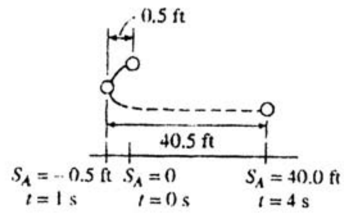
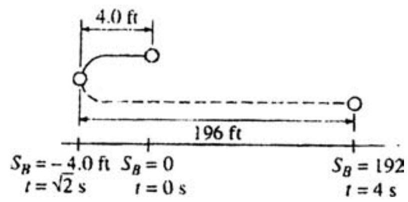
•12–21. Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)\ \text{ft/s}^2$ and $a_B = (12t^2 - 8)\ \text{ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.

$$dv_A = a_A dt$$
$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$
$$v_A = 3t^2 - 3t$$
$$dv_B = a_B dt$$
$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$
$$v_B = 4t^3 - 8t$$





The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determined using Eq. 12-1.

$$ds_A = v_A dt$$
$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$
$$s_A = t^3 - \frac{3}{2}t^2$$
$$ds_B = v_B dt$$
$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$
$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\,s} = 1^3 - \frac{3}{2}(1^2) = -0.500\ \text{ft}$$

$$s_A|_{t=4\,s} = 4^3 - \frac{3}{2}(4^2) = 40.0\ \text{ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0\ \text{ft} \qquad \textbf{Ans.}$$

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4\ \text{ft}$$
$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192\ \text{ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200\ \text{ft} \qquad \textbf{Ans.}$$

At $t = 4$ s the distance beween A and B is

$$\Delta s_{AB} = 192 - 40 = 152\ \text{ft} \qquad \textbf{Ans.}$$

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12–22. A particle moving along a straight line is subjected to a deceleration $a = (-2v^3)$ m/s^2, where v is in m/s. If it has a velocity $v = 8$ m/s and a position $s = 10$ m when $t = 0$, determine its velocity and position when $t = 4$ s.

Velocity: The velocity of the particle can be related to its position by applying Eq. 12–3.

$$ds = \frac{v\,dv}{a}$$

$$\int_{10\text{m}}^{s} ds = \int_{8\text{m/s}}^{v} -\frac{dv}{2v^2}$$

$$s - 10 = \frac{1}{2v} - \frac{1}{16}$$

$$v = \frac{8}{16s - 159} \qquad \textbf{[1]}$$

Position: The position of the particle can be related to the time by applying Eq. 12–1.

$$dt = \frac{ds}{v}$$

$$\int_{0}^{t} dt = \int_{10\text{m}}^{s} \frac{1}{8}(16s - 159)\,ds$$

$$8t = 8s^2 - 159s + 790$$

When $t = 4$ s,

$$8(4) = 8s^2 - 159s + 790$$

$$8s^2 - 159s + 758 = 0$$

Choose the root greater than 10 m $s = 11.94 \text{ m} = 11.9 \text{ m}$ **Ans.**

Substitute $s = 11.94$ m into Eq. [1] yields

$$v = \frac{8}{16(11.94) - 159} = 0.250 \text{ m/s} \qquad \textbf{Ans.}$$

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12–23. A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \text{m/s}^2$, where v is in meters per second. If $v = 20\ \text{m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^{v} \frac{dv}{v} = \int_0^t -2\,dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = (20e^{-2t})\text{m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = (-40e^{-2t})\text{m/s}^2 \qquad \textbf{Ans.}$$

$$\int_0^s ds = \int_0^t v\,dt = \int_0^t (20e^{-2t})dt$$

$$s = -10e^{-2t}\big|_0^t = -10(e^{-2t} - 1)$$

$$s = 10(1 - e^{-2t})\text{m} \qquad \textbf{Ans.}$$

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***12–24.** A particle starts from rest and travels along a straight line with an acceleration $a = (30 - 0.2v)\ \text{ft/s}^2$, where v is in ft/s. Determine the time when the velocity of the particle is $v = 30$ ft/s.

Velocity:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{30 - 0.2v}$$

$$t\Big|_0^t = -\frac{1}{0.2}\ln(30 - 0.2v)\Big|_0^v$$

$$t = 5\ln\frac{30}{30 - 0.2v}$$

$$t = 5\ln\frac{30}{30 - 0.2(50)} = 1.12\ \text{s} \qquad \textbf{Ans.}$$

•12–25. When a particle is projected vertically upwards with an initial velocity of v_0, it experiences an acceleration $a = -(g + kv^2)$, where g is the acceleration due to gravity, k is a constant and v is the velocity of the particle. Determine the maximum height reached by the particle.

Position:

$$(+\uparrow) \qquad ds = \frac{v\,dv}{a}$$

$$\int_0^s ds = \int_{v_0}^v -\frac{v dv}{g + kv^2}$$

$$s\Big|_0^s = -\left[\frac{1}{2k}\ln\left(g + kv^2\right)\right]\Big|_{v_0}^v$$

$$s = \frac{1}{2k}\ln\left(\frac{g + kv_0{}^2}{g + kv^2}\right)$$

The particle achieves its maximum height when $v = 0$. Thus,

$$h_{\text{max}} = \frac{1}{2k}\ln\left(\frac{g + kv_0{}^2}{g}\right)$$

$$= \frac{1}{2k}\ln\left(1 + \frac{k}{g}v_0{}^2\right) \qquad \textbf{Ans.}$$

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12–26. The acceleration of a particle traveling along a straight line is $a = (0.02e^t)\ \mathrm{m/s^2}$, where t is in seconds. If $v = 0$, $s = 0$ when $t = 0$, determine the velocity and acceleration of the particle at $s = 4$ m.

Velocity: $a = 0.02e^{5.329} = 4.13\ \mathrm{m/s^2}$ **Ans.**

$(\overset{+}{\rightarrow})$ $dv = a\,dt$

$$\int_0^v dv = \int_0^t 0.02e^t\,dt$$

$$v\Big|_0^v = 0.02e^t\Big|_0^t$$

$$v = \left[0.02\left(e^t - 1\right)\right]\ \mathrm{m/s} \qquad (1)$$

Position:

$(\overset{+}{\rightarrow})$ $ds = v\,dt$

$$\int_0^s ds = \int_0^t 0.02\left(e^t - 1\right)dt$$

$$s\Big|_0^s = 0.02\left(e^t - t\right)\Big|_0^t$$

$$s = 0.02\left(e^t - t - 1\right)\ \mathrm{m}$$

When $s = 4$ m,

$$4 = 0.02\left(e^t - t - 1\right)$$

$$e^t - t - 201 = 0$$

Solving the above equation by trial and error,

$$t = 5.329\ \mathrm{s}$$

Thus, the velocity and acceleration when $s = 4$ m ($t = 5.329$ s) are

$$v = 0.02\left(e^{5.329} - 1\right) = 4.11\ \mathrm{m/s}$$ **Ans.**

$$a = 0.02e^{5.329} = 4.13\ \mathrm{m/s^2}$$ **Ans.**

12–27. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})\ \mathrm{m/s^2}$, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$$v = 1.29\ \mathrm{m/s}$$ **Ans.**

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***12–28.** If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a very *high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{[1]}$$

a) When $t = 5$ s, then, from Eq. [1]

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\frac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. [1]

$$v_{\max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

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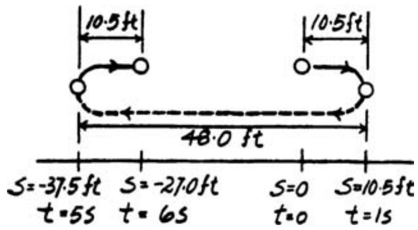
•12–29. The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \qquad \textbf{Ans.}$$

Total Distance Traveled: The velocity of the particle can be determined by applying Eq. 12–1.

$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$



The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0s} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1s} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5s} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{tot} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \qquad \textbf{Ans.}$$

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12–30. The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{v_0 - ks}$$

$$t\Big|_0^t = -\frac{1}{k}\ln(v_0 - ks)\Big|_0^s$$

$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - ks}\right)$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right) \qquad \textbf{Ans.}$$

Velocity:

$$v = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{v_0}{k}\left(1 - e^{-kt}\right)\right]$$

$$v = v_0 e^{-kt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-kt}\right)$$

$$a = -kv_0 e^{-kt} \qquad \textbf{Ans.}$$

12–31. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \mathrm{m/s^2}$, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

$$\int_2^v dv = \int_0^t (2\,t - 1)\,dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2\,t + 1$$

When $t = 6$ s,

$$v = 32\ \mathrm{m/s} \qquad \textbf{Ans.}$$

$$s = 67\ \mathrm{m} \qquad \textbf{Ans.}$$

Since $v \neq 0$ then

$$d = 67 - 1 = 66\ \mathrm{m} \qquad \textbf{Ans.}$$

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***12–32.** Ball *A* is thrown vertically upward from the top of a 30-m-high-building with an initial velocity of 5 m/s. At the same instant another ball *B* is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

Origin at roof:

Ball *A*:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball *B*:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$

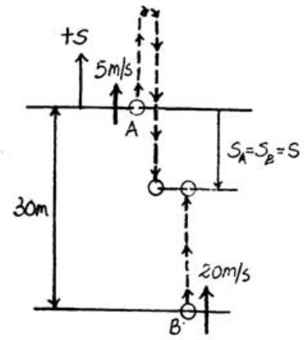
Solving,

$t = 2$ s **Ans.**

$s = 9.62$ m

Distance from ground,

$d = (30 - 9.62) = 20.4$ m **Ans.**



origin at roof

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

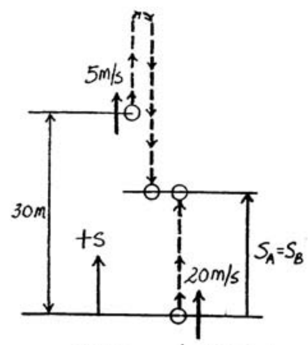
Require

$s_A = s_B$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$t = 2$ s **Ans.**

$s_B = 20.4$ m **Ans.**



origin at ground

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•12–33. A motorcycle starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 50 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, a car located 6000 ft down the road is traveling toward the motorcycle at a constant speed of 30 ft/s. Determine the time and the distance traveled by the motorcycle when they pass each other.

Motorcycle:

$(\xrightarrow{+})$ $\quad v = v_0 + a_c t'$

$50 = 0 + 6t'$

$t' = 8.33$ s

$v^2 = v_0^2 + 2a_c(s - s_0)$

$(50)^2 = 0 + 2(6)(s' - 0)$

$s' = 208.33$ ft

In $t' = 8.33$ s car travels

$s'' = v_0 t' = 30(8.33) = 250$ ft

Distance between motorcycle and car:

$6000 - 250 - 208.33 = 5541.67$ ft

When passing occurs for motorcycle,

$s = v_0 t; \qquad x = 50(t'')$

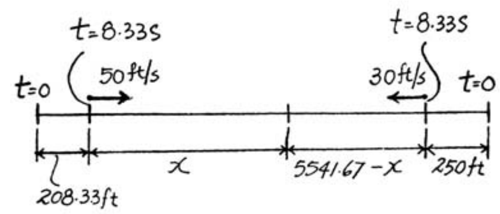
For car:

$s = v_0 t; \qquad 5541.67 - x = 30(t'')$

Solving,

$x = 3463.54$ ft

$t'' = 69.27$ s



Thus, for the motorcycle,

$t = 69.27 + 8.33 = 77.6$ s **Ans.**

$s_m = 208.33 + 3463.54 = 3.67(10)^3$ ft **Ans.**

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12–34. A particle moves along a straight line with a velocity $v = (200s)$ mm/s, where s is in millimeters. Determine the acceleration of the particle at $s = 2000$ mm. How long does the particle take to reach this position if $s = 500$ mm when $t = 0$?

Acceleration:

$$(\overset{+}{\rightarrow}) \qquad \frac{dv}{ds} = 200s$$

Thus, $a = v\dfrac{dv}{ds} = (200s)(200) = 40\left(10^3\right)s \text{ mm/s}^2$

When $s = 2000$ mm,

$$a = 40\left(10^3\right)(2000) = 80\left(10^6\right) \text{ mm/s}^2 = 80 \text{ km/s}^2 \qquad \textbf{Ans.}$$

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_{500\text{ mm}}^{s} \frac{ds}{200s}$$

$$t\Big|_0^t = \frac{1}{200}\ln s\Big|_{500\text{ mm}}^{s}$$

$$t = \frac{1}{200}\ln\frac{s}{500}$$

At $s = 2000$ mm,

$$t = \frac{1}{200}\ln\frac{2000}{500} = 6.93\left(10^{-3}\right)\text{ s} = 6.93 \text{ ms} \qquad \textbf{Ans.}$$

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■**12–35.** A particle has an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)$ m/s^2, where t is in seconds, determine its velocity, after it has traveled 10 m. How much time does this take?

Velocity:

$(\overset{+}{\rightarrow})$ $$dv = a\,dt$$

$$\int_{27}^{t} dv = \int_0^t (-6t)dt$$

$$v\Big|_{27}^{v} = \left(-3t^2\right)\Big|_0^t$$

$$v = (27 - 3t^2) \text{ m/s}$$

$(\overset{+}{\rightarrow})$ $$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right)dt$$

$$s\Big|_0^s = \left(27t - t^3\right)\Big|_0^t$$

$$s = (27t - t^3) \text{ m/s}$$

When $s = 100$ m,

$$t = 0.372 \text{ s} \qquad \textbf{Ans.}$$

$$v = 26.6 \text{ m/s} \qquad \textbf{Ans.}$$

***12–36.** The acceleration of a particle traveling along a straight line is $a = (8 - 2s)$ m/s^2, where s is in meters. If $v = 0$ at $s = 0$, determine the velocity of the particle at $s = 2$ m, and the position of the particle when the velocity is maximum.

Velocity:

$(\overset{+}{\rightarrow})$ $$v\,dv = a\,ds$$

$$\int_0^v v dv = \int_0^s (8 - 2s)\,ds$$

$$\frac{v^2}{2}\Big|_0^v = \left(8s - s^2\right)\Big|_0^s$$

$$v = \sqrt{16s - 2s^2} \text{ m/s}$$

At $s = 2$ m,

$$v\big|_{s=2\text{ m}} = \sqrt{16(2) - 2\left(2^2\right)} = \pm 4.90 \text{ m/s} \qquad \textbf{Ans.}$$

When the velocity is maximum $\frac{dv}{ds} = 0$. Thus,

$$\frac{dv}{ds} = \frac{16 - 4s}{2\sqrt{16s - 2s^2}} = 0$$

$$16 - 4s = 0$$

$$s = 4 \text{ m} \qquad \textbf{Ans.}$$

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•12–37. Ball A is thrown vertically upwards with a velocity of v_0. Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_{0/g}$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h$, $t_A = t'$, and $(a_c)_A = -g$.

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A{}^2$$

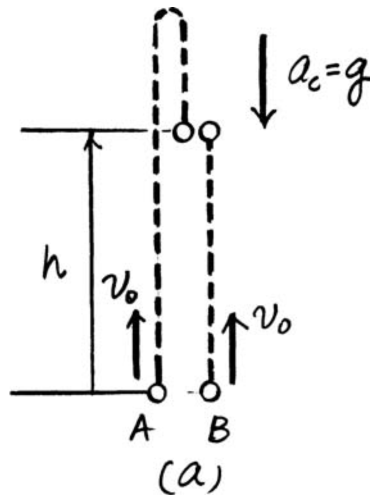
$$h = 0 + v_0 t' + \frac{1}{2}(-g)(t')^2$$

$$h = v_0 t' - \frac{g}{2}t'^2 \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad v_A = (v_A)_0 + (a_c)_A t_A$$

$$v_A = v_0 + (-g)(t')$$

$$v_A = v_0 - gt' \qquad \textbf{(2)}$$



The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B{}^2$$

$$h = 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2$$

$$h = v_0(t' - t) - \frac{g}{2}(t' - t)^2 \qquad \textbf{(3)}$$

$$(+\uparrow) \qquad v_B = (v_B)_0 + (a_c)_B t_B$$

$$v_B = v_0 + (-g)(t' - t)$$

$$v_B = v_0 - g(t' - t) \qquad \textbf{(4)}$$

Solving Eqs. (1) and (3),

$$v_0 t' - \frac{g}{2}t'^2 = v_0(t' - t) - \frac{g}{2}(t' - t)^2$$

$$t' = \frac{2v_0 + gt}{2g} \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$v_A = v_0 - g\left(\frac{2v_0 + gt}{2g}\right)$$

$$= -\frac{1}{2}gt = \frac{1}{2}gt\downarrow \qquad \textbf{Ans.}$$

$$v_B = v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right)$$

$$= \frac{1}{2}gt \uparrow \qquad \textbf{Ans.}$$

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12–38. As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\ \text{m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \to \infty$.

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\frac{v^2}{2}\bigg|_v^0 = \frac{g_0 R^2}{R + y}\bigg|_0^\infty$$

$$v = \sqrt{2g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\ \text{m/s} = 11.2\ \text{km/s}$$ **Ans.**

12–39. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–38), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–38.

From Prob. 12–38,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R + y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R + y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R + y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R + y} - \frac{1}{R + y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R + y)(R + y_0)}}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$$v = -3016\ \text{m/s} = 3.02\ \text{km/s}\ \downarrow$$ **Ans.**

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***12–40.** When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_{f/2}}{v_f - v_{f/2}}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

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•12–41. A particle is moving along a straight line such that its position from a fixed point is $s = (12 - 15t^2 + 5t^3)$ m, where t is in seconds. Determine the total distance traveled by the particle from $t = 1$ s to $t = 3$ s. Also, find the average speed of the particle during this time interval.

Velocity:

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = \frac{d}{dt}\left(12 - 15t^2 + 5t^3\right)$$

$$v = -30t + 15t^2 \text{ m/s}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = -30t + 15t^2 = 0$$

$$t(-30 + 15t) = 0$$

$$t = 0 \text{ and } 2 \text{ s}$$

Position: The positions of the particle at $t = 0$ s, 1 s, 2 s, and 3 s are

$$s\big|_{t=0\text{ s}} = 12 - 15\left(0^2\right) + 5\left(0^3\right) = 12 \text{ m}$$

$$s\big|_{t=1\text{ s}} = 12 - 15\left(1^2\right) + 5\left(1^3\right) = 2 \text{ m}$$

$$s\big|_{t=2\text{ s}} = 12 - 15\left(2^2\right) + 5\left(2^3\right) = -8 \text{ m}$$

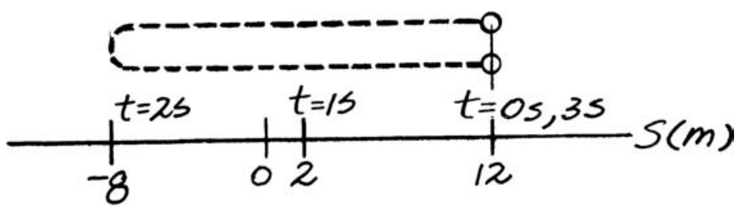
$$s\big|_{t=3\text{ s}} = 12 - 15\left(3^2\right) + 5\left(3^3\right) = 12 \text{ m}$$

Using the above results, the path of the particle is shown in Fig. *a*. From this figure, the distance traveled by the particle during the time interval $t = 1$ s to $t = 3$ s is

$$s_{\text{Tot}} = (2 + 8) + (8 + 12) = 30 \text{ m} \qquad \textbf{Ans.}$$

The average speed of the particle during the same time interval is

$$v_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{30}{3 - 1} = 15 \text{ m/s} \qquad \textbf{Ans.}$$



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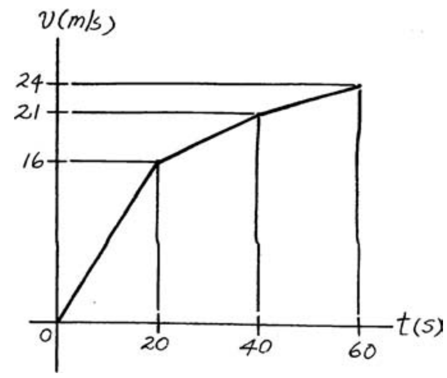
12–42. The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \quad \textbf{Ans.}$$



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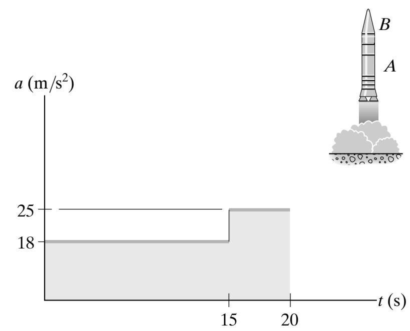
12–43. A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the two-stage motion of the missile for $0 \leq t \leq 20$ s.



Since $v = \int a\,dt$, the constant lines of the a–t graph become sloping lines for the v–t graph.

The numerical values for each point are calculated from the total area under the a–t graph to the point.

At $t = 15$ s, $\quad v = (18)(15) = 270$ m/s

At $t = 20$ s, $\quad v = 270 + (25)(20 - 15) = 395$ m/s

Since $s = \int v\,dt$, the sloping lines of the v–t graph become parabolic curves for the s–t graph.

The numerical values for each point are calculated from the total area under the v–t graph to the point.

At $t = 15$ s, $\quad s = \frac{1}{2}(15)(270) = 2025$ m

At $t = 20$ s, $\quad s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5 \text{ m} = 3.69$ km

Also:

$0 \leq t \leq 15$:

$$a = 18$$

$$v = v_0 + a_c t = 0 + 18t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 0 + 0 + 9t^2$$

At $t = 15$:

$$v = 18(15) = 270$$

$$s = 9(15)^2 = 2025$$

$15 \leq t \leq 20$:

$$a = 25$$

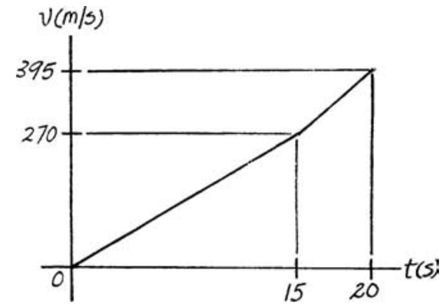
$$v = v_0 + a_c t = 270 + 25(t - 15)$$

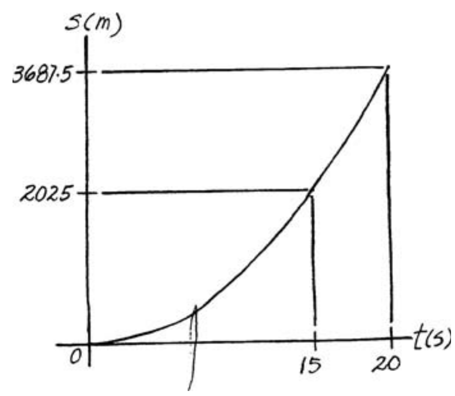
$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 2025 + 270(t - 15) + \frac{1}{2}(25)(t - 15)^2$$

When $t = 20$:

$$v = 395 \text{ m/s}$$

$$s = 3687.5 \text{ m} = 3.69 \text{ km}$$





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***12–44.** A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s². After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$

The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{[1]}$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. [1]). Thus, the distance for this part of motion is

$$(\xrightarrow{+}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{Tot} = 2000$, then

$$s_{Tot} = s_1 + s_2$$

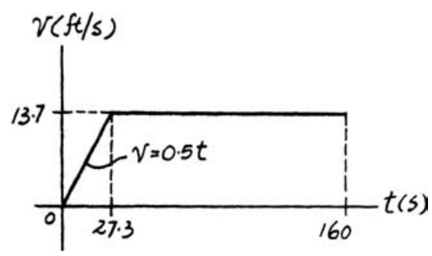
$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

Choose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

***v* – *t* Graph:** The equation for the velocity is given by Eq. [1]. When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.



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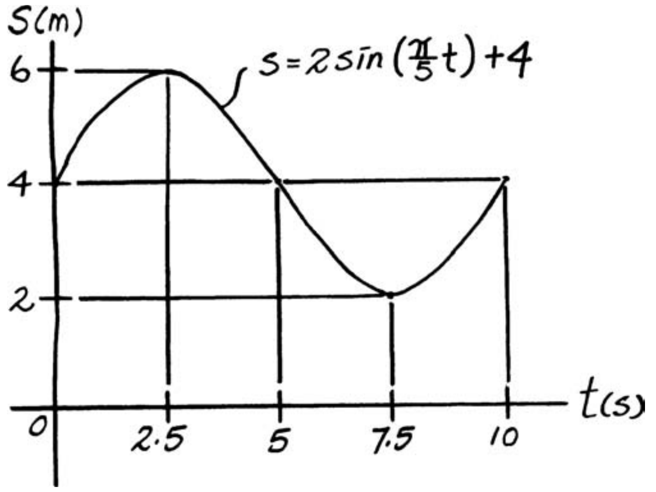
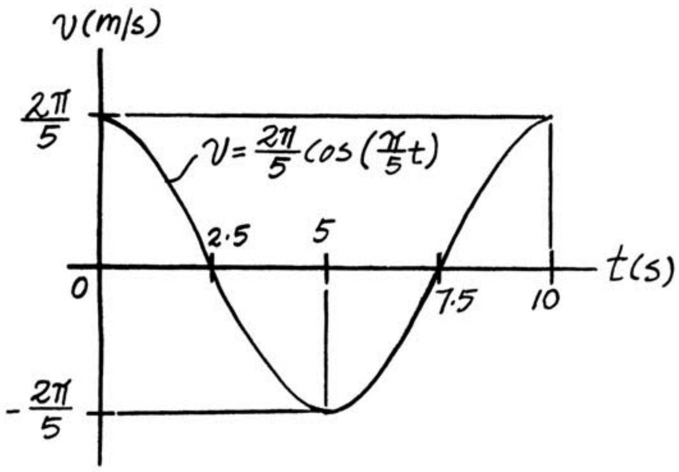
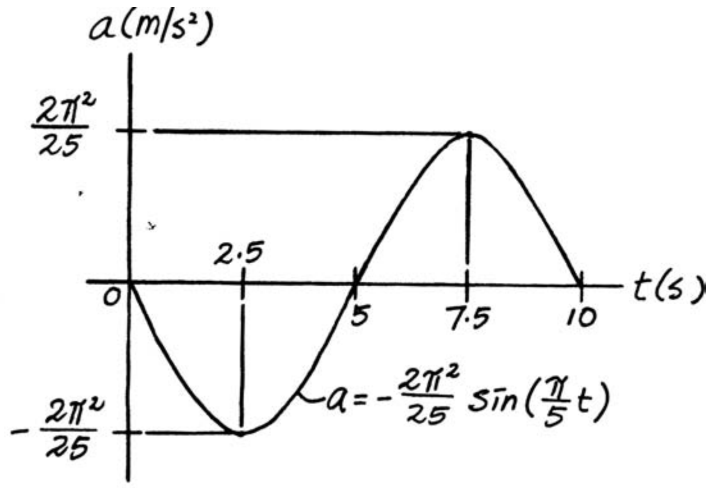
•**12–45.** If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \le t \le 10$ s.







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12–46. A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the v–t graph and determine the maximum speed of the train.

For stage (1) motion,

$(\xrightarrow{+})$ $$v_1 = v_0 + (a_c)_1\, t$$

$$v_{max} = 0 + (a_c)_1\, t_1$$

$$v_{max} = (a_c)_1 t_1 \qquad \textbf{(1)}$$

$(\xrightarrow{+})$ $$v_1{}^2 = v_0{}^2 + 2(a_c)_1(s_1 - s_0)$$

$$v_{max}{}^2 = 0 + 2(a_c)_1(1000 - 0)$$

$$(a_c)_1 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(2)}$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

$$t_1 = \frac{2000}{v_{max}} \qquad \textbf{(3)}$$

For stage (2) motion, the train travels with the constant velocity of v_{max} for $t = (t_2 - t_1)$. Thus,

$(\xrightarrow{+})$ $$s_2 = s_1 + v_1 t + \frac{1}{2}(a_c)_2\, t^2$$

$$1000 + 2000 = 1000 + v_{max}(t_2 - t_1) + 0$$

$$t_2 - t_1 = \frac{2000}{v_{max}} \qquad \textbf{(4)}$$

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$(\xrightarrow{+})$ $$v_3 = v_2 + (a_c)_3 t$$

$$0 = v_{max} - (a_c)_3(360 - t_2)$$

$$v_{max} = (a_c)_3(360 - t_2) \qquad \textbf{(5)}$$

$(\xrightarrow{+})$ $$v_3{}^2 = v_2{}^2 + 2(a_c)_3(s_3 - s_2)$$

$$0 = v_{max}{}^2 + 2\left[-(a_c)_3\right](4000 - 3000)$$

$$(a_c)_3 = \frac{v_{max}{}^2}{2000} \qquad \textbf{(6)}$$

Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

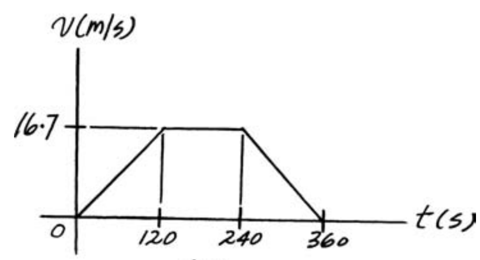
$$360 - t_2 = \frac{2000}{v_{max}} \qquad \textbf{(7)}$$

Solving Eqs. (3), (4), and (7), we have

$$t_1 = 120 \text{ s} \qquad t_2 = 240 \text{ s}$$

$$v_{max} = 16.7 \text{ m/s} \qquad \textbf{Ans.}$$

Based on the above results the $v-t$ graph is shown in Fig. a.



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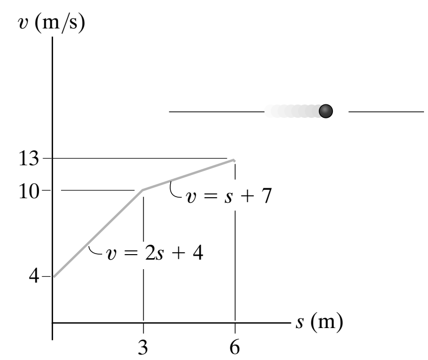
12–47. The particle travels along a straight line with the velocity described by the graph. Construct the $a-s$ graph.



***a – s* Graph:** For $0 \leq s < 3$ m,

$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = (2s + 4)(2) = (4s + 8)\ \text{m/s}^2$$

At $s = 0$ m and 3 m,

$$a|_{s=0\text{ m}} = 4(0) + 8 = 8\ \text{m/s}^2$$

$$a|_{s=3\text{ m}} = 4(3) + 8 = 20\ \text{m/s}^2$$

For $3\text{m} < s \leq 6$ m,

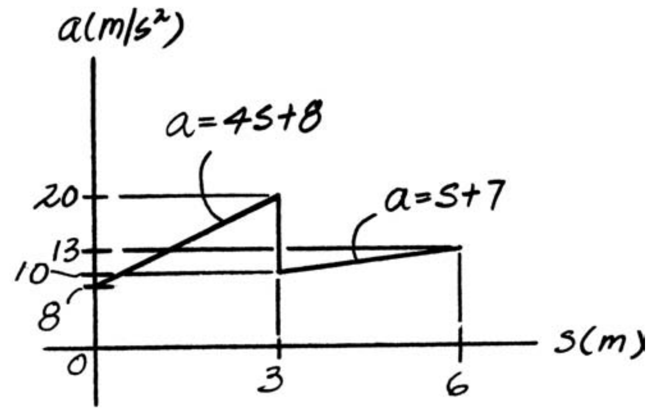
$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = (s + 7)(1) = (s + 7)\ \text{m/s}^2$$

At $s = 3$ m and 6 m,

$$a|_{s=3\text{ m}} = 3 + 7 = 10\ \text{m/s}^2$$

$$a|_{s=6\text{ m}} = 6 + 7 = 13\ \text{m/s}^2$$

The $a-s$ graph is shown in Fig. a.



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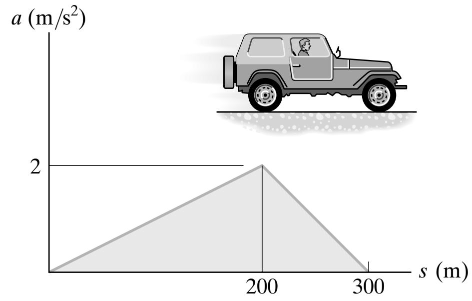
***12–48.** The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0, v = 0$.

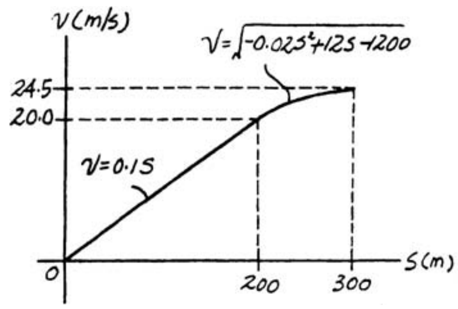


$a-s$ Graph: The function of acceleration a in terms of s for the interval $0\text{ m} \le s < 200\text{ m}$ is

$$\frac{a-0}{s-0} = \frac{2-0}{200-0} \qquad a = (0.01s)\ \text{m/s}^2$$

For the interval $200\text{ m} < s \le 300\text{ m}$,

$$\frac{a-2}{s-200} = \frac{0-2}{300-200} \qquad a = (-0.02s + 6)\ \text{m/s}^2$$



$v-s$ Graph: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval **$0\text{ m} \le s < 200\text{ m}$**,

$$vdv = ds$$

$$\int_0^v vdv = \int_0^s 0.01sds$$

$$v = (0.1s)\ \text{m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0\text{ m/s}$

For the interval **$200\text{ m} < s \le 300\text{ m}$**,

$$vdv = ads$$

$$\int_{20.0\text{m/s}}^{v} vdv = \int_{200m}^{s} (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right)\ \text{m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5\text{ m/s}$

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•12–49. A particle travels along a curve defined by the equation $s = (t^3 - 3t^2 + 2t)$ m. where t is in seconds. Draw the $s - t$, $v - t$, and $a - t$ graphs for the particle for $0 \leq t \leq 3$ s.

$s = t^3 - 3t^2 + 2t$

$v = \dfrac{ds}{dt} = 3t^2 - 6t + 2$

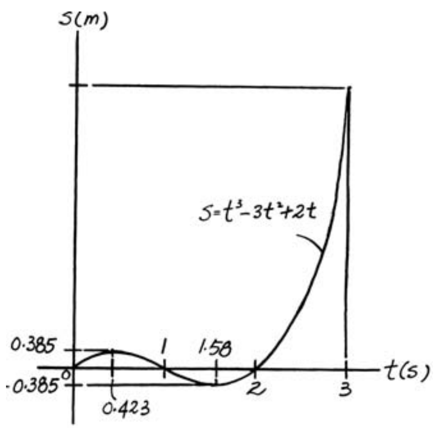
$a = \dfrac{dv}{dt} = 6t - 6$

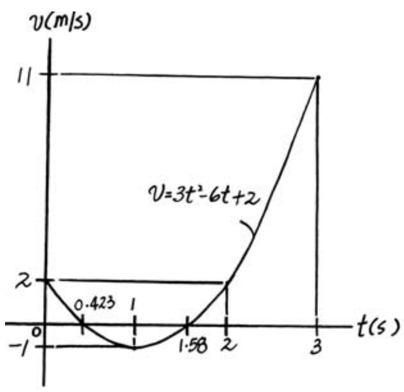
$v = 0$ at $0 = 3t^2 - 6t + 2$

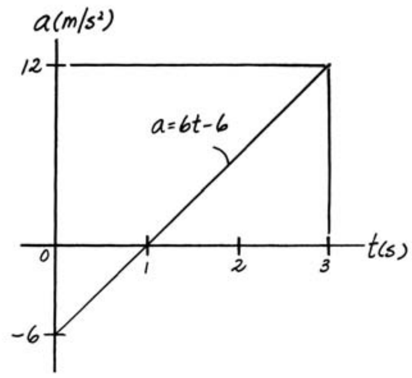
$t = 1.577$ s, and $t = 0.4226$ s,

$s|_{t=1.577} = -0.386$ m

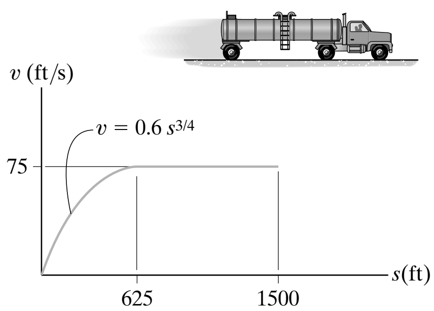
$s|_{t=0.4226} = 0.385$ m







12–50. A truck is traveling along the straight line with a velocity described by the graph. Construct the $a-s$ graph for $0 \leq s \leq 1500$ ft.



***a – s* Graph:** For $0 \leq s < 625$ ft,

$(\overset{+}{\rightarrow}) \qquad a = v\dfrac{dv}{ds} = \left(0.6s^{3/4}\right)\left[\dfrac{3}{4}(0.6)s^{-1/4}\right] = \left(0.27s^{1/2}\right)\text{ft}/\text{s}^2$

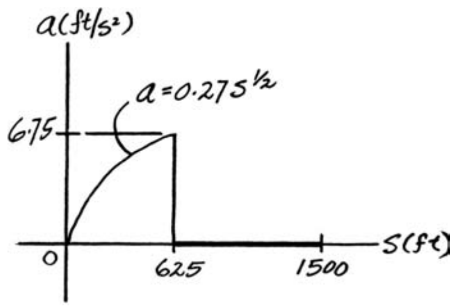
At $s = 625$ft,

$a|_{s=625\text{ ft}} = 0.27\left(625^{1/2}\right) = 6.75\text{ft/s}^2$

For $625\text{ ft} < s < 1500\text{ft}$,

$(\overset{+}{\rightarrow}) \qquad a = v\dfrac{dv}{ds} = 75(0) = 0$

The $a-s$ graph is shown in Fig. a.



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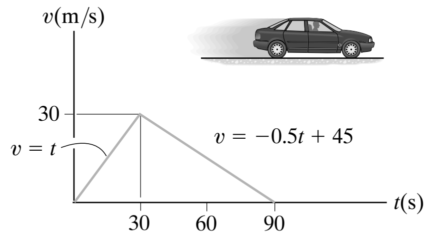
12–51. A car starts from rest and travels along a straight road with a velocity described by the graph. Determine the total distance traveled until the car stops. Construct the *s–t* and *a–t* graphs.



***s − t* Graph:** For the time interval $0 \le t < 30$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$(\xrightarrow{+}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t t\,dt$$

$$s = \left(\frac{t^2}{2}\right) \text{m}$$

When $t = 30$ s,

$$s = \frac{30^2}{2} = 450 \text{ m}$$

For the time interval $30 \text{ s} < t \le 90$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\xrightarrow{+}) \qquad ds = v\,dt$$

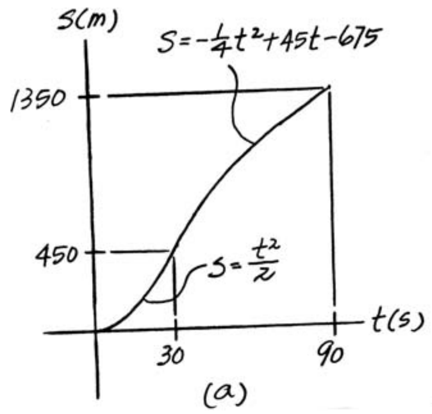
$$\int_{450 \text{ m}}^s ds = \int_{30\text{s}}^t (-0.5t + 45)dt$$

$$s = \left(-\frac{1}{4}t^2 + 45t - 675\right) \text{m}$$

When $t = 90$ s,

$$s\big|_{t=90\text{ s}} = -\frac{1}{4}\left(90^2\right) + 45(90) - 675 = 1350 \text{ m} \qquad \textbf{Ans.}$$

The $s-t$ graph shown is in Fig. *a*.



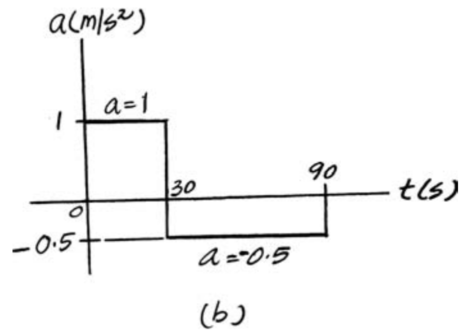
***a − t* Graph:** For the time interval $0 < t < 30$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(t) = 1 \text{ m/s}^2$$

For the time interval $30 \text{ s} < t \le 90$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(-0.5t + 45) = -0.5 \text{ m/s}^2$$

The $a-t$ graph is shown in Fig. *b*.



Note: Since the change in position of the car is equal to the area under the $v-t$ graph, the total distance traveled by the car is

$$\Delta s = \int v\,dt$$

$$s\big|_{t=90\text{ s}} - 0 = \frac{1}{2}(90)(30)$$

$$s\big|_{t=90\text{ s}} = 1350 \text{ s}$$

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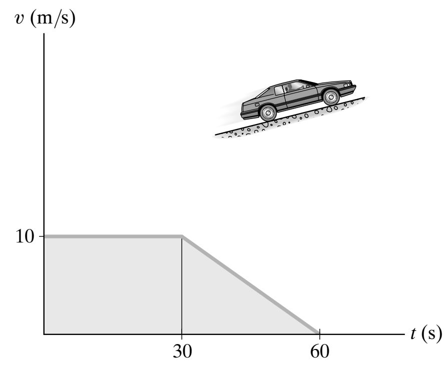
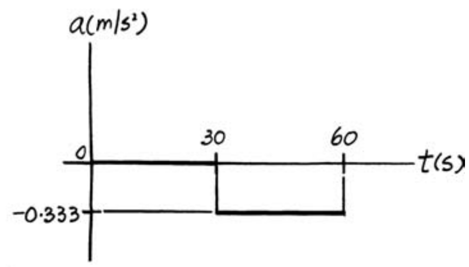
***12–52.** A car travels up a hill with the speed shown. Determine the total distance the car travels until it stops ($t = 60$ s). Plot the $a-t$ graph.

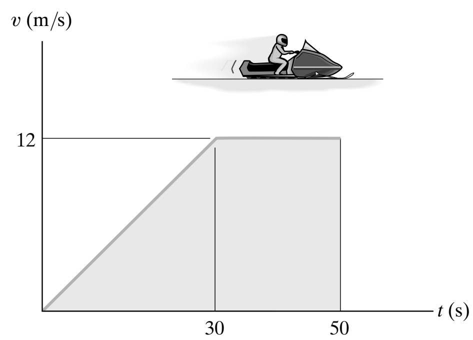


Distance traveled is area under $v-t$ graph.

$$s = (10)(30) + \frac{1}{2}(10)(30) = 450 \text{ m} \qquad \textbf{Ans.}$$



•12–53. The snowmobile moves along a straight course according to the v–t graph. Construct the s–t and a–t graphs for the same 50-s time interval. When $t = 0$, $s = 0$.



***s* – *t* Graph:** The position function in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $0 \text{ s} \le t < 30 \text{ s}$, $v = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s.

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{2}{5}t\,dt$$

$$s = \left(\frac{1}{5}t^2\right) \text{ m}$$

At $t = 30$ s, $\qquad s = \frac{1}{5}\left(30^2\right) = 180 \text{ m}$

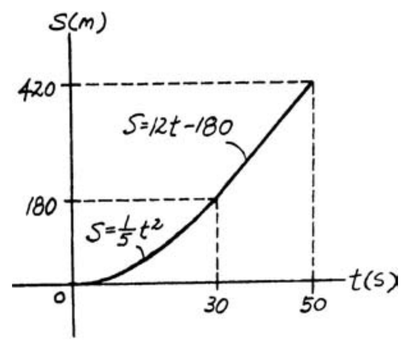
For time interval **30 s < *t* ≤ 50 s**,

$$ds = v\,dt$$

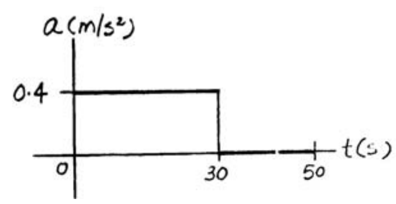
$$\int_{180\text{ m}}^{s} ds = \int_{30\text{ s}}^{t} 12\,dt$$

$$s = (12t - 180) \text{ m}$$

At $t = 50$ s, $\qquad s = 12(50) - 180 = 420 \text{ m}$



***a* – *t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval **0 s ≤ *t* < 30 s** and **30 s < *t* ≤ 50 s**, $a = \frac{dv}{dt} = \frac{2}{5} = 0.4 \text{ m/s}^2$ and $a = \frac{dv}{dt} = 0$, respectively.



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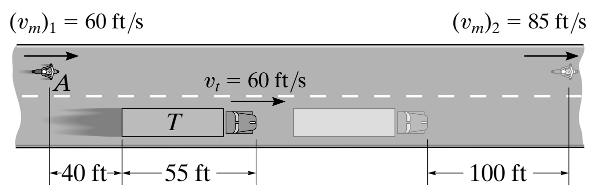
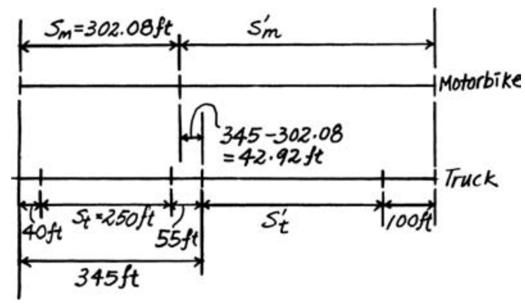
12–54. A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s^2 until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the $v-t$ and $s-t$ graphs for the motorcycle during this time.





Motorcycle:

Time to reach 85 ft/s,

$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

$$t = 4.167 \text{ s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Distance traveled,

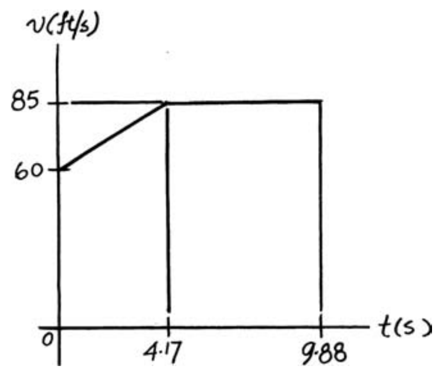
$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

$$s_m = 302.08 \text{ ft}$$

In $t = 4.167$ s, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$

Further distance for motorcycle to travel: $40 + 55 + 250 + 100 - 302.08 = 142.92$ ft



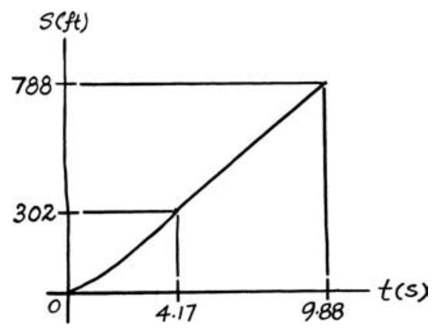
Motorcycle:

$$s = s_0 + v_0 t$$

$$(s + 142.92) = 0 + 85t'$$

Truck:

$$s = 0 + 60t'$$



Thus $t' = 5.717$ s

$$t = 4.167 + 5.717 = 9.88 \text{ s} \qquad \textbf{Ans.}$$

Total distance motorcycle travels

$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$

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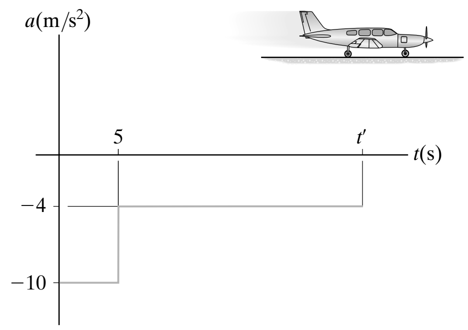
12–55. An airplane traveling at 70 m/s lands on a straight runway and has a deceleration described by the graph. Determine the time t' and the distance traveled for it to reach a speed of 5 m/s. Construct the v–t and s–t graphs for this time interval, $0 \le t \le t'$.



***v* − *t* Graph:** For the time interval $0 \le t < 5$ s, the initial condition is $v = 70$ m/s when $t = 0$ s.

$$\left(\xrightarrow{+}\right) \qquad dv = a\,dt$$

$$\int_{70\text{ m/s}}^{v} dv = \int_0^t -10\,dt$$

$$v = (-10t + 70)\text{ m/s}$$

When $t = 5$ s,

$$v\big|_{t=5\text{ s}} = -10(5) + 70 = 20\text{ m/s}$$

For the time interval $5\text{ s} < t \le t'$, the initial condition is $v = 20$ m/s when $t = 5$ s.

$$\left(\xrightarrow{+}\right) \qquad dv = a\,dt$$

$$\int_{20\text{ m/s}}^{v} dv = \int_{5\text{ s}}^{t} -4\,dt$$

$$v = (-4t + 40)\text{ m/s}$$

When $v = 5$ m/s,

$$5 = -4t' + 40 \qquad\qquad t' = 8.75\text{ s} \qquad\qquad \textbf{Ans.}$$

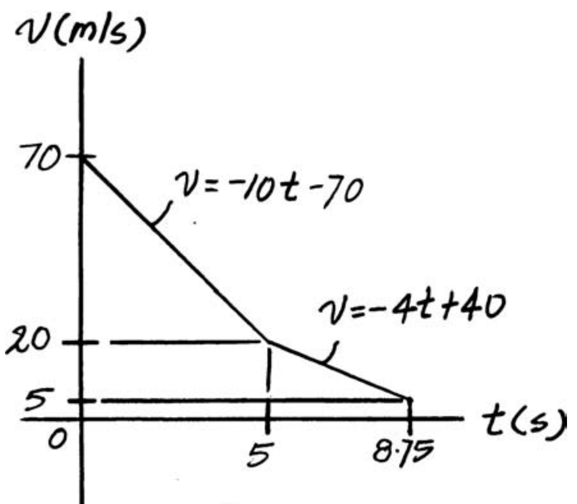
Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a\,dt$$

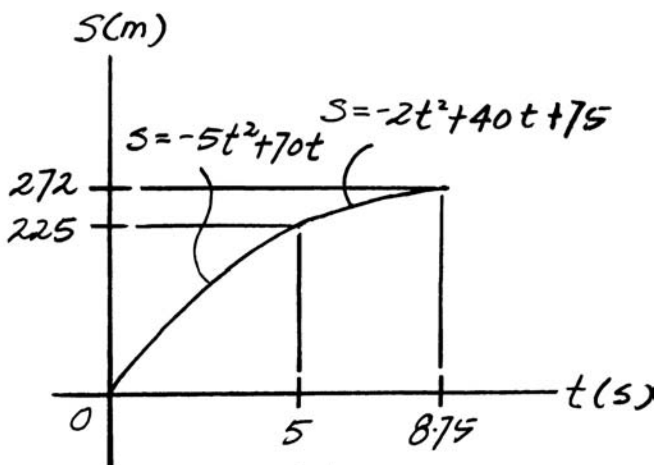
$$5 - 70 = -\left[5(10) + 4(t' - 5)\right]$$

$$t' = 8.75s'$$

The v–t graph is shown in Fig. *a*.



(a)



(b)

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12–57. Continued

***s* − *t* Graph:** For the time interval $0 \leq t < 5$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t (-10t + 70)dt$$

$$s = \left(-5t^2 + 70t\right)\text{m}$$

When $t = 5$ s,

$$s\big|_{t=5\text{ s}} = -5\left(5^2\right) + 70(5) = 225\text{ m}$$

For the time interval $5 < t \leq t' = 8.75$ s the initial condition is $s = 225$ m when $t = 5$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{225\text{ m}}^s ds = \int_5^t (-4t + 40)dt$$

$$s = \left(-2t^2 + 40t + 75\right)\text{m}$$

When $t = t' = 8.75$ s,

$$s\big|_{t=8.75\text{ s}} = -2\left(8.75^2\right) + 40(8.75) + 75 = 271.875\text{ m} = 272\text{ m} \qquad \textbf{Ans.}$$

Also, the change in position is equal to the area under the v–t graph. Referring to Fig. *a*, we have

$$\Delta s = \int v\,dt$$

$$s\big|_{t=8.75\text{ s}} - 0 = \frac{1}{2}(70 + 20)(5) + \frac{1}{2}(20 + 5)(3.75) = 271.875\text{ m} = 272\text{ m} \qquad \textbf{Ans.}$$

The s–t graph is shown in Fig. *b*.

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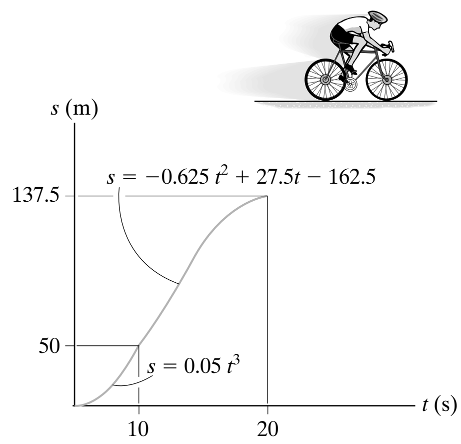
***12–56.** The position of a cyclist traveling along a straight road is described by the graph. Construct the v–t and a–t graphs.



***v* − *t* Graph:** For the time interval $0 \leq t < 10$ s,

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = \frac{d}{dt}\left(0.05t^3\right) = \left(0.15t^2\right)\text{ m/s}$$

When $t = 0$ s and 10 s,

$$v\big|_{t=0} = 0.15\left(0^2\right) = 0 \qquad\qquad v\big|_{t=10\text{ s}} = 0.15\left(10^2\right) = 15\text{ m/s}$$

For the time interval $10\text{ s} < t \leq 20$ s,

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = \frac{d}{dt}\left(-0.625t^2 + 27.5t - 162.5\right) = (-1.25t + 27.5)\text{ m/s}$$

When $t = 10$ s and 20 s,

$$v\big|_{t=10\text{ s}} = -1.25(10) + 27.5 = 15\text{ m/s}$$

$$v\big|_{t=20\text{ s}} = -1.25(20) + 27.5 = 2.5\text{ m/s}$$

The v–t graph is shown in Fig. *a*.

***a* − *t* Graph:** For the time interval $0 \leq t < 10$ s,

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(0.15t^2\right) = (0.3t)\text{ m/s}^2$$

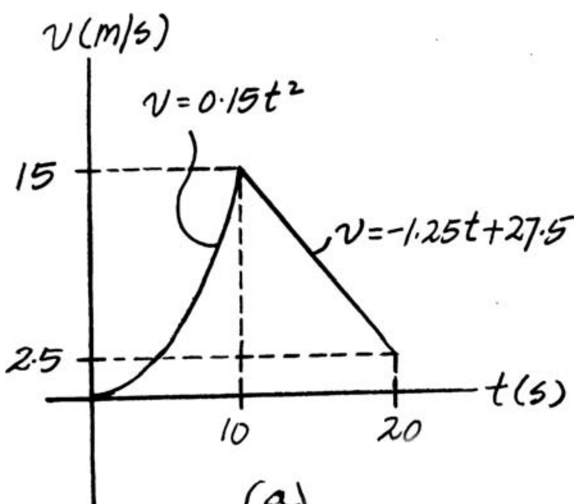
When $t = 0$ s and 10 s,

$$a\big|_{t=0\text{ s}} = 0.3(0) = 0 \qquad\qquad a\big|_{t=10\text{ s}} = 0.3(10) = 3\text{ m/s}^2$$

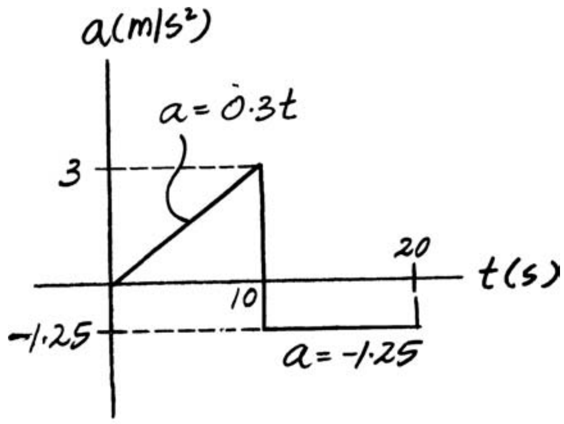
For the time interval $10\text{ s} < t \leq 20$ s,

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(-1.25t + 27.5) = -1.25\text{ m/s}^2$$

the a–t graph is shown in Fig. *b*.



(a)



(b)

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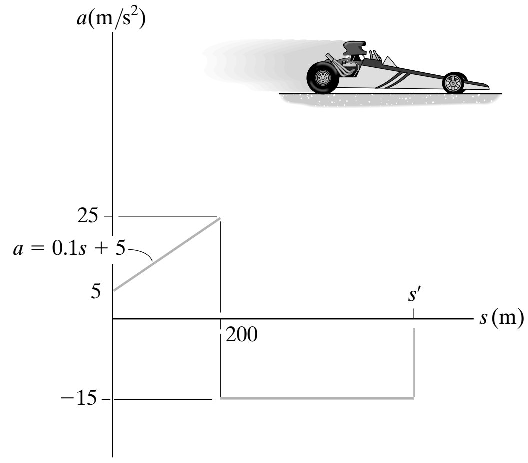
•12–57. The dragster starts from rest and travels along a straight track with an acceleration-deceleration described by the graph. Construct the $v-s$ graph for $0 \le s \le s'$, and determine the distance s' traveled before the dragster again comes to rest.



***v* − *s* Graph:** For $0 \le s < 200$ m, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s (0.1s + 5)ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left(0.05s^2 + 5s\right)\Big|_0^s$$

$$v = \left(\sqrt{0.1s^2 + 10s}\right) \text{ m/s}$$

At $s = 200$ m,

$$v\big|_{s=200\text{ m}} = \sqrt{0.1\left(200^2\right) + 10(200)} = 77.46 \text{ m/s} = 77.5 \text{ m/s}$$

For 200 m $< s \le s'$, the initial condition is $v = 77.46$ m/s at $s = 200$ m.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_{77.46\text{ m/s}}^v v\,dv = \int_{200\text{ m}}^s -15ds$$

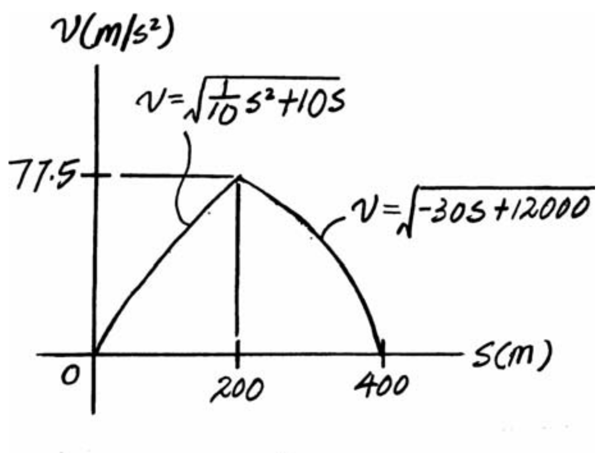
$$\left.\frac{v^2}{2}\right|_{77.46\text{ m/s}}^v = -15s\Big|_{200\text{ m}}^s$$

$$v = \left(\sqrt{-30s + 12000}\right) \text{ m/s}$$

When $v = 0$,

$$0 = \sqrt{-30s' + 12000} \qquad s' = 400 \text{ m} \qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. *a*.



(a)

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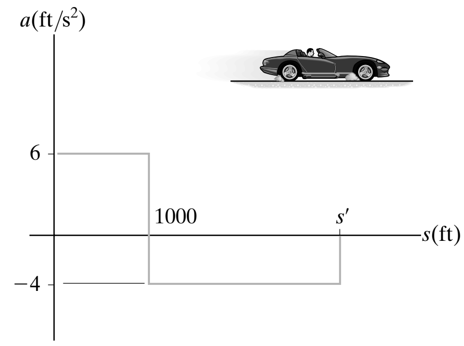
12–58. A sports car travels along a straight road with an acceleration-deceleration described by the graph. If the car starts from rest, determine the distance s' the car travels until it stops. Construct the $v-s$ graph for $0 \leq s \leq s'$.



$v-s$ Graph: For $0 \leq s < 1000$ ft, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 6\,ds$$

$$\frac{v^2}{2} = 6s$$

$$v = \left(\sqrt{12}s^{1/2}\right) \text{ ft/s}$$

When $s = 1000$ ft,

$$v = \sqrt{12}(1000)^{1/2} = 109.54 \text{ ft/s} = 110 \text{ ft/s}$$

For 1000 ft $< s \leq s'$, the initial condition is $v = 109.54$ ft/s at $s = 1000$ ft.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_{109.54 \text{ ft/s}}^{v} v\,dv = \int_{1000 \text{ ft}}^{s} -4\,ds$$

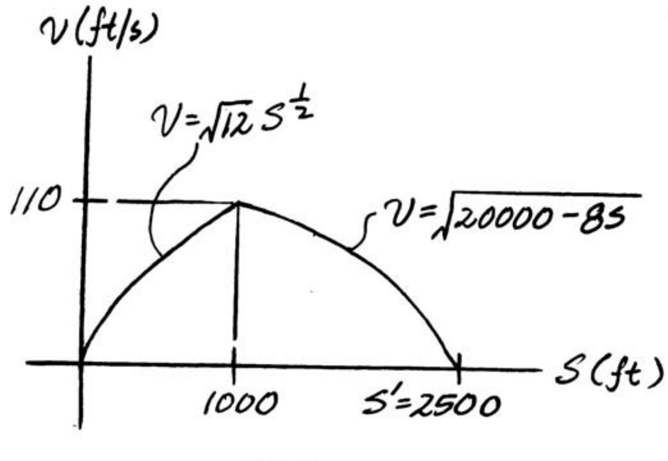
$$\left.\frac{v^2}{2}\right|_{109.54 \text{ ft/s}}^{v} = -4s\Big|_{1000 \text{ ft}}^{s}$$

$$v = \left(\sqrt{20\,000 - 8s}\right) \text{ ft/s}$$

When $v = 0$,

$$0 = \sqrt{20\,000 - 8s'} \qquad s' = 2500 \text{ ft} \qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. *a*.



(a)

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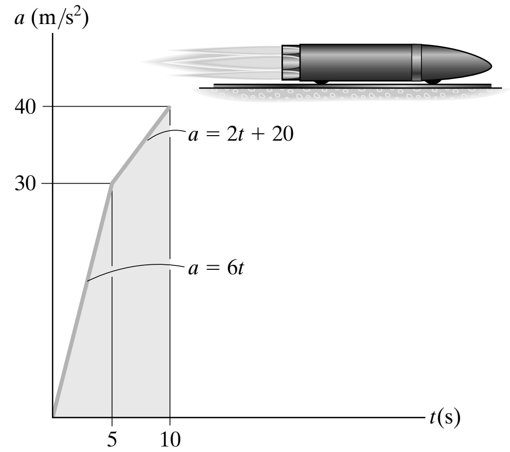
12–59. A missile starting from rest travels along a straight track and for 10 s has an acceleration as shown. Draw the $v-t$ graph that describes the motion and find the distance traveled in 10 s.



For $t \leq 5$ s,

$$a = 6t$$

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 6t\,dt$$

$$v = 3t^2$$

When $t = 5$ s,

$$v = 75 \text{ m/s}$$

For $5 < t < 10$ s,

$$a = 2t + 20$$

$$dv = a\,dt$$

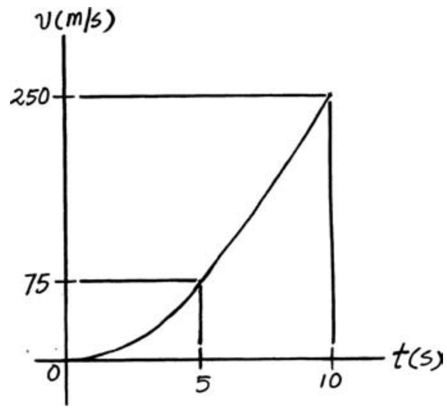
$$\int_{75}^v dv = \int_5^t (2t + 20)\,dt$$

$$v - 75 = t^2 + 20t - 125$$

$$v = t^2 + 20t - 50$$

When $t = 10$ s,

$$v = 250 \text{ m/s}$$



Distance at $t = 5$ s:

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^5 3t^2\,dt$$

$$s = (5)^3 = 125 \text{ m}$$

Distance at $t = 10$ s:

$$ds = v\,dv$$

$$\int_{125}^s ds = \int_5^{10} \left(t^2 + 20t - 50\right)dt$$

$$s - 125 = \frac{1}{3}t^3 + 10t^2 - 50t\Big]_5^{10}$$

$$s = 917 \text{ m} \qquad \textbf{Ans.}$$

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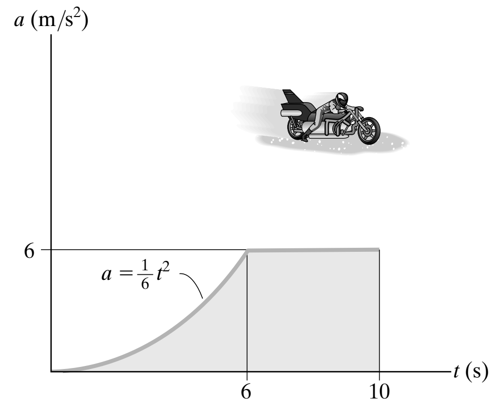
***12–60.** A motorcyclist starting from rest travels along a straight road and for 10 s has an acceleration as shown. Draw the v–t graph that describes the motion and find the distance traveled in 10 s.



For $0 \le t < 6$ $\quad dv = a\,dt$

$$\int_0^v dv = \int_0^t \frac{1}{6} t^2\,dt$$

$$v = \frac{1}{18} t^3$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{1}{18} t^3\,dt$$

$$s = \frac{1}{72} t^4$$

When $t = 6$ s, $\quad v = 12$ m/s $\quad s = 18$ m

For $6 < t \le 10$ $\quad dv = a\,dt$

$$\int_{12}^v dv = \int_6^t 6\,dt$$

$$v = 6t - 24$$

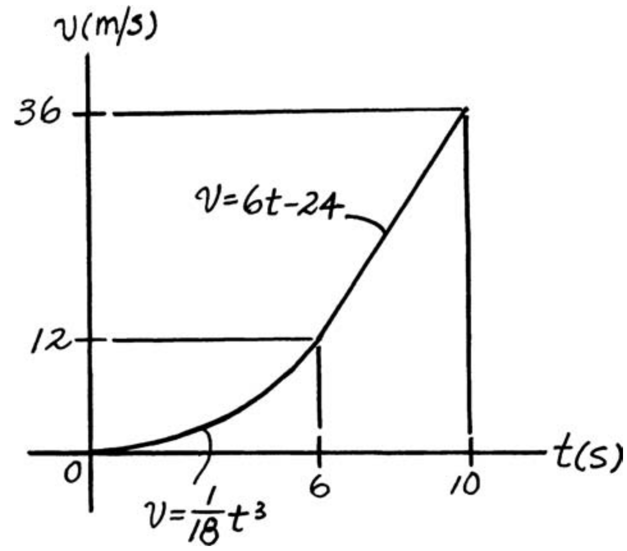
$$ds = v\,dt$$

$$\int_{18}^s ds = \int_6^t (6t - 24)\,dt$$

$$s = 3t^2 - 24t + 54$$

When $t = 10$ s, $\quad v = 36$ m/s

$s = 114$ m **Ans.**



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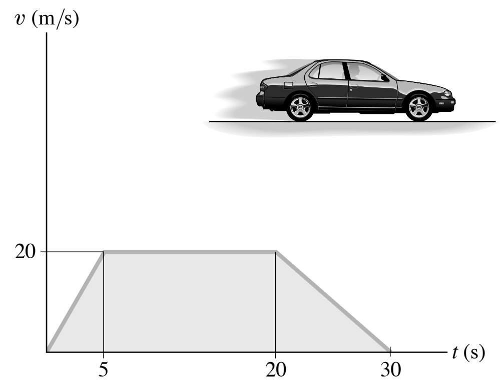
•12–61. The $v-t$ graph of a car while traveling along a road is shown. Draw the $s-t$ and $a-t$ graphs for the motion.



$0 \leq t \leq 5 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{5} = 4 \text{ m/s}^2$

$5 \leq t \leq 20 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20 - 20}{20 - 5} = 0 \text{ m/s}^2$

$20 \leq t \leq 30 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0 - 20}{30 - 20} = -2 \text{ m/s}^2$

From the v–t graph at $t_1 = 5$ s, $t_2 = 20$ s, and $t_3 = 30$ s,

$s_1 = A_1 = \dfrac{1}{2}(5)(20) = 50 \text{ m}$

$s_2 = A_1 + A_2 = 50 + 20(20 - 5) = 350 \text{ m}$

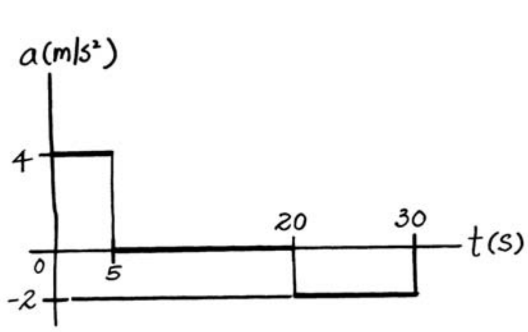
$s_3 = A_1 + A_2 + A_3 = 350 + \dfrac{1}{2}(30 - 20)(20) = 450 \text{ m}$

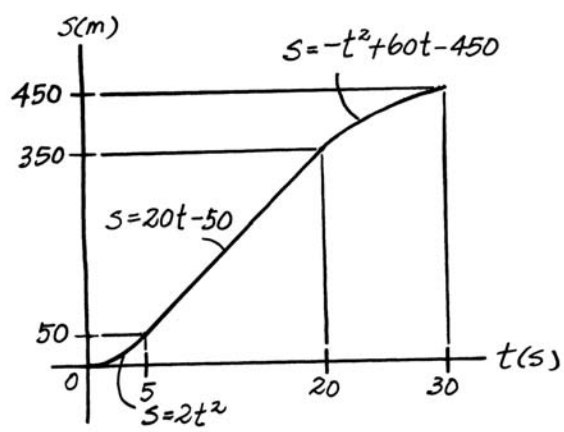
The equations defining the portions of the s–t graph are

$0 \leq t \leq 5 \text{ s} \qquad v = 4t; \qquad ds = v\,dt; \qquad \int_0^s ds = \int_0^t 4t\,dt; \qquad s = 2t^2$

$5 \leq t \leq 20 \text{ s} \qquad v = 20; \qquad ds = v\,dt; \qquad \int_{50}^s ds = \int_5^t 20\,dt; \qquad s = 20t - 50$

$20 \leq t \leq 30 \text{ s} \qquad v = 2(30 - t); \qquad ds = v\,dt; \qquad \int_{350}^s ds = \int_{20}^t 2(30 - t)\,dt; \qquad s = -t^2 + 60t - 450$





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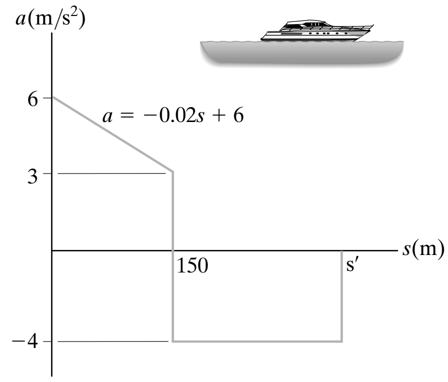
12–62. The boat travels in a straight line with the acceleration described by the $a-s$ graph. If it starts from rest, construct the $v-s$ graph and determine the boat's maximum speed. What distance s' does it travel before it stops?



***v – s* Graph:** For $0 \le s < 150$ m, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s (-0.02s + 6)\,ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left(-0.01s^2 + 6s\right)\Big|_0^s$$

$$v = \left(\sqrt{-0.02s^2 + 12s}\right) \text{ m/s}$$

The maximum velocity of the boat occurs at $s = 150$ m, where its acceleration changes sign. Thus,

$$v_{max} = v\big|_{s=150\text{ m}} = \sqrt{-0.02\left(150^2\right) + 12(150)} = 36.74 \text{ m/s} = 36.7 \text{ m/s} \qquad \textbf{Ans.}$$

For $150 \text{ m} < s < s'$, the initial condition is $v = 36.74$ m/s at $s = 150$ m.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_{36.74\text{ m/s}}^{v} v\,dv = \int_{150\text{ m}}^{s} -4\,ds$$

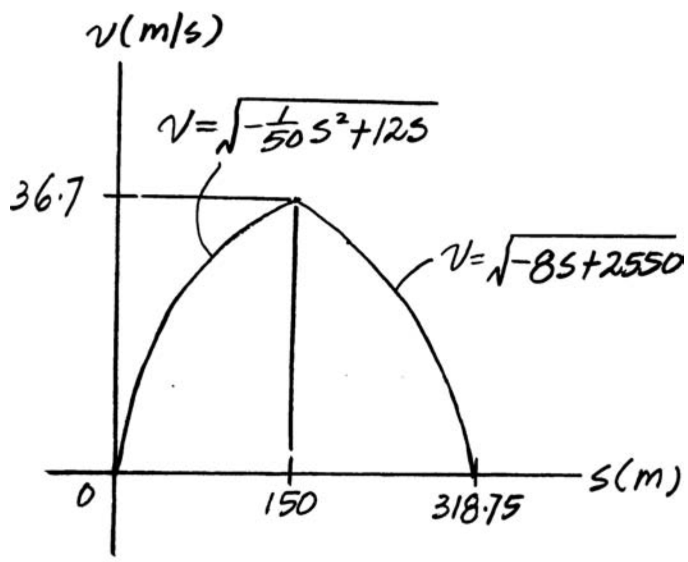
$$\left.\frac{v^2}{2}\right|_{36.74\text{ m/s}}^{v} = -4s\Big|_{150\text{ m}}^{s}$$

$$v = \sqrt{-8s + 2550} \text{ m/s}$$

Thus, when $v = 0$,

$$0 = \sqrt{-8s' + 2550} \qquad s' = 318.7 \text{ m} = 319 \text{ m} \qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. *a*.



(a)

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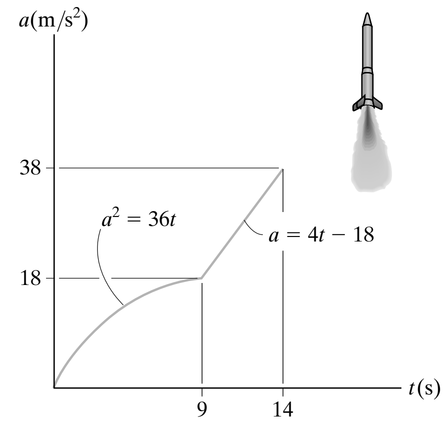
12–63. The rocket has an acceleration described by the graph. If it starts from rest, construct the $v-t$ and $s-t$ graphs for the motion for the time interval $0 \le t \le 14$ s.



$v-t$ Graph: For the time interval $0 \le t < 9$ s, the initial condition is $v = 0$ at $s = 0$.

$(+\uparrow)$ $$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 6t^{1/2}dt$$

$$v = \left(4t^{3/2}\right)\text{m/s}$$

When $t = 9$ s,

$$v\big|_{t=9\text{ s}} = 4\left(9^{3/2}\right) = 108 \text{ m/s}$$

The initial condition is $v = 108$ m/s at $t = 9$ s.

$(+\uparrow)$ $$dv = a\,dt$$

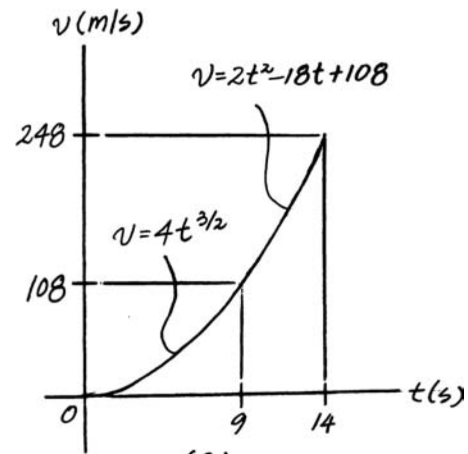
$$\int_{108\text{ m/s}}^v dv = \int_{9\text{ s}}^t (4t - 18)dt$$

$$v = \left(2t^2 - 18t + 108\right) \text{m/s}$$

When $t = 14$ s,

$$v\big|_{t=14\text{ s}} = 2\left(14^2\right) - 18(14) + 108 = 248 \text{ m/s}$$

The v–t graph is shown in Fig. *a*.



(a)

$s-t$ Graph: For the time interval $0 \le t < 9$ s, the initial condition is $s = 0$ when $t = 0$.

$(+\uparrow)$ $$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4t^{3/2}\,dt$$

$$s = \frac{8}{5}t^{5/2}$$

When $t = 9$ s,

$$s\big|_{t=9\text{ s}} = \frac{8}{5}\left(9^{5/2}\right) = 388.8 \text{ m}$$

For the time interval $9\text{ s} < t \le 14$ s, the initial condition is $s = 388.8$ m when $t = 9$ s.

$(+\uparrow)$ $$ds = v\,dt$$

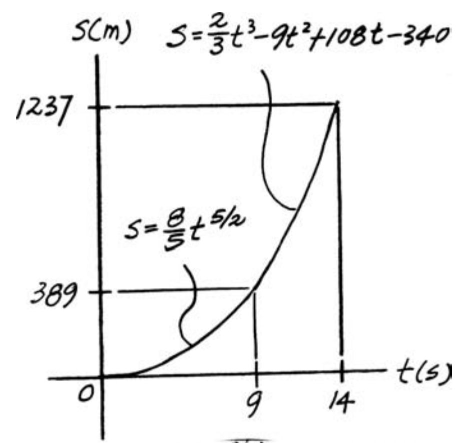
$$\int_{388.8\text{ m}}^s ds = \int_{9\text{ s}}^t \left(2t^2 - 18t + 108\right)dt$$

$$s = \left(\frac{2}{3}t^3 - 9t^2 + 108t - 340.2\right) \text{m}$$

When $t = 14$ s,

$$s\big|_{t=14\text{ s}} = \frac{2}{3}\left(14^3\right) - 9\left(14^2\right) + 108(14) - 340.2 = 1237 \text{ m}$$

The s–t graph is shown in Fig. *b*.



(b)

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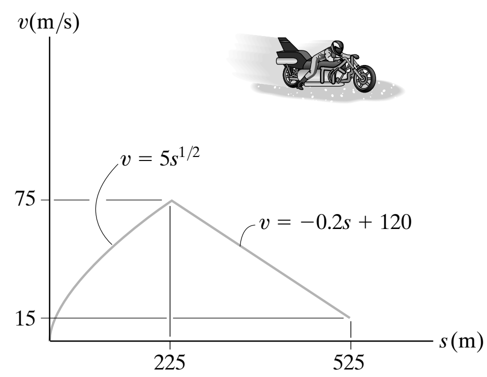
***12–64.** The jet bike is moving along a straight road with the speed described by the $v-s$ graph. Construct the $a-s$ graph.



***a* − *s* Graph:** For $0 \le s < 225$ m,

$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = \left(5s^{1/2}\right)\left(\frac{5}{2}s^{-1/2}\right) = 12.5 \text{ m/s}^2$$

For $225 \text{ m} < s \le 525$ m,

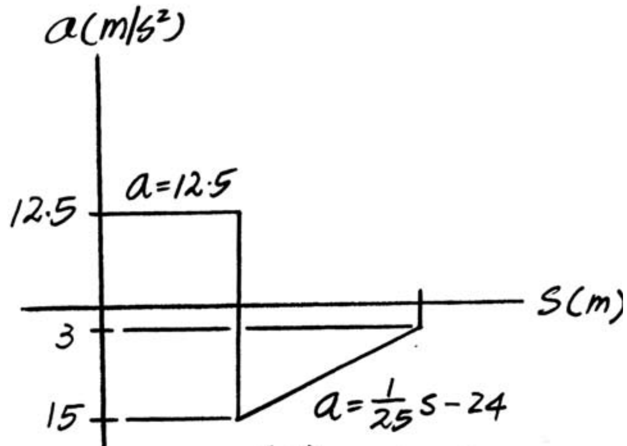
$$(\overset{+}{\rightarrow}) \qquad a = v\frac{dv}{ds} = (-0.2s + 120)(-0.2) = (0.04s - 24) \text{ m/s}^2$$

At $s = 225$ m and 525 m,

$$a\big|_{s=225\text{ m}} = 0.04(225) - 24 = -15 \text{ m/s}^2$$

$$a\big|_{s=525\text{ m}} = 0.04(525) - 24 = -3 \text{ m/s}^2$$

The a–s graph is shown in Fig. a.



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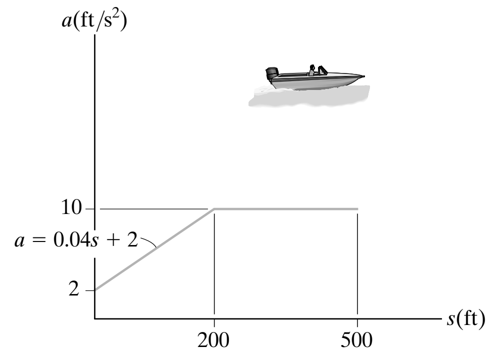
•12–65. The acceleration of the speed boat starting from rest is described by the graph. Construct the $v-s$ graph.



***v* − *s* Graph:** For $0 \leq s < 200$ ft, the initial condition is $v = 0$ at $s = 0$.

$(\overset{+}{\rightarrow})$ $$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s (0.04s + 2)ds$$

$$\left.\frac{v^2}{2}\right|_0^v = 0.02s^2 + 2s\Big|_0^s$$

$$v = \sqrt{0.04s^2 + 4s}\ \text{ft/s}$$

At $s = 200$ ft,

$$v\big|_{s=200\text{ ft}} = \sqrt{0.04\left(200^2\right) + 4(200)} = 48.99\ \text{ft/s} = 49.0\ \text{ft/s}$$

For 200 ft $< s \leq 500$ ft, the initial condition is $v = 48.99$ ft/s at $s = 200$ ft.

$(\overset{+}{\rightarrow})$ $$v\,dv = a\,ds$$

$$\int_{48.99\text{ ft/s}}^v v\,dv = \int_{200\text{ ft}}^s 10ds$$

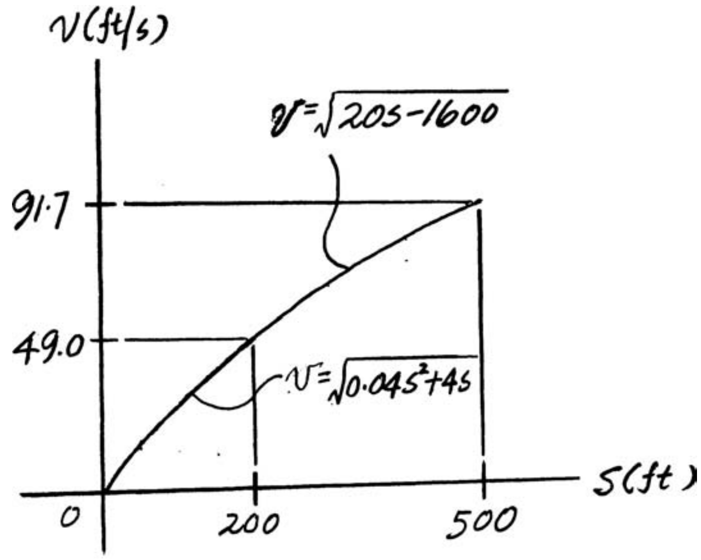
$$\left.\frac{v^2}{2}\right|_{48.99\text{ ft/s}}^v = 10s\Big|_{200\text{ ft}}^s$$

$$v = \sqrt{20s - 1600}\ \text{ft/s}$$

At $s = 500$ ft,

$$v\big|_{s=500\text{ ft}} = \sqrt{20(500) - 1600} = 91.65\ \text{ft/s} = 91.7\ \text{ft/s}$$

The v–s graph is shown in Fig. a.



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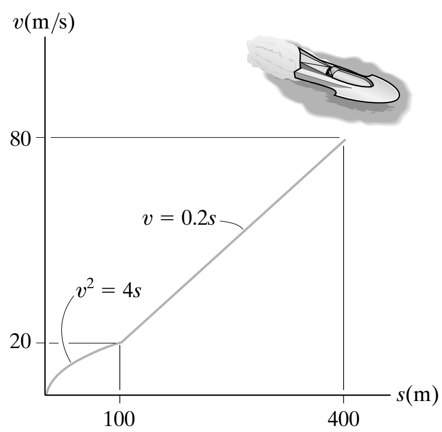
12–66. The boat travels along a straight line with the speed described by the graph. Construct the $s-t$ and $a-s$ graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



$s-t$ Graph: For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right) \text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10 \text{ s}$$

For $100 < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

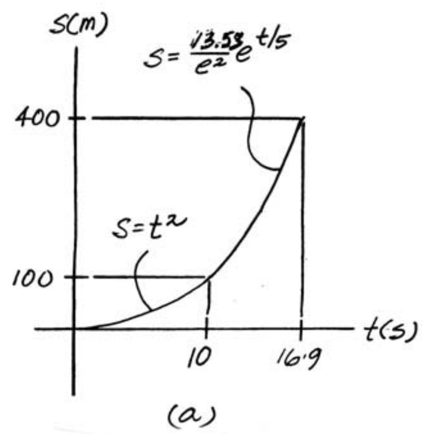
$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

$$s = \left(13.53e^{t/5}\right) \text{ m}$$

When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93 \text{ s} = 16.9 \text{ s} \qquad \textbf{Ans.}$$



The s–t graph is shown in Fig. *a*.

$a-s$ Graph: For $0 \text{ m} \leq s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2 \text{ m/s}^2$$

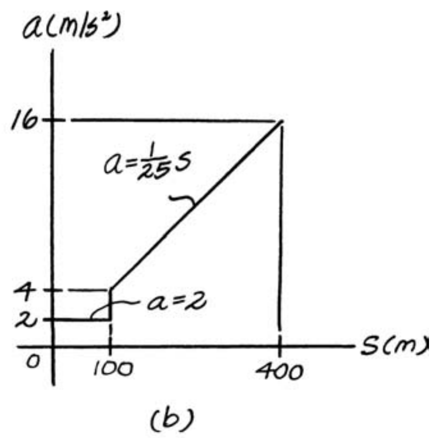
For $100 \text{ m} < s \leq 400$ m,

$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

When $s = 100$ m and 400 m,

$$a\big|_{s=100\text{ m}} = 0.04(100) = 4 \text{ m/s}^2$$

$$a\big|_{s=400\text{ m}} = 0.04(400) = 16 \text{ m/s}^2$$



The a–s graph is shown in Fig. *b*.

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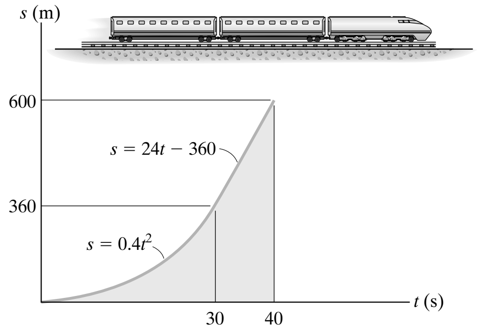
12–67. The s–t graph for a train has been determined experimentally. From the data, construct the v–t and a–t graphs for the motion.



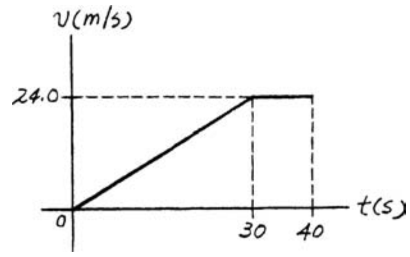
***v* – *t* Graph:** The velocity in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $0 \text{ s} \leq t \leq 30 \text{ s}$,

$$v = \frac{ds}{dt} = 0.8t$$

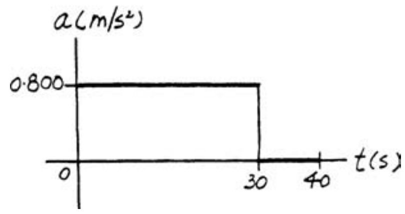
When $t = 30$ s, $\quad v = 0.8(30) = 24.0 \text{ m/s}$

For time interval $30 \text{ s} < t \leq 40 \text{ s}$,

$$v = \frac{ds}{dt} = 24.0 \text{ m/s}$$



***a* – *t* Graph:** The acceleration in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 30 \text{ s}$ and $30 \text{ s} < t \leq 40 \text{ s}$, $a = \frac{dv}{dt} = 0.800 \text{ m/s}^2$ and $a = \frac{dv}{dt} = 0$, respectively.



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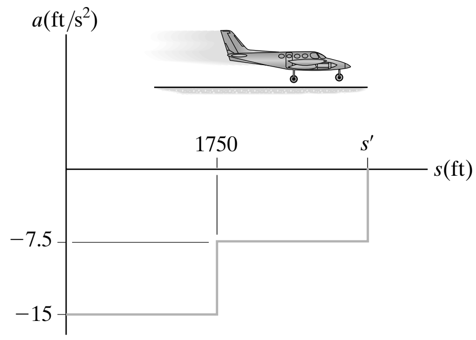
***12–68.** The airplane lands at 250 ft/s on a straight runway and has a deceleration described by the graph. Determine the distance s' traveled before its speed is decreased to 25 ft/s. Draw the $s-t$ graph.



$v-s$ Graph: For $0 \le s < 1750$ ft, the initial condition is $v = 250$ ft/s at $s = 0$ s.

$(\overset{+}{\rightarrow})$ $\quad v\,dv = a\,ds$

$$\int_{250\text{ ft/s}}^{v} v\,dv = \int_0^s -15ds$$

$$\left.\frac{v^2}{2}\right|_{250\text{ ft/s}}^{v} = -15s\Big|_0^s$$

$$v = \left(\sqrt{62\,500 - 30s}\right)\text{ ft/s}$$

At $s = 1750$ ft,

$$v\big|_{s=1750\text{ ft}} = \sqrt{62\,500 - 30(1750)} = 100\text{ ft/s}$$

For $1750\text{ ft} < s < s'$, the initial condition is $v = 100$ ft/s at $s = 1750$ ft.

$(\overset{+}{\rightarrow})$ $\quad v\,dv = a\,ds$

$$\int_{100\text{ ft/s}}^{v} v\,dv = \int_{1750\text{ ft}}^{s} -7.5ds$$

$$\left.\frac{v^2}{2}\right|_{100\text{ ft/s}}^{v} = (-7.5s)\Big|_{1750\text{ ft}}^{s}$$

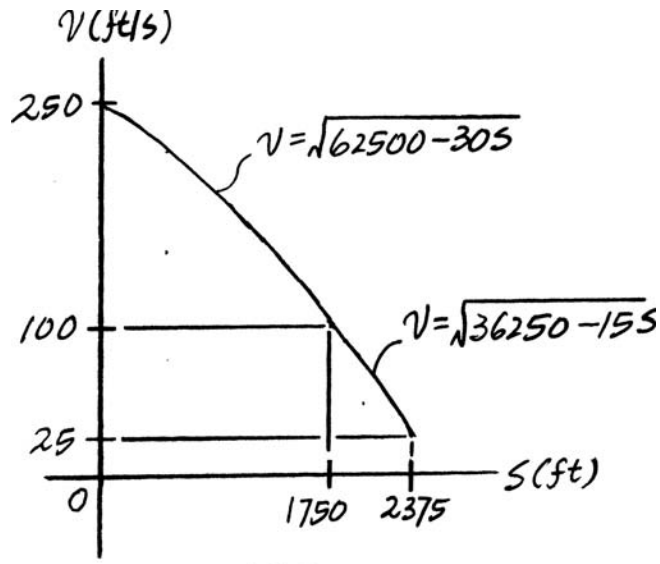
$$v = \sqrt{36\,250 - 15s}$$

When $v = 25$ ft/s

$$25 = \sqrt{36\,250 - 15s}$$

$$s' = 2375\text{ ft} \qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. *a*.



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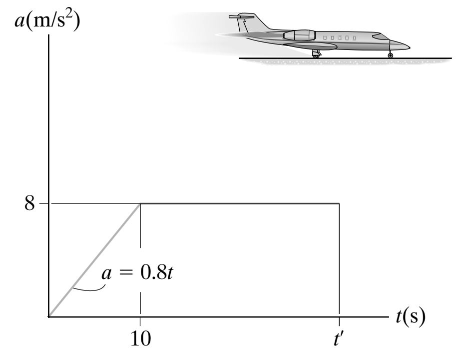
•12–69. The airplane travels along a straight runway with an acceleration described by the graph. If it starts from rest and requires a velocity of 90 m/s to take off, determine the minimum length of runway required and the time t' for take off. Construct the $v-t$ and $s-t$ graphs.



***v* − *t* graph:** For the time interval $0 \leq t < 10$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.8t\,dt$$

$$v = \left(0.4t^2\right) \text{ m/s}$$

When $t = 10$ s,

$$v = 0.4\left(10^2\right) = 40 \text{ m/s}$$

For the time interval $10 \text{ s} < t \leq t'$, the initial condition is $v = 40$ m/s when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_{40 \text{ m/s}}^v dv = \int_{10 \text{ s}}^t 8\,dt$$

$$v\Big|_{40 \text{ m/s}}^{v} = 8t\Big|_{10 \text{ s}}^{t}$$

$$v = (8t - 40) \text{ m/s}$$

Thus, when $v = 90$ m/s,

$$90 = 8t' - 40 \qquad\qquad t' = 16.25 \text{ s} \qquad \textbf{Ans.}$$

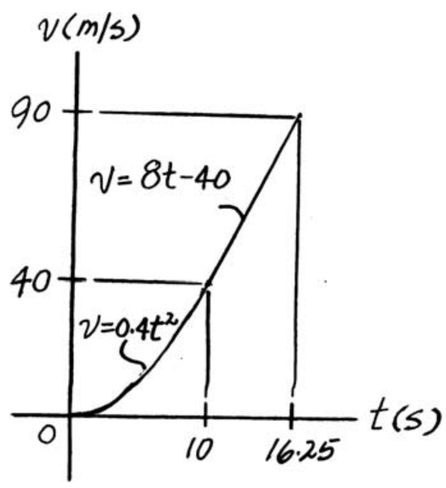
Also, the change in velocity is equal to the area under the $a-t$ graph. Thus,

$$\Delta v = \int a\,dt$$

$$90 - 0 = \frac{1}{2}(8)(10) + 8(t' - 10)$$

$$t' = 16.25 \text{ s} \qquad \textbf{Ans.}$$

The v–t graph is shown in Fig. *a*.



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***s* − *t* Graph:** For the time interval $0 \le t < 10$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2 dt$$

$$s = \left(0.1333t^3\right) \text{ m}$$

When $t = 10$ s,

$$s\big|_{t=10\text{ s}} = 0.1333\left(10^3\right) = 133.33\text{m}$$

For the time interval $10\text{ s} < t \le t' = 16.25$ s, the initial condition is $s = 133.33$m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{133.33\text{ m}}^s ds = \int_{10\text{s}}^t (8t - 40)dt$$

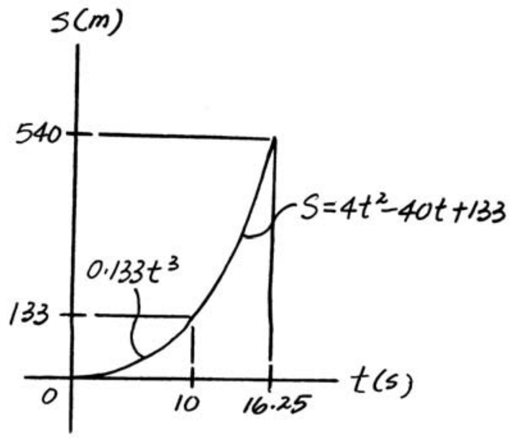
$$s\Big|_{133.33\text{ m}}^s = \left(4t^2 - 40t\right)\Big|_{10\text{ s}}^t$$

$$s = \left(4t^2 - 40t + 133.33\right)\text{m}$$

When $t = t' = 16.25$ s

$$s\big|_{t=16.25\text{ s}} = 4(16.25)^2 - 40(16.25) + 133.33 = 539.58\text{ m} = 540\text{ m} \quad \textbf{Ans.}$$

The *s*–*t* graph is shown in Fig. *b*.



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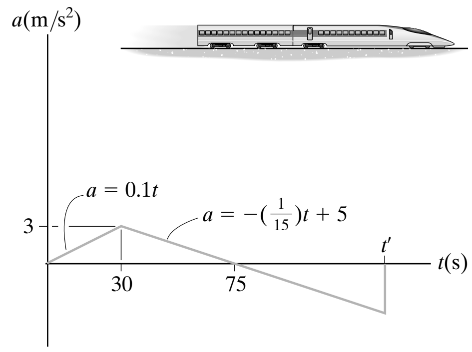
12–70. The a–t graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the v–t and s–t graphs.



$v - t$ Graph: For the time interval $0 \leq t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$\left(\xrightarrow{+}\right) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.1t\,dt$$

$$v = \left(0.05t^2\right) \text{ m/s}$$

When $t = 30$ s,

$$v\big|_{t=30\text{ s}} = 0.05\left(30^2\right) = 45 \text{ m/s}$$

For the time interval $30\text{ s} < t \leq t'$, the initial condition is $v = 45$ m/s at $t = 30$ s.

$$\left(\xrightarrow{+}\right) \qquad dv = a\,dt$$

$$\int_{45\text{ m/s}}^{v} dv = \int_{30\text{ s}}^{t} \left(-\frac{1}{15}t + 5\right)dt$$

$$v = \left(-\frac{1}{30}t^2 + 5t - 75\right) \text{ m/s}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09 \text{ s} = 133 \text{ s} \qquad \textbf{Ans.}$$

Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a\,dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

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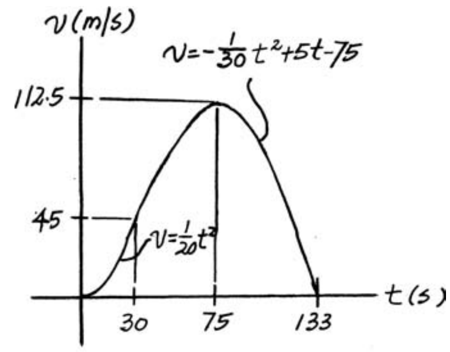
This equation is the same as the one obtained previously.

The slope of the v–t graph is zero when $t = 75$ s, which is the instant $a = \dfrac{dv}{dt} = 0$. Thus,

$$v\big|_{t=75\text{ s}} = -\frac{1}{30}\left(75^2\right) + 5(75) - 75 = 112.5\text{ m/s}$$

The v–t graph is shown in Fig. *a*.



(a)

***s* − *t* Graph:** Using the result of v, the equation of the s–t graph can be obtained by integrating the kinematic equation $ds = v\,dt$. For the time interval $0 \le t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.05t^2\,dt$$

$$s = \left(\frac{1}{60}t^3\right)\text{ m}$$

When $t = 30$ s,

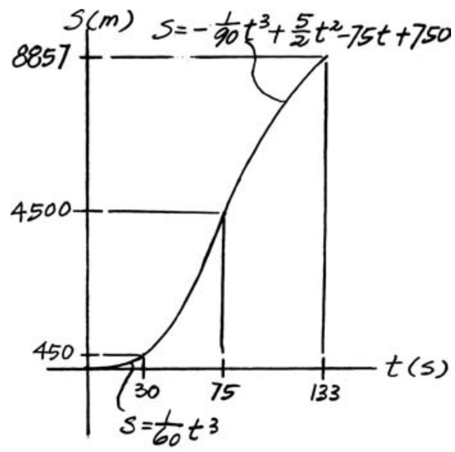
$$s\big|_{t=30\text{ s}} = \frac{1}{60}\left(30^3\right) = 450\text{ m}$$

For the time interval $30\text{ s} < t \le t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right)dt$$

$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right)\text{ m}$$



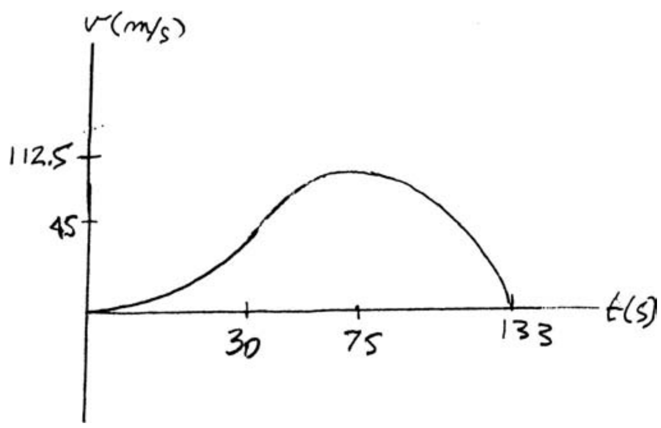
(b)

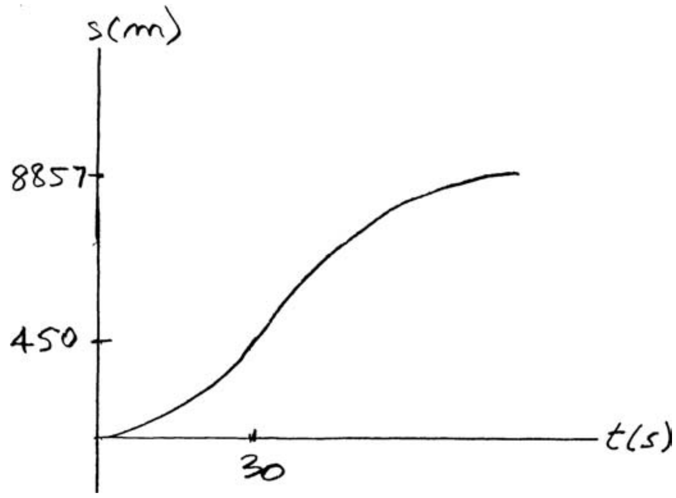
When $t = 75$ s and $t' = 133.09$ s,

$$s\big|_{t=75\text{ s}} = -\frac{1}{90}\left(75^3\right) + \frac{5}{2}\left(75^2\right) - 75(75) + 750 = 4500\text{ m}$$

$$s\big|_{t=133.09\text{ s}} = -\frac{1}{90}\left(133.09^3\right) + \frac{5}{2}\left(133.09^2\right) - 75(133.09) + 750 = 8857\text{ m} \qquad \textbf{Ans.}$$

The s–t graph is shown in Fig. *b*.





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12–71. The position of a particle is $\mathbf{r} = \{(3t^3 - 2t)\mathbf{i} - (4t^{1/2} + t)\mathbf{j} + (3t^2 - 2)\mathbf{k}\}$ m, where t is in seconds. Determine the magnitude of the particle's velocity and acceleration when $t = 2$ s.

Velocity:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}\left[\left(3t^3 - 2t\right)\mathbf{i} - \left(4t^{1/2} + t\right)\mathbf{j} + \left(3t^2 - 2\right)\mathbf{k}\right] = \left[\left(9t^2 - 2\right)\mathbf{i} - \left(2t^{-1/2} + 1\right)\mathbf{j} + (6t)\mathbf{k}\right] \text{ m/s}$$

When $t = 2$ s,

$$\mathbf{v} = \left[\left[9\left(2^2\right) - 2\right]\mathbf{i} - \left[2\left(2^{-1/2}\right) + 1\right]\mathbf{j} + 6(2)\mathbf{k}\right] \text{ m/s}$$

$$= [34\mathbf{i} - 2.414\mathbf{j} + 12\mathbf{k}] \text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2} = \sqrt{34^2 + (-2.414)^2 + 12^2} = 36.1 \text{ m/s}$$ **Ans.**

Acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d}{dt}\left[\left(9t^2 - 2\right)\mathbf{i} - \left(2t^{-1/2} + 1\right)\mathbf{j} + (6t)\mathbf{k}\right] \text{ m/s} = \left[(18t)\mathbf{i} + t^{-3/2}\mathbf{j} + 6\mathbf{k}\right] \text{ m/s}^2$$

When $t = 2$ s,

$$\mathbf{a} = \left[18(2)\mathbf{i} + 2^{-3/2}\mathbf{j} + 6\mathbf{k}\right] \text{ m/s}^2 = [36\mathbf{i} + 0.3536\mathbf{j} + 6\mathbf{k}] \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2} = \sqrt{36^2 + 0.3536^2 + 6^2} = 36.5 \text{ m/s}^2$$ **Ans.**

***12–72.** The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \mathbf{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \mathbf{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}] \text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}] \text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= [6\mathbf{i} + 4\mathbf{j}] \text{ m}$$ **Ans.**

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•12–73. A particle travels along the parabolic path $y = bx^2$. If its component of velocity along the y axis is $v_y = ct^2$, determine the x and y components of the particle's acceleration. Here b and c are constants.

Velocity:

$$dy = v_y\,dt$$

$$\int_0^y dy = \int_0^t ct^2\,dt$$

$$y = \frac{c}{3}t^3$$

Substituting the result of y into $y = bx^2$,

$$\frac{c}{3}t^3 = bx^2$$

$$x = \sqrt{\frac{c}{3b}}\,t^{3/2}$$

Thus, the x component of the particle's velocity can be determined by taking the time derivative of x.

$$v_x = \dot{x} = \frac{d}{dt}\left[\sqrt{\frac{c}{3b}}\,t^{3/2}\right] = \frac{3}{2}\sqrt{\frac{c}{3b}}\,t^{1/2}$$

Acceleration:

$$a_x = \dot{v}_x = \frac{d}{dt}\left(\frac{3}{2}\sqrt{\frac{c}{3b}}t^{1/2}\right) = \frac{3}{4}\sqrt{\frac{c}{3b}}\,t^{-1/2} = \frac{3}{4}\sqrt{\frac{c}{3b}}\,\frac{1}{\sqrt{t}}$$ **Ans.**

$$a_y = \dot{v}_y = \frac{d}{dt}\left(ct^2\right) = 2ct$$ **Ans.**

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12–74. The velocity of a particle is given by $\mathbf{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

$$(42.7, 16.0, 14.0)\ \text{m} \qquad \textbf{Ans.}$$

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12–75. A particle travels along the circular path $x^2 + y^2 = r^2$. If the y component of the particle's velocity is $v_y = 2r \cos 2t$, determine the x and y components of its acceleration at any instant.

Velocity:

$$dy = v_y\, dt$$

$$\int_0^{\mathbf{y}} dy = \int_0^t 2r \cos 2t dt$$

$$y = r \sin 2t$$

Substituting this result into $x^2 + y^2 = r^2$, we obtain

$$x^2 + r^2 \sin^2 2t = r^2$$

$$x^2 = r^2\left(1 - \sin^2 2t\right)$$

$$x = \mp r \cos 2t$$

Thus,

$$v_x = \dot{x} = \frac{d}{dt}(\mp r \cos 2t) = \mp 2r \sin 2t$$

Acceleration:

$$a_x = \dot{v}_x = \frac{d}{dt}(\mp 2r \sin 2t) = \mp 4r \cos 2t \qquad \textbf{Ans.}$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2r \cos 2t) = -4r \sin 2t \qquad \textbf{Ans.}$$

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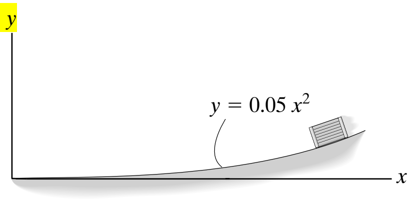
***12–76.** The box slides down the slope described by the equation $y = (0.05x^2)$ m, where x is in meters. If the box has x components of velocity and acceleration of $v_x = -3$ m/s and $a_x = -1.5$ m/s^2 at $x = 5$ m, determine the y components of the velocity and the acceleration of the box at this instant.



Velocity: The x and y components of the box's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = 0.05x^2$$
$$\dot{y} = 0.1x\dot{x}$$

or

$$v_y = 0.1xv_x$$

At $x = 5$ m, $v_x = -3$ m/s. Thus,

$$v_y = 0.1(5)(-3) = -1.5 \text{ m/s} = 1.5 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the box's acceleration can be obtained by taking the second time derivative of the path's equation using the chain rule.

$$\ddot{y} = 0.1[\dot{x}\dot{x} + x\ddot{x}] = 0.1\left(\dot{x}^2 + x\ddot{x}\right)$$

or

$$a_y = 0.1\left(v_x{}^2 + xa_x\right)$$

At $x = 5$ m, $v_x = -3$ m/s and $a_x = -1.5$ m/s^2. Thus,

$$a_y = 0.1\left[(-3)^2 + 5(-1.5)\right] = 0.15 \text{ m/s}^2 \uparrow \qquad \textbf{Ans.}$$

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•12–77. The position of a particle is defined by $\mathbf{r} = \{5\cos 2t\,\mathbf{i} + 4\sin 2t\,\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10\sin 2t\mathbf{i} + 8\cos 2t\mathbf{j}\}\text{ m/s}$$

When $t = 1$ s, $\mathbf{v} = -10\sin 2(1)\mathbf{i} + 8\cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration expressed in Cartesian vector from can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20\cos 2t\mathbf{i} - 16\sin 2t\mathbf{j}\}\text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20\cos 2(1)\mathbf{i} - 16\sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8\text{ m/s}^2 \qquad \textbf{Ans.}$$

Traveling Path: Here, $x = 5\cos 2t$ and $y = 4\sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{[1]}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{[2]}$$

Adding Eqs [1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \qquad \textbf{(Equation of an Ellipse) (Q.E.D.)}$$

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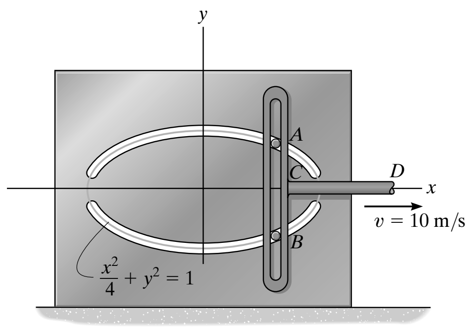
12–78. Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s}\downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left(v_x{}^2 + xa_x\right) + 2\left(v_y{}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2\downarrow$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

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12–79. A particle travels along the path $y^2 = 4x$ with a constant speed of $v = 4\text{ m/s}$. Determine the x and y components of the particle's velocity and acceleration when the particle is at $x = 4\text{ m}$.

Velocity: The x and y components of the particle's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$2y\dot{y} = 4\dot{x}$$

$$\dot{y} = \frac{2}{y}\dot{x}$$

or

$$v_y = \frac{2}{y}v_x \qquad \textbf{(1)}$$

At $x = 4\text{ m}$, $y = \sqrt{4(4)} = 4\text{ m}$. Thus Eq. (1) becomes

$$v_y = \frac{1}{2}v_x \qquad \textbf{(2)}$$

The magnitude of the particle's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \textbf{(3)}$$

Substituting $v = 4\text{ m/s}$ and Eq. (2) into Eq. (3),

$$4 = \sqrt{{v_x}^2 + \left(\frac{1}{2}v_x\right)^2}$$

$$v_x = 3.578\text{ m/s} = 3.58\text{ m/s} \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (2), we obtain

$$v_y = 1.789\text{ m/s} = 1.79\text{ m/s} \qquad \textbf{Ans.}$$

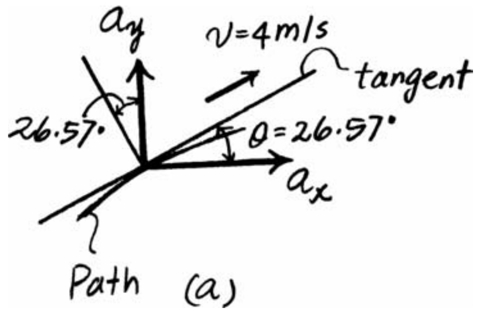
Acceleration: The x and y components of the particle's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.

$$2(\dot{y}\dot{y} + y\overline{y}) = 4\overline{x}$$

$$\dot{y}^2 + y\overline{y} = 2\overline{x}$$

or

$${v_y}^2 + ya_y = 2a_x \qquad \textbf{(4)}$$



When $x = 4\text{ m}$, $y = 4\text{ m}$, and $v_y = 1.789\text{ m/s}$. Thus Eq. (4) becomes

$$1.789^2 + 4a_y = 2a_x$$

$$a_y = 0.5a_x - 0.8 \qquad \textbf{(5)}$$

Since the particle travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 4\text{ m}$ is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\Big|_{x=4\text{ m}} = \tan^{-1}\left(\frac{1}{x^{1/2}}\right)\Big|_{x=4\text{ m}} = \tan^{-1}(0.5) = 26.57°$. Thus, from the diagram shown in Fig. *a*,

$$a_x \cos 26.57° + a_y \sin 26.57° = 0 \qquad \textbf{(6)}$$

Solving Eqs. (5) and (6) yields

$$a_x = 0.32\text{ m/s}^2 \qquad a_y = -0.64\text{ m/s}^2 = 0.64\text{ m/s}^2 \downarrow \qquad \textbf{Ans.}$$

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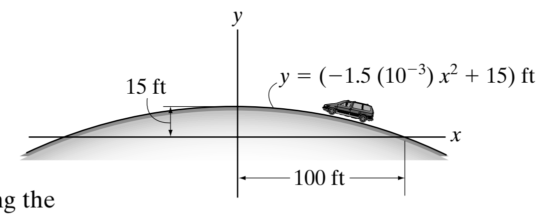
***12–80.** The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5\left(10^{-3}\right)x^2 + 15$$

$$\dot{y} = -3\left(10^{-3}\right)x\dot{x}$$

or

$$v_y = -3\left(10^{-3}\right)xv_x$$

When $x = 50$ ft,

$$v_y = -3\left(10^{-3}\right)(50)v_x = -0.15v_x \qquad \textbf{(1)}$$

The magnitude of the van's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \textbf{(2)}$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

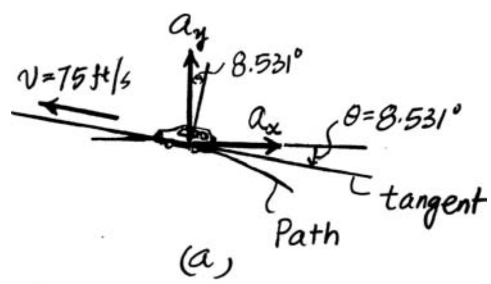
$$75 = \sqrt{{v_x}^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.



$$\ddot{y} = -3\left(10^{-3}\right)(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3\left(10^{-3}\right)\left({v_x}^2 + xa_x\right)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3\left(10^{-3}\right)\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \qquad \textbf{(3)}$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 50$ ft is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\Big|_{x=50\text{ ft}} = \tan^{-1}\left[-3\left(10^{-3}\right)x\right]\Big|_{x=50\text{ ft}} = \tan^{-1}(-0.15) = -8.531°$. Thus, from the diagram shown in Fig. a,

$$a_x \cos 8.531° - a_y \sin 8.531° = 0 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

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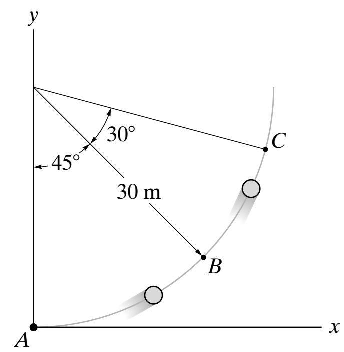
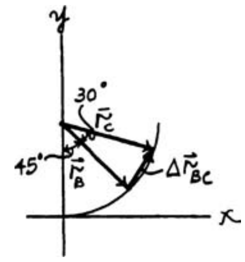
•12–81. A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.





Position: The coordinates for points B and C are [30 sin 45°, 30 − 30 cos 45°] and [30 sin 75°, 30 − 30 cos 75°]. Thus,

$$\mathbf{r}_B = (30\sin 45° - 0)\mathbf{i} + [(30 - 30\cos 45°) - 30]\mathbf{j}$$

$$= \{21.21\mathbf{i} - 21.21\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_C = (30\sin 75° - 0)\mathbf{i} + [(30 - 30\cos 75°) - 30]\mathbf{j}$$

$$= \{28.98\mathbf{i} - 7.765\mathbf{j}\}\text{ m}$$

Average Velocity: The displacement from point B to C is $\Delta\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\}$ m.

$$(\mathbf{v}_{BC})_{\text{avg}} = \frac{\Delta\mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

12–82. A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes, and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Total Distance Traveled and Displacement: The total distance traveled is

$$s = 2 + 3 + 4 = 9\text{ km} \qquad \textbf{Ans.}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{(2 + 4)^2 + 3^2} = 6.708\text{ km} = 6.71\text{ km} \qquad \textbf{Ans.}$$

Average Velocity and Speed: The total time is $\Delta t = 5 + 8 + 10 = 23\text{ min} = 1380\text{ s}$. The magnitude of average velocity is

$$v_{\text{avg}} = \frac{\Delta r}{\Delta t} = \frac{6.708\left(10^3\right)}{1380} = 4.86\text{ m/s} \qquad \textbf{Ans.}$$

and the average speed is

$$\left(v_{sp}\right)_{\text{avg}} = \frac{s}{\Delta t} = \frac{9\left(10^3\right)}{1380} = 6.52\text{ m/s} \qquad \textbf{Ans.}$$

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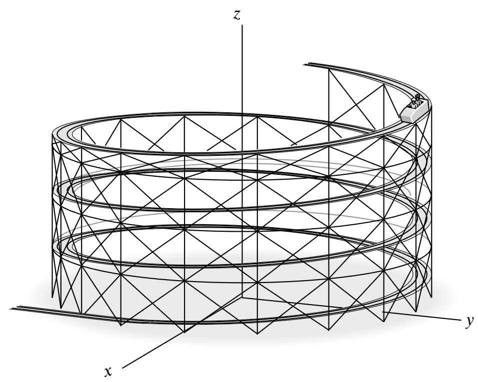
12–83. The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



$x = c \sin kt \qquad \dot{x} = \text{ck} \cos kt \qquad \ddot{x} = -ck^2 \sin kt$

$y = c \cos kt \qquad \dot{y} = -\text{ck} \sin kt \qquad \ddot{y} = -ck^2 \cos kt$

$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2k^2 + b^2}$ **Ans.**

$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$ **Ans.**

***12–84.** The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration when $y = y_0$.

$$y^2 = 4kx$$

$$2yv_y = 4kv_x$$

$$2v_y^2 + 2ya_y = 4ka_x$$

$$v_y = ct$$

$$a_y = c \qquad \textbf{Ans.}$$

$$2(ct)^2 + 2yc = 4ka_x$$

$$a_x = \frac{c}{2k}\left(y + ct^2\right) \qquad \textbf{Ans.}$$

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•12–85. A particle moves along the curve $y = x - (x^2/400)$, where x and y are in ft. If the velocity component in the x direction is $v_x = 2$ ft/s and remains *constant*, determine the magnitudes of the velocity and acceleration when $x = 20$ ft.

Velocity: Taking the first derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\dot{y} = \dot{x} - \frac{1}{400}(2x\dot{x})$$

$$\dot{y} = \dot{x} - \frac{x}{200}\dot{x} \qquad \textbf{[1]}$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq. [1] becomes

$$v_y = v_x - \frac{x}{200}v_x \qquad \textbf{[2]}$$

Here, $v_x = 2$ ft/s at $x = 20$ ft. Then, From Eq. [2]

$$v_y = 2 - \frac{20}{200}(2) = 1.80 \text{ ft/s}$$

Also,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 1.80^2} = 2.69 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Taking the second derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\ddot{y} = \ddot{x} - \frac{1}{200}\left(\dot{x}^2 + x\ddot{x}\right) \qquad \textbf{[3]}$$

However, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. [3] becomes

$$a_y = a_x - \frac{1}{200}\left(v_x^2 + xa_x\right) \qquad \textbf{[4]}$$

Since $v_x = 2$ ft/s is constant, hence $a_x = 0$ at $x = 20$ ft. Then, From Eq. [4]

$$a_y = 0 - \frac{1}{200}\left[2^2 + 20(0)\right] = -0.020 \text{ ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-0.020)^2} = 0.0200 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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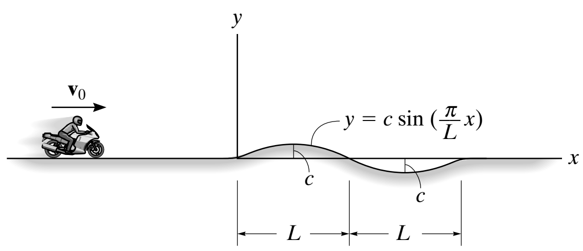
12–86. The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



$$y = c \sin\left(\frac{\pi}{L}x\right)$$

$$y = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\,v_x\left(\cos\frac{\pi}{L}x\right)$$

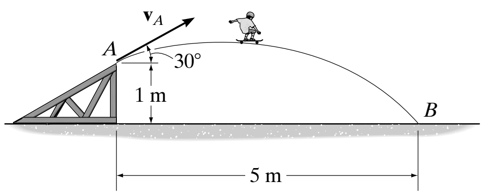
$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

12–87. The skateboard rider leaves the ramp at A with an initial velocity v_A at a 30° angle. If he strikes the ground at B, determine v_A and the time of flight.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = v_A \cos 30°$, $x_A = 0$ and $x_B = 5$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$5 = 0 + v_A \cos 30°\, t$$

$$t = \frac{5}{v_A \cos 30°} \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = v_A \sin 30°$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_B = -1$ m. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$-1 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 30°\, t - 1 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 6.49 \text{ m/s} \qquad t = 0.890 \text{ s} \qquad \textbf{Ans.}$$

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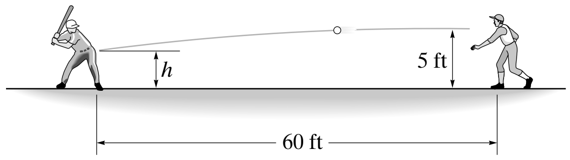
***12–88.** The pitcher throws the baseball horizontally with a speed of 140 ft/s from a height of 5 ft. If the batter is 60 ft away, determine the time for the ball to arrive at the batter and the height h at which it passes the batter.

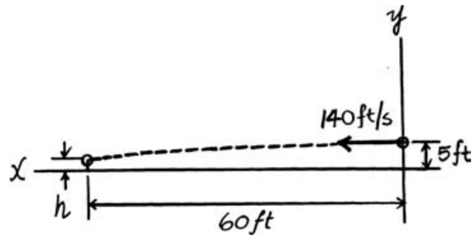


$(\overset{+}{\leftarrow})$ $\quad s = vt; \qquad 60 = 140t$

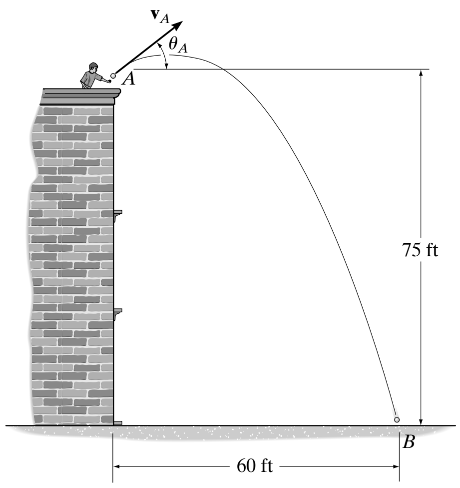
$$t = 0.4286 = 0.429 \text{ s} \qquad \textbf{Ans.}$$

$(+\uparrow)$ $\quad s = s_0 + v_0\, t + \frac{1}{2} a_c\, t^2$

$$h = 5 + 0 + \frac{1}{2}(-32.2)(0.4286)^2 = 2.04 \text{ ft} \qquad \textbf{Ans.}$$



•12–89. The ball is thrown off the top of the building. If it strikes the ground at B in 3 s, determine the initial velocity v_A and the inclination angle θ_A at which it was thrown. Also, find the magnitude of the ball's velocity when it strikes the ground.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = v_A \cos\theta$, $x_A = 0$, and $x_B = 60$ ft, and $t = 3$ s. Thus,

$(\overset{+}{\rightarrow})$ $\quad x_B = x_A + (v_A)_x t$

$$60 = 0 + v_A \cos\theta(3)$$

$$v_A \cos\theta = 20 \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = v_A \sin\theta$, $a_y = -g = -32.2 \text{ ft/s}^2$, $y_A = 0$, and $y_B = -75$ ft, and $t = 3$ s. Thus,

$(+\uparrow)$ $\quad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$

$$-75 = 0 + v_A \sin\theta(3) + \frac{1}{2}(-32.2)\left(3^2\right)$$

$$v_A \sin\theta = 23.3 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta = 49.36° = 49.4° \qquad v_A = 30.71 \text{ ft/s} = 30.7 \text{ ft/s} \qquad \textbf{Ans.}$$

Using the result of θ and v_A, we obtain

$$(v_A)_x = 30.71 \cos 49.36° = 20 \text{ ft/s} \qquad (v_A)_y = 30.71 \sin 49.36° = 23.3 \text{ ft/s}$$

Thus,

$(+\uparrow)$ $\quad (v_B)_y = (v_A)_y + a_y t$

$$(v_B)_y = 23.3 + (-32.2)(3) = -73.3 \text{ ft/s} = 73.3 \text{ ft/s} \downarrow$$

Thus, the magnitude of the ball's velocity when it strikes the ground is

$$v_B = \sqrt{20^2 + 73.3^2} = 76.0 \text{ ft/s} \qquad \textbf{Ans.}$$

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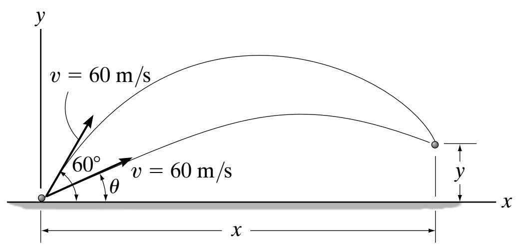
12–90. A projectile is fired with a speed of $v = 60$ m/s at an angle of 60°. A second projectile is then fired with the same speed 0.5 s later. Determine the angle θ of the second projectile so that the two projectiles collide. At what position (x, y) will this happen?



***x*-Motion:** For the motion of the first projectile, $v_x = 60 \cos 60° = 30$ m/s, $x_0 = 0$, and $t = t_1$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + v_x t$$

$$x = 0 + 30t_1 \qquad \textbf{(1)}$$

For the motion of the second projectile, $v_x = 60 \cos \theta$, $x_0 = 0$, and $t = t_1 - 0.5$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + v_x t$$

$$x = 0 + 60 \cos \theta (t_1 - 0.5) \qquad \textbf{(2)}$$

***y*-Motion:** For the motion of the first projectile, $v_y = 60 \sin 60° = 51.96$ m/s, $y_0 = 0$, and $a_y = -g = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad y = y_0 + v_y t + \frac{1}{2} a_y t^2$$

$$y = 0 + 51.96t_1 + \frac{1}{2}(-9.81)t_1{}^2$$

$$y = 51.96t_1 - 4.905t_1{}^2 \qquad \textbf{(3)}$$

For the motion of the second projectile, $v_y = 60 \sin \theta$, $y_0 = 0$, and $a_y = -g = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad y = y_0 + v_y t + \frac{1}{2} a_y t^2$$

$$y = 0 + 60 \sin \theta (t_1 - 0.5) + \frac{1}{2}(-9.81)(t_1 - 0.5)^2$$

$$y = (60 \sin \theta)t_1 - 30 \sin \theta - 4.905\, t_1{}^2 + 4.905t_1 - 1.22625 \qquad \textbf{(4)}$$

Equating Eqs. (1) and (2),

$$30t_1 = 60 \cos \theta (t_1 - 0.5)$$

$$t_1 = \frac{\cos \theta}{2 \cos \theta - 1} \qquad \textbf{(5)}$$

Equating Eqs. (3) and (4),

$$51.96t_1 - 4.905t_1{}^2 = (60 \sin \theta)t_1 - 30 \sin \theta - 4.905t_1{}^2 + 4.905t_1 - 1.22625$$

$$(60 \sin \theta - 47.06)t_1 = 30 \sin \theta + 1.22625$$

$$t_1 = \frac{30 \sin \theta + 1.22625}{60 \sin \theta - 47.06} \qquad \textbf{(6)}$$

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Equating Eqs. (5) and (6) yields

$$\frac{\cos\theta}{2\cos\theta - 1} = \frac{30\sin\theta + 1.22625}{60\sin\theta - 47.06}$$

$$49.51\cos\theta - 30\sin\theta = 1.22625$$

Solving by trial and error,

$$\theta = 57.57° = 57.6° \qquad \textbf{Ans.}$$

Substituting this result into Eq. (5) (or Eq. (6)),

$$t_1 = \frac{\cos 57.57°}{2\cos 57.57° - 1} = 7.3998 \text{ s}$$

Substituting this result into Eqs. (1) and (3),

$$x = 30(7.3998) = 222 \text{ m} \qquad \textbf{Ans.}$$

$$y = 51.96(7.3998) - 4.905\left(7.3998^2\right) = 116 \text{ m} \qquad \textbf{Ans.}$$

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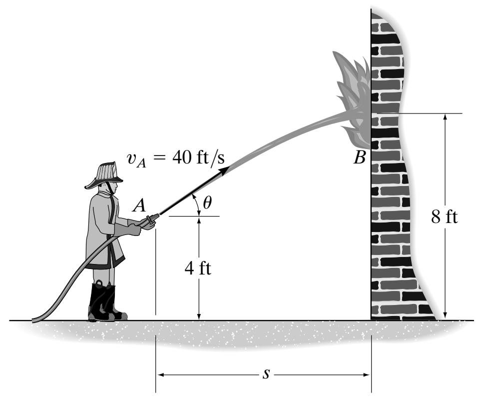
12–91. The fireman holds the hose at an angle $\theta = 30°$ with horizontal, and the water is discharged from the hose at A with a speed of $v_A = 40$ ft/s. If the water stream strikes the building at B, determine his two possible distances s from the building.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = 40\cos 30°\text{ ft/s} = 34.64\text{ ft/s}$, $x_A = 0$, and $x_B = s$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$s = 0 + 34.64t$$

$$s = 34.64t \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 40\sin 30°\text{ ft/s} = 20\text{ ft/s}$, $a_y = -g = -32.2\text{ ft/s}^2$, $y_A = 0$, and $y_B = 8 - 4 = 4$ ft. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$4 = 0 + 20t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 - 20t + 4 = 0$$

$$t = 0.2505\text{ s and }0.9917\text{ s}$$

Substituting these results into Eq. (1), the two possible distances are

$$s = 34.64(0.2505) = 8.68\text{ ft} \qquad \textbf{Ans.}$$

$$s = 34.64(0.9917) = 34.4\text{ ft} \qquad \textbf{Ans.}$$

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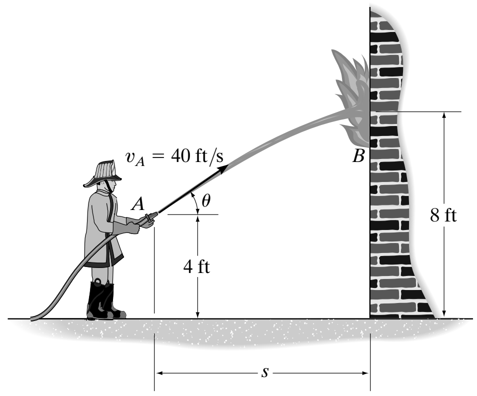
***12–92.** Water is discharged from the hose with a speed of 40 ft/s. Determine the two possible angles θ the fireman can hold the hose so that the water strikes the building at B. Take $s = 20$ ft.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = 40 \cos\theta$, $x_A = 0$, and $x_B = 20$ ft/s. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$20 = 0 + 40\cos\theta t$$

$$t = \frac{1}{2\cos\theta} \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 40 \sin\theta$, $a_y = -g = -32.2 \text{ ft/s}^2$, $y_A = 0$, and $y_B = 8 - 4 = 4$ ft. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$4 = 0 + 40\sin\theta t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 - 40\sin\theta t + 4 = 0 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$16.1\left(\frac{1}{2\cos\theta}\right)^2 - 40\sin\theta\left(\frac{1}{2\cos\theta}\right) + 4 = 0$$

$$20\sin\theta\cos\theta - 4\cos^2\theta = 4.025$$

$$10\sin\theta\cos\theta - 2\cos^2\theta = 2.0125$$

$$5\sin 2\theta - \left(2\cos^2\theta - 1\right) - 1 = 2.0125$$

$$5\sin 2\theta - \cos 2\theta = 3.0125$$

Solving by trial and error,

$$\theta = 23.8^\circ \text{ and } 77.5^\circ \qquad \textbf{Ans.}$$

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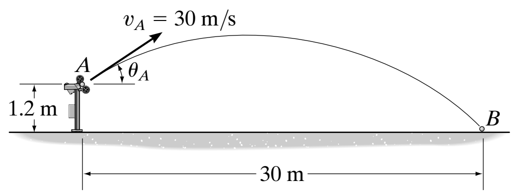
•12–93. The pitching machine is adjusted so that the baseball is launched with a speed of $v_A = 30\,\text{m/s}$. If the ball strikes the ground at B, determine the two possible angles θ_A at which it was launched.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = 30\cos\theta_A$, $x_A = 0$ and $x_B = 30$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$30 = 0 + 30\cos\theta_A t$$

$$t = \frac{1}{\cos\theta_A} \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 30\sin\theta_A$, $a_y = -g = -9.81\ \text{m/s}^2$, and $y_B = -1.2$ m. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$-1.2 = 0 + 30\sin\theta_A t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 30\sin\theta_A\, t - 1.2 = 0 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$4.905\left(\frac{1}{\cos\theta_A}\right)^2 - 30\sin\theta_A\left(\frac{1}{\cos\theta_A}\right) - 1.2 = 0$$

$$1.2\cos^2\theta_A + 30\sin\theta_A\cos\theta_A - 4.905 = 0$$

Solving by trial and error,

$$\theta_A = 7.19^\circ \text{ and } 80.5^\circ \qquad \textbf{Ans.}$$

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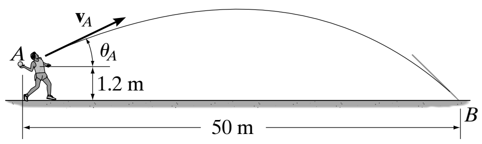
12–94. It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = v_A \cos\theta_A$, $x_A = 0$, $x_B = 50$ m, and $t = 2.5$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$50 = 0 + v_A \cos\theta_A (2.5)$$

$$v_A \cos\theta_A = 20 \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = v_A \sin\theta_A$, $y_A = 0$, $y_B = -1.2$ m, and $a_y = -g = -9.81\ \text{m/s}^2$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

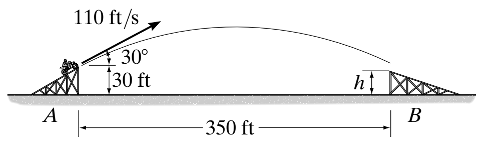
$$-1.2 = 0 + v_A \sin\theta_A (2.5) + \frac{1}{2}(-9.81)\left(2.5^2\right)$$

$$v_A \sin\theta_A = 11.7825 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta_A = 30.5^\circ \qquad v_A = 23.2\ \text{m/s} \qquad \textbf{Ans.}$$

12–95. If the motorcycle leaves the ramp traveling at 110 ft/s, determine the height h ramp B must have so that the motorcycle lands safely.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with the take off point of the motorcycle at ramp A.

***x*-Motion:** Here, $x_A = 0$, $x_B = 350$ ft, and $(v_A)_x = 110 \cos 30^\circ = 95.26$ ft/s. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$350 = 0 + 95.26t$$

$$t = 3.674\ \text{s}$$

***y*-Motion:** Here, $y_A = 0$, $y_B = h - 30$, $(v_A)_y = 110 \sin 30^\circ = 55$ ft/s, and $a_y = -g = -32.2\ \text{ft/s}^2$. Thus, using the result of t, we have

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$h - 30 = 0 + 55(3.674) + \frac{1}{2}(-32.2)\left(3.674^2\right)$$

$$h = 14.7\ \text{ft} \qquad \textbf{Ans.}$$

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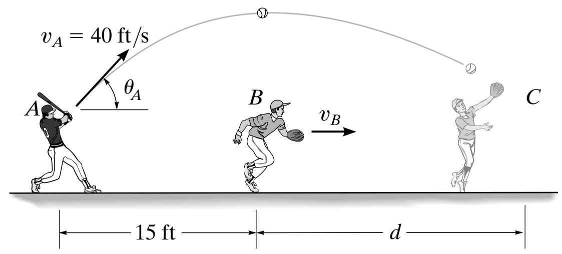
***12–96.** The baseball player A hits the baseball with $v_A = 40$ ft/s and $\theta_A = 60°$. When the ball is directly above of player B he begins to run under it. Determine the constant speed v_B and the distance d at which B must run in order to make the catch at the same elevation at which the ball was hit.



Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60° = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2}(-32.2)t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60° = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player B must travel in order to catch the baseball is

$$d = R - 15 = 43.03 - 15 = 28.0 \text{ ft} \qquad \textbf{Ans.}$$

Player B is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$$v_B = 40 \cos 60° = 20.0 \text{ ft/s} \qquad \textbf{Ans.}$$

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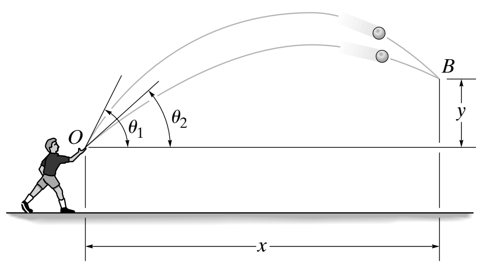
•12–97. A boy throws a ball at O in the air with a speed v_0 at an angle θ_1. If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in mid air at B.



Vertical Motion: For the first ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_1$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_1 t_1 + \frac{1}{2}(-g)t_1^2 \qquad \textbf{[1]}$$

For the second ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_2$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_2 t_2 + \frac{1}{2}(-g)t_2^2 \qquad \textbf{[2]}$$

Horizontal Motion: For the first ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_1$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_1\, t_1 \qquad \textbf{[3]}$$

For the second ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_2\, t_2 \qquad \textbf{[4]}$$

Equating Eqs. [3] and [4], we have

$$t_2 = \frac{\cos\theta_1}{\cos\theta_2} t_1 \qquad \textbf{[5]}$$

Equating Eqs. [1] and [2], we have

$$v_0 t_1 \sin\theta_1 - v_0 t_2 \sin\theta_2 = \frac{1}{2} g\left(t_1^2 - t_2^2\right) \qquad \textbf{[6]}$$

Solving Eq. [5] into [6] yields

$$t_1 = \frac{2v_0 \cos\theta_2 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$t_2 = \frac{2v_0 \cos\theta_1 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

Thus, the time between the throws is

$$\Delta t = t_1 - t_2 = \frac{2v_0 \sin(\theta_1 - \theta_2)(\cos\theta_2 - \cos\theta_1)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$= \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)} \qquad \textbf{Ans.}$$

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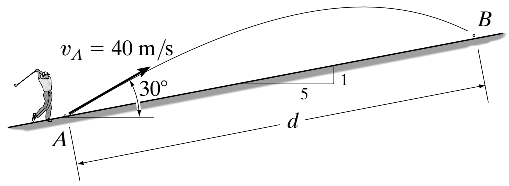
12–98. The golf ball is hit at A with a speed of $v_A = 40\text{ m/s}$ and directed at an angle of 30° with the horizontal as shown. Determine the distance d where the ball strikes the slope at B.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $(v_A)_x = 40\cos 30° = 34.64\text{ m/s}$, $x_A = 0$, and $x_B = d\left(\dfrac{5}{\sqrt{5^2+1}}\right) = 0.9806d$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$0.9806d = 0 + 34.64t$$

$$t = 0.02831d \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 40\sin 30° = 20\text{ m/s}$, $y_A = 0$, $y_B = d\left(\sqrt{\dfrac{1}{\sqrt{5^2+1}}}\right) = 0.1961d$, and $a_y = -g = -9.81\text{ m/s}^2$.

Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$0.1961d = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 20t + 0.1961d = 0 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$4.905(0.02831d)^2 - 20(0.02831d) + 0.1961d = 0$$

$$3.9303\left(10^{-3}\right)d^2 - 0.37002d = 0$$

$$d\left[3.9303\left(10^{-3}\right)d - 0.37002\right] = 0$$

Since $d \neq 0$, then

$$3.9303\left(10^{-3}\right)d = 0.37002 = 0$$

$$d = 94.1\text{ m} \qquad \textbf{Ans.}$$

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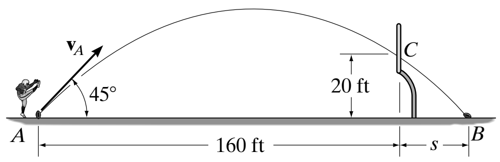
12–99. If the football is kicked at the 45° angle, determine its minimum initial speed v_A so that it passes over the goal post at C. At what distance s from the goal post will the football strike the ground at B?



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** For the motion from A to C, $x_A = 0$, and $x_C = 160$ ft, $(v_A)_x = v_A \cos 45°$, and $t = t_{AC}$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_C = x_A + (v_A)_x t$$

$$160 = 0 + v_A \cos 45° \, t_{AC}$$

$$t_{AC} = \frac{160}{v_A \cos 45°} \qquad \textbf{(1)}$$

For the motion from A to B, $x_A = 0$, and $x_B = 160 + s$, $(v_A)_x = v_A \cos 45°$, and $t = t_{AB}$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$160 + s = 0 + v_A \cos 45° \, t_{AB}$$

$$s = v_A \cos 45°(t_{AB}) - 160 \qquad \textbf{(2)}$$

***y*-Motion:** For the motion from A to C, $y_A = 0$, and $y_C = 20$ ft, $(v_A)_y = v_A \sin 45°$, and $a_y = -g = -32.2$ ft/s^2. Thus,

$$(+\uparrow) \qquad y_C = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$20 = 0 + v_A \sin 45° \, t_{AC} + \frac{1}{2}(-32.2) t_{AC}{}^2$$

$$16.1 t_{AC}{}^2 - v_A \sin 45° \, t_{AC} + 20 = 0 \qquad \textbf{(3)}$$

For the motion from A to B, $y_A = y_B = 0$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$0 = 0 + v_A \sin 45°(t_{AB}) + \frac{1}{2}(-32.2) t_{AB}{}^2$$

$$t_{AB}(16.1 t_{AB} - v_A \sin 45°) = 0$$

Since $t_{AB} \neq 0$, then

$$16.1 t_{AB} - v_A \sin 45° = 0 \qquad \textbf{(4)}$$

Substituting Eq. (1) into Eq. (3) yields

$$16.1\left(\frac{160}{v_A \cos 45°}\right)^2 - v_A \sin 45°\left(\frac{160}{v_A \cos 45°}\right) + 20 = 0$$

$$v_A = 76.73 \text{ ft/s} = 76.7 \text{ ft/s} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (4),

$$16.1 t_{AB} - 76.73 \sin 45° = 0$$

$$t_{AB} = 3.370 \text{ s}$$

Substituting the result of t_{AB} and v_A into Eq. (2),

$$s = 76.73 \cos 45°(3.370) - 160$$

$$= 22.9 \text{ ft} \qquad \textbf{Ans.}$$

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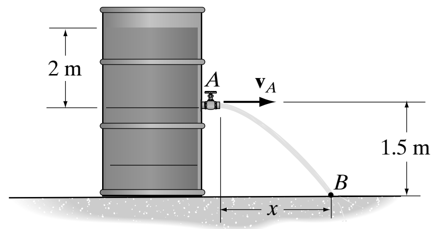
***12–100.** The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2$ m is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A. The speed of the water that the jet discharges from A is

$$v_A = \sqrt{2(9.81)(2)} = 6.264 \text{ m/s}$$

***x*-Motion:** Here, $(v_A)_x = v_A = 6.264$ m/s, $x_A = 0$, $x_B = x$, and $t = t_A$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$x = 0 + 6.264 t_A \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 0$, $a_y = -g = -9.81 \text{ m/s}^2$, $y_A = 0$ m, $y_B = -1.5$ m, and $t = t_A$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

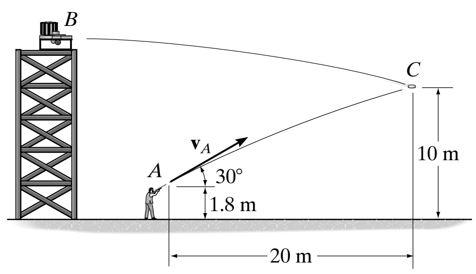
$$-1.5 = 0 + 0 + \frac{1}{2}(-9.81) t_A{}^2$$

$$t_A = 0.553 \text{ s}$$

Thus,

$$x = 0 + 6.264(0.553) = 3.46 \text{ m} \qquad \textbf{Ans.}$$

•12–101. A projectile is fired from the platform at B. The shooter fires his gun from point A at an angle of 30°. Determine the muzzle speed of the bullet if it hits the projectile at C.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

***x*-Motion:** Here, $x_A = 0$ and $x_C = 20$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_C = x_A + (v_A)_x t$$

$$20 = 0 + v_A \cos 30°\, t \qquad \textbf{(1)}$$

***y*-Motion:** Here, $y_A = 1.8$, $(v_A)_y = v_A \sin 30°$, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad y_C = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$10 = 1.8 + v_A \sin 30°(t) + \frac{1}{2}(-9.81)(t)^2$$

Thus,

$$10 - 1.8 = \left(\frac{20 \sin 30°}{\cos 30°(t)}\right)(t) - 4.905(t)^2$$

$$t = 0.8261 \text{ s}$$

So that

$$v_A = \frac{20}{\cos 30°(0.8261)} = 28.0 \text{ m/s} \qquad \textbf{Ans.}$$

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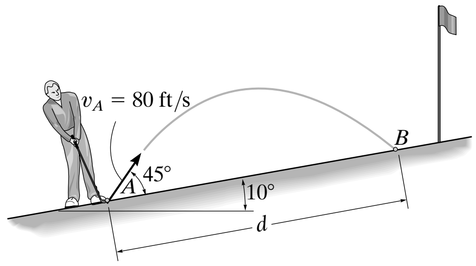
12–102. A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 55° = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d \cos 10°$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d \cos 10° = 0 + 45.89t \qquad \textbf{[1]}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 55° = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d \sin 10°$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

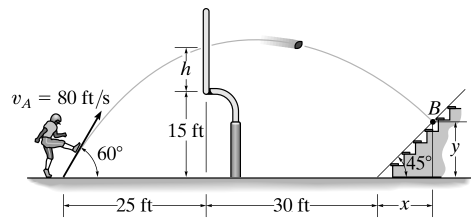
$$d \sin 10° = 0 + 65.53t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$d = 166 \text{ ft} \qquad \textbf{Ans.}$$

$$t = 3.568 \text{ s}$$

12–103. The football is to be kicked over the goalpost, which is 15 ft high. If its initial speed is $v_A = 80$ ft/s, determine if it makes it over the goalpost, and if so, by how much, h.



Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 60° = 40.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 25$ ft, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$25 = 0 + 40.0t$$

$$t = 0.625 \text{ s}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 60° = 69.28$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = H$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$H = 0 + 69.28(0.625) + \frac{1}{2}(-32.2)\left(0.625^2\right)$$

$$H = 37.01 \text{ ft}$$

Since $H > 15$ ft, the football is kicked over the goalpost. **Ans.**

$$h = H - 15 = 37.01 - 15 = 22.0 \text{ ft} \qquad \textbf{Ans.}$$

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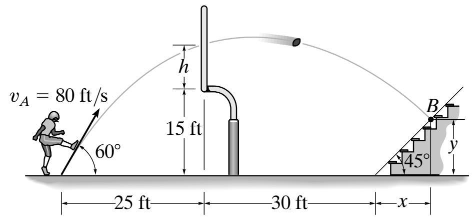
***12–104.** The football is kicked over the goalpost with an initial velocity of $v_A = 80$ ft/s as shown. Determine the point B (x, y) where it strikes the bleachers.



Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 60° = 40.0$ ft/s. The initial and final horzontal positions are $(s_0)_x = 0$ and $s_x = (55 + x)$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$55 + x = 0 + 40.0t \qquad \textbf{[1]}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 60° = 69.28$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y = x \tan 45° = x$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

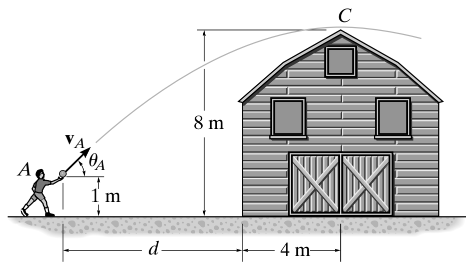
$$x = 0 + 69.28t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$t = 2.969 \text{ s}$$

$$y = x = 63.8 \text{ ft} \qquad \textbf{Ans.}$$

•12–105. The boy at A attempts to throw a ball over the roof of a barn with an initial speed of $v_A = 15$ m/s. Determine the angle θ_A at which the ball must be thrown so that it reaches its maximum height at C. Also, find the distance d where the boy should stand to make the throw.



Vertical Motion: The vertical component, of initial and final velocity are $(v_0)_y = (15 \sin \theta_A)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$$(+\uparrow) \qquad v_y = (v_0) + a_c t$$

$$0 = 15 \sin \theta_A + (-9.81)t \qquad \textbf{[1]}$$

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$8 = 1 + 15 \sin \theta_A t + \frac{1}{2}(-9.81)t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\theta_A = 51.38° = 51.4° \qquad \textbf{Ans.}$$

$$t = 1.195 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos \theta_A = 15 \cos 51.38° = 9.363$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d + 4 = 0 + 9.363(1.195)$$

$$d = 7.18 \text{ m} \qquad \textbf{Ans.}$$

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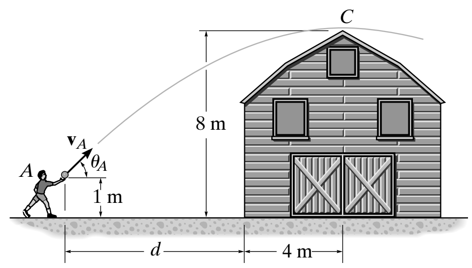
12–106. The boy at A attempts to throw a ball over the roof of a barn such that it is launched at an angle $\theta_A = 40°$. Determine the minimum speed v_A at which he must throw the ball so that it reaches its maximum height at C. Also, find the distance d where the boy must stand so that he can make the throw.



Vertical Motion: The vertical components of initial and final velocity are $(v_0)_y = (v_A \sin 40°)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$$(+\uparrow) \qquad v_y = (v_0) + a_c t$$

$$0 = v_A \sin 40° + (-9.81)\, t \qquad \textbf{[1]}$$

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$8 = 1 + v_A \sin 40° t + \frac{1}{2}(-9.81)\, t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$v_A = 18.23 \text{ m/s} = 18.2 \text{ m/s} \qquad \textbf{Ans.}$$

$$t = 1.195 \text{ s}$$

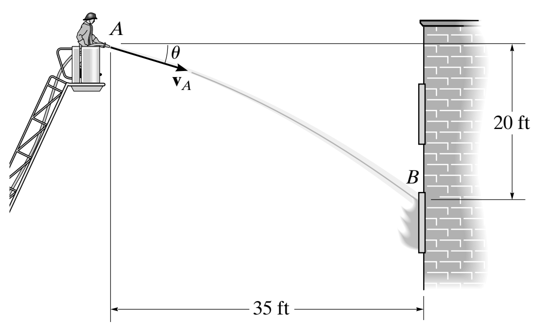
Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos\theta_A = 18.23 \cos 40° = 13.97$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d + 4 = 0 + 13.97(1.195)$$

$$d = 12.7 \text{ m} \qquad \textbf{Ans.}$$

12–107. The fireman wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80$ ft/s.



$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t$$

$$35 = 0 + (80) \cos\theta$$

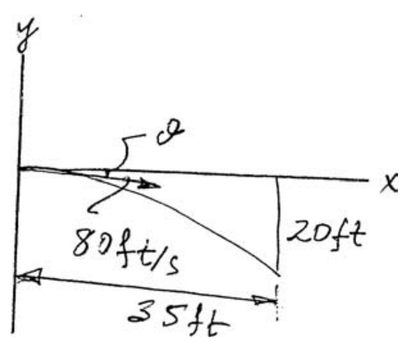
$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-20 = 0 - 80 \sin\theta\, t + \frac{1}{2}(-32.2) t^2$$

Thus,

$$20 = 80 \sin\theta \frac{0.4375}{\cos\theta} t + 16.1\left(\frac{0.1914}{\cos^2\theta}\right)$$

$$20 \cos^2\theta = 17.5 \sin 2\theta + 3.0816$$



Solving,

$$\theta_1 = 25.0° \quad \text{(below the horizontal)} \qquad \textbf{Ans.}$$

$$\theta_2 = 85.2° \quad \text{(above the horizontal)} \qquad \textbf{Ans.}$$

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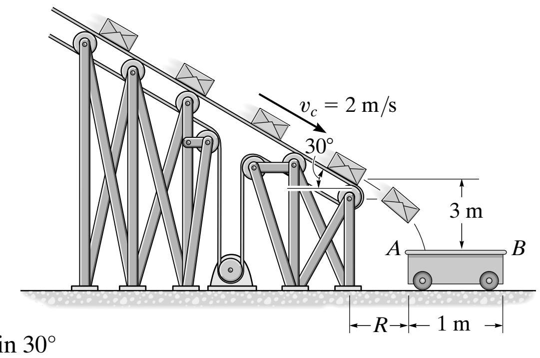
***12–108.** Small packages traveling on the conveyor belt fall off into a l-m-long loading car. If the conveyor is running at a constant speed of $v_C = 2$ m/s, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 2\sin 30° = 1.00$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 3$ m, respectively.

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$3 = 0 + 1.00(t) + \frac{1}{2}(9.81)\left(t^2\right)$$

Choose the positive root $t = 0.6867$ s

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 2\cos 30° = 1.732$ m/s and the initial horizontal position is $(s_0)_x = 0$. If $s_x = R$, then

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 1.732(0.6867) = 1.19 \text{ m} \qquad \textbf{Ans.}$$

If $s_x = R + 1$, then

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$R + 1 = 0 + 1.732(0.6867)$$

$$R = 0.189 \text{ m} \qquad \textbf{Ans.}$$

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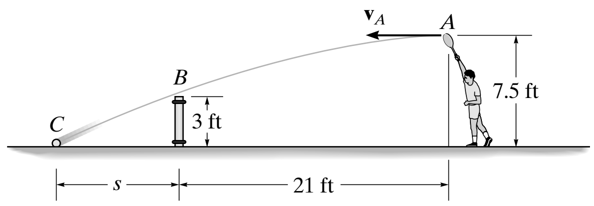
•**12–109.** Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$3 = 7.5 + 0 + \frac{1}{2}(-32.2)t_1^2$$

$$t_1 = 0.5287 \text{ s}$$

For the ball to travel from A to C, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 7.5 + 0 + \frac{1}{2}(-32.2)t_2^2$$

$$t_2 = 0.6825 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A$. For the ball to travel from A to B, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 = 0 + v_A(0.5287)$$

$$v_A = 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \qquad \textbf{Ans.}$$

For the ball to travel from A to C, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 + s = 0 + 39.72(0.6825)$$

$$s = 6.11 \text{ ft} \qquad \textbf{Ans.}$$

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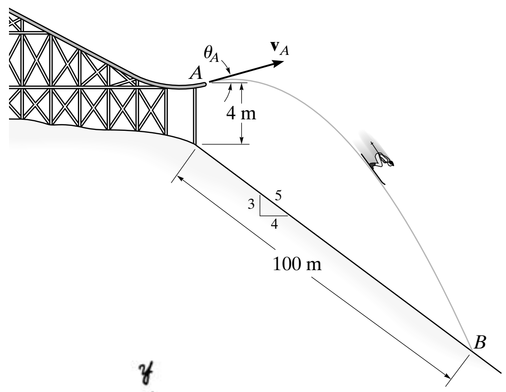
12–110. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



$(\xrightarrow{+})$ $\quad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

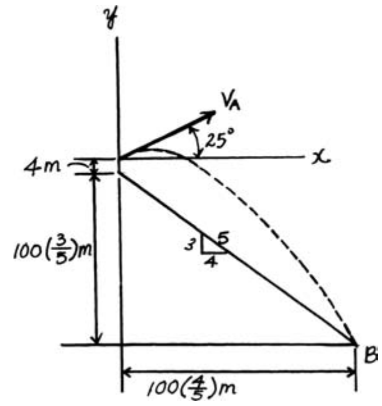
$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$

Solving,

$$v_A = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

$$t_{AB} = 4.54 \text{ s} \qquad \textbf{Ans.}$$



12–111. When designing a highway curve it is required that cars traveling at a constant speed of 25 m/s must not have an acceleration that exceeds 3 m/s^2. Determine the minimum radius of curvature of the curve.

Acceleration: Since the car is traveling with a constant speed, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$3 = \frac{25^2}{\rho}$$

$$\rho = 208 \text{ m} \qquad \textbf{Ans.}$$

***12–112.** At a given instant, a car travels along a circular curved road with a speed of 20 m/s while decreasing its speed at the rate of 3 m/s^2. If the magnitude of the car's acceleration is 5 m/s^2, determine the radius of curvature of the road.

Acceleration: Here, the car's tangential component of acceleration of $a_t = -3$ m/s^2. Thus,

$$a = \sqrt{a_t{}^2 + a_n{}^2}$$

$$5 = \sqrt{(-3)^2 + a_n{}^2}$$

$$a_n = 4 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$4 = \frac{20^2}{\rho}$$

$$\rho = 100 \text{ m} \qquad \textbf{Ans.}$$

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•12–113. Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s} \qquad \textbf{Ans.}$$

12–114. An automobile is traveling on a horizontal circular curve having a radius of 800 ft. If the acceleration of the automobile is 5 ft/s^2, determine the constant speed at which the automobile is traveling.

Acceleration: Since the automobile is traveling at a constant speed, $a_t = 0$. Thus, $a_n = a = 5 \text{ ft/s}^2$. Applying Eq. 12–20, $a_n = \dfrac{v^2}{\rho}$, we have

$$v = \sqrt{\rho a_n} = \sqrt{800(5)} = 63.2 \text{ ft/s} \qquad \textbf{Ans.}$$

12–115. A car travels along a horizontal circular curved road that has a radius of 600 m. If the speed is uniformly increased at a rate of 2000 km/h^2, determine the magnitude of the acceleration at the instant the speed of the car is 60 km/h.

$$a_t = \left(\frac{2000 \text{ km}}{\text{h}^2}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1\text{h}}{3600 \text{ s}}\right)^2 = 0.1543 \text{ m/s}^2$$

$$v = \left(\frac{60 \text{ km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1\text{h}}{3600 \text{ s}}\right) = 16.67 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{600} = 0.4630 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.1543^2 + 0.4630^2} = 0.488 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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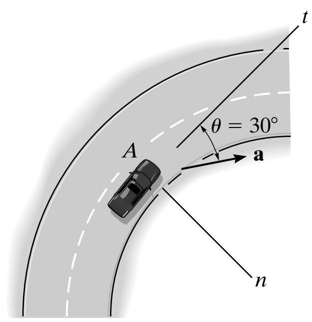
***12–116.** The automobile has a speed of 80 ft/s at point A and an acceleration **a** having a magnitude of 10 ft/s^2, acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.



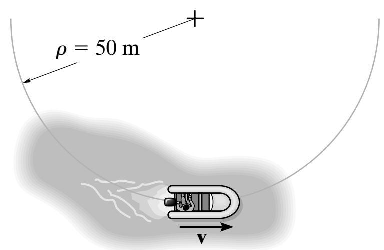
Acceleration: The tangential acceleration is

$$a_t = a\cos 30° = 10\cos 30° = 8.66\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

and the normal acceleration is $a_n = a\sin 30° = 10\sin 30° = 5.00\ \text{ft/s}^2$. Applying Eq. 12–20, $a_n = \dfrac{v^2}{\rho}$, we have

$$\rho = \frac{v^2}{a_n} = \frac{80^2}{5.00} = 1280\ \text{ft} \qquad \textbf{Ans.}$$

•12–117. Starting from rest the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.8t)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration when it has traveled 20 m.



Velocity: The time for which the boat to travel 20 m must be determined first.

$$ds = v\,dt$$

$$\int_0^{20\ \text{m}} ds = \int_0^t 0.8t\,dt$$

$$t = 7.071\ \text{s}$$

The magnitude of the boat's velocity is

$$v = 0.8\,(7.071) = 5.657\ \text{m/s} = 5.66\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential accelerations is

$$a_t = \dot{v} = 0.8\ \text{m/s}^2$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{5.657^2}{50} = 0.640\ \text{m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.8^2 + 0.640^2} = 1.02\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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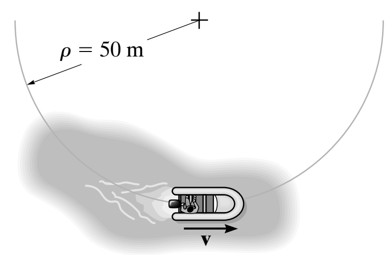
12–118. Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.2t^2)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration at the instant $t = 3$ s.



Velocity: When $t = 3$ s, the boat travels at a speed of

$$v = 0.2\left(3^2\right) = 1.80 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration is $a_t = \dot{v} = (0.4t)$ m/s². When $t = 3$ s,

$$a_t = 0.4\,(3) = 1.20 \text{ m/s}^2$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{1.80^2}{50} = 0.0648 \text{ m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.20^2 + 0.0648^2} = 1.20 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–119. A car moves along a circular track of radius 250 ft, and its speed for a short period of time $0 \le t \le 2$ s is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of the car's acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s?

$$v = 3\left(t + t^2\right)$$

$$a_t = \frac{dv}{dt} = 3 + 6t$$

When $t = 2$ s,

$$a_t = 3 + 6(2) = 15 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{\left[3(2 + 2^2)\right]^2}{250} = 1.296 \text{ ft/s}^2$$

$$a = \sqrt{(15)^2 + (1.296)^2} = 15.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$ds = v\,dt$$

$$\int ds = \int_0^2 3\left(t + t^2\right)dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3\Big]_0^2$$

$$\Delta s = 14 \text{ ft} \qquad \textbf{Ans.}$$

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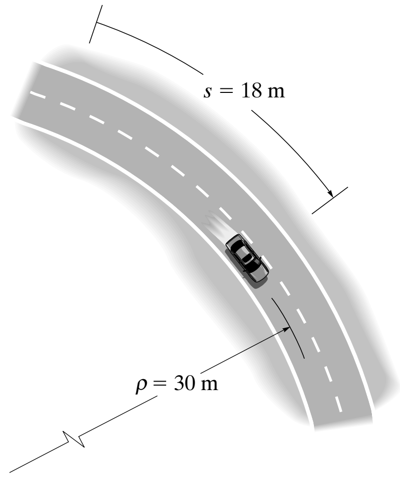
***12–120.** The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)$ m/s², where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064 \text{ s}$$

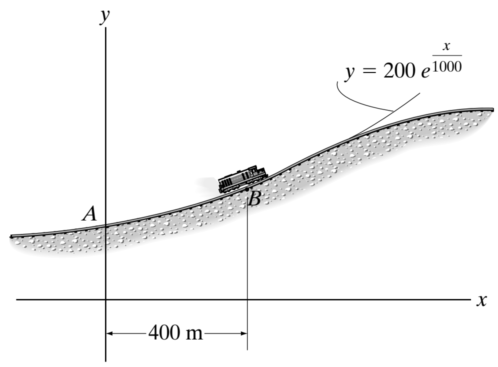
$$v = 0.5(e^{3.7064} - 1) = 19.85 \text{ m/s} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\text{ s}} = 20.35 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2 \text{ m/s}^2 \qquad \textbf{Ans.}$$

•12–121. The train passes point B with a speed of 20 m/s which is decreasing at $a_t = -0.5$ m/s². Determine the magnitude of acceleration of the train at this point.



Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 200\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2e^{\frac{x}{1000}}$$

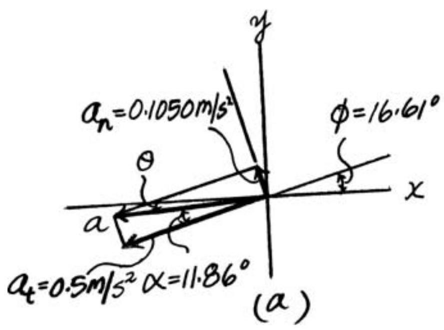
$$\frac{d^2y}{dx^2} = 0.2\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}.$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2\,e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.5 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{20^2}{3808.96} = 0.1050 \text{ m/s}^2$$



The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.1050^2} = 0.511 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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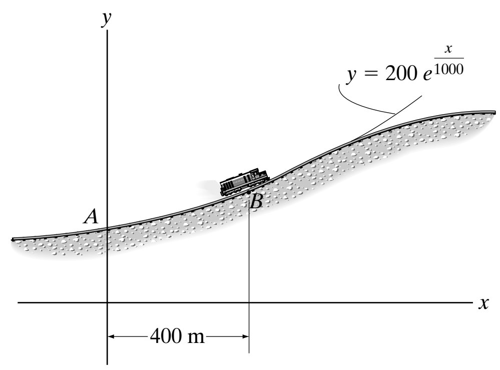
12–122. The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25\ \text{m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B, where $s_{AB} = 412$ m.



Velocity: The speed of the train at B can be determined from

$$v_B^2 = v_A^2 + 2a_t(s_B - s_A)$$

$$v_B^2 = 30^2 + 2(-0.25)(412 - 0)$$

$$v_B = 26.34\ \text{m/s}$$

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = -0.25\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{26.34^2}{3808.96} = 0.1822\ \text{m/s}^2$$

The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-0.5\right)^2 + 0.1822^2} = 0.309\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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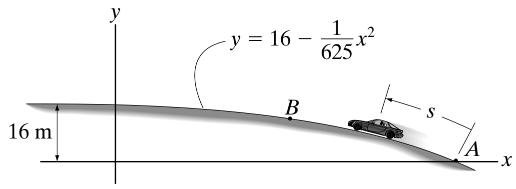
12–123. The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m.



Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B^{\,2}}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = \left(25 - 0.15s\right)\left(-0.15\right) = \left(0.225s - 3.75\right) \text{ m/s}^2$$

When the car is at B $\left(s = 51.5 \text{ m}\right)$

$$a_t = \left[0.225\left(51.5\right) - 3.75\right] = -2.591 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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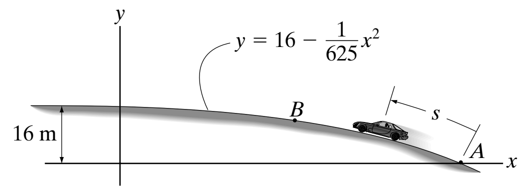
***12–124.** If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 100$ m.



Velocity: The speed of the car at C is

$$v_C{}^2 = v_A{}^2 + 2a_t\,(s_C - s_A)$$

$$v_C{}^2 = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361\ \text{m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}$$

$$a_n = \frac{v_C{}^2}{\rho} = \frac{22.361^2}{312.5} = 1.60\ \text{m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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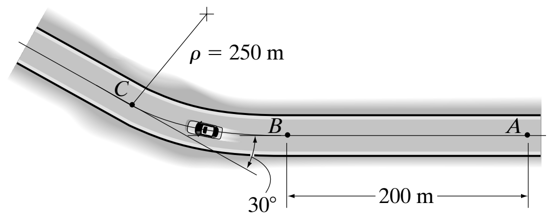
•12–125. When the car reaches point A it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (-\frac{1}{4}t^{1/2})$ m/s^2. Determine the magnitude of acceleration of the car just before it reaches point C.



Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a_t\,dt$$

$$\int_{25\text{ m/s}}^{v} dv = \int_0^t -\frac{1}{4}t^{1/2}\,dt$$

$$v = \left(25 - \frac{1}{6}t^{3/2}\right)\text{ m/s} \qquad \textbf{(1)}$$

Position: Using the initial conditions $s = 0$ when $t = 0$ s,

$$\int ds = \int v\,dt$$

$$\int_0^s ds = \int_0^t \left(25 - \frac{1}{6}t^{3/2}\right)dt$$

$$s = \left(25t - \frac{1}{15}t^{5/2}\right)\text{ m}$$

Acceleration: When the car reaches C, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus,

$$330.90 = 25t - \frac{1}{15}t^{5/2}$$

Solving by trial and error,

$$t = 15.942\text{ s}$$

Thus, using Eq. (1).

$$v_C = 25 - \frac{1}{6}(15.942)^{3/2} = 14.391\text{ m/s}$$

$$(a_t)_C = \dot{v} = -\frac{1}{4}\left(15.942^{1/2}\right) = -0.9982\text{ m/s}^2$$

$$(a_n)_C = \frac{v_C{}^2}{\rho} = \frac{14.391^2}{250} = 0.8284\text{ m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{(a_t)_C{}^2 + (a_n)_C{}^2} = \sqrt{(-0.9982)^2 + 0.8284^2} = 1.30\text{ m/s}^2 \qquad \textbf{Ans.}$$

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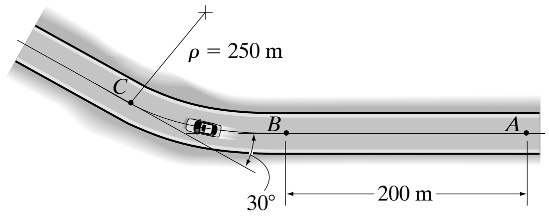
12–126. When the car reaches point A, it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (0.001s - 1)\ \text{m/s}^2$. Determine the magnitude of acceleration of the car just before it reaches point C.



Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$dv = a\,ds$$
$$\int_{25\text{ m/s}}^{v} v\,dv = \int_0^s (0.001s - 1)ds$$
$$v = \sqrt{0.001s^2 - 2s + 625}$$

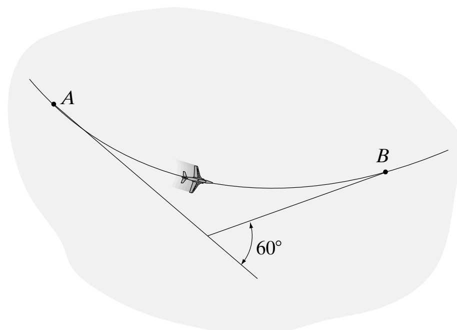
Acceleration: When the car is at point C, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus, the speed of the car at C is

$$v_C = \sqrt{0.001\left(330.90^2\right) - 2(330.90) + 625} = 8.526\ \text{m/s}^2$$
$$(a_t)_C = \dot{v} = [0.001(330.90) - 1] = -0.6691\ \text{m/s}^2$$
$$(a_n)_C = \frac{v_C{}^2}{\rho} = \frac{8.526^2}{250} = 0.2908\ \text{m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{(a_t)_C{}^2 + (a_n)_C{}^2} = \sqrt{(-0.6691)^2 + 0.2908^2} = 0.730\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–127. Determine the magnitude of acceleration of the airplane during the turn. It flies along the horizontal circular path AB in 40 s, while maintaining a constant speed of 300 ft/s.



Acceleration: From the geometry in Fig. a, $2\phi + 60° = 180°$ or $\phi = 60°$. Thus, $\frac{\theta}{2} = 90° - 60°$ or $\theta = 60° = \frac{\pi}{3}$ rad.

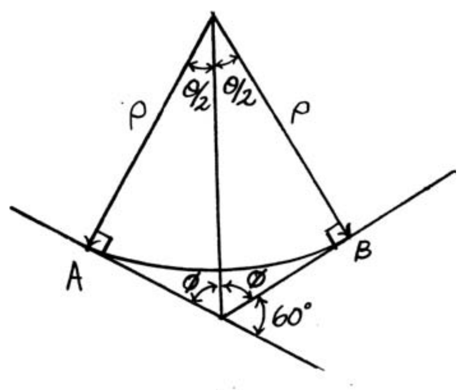
$$s_{AB} = vt = 300\left(40\right) = 12\ 000\ \text{ft}$$

Thus,

$$\rho = \frac{s_{AB}}{\theta} = \frac{12\ 000}{\pi/3} = \frac{36\ 000}{\pi}\ \text{ft}$$
$$a_n = \frac{v^2}{\rho} = \frac{300^2}{36\ 000/\pi} = 7.854\ \text{ft/s}^2$$

Since the airplane travels along the circular path with a constant speed, $a_t = 0$. Thus, the magnitude of the airplane's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{0^2 + 7.854^2} = 7.85\ \text{ft/s}^2 \qquad \textbf{Ans.}$$



(a)

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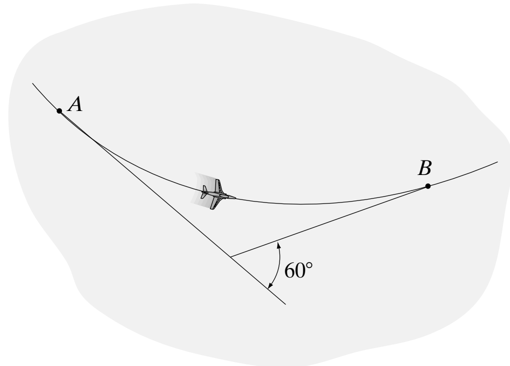
***12–128.** The airplane flies along the horizontal circular path AB in 60 s. If its speed at point A is 400 ft/s, which decreases at a rate of $a_t = (-0.1t)$ ft/s^2, determine the magnitude of the plane's acceleration when it reaches point B.



Velocity: Using the initial condition $v = 400$ ft/s when $t = 0$ s,

$$dv = a_t\,dt$$
$$\int_{400\text{ ft/s}}^{v} dv = \int_0^t -0.1t\,dt$$
$$v = \left(400 - 0.05t^2\right)\text{ ft/s}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$\int ds = \int v\,dt$$
$$\int_0^s ds = \int_0^t \left(400 - 0.05t^2\right)dt$$
$$s = \left(400t - 0.01667\,t^3\right)\text{ ft}$$

Acceleration: From the geometry, $2\phi + 60° = 180°$ or $\phi = 60°$. Thus, $\frac{\theta}{2} = 90° - 60°$ or $\theta = 60° = \frac{\pi}{3}$ rad.

$$s_{AB} = 400(60) - 0.01667\left(60^3\right) = 20\,400\text{ ft}$$
$$\rho = \frac{s_{AB}}{\theta} = \frac{20\,400}{\pi/3} = \frac{61200}{\pi}\text{ ft}$$
$$v_B = 400 - 0.05\left(60^2\right) = 220\text{ ft/s}$$
$$\left(a_n\right)_B = \frac{v_B{}^2}{\rho} = \frac{220^2}{61\,200/\pi} = 2.485\text{ ft/s}^2$$
$$\left(a_t\right)_B = \dot{v} = -0.1(60) = -6\text{ft/s}^2$$

The magnitude of the airplane's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{(-6)^2 + 2.485^2} = 6.49\text{ ft/s}^2 \qquad \textbf{Ans.}$$

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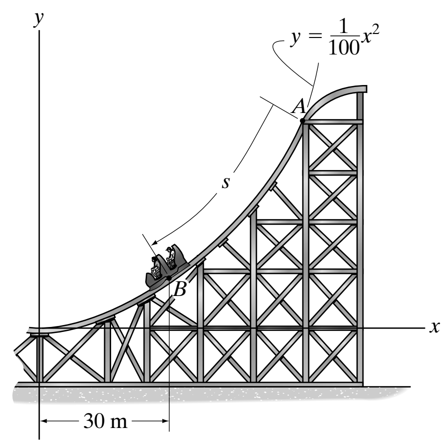
•12–129. When the roller coaster is at B, it has a speed of 25 m/s, which is increasing at $a_t = 3\ \text{m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

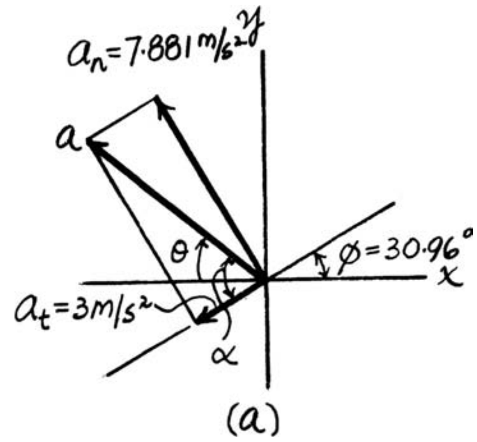
$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\Bigg|_{x=30\text{ m}} = 79.30\text{ m}$$

Acceleration:

$$a_t = \dot{v} = 3\ \text{m/s}^2$$

$$a_n = \frac{v_B{}^2}{\rho} = \frac{25^2}{79.30} = 7.881\ \text{m/s}^2$$



The magnitude of the roller coaster's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{3^2 + 7.881^2} = 8.43\ \text{m/s}^2 \qquad \textbf{Ans.}$$

The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\frac{dy}{dx}\Big|_{x=30\text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96°$.

As shown in Fig. a, $\mathbf{a}_n$ is always directed towards the center of curvature of the path. Here, $\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16°$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

$$\theta = \alpha - \phi = 38.2° \qquad \textbf{Ans.}$$

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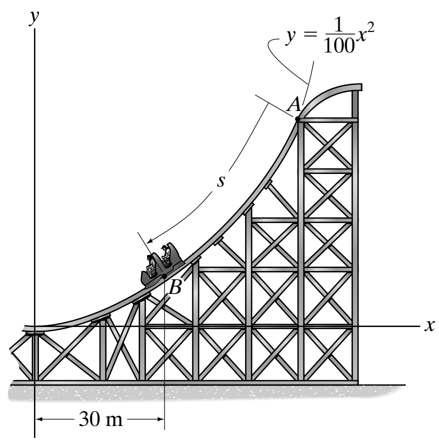
12–130. If the roller coaster starts from rest at A and its speed increases at $a_t = (6 - 0.06s)\ \text{m/s}^2$, determine the magnitude of its acceleration when it reaches B where $s_B = 40$ m.



Velocity: Using the initial condition $v = 0$ at $s = 0$,

$$dv = a_t\,dt$$

$$\int_0^v v\,dv = \int_0^s (6 - 0.06s)\,ds$$

$$v = \left(\sqrt{12s - 0.06s^2}\right)\ \text{m/s} \qquad \textbf{(1)}$$

Thus,

$$v_B = \sqrt{12(40) - 0.06(40)^2} = 19.60\ \text{m/s}$$

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 6 - 0.06(40) = 3.600\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842\ \text{m/s}^2$$

The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{{a_t}^2 + {a_n}^2} = \sqrt{3.600^2 + 4.842^2} = 6.03\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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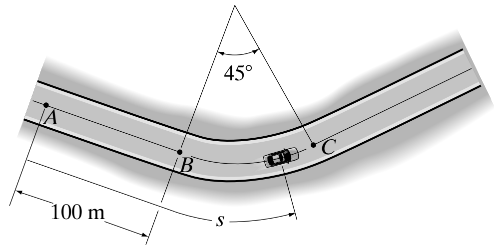
12–131. The car is traveling at a constant speed of 30 m/s. The driver then applies the brakes at A and thereby reduces the car's speed at the rate of $a_t = (-0.08v)\ \text{m/s}^2$, where v is in m/s. Determine the acceleration of the car just before it reaches point C on the circular curve. It takes 15 s for the car to travel from A to C.



Velocity: Using the initial condition $v = 30$ m/s when $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{30\text{ m/s}}^{v} -\frac{dv}{0.08v}$$

$$t = 12.5 \ln \frac{30}{v}$$

$$v = \left(30e^{-0.08t}\right)\text{ m/s} \qquad \textbf{(1)}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t 30e^{-0.08t}\,dt$$

$$s = \left[375\left(1 - e^{-0.08t}\right)\right]\text{ m} \qquad \textbf{(2)}$$

Acceleration:

$$s_C = 375\left(1 - e^{-0.08(15)}\right) = 262.05\text{ m}$$

$$s_{BC} = s_C - s_B = 262.05 - 100 = 162.05\text{ m}$$

$$\rho = \frac{s_{BC}}{\theta} = \frac{162.05}{\pi/4} = 206.33\text{ m}$$

$$v_C = 30e^{-0.08(15)} = 9.036\text{ m/s}$$

$$\left(a_n\right)_C = \frac{v_C{}^2}{\rho} = \frac{9.036^2}{206.33} = 0.3957\text{ m/s}^2$$

$$\left(a_t\right)_C = \dot{v} = -0.08(9.036) = -0.7229\text{ m/s}^2$$

The magnitude of the car's acceleration at point C is

$$a = \sqrt{(a_t)_C{}^2 + (a_n)_C{}^2} = \sqrt{(-0.7229)^2 + 0.3957^2} = 0.824\text{ m/s}^2 \qquad \textbf{Ans.}$$

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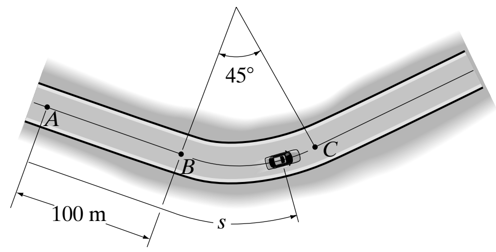
***12–132.** The car is traveling at a speed of 30 m/s. The driver applies the brakes at A and thereby reduces the speed at the rate of $a_t = \left(-\frac{1}{8}t\right)$ m/s^2, where t is in seconds. Determine the acceleration of the car just before it reaches point C on the circular curve. It takes 15 s for the car to travel from A to C.



Velocity: Using the initial condition $v = 30$ m/s when $t = 0$ s,

$$dv = a_t\,dt$$

$$\int_{30\text{ m/s}}^{v} dv = \int_0^t -\frac{1}{8}\,t\,dt$$

$$v = \left(30 - \frac{1}{16}t^2\right)\text{ m/s}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(30 - \frac{1}{16}t^2\right)dt$$

$$s = \left(30t - \frac{1}{48}t^3\right)\text{m}$$

Acceleration:

$$s_C = 30(15) - \frac{1}{48}\left(15^3\right) = 379.6875\text{ m}$$

$$s_{BC} = s_C - s_B = 379.6875 - 100 = 279.6875\text{ m}$$

$$\rho = \frac{s_{BC}}{\theta} = \frac{279.6875}{\pi/4} = 356.11\text{ m}$$

$$v_C = 30 - \frac{1}{16}\left(15^2\right) = 15.9375\text{ m/s}$$

$$\left(a_t\right)_C = \dot{v} = -\frac{1}{8}(15) = -1.875\text{ m/s}^2$$

$$\left(a_n\right)_C = \frac{v_C{}^2}{\rho} = \frac{15.9375^2}{356.11} = 0.7133\text{ m/s}^2$$

The magnitude of the car's acceleration at point C is

$$a = \sqrt{\left(a_t\right)_C{}^2 + \left(a_n\right)_C{}^2} = \sqrt{\left(-1.875\right)^2 + 0.7133^2} = 2.01\text{ m/s}^2 \qquad \textbf{Ans.}$$

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•12–133. A particle is traveling along a circular curve having a radius of 20 m. If it has an initial speed of 20 m/s and then begins to decrease its speed at the rate of $a_t = (-0.25s)\ \text{m/s}^2$, determine the magnitude of the acceleration of the particle two seconds later.

Velocity: Using the initial condition $v = 20$ m/s at $s = 0$.

$$v\,dv = a\,ds$$

$$\int_{20\text{ m/s}}^{v} v\,dv = \int_0^s -0.25\,s\,ds$$

$$v = \left(\sqrt{400 - 0.25s^2}\right)\text{ m/s}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s.

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{\sqrt{400 - 0.25s^2}} = 2\int_0^s \frac{ds}{\sqrt{1600 - s^2}}$$

$$t = 2\sin^{-3}\left(\frac{s}{40}\right)$$

$$s = \left(40\sin(t/2)\right)\text{ m}$$

When $t = 2$ s,

$$s = 40\sin(2/2) = 33.659\text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.25(33.659) = -8.415\text{ m/s}^2$$

$$v = \sqrt{400 - 0.25\left(33.659^2\right)} = 10.81\text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{10.81^2}{20} = 5.8385\text{ m/s}^2$$

The magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-8.415\right)^2 + 5.8385^2} = 10.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

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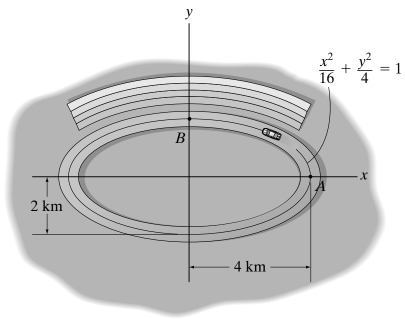
12–134. A racing car travels with a constant speed of 240 km/h around the elliptical race track. Determine the acceleration experienced by the driver at A.



Radius of Curvature:

$$y^2 = 4\left(1 - \frac{x^2}{16}\right)$$

$$2y\frac{dy}{dx} = -\frac{x}{2} \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = -\frac{x}{4y} \qquad \textbf{(2)}$$

Differentiating Eq. (1),

$$2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) + 2y\frac{d^2y}{dx^2} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{2} - 2\left(\frac{dy}{dx}\right)^2}{2y}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{1 + 4\left(\frac{dy}{dx}\right)^2}{4y}\right] \qquad \textbf{(3)}$$

Substituting Eq. (2) into Eq. (3) yields

$$\frac{d^2y}{dx^2} = -\left[\frac{4y^2 + x^2}{16y^3}\right]$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{x}{4y}\right)^2\right]^{3/2}}{\left|\frac{4y^2 + x^2}{16y^3}\right|} = \frac{\left(1 + \frac{x^2}{16y^2}\right)^{3/2}}{\left(\frac{4y^2 + x^2}{16y^3}\right)} = \frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}$$

Acceleration: Since the race car travels with a constant speed along the track, $a_t = 0$. At $x = 4$ km and $y = 0$,

$$\rho_A = \left.\frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}\right|_{\substack{x=4\text{ km}\\ y=0}} = \frac{\left(0 + 4^2\right)^{3/2}}{4\left(0 + 4^2\right)} = 1\text{ km} = 1000\text{ m}$$

The speed of the race car is

$$v = \left(240\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{h}}{3600\text{ s}}\right) = 66.67\text{ m/s}$$

Thus,

$$a_A = \frac{v^2}{\rho_A} = \frac{66.67^2}{1000} = 4.44\text{ m/s}^2 \qquad \textbf{Ans.}$$

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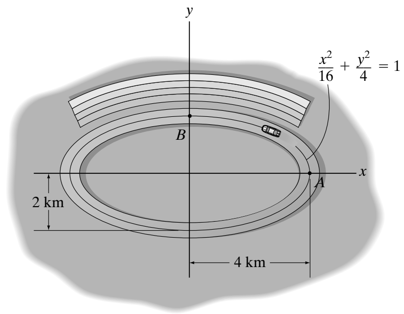
12–135. The racing car travels with a constant speed of 240 km/h around the elliptical race track. Determine the acceleration experienced by the driver at B.



Radius of Curvature:

$$y^2 = 4\left(1 - \frac{x^2}{16}\right)$$

$$2y\frac{dy}{dx} = -\frac{x}{2} \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = -\frac{x}{4y} \qquad \textbf{(2)}$$

Differentiating Eq. (1),

$$2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) + 2y\frac{d^2y}{dx^2} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{2} - 2\left(\frac{dy}{dx}\right)^2}{2y}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{1 + 4\left(\frac{dy}{dx}\right)^2}{4y}\right] \qquad \textbf{(3)}$$

Substituting Eq. (2) into Eq. (3) yields

$$\frac{d^2y}{dx^2} = -\left[\frac{4y^2 + x^2}{16\,y^3}\right]$$

Thus,

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+\left(-\frac{x}{4y}\right)^2\right]^{3/2}}{\left|\frac{4y^2 + x^2}{16y^3}\right|} = \frac{\left(1 + \frac{x^2}{16y^2}\right)^{3/2}}{\left(\frac{4y^2 + x^2}{16y^3}\right)} = \frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}$$

Acceleration: Since the race car travels with a constant speed along the track, $a_t = 0$. At $x = 0$ and $y = 2$ km

$$\rho_B = \left.\frac{\left(16y^2 + x^2\right)^{3/2}}{4\left(4y^2 + x^2\right)}\right|_{\substack{x=0 \\ y=2\text{ km}}} = \frac{\left[16\left(2^2\right) + 0\right]^{3/2}}{4\left[4\left(2^2\right) + 0\right]} = 8\text{ km} = 8000\text{ m}$$

The speed of the car is

$$v = \left(240\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 66.67\text{ m/s}$$

Thus,

$$a_B = \frac{v^2}{\rho_B} = \frac{66.67^2}{8000} = 0.556\text{ m/s}^2 \qquad \textbf{Ans.}$$

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***12–136.** The position of a particle is defined by $\mathbf{r} = \{2\sin(\frac{\pi}{4})t\,\mathbf{i} + 2\cos(\frac{\pi}{4})t\,\mathbf{j} + 3t\,\mathbf{k}\}$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration at any instant.

Velocity:

$$\mathbf{r} = \left[2\sin\left(\frac{\pi}{4}t\right)\mathbf{i} + 2\cos\left(\frac{\pi}{4}t\right)\mathbf{j} + 3t\mathbf{k}\right]\text{ m}$$

$$\mathbf{v} = \frac{dr}{dt} = \left[\frac{\pi}{2}\cos\left(\frac{\pi}{4}t\right)\mathbf{i} - \frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right)\mathbf{j} + 3\mathbf{k}\right]\text{ m/s}$$

The magnitude of the velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2} = \sqrt{\left(\frac{\pi}{2}\cos\frac{\pi}{4}t\right)^2 + \left(-\frac{\pi}{2}\sin\frac{\pi}{4}t\right)^2 + 3^2}$$

$$= \sqrt{\frac{\pi^2}{4} + 9} = 3.39\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left[-\frac{\pi^2}{8}\sin\left(\frac{\pi}{4}t\right)\mathbf{i} - \frac{\pi^2}{8}\cos\left(\frac{\pi}{4}t\right)\mathbf{j}\right]\text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = \sqrt{\left(-\frac{\pi^2}{8}\sin\frac{\pi}{4}t\right)^2 + \left(-\frac{\pi^2}{8}\cos\frac{\pi}{4}t\right)^2} = \frac{\pi^2}{8}\text{ m/s}^2 = 1.23\text{ m/s}^2 \qquad \textbf{Ans.}$$

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•12–137. The position of a particle is defined by $\mathbf{r} = \{t^3\mathbf{i} + 3t^2\mathbf{j} + 8t\mathbf{k}\}$ m, where t is in seconds. Determine the magnitude of the velocity and acceleration and the radius of curvature of the path when $t = 2$ s.

Velocity:

$$\mathbf{r} = \left[t^3\mathbf{i} + 3t^2\mathbf{j} + 8t\mathbf{k}\right] \text{ m}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left[3t^2\mathbf{i} + 6t\mathbf{j} + 8\mathbf{k}\right] \text{ m/s}$$

When $t = 2$ s,

$$\mathbf{v} = \left[3(2^2)\mathbf{i} + 6(2)\mathbf{j} + 8\mathbf{k}\right] = [12\mathbf{i} + 12\mathbf{j} + 8\mathbf{k}] \text{ m/s}$$

The magnitude of the velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2} = \sqrt{12^2 + 12^2 + 8^2} = 18.76 = 18.8 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = [6t\mathbf{i} + 6\mathbf{j}] \text{ m/s}^2$$

When $t = 2$ s,

$$a = \left[6(2)\mathbf{i} + 6\mathbf{j}\right] = [12\mathbf{i} + 6\mathbf{j}] \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = \sqrt{12^2 + 6^2 + 0^2} = 13.42 = 13.4 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Since $\mathbf{a}_t$ is parallel to $\mathbf{v}$, its magnitude can be obtained by the vector dot product $a_t = \mathbf{a} \cdot \mathbf{u}_v$, where $\mathbf{u}_v = \dfrac{\mathbf{v}}{v} = 0.6396\mathbf{i} + 0.6396\mathbf{j} + 0.4264\mathbf{k}$. Thus,

$$a_t = \left(12\mathbf{i} + 6\mathbf{j}\right) \cdot \left(0.6396\mathbf{i} + 0.6396\mathbf{j} + 0.4264\mathbf{k}\right) = 11.51 \text{ m/s}^2$$

Thus,

$$a = \sqrt{a_t{}^2 + a_n{}^2}$$

$$13.42 = \sqrt{11.51^2 + a_n{}^2}$$

$$a_n = 6.889 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$6.889 = \frac{18.76^2}{\rho}$$

$$\rho = 51.1 \text{ m} \qquad \textbf{Ans.}$$

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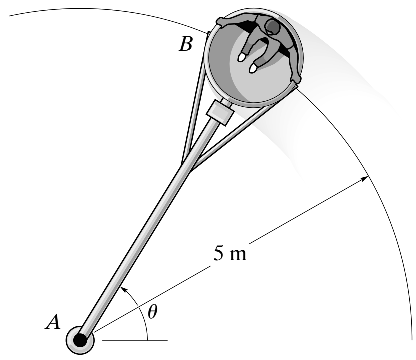
12–138. Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right) \qquad \textbf{[1]}$$

When $\theta = 30°$, the car has traveled a distance of $s = r\theta = 5\left(\dfrac{30°}{180°}\pi\right) = 2.618\ \text{m}$. The time required for the car to travel this distance can be obtained by applying $v = \dfrac{ds}{dt}$.

$$ds = v\,dt$$

$$\int_0^{2.618\ \text{m}} ds = \int_0^t 0.5\left(e^t - 1\right)dt$$

$$2.618 = 0.5\left(e^t - t - 1\right)$$

Solving by trial and error $\quad t = 2.1234\ \text{s}$

Substituting $t = 2.1234\ \text{s}$ into Eq. [1] yields

$$v = 0.5\left(e^{2.1234} - 1\right) = 3.680\ \text{m/s} = 3.68\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for the car at $t = 2.1234\ \text{s}$ is $a_t = 0.5e^{2.1234} = 4.180\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.680^2}{5} = 2.708\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.180^2 + 2.708^2} = 4.98\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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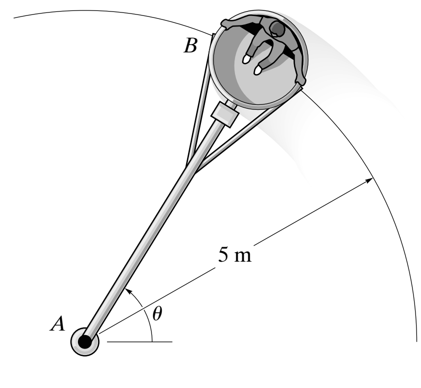
12–139. Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\text{ m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car.



Velocity: The speed v in terms of time t can be obtained by applying $a = \frac{dv}{dt}$.

$$dv = a\,dt$$
$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$
$$v = 0.5\left(e^t - 1\right)$$

When $t = 2$ s, $\quad v = 0.5\left(e^2 - 1\right) = 3.195\text{ m/s} = 3.19\text{ m/s}$ **Ans.**

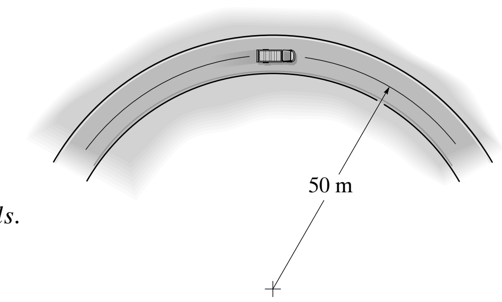
Acceleration: The tangential acceleration of the car at $t = 2$ s is $a_t = 0.5e^2 = 3.695\text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.195^2}{5} = 2.041\text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3.695^2 + 2.041^2} = 4.22\text{ m/s}^2$$ **Ans.**

***12–140.** The truck travels at a speed of 4 m/s along a circular road that has a radius of 50 m. For a short distance from $s = 0$, its speed is then increased by $a_t = (0.05s)\text{ m/s}^2$, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



Velocity: The speed v in terms of position s can be obtained by applying $v\,dv = a\,ds$.

$$v\,dv = a\,ds$$
$$\int_{4\text{ m/s}}^v v\,dv = \int_0^s 0.05s\,ds$$
$$v = \left(\sqrt{0.05s^2 + 16}\right)\text{ m/s}$$

At $s = 10$ m, $\quad v = \sqrt{0.05(10^2) + 16} = 4.583\text{ m/s} = 4.58\text{ m/s}$ **Ans.**

Acceleration: The tangential acceleration of the truck at $s = 10$ m is $a_t = 0.05\,(10) = 0.500\text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{4.583^2}{50} = 0.420\text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.500^2 + 0.420^2} = 0.653\text{ m/s}^2$$ **Ans.**

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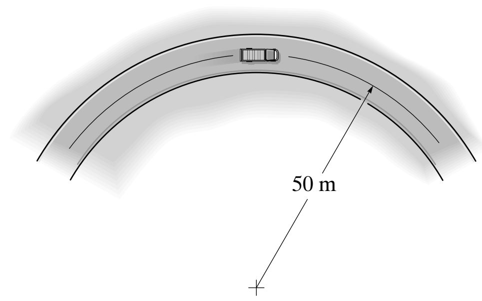
•12–141. The truck travels along a circular road that has a radius of 50 m at a speed of 4 m/s. For a short distance when $t = 0$, its speed is then increased by $a_t = (0.4t)\ \mathrm{m/s^2}$, where t is in seconds. Determine the speed and the magnitude of the truck's acceleration when $t = 4$ s.



Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_{4\ \mathrm{m/s}}^{v} dv = \int_0^t 0.4t\,dt$$

$$v = \left(0.2t^2 + 4\right)\ \mathrm{m/s}$$

When $t = 4$ s, $\quad v = 0.2\left(4^2\right) + 4 = 7.20\ \mathrm{m/s}$ **Ans.**

Acceleration: The tangential acceleration of the truck when $t = 4$ s is $a_t = 0.4(4) = 1.60\ \mathrm{m/s^2}$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{7.20^2}{50} = 1.037\ \mathrm{m/s^2}$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.60^2 + 1.037^2} = 1.91\ \mathrm{m/s^2}$$ **Ans.**

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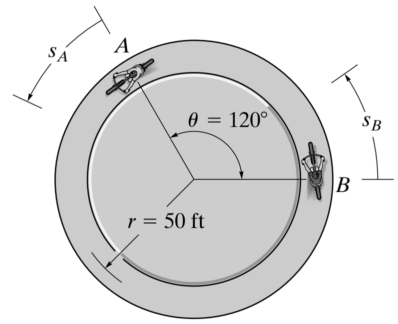
12–142. Two cyclists, A and B, are traveling counterclockwise around a circular track at a constant speed of 8 ft/s at the instant shown. If the speed of A is increased at $(a_t)_A = (s_A)$ ft/s^2, where s_A is in feet, determine the distance measured counterclockwise along the track from B to A between the cyclists when $t = 1$ s. What is the magnitude of the acceleration of each cyclist at this instant?



Distance Traveled: Initially, the distance between the cyclists is $d_0 = \rho\theta = 50\left(\frac{120°}{180°}\pi\right) = 104.72$ ft. When $t = 1$ s, cyclist B travels a distance of $s_B = 8(1) = 8$ ft. The distance traveled by cyclist A can be obtained as follows

$$v_A dv_A = a_A\, ds_A$$

$$\int_{8\text{ ft/s}}^{v_A} v_A\, dv_A = \int_0^{s_A} s_A\, ds_A$$

$$v_A = \sqrt{s_A^2 + 64} \qquad \textbf{[1]}$$

$$dt = \frac{ds_A}{v_A}$$

$$\int_0^{1s} dt = \int_0^{s_A} \frac{ds_A}{\sqrt{s_A^2 + 64}}$$

$$1 = \sinh^{-1}\left(\frac{s_A}{8}\right)$$

$$s_A = 9.402 \text{ ft}$$

Thus, the distance between the two cyclists after $t = 1$ s is

$$d = d_0 + s_A - s_B = 104.72 + 9.402 - 8 = 106 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for cyclist A and B at $t = 1$ s is $(a_t)_A = s_A = 9.402$ ft/s^2 and $(a_t)_B = 0$ (cyclist B travels at constant speed), respectively. At $t = 1$ s, from Eq. [1], $v_A = \sqrt{9.402^2 + 64} = 12.34$ ft/s. To determine normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{12.34^2}{50} = 3.048 \text{ ft/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{50} = 1.28 \text{ ft/s}^2$$

The magnitude of the acceleration for cyclist A and B are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{9.402^2 + 3.048^2} = 9.88 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0^2 + 1.28^2} = 1.28 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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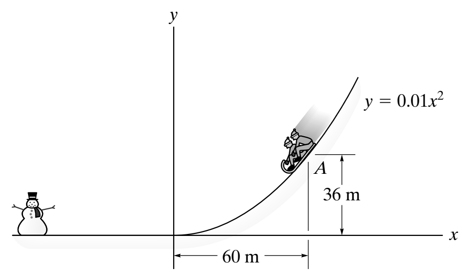
12–143. A toboggan is traveling down along a curve which can be approximated by the parabola $y = 0.01x^2$. Determine the magnitude of its acceleration when it reaches point A, where its speed is $v_A = 10$ m/s, and it is increasing at the rate of $(a_t)_A = 3$ m/s^2.



Acceleration: The radius of curvature of the path at point A must be determined first. Here, $\dfrac{dy}{dx} = 0.02x$ and $\dfrac{d^2y}{dx^2} = 0.02$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (0.02x)^2]^{3/2}}{|0.02|}\right|_{x=60\text{ m}} = 190.57\text{ m}$$

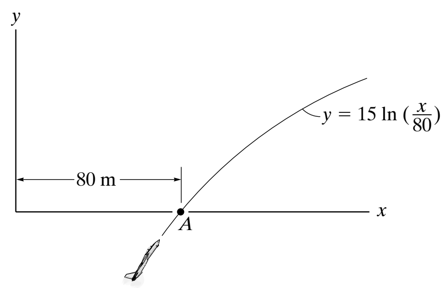
To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{10^2}{190.57} = 0.5247\text{ m/s}^2$$

Here, $a_t = \dot{v}_A = 3$ m/s. Thus, the magnitude of accleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 0.5247^2} = 3.05\text{ m/s}^2 \qquad \textbf{Ans.}$$

***12–144.** The jet plane is traveling with a speed of 120 m/s which is decreasing at 40 m/s^2 when it reaches point A. Determine the magnitude of its acceleration when it is at this point. Also, specify the direction of flight, measured from the x axis.



$$y = 15\ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \left.\frac{15}{x}\right|_{x=80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\left.\frac{15}{x^2}\right|_{x=80\text{ m}} = -0.002344$$

$$\rho\Big|_{x=80\text{ m}} = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=80\text{ m}}$$

$$= \frac{[1 + (0.1875)^2]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(120)^2}{449.4} = 32.04\text{ m/s}^2$$

$$a_t = -40\text{ m/s}^2$$

$$a = \sqrt{(-40)^2 + (32.04)^2} = 51.3\text{ m/s}^2 \qquad \textbf{Ans.}$$

Since

$$\frac{dy}{dx} = \tan\theta = 0.1875$$

$$\theta = 10.6° \qquad \textbf{Ans.}$$

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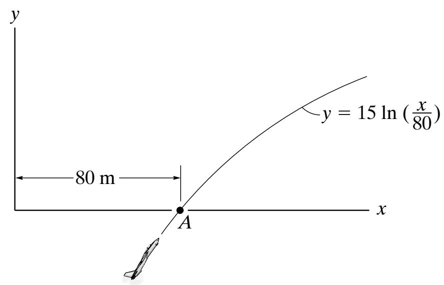
•12–145. The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point A ($y = 0$).



$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x=80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x=80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x=80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=80\text{ m}}$$

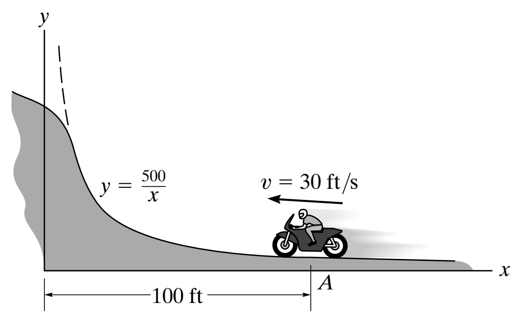
$$= \frac{\left[1 + (0.1875)^2\right]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9\text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 26.9\text{ m/s}^2 \qquad \textbf{Ans.}$$

12–146. The motorcyclist travels along the curve at a constant speed of 30 ft/s. Determine his acceleration when he is located at point A. Neglect the size of the motorcycle and rider for the calculation.



$$\frac{dy}{dx} = -\frac{500}{x^2}\bigg|_{x=100\text{ ft}} = -0.05$$

$$\frac{d^2y}{dx^2} = \frac{1000}{x^3}\bigg|_{x=100\text{ ft}} = 0.001$$

$$\rho\bigg|_{x=100\text{ ft}} = \frac{[1 + \left(\frac{dy}{dx}\right)^2]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ ft}}$$

$$= \frac{[1 + (-0.05)^2]^{3/2}}{|0.001|} = 1003.8\text{ ft}$$

$$a_n = \frac{v^2}{\rho} = \frac{30^2}{1003.8} = 0.897\text{ ft/s}^2$$

Since the motorcyclist travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 0.897\text{ ft/s}^2 \qquad \textbf{Ans.}$$

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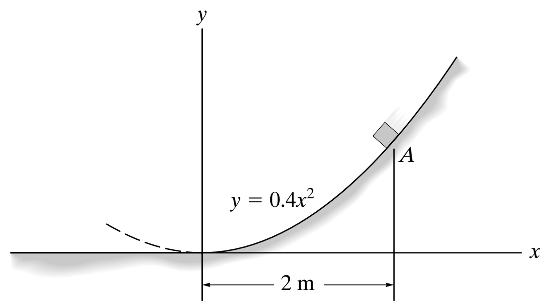
12–147. The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v_B = 8$ m/s and the increase in speed is $dv_B/dt = 4$ m/s^2. Determine the magnitude of the acceleration of the box at this instant.



$$y = 0.4\,x^2$$

$$\left.\frac{dy}{dx}\right|_{x=2\text{ m}} = 0.8x\Big|_{x=2\text{ m}} = 1.6$$

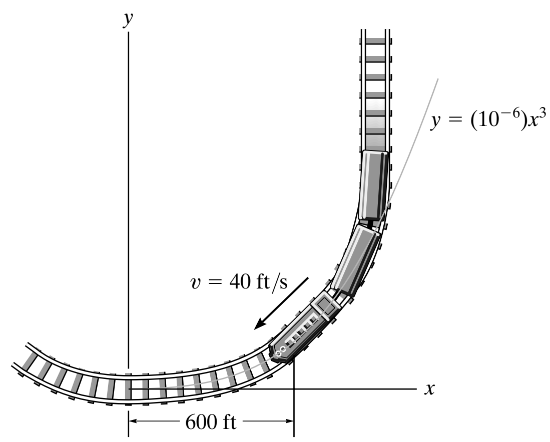
$$\left.\frac{d^2y}{dx^2}\right|_{x=2\text{ m}} = 0.8$$

$$\rho = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=2\text{ m}} = \frac{\left[1 + (1.6)^2\right]^{3/2}}{|0.8|} = 8.396\text{ m}$$

$$a_n = \frac{{v_B}^2}{\rho} = \frac{8^2}{8.396} = 7.622\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61\text{ m/s}^2$$ **Ans.**

***12–148.** A spiral transition curve is used on railroads to connect a straight portion of the track with a curved portion. If the spiral is defined by the equation $y = (10^{-6})x^3$, where x and y are in feet, determine the magnitude of the acceleration of a train engine moving with a constant speed of 40 ft/s when it is at point $x = 600$ ft.



$$y = (10)^{-6}x^3$$

$$\left.\frac{dy}{dx}\right|_{x=600\text{ ft}} = 3(10)^{-6}x^2\Big|_{x=600\text{ ft}} = 1.08$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=600\text{ ft}} = 6(10)^{-6}x\Big|_{x=600\text{ ft}} = 3.6(10)^{-3}$$

$$\rho\Big|_{x=600\text{ ft}} = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=600\text{ ft}} = \frac{[1 + (1.08)^2]^{3/2}}{|3.6(10)^{-3}|} = 885.7\text{ ft}$$

$$a_n = \frac{v^2}{\rho} = \frac{40^2}{885.7} = 1.81\text{ ft/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0 + (1.81)^2} = 1.81\text{ ft/s}^2$$ **Ans.**

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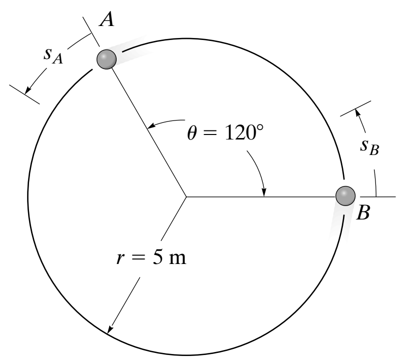
•12–149. Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A)\ \text{m/s}^2$, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



Distance Traveled: Initially the distance between the particles is

$$d_0 = \rho d\theta = 5\left(\frac{120°}{180°}\right)\pi = 10.47 \text{ m}$$

When $t = 1$ s, B travels a distance of

$$d_B = 8(1) = 8 \text{ m}$$

The distance traveled by particle A is determined as follows:

$$vdv = ads$$

$$\int_{8\text{ m/s}}^{v} vdv = \int_0^s 0.4\, sds$$

$$v = 0.6325\sqrt{s^2 + 160} \qquad \textbf{(1)}$$

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.6325\sqrt{s^2 + 160}}$$

$$1 = \frac{1}{0.6325}\left(\text{In}\left[\frac{\sqrt{s^2 + 160} + s}{\sqrt{160}}\right]\right)$$

$$s = 8.544 \text{ m}$$

Thus the distance between the two cyclists after $t = 1$ s is

$$d = 10.47 + 8.544 - 8 = 11.0 \text{ m} \qquad \textbf{Ans.}$$

Acceleration:

For A, when $t = 1$ s,

$$(a_t)_A = \dot{v}_A = 0.4(8.544) = 3.4176 \text{ m/s}^2$$

$$v_A = 0.6325\sqrt{8.544^2 + 160} = 9.655 \text{ m/s}$$

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{9.655^2}{5} = 18.64 \text{ m/s}^2$$

The magnitude of the A's acceleration is

$$a_A = \sqrt{3.4176^2 + 18.64^2} = 19.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

For B, when $t = 1$ s,

$$(a_t)_B = \dot{v}_A = 0$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2$$

The magnitude of the B's acceleration is

$$a_B = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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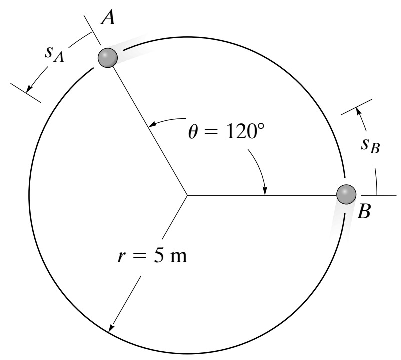
12–150. Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4\ \text{m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t\ \text{m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



Distance Traveled: Initially the distance between the two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)\,t^2 = \left(8t + 2t^2\right)\text{ m} \qquad \textbf{[1]}$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\,dt$$

$$\int_{8\text{ m/s}}^{v_A} dv_A = \int_0^t 0.8\,t dt$$

$$v_A = \left(0.4t^2 + 8\right)\ \text{m/s} \qquad \textbf{[2]}$$

$$ds_A = v_A\,dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right) dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $\quad t = 2.5074\text{ s} = 2.51\text{ s}$ **Ans.**

Note: If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6\text{ s} > 2.51\text{ s}$.

Acceleration: The tangential acceleration for particle A and B when $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8\,(2.5074) = 2.006\ \text{m/s}^2$ and $(a_t)_B = 4\ \text{m/s}^2$, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11\ \text{m/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01\ \text{m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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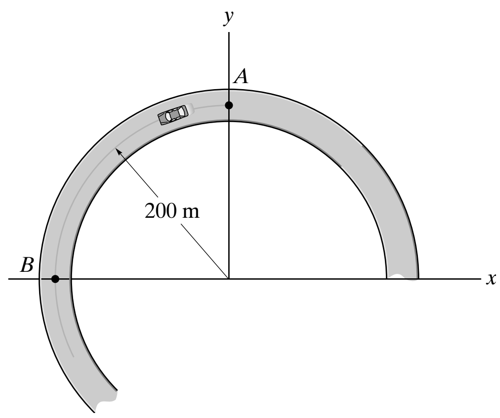
12–151. The race car travels around the circular track with a speed of 16 m/s. When it reaches point A it increases its speed at $a_t = (\frac{4}{3}v^{1/4})$ m/s^2, where v is in m/s. Determine the magnitudes of the velocity and acceleration of the car when it reaches point B. Also, how much time is required for it to travel from A to B?



$$a_t = \frac{4}{3}v^{\frac{1}{4}}$$

$$dv = a_t\ dt$$

$$dv = \frac{4}{3}v^{\frac{1}{4}}\ dt$$

$$\int_{16}^{v} 0.75\frac{dv}{v^{\frac{1}{4}}} = \int_0^t dt$$

$$v^{\frac{1}{4}}\Big|_{16}^{v} = t$$

$$v^{\frac{3}{4}} - 8 = t$$

$$v = (t + 8)^{\frac{4}{3}}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (t + 8)^{\frac{4}{3}}\,dt$$

$$s = \frac{3}{7}(t + 8)^{\frac{2}{3}}\Big|_0^t$$

$$s = \frac{3}{7}(t + 8)^{\frac{2}{3}} - 54.86$$

For $s = \frac{\pi}{2}(200) = 100\pi = \frac{3}{7}(t + 8)^{\frac{7}{3}} - 54.86$

$t = 10.108\ s = 10.1\ s$ **Ans.**

$v = (10.108 + 8)^{\frac{4}{3}} = 47.551 = 47.6$ m/s **Ans.**

$$a_t = \frac{4}{3}(47.551)^{\frac{1}{4}} = 3.501 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(47.551)^2}{200} = 11.305 \text{ m/s}^2$$

$a = \sqrt{(3.501)^2 + (11.305)^2} = 11.8$ m/s^2 **Ans.**

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***12–152.** A particle travels along the path $y = a + bx + cx^2$, where a, b, c are constants. If the speed of the particle is constant, $v = v_0$, determine the x and y components of velocity and the normal component of acceleration when $x = 0$.

$$y = a + bx + cx^2$$

$$\dot{y} = b\dot{x} + 2\,c\,x\,\dot{x}$$

$$\ddot{y} = b\ddot{x} + 2c\,(\dot{x})^2 + 2c\,x\ddot{x}$$

When $x = 0$, $\quad \dot{y} = b\,\dot{x}$

$$v_0^2 + \dot{x}^2 + b^2\,\dot{x}^2$$

$$v_x = \dot{x} = \frac{v_0}{\sqrt{1 + b^2}}$$ **Ans.**

$$v_y = \frac{v_0\,b}{\sqrt{1 + b^2}}$$ **Ans.**

$$a_n = \frac{v_0^2}{\rho}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\frac{dy}{dx} = b + 2c\,x$$

$$\frac{d^2y}{dx^2} = 2\,c$$

At $x = 0$, $\quad \rho = \dfrac{(1 + b^2)^{3/2}}{2\,c}$

$$a_n = \frac{2\,c\,v_0^2}{(1 + b^2)^{3/2}}$$ **Ans.**

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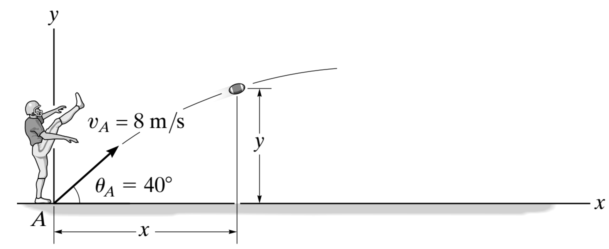
•**12–153.** The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25$ s.



Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8\cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

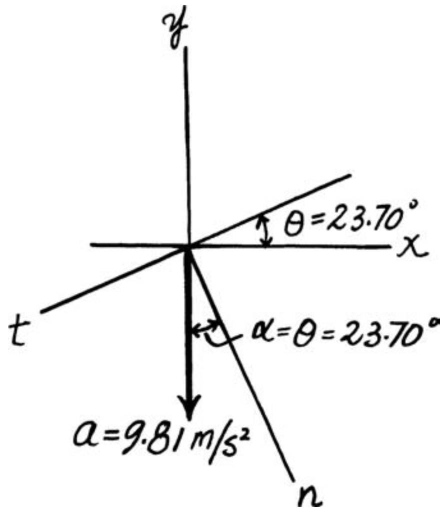
$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad \textbf{[1]}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8\sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)\left(t^2\right) \qquad \textbf{[2]}$$



Eliminate t from Eqs [1] and [2], we have

$$y = \{0.8391x - 0.1306x^2\}\text{ m} = \{0.839x - 0.131x^2\}\text{ m} \qquad \textbf{Ans.}$$

The tangent of the path makes an angle $\theta = \tan^{-1}\frac{3.644}{4} = 42.33°$ with the x axis.

Acceleration: When $t = 0.25$ s, from Eq. [1], $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1}0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s^2 and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = a\sin\alpha = 9.81\sin 23.70° = 3.94\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = a\cos\alpha = 9.81\cos 23.70° = 8.98\text{ m/s}^2 \qquad \textbf{Ans.}$$

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12–154. The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

Velocity: Here, $\mathbf{r} = \left\{\left(2t + t^2\right)\mathbf{i} + t^2\mathbf{j}\right\}$ m. To determine the velocity $\mathbf{v}$, apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\,\mathbf{i} + 2t\mathbf{j}\}\ \text{m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2} = 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \qquad \text{and} \qquad v_t = 7.21\ \text{m/s} \qquad \textbf{Ans.}$$

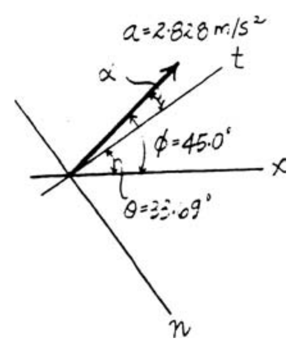
The velocity $\mathbf{v}$ makes an angle $\theta = \tan^{-1}\frac{4}{6} = 33.69°$ with the x axis.

Acceleration: To determine the acceleration $\mathbf{a}$, apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\}\ \text{m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828\ \text{m/s}^2$$



The acceleration $\mathbf{a}$ makes an angle $\phi = \tan^{-1}\frac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a \sin\alpha = 2.828 \sin 11.31° = 0.555\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_t = a \cos\alpha = 2.828 \cos 11.31° = 2.77\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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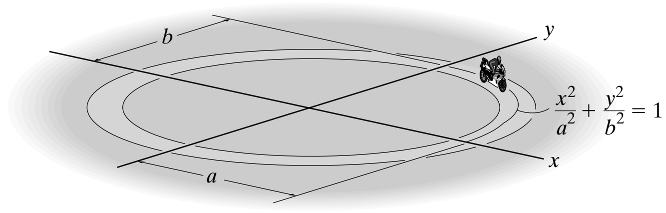
12–155. The motorcycle travels along the elliptical track at a constant speed v. Determine the greatest magnitude of the acceleration if $a > b$.



Acceleration: Differentiating twice the expression $y = \frac{b}{a}\sqrt{a^2 - x^2}$, we have

$$\frac{dy}{dx} = -\frac{bx}{a\sqrt{a^2 - x^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{ab}{(a^2 - x^2)^{3/2}}$$

The radius of curvature of the path is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{bx}{a\sqrt{a^2 - x^2}}\right)^2\right]^{3/2}}{\left|-\frac{ab}{(a^2 - x^2)^{3/2}}\right|} = \frac{\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}}{\frac{ab}{(a^2 - x^2)^{3/2}}} \qquad \textbf{[1]}$$

To have the maximum normal acceleration, the radius of curvature of the path must be a minimum. By observation, this happens when $y = 0$ and $x = a$. When $x \to a$, $\frac{b^2x^2}{a^2(a^2 - x^2)} >>> 1$. Then, $\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} \to \left[\frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} = \frac{b^3x^3}{a^3(a^2 - x^2)^{3/2}}$.

Substituting this value into Eq. [1] yields $\rho = \frac{b^2}{a^4}x^3$. At $x = a$,

$$\rho = \frac{b^2}{a^4}\left(a^3\right) = \frac{b^2}{a}$$

To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_{\max} = \frac{v^2}{\rho} = \frac{v^2}{b^2/a} = \frac{a}{b^2}v^2$$

Since the motorcycle is traveling at a constant speed, $a_t = 0$. Thus,

$$a_{\max} = (a_n)_{\max} = \frac{a}{b^2}v^2 \qquad \textbf{Ans.}$$

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***12–156.** A particle moves along a circular path of radius 300 mm. If its angular velocity is $\dot{\theta} = (2t^2)$ rad/s, where t is in seconds, determine the magnitude of the particle's acceleration when $t = 2$ s.

Time Derivatives:

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = 2t^2\big|_{t=2\text{ s}} = 8 \text{ rad/s} \qquad \ddot{\theta} = 4t\big|_{t=2\text{ s}} = 8 \text{ rad/s}^2$$

Velocity: The radial and transverse components of the particle's velocity are

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 0.3(8) = 2.4 \text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{0^2 + 2.4^2} = 2.4 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.3\left(8^2\right) = -19.2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3(8) + 0 = 2.4 \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-19.2)^2 + 2.4^2} = 19.3 \text{ m/s}^2$$ **Ans.**

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•12–157. A particle moves along a circular path of radius 300 mm. If its angular velocity is $\dot{\theta} = (3t^2)$ rad/s, where t is in seconds, determine the magnitudes of the particle's velocity and acceleration when $\theta = 45°$. The particle starts from rest when $\theta = 0°$.

Time Derivatives: Using the initial condition $\theta = 0°$ when $t = 0$ s,

$$d\theta = 3t^2 dt$$

$$\int_0^\theta d\theta = \int_0^t 3t^2\,dt$$

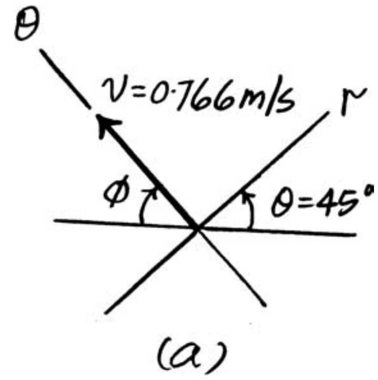
$$\theta = \left(t^3\right)\text{ rad}$$

At $\theta = 45° = \dfrac{\pi}{4}$ rad,

$$\frac{\pi}{4} = t^3 \qquad t = 0.9226\text{ s}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = 3t^2\Big|_{t=0.9226\text{ s}} = 2.554\text{ rad/s} \qquad \ddot{\theta} = 6t\Big|_{t=0.9226\text{ s}} = 5.536\text{ rad/s}^2$$

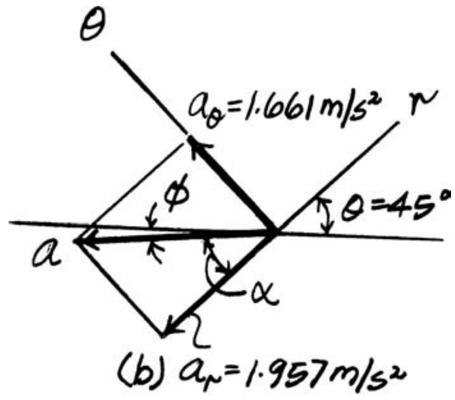


Velocity:

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 0.3(2.554) = 0.7661\text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r{}^2 + v_\theta{}^2} = \sqrt{0^2 + 0.7661^2} = 0.766\text{ m/s} \qquad \textbf{Ans.}$$



Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.3\left(2.554^2\right) = -1.957\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3(5.536) + 0 = 1.661\text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2} = \sqrt{(-1.957)^2 + 1.661^2} = 2.57\text{ m/s}^2 \qquad \textbf{Ans.}$$

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12–158. A particle moves along a circular path of radius 5 ft. If its position is $\theta = (e^{0.5t})$ rad, where t is in seconds, determine the magnitude of the particle's acceleration when $\theta = 90°$.

Time Derivative: At $\theta = 90° = \dfrac{\pi}{2}$ rad,

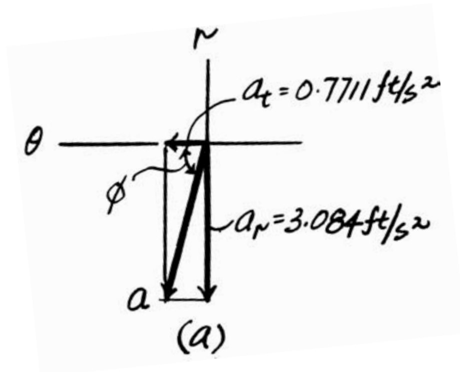
$$\frac{\pi}{2} = e^{0.5t} \qquad\qquad t = 0.9032 \text{ s}$$

Using the result of t, the value of the first and second time derivative of r and θ are

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = 0.5e^{0.5t}\Big|_{t=0.9032\text{ s}} = 0.7854 \text{ rad/s}$$

$$\ddot{\theta} = 0.25e^{0.5t}\Big|_{t=0.9032\text{ s}} = 0.3927 \text{ rad/s}^2$$



Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5\left(0.7854^2\right) = -3.084 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(0.3927) + 0 = 1.963 \text{ ft/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2} = \sqrt{(-3.084)^2 + 1.963^2} = 3.66 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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12–159. The position of a particle is described by $r = (t^3 + 4t - 4)$ mm and $\theta = (t^{3/2})$ rad, where t is in seconds. Determine the magnitudes of the particle's velocity and acceleration at the instant $t = 2$ s.

Time Derivatives: The first and second time derivative of r and θ when $t = 2$ s are

$$r = t^3 + 4t - 4\Big|_{t=2\text{ s}} = 12\text{ m} \qquad \theta = t^{3/2}$$

$$\dot{r} = 3t^2 + 4\Big|_{t=2\text{ s}} = 16\text{ m/s} \qquad \dot{\theta} = \frac{3}{2}t^{1/2}\Bigg|_{t=2\text{ s}} = 2.121\text{ rad/s}$$

$$\ddot{r} = 6t\Big|_{t=2\text{ s}} = 12\text{ m/s}^2 \qquad \ddot{\theta} = \frac{3}{4}t^{-1/2}\Bigg|_{t=2\text{ s}} = 0.5303\text{ rad/s}^2$$

Velocity:

$$v_r = \dot{r} = 16\text{ m/s} \qquad v_\theta = r\dot{\theta} = 12(2.121) = 25.46\text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2} = \sqrt{16^2 + 25.46^2} = 30.1\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 12 - 12\left(2.121^2\right) = -42.0\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 12(0.5303) + 2(16)(2.121) = 74.25\text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2} = \sqrt{(-42.0)^2 + 74.25^2} = 85.3\text{ m/s}^2 \qquad \textbf{Ans.}$$

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***12–160.** The position of a particle is described by $r = (300e^{-0.5t})$ mm and $\theta = (0.3t^2)$ rad, where t is in seconds. Determine the magnitudes of the particle's velocity and acceleration at the instant $t = 1.5$ s.

Time Derivatives: The first and second time derivative of r and θ when $t = 1.5$ s are

$$r = 300e^{-0.5t}\big|_{t-1.5\text{ s}} = 141.77 \text{ mm} \qquad \theta = 0.3t^2 \text{ rad}$$

$$\dot{r} = -150e^{-0.5t}\big|_{t-1.5\text{ s}} = -70.85 \text{ mm/s} \qquad \dot{\theta} = 0.6t\big|_{t-1.5\text{ s}} = 0.9 \text{ rad/s}$$

$$\ddot{r} = 75e^{-0.5t}\big|_{t-1.5\text{ s}} = 35.43 \text{ mm/s}^2 \qquad \ddot{\theta} = 0.6 \text{ rad/s}^2$$

Velocity:

$$v_r = \dot{r} = -70.85 \text{ mm/s} \qquad v_\theta = r\dot{\theta} = 141.71(0.9) = 127.54 \text{ mm/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r{}^2 + v_\theta{}^2} = \sqrt{(-70.85)^2 + 127.54^2} = 146 \text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 35.43 - 141.71\left(0.9^2\right) = -79.36 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 141.71(0.6) + 2(-70.85)(0.9) = -42.51 \text{ mm/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2} = \sqrt{(-79.36)^2 + (-42.51)^2} = 90.0 \text{ mm/s}^2 \qquad \textbf{Ans.}$$

•12–161. An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h^2. If the propeller has a diameter of 6 ft and is rotating at an angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

$$v_{Pl} = \left(\frac{200 \text{ mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 293.3 \text{ ft/s}$$

$$a_{Pl} = \left(\frac{3 \text{ mi}}{\text{h}^2}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 = 0.001\,22 \text{ ft/s}^2$$

$$v_{Pr} = 120(3) = 360 \text{ ft/s}$$

$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464 \text{ ft/s} \qquad \textbf{Ans.}$$

$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43\,200 \text{ ft/s}^2$$

$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.001\,22)^2 + (43\,200)^2} = 43.2(10^3) \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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12–162. A particle moves along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = (\cos 2t)$ rad, where t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

When $\theta = \frac{\pi}{6}$ rad, $\quad \frac{\pi}{6} = \cos 2t \quad t = 0.5099$ s

$$\dot{\theta} = \frac{d\theta}{dt} = -2\sin 2t\Big|_{t=0.5099\text{ s}} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4\cos 2t\Big|_{t=0.5099\text{ s}} = -2.0944 \text{ rad/s}^2$$

$$r = 4 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2$$ **Ans.**

12–163. A particle travels around a limaçon, defined by the equation $r = b - a\cos\theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

$$r = b - a\cos\theta$$

$$\dot{r} = a\sin\theta\dot{\theta}$$

$$\dot{r} = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$

$$v_r = \dot{r} = a\sin\theta\dot{\theta}$$ **Ans.**

$$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta}$$ **Ans.**

$$a_r = \ddot{r} - r\dot{\theta}^2 = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta} - (b - a\cos\theta)\dot{\theta}^2$$

$$= (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\,\ddot{\theta}$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} = (b - a\cos\theta)\ddot{\theta} + 2\left(a\sin\theta\dot{\theta}\right)\dot{\theta}$$

$$= (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta$$ **Ans.**

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***12–164.** A particle travels around a lituus, defined by the equation $r^2\theta = a^2$, where a is a constant. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

$$r^2\theta = a^2$$

$$r = a\theta^{-\frac{1}{2}}$$

$$r = a\left(-\frac{1}{2}\right)\theta^{-\frac{3}{2}}\dot{\theta}.$$

$$\ddot{r} = -\frac{1}{2}a\left(-\frac{3}{2}\theta^{-\frac{3}{2}}\dot{\theta}^2 + \theta^{-\frac{3}{2}}\ddot{\theta}\right)$$

$$v_r = \dot{r} = -\frac{1}{2}a\theta^{-\frac{3}{2}}\dot{\theta} \qquad \textbf{Ans.}$$

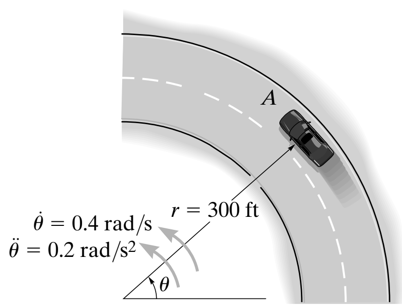
$$v_\theta = r\dot{\theta} = a\theta^{-\frac{1}{2}}\dot{\theta} \qquad \textbf{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{1}{2}a\left(-\frac{3}{2}\theta^{-\frac{3}{2}}\dot{\theta}^2 + \theta^{-\frac{3}{2}}\ddot{\theta}\right) - a\theta^{-\frac{1}{2}}\dot{\theta}^2$$

$$= a\left[\left(\frac{3}{4}\theta^{-2} - 1\right)\theta^{-\frac{1}{2}}\dot{\theta}^2 - \frac{1}{2}\theta^{-\frac{3}{2}}\ddot{\theta}\right] \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = a\theta^{-\frac{1}{2}}\ddot{\theta} + 2(a)\left(-\frac{1}{2}\right)\theta^{-\frac{3}{2}}\dot{\theta}\left(\dot{\theta}\right) = a\left[\ddot{\theta} - \frac{\dot{\theta}^2}{\theta}\right]\theta^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

•12–165. A car travels along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s^2. Determine the magnitudes of the car's velocity and acceleration at this instant.



Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300\left(0.4^2\right) = -48.0 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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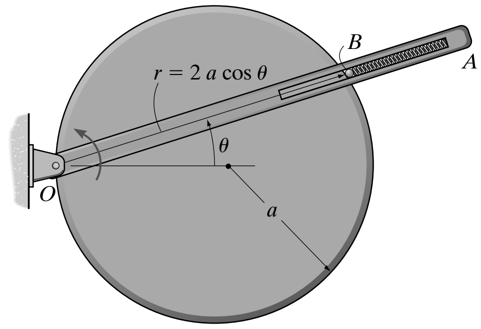
12–166. The slotted arm OA rotates counterclockwise about O with a constant angular velocity of $\dot{\theta}$. The motion of pin B is constrained such that it moves on the fixed circular surface and along the slot in OA. Determine the magnitudes of the velocity and acceleration of pin B as a function of θ.



Time Derivatives:

$$r = 2a\cos\theta$$

$$\dot{r} = -2a\sin\theta\dot{\theta}$$

$$\ddot{r} = -2a\left[\cos\theta\dot{\theta}\dot{\theta} + \sin\theta\ddot{\theta}\right] = -2a\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right]$$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Thus,

$$\ddot{r} = -2a\cos\theta\dot{\theta}^2$$

Velocity:

$$v_r = \dot{r} = -2a\sin\theta\dot{\theta} \qquad v_\theta = r\dot{\theta} = 2a\cos\theta\dot{\theta}$$

Thus, the magnitude of the pin's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{\left(-2a\sin\theta\dot{\theta}\right)^2 + \left(2a\cos\theta\dot{\theta}\right)^2}$$

$$= \sqrt{4a^2\dot{\theta}^2\left(\sin^2\theta + \cos^2\theta\right)} = 2a\dot{\theta} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2a\cos\theta\dot{\theta}^2 - 2a\cos\theta\dot{\theta}^2 = -4a\cos\theta\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\left(-2a\sin\theta\dot{\theta}\right)\dot{\theta} = -4a\sin\theta\dot{\theta}^2$$

Thus, the magnitude of the pin's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{\left(-4a\cos\theta\dot{\theta}^2\right)^2 + \left(-4a\sin\theta\dot{\theta}^2\right)^2}$$

$$= \sqrt{16a^2\dot{\theta}^4\left(\cos^2\theta + \sin^2\theta\right)} = 4a\dot{\theta}^2 \qquad \textbf{Ans.}$$

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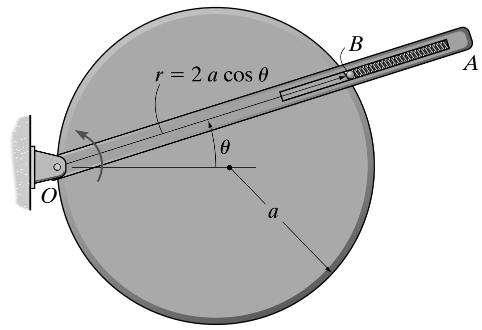
12–167. The slotted arm OA rotates counterclockwise about O such that when $\theta = \pi/4$, arm OA is rotating with an angular velocity of $\dot{\theta}$ and an angular acceleration of $\ddot{\theta}$. Determine the magnitudes of the velocity and acceleration of pin B at this instant. The motion of pin B is constrained such that it moves on the fixed circular surface and along the slot in OA.



Time Derivatives:

$$r = 2a\cos\theta$$

$$\dot{r} = -2a\sin\theta\dot{\theta}$$

$$\ddot{r} = -2a\left[\cos\theta\dot{\theta}\dot{\theta} + \sin\theta\ddot{\theta}\right] = -2a\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right]$$

When $\theta = \dfrac{\pi}{4}$ rad,

$$r\big|_{\theta=\frac{\pi}{4}} = 2a\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}a$$

$$\dot{r}\big|_{\theta=\frac{\pi}{4}} = -2a\left(\frac{1}{\sqrt{2}}\right)\dot{\theta} = -\sqrt{2}a\dot{\theta}$$

$$\ddot{r}\big|_{\theta=\frac{\pi}{4}} = -2a\left(\frac{1}{\sqrt{2}}\dot{\theta}^2 + \frac{1}{\sqrt{2}}\ddot{\theta}\right) = -\sqrt{2}a\left(\dot{\theta}^2 + \ddot{\theta}\right)$$

Velocity:

$$v_r = \dot{r} = -\sqrt{2}a\dot{\theta} \qquad v_\theta = r\dot{\theta} = \sqrt{2}a\dot{\theta}$$

Thus, the magnitude of the pin's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{\left(-\sqrt{2}a\dot{\theta}\right)^2 + \left(\sqrt{2}a\dot{\theta}\right)^2} = 2a\dot{\theta}$$ **Ans.**

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\sqrt{2}a\left(\dot{\theta}^2 + \ddot{\theta}\right) - \sqrt{2}a\dot{\theta}^2 = -\sqrt{2}a\left(2\dot{\theta}^2 + \ddot{\theta}\right)$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \sqrt{2}a\ddot{\theta} + 2\left(-\sqrt{2}a\dot{\theta}\right)\left(\dot{\theta}\right) = \sqrt{2}a\left(\ddot{\theta} - 2\dot{\theta}^2\right)$$

Thus, the magnitude of the pin's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{\left[-\sqrt{2}a\left(2\dot{\theta}^2 + \ddot{\theta}\right)\right]^2 + \left[\sqrt{2}a\left(\ddot{\theta} - 2\dot{\theta}^2\right)\right]^2}$$

$$= 2a\sqrt{4\dot{\theta}^4 + \ddot{\theta}^2}$$ **Ans.**

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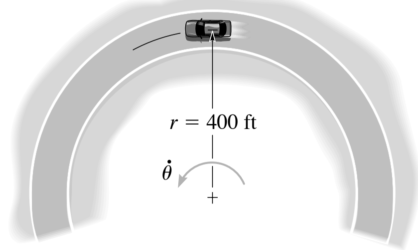
***12–168.** The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s^2. Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.



$r = 400 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

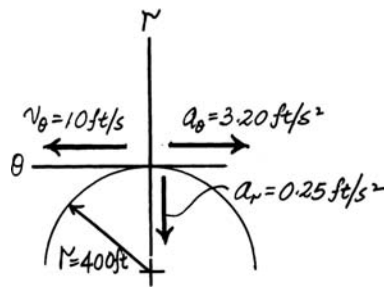
$\dot{\theta} = 0.025 \qquad \theta = -0.008$

$v_r = \dot{r} = 0$ **Ans.**

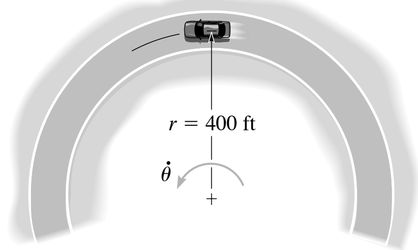
$v_\theta = r\dot{\theta} = 400(0.025) = 10$ ft/s **Ans.**

$a_r = r - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25$ ft/s^2 **Ans.**

$a_\theta = r\,\theta + 2\,r\,\theta = 400(-0.008) + 0 = -3.20$ ft/s^2 **Ans.**



•12–169. The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.



$r = 400 \text{ ft} \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = r = 0 \qquad v_\theta = r\,\theta = 400\left(\theta\right)$

$v = \sqrt{(0)^2 + \left(400\,\dot{\theta}\right)^2} = 30$

$\theta = 0.075$ rad/s **Ans.**

$\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25$ ft/s^2

$a_\theta = r\theta + 2\dot{r}\theta = 400(0) + 2(0)(0.075) = 0$

$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25$ ft/s^2 **Ans.**

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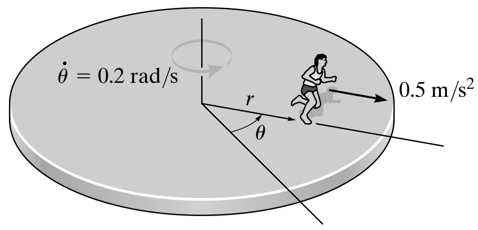
12–170. Starting from rest, the boy runs outward in the radial direction from the center of the platform with a constant acceleration of $0.5\ \text{m/s}^2$. If the platform is rotating at a constant rate $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of the velocity and acceleration of the boy when $t = 3$ s. Neglect his size.



Velocity: When $t = 3$ s, the position of the boy is given by

$$s = (s_0)_r + (v_0)_r\, t + \frac{1}{2}(a_c)_r\, t^2$$

$$r = 0 + 0 + \frac{1}{2}(0.5)\left(3^2\right) = 2.25 \text{ m}$$

The boy's radial component of velocity is given by

$$v_r = (v_0)_r + (a_c)_r\, t$$

$$= 0 + 0.5(3) = 1.50 \text{ m/s} \qquad \textbf{Ans.}$$

The boy's transverse component of velocity is given by

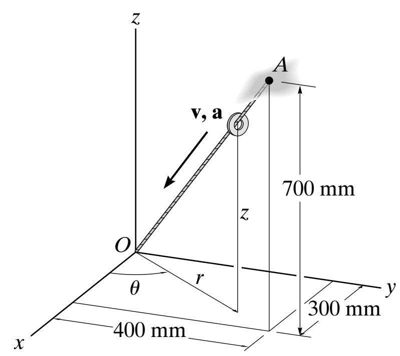
$$v_\theta = r\dot{\theta} = 2.25(0.2) = 0.450 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: When $t = 3$ s, $r = 2.25$ m, $\dot{r} = v_r = 1.50$ m/s, $\ddot{r} = 0.5\ \text{m/s}^2$, $\ddot{\theta} = 0$. Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.5 - 2.25\left(0.2^2\right) = 0.410 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.25(0) + 2(1.50)(0.2) = 0.600 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–171. The small washer slides down the cord OA. When it is at the midpoint, its speed is 200 mm/s and its acceleration is $10\ \text{mm/s}^2$. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



$$OA = \sqrt{(400)^2 + (300)^2 + (700)^2} = 860.23 \text{ mm}$$

$$OB = \sqrt{(400)^2 + (300)^2} = 500 \text{ mm}$$

$$v_r = (200)\left(\frac{500}{860.23}\right) = 116 \text{ mm/s}$$

$$v_\theta = 0$$

$$v_z = (200)\left(\frac{700}{860.23}\right) = 163 \text{ mm/s}$$

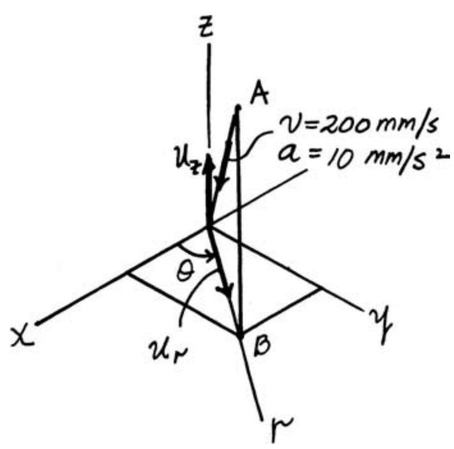
Thus, $\mathbf{v} = \{-116\mathbf{u}_r - 163\mathbf{u}_z\}$ mm/s **Ans.**

$$a_r = 10\left(\frac{500}{860.23}\right) = 5.81$$

$$a_\theta = 0$$

$$a_z = 10\left(\frac{700}{860.23}\right) = 8.14$$

Thus, $\mathbf{a} = \{-5.81\mathbf{u}_r - 8.14\mathbf{u}_z\}\ \text{mm/s}^2$ **Ans.**



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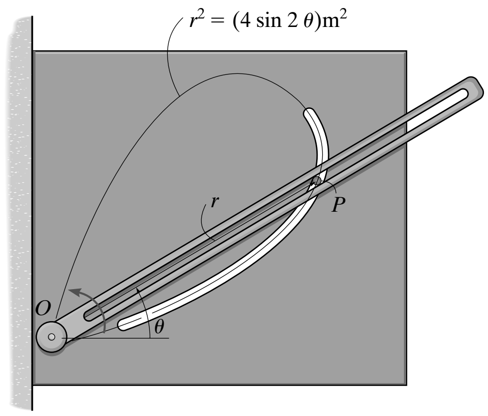
***12–172.** If arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. The peg moves in the fixed groove defined by the lemniscate, and along the slot in the arm.



Time Derivatives:

$$r^2 = 4 \sin 2\theta$$

$$2r\dot{r} = 8 \cos 2\theta\dot{\theta}$$

$$\dot{r} = \left[\frac{4 \cos 2\theta\dot{\theta}}{r}\right] \text{m/s} \qquad \dot{\theta} = 2 \text{ rad/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = 8\left(-2 \sin 2\theta\dot{\theta}^2 + \cos 2\theta\ddot{\theta}\right)$$

$$\ddot{r} = \left[\frac{4\left(\cos 2\theta\ddot{\theta} - 2 \sin 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right] \text{m/s}^2 \qquad \ddot{\theta} = 0$$

At $\theta = 30°$,

$$r\big|_{\theta=30°} = \sqrt{4 \sin 60°} = 1.861 \text{ m}$$

$$\dot{r}\big|_{\theta=30°} = \frac{(4 \cos 60°)(2)}{1.861} = 2.149 \text{ m/s}$$

$$\ddot{r}\big|_{\theta=30°} = \frac{4\left[0 - 2 \sin 60°\left(2^2\right)\right] - (2.149)^2}{1.861} = -17.37 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 2.149 \text{ m/s} \qquad v_\theta = r\dot{\theta} = 1.861(2) = 3.722 \text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r{}^2 + v_\theta{}^2} = \sqrt{2.149^2 + 3.722^2} = 4.30 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -17.37 - 1.861\left(2^2\right) = -24.82 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(2.149)(2) = 8.597 \text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2} = \sqrt{(-24.82)^2 + 8.597^2} = 26.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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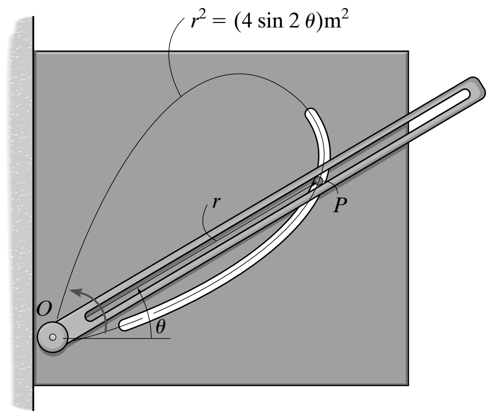
•12–173. The peg moves in the curved slot defined by the lemniscate, and through the slot in the arm. At $\theta = 30°$, the angular velocity is $\dot{\theta} = 2$ rad/s, and the angular acceleration is $\ddot{\theta} = 1.5$ rad/s^2. Determine the magnitudes of the velocity and acceleration of peg P at this instant.



Time Derivatives:

$$2r\dot{r} = 8\cos 2\theta\dot{\theta}$$

$$\dot{r} = \left(\frac{4\cos 2\theta\dot{\theta}}{r}\right)\text{m/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = 8\left(-2\sin 2\theta\ddot{\theta} + \cos 2\theta\dot{\theta}^2\right)$$

$$\ddot{r} = \left[\frac{4\left(\cos 2\theta\ddot{\theta} - 2\sin 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right]\text{m/s}^2 \qquad \ddot{\theta} = 1.5\text{ rad/s}^2$$

At $\theta = 30°$,

$$r\big|_{\theta=30°} = \sqrt{4\sin 60°} = 1.861\text{ m}$$

$$\dot{r}\big|_{\theta=30°} = \frac{(4\cos 60°)(2)}{1.861} = 2.149\text{ m/s}$$

$$\ddot{r}\big|_{\theta=30°} = \frac{4\left[\cos 60°(1.5) - 2\sin 60°\left(2^2\right)\right] - (2.149)^2}{1.861} = -15.76\text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 2.149\text{ m/s} \qquad v_\theta = r\dot{\theta} = 1.861(2) = 3.722\text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{2.149^2 + 3.722^2} = 4.30\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -15.76 - 1.861\left(2^2\right) = -23.20\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.861(1.5) + 2(2.149)(2) = 11.39\text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-23.20)^2 + 11.39^2} = 25.8\text{ m/s}^2 \qquad \textbf{Ans.}$$

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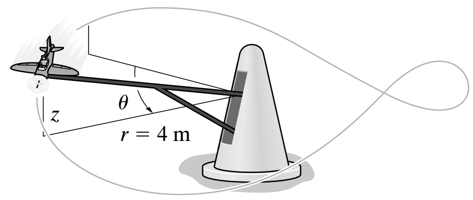
12–174. The airplane on the amusement park ride moves along a path defined by the equations $r = 4$ m, $\theta = (0.2t)$ rad, and $z = (0.5 \cos \theta)$ m, where t is in seconds. Determine the cylindrical components of the velocity and acceleration of the airplane when $t = 6$ s.



$r = 4$ m $\qquad \theta = 0.2t\big|_{t=6\text{ s}} = 1.2$ rad

$\dot{r} = 0 \qquad \dot{\theta} = 0.2$ rad/s

$\ddot{r} = 0 \qquad \ddot{\theta} = 0$

$z = 0.5 \cos \theta \qquad \dot{z} = -0.5 \sin \theta \dot{\theta}\big|_{\theta = 1.2\text{ rad}} = -0.0932$ m/s

$\ddot{z} = -0.5\left[\cos \theta \dot{\theta}^2 + \sin \theta \ddot{\theta}\right]\Big|_{\theta=1.2\text{ rad}} = -0.007247\ \text{m/s}^2$

$v_r = \dot{r} = 0$ **Ans.**

$v_\theta = r\dot{\theta} = 4(0.2) = 0.8$ m/s **Ans.**

$v_z = \dot{z} = -0.0932$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(0.2)^2 = -0.16\ \text{m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(0) + 2(0)(0.2) = 0$ **Ans.**

$a_z = \ddot{z} = -0.00725\ \text{m/s}^2$ **Ans.**

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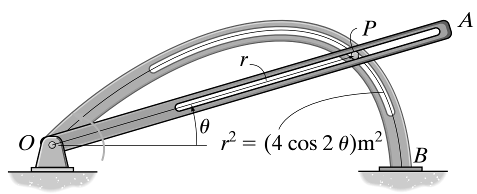
12–175. The motion of peg P is constrained by the lemniscate curved slot in OB and by the slotted arm OA. If OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 3$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$.



Time Derivatives:

$$r^2 = 4\cos 2\theta$$

$$2r\dot{r} = -8\sin 2\theta\dot{\theta}$$

$$\dot{r} = \left(\frac{-4\sin 2\theta\dot{\theta}}{r}\right)\text{ m/s} \qquad \dot{\theta} = 3\text{ rad/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = -8\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\,\dot{\theta}^2\right)$$

$$\ddot{r} = \left[\frac{-4\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right]\text{ m/s}^2 \qquad \ddot{\theta} = 0$$

$\dot{\theta} = 3$ rad/s. Thus, when $\theta = 30°$,

$$r\big|_{\theta=30°} = \sqrt{4\cos 60°} = \sqrt{2}\text{ m}$$

$$\dot{r}\big|_{\theta=30°} = \frac{-4\sin 60°(3)}{\sqrt{2}} = -7.348\text{ m/s}$$

$$\ddot{r}\big|_{\theta=30°} = \frac{-4\left[0 + 2\cos 60°(3)^2\right] - (-7.348)^2}{\sqrt{2}} = -63.64\text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = -7.348\text{ m/s} \qquad v_\theta = r\dot{\theta} = \sqrt{2}(3) = 4.243\text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{7.348^2 + (-4.243)^2} = 8.49\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -63.64 - \sqrt{2}(3)^2 = -76.37\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-7.348)(3) = -44.09\text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-76.37)^2 + (-44.09)^2} = 88.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

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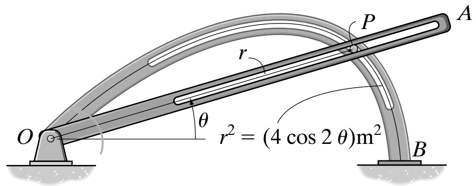
***12–176.** The motion of peg P is constrained by the lemniscate curved slot in OB and by the slotted arm OA. If OA rotates counterclockwise with an angular velocity of $\dot{\theta} = (3t^{3/2})$ rad/s, where t is in seconds, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. When $t = 0, \theta = 0°$.



Time Derivatives:

$$r^2 = 4\cos 2\theta$$

$$2r\dot{r} = -8\sin 2\theta\dot{\theta}$$

$$\dot{r} = \left(\frac{-4\sin 2\theta\dot{\theta}}{r}\right)\text{ m/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = -8\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\,\dot{\theta}^2\right)$$

$$\ddot{r} = \left[\frac{-4\left(\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right]\text{ m/s}^2$$

$$\frac{d\theta}{dt} = \dot{\theta} = 3t^{3/2}$$

$$\int_{0°}^{\theta} d\theta = \int_0^t 3t^{3/2}dt$$

$$\theta = \left(\frac{6}{5}t^{5/2}\right)\text{ rad}$$

At $\theta = 30° = \dfrac{\pi}{6}$ rad,

$$\frac{\pi}{6} = \frac{6}{5}t^{5/2} \qquad t = 0.7177\text{ s}$$

$$\dot{\theta} = 3t^{3/2}\Big|_{t=0.7177\text{ s}} = 1.824\text{ rad/s}$$

$$\ddot{\theta} = \frac{9}{2}t^{1/2}\Big|_{t=0.7177\text{ s}} = 3.812\text{ rad/s}^2$$

Thus,

$$r\big|_{\theta=30°} = \sqrt{4\cos 60°} = \sqrt{2}\text{ m}$$

$$\dot{r}\big|_{\theta=30°} = \frac{-4\sin 60°(1.824)}{\sqrt{2}} = -4.468\text{ m/s}$$

$$\ddot{r}\big|_{\theta=30°} = \frac{-4\left[\sin 60°(3.812) + 2\cos 60°(1.824)^2\right] - (-4.468)^2}{\sqrt{2}} = -32.86\text{ m/s}^2$$

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12–176. Continued

Velocity:

$$v_r = \dot{r} = -4.468 \text{ m/s} \qquad v_\theta = r\dot{\theta} = \sqrt{2}(1.824) = 2.579 \text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2} = \sqrt{(-4.468)^2 + 2.579^2} = 5.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -32.86 - \sqrt{2}(1.824)^2 = -37.57 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \sqrt{2}(3.812) + 2(-4.468)(1.824) = -10.91 \text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2} = \sqrt{(-37.57)^2 + (-10.91)^2} = 39.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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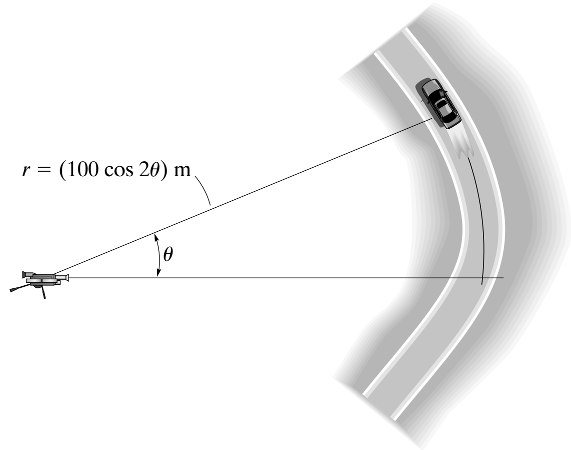
•12–177. The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15°$.



Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 \sin 2\theta\dot{\theta})\ \text{m/s}$$

At $\theta = 15°$,

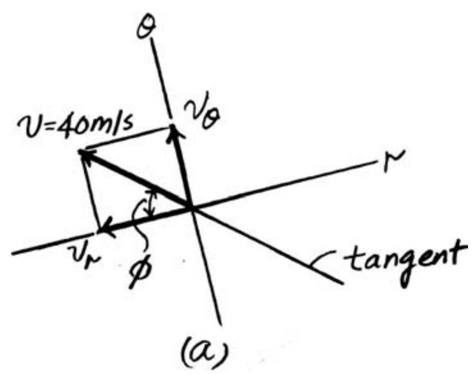
$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60\ \text{m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta}\ \text{m/s}$$

Velocity: Referring to Fig. *a*, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$



and

$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.3024\ \text{rad/s} = 0.302\ \text{rad/s} \qquad \textbf{Ans.}$$

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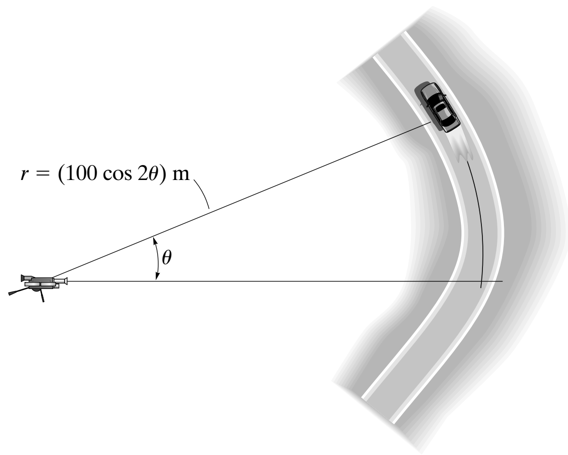
12–178. When $\theta = 15°$, the car has a speed of 50 m/s which is increasing at 6 m/s^2. Determine the angular velocity of the camera tracking the car at this instant.



Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 \sin 2\theta\dot{\theta}) \text{ m/s}$$

$$\ddot{r} = -200\left[\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right] \text{ m/s}^2$$

At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

$$\dot{r}\big|_{\theta=15°} = -200\left[\sin 30°\ddot{\theta} + 2\cos 30°\dot{\theta}^2\right] = \left(-100\ddot{\theta} - 346.41\dot{\theta}^2\right) \text{ m/s}^2$$

Velocity: Referring to Fig. *a*, $v_r = -50 \cos \phi$ and $v_\theta = 50 \sin \phi$. Thus,

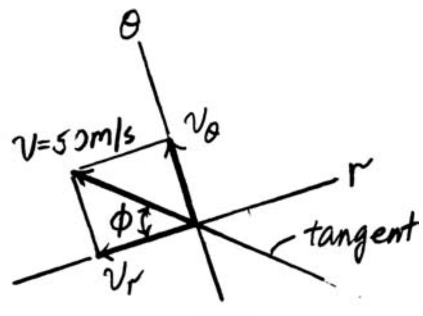
$$v_r = \dot{r}$$

$$-50 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$50 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$



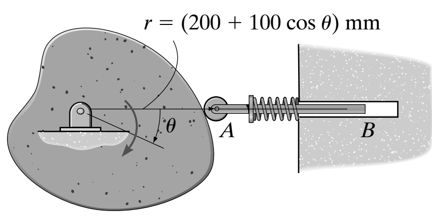
(a)

Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.378 \text{ rad/s} \qquad \textbf{Ans.}$$

12–179. If the cam rotates clockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB at the instant $\theta = 30°$. The surface of the cam has a shape of limaçon defined by $r = (200 + 100 \cos \theta)$ mm.



Time Derivatives:

$$r = (200 + 100 \cos \theta) \text{ mm}$$

$$\dot{r} = (-100 \sin \theta\dot{\theta}) \text{ mm/s} \qquad \dot{\theta} = 5 \text{ rad/s}$$

$$\ddot{r} = -100\left[\sin \theta\ddot{\theta} + \cos \theta\dot{\theta}^2\right] \text{ mm/s}^2 \qquad \ddot{\theta} = 0$$

When $\theta = 30°$,

$$r\big|_{\theta=30°} = 200 + 100 \cos 30° = 286.60 \text{ mm}$$

$$\dot{r}\big|_{\theta=30°} = -100 \sin 30°(5) = -250 \text{ mm/s}$$

$$\dot{r}\big|_{\theta=30°} = -100\left[0 + \cos 30°\left(5^2\right)\right] = -2165.06 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = -250 \text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2156.06 - 286.60(5^2) = -9330 \text{ mm/s}^2 \qquad \textbf{Ans.}$$

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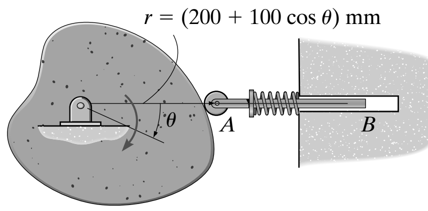
***12–180.** At the instant $\theta = 30°$, the cam rotates with a clockwise angular velocity of $\dot{\theta} = 5$ rad/s and and angular acceleration of $\ddot{\theta} = 6$ rad/s^2. Determine the magnitudes of the velocity and acceleration of the follower rod AB at this instant. The surface of the cam has a shape of a limaçon defined by $r = (200 + 100 \cos \theta)$ mm.



Time Derivatives:

$$r = (200 - 100 \cos \theta) \text{ mm}$$

$$\dot{r} = (-100 \sin \theta \dot{\theta}) \text{ mm/s}$$

$$\dot{r} = -100\left[\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2\right] \text{ mm/s}^2$$

When $\theta = 30°$,

$$r\big|_{\theta=30°} = 200 + 100 \cos 30° = 286.60 \text{ mm}$$

$$\dot{r}\big|_{\theta=30°} = -100 \sin 30°(5) = -250 \text{ mm/s}$$

$$\ddot{r}\big|_{\theta=30°} = -100\left[\sin 30°(6) + \cos 30°\left(5^2\right)\right] = -2465.06 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = -250 \text{ mm/s} \qquad \textbf{Ans.}$$

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2465.06 - 286.60\left(5^2\right) = -9630 \text{ mm/s}^2 \qquad \textbf{Ans.}$$

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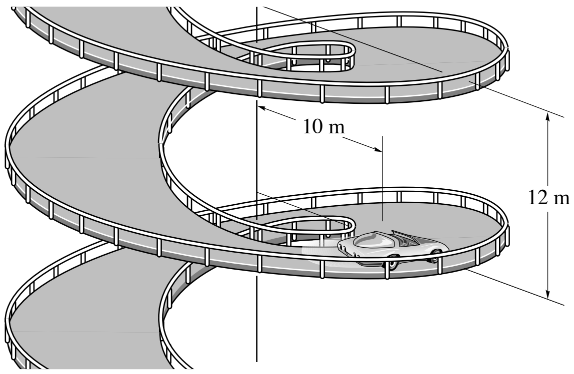
•**12–181.** The automobile travels from a parking deck down along a cylindrical spiral ramp at a constant speed of $v = 1.5$ m/s. If the ramp descends a distance of 12 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the car's acceleration as it moves along the ramp, $r = 10$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle of $\phi = \tan^{-1}(12/[2\pi(10)]) = 10.81°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.



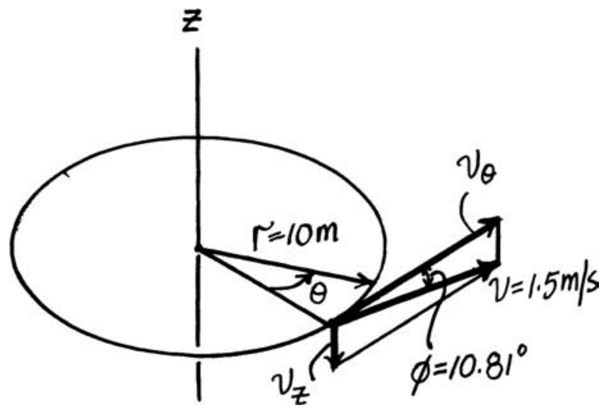
$$\phi = \tan^{-1}\left(\frac{12}{2\pi(10)}\right) = 10.81°$$

$$v = 1.5 \text{ m/s}$$

$$v_r = 0$$

$$v_\theta = 1.5 \cos 10.81° = 1.473 \text{ m/s}$$

$$v_z = -1.5 \sin 10.81° = -0.2814 \text{ m/s}$$



Since

$$r = 10 \qquad \dot{r} = 0 \qquad r = 0$$

$$v_\theta = r\,\dot{\theta} = 1.473 \qquad \theta = \frac{1.473}{10} = 0.1473$$

Since $\theta = 0$

$$a_r = r - \dot{r}\theta^2 = 0 - 10(0.1473)^2 = -0.217$$

$$a_\theta = \dot{r}\ddot{\theta} + 2r\theta = 10(0) + 2(0)(0.1473) = 0$$

$$a_z = \ddot{z} = 0$$

$$a = \sqrt{(-0.217)^2 + (0)^2 + (0)^2} = 0.217 \text{ m/s}^2$$ **Ans.**

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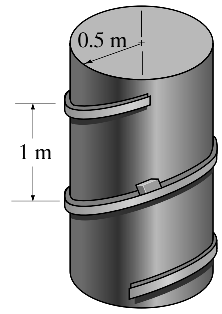
12–182. The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



Velocity: The inclination angle of the ramp is $\phi = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\left[\frac{1}{2\pi(0.5)}\right] = 17.66°$.

Thus, from Fig. a, $v_\theta = 2\cos 17.66° = 1.906$ m/s and $v_z = 2\sin 17.66° = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

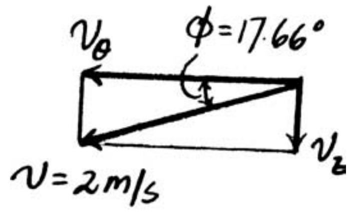
$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$

Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$



(a)

Since $\mathbf{v}_z$ is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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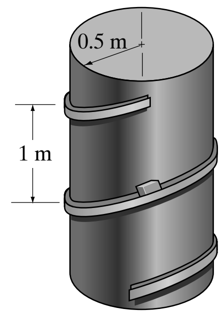
12–183. The box slides down the helical ramp which is defined by $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right)\text{rad/s} \qquad \ddot{\theta} = 3(3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \qquad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}\big|_{t=2.325 \text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}\big|_{t=2.325 \text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}\big|_{t=2.325 \text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}\big|_{t=2.325 \text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2 + {v_z}^2} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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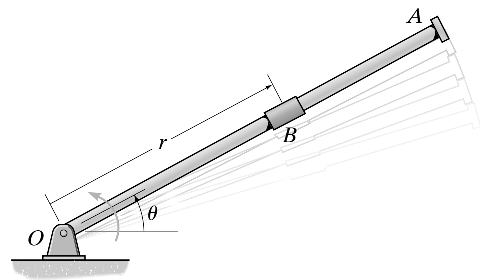
***12–184.** Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 6$ rad/s. Through mechanical means collar B moves along the rod with a speed of $\dot{r} = (4t^2)$ m/s, where t is in seconds. If $r = 0$ when $t = 0$, determine the magnitudes of velocity and acceleration of the collar when $t = 0.75$ s.



Time Derivatives: Using the initial condition $r = 0$ when $t = 0$ s,

$$\frac{dr}{dt} = \dot{r} = 4t^2$$

$$\int_0^r dr = \int_0^t 4t^2\, dt$$

$$r = \left[\frac{4}{3}t^3\right] \text{m}$$

$$r = \frac{4}{3}t^3\bigg|_{t=0.75\text{ s}} = 0.5625 \text{ m}$$

$$\dot{r} = 4t^2\bigg|_{t=0.75\text{ s}} = 2.25 \text{ m/s} \qquad \dot{\theta} = 6 \text{ rad/s}$$

$$\ddot{r} = 8t\bigg|_{t=0.75\text{ s}} = 6 \text{ m/s}^2 \qquad \ddot{\theta} = 0$$

Velocity:

$$v_r = \dot{r} = 2.25 \text{ m/s} \qquad v_\theta = r\dot{\theta} = 0.5625(6) = 3.375 \text{ m/s}$$

Thus, the magnitude of the collar's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2}} = \sqrt{2.25^2 + 3.375^2} = 4.06 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 0.5625\left(6^2\right) = -14.25 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5625(0) + 2(2.25)(6) = 27 \text{ m/s}^2$$

Thus, the magnitude of the collar's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \sqrt{(-14.25)^2 + (-27)^2} = 30.5 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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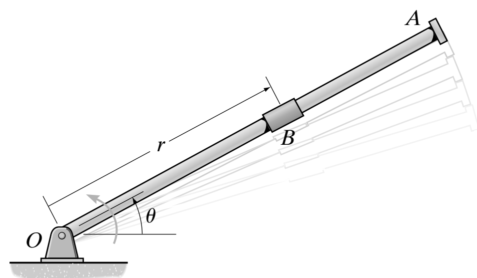
•12–185. Rod OA is rotating counterclockwise with an angular velocity of $\dot{\theta} = (2t^2)$ rad/s. Through mechanical means collar B moves along the rod with a speed of $\dot{r} = (4t^2)$ m/s. If $\theta = 0$ and $r = 0$ when $t = 0$, determine the magnitudes of velocity and acceleration of the collar at $\theta = 60°$.



Position: Using the initial condition $\theta = 0$ when $t = 0$ s,

$$\frac{d\theta}{dt} = \dot{\theta} = 2t^2$$

$$\int_{0°}^{\theta} d\theta = \int_0^t 2t^2 dt$$

$$\theta = \left[\frac{2}{3}t^3\right] \text{rad}$$

At $\theta = 60° = \dfrac{\pi}{3}$ rad,

$$\frac{\pi}{3} = \frac{2}{3}t^3 \qquad t = 1.162 \text{ s}$$

Using the initial condition $r = 0$ when $t = 0$ s,

$$\frac{dr}{dt} = \dot{r} = 4t^2$$

$$\int_0^r dr = \int_0^t 4t^2\, dt$$

$$r = \left[\frac{4}{3}t^3\right] \text{m}$$

When $t = 1.162$ s $\left(\theta = \dfrac{\pi}{3} \text{ rad}\right)$,

$$r = \frac{4}{3}t^3\bigg|_{t=1.162\text{ s}} = 2.094 \text{ m}$$

Time Derivatives:

$$\dot{r} = 4t^2\bigg|_{t=1.162\text{ s}} = 5.405 \text{ m/s} \qquad \dot{\theta} = 2t^2\bigg|_{t=1.162\text{ s}} = 2.703 \text{ rad/s}$$

$$\ddot{r} = 8t\bigg|_{t=1.162\text{ s}} = 9.300 \text{ m/s}^2 \qquad \ddot{\theta} = 4t\bigg|_{t=1.162\text{ s}} = 4.650 \text{ rad/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.405 \text{ m/s} \qquad v_\theta = r\dot{\theta} = 2.094(2.703) = 5.660 \text{ m/s}$$

Thus, the magnitude of the collar's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2} = \sqrt{5.405^2 + 5.660^2} = 7.83 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 9.300 - 2.094\left(2.703^2\right) = -5.998 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.094(4.650) + 2(5.405)(2.703) = 38.95 \text{ m/s}^2$$

Thus, the magnitude of the collar's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2} = \sqrt{(-5.998)^2 + 38.95^2} = 39.4 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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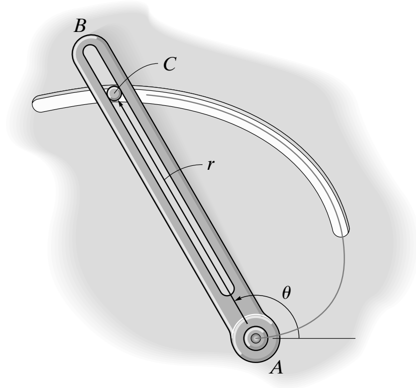
12–186. The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \qquad \dot{r} = a\dot{\theta} \qquad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = a\dot{\theta} \qquad \textbf{Ans.}$$

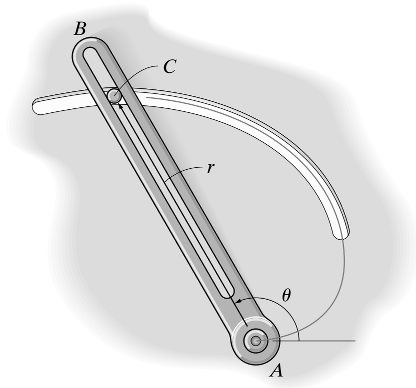
$$v_\theta = r\dot{\theta} = a\theta\dot{\theta} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2 \qquad \textbf{Ans.}$$

12–187. The slotted arm AB drives pin C through the spiral groove described by the equation $r = (1.5\,\theta)$ ft, where θ is in radians. If the arm starts from rest when $\theta = 60°$ and is driven at an angular velocity of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin C when $t = 1$ s.



Time Derivatives: Here, $\dot{\theta} = 4t$ and $\ddot{\theta} = 4$ rad/s^2.

$$r = 1.5\theta \qquad \dot{r} = 1.5\dot{\theta} = 1.5(4t) = 6t \qquad \ddot{r} = 1.5\ddot{\theta} = 1.5(4) = 6 \text{ ft/s}^2$$

Velocity: Integrate the angular rate, $\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$, we have $\theta = \frac{1}{3}(6t^2 + \pi)$ rad.

Then, $r = \left\{\frac{1}{2}(6t^2 + \pi)\right\}$ ft. At $t = 1$ s, $r = \frac{1}{2}\left[6(1^2) + \pi\right] = 4.571$ ft, $\dot{r} = 6(1) = 6.00$ ft/s and $\dot{\theta} = 4(1) = 4$ rad/s. Applying Eq. 12–25, we have

$$v_r = \dot{r} = 6.00 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4.571\,(4) = 18.3 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.571\left(4^2\right) = -67.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.571(4) + 2(6)\,(4) = 66.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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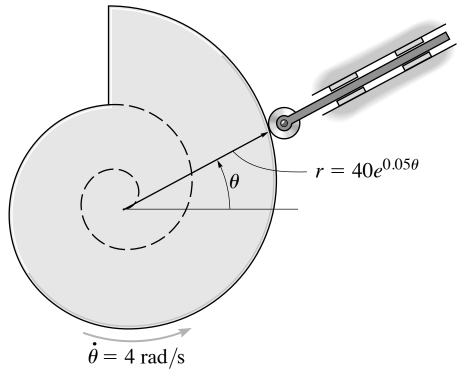
***12–188.** The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam rotates at a constant angular velocity of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the point on the cam that contacts the follower rod at the instant $\theta = 30°$.



$r = 40e^{0.05\,\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.0\,5\,\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\,\theta}\ddot{\theta}$

$\theta = \frac{\pi}{6}$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 40e^{\,0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{\,0.05\left(\frac{\pi}{6}\right)}(4) = 8.2122$

$\ddot{r} = 0.1e^{\,0.05\left(\frac{\pi}{6}\right)}(4)^2 + 0 = 1.64244$

$v_r = \dot{r} = 8.2122$

$v_\theta = r\dot{\theta} = 41.0610(4) = 164.24$

$v = \sqrt{(8.2122)^2 + (164.24)^2} = 164 \text{ mm/s}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 1.642\,44 - 41.0610(4)^2 = -655.33$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(8.2122)(4) = 65.6976$

$a = \sqrt{(-655.33)^2 + (65.6976)^2} = 659 \text{ mm/s}^2$ **Ans.**

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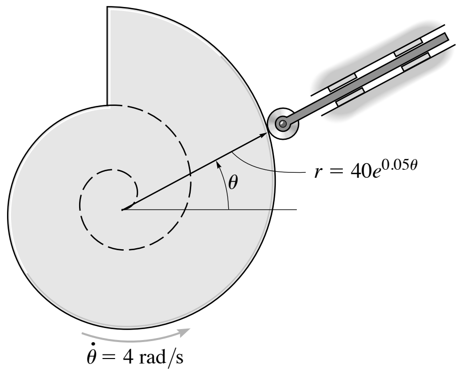
•**12–189.** Solve Prob. 12–188, if the cam has an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$ when its angular velocity is $\dot{\theta} = 4$ rad/s at $\theta = 30°$.



$r = 40e^{0.05\theta}$

$r = 2e^{0.05\,\theta}\dot{\theta}$

$r = 0.1e^{0.05\,\theta}\left(\theta\right)^2 + 2e^{0.05\,\theta}\dot{\theta}$

$\theta = \dfrac{\pi}{6}$

$\theta = 4$

$\theta = 2$

$r = 40e^{\,0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$r = 2e^{\,0.05\left(\frac{\pi}{6}\right)}(4) = 8.2122$

$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(4)^2 + 2e^{\,0.05\left(\frac{\pi}{6}\right)}(2) = 5.749$

$v_r = r = 8.2122$

$v_\theta = r\dot{\theta} = 41.0610(4) = 164.24$

$v = \sqrt{(8.2122)^2 + (164.24)^2} = 164\ \text{mm/s}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 5.749 - 41.0610(4)^2 = -651.2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 41.0610(2) + 2(8.2122)(4) = 147.8197$

$a = \sqrt{(-651.2)^2 + (147.8197)^2} = 668\ \text{mm/s}^2$ **Ans.**

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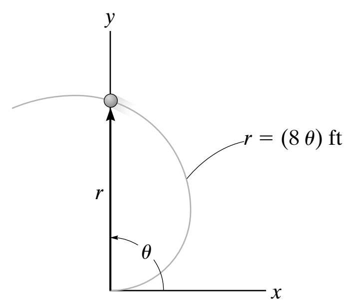
12–190. A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.



Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \qquad \ddot{r} = 8\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

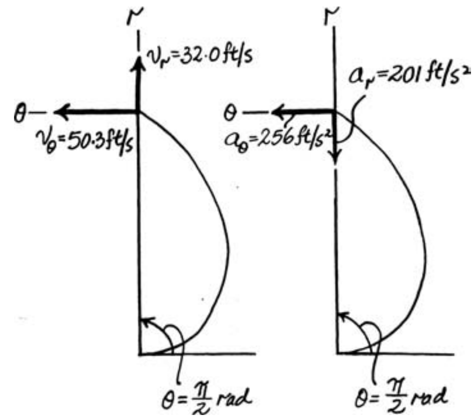
$$v_r = \dot{r} = 32.0 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s} \qquad \textbf{Ans.}$$

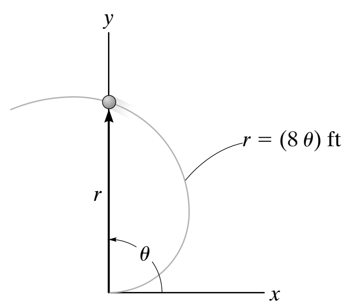
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi\left(4^2\right) = -201 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



12–191. Solve Prob. 12–190 if the particle has an angular acceleration $\ddot{\theta} = 5$ rad/s² when $\dot{\theta} = 4$ rad/s at $\theta = \pi/2$ rad.



Time Derivatives: Here,

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40 \text{ ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

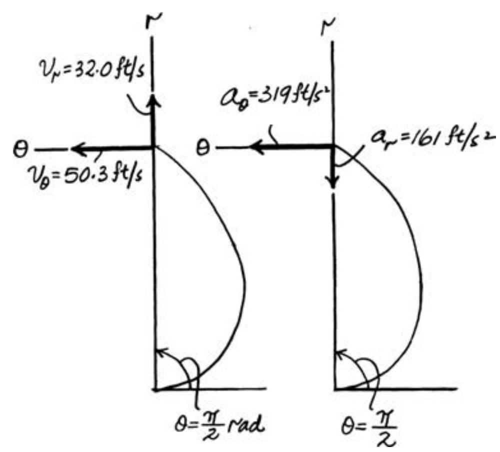
$$v_r = \dot{r} = 32.0 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi\left(4^2\right) = -161 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



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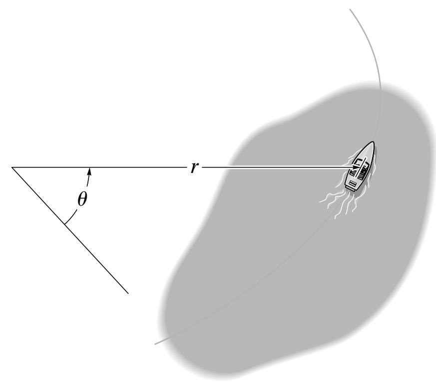
***12–192.** The boat moves along a path defined by $r^2 = [10(10^3)\cos 2\theta]\ \text{ft}^2$, where θ is in radians. If $\theta = (0.4t^2)$ rad, where t is in seconds, determine the radial and transverse components of the boat's velocity and acceleration at the instant $t = 1$ s.



Time Derivatives: Here, $\theta = 0.4t^2$, $\dot{\theta} = 0.8\,t$ and $\ddot{\theta} = 0.8\ \text{rad/s}^2$. When $t = 1$ s, $\theta = 0.4\left(1^2\right) = 0.4$ rad and $\dot{\theta} = 0.8\,(1) = 0.8$ rad/s.

$$r^2 = 10\left(10^3\right)\cos 2\theta \qquad r = 100\sqrt{\cos 2\theta}$$

$$2r\dot{r} = -20\left(10^3\right)\sin 2\theta\dot{\theta} \qquad \dot{r} = -\frac{10(10^2)\sin 2\theta}{r}\dot{\theta}$$

$$2r\ddot{r} + 2\dot{r}^2 = -20\left(10^3\right)\left(2\cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}\right)$$

$$\ddot{r} = \frac{-10(10^3)\left(2\cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}\right) - \dot{r}^2}{r}$$

At $\theta = 0.4$ rad, $r = 100\sqrt{\cos 0.8} = 83.47$ ft, $\dot{r} = -\dfrac{10(10^3)\sin 0.8}{83.47}(0.8) = -68.75$ ft/s

and $\ddot{r} = \dfrac{-10(10^3)\left[2\cos 0.8(0.8^2) + \sin 0.8(0.8)\right] - (-68.75)^2}{83.47} = -232.23\ \text{ft/s}^2$.

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = -68.8\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 83.47(0.8) = 66.8\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -232.23 - 83.47\left(0.8^2\right) = -286\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 83.47(0.8) + 2(-68.75)(0.8) = -43.2\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

•12–193. A car travels along a road, which for a short distance is defined by $r = (200/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 35$ ft/s, determine the radial and transverse components of its velocity when $\theta = \pi/3$ rad.

$$r = \frac{200}{\theta}\bigg|_{\theta=\pi/3\ \text{rad}} = \frac{600}{\pi}\ \text{ft}$$

$$\dot{r} = -\frac{200}{\theta^2}\dot{\theta}\bigg|_{\theta=\pi/3\ \text{rad}} = -\frac{1800}{\pi^2}\dot{\theta}$$

$$v_r = \dot{r} = -\frac{1800}{\pi^2}\dot{\theta} \qquad v_\theta = r\dot{\theta} = \frac{600}{\pi}\dot{\theta}$$

$$v^2 = v_r^2 + v_\theta^2$$

$$35^2 = \left(-\frac{1800}{\pi^2}\dot{\theta}\right)^2 + \left(\frac{600}{\pi}\dot{\theta}\right)^2$$

$$\theta = 0.1325\ \text{rad/s}$$

$$v_r = -\frac{1800}{\pi^2}(0.1325) = -24.2\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = \frac{600}{\pi}(0.1325) = 25.3\ \text{ft/s} \qquad \textbf{Ans.}$$

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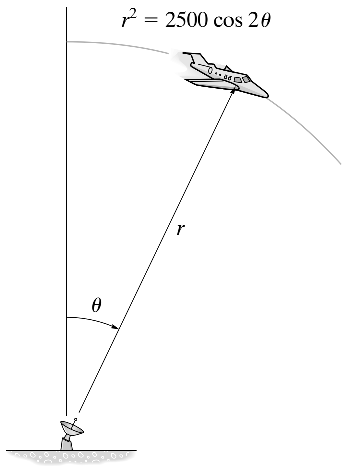
12–194. For a short time the jet plane moves along a path in the shape of a lemniscate, $r^2 = (2500 \cos 2\theta)\ \text{km}^2$. At the instant $\theta = 30°$, the radar tracking device is rotating at $\dot{\theta} = 5(10^{-3})$ rad/s with $\ddot{\theta} = 2(10^{-3})$ rad/s². Determine the radial and transverse components of velocity and acceleration of the plane at this instant.



Time Derivatives: Here, $\dot{\theta} = 5(10^{-3})$ rad/s and $\ddot{\theta} = 2(10^{-3})$ rad/s².

$$r^2 = 2500 \cos 2\theta \qquad r = 50\sqrt{\cos 2\theta}$$

$$2r\dot{r} = -5000 \sin 2\theta\dot{\theta} \qquad \dot{r} = -\frac{2500 \sin 2\theta}{r}\dot{\theta}$$

$$2r\ddot{r} + 2\dot{r}^2 = -5000\left(2 \cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}\right)$$

$$\ddot{r} = \frac{-2500(2 \cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}) - \dot{r}^2}{r}$$

At $\theta = 30°$, $r = 50\sqrt{\cos 60°} = 35.36$ km, $\dot{r} = -\dfrac{2500 \sin 60°}{35.36}\left[5\left(10^{-3}\right)\right] = -0.3062$ km/s

and $\ddot{r} = \dfrac{-2500\{2 \cos 60°\left[5(10^{-3})\right]^2 + \sin 60°\left[2(10^{-3})\right]\} - (-0.3062)^2}{35.36} = -0.1269\ \text{km/s}^2$.

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = -0.3062 \text{ km/s} = 306 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 35.36\left[5\left(10^{-3}\right)\right] = 0.1768 \text{ km/s} = 177 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1269 - 35.36\left[5\left(10^{-3}\right)\right]^2$$

$$= -0.1278 \text{ km/s}^2 = -128 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 35.36\left[2\left(10^{-3}\right)\right] + 2(-0.3062)\left[5\left(10^{-3}\right)\right]$$

$$= 0.06765 \text{ km/s}^2 = 67.7 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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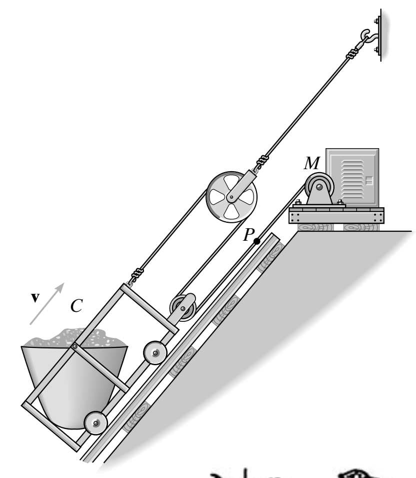
12–195. The mine car C is being pulled up the incline using the motor M and the rope-and-pulley arrangement shown. Determine the speed v_P at which a point P on the cable must be traveling toward the motor to move the car up the plane with a constant speed of $v = 2$ m/s.



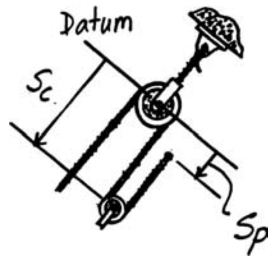
$$2s_C + (s_C - s_P) = l$$

Thus,

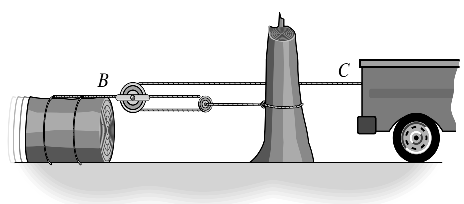
$$3v_C - v_P = 0$$

Hence,

$$v_P = 3(-2) = -6 \text{ m/s} = 6 \text{ m/s} \nearrow \qquad \textbf{Ans.}$$



***12–196.** Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



$$2s_B + (s_B - s_C) = l$$

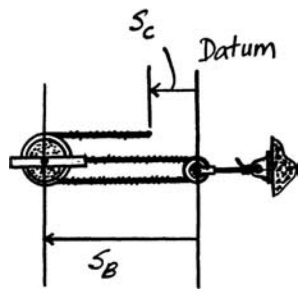
$$3s_B - s_C = l$$

$$3\Delta s_B - \Delta s_C = 0$$

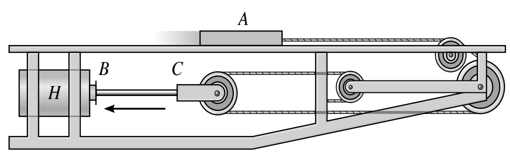
Since $\Delta s_C = -4$, then

$$3\Delta s_B = -4$$

$$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow \qquad \textbf{Ans.}$$



•12–197. If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A.

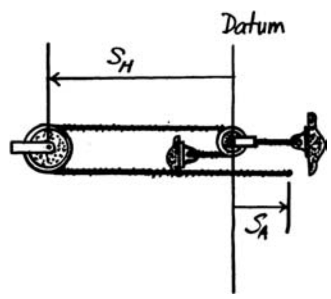


$$2s_H + s_A = l$$

$$2v_H = -v_A$$

$$2(2) = -v_A$$

$$v_A = -4 \text{ ft/s} = 4 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$



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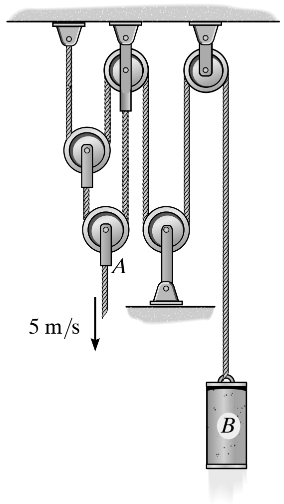
12–198. If end A of the rope moves downward with a speed of 5 m/s, determine the speed of cylinder B.



Position Coordinates: By referring to Fig. a, the length of the two ropes written in terms of the position coordinates s_A, s_B, and s_C are

$$s_B + 2a + 2s_C = l_1$$

$$s_B + 2s_C = l_1 - 2a \qquad \textbf{(1)}$$

and

$$s_A + (s_A - s_C) = l_2$$

$$2s_A - s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields

$$s_B + 4s_A = l_1 - 2a + 2l_2$$

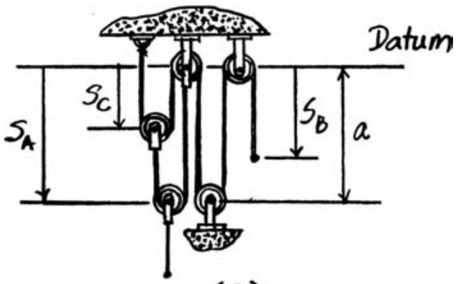
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 4v_A = 0$$

Here, $v_A = 5$ m/s. Thus,

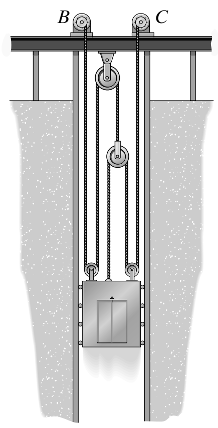
$$v_B + 4(5) = 0$$

$$v_B = -20 \text{ m/s} = 20 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

12–199. Determine the speed of the elevator if each motor draws in the cable with a constant speed of 5 m/s.



Position Coordinates: By referring to Fig. a, the length of the two cables written in terms of the position coordinates are

$$s_E + (s_E - s_A) + s_C = l_1$$

$$2s_E - s_A + s_C = l_1 \qquad \textbf{(1)}$$

and

$$(s_E - s_B) + 2(s_E - s_C) = l_2$$

$$3s_E - s_B - 2s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields

$$7s_E - 2s_A - s_B = 2l_1 + l_2$$

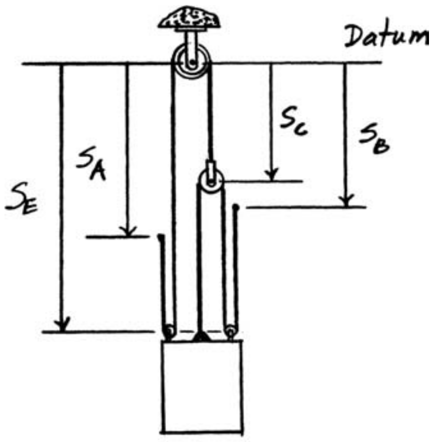
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 7v_E - 2v_A - v_B = 0$$

Here, $v_A = v_B = -5$ m/s. Thus,

$$7v_E - \left[2(-5)\right] - (-5) = 0$$

$$v_E = -2.14 \text{ m/s} = 2.14 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

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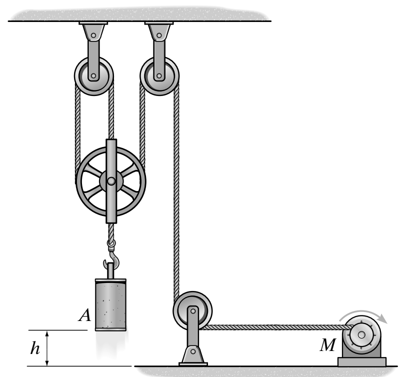
***12–200.** Determine the speed of cylinder A, if the rope is drawn towards the motor M at a constant rate of 10 m/s.



Position Coordinates: By referring to Fig. *a*, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

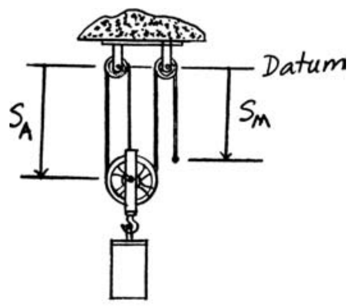
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = 10$ m/s. Thus,

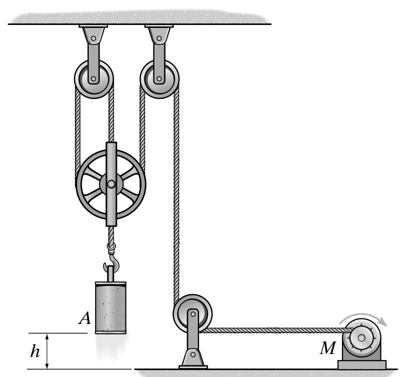
$$3v_A + 10 = 0$$

$$v_A = -3.33 \text{ m/s} = 3.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

•12–201. If the rope is drawn towards the motor M at a speed of $v_M = (5t^{3/2})$ m/s, where t is in seconds, determine the speed of cylinder A when $t = 1$ s.



Position Coordinates: By referring to Fig. *a*, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

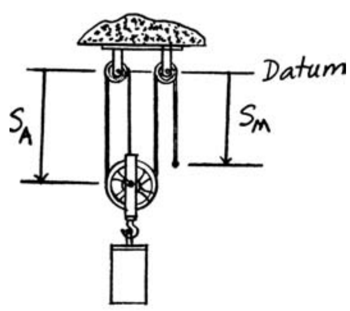
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = \left(5t^{3/2}\right)$ m/s. Thus,

$$3v_A + 5t^{3/2} = 0$$

$$v_A = \left(-\frac{5}{3}t^{3/2}\right) \text{m/s} = \left(\frac{5}{3}t^{3/2}\right) \text{m/s}\bigg|_{t=1\text{ s}} = 1.67 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



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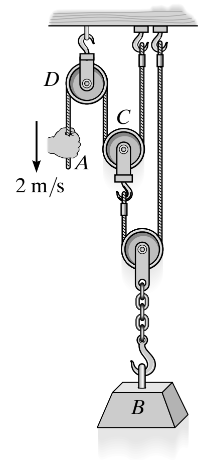
12–202. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B and s_C, respectively. Since the system consists of two cords, two position-coordinate equations can be developed.

$$2s_C + s_A = l_1 \qquad \textbf{[1]}$$

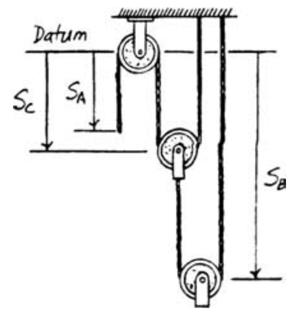
$$s_B + (s_B - s_C) = l_2 \qquad \textbf{[2]}$$

Eliminating s_C from Eqs. [1] and [2] yields

$$s_A + 4s_B = l_1 + 2l_2$$

Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{[3]}$$

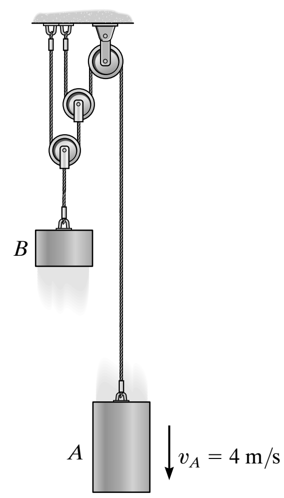


Since $v_A = 2$ m/s, from Eq. [3]

$$(+\downarrow) \qquad 2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

12–203. Determine the speed of B if A is moving downwards with a speed of $v_A = 4$ m/s at the instant shown.



Position Coordinates: By referring to Fig. a, the length of the two ropes written in terms of the position coordinates s_A, s_B, and s_C are

$$s_A + 2s_C = l_1 \qquad \textbf{(1)}$$

and

$$s_B + (s_B - s_C) = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2),

$$4s_B + s_A = 2l_2 + l_1 \qquad \textbf{(3)}$$

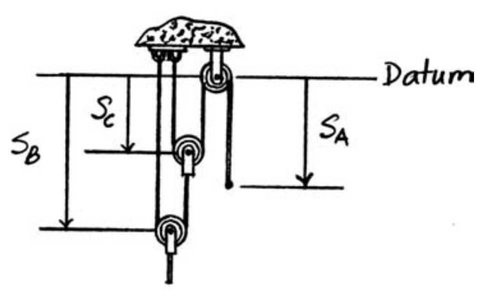
Time Derivative: Taking the time derivative of Eq. (3),

$$(+\downarrow) \qquad 4v_B + v_A = 0$$

Here, $v_A = 4$ m/s. Thus,

$$4v_B + 4 = 0$$

$$v_B = -1 \text{ m/s} = 1 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



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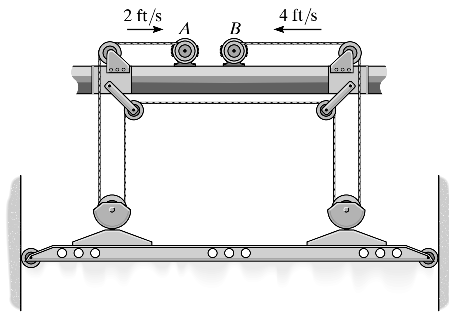
***12–204.** The crane is used to hoist the load. If the motors at A and B are drawing in the cable at a speed of 2 ft/s and 4 ft/s, respectively, determine the speed of the load.

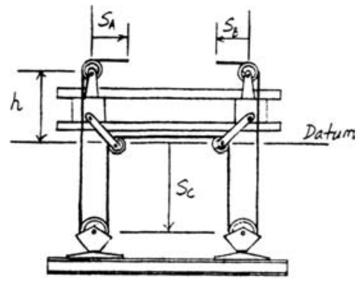


Position-Coordinate Equation: Datum is established as shown. The positon of point A and B and load C with respect to datum are s_A, s_B and s_C, respectively.

$$4s_C + s_A + s_B + 2h = l$$

Time Derivative: Since h is a constant, taking the time derivative of the above equation yields

$$4v_C + v_A + v_B = 0 \qquad \textbf{[1]}$$

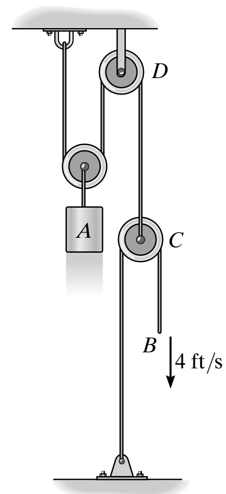


Since $v_A = 2$ ft/s and $v_B = 4$ ft/s, from Eq. [1]

$$4v_C + 2 + 4 = 0$$

$$v_C = -1.50 \text{ ft/s} = 1.50 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

•12–205. The cable at B is pulled downwards at 4 ft/s, and the speed is decreasing at 2 ft/s^2. Determine the velocity and acceleration of block A at this instant.



$2s_A + (h - s_C) = l$

$2v_A = v_C$

$s_C + (s_C - s_B) = l$

$2v_C = v_B$

$v_B = 4v_A$

$a_B = 4a_A$

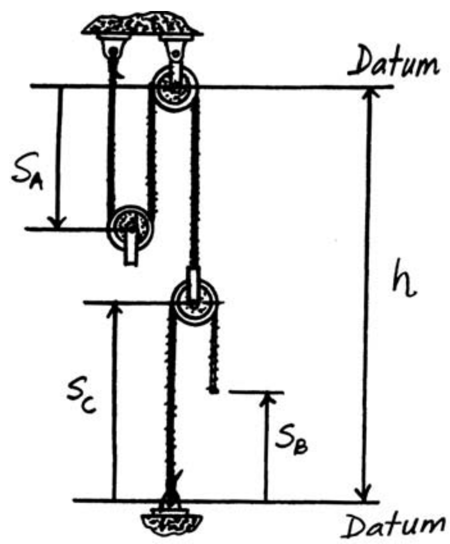
Thus,

$-4 = 4v_A$

$v_A = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow$ **Ans.**

$2 = 4a_A$

$a_A = 0.5 \text{ ft/s} = 0.5 \text{ ft/s}^2 \downarrow$ **Ans.**



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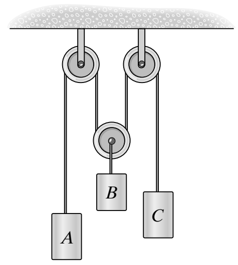
12–206. If block A is moving downward with a speed of 4 ft/s while C is moving up at 2 ft/s, determine the speed of block B.

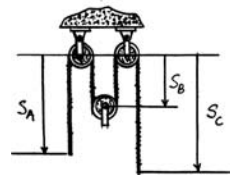


$s_A + 2s_B + s_C = l$

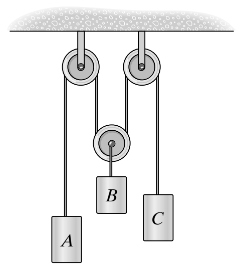
$v_A + 2v_B + v_C = 0$

$4 + 2v_B - 2 = 0$

$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow$ **Ans.**



12–207. If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the speed of block B.

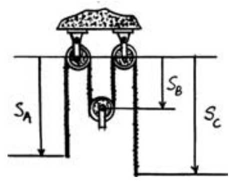


$s_A = 2s_B + s_C = l$

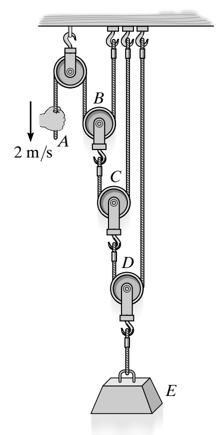
$v_A = 2v_B + v_C = 0$

$6 + 2v_B + 18 = 0$

$v_B = -12 \text{ ft/s} = 12 \text{ ft/s} \uparrow$ **Ans.**



***12–208.** If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block E rises.



Position-Coordinate Equation: Datum is established at fixed pulley. The position of point A, pulley B and C and block E with respect to datum are s_A, s_B, s_C and s_E, respectively. Since the system consists of three cords, three position-coordinate equations can be developed.

$$2s_B + s_A = l_1 \qquad \textbf{[1]}$$

$$s_C + (s_C - s_B) = l_2 \qquad \textbf{[2]}$$

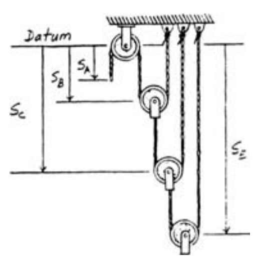
$$s_E + (s_E - s_C) = l_3 \qquad \textbf{[3]}$$

Eliminating s_C and s_B from Eqs. [1], [2] and [3], we have

$$s_A + 8s_E = l_1 + 2l_2 + 4l_3$$

Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 8v_E = 0 \qquad \textbf{[4]}$$



Since $v_A = 2$ m/s, from Eq. [3]

$$(+\downarrow) \qquad 2 + 8v_E = 0$$

$$v_E = -0.250 \text{ m/s} = 0.250 \text{ m/s} \uparrow$$ **Ans.**

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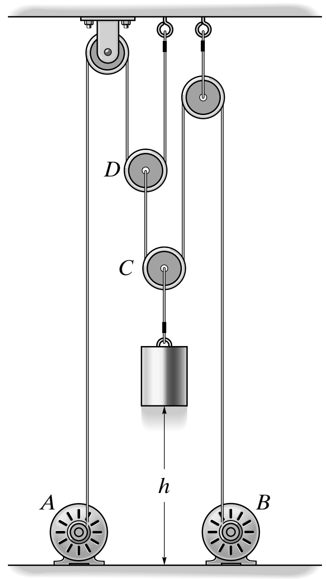
•12–209. If motors at A and B draw in their attached cables with an acceleration of $a = (0.2t)\ \text{m/s}^2$, where t is in seconds, determine the speed of the block when it reaches a height of $h = 4$ m, starting from rest at $h = 0$. Also, how much time does it take to reach this height?



$s_A + 2s_D = l$

$(s_C - s_D) + s_C + s_B = l'$

$\Delta s_A = -2\Delta s_D \qquad 2\Delta s_C - \Delta s_D + \Delta s_B = 0$

If $\Delta s_C = -4$ and $\Delta s_A = \Delta s_B$, then,

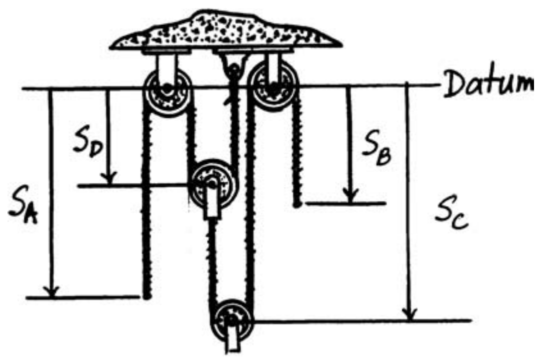
$\Delta s_A = -2\Delta s_D \qquad 2(-4) - \Delta s_D + \Delta s_A = 0$

$\Delta s_D = -2.67\text{ m} \qquad \Delta s_A = \Delta s_B = 5.33\text{ m}$

Thus,

$v_A = -2v_D$ **(1)**

$2v_C - v_D + v_B = 0$ **(2)**



$a = 0.2t$

$dv = a\,dt$

$$\int_0^v dv = \int_0^t 0.2t\,dt$$

$v = 0.1t^2$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t 0.1t^2\,dt$$

$$s = \frac{0.1}{3}t^3 = 5.33$$

$t = 5.428\text{ s} = 5.43\text{ s}$ **Ans.**

$v = 0.1(5.428)^2 = 2.947\text{ m/s}$

$v_A = v_B = 2.947\text{ m/s}$

Thus, from Eqs. (1) and (2):

$2.947 = -2v_D$

$v_D = -1.474$

$2v_C - (-1.474) + 2.947 = 0$

$v_C = -2.21\text{ m/s} = 2.21\text{ m/s} \uparrow$ **Ans.**

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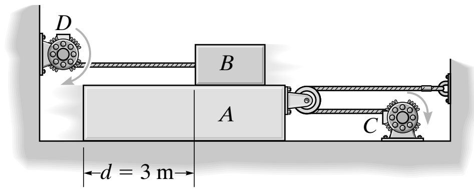
12–210. The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



For A:

$s_A + (s_A - s_C) = l$

$2v_A = v_C$

$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \rightarrow$

$v_A = 0.5t^3 \rightarrow$

$s_A = 0.125t^4 \rightarrow$

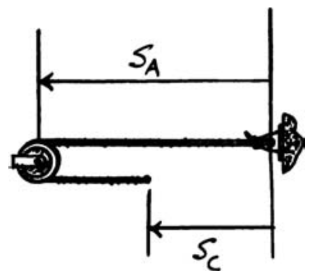
For B:

$a_B = 5\ \text{m/s}^2 \leftarrow$

$v_B = 5t \leftarrow$

$s_B = 2.5t^2 \leftarrow$

Require $s_A + s_B = d$



$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2$ $\quad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$$t = 1.0656 = 1.07\ \text{s} \qquad \textbf{Ans.}$$

$$v_A = .0.5(1.0656)^2 = 0.605\ \text{m/s} \qquad \textbf{Ans.}$$

$$v_B = 5(1.0656) = 5.33\ \text{m/s} \qquad \textbf{Ans.}$$

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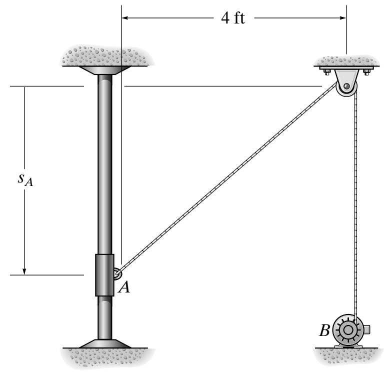
12–211. The motion of the collar at A is controlled by a motor at B such that when the collar is at $s_A = 3$ ft it is moving upwards at 2 ft/s and decreasing at 1 ft/s^2. Determine the velocity and acceleration of a point on the cable as it is drawn into the motor B at this instant.



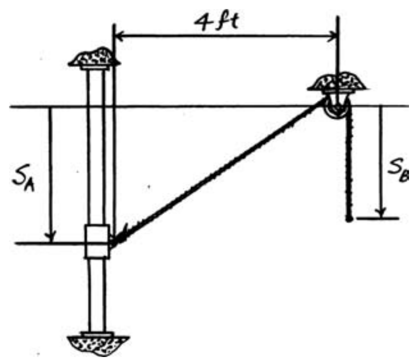
$$\sqrt{s_A^2 + 4^2} + s_B = l$$

$$\frac{1}{2}\left(s_A^2 + 16\right)^{-\frac{1}{2}}(2s_A)\dot{s}_A + s_B = 0$$

$$\dot{s}_B = -s_A\dot{s}_A\left(s_A^2 + 16\right)^{-\frac{1}{2}}$$

$$\dot{s}_B = -\left[\left(\dot{s}_A\right)^2\left(s_A^2 + 16\right)^{-\frac{1}{2}} + s_A\ddot{s}_A\left(s_A^2 + 16\right)^{-\frac{1}{2}} + s_A\dot{s}_A\left(-\frac{1}{2}\right)\left(s_A^2 + 16\right)^{-\frac{3}{2}}\left(2s_A\dot{s}_A\right)\right]$$

$$\dot{s}_B = \frac{\left(s_A\dot{s}_A\right)^2}{\left(s_A^2 + 16\right)^{\frac{3}{2}}} - \frac{\left(s_A\right)^2 + s_A\ddot{s}_A}{\left(s_A^2 + 16\right)^{\frac{1}{2}}}$$

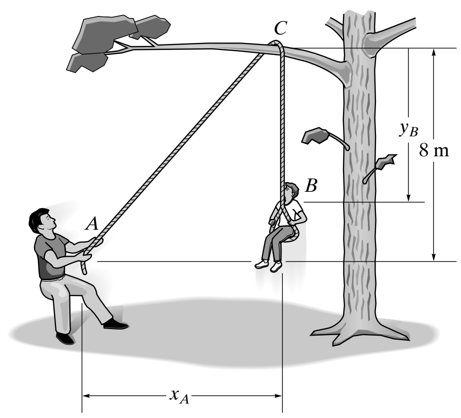


Evaluating these equations:

$$s_B = -3(-2)\left((3)^2 + 16\right)^{-\frac{1}{2}} = 1.20 \text{ ft/s} \downarrow$$ **Ans.**

$$s_B = \frac{((3)(-2))^2}{((3)^2 + 16)^{\frac{3}{2}}} - \frac{(-2)^2 + 3(1)}{((3)^2 + 16)^{\frac{1}{2}}} = -1.11 \text{ ft/s}^2 = 1.11 \text{ ft/s}^2 \uparrow$$ **Ans.**

***12–212.** The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s. Determine the speed at which the boy is being lifted at the instant $x_A = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \quad [1]$$

Time Derivative: Taking the time derivative of Eq. [1] and realizing that $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \quad [2]$$

At the instant $x_A = 4$ m, from Eq. [2]

$$v_B = -\frac{4}{\sqrt{4^2 + 64}}(1.5) = -0.671 \text{ m/s} = 0.671 \text{ m/s} \uparrow$$ **Ans.**

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

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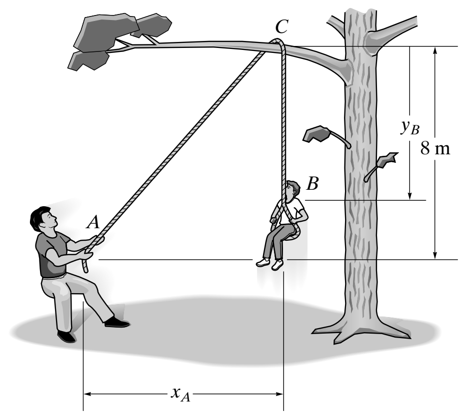
•12–213. The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{[1]}$$

Time Derivative: Taking the time derivative of Eq. [1] Where $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{[2]}$$

At the instant $y_B = 4$ m, from Eq. [1], $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

$$v_A^2 = (v_0)_A^2 + 2(a_c)_A\left[s_A - (s_0)_A\right]$$

$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$

$$v_A = 1.891\ \text{m/s}$$

Substitute the above results into Eq. [2] yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41\ \text{m/s} = 1.41\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

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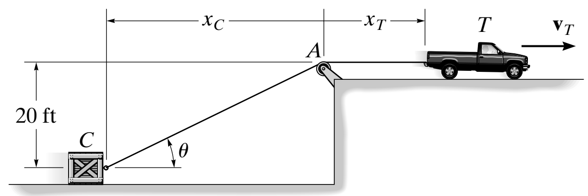
12–214. If the truck travels at a constant speed of $v_T = 6$ ft/s, determine the speed of the crate for any angle θ of the rope. The rope has a length of 100 ft and passes over a pulley of negligible size at A. *Hint:* Relate the coordinates x_T and x_C to the length of the rope and take the time derivative. Then substitute the trigonometric relation between x_C and θ.



$$\sqrt{(20)^2 + x_C^2} + x_T = l = 100$$

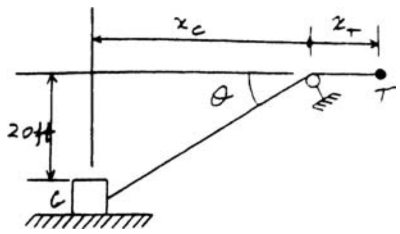
$$\frac{1}{2}\left((20)^2 + (x_C)^2\right)^{-\frac{1}{2}}\left(2x_C\dot{x}_C\right) + \dot{x}_T = 0$$

Since $\dot{x}_T = v_T = 6$ ft/s, $v_C = \dot{x}_C$, and

$$x_C = 20 \text{ ctn } \theta$$

Then,

$$\frac{(20 \text{ ctn } \theta)v_C}{(400 + 400 \text{ ctn}^2 \theta)^{\frac{1}{2}}} = -6$$

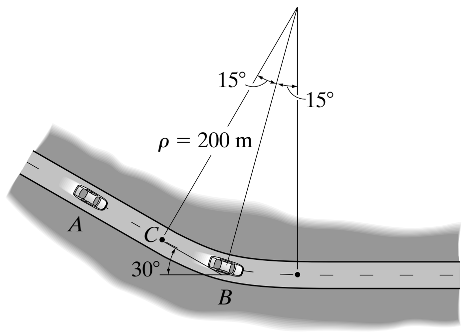


Since $1 + \text{ctn}^2 \theta = \csc^2 \theta$,

$$\left(\frac{\text{ctn } \theta}{\csc \theta}\right)v_C = \cos \theta v_C = -6$$

$$v_C = -6 \sec \theta = (6 \sec \theta) \text{ ft/s} \rightarrow$$ **Ans.**

12–215. At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.



Velocity: Referring to Fig. *a*, the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25 \cos 30° \,\mathbf{i} - 25 \sin 30° \,\mathbf{j}] \text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}] \text{ m/s}$$

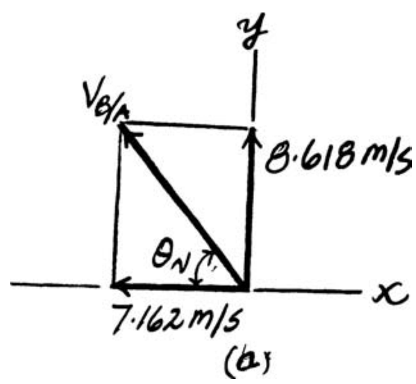
$$\mathbf{v}_B = [15 \cos 15° \,\mathbf{i} - 15 \sin 15° \,\mathbf{j}] \text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}] \text{ m/s}$$



Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2 \text{ m/s}$$ **Ans.**

The direction angle θ_v of $\mathbf{v}_{B/A}$ measured from the x axis, Fig. *a* is

$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3°$$ **Ans.**

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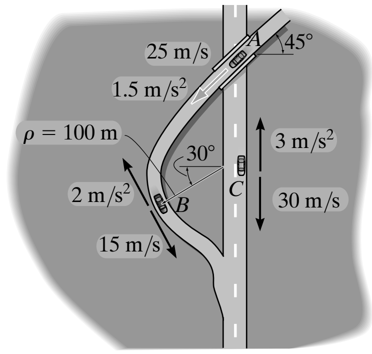
***12–216.** Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



Velocity: The velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45°\mathbf{i} - 25\sin 45°\mathbf{j}]\text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9° \qquad \textbf{Ans.}$$

Acceleration: The acceleration of cars A and B expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45°\mathbf{i} - 1.5\sin 45°\mathbf{j}]\text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4° \qquad \textbf{Ans.}$$

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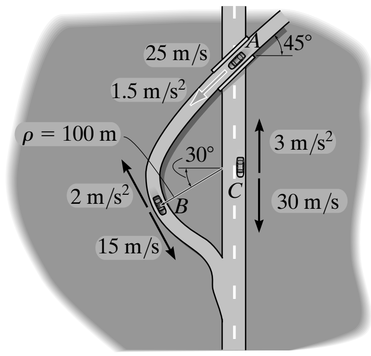
•12–217. Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car B relative to car C.



Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15\cos 60^\circ\,\mathbf{i} - 15\sin 60^\circ\,\mathbf{j}]\text{ m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{BC} = \sqrt{7.5^2 + 17.01^2} = 18.6\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2^\circ \qquad \textbf{Ans.}$$

Acceleration: The normal component of car B's acceleration is $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{100} = 2.25\text{ m/s}^2$. Thus, the tangential and normal components of car B's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2\cos 60^\circ\,\mathbf{i} + 2\sin 60^\circ\mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}]\text{ m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25\cos 30^\circ\,\mathbf{i} + 2.25\sin 30^\circ\,\mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57^\circ \qquad \textbf{Ans.}$$

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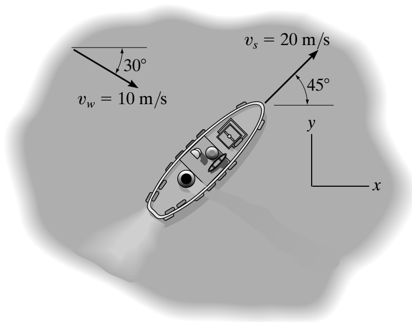
12–218. The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20\cos 45°\,\mathbf{i} + 20\sin 45°\,\mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10\cos 30°\,\mathbf{i} - 10\sin 30°\,\mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9\text{m/s} \qquad \textbf{Ans.}$$

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. a,

$$v_{w/s} = \sqrt{20^2 + 10^2 - 2(20)(10)\cos 75°}$$

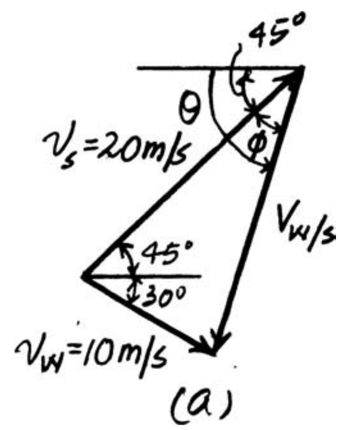
$$= 19.91 \text{ m/s} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of $v_{w/s}$ and applying the law of sines,

$$\frac{\sin\phi}{10} = \frac{\sin 75°}{19.91} \qquad \phi = 29.02°$$

Thus,

$$\theta = 45° + \phi = 74.0° \qquad \textbf{Ans.}$$



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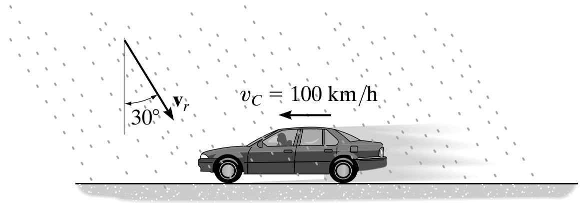
12–219. The car is traveling at a constant speed of 100 km/h. If the rain is falling at 6 m/s in the direction shown, determine the velocity of the rain as seen by the driver.



Solution I

Vector Analysis: The speed of the car is $v_c = \left(100\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ = 27.78 m/s. The velocity of the car and the rain expressed in Cartesian vector form are $\mathbf{v}_c = [-27.78\mathbf{i}]$ m/s and $\mathbf{v}_r = [6\sin 30°\mathbf{i} - 6\cos 30°\mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}]$ m/s. Applying the relative velocity equation, we have

$$\mathbf{v}_r = \mathbf{v}_c + \mathbf{v}_{r/c}$$

$$3\mathbf{i} - 5.196\mathbf{j} = -27.78\mathbf{i} + \mathbf{v}_{r/c}$$

$$\mathbf{v}_{r/c} = [30.78\mathbf{i} - 5.196\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{r/c}$ is given by

$$v_{r/c} = \sqrt{30.78^2 + (-5.196)^2} = 31.2\text{m/s}$$ **Ans.**

and the angle $x_{r/c}$ makes with the x axis is

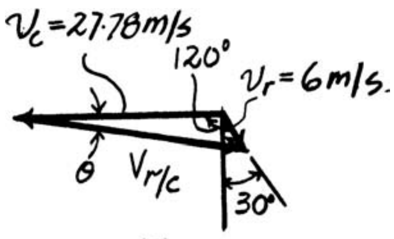
$$\theta = \tan^{-1}\left(\frac{5.196}{30.78}\right) = 9.58°$$ **Ans.**

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$v_{r/c} = \sqrt{27.78^2 + 6^2 - 2(27.78)(6)\cos 120°}$$

$$= 19.91\text{ m/s} = 19.9\text{ m/s}$$ **Ans.**



(a)

Using the result of $v_{r/c}$ and applying the law of sines,

$$\frac{\sin\theta}{6} = \frac{\sin 120°}{31.21}$$

$$\theta = 9.58°$$ **Ans.**

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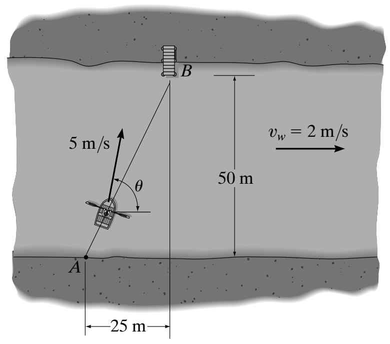
***12–220.** The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle θ he must direct the boat so that it travels from A to B.



Solution I

Vector Analysis: Here, the velocity $\mathbf{v}_b$ of the boat is directed from A to B. Thus, $\phi = \tan^{-1}\left(\frac{50}{25}\right) = 63.43°$. The magnitude of the boat's velocity relative to the flowing river is $v_{b/w} = 5$ m/s. Expressing $\mathbf{v}_b$, $\mathbf{v}_w$, and $\mathbf{v}_{b/w}$ in Cartesian vector form, we have $\mathbf{v}_b = v_b \cos 63.43\mathbf{i} + v_b \sin 63.43\mathbf{j} = 0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j}$, $\mathbf{v}_w = [2\mathbf{i}]$ m/s, and $\mathbf{v}_{b/w} = 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_{b/w}$$

$$0.4472v_b\,\mathbf{i} + 0.8944v_b\,\mathbf{j} = 2\mathbf{i} + 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$$

$$0.4472v_b\,\mathbf{i} + 0.8944v_b\,\mathbf{j} = (2 + 5\cos\theta)\mathbf{i} + 5\sin\theta\mathbf{j}$$

Equating the **i** and **j** components, we have

$$0.4472v_b = 2 + 5\cos\theta \qquad \textbf{(1)}$$

$$0.8944v_b = 5\sin\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_b = 5.56\text{ m/s} \qquad \theta = 84.4° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$5^2 = 2^2 + v_b{}^2 - 2(2)(v_b)\cos 63.43°$$

$$v_b{}^2 - 1.789v_b - 21 = 0$$

$$v_b = \frac{-(-1.789) \pm \sqrt{(-1.789)^2 - 4(1)(-21)}}{2(1)}$$

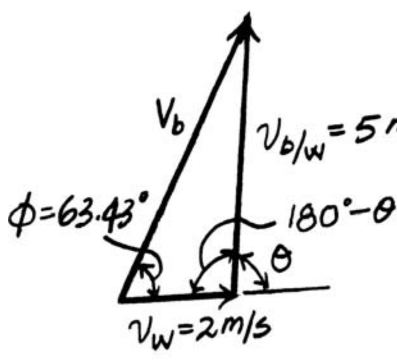
Choosing the positive root,

$$v_b = 5.563\text{ m/s} = 5.56\text{ m/s} \qquad \textbf{Ans.}$$

Using the result of v_b and applying the law of sines,

$$\frac{\sin 180° - \theta}{5.563} = \frac{\sin 63.43°}{5}$$

$$\theta = 84.4° \qquad \textbf{Ans.}$$



(a)

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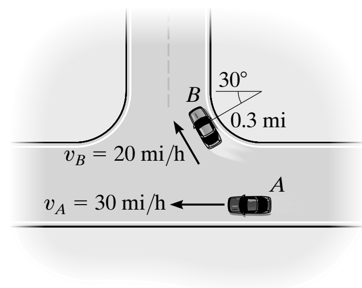
•12–221. At the instant shown, cars A and B travel at speeds of 30 mi/h and 20 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\underset{\nwarrow 30^\circ}{20} = \underset{\leftarrow}{30} + \underset{\rightarrow}{(v_{B/A})_x} + \underset{\uparrow}{(v_{B/A})_y}$$

$(\overset{+}{\rightarrow})$ $\quad -20 \sin 30^\circ = -30 + (v_{B/A})_x$

$(+\uparrow)$ $\quad 20 \cos 30^\circ = (v_{B/A})_y$

Solving

$(v_{B/A})_x = 20 \rightarrow$

$(v_{B/A})_y = 17.32 \uparrow$

$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9^\circ$ **Ans.**

$$(a_B)_n = \frac{(20)^2}{0.3} = 1333.3$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\underset{\nwarrow 30^\circ}{1200} + \underset{30^\circ}{1333.3} = 0 + \underset{\rightarrow}{(a_{B/A})_x} + \underset{\uparrow}{(a_{B/A})_y}$$

$(\overset{+}{\rightarrow})$ $\quad -1200 \sin 30^\circ + 1333.3 \cos 30^\circ = (a_{B/A})_x$

$(+\uparrow)$ $\quad 1200 \cos 30^\circ + 1333.3 \sin 30^\circ = (a_{B/A})_y$

Solving

$(a_{B/A})_x = 554.7 \rightarrow \quad ; \quad (a_{B/A})_y = 1705.9 \uparrow$

$a_{B/A} = \sqrt{(554.7)^2 + 1705.9)^2} = 1.79(10^3) \text{ mi/h}^2$ **Ans.**

$\theta = \tan^{-1}\left(\frac{1705.9}{554.7}\right) = 72.0^\circ$ **Ans.**

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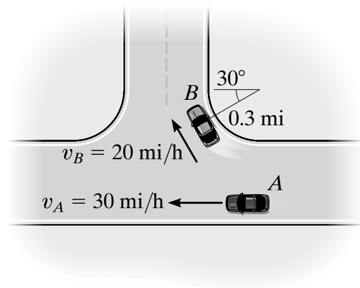
12–222. At the instant shown, cars A and B travel at speeds of 30 m/h and 20 mi/h, respectively. If A is increasing its speed at 400 mi/h^2 whereas the speed of B is decreasing at 800 mi/h^2, determine the velocity and acceleration of B with respect to A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\underset{\nwarrow\,30°}{20} = \underset{\leftarrow}{30} + \underset{\rightarrow}{(v_{B/A})_x} + \underset{\uparrow}{(v_{B/A})_y}$$

$(\overset{+}{\rightarrow})$ $\quad -20 \sin 30° = -30 + (v_{B/A})_x$

$(+\uparrow)$ $\quad 20 \cos 30° = (v_{B/A})_y$

Solving

$(v_{B/A})_x = 20 \rightarrow$

$(v_{B/A})_y = 17.32 \uparrow$

$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9°$ **Ans.**

$$\mathbf{a}_B + \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\left[\frac{20^2}{0.3} = 1333.3\right]_{\theta\,30°} + [800]_{30°} = \underset{\leftarrow}{[400]} + \left[\underset{\rightarrow}{(a_{B/A})_x}\right] + \left[\underset{\uparrow}{(a_{B/A})_y}\right]$$

$(\overset{+}{\rightarrow})$ $\quad 1333.3 \cos 30° + 800 \sin 30° = -400 + (a_{B/A})_x$

$(a_{B/A})_x = 1954.7 \rightarrow$

$(+\uparrow)$ $\quad 1333.3 \sin 30° - 800 \cos 30° = (a_{B/A})_y$

$(a_{B/A})_y = -26.154 = 26.154 \downarrow$

$(a_{B/A}) = \sqrt{(1954.7)^2 + (26.154)^2}$

$a_{B/A} = 1955 \text{ mi/h}^2$ **Ans.**

$\theta = \tan^{-1}\left(\frac{26.154}{1954.7}\right) = 0.767°$ **Ans.**

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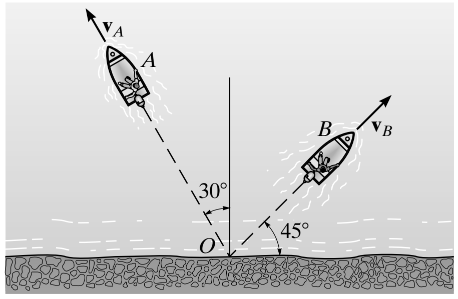
12–223. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20\,\text{ft/s}$ and $v_B = 15\,\text{ft/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



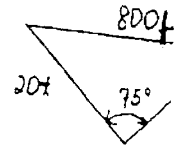
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20 \sin 30°\mathbf{i} + 20 \cos 30°\mathbf{j} = 15 \cos 45°\mathbf{i} + 15 \sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} + 6.714\mathbf{j}\}\ \text{ft/s}$$

$$v_{A/B} = \sqrt{(-20.61)^2 + (+6.714)^2} = 21.7\ \text{ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0°$$ **Ans.**



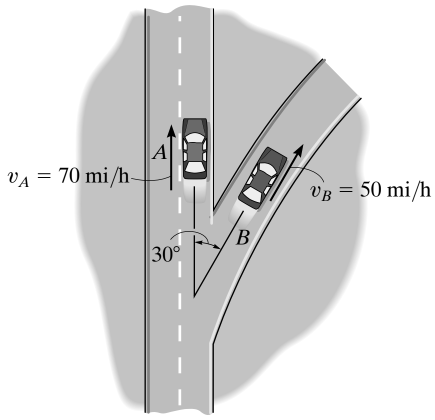
$$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t)\cos 75°$$

$$t = 36.9\ \text{s}$$ **Ans.**

Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9\ \text{s}$$ **Ans.**

***12–224.** At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is increasing its speed by 1100 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



Relative Velocity:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$50 \sin 30°\mathbf{i} + 50 \cos 30°\mathbf{j} = 70\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{25.0\mathbf{i} - 26.70\mathbf{j}\}\ \text{mi/h}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{25.0^2 + (-26.70)^2} = 36.6\ \text{mi/h}$$ **Ans.**

The direction of the relative velocity is the same as the direction of that for relative acceleration. Thus

$$\theta = \tan^{-1}\frac{26.70}{25.0} = 46.9°$$ **Ans.**

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43\ \text{mi/h}^2$. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(1100 \sin 30° + 3571.43 \cos 30°)\mathbf{i} + (1100 \cos 30° - 3571.43 \sin 30°)\mathbf{j} = 0 + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{3642.95\mathbf{i} - 833.09\mathbf{j}\}\ \text{mi/h}^2$$

Thus, the magnitude of the relative velocity $\mathbf{a}_{B/A}$ is

$$a_{B/A} = \sqrt{3642.95^2 + (-833.09)^2} = 3737\ \text{mi/h}^2$$ **Ans.**

And its direction is

$$\phi = \tan^{-1}\frac{833.09}{3642.95} = 12.9°$$ **Ans.**

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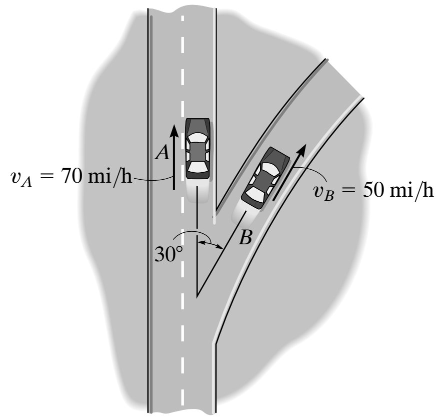
•12–225. At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is decreasing its speed at 1400 mi/h^2 while A is increasing its speed at 800 mi/h^2, determine the acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43$ mi/h^2. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(3571.43 \cos 30^\circ - 1400 \sin 30^\circ)\mathbf{i} + (-1400 \cos 30^\circ - 3571.43 \sin 30^\circ)\mathbf{j} = 800\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{2392.95\mathbf{i} - 3798.15\mathbf{j}\} \text{ mi/h}^2$$

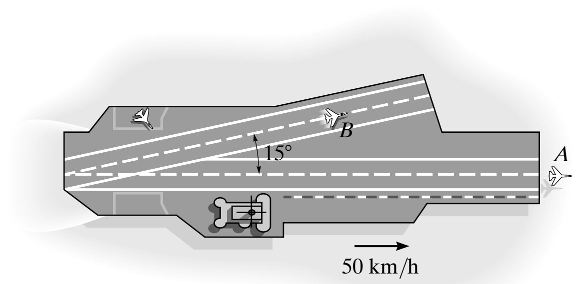
Thus, the magnitude of the relative acc. $\mathbf{a}_{B/A}$ is

$$a_{B/A} = \sqrt{2392.95^2 + (-3798.15)^2} = 4489 \text{ mi/h}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\phi = \tan^{-1}\frac{3798.15}{2392.95} = 57.8^\circ \qquad \textbf{Ans.}$$

12–226. An aircraft carrier is traveling forward with a velocity of 50 km/h. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed of 200 km/h, measured from still water. If the plane at B is traveling along the runway of the carrier at 175 km/h in the direction shown, determine the velocity of A with respect to B.



$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\mathbf{v}_B = 50\mathbf{i} + 175 \cos 15^\circ \mathbf{i} + 175 \sin 15^\circ \mathbf{j} = 219.04\mathbf{i} + 45.293\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$200\mathbf{i} = 219.04\mathbf{i} + 45.293\mathbf{j} + (v_{A/B})_x\mathbf{i} + (v_{A/B})_y\mathbf{j}$$

$$200 = 219.04 + (v_{A/B})_x$$

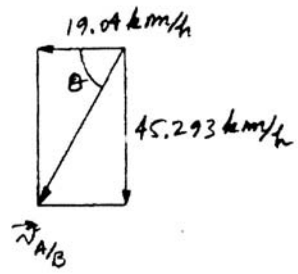
$$0 = 45.293 + (v_{A/B})_y$$

$$(v_{A/B})_x = -19.04$$

$$(v_{A/B})_y = -45.293$$

$$v_{A/B} = \sqrt{(-19.04)^2 + (-45.293)^2} = 49.1 \text{ km/h} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{45.293}{19.04}\right) = 67.2^\circ \qquad \textbf{Ans.}$$



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12–227. A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is directed towards the east. If the car's speed is 80 km/h, the instrument indicates that the wind is directed towards the north-east. Determine the speed and direction of the wind.

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_1\,\mathbf{i}$. Applying the relative velocity equation, we have

$$\mathbf{v}_c = \mathbf{v}_w + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 50\mathbf{j} + (v_{w/c})_1\,\mathbf{i}$$

$$\mathbf{v}_w = (v_{w/c})_1\mathbf{i} + 50\mathbf{j} \qquad \textbf{(1)}$$

For the second case, $v_C = [80\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_2 \cos 45°\mathbf{i} + (v_{W/C})_2 \sin 45°\,\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 80\mathbf{j} + (v_{w/c})_2 \cos 45°\mathbf{i} + (v_{w/c})_2 \sin 45°\,\mathbf{j}$$

$$\mathbf{v}_w = (v_{w/c})_2 \cos 45°\,\mathbf{i} + \left[80 + (v_{w/c})_2 \sin 45°\right]\mathbf{j} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) and then the **i** and **j** components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45° \qquad \textbf{(3)}$$

$$50 = 80 + (v_{w/c})_2 \sin 45° \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \qquad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_W$ is

$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \qquad \textbf{Ans.}$$

and the directional angle θ that $\mathbf{v}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{50}{30}\right) = 59.0° \qquad \textbf{Ans.}$$

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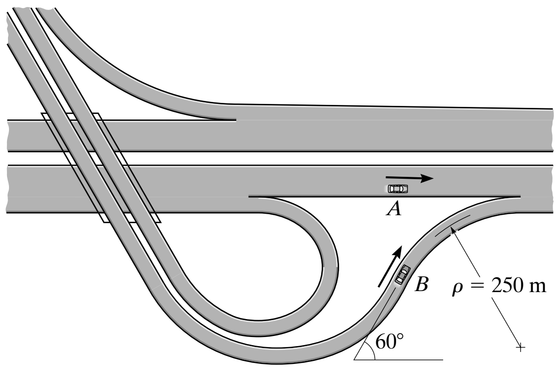
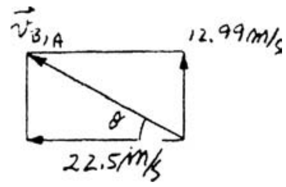
***12–228.** At the instant shown car A is traveling with a velocity of 30 m/s and has an acceleration of 2 m/s^2 along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed of 15 m/s, which is decreasing at 0.8 m/s^2. Determine the relative velocity and relative acceleration of B with respect to A at this instant.



$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$15 \cos 60°\mathbf{i} + 15 \sin 60°\mathbf{j} = 30\mathbf{i} + (v_{B/A})_x\mathbf{i} + (v_{B/A})_y\,\mathbf{j}$



$15 \cos 60° = 30 + (v_{B/A})_x$

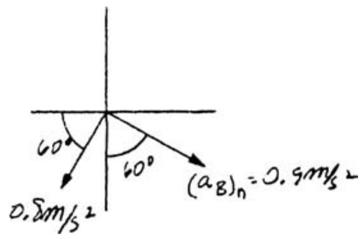
$15 \sin 60° = 0 + (v_{B/A})_y$

$(v_{B/A})_x = -22.5 = 22.5 \text{ m/s} \leftarrow$

$(v_{B/A})_y = 12.99 \text{ m/s} \uparrow$

$v_{B/A} = \sqrt{(22.5)^2 + (12.99)^2} = 26.0 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{12.99}{22.5}\right) = 30°$ **Ans.**



$(a_B)_n = \frac{v^2}{\rho} = \frac{15^2}{250} = 0.9 \text{ m/s}^2$

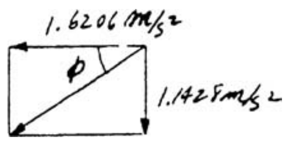
$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$-0.8 \cos 60°\mathbf{i} - 0.8 \sin 60°\mathbf{j} + 0.9 \sin 60°\mathbf{i} - 0.9 \cos 60°\mathbf{j} = 2\mathbf{i} + (a_{B/A})_x\,\mathbf{i} + (a_{B/A})_y\,\mathbf{j}$

$-0.8 \cos 60° + 0.9 \sin 60° = 2 + (a_{B/A})_x$

$-0.8 \sin 60° - 0.9 \cos 60° = (a_{B/A})_y$

$(a_{B/A})_x = -1.6206 \text{ ft/s}^2 = 1.6206 \text{ m/s}^2 \leftarrow$



$(a_{B/A})_y = -1.1428 \text{ ft/s}^2 = 1.1428 \text{ m/s}^2 \downarrow$

$a_{B/A} = \sqrt{(1.6206)^2 + (1.1428)^2} = 1.98 \text{ m/s}^2$ **Ans.**

$\phi = \tan^{-1}\left(\frac{1.1428}{1.6206}\right) = 35.2°$ **Ans.**

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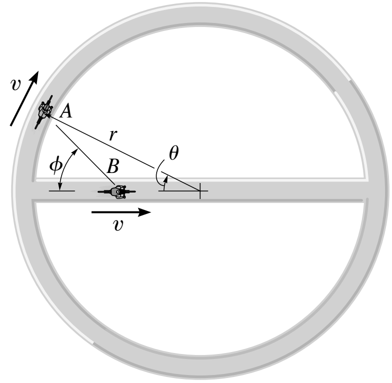
•12–229. Two cyclists A and B travel at the same constant speed v. Determine the velocity of A with respect to B if A travels along the circular track, while B travels along the diameter of the circle.



$$\mathbf{v}_A = v \sin\theta\mathbf{i} + v\cos\theta\,\mathbf{j} \qquad \mathbf{v}_B = v\,\mathbf{i}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (v\sin\theta\mathbf{i} + v\cos\theta\,\mathbf{j}) - v\,\mathbf{i}$$

$$= (v\sin\theta - v)\mathbf{i} + v\cos\theta\mathbf{j}$$

$$v_{A/B} = \sqrt{(v\sin\theta - v)^2 + (v\cos\theta)^2}$$

$$= \sqrt{2v^2 - 2v^2\sin\theta}$$

$$= v\sqrt{2(1-\sin\theta)} \qquad \textbf{Ans.}$$

12–230. A man walks at 5 km/h in the direction of a 20-km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine the direction in which the drops appear to fall with respect to the man. Assume the horizontal speed of the raindrops is equal to that of the wind.

Relative Velocity: The velocity of the rain must be determined first. Applying Eq. 12–34 gives

$$\mathbf{v}_r = \mathbf{v}_w + \mathbf{v}_{r/w} = 20\mathbf{i} + (-7\mathbf{j}) = \{20\mathbf{i} - 7\mathbf{j}\}\text{ km/h}$$

Thus, the relative velocity of the rain with respect to the man is

$$\mathbf{v}_r = \mathbf{v}_m + \mathbf{v}_{r/m}$$

$$20\mathbf{i} - 7\mathbf{j} = 5\mathbf{i} + \mathbf{v}_{r/m}$$

$$\mathbf{v}_{r/m} = \{15\mathbf{i} - 7\mathbf{j}\}\text{ km/h}$$

The magnitude of the relative velocity $\mathbf{v}_{r/m}$ is given by

$$v_{r/m} = \sqrt{15^2 + (-7)^2} = 16.6\text{ km/h} \qquad \textbf{Ans.}$$

And its direction is given by

$$\theta = \tan^{-1}\frac{7}{15} = 25.0^\circ \qquad \textbf{Ans.}$$

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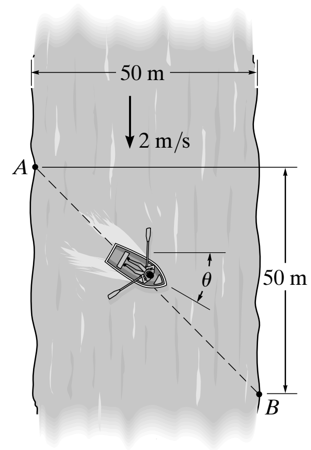
12–231. A man can row a boat at 5 m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.



Relative Velocity:

$$\mathbf{v}_b = \mathbf{v}_r + \mathbf{v}_{b/r}$$

$$v_b \sin 45°\mathbf{i} - v_b \cos 45°\mathbf{j} = -2\mathbf{j} + 5\cos\theta\mathbf{i} - 5\sin\theta\mathbf{j}$$

Equating **i** and **j** component, we have

$$v_b \sin 45° = 5\cos\theta \qquad \textbf{[1]}$$

$$-v_b \cos 45° = -2 - 5\sin\theta \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$\theta = 28.57°$$

$$v_b = 6.210 \text{ m/s} = 6.21 \text{ m/s} \qquad \textbf{Ans.}$$

Thus, the time t required by the boat to travel from point A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{50^2 + 50^2}}{6.210} = 11.4 \text{ s} \qquad \textbf{Ans.}$$

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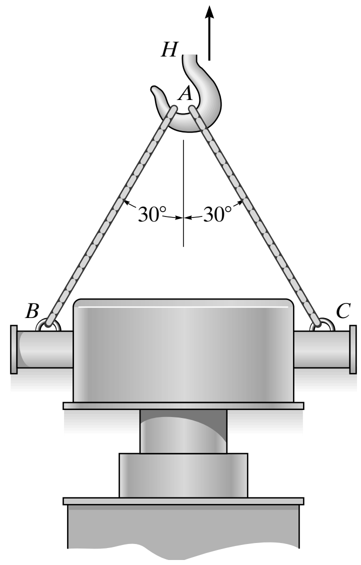
•13–1. The casting has a mass of 3 Mg. Suspended in a vertical position and initially at rest, it is given an upward speed of 200 mm/s in 0.3 s using a crane hook H. Determine the tension in cables AC and AB during this time interval if the acceleration is constant.



Kinematics: Applying the equation $v = v_0 + a_c t$, we have

$(+\uparrow) \qquad 0.2 = 0 + a(0.3) \qquad a = 0.6667 \text{ m/s}^2$

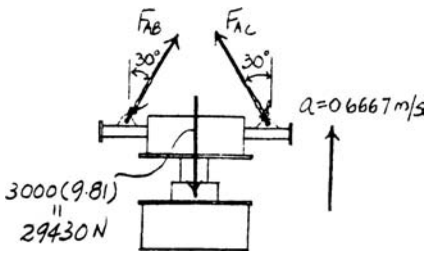
Equations of Motion:

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_{AB} \sin 30 - F_{AC} \sin 30° = 0$

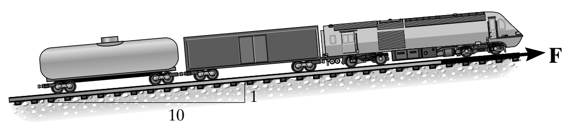
$$F_{AB} = F_{AC} = F$$

$+\uparrow \Sigma F_y = ma_y; \qquad 2F \cos 30° - 29430 = 3000(0.6667)$

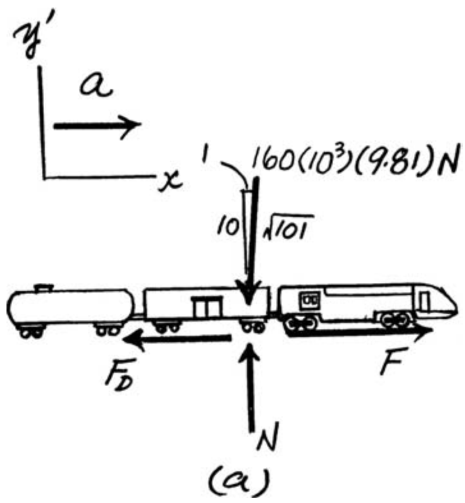
$$F_{AB} = F_{AC} = F = 18146.1 \text{ N} = 18.1 \text{ kN} \qquad \textbf{Ans.}$$



13–2. The 160-Mg train travels with a speed of 80 km/h when it starts to climb the slope. If the engine exerts a traction force **F** of 1/20 of the weight of the train and the rolling resistance $\mathbf{F}_D$ is equal to 1/500 of the weight of the train, determine the deceleration of the train.



Free-Body Diagram: The tractive force and rolling resistance indicated on the free-body diagram of the train, Fig. (a), are $F = \left(\frac{1}{20}\right)(160)(10^3)(9.81) \text{ N} = 78\,480 \text{ N}$ and $F_D = \left(\frac{1}{500}\right)(160)(10^3)(9.81)\text{N} = 3139.2 \text{ N}$, respectively.



Equations of Motion: Here, the acceleration **a** of the train will be assumed to be directed up the slope. By referring to Fig. (a),

$+\nearrow \Sigma F_{x'} = ma_{x'}; \qquad 78\,480 - 3139.2 - 160(10^3)(9.81)\left(\frac{1}{\sqrt{101}}\right) = 160(10^3)a$

$$a = -0.5057 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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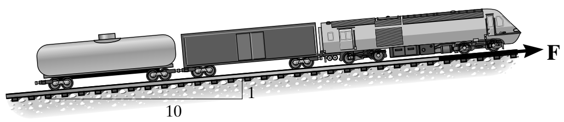
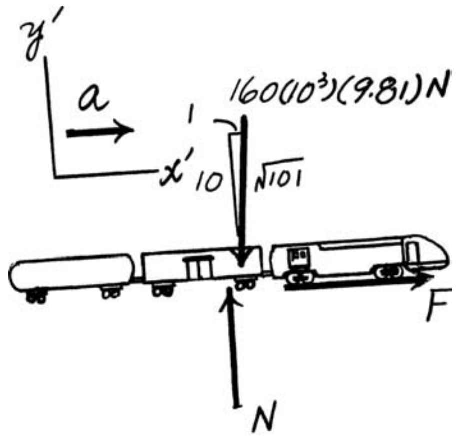
13–3. The 160-Mg train starts from rest and begins to climb the slope as shown. If the engine exerts a traction force **F** of 1/8 of the weight of the train, determine the speed of the train when it has traveled up the slope a distance of 1 km. Neglect rolling resistance.



Free-Body Diagram: Here, the tractive force indicated on the free-body diagram of the train, Fig. (a), is $F = \frac{1}{8}(160)(10^3)(9.81)\text{ N} = 196.2(10^3)\text{ N}$.



(a)

Equations of Motion: Here, the acceleration **a** of the train will be assumed directed up the slope. By referring to Fig. (a),

$$+\nearrow \Sigma F_{x'} = ma_{x'}; \qquad 196.2(10^3) - 160(10^3)(9.81)\left(\frac{1}{\sqrt{101}}\right) = 160(10^3)a$$

$$a = 0.2501\text{ m/s}^2$$

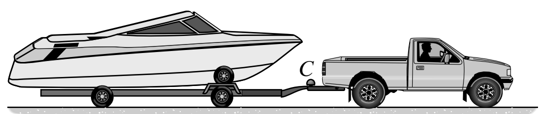
Kinematics: Using the result of **a**,

$$(+\nearrow) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 0 + 2(0.2501)(1000 - 0)$$

$$v = 22.4\text{ m/s} \qquad \textbf{Ans.}$$

***13–4.** The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling *C*, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.

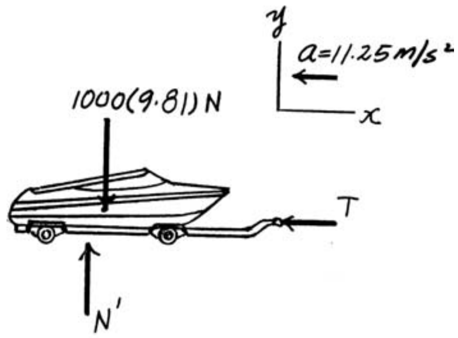


Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$\left(\xrightarrow{+}\right) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25\text{ m/s}^2 = 11.25\text{ m/s}^2 \leftarrow$$



(a)

Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling *C* is represented by **T**.

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

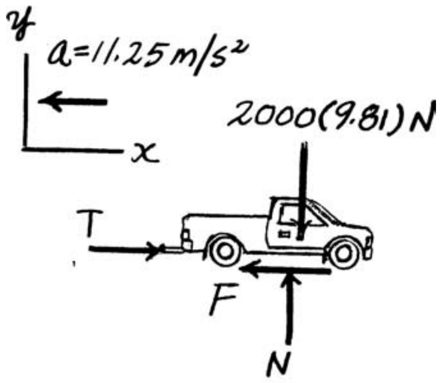
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -T = 1000(-11.25)$$

$$T = 11\,250\text{ N} = 11.25\text{ kN} \qquad \textbf{Ans.}$$

Using the results of **a** and **T** and referring to Fig. (b),

$$+\uparrow \Sigma F_x = ma_x; \qquad 11\,250 - F = 2000(-11.25)$$

$$F = 33\,750\text{ N} = 33.75\text{ kN} \qquad \textbf{Ans.}$$



(b)

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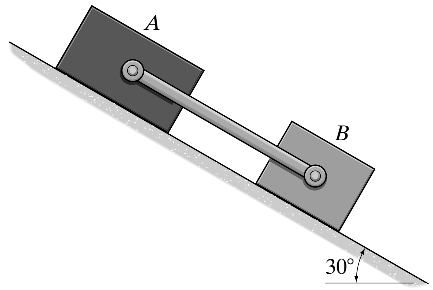
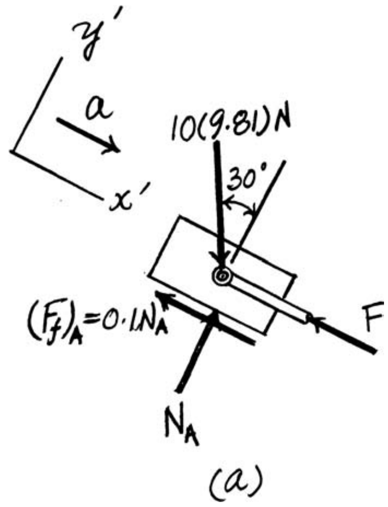
•13–5. If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



Free-Body Diagram: Here, the kinetic friction $(F_f)_A = \mu_A N_A = 0.1 N_A$ and $(F_f)_B = \mu_B N_B = 0.3 N_B$ are required to act up the plane to oppose the motion of the blocks which are down the plane. Since the blocks are connected, they have a common acceleration **a**.

Equations of Motion: By referring to Figs. (a) and (b),



$$+\nearrow\Sigma F_{y'} = ma_{y'}; \qquad N_A - 10(9.81)\cos 30° = 10(0)$$

$$N_A = 84.96 \text{ N}$$

$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad 10(9.81)\sin 30° - 0.1(84.96) - F = 10a$$

$$40.55 - F = 10a \qquad \textbf{(1)}$$

and

$$+\nearrow\Sigma F_{y'} = ma_{y'}; \qquad N_B - 6(9.81)\cos 30° = 6(0)$$

$$N_B = 50.97 \text{ N}$$

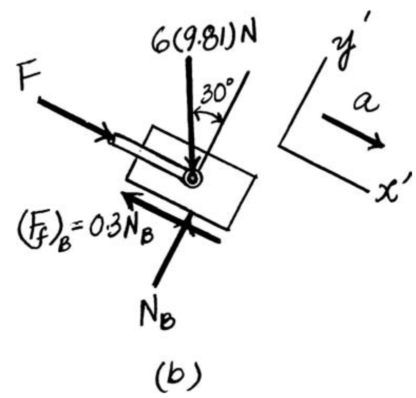
$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad F + 6(9.81)\sin 30° - 0.3(50.97) = 6a$$

$$F + 14.14 = 6a \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a = 3.42 \text{ m/s}^2$$

$$F = 6.37 \text{ N} \qquad \textbf{Ans.}$$



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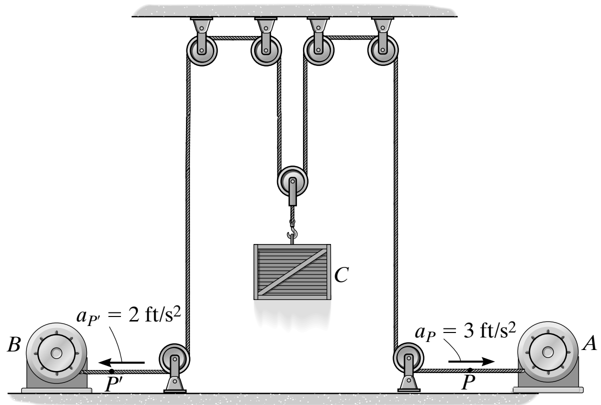
13–6. Motors A and B draw in the cable with the accelerations shown. Determine the acceleration of the 300-lb crate C and the tension developed in the cable. Neglect the mass of all the pulleys.



Kinematics: We can express the length of the cable in terms of s_P, $s_{P'}$, and s_C by referring to Fig. (a).

$$s_P + s_{P'} + 2s_C = l$$

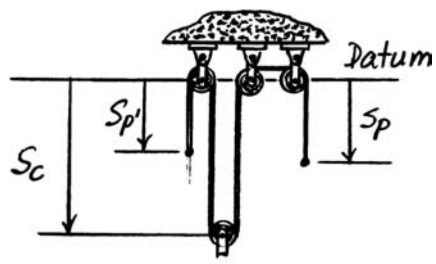
The second time derivative of the above equation gives

$$(+\downarrow) \qquad a_P + a_{P'} + 2a_C = 0 \qquad \textbf{(1)}$$

Here, $a_P = 3\text{ ft/s}^2$ and $a_{P'} = 2\text{ ft/s}^2$. Substituting these values into Eq. (1),

$$3 + 2 + 2a_C = 0$$

$$a_C = -2.5\text{ ft/s}^2 = 2.5\text{ ft/s}^2 \uparrow \qquad \textbf{Ans.}$$



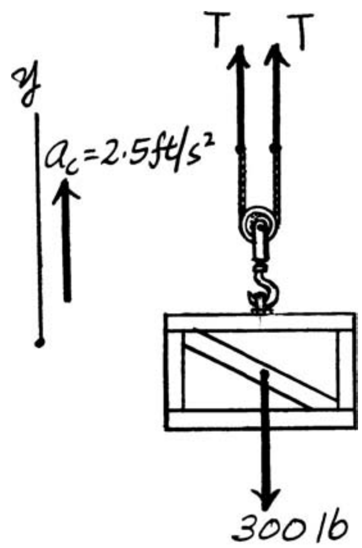
(a)

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. (b).

Equations of Motion: Using the result of $\mathbf{a}_C$ and referring to Fig. (b),

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 300 = \frac{300}{32.2}(2.5)$$

$$T = 162\text{ lb} \qquad \textbf{Ans.}$$



(b)

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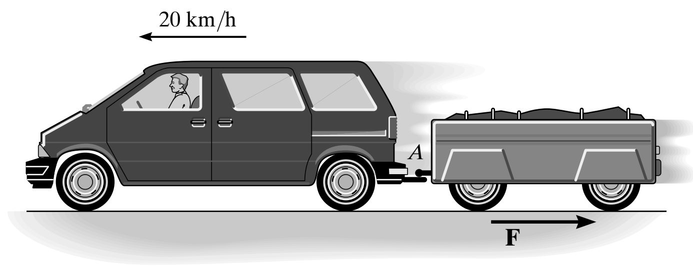
13–7. The van is traveling at 20 km/h when the coupling of the trailer at A fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



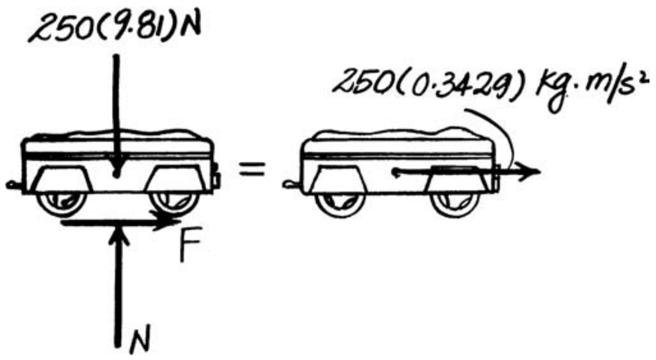
$$20 \text{ km/h} = \frac{20(10^3)}{3600} = 5.556 \text{ m/s}$$

$(\overset{+}{\leftarrow})$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = 5.556^2 + 2(a)(45 - 0)$$

$$a = -0.3429 \text{ m/s}^2 = 0.3429 \text{ m/s}^2 \rightarrow$$

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 250(0.3429) = 85.7 \text{ N}$ **Ans.**



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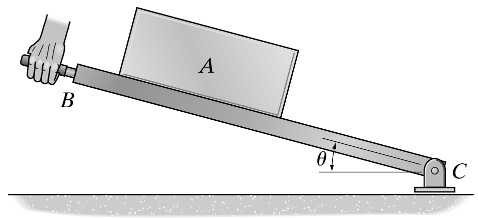
***13–8.** If the 10-lb block A slides down the plane with a constant velocity when $\theta = 30°$, determine the acceleration of the block when $\theta = 45°$.



Free-Body Diagram: The free-body diagrams of the block when $\theta = 30°$ and $\theta = 45°$ are shown in Figs. (a) and (b), respectively. Here, the kinetic friction $F_f = \mu_k N$ and $F_{f'} = \mu_k N'$ are required to act up the plane to oppose the motion of the block which is directed down the plane for both cases.

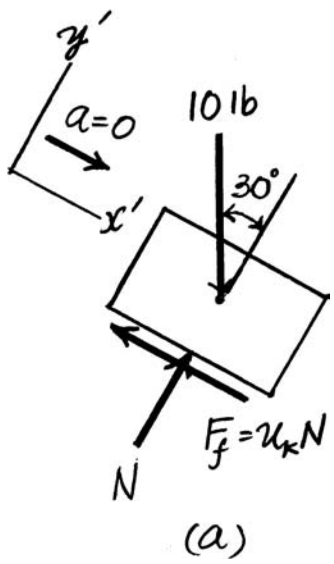
Equations of Motion: Since the block has constant velocity when $\theta = 30°$, $a_{x'} = a = 0$. Also, $a_{y'} = 0$. By referring to Fig. (a), we can write

$$+\nearrow \Sigma F_{y'} = ma_{y'}; \qquad N - 10\cos 30° = \frac{10}{32.2}(0)$$

$$N = 8.660 \text{ lb}$$

$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad 10\sin 30° - \mu_k(8.660) = \frac{10}{32.2}(0)$$

$$\mu_k = 0.5774$$



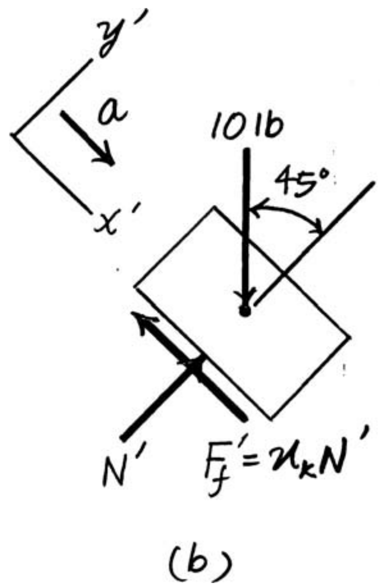
Using the results of μ_k and referring to Fig. (b),

$$+\nearrow \Sigma F_{y'} = ma_{y'}; \qquad N' - 10\cos 45° = \frac{10}{32.2}(0)$$

$$N' = 7.071 \text{ lb}$$

$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad 10\sin 45° - 0.5774(7.071) = \frac{10}{32.2}a$$

$$a = 9.62 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



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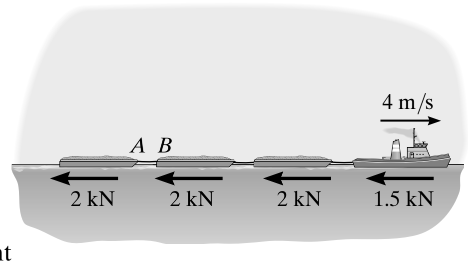
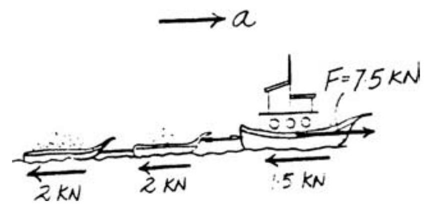
•13–9. Each of the three barges has a mass of 30 Mg, whereas the tugboat has a mass of 12 Mg. As the barges are being pulled forward with a constant velocity of 4 m/s, the tugboat must overcome the frictional resistance of the water, which is 2 kN for each barge and 1.5 kN for the tugboat. If the cable between A and B breaks, determine the acceleration of the tugboat.



Equations of Motion: When the tugboat and barges are travelling at a constant velocity, the driving force F can be determined by applying Eq. 13–7.

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F - 1.5 - 2 - 2 = 0 \qquad F = 7.50 \text{ kN}$$

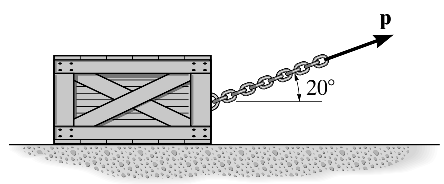


If the cable between barge A and B breaks and the driving force F remains the same, the acceleration of the tugboat and barge is given by

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \qquad (7.50 - 1.5 - 2 - 2)\left(10^3\right) = (12\,000 + 30\,000 + 3000)a$$

$$a = 0.0278 \text{ m/s}^2 \qquad \textbf{Ans.}$$

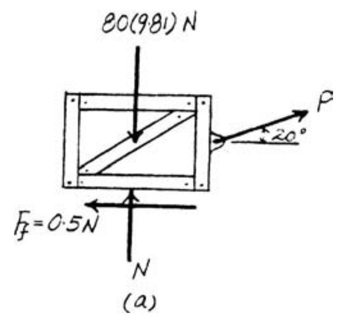
13–10. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **P** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N + P \sin 20° - 80(9.81) = 0 \qquad \textbf{(1)}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad P \cos 20° - 0.5N = 0 \qquad \textbf{(2)}$$



Solving Eqs.(1) and (2) yields

$$P = 353.29 \text{ N} \qquad N = 663.97 \text{ N}$$

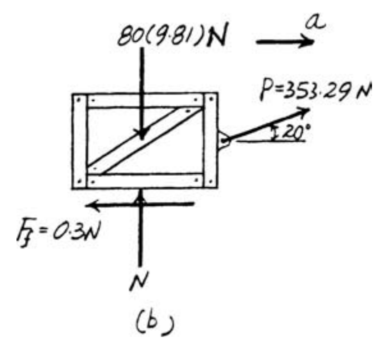
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 353.29 \sin 20° = 80(0)$$

$$N = 663.97 \text{ N}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 353.29 \cos 20° - 0.3(663.97) = 80a$$

$$a = 1.66 \text{ m/s}^2 \qquad \textbf{Ans.}$$



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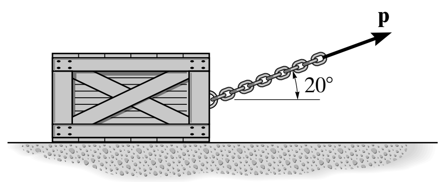
13–11. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



Equations of Equilibrium: At $t = 2$ s, $P = 90\left(2^2\right) = 360$ N. From FBD(a)

$+\uparrow \Sigma F_y = 0; \qquad N + 360 \sin 20° - 80(9.81) = 0 \qquad N = 661.67 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 360 \cos 20° - F_f = 0 \qquad F_f = 338.29\text{N}$

Since $F_f > (F_f)_{\max} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.

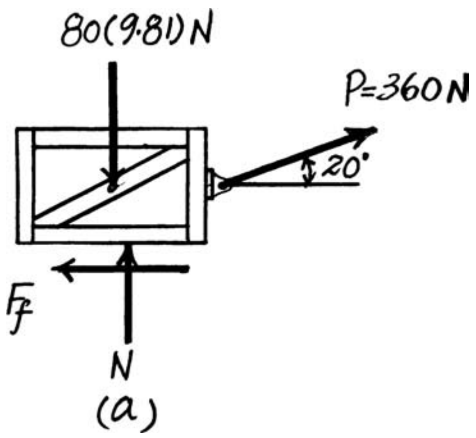
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

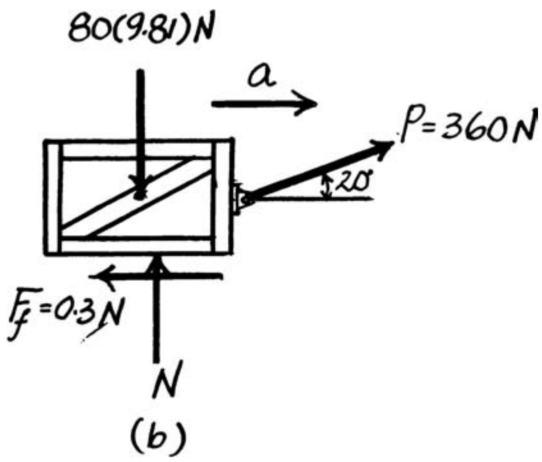
$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 360 \sin 20° = 80(0)$

$N = 661.67 \text{ N}$

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 360 \cos 20° - 0.3(661.67) = 80a$

$a = 1.75 \text{ m/s}^2$ **Ans.**





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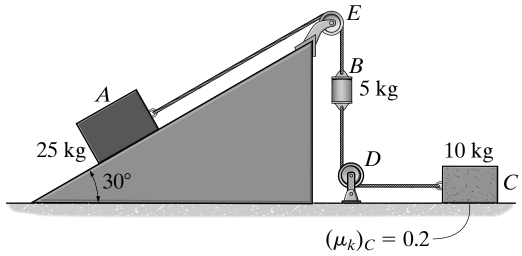
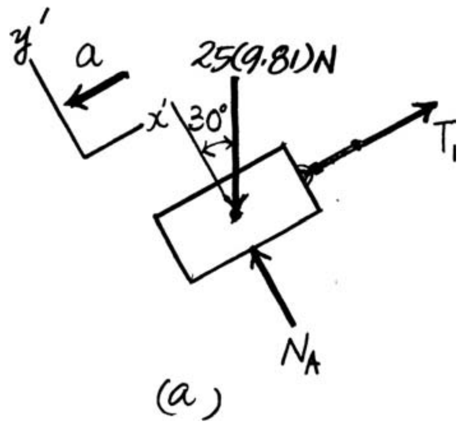
***13–12.** Determine the acceleration of the system and the tension in each cable. The inclined plane is smooth, and the coefficient of kinetic friction between the horizontal surface and block C is $(\mu_k)_C = 0.2$.



Free-Body Diagram: The free-body diagram of block A, cylinder B, and block C are shown in Figs. (a), (b), and (c), respectively. The frictional force $(F_f)_C = (\mu_k)_C N_C = 0.2N_C$ must act to the right to oppose the motion of block C which is to the left.



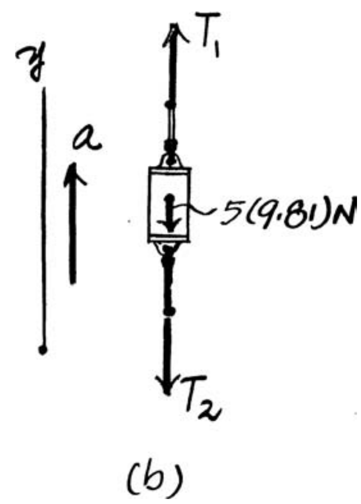
Equations of Motion: Since block A, cylinder B, and block C move together as a single unit, they share a common acceleration **a**. By referring to Figs. (a), (b), and (c),

$$\Sigma F_{x'} = ma_{x'}; \qquad T_1 - 25(9.81)\sin 30° = 25(-a) \qquad \textbf{(1)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T_1 - T_2 - 5(9.81) = 5(a) \qquad \textbf{(2)}$$

and



$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 10(9.81) = 10(0)$$

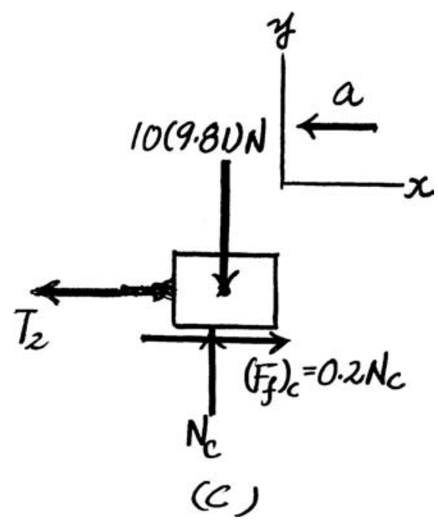
$$N_C = 98.1 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -T_2 + 0.2(98.1) = 10(-a) \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3), yields

$$a = 1.349 \text{ m/s}^2 \qquad T_1 = 88.90 \text{ N} = 88.9\text{N} \qquad \textbf{Ans.}$$

$$T_2 = 33.11 \text{ N} = 33.1 \text{ N} \qquad \textbf{Ans.}$$



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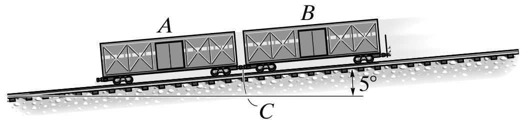
•13–13. The two boxcars A and B have a weight of 20 000 lb and 30 000 lb, respectively. If they coast freely down the incline when the brakes are applied to all the wheels of car A causing it to skid, determine the force in the coupling C between the two cars. The coefficient of kinetic friction between the wheels of A and the tracks is $\mu_k = 0.5$. The wheels of car B are free to roll. Neglect their mass in the calculation. *Suggestion:* Solve the problem by representing single resultant normal forces acting on A and B, respectively.



Car A:

$+\nwarrow \Sigma F_y = 0; \qquad N_A - 20\,000 \cos 5^\circ = 0 \qquad N_A = 19\,923.89 \text{ lb}$

$$+\nearrow \Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - T - 20\,000 \sin 5^\circ = \left(\frac{20\,000}{32.2}\right)a \qquad \textbf{(1)}$$

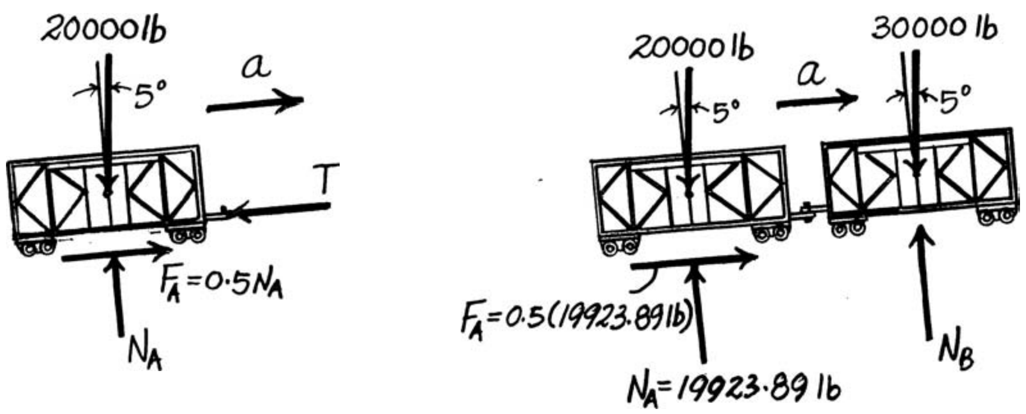
Both cars:

$$+\nearrow \Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - 50\,000 \sin 5^\circ = \left(\frac{50\,000}{32.2}\right)a$$

Solving,

$a = 3.61 \text{ ft/s}^2$

$T = 5.98 \text{ kip}$ **Ans.**



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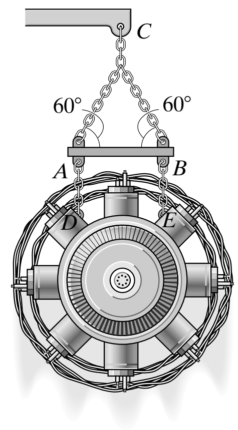
13–14. The 3.5-Mg engine is suspended from a spreader beam AB having a negligible mass and is hoisted by a crane which gives it an acceleration of 4 m/s^2 when it has a velocity of 2 m/s. Determine the force in chains CA and CB during the lift.



System:

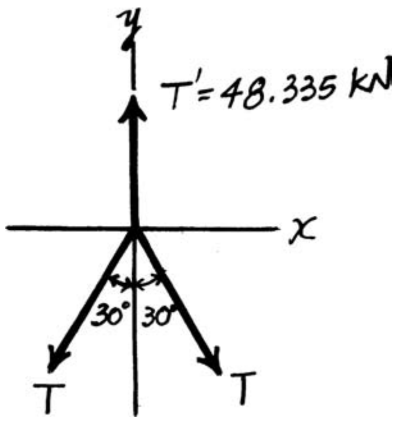
$$+\uparrow \Sigma F_y = ma_y; \qquad T' - 3.5\left(10^3\right)(9.81) = 3.5\left(10^3\right)(4)$$

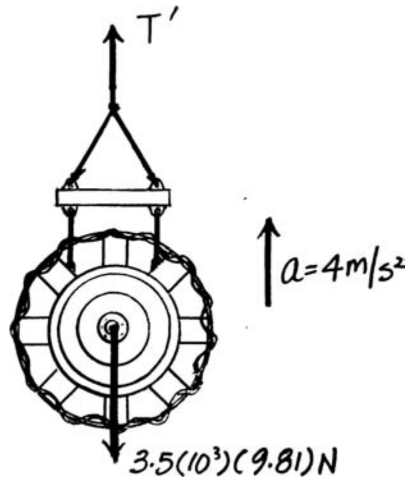
$$T' = 48.335 \text{ kN}$$

Joint C:

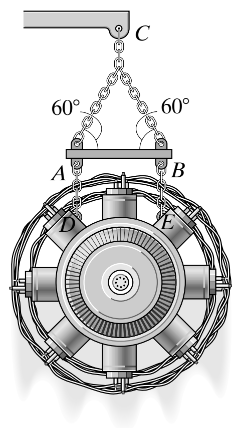
$$+\uparrow \Sigma F_y = ma_y; \qquad 48.335 - 2\,T\cos 30° = 0$$

$$T = T_{CA} = T_{CB} = 27.9 \text{ kN} \qquad \textbf{Ans.}$$





13–15. The 3.5-Mg engine is suspended from a spreader beam having a negligible mass and is hoisted by a crane which exerts a force of 40 kN on the hoisting cable. Determine the distance the engine is hoisted in 4 s, starting from rest.



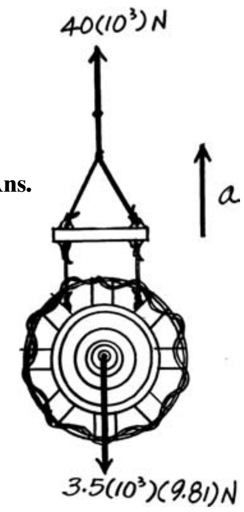
System:

$$+\uparrow \Sigma F_y = ma_y; \qquad 40\left(10^3\right) - 3.5\left(10^3\right)(9.81) = 3.5\left(10^3\right)a$$

$$a = 1.619 \text{ m/s}^2$$

$$(+\uparrow) \qquad s = s_0 + v_0\,t + \frac{1}{2}a_c\,t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.619)(4)^2 = 12.9 \text{ m} \qquad \textbf{Ans.}$$



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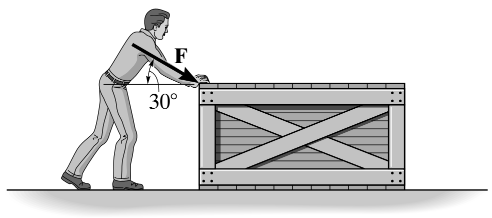
***13–16.** The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



Force to produce motion:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F\cos 30^\circ - 0.6N = 0$$

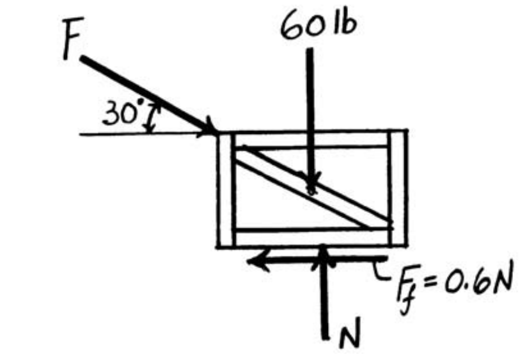
$$+\uparrow \Sigma F_y = 0; \qquad N - 60 - F\sin 30^\circ = 0$$

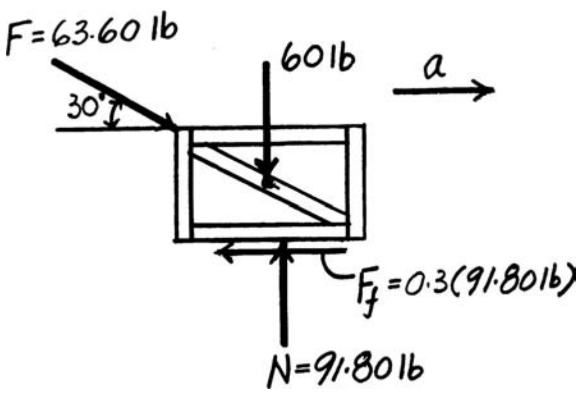
$$N = 91.80 \text{ lb} \qquad F = 63.60 \text{ lb}$$

Since $N = 91.80$ lb,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 63.60\cos 30^\circ - 0.3(91.80) = \left(\frac{60}{32.2}\right)a$$

$$a = 14.8 \text{ ft/s}^2$$ **Ans.**





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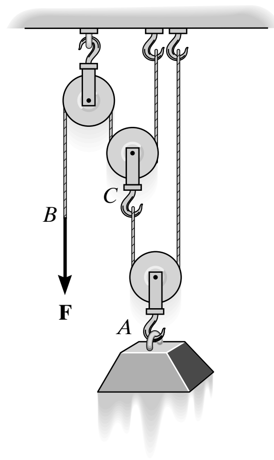
•13–17. A force of $F = 15$ lb is applied to the cord. Determine how high the 30-lb block A rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.



Block:

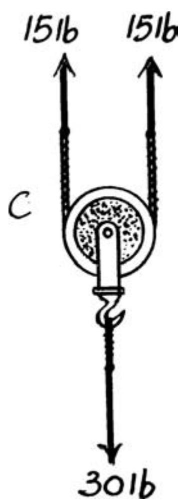
$$+\uparrow \Sigma F_y = ma_y; \qquad -30 + 60 = \left(\frac{30}{32.2}\right)a_A$$

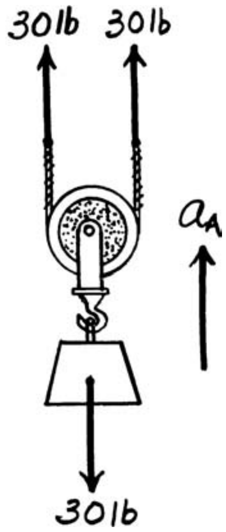
$$a_A = 32.2 \text{ ft/s}^2$$

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

$$s = 64.4 \text{ ft}$$ **Ans.**





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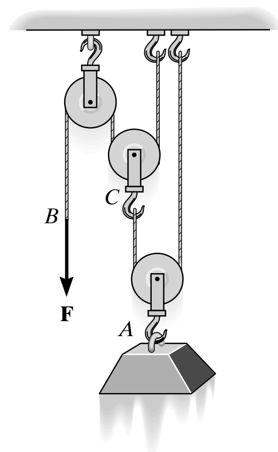
13–18. Determine the constant force **F** which must be applied to the cord in order to cause the 30-lb block A to have a speed of 12 ft/s when it has been displaced 3 ft upward starting from rest. Neglect the weight of the pulleys and cord.



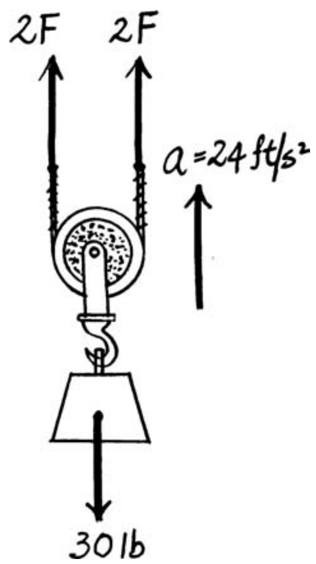
$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(12)^2 = 0 + 2(a)(3)$$

$$a = 24 \text{ ft/s}^2$$

$$+\uparrow \Sigma F_y = ma_y; \qquad -30 + 4F = \left(\frac{30}{32.2}\right)(24)$$

$$F = 13.1 \text{ lb}$$ **Ans.**



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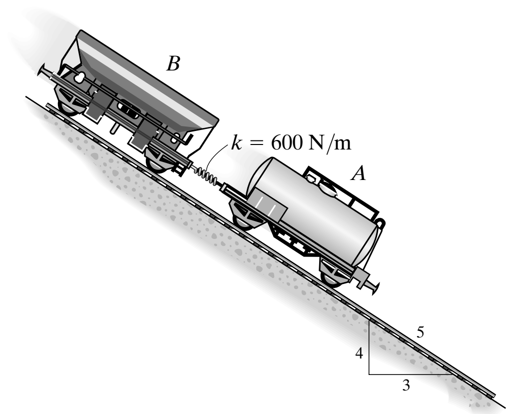
13–19. The 800-kg car at B is connected to the 350-kg car at A by a spring coupling. Determine the stretch in the spring if (a) the wheels of both cars are free to roll and (b) the brakes are applied to all four wheels of car B, causing the wheels to skid. Take $(\mu_k)_B = 0.4$. Neglect the mass of the wheels.



a) Equations of Motion: Applying Eq. 13–7 to FBD(a), we have

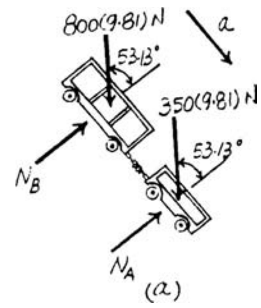
$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad (800 + 350)(9.81)\sin 53.13° = (800 + 350)a$$

$$a = 7.848 \text{ m/s}^2$$

For FBD(b),

$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad 350(9.81)\sin 53.13° + F_{sp} = 350(7.848)$$

$$F_{sp} = 0$$



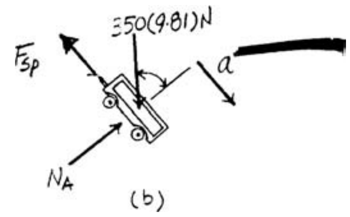
The stretch of spring is given by

$$x = \frac{F_{sp}}{k} = 0 \qquad \textbf{Ans.}$$

b) Equations of Motion: The friction force developed between the wheels of car B and the inclined plane is $(F_f)_B = (\mu_k)_B N_B = 0.4N_B$. For car B only [FBD(c)],

$$+\nearrow \Sigma F_{y'} = ma_{y'}; \qquad N_B - 800(9.81)\cos 53.13° = 800(0)$$

$$N_B = 4708.8 \text{ N}$$



For the whole system (FBD(c)],

$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad (800 + 350)(9.81)\sin 53.13° - 0.4(4708.8) = (800 + 350)a$$

$$a = 6.210 \text{ m/s}^2$$

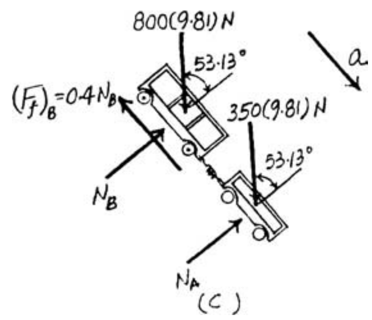
For FBD(b),

$$\searrow + \ \Sigma F_{x'} = ma_{x'}; \qquad 350(9.81)\sin 53.13° - F_{sp} = 350\,(6.210)$$

$$F_{sp} = 573.25 \text{ N}$$

The stretch of spring is given by

$$x = \frac{F_{sp}}{k} = \frac{573.25}{600} = 0.955 \text{ m} \qquad \textbf{Ans.}$$



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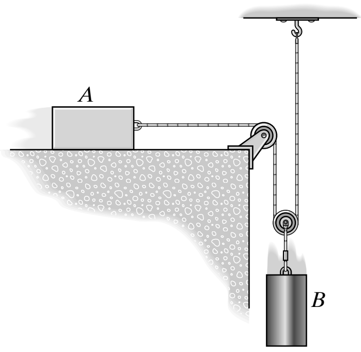
***13–20.** The 10-lb block A travels to the right at $v_A = 2$ ft/s at the instant shown. If the coefficient of kinetic friction is $\mu_k = 0.2$ between the surface and A, determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb.



Block A:

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad -T + 2 = \left(\frac{10}{32.2}\right)a_A \qquad \textbf{(1)}$$

Weight B:

$$+\downarrow \Sigma F_y = ma_y; \qquad 20 - 2T = \left(\frac{20}{32.2}\right)a_B \qquad \textbf{(2)}$$

Kinematics:

$$s_A + 2s_B = l$$

$$a_A = -2a_B \qquad \textbf{(3)}$$

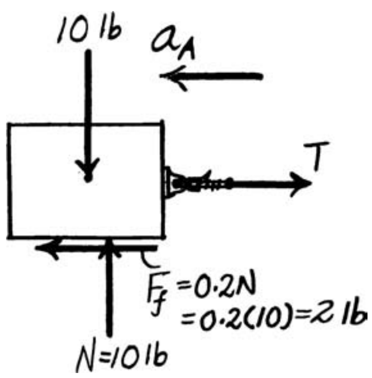
Solving Eqs. (1)–(3):

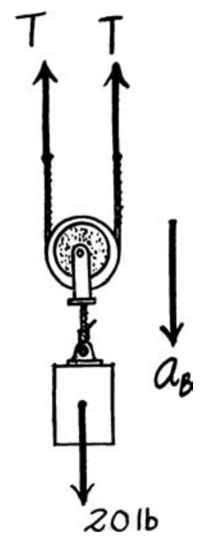
$a_A = -17.173\ \text{ft/s}^2 \qquad a_B = 8.587\ \text{ft/s}^2 \qquad T = 7.33$ lb

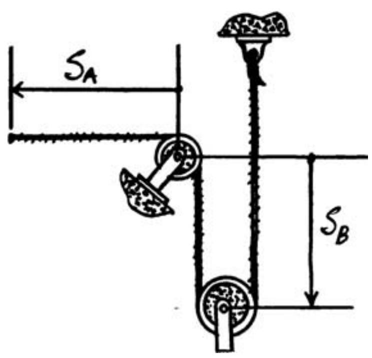
$v^2 = v_0^2 + 2a_c\,(s - s_0)$

$v^2 = (2)^2 + 2(17.173)(4 - 0)$

$$v = 11.9\ \text{ft/s} \qquad \textbf{Ans.}$$







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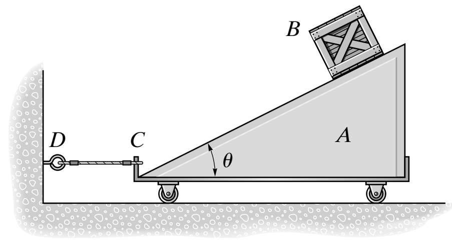
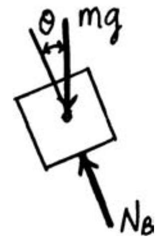
•13–21. Block B has a mass m and is released from rest when it is on top of cart A, which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A. Neglect friction.



Block B:

$+\nwarrow \Sigma F_y = ma_y; \qquad N_B - mg\cos\theta = 0$

$$N_B = mg\cos\theta$$

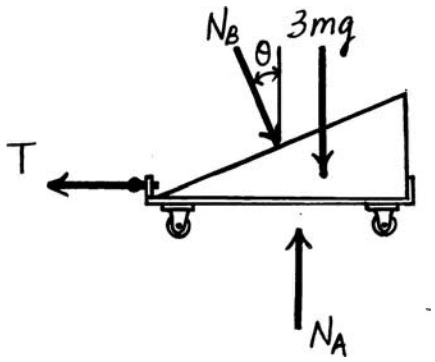


Cart:

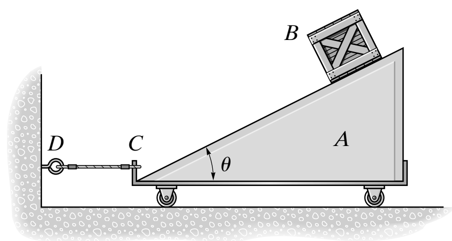
$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -T + N_B \sin\theta = 0$

$$T = mg\sin\theta\cos\theta$$

$$T = \left(\frac{mg}{2}\right)\sin 2\theta \qquad \textbf{Ans.}$$



13–22. Block B has a mass m and is released from rest when it is on top of cart A, which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A. The coefficient of kinetic friction between A and B is μ_k.



Block B:

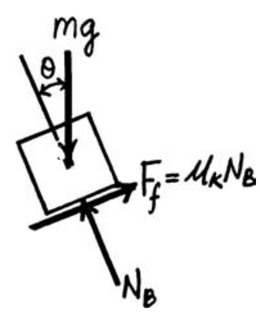
$+\nwarrow \Sigma F_y = ma_y; \qquad N_B - mg\cos\theta = 0$

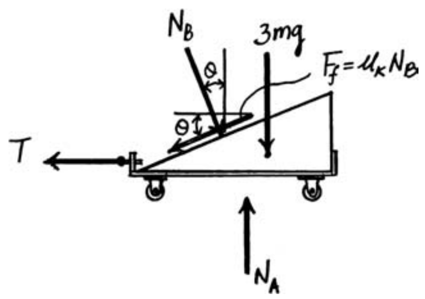
$$N_B = mg\cos\theta$$

Cart:

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -T + N_B\sin\theta - \mu_k N_B\cos\theta = 0$

$$T = mg\cos\theta(\sin\theta - \mu_k\cos\theta) \qquad \textbf{Ans.}$$





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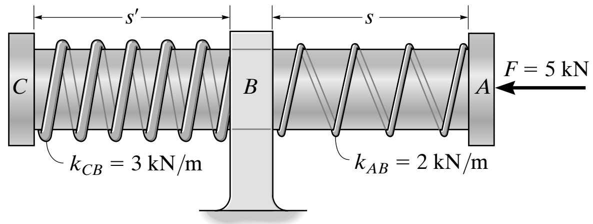
13–23. The 2-kg shaft CA passes through a smooth journal bearing at B. Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' = 250$ mm and the shaft is at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A.



$F_{CB} = k_{CB}x = 3000x \qquad F_{AB} = k_{AB}x = 2000x$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad 5000 - 3000x - 2000x = 2a$

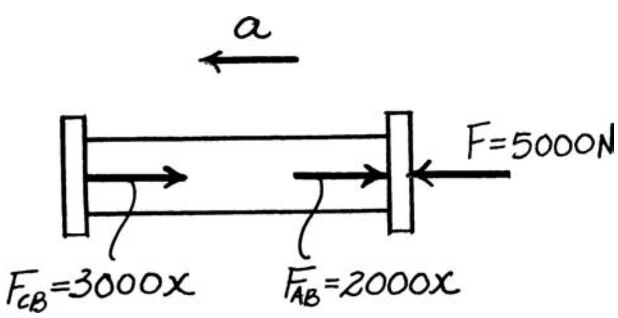
$$2500 - 2500x = a$$

$a\,dx - v\,dv$

$$\int_0^{0.2} (2500 - 2500x)\,dx = \int_0^v v\,dv$$

$$2500(0.2) - \left(\frac{2500(0.2)^2}{2}\right) = \frac{v^2}{2}$$

$v = 30$ m/s **Ans.**



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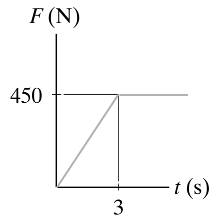
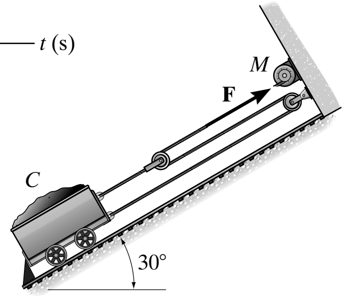
***13–24.** If the force of the motor M on the cable is shown in the graph, determine the velocity of the cart when $t = 3\text{s}$. The load and cart have a mass of 200 kg and the car starts from rest.





Free-Body Diagram: The free-body diagram of the rail car is shown in Fig. (a).

Equations of Motion: For $0 \leq t < 3$ s, $F = \dfrac{450}{3}t = (150t)$ N. By referring to Fig. (a), we can write

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3(150t) - 200(9.81)\sin 30° = 200a$$

$$a = (2.25t - 4.905)\ \text{m/s}^2$$

For $t > 3$ s, $F = 450$ N. Thus,

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3(450) - 200(9.81)\sin 30° = 200a$$

$$a = 1.845\ \text{m/s}^2$$

Equilibrium: For the rail car to move, force **3F** must overcome the weight component of the rail crate. Thus, the time required to move the rail car is given by

$$\Sigma F_{x'} = 0; \quad 3(150t) - 200(9.81)\sin 30° = 0 \qquad t = 2.18\text{ s}$$

Kinematics: The velocity of the rail car can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $2.18\text{ s} \leq t < 3$ s, $v = 0$ at $t = 2.18$ s will be used as the integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

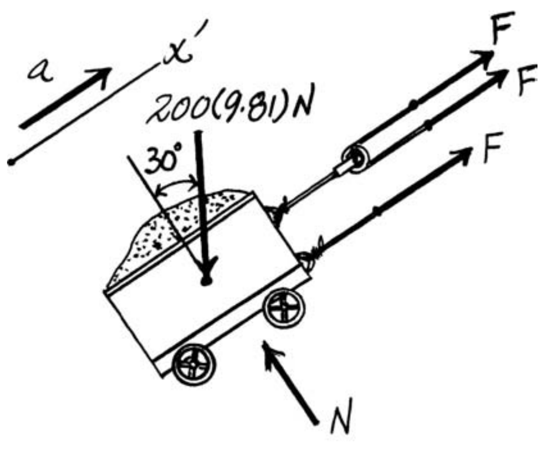
$$\int_0^v dv = \int_{2.18\text{ s}}^t (2.25t - 4.905)dt$$

$$v = \left(1.125t^2 - 4.905t\right)\Big|_{2.18\text{ s}}^t$$

$$= \left(1.125t^2 - 4.905t + 5.34645\right)\text{m/s}$$

When $t = 3$ s,

$$v = 1.125(3)^2 - 4.905(3) + 5.34645 = 0.756\text{m/s} \qquad \textbf{Ans.}$$



(a)

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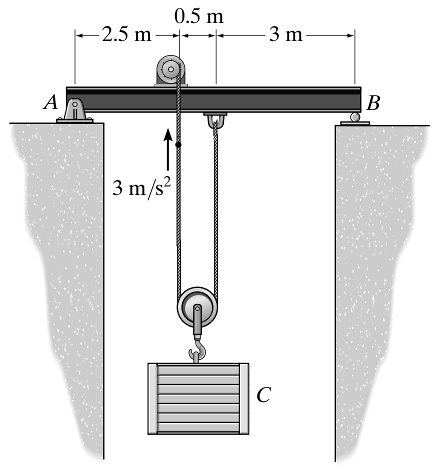
•13–25. If the motor draws in the cable with an acceleration of 3 m/s^2, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.



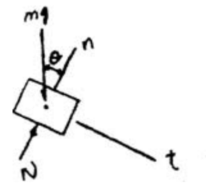
$$+\searrow \Sigma F_t = ma_t; \qquad mg \sin\theta = ma_t \qquad a_t = g\sin\theta$$

$$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad \text{However } dy = ds\sin\theta$$

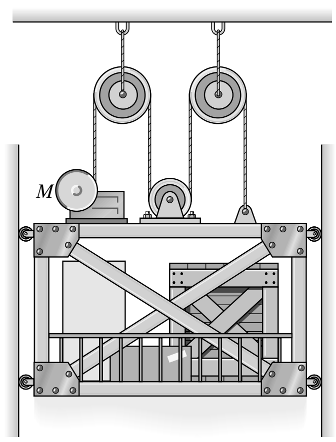
$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

$$v = \sqrt{2gh} \qquad \textbf{Q.E.D.}$$



13–26. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by the track and wheels mounted along its sides. When $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* If it starts from rest, determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys, motor, and cables.



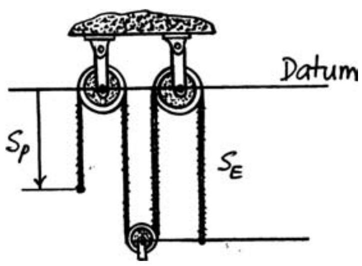
$$3s_E + s_P = l$$

$$3v_E = -v_P$$

$$(+\downarrow) \qquad v_P = v_E + v_{P/E}$$

$$-3v_E = v_E + 6$$

$$v_E = -\frac{6}{4} = -1.5\text{ m/s} = 1.5\text{ m/s}\uparrow$$



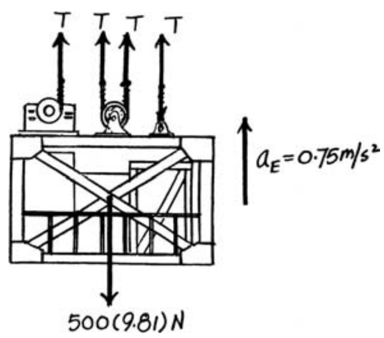
$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$1.5 = 0 + a_E(2)$$

$$a_E = 0.75\text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = ma_y; \qquad 4T - 500(9.81) = 500(0.75)$$

$$T = 1320\text{ N} = 1.32\text{ kN} \qquad \textbf{Ans.}$$



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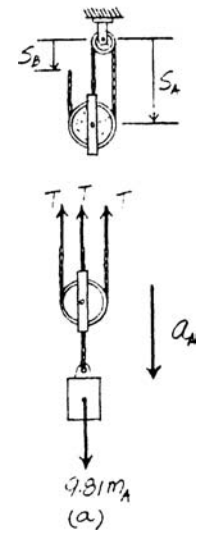
13–27. Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B a distance of 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.

Kinematic: Applying equation $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, we have

$$(\nwarrow +) \qquad 0.75 = 0 + 0 + \frac{1}{2} a_B \left(2^2\right) \qquad a_B = 0.375 \text{ m/s}^2$$

Establishing the position - coordinate equation, we have

$$2s_A + (s_A - s_B) = l \qquad 3s_A - s_B = l$$



Taking time derivative twice yields

$$3a_A - a_B = 0 \tag{1}$$

From Eq.(1),

$$3a_A - 0.375 = 0 \qquad a_A = 0.125 \text{ m/s}^2$$

Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

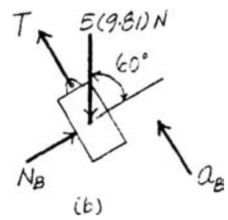
$$\nwarrow + \Sigma F_{y'} = m a_{y'}; \qquad T - 5(9.81) \sin 60° = 5(0.375)$$

$$T = 44.35 \text{ N}$$

From FBD(a),

$$+\uparrow \Sigma F_y = m a_y; \qquad 3(44.35) - 9.81 m_A = m_A(-0.125)$$

$$m_A = 13.7 \text{ kg} \qquad \textbf{Ans.}$$



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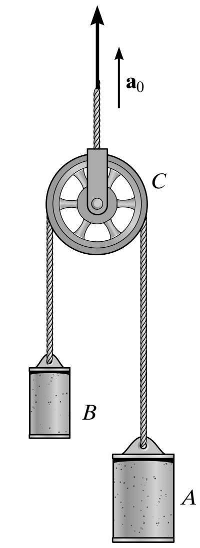
***13–28.** Blocks A and B have a mass of m_A and m_B, where $m_A > m_B$. If pulley C is given an acceleration of $\mathbf{a}_0$, determine the acceleration of the blocks. Neglect the mass of the pulley.



Free-Body Diagram: The free-body diagram of blocks A and B are shown in Figs, (a) and (b), respectively. Here, $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed upwards. Since pulley C is smooth, the tension in the cord remains constant for the entire cord.

Equations of Motion: By referring to Figs. (a) and (b),

$$+\uparrow \Sigma F_y = ma_y; \qquad T - m_A g = m_A a_A \qquad \textbf{(1)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T - m_B g = m_B a_B \qquad \textbf{(2)}$$

Eliminating **T** from Eqs. (1) and (2) yields

$$(m_A - m_B)g = m_B a_B - m_A a_A \qquad \textbf{(3)}$$

Kinematics: The acceleration of blocks A and B relative to pulley C will be of the same magnitude, i.e., $a_{A/C} = a_{B/C} = a_{\text{rel}}$. If we assume that $\mathbf{a}_{A/C}$ is directed downwards, $\mathbf{a}_{B/C}$ must also be directed downwards to be consistent. Applying the relative acceleration equation,

$$(+\uparrow) \qquad \mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$a_A = a_O - a_{\text{rel}} \qquad \textbf{(4)}$$

and

$$(+\uparrow) \qquad \mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$a_B = a_O - a_{\text{rel}} \qquad \textbf{(5)}$$

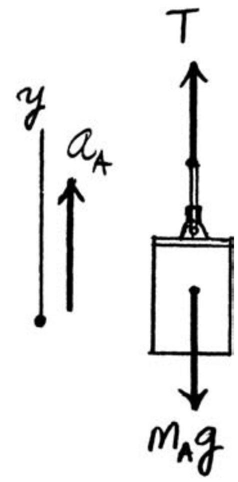
Eliminating a_{rel} from Eqs.(4) and (5),

$$a_A + a_B = 2a_O \qquad \textbf{(6)}$$

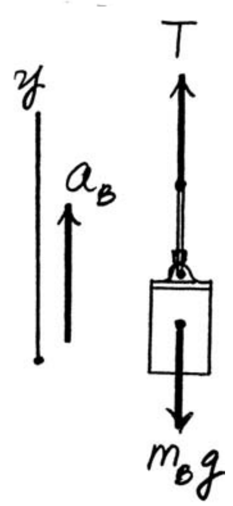
Solving Eqs. (3) and (6), yields

$$a_A = \frac{2mg\,a_O - (m_A - m_B)g}{m_A + m_B} \uparrow \qquad \textbf{Ans.}$$

$$a_B = \frac{2m_A a_O + (m_A - m_B)g}{m_A + m_B} \uparrow \qquad \textbf{Ans.}$$



(a)



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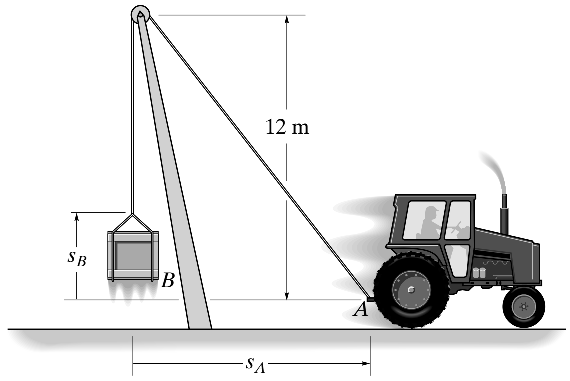
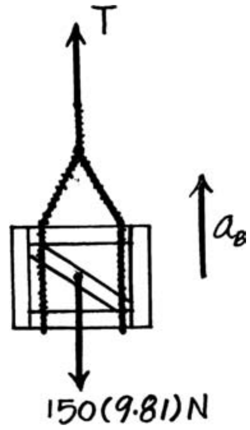
•13–29. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-s_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$



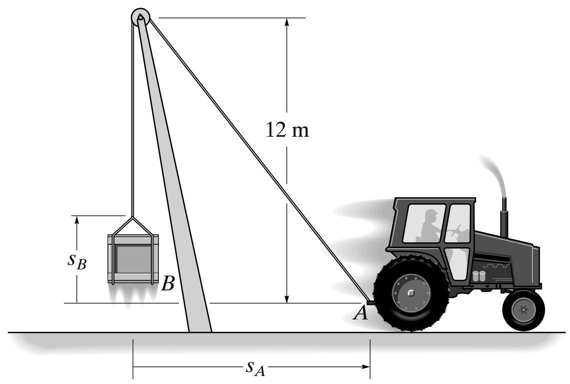
$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

$+\uparrow \Sigma F_y = ma_y;\qquad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans.**

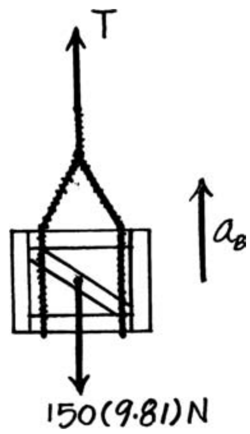
13–30. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



$$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$



$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$

$+\uparrow \Sigma F_y = ma_y;\qquad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans.**

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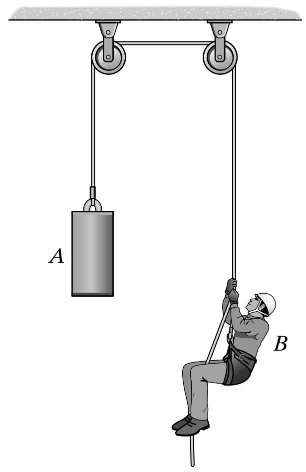
13–31. The 75-kg man climbs up the rope with an acceleration of $0.25\,\text{m/s}^2$, measured relative to the rope. Determine the tension in the rope and the acceleration of the 80-kg block.



Free-Body Diagram: The free-body diagram of the man and block A are shown in Figs. (a) and (b), respectively. Here, the acceleration of the man a_m and the block a_A are assumed to be directed upwards.

Equations of Motion: By referring to Figs. (a) and (b),

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 75(9.81) = 75a_m \qquad \textbf{(1)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 80(9.81) = 80a_A \qquad \textbf{(2)}$$

Kinematics: Here, the rope has an acceleration with a magnitude equal to that of block A, i.e., $a_r = a_A$ and is directed downward. Applying the relative acceleration equation,

$$(+\uparrow) \qquad \mathbf{a}_m = \mathbf{a}_r + \mathbf{a}_{m/r}$$

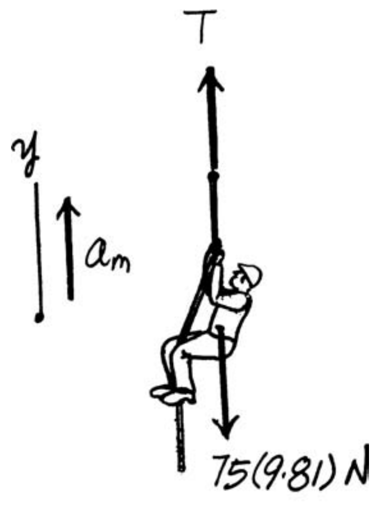
$$a_m = -a_A + 0.25 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

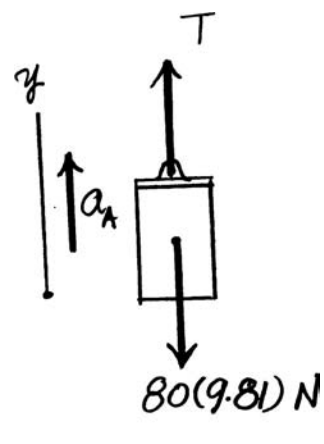
$$a_A = -0.19548\ \text{m/s}^2 = 0.195\ \text{m/s}^2\downarrow \qquad \textbf{Ans.}$$

$$T = 769.16\ \text{N} = 769\ \text{N} \qquad \textbf{Ans.}$$

$$a_m = 0.4455\ \text{m/s}^2$$



(a)



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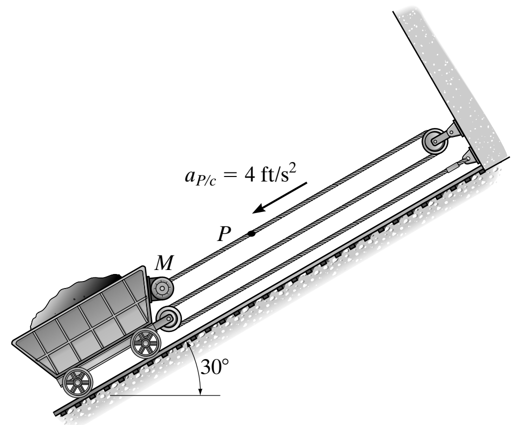
***13–32.** Motor M draws in the cable with an acceleration of $4\,\text{ft/s}^2$, measured relative to the 200-lb mine car. Determine the acceleration of the car and the tension in the cable. Neglect the mass of the pulleys.



Free-Body Diagram: The free-body diagram of the mine car is shown in Fig. (a). Here, its acceleration $\mathbf{a}_C$ is assumed to be directed down the inclined plane so that it is consistent with the position coordinate s_C of the mine car as indicated on Fig. (b).

Equations of Motion: By referring to Fig. (a),

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3T - 200\sin 30^\circ = \frac{200}{32.2}(-a_C) \qquad \textbf{(1)}$$

Kinematics: We can express the length of the cable in terms of s_P and s_C by referring to Fig. (b).

$$s_P + 2s_C = 0$$

The second derivative of the above equation gives

$$a_P + 2a_C = 0 \qquad \textbf{(2)}$$

Applying the relative acceleration equation,

$$\mathbf{a}_P = \mathbf{a}_C + \mathbf{a}_{P/C}$$

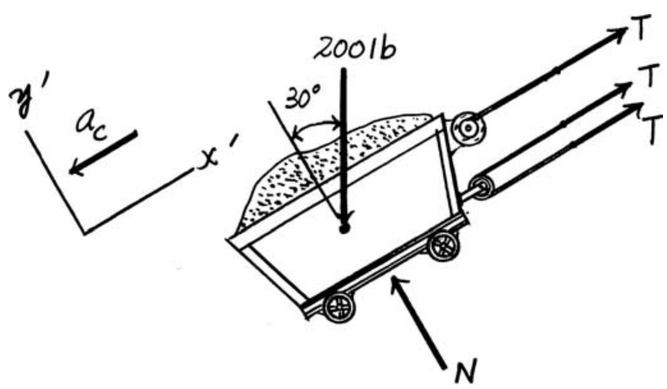
$$a_P = a_C + 4 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

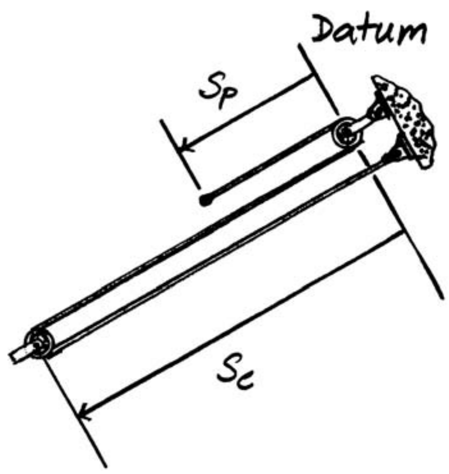
$$a_C = -1.333\ \text{ft/s}^2 = 1.33\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$T = 36.1\ \text{lb} \qquad \textbf{Ans.}$$

$$a_P = 2.667\ \text{ft/s}^2$$



(a)



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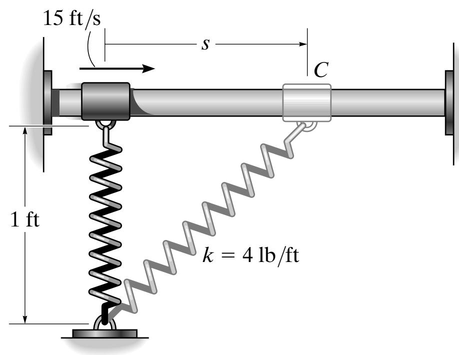
•13–33. The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



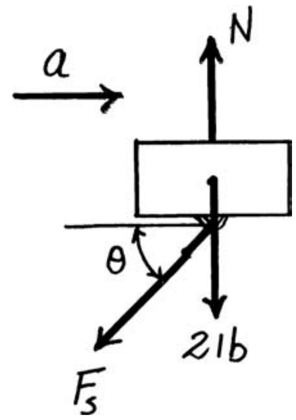
$$F_s = kx; \qquad F_s = 4\left(\sqrt{1 + s^2} - 1\right)$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -4\left(\sqrt{1 + s^2} - 1\right)\left(\frac{s}{\sqrt{1 + s^2}}\right) = \left(\frac{2}{32.2}\right)\left(v\frac{dv}{ds}\right)$$

$$-\int_0^1 \left(4s\,ds - \frac{4s\,ds}{\sqrt{1 + s^2}}\right) = \int_{15}^{v} \left(\frac{2}{32.2}\right) v\,dv$$

$$-\left[2s^2 - 4\sqrt{1 + s^2}\right]_0^1 = \frac{1}{32.2}\left(v^2 - 15^2\right)$$

$v = 14.6$ ft/s **Ans.**



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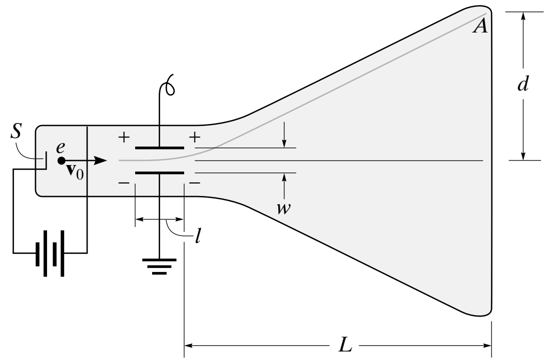
13–34. In the cathode-ray tube, electrons having a mass m are emitted from a source point S and begin to travel horizontally with an initial velocity $\mathbf{v}_0$. While passing between the grid plates a distance l, they are subjected to a vertical force having a magnitude eV/w, where e is the charge of an electron, V the applied voltage acting across the plates, and w the distance between the plates. After passing clear of the plates, the electrons then travel in straight lines and strike the screen at A. Determine the deflection d of the electrons in terms of the dimensions of the voltage plate and tube. Neglect gravity which causes a slight vertical deflection when the electron travels from S to the screen, and the slight deflection between the plates.



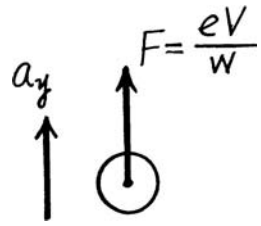
$v_x = v_0$

$t_1 = \dfrac{l}{v_0}$

t_1 is the time between plates.

$t_1 = \dfrac{L}{v_0}$

t_2 is the tune to reach screen.



$+\uparrow \Sigma F_y = ma_y; \qquad \dfrac{eV}{w} = ma_y$

$$a_y = \frac{eV}{mw}$$

During t_1 constant acceleration,

$(+\uparrow) \qquad v = v_0 + a_c t$

$v_y = a_y t_1 = \left(\dfrac{eV}{mw}\right)\left(\dfrac{l}{v_0}\right)$

During time t_2, $a_y = 0$

$d = v_y t_2 = \left(\dfrac{eVl}{mwv_0}\right)\left(\dfrac{L}{v_0}\right)$

$d = \dfrac{eVLl}{v_0^2 wm}$ **Ans.**

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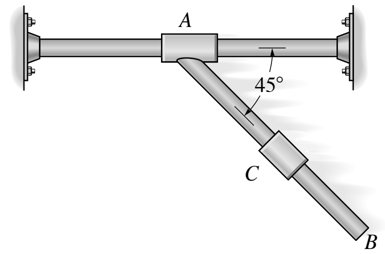
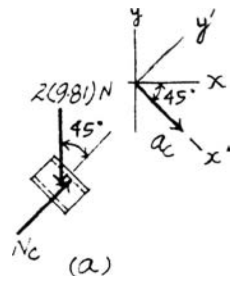
13–35. The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves to the left at constant velocity along the horizontal guide, and (c) collar A is subjected to an acceleration of 2 m/s^2 to the left. In all cases, the motion occurs in the vertical plane.



a, b) Equation of Motion: Applying Eq. 13–7 to FBD(a), we have

$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad 2(9.81)\sin 45° = 2a_C$$

$$a_C = 6.94\text{ m/s}^2 \searrow \qquad \textbf{Ans.}$$



c) Equation of Motion: Applying Eq. 13–7 to FBD(b), we have

$$\searrow + \Sigma F_{x'} = ma_{x'}; \qquad 2(9.81)\sin 45° = 2a_{C/A} + 2(-2\cos 45°)$$

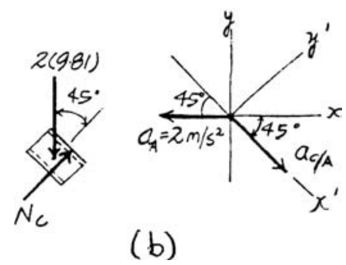
$$a_{C/A} = 8.351\text{ m/s}^2$$

Relative Acceleration:

$$\overset{+}{\rightarrow} \qquad \mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$$

$$= -2\mathbf{i} + 8.351\cos 45°\mathbf{i} - 8.351\sin 45°\mathbf{j}$$

$$= \{3.905\mathbf{i} - 5.905\mathbf{j}\}\text{ m/s}^2$$



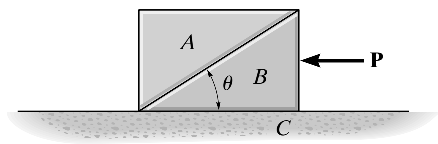
Thus, the magnitude of the acceleration $\mathbf{a}_c$ is

$$a_C = \sqrt{3.905^2 + (-5.905)^2} = 7.08\text{ m/s}^2 \qquad \textbf{Ans.}$$

and its directional angle is

$$\theta = \tan^{-1}\left(\frac{5.905}{3.905}\right) = 56.5° \searrow \qquad \textbf{Ans.}$$

***13–36.** Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not move relative to B. All surfaces are smooth.



Require

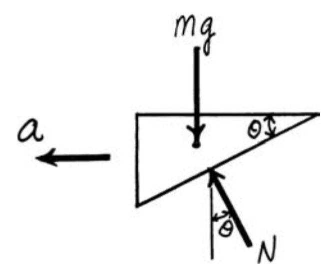
$$a_A = a_B = a$$

Block A:

$$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - mg = 0$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta = ma$$

$$a = g\tan\theta$$

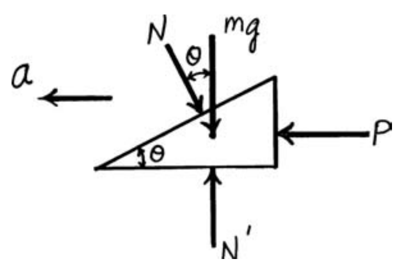


Block B:

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - N\sin\theta = ma$$

$$P - mg\tan\theta = mg\tan\theta$$

$$P = 2mg\tan\theta \qquad \textbf{Ans.}$$



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•13–37. Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.

Require

$a_A = a_B = a$

Block A:

$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - \mu_s N\sin\theta - mg = 0$

$\xleftarrow{+} \Sigma F_x = ma_x; \qquad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

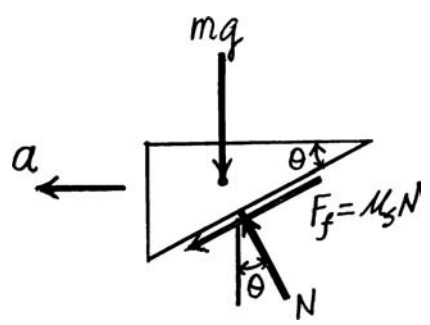
$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

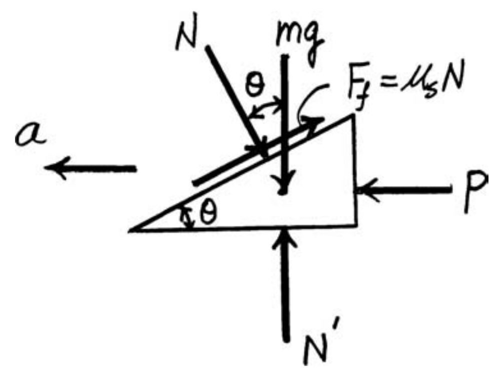
Block B:

$\xleftarrow{+} \Sigma F_x = ma_x; \qquad P - \mu_s N\cos\theta - N\sin\theta = ma$

$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) \qquad \textbf{Ans.}$$





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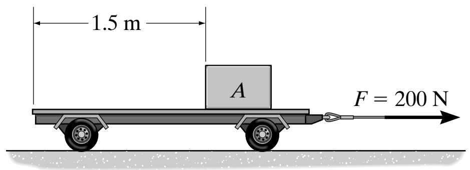
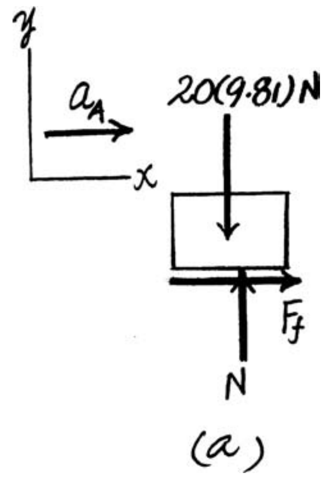
13–38. If a force $F = 200$ N is applied to the 30-kg cart, show that the 20-kg block A will slide on the cart. Also determine the time for block A to move on the cart 1.5 m. The coefficients of static and kinetic friction between the block and the cart are $\mu_s = 0.3$ and $\mu_k = 0.25$. Both the cart and the block start from rest.



Free-Body Diagram: The free-body diagram of block A and the cart are shown in Figs. (a) and (b), respectively.



Equations of Motion: If block A does not slip, it will move together with the cart with a common acceleration, i.e., $a_A = a_C = a$. By referring to Figs. (a) and (b),

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 20(9.81) = 2(0)$$

$$N = 196.2 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_f = 20a \qquad \textbf{(1)}$$

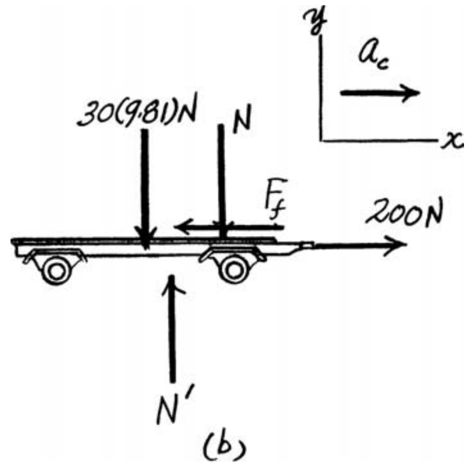
and

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 200 - F_f = 30a \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a = 4 \text{ m/s}^2 \qquad F_f = 80 \text{ N}$$

Since $F_f > (F_f)_{\max} = \mu_S N = 0.3(196.2) = 58.86$ N, the *block A will slide on the cart*. As such $F_f = \mu_k N = 0.25(196.2) = 49.05$ N. Again, by referring to Figs. (a) and (b),



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 49.05 = 20a_A \qquad a_A = 2.4525 \text{m/s}^2$$

and

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 200 - 49.05 = 30a_C \qquad a_C = 5.0317 \text{ m/s}^2$$

Kinematics: The acceleration of block A relative to the cart can be determined by applying the relative acceleration equation

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$(\xrightarrow{+}) \qquad 2.4525 = 5.0317 + a_{A/C}$$

$$a_{A/C} = -2.5792 \text{ m/s}^2 = 2.5792 \text{ m/s}^2 \leftarrow$$

Here, $s_{A/C} = 1.5 \text{ m} \leftarrow$. Thus,

$$s_{A/C} = (s_{A/C})_O + (v_{A/C})_O t + \frac{1}{2} a_{A/C} t^2$$

$$(\xrightarrow{+}) \qquad 1.5 = 0 + 0 + \frac{1}{2}(2.5792)t^2$$

$$t = 1.08 \text{ s} \qquad \textbf{Ans.}$$

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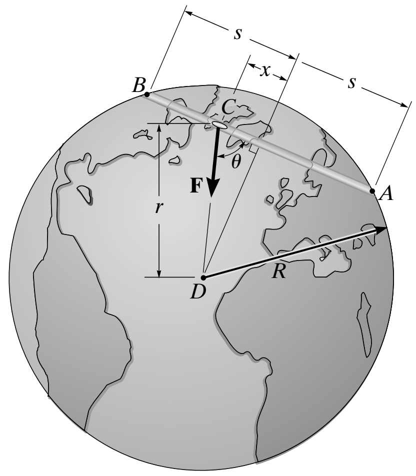
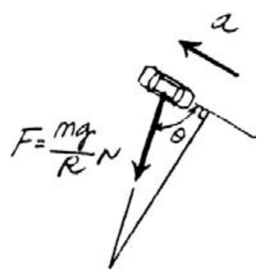
13–39. Suppose it is possible to dig a smooth tunnel through the earth from a city at A to a city at B as shown. By the theory of gravitation, any vehicle C of mass m placed within the tunnel would be subjected to a gravitational force which is always directed toward the center of the earth D. This force $\mathbf{F}$ has a magnitude that is directly proportional to its distance r from the earth's center. Hence, if the vehicle has a weight of $W = mg$ when it is located on the earth's surface, then at an arbitrary location r the magnitude of force $\mathbf{F}$ is $F = (mg/R)r$, where $R = 6328$ km, the radius of the earth. If the vehicle is released from rest when it is at B, $x = s = 2$ Mm, determine the time needed for it to reach A, and the maximum velocity it attains. Neglect the effect of the earth's rotation in the calculation and assume the earth has a constant density. *Hint:* Write the equation of motion in the x direction, noting that $r\cos\theta = x$. Integrate, using the kinematic relation $v\,dv = a\,dx$, then integrate the result using $v = dx/dt$.



Equation of Motion: Applying Eq. 13–7, we have

$$\nwarrow + \Sigma F_{x'} = ma_{x'}; \qquad -\frac{mg}{R} r\cos\theta = ma \qquad a = -\frac{g}{R} r\cos\theta = -\frac{g}{R}x$$



Kinematics: Applying equation $v\,dv = a\,dx$, we have

$$(\nwarrow +) \qquad \int_0^v v\,dv = -\frac{g}{R}\int_s^x x\,dx$$

$$\frac{v^2}{2} = \frac{g}{2R}\left(s^2 - x^2\right)$$

$$v = -\sqrt{\frac{g}{R}\left(s^2 - x^2\right)} \qquad \textbf{(1)}$$

Note: The negative sign indicates that the velocity is in the opposite direction to that of positive x.

Applying equation $dt = dx/v$, we have

$$(\nwarrow +) \qquad \int_0^t dt = -\sqrt{\frac{R}{g}}\int_s^x \frac{dx}{\sqrt{s^2 - x^2}}$$

$$t = \sqrt{\frac{R}{g}}\left(\frac{\pi}{2} - \sin^{-1}\frac{x}{s}\right)$$

At $x = -s$, $$t = \sqrt{\frac{R}{g}}\left(\frac{\pi}{2} - \sin^{-1}\frac{-s}{s}\right) = \pi\sqrt{\frac{R}{g}} \qquad \textbf{(2)}$$

Substituting $R = 6328\left(10^3\right)$ m and $g = 9.81\ \text{m/s}^2$ into Eq.(2) yields

$$t = \pi\sqrt{\frac{6328(10^3)}{9.81}} = 2523.2\ \text{s} = 42.1\ \text{min} \qquad \textbf{Ans.}$$

The maximum velocity occurs at $x = 0$. From Eq.(1)

$$v_{max} = -\sqrt{\frac{g}{R}\left(s^2 - 0^2\right)} = -\sqrt{\frac{g}{R}}s \qquad \textbf{(3)}$$

Substituting $R = 6328\left(10^3\right)$ m, $s = 2\left(10^6\right)$ m, and $g = 9.81\ \text{m/s}^2$ into Eq.(3) yields

$$v_{max} = -\left(\sqrt{\frac{9.81}{6328\left(10^3\right)}}\right)\left[2\left(10^6\right)\right] = -2490.18\ \text{m/s} = 2.49\ \text{km/s} \qquad \textbf{Ans.}$$

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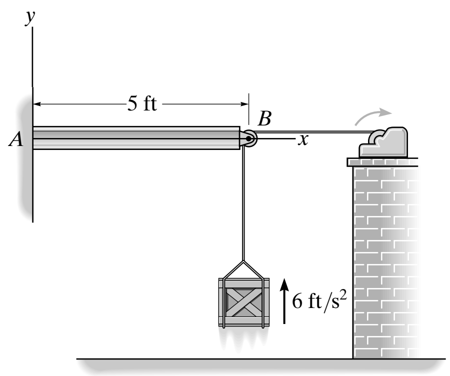
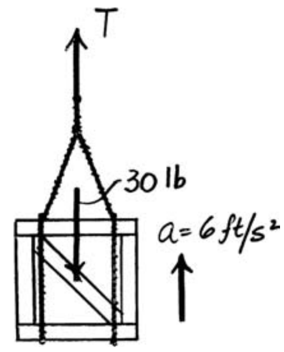
***13–40.** The 30-lb crate is being hoisted upward with a constant acceleration of 6 ft/s^2. If the uniform beam AB has a weight of 200 lb, determine the components of reaction at the fixed support A. Neglect the size and mass of the pulley at B. *Hint:* First find the tension in the cable, then analyze the forces in the beam using statics.



Crate:

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 30 = \left(\frac{30}{32.2}\right)(6) \qquad T = 35.59 \text{ lb}$$

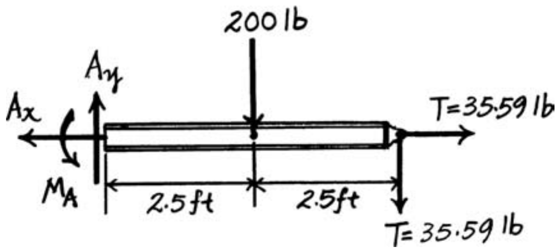


Beam:

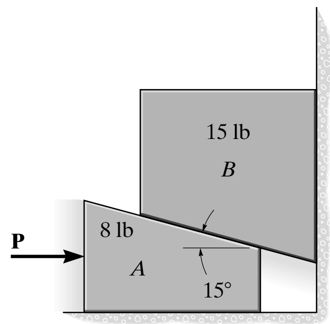
$$\xrightarrow{+} \Sigma F_x = 0; \qquad -A_x + 35.59 = 0 \qquad A_x = 35.6 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 200 - 35.59 = 0 \qquad A_y = 236 \text{ lb} \qquad \textbf{Ans.}$$

$$+\Sigma M_A = 0; \qquad M_A - 200(2.5) - (35.59)(5) = 0 \qquad M_A = 678 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



•13–41. If a horizontal force of $P = 10$ lb is applied to block A, determine the acceleration of block B. Neglect friction. *Hint:* Show that $a_B = a_A \tan 15°$.

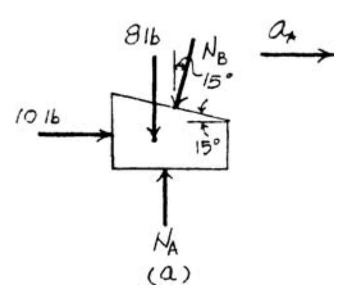


Equations of Motion: Applying Eq. 13–7 to FBD(a), we have

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 10 - N_B \sin 15° = \left(\frac{8}{32.2}\right)a_A \qquad \textbf{(1)}$$

Applying Eq. 13–7 to FBD(b), we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B \cos 15° - 15 = \left(\frac{15}{32.2}\right)a_B \qquad \textbf{(2)}$$



Kinematics: From the geometry of Fig. (c),

$$s_B = s_A \tan 15°$$

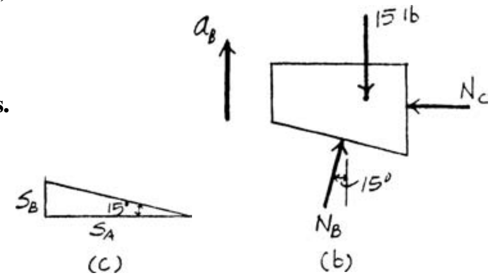
Taking the time derivative twice to the above expression yields

$$a_B = a_A \tan 15° \quad \text{(Q.E.D.)} \qquad \textbf{(3)}$$

Solving Eqs.(1), (2) and (3) yields

$$a_B = 5.68 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_A = 21.22 \text{ ft/s}^2 \qquad N_B = 18.27 \text{ lb}$$



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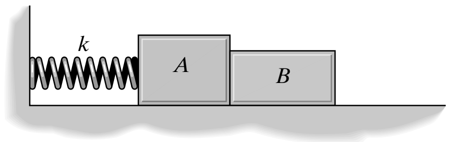
13–42. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -k(x - d) - N = m_A a_A$$

Block B:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad N = m_B a_B$$

Since $a_A = a_B = a$,

$$-k(x - d) - m_B a = m_A a$$

$$a = \frac{k(d - x)}{(m_A + m_B)} \qquad N = \frac{km_B(d - x)}{(m_A + m_B)}$$

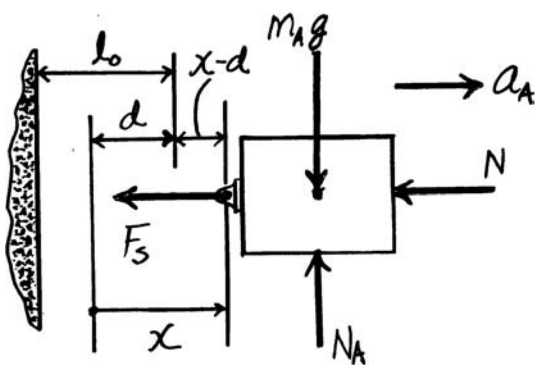
$N = 0$ when $d - x = 0$, or $x = d$ **Ans.**

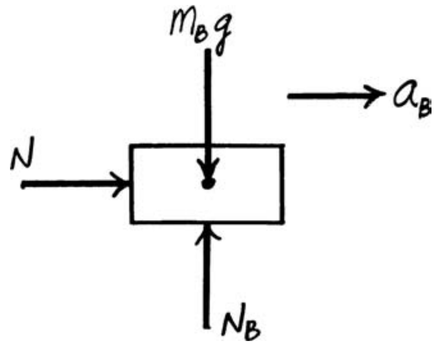
$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \frac{k(d - x)}{(m_A + m_B)}\,dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \frac{1}{2}\frac{kd^2}{(m_A + m_B)}$$

$$v = \sqrt{\frac{kd^2}{(m_A + m_B)}}$$ **Ans.**





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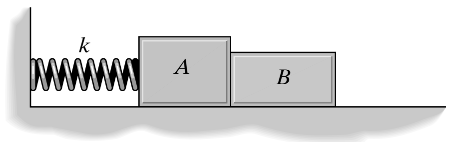
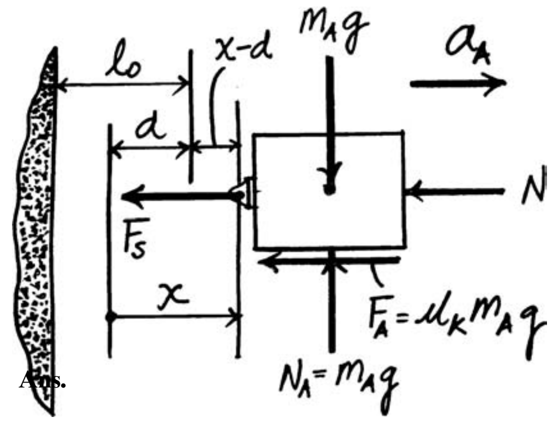
13–43. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



Block A:

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad -k(x-d) - N - \mu_k m_A g = m_A a_A$$



Block B:

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad N - \mu_k m_B g = m_B a_B$$

Since $a_A = a_B = a$,

$$a = \frac{k(d-x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \frac{k(d-x)}{(m_A + m_B)} - \mu_k g$$

$$N = \frac{km_B(d-x)}{(m_A + m_B)}$$

$N = 0$, then $x = d$ for separation. **Ans.**

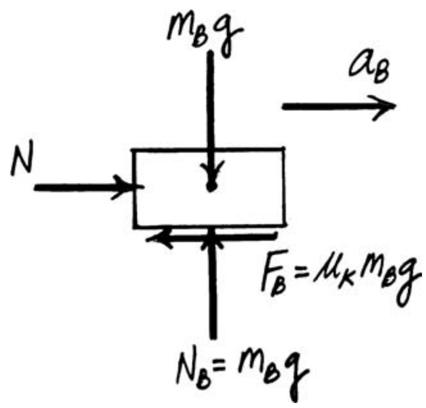
At the moment of separation:

$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \left[\frac{k(d-x)}{(m_A + m_B)} - \mu_k g\right] dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$$



Require $v > 0$, so that

$$kd^2 - 2\mu_k g(m_A + m_B)d > 0$$

Thus,

$$kd > 2\mu_k g(m_A + m_B)$$

$$d > \frac{2\mu_k g}{k}(m_A + m_B)$$ **Q.E.D.**

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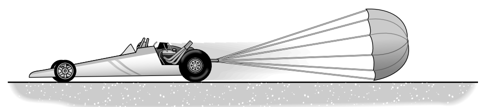
***13–44.** The 600-kg dragster is traveling with a velocity of 125 m/s when the engine is shut off and the braking parachute is deployed. If air resistance imposed on the dragster due to the parachute is $F_D = (6000 + 0.9v^2)$ N, where v is in m/s, determine the time required for the dragster to come to rest.



Free-Body Diagram: The free-body diagram of the dragster is shown in Fig. (a).

Equations of Motion: By referring to Fig. (a),

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 6000 + 0.9v^2 = 600(-a)$$

$$a = -\left[10 + 1.5(10^{-3})v^2\right] \text{ m/s}^2$$

$$= -1.5(10^{-3})\left[6666.67 + v^2\right] \text{ m/s}^2$$

Kinematics: Using the result of **a**, the time the dragster takes to stop can be obtained by integrating.

$$\left(\xleftarrow{+}\right) \qquad \int dt = \int \frac{dv}{a}$$

$$\int_0^t dt = \int_{125 \text{ m/s}}^{v} \frac{dv}{-1.5(10^{-3})(6666.67 + v^2)}$$

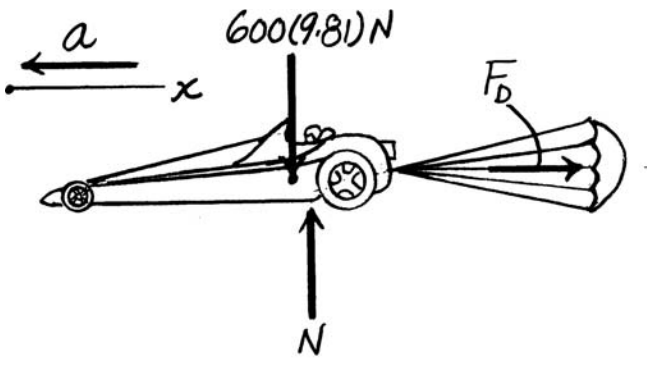
$$t = -666.67 \int_{125 \text{ m/s}}^{v} \frac{dv}{6666.67 + v^2}$$

$$= -666.67\left[\frac{1}{\sqrt{6666.67(1)}} \tan^{-1}\left(\frac{v}{\sqrt{6666.7}}\right)\right]\Bigg|_{125 \text{ m/s}}^{v}$$

$$= 8.165\left[0.9922 - \tan^{-1}(0.01225v)\right]$$

When $v = 0$,

$$t = 8.165\left[0.9922 - \tan^{-1}(0)\right] = 8.10 \text{ s} \qquad \textbf{Ans.}$$



(a)

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•13–45. The buoyancy force on the 500-kg balloon is $F = 6\,\text{kN}$, and the air resistance is $F_D = (100v)\,\text{N}$, where v is in m/s. Determine the terminal or maximum velocity of the balloon if it starts from rest.

Free-Body Diagram: The free-body diagram of the balloon is shown in Fig. (a).

Equations of Motion: By referring to Fig. (a),

$$+\uparrow \Sigma F_y = ma_y; \qquad 6000 - 500(9.81)100v = 500a$$

$$a = (2.19 - 0.2v)\ \text{m/s}^2$$

Kinematics: Using the result of **a**, the velocity of the balloon as a function of t can be determined by integrating the kinematic equation, $dt = \dfrac{dv}{a}$. Here, the initial condition $v = 0$ at $t = 0$ will be used as the integration limit. Thus,

$$(+\uparrow) \qquad \int dt = \int \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{2.19 - 0.2v}$$

$$t = -\frac{1}{0.2}\ln(2.19 - 0.2v)\Big|_0^v$$

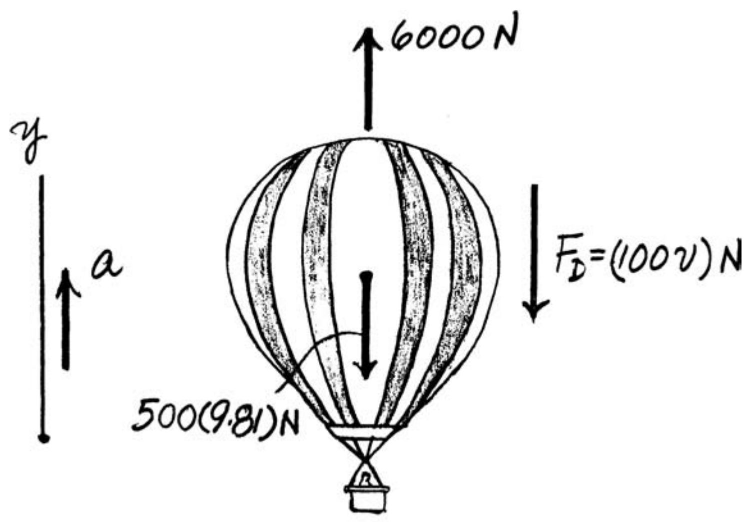
$$t = 5\ln\left(\frac{2.19}{2.19 - 0.2v}\right)$$

$$e^{t/5} = \frac{2.19}{2.19 - 0.2v}$$

$$v = 10.95(1 - e^{-t/5})$$

When $t \to \infty$, the balloon achieves its terminal velocity. Since $e^{-t/5} \to 0$ when $t \to \infty$,

$$v_{max} = 10.95\ \text{m/s} \qquad \textbf{Ans.}$$



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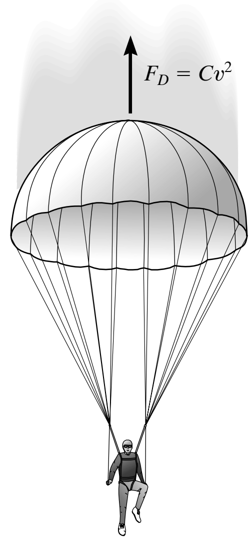
13–46. The parachutist of mass m is falling with a velocity of v_0 at the instant he opens the parachute. If air resistance is $F_D = Cv^2$, determine her maximum velocity (terminal velocity) during the descent.



Free-Body Diagram: The free-body diagram of the parachutist is shown in Fig. (a).

Equations of Motion: By referring to Fig. (a),

$$+\downarrow \Sigma F_y = ma_y; \qquad mg - cv^2 = ma$$

$$a = \frac{mg - cv^2}{m} = g - \frac{c}{m}v^2 \downarrow$$

Kinematics: Using the result of **a**, the velocity of the parachutist as a function of t can be determined by integrating the kinematic equation, $dt = \frac{dv}{a}$. Here, the initial condition $v = v_0$ at $t = 0$ will be used as the integration limit. Thus,

$$(+\downarrow) \qquad \int dt = \int \frac{dv}{a}$$

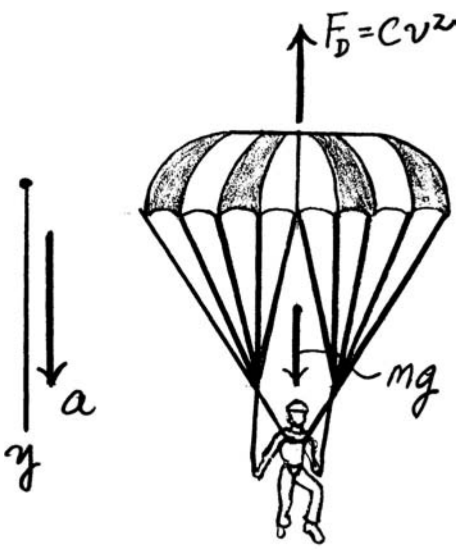
$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{g - \frac{c}{m}v^2}$$

$$t = \frac{1}{2\sqrt{\frac{gc}{m}}} \ln\left(\frac{\sqrt{g} + \sqrt{\frac{c}{m}}v}{\sqrt{g} - \sqrt{\frac{c}{m}}v}\right)\Bigg|_{v_0}^{v}$$

$$t = \frac{1}{2}\sqrt{\frac{m}{gc}} \ln\left[\frac{\sqrt{\frac{mg}{c}} + v}{\sqrt{\frac{mg}{c}} - v}\right]\Bigg|_{v_0}^{v}$$

$$2\sqrt{\frac{m}{gc}}t = \ln\left[\frac{\left(\sqrt{\frac{mg}{c}} + v\right)\left(\sqrt{\frac{mg}{c}} - v\right)}{\left(\sqrt{\frac{mg}{c}} - v\right)\left(\sqrt{\frac{mg}{c}} + v_0\right)}\right]$$

$$e^{2\sqrt{\frac{gc}{m}}t} = \left[\frac{\left(\sqrt{\frac{mg}{c}} + v\right)\left(\sqrt{\frac{mg}{c}} - v_0\right)}{\left(\sqrt{\frac{mg}{c}} - v\right)\left(\sqrt{\frac{mg}{c}} + v_0\right)}\right]$$



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13–46. Continued

$$v = \frac{\sqrt{\frac{mg}{c}}\left[\left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t} - 1\right]}{1 + \left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t}}$$

When $t \rightarrow \infty$, $\left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t} - 1 \cong 1 + \left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t}$. Thus, the terminal velocity of the parachutist is

$$v_{\max} = \sqrt{\frac{mg}{c}} \qquad \textbf{Ans.}$$

Note: The terminal velocity of the parachutist is independent of the initial velocity v_0.

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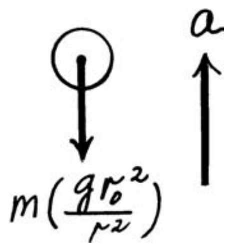
13–47. The weight of a particle varies with altitude such that $W = m(gr_0^2)/r^2$, where r_0 is the radius of the earth and r is the distance from the particle to the earth's center. If the particle is fired vertically with a velocity v_0 from the earth's surface, determine its velocity as a function of position r. What is the smallest velocity v_0 required to escape the earth's gravitational field, what is r_{max}, and what is the time required to reach this altitude?



$$+\uparrow \Sigma F_y = ma_y; \qquad -m\left(\frac{gr_0^2}{r^2}\right) = ma$$

$$a = \frac{gr_0^2}{r^2}$$

$$v\,dv = a\,dr$$

$$\int_{v_0}^{v} v\,dv = \int_{r_0}^{r} -gr_0^2 \frac{dr}{r^2}$$

$$\frac{1}{2}(v^2 - v_0^2) = -gr_0^2\left[-\frac{1}{r}\right]_0^r = gr_0^2\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$v = \sqrt{v_0^2 - 2gr_0\left(1 - \frac{r_0}{r}\right)}$$ **Ans. (1)**

For minimum escape, require $v = 0$,

$$v_0^2 - 2gr_0\left(1 - \frac{r_0}{r}\right) = 0$$

$$r_{max} = \frac{2gr_0^2}{2gr_0 - v_0^2}$$ **Ans.**

$$r_{max} \rightarrow \infty \quad \text{when } v_0^2 \rightarrow 2gr_0$$

Escape velocity is

$$v_{esc} = \sqrt{2gr_0}$$ **Ans. (2)**

From Eq. (1), using the value for v from Eq. (2),

$$v = \frac{dr}{dt} = \sqrt{\frac{2gr_0^2}{r}}$$

$$\int_{r_0}^{r} \frac{dr}{\sqrt{\frac{2gr_0^2}{r}}} = \int_0^t dt$$

$$\frac{1}{\sqrt{2gr_0^2}}\left[\frac{2}{3} r^{\frac{3}{2}}\right]_{r_0}^{r_{max}} = t$$

$$t = \frac{2}{3r_0\sqrt{2g}}\left(r_{max}^{\frac{3}{2}} - r_0^{\frac{3}{2}}\right)$$ **Ans.**

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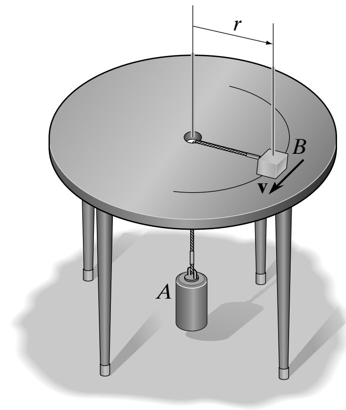
***13–48.** The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.

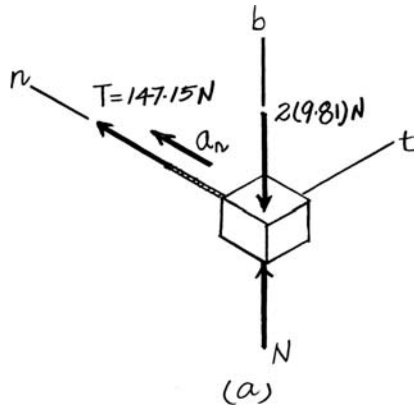


Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

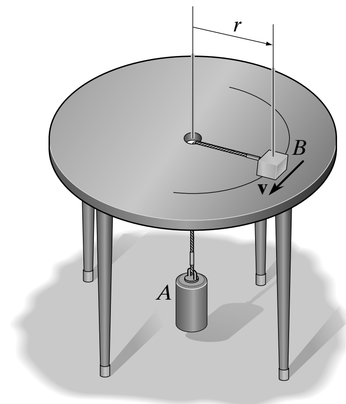
Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

$$r = 1.36\text{ m} \qquad \textbf{Ans.}$$



•13–49. The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.

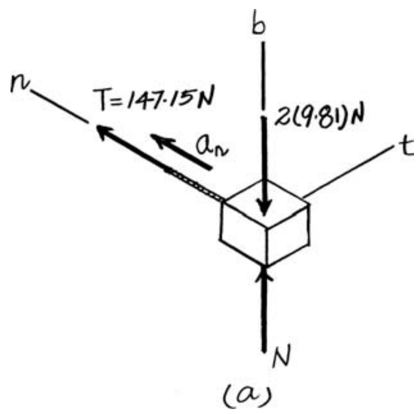


Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{N} = 147.15\text{N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\text{ m/s} \qquad \textbf{Ans.}$$



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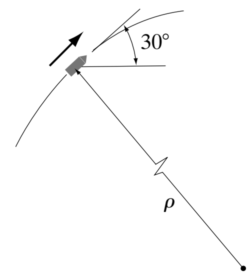
13–50. At the instant shown, the 50-kg projectile travels in the vertical plane with a speed of $v = 40\,\text{m/s}$. Determine the tangential component of its acceleration and the radius of curvature ρ of its trajectory at this instant.



Free-Body Diagram: The free-body diagram of the projectile is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the trajectory (positive n axis).

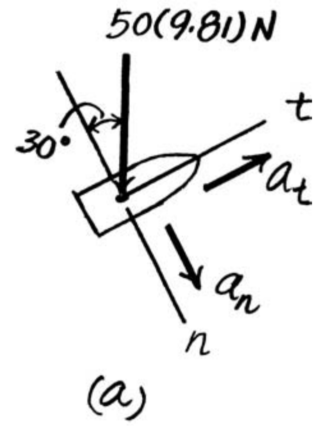
Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{40^2}{\rho}$. By referring to Fig. (a),

$+\nearrow\Sigma F_t = ma_t;\qquad -50(9.81)\sin 30^\circ = 50a_t$

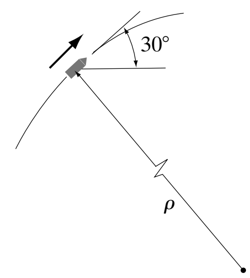
$a_t = -4.905\ \text{m/s}^2$ **Ans.**

$+\searrow\Sigma F_n = ma_n;\qquad 50(9.81)\cos 30^\circ = 50\left(\dfrac{40^2}{\rho}\right)$

$\rho = 188\ \text{m}$ **Ans.**



13–51. At the instant shown, the radius of curvature of the vertical trajectory of the 50-kg projectile is $\rho = 200\,\text{m}$. Determine the speed of the projectile at this instant.

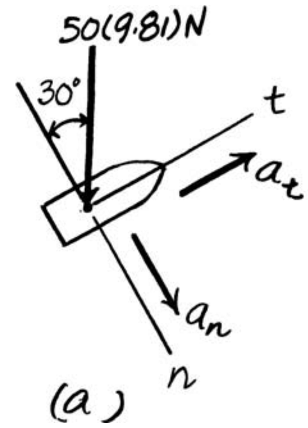


Free-Body Diagram: The free-body diagram of the projectile is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the trajectory (positive n axis).

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{200}$. By referring to Fig. (a),

$\searrow+\Sigma F_n = ma_n;\qquad 50(9.81)\cos 30^\circ = 50\left(\dfrac{v^2}{200}\right)$

$v = 41.2\ \text{m/s}$ **Ans.**



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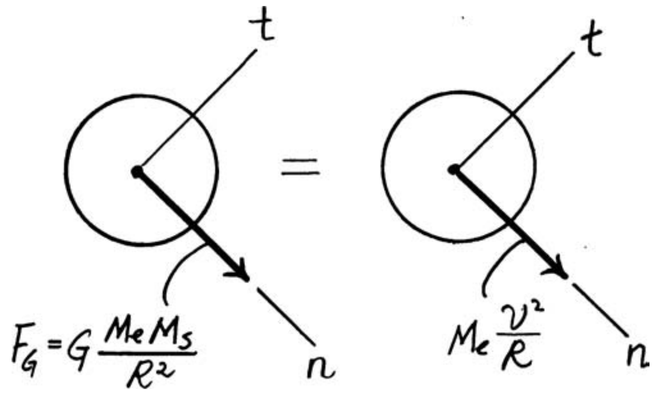
***13–52.** Determine the mass of the sun, knowing that the distance from the earth to the sun is $149.6(10^6)$ km. *Hint:* Use Eq. 13–1 to represent the force of gravity acting on the earth.

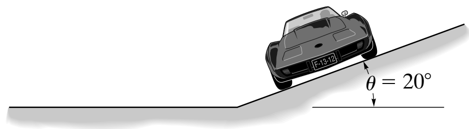
$$\searrow + \Sigma F_n = ma_n; \qquad G\frac{M_e M_s}{R^2} = M_e\frac{v^2}{R} \qquad M_s = \frac{v^2 R}{G}$$

$$v = \frac{s}{t} = \frac{2\pi(149.6)(10^9)}{365(24)(3600)} = 29.81\left(10^3\right)\text{ m/s}$$

$$M_s = \frac{\left[(29.81)(10^3)\right]^2(149.6)(10^9)}{66.73(10^{-12})} = 1.99\left(10^{30}\right)\text{ kg} \qquad \textbf{Ans.}$$



•13–53. The sports car, having a mass of 1700 kg, travels horizontally along a 20° banked track which is circular and has a radius of curvature of $\rho = 100$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the *maximum constant speed* at which the car can travel without sliding up the slope. Neglect the size of the car.

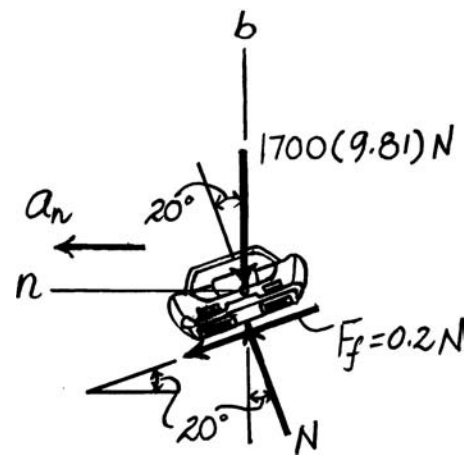


$$+\uparrow \Sigma F_b = 0; \qquad N\cos 20° - 0.2N\sin 20° - 1700(9.81) = 0$$

$$N = 19\,140.6\text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 19\,140.6\sin 20° + 0.2(19\,140.6)\cos 20° = 1700\left(\frac{v_{max}^2}{100}\right)$$

$$v_{max} = 24.4\text{ m/s} \qquad \textbf{Ans.}$$



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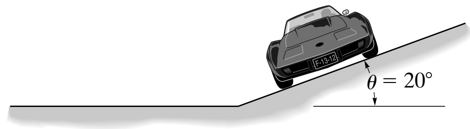
13–54. Using the data in Prob. 13–53, determine the *minimum speed* at which the car can travel around the track without sliding down the slope.

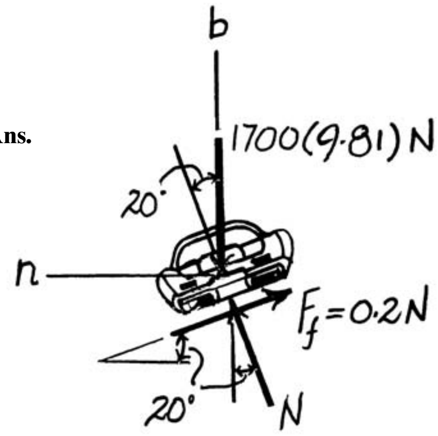


$$+\uparrow \Sigma F_b = 0; \qquad N\cos 20° + 0.2N\sin 20° - 1700(9.81) = 0$$

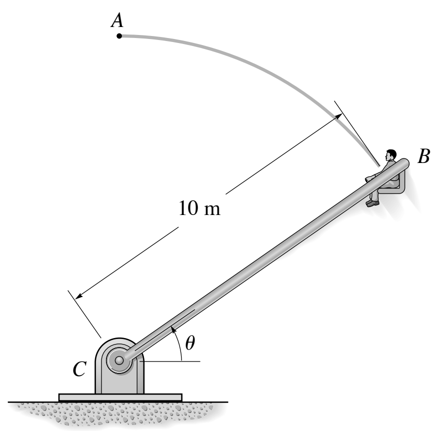
$$N = 16543.1 \text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 16543.1\sin 20° - 0.2(16543.1)\cos 20° = 1700\left(\frac{v_{\min}^2}{100}\right)$$

$$v_{\min} = 12.2 \text{ m/s} \qquad \textbf{Ans.}$$



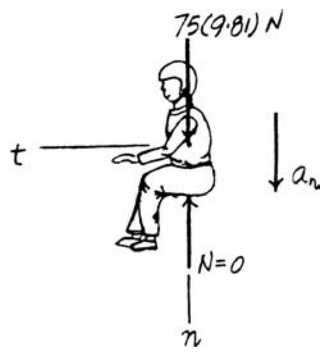
13–55. The device shown is used to produce the experience of weightlessness in a passenger when he reaches point A, $\theta = 90°$, along the path. If the passenger has a mass of 75 kg, determine the minimum speed he should have when he reaches A so that he does not exert a normal reaction on the seat. The chair is pin-connected to the frame BC so that he is always seated in an upright position. During the motion his speed remains constant.



Equation of Motion: If the man is about to fly off from the seat, the normal reaction $N = 0$. Applying Eq. 13–8, we have

$$+\downarrow \Sigma F_n = ma_n; \qquad 75(9.81) = 75\left(\frac{v^2}{10}\right)$$

$$v = 9.90 \text{ m/s} \qquad \textbf{Ans.}$$



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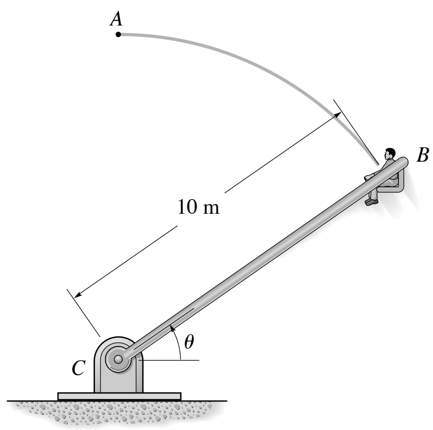
***13–56.** A man having the mass of 75 kg sits in the chair which is pin-connected to the frame BC. If the man is always seated in an upright position, determine the horizontal and vertical reactions of the chair on the man at the instant $\theta = 45°$. At this instant he has a speed of 6 m/s, which is increasing at 0.5 m/s^2.



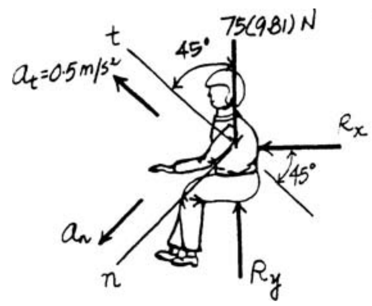
Equation of Motion: Applying Eq. 13–8, we have

$$+\nwarrow \Sigma F_t = ma_t; \qquad R_x \cos 45° + R_y \cos 45° - 75(9.81)\cos 45° = 75(0.5) \qquad \textbf{(1)}$$

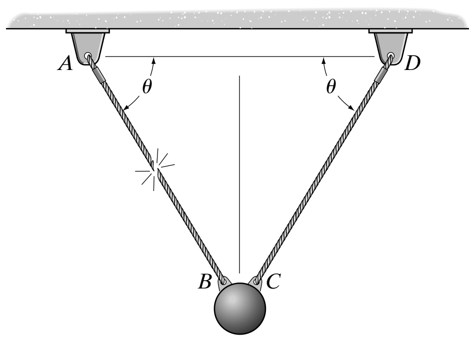
$$+\swarrow \Sigma F_n = ma_n; \qquad R_x \sin 45° - R_y \sin 45° + 75(9.81)\sin 45° = 75\left(\frac{6^2}{10}\right) \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2) yields

$$R_x = 217\text{ N} \qquad R_y = 571\text{ N} \qquad \textbf{Ans.}$$



•13–57. Determine the tension in wire CD just after wire AB is cut. The small bob has a mass m.

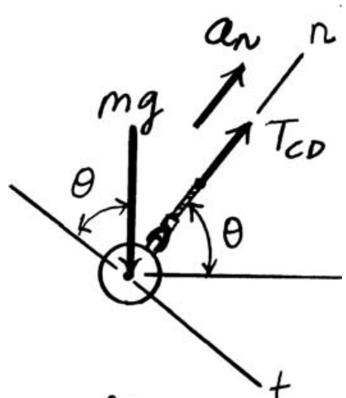


Free-Body Diagram: The free-body diagram of the bob is shown in Fig. (a).

Equations of Motion: Since the speed of the bob is zero just after the wire AB is cut, its normal component of acceleration is $a_n = \dfrac{v^2}{\rho} = 0$. By referring to Fig. (a),

$$+\nearrow \Sigma F_n = ma_n; \qquad T_{CD} - mg \sin\theta = m(0)$$

$$T_{CD} = mg\sin\theta \qquad \textbf{Ans.}$$



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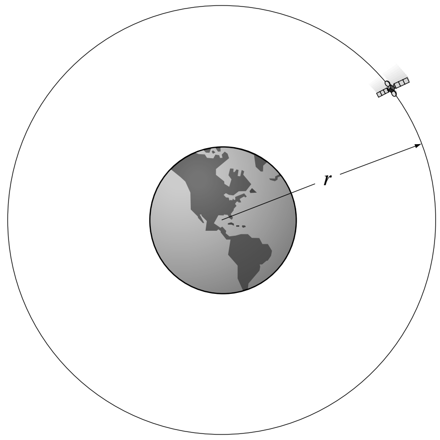
13–58. Determine the time for the satellite to complete its orbit around the earth. The orbit has a radius r measured from the center of the earth. The masses of the satellite and the earth are m_s and M_e, respectively.



Free-Body Diagram: The free-body diagram of the satellite is shown in Fig. (a). The force **F** which is directed towards the center of the orbit (positive n axis) is given by $F = \dfrac{GM_e m_s}{r^2}$ (Eq. 12–1). Also, $\mathbf{a}_n$ must be directed towards the positive n axis.

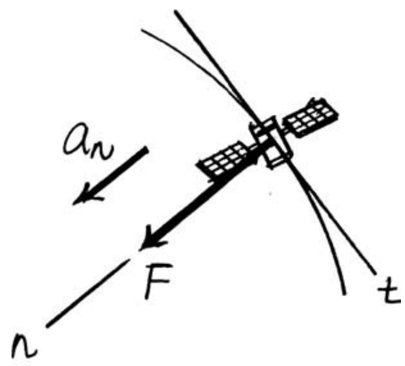
Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{{v_s}^2}{r}$ and referring to Fig. (a),

$$+\swarrow \Sigma F_n = ma_n; \qquad \frac{GM_e m_s}{r^2} = m_s\left(\frac{{v_s}^2}{r}\right)$$

$$v_s = \sqrt{\frac{GM_e}{r}}$$

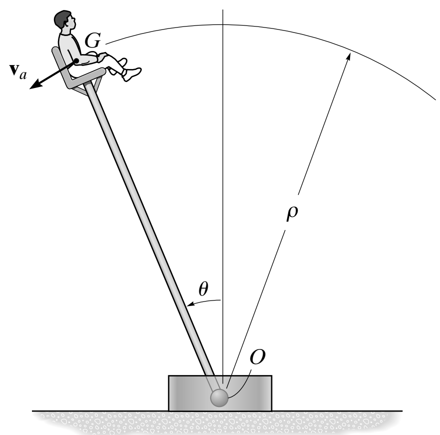
The period is the time required for the satellite to complete one revolution around the orbit. Thus,

$$T = \frac{2\pi r}{v_s} = \frac{2\pi r}{\sqrt{\dfrac{GM_e}{r}}} = 2\pi\sqrt{\frac{r^3}{GM_e}} \qquad \textbf{Ans.}$$



(a)

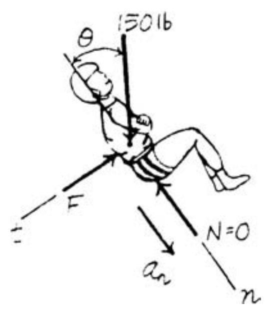
13–59. An acrobat has a weight of 150 lb and is sitting on a chair which is perched on top of a pole as shown. If by a mechanical drive the pole rotates downward at a constant rate from $\theta = 0°$, such that the acrobat's center of mass G maintains a *constant speed* of $v_a = 10$ ft/s, determine the angle θ at which he begins to "fly" out of the chair. Neglect friction and assume that the distance from the pivot O to G is $\rho = 15$ ft.



Equations of Motion: If the acrobat is about to fly off the chair, the normal reaction $N = 0$. Applying Eq. 13–8, we have

$$+\searrow \Sigma F_n = ma_n; \qquad 150\cos\theta = \frac{150}{32.2}\left(\frac{10^2}{15}\right)$$

$$\theta = 78.1° \qquad \textbf{Ans.}$$



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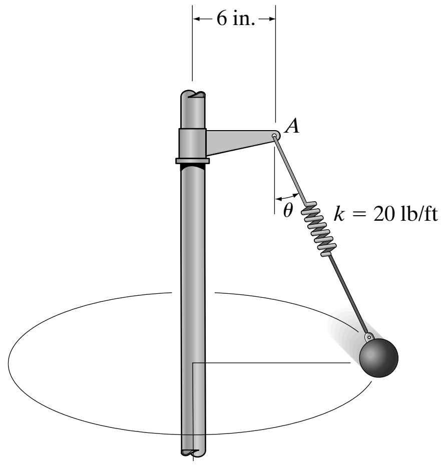
***13–60.** A spring, having an unstretched length of 2 ft, has one end attached to the 10-lb ball. Determine the angle θ of the spring if the ball has a speed of 6 ft/s tangent to the horizontal circular path.



Free-Body Diagram: The free-body diagram of the bob is shown in Fig. (a). If we denote the stretched length of the spring as l, then using the springforce formula, $F_{sp} = ks = 20(l - 2)$ lb. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = 0.5 + l\sin\theta$. Since $a_n = \dfrac{v^2}{r} = \dfrac{6^2}{0.5 + l\sin\theta}$, by referring to Fig. (a),

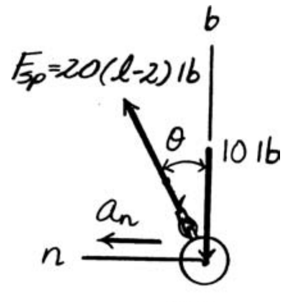
$$+\uparrow \Sigma F_b = 0; \qquad 20(l - 2)\cos\theta - 10 = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 20(l - 2)\sin\theta = \frac{10}{32.2}\left(\frac{6^2}{0.5 + l\sin\theta}\right) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

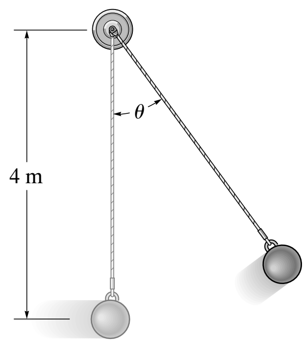
$$\theta = 31.26° = 31.3° \qquad \textbf{Ans.}$$

$$l = 2.585 \text{ ft} \qquad \textbf{Ans.}$$



(a)

•13–61. If the ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cord at this instant. Also, determine the angle θ to which the ball swings and momentarily stops. Neglect the size of the ball.



$$+\uparrow \Sigma F_n = ma_n; \qquad T - 30(9.81) = 30\left(\frac{(4)^2}{4}\right)$$

$$T = 414 \text{ N} \qquad \textbf{Ans.}$$

$$+\nearrow \Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$$

$$a_t = -9.81\sin\theta$$

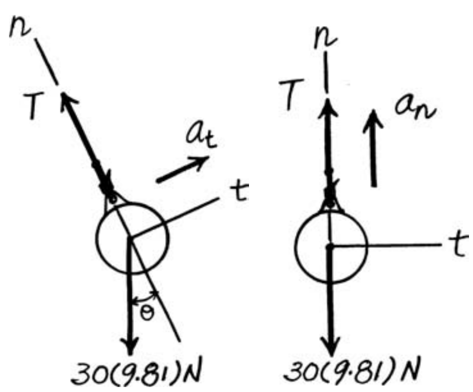
$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^\theta \sin\theta(4\,d\theta) = \int_4^0 v\,dv$$

$$\left[9.81(4)\cos\theta\right]_0^\theta = -\frac{1}{2}(4)^2$$

$$39.24(\cos\theta - 1) = -8$$

$$\theta = 37.2° \qquad \textbf{Ans.}$$



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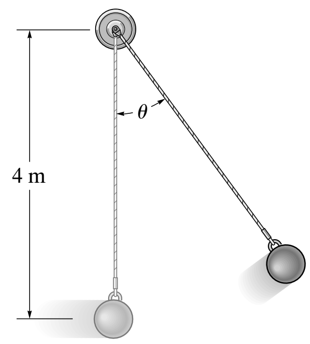
13–62. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$+\nwarrow \Sigma F_n = ma_n; \qquad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$

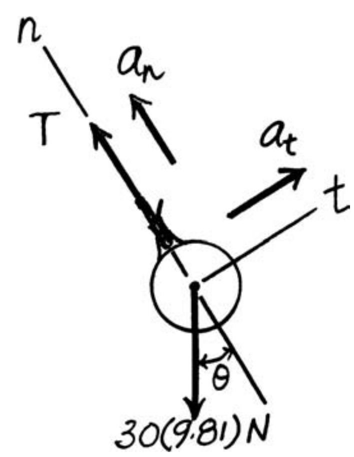
$a_t = -9.81\sin\theta$

$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^\theta \sin\theta\,(4\,d\theta) = \int_4^v v\,dv$$

$$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$



At $\theta = 20°$

$v = 3.357$ m/s

$a_t = -3.36\text{ m/s}^2 = 3.36\text{ m/s}^2 \swarrow$ **Ans.**

$T = 361$ N **Ans.**

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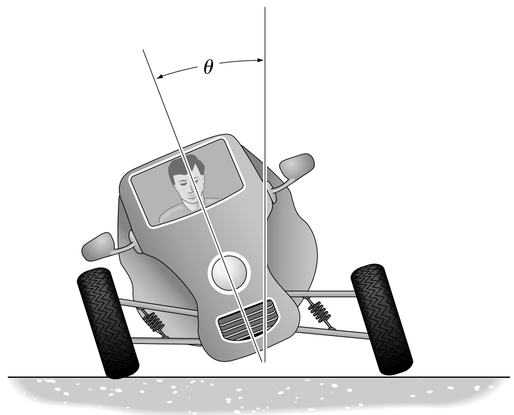
13–63. The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



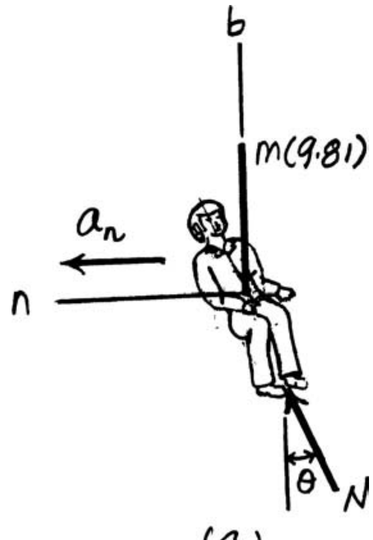
Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: The speed of the passenger is $v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 22.22$ m/s. Thus, the normal component of the passenger's acceleration is given by $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{100} = 4.938\text{ m/s}^2$. By referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad N = \frac{9.81m}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad \frac{9.81m}{\cos\theta}\sin\theta = m(4.938)$$

$$\theta = 26.7° \qquad \textbf{Ans.}$$



(a)

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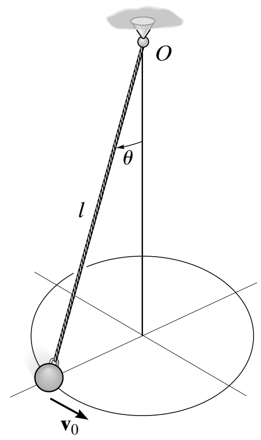
***13–64.** The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



$\xrightarrow{+} \Sigma F_n = ma_n; \qquad T\sin\theta = m\left(\frac{v_0^2}{r}\right)$

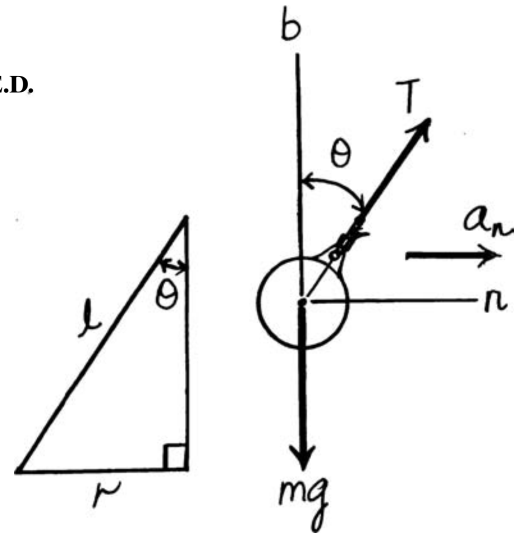
$+\uparrow \Sigma F_b = 0; \qquad T\cos\theta - mg = 0$

Since $r = l\sin\theta \qquad T = \dfrac{mv_0^2}{l\sin^2\theta}$

$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

Q.E.D.



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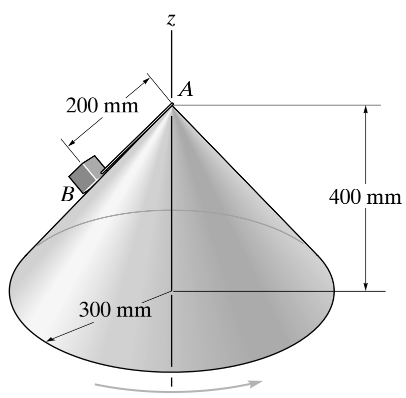
•13–65. The smooth block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. If the block has a speed of 0.5 m/s around the cone, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.



$$\frac{\rho}{200} = \frac{300}{500}; \qquad \rho = 120\text{ mm} = 0.120\text{ m}$$

$$+\nearrow\Sigma F_y = ma_y; \qquad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$$

$$T = 1.82\text{ N} \qquad \textbf{Ans.}$$

$$+\nwarrow\Sigma F_x = ma_x; \qquad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$$

$$N_B = 0.844\text{ N} \qquad \textbf{Ans.}$$

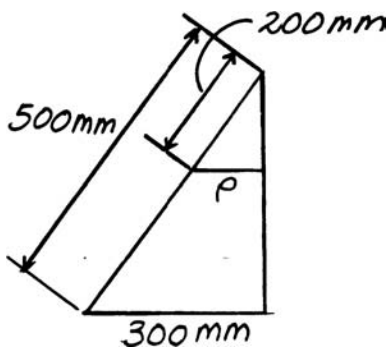
Also,

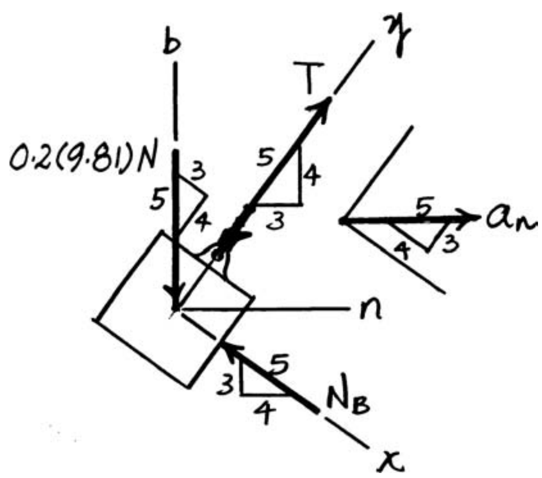
$$\overset{+}{\rightarrow}\Sigma F_n = ma_n; \qquad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$$

$$+\uparrow\Sigma F_b = 0; \qquad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$$

$$T = 1.82\text{ N} \qquad \textbf{Ans.}$$

$$N_B = 0.844\text{ N} \qquad \textbf{Ans.}$$





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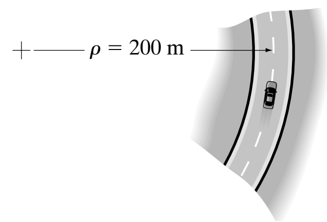
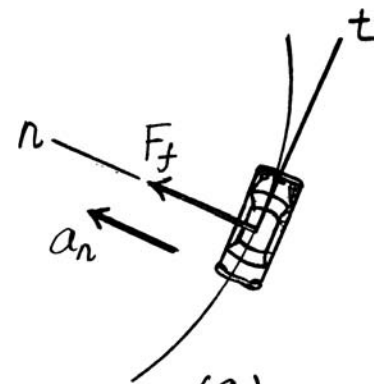
13–66. Determine the minimum coefficient of static friction between the tires and the road surface so that the 1.5-Mg car does not slide as it travels at 80 km/h on the curved road. Neglect the size of the car.



Free-Body Diagram: The frictional force $\mathbf{F}_f$ developed between the tires and the road surface and $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis) as indicated on the free-body diagram of the car, Fig. (a).

Equations of Motion: Here, the speed of the car is $v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 22.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{20} = 2.469\text{ m/s}^2$ and referring to Fig. (a),

$$+\nwarrow\Sigma F_n = ma_n; \qquad F_f = 1500(2.469) = 3703.70\text{ N}$$

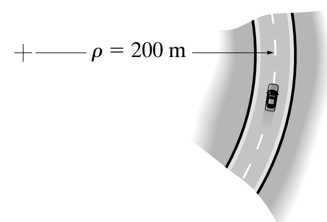


(a)

The normal reaction acting on the car is equal to the weight of the car, i.e., $N = 1500(9.81) = 14\,715$ N. Thus, the required minimum μ_s is given by

$$\mu_s = \frac{F_f}{N} = \frac{370.70}{14\,715} = 0.252 \qquad \textbf{Ans.}$$

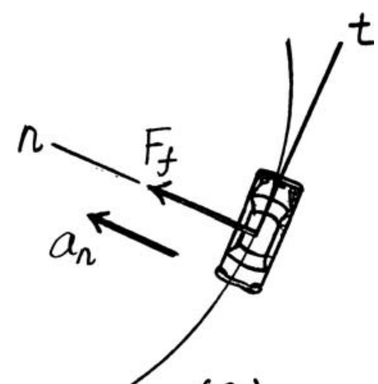
13–67. If the coefficient of static friction between the tires and the road surface is $\mu_s = 0.25$, determine the maximum speed of the 1.5-Mg car without causing it to slide when it travels on the curve. Neglect the size of the car.



Free-Body Diagram: The frictional force $\mathbf{F}_f$ developed between the tires and the road surface and $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis) as indicated on the free-body diagram of the car, Fig. (a).

Equations of Motion: Realizing that $a_n = \frac{v^2}{\rho} = \frac{v^2}{200}$ and referring to Fig. (a),

$$+\nwarrow\Sigma F_n = ma_n; \qquad F_f = 1500\left(\frac{v^2}{200}\right) = 7.5v^2$$



(a)

The normal reaction acting on the car is equal to the weight of the car, i.e., $N = 1500(9.81) = 14\,715$ N. When the car is on the verge of sliding,

$$F_f = \mu_s N$$

$$7.5\,v^2 = 0.25(14\,715)$$

$$v = 22.1\text{ m/s} \qquad \textbf{Ans.}$$

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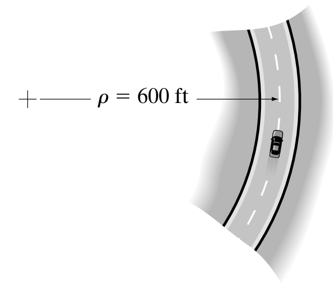
***13–68.** At the instant shown, the 3000-lb car is traveling with a speed of 75 ft/s, which is increasing at a rate of 6 ft/s^2. Determine the magnitude of the resultant frictional force the road exerts on the tires of the car. Neglect the size of the car.



Free-Body Diagram: Here, the force acting on the tires will be resolved into its n and t components $\mathbf{F}_n$ and $\mathbf{F}_t$ as indicated on the free-body diagram of the car, Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the road (positive n axis).

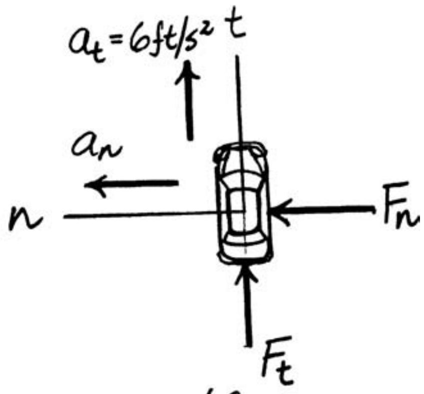
Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{75^2}{600} = 9.375 \text{ ft/s}^2$. By referring to Fig. (a),

$$+\uparrow \Sigma F_t = ma_t; \qquad F_t = \frac{3000}{32.2}(6) = 559.01 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad F_n = \frac{3000}{32.2}(9.375) = 873.45 \text{ lb}$$

Thus, the magnitude of force **F** acting on the tires is

$$F = \sqrt{F_t^{\,2} + F_n^{\,2}} = \sqrt{559.01^2 + 873.45^2} = 1037 \text{ lb} \qquad \textbf{Ans.}$$



(a)

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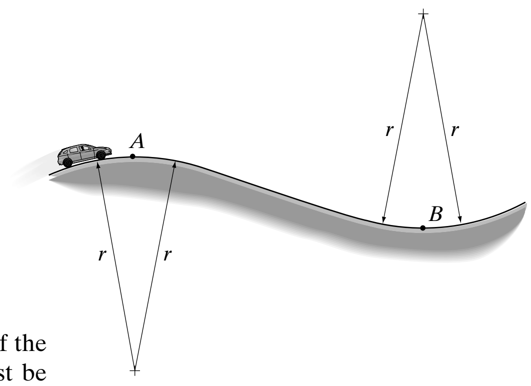
•13–69. Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

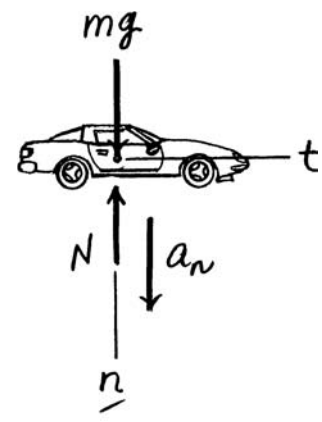
Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \frac{v^2}{\rho} = \frac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad v = \sqrt{gr} \qquad \textbf{Ans.}$$

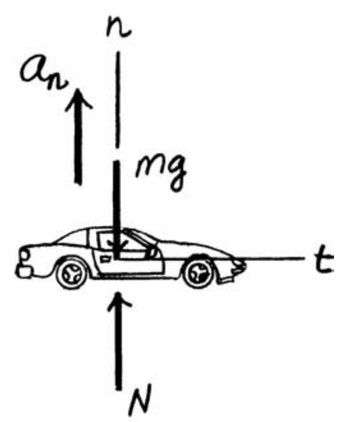
Using the result of v, the normal component of car acceleration is $a_n = \frac{v^2}{\rho} = \frac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

$$N = 2mg \qquad \textbf{Ans.}$$



(a)



(b)

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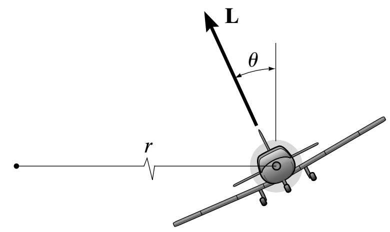
13–70. A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path of radius $r = 3000$ m. Determine the uplift force **L** acting on the airplane and the banking angle θ. Neglect the size of the airplane.



Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

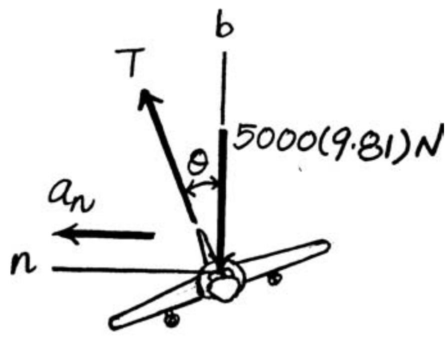
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 97.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{3000} = 3.151\text{ m/s}^2$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\qquad T\cos\theta - 5000(9.81) = 0 \qquad$ **(1)**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\qquad T\sin\theta = 5000(3.151) \qquad$ **(2)**

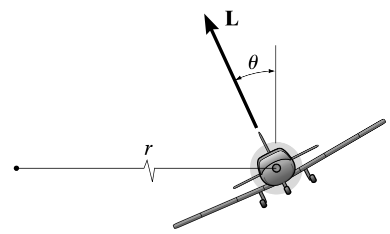
Solving Eqs. (1) and (2) yields

$\theta = 17.8°$ $\qquad T = 51517.75 = 51.5\text{ kN}$ **Ans.**



(a)

13–71. A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle $\theta = 15°$, determine the uplift force **L** acting on the airplane and the radius r of the circular path. Neglect the size of the airplane.



Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

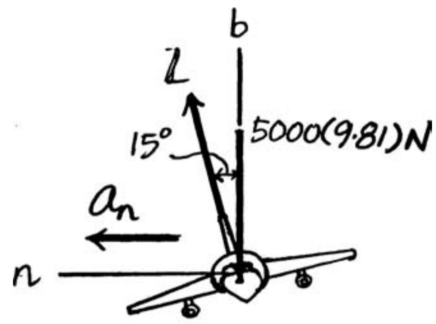
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 97.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{r}$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\qquad L\cos 15° - 5000(9.81) = 0$

$L = 50780.30\text{ N} = 50.8\text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\qquad 50780.30\sin 15° = 5000\left(\frac{97.22^2}{r}\right)$

$r = 3595.92\text{ m} = 3.60\text{ km}$ **Ans.**



(a)

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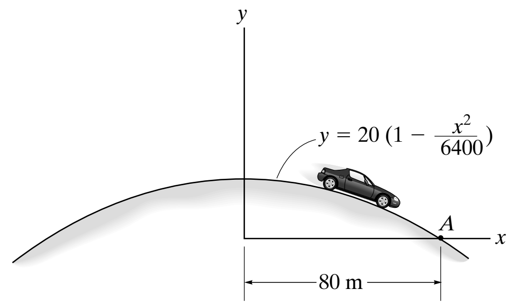
***13–72.** The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

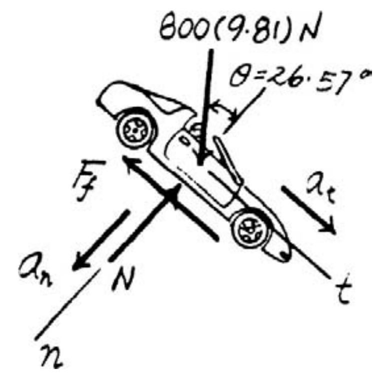


Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$$

$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

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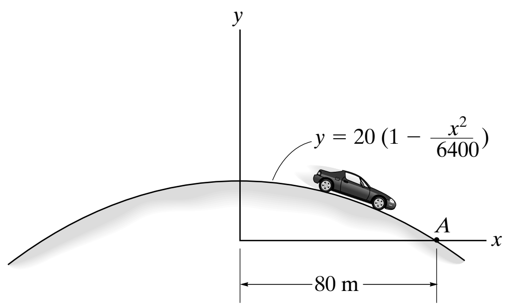
•13–73. The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.

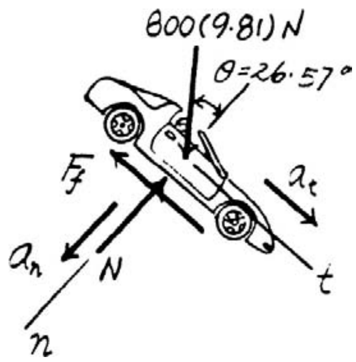


Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$$

$$F_f = 1109.73\text{ N} = 1.11\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

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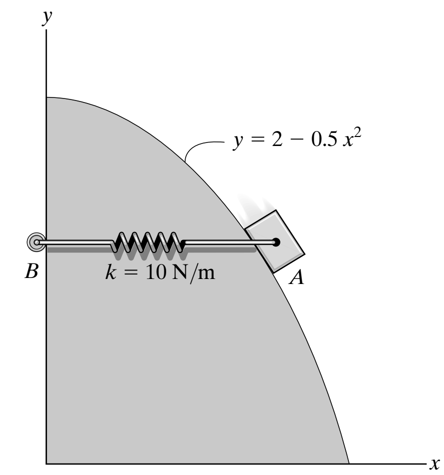
13–74. The 6-kg block is confined to move along the smooth parabolic path. The attached spring restricts the motion and, due to the roller guide, always remains horizontal as the block descends. If the spring has a stiffness of $k = 10$ N/m, and unstretched length of 0.5 m, determine the normal force of the path on the block at the instant $x = 1$ m when the block has a speed of 4 m/s. Also, what is the rate of increase in speed of the block at this point? Neglect the mass of the roller and the spring.



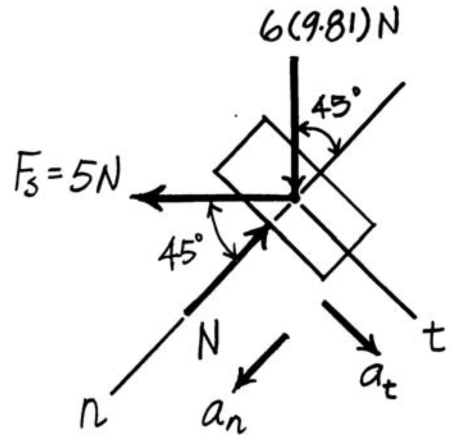
$$v = 2 - 0.5x^2$$

$$\frac{dv}{dx} = \tan\theta = -x\bigg|_{x=1} = -1 \qquad \theta = -45^\circ$$

$$\frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-1)^2\right]^{\frac{3}{2}}}{|-1|} = 2.8284 \text{ m}$$

$$F_s = kx = 10(1 - 0.5) = 5 \text{ N}$$



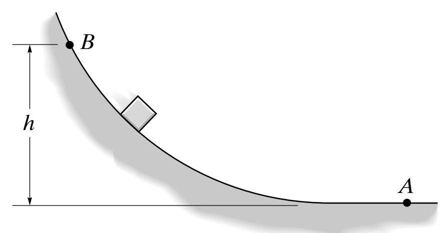
$+\swarrow \Sigma F_n = ma_n;\qquad 6(9.81)\cos 45^\circ - N + 5\cos 45^\circ = 6\left(\frac{(4)^2}{2.8284}\right)$

$N = 11.2 \text{ N}$ **Ans.**

$+\searrow \Sigma F_t = ma_t;\qquad 6(9.81)\sin 45^\circ - 5\sin 45^\circ = 6a_t$

$a_t = 6.35 \text{ m/s}^2$ **Ans.**

13–75. Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



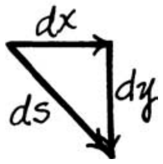
$+\searrow \Sigma F_t = ma_t;\qquad mg\sin\theta = ma_t \qquad a_t = g\sin\theta$

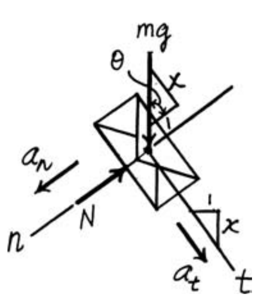
$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad$ However $\quad dy = ds\sin\theta$

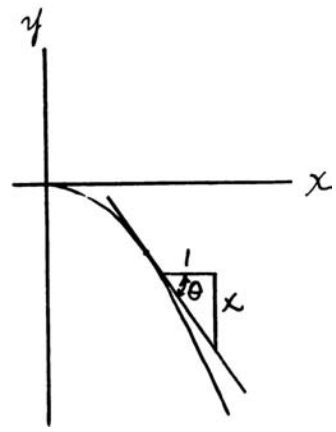
$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

$v = \sqrt{2gh}$ **Q.E.D.**







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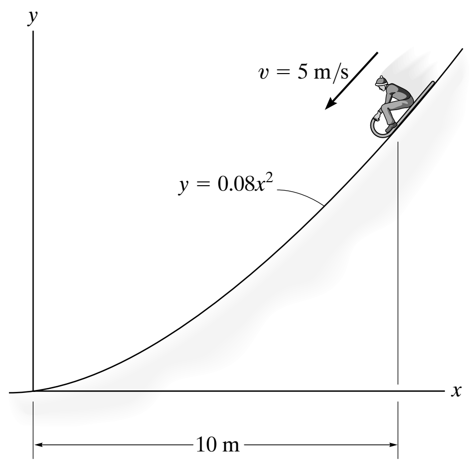
***13–76.** A toboggan and rider of total mass 90 kg travel down along the (smooth) slope defined by the equation $y = 0.08x^2$. At the instant $x = 10$ m, the toboggan's speed is 5 m/s. At this point, determine the rate of increase in speed and the normal force which the slope exerts on the toboggan. Neglect the size of the toboggan and rider for the calculation.



Geometry: Here, $\dfrac{dy}{dx} = 0.16x$ and $\dfrac{d^2y}{dx^2} = 0.16$. The slope angle θ at $x = 10$ m is given by

$$\tan\theta = \frac{dy}{dx}\bigg|_{x=10\text{ m}} = 0.16(10) \qquad \theta = 57.99^\circ$$

and the radius of curvature at $x = 10$ m is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \frac{\left[1 + (0.16x)^2\right]^{3/2}}{|0.16|}\Bigg|_{x=10\text{ m}} = 41.98\text{ m}$$

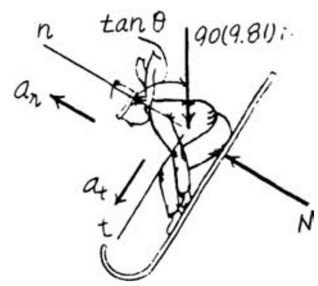
Equations of Motion: Applying Eq. 13–8 with $\theta = 57.99^\circ$ and $\rho = 41.98$ m, we have

$$\Sigma F_t = ma_t; \qquad 90(9.81)\sin 57.99^\circ = 90a_t$$

$$a_t = 8.32\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad -90(9.81)\cos 57.99^\circ + N = 90\left(\frac{5^2}{41.98}\right)$$

$$N = 522\text{ N} \qquad \textbf{Ans.}$$



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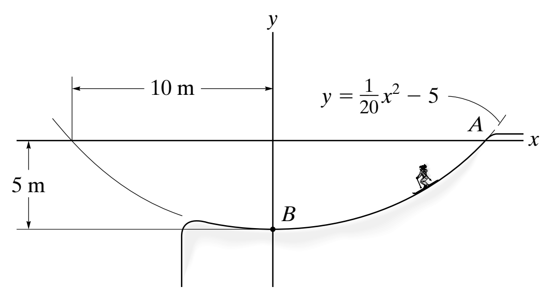
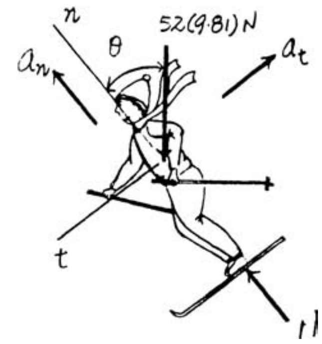
•13–77. The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force the ground exerts on the skier at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–75.



Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{10}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \qquad \theta = 0^\circ$$



and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{|1/10|}\right|_{x=0\text{ m}} = 10.0\text{ m}$$

Equations of Motion:

$+\swarrow\Sigma F_t = ma_t;$ $\qquad 52(9.81)\sin\theta = -52a_t \qquad a_t = -9.81\sin\theta$

$+\nwarrow\Sigma F_n = ma_n;$ $\qquad N - 52(9.81)\cos\theta = m\left(\dfrac{v^2}{\rho}\right)$ **(1)**

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1 + (dy/dx)^2}dx = \sqrt{1 + \frac{1}{100}x^2}dx$

Here, $\tan\theta = \dfrac{1}{10}x$. Then, $\sin\theta = \dfrac{x}{10\sqrt{1 + \frac{1}{100}x^2}}$.

$$(+\ ) \qquad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^{0}\left(\frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}\right)\left(\sqrt{1 + \tfrac{1}{100}x^2}dx\right)$$

$$v^2 = 9.81\text{ m}^2/\text{s}^2$$

Substituting $v^2 = 98.1\text{ m}^2/\text{s}^2$, $\theta = 0^\circ$, and $\rho = 10.0$ m into Eq.(1) yields

$$N - 52(9.81)\cos 0^\circ = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24\text{ N} = 1.02\text{ kN}$$ **Ans.**

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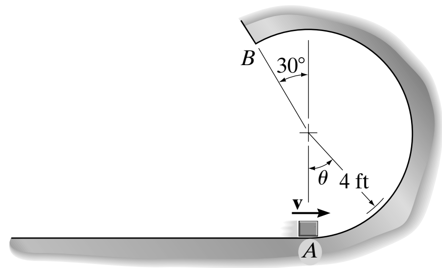
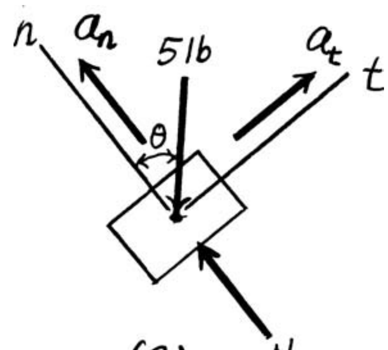
13–78. The 5-lb box is projected with a speed of 20 ft/s at A up the vertical circular smooth track. Determine the angle θ when the box leaves the track.



Free-Body Diagram: The free-body diagram of the box at an arbitrary position θ is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the vertical circular path (positive n axis), while $\mathbf{a}_t$ is assumed to be directed toward the positive t axis.

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{4}$. Also, the box is required to leave the track, so that $N = 0$. By referring to Fig. (a),



(a)

$$+\nearrow\Sigma F_t = ma_t; \qquad -5\sin\theta = \frac{5}{32.2}a_t$$

$$a_t = -(32.2\sin\theta)\ \text{ft/s}^2$$

$$+\nwarrow\Sigma F_n = ma_n; \qquad -5\cos\theta = \frac{5}{32.2}\left(\frac{v^2}{4}\right)$$

$$v^2 = -128.8\cos\theta \qquad \textbf{(1)}$$

Kinematics: Using the result of $\mathbf{a}_t$, the speed of the box can be determined by integrating the kinematic equation $v\,dv = a_t\,ds$, where $ds = r\,d\theta = 4\,d\theta$. Using the initial condition $v = 20$ ft/s at $\theta = 0°$ as the integration limit,

$$\int_{20\text{ft/s}}^{v} v\,dv = \int_{0°}^{\theta} -32.2\sin\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_{20\text{ ft/s}}^{v} = 128.8\cos\theta\Big|_{0°}^{\theta}$$

$$v^2 = 257.6\cos\theta + 142.4 \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$386.4\cos\theta + 142.4 = 0$$

$$\theta = 111.62° = 112° \qquad \textbf{Ans.}$$

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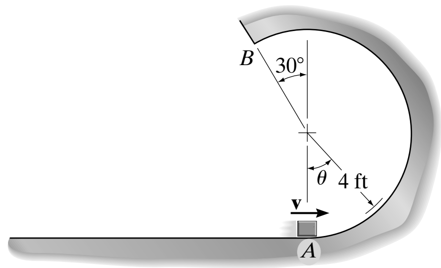
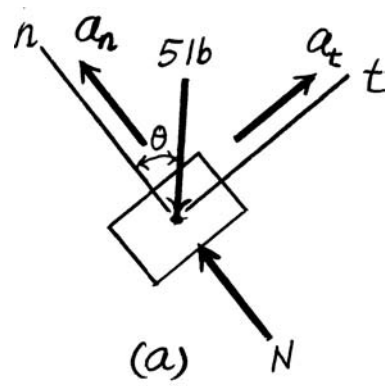
13–79. Determine the minimum speed that must be given to the 5-lb box at A in order for it to remain in contact with the circular path. Also, determine the speed of the box when it reaches point B.



Free-Body Diagram: The free-body diagram of the box at an arbitrary position θ is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the vertical circular path (positive n axis), while $\mathbf{a}_t$ is assumed to be directed toward the positive t axis.



(a)

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{4}$. Also, the box is required to leave the track, so that $N = 0$. By referring to Fig. (a),

$$+\nearrow \Sigma F_t = ma_t; \qquad -5 \sin\theta = \frac{5}{32.2} a_t$$

$$a_t = -(32.2 \sin\theta) \text{ ft/s}^2$$

$$+\nwarrow \Sigma F_n = ma_n; \qquad N - 5\cos\theta = \frac{5}{32.2}\left(\frac{v^2}{4}\right)$$

$$N = 0.03882v^2 + 5\cos\theta \qquad \textbf{(1)}$$

Kinematics: Using the result of $\mathbf{a}_t$, the speed of the box can be determined by intergrating the kinematic equation $v\,dv = a_t\,ds$, where $ds = r\,d\theta = 4\,d\theta$. Using the initial condition $v = v_0$ at $\theta = 0°$ as the integration limit,

$$\int_{v_0}^{v} v\,dv = \int_{0°}^{\theta} -32.2 \sin\theta(4d\theta)$$

$$\left.\frac{v^2}{2}\right|_{v_0}^{v} = 128.8 \cos\theta \Big|_{0°}^{\theta}$$

$$v^2 = 257.6 \cos\theta - 257.6 + {v_0}^2 \qquad \textbf{(2)}$$

Provided the box does not leave the vertical circular path at $\theta = 180°$, then it will remain in contact with the path. Thus, it is required that the box is just about to leave the path at $\theta = 180°$, Thus, $N = 0$. Substituting these two values into Eq. (1),

$$0 = 00.03882v^2 + 5\cos 180°$$

$$v = 11.35 \text{ ft/s}$$

Substituting the result of v and $v_0 = v_{\min}$ into Eq. (2),

$$11.35^2 = 257.6 \cos 180° - 257.6 + {v_{\min}}^2$$

$$v_{\min} = 25.38 \text{ ft/s} = 25.4 \text{ ft/s} \qquad \textbf{Ans.}$$

At point B, $\theta = 210°$. Substituting this value and $v_0 = v_{\min} = 25.38$ ft/s into Eq. (2),

$$v_B{}^2 = 257.6 \cos 210° - 257.6 + 25.38^2$$

$$v_B = 12.8 \text{ ft/s} \qquad \textbf{Ans.}$$

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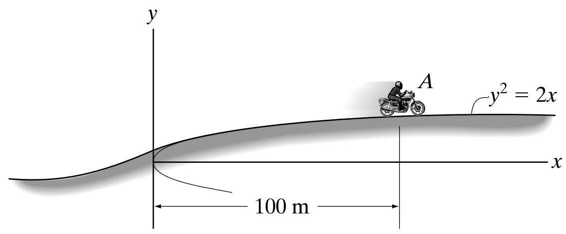
***13–80.** The 800-kg motorbike travels with a constant speed of 80 km/h up the hill. Determine the normal force the surface exerts on its wheels when it reaches point A. Neglect its size.



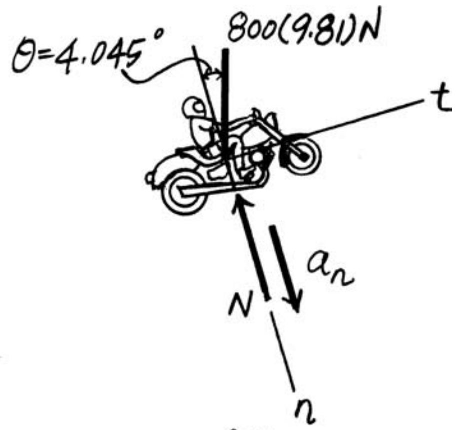
Geometry: Here, $y = \sqrt{2}x^{1/2}$. Thus, $\dfrac{dy}{dx} = \dfrac{\sqrt{2}}{2x^{1/2}}$ and $\dfrac{d^2y}{dx^2} = -\dfrac{\sqrt{2}}{4x^{3/2}}$. The angle that the hill slope at A makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=100\text{ m}} = \tan^{-1}\left(\frac{\sqrt{2}}{2x^{1/2}}\right)\bigg|_{x=100\text{ m}} = 4.045^\circ$$

The radius of curvature of the hill at A is given by

$$\rho_A = \frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ m}} = \frac{\left[1+\left(\dfrac{\sqrt{2}}{2(100^{1/2})}\right)^2\right]^{3/2}}{\left|-\dfrac{\sqrt{2}}{4(100^{3/2})}\right|} = 2849.67\text{ m}$$

Free-Body Diagram: The free-body diagram of the motorcycle is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).



(a)

Equations of Motion: The speed of the motorcycle is

$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$$

Thus, $a_n = \dfrac{v^2}{\rho_A} = \dfrac{22.22^2}{2849.67} = 0.1733\text{ m/s}^2$. By referring to Fig. (a),

$\searrow + \Sigma F_n = ma_n;$ $\quad 800(9.81)\cos 4.045^\circ - N = 800(0.1733)$

$N = 7689.82\text{ N} = 7.69\text{ kN}$ **Ans.**

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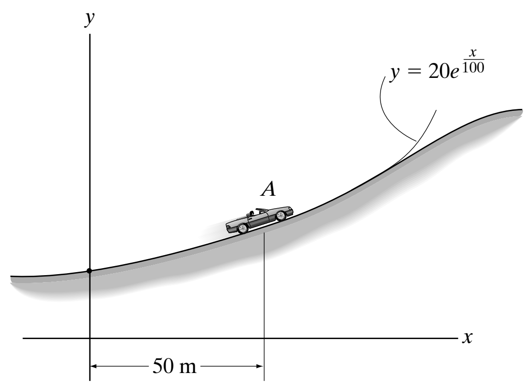
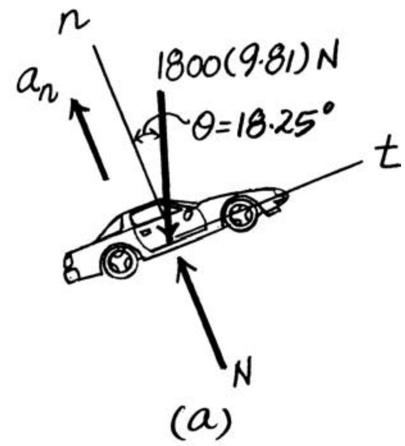
•13–81. The 1.8-Mg car travels up the incline at a constant speed of 80 km/h. Determine the normal reaction of the road on the car when it reaches point A. Neglect its size.



Geometry: Here, $\dfrac{dy}{dx} = 20\left(\dfrac{1}{100}\right)e^{x/100} = 0.2e^{x/100}$ and $\dfrac{d^2y}{dx^2} = 0.2\left(\dfrac{1}{100}\right)e^{x/100} = 0.002e^{x/100}$. The angle that the slope of the road at A makes with the horizontal is $\theta = \tan^{-1}\left(\dfrac{dy}{dx}\right)\Big|_{x=50\text{ m}} = \tan^{-1}\left(0.2e^{50/100}\right) = 18.25°$. The radius of curvature of the road at A is given by

$$\rho_A = \left.\frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\right|_{x=50\text{ m}} = \frac{\left[1+\left(0.2e^{50/100}\right)^2\right]^{3/2}}{\left|0.002e^{50/100}\right|} = 354.05\text{ m}$$



Free-Body Diagram: The free-body diagram of the car is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

Equations of Motion: The speed of the car is

$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}^2$$

Thus, $a_n = \dfrac{v^2}{\rho_A} = \dfrac{22.22^2}{354.05} = 1.395\text{ m/s}^2$. By referring to Fig. (a),

$$+\nwarrow\Sigma F_n = ma_n; \qquad N - 1800(9.81)\cos 18.25° = 1800(1.395)$$

$$N = 19280.46\text{ N} = 19.3\text{ kN} \qquad \textbf{Ans.}$$

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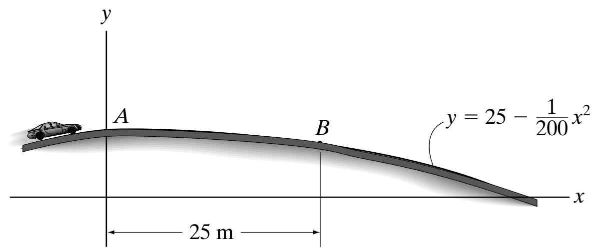
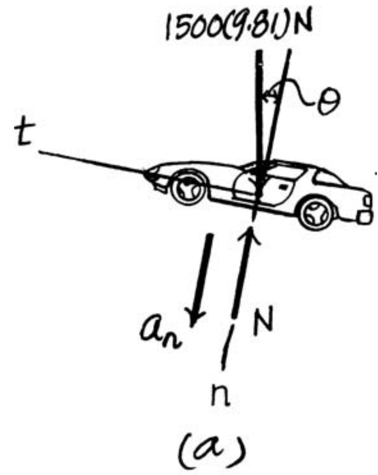
13–82. Determine the maximum speed the 1.5-Mg car can have and still remain in contact with the road when it passes point A. If the car maintains this speed, what is the normal reaction of the road on it when it passes point B? Neglect the size of the car.



Geometry: Here, $\dfrac{dx}{dy} = -0.01x$ and $\dfrac{d^2y}{dx^2} = -0.01$. The angle that the slope of the road makes with the horizontal at A and B are $\theta_A = \tan^{-1}\left(\dfrac{dy}{dx}\right)\bigg|_{x=0\text{ m}}$ $= tan^{-1}(0) = 0°$ and $\theta_B = \tan^{-1}\left(\dfrac{dy}{dx}\right)\bigg|_{x=25\text{ m}} = \tan^{-1}\left(-0.01(25)\right) = -14.04°$. The radius of curvature of the road at A and B are

$$\rho_A = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=0\text{ m}} = \frac{\left[1 + (0)^2\right]^{3/2}}{0.01} = 100\text{ m}$$

$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=25\text{ m}} = \frac{\left[1 + (0.25)^2\right]^{3/2}}{0.01} = 109.52\text{ m}$$



Free-Body Diagram: The free-body diagram of the car at an arbitrary position x is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the road (positive n axis).

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 1500(9.81)\cos\theta - N = 1500\left(\frac{v^2}{\rho}\right)$$

$$N = 14\,715\cos\theta - \frac{1500v^2}{\rho} \qquad \textbf{(1)}$$

Since the car is required to just about lose contact with the road at A, then $N = N_A = 0$, $\theta = \theta_A = 0$ and $\rho = \rho_A = 100$ m. Substituting these values into Eq. (1),

$$0 = 14\,715\cos 0° - \frac{1500v^2}{100}$$

$$v = 31.32\text{ m/s} = 31.3\text{ m/s} \qquad \textbf{Ans.}$$

When the car is at B, $\theta = \theta_B = 14.04°$ and $\rho = \rho_B = 109.52$ m. Substituting these values into Eq. (1), we obtain

$$N_B = 14\,715\cos 14.04° - \frac{1500\left(31.32^2\right)}{109.52}$$

$$= 839.74\text{N} = 840\text{ N} \qquad \textbf{Ans.}$$

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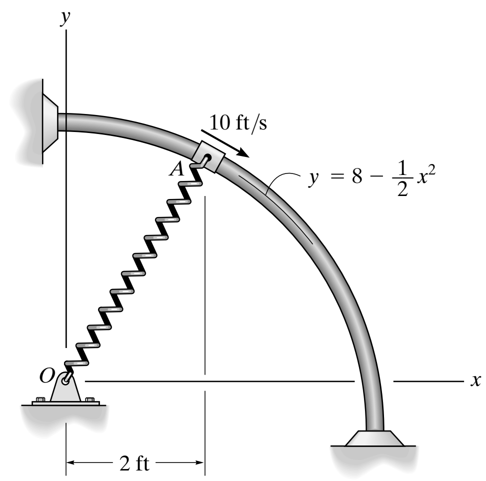
13–83. The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of $k = 10$ lb/ft, determine the normal force on the collar and the acceleration of the collar at this instant.



$$v = 8 - \frac{1}{2}x^2$$

$$\frac{dy}{dx} = \tan\theta = -x\Big|_{x=2} = -2 \qquad \theta = -63.435°$$

$$\frac{d^2y}{dx^2} = -1$$

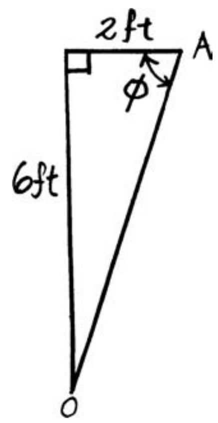
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + (-2)^2)^{\frac{3}{2}}}{|-1|} = 11.18 \text{ ft}$$

$$v = 8 - \frac{1}{2}(2)^2 = 6$$

$$OA = \sqrt{(2)^2 + (6)^2} = 6.3246$$

$$F_s = kx = 10(6.3246 - 3) = 33.246 \text{ lb}$$

$$\tan\phi = \frac{6}{2}\,\phi 1.565°$$



$$+\swarrow\Sigma F_n = ma_n; \qquad 5\cos 63.435° - N + 33.246\cos 8.1301° = \left(\frac{5}{32.2}\right)\left(\frac{(10)^2}{11.18}\right)$$

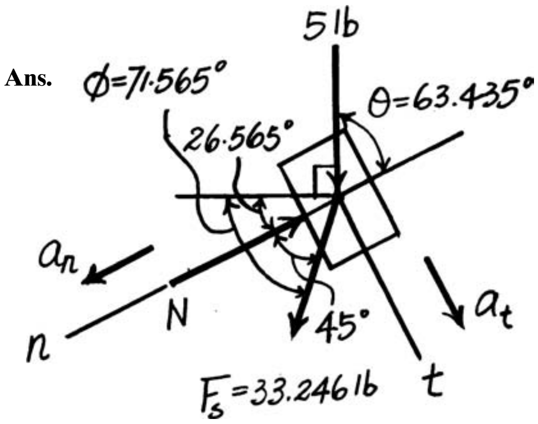
$$N = 33.8 \text{ lb} \qquad \textbf{Ans.}$$

$$+\searrow\Sigma F_t = ma_t; \qquad 5\sin 63.435° + 33.246\sin 8.1301° = \left(\frac{5}{32.2}\right)a_t$$

$$a_t = 59.08 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(10)^2}{11.18} = 8.9443 \text{ ft/s}^2$$

$$a = \sqrt{(59.08)^2 + (8.9443)^2} = 59.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



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***13–84.** The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the resultant force acting on the particle when $t = 2$ s.

$r = 2t + 1|_{t=2\,s} = 5\text{ ft} \qquad \dot{r} = 2\text{ ft/s} \qquad \ddot{r} = 0$

$\theta = 0.5t^2 - t|_{t=2\,s} = 0\text{ rad} \qquad \dot{\theta} = t - 1|_{t=2\,s} = 1\text{ rad/s} \qquad \ddot{\theta} = 1\text{ rad/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5\text{ ft/s}^2$

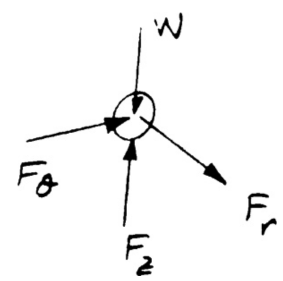
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9\text{ ft/s}^2$

$\Sigma F_r = ma_r; \qquad F_r = \dfrac{5}{32.2}(-5) = -0.7764\text{ lb}$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = \dfrac{5}{32.2}(9) = 1.398\text{ lb}$

$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60\text{ lb}$ **Ans.**

•13–85. Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.



$r = 2t + 10|_{t=2\,s} = 14$

$\dot{r} = 2$

$\ddot{r} = 0$

$\theta = 1.5t^2 - 6t$

$\dot{\theta} = 3t - 6\Big|_{t=2\,s} = 0$

$\ddot{\theta} = 3$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 14(3) + 0 = 42$

Hence,

$\Sigma F_r = ma_r; \qquad F_r = 5(0) = 0$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 5(42) = 210\text{ N}$

$F = \sqrt{(F_r)^2 + (F_\theta)^2} = 210\text{ N}$ **Ans.**

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13–86. A 2-kg particle travels along a horizontal smooth path defined by

$$r = \left(\frac{1}{4}t^3 + 2\right)\text{m}, \ \theta = \left(\frac{t^2}{4}\right)\text{rad},$$

where t is in seconds. Determine the radial and transverse components of force exerted on the particle when $t = 2\,\text{s}$.

Kinematics: Since the motion of the particle is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of **r** and the time derivative of r and θ evaluated at $t = 2$ s are

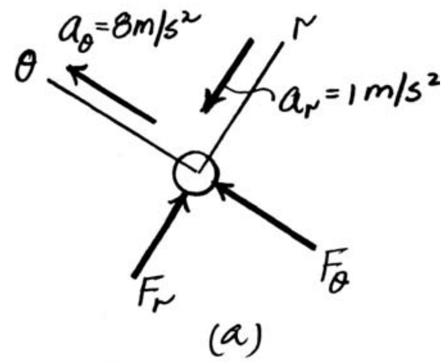
$$r|_{t=2\,\text{s}} = \frac{1}{4}t^3 + 2\bigg|_{t=2\,\text{s}} = 4\text{ m} \qquad \dot{r}|_{t=2\,\text{s}} = \frac{3}{4}t^2\bigg|_{t=2\,\text{s}} = 3\text{ m/s} \qquad \ddot{r}|_{t=2\,\text{s}} = \frac{3}{2}t\bigg|_{t=2\,\text{s}} = 3\text{ m/s}^2$$

$$\dot{\theta} = \frac{t}{2}\bigg|_{t=2\,\text{s}} = 1\text{ rad/s} \qquad \ddot{\theta}\big|_{t=2\,\text{s}} = 0.5\text{ rad/s}^2$$

Using the above time derivative,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3 - 4(1^2) = -1\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(0.5) + 2(3)(1) = 8\text{ m/s}^2$$



Equations of Motion: By referring to the free-body diagram of the particle in Fig. (a),

$\Sigma F_r = ma_r;$ $\quad F_r = 2(-1) = -2\text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta;$ $\quad F_\theta = 2(8) = 16\text{ N}$ **Ans.**

Note: The negative sign indicates that $\mathbf{F}_r$ acts in the opposite sense to that shown on the free-body diagram.

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13–87. A 2-kg particle travels along a path defined by

$$r = (3 + 2t^2)\,\text{m},\ \theta = \left(\frac{1}{3}t^3 + 2\right)\text{rad}$$

and $z = (5 - 2t^2)$ m, where t is in seconds. Determine the r, θ, z components of force that the path exerts on the particle at the instant $t = 1$ s.

Kinematics: Since the motion of the particle is known, $\mathbf{a}_r$, $\mathbf{a}_\theta$, and $\mathbf{a}_z$ will be determined first. The values of r and the time derivative of r, θ, and z evaluated at $t = 1$ s are

$$r\big|_{t=1\text{ s}} = 3 + 2t^2\Big|_{t=1\text{ s}} = 5\text{ m} \qquad \dot{r}\big|_{t=1\text{ s}} = 4t\big|_{t=1\text{ s}} = 4\text{ m/s} \qquad \ddot{r}\big|_{t=1\text{ s}} = 4\text{ m/s}^2$$

$$\dot{\theta}\big|_{t=1\text{ s}} = t^2\Big|_{t=1\text{ s}} = 1\text{ rad/s} \qquad \ddot{\theta}\big|_{t=1\text{ s}} = 2t\big|_{t=1\text{ s}} = 2\text{ rad/s}^2$$

$$\dot{z} = -4t \qquad \ddot{z} = -4\text{ m/s}^2$$

Using the above time derivative,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - 5(1^2) = -1\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(2) + 2(4)(1) = 18\text{ m/s}^2$$

$$a_z = \ddot{z} = -4\text{ m/s}^2$$

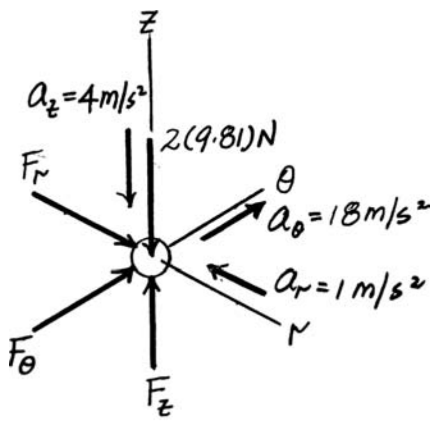
Equations of Motion: By referring to the free-body diagram of the particle in Fig. (a),

$\Sigma F_r = ma_r$; $\quad F_r = 2(-1) = -2\text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta$; $\quad F_\theta = 2(18) = 36\text{ N}$ **Ans.**

$\Sigma Fz = ma_z$; $\quad F_z - 2(9.81) = 2(-4)$; $\quad F_z = 11.6\text{ N}$ **Ans.**

Note: The negative sign indicates that $\mathbf{F}_r$ acts in the opposite sense to that shown on the free-body diagram.



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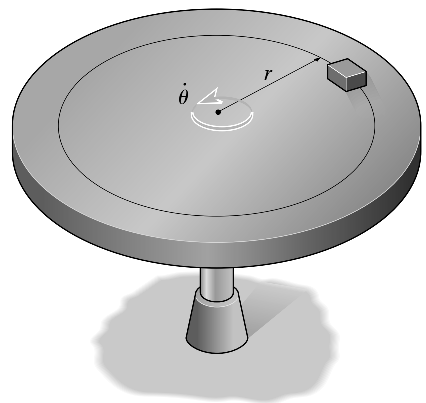
***13–88.** If the coefficient of static friction between the block of mass m and the turntable is μ_s, determine the maximum constant angular velocity of the platform without causing the block to slip.



Free-Body Diagram: The free-body diagram of the block is shown in Fig. (a). Here, the frictional force developed is resolved into its radial and transversal components $\mathbf{F}_r$, $\mathbf{F}_\theta$, $\mathbf{a}_r$, and $\mathbf{a}_\theta$ are assumed to be directed towards their positive axes.

Equations of Motion: By referring to Fig. (a),

$$\Sigma F_r = ma_r; \qquad -F_r = ma_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad F_\theta = ma_\theta \qquad \textbf{(2)}$$

$$\Sigma F_z = ma_z; \qquad N - mg = m(0) \qquad N = mg$$

Kinematics: Since r and $\dot{\theta}$ are constant, $\dot{r} = \ddot{r} = 0$ and $\ddot{\theta} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\dot{\theta}^2 = -r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$$

Substituting the results of $\mathbf{a}_r$ and $\mathbf{a}_\theta$ into Eqs. (1) and (2),

$$F_r = mr\,\dot{\theta}^2 \qquad F_\theta = 0$$

Thus, the magnitude of the frictional force is given by

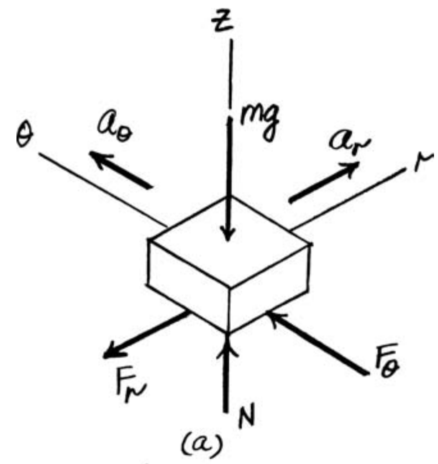
$$F = \sqrt{F_r{}^2 + F_\theta{}^2} = \sqrt{(mr\dot{\theta})^2 + 0} = mr\dot{\theta}^2$$

Since the block is required to be on the verge of slipping,

$$F = \mu_s N$$

$$mr\dot{\theta}^2 = \mu_s mg$$

$$\dot{\theta} = \sqrt{\frac{\mu_s g}{r}} \qquad \textbf{Ans.}$$



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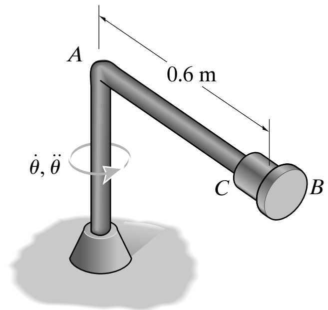
•13–89. The 0.5-kg collar C can slide freely along the smooth rod AB. At a given instant, rod AB is rotating with an angular velocity of $\dot{\theta} = 2$ rad/s and has an angular acceleration of $\ddot{\theta} = 2$ rad/s^2. Determine the normal force of rod AB and the radial reaction of the end plate B on the collar at this instant. Neglect the mass of the rod and the size of the collar.



Free-Body Diagram: The free-body diagram of the collar is shown in Fig. (a). Here, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to be directed towards the positive of their respective axes.

Equations of Motion: By referring to Fig. (a),

$$\Sigma F_r = ma_r; \qquad -N_B = 0.5a_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad F_{AB} = 0.5a_\theta \qquad \textbf{(2)}$$

Kinematics: Since $r = 0.6$ m is constant, $\dot{r} = \ddot{r} = 0$.

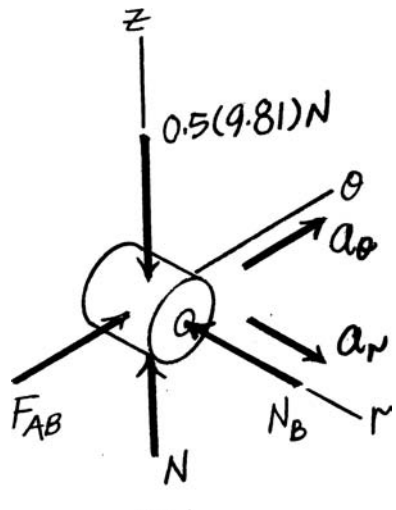
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (0.6)(2^2) = -2.4 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6(2) + 0 = 1.2 \text{ m/s}^2$$

Substituting the results of $\mathbf{a}_r$ and $\mathbf{a}_\theta$ into Eqs. (1) and (2) yields

$$N_B = 1.20 \text{ N} \qquad \textbf{Ans.}$$

$$F_{AB} = 0.6 \text{ N} \qquad \textbf{Ans.}$$



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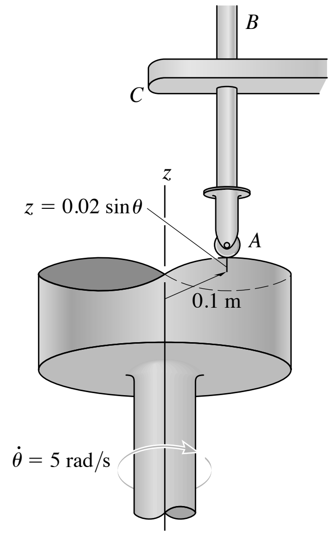
13–90. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin \theta)$ m. If the cam is rotating with a constant angular velocity of 5 rad/s, determine the force on the roller A when $\theta = 90°$. Neglect friction at the bearing C and the mass of the roller.



Kinematics: Taking the required time derivatives, we have

$$\dot{\theta} = 5 \text{ rad/s} \qquad \ddot{\theta} = 0$$

$$z = 0.02 \sin \theta \qquad \dot{z} = 0.02 \cos \theta \dot{\theta} \qquad \ddot{z} = 0.02\left(\cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2\right)$$

Thus,

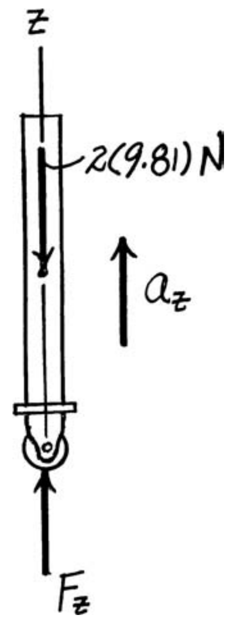
$$a_z = \ddot{z} = 0.02\left[\cos \theta(0) - \sin \theta\left(5^2\right)\right] = -0.5 \sin \theta$$

At $\theta = 90°$, $\qquad a_z = -0.5 \sin 90° = -0.500 \text{ m/s}^2$

Equations of Motion:

$\Sigma F_z = ma_z;$ $\qquad F_z - 2(9.81) = 2(-0.500)$

$$F_z = 18.6 \text{ N}$$ **Ans.**



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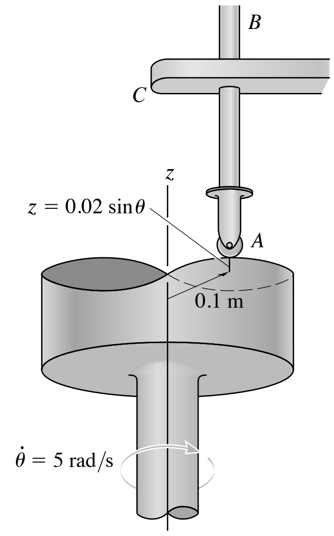
13–91. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin\theta)$ m. If the cam is rotating at a constant angular velocity of 5 rad/s, determine the maximum and minimum force the cam exerts on the roller at A. Neglect friction at the bearing C and the mass of the roller.



Kinematics: Taking the required time derivatives, we have

$$\dot{\theta} = 5 \text{ rad/s} \qquad \ddot{\theta} = 0$$

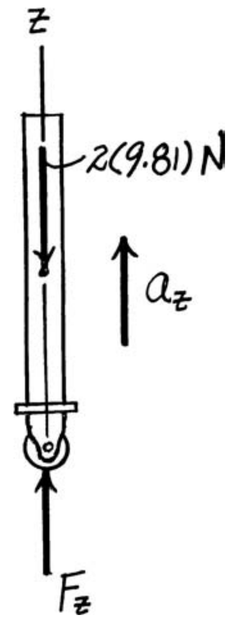
$$z = 0.02 \sin\theta \qquad \dot{z} = 0.02\cos\theta\dot{\theta} \qquad \ddot{z} = 0.02\left(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2\right)$$

Thus,

$$a_z = \ddot{z} = 0.02\left[\cos\theta(0) - \sin\theta\left(5^2\right)\right] = -0.5\sin\theta$$

At $\theta = 90°$, $\qquad a_z = -0.5\sin 90° = -0.500 \text{ m/s}^2$

At $\theta = -90°$, $\qquad a_z = -0.5\sin(-90°) = 0.500 \text{ m/s}^2$



Equations of Motion: At $\theta = 90°$, applying Eq. 13–9, we have

$$\Sigma F_z = ma_z; \qquad (F_z)_{\min} - 2(9.81) = 2(-0.500)$$

$$(F_z)_{\min} = 18.6 \text{ N} \qquad \textbf{Ans.}$$

At $\theta = -90°$, we have

$$\Sigma F_z = ma_z; \qquad (F_z)_{\max} - 2(9.81) = 2(0.500)$$

$$(F_z)_{\max} = 20.6 \text{ N} \qquad \textbf{Ans.}$$

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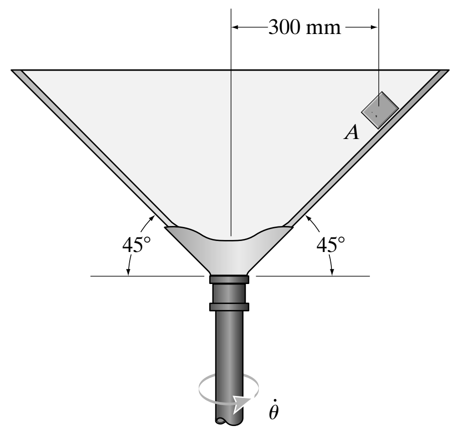
***13–92.** If the coefficient of static friction between the conical surface and the block of mass m is $\mu_s = 0.2$, determine the minimum constant angular velocity $\dot{\theta}$ so that the block does not slide downwards.



Free-Body Diagram: The free-body diagram of the block is shown in Fig. (a). Since the block is required to be on the verge of sliding down the conical surface, $F_f = \mu_k N = 0.2N$ must be directed up the conical surface. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

$+\uparrow \Sigma F_z = ma_z;\quad N \cos 45° + 0.2N \sin 45° - m(9.81) = m(0) \qquad N = 11.56m$

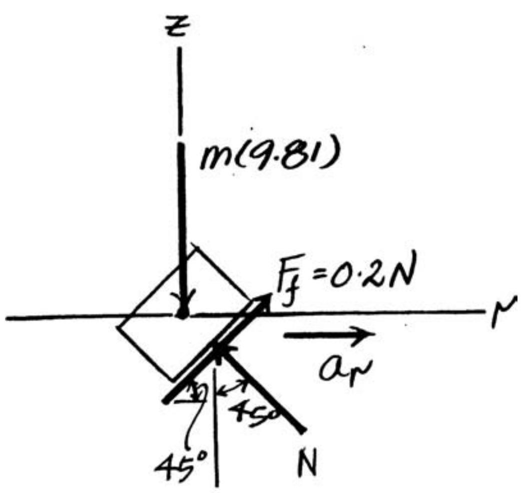
$\overset{+}{\rightarrow} \Sigma F_r = ma_r;\quad 0.2(11.56m) \cos 45° - (11.56m) \sin 45° = ma_r \quad a_r = -6.54 \text{ m/s}^2$

Kinematics: Since $r = 0.3$ m is constant, $\dot{r} = \ddot{r} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$-6.54 = 0 - 0.3\dot{\theta}^2$$

$$\dot{\theta} = 4.67 \text{ rad/s}$$ **Ans.**



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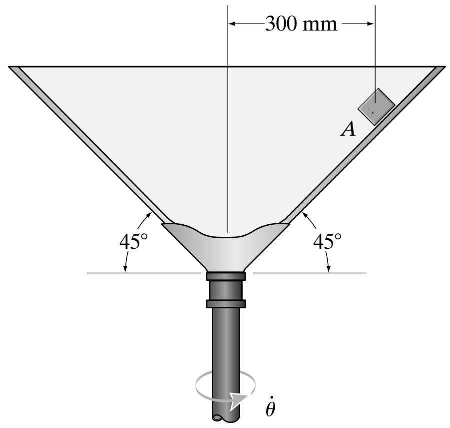
•13–93. If the coefficient of static friction between the conical surface and the block is $\mu_s = 0.2$, determine the maximum constant angular velocity $\dot{\theta}$ without causing the block to slide upwards.



Free-Body Diagram: The free-body diagram of the block is shown in Fig. (a). Since the block is required to be on the verge of sliding up the conical surface, $F_f = \mu_k N = 0.2N$ must be directed down the conical surface. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

$$+\uparrow \Sigma F_z = ma_z; \quad N\cos 45° - 0.2N\sin 45° - m(9.81) = m(0) \quad N = 17.34m$$

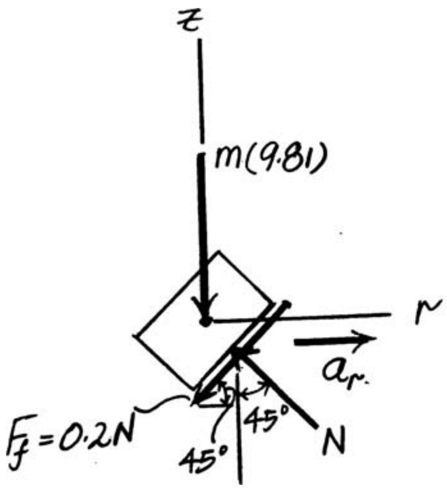
$$\xrightarrow{+} \Sigma F_r = ma_r; \quad -17.34m\sin 45° - 0.2(17.34m)\cos 45° = ma_r \quad a_r = -14.715\ \text{m/s}^2$$

Kinematics: Since $r = 0.3$ m is constant, $\dot{r} = \ddot{r} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$-14.715 = 0 - 0.3\dot{\theta}^2$$

$$\dot{\theta} = 7.00\ \text{rad/s} \qquad \textbf{Ans.}$$



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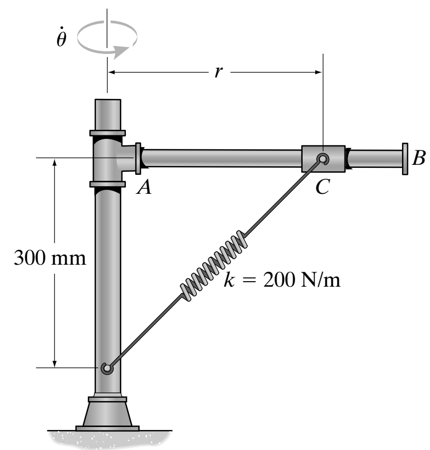
13–94. If the position of the 3-kg collar C on the smooth rod AB is held at $r = 720$ mm, determine the constant angular velocity $\dot{\theta}$ at which the mechanism is rotating about the vertical axis. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.



Free-Body Diagram: The free-body diagram of the collar is shown in Fig. (a). The force in the spring is given by $F_{sp} = ks = 200\left(\sqrt{0.72^2 + 0.3^2} - 0.4\right) = 76$ N. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

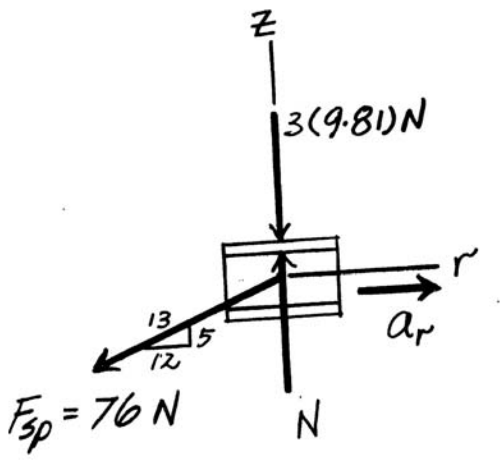
$$\xrightarrow{+} \Sigma F_r = ma_r; \qquad -76\left(\frac{12}{13}\right) = 3a_r \qquad a_r = -23.38 \text{ m/s}^2$$

Kinematics: Since $r = 0.72$ m is constant, $\dot{r} = \ddot{r} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$-23.38 = 0 - 0.72\dot{\theta}^2$$

$$\dot{\theta} = 5.70 \text{ rad/s}$$ **Ans.**



(a)

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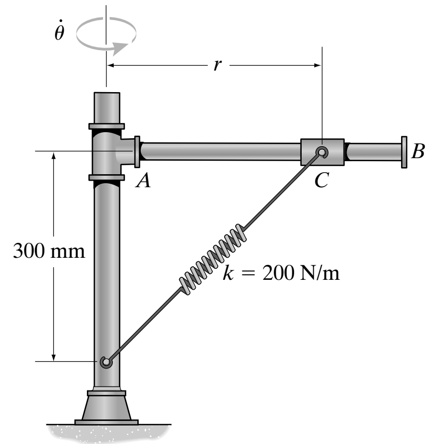
13–95. The mechanism is rotating about the vertical axis with a constant angular velocity of $\dot{\theta} = 6\,\text{rad/s}$. If rod AB is smooth, determine the constant position r of the 3-kg collar C. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.



Free-Body Diagram: The free-body diagram of the collar is shown in Fig. (a). The force in the spring is given by $F_{sp} = ks = 200\left(\sqrt{r^2 + 0.3^2} - 0.4\right)$. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

$$\overset{+}{\rightarrow} \Sigma F_r = ma_r; \qquad -200\left(\sqrt{r^2 + 0.3^2} - 0.4\right)\cos\alpha = 3a_r \qquad \textbf{(1)}$$

However, from the geometry shown in Fig. (b),

$$\cos\alpha = \frac{r}{\sqrt{r^2 + 0.3^2}}$$

Thus, Eq. (1) can be rewritten as

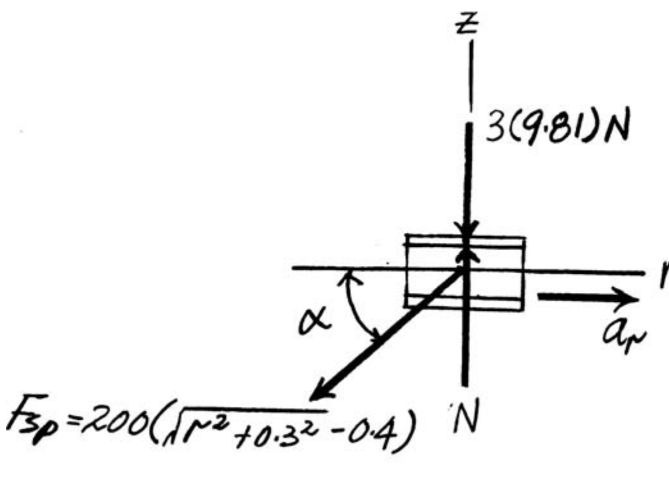
$$-200\left(r - \frac{0.4r}{\sqrt{r^2 + 0.3^2}}\right) = 3a_r \qquad \textbf{(2)}$$

Kinematics: Since r is constant, $\dot{r} = \ddot{r} = 0$.

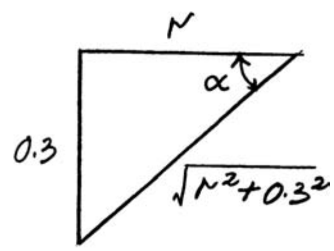
$$a_r = \ddot{r} - r\dot{\theta}^2 = -r(6^2) \qquad \textbf{(3)}$$

Substituting Eq. (3) into Eq. (2) and solving,

$$r = 0.8162 \text{ m} = 816 \text{ mm} \qquad \textbf{Ans.}$$



(a)



(b)

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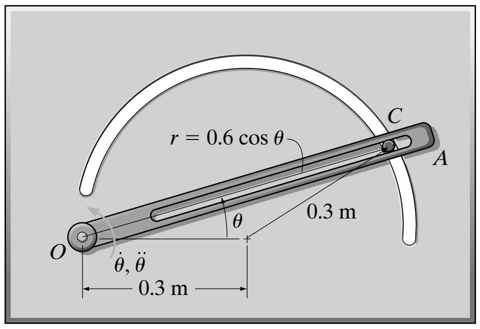
***13–96.** Due to the constraint, the 0.5-kg cylinder C travels along the path described by $r = (0.6 \cos \theta)$ m. If arm OA rotates counterclockwise with an angular velocity of $\dot{\theta} = 2$ rad/s and an angular acceleration of $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force exerted by the arm on the cylinder at this instant. The cylinder is in contact with only one edge of the smooth slot, and the motion occurs in the horizontal plane.



Kinematics: Since the motion of cylinder C is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of r and the time derivatives at the instant $\theta = 30°$ are evaluated below.

$$r = 0.6 \cos \theta\big|_{\theta=30°} = 0.6 \cos 30° = 0.5196 \text{ m}$$

$$\dot{r} = -0.6 \sin \theta\dot{\theta}\big|_{\theta=30°} = -0.6 \sin 30°(2) = -0.6 \text{ m/s}$$

$$\ddot{r} = -0.6(\cos \theta\dot{\theta}^2 + \sin \theta\ddot{\theta})\big|_{\theta=30°} = -0.6\left[\cos 30°\left(2^2\right) + \sin 30°(0.8)\right] = -2.318 \text{ m/s}^2$$

Using the above time derivatives, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2.318 - 0.5196\left(2^2\right) = -4.397 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5196(0.8) + 2(-0.6)(2) = -1.984 \text{ m/s}^2$$

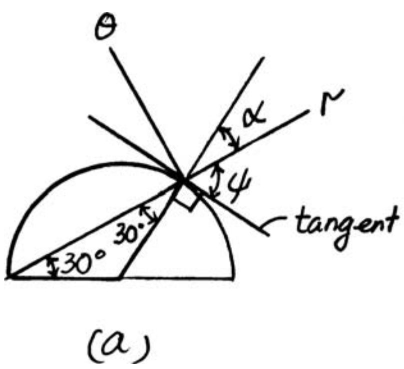
Free-Body Diagram: From the geometry shown in Fig. (a), we notice that $\alpha = 30°$. The free-body diagram of the cylinder C is shown in Fig. (b).

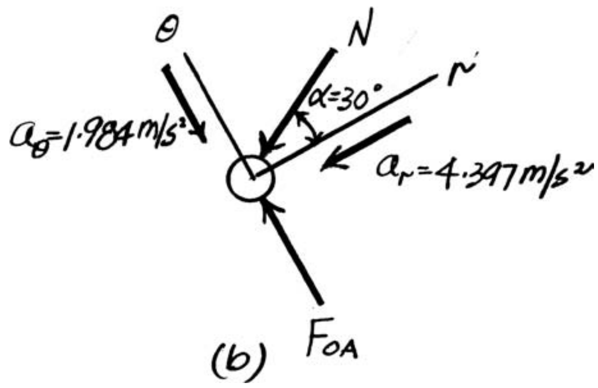
Equations of Motion: By referring to Fig. (b),

$+\nearrow\Sigma F_r = ma_r;$ $\quad -N \cos 30° = 0.5(-4.397)$ $\quad N = 2.539$ N

$+\nwarrow\Sigma F_\theta = ma_\theta;$ $\quad F_{OA} - 2.539 \sin 30° = 0.5(-1.984)$ $\quad F_{OA} = 0.277$ N **Ans.**

Kinematics: The values of r and the time derivatives at the instant $\theta = 30°$ are evaluated below.





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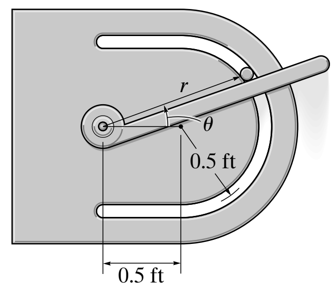
•13–97. The 0.75-lb smooth can is guided along the circular path using the arm guide. If the arm has an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 0.4$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the can. Motion occurs in the *horizontal plane.*



$$r = \cos\theta\big|_{\theta=30°} = 0.8660 \text{ ft}$$

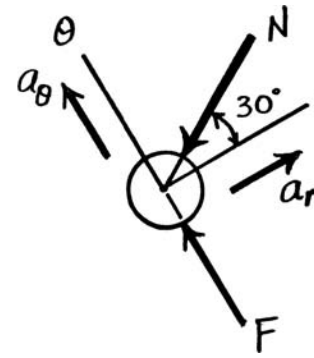
$$\dot{r} = -\sin\theta\dot{\theta}\big|_{\theta=30°} = -1.00 \text{ ft/s}$$

$$\ddot{r} = -(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -3.664 \text{ ft/s}^2$$

Using the above time derivative, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.664 - 0.8660\left(2^2\right) = -7.128 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(4) + 2(-1)(2) = -0.5359 \text{ ft/s}^2$$

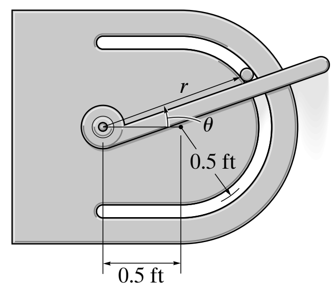


Equations of Motion: By referring to Fig. (a),

$\Sigma F_r = ma_r$; $\quad -N\cos 30° = \dfrac{0.75}{32.2}(-7.128) \qquad N = 0.1917$ lb

$\Sigma F_\theta = ma_\theta$; $\quad F - 0.1917\sin 30° = \dfrac{0.75}{32.2}(-0.5359) \qquad F = 0.0835$ lb **Ans.**

13–98. Solve Prob. 13–97 if motion occurs in the *vertical plane*.



Kinematics: The values of r and the time derivatives at the instant $\theta = 30°$ are evaluated below.

$$r = \cos\theta\big|_{\theta=30°} = 0.8660 \text{ ft}$$

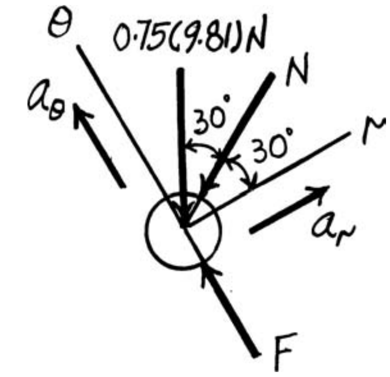
$$\dot{r} = -\sin\theta\dot{\theta}\big|_{\theta=30°} = -1.00 \text{ ft/s}$$

$$\ddot{r} = -(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -3.664 \text{ ft/s}^2$$

Using the above time derivative, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.664 - 0.8660\left(2^2\right) = -7.128 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(4) + 2(-1)(2) = -0.5359 \text{ ft/s}^2$$



Equations of Motion: By referring to Fig. (a),

$\Sigma F_r = ma_r$; $\quad -N\cos 30° - 0.75\cos 60° = \dfrac{0.75}{32.2}(-7.128)$

$N = -0.2413$ lb

$\Sigma F_\theta = ma_\theta$; $\quad F + 0.2413\sin 30° - 0.75\sin 60° = \dfrac{0.75}{32.2}(-0.5359)$

$F = 0.516$ lb **Ans.**

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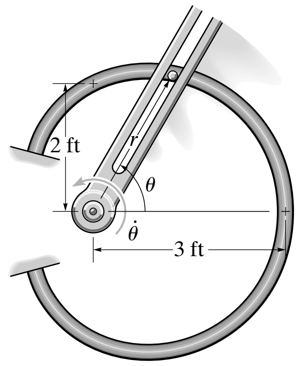
13–99. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 90°$. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 90°, \dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0$

$r = 2 + \cos 90° = 2$ ft

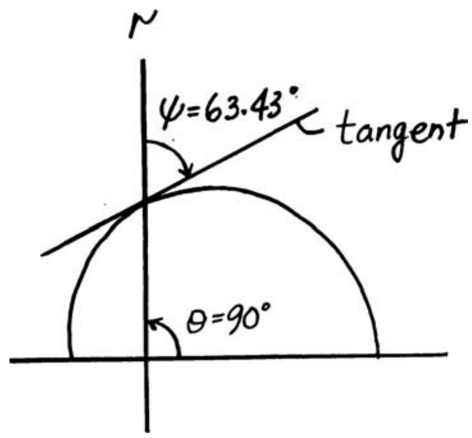
$\dot{r} = -\sin 90°(0.5) = -0.5$ ft/s

$\ddot{r} = -\cos 90°(0.5)^2 - \sin 90°(0) = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(0.5)^2 = -0.5 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(-0.5)(0.5) = -0.5 \text{ ft/s}^2$

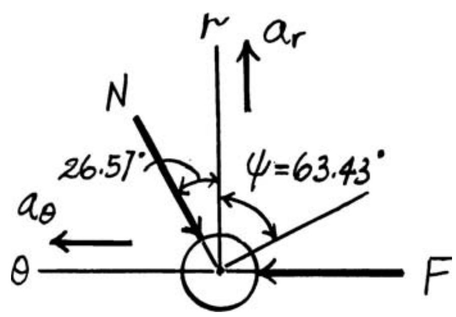
$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{2 + \cos\theta}{-\sin\theta}\Bigg|_{\theta=90°} = -2 \qquad \psi = -63.43°$



$+\uparrow\Sigma F_r = ma_r; \qquad -N\cos 26.57° = \dfrac{2}{32.2}(-0.5) \qquad N = 0.03472$ lb

$\overset{+}{\leftarrow}\Sigma F_\theta = ma_\theta; \qquad F - 0.03472\ \sin 26.57° = \dfrac{2}{32.2}(-0.5)$

$F = -0.0155$ lb **Ans.**



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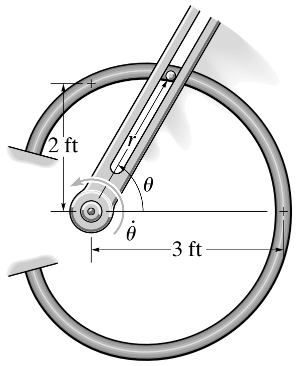
***13–100.** Solve Prob. 13–99 at the instant $\theta = 60°$.



$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 60°, \dot{\theta} = 0.5\text{ rad/s}$, and $\ddot{\theta} = 0$

$r = 2 + \cos 60° = 2.5\text{ ft}$

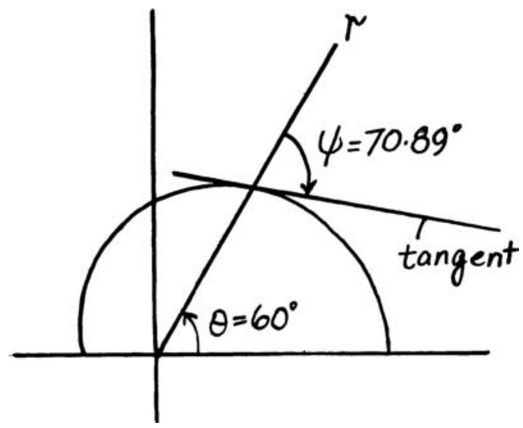
$\dot{r} = -\sin 60°(0.5) = -0.4330\text{ ft/s}$

$\ddot{r} = -\cos 60°(0.5)^2 - \sin 60°(0) = -0.125\text{ ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.125 - 2.5(0.5)^2 = -0.75\text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(0) + 2(-0.4330)(0.5) = -0.4330\text{ ft/s}^2$

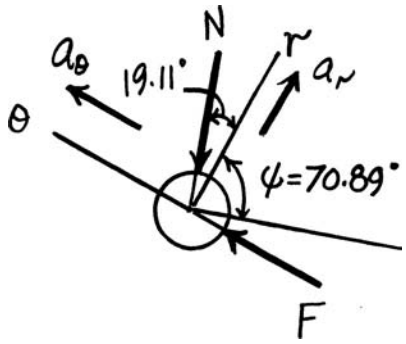
$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2 + \cos\theta}{-\sin\theta}\right|_{\theta=60°} = -2.887 \qquad \psi = -70.89°$$



$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 19.11° = \frac{2}{32.2}(-0.75) \qquad N = 0.04930\text{ lb}$

$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.04930\sin 19.11° = \frac{2}{32.2}(-0.4330)$

$F = -0.0108\text{ lb}$ **Ans.**



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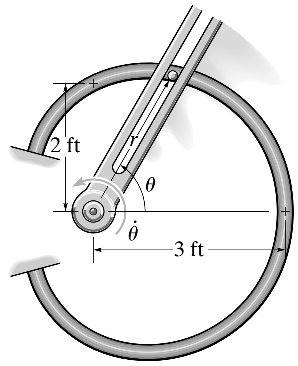
•13–101. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\theta \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

At $t = 1$ s, $\theta = 0.5$ rad, $\theta = 1$ rad/s, and $\ddot{\theta} = 1$ rad/s^2

$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r} = -\sin 0.5(1) = -0.4974 \text{ ft/s}^2$

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357 \text{ ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2 = -4.2346 \text{ ft/s}^2$

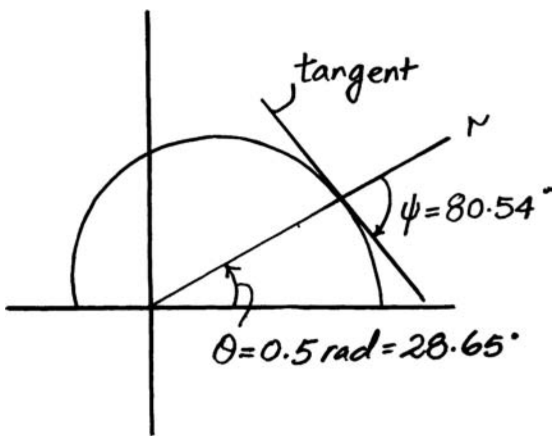
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187 \text{ ft/s}^2$

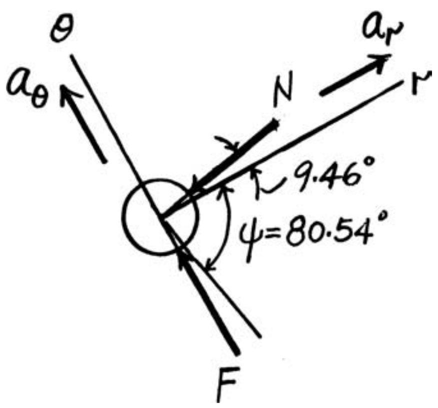
$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta = 0.5 \text{ rad}} = -6.002 \qquad \psi = -80.54°$$

$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 9.46° = \frac{2}{32.2}(-4.2346) \qquad N = 0.2666$ lb

$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46° = \frac{2}{32.2}(1.9187)$

$F = 0.163$ lb **Ans.**





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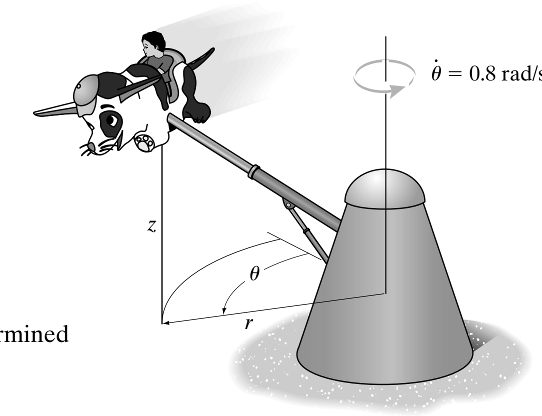
13–102. The amusement park ride rotates with a constant angular velocity of $\dot{\theta} = 0.8$ rad/s. If the path of the ride is defined by $r = (3\sin\theta + 5)$ m and $z = (3\cos\theta)$ m, determine the r, θ, and z components of force exerted by the seat on the 20-kg boy when $\theta = 120°$.



Kinematics: Since the motion of the boy is known, $\mathbf{a}_r$, $\mathbf{a}_\theta$, and $\mathbf{a}_z$ will be determined first. The value of r and its time derivatives at the instant $\theta = 120°$ are

$$r = (3\sin\theta + 5)\big|_{\theta=120°} = 3\sin 120° + 5 = 7.598 \text{ m}$$

$$\dot{r} = 3\cos\theta\dot{\theta}\Big|_{\theta=120°} = 3\cos 120°(0.8) = -1.2 \text{ m/s}$$

$$\ddot{r} = 3(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2)\Big|_{\theta=120°}$$

$$= 3\left[\cos 120°(0) - \sin 120°(0.8^2)\right] = -1.663 \text{ m/s}^2$$

Using the above time derivatives, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -1.663 - 7.598(0.8^2) = -6.526 \text{ m/s}^2$$

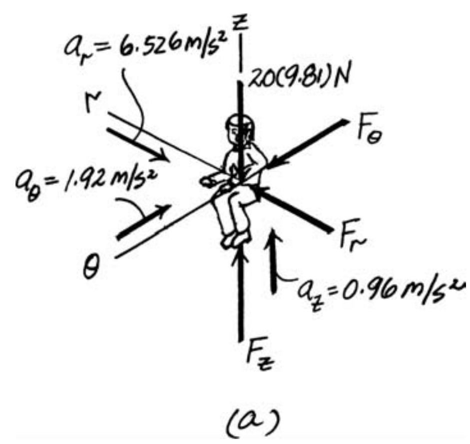
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.598(0) + 2(-1.2)(0.8) = -1.92 \text{ m/s}^2$$

Also,

$$z = 3\cos\theta \text{ m} \qquad \dot{z} = -3\sin\theta\dot{\theta} \text{ m/s}$$

$$a_z = \ddot{z} = -3(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)\Bigg|_{\theta=120°} = -3\left[\sin 120°(0) + \cos 120°(0.8^2)\right]$$

$$= 0.96 \text{ m/s}^2$$



(a)

Equations of Motion: By referring to the free-body diagram of the boy shown in Fig. (a),

$\Sigma F_r = ma_r;$ $\quad F_r = 20(-6.526) = -131$ N **Ans.**

$\Sigma F_\theta = ma_\theta;$ $\quad F_\theta = 20(-1.92) = -38.4$ N **Ans.**

$\Sigma F_z = ma_z;$ $\quad F_z - 20(9.81) = 20(0.96)$ $\quad F_z = 215$ N **Ans.**

Note: The negative signs indicate that $\mathbf{F}_r$ and $\mathbf{F}_\theta$ act in the opposite sense to those shown on the free-body diagram.

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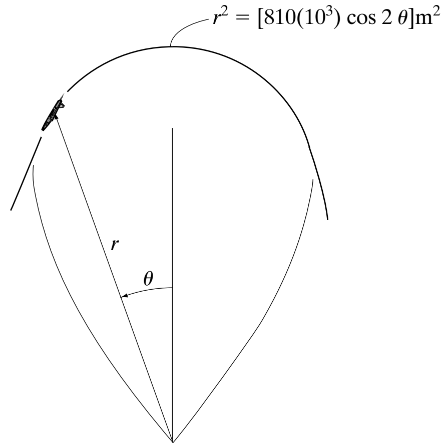
13–103. The airplane executes the vertical loop defined by $r^2 = [810(10^3)\cos 2\theta]\,\text{m}^2$. If the pilot maintains a constant speed $v = 120\,\text{m/s}$ along the path, determine the normal force the seat exerts on him at the instant $\theta = 0°$. The pilot has a mass of 75 kg.



Kinematics: Since the motion of the airplane is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The value of r and θ at $\theta = 0°$ are

$$r^2 = 810(10^3)\cos 2\theta\Big|_{\theta=0°} = 810(10^3)\cos 0°$$

$$r = 900\text{ m}$$

and

$$2r\dot{r} = -810(10^3)\sin 2\theta(2\dot{\theta})$$

$$\dot{r} = \frac{-810(10^3)\sin 2\theta\dot{\theta}}{r}\Bigg|_{\theta=0°} = \frac{-810(10^3)\sin 0°\theta}{900} = 0$$

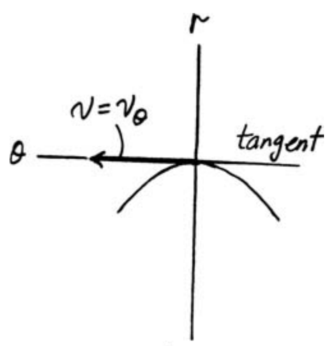
and

$$r\ddot{r} + \dot{r}^2 = -810(10^3)\left[\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right]$$

$$\ddot{r} = \frac{-810(10^3)\left[\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right] - \dot{r}^2}{r}\Bigg|_{\theta=0°}$$

$$= \frac{-810(10^3)\left[\sin 0°\ddot{\theta} + 2\cos 0°\dot{\theta}^2\right] - 0}{900}$$

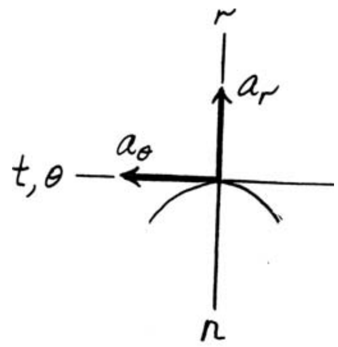
$$= -1800\dot{\theta}^2$$



(a)

The radial and transversal components of the airplane's velocity are given by

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 900\dot{\theta}$$



(b)

Thus,

$$v = v_\theta$$

$$120 = 900\dot{\theta}$$

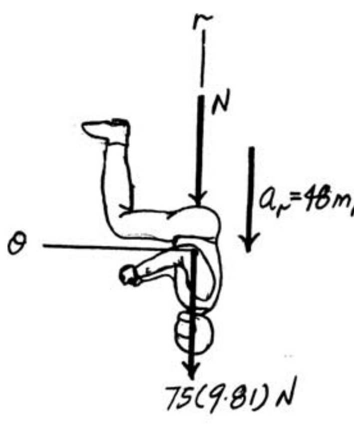
$$\dot{\theta} = 0.1333\text{ rad/s}^2$$

Substituting the result of $\dot{\theta}$ into $\ddot{r}$, we obtain

$$\ddot{r} = -1800(0.1333^2) = -32\text{ m/s}^2$$

Since $v = v_\theta$ and v are always directed along the tangent, then the tangent of the path at $\theta = 0°$ coincide with the θ axis, Fig. (a). As a result $a_\theta = a_t = 0$, Fig. (b), because v is constant. Using the results of $\ddot{r}$ and $\dot{\theta}$, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -32 - 900(1.1333^2) = -48\text{ m/s}^2$$



(c)

Equations of Motion: By referring to the free-body diagram of the pilot shown in Fig. (c),

$$+\uparrow\Sigma F_r = ma_r; \qquad -N - 75(9.81) = 75(-48)$$

$$N = 2864.25\text{ N} = 2.86\text{ kN} \qquad \textbf{Ans.}$$

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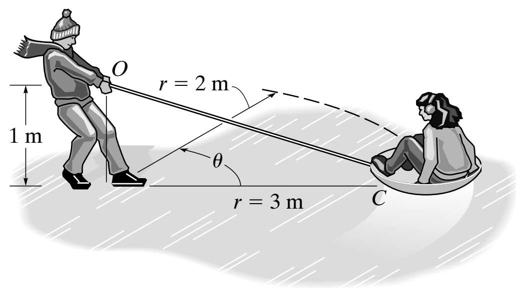
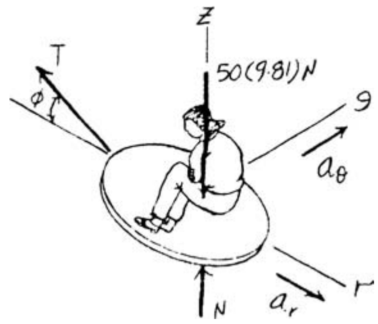
***13–104.** A boy standing firmly spins the girl sitting on a circular "dish" or sled in a circular path of radius $r_0 = 3$ m such that her angular velocity is $\dot{\theta}_0 = 0.1$ rad/s. If the attached cable OC is drawn inward such that the radial coordinate r changes with a constant speed of $\dot{r} = -0.5$ m/s, determine the tension it exerts on the sled at the instant $r = 2$ m. The sled and girl have a total mass of 50 kg. Neglect the size of the girl and sled and the effects of friction between the sled and ice. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)\,d/dt(r^2\dot{\theta}) = 0$. When integrated, $r^2\dot{\theta} = C$, where the constant C is determined from the problem data.



Equations of Motion: Applying Eq.13–9, we have

$$\Sigma F_\theta = ma_\theta; \qquad 0 = 50\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = \frac{1}{r}\frac{d\left(r^2\dot{\theta}\right)}{dt} = 0 \qquad \textbf{Q.E.D.}$$



Thus, $\displaystyle\int \frac{d\left(r^2\dot{\theta}\right)}{dt} = C$. Then, $r^2\,\dot{\theta} = C$. At $r = r_0 = 3$ m, $\dot{\theta} = \dot{\theta}_0 = 0.1$ rad/s.

Hence, $r = 2$ m,

$$\left(2^2\right)\dot{\theta} = \left(3^2\right)(0.1) \qquad \dot{\theta} = 0.225 \text{ rad/s}$$

Here, $\dot{r} = -0.5$ m/s and $\ddot{r} = 0$. Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2\left(0.225^2\right) = -0.10125 \text{ m/s}^2$$

At $r = 2$ m, $\phi = \tan^{-1}\left(\dfrac{1}{2}\right) = 26.57°$. Then

$$\Sigma F_r = ma_r; \qquad -T\cos 26.57° = 50(-0.10125)$$

$$T = 5.66 \text{ N} \qquad \textbf{Ans.}$$

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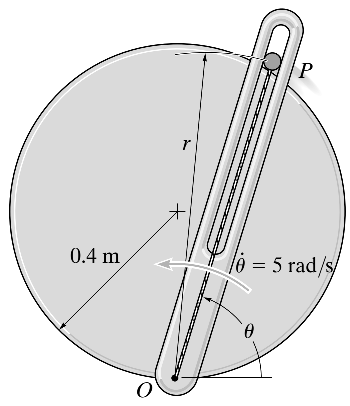
13–105. The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin \theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$. The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.



$r = 0.8 \sin \theta$

$\dot{r} = 0.8 \cos \theta \, \dot{\theta}$

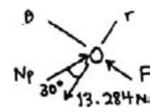
$\ddot{r} = -0.8 \sin \theta \, (\dot{\theta})^2 + 0.8 \cos \theta \ddot{\theta}$

$\dot{\theta} = 5, \qquad \ddot{\theta} = 0$

At $\theta = 60°$, $\qquad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -17.321$



$a_r = \ddot{r} - r(\dot{\theta})^2 = -17.321 - 0.6928(5)^2 = -34.641$

$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0 + 2(2)(5) = 20$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-34.641)$

$\nwarrow + \Sigma F_\theta = m a_\theta; \qquad F - N_P \sin 30° = 0.08(20)$

$F = 7.67$ N **Ans.**

$N_P = 12.1$ N

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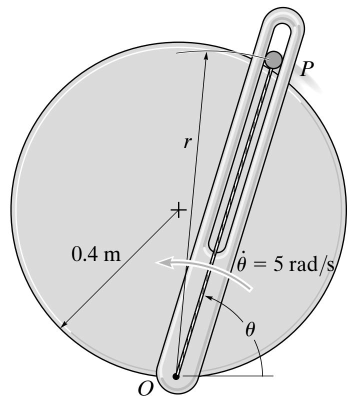
13–106. Solve Prob. 13–105 if $\ddot{\theta} = 2\ \text{rad/s}^2$ when $\dot{\theta} = 5$ rad/s and $\theta = 60°$.



$r = 0.8 \sin\theta$

$\dot{r} = 0.8 \cos\theta\,\dot{\theta}$

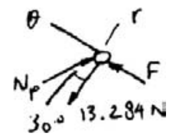
$\ddot{r} = -0.8 \sin\theta\,(\dot{\theta})^2 + 0.8 \cos\theta\ddot{\theta}$

$\dot{\theta} = 5, \qquad \ddot{\theta} = 2$

At $\theta = 60°$, $\quad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -16.521$



$a_r = \ddot{r} - r(\dot{\theta})^2 = -16.521 - 0.6928(5)^2 = -33.841$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6925(2) + 2(2)(5) = 21.386$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284\ \text{N}$

$\nearrow + \Sigma F_r = m\,a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-33.841)$

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - N_P \sin 30° = 0.08(21.386)$

$F = 7.82\ \text{N}$ **Ans.**

$N_P = 12.2\ \text{N}$

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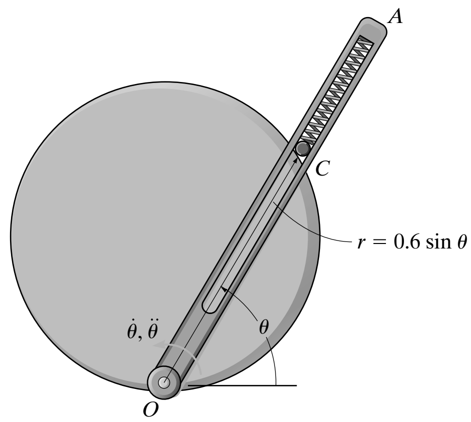
13–107. The 1.5-kg cylinder C travels along the path described by $r = (0.6 \sin \theta)$ m. If arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 3$ rad/s, determine the force exerted by the smooth slot in arm OA on the cylinder at the instant $\theta = 60°$. The spring has a stiffness of 100 N/m and is unstretched when $\theta = 30°$. The cylinder is in contact with only one edge of the slotted arm. Neglect the size of the cylinder. Motion occurs in the horizontal plane.



Kinematics: Since the motion of cylinder C is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of r and its time derivatives at the instant $\theta = 60°$ are evaluated below.

$$r = 0.6 \sin \theta\big|_{\theta=60°} = 0.6 \sin 60° = 0.5196 \text{ m}$$

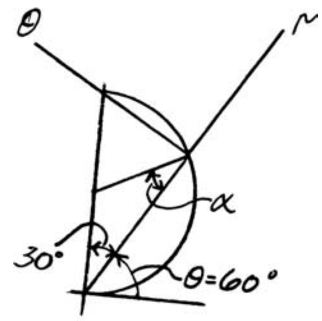
$$\dot{r} = 0.6 \cos \theta\dot{\theta}\big|_{\theta=60°} = 0.6 \cos 60°(3) = 0.9 \text{ m/s}$$

$$\ddot{r} = 0.6(\cos \theta\ddot{\theta} - \sin \theta(\dot{\theta})^2)\bigg|_{\theta=60°} = 0.6\left[\cos 60°(0) - \sin 60°(3^2)\right] = -4.677 \text{ m/s}^2$$

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -4.677 - 0.5196(3^2) = -9.353 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5196(0) + 2(0.9)(3) = 5.4 \text{ m/s}^2$$



(a)

Free-Body Diagram: From the geometry shown in Fig. (a), we notice that $\alpha = 30°$. The force developed in the spring is given by $F_{sp} = ks = 100(0.6 \sin 60° - 0.6 \sin 30°) = 21.96$ N. The free-body diagram of the cylinder C is shown in Fig. (b).

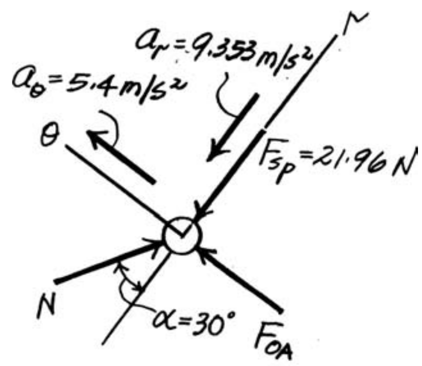
Equations of Motion: By referring to Fig. (a),

$$+\nearrow\Sigma F_r = ma_r; \qquad N \cos 30° - 21.96 = 1.5(-9.353)$$

$$N = 9.159 \text{ N}$$

$$\nwarrow+\Sigma F_\theta = ma_\theta; \qquad F_{OA} - 9.159 \sin 30° = 1.5(5.4)$$

$$F_{OA} = 12.68 \text{ N} = 12.7 \text{ N} \qquad \textbf{Ans.}$$



(b)

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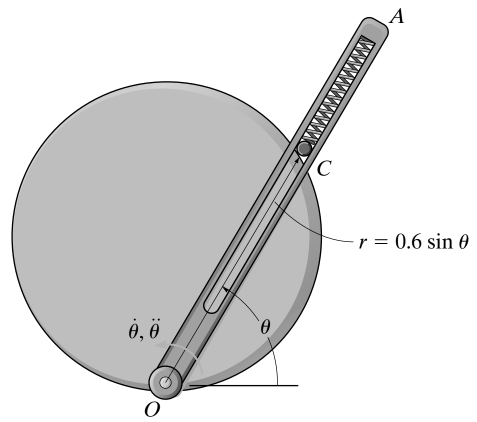
***13–108.** The 1.5-kg cylinder C travels along the path described by $r = (0.6 \sin \theta)$ m. If arm OA is rotating counterclockwise with an angular velocity of $\dot{\theta} = 3$ rad/s, determine the force exerted by the smooth slot in arm OA on the cylinder at the instant $\theta = 60°$. The spring has a stiffness of 100 N/m and is unstretched when $\theta = 30°$. The cylinder is in contact with only one edge of the slotted arm. Neglect the size of the cylinder. Motion occurs in the vertical plane.



Kinematics: Since the motion of cylinder C is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of r and its time derivatives at the instant $0 = 60°$ are evaluated below.

$$r = 0.6 \sin \theta\big|_{\theta=60°} = 0.6 \sin 60° = 0.5196 \text{ m}$$

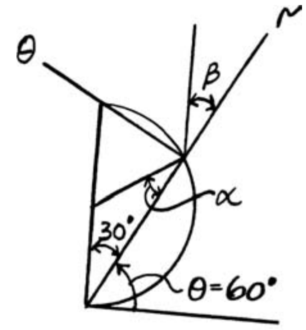
$$\dot{r} = 0.6 \cos \theta\dot{\theta}\big|_{\theta=60°} = 0.6 \cos 60°(3) = 0.9 \text{ m/s}$$

$$\ddot{r} = 0.6(\cos \theta\ddot{\theta} - \sin \theta\left(\dot{\theta}\right)^2)\bigg|_{\theta=60°} = 0.6\left[\cos 60°(0) - \sin 60°\left(3^2\right)\right] = -4.677 \text{ m/s}^2$$

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -4.677 - 0.5196(3^2) = -9.353 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5196(0) + 2(0.9)(3) = 5.4 \text{ m/s}^2$$



(a)

Free-Body Diagram: From the geometry shown in Fig. (a), we notice that $\alpha = 30°$ and $\beta = 30°$. The force developed in the spring is given by $F_{sp} = ks = 100(0.6 \sin 60° - 0.6 \sin 30°) = 21.96$ N. The free-body diagram of the cylinder C is shown in Fig. (b).

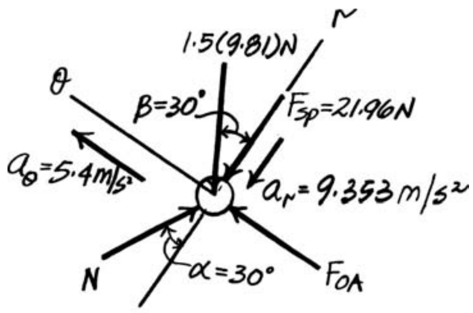
Equations of Motion: By referring to Fig. (a),

$+\nearrow\Sigma F_r = ma_r;$ $\qquad N \cos 30° - 21.96 - 1.5(9.81) \cos 30° = 1.5(-9.353)$

$$N = 23.87 \text{ N}$$

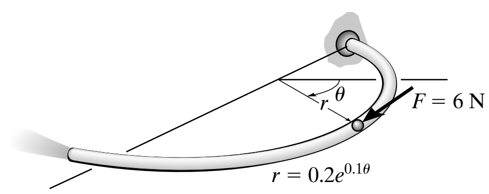
$\nwarrow+\Sigma F_\theta = ma_\theta;$ $\qquad F_{OA} - 1.5(9.81) \sin 30° - 23.87 \sin 30° = 1.5(5.4)$

$$F_{OA} = 27.4 \text{ N} \qquad \textbf{Ans.}$$



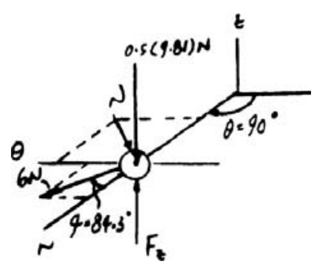
(b)

•13–109. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the horizontal plane and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to air pressure is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. Also, what is the angle ψ from the extended radial coordinate r to the line of action of the 6-N force?



$$\tan \psi = \frac{r}{dr/d\theta} = \frac{0.2e^{0.1\,\theta}}{0.02e^{0.1\theta}} = 10 \qquad \psi = 84.3° \qquad \textbf{Ans.}$$

$$\Sigma F_t = ma_t; \qquad 6 = 0.5a_t \qquad a_t = 12 \text{ m/s}^2 \qquad \textbf{Ans.}$$



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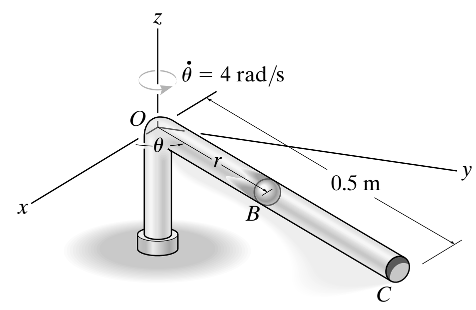
13–110. The tube rotates in the horizontal plane at a constant rate of $\dot{\theta} = 4$ rad/s. If a 0.2-kg ball B starts at the origin O with an initial radial velocity of $\dot{r} = 1.5$ m/s and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C, $r = 0.5$ m. *Hint:* Show that the equation of motion in the r direction is $\ddot{r} - 16r = 0$. The solution is of the form $r = Ae^{-4t} + Be^{4t}$. Evaluate the integration constants A and B, and determine the time t when $r = 0.5$ m. Proceed to obtain v_r and v_θ.



$$\dot{\theta} = 4 \qquad \ddot{\theta} = 0$$

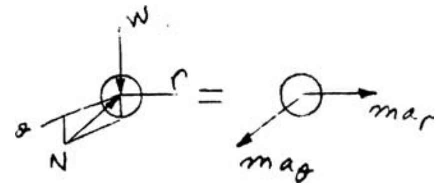
$$\Sigma F_r = ma_r; \qquad 0 = 0.2\left[\ddot{r} - r(4)^2\right]$$

$$\ddot{r} - 16r = 0$$

Solving this second-order differential equation,

$$r = Ae^{-4t} + Be^{4t} \qquad \textbf{(1)}$$

$$\dot{r} = -4Ae^{-4t} + 4Be^{4t} \qquad \textbf{(2)}$$



At $t = 0, r = 0, \dot{r} = 1.5$:

$$0 = A + B \qquad \frac{1.5}{4} = -A + B$$

$$A = -0.1875 \qquad B = 0.1875$$

From Eq. (1) at $r = 0.5$ m,

$$0.5 = 0.1875\left(-e^{-4t} + e^{4t}\right)$$

$$\frac{2.667}{2} = \frac{(-e^{-4t} + e^{4t})}{2}$$

$$1.333 = \sin\text{h}(4t)$$

$$t = \frac{1}{4}\sin\text{h}^{-1}(1.333) \qquad t = 0.275 \text{ s}$$

Using Eq. (2),

$$\dot{r} = 4(0.1875)\left(e^{-4t} + e^{4t}\right)$$

$$\dot{r} = 8(0.1875)\left(\frac{e^{-4t} + e^{4t}}{2}\right) = 8(0.1875)(\cos\text{h}(4t))$$

At $t = 0.275$ s:

$$\dot{r} = 1.5\cos\text{h}[4(0.275)]$$

$$v_t = r = 2.50 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 0.5(4) = 2 \text{ m/s} \qquad \textbf{Ans.}$$

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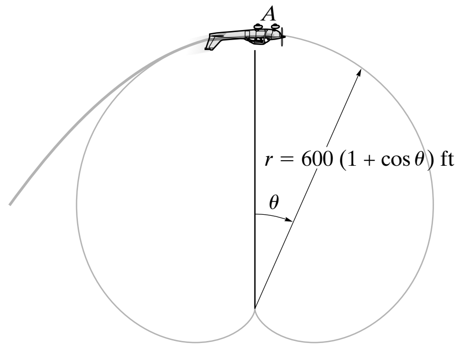
13–111. The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft. If his speed at A $(\theta = 0°)$ is a constant $v_P = 80$ ft/s, determine the vertical force the seat belt must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb.



$$r = 600(1 + \cos\theta)\big|_{\theta=0°} = 1200 \text{ ft}$$

$$\dot{r} = -600 \sin\theta\dot{\theta}\big|_{\theta=0°} = 0$$

$$\ddot{r} = -600 \sin\theta\ddot{\theta} - 600\cos\theta\dot{\theta}^2\big|_{\theta=0°} = -600\dot{\theta}^2$$

$$v_p^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$$

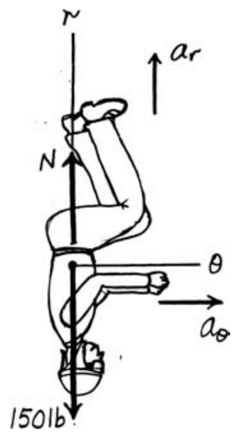
$$(80)^2 = 0 + \left(1200\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.06667$$

$$2v_p v_p = 2r\ddot{r} + 2\left(r\dot{\theta}\right)\left(\dot{r}\theta + r\ddot{\theta}\right)$$

$$0 = 0 + 0 + 2r^2\theta\ddot{\theta} \qquad \ddot{\theta} = 0$$

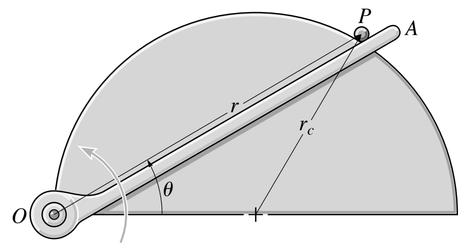
$$a_r = \ddot{r} - r\dot{\theta}^2 = -600(0.06667)^2 - 1200(0.06667)^2 = -8 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$



$+\uparrow\Sigma F_r = ma_r;\qquad N - 150 = \left(\dfrac{150}{32.2}\right)(-8) \qquad N = 113$ lb **Ans.**

***13–112.** The 0.5-lb ball is guided along the vertical circular path $r = 2r_c\cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.



$$r = 2(0.4)\cos\theta = 0.8\cos\theta$$

$$\dot{r} = -0.8\sin\theta\dot{\theta}$$

$$\ddot{r} = -0.8\cos\theta\dot{\theta}^2 - 0.8\sin\theta\ddot{\theta}$$

At $\theta = 30°, \dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s^2

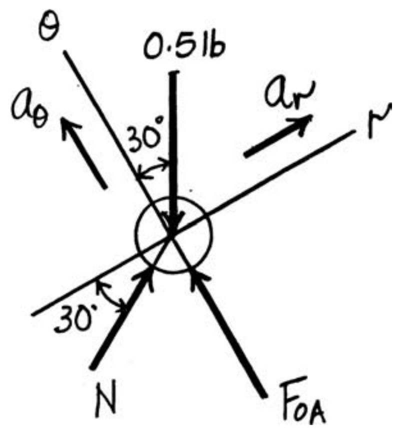
$$r = 0.8\cos 30° = 0.6928 \text{ ft}$$

$$\dot{r} = -0.8\sin 30°(0.4) = -0.16 \text{ ft/s}$$

$$\ddot{r} = -0.8\cos 30°(0.4)^2 - 0.8\sin 30°(0.8) = -0.4309 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4)^2 = -0.5417 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(0.8) + 2(-0.16)(0.4) = 0.4263 \text{ ft/s}^2$$



$+\nearrow\Sigma F_r = ma_r;\qquad N\cos 30° - 0.5\sin 30° = \dfrac{0.5}{32.2}(-0.5417) \qquad N = 0.2790$ lb

$\nwarrow+\Sigma F_\theta = ma_\theta;\qquad F_{OA} + 0.2790\sin 30° - 0.5\cos 30° = \dfrac{0.5}{32.2}(0.4263)$

$F_{OA} = 0.300$ lb **Ans.**

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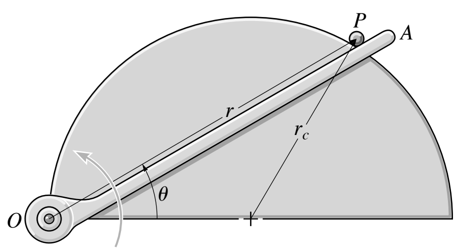
•13–113. The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



$$r = 2r_c \cos\theta$$

$$\dot{r} = -2r_c \sin\theta\dot{\theta}$$

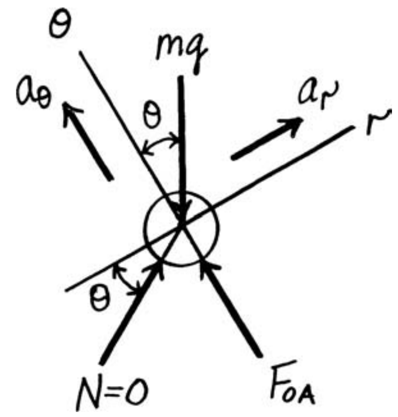
$$\ddot{r} = -2r_c \cos\theta\dot{\theta}^2 - 2r_c \sin\theta\ddot{\theta}$$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

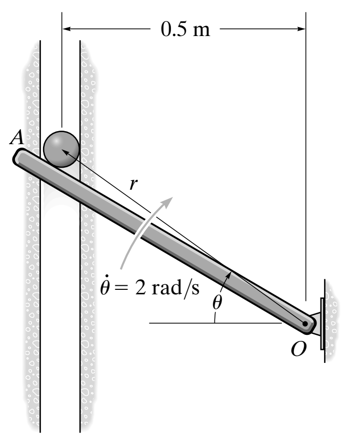
$$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos\theta\dot{\theta}_0^2 - 2r_c \cos\theta\dot{\theta}_0^2 = -4r_c \cos\theta\dot{\theta}_0^2$$

$+\nearrow\Sigma F_r = ma_r; \qquad -mg\sin\theta = m(-4r_c\cos\theta\dot{\theta}_0^2)$

$$\tan\theta = \frac{4r_c\,\dot{\theta}_0^2}{g} \qquad \theta = \tan^{-1}\left(\frac{4r_c\,\dot{\theta}_0^2}{g}\right)$$ **Ans.**



13–114. The ball has a mass of 1 kg and is confined to move along the smooth vertical slot due to the rotation of the smooth arm OA. Determine the force of the rod on the ball and the normal force of the slot on the ball when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 3$ rad/s. Assume the ball contacts only one side of the slot at any instant.



Kinematics: Here, $\dot{\theta} = 3$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 30°$, we have

$$r = \frac{0.5}{\cos\theta}\bigg|_{\theta=30°} = 0.5774 \text{ m}$$

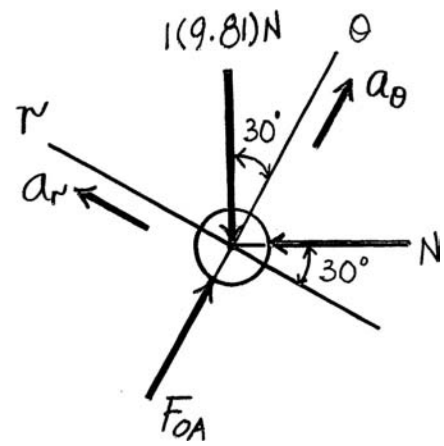
$$\dot{r} = \frac{0.5\sin\theta}{\cos^2\theta}\dot{\theta} = 0.5\tan\theta\sec\theta\dot{\theta}\bigg|_{\theta=30°} 1.00 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\tan\theta\sec\theta\ddot{\theta} + \left(\sec^3\theta + \tan^2\theta\sec\theta\right)\dot{\theta}^2\right]\bigg|_{\theta=30°} = 8.660 \text{ m/s}^2$$

Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 8.660 - 0.5774\left(3^2\right) = 3.464 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(1.00)(3) = 6.00 \text{m/s}^2$$



Equations of Motion:

$\nwarrow+\Sigma F_r = ma_r; \qquad N\cos 30° - 1(9.81)\sin 30° = 1(3.464)$

$$N = 9.664 \text{ N} = 9.66 \text{ N}$$ **Ans.**

$+\nearrow\Sigma F_\theta = ma_\theta; \qquad F_{OA} - 1(9.81)\cos 30° - 9.664\sin 30° = 1(6.00)$

$$F_{OA} = 19.3 \text{ N}$$ **Ans.**

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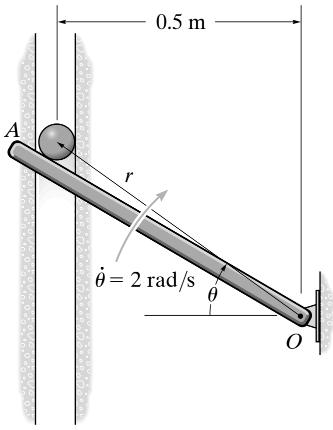
13–115. Solve Prob. 13–114 if the arm has an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$ when $\dot{\theta} = 3\ \text{rad/s}$ at $\theta = 30°$.



Kinematics: Here, $\dot{\theta} = 3\ \text{rad/s}$ and $\ddot{\theta} = 2\ \text{rad/s}^2$. Taking the required time derivatives at $\theta = 30°$, we have

$$r = \frac{0.5}{\cos\theta}\bigg|_{\theta=30°} = 0.5774\ \text{m}$$

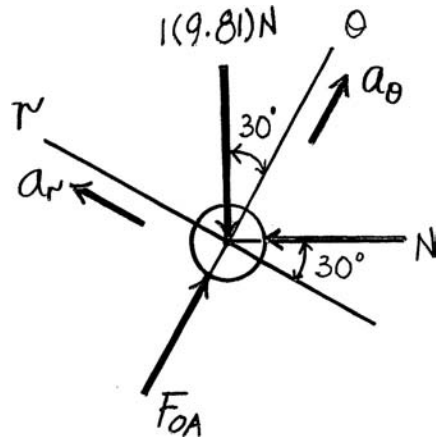
$$\dot{r} = \frac{0.5\sin\theta}{\cos^2\theta}\dot{\theta} = 0.5\tan\theta\sec\theta\dot{\theta}\bigg|_{\theta=30°} = 1.00\ \text{m/s}$$

$$\ddot{r} = 0.5\left[\tan\theta\sec\theta\ddot{\theta} + \left(\sec^2\theta + \tan^2\theta\sec\theta\right)\dot{\theta}^2\right]\bigg|_{\theta=30°} = 9.327\ \text{m/s}^2$$

Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 9.327 - 0.5774\left(3^2\right) = 4.131\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(2) + 2(1.00)(3) = 7.155\ \text{m/s}^2$$



Equations of Motion:

$\Sigma F_r = ma_r;$ $\quad N\cos 30° - 1(9.81)\sin 30° = 1(4.131)$

$N = 10.43\ \text{N} = 10.4\ \text{N}$ **Ans.**

$\Sigma F_\theta = ma_\theta;$ $\quad F_{OA} - 1(9.81)\cos 30° - 10.43\sin 30° = 1(7.155)$

$F_{OA} = 20.9\ \text{N}$ **Ans.**

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***13–116.** Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

From Eq. 13–19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–28 and 13–29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–31,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$ **Q.E.D.**

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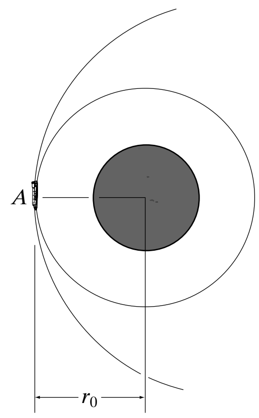
•13–117. The Viking explorer approaches the planet Mars on a parabolic trajectory as shown. When it reaches point A its velocity is 10 Mm/h. Determine r_0 and the required velocity at A so that it can then maintain a circular orbit as shown. The mass of Mars is 0.1074 times the mass of the earth.



When the Viking explorer approaches point A on a parabolic trajectory, its velocity at point A is given by

$$v_A = \sqrt{\frac{2GM_M}{r_0}}$$

$$\left[10(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \sqrt{\frac{2(66.73)(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{r_0}}$$

$$r_0 = 11.101(10^6)\text{ m} = 11.1\text{ Mm} \qquad \textbf{Ans.}$$

When the explorer travels along a circular orbit of $r_0 = 11.101(10^6)$ m, its velocity is

$$v_{A'} = \sqrt{\frac{GM_r}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{11.101(10^6)}}$$

$$= 1964.19\text{ m/s}$$

Thus, the required sudden decrease in the explorer's velocity is

$$\Delta v_A = v_A - v_{A'}$$

$$= 10(10^6)\left(\frac{1}{3600}\right) - 1964.19$$

$$= 814\text{ m/s} \qquad \textbf{Ans.}$$

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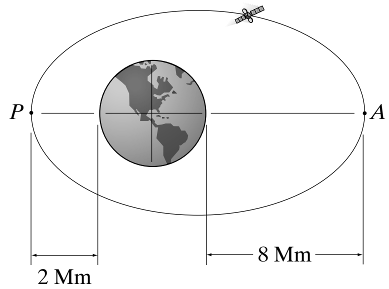
13–118. The satellite is in an elliptical orbit around the earth as shown. Determine its velocity at perigee P and apogee A, and the period of the satellite.



Here,

$$r_0 = r_p = 2(10^6) + 6378(10^3) = 8.378(10^6)\text{ m}$$

and

$$r_a = 8(10^6) + 6378(10^3)$$

$$= 14.378(10^6)\text{ m}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1}$$

$$14.378(10^6) = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)v_0{}^2} - 1}$$

$$v_p = v_0 = 7755.53\text{ m/s} = 7.76\text{ km/s} \qquad \textbf{Ans.}$$

Using the result of v_p, we have

$$h = r_p v_p = r_a v_a$$

$$h = 8.378(10^6)(7755.53\text{ m/s}) = 14.378(10^6)v_a$$

$$v_A = v_a = 4519.12\text{ m/s} = 4.52\text{ km/s} \qquad \textbf{Ans.}$$

$$h = r_p v_p = 8.378(10^6)(7755.53)$$

$$= 64.976(10^9)\text{ m}^2/\text{s}$$

Thus,

$$T = \frac{\pi}{h}\left(r_p + r_a\right)\sqrt{r_p r_a}$$

$$= \frac{\pi}{64.976(10^9)}\left[8.378(10^6) + 14.378(10^6)\right]\sqrt{8.378(10^6)(14.378)(10^6)}$$

$$= 12\,075.71\text{ s} = 3.35\text{ hr} \qquad \textbf{Ans.}$$

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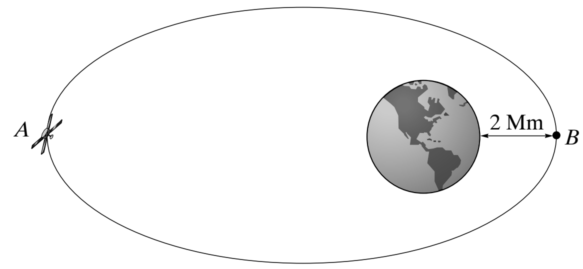
13–119. The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



$$e = \frac{Ch^2}{GM_e}$$

where $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0 v_0$.

$$e = \frac{1}{GM_e\, r_0}\left(1 - \frac{GM_e}{r_0\, v_0^2}\right)(r_0\, v_0)^2$$

$$e = \left(\frac{r_0\, v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e+1)}{r_0}}$$

where $r_0 = r_p = 2\left(10^6\right) + 6378\left(10^3\right) = 8.378\left(10^6\right)$ m.

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25+1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \qquad \textbf{Ans.}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0} - 1} = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96\left(10^6\right) \text{ m}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

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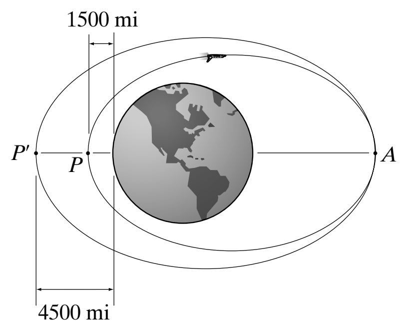
***13–120.** The space shuttle is launched with a velocity of 17 500 mi/h parallel to the tangent of the earth's surface at point P and then travels around the elliptical orbit. When it reaches point A, its engines are turned on and its velocity is suddenly increased. Determine the required increase in velocity so that it enters the second elliptical orbit. Take $G = 34.4(10^{-9})\,\text{ft}^4/\text{lb}\cdot\text{s}^4$, $M_e = 409(10^{21})$ slug, and $r_e = 3960\,\text{mi}$, where 5280 ft = mi.



For the first elliptical orbit,

$$r_P = 1500 + 3960 = (5460\text{ mi})\left(\frac{5280\text{ ft}}{1\text{ mi}}\right) = 28.8288(10^6)\text{ ft}$$

and

$$v_P = \left(17500\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{h}}{3600\text{ s}}\right) = 25\,666.67\text{ ft/s}$$

Using the results of r_p and v_p,

$$r_a = \frac{r_P}{\dfrac{2GM_e}{r_P\,v_P{}^2} - 1} = \frac{28.8288(10^6)}{\dfrac{2(34.4)(10^{-9})(409)(10^{21})}{28.8288(10^6)(25666.67^2)} - 1}$$

$$= 59.854(10^6)\text{ ft/s}$$

Since $h = r_P\,v_P = 28.8288(10^6)(25666.67) = 739.94(10^9)\text{ ft}^2/\text{s}$ is constant,

$$r_a\,v_a = h$$

$$59.854(10^6)v_a = 739.94(10^9)$$

$$v_a = 12\,362.40\text{ ft/s}$$

When the shuttle enters the second elliptical orbit, $r_P{}' = 4500 + 3960 = 8460\text{ mi}\left(\frac{5280\text{ ft}}{1\text{ mi}}\right) = 44.6688(10^6)\text{ ft}$ and $r_a{}' = r_a = 59.854(10^6)$ ft.

$$r_a{}' = \frac{r_P{}'}{\dfrac{2GM_e}{r_P{}'\left(v_P{}'\right)^2} - 1}$$

$$59.854(10^6) = \frac{44.6688(10^6)}{\dfrac{2(34.4)(10^{-9})(409)(10^{21})}{44.6688(10^6)\left(v_P{}'\right)^2} - 1}$$

$$v_P{}' = 18\,993.05\text{ ft/}s$$

Since $h' = r_P{}'\,v_P{}' = 44.6688(10^6)(18\,993.05) = 848.40(10^9)\text{ ft}^2/\text{s}$ is constant,

$$r_a{}'\,v_a{}' = h'$$

$$59.854(10^6)v_a{}' = 848.40(10^9)$$

$$v_a{}' = 14\,174.44\text{ ft/s}$$

Thus, the required increase in the shuttle's speed at point A is

$$\Delta v_A = v_{A'} - v_A = 14\,174.44 - 12\,362.40$$

$$= 1812.03\text{ ft/s} = 1812\text{ ft/s}$$ **Ans.**

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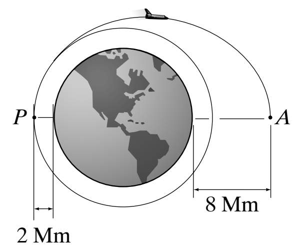
•13–121. Determine the increase in velocity of the space shuttle at point P so that it travels from a circular orbit to an elliptical orbit that passes through point A. Also, compute the speed of the shuttle at A.



When the shuttle is travelling around the circular orbit of radius $r_o = 2(10^6) + 6378(10^3) = 8.378(10^6)$ m, its speed is

$$v_o = \sqrt{\frac{GM_e}{r_o}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{8.378(10^6)}} = 6899.15 \text{ m/s}$$

When the shuttle enters the elliptical orbit, $r_p = r_o = 8.378(10^6)$ m and $r_a = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m.

$$r_a = \frac{r_p}{\dfrac{2GM_e}{r_p {v_p}^2} - 1}$$

$$14.378(10^6) = \frac{8.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6){v_p}^2} - 1}$$

$$v_p = 7755.54 \text{ m/s}$$

Thus, the required increase in speed for the shuttle at point P is

$$\Delta v_p = v_p - v_o = 7755.54 - 6899.15 = 856.39 \text{ m/s} = 856 \text{ m/s} \qquad \textbf{Ans.}$$

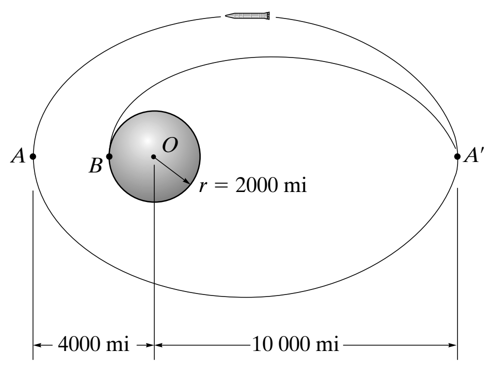
Since $h = r_p v_p = 8.378(10^6)(7755.54) = 64.976(10^9)$ m^2/s is constant,

$$r_a v_a = h$$

$$14.378(10^6)v_a = 64.976(10^9)$$

$$v_A = 4519.11 \text{ m/s} = 4.52 \text{ km/s} \qquad \textbf{Ans.}$$

13–122. The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the apoapsis and periapsis shown, determine the rocket's velocity when it is at point A. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, 1 mi = 5280 ft.



$$r_0 = OA = (4000)(5280) = 21.12\left(10^6\right) \text{ ft}$$

$$OA' = (10\,000)(5280) = 52.80\left(10^6\right)\text{ft}$$

$$M_P = \left(409\left(10^{21}\right)\right)(0.6) = 245.4\left(10^{21}\right) \text{ slug}$$

$$OA' = \frac{OA}{\left(\dfrac{2GM_p}{OAv_0^2} - 1\right)}$$

$$v_0 = \sqrt{\frac{2GM_P}{OA\left(\dfrac{OA}{OA'} + 1\right)}} = \sqrt{\frac{2(34.4)(10^{-9})(245.4)(10^{21})}{21.12(10^6)\left(\dfrac{21.12}{52.80} + 1\right)}}$$

$$v_0 = 23.9\left(10^3\right) \text{ ft/s} \qquad \textbf{Ans.}$$

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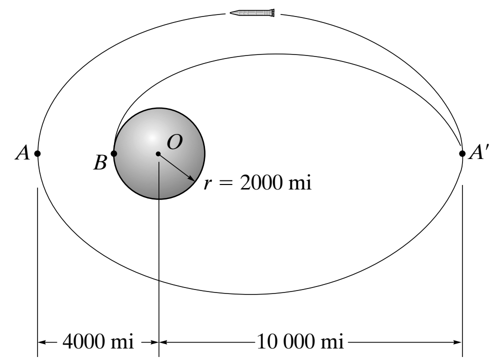
13–123. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, in going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, 1 mi = 5280 ft.



$$M_P = 409\left(10^{21}\right)(0.6) = 245.4\left(10^{21}\right)\text{ slug}$$

$$OA' = (10\,000)(5280) = 52.80\left(10^6\right)\text{ ft} \qquad OB = (2000)(5280) = 10.56\left(10^6\right)\text{ ft}$$

$$OA' = \frac{OB}{\left(\dfrac{2GM_P}{OBv_0^2} - 1\right)}$$

$$v_0 = \sqrt{\frac{2GM_P}{OB\left(\dfrac{OB}{OA'} + 1\right)}} = \sqrt{\frac{2(34.4(10^{-9}))245.4(10^{21})}{10.56(10^6)\left(\dfrac{10.56}{52.80} + 1\right)}}$$

$$v_0 = 36.50\left(10^3\right)\text{ ft/s} \qquad \text{(speed at } B\text{)}$$

$$v_{A'} = \frac{OBv_0}{OA'}$$

$$v_{A'} = \frac{10.56(10^6)36.50(10^3)}{52.80(10^6)}$$

$$v_{A'} = 7.30\left(10^3\right)\text{ ft/s} \qquad \textbf{Ans.}$$

$$T = \frac{\pi}{h}(OB + OA')\sqrt{(OB)(OA')}$$

$$h = (OB)(v_0) = 10.56\left(10^6\right)36.50\left(10^3\right) = 385.5\left(10^9\right)$$

Thus,

$$T = \frac{\pi(10.56 + 52.80)(10^6)}{385.5(10^9)}\left(\sqrt{(10.56)(52.80)}\right)\left(10^6\right)$$

$$T = 12.20\left(10^3\right)\text{ s}$$

$$t = \frac{T}{2} = 6.10\left(10^3\right)\text{ s} = 1.69\text{ h} \qquad \textbf{Ans.}$$

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***13–124.** A communications satellite is to be placed into an equatorial circular orbit around the earth so that it always remains directly over a point on the earth's surface. If this requires the period to be 24 hours (approximately), determine the radius of the orbit and the satellite's velocity.

$$\frac{GM_eM_s}{r^2} = \frac{M_s v^2}{r}$$

$$\frac{GM_e}{r} = v^2$$

$$\frac{GM_e}{r} = \left[\frac{2\pi r}{24(3600)}\right]^2$$

$$\frac{66.73(10^{-12})(5.976)(10^{24})}{\left[\frac{2\pi}{24(3600)}\right]^2} = r^3$$

$$r = 42.25(10^6)\text{ m} = 42.2\text{ Mm} \qquad \textbf{Ans.}$$

$$v = \frac{2\pi(42.25)(10^6)}{24(3600)} = 3.07\text{ km/s} \qquad \textbf{Ans.}$$

•13–125. The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

For a 800-km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6\text{ m/s} = 7.45\text{ km/s} \qquad \textbf{Ans.}$$

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13–126. The earth has an orbit with eccentricity $e = 0.0821$ around the sun. Knowing that the earth's minimum distance from the sun is $151.3(10^6)$ km, find the speed at which a rocket travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

$$e = \frac{Ch^2}{GM_S} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\,v_0^2}\right) \text{ and } h = r_0\,v_0$$

$$e = \frac{1}{GM_S\,r_0}\left(1 - \frac{GM_S}{r_0\,v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\frac{r_0\,v_0^2}{GM_S} - 1\right) \qquad \frac{r_0 v_0^2}{GM_S} = e + 1$$

$$v_0 = \sqrt{\frac{GM_S\,(e+1)}{r_0}}$$

$$= \sqrt{\frac{66.73(10^{-12})(1.99)(10^{30})(0.0821 + 1)}{151.3(10^9)}} = 30818 \text{ m/s} = 30.8 \text{ km/s} \qquad \textbf{Ans.}$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\,v_0^2}\right)\cos\theta + \frac{GM_S}{r_0^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{151.3(10^9)}\left(1 - \frac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30818)^2}\right)\cos\theta + \frac{66.73(10^{-12})(1.99)(10^{30})}{\left[151.3(10^9)\right]^2(30818)^2}$$

$$\frac{1}{r} = 0.502(10^{-12})\cos\theta + 6.11(10^{-12}) \qquad \textbf{Ans.}$$

13–127. A rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.

To escape the earth's gravitational field, the rocket has to make a parabolic trajectory.

Parabolic Trajectory:

$$v_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical Orbit:

$$e = \frac{Ch^2}{GM_e} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0\,v_0$$

$$e = \frac{1}{GM_e\,r_0}\left(1 - \frac{GM_e}{r_0\,v_0^2}\right)(r_0\,v_0)^2$$

$$e = \left(\frac{r_0\,v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\,v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e+1)}{r_0}}$$

$$\Delta v = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e\,(e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \textbf{Ans.}$$

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***13–128.** A rocket is in circular orbit about the earth at an altitude of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.

Circular Orbit:

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 6198.8 \text{ m/s}$$

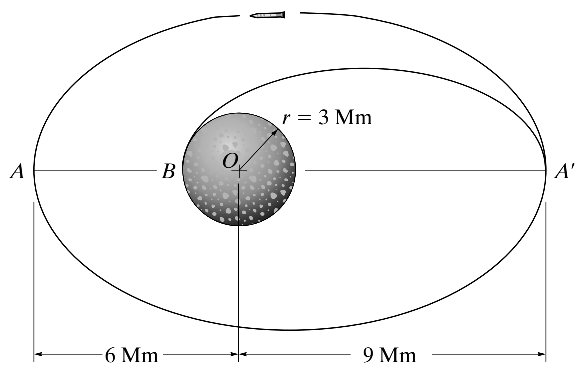
Parabolic Orbit:

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 8766.4 \text{ m/s}$$

$$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6 \text{ m/s}$$

$$\Delta v = 2.57 \text{ km/s}$$ **Ans.**

•13–129. The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



Central-Force Motion: Use $r_a = \dfrac{r_0}{(2\,GM/r_0\,v_0^2)-1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70M_e$, we have

$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_P = 7471.89 \text{ m/s} = 7.47 \text{ km/s}$$ **Ans.**

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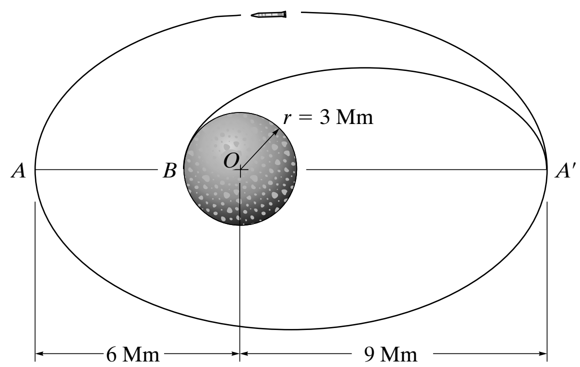
13–130. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that it strikes the planet at B. How long does it take for the rocket to land, going from A' to B along an elliptical path? The planet has no atmosphere, and its mass is 0.70 times that of the earth.



Central-Force Motion: Use $r_A = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right) - 1}$, with $r_A = 9\left(10^6\right)$ m, $r_0 = r_P = 3\left(10^6\right)$ m, and $M = 0.70M_e$. We have

$$9\left(10^6\right) = \frac{3(10^6)}{\left(\dfrac{2(66.73)(10^{-12})\,(0.7)[5.976(10^{24})]}{3(10^6)v_P^2}\right) - 1}$$

$$v_P = 11814.08 \text{ m/s}$$

Applying Eq. 13–20, we have

$$v_A = \left(\frac{r_P}{r_A}\right)v_P = \left[\frac{3(10^6)}{9(10^6)}\right](11814.08) = 3938.03 \text{ m/s} = 3.94 \text{ km/s} \qquad \textbf{Ans.}$$

Eq. 13–20 gives $h = r_p\, v_p = 3\left(10^6\right)(11814.08) = 35.442\left(10^9\right)\ \text{m}^2/\text{s}$. Thus, applying Eq.13–31, we have

$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P\, r_A}$$

$$= \frac{\pi}{35.442(10^9)}\left[(9 + 3)\left(10^6\right)\right]\sqrt{3(10^6)\,9\,(10^6)}$$

$$= 5527.03 \text{ s}$$

The time required for the rocket to go from A' to B (half the orbit) is given by

$$t = \frac{T}{2} = 2763.51 \text{ s} = 46.1 \text{ min} \qquad \textbf{Ans.}$$

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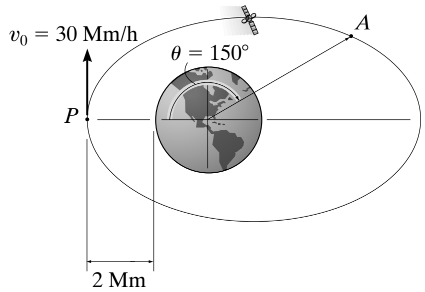
13–131. The satellite is launched parallel to the tangent of the earth's surface with a velocity of $v_0 = 30$ Mm/h from an altitude of 2 Mm above the earth as shown. Show that the orbit is elliptical, and determine the satellite's velocity when it reaches point A.



Here,

$$r_0 = 2(10^6) + 6378(10^3) = 8.378(10^6)\text{ m}$$

and

$$v_0 = \left[30(10^6)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 8333.33\text{m/s}$$

$$h = r_0 v_0 = 8.378(10^6)(8333.33) = 69.817(10^9)\text{ m}^2/\text{s}$$

and

$$C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0\, v_0{}^2}\right)$$

$$= \frac{1}{8.378(10^6)}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{8.378(10^6)(8333.33^2)}\right]$$

$$= 37.549(10^{-9})\text{ m}^{-1}$$

The eccentricity of the satellite orbit is given by

$$e = \frac{Ch^2}{GM_e} = \frac{37.549(10^{-9})\left[69.817(10^9)\right]^2}{66.73(10^{-12})(5.976)(10^{24})} = 0.459$$

Since $e < 1$, the satellite orbit is *elliptical* (Q.E.D.). $r = r_A$ at $\theta = 150°$, we obtain

$$\frac{1}{r} = C\cos\theta + \frac{GM_e}{h^2}$$

$$\frac{1}{r_A} = 37.549(10^{-9})\cos 150° + \frac{66.73(10^{-12})(5.976)(10^{24})}{\left[69.817(10^9)\right]^2}$$

$$r_A = 20.287(10^6)\text{ m}$$

Since h is constant,

$$r_A v_A = h$$

$$20.287(10^6)v_A = 69.817(10^9)$$

$$v_A = 3441.48\text{ m/s} = 3.44\text{ km/s} \qquad \textbf{Ans.}$$

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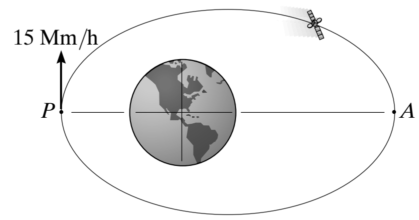
***13–132.** The satellite is in an elliptical orbit having an eccentricity of $e = 0.15$. If its velocity at perigee is $v_P = 15\,\text{Mm/h}$, determine its velocity at apogee A and the period of the satellite.



Here, $v_P = \left[15(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4166.67\text{ m/s}.$

$$h = r_P v_P$$

$$h = r_P\,(4166.67) = 4166.67 r_p \qquad \textbf{(1)}$$

and

$$C = \frac{1}{r_P}\left(1 - \frac{GM_e}{r_P\,{v_P}^2}\right)$$

$$C = \frac{1}{r_p}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{r_p(4166.67^2)}\right]$$

$$C = \frac{1}{r_P}\left[1 - \frac{22.97(10^6)}{r_P}\right] \qquad \textbf{(2)}$$

$$e = \frac{Ch^2}{GM_e}$$

$$0.15 = \frac{\frac{1}{r_P}\left[1 - \frac{22.97(10^6)}{r_P}\right](4166.67\,r_P)^2}{66.73(10^{-12})(5.976)(10^{24})}$$

$$r_P = 26.415(10^6)\text{ m}$$

Using the result of r_p

$$r_A = \frac{r_P}{\frac{2GM_e}{r_P\,{v_P}^2} - 1}$$

$$= \frac{26.415(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{26.415(10^6)(4166.67^2)} - 1}$$

$$= 35.738(10^6)\text{ m}$$

Since $h = r_P\,v_P = 26.415(10^6)(4166.67^2) = 110.06(10^9)\text{ m}^2\text{/s}$ is constant,

$$r_A\,v_A = h$$

$$35.738(10^6)v_A = 110.06(10^9)$$

$$v_A = 3079.71\text{ m/s} = 3.08\text{ km/s} \qquad \textbf{Ans.}$$

Using the results of h, r_A, and r_P,

$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P\,r_A}$$

$$= \frac{\pi}{110.06(10^9)}\left[26.415(10^6) + 35.738(10^6)\right]\sqrt{26.415(10^6)(35.738)(10^6)}$$

$$= 54\,508.43\text{ s} = 15.1\text{ hr} \qquad \textbf{Ans.}$$

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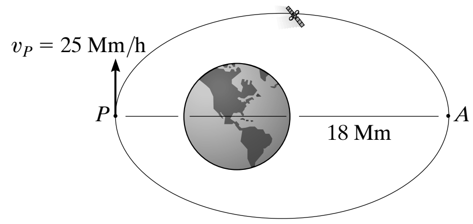
•13–133. The satellite is in an elliptical orbit. When it is at perigee P, its velocity is $v_P = 25\,\text{Mm/h}$, and when it reaches point A, its velocity is $v_A = 15\,\text{Mm/h}$ and its altitude above the earth's surface is 18 Mm. Determine the period of the satellite.



Here,

$$v_A = \left[15\left(10^6\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4166.67\text{ m/s}$$

and

$$r_A = 18\left(10^6\right) + 6378\left(10^3\right) = 24.378\left(10^6\right)\text{ m}$$

$$h = r_A\,v_A\left[24.378\left(10^6\right)\right](4166.67) = 101.575\left(10^9\right)\text{ m}^2/\text{s}$$

Since h is constant and $v_P = \left[25\left(10^6\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 6944.44\text{ m/s}$,

$$r_P v_P = h$$

$$r_P\,(6944.44) = 101.575(10^9)$$

$$r_P = 14.6268(10^6)\text{ m}$$

Using the results of h, r_A, and r_P,

$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P r_A}$$

$$= \frac{\pi}{101.575\left(10^9\right)}\left[14.6268\left(10^6\right) + 24.378\left(10^6\right)\right]\sqrt{14.6268\left(10^6\right)(24.378)\left(10^6\right)}$$

$$= 430158.48\text{ s} = 119\text{ h} \qquad \textbf{Ans.}$$

13–134. A satellite is launched with an initial velocity $v_0 = 4000$ km/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic.

$$v_0 = \frac{4000(10^3)}{3600} = 1111\text{ m/s}$$

(a) For circular trajectory, $e = 0$

$$v_0 = \sqrt{\frac{GM_e}{r_0}} \qquad r_0 = \frac{GM_e}{v_0^2} = \frac{(66.73)(10^{-12})(5.976)(10^{24})}{(1111)^2} = 323(10^3)\text{ km}$$

$$r = r_0 - 6378\text{ km} = 317\left(10^3\right)\text{ km} = 317\text{ Mm} \qquad \textbf{Ans.}$$

(b) For parabolic trajectory, $e = 1$

$$v_0 = \sqrt{\frac{2GM_e}{r_0}} \qquad r_0 = \frac{2GM_e}{v_0^2} = \frac{2(66.73)(10^{-12})(5.976)(10^{24})}{1111^2} = 646\left(10^3\right)\text{ km}$$

$$r = r_0 - 6378\text{ km} = 640\left(10^3\right)\text{ km} = 640\text{ Mm} \qquad \textbf{Ans.}$$

(c) For elliptical trajectory, $e < 1$

$$317\text{ Mm} < r < 640\text{ Mm} \qquad \textbf{Ans.}$$

(d) For hyperbolic trajectory, $e > 1$

$$r > 640\text{ Mm} \qquad \textbf{Ans.}$$

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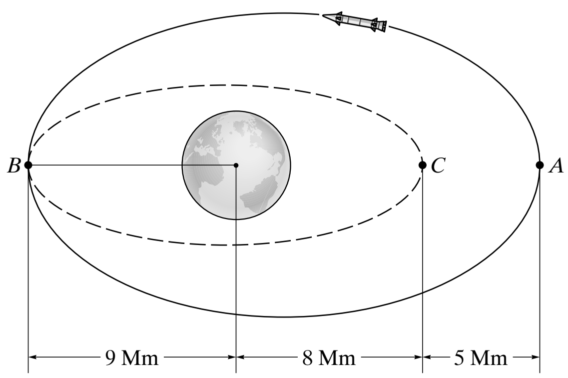
13–135. The rocket is in a free-flight elliptical orbit about the earth such that $e = 0.76$ as shown. Determine its speed when it is at point A. Also determine the sudden change in speed the rocket must experience at B in order to travel in free flight along the orbit indicated by the dashed path.



$$e = \frac{Ch^2}{GM_e} \qquad \text{where} \quad C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad \text{or} \qquad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \tag{1}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} \tag{2}$$

Substituting Eq.(1) into (2) yields:

$$r_a = \frac{r_0}{2\left(\frac{1}{e+1}\right) - 1} = \frac{r_0 (e+1)}{1 - e} \tag{3}$$

From Eq.(1),

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.76+1)}{9(10^6)}} = 8831 \text{ m/s}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{9(10^6)}{13(10^6)}(8831) = 6113 \text{ m/s} = 6.11 \text{ km/s} \qquad \textbf{Ans.}$$

From Eq.(3),

$$r_a = \frac{r_0(e+1)}{1-e}$$

$$9(10)^6 = \frac{8(10^6)(e+1)}{1-e} \qquad e = 0.05882$$

From Eq. (1),

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$v_C = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.05882+1)}{8(10^6)}} = 7265 \text{ m/s}$$

$$v_B = \frac{r_p}{r_a} v_C = \frac{8(10^6)}{9(10^6)}(7265) = 6458 \text{ m/s}$$

$$\Delta v_B = 6458 - 8831 = -2374 \text{ m/s} = -2.37 \text{ km/s} \qquad \textbf{Ans.}$$

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***13–136.** A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right.}{86.4(10^3)} \qquad \textbf{(1)}$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

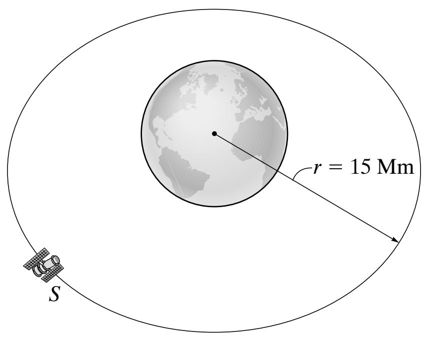
$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2),

$$h = 35.87(10^6)\text{ m} = 35.9\text{ Mm} \qquad v_S = 3072.32\text{ m/s} = 3.07\text{ km/s} \quad \textbf{Ans.}$$

•13–137. Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



$$F = G\frac{m_s m_e}{r^2} \qquad \text{Also} \qquad F = m_s\left(\frac{v_s^2}{r}\right) \qquad \text{Hence}$$

$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156\text{ m/s} = 5.16\text{ km/s} \qquad \textbf{Ans.}$$

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•14–1. A 1500-lb crate is pulled along the ground with a constant speed for a distance of 25 ft, using a cable that makes an angle of 15° with the horizontal. Determine the tension in the cable and the work done by this force. The coefficient of kinetic friction between the ground and the crate is $\mu_k = 0.55$.

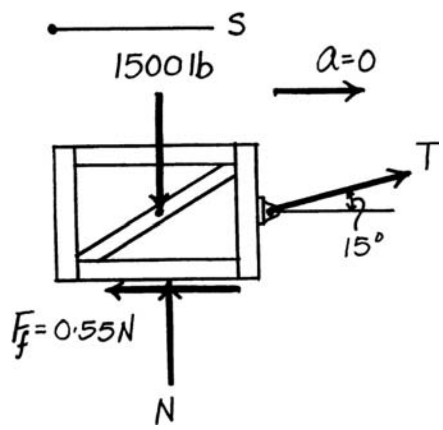
$\xrightarrow{+} \Sigma F_x = 0; \qquad T\cos 15° - 0.55N = 0$

$+\uparrow \Sigma F_y = 0; \qquad N + T\sin 15° - 1500 = 0$

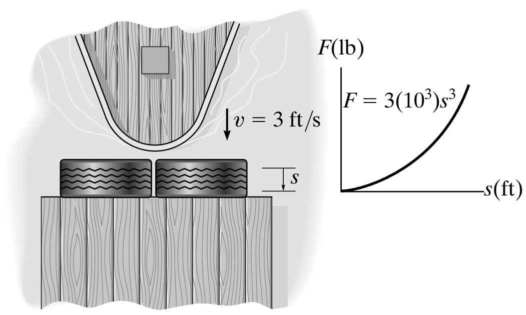
$N = 1307$ lb

$T = 744.4 \text{ lb} = 744 \text{ lb}$ **Ans.**

$U_T = (744.4 \cos 15°)(25) = 18.0(10^3) \text{ ft} \cdot \text{lb}$ **Ans.**



14–2. The motion of a 6500-lb boat is arrested using a bumper which provides a resistance as shown in the graph. Determine the maximum distance the boat dents the bumper if its approaching speed is 3 ft/s.



Principle of Work and Energy: Here, the bumper resisting force F does *negative* work since it acts in the opposite direction to that of displacement. Since the boat is required to stop, $T_2 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{6500}{32.2}\right)(3^2) + \left[-\int_0^s 3(10^3)\, s^3 ds\right] = 0$$

$$s = 1.05 \text{ ft}$$ **Ans.**

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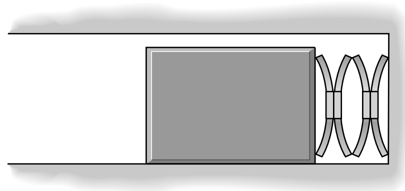
14–3. The smooth plug has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the plug is $F = (3s^{1/3})$ lb, where s is given in feet, determine the speed of the plug after it moves away from the spring. Neglect friction.

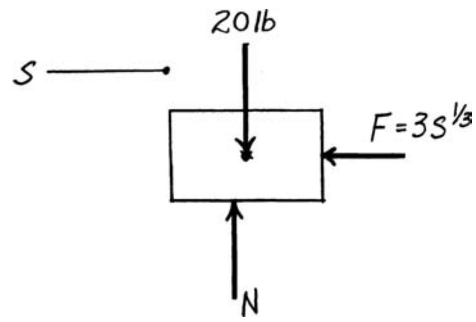


$$T_1 + \Sigma U_{1-2} = T_2$$

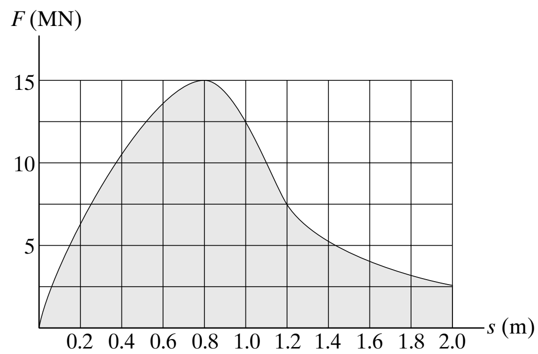
$$0 + \int_0^{0.05} 3s^{\frac{1}{3}}\,ds = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$$3\left(\frac{3}{4}\right)(0.05)^{\frac{4}{3}} = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

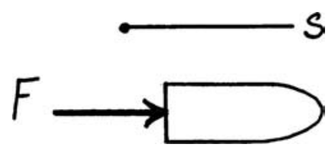
$v = 0.365$ ft/s **Ans.**



***14–4.** When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.



The work done is measured as the area under the force–displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5\left(10^6\right)(0.2)$,



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[(31.5)(2.5)\left(10^6\right)(0.2)\right] = \frac{1}{2}(7)(v_2)^2$$

$v_2 = 2121$ m/s $= 2.12$ km/s (approx.) **Ans.**

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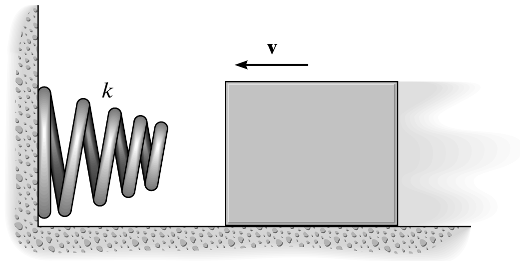
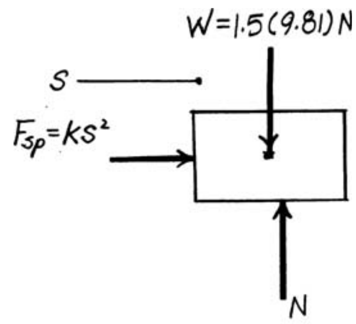
•14–5. The 1.5-kg block slides along a smooth plane and strikes a *nonlinear spring* with a speed of $v = 4$ m/s. The spring is termed "nonlinear" because it has a resistance of $F_s = ks^2$, where $k = 900\ \text{N/m}^2$. Determine the speed of the block after it has compressed the spring $s = 0.2$ m.



Principle of Work and Energy: The spring force F_{sp} which acts in the opposite direction to that of displacement does *negative* work. The normal reaction N and the weight of the block do not displace hence do no work. Applying Eq. 14–7, we have

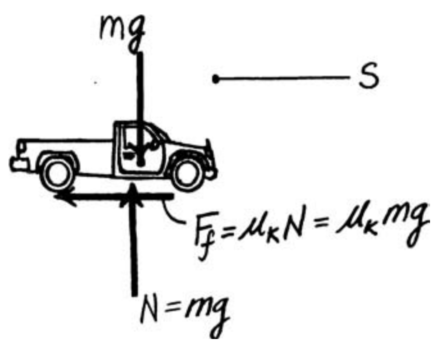


$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(1.5)\left(4^2\right) + \left[-\int_0^{0.2\text{ m}} 900 s^2\, ds\right] = \frac{1}{2}(1.5)\, v^2$$

$$v = 3.58 \text{ m/s} \qquad \textbf{Ans.}$$

14–6. When the driver applies the brakes of a light truck traveling 10 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



$$10 \text{ km/h} = \frac{10\left(10^3\right)}{3600} = 2.778 \text{ m/s} \qquad 80 \text{ km/h} = 22.22 \text{ m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2} m(2.778)^2 - \mu_k mg(3) = 0$$

$$\mu_k g = 1.286$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2} m(22.22)^2 - (1.286) m(d) = 0$$

$$d = 192 \text{ m} \qquad \textbf{Ans.}$$

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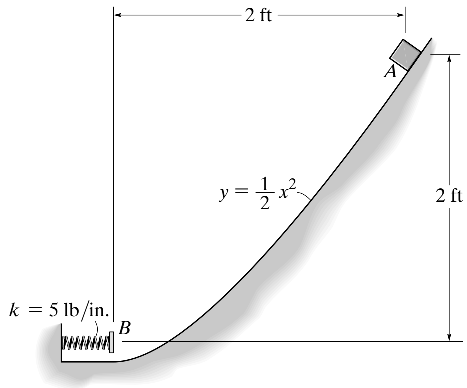
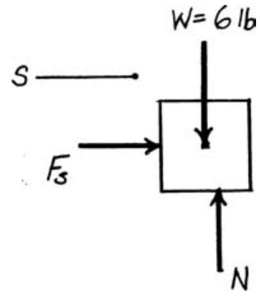
14–7. The 6-lb block is released from rest at A and slides down the smooth parabolic surface. Determine the maximum compression of the spring.



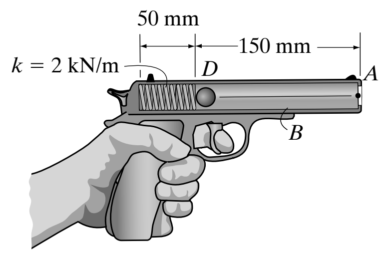
$T_1 + \Sigma U_{1-2} = T_2$

$0 + 2(6) - \frac{1}{2}(5)(12)s^2 = 0$

$s = 0.632 \text{ ft} = 7.59 \text{ in.}$ **Ans.**



***14–8.** The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



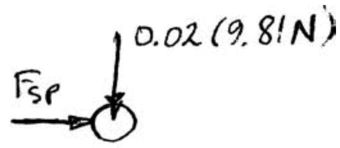
Principle of Work and Energy: Referring to the free-body diagram of the ball bearing shown in Fig. *a*, notice that $\mathbf{F}_{sp}$ does positive work. The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[\frac{1}{2}ks_1^{\,2} - \frac{1}{2}ks_2^{\,2}\right] = \frac{1}{2}mv_A^{\,2}$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2 - \frac{1}{2}(2000)(0.0375^2)\right] = \frac{1}{2}(0.02)v_A^{\,2}$$

$v_A = 10.5$ m/s **Ans.**



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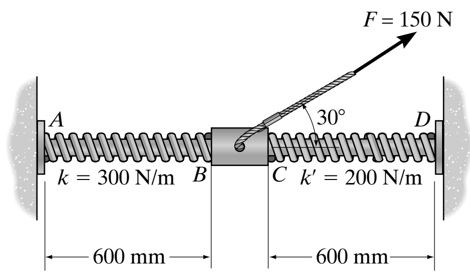
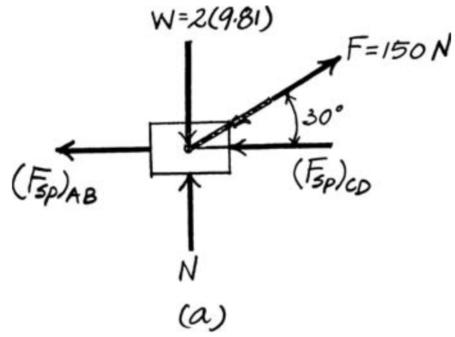
•14–9. Springs AB and CD have a stiffness of $k = 300$ N/m and $k' = 200$ N/m, respectively, and both springs have an unstretched length of 600 mm. If the 2-kg smooth collar starts from rest when the springs are unstretched, determine the speed of the collar when it has moved 200 mm.



Principle of Work and Energy: By referring to the free-body diagram of the collar, notice that **W**, **N**, and $F_y = 150 \sin 30°$ do no work. However, $F_x = 150 \cos 30°$ N does positive work and $(\mathbf{F}_{sp})_{AB}$ and $(\mathbf{F}_{sp})_{CD}$ do negative work.

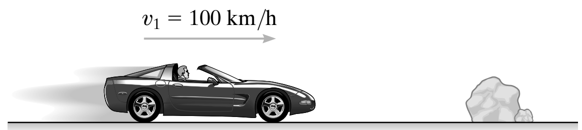


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150 \cos 30°(0.2) + \left[-\frac{1}{2}(300)(0.2^2)\right] + \left[-\frac{1}{2}(200)(0.2^2)\right] = \frac{1}{2}(2)v^2$$

$$v = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$

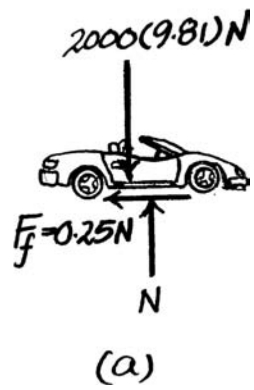
14–10. The 2-Mg car has a velocity of $v_1 = 100$ km/h when the driver sees an obstacle in front of the car. If it takes 0.75 s for him to react and lock the brakes, causing the car to skid, determine the distance the car travels before it stops. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.25$.



Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis. By referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620 \text{ N}$$

Since the car skids, the frictional force acting on the car is $F_f = \mu_k N = 0.25(19620) = 4905N$.



Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\frac{\text{m}}{\text{h}}\right]\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is denoted as s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + (-4905s') = 0$$

$$s' = 157.31 \text{ m}$$

The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

$$s = s' + s'' = 157.31 + 20.83 = 178.14 \text{ m} = 178 \text{ m} \qquad \textbf{Ans.}$$

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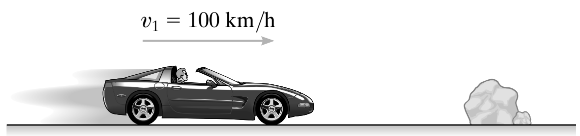
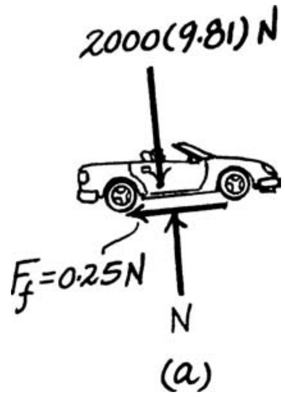
14–11. The 2-Mg car has a velocity of $v_1 = 100\,\text{km/h}$ when the driver sees an obstacle in front of the car. It takes 0.75 s for him to react and lock the brakes, causing the car to skid. If the car stops when it has traveled a distance of 175 m, determine the coefficient of kinetic friction between the tires and the road.



Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis and referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620\text{ N}$$

Since the car skids, the frictional force acting on the car can be computed from $F_f = \mu_k N = \mu_k(19\,620)$.



Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + \left[-\mu_k(19\,620)s'\right] = 0$$

$$s' = \frac{39.327}{\mu_k}$$

The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

$$s = s' + s''$$

$$175 = \frac{39.327}{\mu_k} + 20.83$$

$$\mu_k = 0.255 \qquad \textbf{Ans.}$$

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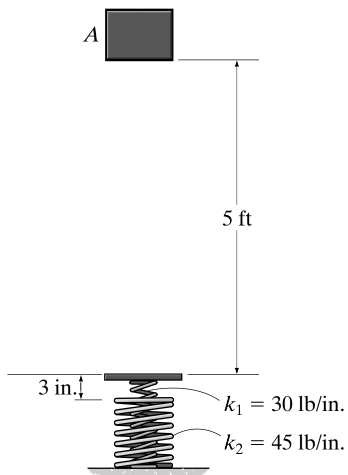
***14–12.** The 10-lb block is released from rest at A. Determine the compression of each of the springs after the block strikes the platform and is brought momentarily to rest. Initially both springs are unstretched. Assume the platform has a negligible mass.



Free-Body Diagram: The free-body diagram of the block in contact with both springs is shown in Fig. a. When the block is brought momentarily to rest, springs (1) and (2) are compressed by $s_1 = y$ and $s_2 = (y - 3)$, respectively.

Principle of Work and Energy: When the block is momentarily at rest, **W** which displaces downward $h = [5(12) + y]$in. $= (60 + y)$ in., does positive work, whereas $(\mathbf{F}_{sp})_1$ and $(\mathbf{F}_{sp})_2$ both do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 10(60 + y) + \left[-\frac{1}{2}(30)y^2\right] + \left[-\frac{1}{2}(45)(y - 3)^2\right] = 0$$

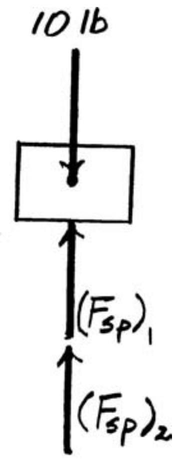
$$37.5y^2 - 145y - 397.5 = 0$$

Solving for the positive root of the above equation,

$$y = 5.720 \text{ in.}$$

Thus,

$$s_1 = 5.72 \text{ in.} \qquad s_2 = 5.720 - 3 = 2.72 \text{ in.} \qquad \textbf{Ans.}$$



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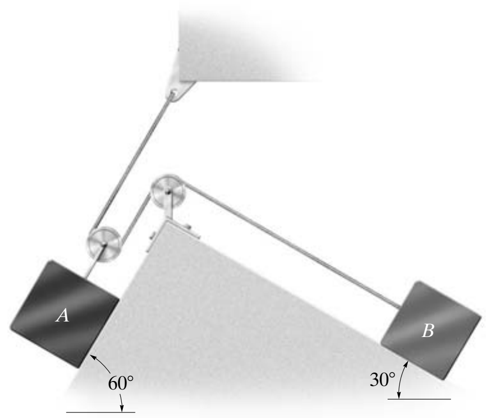
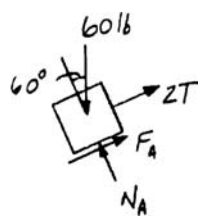
14–13. Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



Block A:

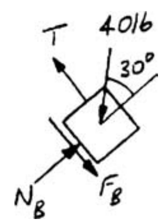


$+\nwarrow\Sigma F_y = ma_y;\qquad N_A - 60\cos 60° = 0$

$N_A = 30$ lb

$F_A = 0.1(30) = 3$ lb

Block B:



$+\nearrow\Sigma F_y = ma_y;\qquad N_B - 40\cos 30° = 0$

$N_B = 34.64$ lb

$F_B = 0.1(34.64) = 3.464$ lb

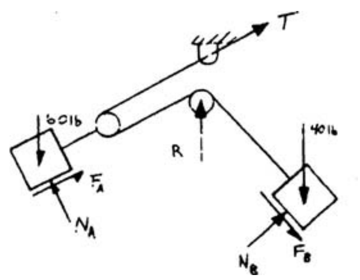
Use the system of both blocks. N_A, N_B, T, and R do no work.

$T_1 + \Sigma U_{1-2} = T_2$

$$(0+0) + 60\sin 60°|\Delta s_A| - 40\sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$



When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

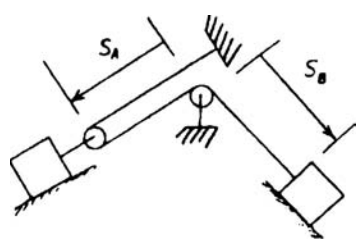
Also,

$2v_A = -v_B$

Substituting and solving,

$v_A = 0.771$ ft/s **Ans.**

$v_B = -1.54$ ft/s



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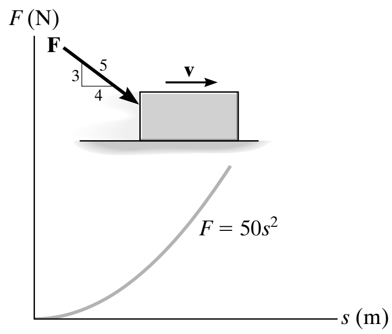
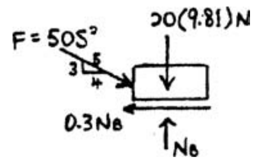
14–14. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block slides before its velocity becomes 5 m/s. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$+\uparrow \Sigma F_y = 0; \qquad N_B - 20(9.81) - \frac{3}{5}(50\, s^2) = 0$

$$N_B = 196.2 + 30\, s^2$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^s 50\, s^2\, ds - 0.3(196.2)(s) - 0.3\int_0^s 30\, s^2\, ds = \frac{1}{2}(20)(5)^2$$

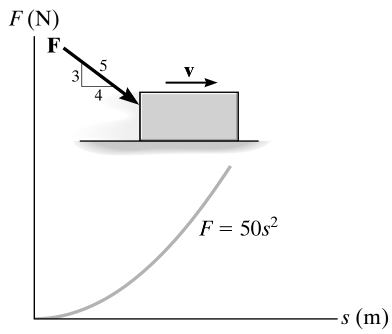
$$40 + 13.33\, s^3 - 58.86\, s - 3\, s^3 = 250$$

$$s^3 - 5.6961\, s - 20.323 = 0$$

Solving for the real root yields

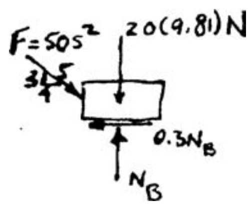
$s = 3.41$ m **Ans.**

14–15. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with position s of the block. Determine the speed of the block after it slides 3 m. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$+\uparrow \Sigma F_y = 0; \qquad N_B - 20(9.81) - \frac{3}{5}(50\, s^2) = 0$

$$N_B = 196.2 + 30\, s^2$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^3 50\, s^2\, ds - 0.3(196.2)(3) - 0.3\int_0^3 30\, s^2\, ds = \frac{1}{2}(20)(v)^2$$

$$40 + 360 - 176.58 - 81 = 10\, v^2$$

$v = 3.77$ m/s **Ans.**

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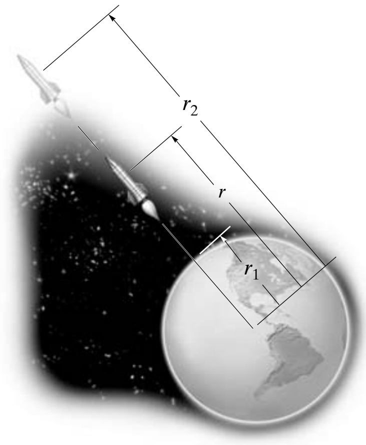
14–16. A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.

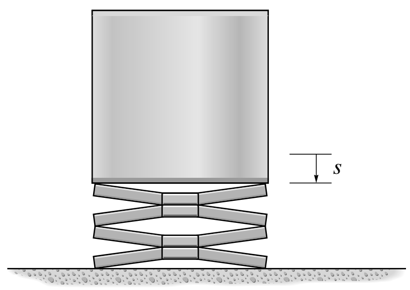


$$F = G\frac{M_e m}{r^2}$$

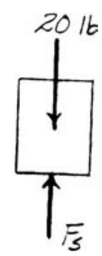
$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \textbf{Ans.}$$

•**14–17.** The cylinder has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the cylinder is $F = (100s^{1/3})$ lb, where s is given in feet, determine the speed of the cylinder just after it moves away from the spring, i.e., at $s = 0$.



Principle of Work and Energy: The spring force which acts in the direction of displacement does *positive* work, whereas the weight of the block does *negative* work since it acts in the opposite direction to that of displacement. Since the block is initially at rest, $T_1 = 0$. Applying Eq. 14–7, we have



$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \int_0^{0.05\text{ ft}} 100s^{1/3}\,ds - 20(0.05) = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$$v = 1.11\text{ ft/s} \qquad \textbf{Ans.}$$

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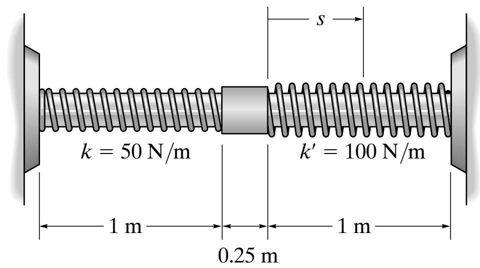
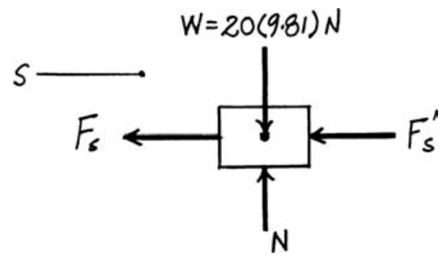
14–18. The collar has a mass of 20 kg and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length of 1 m. If the collar is displaced $s = 0.5$ m and released from rest, determine its velocity at the instant it returns to the point $s = 0$.



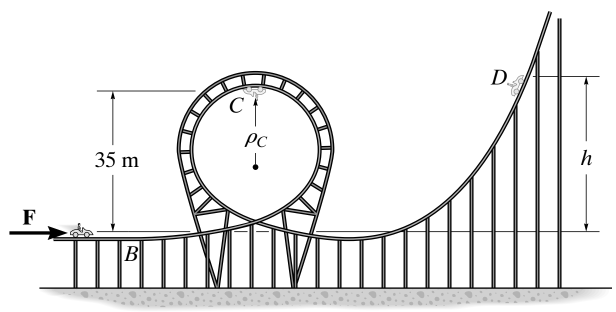
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \frac{1}{2}(50)(0.5)^2 + \frac{1}{2}(100)(0.5)^2 = \frac{1}{2}(20)v_C^2$$

$$v_C = 1.37 \text{ m/s} \qquad \textbf{Ans.}$$

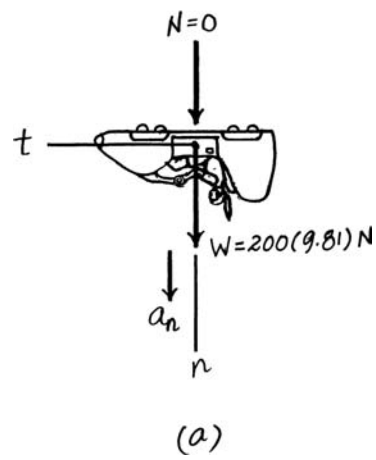


14–19. Determine the height h of the incline D to which the 200-kg roller coaster car will reach, if it is launched at B with a speed just sufficient for it to round the top of the loop at C without leaving the track. The radius of curvature at C is $\rho_c = 25$ m.



Equations of Motion: Here, it is required that $N = 0$. Applying Eq. 13–8 to FBD(a), we have

$$\Sigma F_n = ma_n; \qquad 200(9.81) = 200\left(\frac{v_C^2}{25}\right) \qquad v_C^2 = 245.25 \text{ m}^2/\text{s}^2$$



Principle of Work and Energy: The weight of the roller coaster car and passengers do *negative* work since they act in the opposite direction to that of displacement. When the roller coaster car travels from B to C, applying Eq. 14–7, we have

$$T_B + \sum U_{B-C} = T_C$$

$$\frac{1}{2}(200)\,v_B^2 - 200(9.81)\,(35) = \frac{1}{2}(200)(245.25)$$

$$v_B = 30.53 \text{ m/s}$$

When the roller coaster car travels from B to D, it is required that the car stops at D, hence $T_D = 0$.

$$T_B + \sum U_{B-D} = T_D$$

$$\frac{1}{2}(200)\left(30.53^2\right) - 200(9.81)(h) = 0$$

$$h = 47.5 \text{ m} \qquad \textbf{Ans.}$$

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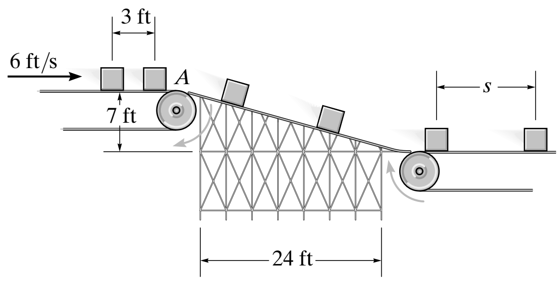
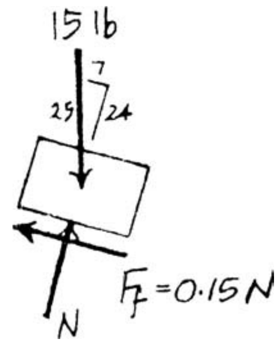
***14–20.** Packages having a weight of 15 lb are transferred horizontally from one conveyor to the next using a ramp for which $\mu_k = 0.15$. The top conveyor is moving at 6 ft/s and the packages are spaced 3 ft apart. Determine the required speed of the bottom conveyor so no sliding occurs when the packages come horizontally in contact with it. What is the spacing s between the packages on the bottom conveyor?



Equations of Motion:

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 15\left(\frac{24}{25}\right) = \frac{15}{32.2}(0) \qquad N = 14.4 \text{ lb}$$



Principle of Work and Energy: Only force components parallel to the inclined plane which are in the direction of displacement [15(7/25) lb and $F_f = \mu_k N = 0.15(14.4) = 2.16$ lb] do work, whereas the force components perpendicular to the inclined plane [15(24/25) lb and normal reaction N] do no work since no displacement occurs in this direction. Here, the 15(7/25) lb force does *positive* work and $F_f = 2.16$ lb does *negative* work. Slipping at the contact surface between the package and the belt will not occur if the speed of belt is the same as the speed of the package at B. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

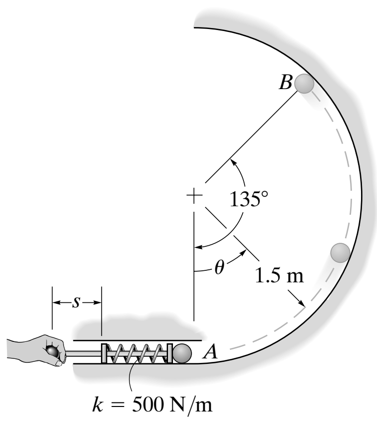
$$\frac{1}{2}\left(\frac{15}{32.2}\right)(6^2) + 15\left(\frac{7}{25}\right)(25) - 2.16(25) = \frac{1}{2}\left(\frac{15}{32.2}\right)v^2$$

$$v = 15.97 \text{ ft/s} = 16.0 \text{ ft/s} \qquad \textbf{Ans.}$$

The time between two succesive packages to reach point B is $t = \frac{3}{6} = 0.5$ s. Hence, the distance between two succesive packages on the lower belt is

$$s = vt = 15.97(0.5) = 7.98 \text{ ft} \qquad \textbf{Ans.}$$

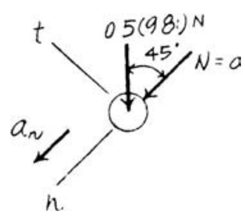
•14–21. The 0.5-kg ball of negligible size is fired up the smooth vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.



Equations of Motion:

$$\Sigma F_n = ma_n; \qquad 0.5(9.81)\cos 45° = 0.5\left(\frac{v_B^2}{1.5}\right) \qquad v_B^2 = 10.41 \text{ m}^2/\text{s}^2$$

Principle of Work and Energy: Here, the weight of the ball is being displaced vertically by $s = 1.5 + 1.5\sin 45° = 2.561$ m and so it does *negative* work. The spring force, given by $F_{sp} = 500(s + 0.08)$, does positive work. Since the ball is at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have



$$T_A + \sum U_{A-B} = T_B$$

$$0 + \int_0^s 500(s + 0.08)\,ds - 0.5(9.81)(2.561) = \frac{1}{2}(0.5)(10.41)$$

$$s = 0.1789 \text{ m} = 179 \text{ mm} \qquad \textbf{Ans.}$$

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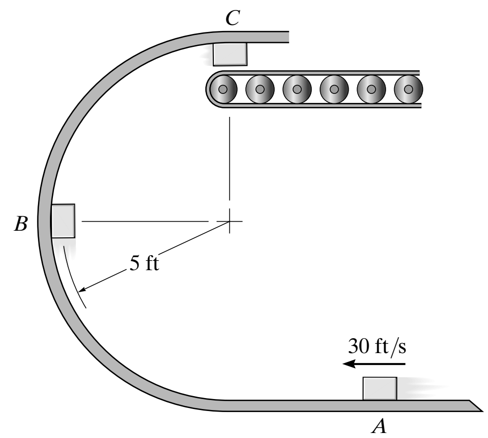
14–22. The 2-lb box slides on the smooth circular ramp. If the box has a velocity of 30 ft/s at A, determine the velocity of the box and normal force acting on the ramp when the box is located at B and C. Assume the radius of curvature of the path at C is still 5 ft.



Point B:

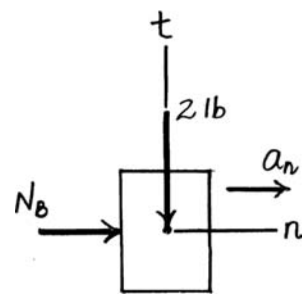
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 - 2(5) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)^2$$

$v_B = 24.0$ ft/s **Ans.**

$\rightarrow \Sigma F_n = ma_n;\qquad N_B = \left(\frac{2}{32.2}\right)\left(\frac{(24.0)^2}{5}\right)$

$N_B = 7.18$ lb **Ans.**



Point C:

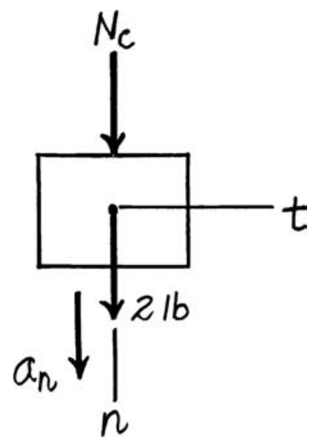
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 - 2(10) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2$$

$v_C = 16.0$ ft/s **Ans.**

$+\downarrow \Sigma F_n = ma_n;\qquad N_C + 2 = \left(\frac{2}{32.2}\right)\left(\frac{(16.0)^2}{5}\right)$

$N_C = 1.18$ lb **Ans.**



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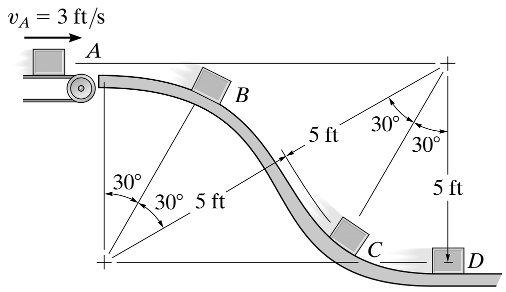
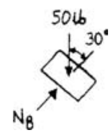
14–23. Packages having a weight of 50 lb are delivered to the chute at $v_A = 3$ ft/s using a conveyor belt. Determine their speeds when they reach points B, C, and D. Also calculate the normal force of the chute on the packages at B and C. Neglect friction and the size of the packages.



$T_A + \Sigma U_{A-B} = T_B$

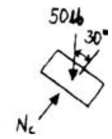
$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5)(1 - \cos 30^\circ) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_B^2$$

$v_B = 7.221 = 7.22$ ft/s **Ans.**



$$+\swarrow\Sigma F_n = ma_n; \qquad -N_B + 50\cos 30^\circ = \left(\frac{50}{32.2}\right)\left[\frac{(7.221)^2}{5}\right]$$

$N_B = 27.1$ lb **Ans.**



$T_A + \Sigma U_{A-C} = T_C$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5\cos 30^\circ) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_C^2$$

$v_C = 16.97 = 17.0$ ft/s **Ans.**

$$+\nearrow\Sigma F_n = ma_n; \qquad N_C - 50\cos 30^\circ = \left(\frac{50}{32.2}\right)\left[\frac{(16.97)^2}{5}\right]$$

$N_C = 133$ lb **Ans.**

$T_A + \Sigma U_{A-D} = T_D$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_D^2$$

$v_D = 18.2$ ft/s **Ans.**

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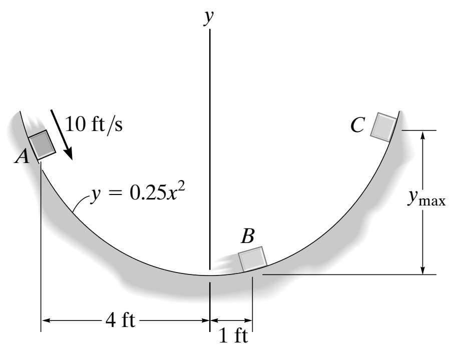
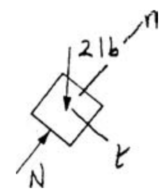
***14–24.** The 2-lb block slides down the smooth parabolic surface, such that when it is at A it has a speed of 10 ft/s. Determine the magnitude of the block's velocity and acceleration when it reaches point B, and the maximum height $y_{\max}$ reached by the block.



$$y = 0.25x^2$$

$$y_A = 0.25(-4)^2 = 4\text{ ft}$$

$$y_B = 0.25(1)^2 = 0.25\text{ ft}$$



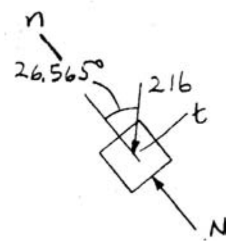
$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(10)^2 + 2(4-0.25) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$$v_B = 18.48\text{ ft/s} = 18.5\text{ ft/s} \qquad \textbf{Ans.}$$

$$\frac{dy}{dx} = \tan\theta = 0.5x\bigg|_{x=1} = 0.5 \qquad \theta = 26.565^\circ$$

$$\frac{d^2y}{dx^2} = 0.5$$



$$+\nearrow\Sigma F_t = ma_t; \qquad -2\sin 26.565^\circ = \left(\frac{2}{32.2}\right)a_t$$

$$a_t = -14.4\text{ ft/s}^2$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (0.5)^2\right]^{\frac{3}{2}}}{|0.5|} = 2.795\text{ ft}$$

$$a_n = \frac{v_B^2}{\rho} = \frac{(18.48)^2}{2.795} = 122.2\text{ ft/s}^2$$

$$a_B = \sqrt{(-14.4)^2 + (122.2)^2} = 123\text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(10)^2 - 2(v_{\max} - 4) = 0 \qquad y_{\max} = 5.55\text{ ft} \qquad \textbf{Ans.}$$

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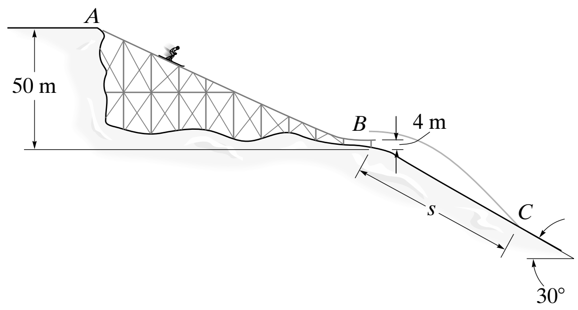
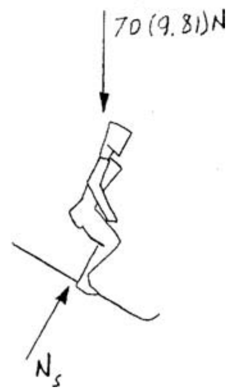
•14–25. The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$$

$$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$$ **Ans.**



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$s \cos 30° = 0 + 30.04t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s \sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$$

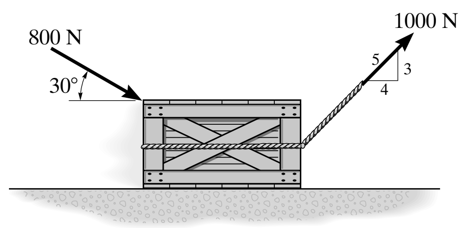
Eliminating t,

$$s^2 - 122.67s - 981.33 = 0$$

Solving for the positive root

$$s = 130 \text{ m}$$ **Ans.**

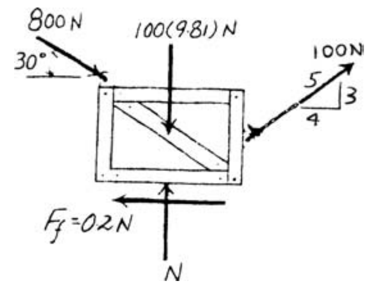
14–26. The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30° - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$



Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800 \cos 30°(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$$s = 1.35\text{m}$$ **Ans.**

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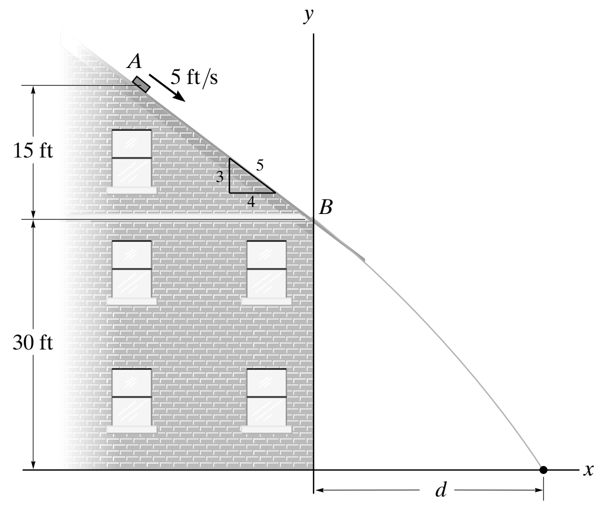
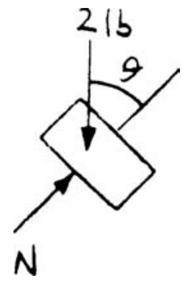
14–27. The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



$T_A + \Sigma U_{A-B} = T_B$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s}$ **Ans.**



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t - \frac{1}{2}a_c t^2$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$

$16.1t^2 + 18.888t - 30 = 0$

Solving for the positive root,

$t = 0.89916 \text{ s}$

$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft}$ **Ans.**

$T_A + \Sigma U_{A-C} = T_C$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$v_C = 54.1 \text{ ft/s}$ **Ans.**

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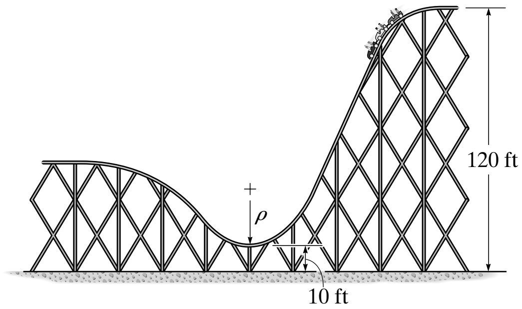
***14–28.** Roller coasters are designed so that riders will not experience a normal force that is more than 3.5 times their weight against the seat of the car. Determine the smallest radius of curvature ρ of the track at its lowest point if the car has a speed of 5 ft/s at the crest of the drop. Neglect friction.



Principle of Work and Energy: Here, the rider is being displaced vertically (downward) by $s = 120 - 10 = 110$ ft and does *positive* work. Applying Eq. 14–7 we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{W}{32.2}\right)(5^2) + W(110) = \frac{1}{2}\left(\frac{W}{32.2}\right)v^2$$

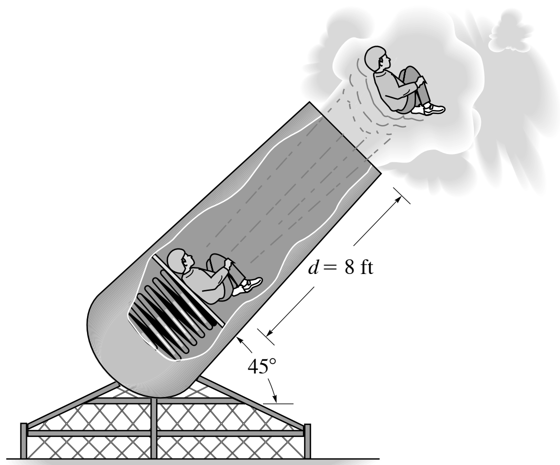
$$v^2 = 7109 \text{ ft}^2/\text{s}^2$$

Equations of Motion: It is required that $N = 3.5W$. Applying Eq. 13–7, we have

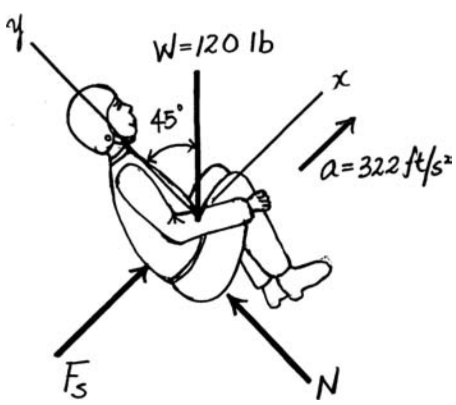
$$\Sigma F_n = ma_n; \qquad 3.5W - W = \left(\frac{W}{32.2}\right)\left(\frac{7109}{\rho}\right)$$

$$\rho = 88.3 \text{ ft} \qquad \textbf{Ans.}$$

•14–29. The 120-lb man acts as a human cannonball by being "fired" from the spring-loaded cannon shown. If the greatest acceleration he can experience is $a = 10g = 322 \text{ ft/s}^2$, determine the required stiffness of the spring which is compressed 2 ft at the moment of firing. With what velocity will he exit the cannon barrel, $d = 8$ ft, when the cannon is fired? When the spring is compressed $s = 2$ ft then $d = 8$ ft. Neglect friction and assume the man holds himself in a rigid position throughout the motion.



Initial acceleration is $10g = 322 \text{ ft/s}^2$



$$+\nearrow\Sigma F_x = ma_x; \qquad F_s - 120\sin 45° = \left(\frac{120}{32.2}\right)(322), \qquad F_s = 1284.85 \text{ lb}$$

For $s = 2$ ft; $\quad 1284\ 85 = k(2) \qquad k = 642.4 = 642 \text{ lb/ft}$ **Ans.**

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[\frac{1}{2}(642.2)(2)^2 - 120(8)\sin 45°\right] = \frac{1}{2}\left(\frac{120}{32.2}\right)v_2^2$$

$v_2 = 18.0$ ft/s **Ans.**

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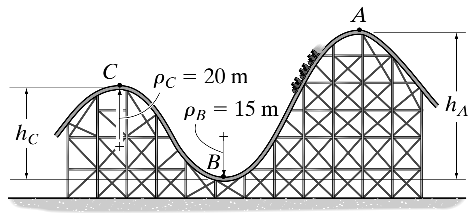
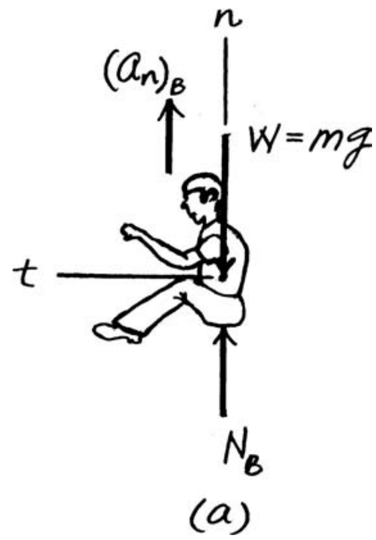
14–30. If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.



Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B{}^2}{15}\right)$$

$$v_B{}^2 = 45g$$

At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C{}^2}{20}\right)$$

$$v_C{}^2 = 20g$$

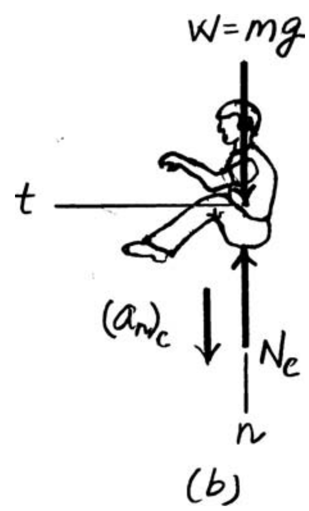
Principle of Work and Energy: The normal reaction **N** does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, **W** displaces vertically downward $h = h_A$ and does positive work.

We have

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$



When the rollercoaster moves from position A to C, **W** displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

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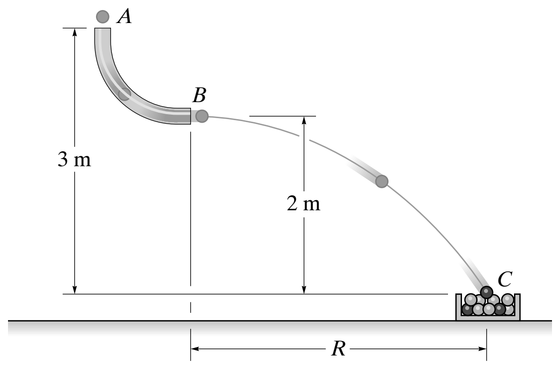
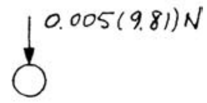
14–31. Marbles having a mass of 5 g fall from rest at A through the glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.



$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$

$$v_B = 4.429 \text{ m/s}$$



$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t$$

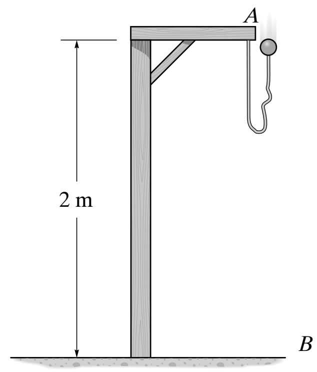
$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_1$$

$$0 + [0.005(9.81)(3) = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

***14–32.** The ball has a mass of 0.5 kg and is suspended from a rubber band having an unstretched length of 1 m and a stiffness $k = 50$ N/m. If the support at A to which the rubber band is attached is 2 m from the floor, determine the greatest speed the ball can have at A so that it does not touch the floor when it reaches its lowest point B. Neglect the size of the ball and the mass of the rubber band.



Principle of Work and Energy: The weight of the ball, which acts in the direction of displacement, does *positive* work, whereas the force in the rubber band does *negative* work since it acts in the opposite direction to that of displacement. Here it is required that the ball displace 2 m downward and stop, hence $T_2 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(0.5)v^2 + 0.5(9.81)(2) - \frac{1}{2}(50)(2 - 1)^2 = 0$$

$$v = 7.79 \text{ m/s}$$ **Ans.**

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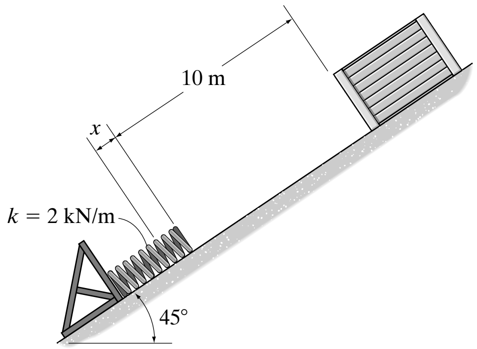
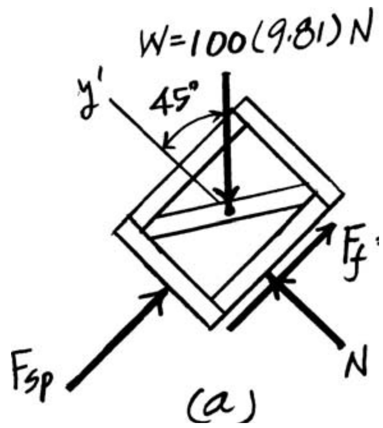
•14–33. If the coefficient of kinetic friction between the 100-kg crate and the plane is $\mu_k = 0.25$, determine the compression x of the spring required to bring the crate momentarily to rest. Initially the spring is unstretched and the crate is at rest.



Free-Body Diagram: The normal reaction **N** on the crate can be determined by writing the equation of motion along the y' axis and referring to the free-body diagram of the crate when it is in contact with the spring, Fig. *a*.

$\nwarrow + F_{y'} = ma_{y'}$; $\quad N - 100(9.81)\cos 45° = 100(0) \quad N = 693.67 \text{ N}$

Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.25(693.67) \text{ N} = 173.42 \text{ N}$.



Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work. Here, **W** which displaces downward through a distance of $h = (10 + x)\sin 45°$ does positive work, whereas $\mathbf{F}_f$ and $\mathbf{F}_{sp}$ do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

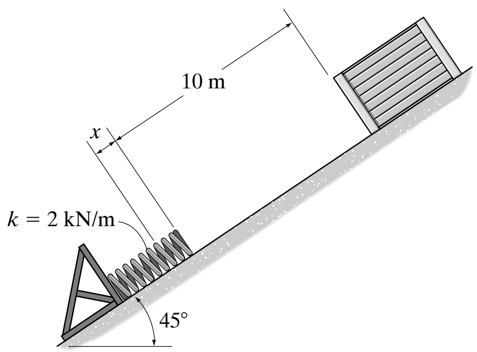
$$0 + 100(9.81)\left[(10 + x)\sin 45°\right] + \left[-173.42(10 + x)\right] + \left[-\frac{1}{2}(2000)x^2\right] = 0$$

$$1000x^2 - 520.25x - 5202.54 = 0$$

Solving for the positive root

$$x = 2.556 \text{ m} = 2.57 \text{ m}$$ **Ans.**

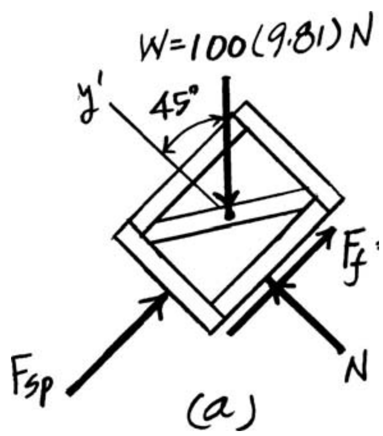
14–34. If the coefficient of kinetic friction between the 100-kg crate and the plane is $\mu_k = 0.25$, determine the speed of the crate at the instant the compression of the spring is $x = 1.5$ m. Initially the spring is unstretched and the crate is at rest.



Free-Body Diagram: The normal reaction **N** on the crate can be determined by writing the equation of motion along the y' axis and referring to the free-body diagram of the crate when it is in contact with the spring, Fig. *a*.

$\nwarrow + F_{y'} = ma_{y'}$; $\quad N - 100(9.81)\cos 45° = 100(0) \quad N = 693.67 \text{ N}$

Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.25(693.67) \text{ N} = 173.42 \text{ N}$. The force developed in the spring is $F_{sp} = kx = 2000x$.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N** does no work. Here, **W** which displaces downward through a distance of $h = (10 + 1.5)\sin 45° = 8.132$ m does positive work, whereas $\mathbf{F}_f$ and $\mathbf{F}_{sp}$ do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 100(9.81)(8.132) + \left[-173.42(10 + 1.5)\right] + \left[-\frac{1}{2}(2000)(1.5^2)\right] = \frac{1}{2}(100)v^2$$

$$v = 8.64 \text{m/s}$$ **Ans.**

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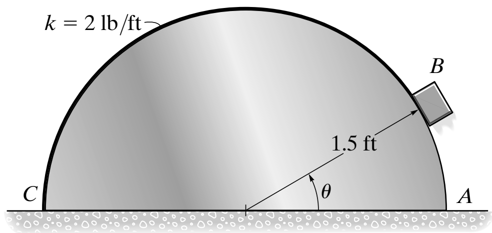
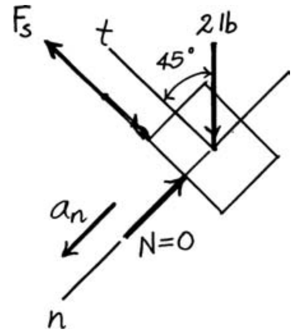
14–35. A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.





$$+\swarrow \Sigma F_n = ma_n; \qquad 2 \sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$$

$$v = 5.844 \text{ ft/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5 \sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft **Ans.**

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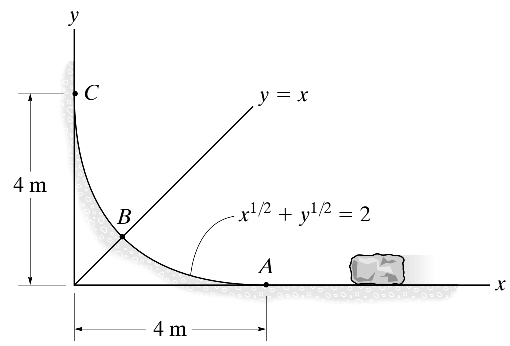
***14–36.** The 50-kg stone has a speed of $v_A = 8$ m/s when it reaches point A. Determine the normal force it exerts on the incline when it reaches point B. Neglect friction and the stone's size.

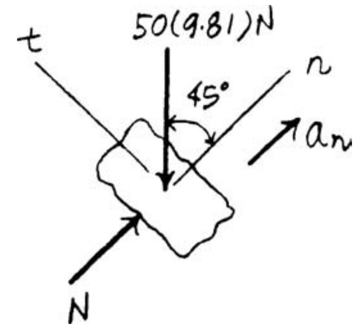


Geometry: Here, $x^{1/2} + y^{1/2} = 2$. At point B, $y = x$, hence $2x^{1/2} = 2$ and $x = y = 1$ m.

$$y^{-1/2}\frac{dy}{dx} = -x^{-1/2} \qquad \frac{dy}{dx} = \left.\frac{x^{-1/2}}{y^{-1/2}}\right|_{x=1\text{ m},\, y=1\text{ m}} = -1$$

$$y^{-1/2}\frac{d^2y}{dx^2} + \left(-\frac{1}{2}\right)y^{-3/2}\left(\frac{dy}{dx}\right)^2 = -\left(-\frac{1}{2}x^{-3/2}\right)$$

$$\frac{d^2y}{dx^2} = y^{1/2}\left[\frac{1}{2y^{3/2}}\left(\frac{dy}{dx}\right)^2 + \frac{1}{2x^{3/2}}\right]\Bigg|_{x=1\text{ m},\, y=1\text{ m}} = 1$$



The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=1\text{ m},\, y=1\text{ m}} = -1 \qquad \theta = -45.0^\circ$$

and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left|\frac{\left[1 + (-1)^2\right]^{3/2}}{|1|}\right| = 2.828\text{ m}$$

Principle of Work and Energy: The weight of the block which acts in the opposite direction to that of the vertical displacement does *negative* work when the block displaces 1 m vertically. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-B} = T_B$$

$$\frac{1}{2}(50)\left(8^2\right) - 50(9.81)(1) = \frac{1}{2}(50)\,v_B^2$$

$$v_B^2 = 44.38\text{ m}^2/\text{s}^2$$

Equations of Motion: Applying Eq. 13–8 with $\theta = 45.0^\circ$, $v_B^2 = 44.38\text{ m}^2/\text{s}^2$ and $\rho = 2.828$ m, we have

$$+\nearrow\Sigma F_n = ma_n; \qquad N - 50(9.81)\cos 45^\circ = 50\left(\frac{44.38}{2.828}\right)$$

$$N = 1131.37\text{ N} = 1.13\text{ kN} \qquad \textbf{Ans.}$$

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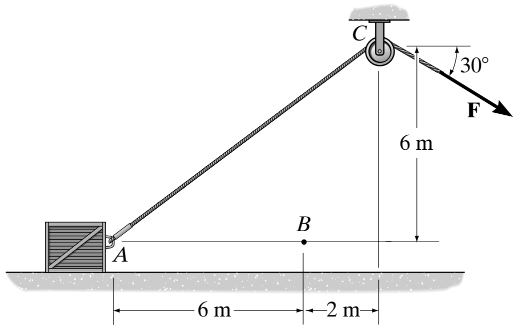
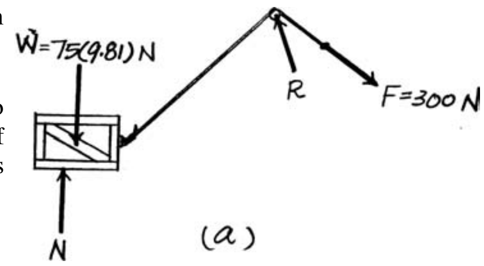
•14–37. If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.



Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



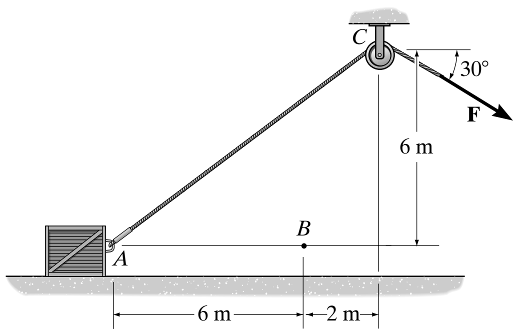
Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

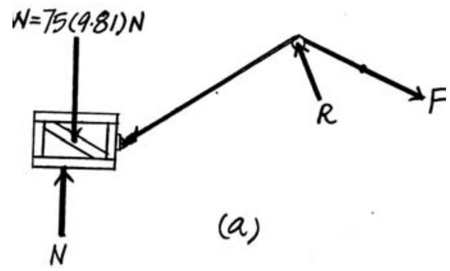
$$0 + 300(3.675) = \frac{1}{2}(75)v_B{}^2$$

$$v_B = 5.42 \text{ m/s}$$ **Ans.**

14–38. If the 75-kg crate starts from rest at A, and its speed is 6 m/s when it passes point B, determine the constant force **F** exerted on the cable. Neglect friction and the size of the pulley.



Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + F(3.675) = \frac{1}{2}(75)(6^2)$$

$$F = 367 \text{ N}$$ **Ans.**

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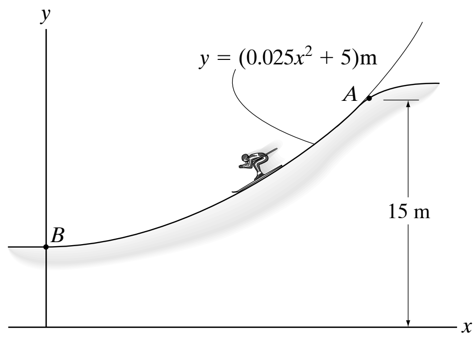
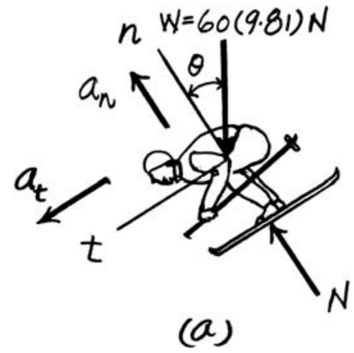
14–39. If the 60-kg skier passes point A with a speed of $5\,\mathrm{m/s}$, determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



Free-Body Diagram: The free-body diagram of the skier at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 15 - \left[0.025\left(0^2\right) + 5\right] = 10\text{ m}$ and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(60)(5^2) + \left[60(9.81)(10)\right] = \frac{1}{2}(60)v_B{}^2$$

$$v_B = 14.87\text{ m/s} = 14.9\text{ m/s} \qquad \textbf{Ans.}$$

$$dy/dx = 0.05x$$

$$d^2y/dx^2 = 0.05$$

$$\rho = \frac{[1 + 0]^{3/2}}{0.5} = 20\text{ m}$$

$$+\uparrow \Sigma F_n = ma_n; \qquad N - 60(9.81) = 60\left(\frac{(14.87)^2}{20}\right)$$

$$N = 1.25\text{ kN} \qquad \textbf{Ans.}$$

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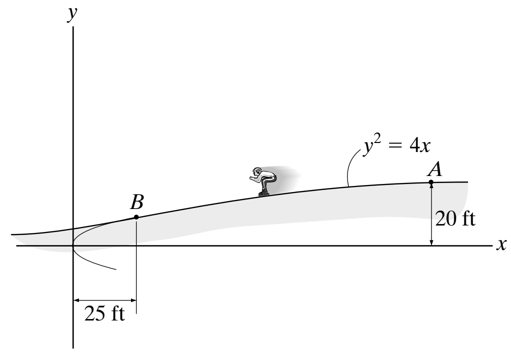
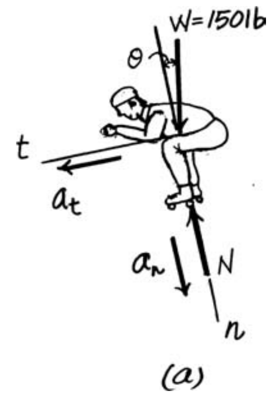
***14–40.** The 150-lb skater passes point A with a speed of 6 ft/s. Determine his speed when he reaches point B and the normal force exerted on him by the track at this point. Neglect friction.



Free-Body Diagram: The free-body diagram of the skater at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 20 - \left[2(25)^{1/2}\right] = 10$ ft and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)\left(6^2\right) + \left[150(10)\right] = \frac{1}{2}\left(\frac{150}{32.2}\right)v_B{}^2$$

$$v_B = 26.08 \text{ ft/s} = 26.1 \text{ ft/s} \qquad \textbf{Ans.}$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. *a*,

$$\searrow + \Sigma F_n = ma_n; \qquad 150\cos\theta - N = \frac{150}{32.2}\left(\frac{v^2}{\rho}\right)$$

$$N = 150\cos\theta - \frac{150}{32.2}\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

Geometry: Here, $y = 2x^{1/2}$, $\dfrac{dy}{dx} = \dfrac{1}{x^{1/2}}$, and $\dfrac{d^2y}{dx^2} = -\dfrac{1}{2x^{3/2}}$. The slope that the track at position B makes with the horizontal is $\theta_B = \tan^{-1}\left(\dfrac{dx}{dy}\right)\Big|_{x=25\text{ ft}}$ $= \tan\left(\dfrac{1}{x^{1/2}}\right)\Big|_{x=25\text{ ft}} = 11.31°$. The radius of curvature of the track at position B is given by

$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\dfrac{1}{x^{1/2}}\right)^2\right]^{3/2}}{\left|-\dfrac{1}{2x^{3/2}}\right|}\right|_{x=25\text{ ft}} = 265.15 \text{ ft}$$

Substituting $\theta = \theta_B = 11.31°$, $v = v_B = 26.08$ ft/s, and $\rho = \rho_B = 265.15$ ft into Eq. (1),

$$N_B = 150\cos 11.31° - \frac{150}{32.2}\left(\frac{26.08^2}{265.15}\right)$$

$$= 135 \text{ lb} \qquad \textbf{Ans.}$$

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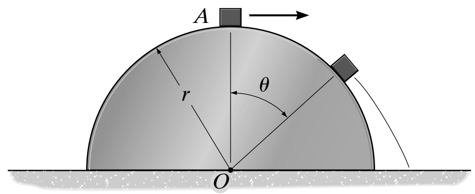
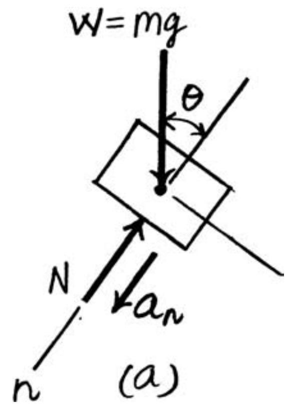
•14–41. A small box of mass m is given a speed of $v = \sqrt{\frac{1}{4}gr}$ at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.



Principle of Work and Energy: By referring to the free-body diagram of the block, Fig. a, notice that **N** does no work, while **W** does positive work since it displaces downward though a distance of $h = r - r\cos\theta$.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mg(r - r\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = gr\left(\frac{9}{4} - 2\cos\theta\right) \tag{1}$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{gr\left(\frac{9}{4} - 2\cos\theta\right)}{r} = g\left(\frac{9}{4} - 2\cos\theta\right)$. By referring to Fig. a,

$$\Sigma F_n = ma_n; \qquad mg\cos\theta - N = m\left[g\left(\frac{9}{4} - 2\cos\theta\right)\right]$$

$$N = mg\left(3\cos\theta - \frac{9}{4}\right)$$

It is required that the block leave the track. Thus, $N = 0$.

$$0 = mg\left(3\cos\theta - \frac{9}{4}\right)$$

Since $mg \neq 0$,

$$3\cos\theta - \frac{9}{4} = 0$$

$$\theta = 41.41° = 41.4° \qquad \textbf{Ans.}$$

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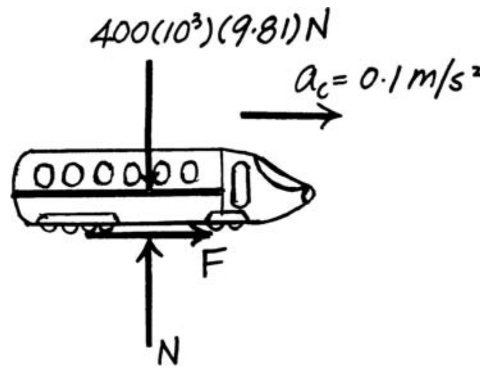
14–42. The diesel engine of a 400-Mg train increases the train's speed uniformly from rest to 10 m/s in 100 s along a horizontal track. Determine the average power developed.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + U_{1-2} = \frac{1}{2}(400)\left(10^3\right)(10^2)$$

$$U_{1-2} = 20\left(10^6\right)\text{ J}$$

$$P_{\text{avg}} = \frac{U_{1-2}}{t} = \frac{20(10^6)}{100} = 200\text{ kW}$$ **Ans.**

Also,

$$v = v_0 + a_c t$$

$$10 = 0 + a_c(100)$$

$$a_c = 0.1\text{ m/s}^2$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F = 400\left(10^3\right)(0.1) = 40\left(10^3\right)\text{ N}$$

$$P_{\text{avg}} = \mathbf{F}\cdot\mathbf{v}_{\text{avg}} = 40\left(10^3\right)\left(\frac{10}{2}\right) = 200\text{ kW}$$ **Ans.**

14–43. Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

Power: The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 300(5) = 1500\text{ ft}\cdot\text{lb/s}$$

Using Eq. 14–11, the required power input for the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{1500}{0.65} = 2307.7\text{ ft}\cdot\text{lb/s} = 4.20\text{ hp}$$ **Ans.**

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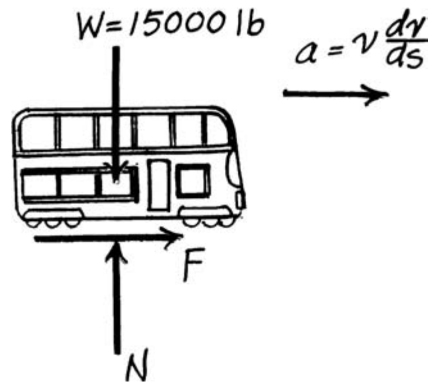
***14–44.** An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest so that the power is always 100 hp. Determine how far it must travel to reach a speed of 40 ft/s.



$$F = ma = \frac{W}{g}\left(\frac{v\,dv}{ds}\right)$$

$$P = Fv = \left[\left(\frac{W}{g}\right)\left(\frac{v\,dv}{ds}\right)\right]v$$

$$\int_0^s P\,ds = \int_0^v \frac{W}{g}v^2\,dv$$

P = constant

$$Ps = \frac{W}{g}\left(\frac{1}{3}\right)v^3 \qquad s = \frac{W}{3gP}v^3$$

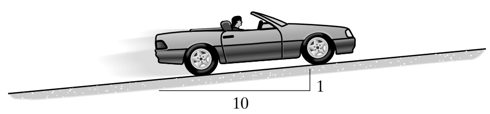
$$s = \frac{(15\,000)(40)^3}{3(32.2)(100)(550)} = 181 \text{ ft}$$ **Ans.**

•14–45. The Milkin Aircraft Co. manufactures a turbojet engine that is placed in a plane having a weight of 13000 lb. If the engine develops a constant thrust of 5200 lb, determine the power output of the plane when it is just ready to take off with a speed of 600 mi/h.

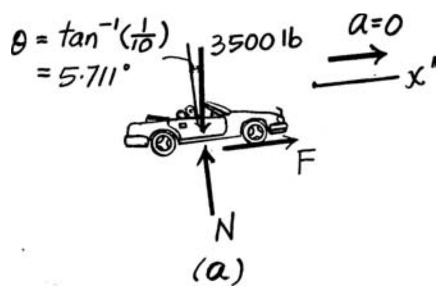
At 600 ms/h.

$$P = 5200(600)\left(\frac{88 \text{ ft/s}}{60 \text{ m/h}}\right)\frac{1}{550} = 8.32\,(10^3) \text{ hp}$$ **Ans.**

14–46. The engine of the 3500-lb car is generating a constant power of 50 hp while the car is traveling up the slope with a constant speed. If the engine is operating with an efficiency of $\epsilon = 0.8$, determine the speed of the car. Neglect drag and rolling resistance.



Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,



$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad F - 3500\sin 5.711° = \frac{3500}{32.2}(0) \qquad F = 348.26 \text{ lb}$$

Power: The power input of the car is $P_{in} = (50 \text{ hp})\left(\frac{550 \text{ ft}\cdot\text{lb/s}}{1 \text{ hp}}\right) = 27\,500 \text{ ft}\cdot\text{lb/s}$. Thus, the power output is given by $P_{out} = \varepsilon P_{in} = 0.8(27\,500) = 22\,000 \text{ ft}\cdot\text{lb/s}$.

$$P_{out} = \mathbf{F}\cdot\mathbf{v}$$

$$22\,000 = 348.26v$$

$$v = 63.2 \text{ ft/s}$$ **Ans.**

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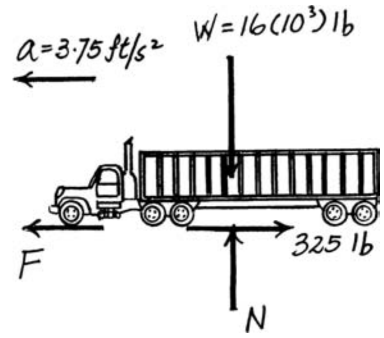
14–47. A loaded truck weighs $16(10^3)$ lb and accelerates uniformly on a level road from 15 ft/s to 30 ft/s during 4 s. If the frictional resistance to motion is 325 lb, determine the maximum power that must be delivered to the wheels.

$$a = \frac{\Delta v}{\Delta t} = \frac{30 - 15}{4} = 3.75 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad F - 325 = \left(\frac{16(10^3)}{32.2}\right)(3.75)$$

$$F = 2188.35 \text{ lb}$$

$$P_{\max} = \mathbf{F} \cdot \mathbf{v}_{\max} = \frac{2188.35(30)}{550} = 119 \text{ hp} \qquad \textbf{Ans.}$$



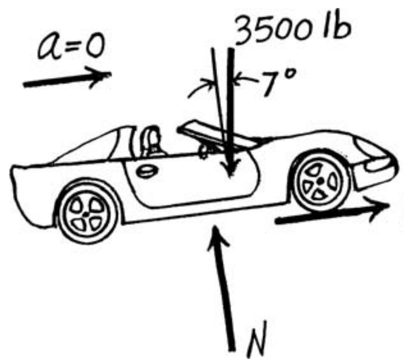
***14–48.** An automobile having a weight of 3500 lb travels up a 7° slope at a constant speed of $v = 40$ ft/s. If friction and wind resistance are neglected, determine the power developed by the engine if the automobile has a mechanical efficiency of $\epsilon = 0.65$.

$$s = vt = 40(1) = 40 \text{ ft}$$

$$U_{1-2} = (3500)(40 \sin 7°) = 17.062\left(10^3\right) \text{ ft} \cdot \text{lb}$$

$$P_{in} = \frac{P_o}{e} = \frac{17.062(10^3) \text{ ft} \cdot \text{lb/s}}{0.65} = 26.249\left(10^3\right) \text{ ft} \cdot \text{lb/s} = 47.7 \text{ hp}$$

$$P_{out} = \frac{U_{1-2}}{t} = \frac{17.602(10^3)}{1} = 17.062\left(10^3\right) \text{ ft} \cdot \text{lb/s} \qquad \textbf{Ans.}$$



Also,

$$F = 3500 \sin 7° = 426.543 \text{ lb}$$

$$P_{out} = \mathbf{F} \cdot \mathbf{v} = 426.543\,(40) = 17.062\left(10^3\right) \text{ ft} \cdot \text{lb/s}$$

$$P_{in} = \frac{P_o}{e} = \frac{17.062\,(10^3) \text{ ft} \cdot \text{lb/s}}{0.65} = 26.249\left(10^3\right) \text{ ft} \cdot \text{lb/s} = 47.7 \text{ hp} \qquad \textbf{Ans.}$$

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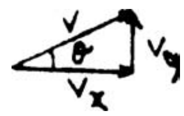
•14–49. An escalator step moves with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



Step height: 0.125 m

The number of steps: $\frac{4}{0.125} = 32$

Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \frac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_s = v \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

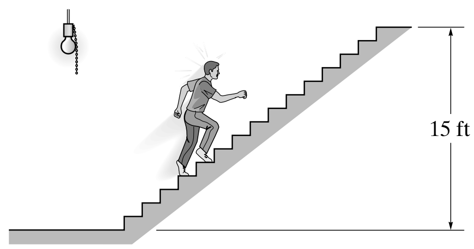
$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW} \qquad \textbf{Ans.}$$

Also,

$$P = \mathbf{F}\cdot\mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW} \qquad \textbf{Ans.}$$

14–50. The man having the weight of 150 lb is able to run up a 15-ft-high flight of stairs in 4 s. Determine the power generated. How long would a 100-W light bulb have to burn to expend the same amount of energy? *Conclusion:* Please turn off the lights when they are not in use!



Power: The work done by the man is

$$U = Wh = 150(15) = 2250 \text{ ft}\cdot\text{lb}$$

Thus, the power generated by the man is given by

$$P_{\max} = \frac{U}{t} = \frac{2250}{4} = 562.5 \text{ ft}\cdot\text{lb/s} = 1.02 \text{ hp} \qquad \textbf{Ans.}$$

The power of the bulb is $P_{\text{bulb}} = 100\,W \times \left(\frac{1 \text{ hp}}{746 \text{ W}}\right) \times \left(\frac{550 \text{ ft}\cdot\text{lb/s}}{1 \text{ hp}}\right) = 73.73 \text{ ft}\cdot\text{lb/s}$. Thus,

$$t = \frac{U}{P_{\text{bulb}}} = \frac{2250}{73.73} = 30.5 \text{ s} \qquad \textbf{Ans.}$$

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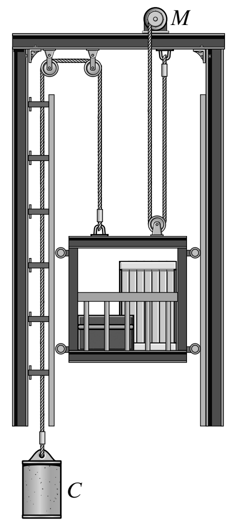
14–51. The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. At a given instant, the hoist has an upward velocity of 2 m/s and an acceleration of 1.5 m/s^2. Determine the power generated by the motor M at this instant if it operates with an efficiency of $\epsilon = 0.8$.



Equations of Motion: Here, $a = 1.5\text{ m/s}^2$. By referring to the free-body diagram of the hoist and counterweight shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(1.5) \qquad \textbf{(1)}$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(1.5)$$

Solving,

$$T' = 1246.5\text{ N}$$

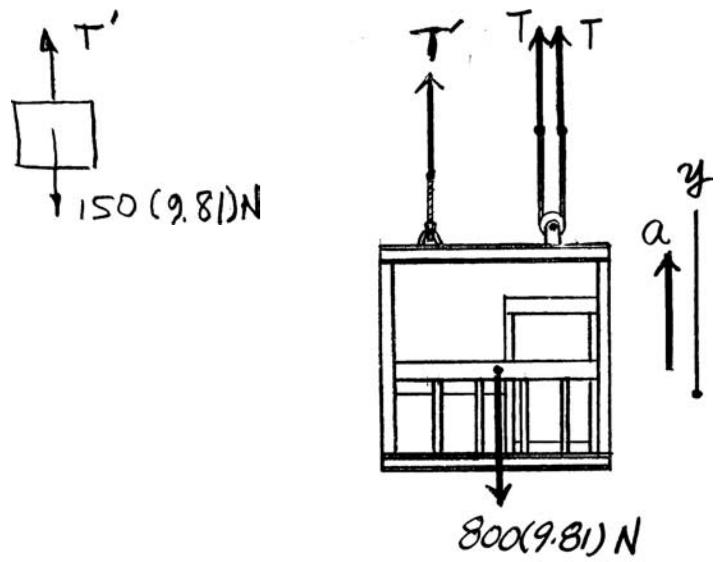
$$T = 3900.75\text{ N}$$

Power:

$$P_{\text{out}} = 2\mathbf{T}\cdot\mathbf{v} = 2(3900.75)(2) = 15\,603\text{ W}$$

Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{15603}{0.8} = 19.5(10^3)\text{ W} = 19.5\text{ kW} \qquad \textbf{Ans.}$$



(a)

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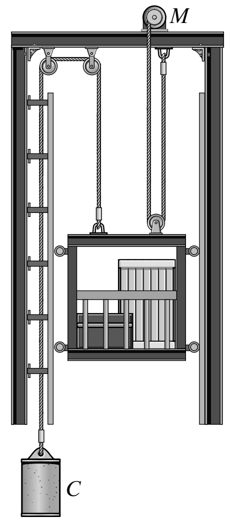
***14–52.** The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. If the upward speed of the hoist increases uniformly from 0.5 m/s to 1.5 m/s in 1.5 s, determine the average power generated by the motor M during this time. The motor operates with an efficiency of $\epsilon = 0.8$.



Kinematics: The acceleration of the hoist can be determined from

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$1.5 = 0.5 + a(1.5)$$

$$a = 0.6667 \text{ m/s}^2$$

Equations of Motion: Using the result of **a** and referring to the free-body diagram of the hoist and block shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(0.6667)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(0.6667)$$

Solving,

$$T' = 1371.5 \text{ N}$$

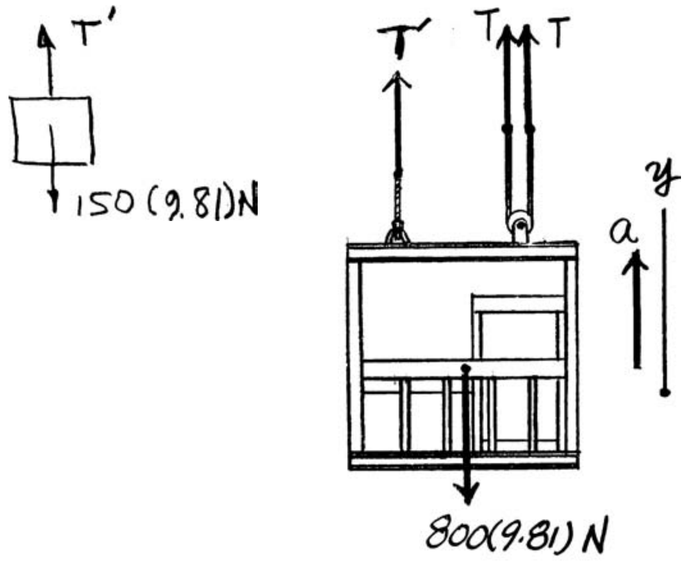
$$T = 3504.92 \text{ N}$$

Power:

$$(P_{\text{out}})_{\text{avg}} = 2\mathbf{T} \cdot \mathbf{v}_{\text{avg}} = 2(3504.92)\left(\frac{1.5 + 0.5}{2}\right) = 7009.8 \text{ W}$$

Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{7009.8}{0.8} = 8762.3 \quad \text{W} = 8.76 \text{ kW} \qquad \textbf{Ans.}$$



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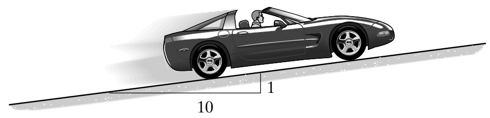
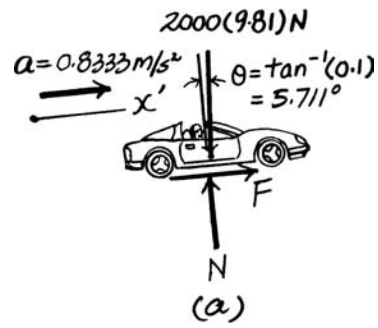
•14–53. The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.



Kinematics: The constant acceleration of the car can be determined from



$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\text{max}} = \mathbf{F} \cdot \mathbf{v}_{\text{max}} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

$$(P_{\text{out}})_{\text{avg}} = \mathbf{F} \cdot \mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \textbf{Ans.}$$

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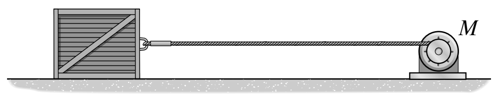
14–54. Determine the velocity of the 200-lb crate in 15 s if the motor operates with an efficiency of $\epsilon = 0.8$. The power input to the motor is 2.5 hp. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.2$.

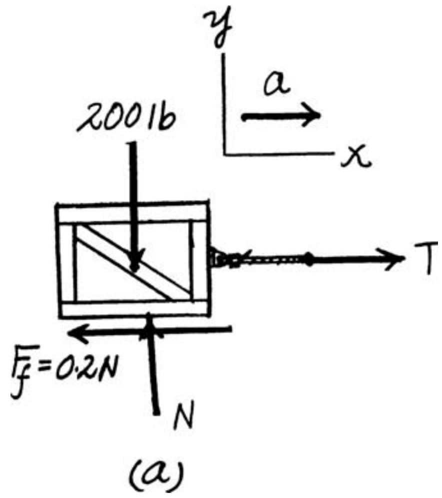


Equations of Motion: By referring to the free-body diagram of the crate shown in Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 200 = \frac{200}{32.2}(0) \qquad N = 200 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad T - 0.2(200) = \frac{200}{32.2}(a)$$

$$T = \left(\frac{200}{32.2}a + 40\right) \text{lb} \qquad \textbf{(1)}$$



Power: Here, the power input is $P_{in} = (2.5 \text{ hp})\left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right) = 1375 \text{ ft} \cdot \text{lb/s}$. Thus,

$P_{out} = \varepsilon P_{in} = 0.8(1375) = 1100 \text{ ft} \cdot \text{lb/s}$.

$$P_{out} = \mathbf{T} \cdot \mathbf{v}$$

$$1100 = Tv \qquad \textbf{(2)}$$

Kinematics: The speed of the crate is

$$\left(\overset{+}{\rightarrow}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + a(15)$$

$$v = 15a \qquad \textbf{(3)}$$

Substituting Eq. (3) into Eq. (2) yields

$$T = \frac{73.33}{a} \qquad \textbf{(4)}$$

Substituting Eq. (4) into Eq. (1) yields

$$\frac{73.33}{a} = \frac{200}{32.2}a + 40$$

Solving for the positive root,

$$a = 1.489 \text{ ft/s}^2$$

Substituting the result into Eq. (3),

$$v = 15(1.489) = 22.3 \text{ ft/s} \qquad \textbf{Ans.}$$

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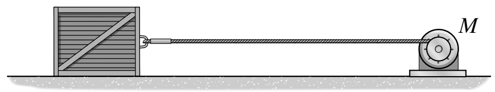
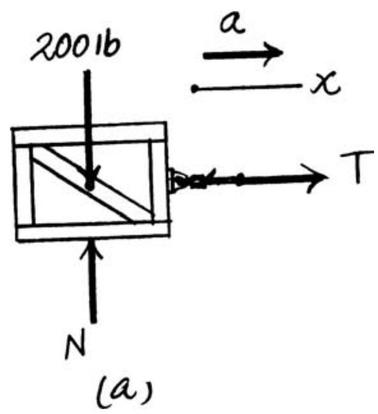
14–55. A constant power of 1.5 hp is supplied to the motor while it operates with an efficiency of $\epsilon = 0.8$. Determine the velocity of the 200-lb crate in 15 seconds, starting from rest. Neglect friction.



Equations of Motion: Here, $a = \dfrac{dv}{dt}$. By referring to the free-body diagram of the crate shown in Fig. *a*,



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad T = \frac{200}{32.2}\left(\frac{dv}{dt}\right) \qquad \textbf{(1)}$$

Power: Here, the power input is $P_{\text{in}} = (1.5\text{ hp})\left(\dfrac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 825\text{ ft}\cdot\text{lb/s}$. Thus,

$$P_{\text{out}} = \varepsilon P_{\text{in}} = 0.8(825) = 660\text{ ft}\cdot\text{lb/s}.$$

$$P_{\text{out}} = \mathbf{T}\cdot\mathbf{v}$$

$$660 = \frac{200}{32.2}\left(\frac{dv}{dt}\right)v$$

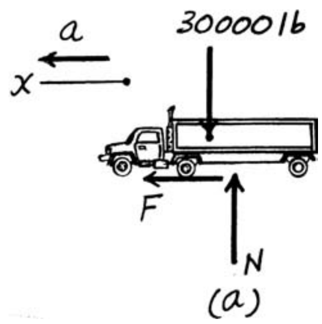
$$\int_0^v v\,dv = \int_0^{15\text{ s}} 106.26\,dt$$

$$\frac{v^2}{2}\bigg|_0^v = 106.26\,dt\bigg|_0^{15\text{ s}}$$

$$v = 56.5\text{ ft/s} \qquad \textbf{Ans.}$$

***14–56.** The fluid transmission of a 30 000-lb truck allows the engine to deliver constant power to the rear wheels. Determine the distance required for the truck traveling on a level road to increase its speed from 35 ft/s to 60 ft/s if 90 hp is delivered to the rear wheels. Neglect drag and rolling resistance.

Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the fre -body diagram of the truck shown in Fig. *a*,



$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad F = \left(\frac{30000}{32.2}\right)\left(v\frac{dv}{ds}\right) \qquad \textbf{(1)}$$

Power: Here, the power output is $P_{\text{out}} = (90\text{ hp})\left(\dfrac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 49500\text{ ft}\cdot\text{lb/s}$. Using Eq. (1),

$$P_{\text{out}} = \mathbf{F}\cdot\mathbf{v}$$

$$49500 = \left(\frac{30000}{32.2}\right)\left(v\frac{dv}{ds}\right)v$$

$$\int_0^s 53.13ds = \int_{35\text{ ft/s}}^{60\text{ ft/s}} v^2\,dv$$

$$53.13s\Big|_0^s = \frac{v^3}{3}\bigg|_{35\text{ ft/s}}^{60\text{ ft/s}}$$

$$s = 1086\text{ ft} \qquad \textbf{Ans.}$$

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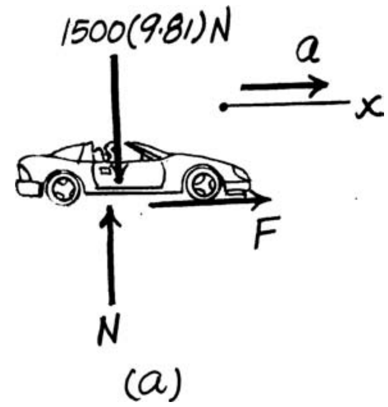
•14–57. If the engine of a 1.5-Mg car generates a constant power of 15 kW, determine the speed of the car after it has traveled a distance of 200 m on a level road starting from rest. Neglect friction.



Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the car shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 1500\left(v\frac{dv}{ds}\right)$$

Power:

$$P_{\text{out}} = \mathbf{F}\cdot\mathbf{v}$$

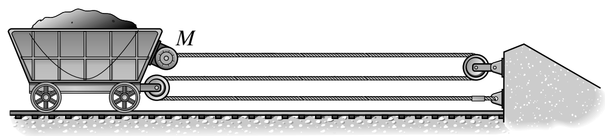
$$15(10^3) = 1500\left(v\frac{dv}{ds}\right)v$$

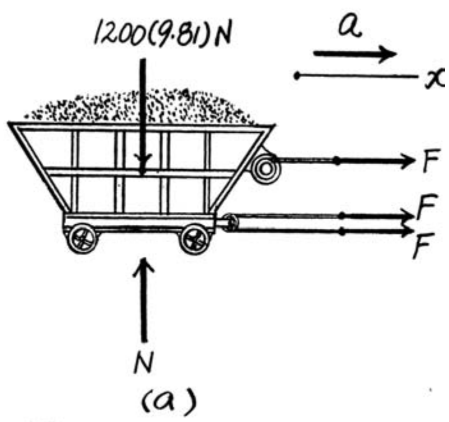
$$\int_0^{200\text{ m}} 10ds = \int_0^v v^2\,dv$$

$$10s\Big|_0^{200\text{ m}} = \frac{v^3}{3}\Big|_0^v$$

$$v = 18.7\text{ m/s} \qquad \textbf{Ans.}$$

14–58. The 1.2-Mg mine car is being pulled by the winch M mounted on the car. If the winch exerts a force of $F = (150t^{3/2})$ N on the cable, where t is in seconds, determine the power output of the winch when $t = 5$ s, starting from rest.





Equations of Motion: By referring to the free-body diagram of the mine car shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 3\left(150t^{3/2}\right) = 1200a$$

$$a = \left(0.375t^{3/2}\right)\text{ m/s}^2$$

Kinematics: The speed of the mine car at $t = 5$ s can be determined by integrating the kinematic equation $dv = adt$ and using the result of **a**.

$$\left(\overset{+}{\rightarrow}\right) \qquad \int_0^v dv = \int_0^{5\text{ s}} 0.375t^{3/2}\,dt$$

$$v = 0.15t^{5/2}\Big|_0^{5\text{ s}} = 8.385\text{ m/s}$$

Power: At $t = 5$ s, $F = 150(5^{3/2}) = 1677.05$ N.

$$P_{\text{out}} = 3\mathbf{F}\cdot\mathbf{v}$$

$$= 3(1677.05)(8.385)$$

$$= 42.1875(10^3)\text{ W}$$

$$= 42.2\text{ kW} \qquad \textbf{Ans.}$$

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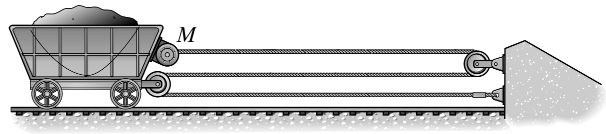
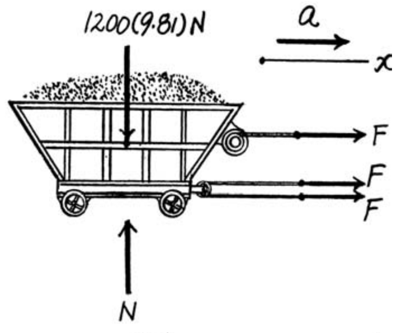
14–59. The 1.2-Mg mine car is being pulled by the winch M mounted on the car. If the winch generates a constant power output of 30 kW, determine the speed of the car at the instant it has traveled a distance of 30 m, starting from rest.



Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the mine car shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 3F = 1200\left(v\frac{dv}{ds}\right) \qquad \textbf{(1)}$$



(a)

Power:

$$P_{\text{out}} = 3\mathbf{F}\cdot\mathbf{v}$$

$$30(10^3) = 3Fv \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$30(10^3) = 1200\left(v\frac{dv}{ds}\right)v$$

$$\int_0^v v^2 dv = \int_0^{30\text{ m}} 25 ds$$

$$\frac{v^3}{3}\bigg|_0^v = 25s\bigg|_0^{30\text{ m}}$$

$$v = 13.1 \text{ m/s} \qquad \textbf{Ans.}$$

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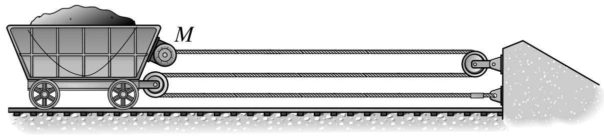
***14–60.** The 1.2-Mg mine car is being pulled by winch M mounted on the car. If the winch generates a constant power output of 30 kW, and the car starts from rest, determine the speed of the car when $t = 5$ s.



Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the mine car shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 3F = 1200\left(v\frac{dv}{ds}\right) \qquad \textbf{(1)}$$

Power:

$$P_{\text{out}} = 3\mathbf{F} \cdot \mathbf{v}$$

$$30(10^3) = 3Fv \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$30(10^3) = \left(1200\frac{dv}{ds}\right)v$$

$$\int_0^v v\,dv = \int_0^{5\text{ s}} 25dt$$

$$\left.\frac{v^2}{2}\right|_0^v = 25t\Big|_0^{5\text{ s}}$$

$$v = 15.8 \text{ m/s} \qquad \textbf{Ans.}$$

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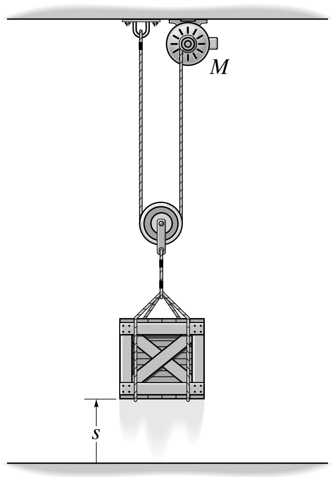
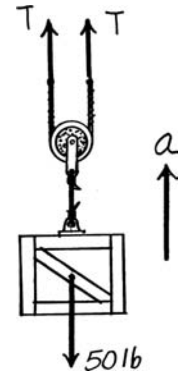
•14–61. The 50-lb crate is hoisted by the motor M. If the crate starts from rest and by constant acceleration attains a speed of 12 ft/s after rising $s = 10$ ft, determine the power that must be supplied to the motor at the instant $s = 10$ ft. The motor has an efficiency $\epsilon = 0.65$. Neglect the mass of the pulley and cable.



$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 50 = \frac{50}{32.2} a$



$(+\uparrow)\; v^2 = v_0^2 + 2\,a_c\,(s - s_0)$

$(12)^2 = 0 + 2(a)(10 - 0)$

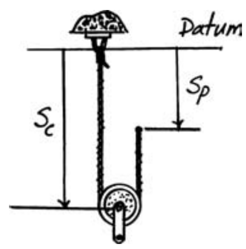
$a = 7.20 \text{ ft/s}^2$

Thus, $T = 30.59$ lb

$s_C + (s_C - s_P) = l$

$2\,v_C = v_P$

$2(12) = v_P = 24$ ft/s



$P_o = 30.59(24) = 734.16$

$P_i = \frac{734.16}{0.65} = 1129.5 \text{ ft} \cdot \text{lb/s}$

$P_i = 2.05$ hp **Ans.**

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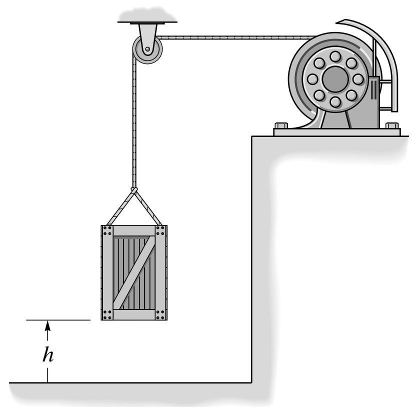
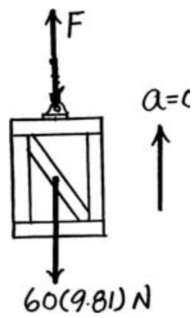
14–62. A motor hoists a 60-kg crate at a constant velocity to a height of $h = 5$ m in 2 s. If the indicated power of the motor is 3.2 kW, determine the motor's efficiency.



Equations of Motion:

$$+\uparrow \Sigma F_y = ma_y; \qquad F - 60(9.81) = 60(0) \qquad F = 588.6 \text{ N}$$

Power: The crate travels at a constant speed of $v = \frac{5}{2} = 2.50$ m/s. The power output can be obtained using Eq. 14–10.

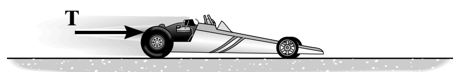


$$P = \mathbf{F} \cdot \mathbf{v} = 588.6\,(2.50) = 1471.5 \text{ W}$$

Thus, from Eq. 14–11, the efficiency of the motor is given by

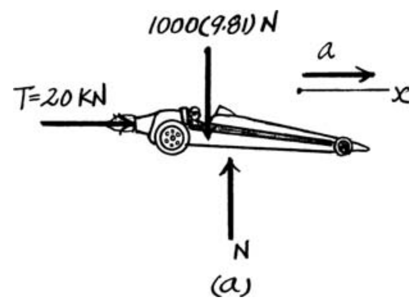
$$\varepsilon = \frac{\text{power output}}{\text{power input}} = \frac{1471.5}{3200} = 0.460$$ **Ans.**

14–63. If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.



Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 20(10^3) = 1000(a) \qquad a = 20 \text{ m/s}^2$$



Kinematics: The velocity of the dragster can be determined from

$$\left(\overset{+}{\rightarrow}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + 20t = (20t) \text{ m/s}$$

Power:

$$P = \mathbf{F} \cdot \mathbf{v} = 20(10^3)(20t)$$

$$= \left[400(10^3)t\right] \text{ W}$$ **Ans.**

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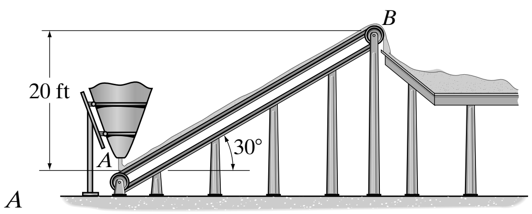
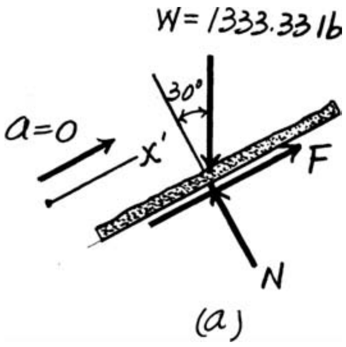
***14–64.** Sand is being discharged from the silo at A to the conveyor and transported to the storage deck at the rate of 360 000 lb/h. An electric motor is attached to the conveyor to maintain the speed of the belt at 3 ft/s. Determine the average power generated by the motor.



Equations of Motion: The time required for the conveyor to move from point A point B is $t_{AB} = \frac{s_{AB}}{v} = \frac{20/\sin 30^\circ}{3} = 13.33$ s. Thus, the weight of the sand on the conveyor at any given instant is $W = \left(360\,000 \text{ lb/h}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(13.33 \text{ s}\right) = 1333.33$ lb. By referring to the free-body diagram of the sand shown in Fig. a,



(a)

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad F - 1333.3 \sin 30^\circ = \frac{1333.33}{32.2}(0)$$

$$F = 666.67 \text{ lb}$$

Power: Using the result of **F**,

$$P = \mathbf{F} \cdot \mathbf{v} = 666.67(3) = 2000 \text{ ft} \cdot \text{lb/s}$$

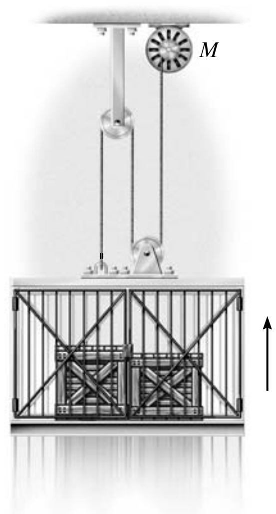
Thus,

$$P = \left(2000 \text{ ft} \cdot \text{lb/s}\right)\left(\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb/s}}\right) = 3.64 \text{ hp} \qquad \textbf{Ans.}$$

Note that P can also be determined in a more direct manner using

$$P_{\text{out}} = \frac{dW}{dt}(h) = \left(360\,000 \frac{\text{lb}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(20 \text{ ft}\right) = 2000 \text{ ft} \cdot \text{lb/s}$$

14–65. The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2 \text{ m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



$$+\uparrow\Sigma F_y = m\,a_y; \qquad 3T - 500(9.81) = 500(2)$$

$$T = 1968.33 \text{ N}$$

$3s_E - s_P = l$

$3\,v_E = v_P$

When $t = 3$ s,

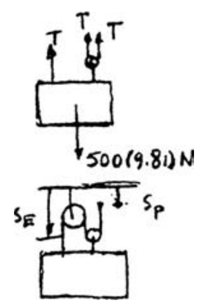
$(+\uparrow)\ v_0 + a_c\,t$

$v_E = 0 + 2(3) = 6$ m/s

$v_P = 3(6) = 18$ m/s

$P_O = 1968.33(18)$

$P_O = 35.4$ kW **Ans.**



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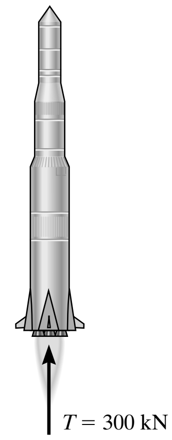
14–66. A rocket having a total mass of 8 Mg is fired vertically from rest. If the engines provide a constant thrust of $T = 300$ kN, determine the power output of the engines as a function of time. Neglect the effect of drag resistance and the loss of fuel mass and weight.

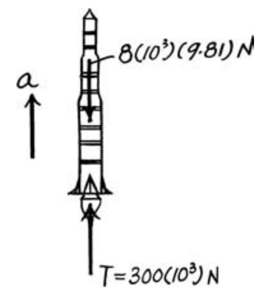


$+\uparrow \Sigma F_y = ma_y; \qquad 300(10^3) - 8(10^3)(9.81) = 8(10^3)a \qquad a = 27.69 \text{ m/s}^2$

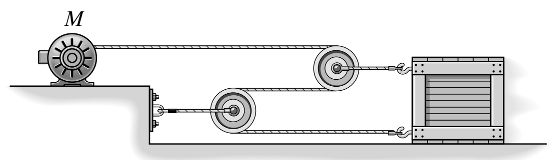
$(+\uparrow) \qquad v = v_0 + a_c t$

$\qquad = 0 + 27.69t = 27.69t$

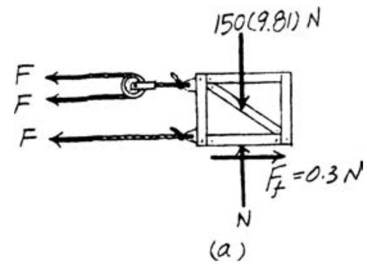
$P = \mathbf{T} \cdot \mathbf{v} = 300\,(10^3)\,(27.69t) = 8.31\,t \text{ MW}$ **Ans.**



14–67. The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.3N$. From FBD(a),



$+\uparrow \Sigma F_y = 0; \qquad N - 150(9.81) = 0 \qquad N = 1471.5 \text{ N}$

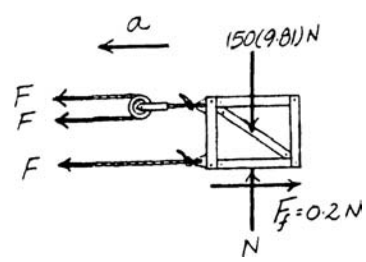
$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - 3\left(8t^2 + 20\right) = 0 \qquad t = 3.9867 \text{ s}$

Equations of Motion: Since the crate moves 3.9867 s later, $F_f = \mu_k N = 0.2N$. From FBD(b),

$+\uparrow \Sigma F_y = ma_y; \qquad N - 150(9.81) = 150\,(0) \qquad N = 1471.5 \text{ N}$

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.2\,(1471.5) - 3\left(8t^2 + 20\right) = 150\,(-a)$

$$a = \left(0.160t^2 - 1.562\right) \text{ m/s}^2$$



Kinematics: Applying $dv = a\,dt$, we have

$$\int_0^v dv = \int_{3.9867 \text{ s}}^{5} \left(0.160t^2 - 1.562\right) dt$$

$$v = 1.7045 \text{ m/s}$$

Power: At $t = 5$ s, $F = 8\left(5^2\right) + 20 = 220$ N. The power can be obtained using Eq. 14–10.

$P = \mathbf{F} \cdot \mathbf{v} = 3\,(220)\,(1.7045) = 1124.97 \text{ W} = 1.12 \text{ kW}$ **Ans.**

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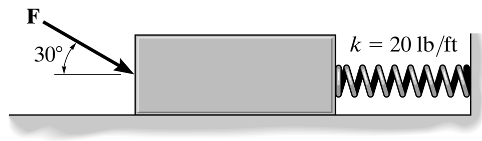
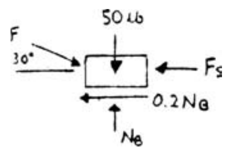
***14–68.** The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.





$+\uparrow \Sigma F_y = 0; \qquad N_B - \left(40 + s^2\right) \sin 30° - 50 = 0$

$N_B = 70 + 0.5s^2$

$T_1 + \Sigma U_{1-2} + T_2$

$$0 + \int_0^{1.5} \left(40 + s^2\right) \cos 30° \, ds - \frac{1}{2}(20)(1.5)^2 - 0.2 \int_0^{1.5} \left(70 + 0.5s^2\right) ds = \frac{1}{2}\left(\frac{50}{32.2}\right) v_2^2$$

$0 + 52.936 - 22.5 - 21.1125 = 0.7764\, v_2^2$

$v_2 = 3.465$ ft/s

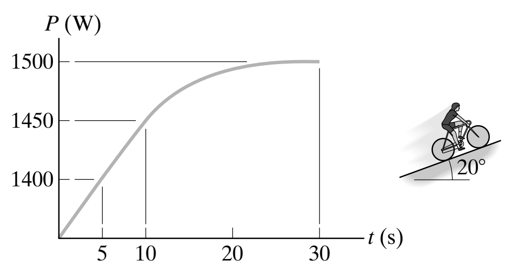
When $s = 1.5$ ft,

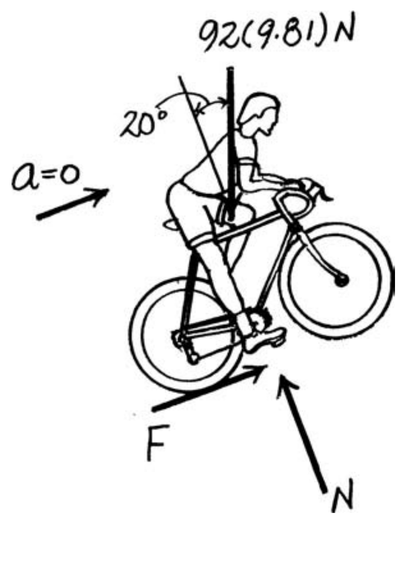
$F = 40 + (1.5)^2 = 42.25$ lb

$P = \mathbf{F} \cdot \mathbf{v} = (42.25 \cos 30°)(3.465)$

$P = 126.79$ ft · lb/s $= 0.231$ hp **Ans.**

•14–69. Using the biomechanical power curve shown, determine the maximum speed attained by the rider and his bicycle, which have a total mass of 92 kg, as the rider ascends the 20° slope starting from rest.





$F = 92(9.81) \sin 20° = 308.68$ N

$P = \mathbf{F} \cdot \mathbf{v}; \qquad 1500 = 308.68\, v$

$v = 4.86$ m/s **Ans.**

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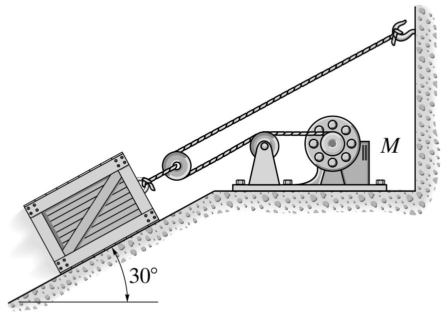
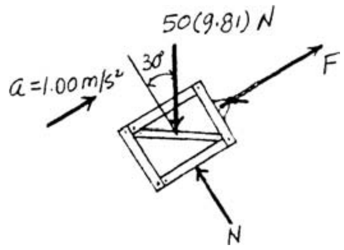
14–70. The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at the instant the crate has moved 8 m. Neglect friction along the plane. The motor has an efficiency of $\epsilon = 0.74$.



Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00\ \text{m/s}^2$$



Equations of Motion:

$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 50(9.81)\sin 30^\circ = 50(1.00) \qquad F = 295.25\ \text{N}$$

Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

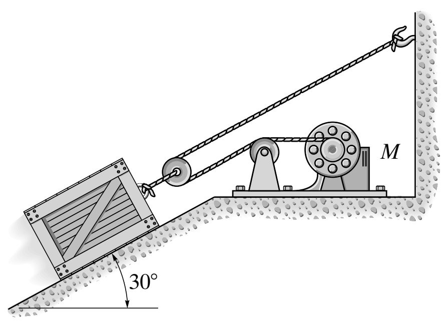
$$P = \mathbf{F}\cdot\mathbf{v} = 295.25\,(4) = 1181\ \text{W} = 1.181\ \text{kW}$$

Using Eq. 14–11, the required power input to the motor in order to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.181}{0.74} = 1.60\ \text{kW} \qquad \textbf{Ans.}$$

14–71. Solve Prob. 14–70 if the coefficient of kinetic friction between the plane and the crate is $\mu_k = 0.3$.



Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

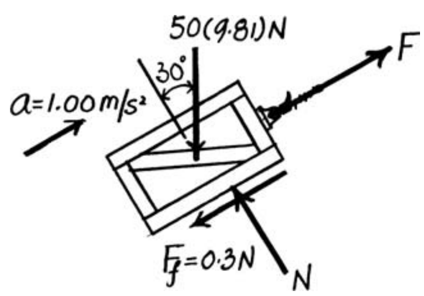
$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00\ \text{m/s}^2$$

Equations of Motion:

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 50(9.81)\cos 30^\circ = 50(0) \qquad N = 424.79\ \text{N}$$

$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 0.3\,(424.79) - 50(9.81)\sin 30^\circ = 50(1.00)$$

$$F = 422.69\ \text{N}$$



Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 422.69\,(4) = 1690.74\ \text{W} = 1.691\ \text{kW}$$

Using Eq. 14–11, the required power input to the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.691}{0.74} = 2.28\ \text{kW} \qquad \textbf{Ans.}$$

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***14–72.** Solve Prob. 14–12 using the conservation of energy equation.

Put Datum at center of block at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(60 + y) = 0 + \left[\frac{1}{2}(30)y^2\right] + \left[\frac{1}{2}(45)(y - 3)^2\right] = 0$$

$$37.5y^2 - 145y - 397.5 = 0$$

Solving for the positive root of the above equation,

$$y = 5.720 \text{ in.}$$

Thus,

$s_1 = 5.72$ in. $\qquad s_2 = 5.720 - 3 = 2.72$ in. **Ans.**

•14–73. Solve Prob. 14–7 using the conservation of energy equation.

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$0 + 6(2) = 0 + \frac{1}{2}(5)(12)(x)^2$$

$x = 0.6325 \text{ ft} = 7.59 \text{ in.}$ **Ans.**

14–74. Solve Prob. 14–8 using the conservation of energy equation.

The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \left[\frac{1}{2}ks_1{}^2\right] + \left[-Wh\right] = \frac{1}{2}mv_A{}^2 + \frac{1}{2}ks_2{}^2 + 0$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2\right] = \frac{1}{2}(0.02)v_A{}^2 + \frac{1}{2}(2000)(0.0375^2)$$

$v_A = 10.5$ m/s **Ans.**

14–75. Solve Prob. 14–18 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(100)(0.5)^2 + \frac{1}{2}(50)(0.5)^2 = \frac{1}{2}(20)v^2 + 0$$

$v = 1.37$ m/s **Ans.**

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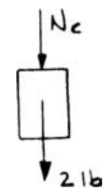
***14–76.** Solve Prob. 14–22 using the conservation of energy equation.

Datum at A

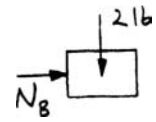


$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 + 2(5)$$

$v_B = 24.042 = 24.0 \text{ ft/s}$ **Ans.**

$$\overset{+}{\rightarrow} \Sigma F_n = ma_n; \qquad N_B = \left(\frac{2}{32.2}\right)\left(\frac{(24.042)^2}{5}\right)$$



$N_B = 7.18 \text{ lb}$ **Ans.**

$$T_A + V_A = T_C + V_C$$

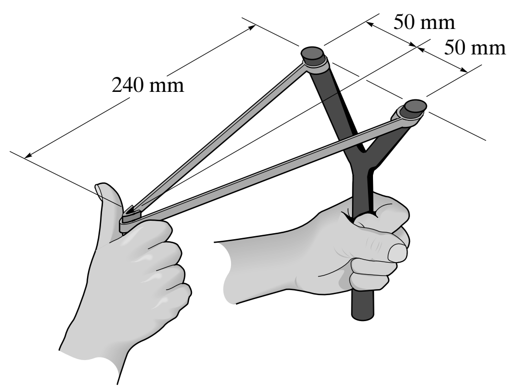
$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + 2(10)$$

$v_C = 16.0 \text{ ft/s}$ **Ans.**

$$+\downarrow \Sigma F_n = ma_n; \qquad N_C + 2 = \left(\frac{2}{32.2}\right)\left(\frac{(16.0)^2}{5}\right)$$

$N_C = 1.18 \text{ lb}$ **Ans.**

•14–77. Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the speed of the 25-g pellet just after the rubber bands become unstretched. Neglect the mass of the rubber bands. Each rubber band has a stiffness of $k = 50$ N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + (2)\left(\frac{1}{2}\right)(50)[\sqrt{(0.05)^2 + (0.240)^2} - 0.2]^2 = \frac{1}{2}(0.025)v^2$$

$v = 2.86 \text{ m/s}$ **Ans.**

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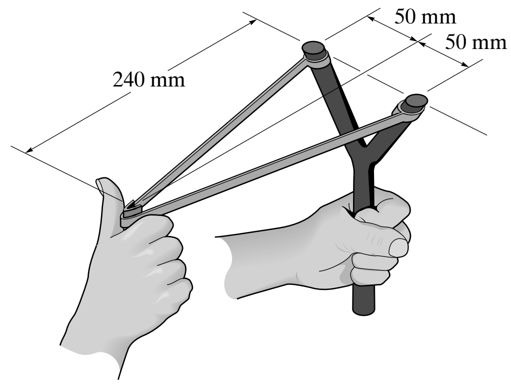
14–78. Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the maximum height the 25-g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness $k = 50$ N/m.

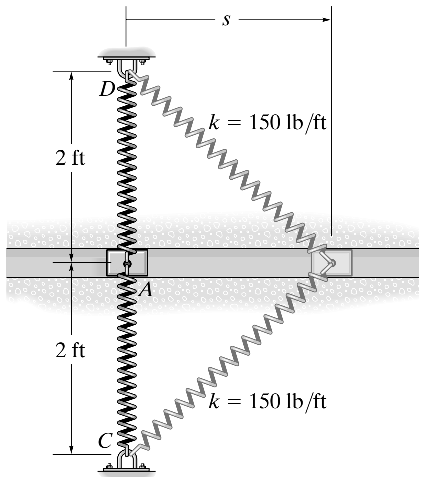


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left(\frac{1}{2}\right)(50)\left[\sqrt{(0.05)^2 + (0.240)^2} - 0.2\right]^2 = 0 + 0.025(9.81)h$$

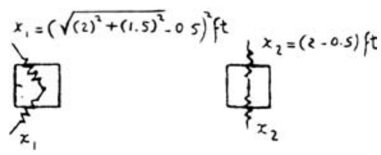
$h = 0.416$ m $= 416$ mm **Ans.**

14–79. Block A has a weight of 1.5 lb and slides in the smooth horizontal slot. If the block is drawn back to $s = 1.5$ ft and released from rest, determine its speed at the instant $s = 0$. Each of the two springs has a stiffness of $k = 150$ lb/ft and an unstretched length of 0.5 ft.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(150)\left(\sqrt{(2)^2 + (1.5)^2} - 0.5\right)^2\right] = \frac{1}{2}\left(\frac{1.5}{32.2}\right)(v_2)^2 + 2\left[\frac{1}{2}(150)(2 - 0.5)^2\right]$$



$v_2 = 106$ ft/s **Ans.**

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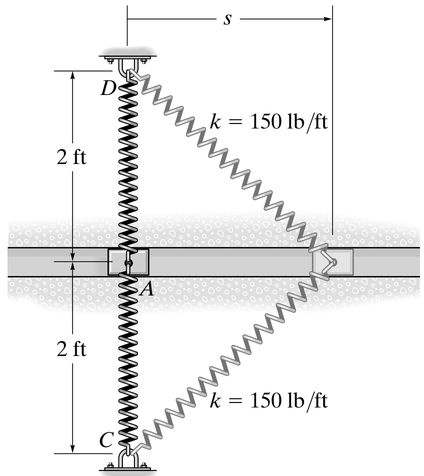
***14–80.** The 2-lb block A slides in the smooth horizontal slot. When $s = 0$ the block is given an initial velocity of 60 ft/s to the right. Determine the maximum horizontal displacement s of the block. Each of the two springs has a stiffness of $k = 150$ lb/ft and an unstretched length of 0.5 ft.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(60)^2 + 2\left[\frac{1}{2}(150)(2 - 0.5)^2\right] = 0 + 2\left[\frac{1}{2}(150)\left(\sqrt{(2)^2 + s^2} - 0.5\right)^2\right]$$

Set $d = \sqrt{(2)^2 + s^2}$ then

$$d^2 - d - 2.745 = 0$$

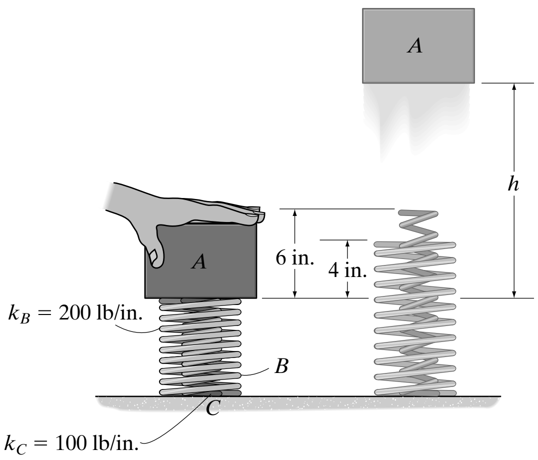
Solving for the positive root,

$$d = 2.231$$

$$(2.231)^2 = (2)^2 + s^2$$

$s = 0.988$ ft **Ans.**

•14–81. The 30-lb block A is placed on top of two nested springs B and C and then pushed down to the position shown. If it is then released, determine the maximum height h to which it will rise.



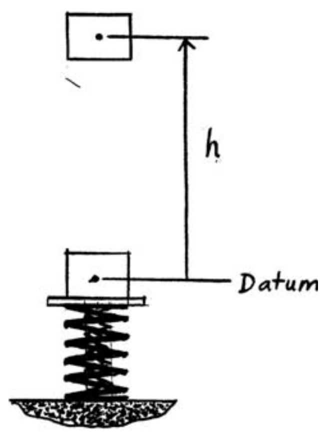
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + 0 + \frac{1}{2}(200)(4)^2 + \frac{1}{2}(100)(6)^2 = 0 + h(30) + 0$$

$h = 113$ in. **Ans.**



(a)

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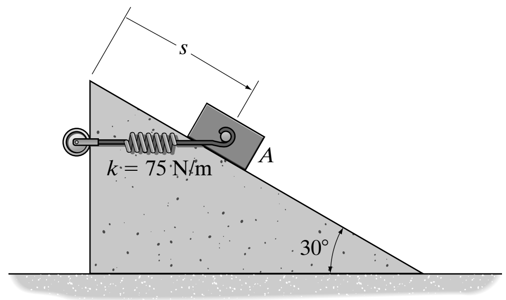
14–82. The spring is unstretched when $s = 1\text{ m}$ and the 15-kg block is released from rest at this position. Determine the speed of the block when $s = 3\text{ m}$. The spring remains horizontal during the motion, and the contact surfaces between the block and the inclined plane are smooth.



Potential Energy: With reference to the datum set in Fig. *a,* the gravitational potential energy of the block at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 15(9.81)(0) = 0$ and $\left(V_g\right)_2 = mgh_2 = 15(9.81)\left[-2 \sin 30°\right] = -147.15\text{ J}$. When the block is at position (1) the spring is unstretched. Thus, the elastic potential energy of the spring at this instant is $\left(V_e\right)_1 = \frac{1}{2}ks_1{}^2 = 0$. The spring is stretched $s_2 = 2 \cos 30°$ m when the block is at position (2). Thus, $\left(V_e\right)_2 = \frac{1}{2}ks_2{}^2 = \frac{1}{2}(75)(2 \cos 30°)^2 = 112.5\text{ J}$ since it is being stretched $s_2 = x$.

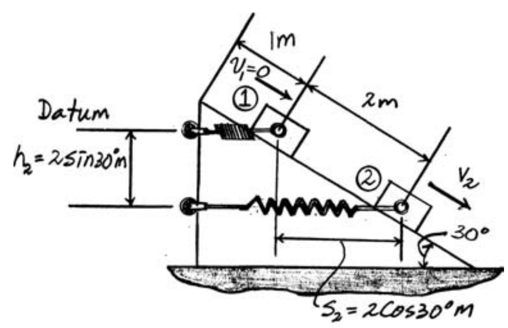
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 0\right) = \frac{1}{2}(15)v_2{}^2 + \left[-147.15 + 112.5\right]$$

$$v_2 = 2.15\text{ m/s} \qquad \textbf{Ans.}$$



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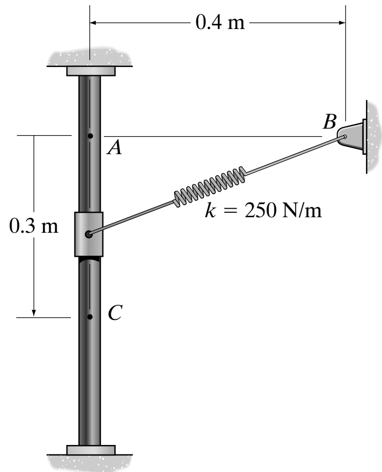
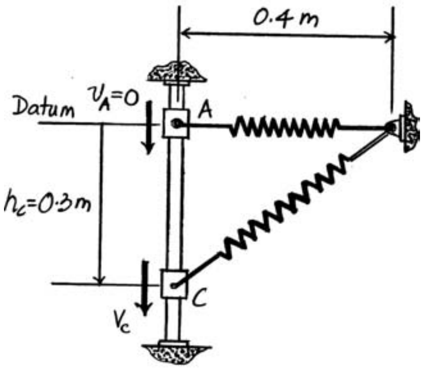
14–83. The vertical guide is smooth and the 5-kg collar is released from rest at A. Determine the speed of the collar when it is at position C. The spring has an unstretched length of 300 mm.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the collar at positions A and C are $(V_g)_A = mgh_A = 5(9.81)(0) = 0$ and $(V_g)_C = mgh_C = 5(9.81)(-0.3) = -14.715$ J. When the collar is at positions A and C, the spring stretches $s_A = 0.4 - 0.3 = 0.1$ m and $s_C = \sqrt{0.4^2 + 0.3^2} - 0.3 = 0.2$ m. The elastic potential energy of the spring when the collar is at these two positions are $(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(250)(0.1^2) = 1.25$ J and

$(V_e)_C = \frac{1}{2}ks_C{}^2 = \frac{1}{2}(250)(0.2^2) = 5$ J.



(a)

Conservation of Energy:

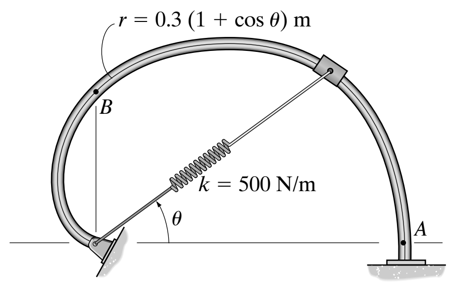
$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A{}^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2}mv_C{}^2 + \left[(V_g)_C + (V_e)_C\right]$$

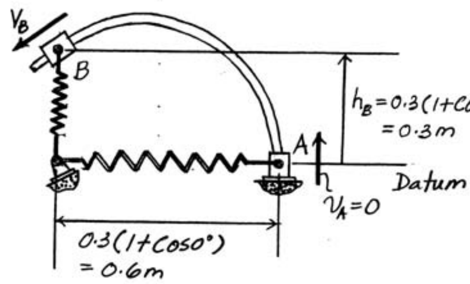
$$0 + (0 + 1.25) = \frac{1}{2}(5)v_C{}^2 + (-14.715 + 5)$$

$$v_C = 2.09 \text{ m/s}$$ **Ans.**

***14–84.** The 5-kg collar slides along the smooth vertical rod. If the collar is nudged from rest at A, determine its speed when it passes point B. The spring has an unstretched length of 200 mm.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the collar at positions A and B are $(V_g)_A = mgh_A = 5(9.81)(0) = 0$ and $(V_g)_B = mgh_B = 5(9.81)(0.3) = 14.715$ J. The spring stretches $s_A = 0.6 - 0.2 = 0.4$ m and $s_B = 0.3 - 0.2 = 0.1$ m when the collar is at positions A and B, respectively. Thus, the elastic potential energy of the spring when the collar is at these two positions are $(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(500)(0.4^2) = 40$ J and $(V_e)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(500)(0.1^2) = 2.5$ J.



(a)

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2}mv_B{}^2 + \left[(V_g)_B + (V_e)_B\right]$$

$$0 + (0 + 40) = \frac{1}{2}(5)v_B{}^2 + (14.715 + 2.5)$$

$$v_B = 3.02 \text{ m/s}$$ **Ans.**

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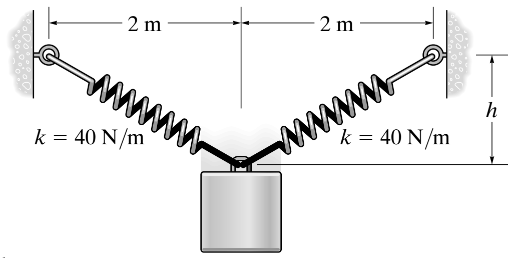
•14–85. The cylinder has a mass of 20 kg and is released from rest when $h = 0$. Determine its speed when $h = 3$ m. The springs each have an unstretched length of 2 m.



Potential Energy: Datum is set at the cylinder position when $h = 0$. When the cylinder moves to a position $h = 3$ m *below* the datum, its gravitational potential energy at this position is $20(9.81)(-3) = -588.6$ J. The initial and final elastic potential energy are $2\left[\frac{1}{2}(40)(2-2)^2\right] = 0$ and $2\left[\frac{1}{2}(40)\left(\sqrt{2^2+3^2}-2\right)^2\right] = 103.11$ J, respectively.

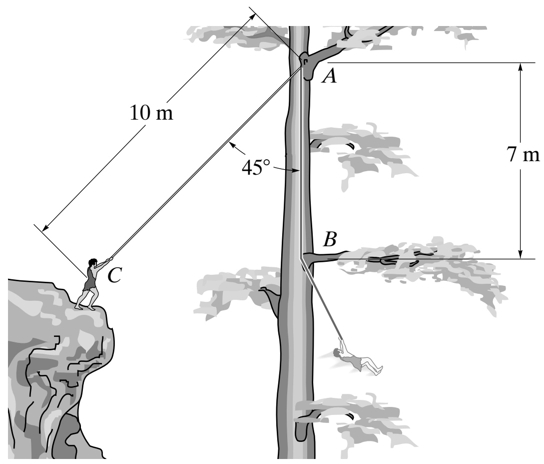
Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 0 = \frac{1}{2}(20)\,v^2 + 103.11 + (-588.6)$$

$$v = 6.97 \text{ m/s}$$ **Ans.**

14–86. Tarzan has a mass of 100 kg and from rest swings from the cliff by rigidly holding on to the tree vine, which is 10 m measured from the supporting limb A to his center of mass. Determine his speed just after the vine strikes the lower limb at B. Also, with what force must he hold on to the vine just before and just after the vine contacts the limb at B?

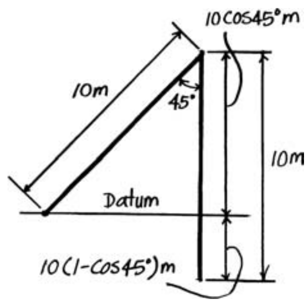


Datum at C

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}(100)(v_C)^2 - 100(9.81)(10)(1 - \cos 45°)$

$v_C = 7.581 = 7.58$ m/s **Ans.**



Just before striking B, $\rho = 10$ m:

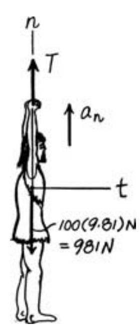
$+\uparrow \Sigma F_n = ma_n; \qquad T - 981 = 100\left(\frac{(7.581)^2}{10}\right)$

$T = 1.56$ kN **Ans.**

Just after striking B, $\rho = 3$ m:

$+\uparrow \Sigma F_n = ma_n; \qquad T - 981 = 100\left(\frac{(7.581)^2}{3}\right)$

$T = 2.90$ kN **Ans.**



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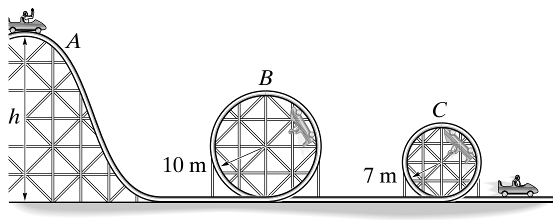
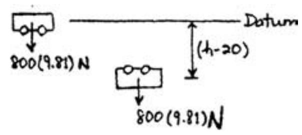
14–87. The roller-coaster car has a mass of 800 kg, including its passenger, and starts from the top of the hill A with a speed $v_A = 3$ m/s. Determine the minimum height h of the hill so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and at C?



$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h - 20)$$

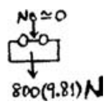


$$+\downarrow \Sigma F_n = m\,a_n; \qquad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$$

Thus,

$$v_B = 9.90 \text{ m/s}$$

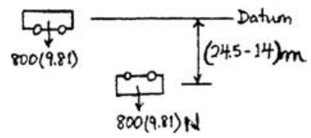
$$h = 24.5 \text{ m} \qquad \textbf{Ans.}$$



At B: $\quad N_B = 0 \quad$ (For h to be minimum.) **Ans.**

$$T_A + V_A = T_C + V_C$$

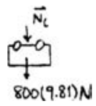
$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(24.5 - 14)$$



$$v_C = 14.69 \text{ m/s}$$

$$a_n = \frac{14.69^2}{7}$$

$$+\downarrow \Sigma F_n = m\,a_n; \qquad N_C + 800(9.81) = 800\left(\frac{14.69^2}{7}\right)$$



$$N_C = 16.8 \text{ kN} \qquad \textbf{Ans.}$$

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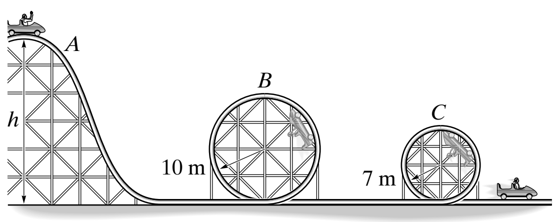
***14–88.** The roller-coaster car has a mass of 800 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and at C?

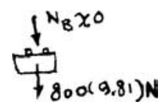


Since friction is neglected, the car will travel around the 7-m loop provided it first travels around the 10-m loop.

$$T_A + V_A = T_B + V_B$$

$$0 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h - 20)$$

$$+\downarrow \Sigma F_n = m\,a_n; \qquad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$$



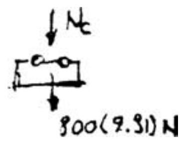
Thus,

$$v_B = 9.90 \text{ m/s}$$

$$h = 25.0 \text{ m} \qquad \textbf{Ans.}$$

At B: $\qquad N_B = 0 \qquad$ (For h to be minimum.) **Ans.**

$$T_A + V_A = T_C + V_C$$



$$0 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(25 - 14)$$

$$v_C = 14.69 \text{ m/s}$$

$$+\downarrow \Sigma F_n = m\,a_n; \qquad N_C + 800(9.81) = 800\left(\frac{(14.69)^2}{7}\right)$$

$$N_C = 16.8 \text{ kN} \qquad \textbf{Ans.}$$

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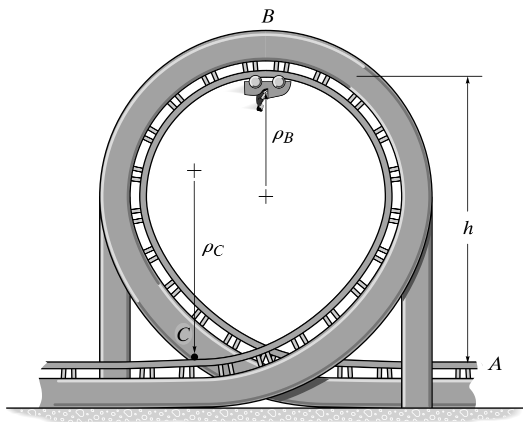
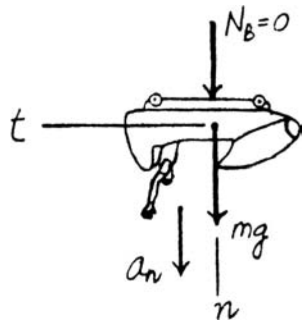
•14–89. The roller coaster and its passenger have a total mass m. Determine the smallest velocity it must have when it enters the loop at A so that it can complete the loop and not leave the track. Also, determine the normal force the tracks exert on the car when it comes around to the bottom at C. The radius of curvature of the tracks at B is ρ_B, and at C it is ρ_C. Neglect the size of the car. Points A and C are at the same elevation.



Equations of Motion: In order for the roller coaster to just pass point B without falling off the track, it is required that $N_B = 0$. Applying Eq. 13–8, we have



$$\Sigma F_n = ma_n; \qquad mg = m\left(\frac{v_B^2}{\rho_B}\right) \qquad v_B^2 = \rho_B\, g$$

Potential Energy: Datum is set at lowest point A. When the roller coaster is at point B, its position is h *above* the datum. Thus, the gravitational potential energy at this point is mgh.

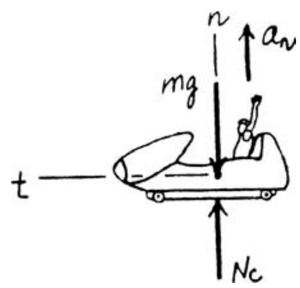
Conservation of Energy: When the roller coaster travels from A to B, we have

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}m(\rho_B\, g) + mgh$$

$$v_A = \sqrt{\rho_B\, g + 2gh} \qquad \textbf{Ans.}$$

When the roller coaster travels from A to C, we have



$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}m(\rho_B\, g + 2gh) + 0 = \frac{1}{2}mv_C^2 + 0$$

$$v_C^2 = \rho_B\, g + 2gh$$

Equations of Motion:

$$\Sigma F_n = ma_n; \qquad N_C - mg = m\left(\frac{\rho_B\, g + 2gh}{\rho_C}\right)$$

$$N_C = \frac{mg}{\rho_C}(\rho_B + \rho_C + 2h) \qquad \textbf{Ans.}$$

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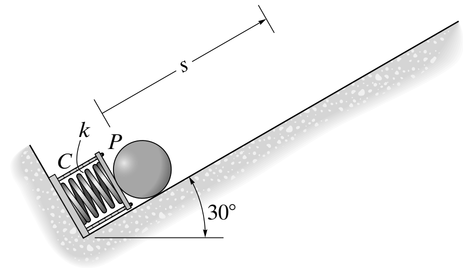
14–90. The 0.5-lb ball is shot from the spring device. The spring has a stiffness $k = 10$ lb/in. and the four cords C and plate P keep the spring compressed 2 in. when no load is on the plate. The plate is pushed back 3 in. from its initial position. If it is then released from rest, determine the speed of the ball when it reaches a position $s = 30$ in. on the smooth inclined plane.



Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(120)\left(\frac{2+3}{12}\right)^2 = 10.42$ ft·lb and $\frac{1}{2}(120)\left(\frac{2}{12}\right)^2 = 1.667$ ft·lb, respectively.

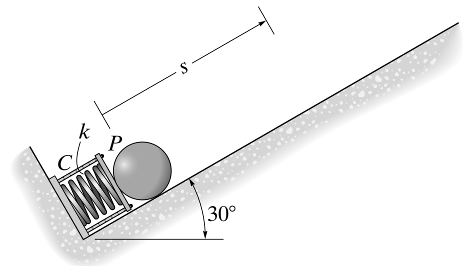
Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 10.42 = \frac{1}{2}\left(\frac{0.5}{32.2}\right)v^2 + 0.625 + 1.667$$

$$v = 32.3 \text{ ft/s}$$ **Ans.**

14–91. The 0.5-lb ball is shot from the spring device shown. Determine the smallest stiffness k which is required to shoot the ball a maximum distance $s = 30$ in. up the plane after the spring is pushed back 3 in. and the ball is released from rest. The four cords C and plate P keep the spring compressed 2 in. when no load is on the plate.



Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(k)\left(\frac{2+3}{12}\right)^2 = 0.08681k$ and $\frac{1}{2}(k)\left(\frac{2}{12}\right)^2 = 0.01389k$, respectively.

Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 0.08681k = 0 + 0.625 + 0.01389k$$

$$k = 8.57 \text{ lb/ft}$$ **Ans.**

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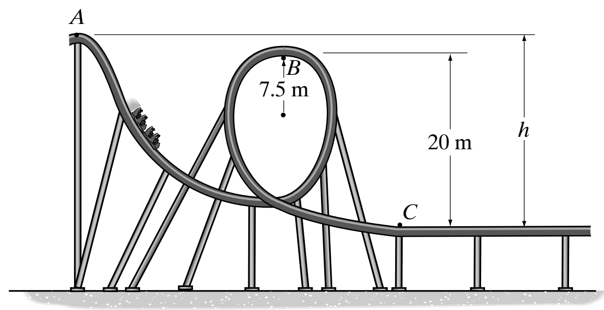
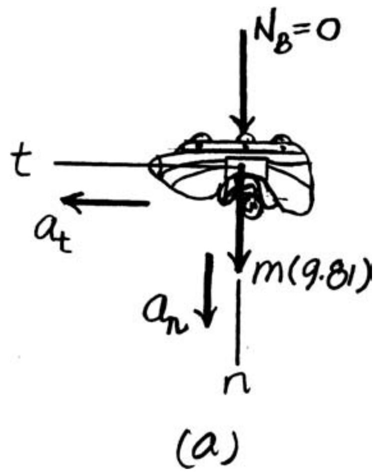
***14–92.** The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \frac{v_B^2}{\rho_B} = \frac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \quad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $(V_g)_A = mgh_A = m(9.81)h = 9.81mh$, $(V_g)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $(V_g)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + (V_g)_A = \frac{1}{2}mv_B^2 + (V_g)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m} \qquad \textbf{Ans.}$$

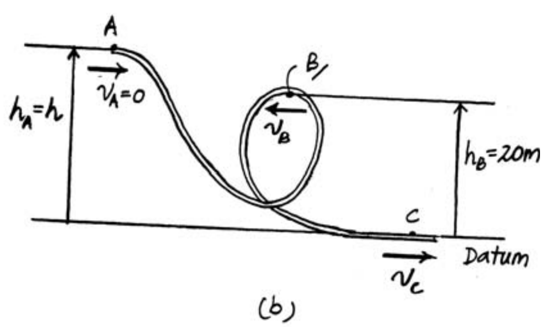
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + (V_g)_B = \frac{1}{2}mv_C^2 + (V_g)_C$$

$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

$$v_C = 21.6 \text{ m/s} \qquad \textbf{Ans.}$$



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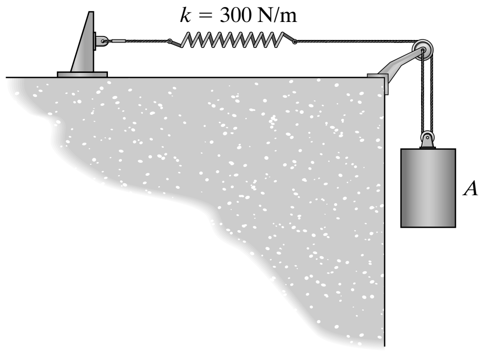
•14–93. When the 50-kg cylinder is released from rest, the spring is subjected to a tension of 60 N. Determine the speed of the cylinder after it has fallen 200 mm. How far has it fallen when it momentarily stops?

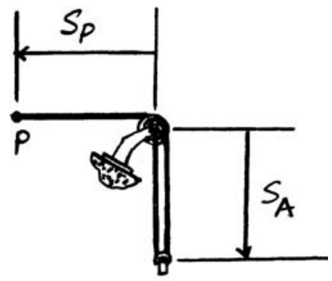


Kinematics: We can express the length of the cord in terms of the position coordinates s_A and s_P. By referring to Fig. *a*,

$$s_P + 2s_A = l$$

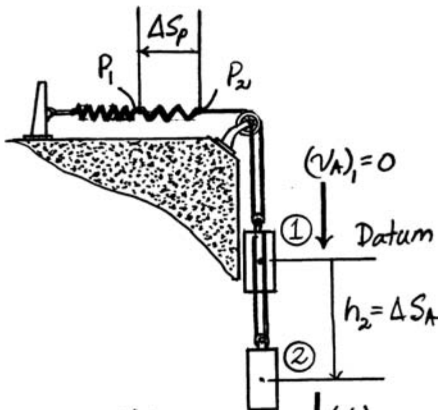
Thus,

$$\Delta s_P + 2\Delta s_A = 0 \qquad \textbf{(1)}$$



(a)

Potential Energy: By referring to the datum set in Fig. *b*, the gravitational potential energy of the cylinder at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 50(9.81)(0) = 0$ and $\left(V_g\right)_2 = mgh_2 = 50(9.81)(-\Delta s_A) = -490.5\Delta s_A$. When the cylinder is at positions (1) and (2), the stretch of the springs are $s_1 = \dfrac{F}{k} = \dfrac{60}{300} = 0.2$ m and $s_2 = s_1 + \Delta s_P = (0.2 + \Delta s_P)$ m. Thus, the elastic potential energy of the spring at these two instances are $\left(V_e\right)_1 = \dfrac{1}{2}ks_1{}^2 = \dfrac{1}{2}(300)(0.2^2) = 6$ J and $\left(V_e\right)_2 = \dfrac{1}{2}ks_2{}^2 = \dfrac{1}{2}(300)(0.2 + \Delta s_P)^2 = 150(0.2 + \Delta s_P)^2$.



(b)

Conservation of Energy: For the case when $\Delta s_A = 0.2$ m, from Eq. (1), we obtain $\Delta s_P + 2(0.2) = 0$ or $\Delta s_P = -0.4 \text{ m} = 0.4 \text{ m} \rightarrow$. We have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A\left(v_A\right)_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}m_A\left(v_A\right)_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 6\right) = \frac{1}{2}(50)(v_A)_2{}^2 + \left[-490.5(0.2) + 150\left(0.2 + 0.4\right)^2\right]$$

$$(v_A)_2 = 1.42 \text{ m/s} \qquad \textbf{Ans.}$$

For the case when the cylinder momentarily stops at position (2), from Eq. (1), $\Delta s_P = |-2\Delta s_A|$ Also, $(v_A)_2 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A\left(v_A\right)_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}m_A\left(v_A\right)_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 6\right) = 0 + \left[(-490.5\Delta s_A) + 150\left(0.2 + 2\Delta s_A\right)^2\right]$$

$$\Delta s_A = 0.6175 \text{ m} = 617.5 \text{ mm} \qquad \textbf{Ans.}$$

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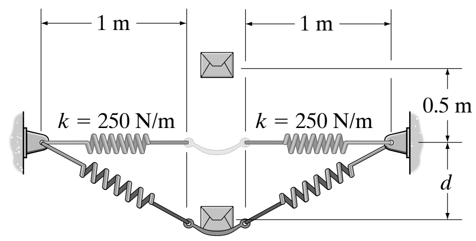
14–94. A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the box at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0) = 0$ and $\left(V_g\right)_2 = mgh_2 = 10(9.81)\left[-\left(0.5 + d\right)\right] = -98.1\left(0.5 + d\right)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $\left(V_e\right)_1 = (2)\dfrac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$\left(V_e\right)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2 + 1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

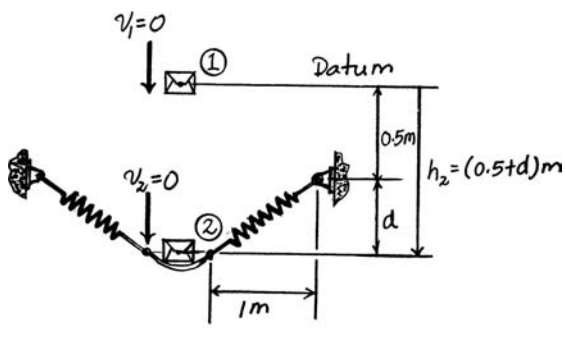
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 10\right) = 0 + \left[-98.1\left(0.5 + d\right) + 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2 + 1} + 350.95 = 0$$

Solving the above equation by trial and error,

$$d = 1.34 \text{ m} \qquad \textbf{Ans.}$$



(a)

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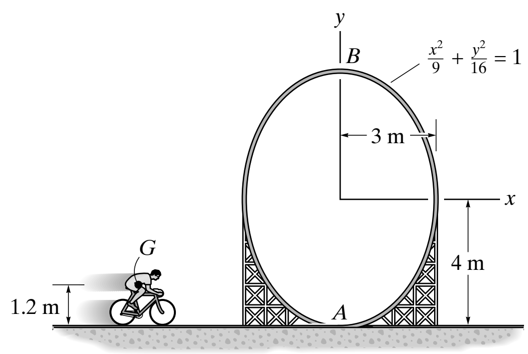
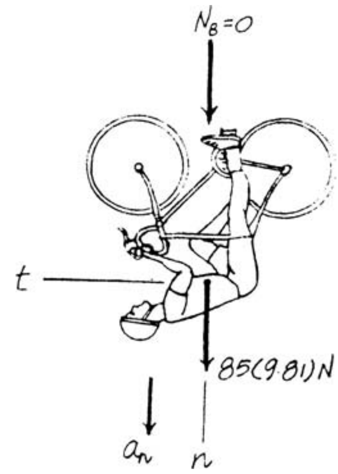
14–95. The man on the bicycle attempts to coast around the ellipsoidal loop without falling off the track. Determine the speed he must maintain at A just before entering the loop in order to perform the stunt. The bicycle and man have a total mass of 85 kg and a center of mass at G. Neglect the mass of the wheels.





Geometry: Here, $y = \frac{4}{3}\sqrt{9 - x^2}$.

$$\frac{dy}{dx} = -\frac{4}{3}x\left(9 - x^2\right)^{-1/2}\Big|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = -\frac{4}{3}\left[x^2\left(9 - x^2\right)^{-3/2} + (9 - x)^{-1/2}\right]\Bigg|_{x=0} = -0.4444$$

The slope angle θ at point B is given by

$$\tan\theta = \frac{dy}{dx}\bigg|_{x=0} = 0 \qquad \theta = 0°$$

and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left|\frac{\left[1 + 0^2\right]^{3/2}}{|-0.4444|}\right| = 2.25 \text{ m}$$

Since the center of mass for the cyclist is 1.2 m off the track, the radius of curvature for the cyclist is

$$\rho' = \rho - 1.2 = 1.05 \text{ m}$$

Equations of Motion: In order for the cyclist to just pass point B without falling off the track, it is required that $N_B = 0$. Applying Eq. 13–8 with $\theta = 0°$ and $\rho = 1.05$ m, we have

$$\Sigma F_n = ma_n; \qquad 85(9.81) = 85\left(\frac{v_B^2}{1.05}\right) \qquad v_B^2 = 10.30 \text{ m}^2/\text{s}^2$$

Potential Energy: Datum is set at the center of mass of the cyclist before he enters the track. When the cyclist is at point B, his position is $(8 - 1.2 - 1.2) = 5.6$ m *above* the datum. Thus, his gravitational potential energy at this point is $85(9.81)(5.6) = 4669.56$ J.

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(85)v_A^2 + 0 = \frac{1}{2}(85)(10.30) + 4669.56$$

$$v_A = 11.0 \text{ m/s}$$ **Ans.**

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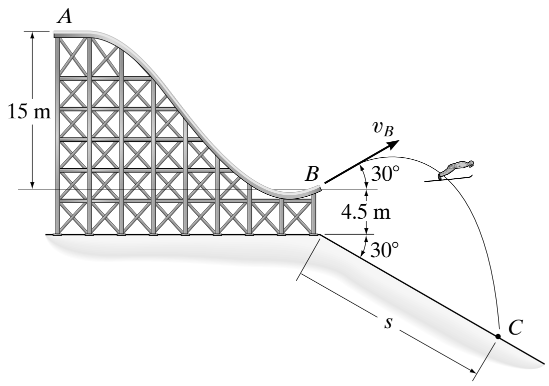
***14–96.** The 65-kg skier starts from rest at A. Determine his speed at B and the distance s where he lands at C. Neglect friction.



Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the skier at positions A and B are $(V_g)_A = mgh_A = 65(9.81)(0) = 0$ and $(V_g)_B = mgh_B = 65(9.81)(-15) = -9564.75$ J.

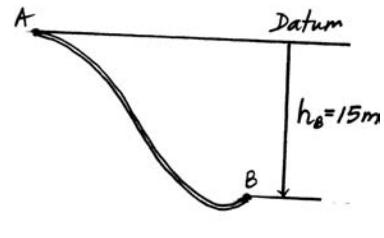
Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + (V_g)_A = \frac{1}{2}mv_B{}^2 + (V_g)_B$$

$$0 + 0 = \frac{1}{2}(65)v_B{}^2 + (-9564.75)$$

$$v_B = 17.16 \text{ m/s} = 17.2 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Kinematics: By considering the x-motion of the skier, Fig. b,

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_B)_x + (v_B)_x t$$

$$s \cos 30° = 0 + 17.16 \cos 30°(t)$$

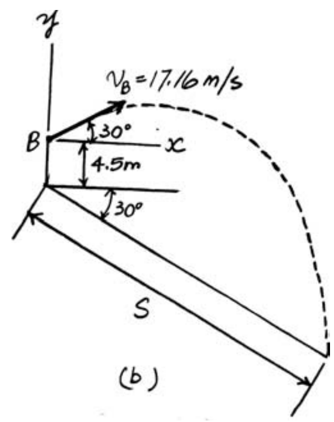
$$s = 17.16t \qquad \textbf{(1)}$$

By considering the y-motion of the skier, Fig. a,

$$(+\uparrow) \qquad s_y = (s_B)_y + (v_B)_y t + \frac{1}{2}a_y t^2$$

$$-(4.5 + s \sin 30°) = 0 + 17.16 \sin 30°\, t + \frac{1}{2}(-9.81)t^2$$

$$0 = 4.5 + 0.5s + 8.5776t - 4.905t^2 \qquad \textbf{(2)}$$



(b)

Solving Eqs. (1) and (2) yields

$$s = 64.2 \text{ m} \qquad \textbf{Ans.}$$

$$t = 3.743 \text{ s}$$

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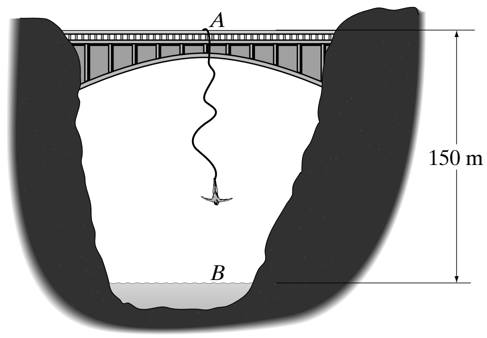
•**14–97.** The 75-kg man bungee jumps off the bridge at A with an initial downward speed of 1.5 m/s. Determine the required unstretched length of the elastic cord to which he is attached in order that he stops momentarily just above the surface of the water. The stiffness of the elastic cord is $k = 3$ kN/m. Neglect the size of the man.



Potential Energy: With reference to the datum set at the surface of the water, the gravitational potential energy of the man at positions A and B are $\left(V_g\right)_A = mgh_A = 75(9.81)(150) = 110362.5$ J and $\left(V_g\right)_B = mgh_B = 75(9.81)(0) = 0$. When the man is at position A, the elastic cord is unstretched ($s_A = 0$), whereas the elastic cord stretches $s_B = \left(150 - l_0\right)$ m, where l_0 is the unstretched length of the cord. Thus, the elastic potential energy of the elastic cord when the man is at these two positions are $\left(V_e\right)_A = \frac{1}{2}ks_A{}^2 = 0$ and $\left(V_e\right)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(3000)(150 - l_0)^2 = 1500(150 - l_0)^2$.

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + \left[\left(V_g\right)_A + \left(V_e\right)_A\right] = \frac{1}{2}mv_B{}^2 + \left[\left(V_g\right)_B + \left(V_e\right)_B\right]$$

$$\frac{1}{2}(75)(1.5^2) + \left(110362.5 + 0\right) = 0 + \left[0 + 1500(150 - l_0)^2\right]$$

Ans.

$$l_0 = 141 \text{ m}$$

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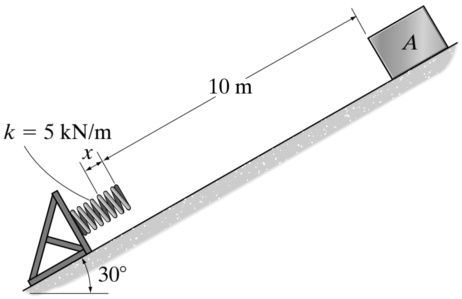
14–98. The 10-kg block A is released from rest and slides down the smooth plane. Determine the compression x of the spring when the block momentarily stops.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the block at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 10(9.81)\left[-(10 + x)\sin 30°\right] = -49.05(10 + x)$, respectively. The spring is unstretched initially, thus the initial elastic potential energy of the spring is $(V_e)_1 = 0$. The final elastic energy of the spring is $(V_e)_2 = \frac{1}{2}ks_2{}^2 = \frac{1}{2}(5)(10^3)x^2$ since it is being compressed $s_2 = x$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

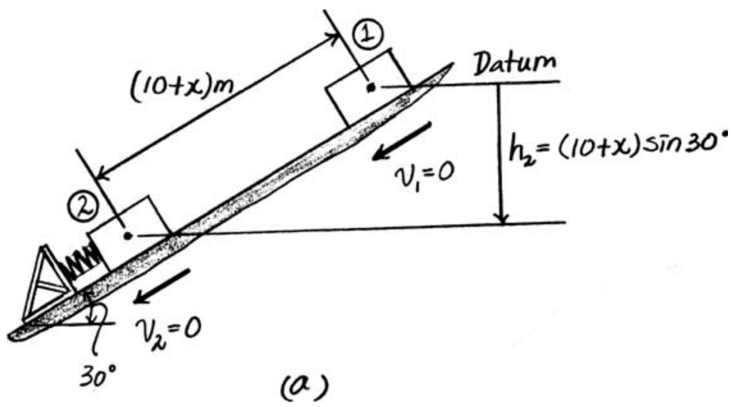
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + (V_e)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + (V_e)_2\right]$$

$$0 + (0 + 0) = 0 + \left[-49.05(10 + x)\right] + \frac{1}{2}(5)(10^3)x^2$$

$$2500x^2 - 49.05x - 490.5 = 0$$

Solving for the positive root of the above equation,

$$x = 0.4529 \text{ m} = 453 \text{ mm} \qquad \textbf{Ans.}$$



(a)

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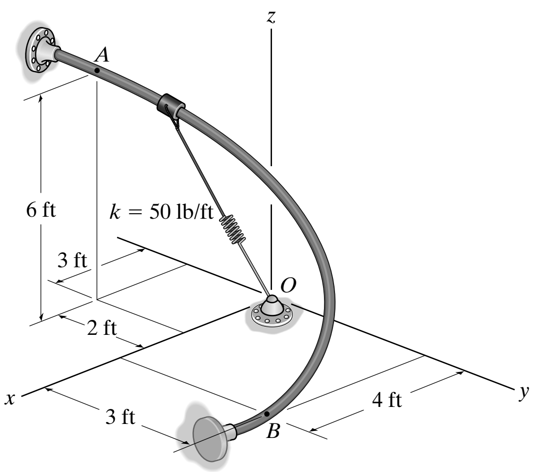
14–99. The 20-lb smooth collar is attached to the spring that has an unstretched length of 4 ft. If it is released from rest at position A, determine its speed when it reaches point B.



Potential Energy: With reference to the datum set at the x–y plane, the gravitational potential energy of the collar at positions A and B are $\left(V_g\right)_A = Wh_A = 20(6) = 120 \text{ ft} \cdot \text{lb}$ and $\left(V_g\right)_B = Wh_B = 20(0) = 0$. The stretch of the spring when the collar is at positions A and B are $s_A = OA - l_0 = \sqrt{(3-0)^2 + (-2-0)^2 + (6-0)^2} - 4 = 3 \text{ ft}$ and $s_B = OB - l_0 = \sqrt{(4-0)^2 + (3-0)^2} - 4 = 1 \text{ ft}$. Thus, the elastic potential energy of the spring when the collar is at these two positions are $\left(V_e\right)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(50)(3^2) = 225 \text{ ft} \cdot \text{lb}$ and $\left(V_e\right)_B = \frac{1}{2}ks B^2 = \frac{1}{2}(50)(1^2) = 25 \text{ ft} \cdot \text{lb}$.

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + \left[\left(V_g\right)_A + \left(V_e\right)_A\right] = \frac{1}{2}mv_B{}^2 + \left[\left(V_g\right)_B + \left(V_e\right)_B\right]$$

$$0 + \left(120 + 225\right) = \frac{1}{2}\left(\frac{20}{32.2}\right)v_B{}^2 + \left(0 + 25\right)$$

$$v_B = 32.1 \text{ ft/s} \qquad \textbf{Ans.}$$

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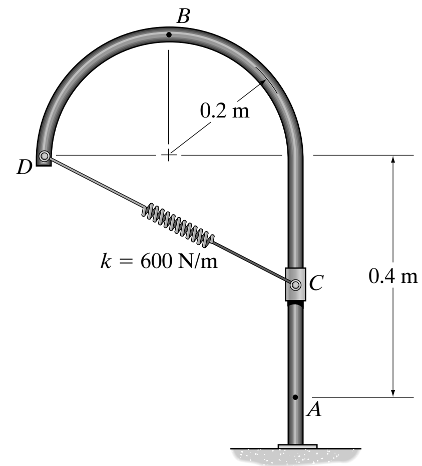
***14–100.** The 2-kg collar is released from rest at A and travels along the smooth vertical guide. Determine the speed of the collar when it reaches position B. Also, find the normal force exerted on the collar at this position. The spring has an unstretched length of 200 mm.

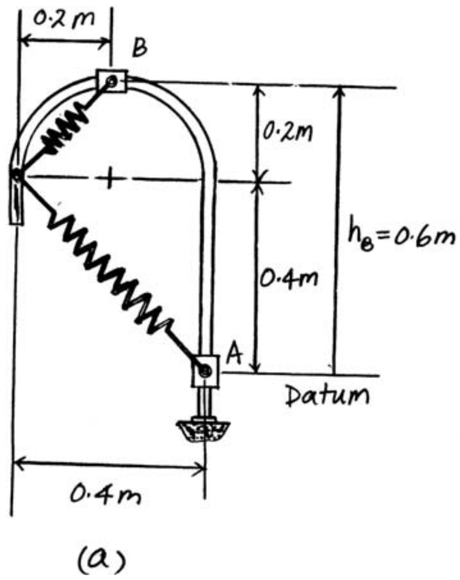


Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the collar at positions A and B are $(V_g)_A = mgh_A = 2(9.81)(0) = 0$ and $(V_g)_B = mgh_B = 2(9.81)(0.6) = 11.772$ J. When the collar is at positions A and B, the spring stretches $s_A = \sqrt{0.4^2 + 0.4^2} - 0.2 = 0.3657$ m and $s_B = \sqrt{0.2^2 + 0.2^2} - 0.2 = 0.08284$ m. Thus, the elastic potential energy of the spring when the collar is at these two positions are

$$(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(600)(0.3657^2) = 40.118 \text{ J}$$

and

$$(V_e)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(600)(0.08284^2) = 2.0589 \text{ J.}$$



Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

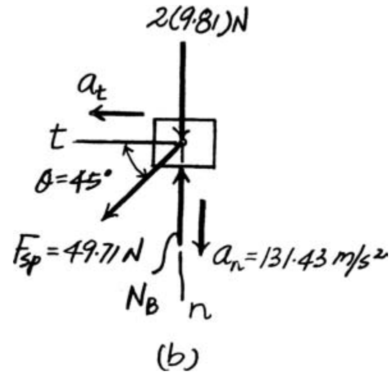
$$\frac{1}{2}mv_A{}^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2}mv_B{}^2 + \left[(V_g)_B + (V_e)_B\right]$$

$$0 + (0 + 40.118) = \frac{1}{2}(2)v_B{}^2 + (11.772 + 2.0589)$$

$$v_B = 5.127 \text{ m/s} = 5.13 \text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion: When the collar is at position B, $\theta = \tan^{-1}\left(\frac{0.2}{0.2}\right) = 45°$ and $F_{sp} = ks_B = 600(0.08284) = 49.71$ N. Here,

$$a_n = \frac{v^2}{\rho} = \frac{v_B{}^2}{0.2} = \frac{(5.127)^2}{0.2} = 131.43 \text{ m/s}^2.$$



By referring to the free-body diagram of the collar shown in Fig. b,

$$\Sigma F_n = ma_n; \qquad 2(9.81) + 49.71 \sin 45° - N_B = 2(131.43)$$

$$N_B = -208.09 \text{ N} = 208 \text{ N}\downarrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that $\mathbf{N}_B$ acts in the opposite sense to that shown on the free-body diagram.

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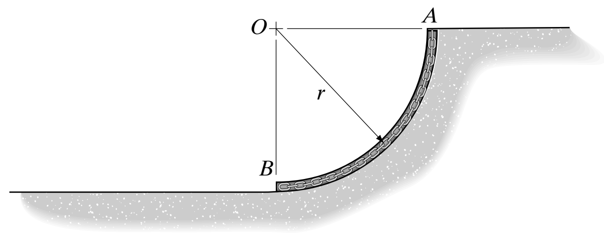
•14–101. A quarter-circular tube AB of mean radius r contains a smooth chain that has a mass per unit length of m_0. If the chain is released from rest from the position shown, determine its speed when it emerges completely from the tube.



Potential Energy: The location of the center of gravity G of the chain at positions (1) and (2) are shown in Fig. *a*. The mass of the chain is $m = m_0\left(\frac{\pi}{2}r\right) = \frac{\pi}{2}m_0 r$. Thus, the center of mass is at $h_1 = r - \frac{2r}{\pi} = \left(\frac{\pi - 2}{\pi}\right)r$. With reference to the datum set in Fig. *a* the gravitational potential energy of the chain at positions (1) and (2) are

$$\left(V_g\right)_1 = mgh_1 = \left(\frac{\pi}{2}m_0 rg\right)\left(\frac{\pi - 2}{\pi}\right)r = \left(\frac{\pi - 2}{2}\right)m_0 r^2 g$$

and

$$\left(V_g\right)_2 = mgh_2 = 0$$

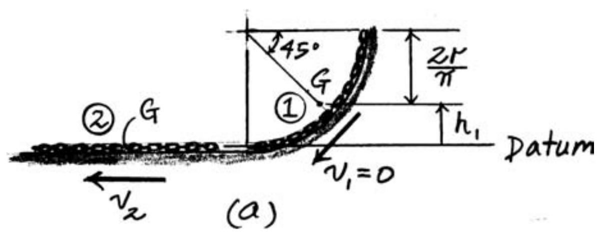
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^{\,2} + \left(V_g\right)_1 = \frac{1}{2}mv_2^{\,2} + \left(V_g\right)_2$$

$$0 + \left(\frac{\pi - 2}{2}\right)m_0 r^2 g = \frac{1}{2}\left(\frac{\pi}{2}m_0 r\right)v_2^{\,2} + 0$$

$$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr} \qquad \textbf{Ans.}$$



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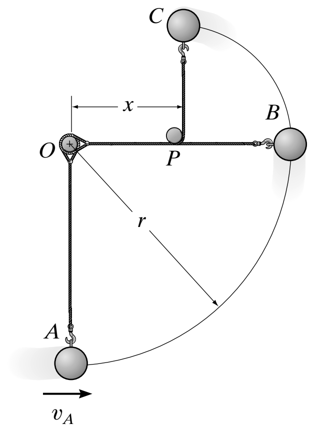
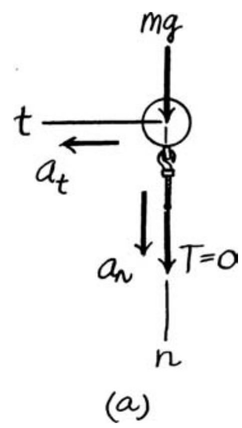
14–102. The ball of mass m is given a speed of $v_A = \sqrt{3gr}$ at position A. When it reaches B, the cord hits the small peg P, after which the ball describes a smaller circular path. Determine the position x of P so that the ball will just be able to reach point C.



Equation of Motion: If the ball is just about to complete the small circular path, the cord will become slack at position C, i.e., $T = 0$. Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v_C{}^2}{r - x}$. By referring to the free-body diagram of the ball shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad mg = m\left(\frac{v_C{}^2}{r - x}\right) \qquad v_C{}^2 = g(r - x) \qquad \textbf{(1)}$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the ball at positions A and C are $\left(V_g\right)_A = mgh_A = mg(0) = 0$ and $\left(V_g\right)_C = mgh_C = mg(2r - x)$.

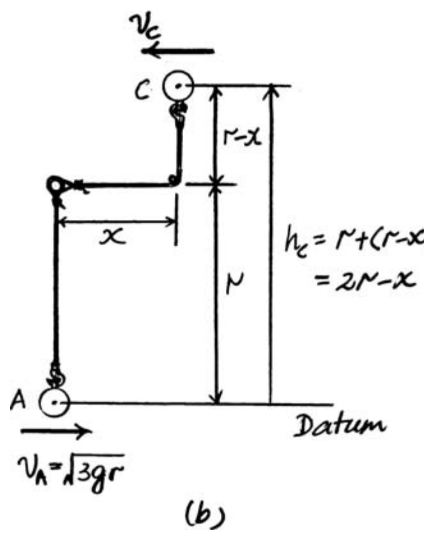
Conservation of Energy:

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A{}^2 + \left(V_g\right)_A = \frac{1}{2}mv_C{}^2 + \left(V_g\right)_C$$

$$\frac{1}{2}m(3gr) + 0 = \frac{1}{2}mv_C{}^2 + mg(2r - x)$$

$$v_C{}^2 = g(2x - r) \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$x = \frac{2}{3}r \qquad \textbf{Ans.}$$

$$v_C = \sqrt{\frac{1}{2}gr}$$

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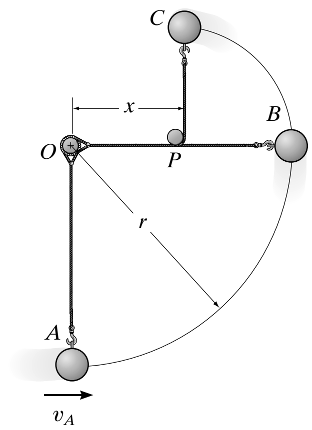
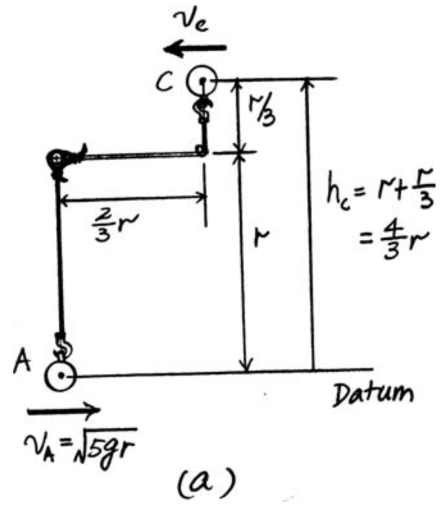
14–103. The ball of mass m is given a speed of $v_A = \sqrt{5gr}$ at position A. When it reaches B, the cord hits the peg P, after which the ball describes a smaller circular path. If $x = \frac{2}{3}r$, determine the speed of the ball and the tension in the cord when it is at the highest point C.



Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the ball at positions A and C are $\left(V_g\right)_A = mgh_A = mg(0) = 0$ and $\left(V_g\right)_C = mgh_C = mg\left(\frac{4}{3}r\right) = \frac{4}{3}mgr$.



Conservation of Energy:

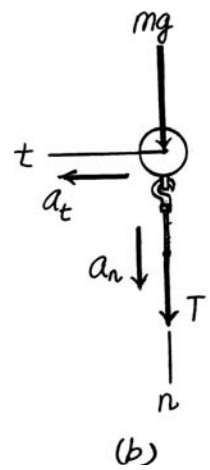
$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A{}^2 + \left(V_g\right)_A = \frac{1}{2}mv_C{}^2 + \left(V_g\right)_C$$

$$\frac{1}{2}m(5gr) + 0 = \frac{1}{2}mv_C{}^2 + \frac{4}{3}mgr$$

$$v_C = \sqrt{\frac{7}{3}gr} \qquad \textbf{Ans.}$$

Equations of Motion: Here, $a_n = \dfrac{v_C{}^2}{\rho} = \dfrac{\frac{7}{3}gr}{r/3}$. By referring to the free-body diagram of the ball shown in Fig. b,



$\Sigma F_n = ma_n;\qquad T + mg = m(7g)$

$$T = 6mg \qquad \textbf{Ans.}$$

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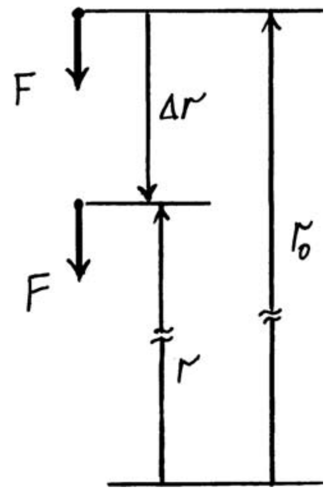
***14–104.** If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum an "infinite" distance from the earth. Also, prove that **F** is a conservative force.



The work is computed by moving F from position r_1 to a farther position r_2.

$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m\int_{r1}^{r2}\frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As $r_1 \rightarrow \infty$, let $r_2 = r_1$, $F_2 = F_1$, then

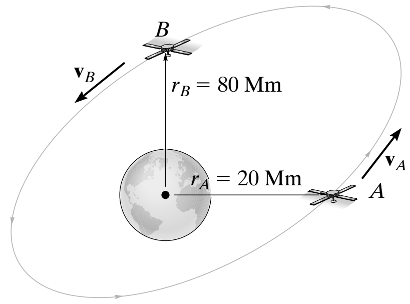
$$V_g \rightarrow \frac{-G\,M_e\,m}{r}$$

To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

$$= \frac{-G\,M_e\,m}{r^2} \qquad \textbf{Q.E.D.}$$

•14–105. A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–104, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.



$v_A = 40\ \text{Mm/h} = 11\,111.1\ \text{m/s}$

Since $V = -\dfrac{GM_e\,m}{r}$

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$v_B = 9672\ \text{m/s} = 34.8\ \text{Mm/h}$ **Ans.**

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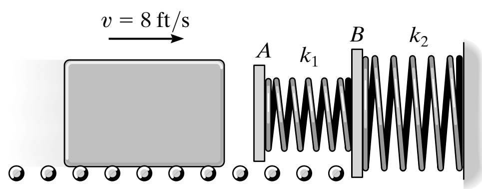
14–106. The double-spring bumper is used to stop the 1500-lb steel billet in the rolling mill. Determine the maximum displacement of the plate A if the billet strikes the plate with a speed of 8 ft/s. Neglect the mass of the springs, rollers and the plates A and B. Take $k_1 = 3000$ lb/ft, $k_2 = 45000$ lb/ft.



$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{1500}{32.2}\right)(8)^2 + 0 = 0 + \frac{1}{2}(3000)s_L^2 + \frac{1}{2}(4500)s\frac{2}{2} \qquad \textbf{(1)}$$

$$F_s = 3000s_1 = 4500s_2; \qquad \textbf{(2)}$$

$s_1 = 1.5s_2$

Solving Eqs. (1) and (2) yields:

$s_2 = 0.5148$ ft $\qquad s_1 = 0.7722$ ft

$s_A = s_1 + s_2 = 0.7722 + 0.5148 = 1.29$ ft **Ans.**

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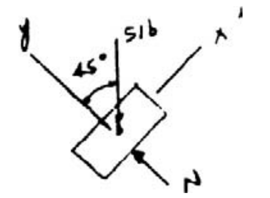
•15–1. A 5-lb block is given an initial velocity of 10 ft/s up a 45° smooth slope. Determine the time for it to travel up the slope before it stops.

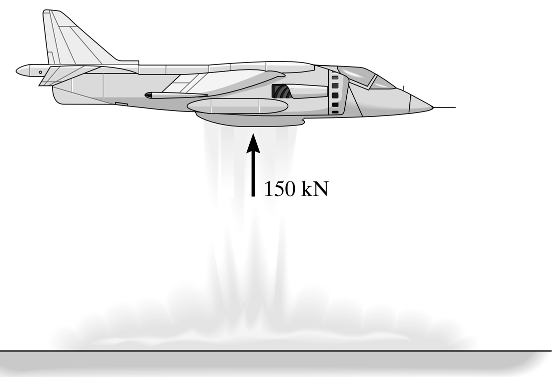


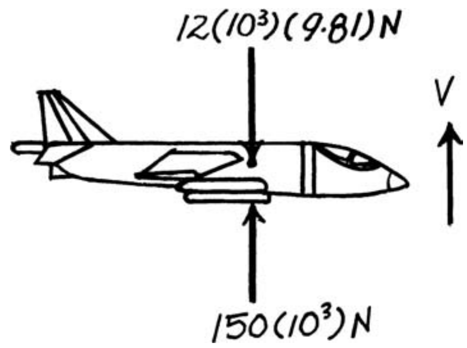
$(\nearrow+)\qquad m(v_{x'})_1 + \Sigma \int_{t_1}^{-t_2} F_x\, dt = m(v_{x'})_2$

$\frac{5}{32.2}(10) + (-5 \sin 45°)t = 0$

$t = 0.439$ s **Ans.**

15–2. The 12-Mg "jump jet" is capable of taking off vertically from the deck of a ship. If its jets exert a constant vertical force of 150 kN on the plane, determine its velocity and how high it goes in $t = 6$ s, starting from rest. Neglect the loss of fuel during the lift.





$(+\uparrow)\qquad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$

$0 + 150(10^3)(6) - 12(10^3)(9.81)(6) = 12(10^3)v$

$v = 16.14 \text{ m/s} = 16.1 \text{ m/s}$ **Ans.**

$(+\uparrow)\qquad v = v_0 + a_c t$

$16.14 = 0 + a(6)$

$a = 2.690 \text{ m/s}^2$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s = 0 + 0 + \frac{1}{2}(2.690)(6)^2$

$s = 48.4$ m **Ans.**

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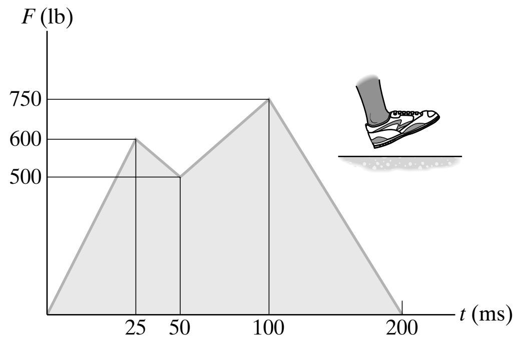
15–3. The graph shows the vertical reactive force of the shoe-ground interaction as a function of time. The first peak acts on the heel, and the second peak acts on the forefoot. Determine the total impulse acting on the shoe during the interaction.



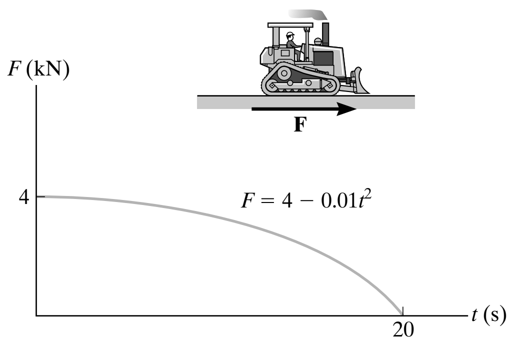
Impulse: The total impluse acting on the shoe can be obtained by evaluating the area under the $F-t$ graph.

$$I = \frac{1}{2}(600)\left[25\left(10^{-3}\right)\right] + \frac{1}{2}(500 + 600)(50 - 25)\left(10^{-3}\right)$$

$$+ \frac{1}{2}(500 + 750)(100 - 50)\left(10^{-3}\right) + \frac{1}{2}(750)\left[(200 - 100)\left(10^{-3}\right)\right]$$

$$= 90.0 \text{ lb} \cdot \text{s}$$ **Ans.**

***15–4.** The 28-Mg bulldozer is originally at rest. Determine its speed when $t = 4$ s if the horizontal traction $\mathbf{F}$ varies with time as shown in the graph.



$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + \int_0^4 (4 - 0.01t^2)(10^3)dt = 28(10^3)v$$

$$v = 0.564 \text{ m/s}$$ **Ans.**

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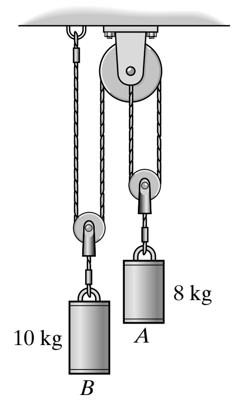
•15–5. If cylinder A is given an initial downward speed of 2 m/s, determine the speed of each cylinder when $t = 3$ s. Neglect the mass of the pulleys.



Free-Body Diagram: The free-body diagram of blocks A and B are shown in Figs. *b* and *c*, respectively. Here, the final velocity of blocks A and B, $(\mathbf{v}_A)_2$ and $(\mathbf{v}_B)_2$ must be assumed to be directed downward so that they are consistent with the positive sense of s_A and s_B shown in Fig. *a*.

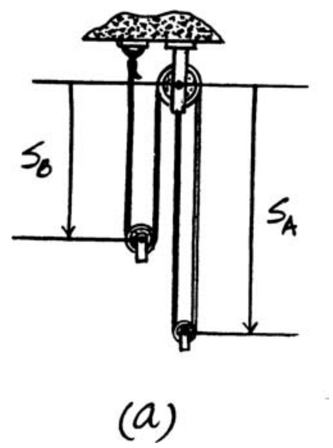
Kinematics: Expressing the length of the cable in terms of s_A and s_B by referring to Fig. *a*,

$$2s_A + 2s_B = l$$

$$s_A + s_B = l/2 \qquad \textbf{(1)}$$

Taking the time derivative of Eq. (1), we obtain

$$(+\downarrow) \qquad v_A + v_B = 0 \qquad \textbf{(2)}$$



Principle of Impulse and Momentum: Initially, the velocity of block A is directed downward. Thus, $(v_A)_1 = 2$ m/s $\downarrow$.

From Eq. (2),

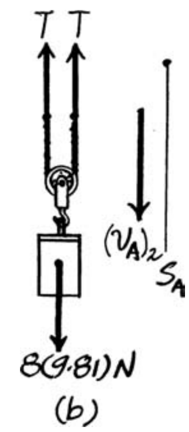
$$(+\downarrow) \qquad 2 + (v_B)_1 = 0 \qquad (v_B)_1 = -2 \text{ m/s} = 2 \text{ m/s} \uparrow$$

By referring to Fig. *b*,

$$(+\uparrow) \qquad m(v_A)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_A)_2$$

$$8(-2) + 2T(3) - 8(9.81)(3) = 8\left[-(v_A)_2\right]$$

$$6T = 251.44 - 8(v_A)_2 \qquad \textbf{(3)}$$



By referring Fig. *c*,

$$(+\uparrow) \qquad m(v_B)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_B)_2$$

$$10(2) + 2T(3) - 10(9.81)(3) = 10\left[-(v_B)_2\right]$$

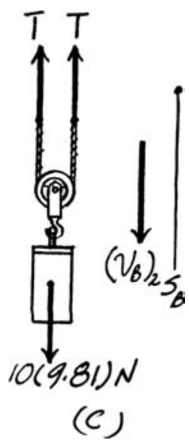
$$6T = 274.3 - 10(v_B)_2 \qquad \textbf{(4)}$$

Solving Eqs. (2), (3), and (4),

$$(v_A)_2 = -1.27 \text{ m/s} = 1.27 \text{ m/s}\uparrow \qquad \textbf{Ans.}$$

$$(v_B)_2 = 1.27 \text{ m/s}\downarrow \qquad \textbf{Ans.}$$

$$T = 43.6 \text{ N}$$



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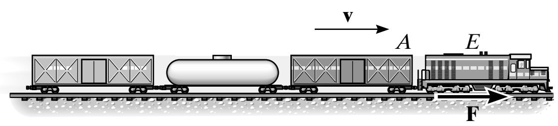
15–6. A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.



$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

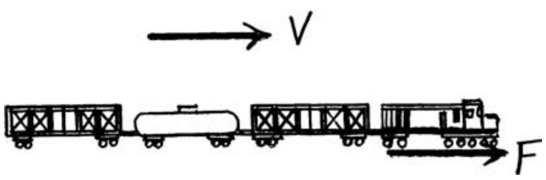
$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

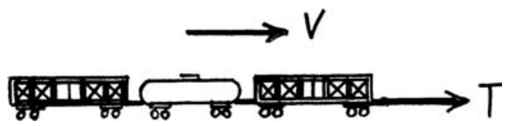
$$F = 19.4 \text{ kN} \qquad \textbf{Ans.}$$

Three cars:

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN} \qquad \textbf{Ans.}$$





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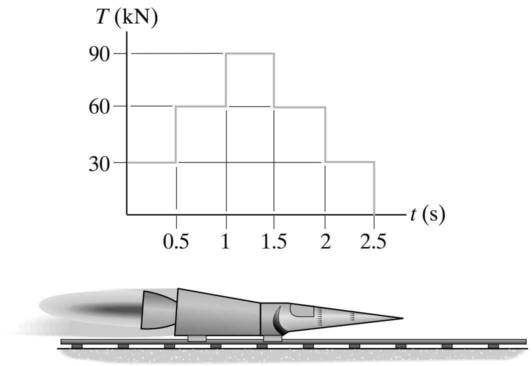
15–7. Determine the maximum speed attained by the 1.5-Mg rocket sled if the rockets provide the thrust shown in the graph. Initially, the sled is at rest. Neglect friction and the loss of mass due to fuel consumption.



Principle of Impulse and Momentum: The graph of thrust **T** vs. time t due to the successive ignition of the rocket is shown in Fig. *a*. The sled attains its maximum speed at the instant that all the rockets burn out their fuel, that is, at $t = 2.5$ s. The impulse generated by **T** during $0 \leq t \leq 2.5$ s is equal to the area under the T vs t graphs. Thus,

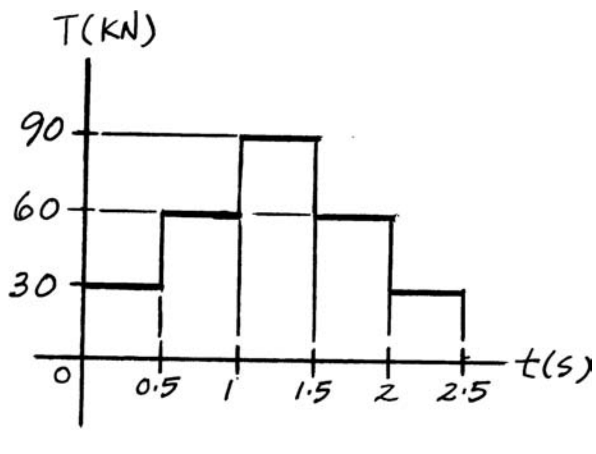
$$I = \int T\,dt = 30(10^3)(0.5 - 0) + 60(10^3)(1 - 0.5) + 90(10^3)(1.5 - 1)$$
$$+ 60(10^3)(2 - 1.5) + 30(10^3)(25 - 2) = 135\,000 \text{ N} \cdot \text{s}$$

By referring to the free-body diagram of the sled shown in Fig. *a*,

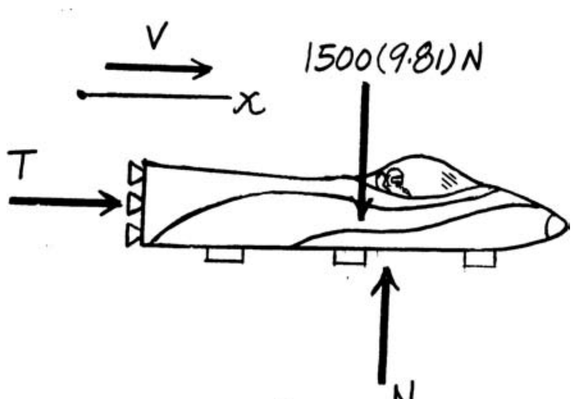
$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int F_x dt = m(v_2)_x$$

$$1500(0) + 135000 = 1500 v_{\max}$$

$$v_{\max} = 90 \text{ m/s} \qquad \textbf{Ans.}$$



(a)



(b)

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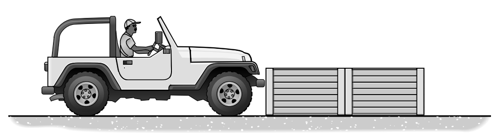
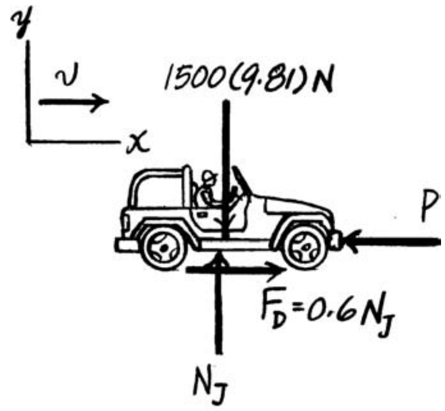
***15–8.** The 1.5-Mg four-wheel-drive jeep is used to push two identical crates, each having a mass of 500 kg. If the coefficient of static friction between the tires and the ground is $\mu_s = 0.6$, determine the maximum possible speed the jeep can achieve in 5 s without causing the tires to slip. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.3$.



Free-Body Diagram: The free-body diagram of the jeep and crates are shown in Figs. *a* and *b*, respectively. Here, the maximum driving force for the jeep is equal to the maximum static friction between the tires and the ground, i.e., $F_D = \mu_s N_J = 0.6N_J$. The frictional force acting on the crate is $(F_f)_C = \mu_k N_C = 0.3N_C$.



(a)

Principle of Impulse and Momentum: By referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$1500(0) + N_J(5) - 1500(9.81)(5) = 1500(0)$$

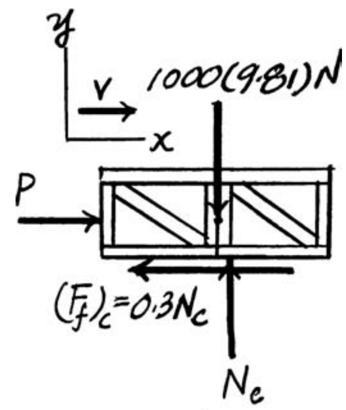
$$N_J = 14715 \text{ N}$$

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$1500(0) + 0.6(14715)(5) - P(5) = 1500v$$

$$v = 29.43 - 3.333(10^{-3})P \qquad \textbf{(1)}$$

By considering Fig. *b*,



(b)

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$1000(0) + N_C(5) - 1000(9.81)(5) = 1000(0)$$

$$N_C = 9810 \text{ N}$$

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$1000(0) + P(5) - 0.3(9810)(5) = 1000v$$

$$v = 0.005P - 14.715 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v = 11.772 \text{ m/s} = 11.8 \text{ m/s} \qquad \textbf{Ans.}$$

$$P = 5297.4 \text{ N}$$

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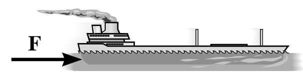
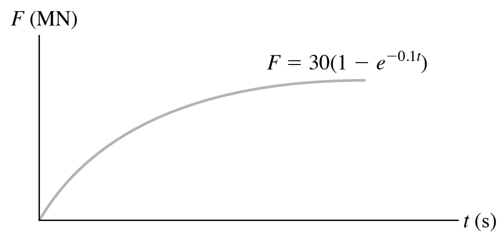
•15–9. The tanker has a mass of 130 Gg. If it is originally at rest, determine its speed when $t = 10$ s. The horizontal thrust provided by its propeller varies with time as shown in the graph. Neglect the effect of water resistance.



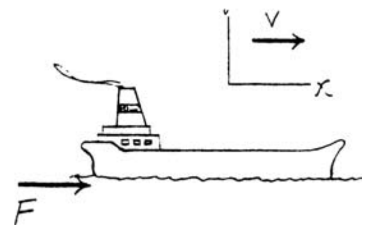


Principle of Linear Impulse and Momentum: Applying Eq. 15–4, we have

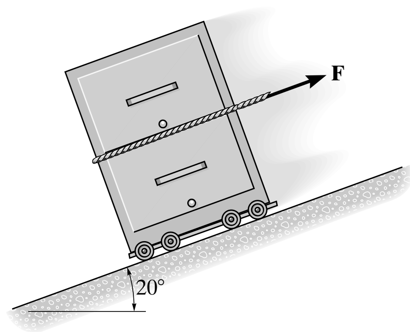
$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 0 + \int_0^{10s} 30\left(10^6\right)\left(1 - e^{-0.1t}\right)dt = 0.130\left(10^9\right) v$$

$$v = 0.849 \text{ m/s}$$ **Ans.**



15–10. The 20-lb cabinet is subjected to the force $F = (3 + 2t)$ lb, where t is in seconds. If the cabinet is initially moving down the plane with a speed of 6 ft/s, determine how long for the force to bring the cabinet to rest. **F** always acts parallel to the plane.



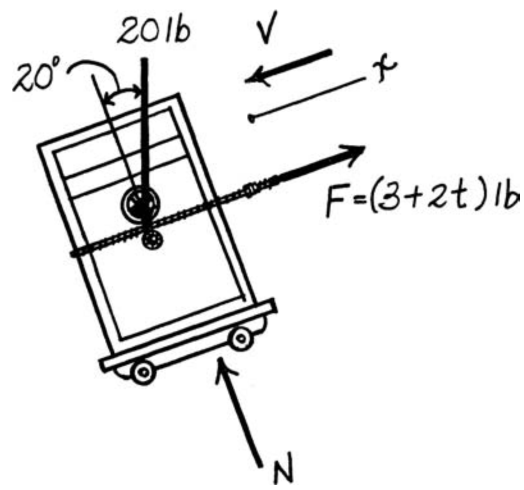
$$(+\swarrow) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$\left(\frac{20}{32.2}\right)(6) + 20(\sin 20°)t - \int_0^t (3 + 2t)\, dt = 0$$

$$3.727 + 3.840t - t^2 = 0$$

Solving for the positive root,

$t = 4.64$ s **Ans.**



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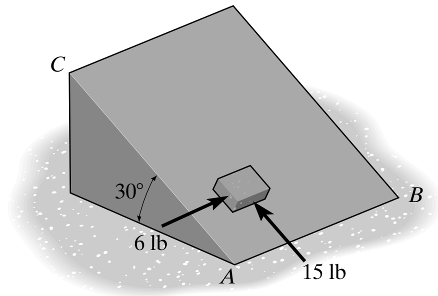
15–11. The small 20-lb block is placed on the inclined plane and subjected to 6-lb and 15-lb forces that act parallel with edges AB and AC, respectively. If the block is initially at rest, determine its speed when $t = 3$ s. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



Free-Body Diagram: Here, the x–y plane is set parallel with the inclined plane. Thus, the z axis is perpendicular to the inclined plane. The frictional force will act along but in the opposite sense to that of the motion, which makes an angle θ with the x axis. Its magnitude is $F_f = \mu_k N = 0.2N$.

Principle of Impulse and Momentum: By referring to Fig. a,

$$m\left(v_1\right)_z + \Sigma \int_{t_1}^{t_2} F_z\, dt = m\left(v_2\right)_z$$

$$\frac{20}{32.2}(0) + N(3) - 20\cos 30°(3) = \frac{20}{32.2}(0)$$

$$N = 17.32 \text{ lb}$$

and

$$m\left(v_1\right)_x + \Sigma \int_{t_1}^{t_2} F_x\, dt = m\left(v_2\right)_x$$

$$\frac{20}{32.2}(0) + 6(3) - \left[0.2(17.32)\cos\theta\right](3) = \; = \frac{20}{32.2}(v\cos\theta)$$

$$\cos\theta(v + 16.73) = 28.98 \qquad \textbf{(1)}$$

and

$$m\left(v_1\right)_y + \Sigma \int_{t_1}^{t_2} F_y\, dt = m\left(v_2\right)_y$$

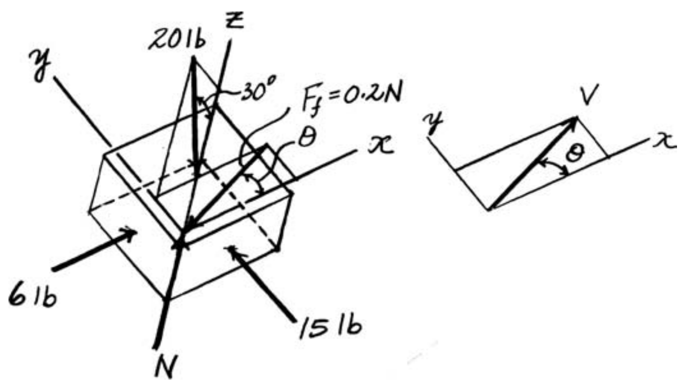
$$\frac{20}{32.2}(0) + 15(3) - (20\sin 30°)(3) - \left[0.2(17.32)\sin\theta\right](3) = \frac{20}{32.2}(v\sin\theta)$$

$$\sin\theta(v + 16.73) = 24.15 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$\theta = 39.80°$$

$$v = 20.99 \text{ ft/s} = 21.0 \text{ ft/s} \qquad \textbf{Ans.}$$



(a)

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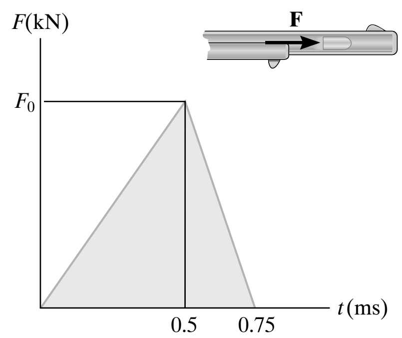
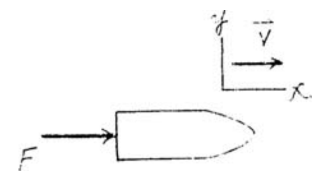
***15–12.** Assuming that the force acting on a 2-g bullet, as it passes horizontally through the barrel of a rifle, varies with time in the manner shown, determine the maximum net force F_0 applied to the bullet when it is fired. The muzzle velocity is 500 m/s when $t = 0.75$ ms. Neglect friction between the bullet and the rifle barrel.



Principle of Linear Impulse and Momentum: The total impluse acting on the bullet can be obtained by evaluating the area under the F–t graph. Thus, $I = \Sigma\int_{t_1}^{t_2} F_x\,dt = \frac{1}{2}(F_0)\left[0.5\left(10^{-3}\right)\right] + \frac{1}{2}(F_0)\left[(0.75 - 0.5)\left(10^{-3}\right)\right]$ $= 0.375\left(10^{-3}\right) F_0$. Applying Eq. 15–4, we have

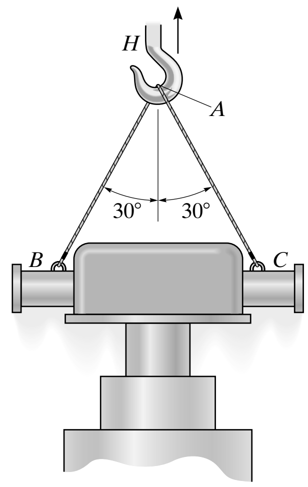


$$m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$(\xrightarrow{+})\qquad 0 + 0.375\left(10^{-3}\right) F_0 = 2\left(10^{-3}\right)(500)$$

$$F_0 = 2666.67\text{ N} = 2.67\text{ kN}$$ **Ans.**

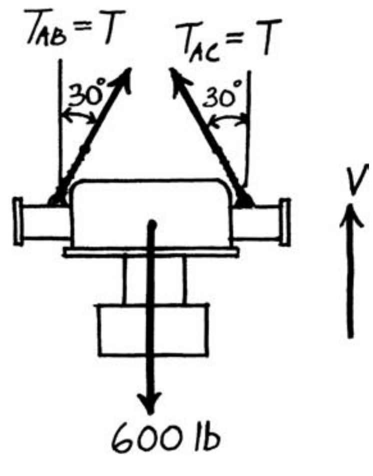
•15–13. The fuel-element assembly of a nuclear reactor has a weight of 600 lb. Suspended in a vertical position from H and initially at rest, it is given an upward speed of 5 ft/s in 0.3 s. Determine the average tension in cables AB and AC during this time interval.



$$(+\uparrow)\qquad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$$

$$0 + 2(T\cos 30°)(0.3) - 600(0.3) = \left(\frac{600}{32.2}\right)(5)$$

$$T = 526\text{ lb}$$ **Ans.**



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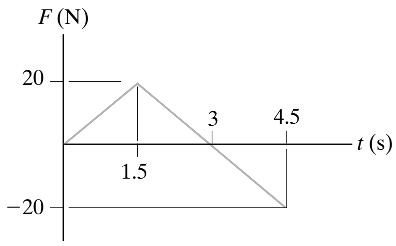
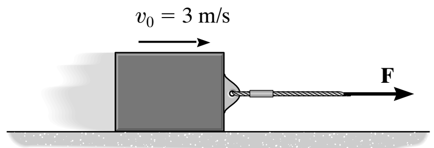
15–14. The 10-kg smooth block moves to the right with a velocity of $v_0 = 3$ m/s when force **F** is applied. If the force varies as shown in the graph, determine the velocity of the block when $t = 4.5$ s.



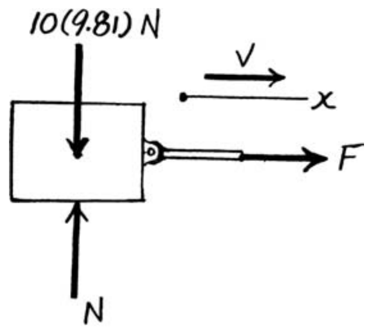


Principle of Impulse and Momentum: The impulse generated by force **F** during $0 \le t \le 4.5$ is equal to the area under the **F** vs. t graph, i.e., $I = \int F dt = \frac{1}{2}(20)(3-0) + \left[-\frac{1}{2}(20)(4.5-3)\right] = 15 \text{ N} \cdot \text{s}$. Referring to the free-body diagram of the block shown in Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

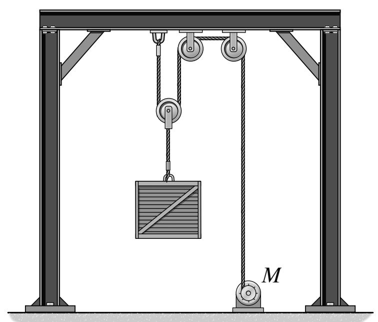
$$10(3) + 15 = 10v$$

$$v = 4.50 \text{ m/s}$$ **Ans.**



(a)

15–15. The 100-kg crate is hoisted by the motor M. If the velocity of the crate increases uniformly from 1.5 m/s to 4.5 m/s in 5 s, determine the tension developed in the cable during the motion.

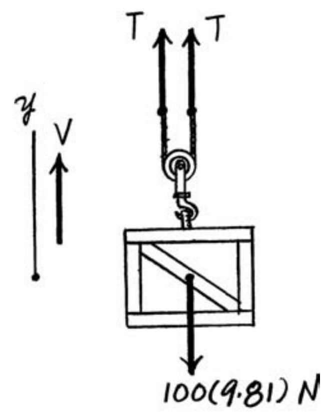


Principle of Impulse and Momentum: By referring to the free-body diagram of the crate shown in Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$100(1.5) + 2T(5) - 100(9.81)(5) = 100(4.5)$$

$$T = 520.5 \text{ N}$$ **Ans.**



(a)

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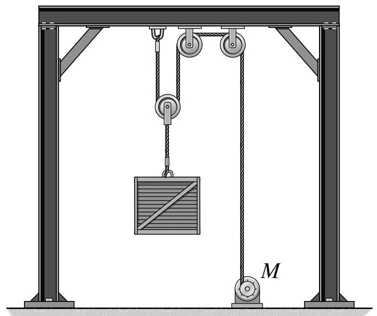
***15–16.** The 100-kg crate is hoisted by the motor M. The motor exerts a force on the cable of $T = (200t^{1/2} + 150)$ N, where t is in seconds. If the crate starts from rest at the ground, determine the speed of the crate when $t = 5$ s.



Free-Body Diagram: Here, force 2**T** must overcome the weight of the crate before it moves. By considering the equilibrium of the free-body diagram of the crate shown in Fig. a,

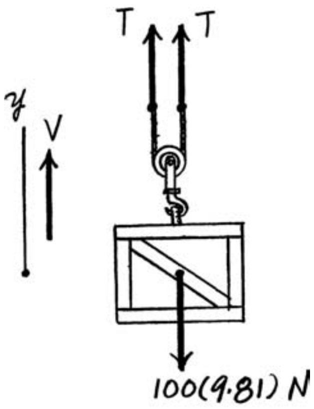
$+\uparrow \Sigma F_y = 0; \qquad 2\left(200t^{1/2} + 150\right) - 100(9.81) = 0 \qquad t = 2.8985 \text{ s}$

Principle of Impulse and Momentum: Here, only the impulse generated by force 2**T** after $t = 2.8186$ s contributes to the motion. Referring to Fig. a,

$$(+\uparrow) \qquad m\left(v_1\right)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m\left(v_2\right)_y$$

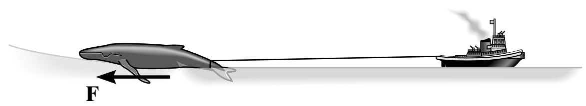
$$100(0) + 2\int_{2.898\text{ s}}^{5\text{s}}\left(200t^{1/2} + 150\right)dt - 100(9.81)(5 - 2.8985) = 100v$$

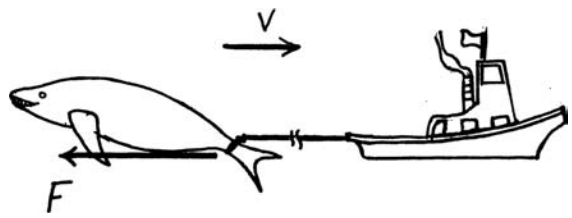
$$v = 2.34 \text{ m/s} \qquad \textbf{Ans.}$$

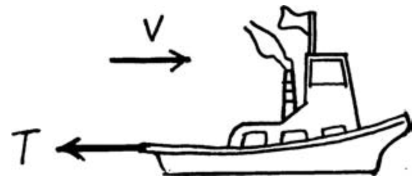


(a)

•15–17. The 5.5-Mg humpback whale is stuck on the shore due to changes in the tide. In an effort to rescue the whale, a 12-Mg tugboat is used to pull it free using an inextensible rope tied to its tail. To overcome the frictional force of the sand on the whale, the tug backs up so that the rope becomes slack and then the tug proceeds forward at 3 m/s. If the tug then turns the engines off, determine the average frictional force **F** on the whale if sliding occurs for 1.5 s before the tug stops after the rope becomes taut. Also, what is the average force on the rope during the tow?







$$\left(\xrightarrow{+}\right) \qquad m_1\,(v_x)_1 + \Sigma\int F_x\,dt = m_2(v_x)_2$$

$$0 + 12\left(10^3\right)(3) - F(1.5) = 0 + 0$$

$$F = 24 \text{ kN} \qquad \textbf{Ans.}$$

Tug:

$$\left(\xrightarrow{+}\right) \qquad m\,(v_x)_1 + \Sigma\int F_x\,dt = m\,(v_x)_2$$

$$12\left(10^3\right)(3) - T(1.5) = 0$$

$$T = 24 \text{ kN} \qquad \textbf{Ans.}$$

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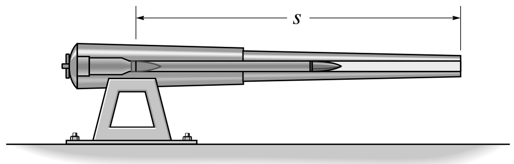
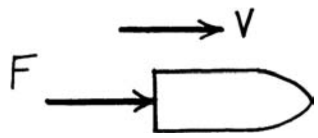
15–18. The force acting on a projectile having a mass m as it passes horizontally through the barrel of the cannon is $F = C \sin(\pi t/t')$. Determine the projectile's velocity when $t = t'$. If the projectile reaches the end of the barrel at this instant, determine the length s.





$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + \int_0^t C \sin\left(\frac{\pi t}{t'}\right) = \text{m}v$$

$$-C\left(\frac{t'}{\pi}\right)\cos\left(\frac{\pi t}{t'}\right)\Big|_0^t = \text{m}v$$

$$v = \frac{Ct'}{\pi m}\left(1 - \cos\left(\frac{\pi t}{t'}\right)\right)$$

When $t = t'$,

$$v_2 = \frac{2C\,t'}{\pi m} \qquad \text{`Ans.}$$

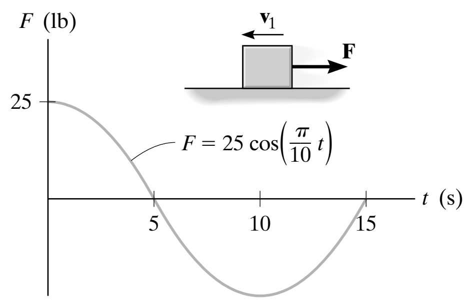
$$ds = v\,dt$$

$$\int_0^s ds = \int_0^{t'}\left(\frac{C\,t'}{\pi\,m}\right)\left(1 - \cos\left(\frac{\pi t}{t'}\right)\right)\text{dt}$$

$$s = \left(\frac{C\,t'}{\pi m}\right)\left[t - \frac{t'}{\pi}\sin\left(\frac{\pi t}{t'}\right)\right]_0^{t'}$$

$$s = \frac{Ct'^2}{\pi m} \qquad \textbf{Ans.}$$

15–19. A 30-lb block is initially moving along a smooth horizontal surface with a speed of $v_1 = 6$ ft/s to the left. If it is acted upon by a force **F**, which varies in the manner shown, determine the velocity of the block in 15 s.



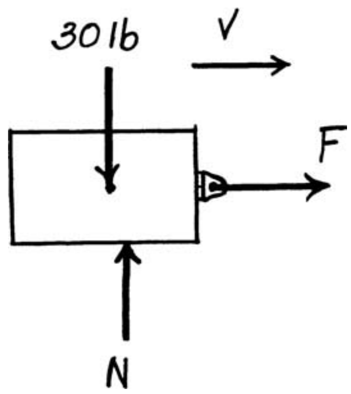
$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$-\left(\frac{30}{32.2}\right)(6) + \int_0^{15} 25\cos\left(\frac{\pi}{10}t\right)dt = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$-5.59 + (25)\left[\sin\left(\frac{\pi}{10}t\right)\right]_0^{15}\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$-5.59 + (25)[-1]\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$(v_x)_2 = -91.4 = 91.4 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$



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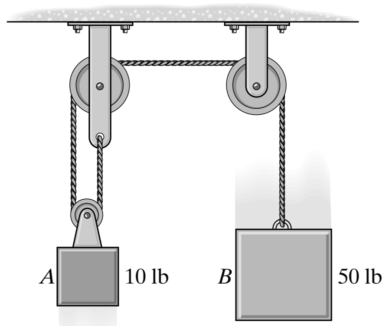
***15–20.** Determine the velocity of each block 2 s after the blocks are released from rest. Neglect the mass of the pulleys and cord.



Kinematics: The speed of block A and B can be related by using the position coordinate equation.

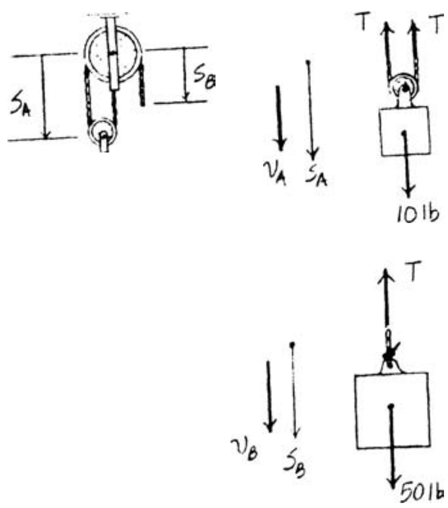
$$2s_A + s_B = l$$

$$2v_A + v_B = 0 \qquad \textbf{[1]}$$

Principle of Linear Impulse and Momentum: Applying Eq. 15–4 to block A, we have

$$m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$(+\uparrow) \qquad -\left(\frac{10}{32.2}\right)(0) + 2T(2) - 10(2) = -\left(\frac{10}{32.2}\right)(v_A) \qquad \textbf{[2]}$$



Applying Eq. 15–4 to block B, we have

$$m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

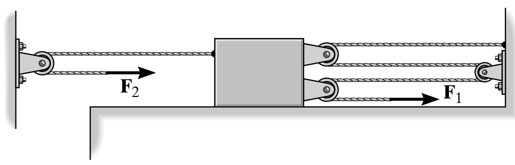
$$(+\uparrow) \qquad -\left(\frac{50}{32.2}\right)(0) + T(2) - 50(2) = -\left(\frac{50}{32.2}\right)(v_B) \qquad \textbf{[3]}$$

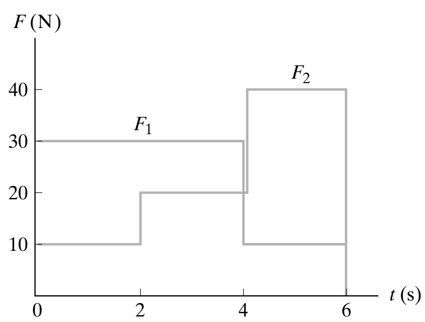
Solving Eqs. [1], [2] and [3] yields

$$v_A = -27.6 \text{ ft/s} = 27.6 \text{ ft/s} \uparrow \qquad v_B = 55.2 \text{ ft/s} \downarrow \qquad \textbf{Ans.}$$

$$T = 7.143 \text{ lb}$$

•15–21. The 40-kg slider block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.



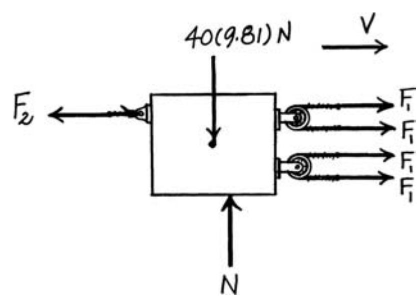


The impulses acting on the block are equal to the areas under the graph.

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$40(1.5) + 4[(30)4 + 10(6-4)] - [10(2) + 20(4-2) + 40(6-4)] = 40v_2$$

$$v_2 = 12.0 \text{ m/s } (\rightarrow) \qquad \textbf{Ans.}$$



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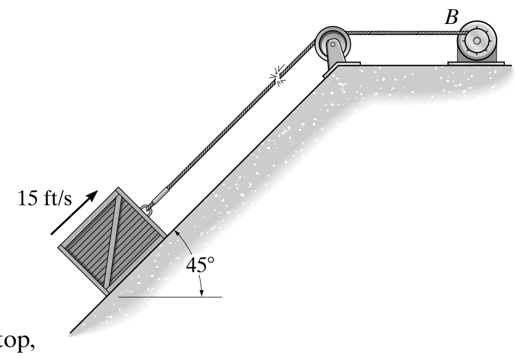
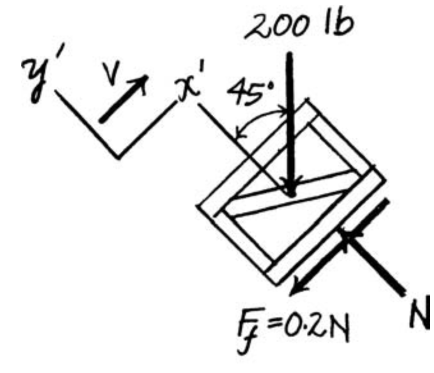
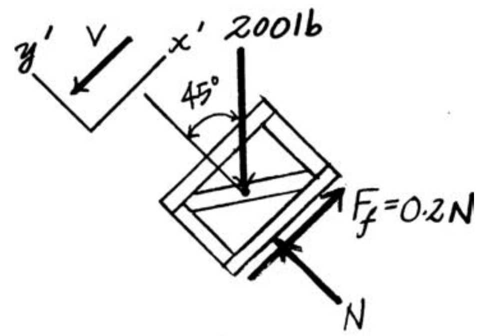
15–22. At the instant the cable fails, the 200-lb crate is traveling up the plane with a speed of 15 ft/s. Determine the speed of the crate 2 s afterward. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.20$.



Free-Body Diagram: When the cable snaps, the crate will slide up the plane, stop, and then slide down the plane. The free-body diagram of the crate in both cases are shown in Figs. *a* and *b*. The frictional force acting on the crate in both cases can be computed from $F_f = \mu_k N = 0.2N$.



(a)



(b)

Principle of Impulse and Momentum: By referring to Fig. *a*,

$$+\nwarrow \quad m(v_1)_{y'} + \Sigma\int_{t_1}^{t_2} F_{y'}\,dt = m(v_2)_{y'}$$

$$\frac{200}{32.2}(0) + N(t') - 200\cos 45°(t') = \;= \frac{200}{32.2}(0)$$

$$N = 141.42 \text{ lb}$$

$$+\nearrow \quad m(v_1)_{x'} + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_2)_{x'}$$

$$\frac{200}{32.2}(15) - 200\sin 45°(t') - 0.2(141.42)(t') = \frac{200}{32.2}(0)$$

$$t' = 0.5490 \text{ s}$$

Thus, the time the crate takes to slide down the plane is $t'' = 2 - 0.5490 = 1.451$ s. Here, $N = 141.42$ for both cases. By referring to Fig. *b*,

$$+\nearrow \quad m(v_1)_{x'} + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_2)_{x'}$$

$$\frac{200}{32.2}(0) + 0.2(141.42)(1.451) - 200\sin 45°(1.451) = \frac{200}{32.2}(-v)$$

$$v = 26.4 \text{ ft/s} \qquad \textbf{Ans.}$$

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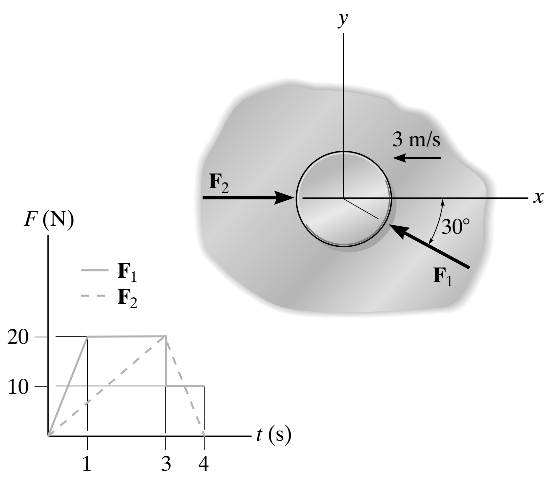
15–23. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ vary as shown by the graph. The 5-kg smooth disk is traveling to the left with a speed of 3 m/s when $t = 0$. Determine the magnitude and direction of the disk's velocity when $t = 4$ s.



Principle of Impulse and Momentum: The impulse generated by $\mathbf{F}_1$ and $\mathbf{F}_2$ during the time period $0 \le t \le 4$ s is equal to the area under the F_1 vs t and F_2 vs t graphs, i.e., $I_1 = \frac{1}{2}(20)(1) + 20(3 - 1) + 10(4 - 3) = 60 \text{N} \cdot \text{s}$ and $I_2 = \frac{1}{2}(20)(3 - 0) + \frac{1}{2}(20)(4 - 3) = 40 \text{ N} \cdot \text{s}$. By referring to the impulse and momentum diagram shown in Fig. *a*

$$\left(\xrightarrow{+}\right) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t^2} F_x\,dt = m(v_2)_x$$

$$-5(3) + 40 - 60\cos 30° = 5v_x$$

$$v_x = -5.392 \text{ m/s} = 5.392 \text{ m/s} \leftarrow$$

$$\left(+\uparrow\right) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$

$$0 + 60\sin 30° = 5v_y$$

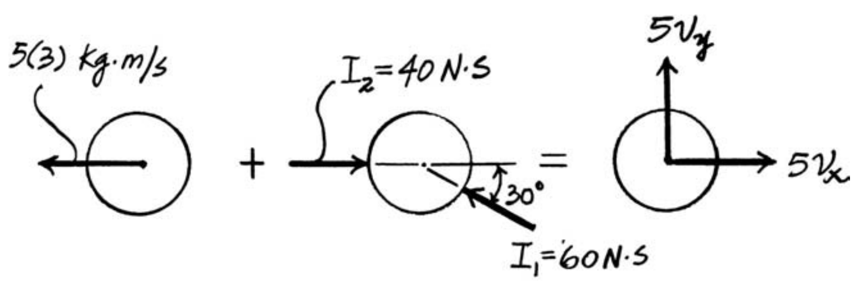
$$v_y = 6 \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}$,

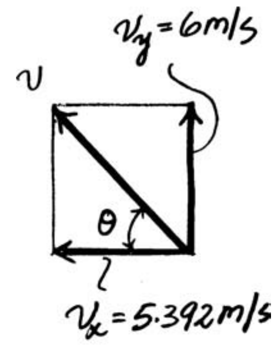
$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{5.392^2 + 6^2} = 8.07 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6}{5.392}\right) = 48.1° \qquad \textbf{Ans.}$$



(a)



(b)

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***15–24.** A 0.5-kg particle is acted upon by the force $\mathbf{F} = \{2t^2\mathbf{i} - (3t + 3)\mathbf{j} + (10 - t^2)\mathbf{k}\}$ N, where t is in seconds. If the particle has an initial velocity of $\mathbf{v}_0 = \{5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}\}$ m/s, determine the magnitude of the velocity of the particle when $t = 3$ s.

Principle of Impulse and Momentum:

$$m\mathbf{v}_1 + \Sigma\int_{t_1}^{t_2}\mathbf{F}dt = m\mathbf{v}_2$$

$$0.5(5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}) + \int_0^{3\text{ s}}\left[2t^2\mathbf{i} - \left(3t + 3\right)\mathbf{j} + \left(10 - t^2\right)\mathbf{k}\right] = 0.5\mathbf{v}_2$$

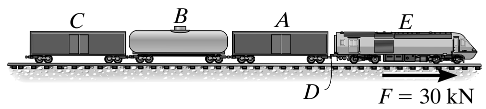
$$\mathbf{v}_2 = \left\{41\mathbf{i} - 35\mathbf{j} + 62\mathbf{k}\right\}\text{ m/s}$$

The magnitude of $\mathbf{v}_2$ is given by

$$v_2 = \sqrt{\left(v_2\right)_x{}^2 + \left(v_2\right)_y{}^2 + \left(v_2\right)_z{}^2} = \sqrt{\left(41\right)^2 + \left(-35\right)^2 + \left(62\right)^2}$$

$$= 82.2\text{ m/s} \qquad \textbf{Ans.}$$

•15–25. The train consists of a 30-Mg engine E, and cars A, B, and C, which have a mass of 15 Mg, 10 Mg, and 8 Mg, respectively. If the tracks provide a traction force of $F = 30$ kN on the engine wheels, determine the speed of the train when $t = 30$ s, starting from rest. Also, find the horizontal coupling force at D between the engine E and car A. Neglect rolling resistance.

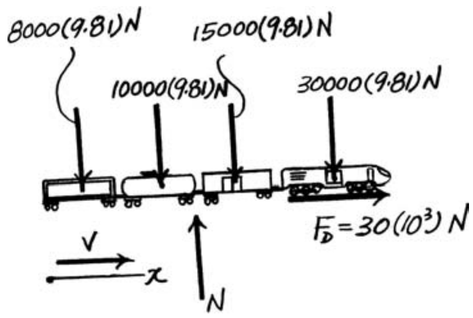


Principle of Impulse and Momentum: By referring to the free-body diagram of the entire train shown in Fig. a, we can write

$$\left(\overset{+}{\rightarrow}\right) \qquad m\left(v_1\right)_x + \Sigma\int_{t_1}^{t_2}F_x dt = m\left(v_2\right)_x$$

$$63\,000(0) + 30(10^3)(30) = 63\,000v$$

$$v = 14.29\text{ m/s} \qquad \textbf{Ans.}$$



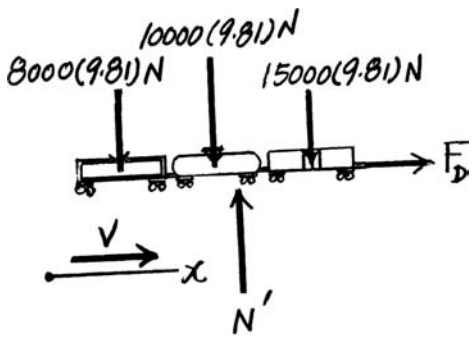
(a)

Using this result and referring to the free-body diagram of the train's car shown in Fig. b,

$$\left(\overset{+}{\rightarrow}\right) \qquad m\left(v_1\right)_x + \Sigma\int_{t_1}^{t_2}F_x\, dt = m\left(v_2\right)_x$$

$$33000(0) + F_D(30) = 33\,000\left(14.29\right)$$

$$F_D = 15\,714.29\text{ N} = 15.7\text{ kN} \qquad \textbf{Ans.}$$



(b)

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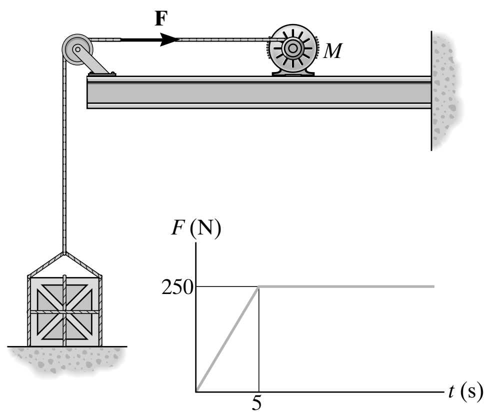
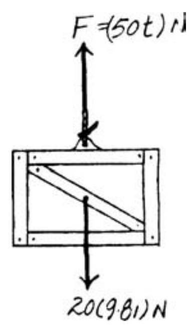
15–26. The motor M pulls on the cable with a force of $\mathbf{F}$, which has a magnitude that varies as shown on the graph. If the 20-kg crate is originally resting on the floor such that the cable tension is zero at the instant the motor is turned on, determine the speed of the crate when $t = 6$ s. *Hint*: First determine the time needed to begin lifting the crate.



Equations of Equilibrium: For the period $0 \leq t < 5$ s, $F = \frac{250}{5}t = (50t)$ N. The time needed for the motor to move the crate is given by



$$+\uparrow \Sigma F_y = 0; \qquad 50t - 20(9.81) = 0 \qquad t = 3.924 \text{ s} < 5 \text{ s}$$

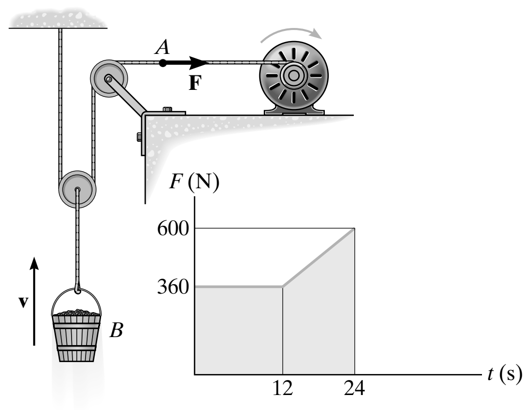
Principle of Linear Impulse and Momentum: The crate starts to move 3.924 s after the motor is turned on. Applying Eq. 15–4, we have

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

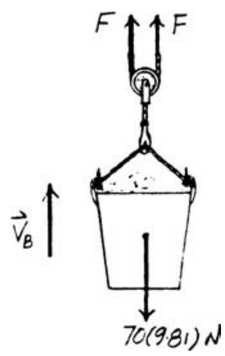
$$(+\uparrow) \qquad 20(0) + \int_{3.924 \text{ s}}^{5 \text{ s}} 50t\,dt + 250(6 - 5) - 20(9.81)(6 - 3.924) = 20v$$

$$v = 4.14 \text{ m/s} \qquad \textbf{Ans.}$$

15–27. The winch delivers a horizontal towing force $\mathbf{F}$ to its cable at A which varies as shown in the graph. Determine the speed of the 70-kg bucket when $t = 18$ s. Originally the bucket is moving upward at $v_1 = 3$ m/s.



Principle of Linear Impulse and Momentum: For the time period $12 \text{ s} \leq t < 18$ s, $\frac{F - 360}{t - 12} = \frac{600 - 360}{24 - 12}$, $F = (20t + 120)$ N. Applying Eq. 15–4 to bucket B, we have



$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$(+\uparrow) \qquad 70(3) + 2\left[360(12) + \int_{12 s}^{18 s} (20t + 120)dt\right] - 70(9.81)(18) = 70v_2$$

$$v_2 = 21.8 \text{ m/s} \qquad \textbf{Ans.}$$

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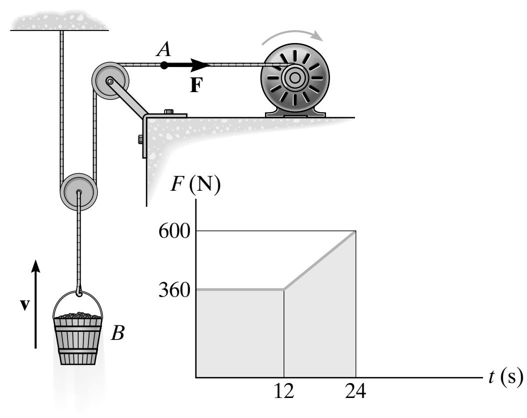
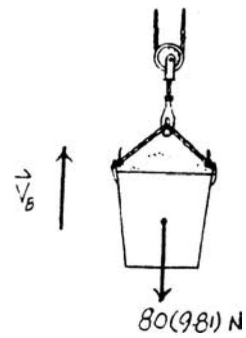
***15–28.** The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 80-kg bucket when $t = 24$ s. Originally the bucket is released from rest.



Principle of Linear Impulse and Momentum: The total impluse exerted on bucket B can be obtained by evaluating the area under the F–t graph. Thus, $I = \Sigma \int_{t_1}^{t_2} F_y\,dt = 2\left[360(12) + \frac{1}{2}(360 + 600)(24 - 12)\right] = 20160\ \text{N}\cdot\text{s}$. Applying Eq. 15–4 to the bucket B, we have



$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$(+\uparrow) \qquad 80(0) + 20160 - 80(9.81)(24) = 80v_2$$

$$v_2 = 16.6\text{m/s} \qquad \textbf{Ans.}$$

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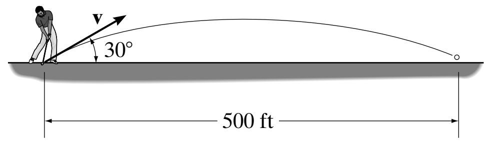
•15–29. The 0.1-lb golf ball is struck by the club and then travels along the trajectory shown. Determine the average impulsive force the club imparts on the ball if the club maintains contact with the ball for 0.5 ms.



Kinematics: By considering the x-motion of the golf ball, Fig. a,

$$\left(\xrightarrow{+}\right) \qquad s_x = (s_0) + (v_0)_x t$$

$$500 = 0 + v\cos 30^\circ\, t$$

$$t = \frac{500}{v\cos 30^\circ}$$

Subsequently, using the result of t and considering the y-motion of the golf ball,

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$0 = 0 + v\sin 30^\circ\left(\frac{500}{v\cos 30^\circ}\right) + \frac{1}{2}(-32.2)\left(\frac{500}{v\cos 30^\circ}\right)^2$$

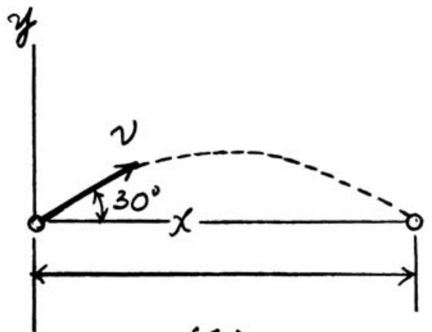
$$v = 136.35 \text{ ft/s}$$

Principle of Impulse and Momentum: Here, the impulse generated by the weight of the golf ball is very small compared to that generated by the force of the impact. Hence, it can be neglected. By referring to the impulse and momentum diagram shown in Fig. b,

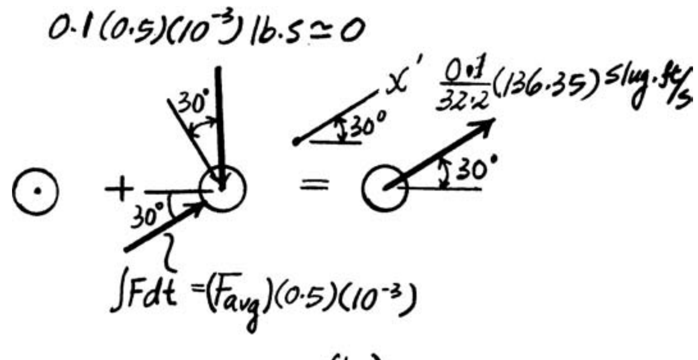
$$(\;) \qquad m(v_1)_{x'} + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_2)_{x'}$$

$$0 + F_{\text{avg}}(0.5)(10^{-3}) = \frac{0.1}{32.2}(136.35)$$

$$F_{\text{avg}} = 847 \text{ lb} \qquad \textbf{Ans.}$$



(a)



(b)

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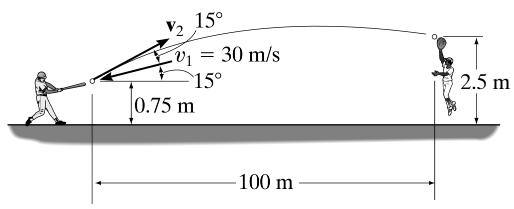
15–30. The 0.15-kg baseball has a speed of $v = 30$ m/s just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.



$(\overset{+}{\rightarrow})$ $\quad s_x = (s_0)_x + (v_0)_x t$

$$100 = 0 + v\cos 30°\, t$$

$$t = \frac{100}{v\cos 30°}$$

Subsequently, using the result of t and considering the y-motion of the golf ball.

$(+\uparrow)$ $\quad x_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2$

$$1.75 = 0 + v\sin 30°\left(\frac{100}{v\cos 30°}\right) + \frac{1}{2}(-9.81)\left(\frac{100}{v\cos 30°}\right)^2$$

$$v = 34.18 \text{ m/s}$$

Principle of Impulse and Momentum: Here, the impulse generated by the weight of the baseball is very small compared to that generated by the force of the impact. Hence, it can be neglected. By referring to the impulse and momentum diagram shown in Fig. *b*,

$(\overset{+}{\rightarrow})$ $\quad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$

$$-0.15(30)\cos 15° + \left(F_{\text{avg}}\right)_x(0.75)\left(10^{-3}\right) = 0.15(34.18)\cos 30°$$

$$\left(F_{\text{avg}}\right)_x = 11\,715.7 \text{ N}$$

$(+\uparrow)$ $\quad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$

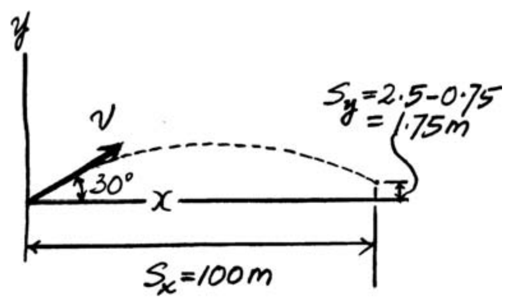
$$-0.15(30)\sin 15° + \left(F_{\text{avg}}\right)_y(0.75)\left(10^{-3}\right) = 0.15(34.18)\sin 30°$$

$$\left(F_{\text{avg}}\right)_y = 4970.9 \text{ N}$$

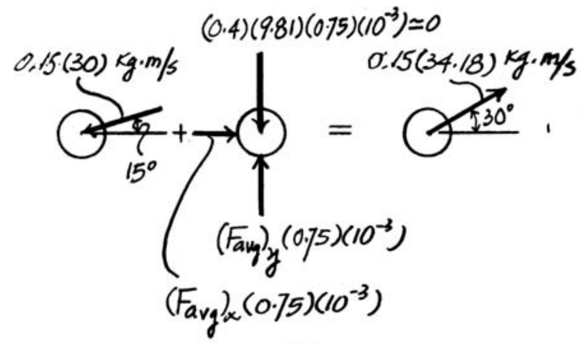
Thus,

$$F_{\text{avg}} = \sqrt{\left(F_{\text{avg}}\right)_x^2 + \left(F_{\text{avg}}\right)_y^2} = \sqrt{11715.7^2 + 4970.9^2}$$

$$= 12.7 \text{ kN} \qquad \textbf{Ans.}$$



(a)



(b)

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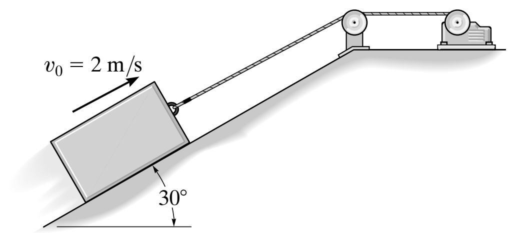
15–31. The 50-kg block is hoisted up the incline using the cable and motor arrangement shown. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.4$. If the block is initially moving up the plane at $v_0 = 2$ m/s, and at this instant ($t = 0$) the motor develops a tension in the cord of $T = (300 + 120\sqrt{t})$ N, where t is in seconds, determine the velocity of the block when $t = 2$ s.

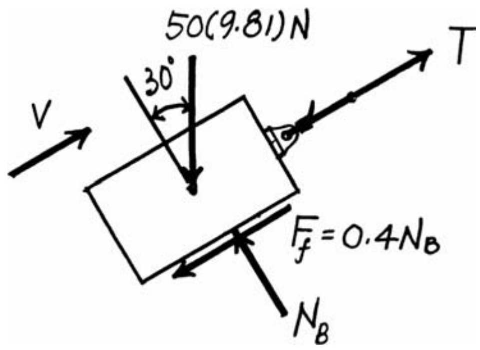


$+\nwarrow \Sigma F_x = 0; \qquad N_B - 50(9.81)\cos 30° = 0 \qquad N_B = 424.79$ N

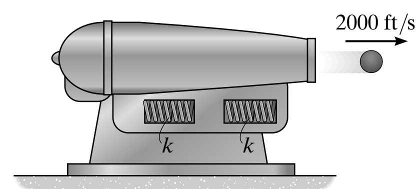
$$(+\nearrow) \qquad m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$$

$$50(2) + \int_0^2 \left(300 + 120\sqrt{t}\right)dt - 0.4(424.79)(2) - 50(9.81)\sin 30°(2) = 50v_2$$

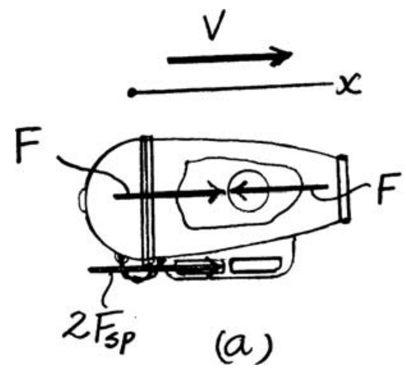
$$v_2 = 192 \text{ m/s} \qquad \textbf{Ans.}$$



***15–32.** The 10-lb cannon ball is fired horizontally by a 500-lb cannon as shown. If the muzzle velocity of the ball is 2000 ft/s, measured relative to the ground, determine the recoil velocity of the cannon just after firing. If the cannon rests on a smooth support and is to be stopped after it has recoiled a distance of 6 in., determine the required stiffness k of the two identical springs, each of which is originally unstretched.



Free-Body Diagram: The free-body diagram of the cannon and ball system is shown in Fig. *a*. Here, the spring force $2\mathbf{F}_{sp}$ is nonimpulsive since the spring acts as a shock absorber. The pair of impulsive forces $\mathbf{F}$ resulting from the explosion cancel each other out since they are internal to the system



Conservation of Linear Momentum: Since the resultant of the impulsice force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$\left(\overset{+}{\rightarrow}\right) \qquad m_C\left(v_C\right)_1 + m_b\left(v_b\right)_1 = m_C\left(v_C\right)_2 + m_b\left(v_b\right)_2$$

$$\frac{500}{32.2}(0) + \frac{10}{32.2}(0) = \frac{500}{32.2}\left(v_C\right)_2 + \frac{10}{32.2}(2000)$$

$$\left(v_C\right)_2 = -40 \text{ ft/s} = 40 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Conservation of Energy: The initial and final elastic potential energy in each spring are $\left(V_e\right)_i = \frac{1}{2}ks_i^{\,2} = 0$ and $\left(V_e\right)_f = \frac{1}{2}ks_f^{\,2} = \frac{1}{2}k(0.5^2) = 0.125k$. By referring to Fig. *a*,

$$T_i + V_i = T_f + V_f$$

$$\frac{1}{2}m_C\left(v_C\right)_i^{\,2} + 2\left(V_e\right)_i = \frac{1}{2}m_C\left(v_C\right)_f^{\,2} + 2\left(V_e\right)_f$$

$$\frac{1}{2}\left(\frac{500}{32.2}\right)\left(40^2\right) + 2\left(0\right) = 0 + 2\left(0.125k\right)$$

$$k = 49\,689.44 \text{ lb/ft} = 49.7 \text{ kip/ft} \qquad \textbf{Ans.}$$

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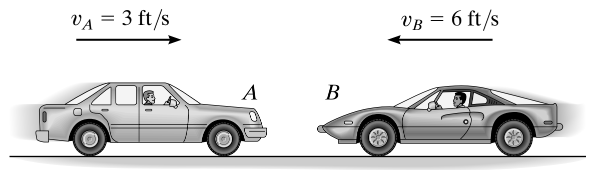
15–33. A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.



$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$$

$$v_2 = 0.5 \text{ m/s} \qquad \textbf{Ans.}$$

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

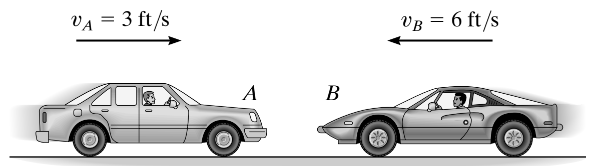
$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

$$\Delta T = T_1 - T_2$$

$$= 20.25 - 3.375 = 16.9 \text{ kJ} \qquad \textbf{Ans.}$$

This energy is dissipated as noise, shock, and heat during the coupling.

15–34. The car A has a weight of 4500 lb and is traveling to the right at 3 ft/s. Meanwhile a 3000-lb car B is traveling at 6 ft/s to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.



$(\overset{+}{\rightarrow})$ $\quad m_A (v_A)_1 + m_B (v_B)_1 = (m_A + m_B) v_2$

$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2} v_2$$

$$v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

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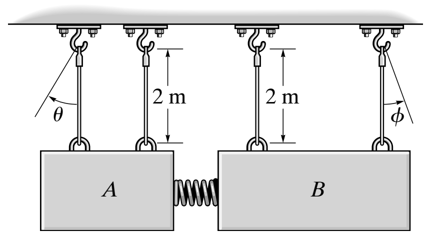
15–35. The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60$ N/m, is attached to B and is compressed 0.3 m against A as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



$(\xrightarrow{+})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$$v = 0.7348 \text{ m/s}$$

For A or B: Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\theta)$$

$\theta = \phi = 9.52°$ **Ans.**

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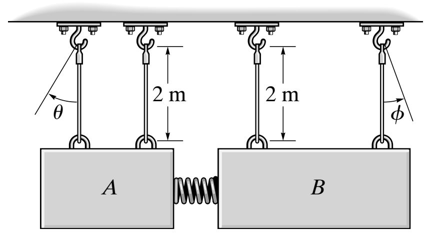
***15–36.** Block A has a mass of 4 kg and B has a mass of 6 kg. A spring, having a stiffness of $k = 40$ N/m, is attached to B and is compressed 0.3 m against A as shown. Determine the maximum angles θ and ϕ of the cords after the blocks are released from rest and the spring becomes unstretched.



$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$0 + 0 = 6v_B - 4v_A$$

$$v_A = 1.5v_B$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(40)(0.3)^2 = \frac{1}{2}(4)(v_A)^2 + \frac{1}{2}(6)(v_B)^2 + 0$$

$$3.6 = 4v_A^2 + 6v_B^2$$

$$3.6 = 4(1.5v_B)^2 + 6v_B^2$$

$$v_B = 0.4899 \text{ m/s} \qquad v_A = 0.7348 \text{ m/s}$$

For A:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(4)(0.7348)^2 + 0 = 0 + 4(9.81)(2)(1 - \cos\theta)$$

$\theta = 9.52°$ **Ans.**

For B:

Datum at lowest point

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(6)(0.4899)^2 + 0 = 0 + 6(9.81)(2)(1 - \cos\phi)$$

$\phi = 6.34°$ **Ans.**

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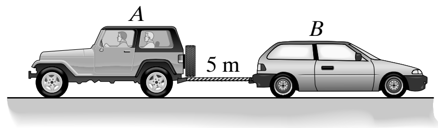
•15–37. The winch on the back of the Jeep A is turned on and pulls in the tow rope at 2 m/s measured relative to the Jeep. If both the 1.25-Mg car B and the 2.5-Mg Jeep A are free to roll, determine their velocities at the instant they meet. If the rope is 5 m long, how long will this take?



$(\overset{+}{\rightarrow})$ $\quad 0 + 0 = m_A v_A - m_B v_B$ **(1)**

$$0 = 2.5(10^3)\, v_A - 1.25(10^3)\, v_B$$

However, $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$(\overset{+}{\rightarrow})$ $\quad v_A = -v_B + 2$ **(2)**

Substituting Eq. (2) into (1) yields:

$v_B = 1.33\text{m/s}$ **Ans.**

$v_A = 0.667 \text{ m/s}$ **Ans.**

Kinematics:

$(\overset{+}{\rightarrow})$ $\quad s_{A/B} = v_{A/B}\, t$

$$5 = 2t$$

$t = 2.5 \text{ s}$ **Ans.**

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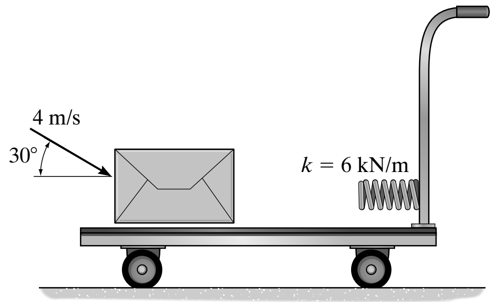
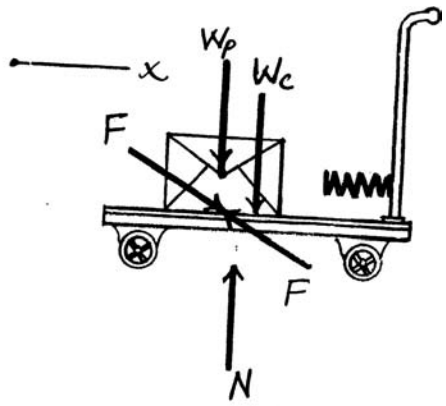
15–38. The 40-kg package is thrown with a speed of 4 m/s onto the cart having a mass of 20 kg. If it slides on the smooth surface and strikes the spring, determine the velocity of the cart at the instant the package fully compresses the spring. What is the maximum compression of the spring? Neglect rolling resistance of the cart.



Conservation of Linear Momentum: By referring to the free-body diagram of the package and cart system shown in Fig. *a*, we notice the pair of impulsive forces **F** generated during the impact cancel each other since they are internal to the system. Thus, the resultant of the impulsive forces along the *x* axis is zero. As a result, the linear momentum of the system is conserved along the *x* axis. The cart does not move after the impact until the package strikes the spring. Thus,



(a)

$$\left(\overset{+}{\rightarrow}\right) \qquad m_p\left[\left(v_p\right)_1\right]_x + m_c\left(v_c\right)_1 = m_p\left(v_p\right)_2 + m_c\left(v_c\right)_2$$

$$40\left(4\cos 30^\circ\right) + 0 = 40\left(v_p\right)_2 + 0$$

$$\left(v_p\right)_2 = 3.464 \text{ m/s} \rightarrow$$

When the spring is fully compressed, the package momentarily stops sliding on the cart. At this instant, the package and the cart move with a common speed.

$$\left(\overset{+}{\rightarrow}\right) \qquad m_p\left(v_p\right)_2 + m_c\left(v_c\right)_2 = \left(m_p + m_c\right)v_3$$

$$40(3.464) + 0 = \left(40 + 20\right)v_3$$

$$v_3 = 2.309 \text{ m/s} = 2.31 \text{ m/s} \qquad \textbf{Ans.}$$

Conservation of Energy: We will consider the conservation of energy of the system. The initial and final elastic potential energies of the spring are $\left(V_e\right)_2 = \frac{1}{2}ks_2{}^2 = 0$ and $\left(V_e\right)_3 = \frac{1}{2}ks_3{}^2 = \frac{1}{2}\left(6000\right)s_{\max}{}^2 = 3000s_{\max}{}^2$.

$$T_2 + V_2 = T_3 + V_3$$

$$\left[\frac{1}{2}m_p\left(v_p\right)^2 + \frac{1}{2}m_c\left(v_c\right)_2{}^2\right] + \left(V_e\right)_2 = \frac{1}{2}\left(m_p + m_p\right)v_3{}^2 + \left(V_e\right)_3$$

$$\left[\frac{1}{2}(40)(3.464^2) + 0\right] + 0 = \frac{1}{2}\left(40 + 20\right)\left(2.309^2\right) + 3000s_{\max}{}^2$$

$$s_{\max} = 0.1632 \text{ m} = 163 \text{ mm} \qquad \textbf{Ans.}$$

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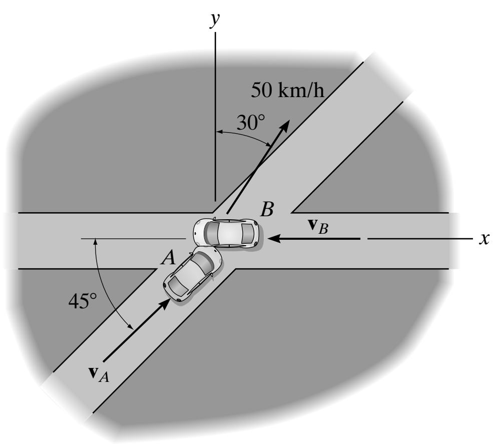
15–39. Two cars A and B have a mass of 2 Mg and 1.5 Mg, respectively. Determine the magnitudes of $\mathbf{v}_A$ and $\mathbf{v}_B$ if the cars collide and stick together while moving with a common speed of 50 km/h in the direction shown.



Conservation of Linear Momentum: Since the pair of impulsice forces **F** generated during the impact are internal to the system of cars A and B, they cancel each other out. Thus, the resultant impulsive force along the x and y axes are zero. Consequently, the linear momentum of the system is conserved along the x and y axes. The common speed of the system just after the impact is $v_2 = \left[50(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 13.89\text{ m/s}$. Thus, we can write

$$\left(\overset{+}{\rightarrow}\right) \qquad m_A(v_A)_x + \left[-m_B(v_B)_x\right] = (m_A + m_B)(v_2)_x$$

$$2000v_A \cos 45° - 1500v_B = (2000 + 1500)(13.89 \sin 30°)$$

$$1414.21v_A - 1500v_B = 24305.56 \qquad \textbf{(1)}$$

and

$$\left(+\uparrow\right) \qquad m_A(v_A)_y + m_B(v_B)_y = (m_A + m_B)(v_2)_y$$

$$2000v_A \sin 45° + 0 = (2000 + 1500)(13.89 \cos 30°)$$

$$v_A = 29.77\text{ m/s} = 29.8\text{ m/s} \qquad \textbf{Ans.}$$

Substituting the result of $\mathbf{v}_A$ into Eq. (1),

$$v_B = 11.86\text{ m/s} = 11.9\text{ m/s} \qquad \textbf{Ans.}$$

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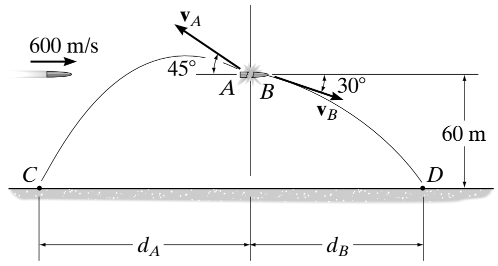
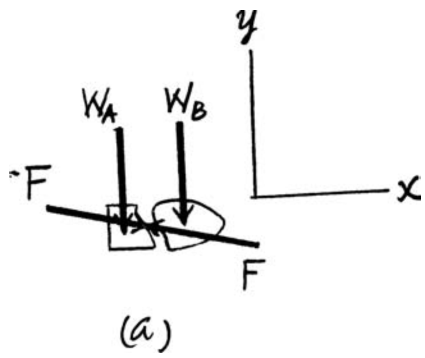
***15–40.** A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_A where segment A strikes the ground at C.



Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. *a*, we notice that the pair of impulsive forces **F** generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



$(\xrightarrow{+})$ $$mv_x = m_A(v_A)_x + m_B(v_B)_x$$

$$4(600) = -1.5v_A \cos 45° + 2.5v_B \cos 30°$$

$$2.165v_B - 1.061v_A = 2400 \quad \textbf{(1)}$$

$(+\uparrow)$ $$mv_y = m_A(v_A)_y + m_B(v_B)_y$$

$$0 = 1.5v_A \sin 45° - 2.5v_B \sin 30°$$

$$v_B = 0.8485v_A \quad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96 \text{ m/s} = 3.09(10^3) \text{ m/s} \quad \textbf{Ans.}$$

$$v_B = 2622.77 \text{ m/s} = 2.62(10^3) \text{ m/s} \quad \textbf{Ans.}$$

By considering the x and y motion of segment A,

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a_y t^2$$

$$-60 = 0 + 3090.96 \sin 45° \, t_{AC} + \frac{1}{2}(-9.81)t_{AC}{}^2$$

$$4.905t_{AC}{}^2 - 2185.64t_{AC} - 60 = 0$$

Solving for the positive root of this equation,

$$t_{AC} = 445.62 \text{ s}$$

and

$(\xleftarrow{+})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$d_A = 0 + 3090.96 \cos 45°(445.62)$$

$$= 973.96\left(10^3\right) \text{ m} = 974 \text{ km} \quad \textbf{Ans.}$$

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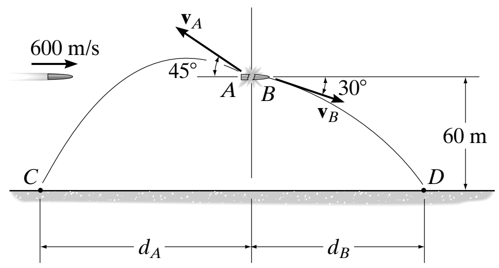
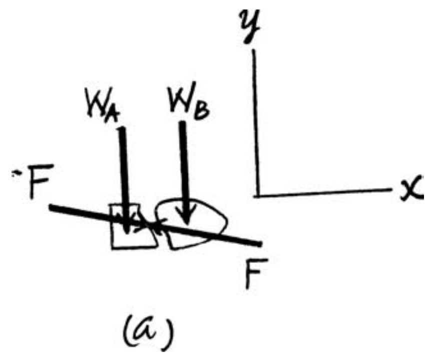
•15–41. A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_B where segment B strikes the ground at D.



Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. a, we notice that the pair of impulsive forces $\mathbf{F}$ generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



$(\xrightarrow{+})$ $$mv_x = m_A(v_A)_x + m_B(v_B)_x$$

$$4(600) = -1.5v_A \cos 45° + 2.5v_B \cos 30°$$

$$2.165v_B - 1.061v_A = 2400 \qquad \textbf{(1)}$$

$(+\uparrow)$ $$mv_y = m_A(v_A)_y + m_B(v_B)_y$$

$$0 = 1.5v_A \sin 45° - 2.5v_B \sin 30°$$

$$v_B = 0.8485v_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96 \text{ m/s} = 3.09(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

$$v_B = 2622.77 \text{ m/s} = 2.62(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

By considering the x and y motion of segment B,

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a_y t^2$$

$$-60 = 0 - 2622.77 \sin 30°\, t_{BD} + \frac{1}{2}(-9.81)t_{BD}^2$$

$$4.905t_{BD}^2 + 1311.38t_{BD} - 60 = 0$$

Solving for the positive root of the above equation,

$$t_{BD} = 0.04574 \text{ s}$$

and

$(\xrightarrow{+})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$d_B = 0 + 2622.77 \cos 30°(0.04574)$$

$$= 103.91 \text{ m} = 104 \text{ m} \qquad \textbf{Ans.}$$

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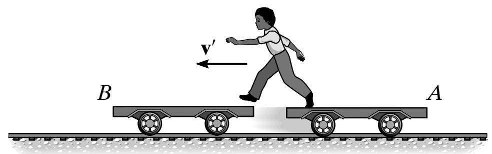
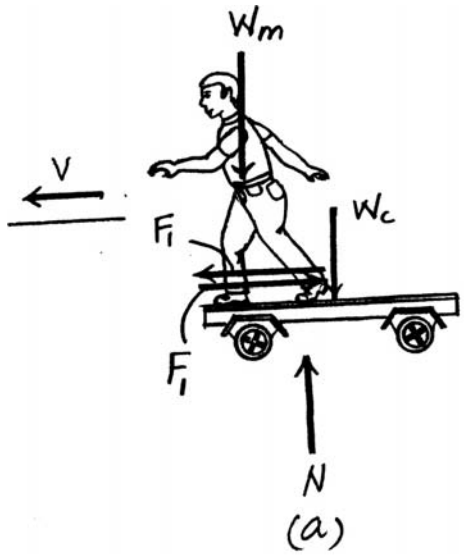
15–42. The 75-kg boy leaps off cart A with a horizontal velocity of $v' = 3\text{ m/s}$ measured relative to the cart. Determine the velocity of cart A just after the jump. If he then lands on cart B with the same velocity that he left cart A, determine the velocity of cart B just after he lands on it. Carts A and B have the same mass of 50 kg and are originally at rest.



Free-Body Diagram: The free-body diagram of the man and cart system when the man leaps off and lands on the cart are shown in Figs. *a* and *b,* respectively. The pair of impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the leap and landing are internal to the system and thus cancel each other.



Kinematics: Applying the relative velocity equation, the relation between the velocity of the man and cart A just after leaping can be determined.

$$\mathbf{v}_m = \mathbf{v}_A + \mathbf{v}_{m/A}$$

$$(\overset{+}{\leftarrow}) \qquad (v_m)_2 = (v_A)_2 + 3 \qquad \textbf{(1)}$$

Conservation of Linear Momentum: Since the resultant of the impulse forces along the x axis is zero, the linear momentum of the system is conserved along the x axis for both cases. When the man leaps off cart A,

$$(\overset{+}{\leftarrow}) \qquad m_m(v_m)_1 + m_A(v_A)_1 = m_m(v_m)_2 + m_A(v_A)_2$$

$$0 + 0 = 75(v_m)_2 + 50(v_A)_2$$

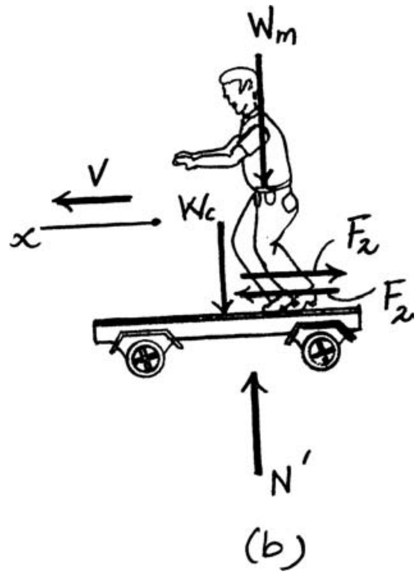
$$(v_m)_2 = -0.6667(v_A)_2$$

Solving Eqs. (1) and (2) yields

$$(v_A)_2 = -1.80\text{ m/s} = 1.80\text{ m/s} \rightarrow$$

$$(v_m)_2 = 1.20\text{ m/s} \leftarrow$$

Using the result of $(v_m)_2$ and considering the man's landing on cart B,



$$(\overset{+}{\leftarrow}) \qquad m_m(v_m)_2 + m_B(v_B)_1 = (m_m + m_B)v$$

$$75(1.20) + 0 = (75 + 50)v$$

$$v = 0.720\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

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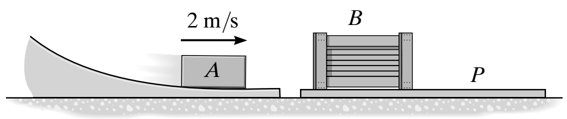
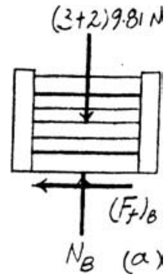
15–43. Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.4$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.5$.



Equations of Equilibrium: From FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad N_B - (3 + 2)(9.81) = 0 \qquad N_B = 49.05\text{ N}$$



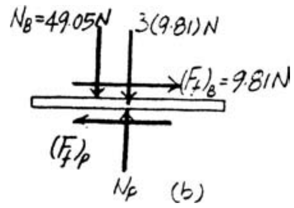
When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).

$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81\text{ N}$$

Since $(F_f)_P < \left[(F_f)_P\right]_{\max} = \mu_s' N_P = 0.5(78.48) = 39.24$ N, plate P does not move. Thus

$$s_P = 0 \qquad \textbf{Ans.}$$

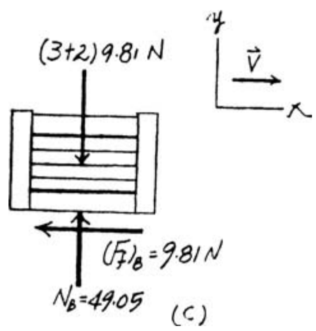


Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along the x axis.

$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R)\, v_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(2) + 0 = (2 + 3)\, v_2$$

$$v_2 = 0.800\text{ m/s} \rightarrow$$



Principle of Linear Impulse and Momentum: Applying Eq. 15–4, we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(0.8) + \left[-9.81(t)\right] = 5(0)$$

$$t = 0.408\text{ s} \qquad \textbf{Ans.}$$

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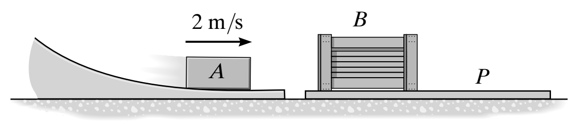
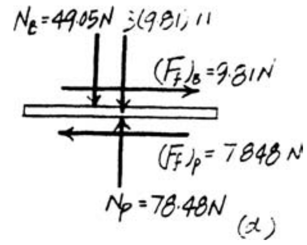
***15–44.** Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.1$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.12$.



Equations of Equilibrium: From FBD(a),



$$+\uparrow \Sigma F_x = 0; \qquad N_B - (3 + 2)(9.81) = 0 \qquad N_B = 49.05 \text{ N}$$

When box B slides on top of plate P. $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).

$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P > \left[(F_f)_P\right]_{\text{max}} = \mu_s{}' N_P = 0.12(78.48) = 9.418\text{N}$, plate P slides. Thus, $(F_f)_P = \mu_k{}' N_P = 0.1(78.48) = 7.848$ N.

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along x axis.

$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R) v_2$$

$$(\xrightarrow{+}) \qquad 2(2) + 0 = (2 + 3) v_2$$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: In order for box B to stop sliding on plate P, both box B and plate P must have same speed v_3. Applying Eq. 15–4 to box B (FBD(c)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$(\xrightarrow{+}) \qquad 5(0.8) + \left[-9.81(t_1)\right] = 5v_3 \qquad \textbf{[1]}$$

Applying Eq. 15–4 to plate P[FBD(d)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$(\xrightarrow{+}) \qquad 3(0) + 9.81(t_1) - 7.848(t_1) = 3v_3 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$t_1 = 0.3058 \text{ s} \qquad v_3 = 0.200 \text{ m/s}$$

Equation of Motion: From FBD(d), the acceleration of plate P when box B still slides on top of it is given by

$$\xrightarrow{+} \Sigma Fx = ma_x; \qquad 9.81 - 7.848 = 3(a_P)_1 \qquad (a_P)_1 = 0.654 \text{ m/s}^2$$

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***15–44. Continued**

When box B stop slid ling on top of box B, $(F_f)_B = 0$. From this instant onward plate P and box B act as a unit and slide together. From FBD(d), the acceleration of plate P and box B is given by

$\overset{+}{\rightarrow} \Sigma Fx = ma_x; \qquad -7.848 = 8(a_P)_2 \qquad (a_P)_2 = -0.981 \text{ m/s}^2$

Kinematics: Plate P travels a distance s_1 before box B stop sliding.

$$\left(\overset{+}{\rightarrow}\right) \qquad s_1 = (v_0)_P\, t_1 + \frac{1}{2}(a_P)_1\, t_1^2$$

$$= 0 + \frac{1}{2}(0.654)\left(0.3058^2\right) = 0.03058 \text{ m}$$

The time t_2 for plate P to stop after box B stop slidding is given by

$$\left(\overset{+}{\rightarrow}\right) \qquad v_4 = v_3 + (a_P)_2\, t_2$$

$$0 = 0.200 + (-0.981)t_2 \qquad t_2 = 0.2039 \text{ s}$$

The distance s_2 traveled by plate P after box B stop sliding is given by

$$\left(\overset{+}{\rightarrow}\right) \qquad v_4^2 = v_3^2 + 2(a_P)_2\, s_2$$

$$0 = 0.200^2 + 2(-0.981)s_2 \qquad s_2 = 0.02039 \text{ m}$$

The total distance travel by plate P is

$$s_P = s_1 + s_2 = 0.03058 + 0.02039 = 0.05097 \text{ m} = 51.0 \text{ mm}$$ **Ans.**

The total time taken to cease all the motion is

$$t_{\text{Tot}} = t_1 + t_2 = 0.3058 + 0.2039 = 0.510 \text{ s}$$ **Ans.**

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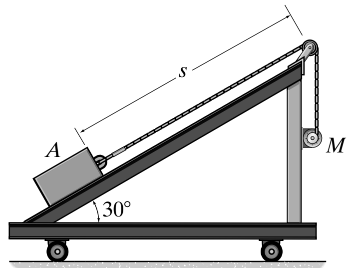
•15–45. The 20-kg block A is towed up the ramp of the 40-kg cart using the motor M mounted on the side of the cart. If the motor winds in the cable with a constant velocity of 5 m/s, measured relative to the cart, determine how far the cart will move when the block has traveled a distance $s = 2$ m up the ramp. Both the block and cart are at rest when $s = 0$. The coefficient of kinetic friction between the block and the ramp is $\mu_k = 0.2$. Neglect rolling resistance.



Conservation of Linear Momentum: The linear momentum of the block and cart system is conserved along the x axis since no impulsive forces act along the x axis.

$$\left(\xrightarrow{+}\right) \qquad m_B\left[(v_B)_x\right]_1 + m_C(v_C)_1 = m_B\left[(v_B)_x\right]_2 + m_C(v_C)_2$$

$$0 + 0 = 20(v_B)_x + 40v_C \qquad \textbf{(1)}$$

Kinematics: Here, the velocity of the block relative to the cart is directed up the ramp with a magnitude of $v_{B/C} = 5$ m/s. Applying the relative velocity equation and considering the motion of the block.

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\begin{bmatrix} v_B \end{bmatrix} = \begin{bmatrix} \underset{\rightarrow}{v_C} \end{bmatrix} + \begin{bmatrix} 5 \end{bmatrix}$$

$$\left(\xrightarrow{+}\right) \qquad (v_B)_x = v_C + 5\cos 30° \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_C = -1.443 \text{ m/s} = 1.443 \text{ m/s} \leftarrow \qquad (v_B)_x = 2.887 \text{ m/s}$$

The time required for the block to travel up the ramp a relative distance of $s_{B/C} = 2$ m is

$$(\ ) \qquad s_{B/C} = (s_{B/C})_0 + (v_{B/C})t$$

$$2 = 0 + 5t$$

$$t = 0.4 \text{ s}$$

Thus, the distance traveled by the cart during time t is

$$\left(\xleftarrow{+}\right) \qquad s_C = v_C t = 1.443(0.4) = 0.577 \text{ m} \leftarrow \qquad \textbf{Ans.}$$

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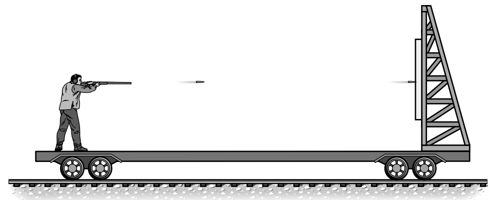
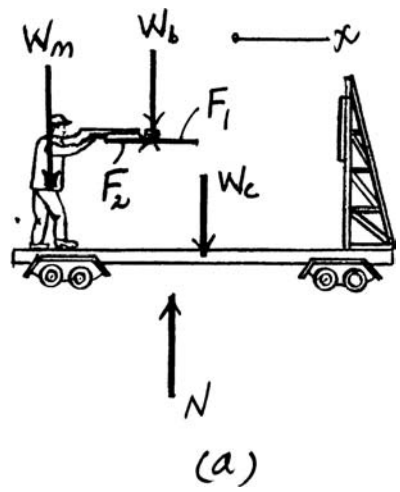
15–46. If the 150-lb man fires the 0.2-lb bullet with a horizontal muzzle velocity of 3000 ft/s, measured relative to the 600-lb cart, determine the velocity of the cart just after firing. What is the velocity of the cart when the bullet becomes embedded in the target? During the firing, the man remains at the same position on the cart. Neglect rolling resistance of the cart.



Free-Body Diagram: The free-body diagram of the bullet, man, and cart just after firing and at the instant the bullet hits the target are shown in Figs., *a* and *b*, respectively. The pairs of impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the firing and impact are internal to the system and thus cancel each other.



Kinematics: Applying the relative velocity equation, the relation between the velocity of the bullet and the cart just after firing can be determined

$$(\stackrel{+}{\rightarrow}) \qquad \mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

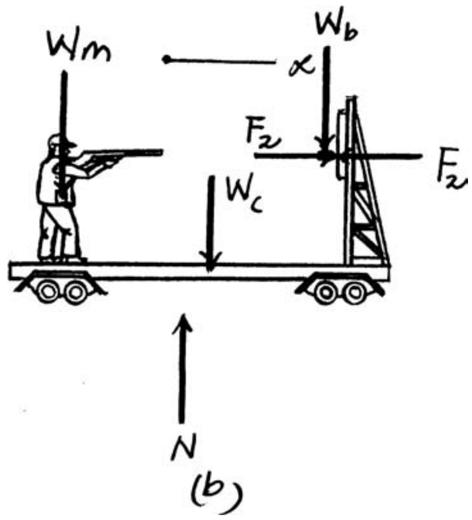
$$(v_b)_2 = (v_c)_2 + 3000 \qquad \textbf{(1)}$$

Conservation of Linear Momentum: Since the pair of resultant impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the firing and impact is zero along the x axis, the linear momentum of the system for both cases are conserved along the. x axis. For the case when the bullet is fired, momentum is conserved along the x' axis.

$$(\stackrel{+}{\rightarrow}) \qquad m_b(v_b)_1 + m_c(v_c)_1 = m_b(v_b)_2 + m_c(v_c)_2$$

$$0 + 0 = \left(\frac{0.2}{32.2}\right)(v_b)_2 + \left(\frac{150 + 600}{32.2}\right)(v_c)_2$$

$$(v_b)_2 = -3750(v_c)_2 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$(v_c)_2 = -0.7998 \text{ ft/s} = 0.800 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

$$(v_b)_2 = 2999.20 \text{ ft/s} \rightarrow$$

Using the results of $(v_c)_2$ and $(v_b)_2$ and considering the case when the bullet hits the target,

$$(\stackrel{+}{\rightarrow}) \qquad m_b(v_b)_2 + m_c(v_c)_2 = (m_b + m_c)v_3$$

$$\frac{0.2}{32.2}(2999.20) + \left[-\left(\frac{150 + 600}{32.2}\right)(0.7998)\right] = \left(\frac{150 + 600 + 0.2}{32.2}\right)v_3$$

$$v_3 = 0 \qquad \textbf{Ans.}$$

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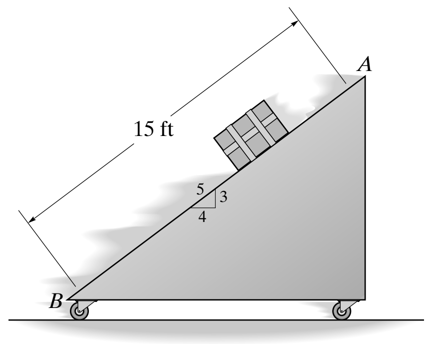
15–47. The free-rolling ramp has a weight of 120 lb. The crate whose weight is 80 lb slides from rest at A, 15 ft down the ramp to B. Determine the ramp's speed when the crate reaches B. Assume that the ramp is smooth, and neglect the mass of the wheels.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 80\left(\frac{3}{5}\right)(15) = \frac{1}{2}\left(\frac{80}{32.2}\right)v_B^2 + \frac{1}{2}\left(\frac{120}{32.2}\right)v_r^2 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow})\Sigma\, m\, v_1 = \Sigma\, m\, v_2$$

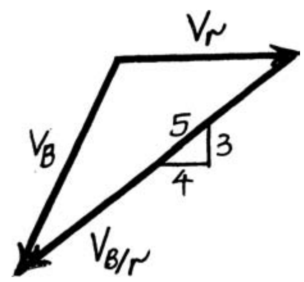
$$0 + 0 = \frac{120}{32.2}v_r - \frac{80}{32.2}(v_B)_x$$

$$(v_B)_x = 1.5v_r$$

$$\mathbf{v}_B = \mathbf{v}_r + \mathbf{v}_{B/r}$$

$$\left(\overset{+}{\rightarrow}\right) - (v_B)_x = vr - \frac{4}{5}v_{B/r} \qquad \textbf{(2)}$$

$$(+\uparrow)(-v_B)_y = 0 - \frac{3}{5}v_{B/r} \qquad \textbf{(3)}$$



Eliminating $(v_B)_{/r}$, from Eqs. (2) and (3) and substituting $(v_B)_y = 1.875\, v_r$, results in

$$v_B^2 = (v_B)_x^2 + (v_B)_y^2 = (1.5v_r)^2 + (1.875v_r)^2 = 5.7656v_r^2 \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

$$80\left(\frac{3}{5}\right)(15) = \frac{1}{2}\left(\frac{80}{32.2}\right)\left(5.7656v_r^2\right) + \frac{1}{2}\left(\frac{120}{32.2}\right)v_r^2$$

$v_r = 8.93$ ft/s **Ans.**

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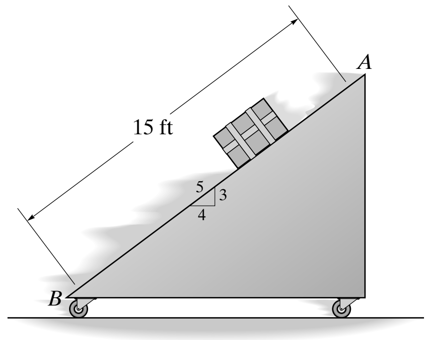
***15–48.** The free-rolling ramp has a weight of 120 lb. If the 80-lb crate is released from rest at A, determine the distance the ramp moves when the crate slides 15 ft down the ramp to the bottom B.



$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 = \frac{120}{32.2} v_r - \frac{80}{32.2}(v_B)_x$$

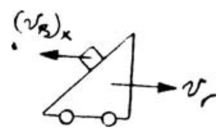
$$(v_B)_x = 1.5\, v_r$$

$$\mathbf{v}_B = \mathbf{v}_r + \mathbf{v}_{B/r}$$

$$-(v_B)_x = v_r - (v_{B/r})_x \left(\frac{4}{5}\right)$$

$$-1.5 v_r = v_r - (v_{B/r})_x \left(\frac{4}{5}\right)$$

$$2.5\, v_r = (v_{B/r})_x \left(\frac{4}{5}\right)$$



Integrate

$$2.5\, s_r = (s_{B/r})_x \left(\frac{4}{5}\right)$$

$$2.5\, s_r = \left(\frac{4}{5}\right)(15)$$

$s_r = 4.8$ ft **Ans.**

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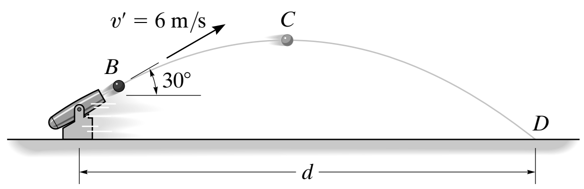
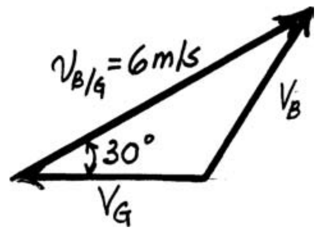
•15–49. The 5-kg spring-loaded gun rests on the smooth surface. It fires a ball having a mass of 1 kg with a velocity of $v' = 6$ m/s relative to the gun in the direction shown. If the gun is originally at rest, determine the horizontal distance d the ball is from the initial position of the gun at the instant the ball strikes the ground at D. Neglect the size of the gun.





$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = 1(v_B)_x - 5v_G$$

$$(v_B)_x = 5v_G$$

$(\overset{+}{\rightarrow})$ $\quad v_B = v_G + v_{B/G}$

$$5v_G = -v_G + 6\cos 30^\circ$$

$$v_G = 0.8660 \text{ m/s} \leftarrow$$

So that,

$(v_B)_x = 4.330 \text{ m/s} \rightarrow$

$(v_B)_y = 4.330 \tan 30^\circ = 2.5 \text{ m/s} \uparrow$

Time of flight for the ball:

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$-2.5 = 2.5 - 9.81t$$

$$t = 0.5097 \text{ s}$$

Distance ball travels:

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$$s = 4.330(0.5097) = 2.207 \text{ m} \rightarrow$$

Distance gun travels:

$(\overset{+}{\leftarrow})$ $\quad s = v_0 t$

$$s' = 0.8660(0.5097) = 0.4414 \text{ m} \leftarrow$$

Thus,

$d = 2.207 + 0.4414 = 2.65$ m **Ans.**

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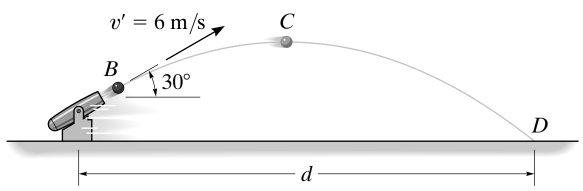
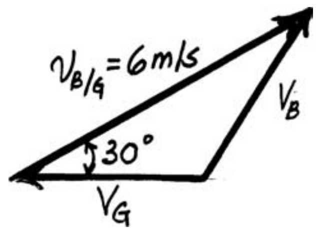
15–50. The 5-kg spring-loaded gun rests on the smooth surface. It fires a ball having a mass of 1 kg with a velocity of $v' = 6$ m/s relative to the gun in the direction shown. If the gun is originally at rest, determine the distance the ball is from the initial position of the gun at the instant the ball reaches its highest elevation C. Neglect the size of the gun.





$(\xrightarrow{+})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = 1(v_B)_x - 5v_G$$

$$(v_B)_x = 5v_G$$

$(\xrightarrow{+})$ $\quad v_B = v_G + v_{B/G}$

$$5v_G = -v_G + 6\cos 30^\circ$$

$$v_G = 0.8660 \text{ m/s} \leftarrow$$

So that,

$(v_B)_x = 4.330 \text{ m/s} \rightarrow$

$(v_B)_y = 4.330 \tan 30^\circ = 2.5 \text{ m/s} \uparrow$

Time of flight for the ball:

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$0 = 2.5 - 9.81t$$

$$t = 0.2548 \text{ s}$$

Height of ball:

$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (2.5)^2 - 2(9.81)(h - 0)$$

$$h = 0.3186 \text{ m}$$

Distance ball travels:

$(\xrightarrow{+})$ $\quad s = v_0 t$

$$s = 4.330(0.2548) = 1.103 \text{ m} \rightarrow$$

Distance gun travels:

$(\xleftarrow{+})$ $\quad s = v_0 t$

$$s' = 0.8660(0.2548) = 0.2207 \text{ m} \leftarrow$$

$1.103 + 0.2207 = 1.324$ m

Distance from cannon to ball:

$\sqrt{(0.4587)^2 + (1.324)^2} = 1.36$ m **Ans.**

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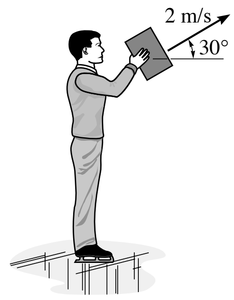
15–51. A man wearing ice skates throws an 8-kg block with an initial velocity of 2 m/s, measured relative to himself, in the direction shown. If he is originally at rest and completes the throw in 1.5 s while keeping his legs rigid, determine the horizontal velocity of the man just after releasing the block. What is the vertical reaction of both his skates on the ice during the throw? The man has a mass of 70 kg. Neglect friction and the motion of his arms.

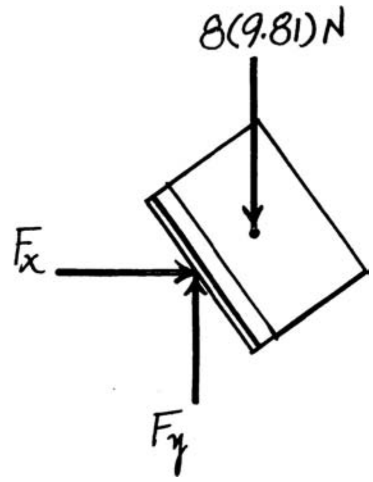


$\left(\overset{+}{\rightarrow}\right)$ $\quad 0 = -m_M\, v_M + m_B\,(v_B)_x$ **(1)**

However, $\mathbf{v}_B = \mathbf{v}_M + \mathbf{v}_{B/M}$

$\left(\overset{+}{\rightarrow}\right)$ $\quad (v_B)_x = -v_M + 2\cos 30°$ **(2)**

$(+\uparrow)$ $\quad (v_B)_y = 0 + 2\sin 30° = 1\text{ m/s}$



Substituting Eq. (2) into (1) yields:

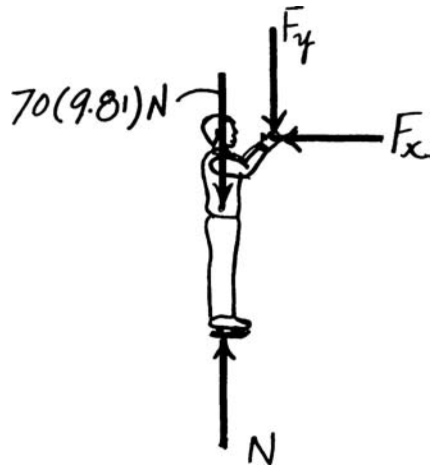
$$0 = -m_M\, v_M + m_B\,(-v_M + 2\cos 30°)$$

$$v_M = \frac{2m_B\cos 30°}{m_B + m_M} = \frac{2(8)\cos 30°}{8 + 70} = 0.178\text{m/s}$$ **Ans.**

For the block:

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + F_y(1.5) - 8(9.81)(1.5) = 8(2\sin 30°) \qquad F_y = 83.81\text{ N}$$

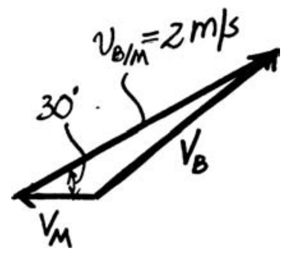


For the man:

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + N(1.5) - 70(9.81)(1.5) - 83.81(1.5) = 0$$

$$N = 771\text{ N}$$ **Ans.**



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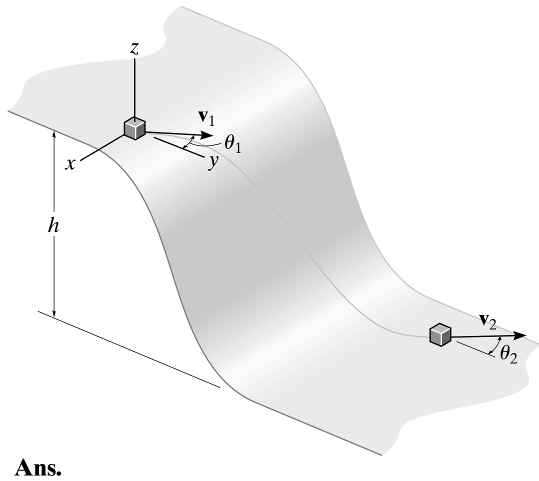
***15–52.** The block of mass m travels at v_1 in the direction θ_1 shown at the top of the smooth slope. Determine its speed v_2 and its direction θ_2 when it reaches the bottom.



There are no impulses in the : direction:

$$mv_1 \sin\theta_1 = mv_2 \sin\theta_2$$

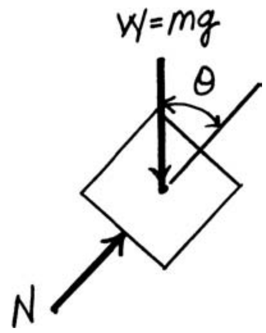
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2 + 0$$

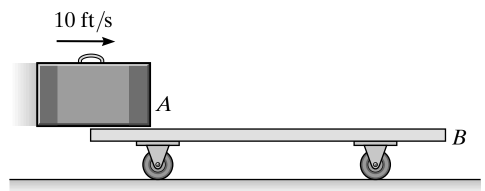
$$v_2 = \sqrt{v_1^2 + 2gh} \qquad \textbf{Ans.}$$

$$\sin\theta_2 = \frac{v_t \sin\theta_1}{\sqrt{v_1^2 + 2gh}}$$

$$\theta_2 = \sin^{-1}\left(\frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}\right) \qquad \textbf{Ans.}$$



•15–53. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally onto the cart at 10 ft/s when it is at rest, determine the length of time that A slides relative to B, and the final velocity of A and B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.



System

$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10 + 20}{32.2}\right)v$$

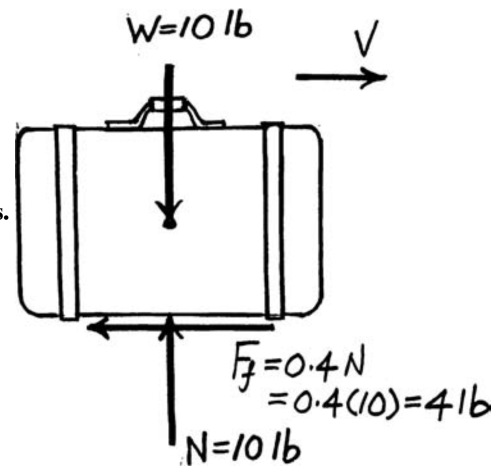
$$v = 3.33 \text{ ft/s} \qquad \textbf{Ans.}$$

For A:

$$m v_1 + \Sigma \int F\, dt = m v_2$$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$$t = 0.5176 = 0.518 \text{ s} \qquad \textbf{Ans.}$$



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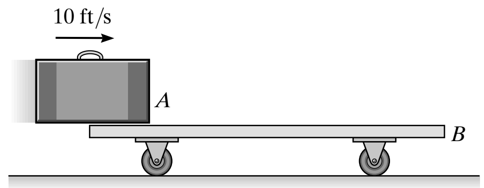
15–54. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally onto the cart at 10 ft/s when it is at rest, determine the time t and the distance B moves at the instant A stops relative to B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.

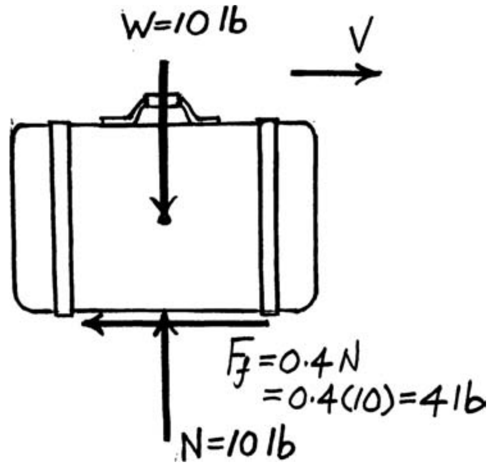


System:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10 + 20}{32.2}\right)v$$

$$v = 3.33 \text{ ft/s}$$



For A:

$$m v_1 + \Sigma \int F\,dt = m v_2$$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$$t = 0.5176 = 0.518 \text{ s}$$ **Ans.**

For B:

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$3.33 = 0 + a_c\,(0.5176)$$

$$a_c = 6.440 \text{ ft/s}^2$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(6.440)(0.5176)^2 = 0.863 \text{ ft}$$ **Ans.**

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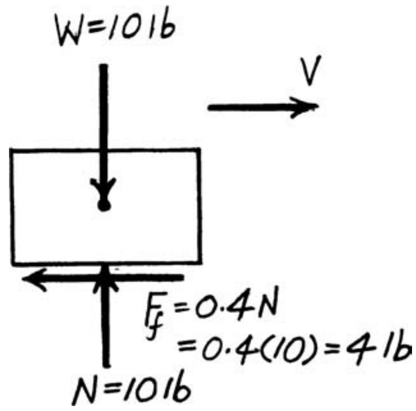
15–55. A 1-lb ball A is traveling horizontally at 20 ft/s when it strikes a 10-lb block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the time for the block B to stop sliding.



$$\left(\overset{+}{\rightarrow}\right) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$$\left(\overset{+}{\rightarrow}\right) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$(v_B)_2 = 2.909 \text{ ft/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow$

Block B:

$$\left(\overset{+}{\rightarrow}\right) \qquad m\,v_1 + \Sigma \int F\,dt = m\,v_2$$

$$\left(\frac{10}{32.2}\right)(2.909) - 4t = 0$$

$$t = 0.226 \text{ s}$$ **Ans.**

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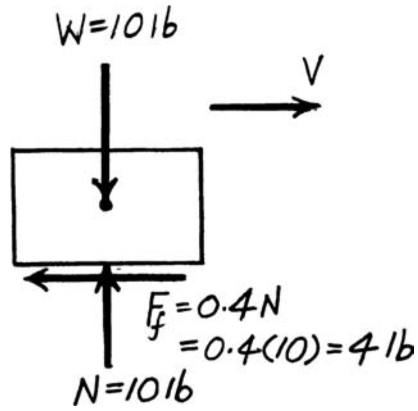
***15–56.** A 1-lb ball A is traveling horizontally at 20 ft/s when it strikes a 10-lb block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the distance block B slides on the plane before it stops sliding.



$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$(v_B)_2 = 2.909 \text{ ft/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow$

Block B:

$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(2.909)^2 - 4d = 0$$

$d = 0.329$ ft **Ans.**

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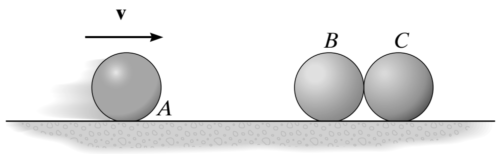
•15–57. The three balls each have a mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each ball is e. Neglect the size of each ball.



Conservation of Momentum: When ball A strikes ball B, we have

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = \frac{v(1 - e)}{2} \qquad (v_B)_2 = \frac{v(1 + e)}{2}$$

Conservation of Momentum: When ball B strikes ball C, we have

$$m_B (v_B)_2 + m_C (v_C)_1 = m_B (v_B)_3 + m_C (v_C)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\left[\frac{v(1 + e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad \textbf{[3]}$$

Coefficient of Restitution:

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\dfrac{v(1 + e)}{2} - 0} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

$$(v_C)_2 = \frac{v(1 + e)^2}{4} \qquad \textbf{Ans.}$$

$$(v_B)_3 = \frac{v(1 - e^2)}{4}$$

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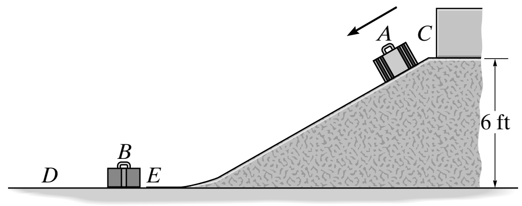
15–58. The 15-lb suitcase A is released from rest at C. After it slides down the smooth ramp, it strikes the 10-lb suitcase B, which is originally at rest. If the coefficient of restitution between the suitcases is $e = 0.3$ and the coefficient of kinetic friction between the floor DE and each suitcase is $\mu_k = 0.4$, determine (a) the velocity of A just before impact, (b) the velocities of A and B just after impact, and (c) the distance B slides before coming to rest.



Conservation of Energy: The datum is set at lowest point E. When the suitcase A is at point C it is 6 ft *above* the datum. Its gravitational potential energy is $15(6) = 90.0\ \text{ft}\cdot\text{lb}$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 90.0 = \frac{1}{2}\left(\frac{15}{32.2}\right)(v_A)_1^2 + 0$$

$$(v_A)_1 = 19.66\ \text{ft/s} = 19.7\ \text{ft/s} \qquad \textbf{Ans.}$$

Conservation of Momentum:

$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$(\overset{+}{\leftarrow}) \qquad \left(\frac{15}{32.2}\right)(19.66) + 0 = \left(\frac{15}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.3 = \frac{(v_B)_2 - (v_A)_2}{19.66 - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = 9.435\ \text{ft/s} = 9.44\ \text{ft/s} \leftarrow \qquad \textbf{Ans.}$$

$$(v_B)_2 = 15.33\ \text{ft/s} = 15.3\ \text{ft/s} \leftarrow \qquad \textbf{Ans.}$$

Principle of Work and Energy: $N_B = 10.0$ lb. Thus, the friction $F_f = \mu_k N_B = 0.4(10.0) = 4.00$ lb. The friction F_f which acts in the opposite direction to that of displacement does *negative* work. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)\left(15.33^2\right) + (-4.00 s_B) = 0$$

$$s_B = 9.13\ \text{ft} \qquad \textbf{Ans.}$$

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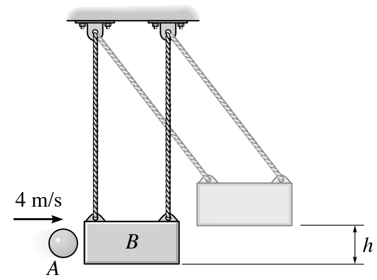
15–59. The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e = 0.8$, determine the maximum height h to which the block will swing before it momentarily stops.



System:

$$\left(\xrightarrow{+}\right) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$$\left(\xrightarrow{+}\right) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving:

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

Block:

Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$

$h = 0.0218 \text{ m} = 21.8 \text{ mm}$ **Ans.**

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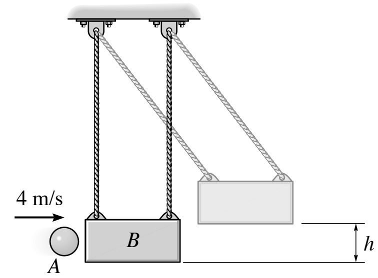
***15–60.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the time of impact between the ball and the block is 0.005 s, determine the average normal force exerted on the block during this time. Take $e = 0.8$.



System:

$$\left(\xrightarrow{+}\right) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

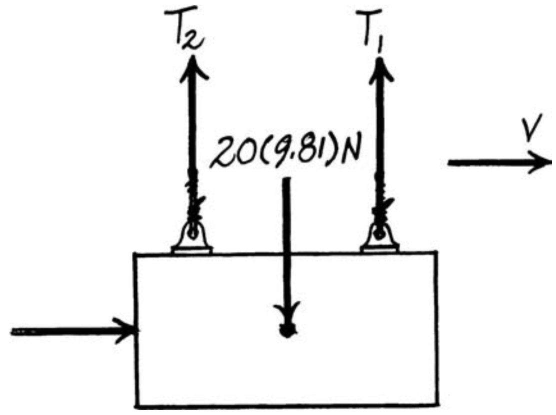
$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$$\left(\xrightarrow{+}\right) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 + (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$



Solving:

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

Block:

$$\left(\xrightarrow{+}\right) \qquad m v_1 + \Sigma \int F\,dt = m v_2$$

$$0 + F(0.005) = 20(0.6545)$$

$$F = 2618\text{ N} = 2.62\text{ kN} \qquad \textbf{Ans.}$$

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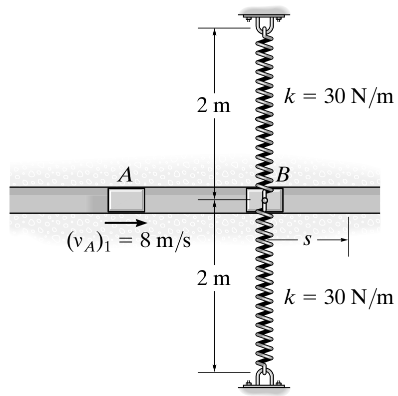
•15–61. The slider block B is confined to move within the smooth slot. It is connected to two springs, each of which has a stiffness of $k = 30$ N/m. They are originally stretched 0.5 m when $s = 0$ as shown. Determine the maximum distance, s_{max}, block B moves after it is hit by block A which is originally traveling at $(v_A)_1 = 8$ m/s. Take $e = 0.4$ and the mass of each block to be 1.5 kg.



$$\left(\xrightarrow{+}\right) \qquad \Sigma m\, v_1 = \Sigma m\, v_2$$

$$(1.5)(8) + 0 = (1.5)(v_A)_2 + (1.5)(v_B)_2$$

$$\left(\xrightarrow{+}\right) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.4 = \frac{(v_B)_2 - (v_A)_2}{8 - 0}$$

Solving:

$(v_A)_2 = 2.40$ m/s

$(v_B)_2 = 5.60$ m/s

$T_1 + V_1 + T_2 + V_2$

$$\frac{1}{2}(1.5)(5.60)^2 + 2\left[\frac{1}{2}(30)(0.5)^2\right] = 0 + 2\left[\frac{1}{2}(3)\left(\sqrt{s_{max}^2 + 2^2} - 1.5\right)^2\right]$$

$s_{max} = 1.53$ m **Ans.**

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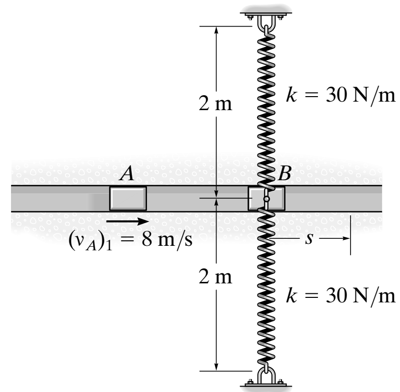
15–62. In Prob. 15–61 determine the average net force between blocks A and B during impact if the impact occurs in 0.005 s.



$$\left(\overset{+}{\rightarrow}\right) \qquad \Sigma m\, v_1 = \Sigma m\, v_2$$

$$(1.5)(8) + 0 = (1.5)(v_A)_2 + (1.5)(v_B)_2$$

$$\left(\overset{+}{\rightarrow}\right) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.4 = \frac{(v_B)_2 - (v_A)_2}{8 - 0}$$

Solving:

$(v_A)_2 = 2.40$ m/s

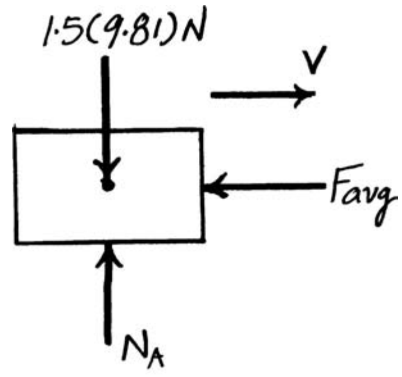
$(v_B)_2 = 5.60$ m/s

Choosing block A:

$$\left(\overset{+}{\rightarrow}\right) \qquad m v_1 + \Sigma \int F\, dt = m v_2$$

$$(1.5)(8) - F_{avg}\,(0.005) = 1.5(2.40)$$

$F_{avg} = 1.68$ kN **Ans.**

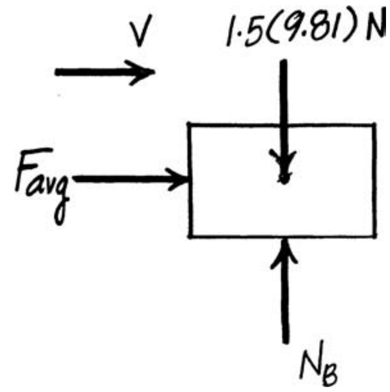


Choosing block B:

$$\left(\overset{+}{\rightarrow}\right) \qquad m v_1 + \Sigma \int F\, dt = m v_2$$

$$0 + F_{avg}\,(0.005) = 1.5(5.60)$$

$F_{avg} = 1.68$ kN **Ans.**



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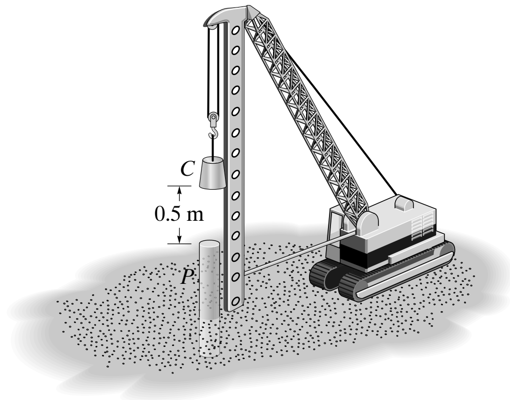
15–63. The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the initial speed of the pile just after it is struck by the hammer. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,

Counter weight: Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

$$(v_P)_2 - (v_C)_2 = 0.31321$$

Solving:

$(v_P)_2 = 0.940$ m/s **Ans.**

$(v_C)_2 = 0.626$ m/s

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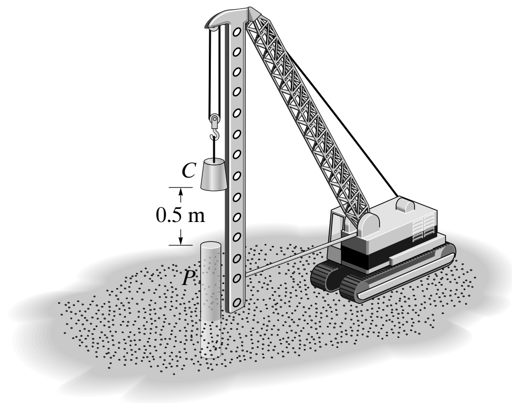
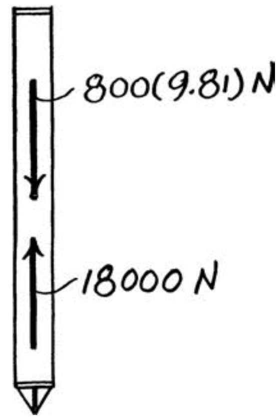
***15–64.** The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the distance the pile is driven into the sand after one blow if the sand offers a frictional resistance against the pile of 18 kN. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,



Counter weight: Datum at lowest point,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m\, v_1 = \Sigma m\, v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

$$(v_P)_2 - (v_C)_2 = 0.31321$$

Solving:

$$(v_P)_2 = 0.9396 \text{ m/s}$$

$$(v_C)_2 = 0.6264 \text{ m/s}$$

Pile:

$$T_2 + \Sigma U_{2-3} = T_3$$

$$\frac{1}{2}(800)(0.9396)^2 + 800(9.81)d - 18\,000d = 0$$

$$d = 0.0348 \text{ m} = 34.8 \text{ mm} \qquad \textbf{Ans.}$$

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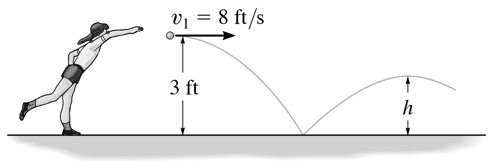
•15–65. The girl throws the ball with a horizontal velocity of $v_1 = 8$ ft/s. If the coefficient of restitution between the ball and the ground is $e = 0.8$, determine (a) the velocity of the ball just after it rebounds from the ground and (b) the maximum height to which the ball rises after the first bounce.



Kinematics: By considering the vertical motion of the falling ball, we have

$$(+\downarrow) \qquad (v_1)_y^2 = (v_0)_y^2 + 2a_c\left[s_y - (s_0)_y\right]$$

$$(v_1)_y^2 = 0^2 + 2(32.2)(3 - 0)$$

$$(v_1)_y = 13.90 \text{ ft/s}$$

Coefficient of Restitution (y):

$$e = \frac{\left(v_g\right)_2 - (v_2)_y}{(v_1)_y - \left(v_g\right)_1}$$

$$(+\uparrow) \qquad 0.8 = \frac{0 - (v_2)_y}{-13.90 - 0}$$

$$(v_2)_y = 11.12 \text{ ft/s}$$

Conservation of "x" Momentum: The momentum is conserved along the x axis.

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 = m(v_x)_2\,; \qquad (v_x)_2 = 8 \text{ ft/s} \rightarrow$$

The magnitude and the direction of the rebounding velocity for the ball is

$$v_2 = \sqrt{(v_x)_2^2 + \left(v_y\right)_2^2} = \sqrt{8^2 + 11.12^2} = 13.7 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{11.12}{8}\right) = 54.3° \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of tfie ball after it rebounds from the ground, we have

$$(+\uparrow) \qquad (v)_y^2 = (v_2)_y^2 + 2a_c\left[s_y - (s_2)_y\right]$$

$$0 = 11.12^2 + 2(-32.2)(h - 0)$$

$$h = 1.92 \text{ ft} \qquad \textbf{Ans.}$$

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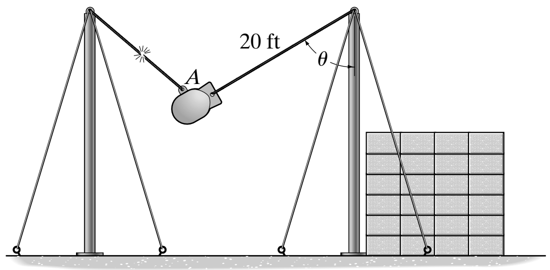
15–66. During an impact test, the 2000-lb weight is released from rest when $\theta = 60°$. It swings downwards and strikes the concrete blocks, rebounds and swings back up to $\theta = 15°$ before it momentarily stops. Determine the coefficient of restitution between the weight and the blocks. Also, find the impulse transferred between the weight and blocks during impact. Assume that the blocks do not move after impact.



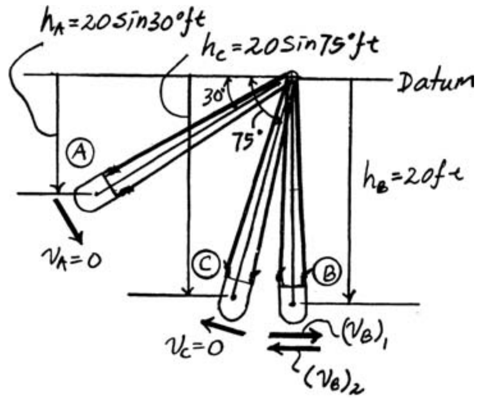
Conservation of Energy: First, consider the weight's fall from position A to position B as shown in Fig. a,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_A{v_A}^2 + \left(V_g\right)_A = \frac{1}{2}m(v_B)_1{}^2 + \left(V_g\right)_B$$

$$0 + [-2000(20\sin 30°)] = \frac{1}{2}\left(\frac{2000}{32.2}\right)(v_B)_1{}^2 + [-2000(20)]$$

$$(v_B)_1 = 25.38\,\text{ft/s} \rightarrow$$



(a)

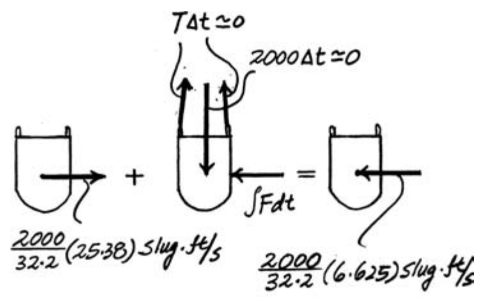
Subsequently, we will consider the weight rebounds from position B to position C.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}m(v_B)_1{}^2 + \left(V_g\right)_B = \frac{1}{2}m{v_C}^2 + \left(V_g\right)_C$$

$$\frac{1}{2}\left(\frac{2000}{32.2}\right)(v_B)_1{}^2 + [-2000(20)] = 0 + [-2000(20\sin 75°)]$$

$$(v_B)_2 = 6.625\text{ ft/s} \leftarrow$$



(b)

Coefficient of Restitution: Since the concrete blocks do not move, the coefficient of restitution can be written as

$$(\overset{+}{\rightarrow}) \qquad e = -\frac{(v_B)_2}{(v_B)_1} = \frac{(-6.625)}{25.38} = 0.261 \qquad \textbf{Ans.}$$

Principle of Impulse and Momentum: By referring to the Impulse and momentum diagrams shown in Fig. b,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_2}^{t_1} F_x dt = m(v_2)_x$$

$$\frac{2000}{32.2}(25.38) - \int F dt = -\frac{2000}{32.2}(6.625)$$

$$\int F dt = 1987.70\text{ lb}\cdot\text{s} = 1.99\text{ kip}\cdot\text{s} \qquad \textbf{Ans.}$$

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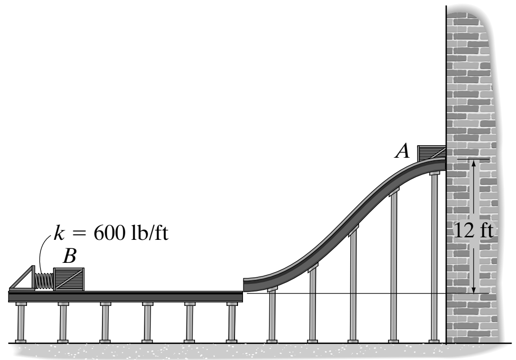
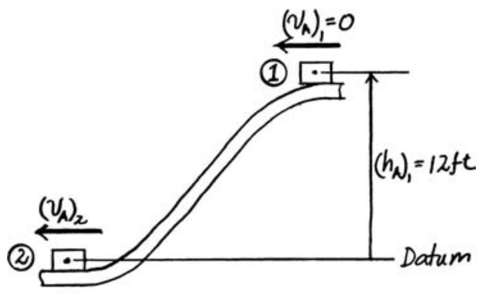
15–67. The 100-lb crate A is released from rest onto the smooth ramp. After it slides down the ramp it strikes the 200-lb crate B that rests against the spring of stiffness $k = 600$ lb/ft. If the coefficient of restitution between the crates is $e = 0.5$, determine their velocities just after impact. Also, what is the spring's maximum compression? The spring is originally unstretched.



Conservation of Energy: By considering crate A's fall from position (1) to position (2) as shown in Fig. a,



(a)

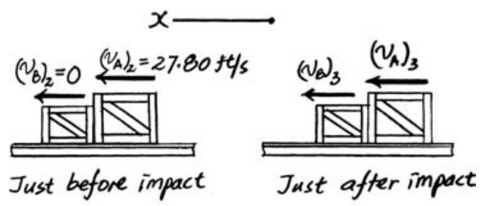
$$(T_A)_1 + (V_A)_1 = (T_A)_2 + (V_A)_2$$

$$\frac{1}{2}m_A(v_A)_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}m_A(v_A)_2{}^2 + \left(V_g\right)_2$$

$$0 + 100(12) = \frac{1}{2}\left(\frac{100}{32.2}\right)(v_A)_2{}^2 + 0$$

$$(v_A)_2 = 27.80 \text{ ft/s}$$

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact). By referring to Fig. b,



(b)

$$(\overset{+}{\leftarrow}) \qquad m_A(v_A)_2 + m_B(v_B)_2 = m_A(v_A)_3 + m_B(v_B)_3$$

$$\left(\frac{100}{32.2}\right)(27.80) + 0 = \left(\frac{100}{32.2}\right)(v_A)_3 + \left(\frac{200}{32.2}\right)(v_B)_3$$

$$100(v_A)_3 + 200(v_B)_3 = 2779.93 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$(\overset{+}{\leftarrow}) \qquad e = \frac{(v_B)_3 - (v_A)_3}{(v_A)_2 - (v_B)_2}$$

$$0.5 = \frac{(v_B)_3 - (v_A)_3}{27.80 - 0}$$

$$(v_B)_3 - (v_A)_3 = 13.90 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$(v_B)_3 = 13.90 \text{ ft/s} = 13.9 \text{ ft/s} \leftarrow \qquad (v_A)_3 = 0 \qquad \textbf{Ans.}$$

Conservation of Energy: The maximum compression of the spring occurs when crate B momentarily stops. By considering the conservation of energy of crate B,

$$(T_B)_3 + (V_B)_3 = (T_B)_4 + (V_B)_4$$

$$\frac{1}{2}m_B(v_B)_3{}^2 + \frac{1}{2}ks_3{}^2 = \frac{1}{2}m_B(v_B)_4{}^2 + \frac{1}{2}ks_{\max}{}^2$$

$$\frac{1}{2}\left(\frac{200}{32.2}\right)(13.90^2) + 0 = 0 + \frac{1}{2}(600)s_{\max}{}^2$$

$$s_{\max} = 1.41 \text{ ft} \qquad \textbf{Ans.}$$

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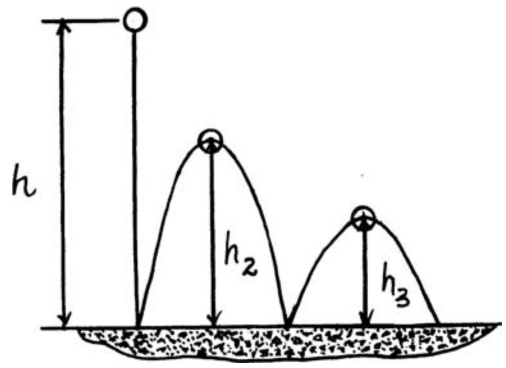
***15–68.** A ball has a mass m and is dropped onto a surface from a height h. If the coefficient of restitution is e between the ball and the surface, determine the time needed for the ball to stop bouncing.



Just before impact:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{2gh}$$

Time to fall:

$$(+\downarrow) \qquad v = v_0 + a_c t$$

$$v = v_0 + gt_1$$

$$\sqrt{2gh} = 0 + gt_1$$

$$t_1 = \sqrt{\frac{2h}{g}}$$

After impact:

$$(+\uparrow) \qquad e = \frac{v^2}{v}$$

$$v_2 = e\sqrt{2gh}$$

Height after first bounce: Datum at lowest point

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}m\left(e\sqrt{2gh}\right)^2 + 0 = 0 + mgh_2$$

$$h_2 = \frac{1}{2}e^2\left(\frac{2\,gh}{g}\right) = e^2 h$$

Time to rise to h_2:

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$v_3 = v_2 - gt_2$$

$$0 = e\sqrt{2gh} - gt_2$$

$$t_2 = e\sqrt{\frac{2h}{g}}$$

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***15–68. Continued**

Total time for first bounce

$$t_{1b} = t_1 + t_2 = \sqrt{\frac{2h}{g}} + e\sqrt{\frac{2h}{g}} = \sqrt{\frac{2h}{g}}\,(1 + e)$$

For the second bounce.

$$t_{2b} = \sqrt{\frac{2h_2}{g}}\,(1 + e) = \sqrt{\frac{2gh}{g}}\,(1 + e)e$$

For the third bounce.

$$h_3 = e^2\,h_2 = e^2\left(e^2\,h\right) = e^4\,h$$

$$t_{3b} = \sqrt{\frac{2h_3}{g}}\,(1 + e) = \sqrt{\frac{2h}{g}}\,(1 + e)e^2$$

Thus the total time for an infinite number of bounces:

$$t_{tot} = \sqrt{\frac{2h}{g}}\,(1 + e)\left(1 + e + e^2 + e^3 + \ldots\right)$$

$$t_{tot} = \sqrt{\frac{2h}{g}}\left(\frac{1 + e}{1 - e}\right)$$ **Ans.**

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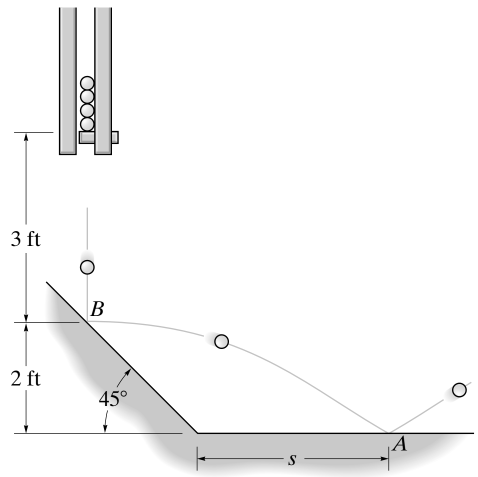
•15–69. To test the manufactured properties of 2-lb steel balls, each ball is released from rest as shown and strikes the 45° smooth inclined surface. If the coefficient of restitution is to be $e = 0.8$, determine the distance s to where the ball strikes the horizontal plane at A. At what speed does the ball strike point A?



Just before impact Datum at lowest point

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (2)(3) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)_1^2 + 0$$

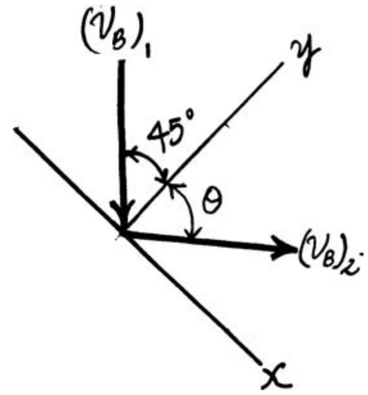
$$(v_B)_1 = 13.900 \text{ ft/s}$$

At B:

$$(+\searrow) \qquad \Sigma m(v_B)_{x1} = \Sigma m(v_B)_{x2}$$

$$\left(\frac{2}{32.2}\right)(13.900) \sin 45° = \left(\frac{2}{32.2}\right)(v_B)_2 \sin \theta$$

$$(v_B)_2 \sin \theta = 9.829 \text{ ft/s} \qquad \textbf{(1)}$$



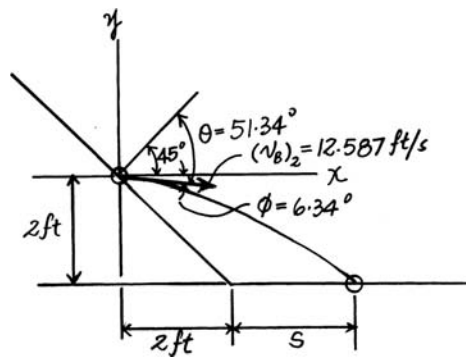
$$(+\nearrow) \qquad e = \frac{(v_B)_{y2} - 0}{0 - (v_B)_{y1}}$$

$$0.8 = \frac{(v_B)_2 \cos \theta - 0}{0 - (-13.900) \cos 45°}$$

$$(v_B)_2 \cos \theta = 7.863 \text{ ft/s} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2):

$$(v_B)_2 = 12.587 \text{ ft/s} \qquad \theta = 51.34°$$



$$\phi = 51.34° - 45° = 6.34°$$

$$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$\left(v_{A_r}\right)^2 = \left[12.587 \sin 6.34°\right]^2 + 2(32.2)(2 - 0)$$

$$v_{A_r} = 11.434 \text{ ft/s}$$

$$(+\downarrow) \qquad v = v_0 + a_c t$$

$$11.434 = 12.587 \sin 6.34° + 32.2t$$

$$t = 0.3119 \text{ s}$$

$$\left(\overset{+}{\rightarrow}\right) \qquad v_{Ax} = 12.587 \cos 6.34° = 12.510 \text{ ft/s}$$

$$s_t = v_B t$$

$$s + \frac{2}{\tan 45°} = (12.51)(0.3119)$$

$$s = 1.90 \text{ ft} \qquad \textbf{Ans.}$$

$$v_A = \sqrt{(12.510)^2 + (11.434)^2} = 16.9 \text{ ft/s} \qquad \textbf{Ans.}$$

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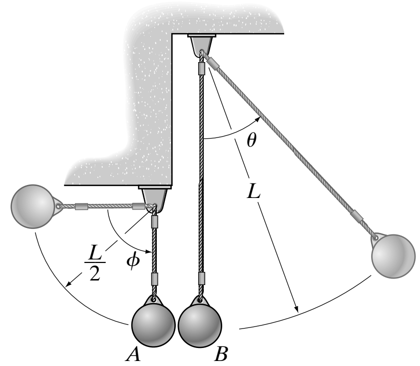
15–70. Two identical balls A and B of mass m are suspended from cords of length $L/2$ and L, respectively. Ball A is released from rest when $\phi = 90°$ and swings down to $\phi = 0°$, where it strikes B. Determine the speed of each ball just after impact and the maximum angle θ through which B will swing. The coefficient of restitution between the balls is e.



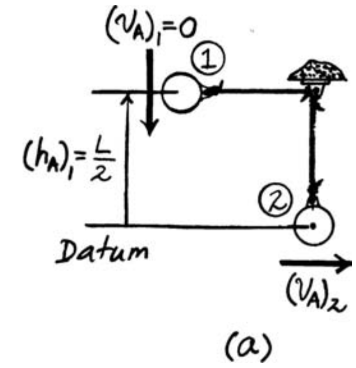
Conservation of Energy: First, we will consider bob A's swing from position (1) to position (2) as shown in Fig. a,

$$(T_A)_1 + (V_A)_1 = (T_A)_2 + (V_A)_2$$

$$\frac{1}{2}m_A(v_A)_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}m_A(v_A)_2{}^2 + \left(V_g\right)_2$$

$$0 + mg\left(\frac{L}{2}\right) = \frac{1}{2}m(v_A)_2{}^2 + 0$$

$$(v_A)_2 = \sqrt{gL}$$



Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact). By referring to Fig. b,

$$(\xrightarrow{+}) \qquad m_A(v_A)_2 + m_B(v_B)_2 = m_A(v_A)_3 + m_B(v_B)_3$$

$$m\sqrt{gL} + 0 = m(v_A)_3 + m(v_B)_3$$

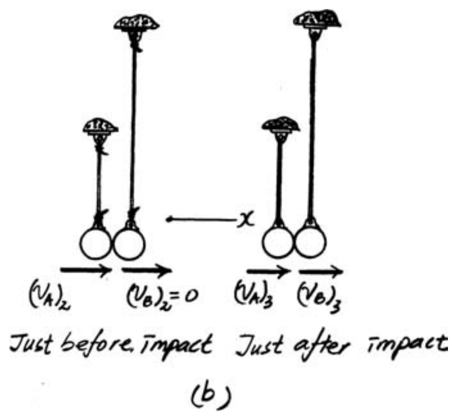
$$(v_A)_3 + (v_B)_3 = \sqrt{gL} \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 15–11 we have

$$(\xrightarrow{+}) \qquad e = \frac{(v_B)_3 - (v_A)_3}{(v_A)_2 - (v_B)_2}$$

$$e = \frac{(v_B)_3 + (v_A)_3}{\sqrt{gL} - 0}$$

$$(v_B)_3 - (v_A)_3 = e\sqrt{gL} \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2), yields

$$(v_A)_3 = \left(\frac{1-e}{2}\right)\sqrt{gL} \qquad \textbf{Ans.}$$

$$(v_B)_3 = \left(\frac{1+e}{2}\right)\sqrt{gL}$$

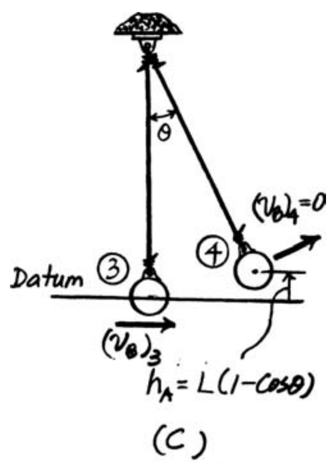
Conservation of Energy: We will now consider the swing of B from position (3) to position (4) as shown in Fig. c. Using the result of $(v_B)_3$,

$$(T_B)_3 + (V_B)_3 = (T_B)_4 + (V_B)_4$$

$$\frac{1}{2}m_B(v_B)_3{}^2 + \left(V_g\right)_3 = \frac{1}{2}m_B(v_B)_4{}^2 + \left(V_g\right)_4$$

$$\frac{1}{2}m\left[\left(\frac{1+e}{2}\right)\sqrt{gL}\right]^2 + 0 = 0 + mg\left[L(1-\cos\theta)\right]$$

$$\theta = \cos^{-1}\left[1 - \frac{(1+e)^2}{8}\right] \qquad \textbf{Ans.}$$



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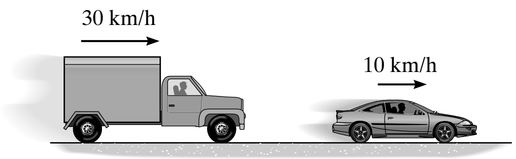
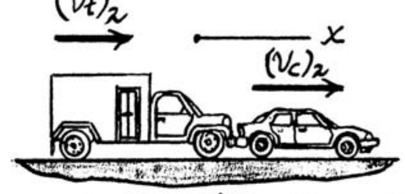
15–71. The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.



Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are $(v_t)_1 = \left[30\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 8.333\text{ m/s}$ and $(v_c)_1 = \left[10\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\,h}{3600\text{ s}}\right) = 2.778\text{ m/s}$.



By referring to Fig. *a*,

$$\left(\overset{+}{\rightarrow}\right) \quad m_t\left(v_t\right)_1 + m_c\left(v_c\right)_1 = m_t\left(v_t\right)_2 + m_c\left(v_c\right)_2$$

$$5000(8.333) + 2000(2.778) = 5000\left(v_t\right)_2 + 2000\left(v_c\right)_2$$

$$5\left(v_t\right)_2 + 2\left(v_c\right)_2 = 47.22 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_{c/t}) = \left[15\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4.167\text{ m/s} \rightarrow$. Applying the relative velocity equation,

$$(\mathbf{v}_c)_2 = (\mathbf{v}_t)_2 + (\mathbf{v}_{c/t})_2$$

$$\left(\overset{+}{\rightarrow}\right) \quad (v_c)_2 = (v_t)_2 + 4.167$$

$$(v_c)_2 - (v_t)_2 = 4.167 \qquad \textbf{(2)}$$

Applying the coefficient of restitution equation,

$$\left(\overset{+}{\rightarrow}\right) \quad e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_1 - (v_c)_1}$$

$$e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \qquad \textbf{(3)}$$

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15–71. Continued

Substituting Eq. (2) into Eq. (3),

$$e = \frac{4.167}{8.333 - 2.778} = 0.75 \qquad \textbf{Ans.}$$

Solving Eqs. (1) and (2) yields

$$(v_t)_2 = 5.556 \text{ m/s}$$

$$(v_c)_2 = 9.722 \text{ m/s}$$

Kinetic Energy: The kinetic energy of the system just before and just after the collision are

$$T_1 = \frac{1}{2} m_t (v_t)_1{}^2 + \frac{1}{2} m_c (v_c)_1{}^2$$

$$= \frac{1}{2}(5000)(8.333^2) + \frac{1}{2}(2000)(2.778^2)$$

$$= 181.33\left(10^3\right) \text{ J}$$

$$T_2 = \frac{1}{2} m_t (v_t)_2{}^2 + \frac{1}{2} m_c (v_c)_2{}^2$$

$$= \frac{1}{2}(5000)(5.556^2) + \frac{1}{2}(2000)(9.722^2)$$

$$= 171.68\left(10^3\right)\text{J}$$

Thus,

$$\Delta E = T_1 - T_2 = 181.33\left(10^3\right) - 171.68\left(10^3\right)$$

$$= 9.645\left(10^3\right) \text{ J}$$

$$= 9.65\text{kJ} \qquad \textbf{Ans.}$$

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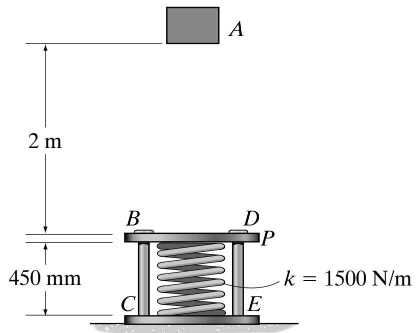
***15–72.** A 10-kg block A is released from rest 2 m above the 5-kg plate P, which can slide freely along the smooth vertical guides BC and DE. Determine the velocity of the block and plate just after impact. The coefficient of restitution between the block and the plate is $e = 0.75$. Also, find the maximum compression of the spring due to impact. The spring has an unstretched length of 600 mm.



Conservation of Energy: By considering block A's fall from position (1) to position (2) as shown in Fig. a,

$$(T_A)_1 + (V_A)_1 = (T_A)_2 + (V_A)_2$$

$$\frac{1}{2}m_A(v_A)_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}m_A(v_A)_2{}^2 + \left(V_g\right)_2$$

$$0 + 10(9.81)(2) = \frac{1}{2}(10)\left(v_A\right)_2{}^2 + 0$$

$$(v_A)_2 = 6.264 \text{ m/s}$$

Conservation of Linear Momentum: Since the weight of block A and plate **P** and the force developed in the spring are nonimpulsive, the linear momentum of the system is conserved along the line of impact (y axis). By referring to Fig. b,

$$(+\downarrow) \qquad m_A(v_A)_2 + m_P(v_P)_2 = m_A(v_A)_3 + m_P(v_P)_2$$

$$10(6.262) + 0 = 10(v_A)_3 + 5(v_P)_2$$

$$(v_P)_2 + 2(v_A)_3 = 12.528 \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 15–11 we have

$$(+\downarrow) \qquad e = \frac{(v_P)_3 - (v_A)_3}{(v_A)_2 - (v_P)_2}$$

$$0.75 = \frac{(v_P)_3 - (v_A)_3}{6.264 - 0}$$

$$(v_P)_3 - (v_A)_3 = 4.698 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_A)_3 = 2.610 \text{ m/s} \qquad (v_P)_3 = 7.308 \text{ m/s}$$

Conservation of Energy: The maximum compression of the spring occurs when plate P momentarily stops. If we consider the plate's fall fromposition (3) to position (4) as shown in Fig. c,

$$(T_P)_3 + (V_P)_3 = (T_P)_4 + (V_P)_4$$

$$\frac{1}{2}m_P(v_P)_3{}^2 + \left[\left(V_g\right)_3 + (V_e)_3\right] = \frac{1}{2}m_P(v_P)_4{}^2 + \left[\left(V_g\right)_4 + (V_e)_4\right]$$

$$\frac{1}{2}(5)(7.308^2) + \left[5(9.81)s_{\text{max}} + \frac{1}{2}(1500)(0.6 - 0.45)^2\right]$$

$$= 0 + \left[0 + \frac{1}{2}(1500)\left[s_{\text{max}} + (0.6 - 0.45)\right]^2\right]$$

$$750s^2{}_{\text{max}} + 175.95s_{\text{max}} - 133.25 = 0$$

$$s_{\text{max}} = 0.3202 \text{ m} = 320 \text{ mm} \qquad \textbf{Ans.}$$

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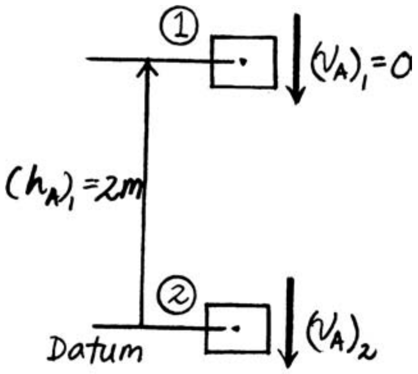
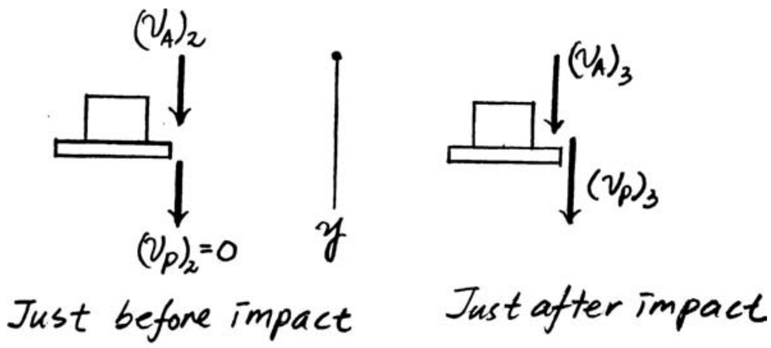
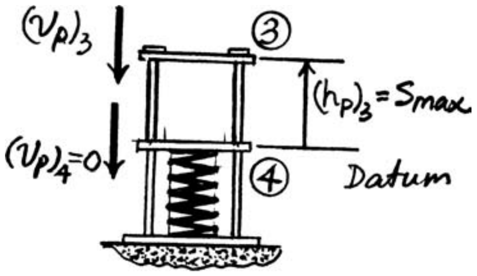


(a)



(b)



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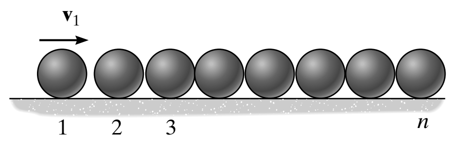
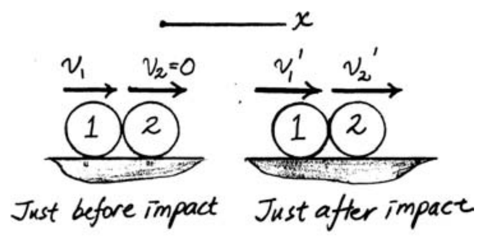
•15–73. A row of n similar spheres, each of mass m, are placed next to each other as shown. If sphere 1 has a velocity of v_1, determine the velocity of the nth sphere just after being struck by the adjacent $(n-1)$th sphere. The coefficient of restitution between the spheres is e.



When sphere (1) strikes sphere (2), the linear momentum of the system is conserved along the x axis (line of impact). By referring to Fig. a,



(a)

$$\left(\xrightarrow{+}\right) \qquad mv_1 + 0 = mv_1' + mv_2'$$

$$v_1' + v_2' = v_1 \tag{1}$$

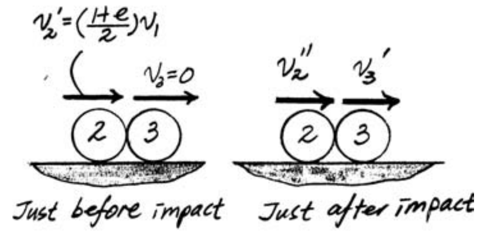
$$\left(\xrightarrow{+}\right) \qquad e = \frac{v_2' - v_1'}{v_1 - 0}$$

$$v_2' - v_1' = ev_1 \tag{2}$$

Eliminating v_1' from Eqs. (1) and (2), we obtain

$$v_2' = \left(\frac{1+e}{2}\right)v_1$$

Subsequently, sphere (2) strikes sphere (3). By referring to Fig. b and using the result of v_2',



(b)

$$\left(\xrightarrow{+}\right) \qquad m\left(\frac{1+e}{2}\right)v_1 + 0 = mv_2'' + mv_3'$$

$$v_2'' + v_3' = \left(\frac{1+e}{2}\right)v_1 \tag{3}$$

Applying Eq. 15–11 we have

$$\left(\xrightarrow{+}\right) \qquad e = \frac{v_3' - v_2''}{\left(\frac{1+e}{2}\right)v_1 - 0}$$

$$v_3' - v_2'' = \left[\frac{e(1+e)}{2}\right]v_1 \tag{4}$$

Eliminating v_2'' from Eqs. (3) and (4), we obtain

$$v_3' = \left(\frac{1+e}{2}\right)^2 v_1$$

If we continue to use the above procedures to analyse the impact between spheres (3) and (4), the speed of sphere (4) after the impact.

$$v_4' = \left(\frac{1+e}{2}\right)^3 v_1$$

Thus, when sphere $(n-1)$ strikes sphere n, the speed of sphere n just after impact is

$$v_n' = \left(\frac{1+e}{2}\right)^{n-1} v_1 \qquad \textbf{Ans.}$$

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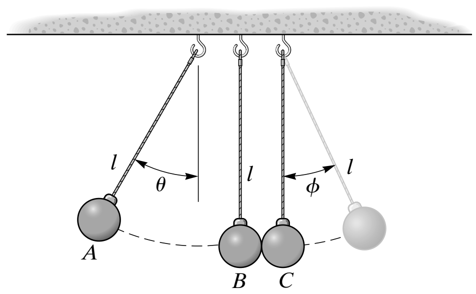
15–74. The three balls each have a mass of m. If A is released from rest at θ, determine the angle ϕ to which C rises after collision. The coefficient of restitution between each ball is e.



Conservation of Energy: The datum is set at the initial position of ball B. When ball A is $l(1 - \cos\theta)$ *above* the datum its gravitational potential energy is $mg[l(l - \cos\theta)]$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mg[l(1 - \cos\theta)] = \frac{1}{2}m(v_A)_1^2 + 0$$

$$(v_A)_1 = \sqrt{2gl(1 - \cos\theta)}$$

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$\left(\xrightarrow{+}\right) \quad m\sqrt{2gl(1 - \cos\theta)} + 0 = m(v_A)_2 + m(v_B)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$\left(\xrightarrow{+}\right) \quad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gl(1 - \cos\theta)} - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = \frac{(1 - e)\sqrt{2gl(1 - \cos\theta)}}{2}$$

$$(v_B)_2 = \frac{(1 + e)\sqrt{2gl(1 - \cos\theta)}}{2}$$

Conservation of Momentum: When ball B strikes ball C, we have

$$m_B(v_B)_2 + m_C(v_C)_1 = m_B(v_B)_3 + m_C(v_C)_2$$

$$\left(\xrightarrow{+}\right) \quad m\left[\frac{(1 + e)\sqrt{2gl(1 - \cos\theta)}}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad \textbf{[3]}$$

Coefficient of Restitution:

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

$$\left(\xrightarrow{+}\right) \quad e = \frac{(v_C)_2 - (v_B)_3}{\dfrac{(1 + e)\sqrt{2gl(1 - \cos\theta)}}{2} - 0} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

$$(v_C)_2 = \frac{(1 + e)^2}{4}\sqrt{2gl(1 - \cos\theta)}$$

$$(v_B)_3 = \frac{(1 - e^2)}{4}\sqrt{2gl(1 - \cos\theta)}$$

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15–74. Continued

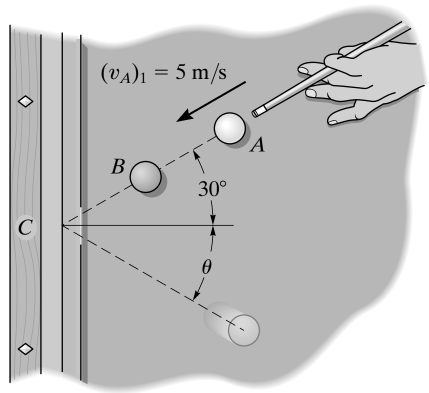
Conservation of Energy: The datum is set at the initial position of ball C. When ball C is $l(1 - \cos\phi)$ *above* the datum its gravitational potential energy is $mg[l(1 - \cos\phi)]$. Applying Eq. 14–21, we have

$$T_2 + V_2 = T_3 + V_3$$

$$0 + \frac{1}{2}m\left[\frac{(1+e)^2}{4}\sqrt{2gl(1-\cos\theta)}\right]^2 = 0 + mgl(1 - \cos\phi)$$

$$\phi = \cos^{-1}\left[1 - \frac{(1+e)^4}{16}(1 - \cos\theta)\right] \qquad \textbf{Ans.}$$

15–75. The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball B ($e = 0.8$), determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 kg. Neglect the size of each ball.



Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(+\ ) \qquad 0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$\left(\overset{+}{\leftarrow}\right) \qquad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = 0.500 \text{ m/s} \qquad (v_B)_2 = 4.50 \text{ m/s}$$

Conservation of "*y*" Momentum: When ball B strikes the cushion at C, we have

$$m_B\left(v_{B_y}\right)_2 = m_B\left(v_{B_y}\right)_3$$

$$(+\downarrow) \qquad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$$

$$(v_B)_3 \sin\theta = 2.25 \qquad \textbf{[3]}$$

Coefficient of Restitution (*x*):

$$e = \frac{(v_C)_2 - \left(v_{B_x}\right)_3}{\left(v_{B_x}\right)_2 - (v_C)_1}$$

$$\left(\overset{+}{\leftarrow}\right) \qquad 0.6 = \frac{0 - \left[-(v_B)_3 \cos\theta\right]}{4.50 \cos 30° - 0} \qquad \textbf{[4]}$$

Solving Eqs. [1] and [2] yields

$$(v_B)_3 = 3.24 \text{ m/s} \qquad \theta = 43.9° \qquad \textbf{Ans.}$$

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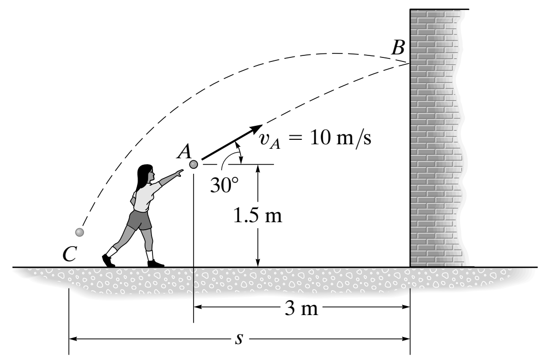
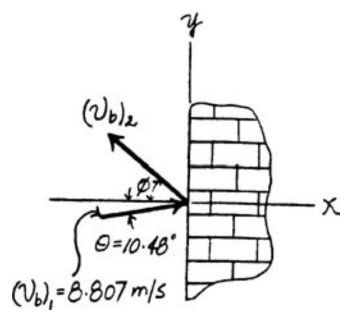
***15–76.** The girl throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



Kinematics: By considering the horizontal motion of the ball before the impact, we have



$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$3 = 0 + 10 \cos 30° t \qquad t = 0.3464 \text{ s}$$

By considering the vertical motion of the ball before the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + (a_c)_y t$$

$$= 10 \sin 30° + (-9.81)(0.3464)$$

$$= 1.602 \text{ m/s}$$

The vertical position of point B above the ground is given by

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$(s_B)_y = 1.5 + 10 \sin 30°(0.3464) + \frac{1}{2}(-9.81)\left(0.3464^2\right) = 2.643 \text{ m}$$

Thus, the magnitude of the velocity and its directional angle are

$$(v_b)_1 = \sqrt{(10 \cos 30°)^2 + 1.602^2} = 8.807 \text{ m/s} = 8.81 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1} \frac{1.602}{10 \cos 30°} = 10.48° = 10.5° \qquad \textbf{Ans.}$$

Conservation of "y" Momentum: When the ball strikes the wall with a speed of $(v_b)_1 = 8.807$ m/s, it rebounds with a speed of $(v_b)_2$.

$$m_b \left(v_{b_y}\right)_1 = m_b \left(v_{b_y}\right)_2$$

$$(\overset{+}{\leftarrow}) \qquad m_b (1.602) = m_b \left[(v_b)_2 \sin \phi\right]$$

$$(v_b)_2 \sin \phi = 1.602 \qquad \textbf{[1]}$$

Coefficient of Restitution (x):

$$e = \frac{(v_w)_2 - \left(v_{b_x}\right)_2}{\left(v_{b_x}\right)_1 - (v_w)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.5 = \frac{0 - \left[-(v_b)_2 \cos \phi\right]}{10 \cos 30° - 0} \qquad \textbf{[2]}$$

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***15–76. Continued**

Solving Eqs. [1] and [2] yields

$$\phi = 20.30° = 20.3° \qquad (v_b)_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the ball after the impact, we have

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$-2.643 = 0 + 4.617 \sin 20.30° t_1 + \frac{1}{2}(-9.81)t_1^2$$

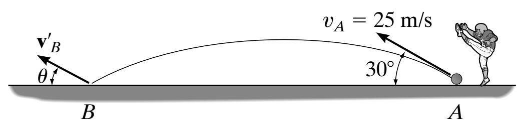
$$t_1 = 0.9153 \text{ s}$$

By considering the horizontal motion of the ball after the impact, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$s = 0 + 4.617 \cos 20.30°(0.9153) = 3.96 \text{ m} \qquad \textbf{Ans.}$$

•15–77. A 300-g ball is kicked with a velocity of $v_A = 25$ m/s at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



Kinematics: The parabolic trajectory of the football is shown in Fig. *a*. Due to the symmetrical properties of the trajectory, $v_B = v_A = 25$ m/s and $\phi = 30°$.

Conservation of Linear Momentum: Since no impulsive force acts on the football along the x axis, the linear momentum of the football is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m(v_B)_x = m(v'_B)_x$$

$$0.3(25 \cos 30°) = 0.3(v'_B)_x$$

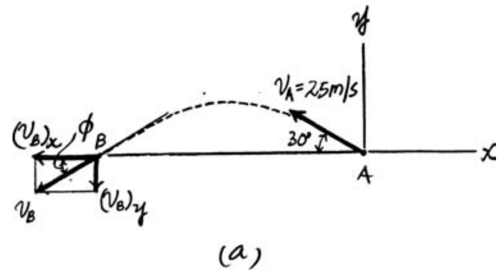
$$(v'_B)_x = 21.65 \text{ m/s} \leftarrow$$

Coefficient of Restitution: Since the ground does not move during the impact, the coefficient of restitution can be written as

$$(+\uparrow) \qquad e = \frac{0 - (v'_B)_y}{(v_B)_y - 0}$$

$$0.4 = \frac{-(v'_B)_y}{-25 \sin 30°}$$

$$(v'_B)_y = 5 \text{ m/s} \uparrow$$



Thus, the magnitude of $\mathbf{v}'_B$ is

$$v'_B = \sqrt{(v'_B)_x + (v'_B)_y} = \sqrt{21.65^2 + 5^2} = 22.2 \text{ m/s} \qquad \textbf{Ans.}$$

and the angle of $\mathbf{v}'_B$ is

$$\theta = \tan^{-1}\left[\frac{(v'_B)_y}{(v'_B)_x}\right] = \tan^{-1}\left(\frac{5}{21.65}\right) = 13.0° \qquad \textbf{Ans.}$$

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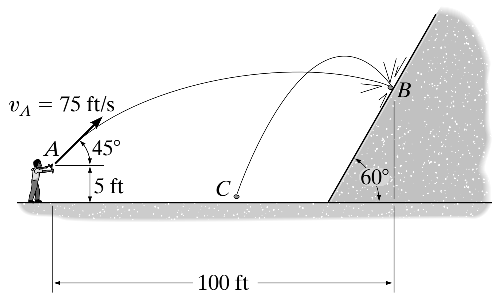
15–78. Using a slingshot, the boy fires the 0.2-lb marble at the concrete wall, striking it at B. If the coefficient of restitution between the marble and the wall is $e = 0.5$, determine the speed of the marble after it rebounds from the wall.



Kinematics: By considering the x and y motion of the marble from A to B, Fig. *a,*

$$\left(\xrightarrow{+}\right) \qquad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$100 = 0 + 75 \cos 45°\, t$$

$$t = 1.886 \text{ s}$$

and

$$(+\uparrow) \qquad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$(s_B)_y = 0 + 75 \sin 45°(1.886) + \frac{1}{2}(-32.2)(1.886^2)$$

$$= 42.76 \text{ ft}$$

and

$$(+\uparrow) \qquad (v_B)_y = (v_A)_y + a_y t$$

$$(v_B)_y = 75 \sin 45° + (-32.2)(1.886) = -7.684 \text{ ft/s} = 7.684 \text{ ft/s} \downarrow$$

Since $(v_B)_x = (v_A)_x = 75 \cos 45° = 53.03$ ft/s, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{(v_B)_x{}^2 + (v_B)_y{}^2} = \sqrt{53.03^2 + 7.684^2} = 53.59 \text{ ft/s}$$

and the direction angle of $\mathbf{v}_B$ is

$$\theta = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{7.684}{53.03}\right) = 8.244°$$

Conservation of Linear Momentum: Since no impulsive force acts on the marble along the inclined surface of the concrete wall (x' axis) during the impact, the linear momentum of the marble is conserved along the x' axis. Referring to Fig. *b,*

$$(+\nearrow) \qquad m_B(v_B')_{x'} = m_B(v_B')_{x'}$$

$$\frac{0.2}{32.2}(53.59 \sin 21.756°) = \frac{0.2}{32.2}(v_B' \cos \phi)$$

$$v_B' \cos \phi = 19.862 \qquad \textbf{(1)}$$

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15–78. Continued

Coefficient of Restitution: Since the concrete wall does not move during the impact, the coefficient of restitution can be written as

$$(+\nwarrow) \qquad e = \frac{0 - \left(v_B'\right)_{y'}}{\left(v_B\right)_{y'} - 0}$$

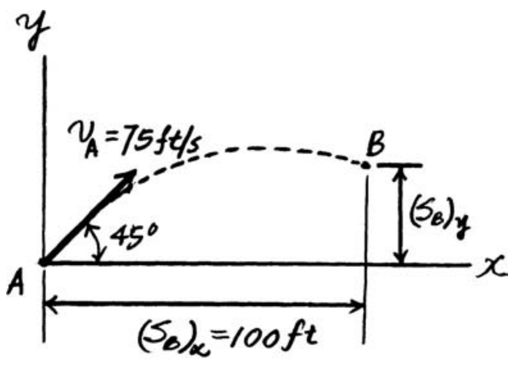
$$0.5 = \frac{-v_B' \sin \phi}{-53.59 \cos 21.756°}$$

$$v_B' \sin \phi = 24.885 \qquad \textbf{(2)}$$

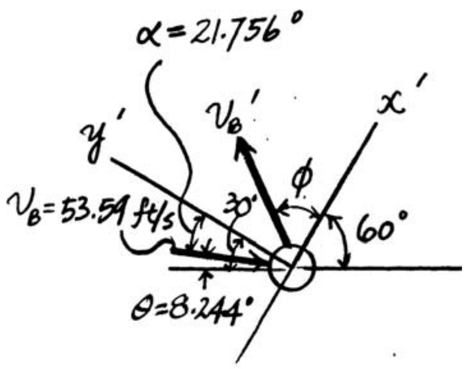
Solving Eqs. (1) and (2) yields

$$v_B' = 31.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\phi = 51.40°$$



(a)



(b)

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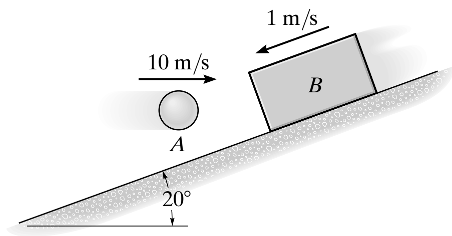
15–79. The 2-kg ball is thrown so that it travels horizontally at 10 m/s when it strikes the 6-kg block as it is traveling down the inclined plane at 1 m/s. If the coefficient of restitution between the ball and the block is $e = 0.6$, determine the speeds of the ball and the block just after the impact. Also, what distance does B slide up the plane before it momentarily stops? The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.4$.



System:

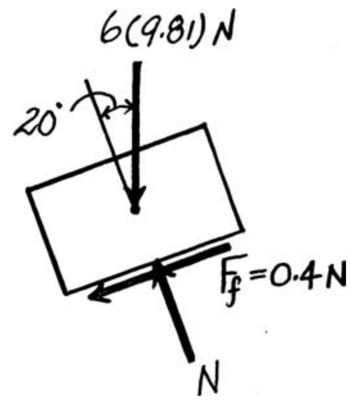
$$(+\nearrow) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$2(10 \cos 20°) - 6(1) = 2(v_{A_x})_2 + 6(v_{B_x})_2$$

$$(v_{A_x})_2 + 3(v_{B_x})_2 = 6.3969$$

$$(+\nearrow) \qquad e = \frac{(v_{B_x})_2 - (v_{A_x})_2}{(v_{A_x})_1 - v_{B_x})_1}; \qquad 0.6 = \frac{(v_{B_x})_2 - (v_{A_x})_2}{10 \cos 20° - (-1)}$$

$$(v_{B_x})_2 - (v_{A_x})_2 = 6.23816$$



Solving:

$$(v_{A_x})_2 = -3.0794 \text{ m/s}$$

$$(v_{B_x})_2 = 3.1588 \text{ m/s}$$

Ball A:

$$(+\nwarrow) \qquad m_A \left(v_{A_y}\right)_1 = m_A \left(v_{A_y}\right)_2$$

$$m_A(-10 \sin 20°) = m_A \left(v_{A_y}\right)_2$$

$$\left(v_{A_y}\right)_2 = -3.4202 \text{ m/s}$$

Thus,

$$(v_A)_2 = \sqrt{(-3.0794)^2 + (-3.4202)^2} = 4.60 \text{ m/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = 3.1588 = 3.16 \text{ m/s} \qquad \textbf{Ans.}$$

$$+\nwarrow \Sigma F_y = 0; \qquad -6(9.81) \cos 20° + N = 0 \qquad N = 55.31 \text{ N}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(6)(3.1588)^2 - 6(9.81) \sin 20° d - 0.4(55.31)d = 0$$

$$d = 0.708 \text{ m} \qquad \textbf{Ans.}$$

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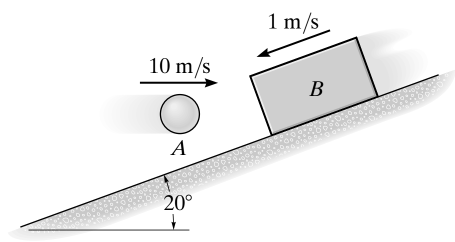
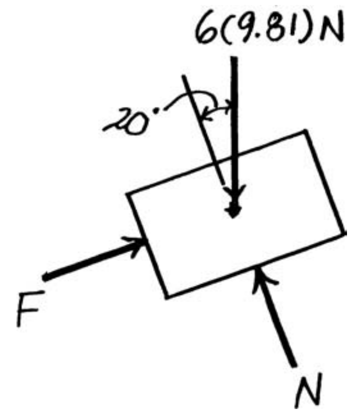
***15–80.** The 2-kg ball is thrown so that it travels horizontally at 10 m/s when it strikes the 6-kg block as it travels down the smooth inclined plane at 1 m/s. If the coefficient of restitution between the ball and the block is $e = 0.6$, and the impact occurs in 0.006 s, determine the average impulsive force between the ball and block.



System:



$(+\nearrow)\qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$2(10\cos 20^\circ) - 6(1) = 2(v_{A_x})_2 + 6(v_{B_x})_2$$

$$(v_{A_x})_2 + 3(v_{B_x})_2 = 6.3969$$

$(+\nearrow)\qquad e = \dfrac{(v_{B_x}) - (v_{A_x})_2}{(v_{A_x})_1 - (v_{B_x})_1};\qquad 0.6 = \dfrac{(v_{B_x})_2 - (v_{A_x})_2}{10\cos 20^\circ - (-1)}$

$$(v_{B_x})_2 - (v_{A_x})_2 = 6.23816$$

Solving:

$(v_{A_x})_2 = -3.0794$ m/s

$(v_{B_x})_2 = 3.1588$ m/s

Block B.

Neglect impulse of weight.

$(+\nearrow)\qquad mv_1 + \Sigma \int F\,dt = mv_2$

$$-6(1) + F(0.006) = 6(3.1588)$$

$F = 4.16$ kN **Ans.**

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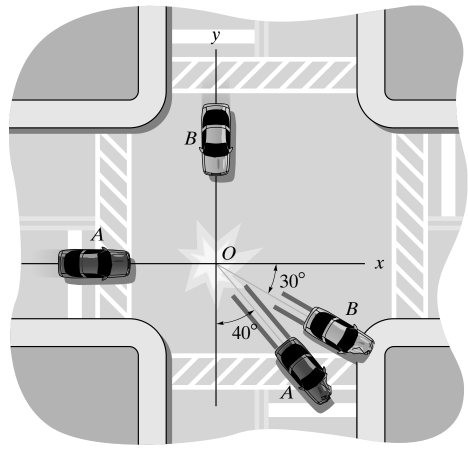
•15–81. Two cars A and B each have a weight of 4000 lb and collide on the icy pavement of an intersection. The direction of motion of each car after collision is measured from snow tracks as shown. If the driver in car A states that he was going 44 ft/s (30 mi/h) just before collision and that after collision he applied the brakes so that his car skidded 10 ft before stopping, determine the approximate speed of car B just before the collision. Assume that the coefficient of kinetic friction between the car wheels and the pavement is $\mu_k = 0.15$. *Note:* The line of impact has not been defined; however, this information is not needed for the solution.

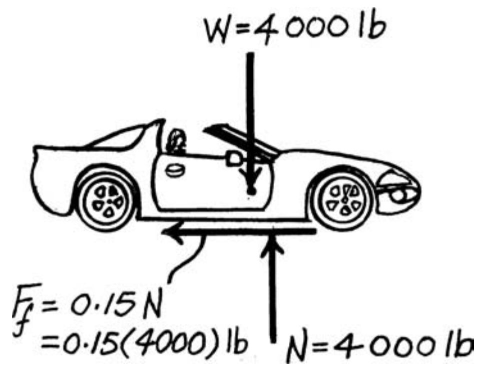


$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(v_A)_2^2 - (0.15)(4000)(10) = 0$$

$$(v_A)_2 = 9.829 \text{ ft/s}$$

$$(v_A)_1 = 44 \text{ ft/s}$$



$$\left(\xrightarrow{+}\right) \qquad \Sigma m_1 (v_x)_1 = \Sigma m_2 (v_x)_2$$

$$\left(\frac{4000}{32.2}\right)(44) + 0 = \left(\frac{4000}{32.2}\right)(9.829) \sin 40° + \left(\frac{4000}{32.2}\right)(v_B)_2 \cos 30°$$

$$(v_B)_2 = 43.51 \text{ ft/s}$$

$$(+\uparrow) \qquad \Sigma m_1 (v_y)_1 = \Sigma m_2 (v_y)_2$$

$$0 - \left(\frac{4000}{32.2}\right)(v_B)_1 = -\left(\frac{4000}{32.2}\right)(9.829) \cos 40° - \left(\frac{4000}{32.2}\right)(43.51) \sin 30°$$

$$(v_B)_1 = 29.3 \text{ ft/s}$$ **Ans.**

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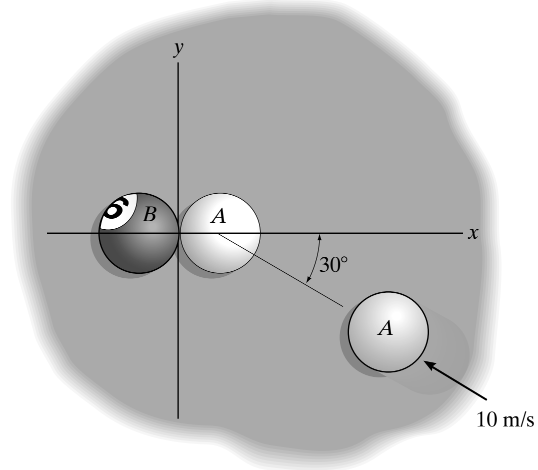
15–82. The pool ball A travels with a velocity of 10 m/s just before it strikes ball B, which is at rest. If the masses of A and B are each 200 g, and the coefficient of restitution between them is $e = 0.8$, determine the velocity of both balls just after impact.



Conservation of Linear Momentum: By referring to the impulse and momentum of the system of billiard balls shown in Fig. a, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$\left(\xleftarrow{+}\right) \qquad m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v'_A\right)_n + m_B\left(v'_B\right)_n$$

$$0.2(10)\cos 30° = 0.2v'_A \cos\theta_A + 0.2v'_B \cos\theta_B$$

$$v'_A \cos\theta_A + v'_B \cos\theta_B = 8.6603 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of each ball A and B is conserved along the t axis (tangent of plane impact). Thus,

$$\left(+\uparrow\right) \qquad m_A\left(v_A\right)_t = m_A\left(v'_A\right)_t$$

$$0.2(10)\sin 30° = 0.2v'_A \sin\theta_A$$

$$v'_A \sin\theta_A = 5 \qquad \textbf{(2)}$$

and

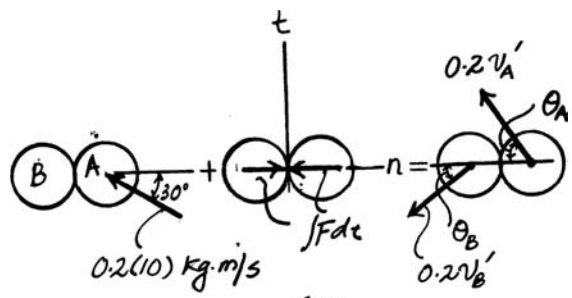
$$\left(+\uparrow\right) \qquad m_B\left(v\right)_t = m_B\left(v'_B\right)_t$$

$$0 = 0.2v'_B \sin\theta_B$$

$$v'_B \sin\theta_B = 0$$

Since $v'_B \neq 0$, then $\sin\theta_B = 0$. Thus

$\theta_B = 0$



(a)

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$\left(\xleftarrow{+}\right) \qquad e = \frac{\left(v'_B\right)_n - \left(v'_A\right)_n}{\left(v_A\right)_n - \left(v_B\right)_n}$$

$$0.8 = \frac{v'_B\cos\theta_B - v'_A\cos\theta_A}{10\cos 30° - 0}$$

$$v'_B\cos\theta_B - v'_A\cos\theta_A = 6.928 \qquad \textbf{(3)}$$

Using the result of θ_B and solving Eqs. (1), (2), and (3),

$$v'_A = 5.07\text{m/s} \qquad \theta_A = 80.2° \qquad \textbf{Ans.}$$

$$v'_B = 7.79 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

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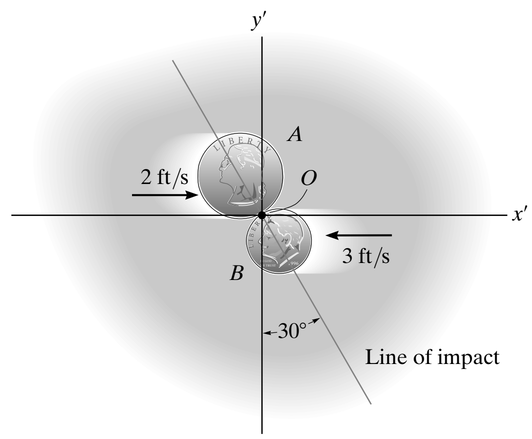
15–83. Two coins A and B have the initial velocities shown just before they collide at point O. If they have weights of $W_A = 13.2(10^{-3})$ lb and $W_B = 6.60(10^{-3})$ lb and the surface upon which they slide is smooth, determine their speeds just after impact. The coefficient of restitution is $e = 0.65$.



$$(+\searrow) \qquad m_A(v_{A_x})_1 + m_B(v_{B_x})_1 = m_A(v_{A_x})_2 + m_B(v_{B_x})_2$$

$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\sin 30° - \left(\frac{6.6(10^{-3})}{32.2}\right)3\sin 30°$$

$$= \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_x})_2 + \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_x})_2$$

$$(+\searrow) \qquad e = \frac{(v_{B_x})_2 - (v_{A_x})_2}{(v_{A_x})_1 - (v_{B_x})_1} \qquad\qquad 0.65 = \frac{(v_{B_x})_2 - (v_{A_x})_2}{2\sin 30° - (-3\sin 30°)}$$

Solving:

$(v_{A_x})_2 = -0.3750$ ft/s

$(v_{B_x})_2 = 1.250$ ft/s

$$(+\nearrow) \qquad m_A(v_{A_y})_2 = m_A(v_{A_y})_2$$

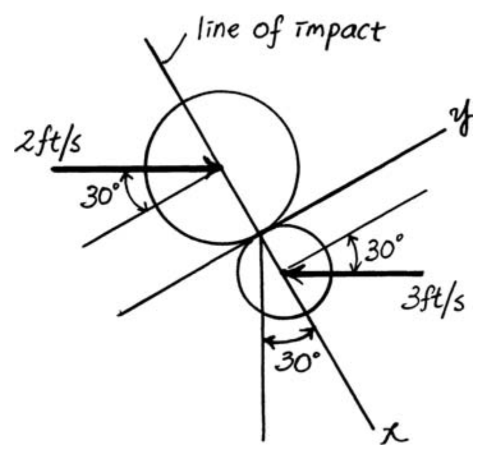
$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\cos 30° = \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_y})_2$$

$(v_{A_y})_2 = 1.732$ ft/s

$$(+\swarrow) \qquad m_B(v_{B_y})_1 = m_B(v_{B_y})_2$$

$$\left(\frac{6.6(10^{-3})}{32.2}\right)3\cos 30° = \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_y})_2$$

$(v_{B_y})_2 = 2.598$ ft/s



Thus:

$(v_B)_2 = \sqrt{(1.250)^2 + (2.598)^2} = 2.88$ ft/s **Ans.**

$(v_A)_2 = \sqrt{(-0.3750)^2 + (1.732)^2} = 1.77$ ft/s **Ans.**

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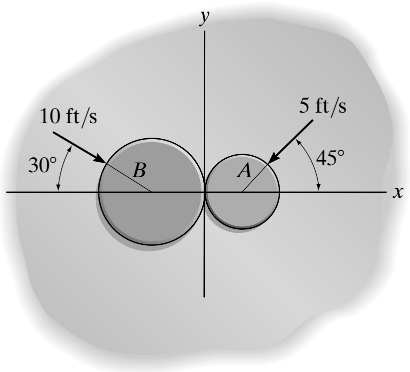
***15–84.** Two disks A and B weigh 2 lb and 5 lb, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their velocities just after impact. The coefficient of restitution between the disks is $e = 0.6$.



Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$\left(\xrightarrow{+}\right) \qquad m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v'_A\right)_n + m_B\left(v'_B\right)_n$$

$$-\frac{2}{32.2}(5)\cos 45° + \frac{5}{32.2}(10)\cos 30° = \frac{2}{32.2}v'_A\cos\theta_A + \frac{5}{32.2}v'_B\cos\theta_B$$

$$2v'_A\cos\theta_A + 5v'_B\cos\theta_B = 36.23 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum a of disks A and B are conserved along the t axis (tangent to the plane of impact). Thus,

$$\left(+\downarrow\right) \qquad m_A\left(v_A\right)_t = m_A\left(v'_A\right)_t$$

$$\frac{2}{32.2}(5)\sin 45° = \frac{2}{32.2}v'_A\sin\theta_A$$

$$v'_A\sin\theta_A = 3.5355 \qquad \textbf{(2)}$$

and

$$\left(+\downarrow\right) \qquad m_B\left(v_B\right)_t = m_B\left(v'_B\right)_t$$

$$\frac{2}{32.2}(10)\sin 30° = \ = \frac{2}{32.2}v'_B\sin\theta_B$$

$$v'_B\sin\theta_B = 5 \qquad \textbf{(3)}$$

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$\left(\xrightarrow{+}\right) \qquad e = \frac{\left(v'_A\right)_n - \left(v'_B\right)_n}{\left(v_B\right)_n - \left(v_A\right)_n}$$

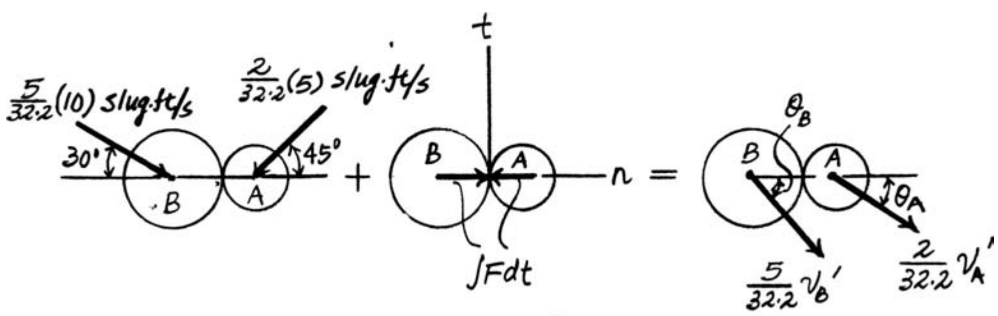
$$0.6 = \frac{v'_A\cos\theta_A - v'_B\cos\theta_B}{10\cos 30° - \left(-5\cos 45°\right)}$$

$$v'_A\cos\theta_A - v'_B\cos\theta_B = 7.317 \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), (3), and (4), yields

$v'_A = 11.0$ ft/s $\qquad \theta_A = 18.8°$ **Ans.**

$v'_B = 5.88$ ft/s $\qquad \theta_B = 58.3°$ **Ans.**



(a)

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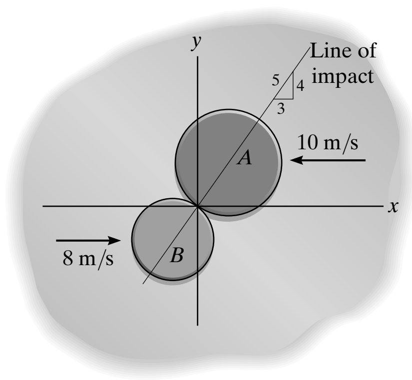
•**15–85.** Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$+\nearrow\ m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v'_A\right)_n + m_B\left(v'_B\right)_n$$

$$15(10)\left(\frac{3}{5}\right) - 10(8)\left(\frac{3}{5}\right) = 15v'_A\cos\phi_A + 10v'_B\cos\phi_B$$

$$15v'_A\cos\phi_A + 10v'_B\cos\phi_B = 42 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis (tangent to? plane of impact). Thus,

$$+\nwarrow\ m_A\left(v_A\right)_t = m_A\left(v'_A\right)_t$$

$$15(10)\left(\frac{4}{5}\right) = 15v'_A\sin\phi_A$$

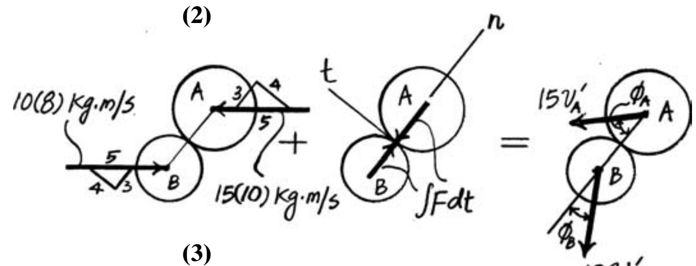
$$v'_A\sin\phi_A = 8 \qquad \textbf{(2)}$$

and

$$+\nwarrow\ m_B\left(v_B\right)_t = m_B\left(v'_B\right)_t$$

$$10(8)\left(\frac{4}{5}\right) = 10\,v'_B\sin\phi_B$$

$$v'_B\sin\phi_B = 6.4 \qquad \textbf{(3)}$$



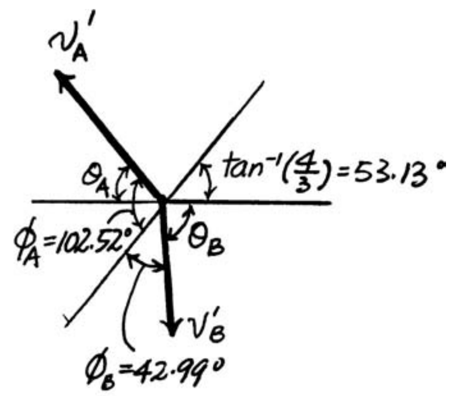
(a)

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$+\nearrow\ e = \frac{(v'_B)_n - (v'_A)_n}{(v_A)_n - (v_B)_n}$$

$$0.8 = \frac{v'_B\cos\phi_B - v'_A\cos\phi_A}{10\left(\frac{3}{5}\right) - \left[-8\left(\frac{3}{5}\right)\right]}$$

$$v'_B\cos\phi_B - v'_A\cos\phi_A = 8.64 \qquad \textbf{(4)}$$



(b)

Solving Eqs. (1), (2), (3), and (4), yeilds

$$v'_A = 8.19\text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_A = 102.52°$$

$$v'_B = 9.38\text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_B = 42.99°$$

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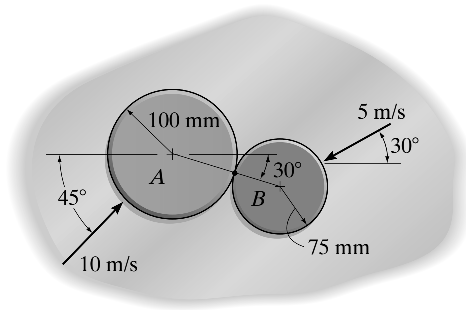
15–86. Disks A and B have a mass of 6 kg and 4 kg, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between the disks is $e = 0.6$.



Conservation of Linear Momentum: The orientation of the line of impact (n axis) and the tangent of the plane of contact (t axis) are shownn in Fig. *a.* By referring to the impulse and momentum of the system of disks shown in Fig. *b,* we notice that the linear momentum of the system is conserved along the n axis. Thus,

$$(\searrow +) \qquad m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$-6(10)\cos 75° + 4(5)\cos 60° = 6(v'_A \cos \phi_A) - 4(v'_B \cos \phi_B)$$

$$4v'_A \cos \phi_A - 6v'_B \cos \phi_B = 5.529 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis. Thus,

$$(+\nearrow) \qquad m_A(v_A)_t = m_A(v'_A)_t$$

$$6(10)\sin 75° = 6\,v'_A \sin \phi_A$$

$$v'_A \sin \phi_A = 9.659 \qquad \textbf{(2)}$$

and

$$(+\nearrow) \qquad m_B(v_B)_t = m_B(v'_B)_t$$

$$4(5)\sin 60° = 4\,v'_B \sin \phi_B$$

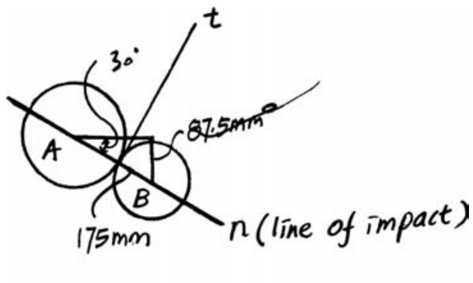
$$v'_B \sin \phi_B = 4.330 \qquad \textbf{(3)}$$

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$(\searrow +) \qquad e = \frac{(v'_B)_n - (v'_A)_n}{(v_A)_n - (v_B)_n}$$

$$0.6 = \frac{-v'_B \cos \phi_B - v'_A \cos \phi_A}{-10\cos 75° - 5\cos 60°}$$

$$v'_B \cos \phi_B + v'_A \cos \phi_A = 3.053 \qquad \textbf{(4)}$$



(a)

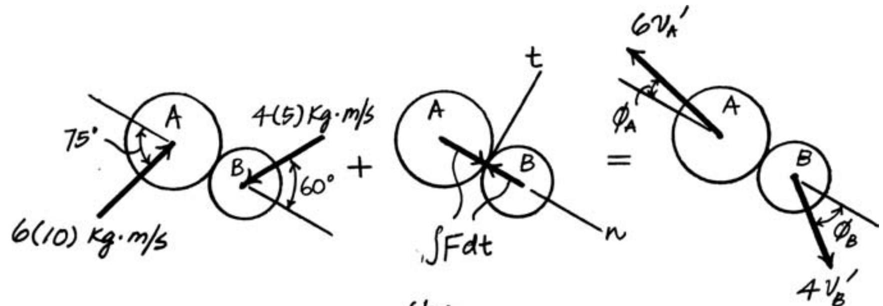
Solving Eqs. (1), (2), (3), and (4), yields

$$v'_A = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_A = 86.04°$$

$$v'_B = 4.94 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_B = 61.16°$$



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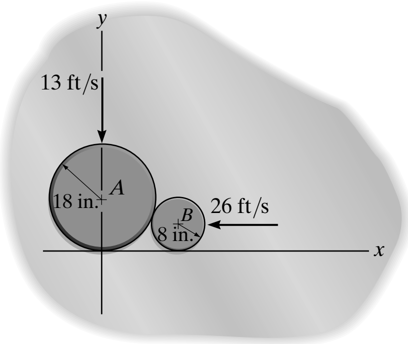
15–87. Disks A and B weigh 8 lb and 2 lb, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.5$.



Conservation of Linear Momentum: The orientation of the line of impact (n axis) and the tangent of the plane of contact, (t axis) are shown in Fig. *a*. By referring to the impulse and momentum of the system of disks shown in Fig. *b*, notice that the linear momentum of the system is conserved along the n axis. Thus,

$$m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v'_A\right)_n + m_B\left(v'_B\right)_n$$

$$-\frac{8}{32.2}(13)\left(\frac{5}{13}\right) + \frac{2}{32.2}(26)\left(\frac{12}{13}\right) = \frac{8}{32.2}\left(v'_A\cos\phi_A\right) - \frac{2}{32.2}\left(v'_B\cos\phi_B\right)$$

$$8v'_A\cos\phi_A - 2v'_B\cos\phi_B = 8 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis. Thus,

$$m_A\left(v_A\right)_t = m_A\left(v'_A\right)_t$$

$$\frac{8}{32.2}(13)\left(\frac{12}{13}\right) = \frac{8}{32.2}v'_A\sin\phi_A$$

$$v'_B\sin\phi_A = 12 \qquad \textbf{(2)}$$

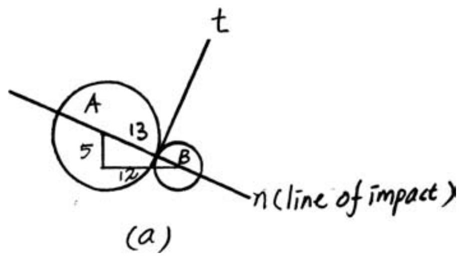
and

$$m_B\left(v_B\right)_t = m_B\left(v'_B\right)_t$$

$$\frac{2}{32.2}(26)\left(\frac{5}{13}\right) = \frac{2}{32.2}v'_B\sin\phi_B$$

$$v'_B\sin\phi_B = 10 \qquad \textbf{(3)}$$

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives



$$e = \frac{\left(v'_B\right)_n - \left(v'_A\right)_n}{\left(v_A\right)_n - \left(v_A\right)_n}$$

$$0.5 = \frac{-v'_B\cos\phi_B - v'_A\cos\phi_A}{-13\left(\frac{5}{13}\right) - 26\left(\frac{12}{13}\right)}$$

$$v'_A\cos\phi_A + v'_B\cos\phi_B = 14.5 \qquad \textbf{(4)}$$

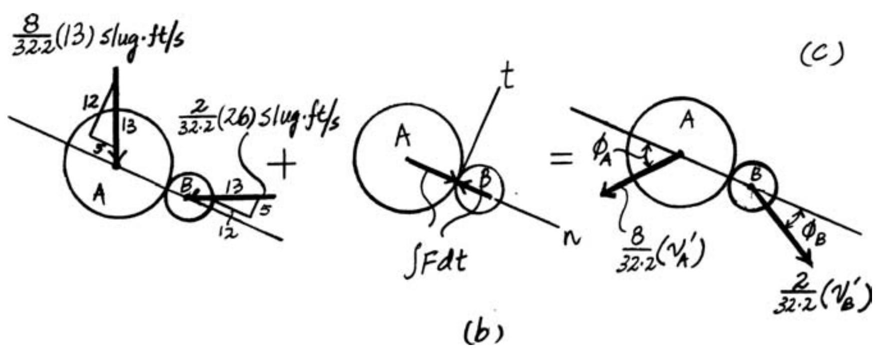
Solving Eqs. (1), (2), (3), and (4), yields

$v'_A = 12.6$ ft/s **Ans.**

$\phi_A = 72.86°$

$v'_B = 14.7$ ft/s **Ans.**

$\phi_B = 42.80°$



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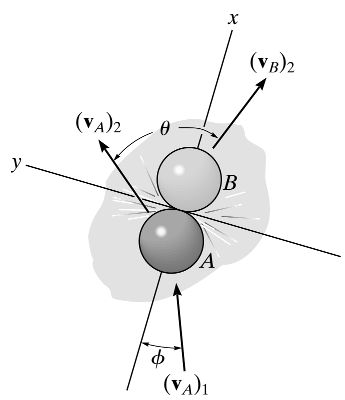
***15–88.** Ball A strikes ball B with an initial velocity of $(\mathbf{v}_A)_1$ as shown. If both balls have the same mass and the collision is perfectly elastic, determine the angle θ after collision. Ball B is originally at rest. Neglect the size of each ball.



Velocity before impact:

$$(v_{Ax})_1 = (v_A)_1 \cos\phi \qquad (v_{Ay})_1 = (v_A)_1 \sin\phi$$

$$(v_{Bx})_1 = 0 \qquad (v_{By})_1 = 0$$

Velocity after impact

$$(v_{Ax})_2 = (v_A)_2 \cos\theta_1 \qquad (v_{Ay})_2 = (v_A)_2 \sin\theta_1$$

$$(v_{Bx})_2 = (v_B)_2 \cos\theta_2 \qquad (v_{By})_2 = -(v_B)_2 \sin\theta_2$$

Conservation of "y" momentum:

$$m_B\,(v_{By})_1 = m_B\,(v_{By})_2$$

$$0 = m\big[-(v_\theta)_2 \sin\theta_2\big] \qquad \theta_2 = 0°$$

Conservation of "x" momentum:

$$m_A\,(v_{Ax})_1 + m_B\,(v_{Bx})_1 = m_A\,(v_{Ax})_2 + m_B\,(v_{Bx})_2$$

$$m\,(v_A)_1 \cos\phi + 0 = m\,(v_A)_2 \cos\theta_1 + m\,(v_B)_2 \cos 0°$$

$$(v_A)_1 \cos\phi = (v_A)_2 \cos\theta_1 + (v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution (x direction):

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}; \qquad 1 = \frac{(v_B)_2 \cos 0° - (v_A)_2 \cos\theta_1}{(v_A)_1 \cos\phi - 0}$$

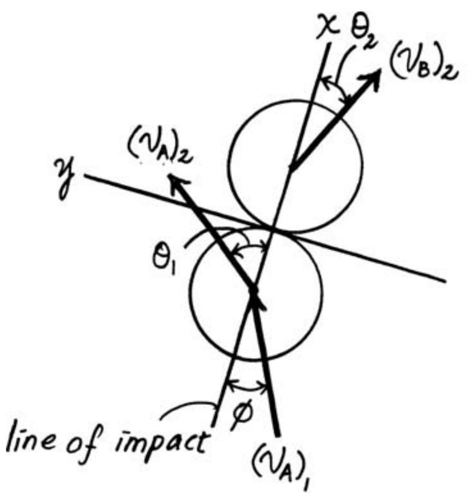
$$(v_A)_1 \cos\phi = -(v_A)_2 \cos\theta_1 + (v_B)_2 \qquad \textbf{(2)}$$

Subtracting Eq. (1) from Eq. (2) yields:

$$2\,(v_A)_2 \cos\theta_1 = 0 \qquad \text{Since } 2(v_A)_2 \neq 0$$

$$\cos\theta_1 = 0 \qquad \theta_1 = 90°$$

$$\theta = \theta_1 + \theta_2 = 90° + 0° = 90° \qquad \textbf{Ans.}$$



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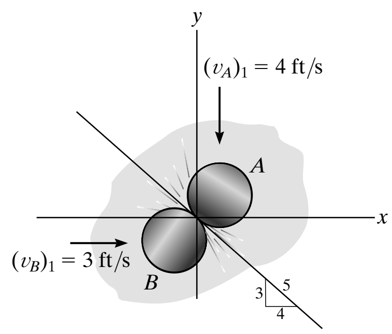
•15–89. Two disks A and B each have a weight of 2 lb and the initial velocities shown just before they collide. If the coefficient of restitution is $e = 0.5$, determine their speeds just after impact.



System:

$(+\nearrow)$ $\quad \Sigma\, mv_{1x} = \Sigma mv_{2x}$

$$\frac{2}{32.2}(3)\left(\frac{3}{5}\right) - \frac{2}{32.2}(4)\left(\frac{4}{5}\right) = \frac{2}{32.2}(v_B)_{2x} + \frac{2}{32.2}(v_A)_{2\,x}$$

$(+\nearrow)$ $\quad e = \dfrac{(v_{Ax})_2 - (v_{Bx})_2}{(v_{Bx})_1 - (v_{Ax})_1}; \qquad 0.5 = \dfrac{(v_{Ax})_2 - (v_{Bx})_2}{3\left(\frac{3}{5}\right) - \left[-4\left(\frac{4}{5}\right)\right]}$

Solving,

$(v_{A\,x})_2 = 0.550$ ft/s

$(v_{Bx})_2 = -1.95$ ft/s $= 1.95$ ft/s $\leftarrow$

Ball A:

$(+\nwarrow)$ $\quad m_A\left(v_{Ay}\right)_1 = m_A\left(v_{Ay}\right)_2$

$$-\frac{2}{32.2}(4)\left(\frac{3}{5}\right) = \frac{2}{32.2}\left(v_{Ay}\right)_2$$

$$\left(v_{Ay}\right)_2 = -2.40 \text{ ft/s}$$

Ball B:

$(+\nwarrow)$ $\quad m_B\left(v_{By}\right)_1 = m_B\left(v_{By}\right)_2$

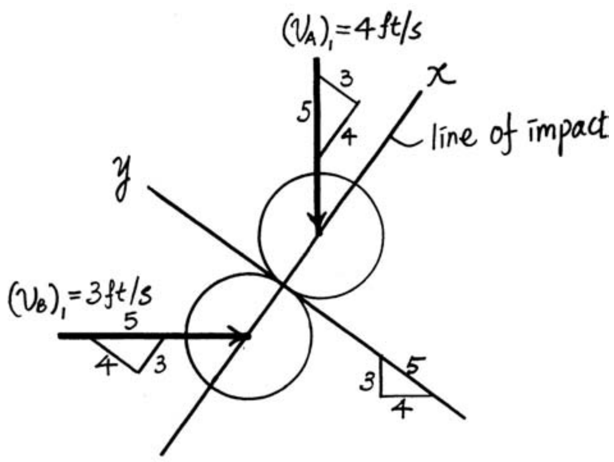
$$-\frac{2}{32.2}(3)\left(\frac{4}{5}\right) = \frac{2}{32.2}\left(v_{By}\right)_2$$

$$(v_{B_y})_2 = -2.40 \text{ ft/s}$$

Thus,

$(v_A)_2 = \sqrt{(0.550)^2 + (2.40)^2} = 2.46$ ft/s **Ans.**

$(v_B)_2 = \sqrt{(1.95)^2 + (2.40)^2} = 3.09$ ft/s **Ans.**



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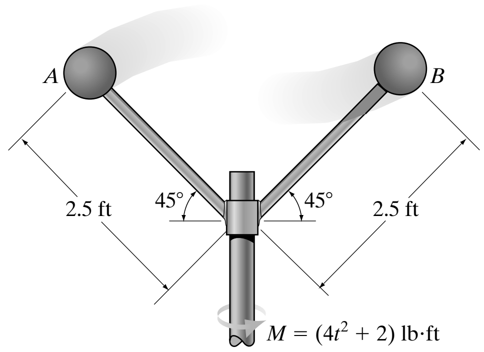
15–90. The spheres A and B each weighing 4 lb, are welded to the light rods that are rigidly connected to a shaft as shown. If the shaft is subjected to a couple moment of $M = (4t^2 + 2)$ lb · ft, where t is in seconds, determine the speed of A and B when $t = 3$ s. The system starts from rest. Neglect the size of the spheres.



Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_C$ has no component about the z axis, the force reaction $\mathbf{F}_C$ acts through the z axis, and the line of action of $\mathbf{W}_A$ and $\mathbf{W}_B$ are parallel to the z axis, they produce no angular impulse about the z axis.

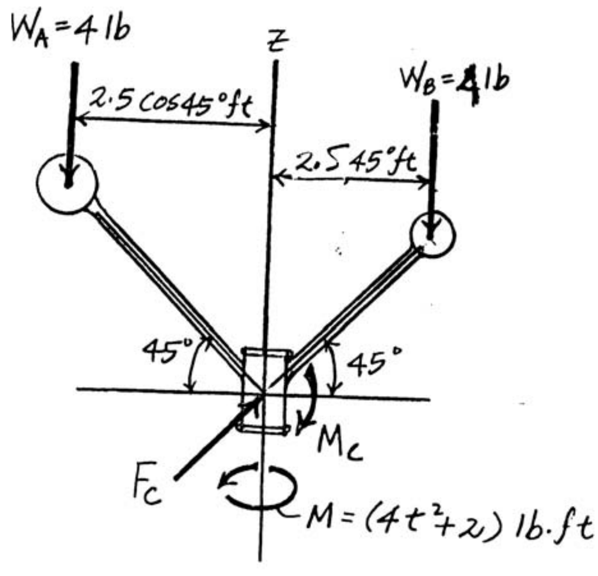
Principle of Angular Impulse and Momentum:

$$(H_1)_z + \Sigma \int_{t_2}^{t_1} M_z\, dt = (H_2)$$

$$0 + \int_0^{3\text{ s}} \left(4t^2 + 2\right)dt = 2\left(\frac{4}{32.2}(v)(2.5\cos 45^\circ)\right)$$

$$\left.\frac{4t^3}{3} + 2t\right|_0^{3\text{ s}} = 0.4392\, v$$

$v = 95.6$ ft/s **Ans.**



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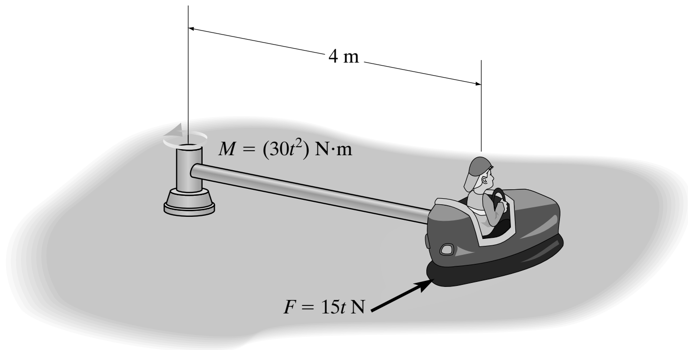
15–91. If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)\ \text{N}\cdot\text{m}$ and the engine of the car supplies a traction force of $F = (15t)$ N to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



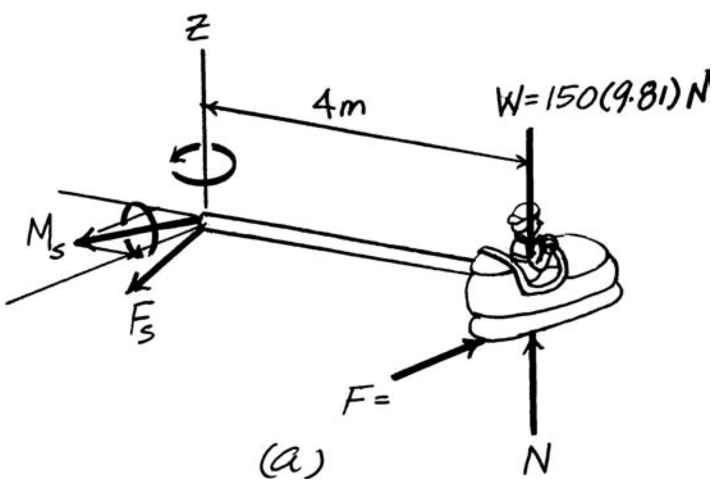
Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_S$ has no component about the z axis, the force reaction $\mathbf{F}_S$ acts through the z axis, and the line of action of **W** and **N** are parallel to the z axis, they produce no angular impulse about the z axis.

Principle of Angular Impulse and Momentum:

$$\left(H_1\right)_z + \Sigma\int_{t_2}^{t_1} M_z\,dt = \left(H_2\right)_z$$

$$0 + \int_0^{5\text{ s}} 30t^2\,dt + \int_0^{5\text{ s}} 15t(4)dt = 150v(4)$$

$v = 3.33$ m/s **Ans.**



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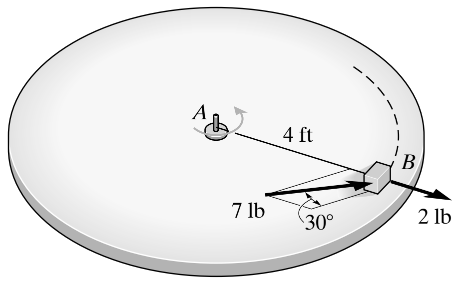
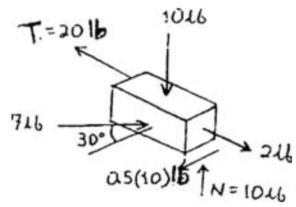
15–92. The 10-lb block rests on a surface for which $\mu_k = 0.5$. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. If the block is initially moving in a circular path with a speed $v_1 = 2$ ft/s at the instant the forces are applied, determine the time required before the tension in cord AB becomes 20 lb. Neglect the size of the block for the calculation.



$\Sigma F_n = ma_n;$

$$20 - 7\sin 30° - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$

$v = 13.67$ ft/s

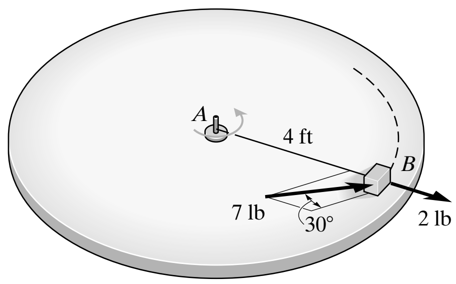


$$(H_A)_t + \Sigma\int M_A\,dt = (H_A)_2$$

$$\left(\frac{10}{32.2}\right)(2)(4) + (7\cos 30°)(4)(t) - 0.5(10)(4)\,t = \frac{10}{32.2}(13.67)(4)$$

$t = 3.41$ s **Ans.**

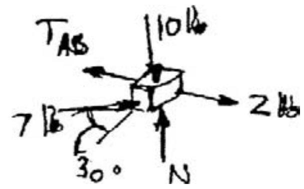
15–93. The 10-lb block is originally at rest on the smooth surface. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. Determine the time required to break the cord, which requires a tension $T = 30$ lb. What is the speed of the block when this occurs? Neglect the size of the block for the calculation.



$\Sigma F_n = ma_n;$

$$30 - 7\sin 30° - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$

$v = 17.764$ ft/s



$$(H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

$$0 + (7\cos 30°)(4)(t) = \frac{10}{32.2}(17.764)(4)$$

$t = 0.910$ s **Ans.**

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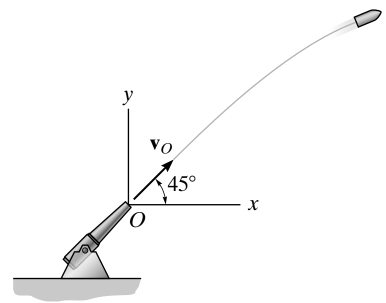
15–94. The projectile having a mass of 3 kg is fired from a cannon with a muzzle velocity of $v_0 = 500\,\text{m/s}$. Determine the projectile's angular momentum about point O at the instant it is at the maximum height of its trajectory.



At the maximum height, the projectile travels with a horizontal speed of $v = v_x = 500 \cos 45° = 353.6\ \text{m/s}^2$.

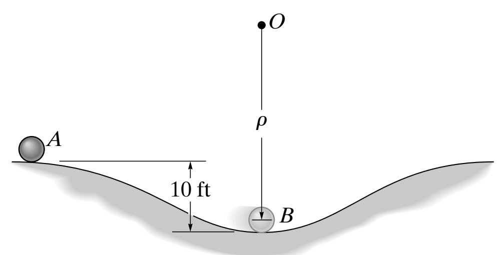
$$(+\uparrow) \qquad v_y^2 = (v_0)_y^2 + 2a_c\left[s_y - (s_0)_y\right]$$

$$0 = (500 \sin 45°)^2 + 2(-9.81)\left[(s_y)_{max} - 0\right]$$

$$(s_y)_{max} = 6371\ \text{m}$$

$$H_O = (d)(mv) = 6371(3)(353.6) = 6.76(10^6)\ \text{kg} \cdot \text{m}^2/\text{s}$$ **Ans.**

15–95. The 3-lb ball located at A is released from rest and travels down the curved path. If the ball exerts a normal force of 5 lb on the path when it reaches point B, determine the angular momentum of the ball about the center of curvature, point O. *Hint:* Neglect the size of the ball. The radius of curvature at point B must first be determined.



Datum at B:

$$T_1 + V_1 = T_2 + V_2$$

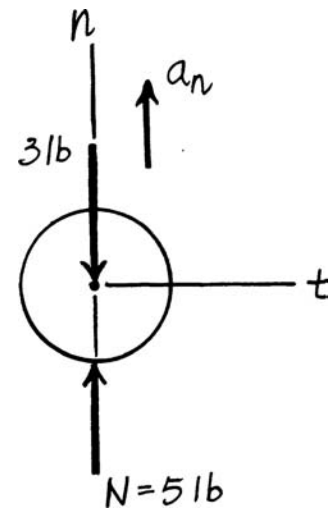
$$0 + 3(10) = \frac{1}{2}\left(\frac{3}{32.2}\right)(v_B)^2 + 0$$

$$v_B = 25.38\ \text{ft/s}$$

$$(+\uparrow)\Sigma F_n = ma_n; \qquad 5 - 3 = \left(\frac{3}{32.2}\right)\left(\frac{(25.38)^2}{\rho}\right)$$

$$\rho = 30\ \text{ft}$$

$$H_B = 30\left(\frac{3}{32.2}\right)(25.38) = 70.9\ \text{slug} \cdot \text{ft}^2/\text{s}$$ **Ans.**



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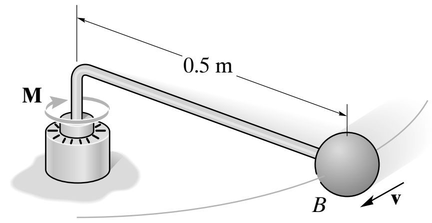
***15–96.** The ball B has a mass of 10 kg and is attached to the end of a rod whose mass can be neglected. If the shaft is subjected to a torque $M = (2t^2 + 4)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.

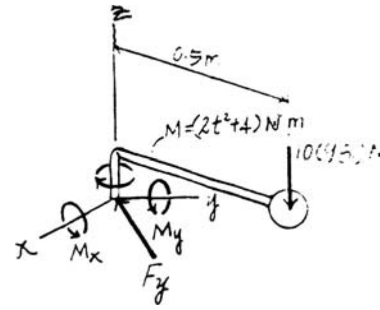


Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

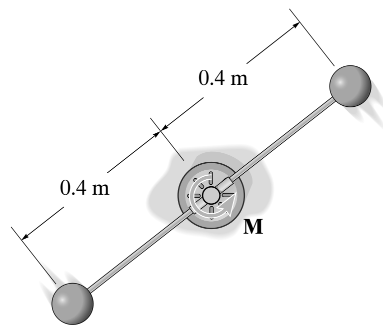
$$(H_z)_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

$$0.5(10)(2) + \int_0^{2\text{ s}} \left(2\,t^2 + 4\right) dt = 0.5(10)\,v$$

$$v = 4.67\text{ m/s} \qquad \textbf{Ans.}$$



•15–97. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. If a torque $M = (6e^{0.2t})\ \text{N}\cdot\text{m}$, where t is in seconds, is applied to the rod as shown, determine the speed of each of the spheres in 2 s, starting from rest.

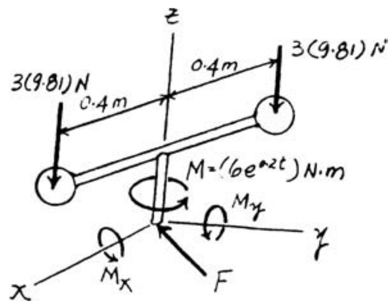


Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

$$(H_z)_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

$$2[0.4\,(3)\,(0)] + \int_0^{2\text{ s}} \left(6e^{0.2\,t}\right)\,dt = 2\,[0.4\,(3)\,v]$$

$$v = 6.15\text{ m/s} \qquad \textbf{Ans.}$$



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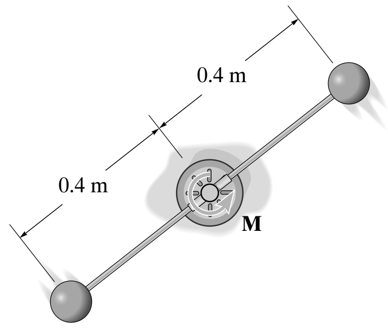
15–98. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. Determine the time the torque $M = (8t)$ N · m, where t is in seconds, must be applied to the rod so that each sphere attains a speed of 3 m/s starting from rest.

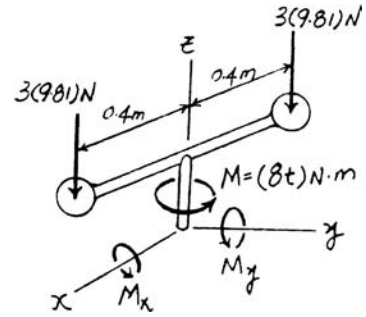


Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

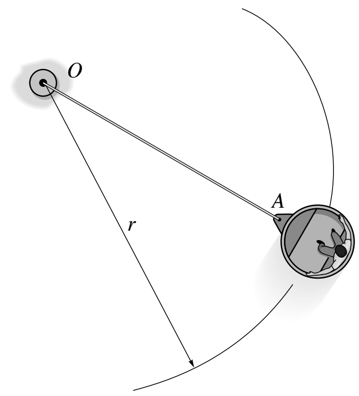
$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

$$2[0.4\,(3)\,(0)] + \int_0^t (8t)\, dt = 2[0.4\,(3)\,(3)]$$

$$t = 1.34 \text{ s} \qquad \textbf{Ans.}$$



15–99. An amusement park ride consists of a car which is attached to the cable OA. The car rotates in a horizontal circular path and is brought to a speed $v_1 = 4$ ft/s when $r = 12$ ft. The cable is then pulled in at the constant rate of 0.5 ft/s. Determine the speed of the car in 3 s.



Conservation of Angular Momentum: Cable OA is shorten by $\Delta r = 0.5(3) = 1.50$ ft. Thus, at this instant $r_2 = 12 - 1.50 = 10.5$ ft. Since no force acts on the car along the tangent of the moving path, the angular momentum is conserved about point O. Applying Eq. 15–23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1\, m v_1 = r_2\, m v'$$

$$12(m)(4) = 10.5(m)\, v'$$

$$v' = 4.571 \text{ ft/s}$$

The speed of car after 3 s is

$$v_2 = \sqrt{0.5^2 + 4.571^2} = 4.60 \text{ ft/s} \qquad \textbf{Ans.}$$

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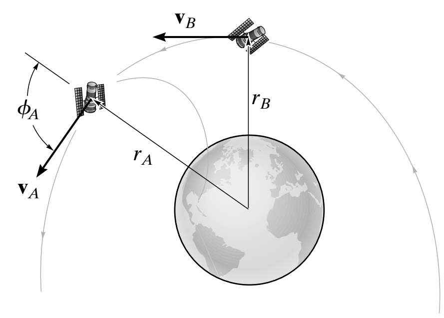
***15–100.** An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.



$(H_O)_1 = (H_O)_2$

$m_s\,(v_A \sin\phi_A) r_A = m_s\,(v_B) r_B$

$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B)$ **(1)**

$T_A + V_A = T_B + V_B$

$$\frac{1}{2}m_s\,(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2}m_s\,(v_B)^2 - \frac{GM_e m_s}{r_B}$$

$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad \textbf{(2)}$$

Solving,

$v_\theta = 10.2$ km/s **Ans.**

$r_\theta = 13.8$ Mm **Ans.**

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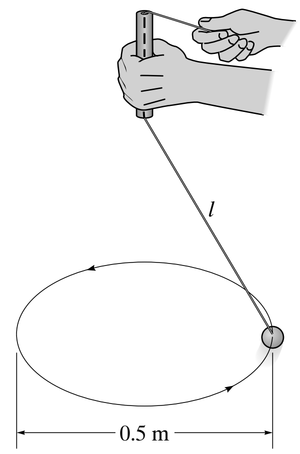
•15–101. The 2-kg ball rotates around a 0.5-m-diameter circular path with a constant speed. If the cord length is shortened from $l = 1$ m to $l' = 0.5$ m, by pulling the cord through the tube, determine the new diameter of the path d'. Also, what is the tension in the cord in each case?



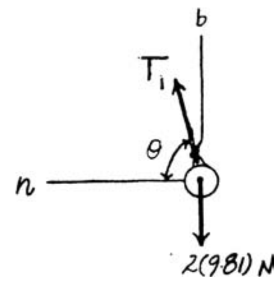
Equation of Motion: When the ball is travelling around the 0.5 m diameter circular path, $\cos\theta = \dfrac{0.25}{1} = 0.25$ and $\sin\theta = \dfrac{\sqrt{0.9375}}{1} = \sqrt{0.9375}$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad T_1\left(\sqrt{0.9375}\right) - 2(9.81) = 0$$

$$T_1 = 20.26\text{ N} = 20.3\text{ N} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 20.26(0.25) = 2\left(\frac{v_1^2}{0.25}\right)$$

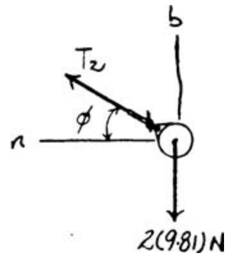
$$v_1 = 0.7958\text{ m/s}$$



When the ball is travelling around d' the diameter circular path, $\cos\phi = \dfrac{d'/2}{0.5} = \text{d}'$ and $\sin\phi = \dfrac{\sqrt{0.25 - 0.25\,d'^2}}{0.5} = \sqrt{1 - d'^2}$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad T_2\left(\sqrt{1 - d^2}\right) - 2(9.81) = 0 \qquad \textbf{[1]}$$

$$\Sigma F_n = ma_n; \qquad T_2\,(d') = 2\left(\frac{v_2^2}{d'/2}\right) \qquad \textbf{[2]}$$



Conservation of Angular Momentum: Since no force acts on the ball along the tangent of the circular path, the angular momentum is conserved about z axis. Applying Eq. 15– 23, we have

$$(\mathbf{H}_z)_1 = (\mathbf{H}_z)_2$$

$$r_1\,mv_1 = r_2\,mv_2$$

$$0.25\,(2)\,(0.7958) = \frac{d'}{2}\,(2)\,v_2 \qquad \textbf{[3]}$$

Solving Eqs. [1], [2] and [3] yields

$$d' = 0.41401\text{ m} = 0.414\text{ m} \qquad T_2 = 21.6\text{ N} \qquad \textbf{Ans.}$$

$$v_2 = 0.9610\text{ m/s}$$

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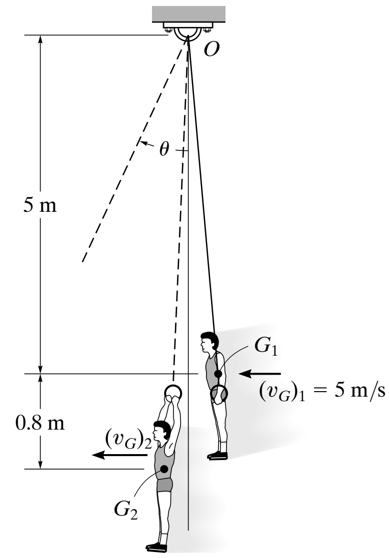
15–102. A gymnast having a mass of 80 kg holds the two rings with his arms down in the position shown as he swings downward. His center of mass is located at point G_1. When he is at the lowest position of his swing, his velocity is $(v_G)_1 = 5$ m/s. At this position he *suddenly* lets his arms come up, shifting his center of mass to position G_2. Determine his new velocity in the upswing and the angle θ to which he swings before momentarily coming to rest. Treat his body as a particle.



$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$

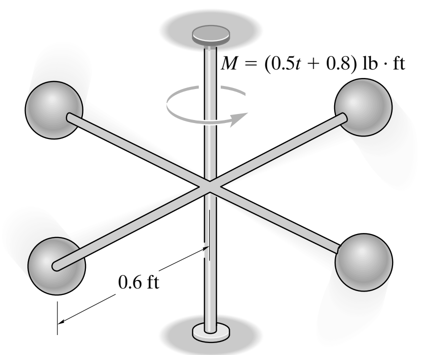
$5\,(80)(5) = 5.8\,(80)\,v_2 \qquad v_2 = 4.310 \text{ m/s} = 4.31 \text{ m/s}$ **Ans.**

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(80)(4.310)^2 + 0 = 0 + 80(9.81)\left[5.8\,(1 - \cos\theta)\right]$$

$\theta = 33.2°$ **Ans.**

15–103. The four 5-lb spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment $M = (0.5t + 0.8)$ lb·ft, where t is in seconds, is applied as shown, determine the speed of each of the spheres in 4 seconds starting from rest. Neglect the size of the spheres.

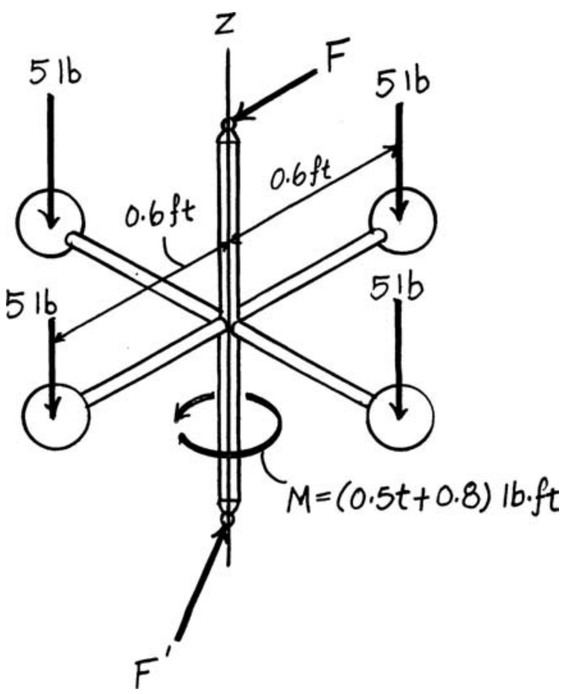


$$(H_z)_1 + \Sigma\int M_z\,dt = (H_z)_2$$

$$0 + \int_0^4 (0.5\,t + 0.8)\,dt = 4\left[\left(\frac{5}{32.2}\right)(0.6\,v_2)\right]$$

$7.2 = 0.37267\,v_2$

$v_2 = 19.3$ ft/s **Ans.**



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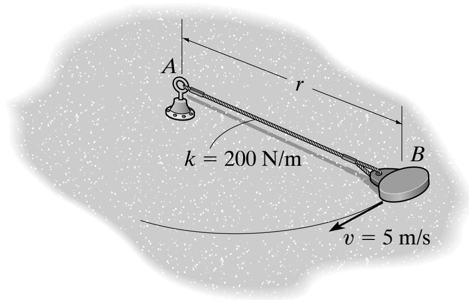
***15–104.** At the instant $r = 1.5$ m, the 5-kg disk is given a speed of $v = 5$ m/s, perpendicular to the elastic cord. Determine the speed of the disk and the rate of shortening of the elastic cord at the instant $r = 1.2$ m. The disk slides on the smooth horizontal plane. Neglect its size. The cord has an unstretched length of 0.5 m.



Conservation of Energy: The intial and final stretch of the elastic cord is $s_1 = 1.5 - 0.5 = 1$ m and $s_2 = 1.2 - 0.5 = 0.7$ m. Thus,

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1{}^2 + \frac{1}{2}ks_1{}^2 = \frac{1}{2}mv_2{}^2 = \frac{1}{2}ks_2{}^2$$

$$\frac{1}{2}5(5^2) + \frac{1}{2}(200)(1^2) = \frac{1}{2}(5)v_2{}^2 + \frac{1}{2}(200)(0.7^2)$$

$$v_2 = 6.738 \text{ m/s} \qquad \textbf{Ans.}$$

Conservation of Angular Momentum: Since no angular impulse acts on the disk about an axis perpendicular to the page passing through point O, its angular momentum of the system is conserved about this z axis. Thus,

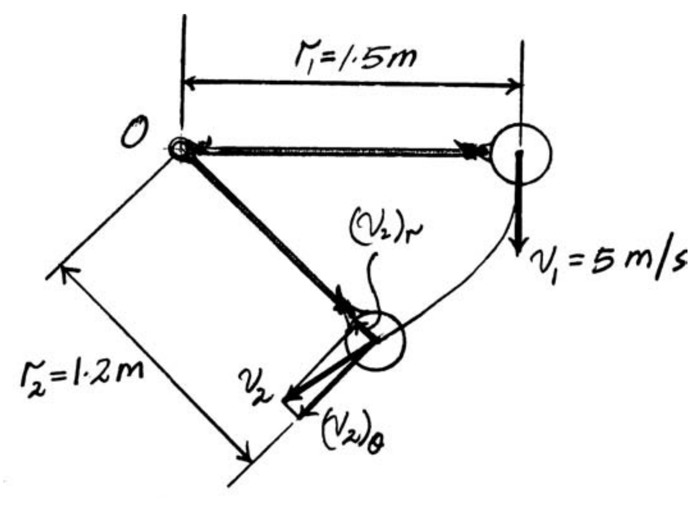
$$\left(H_O\right)_1 = \left(H_O\right)_2$$

$$r_1 m v_1 = r_2 m\left(v_2\right)_\theta$$

$$\left(v_2\right)_\theta = \frac{r_1 v_1}{r_2} = \frac{1.5(5)}{1.2} = 6.25 \text{ m/s}$$

Since $v_2{}^2 = \left(v_2\right)_\theta{}^2 + \left(v_2\right)_r{}^2$, then

$$\left(v_2\right)_r = \sqrt{v_2{}^2 - \left(v_2\right)_\theta{}^2} = \sqrt{6.738^2 - 6.25^2} = 2.52 \text{ m/s} \qquad \textbf{Ans.}$$



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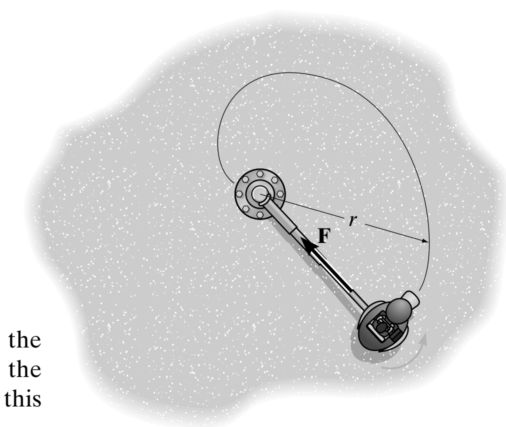
•15–105. The 150-lb car of an amusement park ride is connected to a rotating telescopic boom. When $r = 15$ ft, the car is moving on a horizontal circular path with a speed of 30 ft/s. If the boom is shortened at a rate of 3 ft/s, determine the speed of the car when $r = 10$ ft. Also, find the work done by the axial force **F** along the boom. Neglect the size of the car and the mass of the boom.



Conservation of Angular Momentum: By referring to Fig. *a*, we notice that the angular momentum of the car is conserved about an axis perpendicular to the page passing through point O, since no angular impulse acts on the car about this axis. Thus,

$$(H_O)_1 = (H_O)_2$$

$$r_1 m v_1 = r_2 m (v_2)_\theta$$

$$(v_2)_\theta = \frac{r_1 v_1}{r_2} = \frac{15(30)}{10} = 45 \text{ ft/s}$$

Thus, the magnitude of $\mathbf{v}_2$ is

$$v_2 = \sqrt{(v_2)_r{}^2 - (v_2)_\theta{}^2} = \sqrt{3^2 + 45^2} = 45.10 \text{ ft/s} = 45.1 \text{ ft/s} \qquad \textbf{Ans.}$$

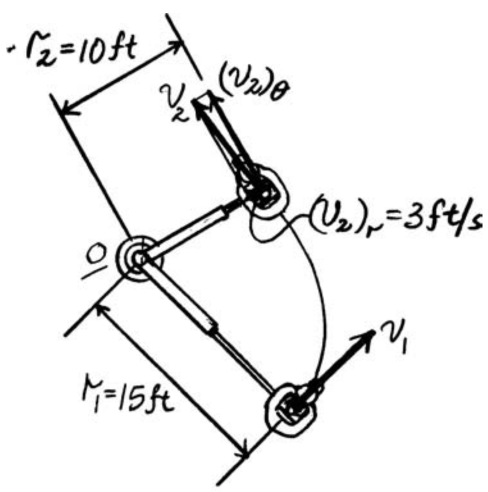
Principle of Work and Energy: Using the result of v_2,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2} m v_1{}^2 + U_F = \frac{1}{2} m v_2{}^2$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)(30^2) + U_F = \frac{1}{2}\left(\frac{150}{32.2}\right)(45.10^2)$$

$$U_F = 2641 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans.}$$



(a)

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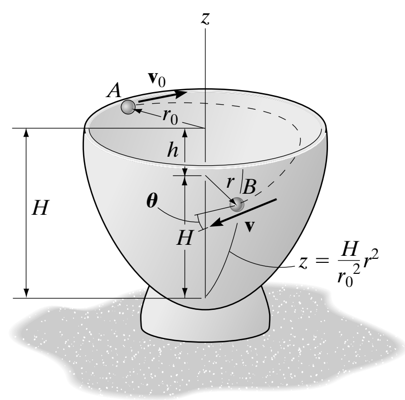
15–106. A small ball bearing of mass m is given a velocity of v_0 at A parallel to the horizontal rim of a smooth bowl. Determine the magnitude of the velocity **v** of the ball when it has fallen through a vertical distance h to reach point B. Angle θ is measured from **v** to the horizontal at point B.



Conservation of Angular Momentum: By observing the free-body diagram of the ball shown in Fig. a, notice that the weight **W** of the ball is parallel to the z axis and the line of action of the normal reaction **N** always intersect the z axis, and they produce no angular impulse about the z axis. Thus, the angular momentum of the ball is conserved about the z axis. At point B, $z = H - h$. Thus, $H - h = \frac{H}{r_0^{\,2}} r^2$ or $r = \sqrt{\frac{H - h}{H}}\, r_0$. Thus, we can write

$$\left(H_1\right)_z = \left(H_2\right)_z$$

$$r_0 m v_0 = r m v \cos\theta$$

$$r_0 v_0 = \left(\sqrt{\frac{H - h}{H}}\, r_0\right) v \cos\theta$$

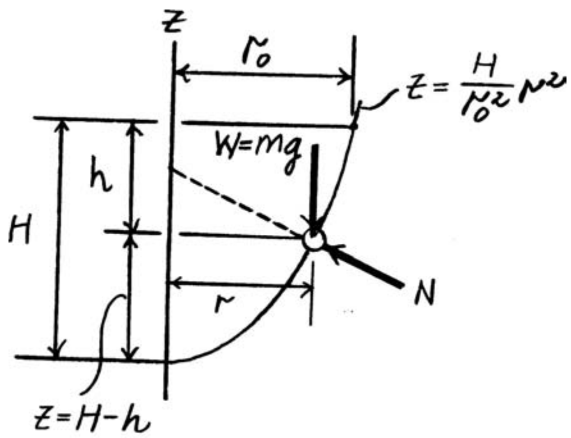
$$\cos\theta = \frac{v_0}{v}\sqrt{\frac{H}{Hh}} \qquad \textbf{(1)}$$

Conservation of Energy: By setting the datum at point B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m v_0^{\,2} + mgh = \frac{1}{2} m v^2 + 0$$

$$v = \sqrt{v_0^{\,2} + 2gh} \qquad \textbf{Ans.}$$



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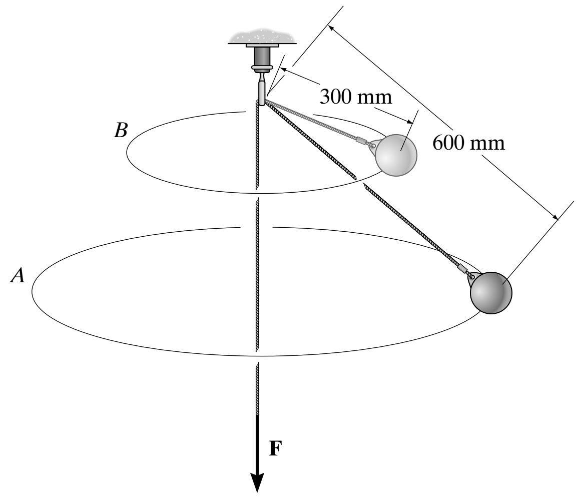
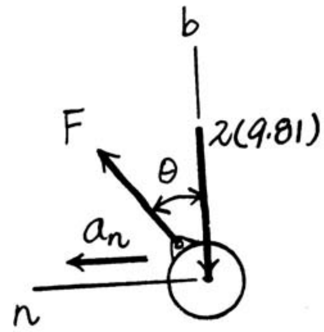
15–107. When the 2-kg bob is given a horizontal speed of 1.5 m/s, it begins to rotate around the horizontal circular path A. If the force $\mathbf{F}$ on the cord is increased, the bob rises and then rotates around the horizontal circular path B. Determine the speed of the bob around path B. Also, find the work done by force $\mathbf{F}$.



Equations of Motion: By referring to the free-body diagram of the bob shown in Fig. a,

$$+\uparrow \Sigma F_b = 0; \qquad F\cos\theta - 2(9.81) = 0 \qquad \textbf{(1)}$$

$$\leftarrow \Sigma F_n = ma_n; \qquad F\sin\theta = 2\left(\frac{v^2}{l\sin\theta}\right) \qquad \textbf{(2)}$$



(a)

Eliminating F from Eqs. (1) and (2) yields

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{9.81l}$$

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{9.81l} \qquad \textbf{(3)}$$

When $l = 0.6$ m, $v = v_1 = 5$ m/s. Using Eq. (3), we obtain

$$\frac{1-\cos^2\theta_1}{\cos\theta_1} = \frac{1.5^2}{9.81(0.6)}$$

$$\cos^2\theta_1 + 0.3823\cos\theta_1 - 1 = 0$$

Solving for the root < 1, we obtain

$$\theta_1 = 34.21°$$

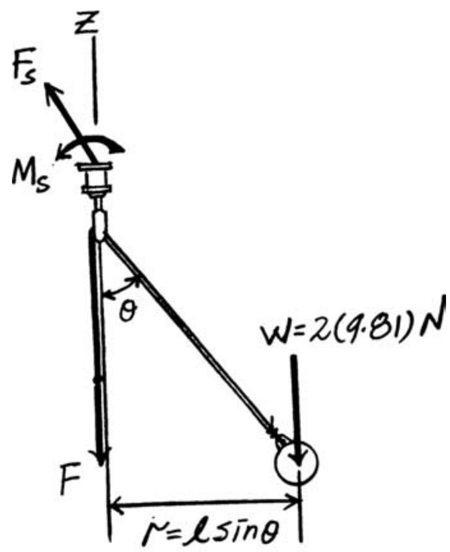
Conservation of Angular Momentum: By observing the free-body diagram of the system shown in Fig. b, notice that $\mathbf{W}$ and $\mathbf{F}$ are parallel to the z axis, $\mathbf{M}_S$ has no z component, and $\mathbf{F}_S$ acts through the z axis. Thus, they produce no angular impulse about the z axis. As a result, the angular momentum of the system is conserved about the z axis. When $\theta = \theta_1 = 34.21°$ and $\theta = \theta_2$, $r = r_1 = 0.6\sin 34.21° = 0.3373$ m and $r = r_2 = 0.3\sin\theta_2$. Thus,

$$(H_z)_1 = (H_z)_2$$

$$r_1 m v_1 = r_2 m v_2$$

$$0.3373(2)(1.5) = 0.3\sin\theta_2\,(2)v_2$$

$$v_2\sin\theta_2 = 1.6867 \qquad \textbf{(4)}$$



(b)

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15–107. Continued

Substituting $l = 0.3$ and $\theta = \theta_2$ $v = v_2$ into Eq. (3) yields

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{v^2}{9.81(0.3)}$$

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{{v_2}^2}{2.943} \qquad \textbf{(5)}$$

Eliminating v_2 from Eqs. (4) and (5),

$$\sin^3\theta_2 \tan\theta_2 - 0.9667 = 0$$

Solving the above equation by trial and error, we obtain

$$\theta_2 = 57.866°$$

Substituting the result of θ_2 into Eq. (4), we obtain

$$v_2 = 1.992 \text{ m/s} = 1.99 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy: When θ changes from θ_1 to θ_2, **W** displaces vertically upward $h = 0.6\cos 34.21° - 0.3\cos 57.866° = 0.3366$ m. Thus, **W** does negatives work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m{v_1}^2 + U_F + \left(-Wh\right) = \frac{1}{2}m{v_2}^2$$

$$\frac{1}{2}(2)(1.5^2) + U_F - 2(9.81)(0.3366) = \frac{1}{2}(2)(1.992)^2$$

$$U_F = 8.32 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

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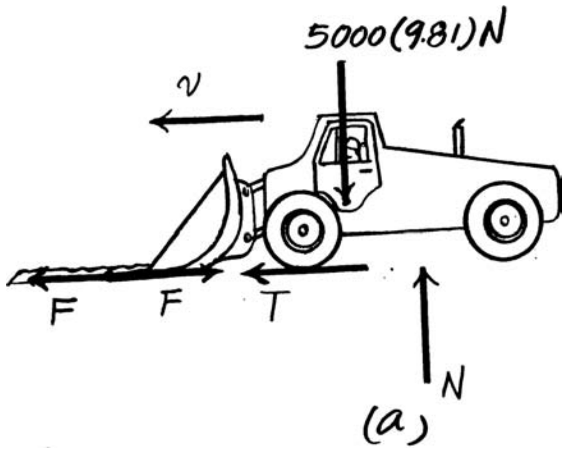
***15–108.** A scoop in front of the tractor collects snow at a rate of 200 kg/s. Determine the resultant traction force **T** that must be developed on all the wheels as it moves forward on level ground at a constant speed of 5 km/h. The tractor has a mass of 5 Mg.

Here, the tractor moves with the constant speed of $v = \left[5\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right]$ $= 1.389$ m/s. Thus, $v_{D/s} = v = 1.389$ m/s since the snow on the ground is at rest. The rate at which the tractor gains mass is $\frac{dm_s}{dt} = 200$ kg/s. Since the tractor is moving with a constant speeds $\frac{dv}{dt} = 0$. Referring to Fig. *a*,

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad T = 0 + 1.389(200)$$

$$T = 278 \text{ N} \qquad \textbf{Ans.}$$



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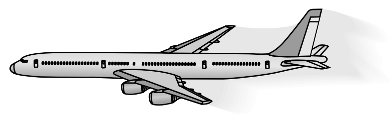
•15–109. A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. Neglect the loss of mass due to fuel consumption.



Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is

$$(\overset{+}{\rightarrow}) \qquad v_e + v_p + v_{e/p}$$

When the four engines are in operation, the airplane has a constant speed of $v_p = \left[800(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 222.22$ m/s. Thus,

$$(\overset{+}{\rightarrow}) \qquad v_e = -222.22 + 775 = 552.78 \text{ m/s} \rightarrow$$

Referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad C(222.22^2) = 4\,\frac{dm}{dt}\,(552.78 - 0)$$

$$C = 0.44775\,\frac{dm}{dt}$$

When only two engines are in operation, the exit speed of the air is

$$(\overset{+}{\rightarrow}) \qquad v_e = -v_p + 775$$

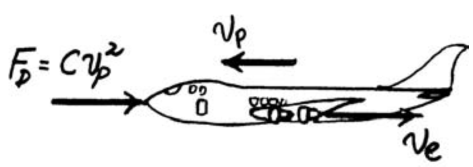
Using the result for C,

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \quad \left(0.044775\,\frac{dm}{dt}\right)\left(v_p{}^2\right) = 2\frac{dm}{dt}\left[-v_p + 775\right) - 0\right]$$

$$0.044775 v_p{}^2 + 2v_p - 1550 = 0$$

Solving for the positive root,

$v_p = 165.06$ m/s $= 594$ km/h **Ans.**



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15–110. The jet dragster when empty has a mass of 1.25 Mg and carries 250 kg of solid propellent fuel. Its engine is capable of burning the fuel at a constant rate of 50 kg/s, while ejecting it at 1500 m/s relative to the dragster. Determine the maximum speed attained by the dragster starting from rest. Assume air resistance is $F_D = (10v^2)$ N, where v is the dragster's velocity in m/s. Neglect rolling resistance.

The free-body diagram of the dragster and exhaust system is shown in Fig. *a*, The pair of thrust **T** cancel each other since they are internal to the system. The mass of the dragster at any instant t is $m = (1250 + 250) - 50t = (1500 - 50t)$ kg.

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad -10v^2 = \left(1500 - 50t\right)\frac{dv}{dt} - 1500(50)$$

$$\frac{dt}{1500 - 50t} = \frac{dv}{75000 - 10v^2} \qquad \textbf{(1)}$$

The dragster acheives its maximum speed when all the fuel is consumed. The time it takes for this to occur is $t = \dfrac{250}{50} = 5$ s. Integratiing Eq. (1),

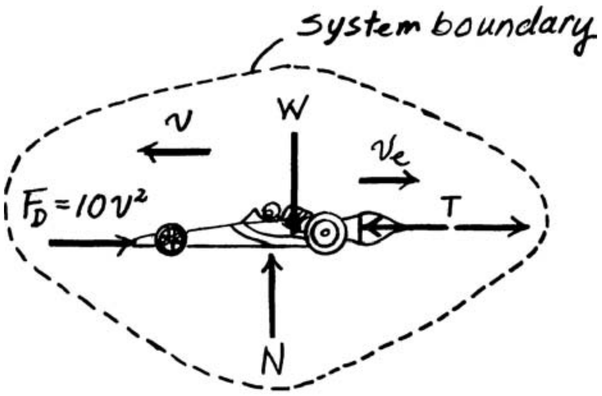
$$\int_0^{5\text{ s}} \frac{dt}{1500 - 50t} = \int_0^v \frac{dv}{75000 - 10v^2}$$

$$-\frac{1}{50}\ln\left(1500 - 50t\right)\Big|_0^{5\text{ s}} = \frac{1}{2\sqrt{75000(10)}}\ln\frac{\sqrt{75000} + \sqrt{10}v}{\sqrt{75000} - \sqrt{10}v}\Bigg|_0^{v}$$

$$\ln\frac{\sqrt{75000} + \sqrt{10}v}{\sqrt{75000} - \sqrt{10}v} = 6.316$$

$$\frac{\sqrt{75000} + \sqrt{10}v}{\sqrt{75000} - \sqrt{10}v} = e^{6.316}$$

$v = 86.3$ m/s **Ans.**



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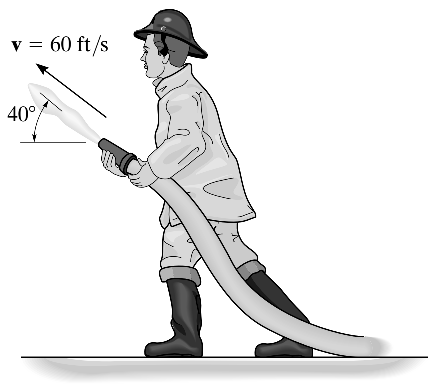
15–111. The 150-lb fireman is holding a hose which has a nozzle diameter of 1 in. and hose diameter of 2 in. If the velocity of the water at discharge is 60 ft/s, determine the resultant normal and frictional force acting on the man's feet at the ground. Neglect the weight of the hose and the water within it. $\gamma_w = 62.4\ \text{lb/ft}^3$.



Originally, the water flow is horizontal. The fireman alters the direction of flow to 40° from the horizontal.

$$\frac{dm}{dt} = \rho v_B A_B = \frac{62.4}{32.2}(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right) = 0.6342\ \text{slug/s}$$

Also, the velocity of the water through the hose is

$$\rho v_A A_A = \rho v_B A_B$$

$$\rho v_A \left(\frac{\pi(1)^2}{(12)^2}\right) = \rho\,(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right)$$

$$v_A = 15\ \text{ft/s}$$

$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}\left((v_B)_x - (v_A)_x\right)$$

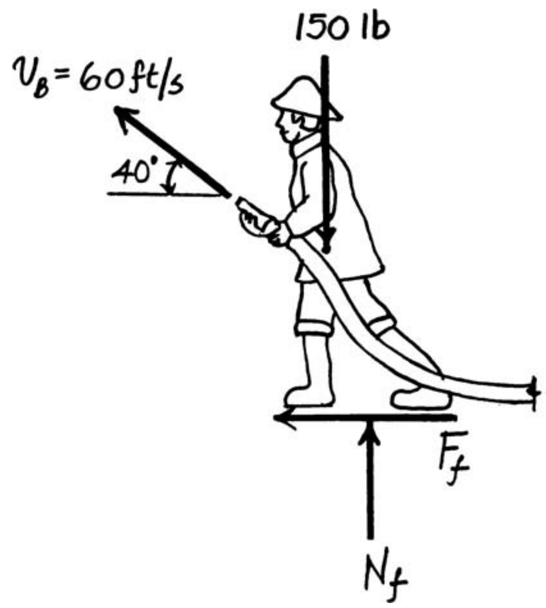
$$F_f = 0.6342\left[60 \cos 40^\circ - 15\right]$$

$F_f = 19.6$ lb **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm}{dt}\left((v_B)_y - (v_A)_y\right)$$

$$N_f - 150 = 0.6342\left[60 \sin 40^\circ - 0\right]$$

$N_f = 174$ lb **Ans.**



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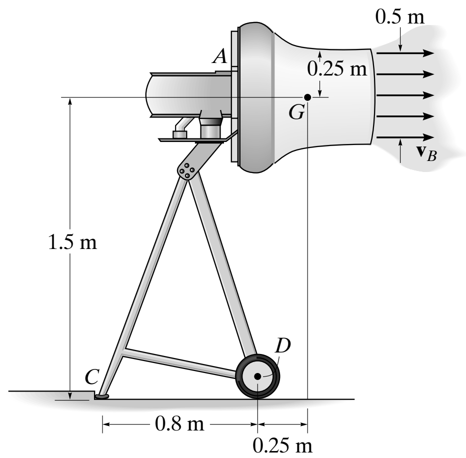
***15–112.** When operating, the air-jet fan discharges air with a speed of $v_B = 20$ m/s into a slipstream having a diameter of 0.5 m. If air has a density of 1.22 kg/m^3, determine the horizontal and vertical components of reaction at C and the vertical reaction at each of the two wheels, D, when the fan is in operation. The fan and motor have a mass of 20 kg and a center of mass at G. Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



$$\frac{dm}{dt} = \rho v A = 1.22(20)(\pi)(0.25)^2 = 4.791 \text{ kg/s}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x})$$

$$C_x = 4.791(20 - 0)$$

$$C_x = 95.8 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y + 2D_y - 20(9.81) = 0$$

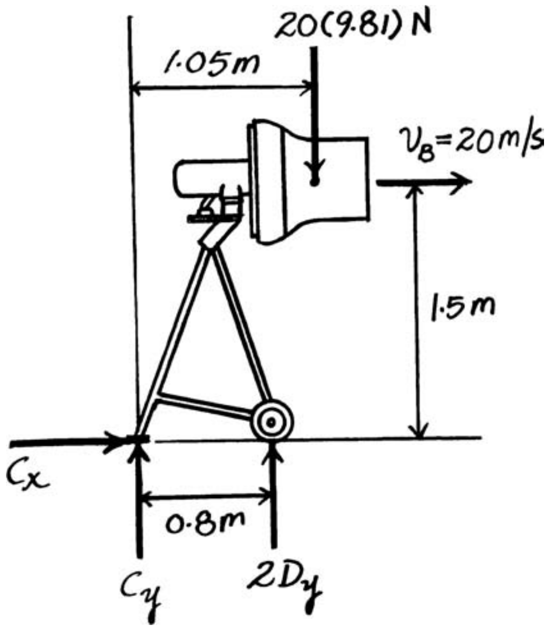
$$\circlearrowleft + \Sigma M_C = \frac{dm}{dt}(d_{CG}\, v_B - d_{CG}\, v_A)$$

$$2D_y(0.8) - 20(9.81)(1.05) = 4.791(-1.5(20) - 0)$$

Solving:

$$D_y = 38.9 \text{ N} \qquad \textbf{Ans.}$$

$$C_y = 118 \text{ N} \qquad \textbf{Ans.}$$



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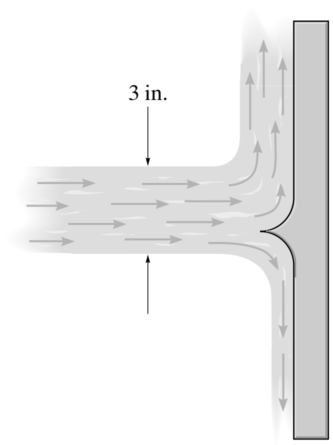
•15–113. The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upwards, and the total flow is $Q = 0.5\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force exerted on the blade by the jet, $\gamma_w = 62.4\ \text{lb/ft}^3$.



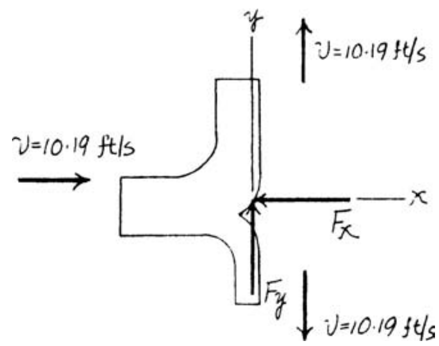
Equations of Steady Flow: Here, the flow rate $Q = 0.5\ \text{ft}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.5}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 10.19\ \text{ft/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = \dfrac{62.4}{32.2}(0.5) = 0.9689\ \text{slug/s}$.

Applying Eq. 15–25 we have

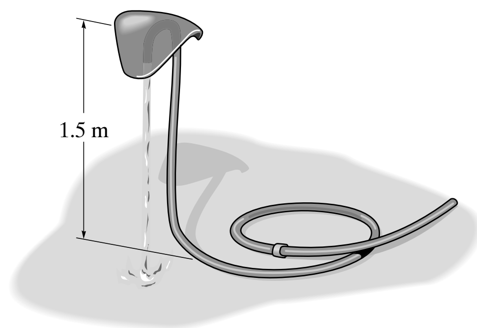
$$\Sigma F_x = \Sigma\frac{dm}{dt}\left(v_{\text{out}_s} - v_{\text{in}_s}\right); \quad -F_x = 0 - 0.9689\,(10.19) \qquad F_x = 9.87\ \text{lb} \qquad \textbf{Ans.}$$

$$\Sigma F_y = \Sigma\frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right); \quad F_y = \frac{3}{4}(0.9689)(10.19) + \frac{1}{4}(0.9689)(-10.19)$$

$$F_y = 4.93\ \text{lb} \qquad \textbf{Ans.}$$



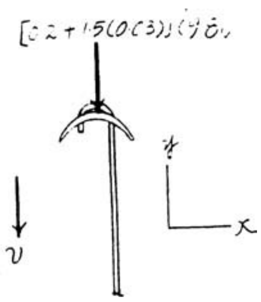
15–114. The toy sprinkler for children consists of a 0.2-kg cap and a hose that has a mass per length of 30 g/m. Determine the required rate of flow of water through the 5-mm-diameter tube so that the sprinkler will lift 1.5 m from the ground and hover from this position. Neglect the weight of the water in the tube. $\rho_w = 1\ \text{Mg/m}^3$.



Equations of Steady Flow: Here, $v = \dfrac{Q}{\frac{\pi}{4}(0.005^2)} = \dfrac{Q}{6.25(10^{-6})\pi}$ and $\dfrac{dm}{dt} = \rho_w Q = 1000Q$. Applying Eq. 15–25, we have

$$\Sigma F_y = \frac{dm}{dt}\left(v_{B_y} - v_{A_y}\right); \quad -[0.2 + 1.5\,(0.03)]\,(9.81) = 1000Q\left(-\frac{Q}{6.25\,(10^{-6})\ \pi} - 0\right)$$

$$Q = 0.217\left(10^{-3}\right)\ \text{m}^3/\text{s} \qquad \textbf{Ans.}$$



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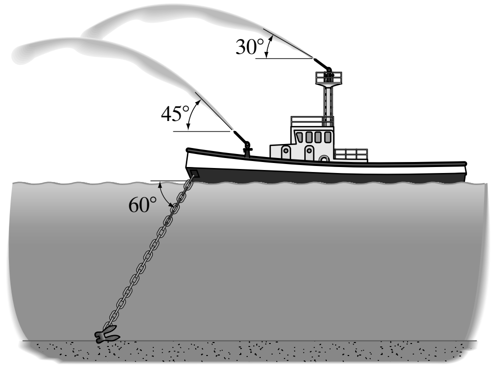
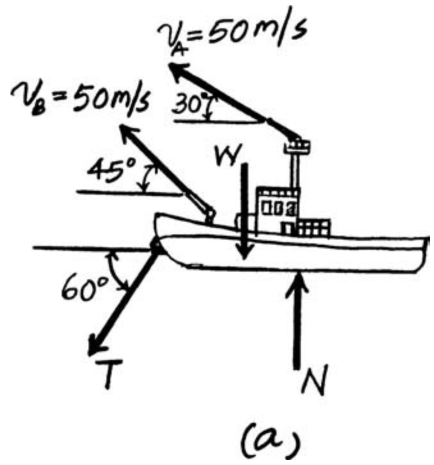
15–115. The fire boat discharges two streams of seawater, each at a flow of $0.25\ \mathrm{m^3/s}$ and with a nozzle velocity of $50\ \mathrm{m/s}$. Determine the tension developed in the anchor chain, needed to secure the boat. The density of seawater is $\rho_{sw} = 1020\ \mathrm{kg/m^3}$.



Steady Flow Equation: Here, the mass flow rate of the sea water at nozzles A and B are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_{sw} Q = 1020(0.25) = 225\ \mathrm{kg/s}$. Since the sea water is collected from the large reservoir (the sea), the velocity of the sea water entering the control volume can be considered zero. By referring to the free-body diagram of the control volume(the boat),



$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm_A}{dt}\left(v_A\right)_x + \frac{dm_B}{dt}\left(v_B\right)_x;$$

$$T \cos 60° = 225\left(50 \cos 30°\right) + 225\left(50 \cos 45°\right)$$

$$T = 40\,114.87\ \mathrm{N} = 40.1\ \mathrm{kN}$$ **Ans.**

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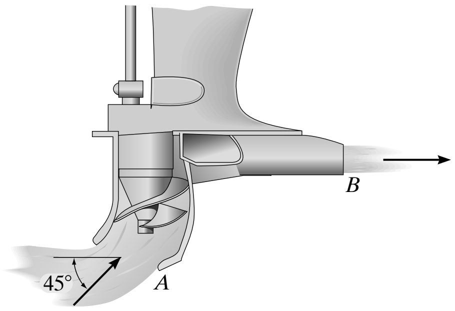
***15–116.** A speedboat is powered by the jet drive shown. Seawater is drawn into the pump housing at the rate of 20 ft^3/s through a 6-in.-diameter intake A. An impeller accelerates the water flow and forces it out horizontally through a 4-in.- diameter nozzle B. Determine the horizontal and vertical components of thrust exerted on the speedboat. The specific weight of seawater is $\gamma_{sw} = 64.3\ \text{lb/ft}^3$.



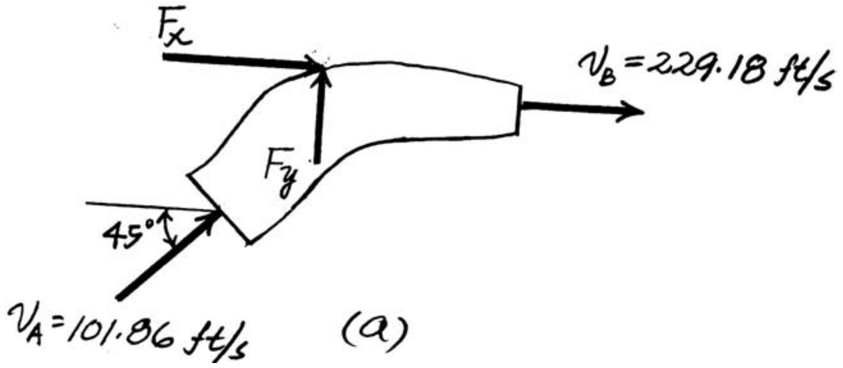
Steady Flow Equation: The speed of the sea water at the hull bottom A and B are $v_A = \dfrac{Q}{A_A} = \dfrac{20}{\frac{\pi}{4}\left(\frac{6}{12}\right)^2} = 101.86\ \text{ft/s}$ and $v_B = \dfrac{Q}{A_B} = \dfrac{20}{\frac{\pi}{4}\left(\frac{4}{12}\right)^2} = 229.18\ \text{ft/s}$ and the mass flow rate at the hull bottom A and nozle B are the same, i.e., $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \dfrac{dm}{dt} = \rho_{sw}\,Q = \left(\dfrac{64.3}{32.2}\right)(20) = 39.94\ \text{slug/s}$. By referring to the free-body diagram of the control volume shown in Fig. *a*,

$$\left(\xrightarrow{+}\right)\Sigma F_x = \frac{dm}{dt}\left[\left(v_B\right)_x - \left(v_A\right)_x\right]; \qquad F_x = 39.94\left(229.18 - 101.86\cos 45°\right)$$

$$= 6276.55\ \text{lb} = 6.28\ \text{kip} \qquad \textbf{Ans.}$$

$$\left(+\uparrow\right)\Sigma F_y = \frac{dm}{dt}\left[\left(v_B\right)_y - \left(v_A\right)_y\right]; \qquad F_y = 39.94\left(101.86\sin 45° - 0\right)$$

$$= 2876.53\ \text{lb} = 2.28\ \text{kip} \qquad \textbf{Ans.}$$



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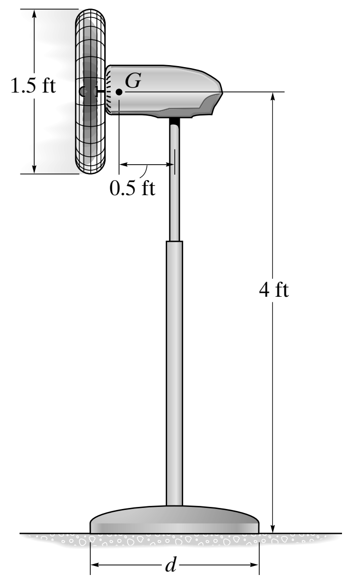
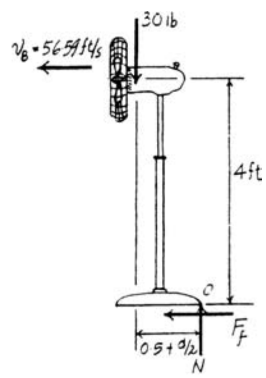
•15–117. The fan blows air at 6000 ft^3/min. If the fan has a weight of 30 lb and a center of gravity at G, determine the smallest diameter d of its base so that it will not tip over. The specific weight of air is $\gamma = 0.076\ lb/ft^3$.



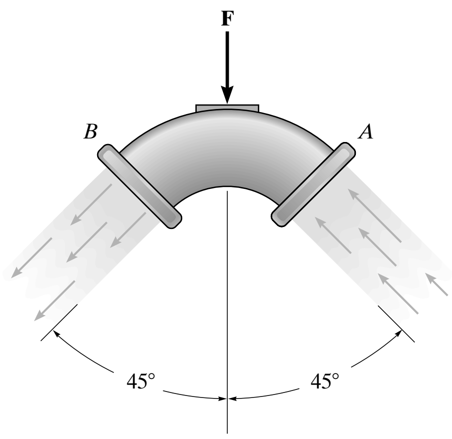
Equations of Steady Flow: Here $Q = \left(\frac{6000\ ft^3}{min}\right) \times \left(\frac{1\ min}{60\ s}\right) = 100\ ft^3/s$. Then, $v = \frac{Q}{A} = \frac{100}{\frac{\pi}{4}(1.5^2)} = 56.59$ ft/s. Also, $\frac{dm}{dt} = \rho_a Q = \frac{0.076}{32.2}(100) = 0.2360$ slug/s. Applying Eq. 15–26 we have

$$\zeta + \Sigma M_O = \frac{dm}{dt}(d_{OB} v_B - d_{OA} v_A); \quad 30\left(0.5 + \frac{d}{2}\right) = 0.2360\,[4(56.59) - 0]$$

$$d = 2.56 \text{ ft} \qquad \textbf{Ans.}$$



15–118. The elbow for a 5-in-diameter buried pipe is subjected to a static pressure of 10 lb/in^2. The speed of the water passing through it is $v = 8$ ft/s. Assuming the pipe connections at A and B do not offer any vertical force resistance on the elbow, determine the resultant vertical force **F** that the soil must then exert on the elbow in order to hold it in equilibrium. Neglect the weight of the elbow and the water within it. $\gamma_w = 62.4\ lb/ft^3$.



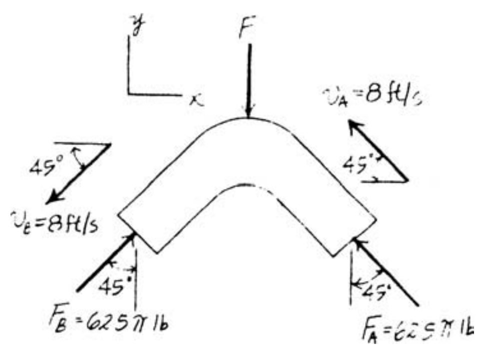
Equations of Steady Flow: Here. $Q = vA = 8\left[\frac{\pi}{4}\left(\frac{5}{12}\right)^2\right] = 1.091\ ft^3/s$. Then, the mass flow rate is $\frac{dm}{dt} = \rho_w Q = \frac{62.4}{32.2}(1.091) = 2.114$ slug/s. Also, the force induced by the water pressure at A is $F = \rho A = 10\left[\frac{\pi}{4}(5^2)\right] = 62.5\pi$ lb. Applying Eq. 15–26, we have

$$\Sigma F_y = \frac{dm}{dt}\left(v_{B_y} - v_{A_y}\right); -F + 2(62.5\,\pi \cos 45°) = 2.114(-8 \sin 45° - 8 \sin 45°)$$

$$F = 302 \text{ lb} \qquad \textbf{Ans.}$$

$$\Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right); 62.5\,\pi \sin 45° - 62.5\,\pi \sin 45°$$
$$= 2.114[-8 \cos 45° - (-8 \cos 45°)]$$

$$0 = 0 \qquad \textbf{(Check!)}$$



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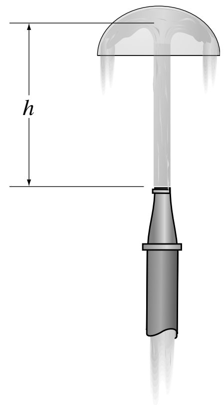
15–119. The hemispherical bowl of mass m is held in equilibrium by the vertical jet of water discharged through a nozzle of diameter d. If the discharge of the water through the nozzle is Q, determine the height h at which the bowl is suspended. The water density is ρ_w. Neglect the weight of the water jet.



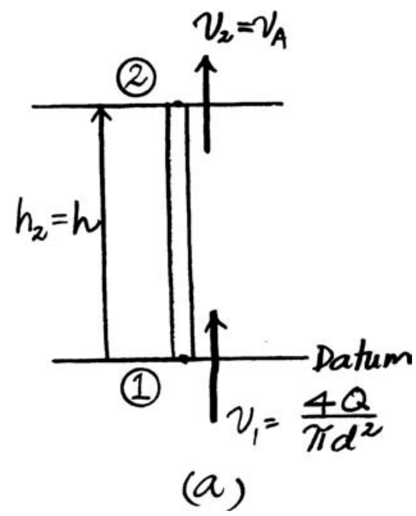
Conservation of Energy: The speed at which the water particle leaves the nozzle is $v_1 = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$. The speed of particle v_A when it comes in contact with the bowl can be determined using conservation of energy. With reference to the datum set in Fig. a,

$$T_1 + V_1 = T_2 + V_2 0$$

$$\frac{1}{2}mv_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}mv_2{}^2 + \left(V_g\right)_2$$

$$\frac{1}{2}m\left(\frac{4Q}{\pi d^2}\right)^2 + 0 = \frac{1}{2}mv_A{}^2 + mgh$$

$$v_A = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$$



Steady Flow Equation: The mass flow rate of the water jet that enters the control volume at A is $\frac{dm_A}{dt} = \rho_w Q$, and exits from the control volume at B is $\frac{dm_B}{dt} = \frac{dm_A}{dt} = \rho_w Q$. Thus, $v_B = v_A = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$. Here, the vertical force acting on the control volume is equal to the weight of the bowl. By referring to the free - body diagram of the control volume, Fig. b,

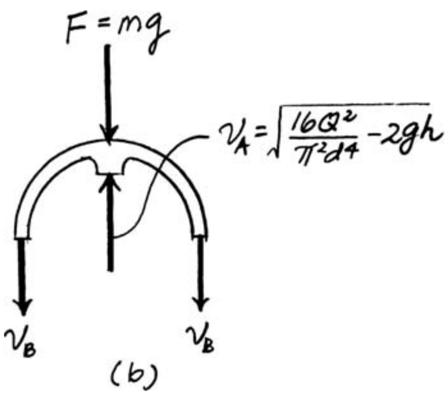
$$+\uparrow \Sigma F_y = 2\frac{dm_B}{dt}v_B - \frac{dm_A}{dt}v_A;$$

$$-mg = -(\rho_w Q)\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right) - \rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

$$mg = 2\rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

$$m^2g^2 = 4\rho_w{}^2 Q^2\left(\frac{16Q^2}{\pi^2 d^4} - 2gh\right)$$

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w{}^2 Q^2}$$ **Ans.**



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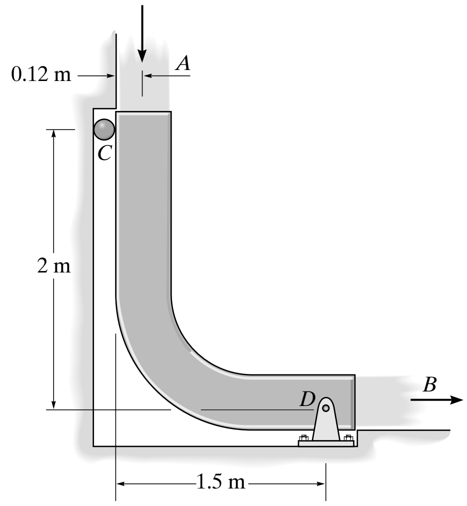
***15–120.** The chute is used to divert the flow of water, $Q = 0.6\ \text{m}^3/\text{s}$. If the water has a cross-sectional area of $0.05\ \text{m}^2$, determine the force components at the pin D and roller C necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute. $\rho_w = 1\ \text{Mg/m}^3$.



Equations of Steady Flow: Here, the flow rate $Q = 0.6\ \text{m}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.6}{0.05} = 12.0\ \text{m/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = 1000\,(0.6) = 600\ \text{kg/s}$. Applying Eqs. 15–26 and 15–28, we have

$$\zeta + \Sigma M_A = \frac{dm}{dt}\left(d_{DB}\, v_B - d_{DA}\, v_A\right);$$

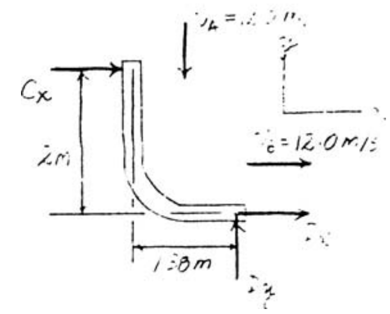
$$-C_x\,(2) = 600\,[0 - 1.38(12.0)] \qquad C_x = 4968\ \text{N} = 4.97\ \text{kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right);$$

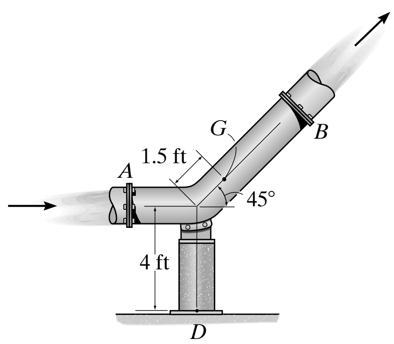
$$D_x + 4968 = 600\,(12.0 - 0) \qquad D_x = 2232\text{N} = 2.23\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \Sigma\frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right);$$

$$D_y = 600[0 - (-12.0)] \qquad D_y = 7200\ \text{N} = 7.20\ \text{kN} \qquad \textbf{Ans.}$$



•15–121. The bend is connected to the pipe at flanges A and B as shown. If the diameter of the pipe is 1 ft and it carries a discharge of $50\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force reaction and the moment reaction exerted at the fixed base D of the support. The total weight of the bend and the water within it is 500 lb, with a mass center at point G. The gauge pressure of the water at the flanges at A and B are 15 psi and 12 psi, respectively. Assume that no force is transferred to the flanges at A and B. The specific weight of water is $\gamma_w = 62.4\ \text{lb/ft}^3$.



Free-Body Diagram: The free-body of the control volume is shown in Fig. a. The force exerted on sections A and B due to the water pressure is $F_A = P_A A_A = 15\left[\dfrac{\pi}{4}\left(12^2\right)\right] = 1696.46\ \text{lb}$ and $F_B = P_B A_B = 12\left[\dfrac{\pi}{4}\left(12^2\right)\right] = 1357.17\ \text{lb}$. The speed of the water at, sections A and B are $v_A = v_B = \dfrac{Q}{A} = \dfrac{50}{\frac{\pi}{4}\left(1^2\right)} = 63.66\ \text{ft/s}$. Also, the mass flow rate at these two sections are $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \rho_W Q = \left(\dfrac{62.4}{32.2}\right)(50) = 96.894\ \text{slug/s}$.

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Steady Flow Equation: The moment steady flow equation will be written about point D to eliminate D_x and D_y.

$$\zeta + \Sigma M_D = \frac{dm_B}{dt} dv_B - \frac{dm_A}{dt} dv_A;$$

$$M_D + 1357.17 \cos 45°(4) - 500\left(1.5 \cos 45°\right) - 1696.46(4)$$

$$= -96.894(4)\left(63.66 \cos 45°\right) - [-96.894(4)(63.66)]$$

$$M_D = 10\,704.35 \text{ lb} \cdot \text{ft} = 10.7 \text{ kip} \cdot \text{ft}$$ **Ans.**

Writing the force steady flow equation along the x and y axes,

$$(+\uparrow)\Sigma F_y = \frac{dm}{dt}\left[\left(v_B\right)_y - \left(v_A\right)_y\right];$$

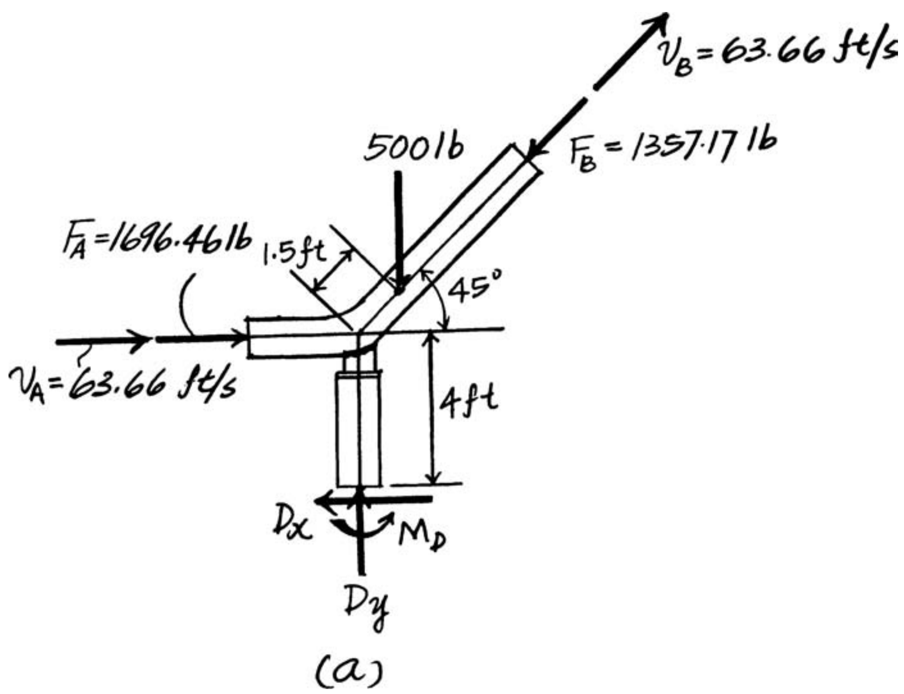
$$D_y - 500 - 1357.17 \sin 45° = 96.894(63.66 \sin 45° - 0)$$

$$D_y = 5821.44 \text{ lb} = 5.82 \text{ kip}$$ **Ans.**

$$\left(\xrightarrow{+}\right)\Sigma F_x = \frac{dm}{dt}\left[\left(v_B\right)_x - \left(v_A\right)_x\right];$$

$$1696.46 - 1357.17 \cos 45° - D_x = 96.894[63.66 \cos 45° - 63.66]$$

$$D_x = 2543.51 \text{ lb} = 2.54 \text{ kip}$$ **Ans.**



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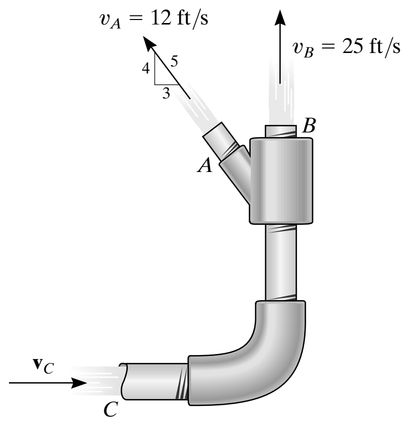
15–122. The gauge pressure of water at C is 40 lb/in^2. If water flows out of the pipe at A and B with velocities $v_A = 12$ ft/s and $v_B = 25$ ft/s, determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at C, and at A and B the diameter is 0.5 in. $\gamma_w = 62.4$ lb/ft^3.



$$\frac{dm_A}{dt} = \frac{63.4}{32.2}(25)(\pi)\left(\frac{025}{15}\right)^2 = 0.03171 \text{ slug/s}$$

$$\frac{dm_B}{dt} = \frac{62.4}{32.2}(25)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.06606 \text{ slug/s}$$

$$\frac{dm_C}{dt} = 0.03171 + 0.06606 = 0.09777 \text{ slug/s}$$

$$v_C A_C = v_A A_A + v_B A_B$$

$$v_C(\pi)\left(\frac{0.375}{12}\right)^2 = 12(\pi)\left(\frac{0.25}{12}\right)^2 + 25(\pi)\left(\frac{0.25}{12}\right)^2$$

$$v_C = 16.44 \text{ ft/s}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm_B}{dt} v_{B_s} + \frac{dm_A}{dt} v_{A_s} - \frac{dm_C}{dt} v_{C_s}$$

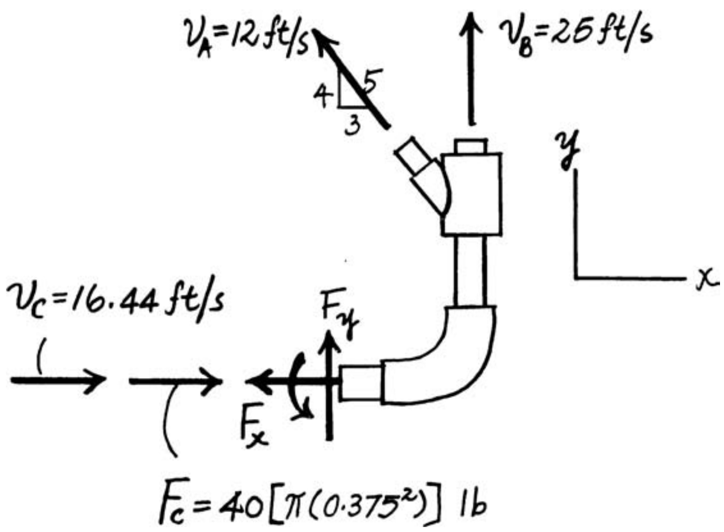
$$40(\pi)(0.375)^2 - F_x = 0 - 0.03171(12)\left(\frac{3}{5}\right) - 0.09777(16.44)$$

$$F_x = 19.5 \text{ lb}$$ **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm_B}{dt} v_{B_y} + \frac{dm_A}{dt} v_{A_y} - \frac{dm_C}{dt} v_{C_y}$$

$$F_y = 0.06606(25) + 0.03171\left(\frac{4}{5}\right)(12) - 0$$

$$F_y = 1.9559 = 1.96 \text{ lb}$$ **Ans.**



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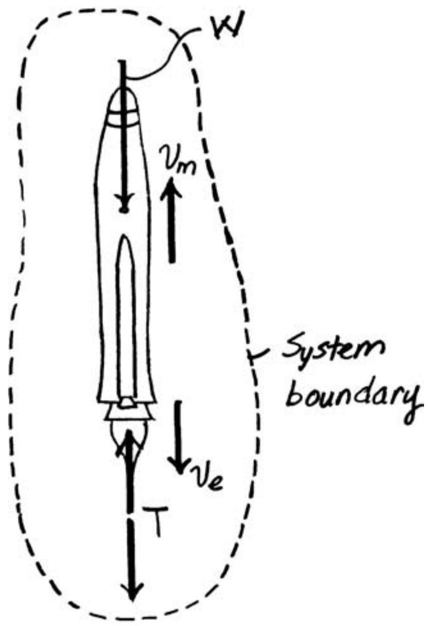
15–123. A missile has a mass of 1.5 Mg (without fuel). If it consumes 500 kg of solid fuel at a rate of 20 kg/s and ejects it with a velocity of 2000 m/s relative to the missile, determine the velocity and acceleration of the missile at the instant all the fuel has been consumed. Neglect air resistance and the variation of its weight with altitude. The missile is launched vertically starting from rest.

By referring to the free-body diagram of the missile system in Fig. *a*, notice that the pair of thrust **T** cancel each other since they are internal to the system. The mass of the missile at any instant t after lauch is given by $m = (1500 + 500) - 20t = (2000 - 20t)$kg. Thus, the weight at the same instant is $W = (2000 - 20t)(9.81)$.



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad -(200-20t)(9.81) = (2000-20t)\frac{dv}{dt} - 2000(20)$$

$$a = \frac{dv}{dt} = \frac{2000}{100-t} - 9.81 \qquad \textbf{(1)}$$

The time taken for all the fuel to the consumed is $t = \frac{500}{20} = 25$ s. Substituting the result of t into Eq. (1),

$$a = \frac{2000}{100-25} - 9.81 = 16.9 \text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$

Integrating Eq. (1),

$$\int_0^v dv = \int_0^{25\,s}\left(\frac{2000}{100-t} - 9.81\right)dt$$

$$v = \left(-2000\ln(100 - t) - 9.81t\right)\Big|_0^{25\text{ s}}$$

$$= 330 \text{ m/s} \qquad \textbf{Ans.}$$

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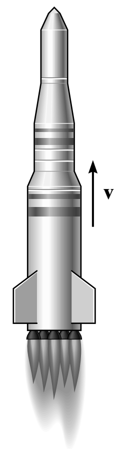
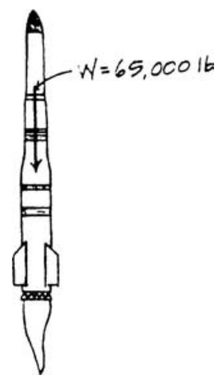
***15–124.** The rocket has a weight of 65 000 lb including the solid fuel. Determine the constant rate at which the fuel must be burned so that its thrust gives the rocket a speed of 200 ft/sin 10 s starting from rest. The fuel is expelled from the rocket at a relative speed of 3000 ft/s relative to the rocket. Neglect the effects of air resistance and assume that g is constant.





A System That Loses Mass: Here, $W = \left(m_0 - \frac{dm_r}{dt}t\right)g$. Applying Eq. 15–28, we have

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$-\left(m_0 - \frac{dm_e}{dt}t\right)g = \left(m_0 - \frac{dm_e}{dt}t\right)\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$\frac{dv}{dt} = \frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g$$

$$\int_0^v dv = \int_0^t \left(\frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g\right)dt$$

$$v = \left[-v_{D/e}\,1\text{n}\left(m_0 - \frac{dm_e}{dt}\text{t}\right) - \text{gt}\right]\Bigg|_0^{\text{t}}$$

$$v = v_{D/e}\,1\text{n}\left(\frac{m_0}{m_0 - \frac{dm_e}{dt}t}\right) - \text{gt} \qquad \textbf{[1]}$$

Substitute Eq. [1] with $m_0 = \frac{65\,000}{32.2} = 2018.63$ slug, $v_{D/e} = 3000$ ft/s, $v = 200$ ft/s and $t = 10\ s$, we have

$$200 = 3000\ 1\text{n}\left[\frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}\right] - 32.2(10)$$

$$e^{0\ 174} = \frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}$$

$$\frac{dm_e}{dt} = 32.2\ \text{slug/s} \qquad \textbf{Ans.}$$

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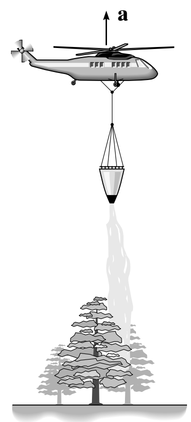
•15–125. The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration the helicopter experiences as the water is being released.



$$+\uparrow \Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

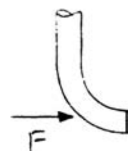
Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

$$0 = 10.5(10^3)a - (10)(50)$$

$a = 0.0476\ \text{m/s}^2$ **Ans.**

15–126. A plow located on the front of a locomotive scoops up snow at the rate of 10 ft^3/s and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is $\gamma_s = 6$ lb/ft^3.

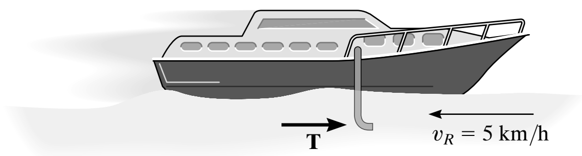
$$\Sigma F_x = m\frac{dv}{dt} + v_{D/t}\frac{dm_t}{dt}$$



$$F = 0 + (12 - 0)\left(\frac{10(6)}{32.2}\right)$$

$F = 22.4$ lb **Ans.**

15–127. The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1$ Mg/m^3.



$$\frac{dm}{dt} = \frac{40}{80} = 0.5\ \text{kg/s}$$

$$v_{D/t} = (70)\left(\frac{1000}{3600}\right) = 19.444\ \text{m/s}$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$T = 0 + 19.444(0.5) = 9.72$ N **Ans.**

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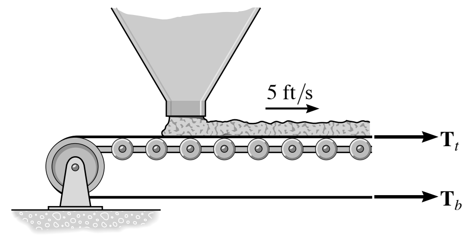
***15–128.** The bin deposits gravel onto the conveyor belt at the rate of 1500 lb/min. If the speed of the belt is 5 ft/s, determine how much greater the tension in the top portion of the belt must be than that in the bottom portion in order to pull the belt forward.

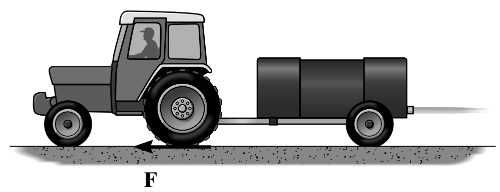


A System That Gains Mass: Here, $v_{D/t} = 5$ ft/s, $\frac{dv}{dt} = 0$ and $\frac{dm_t}{dt} = \left(\frac{1500\text{ lb}}{\text{min}}\right) \times \left(\frac{1\text{ slug}}{32.2\text{ lb}}\right) \times \left(\frac{1\text{ sin}}{60\text{ s}}\right) = 0.7764$ slug/s. Applying Eq. 15–29, we have

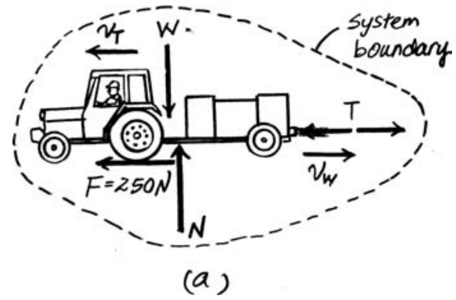
$$\overset{+}{\rightarrow} \Sigma F_s = m\frac{dv}{dt} + v_{D/t}\frac{dm_s}{dt}; \qquad T_t - T_b = 0 + 5(0.7764)$$

$$\Delta T = 3.88\text{ lb} \qquad \textbf{Ans.}$$

•15–129. The tractor together with the empty tank has a total mass of 4 Mg. The tank is filled with 2 Mg of water. The water is discharged at a constant rate of 50 kg/s with a constant velocity of 5 m/s, measured relative to the tractor. If the tractor starts from rest, and the rear wheels provide a resultant traction force of 250 N, determine the velocity and acceleration of the tractor at the instant the tank becomes empty.



The free-body diagram of the tractor and water jet is shown in Fig. *a*. The pair of thrust **T** cancel each other since they are internal to the system. The mass of the tractor and the tank at any instant t is given by $m = (4000 + 2000) - 50t = (6000 - 50t)$kg.



$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 250 = (6000 - 50t)\frac{dv}{dt} - 5(50)$$

$$a = \frac{dv}{dt} = \frac{10}{120 - t} \qquad \textbf{(1)}$$

The time taken to empty the tank is $t = \frac{2000}{50} = 40$ s. Substituting the result of t into Eq. (1),

$$a = \frac{10}{120 - 40} = 0.125\text{m/s}^2 \qquad \textbf{Ans.}$$

Integrating Eq. (1),

$$\int_0^v dv = \int_0^{40\text{ s}} \frac{10}{120 - t}dt$$

$$v = -10\ln(120 - t)\Big|_0^{40\text{ s}}$$

$$= 4.05\text{ m/s} \qquad \textbf{Ans.}$$

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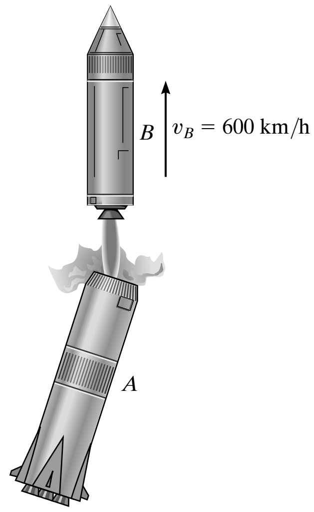
15–130. The second stage B of the two-stage rocket has a mass of 5 Mg (empty) and is launched from the first stage A with an initial velocity of 600 km/h. The fuel in the second stage has a mass of 0.7 Mg and is consumed at the rate of 4 kg/s. If it is ejected from the rocket at the rate of 3 km/s, measured relative to B, determine the acceleration of B at the instant the engine is fired and just before all the fuel is consumed. Neglect the effects of gravitation and air resistance.



A System That Loses Mass: At the instant when stage B of rocket is launched, the total mass of the rocket is $m = 5000 + 5700$ kg. Applying Eq. 15–29, we have

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$0 = (5700)\frac{dv}{dt} - 3(10^3)(4) \qquad a = \frac{dv}{dt} = 2.11 \text{ m/s}^2$$ **Ans.**

At the instant just before all the fuel being consumed, the mass of the rocket is $m = 5000$ kg. Applying Eq. 15–29, we have

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$0 = (5000)\frac{dv}{dt} - 3(10^2)(4) \qquad a = \frac{dv}{dt} = 2.40 \text{ m/s}^2$$ **Ans.**

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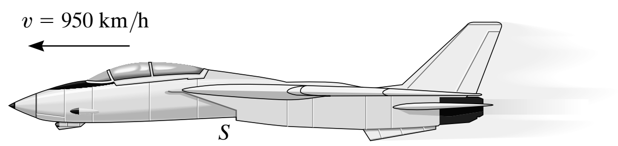
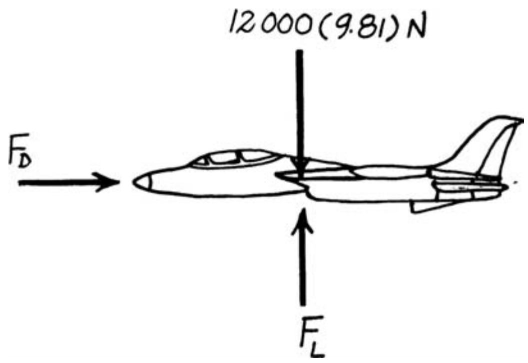
15–131. The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield.

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}.$$



$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad \textbf{(1)}$$



$$v = 950 \text{ km/h} = 0.2639 \text{ km/s}, \qquad \frac{dv}{dt} = 0$$

$$v_{D/E} = 0.45 \text{ km/s}$$

$$v_{D/t} = 0.2639 \text{ km/s}$$

$$\frac{dm_t}{dt} = 50(1.22) = 61.0 \text{ kg/s}$$

$$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}$$

Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

$$(\overset{+}{\leftarrow}) \quad -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$$

$$F_D = 11.5 \text{ kN} \qquad \textbf{Ans.}$$

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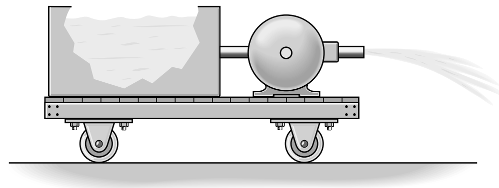
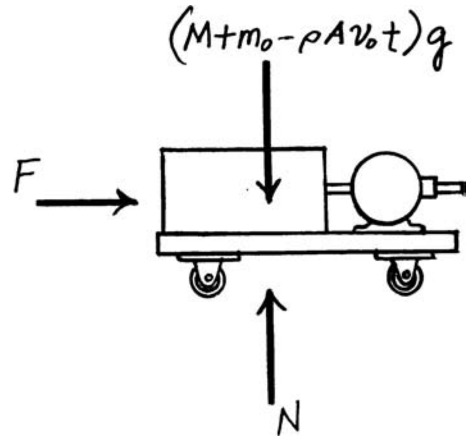
***15–132.** The cart has a mass M and is filled with water that has a mass m_0. If a pump ejects the water through a nozzle having a cross-sectional area A at a constant rate of v_0 relative to the cart, determine the velocity of the cart as a function of time. What is the maximum speed of the cart assuming all the water can be pumped out? The frictional resistance to forward motion is F. The density of the water is ρ.





$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\left(\frac{dm_e}{dt}\right)$$

$$\frac{dm_e}{dt} = \rho A v_0$$

$$-F = (M + m_0 - \rho A v_0 t)\frac{dv}{dt} - v_0(\rho A v_0)$$

$$\int_0^t \frac{dt}{M + m_0 - \rho A v_0 t} = \int_0^v \frac{dv}{\rho A v_0^2 - F}$$

$$-\left(\frac{1}{\rho A v_0}\right)\ln(M + m_0) = \rho A v_0 t) + \left(\frac{1}{\rho A v_0}\right)\ln(M + m_0) = \frac{v}{\rho A v_0^2 - F}$$

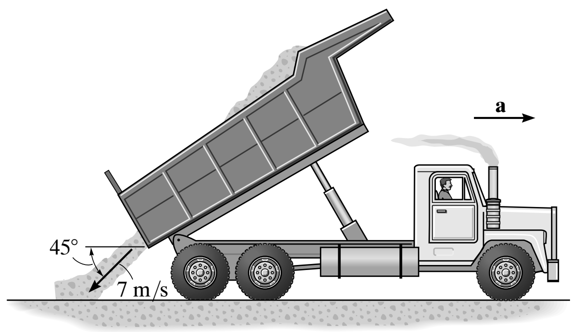
$$\left(\frac{1}{\rho A v_0}\right)\ln\left(\frac{M + m_0}{M + m_0 - \rho A v_0 t}\right) = \frac{v}{\rho A v_0^2 - F}$$

$$v = \left(\frac{\rho A v_0^2 - F}{\rho A v_0}\right)\ln\left(\frac{M + m_0}{M + m_0 - \rho A v_0 t}\right) \qquad \textbf{Ans.}$$

v_{max} occurs when $t = \dfrac{m_0}{\rho A v_0}$, or,

$$v_{max} = \left(\frac{\rho A v_0^2 - F}{\rho A v_0}\right)\ln\left(\frac{M + m_0}{M}\right) \qquad \textbf{Ans.}$$

•15–133. The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the sand begins to fall out. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520$ kg/m^3.



A System That Loses Mass: Initially, the total mass of the truck is $m = 50\left(10^3\right) + 5(1520) = 57.6\left(10^3\right)$ kg and $\dfrac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s. Applying Eq. 15–29, we have

$$\overset{+}{\rightarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \quad 0 = 57.6\left(10^3\right)a - (0.8\cos 45°)(1216)$$

$$a = 0.104 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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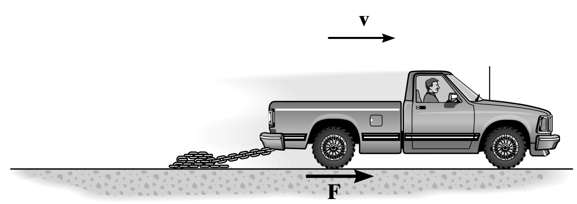
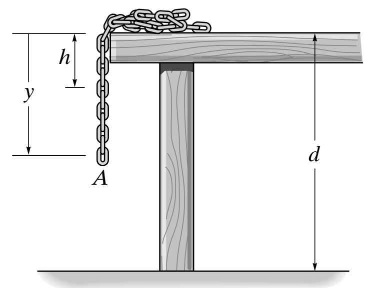
15–134. The truck has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force **F** that must be supplied by the rear wheels of the truck necessary to maintain a constant speed v while the chain is being drawn out.



A System That Loses Mass: Here, $v_{D/t} = v$, $\dfrac{dv}{dt} = 0$ and $\dfrac{dm_t}{dt} = m'v$. Applying Eq. 15–29, we have

$$\xrightarrow{+} \Sigma F_S = m\frac{dv}{dt} + v_{D/t}\frac{dm_t}{dt}; \qquad F = 0 + v(m'v) = m'v^2 \qquad \textbf{Ans.}$$

15–135. The chain has a total length $L < d$ and a mass per unit length of m'. If a portion h of the chain is suspended over the table and released, determine the velocity of its end A as a function of its position y. Neglect friction.



$$\Sigma F_s = m\frac{dv}{dt} + v_{D/e}\frac{dm_e}{dt}$$

$$m'gy = m'y\frac{dv}{dt} + v(m'v)$$

$$m'gy = m'\left(y\frac{dv}{dt} + v^2\right)$$

Since $dt = \dfrac{dy}{v}$, we have

$$gy = vy\frac{dv}{dy} + v^2$$

Multiply by $2y$ and integrate:

$$\int 2gy^2\,dy = \int\left(2vy^2\frac{dv}{dy} + 2yv^2\right)dy$$

$$\frac{2}{3}g^3y^3 + C = v^2y^2$$

when $v = 0$, $y = h$, so that $C = -\dfrac{2}{3}gh^3$

Thus, $v^2 = \dfrac{2}{3}g\left(\dfrac{y^3 - h^3}{y^2}\right)$

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)} \qquad \textbf{Ans.}$$

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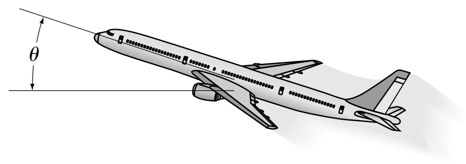
***15–136.** A commercial jet aircraft has a mass of 150 Mg and is cruising at a constant speed of 850 km/h in level flight ($\theta = 0°$). If each of the two engines draws in air at a rate of 1000 kg/s and ejects it with a velocity of 900 m/s, relative to the aircraft, determine the maximum angle of inclination θ at which the aircraft can fly with a constant speed of 750 km/h. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. The engines are operating with the same power in both cases. Neglect the amount of fuel consumed.



Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is given by

$$v_e = v_p + v_{e/p}$$

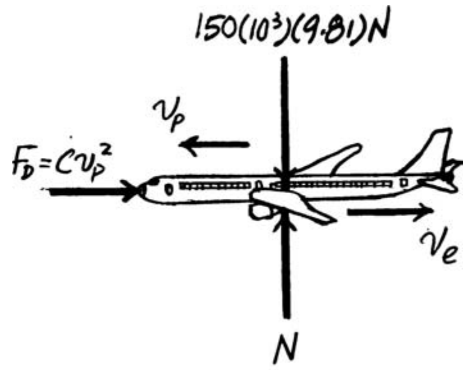
When the airplane is in level flight, it has a constant speed of $v_p = \left[850(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 236.11\text{m/s}$. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad v_e = -236.11 + 900 = 663.89 \text{ m/s} \rightarrow$$

By referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\left(\overset{+}{\rightarrow}\right)\Sigma F_x = \frac{dm}{dt}\left[\left(v_B\right)_x - \left(v_A\right)_x\right]; \qquad C(236.11^2) = 2(1000)(663.89 - 0)$$

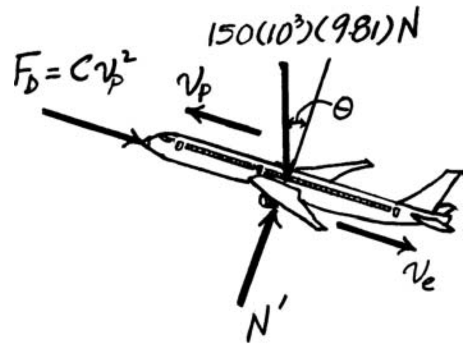
$$C = 23.817 \text{ kg}\cdot\text{s/m}$$



(a)

When the airplane is in the inclined position, it has a constant speed of $v_p = \left[750\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 208.33$ m/s. Thus,

$$v_e = -208.33 + 900 = 691.67 \text{ m/s}$$



(b)

By referring to the free-body diagram of the airplane shown in Fig. *b* and using the result of C, we can write

$$\nwarrow\!+\Sigma F_{x'} = \frac{dm}{dt}\left[\left(v_B\right)_{x'} - \left(v_A\right)_{x'}\right]; \qquad 23.817\left(208.33^2\right) + 150\left(10^3\right)(9.81)\sin\theta$$

$$= 2(1000)(691.67 - 0)$$

$$\theta = 13.7° \qquad \textbf{Ans.}$$

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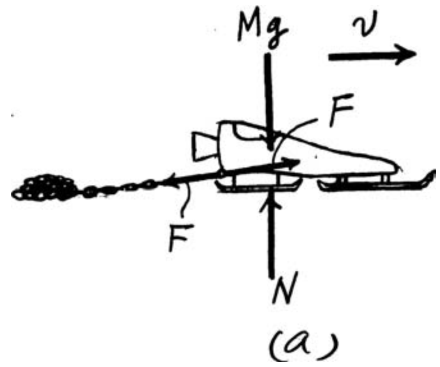
•15–137. A coil of heavy open chain is used to reduce the stopping distance of a sled that has a mass M and travels at a speed of v_0. Determine the required mass per unit length of the chain needed to slow down the sled to $(1/2)v_0$ within a distance $x = s$ if the sled is hooked to the chain at $x = 0$. Neglect friction between the chain and the ground.

Observing the free-body diagram of the system shown in Fig. a, notice that the pair of forces **F,** which develop due to the change in momentum of the chain, cancel each other since they are internal to the system. Here, $v_{D/s} = v$ since the chain on the ground is at rest. The rate at which the system gains mass is $\frac{dm_s}{dt} = m'v$ and the mass of the system is $m = m'x + M$. Referring to Fig. a,



$$\left(\xrightarrow{+}\right)\Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 0 = \left(m'x + M\right)\frac{dv}{dt} + v\left(m'v\right)$$

$$0 = \left(m'x + M\right)\frac{dv}{dt} + m'v^2 \qquad \textbf{(1)}$$

Since $\frac{dx}{dt} = v$ or $dt = \frac{dx}{v}$,

$$\left(m'x + M\right)v\frac{dv}{dx} + m'v^2 = 0$$

$$\frac{dv}{v} = -\left(\frac{m'}{m'x + M}\right)dx \qquad \textbf{(2)}$$

Integrating using the limit $v = v_0$ at $x = 0$ and $v = \frac{1}{2}v_0$ at $x = s$,

$$\int_{v_0}^{\frac{1}{2}v_0}\frac{dv}{v} = -\int_0^s\left(\frac{m'}{m'x + M}\right)dx$$

$$1\text{n}\, v\Big|_{v_0}^{\frac{1}{2}v_0} = -1\text{n}\left(m'x + M\right)\Big|_0^s$$

$$\frac{1}{2} = \frac{M}{m's + M}$$

$$m' = \frac{M}{s} \qquad \textbf{Ans.}$$

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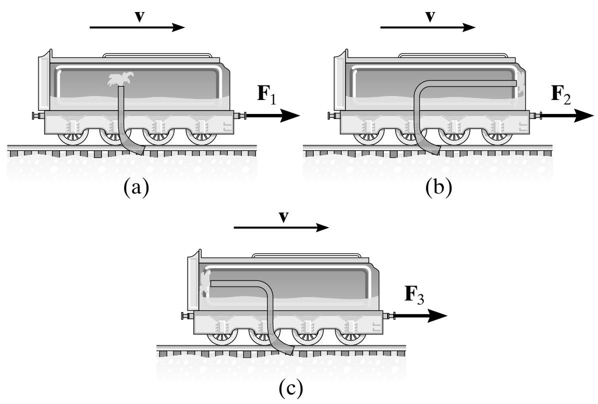
15–138. The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



(a) (b) (c)

The system consists of the car and the scoop. In all cases

$$\Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$F = 0 - v(\rho)(A)\,v$$

$$F = v^2\,\rho\,A$$ **Ans.**

15–139. A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 1.5 lb/s and ejected with a velocity of 4400 ft/s relative to the rocket, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

$$+\uparrow \Sigma F_s = \frac{dv}{dt} - v_{\,D/e}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$. In space the weight of the rocket is zero.

$$0 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}\,c$$

$$\int_0^v dv = \int_0^t \left(\frac{cv_{D/e}}{m_0 - ct}\right)dt$$

$$v = v_{D/e}\,1\text{n}\left(\frac{m_0}{m_0 - ct}\right) \qquad [1]$$

The maximum speed occurs when all the fuel is consumed, that is, when $t = \frac{300}{15} = 20$ s.

Here, $m_0 = \frac{500\,+\,300}{32.2} = 24.8447$ slug, $c = \frac{15}{32.2} = 0.4658$ slug/s, $v_{D/e} = 4400$ ft/s.

Substitute the numerical into Eq. [1]:

$$v_{\max} = 4400\,1\text{n}\left(\frac{24.8447}{24.8447 - (0.4658(20))}\right)$$

$$v_{\max} = 2068\text{ ft/s}$$ **Ans.**

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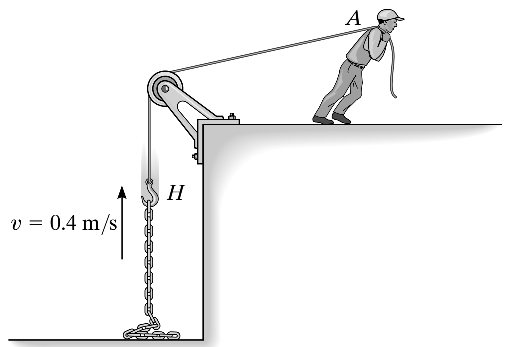
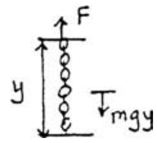
***15–140.** Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.



$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$

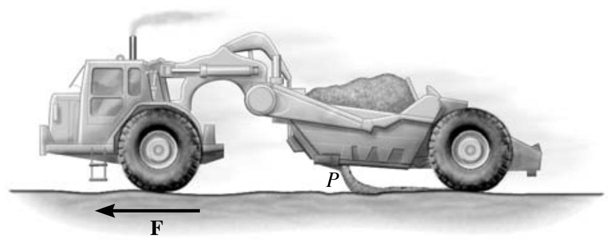
$$F - mgvt = 0 + v(mv)$$

$$F = m(gvt + v^2)$$

$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320)\text{ N}$$ **Ans.**

•15–141. The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5-m^2 dumping port P at a rate of 900 kg/s measured relative to the port. If the earthmover maintains a constant resultant tractive force $F = 4$ kN at its front wheels to provide forward motion, determine its acceleration when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600\text{ kg}$$

$$\frac{dm}{dt} = 900\text{ kg/s} = \rho\, v_{D/e} A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237\text{ m/s}$$

$$a = \frac{dv}{dt} = 0.1$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

$$F = 37\,600(0.1) - 900(0.237)$$

$$F = 3.55\text{ kN}$$ **Ans.**

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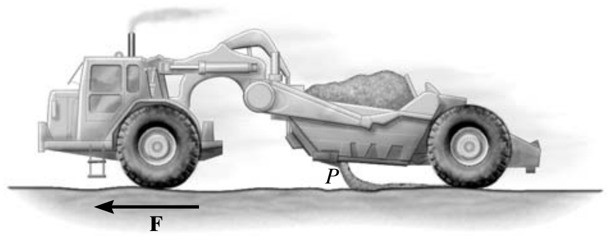
15–142. The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5-m^2 dumping port P at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is 0.1 m/s^2 when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho\, v_{D/e}\, A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237 \text{ m/s}$$

$$a = \frac{dv}{dt} = 0.1$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}\text{v}$$

$$F = 37\,600(0.1) - 900(0.237)$$

$$F = 3.55 \text{ kN}$$ **Ans.**

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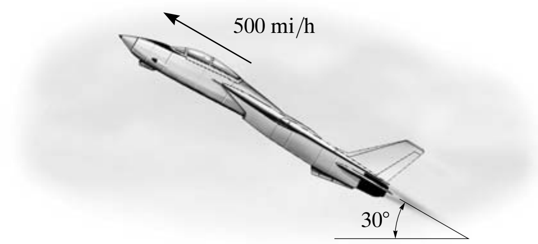
15–143. The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* See Prob. 15–131.

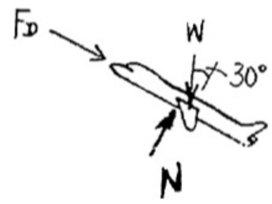


$$\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42 \text{ slug/s}$$

$$\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52 \text{ slug/s}$$

$$v = v_{D/i} = 500 \text{ mi/h} = 733.3 \text{ ft/s}$$

$$\nwarrow + \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$



$$-(15\,000)\sin 30° - 0.7(733.3)^2 = \frac{15\,000}{32.2}\frac{dv}{dt} - 32\,800(12.52) + 733.3(12.42)$$

$$a = \frac{dv}{dt} = 37.5 \text{ ft/s}^2$$ **Ans.**

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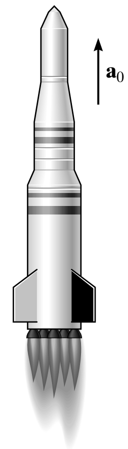
***15–144.** The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed $v_{e/r}$ determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.



$$a_0 = \frac{dv}{dt}$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - \mathrm{v}_{D/e}\frac{dm_e}{dt}$$

$$-mg = ma_0 - v_{e/r}\frac{dm}{dt}$$

$$v_{e/r}\frac{dm}{m} = (a_0 + g)\,dt \qquad \textbf{(1)}$$

Since $v_{c/r}$ is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{e/r}\ln\left(\frac{m}{m_0}\right) = (a_0 + g)t$$

$$\frac{m}{m_0} = e^{[(a_0+g)/v_{e/r}]t}$$

The time rate of fuel consumption is determined from Eq. (1).

$$\frac{dm}{dt} = m\left(\frac{(a_0 + g)}{v_{e/r}}\right)$$

$$\frac{dm}{dt} = m_0\left(\frac{(a_0 + g)}{v_{e/r}}\right)e^{[(a_0+g)v_{e/r}]t} \qquad \textbf{Ans.}$$

Note: $v_{c/r}$ must be considered a negative quantity.

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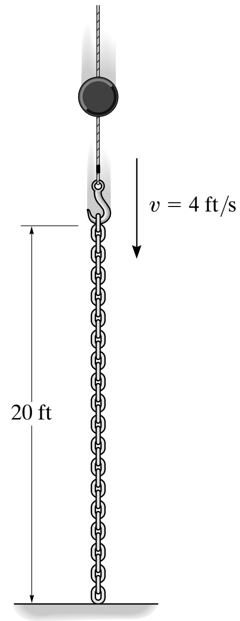
•15–145. If the chain is lowered at a constant speed, determine the normal reaction exerted on the floor as a function of time. The chain has a weight of 5 lb/ft and a total length of 20 ft.



At time t, the weight of the chain on the floor is $W = mg(vt)$

$$\frac{dv}{dt} = 0, \qquad m_i = m(vt)$$

$$\frac{dm_i}{dt} = mv$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$R - mg(vt) = 0 + v(mv)$$

$$R = m(gvt + v^2)$$

$$R = \frac{5}{32.2}(32.2(4)(t) + (4)^2)$$

$$R = (20t + 2.48)\text{ lb}$$ **Ans.**

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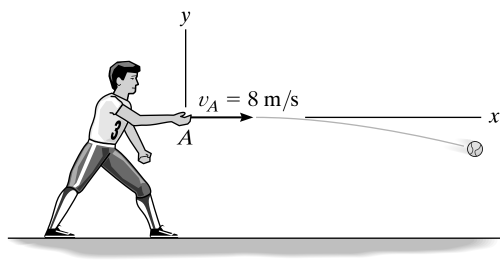
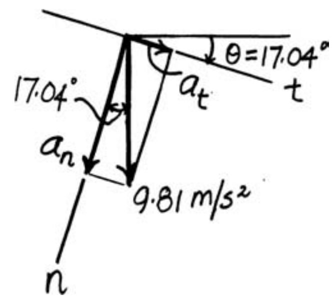
R1–1. The ball is thrown horizontally with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then determine the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.





Horizontal Motion: The horizontal component of velocity is $v_x = 8$ m/s and the initial horizontal position is $(s_0)_x = 0$.

$(\xrightarrow{+})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 8t \qquad \textbf{[1]}$$

Vertical Motion: The vertical component of initial velocity $(v_0)_y = 0$ and the initial vertical position are $(s_0)_y = 0$.

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 0 + \frac{1}{2}(-9.81)t^2 \qquad \textbf{[2]}$$

Eliminate t from Eqs. [1] and [2] yields

$$y = -0.0766x^2 \qquad \textbf{Ans.}$$

The vertical component of velocity when $t = 0.25$ s is given by

$(+\uparrow)$ $$v_y = (v_0)_y + (a_c)_y t$$

$$v_y = 0 + (-9.81)(0.25) = -2.4525 \text{ m/s} = 2.4525 \text{ m/s} \downarrow$$

The magnitude and direction angle when $t = 0.25$ s are

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{8^2 + 2.4525^2} = 8.37 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{2.4525}{8} = 17.04° = 17.0° \qquad \textbf{Ans.}$$

Since the velocity is always directed along the tangent of the path and the acceleration $a = 9.81 \text{ m/s}^2$ is directed downward, then tangential and normal components of acceleration are

$$a_t = 9.81 \sin 17.04° = 2.88 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = 9.81 \cos 17.04° = 9.38 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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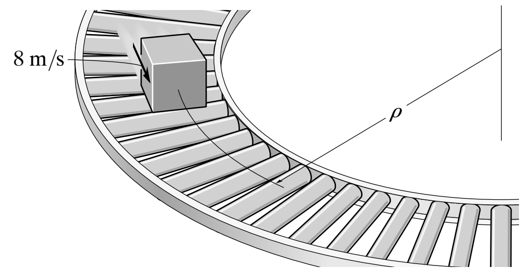
R1–2. Cartons having a mass of 5 kg are required to move along the assembly line with a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



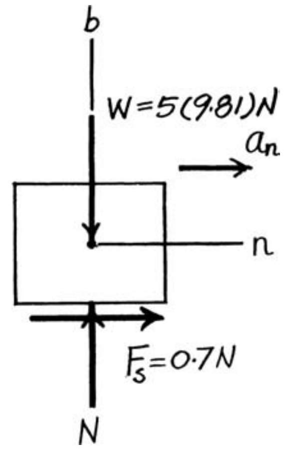
$+\uparrow \Sigma F_b = m\,a_b;\qquad N - W = 0$

$N = W$

$F_s = 0.7W$

$\xrightarrow{+} \Sigma F_n = m\,a_n;\qquad 0.7W = \dfrac{W}{9.81}\left(\dfrac{8^2}{\rho}\right)$

$\rho = 9.32\text{ m}$ **Ans.**



R1–3. A small metal particle travels downward through a fluid medium while being subjected to the attraction of a magnetic field such that its position is $s = (15t^3 - 3t)$ mm, where t is in seconds. Determine (a) the particle's displacement from $t = 2$ s to $t = 4$ s, and (b) the velocity and acceleration of the particle when $t = 5$ s.

a) $s = 15t^3 - 3t$

At $t = 2$ s, $s_1 = 114$ mm

At $t = 4$ s, $s_3 = 948$ mm

$\Delta s = 948 - 114 = 834$ mm **Ans.**

b) $v = \dfrac{ds}{dt} = 45t^2 - 3\Big|_{t=5} = 1122\text{ mm/s} = 1.12\text{ m/s}$ **Ans.**

$a = \dfrac{dv}{dt} = 90t\Big|_{t=5} = 450\text{ mm/s}^2 = 0.450\text{ m/s}^2$ **Ans.**

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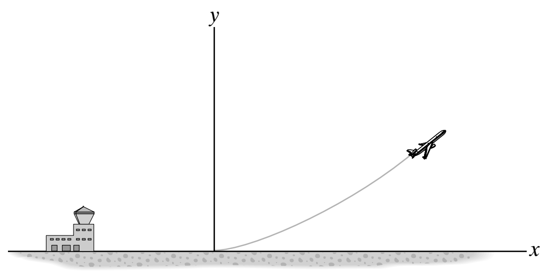
***R1–4.** The flight path of a jet aircraft as it takes off is defined by the parametric equations $x = 1.25t^2$ and $y = 0.03t^3$, where t is the time after take-off, measured in seconds, and x and y are given in meters. If the plane starts to level off at $t = 40$ s, determine at this instant (a) the horizontal distance it is from the airport, (b) its altitude, (c) its speed, and (d) the magnitude of its acceleration.



Position: When $t = 40\ s$, its horizontal position is given by

$$x = 1.25\left(40^2\right) = 2000\text{ m} = 2.00\text{ km}$$ **Ans.**

and its altitude is given by

$$y = 0.03\left(40^3\right) = 1920\text{ m} = 1.92\text{ km}$$ **Ans.**

Velocity: When $t = 40$ s, the horizontal component of velocity is given by

$$v_x = \dot{x} = 2.50t\big|_{t=40\text{ s}} = 100\text{ m/s}$$

The vertical component of velocity is

$$v_y = \dot{y} = 0.09t^2\big|_{t=40\text{ s}} = 144\text{ m/s}$$

Thus, the plane's speed at $t = 40$ s is

$$v_y = \sqrt{v_x^2 + v_y^2} = \sqrt{100^2 + 144^2} = 175\text{ m/s}$$ **Ans.**

Acceleration: The horizontal component of acceleration is

$$a_x = \ddot{x} = 2.50\text{ m/s}^2$$

and the vertical component of acceleration is

$$a_y = \ddot{y} = 0.18t\big|_{t=40\text{ s}} = 7.20\text{ m/s}^2$$

Thus, the magnitude of the plane's acceleration at $t = 40$ s is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{2.50^2 + 7.20^2} = 7.62\text{ m/s}^2$$ **Ans.**

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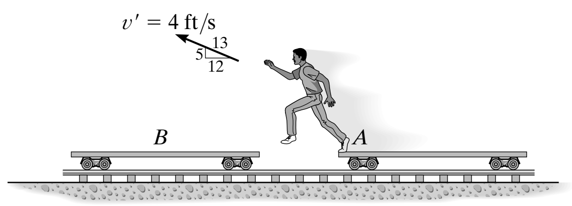
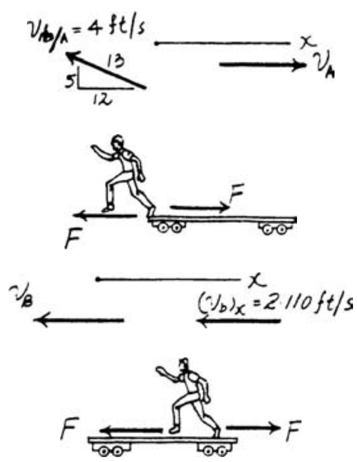
R1–5. The boy jumps off the flat car at A with a velocity of $v' = 4$ ft/s relative to the car as shown. If he lands on the second flat car B, determine the final speed of both cars after the motion. Each car has a weight of 80 lb. The boy's weight is 60 lb. Both cars are originally at rest. Neglect the mass of the car's wheels.



Relative Velocity: The horizontal component of the relative velocity of the boy with respect to the car A is $(v_{b/A})_x = 4\left(\frac{12}{13}\right) = 3.692$ ft/s. Thus, the horizontal component of the velocity of the boy is

$$(v_b)_x = v_A + (v_{b/A})_x$$

$$(\overset{+}{\leftarrow}) \qquad (v_b)_x = -v_A + 3.692 \qquad \textbf{[1]}$$



Conservation of Linear Momentum: If we consider the boy and the car as a system, then the impulsive force caused by traction of the shoes is *internal* to the system. Therefore, they will cancel out. As the result, the linear momentum is conserved along x axis. For car A

$$0 = m_b(v_b)_x + m_A v_A$$

$$(\overset{+}{\leftarrow}) \qquad 0 = \left(\frac{60}{32.2}\right)(v_b)_x + \left(\frac{80}{32.2}\right)(-v_A) \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$v_A = 1.58 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_b)_x = 2.110 \text{ ft/s}$$

For car B

$$m_b\,(v_b)_x = (m_b + m_B)v_B$$

$$(\overset{+}{\leftarrow}) \qquad \left(\frac{60}{32.2}\right)(2.110) = \left(\frac{60+80}{32.2}\right)v_B$$

$$v_B = 0.904 \text{ ft/s} \qquad \textbf{Ans.}$$

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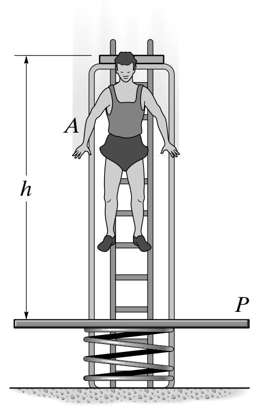
R1-6. The man A has a weight of 175 lb and jumps from rest at a height $h = 8$ ft onto a platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. Determine (a) the velocities of A and P just after impact and (b) the maximum compression imparted to the spring by the impact. Assume the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion.



Conservation of Energy: The datum is set at the initial position of platform P. When the man falls from a height of 8 ft *above* the datum, his initial gravitational potential energy is $175(8) = 1400$ ft · lb. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 1400 = \frac{1}{2}\left(\frac{175}{32.2}\right)(v_M)_1^2 + 0$$

$$(v_H)_1 = 22.70 \text{ ft/s}$$

Conservation of Momentum:

$$m_M\,(v_M)_1 + m_P(v_P)_1 = m_M(v_M)_2 + m_p(v_p)_2$$

$$(+\downarrow) \qquad \left(\frac{175}{32.2}\right)(22.70) + 0 = \left(\frac{175}{32.2}\right)(v_M)_2 + \left(\frac{60}{32.2}\right)(v_p)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_P)_2 - (v_M)_2}{(v_M)_1 - (v_p)_1}$$

$$(+\downarrow) \qquad 0.6 = \frac{(v_P)_2 - (v_P)_2}{22.70 - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_p)_2 = 27.04 \text{ ft/s} \downarrow = 27.0 \text{ ft/s} \downarrow \qquad (v_M)_2 = 13.4 \text{ ft/s} \downarrow \qquad \textbf{Ans.}$$

Conservation of Energy: The datum is set at the spring's compressed position. Initially, the spring has been compressed $\frac{60}{200} = 0.3$ ft and the elastic potential energy is $\frac{1}{2}(200)\left(0.3^2\right) = 9.00$ ft · lb. When platform P is at a height of s *above* the datum, its initial gravitational potential energy is $60s$. When platform P stops momentary, the spring has been compressed to its maximum and the elastic potential energy at this instant is $\frac{1}{2}(200)(s + 0.3)^2 = 100s^2 + 60s + 9$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)\left(27.04^2\right) + 60s + 9.00 = 100s^2 + 60s + 9$$

$$s = 2.61 \text{ ft} \qquad \textbf{Ans.}$$

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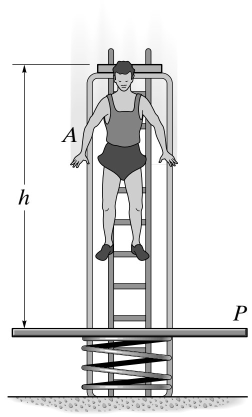
R1–7. The man A has a weight of 100 lb and jumps from rest onto the platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. If the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion, determine the required height h of the jump if the maximum compression of the spring is 2 ft.



Conservation of Energy: The datum is set at the initial position of platform P. When the man falls from a height of h *above* the datum, his initial gravitational potential energy is $100h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 100h = \frac{1}{2}\left(\frac{100}{32.2}\right)(v_M)_1^2 + 0$$

$$(v_H)_1 = \sqrt{64.4h}$$

Conservation of Momentum:

$$m_M\,(v_M)_1 + m_P\,(v_P)_1 = m_M\,(v_M)_2 + m_P\,(v_P)_2$$

$$(+\downarrow) \qquad \left(\frac{100}{32.2}\right)(\sqrt{64.4h}) + 0 = \left(\frac{100}{32.2}\right)(v_M)_2 + \left(\frac{60}{32.2}\right)(v_P)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_p)_2 - (v_M)_2}{(v_M)_1 - (v_p)_1}$$

$$(+\downarrow) \qquad 0.6 = \frac{(v_p)_2 - (v_M)_2}{\sqrt{64.4h} - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_p)_2 = \sqrt{64.4h}\ \downarrow \qquad (v_M)_2 = 0.4\sqrt{64.4h}\ \downarrow$$

Conservation of Energy: The datum is set at the spring's compressed position. Initially, the spring has been compressed $\frac{60}{200} = 0.3$ ft and the elastic potential energy is $\frac{1}{2}(200)\left(0.3^2\right) = 9.00$ ft·lb. Here, the compression of the spring caused by impact is $(2 - 0.3)$ ft $= 1.7$ ft. When platform P is at a height of 1.7 ft *above* the datum, its initial gravitational potential energy is $60(1.7) = 102$ ft·lb. When platform P stops momentary, the spring has been compressed to its maximum and the elastic potential energy at this instant is $\frac{1}{2}(200)\left(2^2\right) = 400$ ft·lb. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)\left(\sqrt{64.4h}\right)^2 + 102 + 9.00 = 400$$

$$h = 4.82 \text{ ft} \qquad \textbf{Ans.}$$

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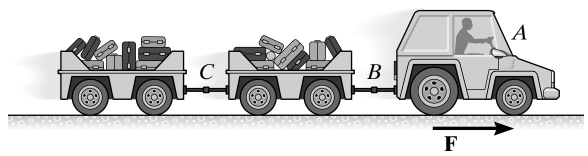
***R1–8.** The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. Determine the tension in the couplings at B and C if the tractive force **F** on the truck is $F = 480$ N. What is the speed of the truck when $t = 2$ s, starting from rest? The car wheels are free to roll. Neglect the mass of the wheels.



$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 480 = [800 + 2(300)]a$

$a = 0.3429 \text{ m/s}^2$

$(\xrightarrow{+})\qquad v = v_0 + a_c t$

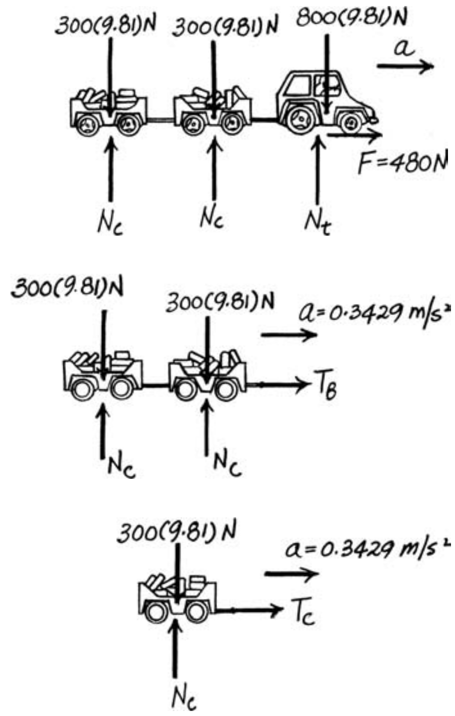
$v = 0 + 0.3429(2) = 0.686 \text{ m/s}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad T_B = 2(300)(0.3429)$

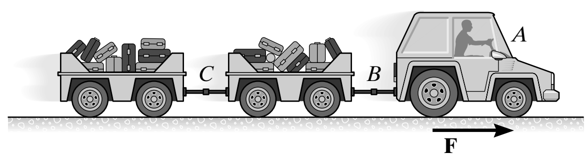
$T_B = 205.71 = 206 \text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad T_C = (300)(0.3429)$

$T_C = 102.86 = 103 \text{ N}$ **Ans.**



R1–9. The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. If the tractive force **F** on the truck is $F = 480$ N, determine the acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

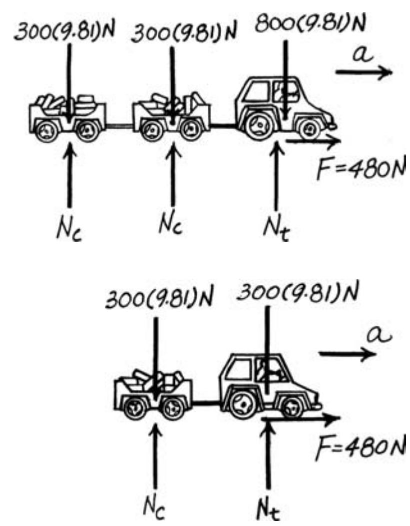


$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 480 = [800 + 2(300)]a$

$a = 0.3429 = 0.343 \text{ m/s}^2$ **Ans.**

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 480 = (800 + 300)a$

$a = 0.436 \text{ m/s}^2$ **Ans.**



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R1–10. A car travels at 80 ft/s when the brakes are suddenly applied, causing a constant deceleration of 10 ft/s². Determine the time required to stop the car and the distance traveled before stopping.

$\left(\xrightarrow{+}\right)$ $\quad v = v_0 + a_c t$

$0 = 80 + (-10)t$

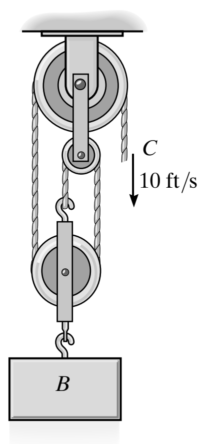
$t = 8\text{ s}$ **Ans.**

$\left(\xrightarrow{+}\right)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (80)^2 + 2(-10)(s - 0)$

$s = 320\text{ ft}$ **Ans.**

R1–11. Determine the speed of block B if the end of the cable at C is pulled downward with a speed of 10 ft/s. What is the relative velocity of the block with respect to C?



$3s_B + s_C = l$

$3v_B = -v_C$

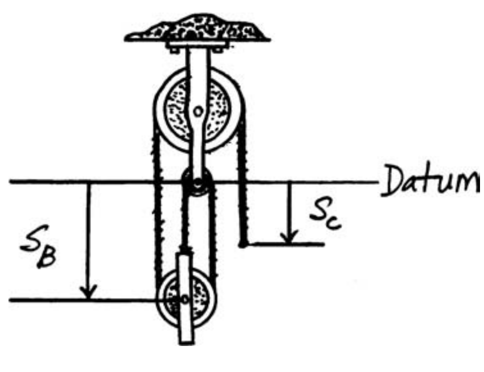
$3v_B = -(10)$

$v_B = -3.33\text{ ft/s} = 3.33\text{ ft/s}\uparrow$ **Ans.**

$(+\downarrow)$ $\quad v_B = v_C + v_{B/C}$

$-3.33 = 10 + v_{B/C}$

$v_{B/C} = -13.3\text{ ft/s} = 13.3\text{ ft/s}\uparrow$ **Ans.**



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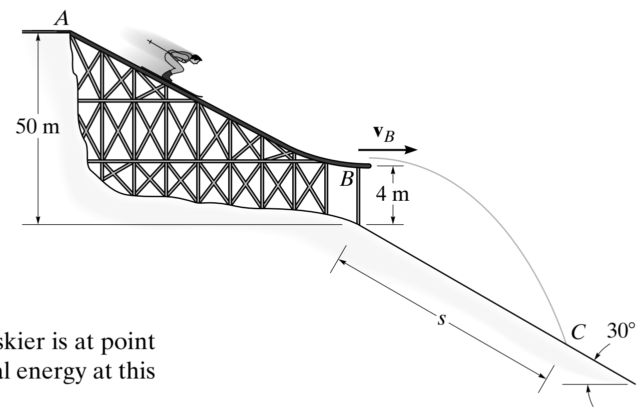
***R1–12.** The skier starts fom rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, compute the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



Potential Energy: The datum is set at the lowest point B. When the skier is at point A, he is $(50 - 4) = 46$ m *above* the datum. His gravitational potential energy at this position is $70(9.81)(46) = 31588.2$ J.

Conservation of Energy: Applying Eq. 14–21, we have

$$T_A + V_A = T_B + V_B$$

$$0 + 31588.2 = \frac{1}{2}(70)\,v_B^2$$

$$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the skier, we have

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y\,t + \frac{1}{2}(a_c)_y\,t^2$$

$$4 + s\sin 30^\circ = 0 + 0 + \frac{1}{2}(9.81)\,t^2 \qquad \textbf{[1]}$$

By considering the horizontal motion of the skier, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x\,t$$

$$s\cos 30^\circ = 0 + 30.04t \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$s = 130 \text{ m} \qquad \textbf{Ans.}$$

$$t = 3.753 \text{ s}$$

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R1–13. The position of a particle is defined by $r = \{5(\cos 2t)\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2r\mathbf{i} + 8 \cos 2r\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $\mathbf{v} = -10 \sin 2(1)\mathbf{i} + 8 \cos 2(1)\,\mathbf{j} = (-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2r\mathbf{i} - 16 \sin 2r\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1)\,\mathbf{i} - 16 \sin 2(1)\,\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Travelling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{[1]}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{[2]}$$

Adding Eqs [1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2r + \sin^2 2t$$

However, $\cos^2 2r + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \quad (\textbf{Equation of an Ellipse}) \qquad \textbf{(Q.E.D.)}$$

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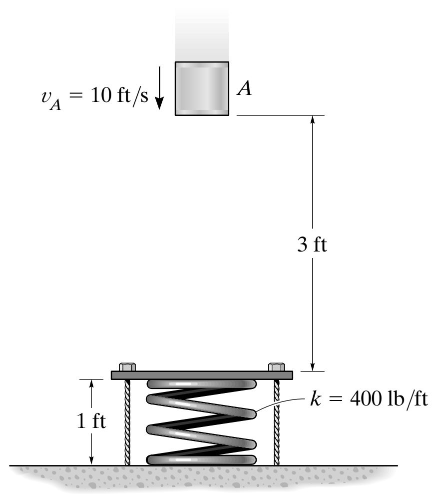
R1–14. The 5-lb cylinder falls past A with a speed $v_A = 10$ ft/s onto the platform. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 1.75 ft and is originally kept in compression by the 1-ft-long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.



Potential Energy: Datum is set at the final position of the platform. When the cylinder is at point A, its position is $(3 + s)$ *above* the datum where s is the maximum displacement of the platform when the cylinder stops momentary. Thus, its gravitational potential energy at this position is $5(3 + s) = (15 + 5s)$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(400)(1.75 - 1)^2 = 112.5$ ft · lb and $\frac{1}{2}(400)(s + 0.75)^2 = 200s^2 + 300s + 112.5$, respectively.

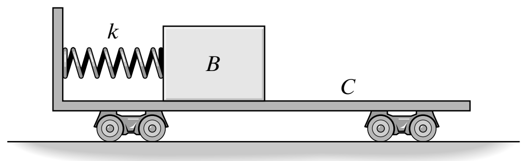
Conservation of Energy: Applying Eq. 14–22, we have

$$\Sigma T_A + \Sigma V_A = \Sigma T_B + \Sigma V_B$$

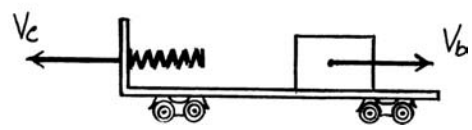
$$\frac{1}{2}\left(\frac{5}{32.2}\right)\left(10^2\right) + (15 + 5s) + 112.5 = 0 + 200s^2 + 300s + 112.5$$

$$s = 0.0735 \text{ ft} \qquad \textbf{Ans.}$$

R1–15. The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block after the spring becomes undeformed. Neglect the mass of the cart's wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.



$$T_1 + V_1 = T_2 + V_2$$



$$[0 + 0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)\,v_b^2 + \frac{1}{2}(75)\,v_e^2$$

$$12 = 50\ v_b^2 + 75\ v_e^2$$

$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0 + 0 = 50\,v_b - 75\,v_e$$

$$v_b = 1.5 v_e$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

$$v_b = 0.379 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

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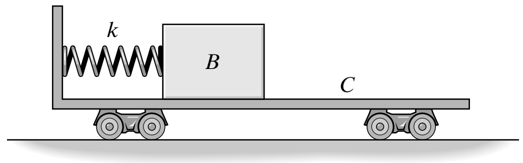
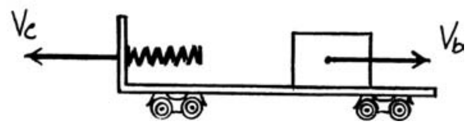
***R1–16.** The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block with respect to the cart after the spring becomes undeformed. Neglect the mass of the cart's wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)v_b^2 + \frac{1}{2}(75)\,v_e^2$$

$$12 = 50\,v_b^2 + 75\,v_e^2$$



$(\xrightarrow{+})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 50\,v_b - 75\,v_e$$

$$v_b = 1.5\,v_e$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

$$v_b = 0.379 \text{ m/s} \rightarrow$$

$$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

$(\xrightarrow{+})$ $\quad 0.379 = -0.253 + \mathbf{v}_{b/c}$

$v_{b/c} = 0.632 \text{ m/s} \rightarrow$ **Ans.**

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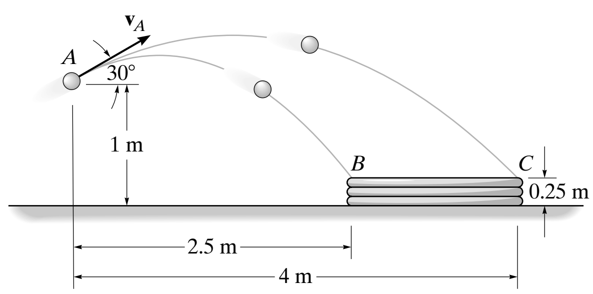
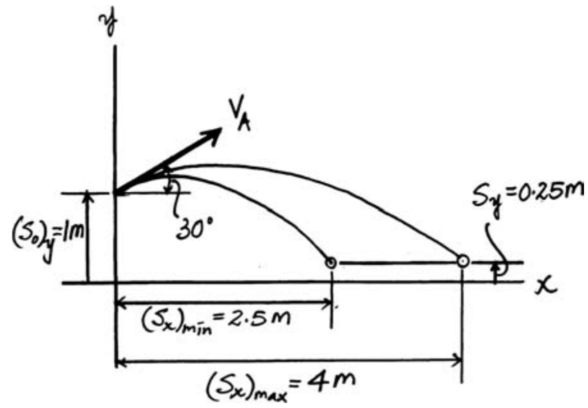
R1–17. A ball is launched from point A at an angle of 30°. Determine the maximum and minimum speed v_A it can have so that it lands in the container.





Min. speed:

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$2.5 = 0 + v_A \cos 30° t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0.25 = 1 + v_A \sin 30° t - \frac{1}{2}(9.81)t^2$$

Solving

$t = 0.669$ s

$v_A = (v_A)_{\min} = 4.32$ m/s **Ans.**

Max. speed:

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$4 = 0 + v_A \cos 30° t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0.25 = 1 + v_A \sin 30° t - \frac{1}{2}(9.81)t^2$$

Solving:

$t = 0.790$ s

$v_A = (v_A)_{\max} = 5.85$ m/s **Ans.**

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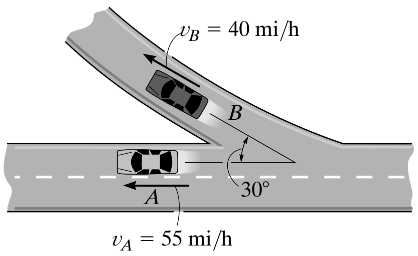
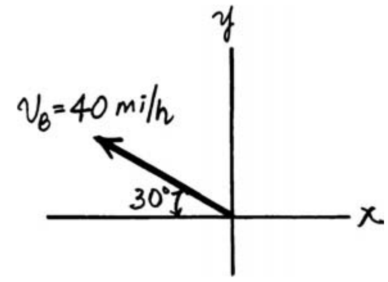
R1–18. At the instant shown, cars A and B travel at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains its constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.





$$\mathbf{v}_B = -40\cos 30°\mathbf{i} + 40\sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$\mathbf{v}_A = \{-55\mathbf{i}\}\text{ mi/h}$$

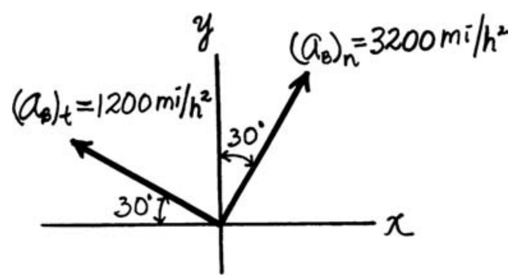
$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5\text{ mi/h}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{20}{20.36}\right) = 44.5° \measuredangle$$ **Ans.**

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{40^2}{0.5} = 3200\text{ mi/h}^2 \qquad (a_B)_t = 1200\text{ mi/h}^2$$



$$\mathbf{a}_B = (3200\sin 30° - 1200\cos 30°)\mathbf{i} + (3200\cos 30° + 1200\sin 30°)\mathbf{j}$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\text{ mi/h}^2$$

$$\mathbf{a}_A = \mathbf{0}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$560.77\mathbf{i} + 3371.28\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\text{ mi/h}^2$$

$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418\text{ mi/h}^2 = 3.42\left(10^3\right)\text{ mi/h}^2$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{3371.28}{560.77}\right) = 80.6° \measuredangle$$ **Ans.**

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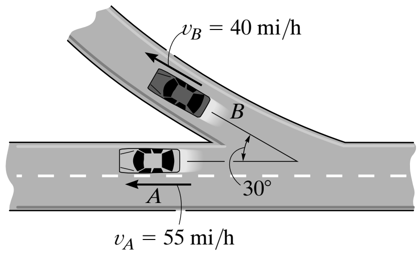
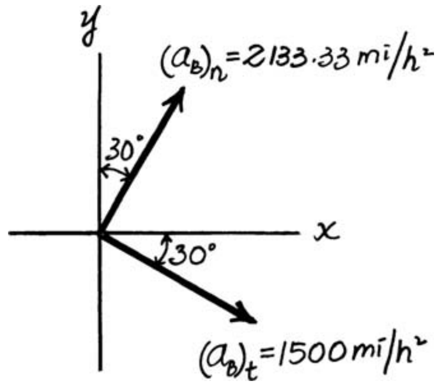
R1–19. At the instant shown, cars A and B travel at speeds of 55 mi/h and 40 mi/h, respectively. If B is decreasing its speed at 1500 mi/h^2 while A is increasing its speed at 800 mi/h^2, determine the acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.75 mi.



$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(40)^2}{0.75} = 2133.33 \text{ mi/h}^2$$



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$2133.33 \sin 30°\mathbf{i} + 2133.33 \cos 30°\mathbf{j} + 1500 \cos 30°\mathbf{i} - 1500 \sin 30°\mathbf{j}$$

$$= -800\mathbf{i} + (a_{B/A})_x\,\mathbf{i} + (a_{B/A})_y\,\mathbf{j}$$

$(\overset{+}{\rightarrow})$ $\quad 2133.33 \sin 30° + 1500 \cos 30° = -800 + (a_{B/A})_x$

$$(a_{B/A})_x = 3165.705 \rightarrow$$

$(+\uparrow)$ $\quad 2133.33 \cos 30° - 1500 \sin 30° = (a_{B/A})_y$

$$(a_{B/A})_y = 1097.521 \uparrow$$

$$(a_{B/A}) = \sqrt{(1097.521)^2 + (3165.705)^2}$$

$$a_{B/A} = 3351 \text{ mi/h}^2 = 3.35\left(10^3\right) \text{ mi/h}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{1097.521}{3165.705}\right) = 19.1° \measuredangle \qquad \textbf{Ans.}$$

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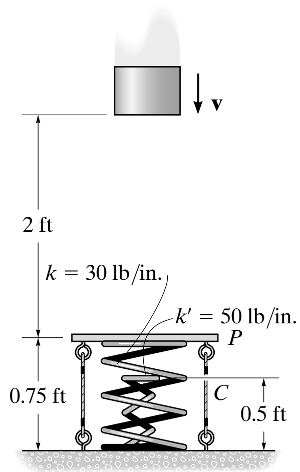
***R1–20.** Four inelastic cables C are attached to a plate P and hold the 1-ft-long spring 0.25 ft in compression when *no weight* is on the plate. There is also an undeformed spring nested within this compressed spring. If the block, having a weight of 10 lb, is moving downward at $v = 4$ ft/s, when it is 2 ft above the plate, determine the maximum compression in each spring after it strikes the plate. Neglect the mass of the plate and springs and any energy lost in the collision.



$k = 30(12) = 360$ lb/ft

$k' = 50(12) = 600$ lb/ft

Assume both springs compress;

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2 + 0 + \frac{1}{2}(360)(0.25)^2 = 0 + \frac{1}{2}(360)(s + 0.25)^2 + \frac{1}{2}(600)(s - 0.25)^2 - 10(s + 2)$$

$13.73 = 180(s + 0.25)^2 + 300(s - 0.25)^2 - 10s - 20$ **(1)**

$33.73 = 180(s + 0.25)^2 + 300(s - 0.25)^2 - 10s$

$480s^2 - 70s - 3.73 = 0$

Choose the positive root;

$s = 0.1874 \text{ ft} < 0.25 \text{ ft}$ **NG!**

The nested spring does not deform.

Thus Eq. (1) becomes

$13.73 = 180(s + 0.25)^2 - 10s - 20$

$180\, s^2 + 80\, s - 22.48 = 0$

$s = 0.195$ ft **Ans.**

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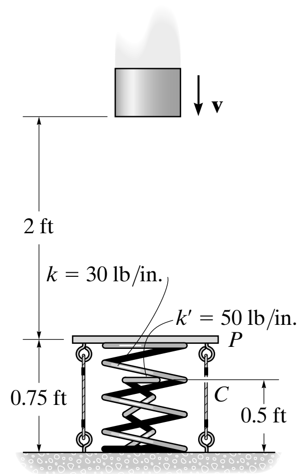
R1–21. Four inelastic cables C are attached to plate P and hold the 1-ft-long spring 0.25 ft in compression when *no weight* is on the plate. There is also a 0.5-ft-long undeformed spring nested within this compressed spring. Determine the speed v of the 10-lb block when it is 2 ft above the plate, so that after it strikes the plate, it compresses the nested spring, having a stiffness of 50 lb/in., an amount of 0.20 ft. Neglect the mass of the plate and springs and any energy lost in the collision.



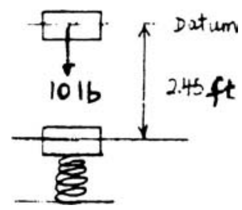
$$k = 30(12) = 360 \text{ lb/ft}$$

$$k' = 50(12) = 600 \text{ lb/ft}$$

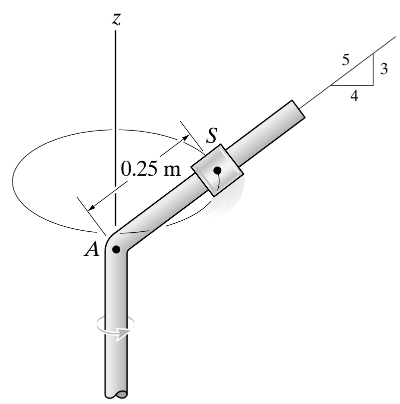
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)v^2 + \frac{1}{2}(360)(0.25)^2 = \frac{1}{2}(360)(0.25 + 0.25 + 0.20)^2 + \frac{1}{2}(600)(0.20)^2 - 10(2 + 0.25 + 0.20)$$

$v = 20.4$ ft/s **Ans.**



R1–22. The 2-kg spool S fits loosely on the rotating inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



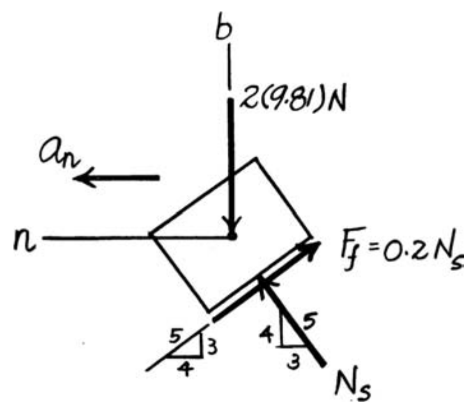
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$$\stackrel{+}{\leftarrow}\Sigma F_n = ma_n; \qquad N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

$$+\uparrow \Sigma F_b = m\,a_b; \qquad N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 21.3 \text{ N}$$

$v = 0.969$ m/s **Ans.**



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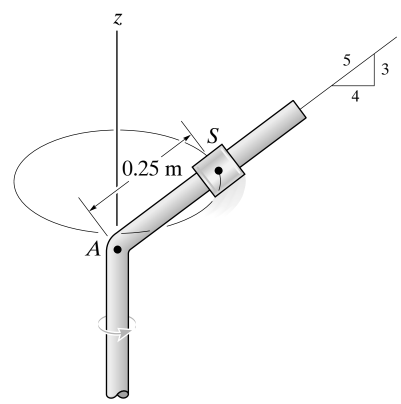
R1–23. The 2-kg spool S fits loosely on the rotating inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



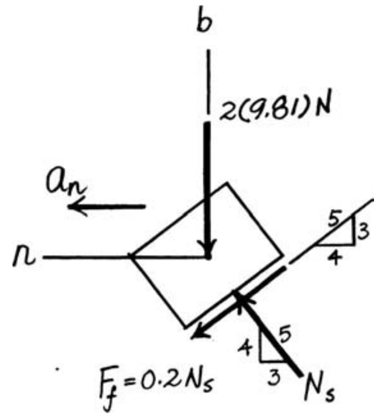
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$$\overset{+}{\leftarrow}\Sigma F_n = ma_n; \qquad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

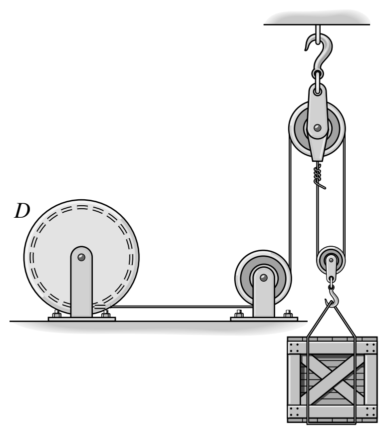
$$+\uparrow\Sigma F_b = m\,a_b; \qquad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 28.85 \text{ N}$$

$$v = 1.48 \text{ m/s}$$ **Ans.**



***R1–24.** The winding drum D draws in the cable at an accelerated rate of 5 m/s². Determine the cable tension if the suspended crate has a mass of 800 kg.



$$s_A + 2\,s_B = l$$

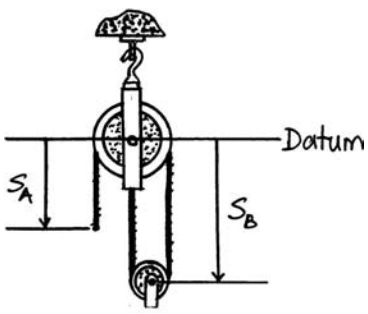
$$a_A = -2\,a_B$$

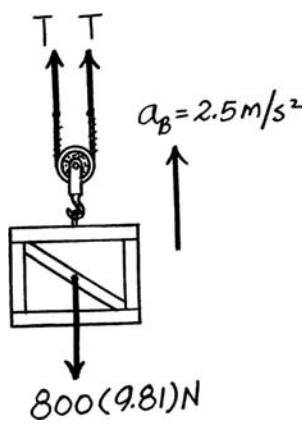
$$5 = -2\,a_B$$

$$a_B = -2.5 \text{ m/s}^2 = 2.5 \text{ m/s}^2 \uparrow$$

$$+\uparrow\Sigma F_y = ma_y; \qquad 2T - 800(9.81) = 800(2.5)$$

$$T = 4924 \text{ N} = 4.92 \text{ kN}$$ **Ans.**





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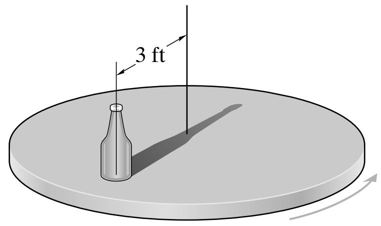
R1–25. The bottle rests at a distance of 3 ft from the center of the horizontal platform. If the coefficient of static friction between the bottle and the platform is $\mu_s = 0.3$, determine the maximum speed that the bottle can attain before slipping. Assume the angular motion of the platform is slowly increasing.

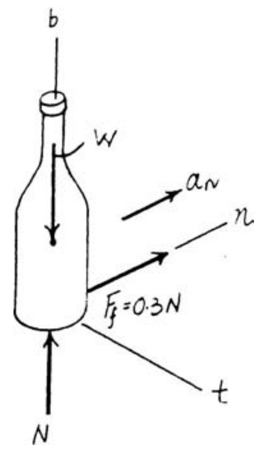


$\Sigma F_b = 0;$ $\qquad N - W = 0 \qquad N = W$

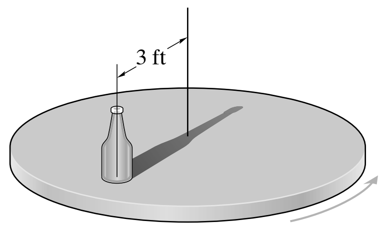
Since the bottle is on the verge of slipping, then $F_f = \mu_s N = 0.3W$.

$\Sigma F_n = ma_n;$ $\qquad 0.3W = \left(\frac{W}{32.2}\right)\left(\frac{v^2}{3}\right)$

$v = 5.38 \text{ ft/s}$ **Ans.**



R1–26. Work Prob. R1–25 assuming that the platform starts rotating from rest so that the speed of the bottle is increased at 2 ft/s^2.



Applying Eq. 13–8, we have

$\Sigma F_b = 0;$ $\qquad N - W = 0 \qquad N = W$

Since the bottle is on the verge of slipping, then $F_f = \mu_s N = 0.3W$.

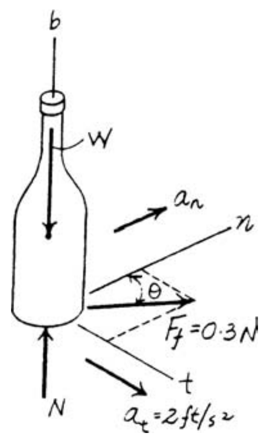
$\Sigma F_t = ma_t;$ $\qquad 0.3W \sin\theta = \left(\frac{W}{32.2}\right)(2)$ **[1]**

$\Sigma F_n = ma_n;$ $\qquad 0.3W \cos\theta = \left(\frac{W}{32.2}\right)\left(\frac{v^2}{3}\right)$ **[2]**

Solving Eqs. [1] and [2] yields

$v = 5.32 \text{ ft/s}$ **Ans.**

$\theta = 11.95^\circ$



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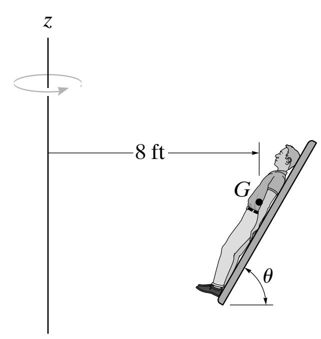
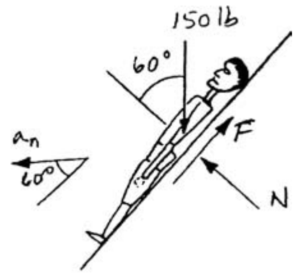
R1–27. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.



$$+\nwarrow \Sigma F_y = m(a_n)_y; \qquad N - 150\cos 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\sin 60°$$

$$N = 277 \text{ lb} \qquad \textbf{Ans.}$$



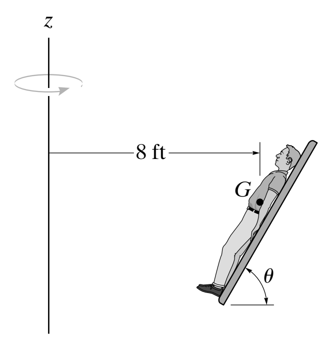
$$+\swarrow \Sigma F_x = m(a_n)_x; \qquad -F + 150\sin 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\cos 60°$$

$$F = 13.4 \text{ lb} \qquad \textbf{Ans.}$$

Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4 \text{ lb}$

***R1–28.** The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip up the cushion.



$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 0.5N\cos\theta + N\sin\theta = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

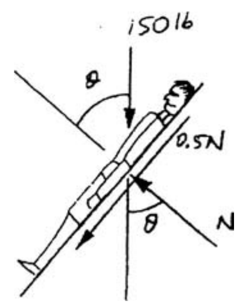
$$+\uparrow \Sigma F_b = 0; \qquad -150 + N\cos\theta - 0.5\,N\sin\theta = 0$$

$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

$$0.5\cos\theta + \sin\theta = 3.493\,79\cos\theta - 1.746\,89\sin\theta$$

$$\theta = 47.5° \qquad \textbf{Ans.}$$



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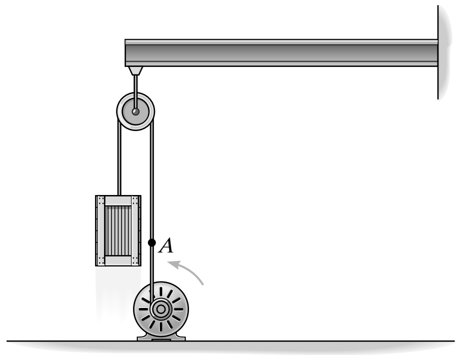
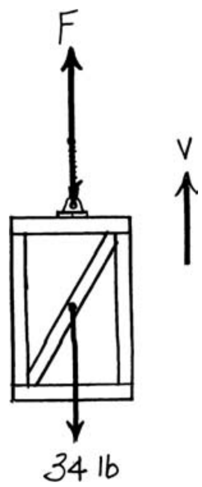
R1–29. The motor pulls on the cable at A with a force $F = (30 + t^2)$ lb, where t is in seconds. If the 34-lb crate is originally at rest on the ground when $t = 0$, determine its speed when $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



$$30 + t^2 = 34$$

$t = 2$ s for crate to start moving



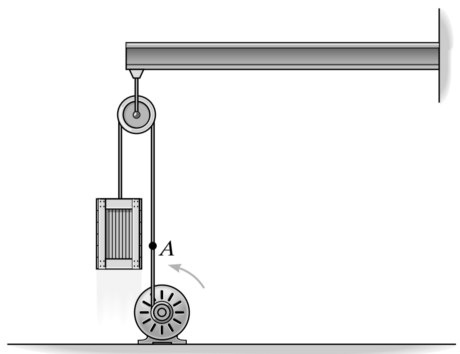
$$(+\uparrow) \qquad mv_1 + \Sigma \int F dt = mv_2$$

$$0 + \int_2^4 (30 + t^2)dt - 34(4 - 2) = \frac{34}{32.2} v_2$$

$$[30t + \frac{1}{3}t^3]_2^4 - 68 = \frac{34}{32.2} v_2$$

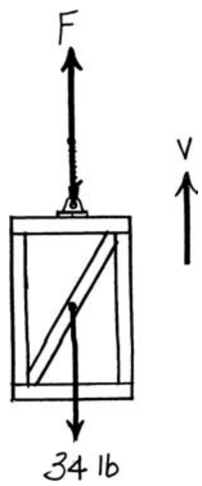
$$v_2 = 10.1 \text{ ft/s}$$ **Ans.**

R1–30. The motor pulls on the cable at A with a force $F = (e^{2t})$ lb, where t is in seconds. If the 34-lb crate is originally at rest on the ground when $t = 0$, determine the crate's velocity when $t = 2$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



$$F = e^{2t} = 34$$

$t = 1.7632$ s for crate to start moving



$$(+\uparrow) \qquad mv_1 + \Sigma \int F\, dt = mv_2$$

$$0 + \int_{1.7632}^{2} e^{2t}\, dt - 34(2 - 1.7632) = \frac{34}{32.2} v_2$$

$$\frac{1}{2} e^{2t} \Big|_{1.7632}^{2} - 8.0519 = 1.0559\, v_2$$

$$v_2 = 2.13 \text{ ft/s}$$ **Ans.**

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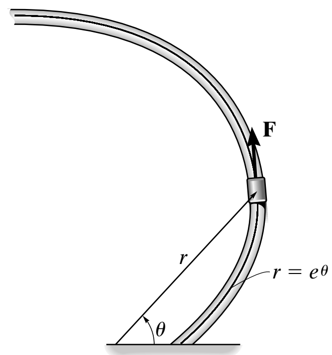
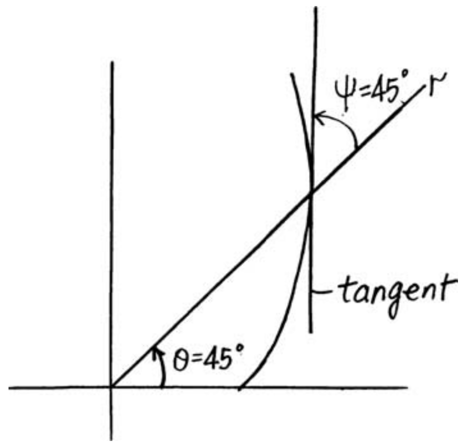
R1–31. The collar has a mass of 2 kg and travels along the smooth *horizontal* rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force **F** and the normal force **N** acting on the collar when $\theta = 45°$, if force **F** maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



$$r = e^{\theta}$$

$$\dot{r} = e^{\theta}\dot{\theta}$$

$$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\dot{\theta}$$



At $\theta = 45°$

$$\dot{\theta} = 2 \text{ rad/s}$$

$$\theta = 0$$

$$r = 2.1933 \text{ m}$$

$$r = 4.38656 \text{ m/s}$$

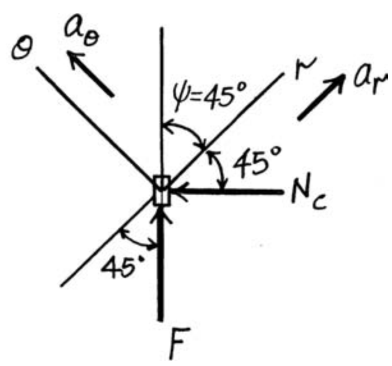
$$\ddot{r} = 8.7731 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\ (\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$$

$$a_{\theta} = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$$\psi = \theta = 45°$$



$\Sigma F_r = m\,a_r$; $-N_r \cos 45° + F \cos 45° = 2(0)$

$\Sigma F_{\theta} = m\,a_{\theta}$; $F \sin 45° + N_{\theta} \sin 45° = 2(17.5462)$

$N = 24.8$ N **Ans.**

$F = 24.8$ N **Ans.**

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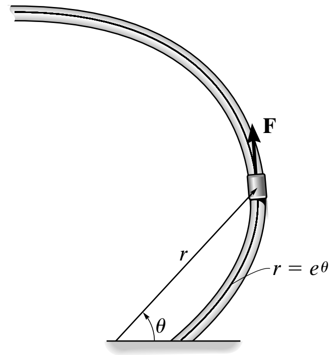
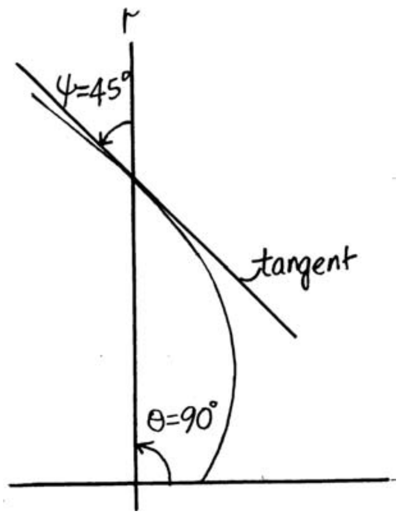
***R1–32.** The collar has a mass of 2 kg and travels along the smooth *horizontal* rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force **F** and the normal force **N** acting on the collar when $\theta = 90°$, if force **F** maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



$$r = e^{\theta}$$

$$\dot{r} = e^{\theta}\,\dot{\theta}$$

$$\ddot{r} = e^{\theta}\,(\dot{\theta})^2 + e^{\theta}\,\ddot{\theta}$$



At $\theta = 90°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 4.8105$ m

$\dot{r} = 9.6210$ m/s

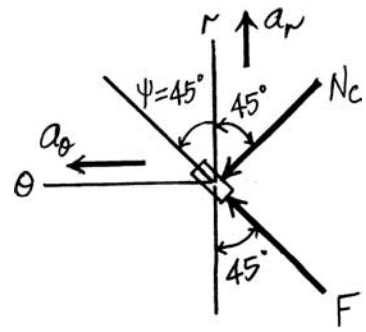
$\ddot{r} = 19.242$ m/s^2

$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$

$a_{\theta} = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838$ m/s^2

$\tan\psi = \dfrac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\psi = \theta = 45°$



$+\uparrow \Sigma F_t = m\,a_t;$ $\quad -N\cos 45° + F\cos 45° = 2(0)$

$\overset{+}{\leftarrow} \Sigma F_{\theta} = m\,a_{\theta};$ $\quad F\sin 45° + N\sin 45° = 2(38.4838)$

$N_t = 54.4$ N **Ans.**

$F = 54.4$ N **Ans.**

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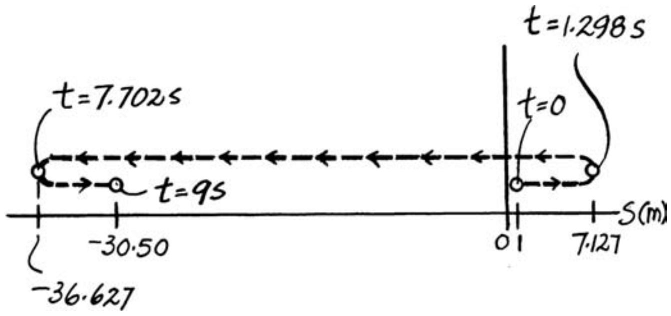
R1–33. The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. When $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity. Assume the positive direction is to the right.



$$a = (2t - 9)$$

$$dv = a\,dt$$

$$\int_{10}^{v} dv = \int_0^t (2t - 9)\,dt$$

$$v - 10 = t^2 - 9t$$

$$v = t^2 - 9t + 10$$

$$ds = v\,dt$$

$$\int_1^s ds = \int_0^t \left(t^2 - 9t + 10\right)dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5t^2 + 10t$$

$$s = \frac{1}{3}t^3 - 4.5t^2 + 10t + 1$$

Note $v = 0$ at $t^2 - 9t + 10 = 0$

$t = 1.298$ s and $t = 7.702$ s

At $t = 1.298$ s, $s = 7.127$ m

At $t = 7.702$ s, $s = -36.627$ m

At $t = 9$ s, $s = -30.50$ m

a) $s = -30.5$ m **Ans.**

b) $s_{\text{tot}} = (7.127 - 1) + 7.127 + 36.627 + (36.627 - 30.50) = 56.0$ m **Ans.**

c) $v|_{t=9} = (9)^2 - 9(9) + 10 = 10$ m/s **Ans.**

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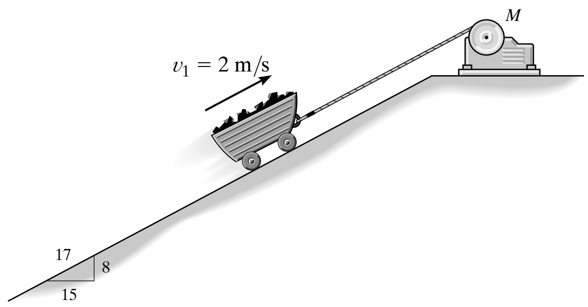
R1–34. The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.



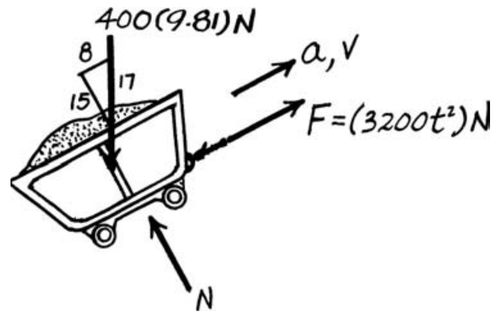
$$+\nearrow \Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$

$$dv = a\,dt$$

$$\int_2^v dv = \int_0^2 \left(8t^2 - 4.616\right)dt$$

$$v = 14.1 \text{ m/s}$$ **Ans.**

Also,



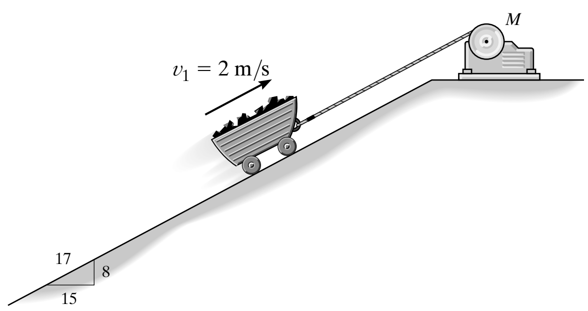
$$m\mathbf{v}_1 + \Sigma \int \mathbf{F}\,dt = m\mathbf{v}_2$$

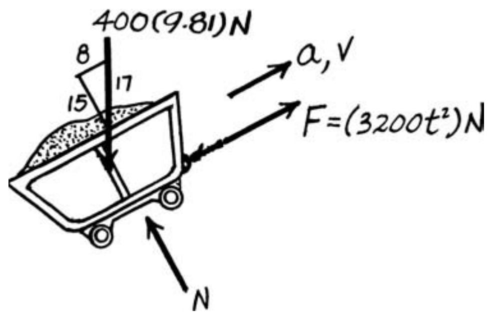
$$+\nearrow \qquad 400(2) + \int_0^2 3200\,t^2\,dt - 400(9.81)(2-0)\left(\frac{8}{17}\right) = 400v_2$$

$$800 + 8533.33 - 3693.18 = 400v_2$$

$$v_2 = 14.1 \text{ m/s}$$

R1–35. The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.





$$\Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$

$$dv = a\,dt$$

$$\int_2^v dv = \int_0^t \left(8t^2 - 4.616\right)dt$$

$$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

$$\int_2^s ds = \int_0^2 \left(2.667t^3 - 4.616t + 2\right)dt$$

$$s = 5.43 \text{ m}$$ **Ans.**

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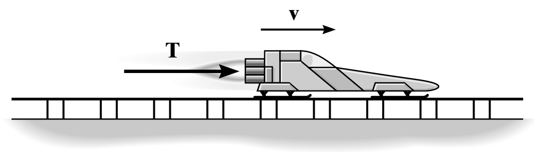
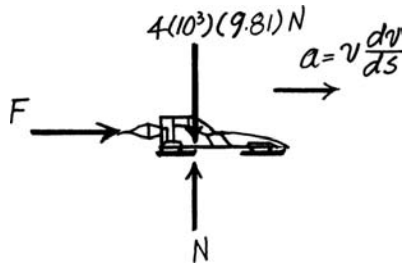
***R1–36.** The rocket sled has a mass of 4 Mg and travels along the smooth horizontal track such that it maintains a constant power output of 450 kW. Neglect the loss of fuel mass and air resistance, and determine how far the sled must travel to reach a speed of $v = 60$ m/s starting from rest.



$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad F = m\,a = m\left(\dfrac{v\,dv}{ds}\right)$



$$P = F\,v = m\left(\frac{v^2\,dv}{ds}\right)$$

$$\int P\,ds = m\int v^2\,dv$$

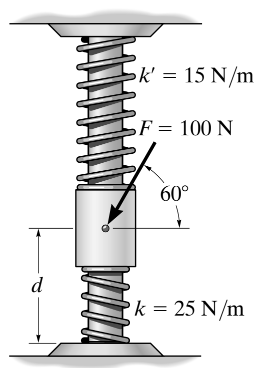
$$P\int_0^s ds = m\int_0^v v^2\,dv$$

$$Ps = \frac{m\,v^3}{3}$$

$$s = \frac{m\,v^3}{3\,P}$$

$$s = \frac{4(10^3)(60)^3}{3(450)(10^3)} = 640 \text{ m}$$ **Ans.**

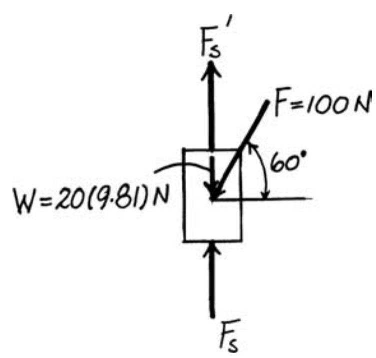
R1–37. The collar has a mass of 20 kg and can slide freely on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 100\sin 60°(0.5 - 0.3) + 20(9.81)(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2 - \frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$

$v_C = 2.36$ m/s **Ans.**



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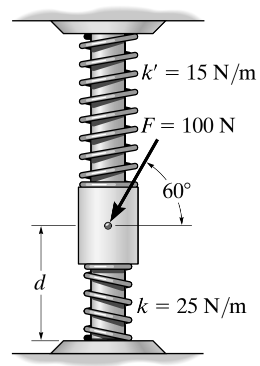
R1–38. The collar has a mass of 20 kg and can slide freely on the smooth rod. The attached springs are both compressed 0.4 m when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.

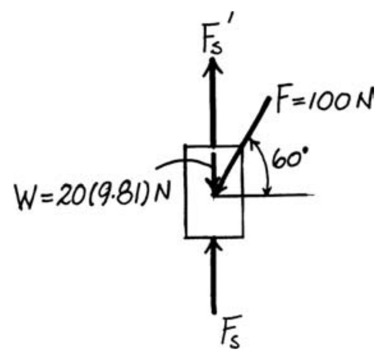


$$T_1 + \Sigma U_{1-2} = T_2$$

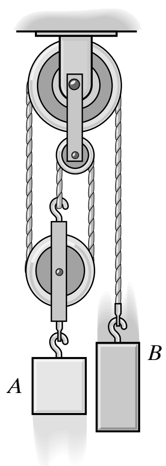
$$0 + 100 \sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - [\frac{1}{2}(25)[0.4 + 0.2]^2 - \frac{1}{2}(25)(0.4)^2]$$

$$- [\frac{1}{2}(15)[0.4 - 0.2]^2 - \frac{1}{2}(15)(0.4)^2] = \frac{1}{2}(20)v_C^2$$

$v_C = 2.34$ m/s **Ans.**



R1–39. The assembly consists of two blocks A and B which have masses of 20 kg and 30 kg, respectively. Determine the speed of each block when B descends 1.5 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



$$3\, s_A + s_B = l$$

$$3\, \Delta s_A = -\Delta s_B$$

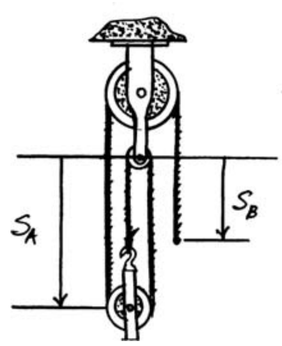
$$3\, v_A = -v_B$$

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(v_A)^2 + \frac{1}{2}(30)(-3v_A)^2 + 20(9.81)\left(\frac{1.5}{3}\right) - 30(9.81)(1.5)$$

$v_A = 1.54$ m/s **Ans.**

$v_B = 4.62$ m/s **Ans.**



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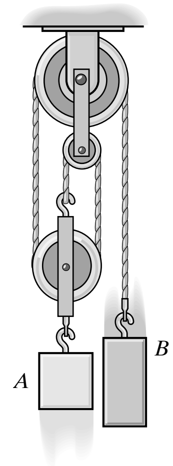
***R1–40.** The assembly consists of two blocks A and B, which have masses of 20 kg and 30 kg, respectively. Determine the distance B must descend in order for A to achieve a speed of 3 m/s starting from rest.



$$3\, s_A + s_B - l$$

$$3\, \Delta s_A = -\Delta s_B$$

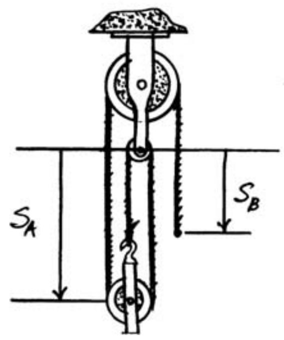
$$3\, v_A = -v_B$$

$$v_B = -9 \text{ m/s}$$

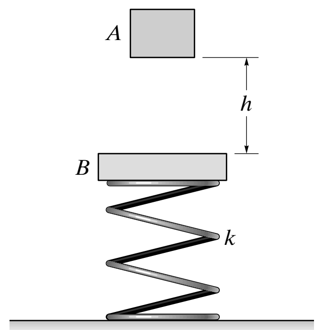
$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(3)^2 + \frac{1}{2}(30)(-9)^2 + 20(9.81)\left(\frac{s_B}{3}\right) - 30(9.81)(s_B)$$

$$s_B = 5.70 \text{ m} \qquad \textbf{Ans.}$$



R1–41. Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2} m v_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$$(+\downarrow) \qquad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad \textbf{(1)}$$

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_B)_2$ yields

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}\,(1 + e) \qquad \textbf{Ans.}$$

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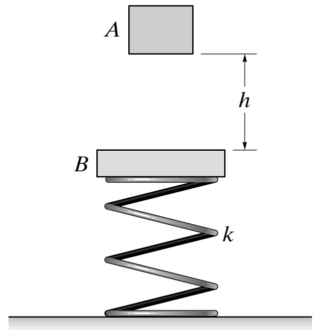
R1–42. Block A, having a mass of 2 kg, is released from rest, falls a distance $h = 0.5$ m, and strikes the plate B having a mass of 3 kg. If the coefficient of restitution between A and B is $e = 0.6$, determine the velocity of the block just after collision. The spring has a stiffness $k = 30$ N/m.



Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(2)(v_A)_2^2 - 2(9.81)(0.5)$$

$$(v_A)_2 = \sqrt{2(9.81)(0.5)} = 3.132 \text{ m/s}$$

$$(+\downarrow) \qquad e = \frac{(v_B)_3 - (v_A)_3}{3.132 - 0}$$

$$0.6(3.132) = (v_B)_3 - (v_A)_3$$

$$1.879 = (v_B)_3 - (v_A)_3 \qquad \textbf{(1)}$$

$$(+\downarrow) \qquad \Sigma m v_2 = \Sigma m v_3$$

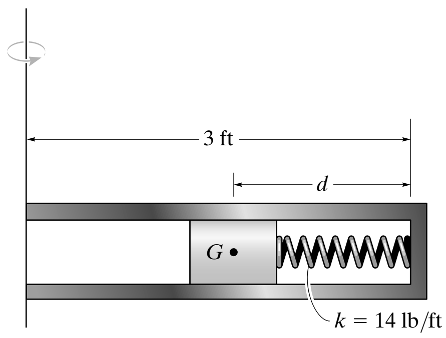
$$2(3.132) + 0 = 2(v_A)_3 + 3(v_B)_3 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_A)_3$ yields

$$(v_B)_3 = 2.00 \text{ m/s}$$

$$(v_A)_3 = 0.125 \text{ m/s} \qquad \textbf{Ans.}$$

R1–43. The cylindrical plug has a weight of 2 lb and it is free to move within the confines of the smooth pipe. The spring has a stiffness $k = 14$ lb/ft and when no motion occurs the distance $d = 0.5$ ft. Determine the force of the spring on the plug when the plug is at rest with respect to the pipe. The plug travels in a circle with a constant speed of 15 ft/s, which is caused by the rotation of the pipe about the vertical axis. Neglect the size of the plug.



$$\xleftarrow{+} \Sigma F_n = m\, a_n; \qquad F_s = \frac{2}{32.2}\left[\frac{(15)^2}{3 - d}\right]$$

$$F_s = ks; \qquad F_s = 14(0.5 - d)$$

Thus,

$$14(0.5 - d) = \frac{2}{32.2}\left[\frac{(15)^2}{3 - d}\right]$$

$$(0.5 - d)(3 - d) = 0.9982$$

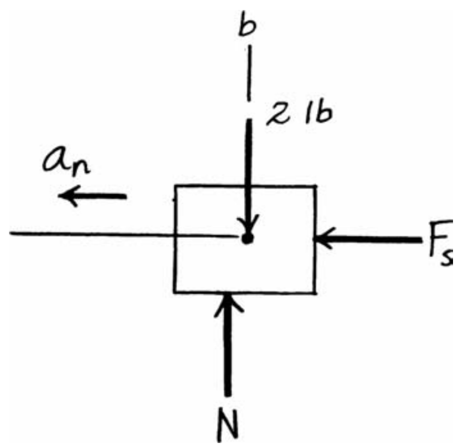
$$1.5 - 3.5d + d^2 = 0.9982$$

$$d^2 - 3.5d + 0.5018 = 0$$

Choosing the root < 0.5 ft

$$d = 0.1498 \text{ ft}$$

$$F_s = 14(0.5 - 0.1498) = 4.90 \text{ lb} \qquad \textbf{Ans.}$$



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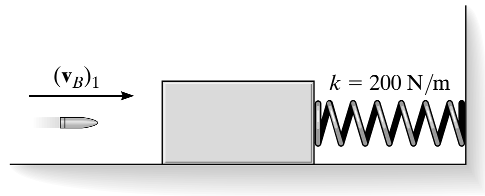
***R1–44.** A 20-g bullet is fired horizontally into the 300-g block which rests on the smooth surface. After the bullet becomes embedded into the block, the block moves to the right 0.3 m before momentarily coming to rest. Determine the speed $(v_B)_1$ of the bullet. The spring has a stiffness $k = 200$ N/m and is originally unstretched.

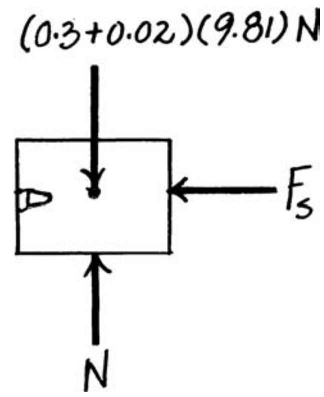


After collision

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(0.320)(v_2)^2 - \frac{1}{2}(200)(0.3)^2 = 0$$

$$v_2 = 7.50 \text{ m/s}$$



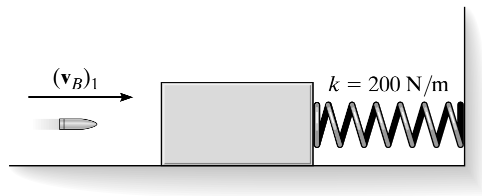
Impact

$$\Sigma m v_1 = \Sigma m v_2$$

$$0.02(v_B)_1 + 0 = 0.320(7.50)$$

$$(v_B)_1 = 120 \text{ m/s} \qquad \textbf{Ans.}$$

R1–45. The 20-g bullet is fired horizontally at $(v_B)_1 = 1200$ m/s into the 300-g block which rests on the smooth surface. Determine the distance the block moves to the right before momentarily coming to rest. The spring has a stiffness $k = 200$ N/m and is originally unstretched.

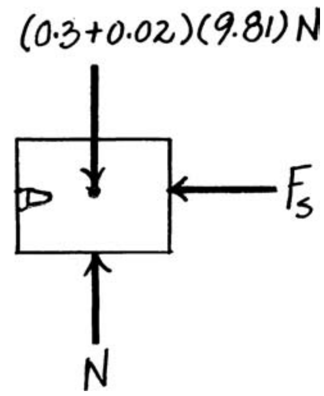


Impact

$$\Sigma m v_1 = \Sigma m v_2$$

$$0.02(1200) + 0 = 0.320(v_2)$$

$$v_2 = 75 \text{ m/s}$$



After collision;

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(0.320)(75)^2 - \frac{1}{2}(200)(x^2) = 0$$

$$x = 3 \text{ m} \qquad \textbf{Ans.}$$

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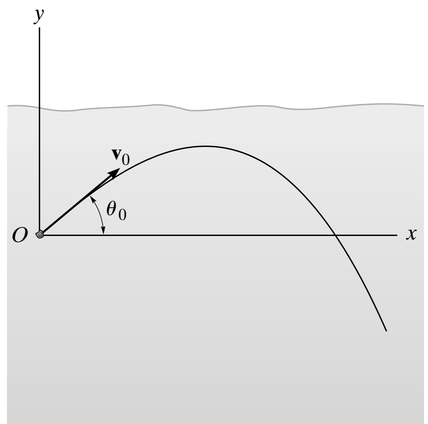
R1–46. A particle of mass m is fired at an angle θ_0 with a velocity $\mathbf{v}_0$ in a liquid that develops a drag resistance $F = -kv$, where k is a constant. Determine the maximum or terminal speed reached by the particle.



Equation of Motion: Applying Eq. 13–7, we have

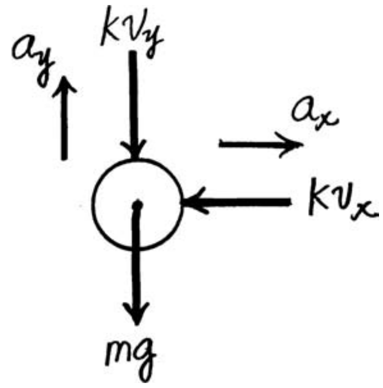
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -kv_x = ma_x \qquad a_x = -\frac{k}{m}v_x \qquad \textbf{[1]}$$

$$+\uparrow \Sigma F_y = ma_y; \qquad -mg - kv_y = ma_y \qquad a_y = -g - \frac{k}{m}v_y \qquad \textbf{[2]}$$

However, $v_x = \frac{dx}{dt}$, $a_x = \frac{d^2x}{dt^2}$, $v_y = \frac{dy}{dt}$ and $a_y = \frac{d^2y}{dt^2}$. Substitute these values into Eqs. [1] and [2], we have

$$\frac{d^2x}{dt^2} + \frac{k}{m}\frac{dx}{dt} = 0 \qquad \textbf{[3]}$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}\frac{dy}{dt} = -g \qquad \textbf{[4]}$$



The solution for the differential equation, Eq. [3], is in the form of

$$x = C_1 e^{-\frac{k}{m}t} + C_2 \qquad \textbf{[5]}$$

Thus,

$$\dot{x} = -\frac{C_1 k}{m} e^{-\frac{k}{m}t} \qquad \textbf{[6]}$$

However, at $t = 0$, $x = 0$ and $\dot{x} = v_0 \cos\theta_0$. Substitute these values into Eqs. [5] and [6], one obtains $C_1 = -\frac{m}{k} v_0 \cos\theta_0$ and $C_2 = \frac{m}{k} v_0 \cos\theta_0$. Substitute C_1 into Eq. [6] and rearrange. This yields

$$\dot{x} = e^{-\frac{k}{m}t}(v_0 \cos\theta_0) \qquad \textbf{[7]}$$

The solution for the differential equation, Eq. [4], is in the form of

$$y = C_3 e^{-\frac{k}{m}t} + C_4 - \frac{mg}{k}t \qquad \textbf{[8]}$$

Thus,

$$\dot{y} = -\frac{C_3 k}{m} e^{-\frac{k}{m}t} - \frac{mg}{k} \qquad \textbf{[9]}$$

However, at $t = 0$, $y = 0$ and $\dot{y} = v_0 \sin\theta_0$. Substitute these values into Eq. [8] and [9], one obtains $C_3 = -\frac{m}{k}\left(v_0 \sin\theta_0 + \frac{mg}{k}\right)$ and $C_4 = \frac{m}{k}\left(v_0 \sin\theta_0 + \frac{mg}{k}\right)$. Substitute C_3 into Eq. [9] and rearrange. This yields

$$\dot{y} = e^{-\frac{k}{m}t}\left(v_0 \sin\theta_0 + \frac{mg}{k}\right) - \frac{mg}{k} \qquad \textbf{[10]}$$

For the particle to achieve terminal speed, $t \to \infty$ and $e^{-\frac{k}{m}t} \to 0$. When this happen, from Eqs. [7] and [10], $v_x = \dot{x} = 0$ and $v_y = \dot{y} = -\frac{mg}{k}$. Thus,

$$v_{max} = \sqrt{v_x^2 + v_y^2} = \sqrt{0^2 + \left(-\frac{mg}{k}\right)^2} = \frac{mg}{k} \qquad \textbf{Ans.}$$

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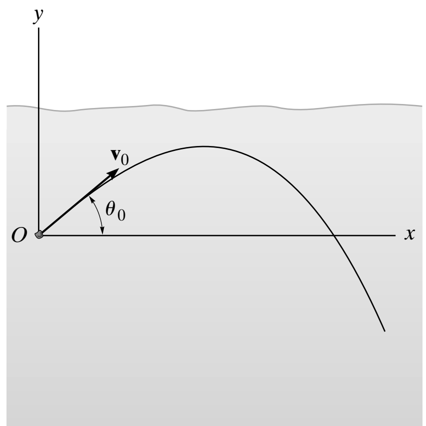
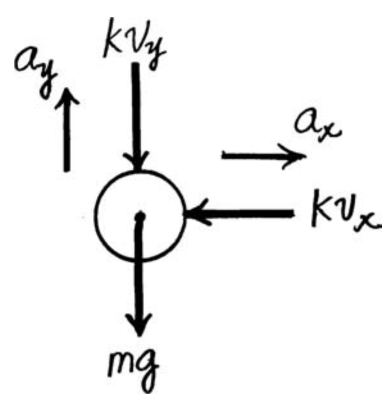
R1–47. A projectile of mass m is fired into a liquid at an angle θ_0 with an initial velocity $\mathbf{v}_0$ as shown. If the liquid develops a friction or drag resistance on the projectile which is proportional to its velocity, i.e., $F = -kv$, where k is a constant, determine the x and y components of its position at any instant. Also, what is the maximum distance x_{max} that it travels?





Equation of Motion: Applying Eq. 13–7, we have

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -kv_x = ma_x \qquad a_x = -\frac{k}{m}v_x \qquad \textbf{[1]}$$

$$+\uparrow \Sigma F_y = ma_y; \qquad -mg - kv_y = ma_y \qquad a_y = -g - \frac{k}{m}v_y \qquad \textbf{[2]}$$

However, $v_x = \frac{dx}{dt}, a_x = \frac{d^2x}{dt^2}, v_y = \frac{dy}{dt}$ and $a_y = \frac{d^2y}{dt^2}$. Substituting these values into Eqs. [1] and [2], we have

$$\frac{d^2x}{dt^2} + \frac{k}{m}\frac{dx}{dt} = 0 \qquad \textbf{[3]}$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}\frac{dy}{dt} = -g \qquad \textbf{[4]}$$

The solution for the differential equation, Eq. [3], is in the form of

$$x = C_1 e^{-\frac{k}{m}t} + C_2 \qquad \textbf{[5]}$$

Thus,

$$\dot{x} = -\frac{C_1 k}{m} e^{-\frac{k}{m}t} \qquad \textbf{[6]}$$

However, at $t = 0$, $x = 0$ and $\dot{x} = v_0 \cos\theta_0$. Substituting these values into Eq. [5] and [6], one can obtain $C_1 = -\frac{m}{k}v_0\cos\theta_0$ and $C_2 = \frac{m}{k}v_0\cos\theta_0$. Substituting C_1 and C_2 into Eq. [5] and rearrange yields

$$x = \frac{m}{k}v_0\cos\theta_0\left(1 - e^{-\frac{k}{m}t}\right) \qquad \textbf{Ans.}$$

When $t \to \infty$, $e^{-\frac{k}{m}t} \to 0$ and $x = x_{max}$. Then,

$$x_{max} = \frac{m}{k}v_0\cos\theta_0 \qquad \textbf{Ans.}$$

The solution for the differential equation. Eq. [4], is in the form of

$$y = C_3 e^{-\frac{k}{m}t} + C_4 - \frac{mg}{k}t \qquad \textbf{[7]}$$

Thus,

$$\dot{y} = -\frac{C_3 k}{m} e^{-\frac{k}{m}t} - \frac{mg}{k} \qquad \textbf{[8]}$$

However, at $t = 0$, $y = 0$ and $\dot{y} = v_0 \sin\theta_0$. Substitute these values into Eq. [7] and [8], one can obtain $C_3 = -\frac{m}{k}\left(v_0\sin\theta_0 + \frac{mg}{k}\right)$ and $C_4 = \frac{m}{k}\left(v_0\sin\theta_0 + \frac{mg}{k}\right)$. Substitute C_3 and C_4 into Eq. [7] and rearrange yields

$$y = \frac{m}{k}\left(v_0\sin\theta_0 + \frac{mg}{k}\right)\left(1 - e^{-\frac{k}{m}t}\right) - \frac{mg}{k}t \qquad \textbf{Ans.}$$

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***R1–48.** The position of particles A and B are $\mathbf{r}_A = \{3t\mathbf{i} + 9t(2 - t)\mathbf{j}\}$ m and $\mathbf{r}_B = \{3(t^2 - 2t + 2)\mathbf{i} + 3(t - 2)\mathbf{j}\}$ m, respectively, where t is in seconds. Determine the point where the particles collide and their speeds just before the collision. How long does it take before the collision occurs?

When collision occurs, $\mathbf{r}_A = \mathbf{r}_B$.

$$3t = 3(t^2 - 2t + 2)$$

$$t^2 - 3t + 2 = 0$$

$$t = 1\text{ s}, \qquad t = 2\text{ s}$$

Also,

$$9t(2 - t) = 3(t - 2)$$

$$3t^2 - 5t - 2 = 0$$

The positive root is $t = 2$ s

Thus,

$$t = 2\text{ s} \qquad \textbf{Ans.}$$

$$x = 3(2) = 6\text{ m} \qquad y = 9(2)(2 - 2) = 0$$

Hence, (6 m, 0) **Ans.**

$$\mathbf{v}_A = \frac{d\mathbf{r}_A}{dt} = 3\mathbf{i} + (18 - 18t)\mathbf{j}$$

$$\mathbf{v}_A\big|_{t=2} = \{3\mathbf{i} - 18\mathbf{j}\}\text{ m/s}$$

$$v_A = \sqrt{(3)^2 + (-18)^2} = 18.2\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \frac{d\mathbf{r}_B}{dt} = 3(2t - 2)\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{v}_B\big|_{t=2} = \{6\mathbf{i} + 3\mathbf{j}\}\text{ m/s}$$

$$v_B = \sqrt{(6)^2 + (3)^2} = 6.71\text{ m/s} \qquad \textbf{Ans.}$$

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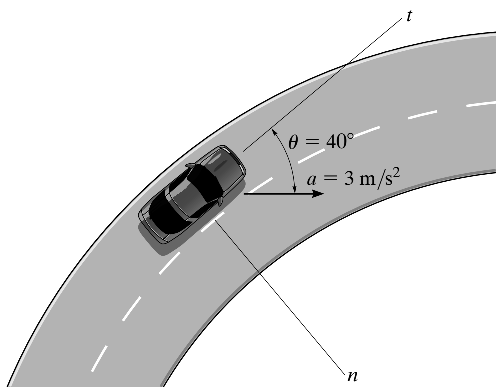
R1–49. Determine the speed of the automobile if it has the acceleration shown and is traveling on a road which has a radius of curvature of $\rho = 50$ m. Also, what is the automobile's rate of increase in speed?



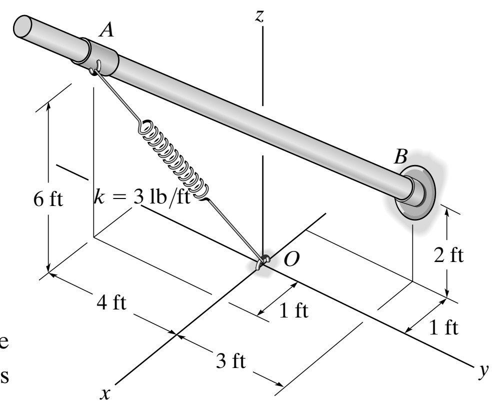
$$a_n = \frac{v^2}{\rho}$$

$$3 \sin 40° = \frac{v^2}{50}$$

$v = 9.82$ m/s **Ans.**

$a_t = 3 \cos 40° = 2.30$ m/s^2 **Ans.**

R1–50. The spring has a stiffness $k = 3$ lb/ft and an unstretched length of 2 ft. If it is attached to the 5-lb smooth collar and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.



Potential Energy: Datum is set at point B. The collar is $(6 - 2) = 4$ ft *above* the datum when it is at A. Thus, its gravitational potential energy at this point is $5(4) = 20.0$ ft · lb. The length of the spring when the collar is at points A and B are calculated as $l_{OA} = \sqrt{1^2 + 4^2 + 6^2} = \sqrt{53}$ ft and $l_{OB} = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{14}$ ft, respectively. The initial and final elastic potential energy are $\frac{1}{2}(3)\left(\sqrt{53} - 2\right)^2 = 41.82$ ft · lb and $\frac{1}{2}(3)\left(\sqrt{14} - 2\right)^2 = 4.550$ ft · lb, respectively.

Conservation of Energy: Applying Eq. 14–22, we have

$$\Sigma T_A + \Sigma V_A = \Sigma T_B + \Sigma V_B$$

$$0 + 20.0 + 41.82 = \frac{1}{2}\left(\frac{5}{32.2}\right)v_B^2 + 4.550$$

$$v_B = 27.2 \text{ ft/s}$$ **Ans.**

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•16–1. A disk having a radius of 0.5 ft rotates with an initial angular velocity of 2 rad/s and has a constant angular acceleration of 1 rad/s^2. Determine the magnitudes of the velocity and acceleration of a point on the rim of the disk when $t = 2$ s.

$\omega = \omega_0 + \alpha_c t;$

$\omega = 2 + 1(2) = 4$ rad/s

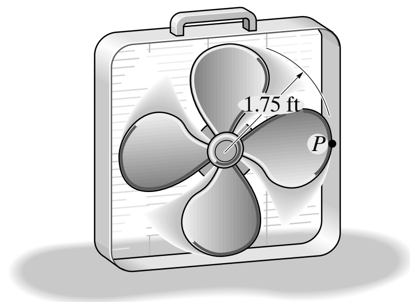
$v = r\omega; \qquad v = 0.5(4) = 2$ ft/s **Ans.**

$a_t = r\alpha; \qquad a_t = 0.5(1) = 0.5$ ft/s^2

$a_n = \omega^2 r; \qquad a_n = (4)^2(0.5) = 8$ ft/s^2

$a = \sqrt{8^2 + (0.5)^2} = 8.02$ ft/s^2 **Ans.**

16–2. Just after the fan is turned on, the motor gives the blade an angular acceleration $\alpha = (20e^{-0.6t})$ rad/s^2, where t is in seconds. Determine the speed of the tip P of one of the blades when $t = 3$ s. How many revolutions has the blade turned in 3 s? When $t = 0$ the blade is at rest.



$d\omega = \alpha\, dt$

$$\int_0^{\omega} d\omega = \int_0^t 20e^{-0.6t}\, dt$$

$$\omega = -\frac{20}{0.6} e^{-0.6t}\Big|_0^t = 33.3\left(1 - e^{-0.6t}\right)$$

$\omega = 27.82$ rad/s when $t = 3$s

$v_p = \omega r = 27.82(1.75) = 48.7$ ft/s **Ans.**

$d\theta = \omega\, dt$

$$\int_0^{\theta} d\theta = \int_0^t 33.3\left(1 - e^{-0.6t}\right) dt$$

$$\theta = 33.3\left(t + \left(\frac{1}{0.6}\right)e^{-0.6t}\right)\Big|_0^3 = 33.3\left[3 + \left(\frac{1}{0.6}\right)\left(e^{-0.6(3)} - 1\right)\right]$$

$\theta = 53.63$ rad $= 8.54$ rev **Ans.**

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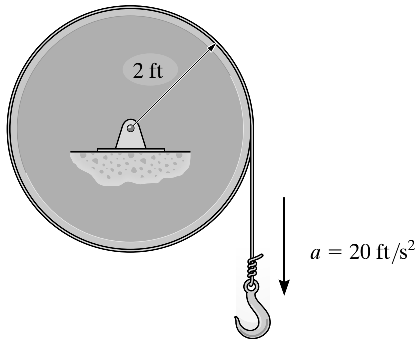
16–3. The hook is attached to a cord which is wound around the drum. If it moves from rest with an acceleration of 20 ft/s^2, determine the angular acceleration of the drum and its angular velocity after the drum has completed 10 rev. How many more revolutions will the drum turn after it has first completed 10 rev and the hook continues to move downward for 4 s?



Angular Motion: The angular acceleration of the drum can be determine by applying Eq. 16–11.

$$a_t = \alpha r; \qquad 20 = \alpha(2) \qquad \alpha = 10.0 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Applying Eq. 16–7 with $\alpha_c = \alpha = 10.0 \text{ rad/s}^2$ and $\theta = (10 \text{ rev}) \times \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$ $= 20\pi$ rad, we have

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = 0 + 2(10.0)(20\pi - 0)$$

$$\omega = 35.45 \text{ rad/s} = 35.4 \text{ rad/s} \qquad \textbf{Ans.}$$

The angular displacement of the drum 4 s after it has completed 10 revolutions can be determined by applying Eq. 16–6 with $\omega_0 = 35.45$ rad/s.

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$= 0 + 35.45(4) + \frac{1}{2}(10.0)\left(4^2\right)$$

$$= (221.79 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 35.3\text{rev} \qquad \textbf{Ans.}$$

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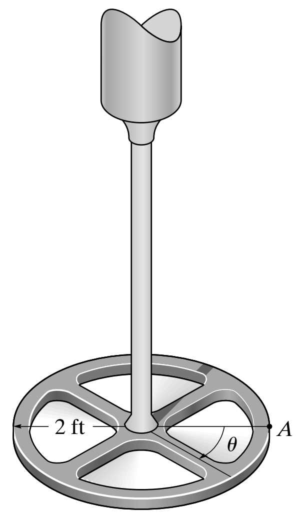
***16–4.** The torsional pendulum (wheel) undergoes oscillations in the horizontal plane, such that the angle of rotation, measured from the equilibrium position, is given by $\theta = (0.5 \sin 3t)$ rad, where t is in seconds. Determine the maximum velocity of point A located at the periphery of the wheel while the pendulum is oscillating. What is the acceleration of point A in terms of t?



Angular Velocity: Here, $\theta = (0.5 \sin 3t)$ rad/s. Applying Eq. 16–1, we have

$$\omega = \frac{d\theta}{dt} = (1.5 \cos 3t) \text{ rad/s}$$

By observing the above equation, the angular velocity is maximum if $\cos 3t = 1$. Thus, the maximum angular velocity is $\omega_{max} = 1.50$ rad/s. The maximum speed of point A can be obtained by applying Eq. 16–8.

$$(v_A)_{max} = \omega_{max}\, r = 1.50(2) = 3.00 \text{ ft/s} \qquad \textbf{Ans.}$$

Angular Acceleration: Applying Eq. 16–2, we have

$$\alpha = \frac{d\omega}{dt} = (-4.5 \sin 3t) \text{ rad/s}^2$$

The tangential and normal components of the acceleration of point A can be determined using Eqs. 16–11 and 16–12, respectively.

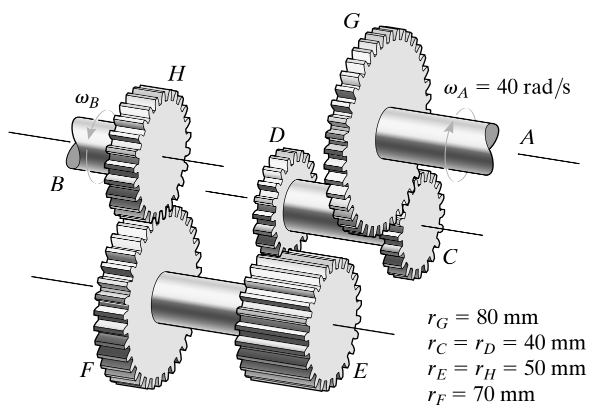
$$a_t = \alpha r = (-4.5 \sin 3t)(2) = (-9 \sin 3t) \text{ ft/s}^2$$

$$a_n = \omega^2 r = (1.5 \cos 3t)^2 (2) = \left(4.5 \cos^2 3t\right) \text{ ft/s}^2$$

Thus,

$$\mathbf{a}_A = \left(-9 \sin 3t\mathbf{u}_t + 4.5 \cos^2 3t\mathbf{u}_n\right) \text{ ft/s}^2 \qquad \textbf{Ans.}$$

•16–5. The operation of reverse gear in an automotive transmission is shown. If the engine turns shaft A at $\omega_A = 40$ rad/s, determine the angular velocity of the drive shaft, ω_B. The radius of each gear is listed in the figure.



$r_A \omega_A = r_C \omega_C$: $\quad 80(40) = 40\omega_C \quad \omega_C = \omega_D = 80$ rad/s

$\omega_E r_E = \omega_D r_D$: $\quad \omega_E(50) = 80(40) \quad \omega_E = \omega_F = 64$ rad/s

$\omega_F r_F = \omega_B r_B$: $\quad 64(70) = \omega_B (50) \quad \omega_B = 89.6$ rad/s

$\omega_B = 89.6$ rad/s **Ans.**

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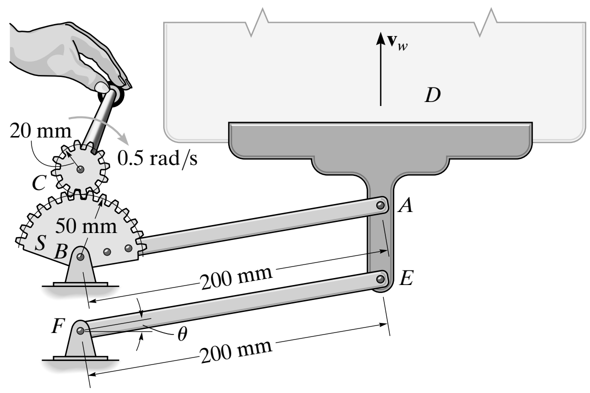
16–6. The mechanism for a car window winder is shown in the figure. Here the handle turns the small cog C, which rotates the spur gear S, thereby rotating the fixed-connected lever AB which raises track D in which the window rests. The window is free to slide on the track. If the handle is wound at 0.5 rad/s, determine the speed of points A and E and the speed v_w of the window at the instant $\theta = 30°$.



$$v_C = \omega_C\, r_C = 0.5(0.02) = 0.01 \text{ m/s}$$

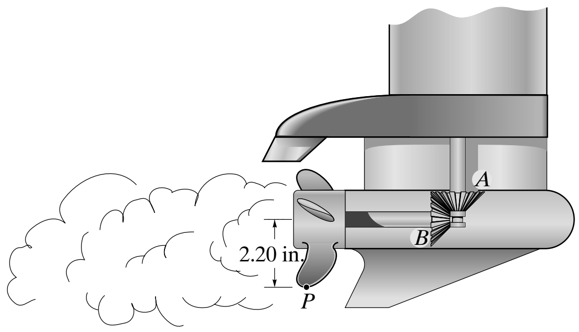
$$\omega_S = \frac{v_C}{r_S} = \frac{0.01}{0.05} = 0.2 \text{ rad/s}$$

$$v_A = v_E = \omega_S\, r_A = 0.2(0.2) = 0.04 \text{ m/s} = 40 \text{ mm/s} \qquad \textbf{Ans.}$$

Points A and E move along circular paths. The vertical component closes the window.

$$v_w = 40 \cos 30° = 34.6 \text{ mm/s} \qquad \textbf{Ans.}$$

16–7. The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.5$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.2$ in. Determine the angular velocity of the propeller in $t = 1.5$ s, if the drive shaft rotates with an angular acceleration $\alpha = (400t^3)$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



Angular Motion: The angular velocity of gear A at $t = 1.5$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{1.5\,s} 400t^3\, dt$$

$$\omega_A = 100t^4\Big|_0^{1.5\,s} = 506.25 \text{ rad/s}$$

However, $\omega_A\, r_A = \omega_B\, r_B$ where ω_B is the angular velocity of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\,\omega_A = \left(\frac{0.5}{1.2}\right)(506.25) = 211 \text{ rad/s} \qquad \textbf{Ans.}$$

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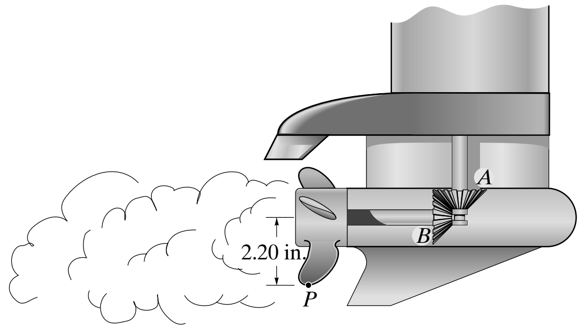
***16–8.** For the outboard motor in Prob. 16–7, determine the magnitude of the velocity and acceleration of point P located on the tip of the propeller at the instant $t = 0.75$ s.



Angular Motion: The angular velocity of gear A at $t = 0.75$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{0.75\,s} 400t^3\,dt$$

$$\omega_A = 100t^4\Big|_0^{0.75\,s} = 31.64 \text{ rad/s}$$

The angular acceleration of gear A at $t = 0.75$ s is given by

$$\alpha_A = 400\left(0.75^3\right) = 168.75 \text{ rad/s}^2$$

However, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$ where ω_B and α_B are the angular velocity and acceleration of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{0.5}{1.2}\right)(31.64) = 13.18 \text{ rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{0.5}{1.2}\right)(168.75) = 70.31 \text{ rad/s}^2$$

Motion of *P*: The magnitude of the velocity of point P can be determined using Eq. 16–8.

$$v_P = \omega_B r_P = 13.18\left(\frac{2.20}{12}\right) = 2.42 \text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point P can be determined using Eqs. 16–11 and 16–12, respectively.

$$a_r = \alpha_B r_P = 70.31\left(\frac{2.20}{12}\right) = 12.89 \text{ ft/s}^2$$

$$a_n = \omega_B^2 r_P = \left(13.18^2\right)\left(\frac{2.20}{12}\right) = 31.86 \text{ ft/s}^2$$

The magnitude of the acceleration of point P is

$$a_P = \sqrt{a_r^2 + a_n^2} = \sqrt{12.89^2 + 31.86^2} = 34.4 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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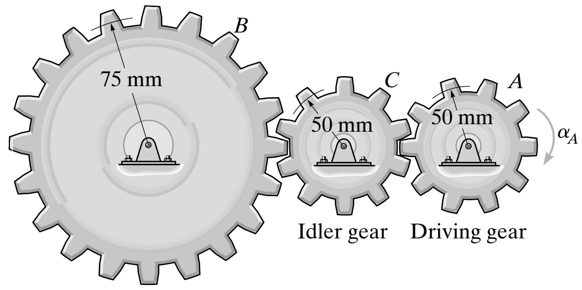
•16–9. When only two gears are in mesh, the driving gear A and the driven gear B will always turn in opposite directions. In order to get them to turn in the *same direction* an idler gear C is used. In the case shown, determine the angular velocity of gear B when $t = 5$ s, if gear A starts from rest and has an angular acceleration of $\alpha_A = (3t + 2)$ rad/s^2, where t is in seconds.



$d\omega = \alpha\, dt$

$$\int_0^{\omega_A} d\omega_A = \int_0^t (3t + 2)\, dt$$

$\omega_A = 1.5t^2 + 2t|_{t=5} = 47.5$ rad/s

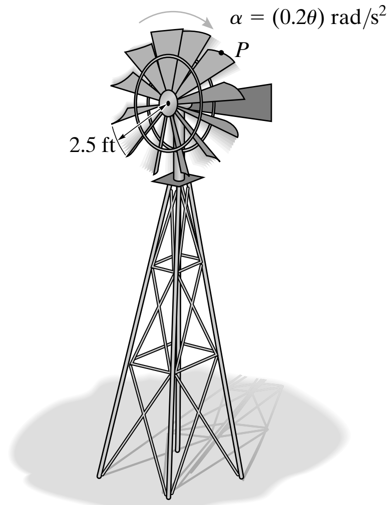
$(47.5)(50) = \omega_C\,(50)$

$\omega_C = 47.5$ rad/s

$\omega_B\,(75) = 47.5(50)$

$\omega_B = 31.7$ rad/s **Ans.**

16–10. During a gust of wind, the blades of the windmill are given an angular acceleration of $\alpha = (0.2\theta)$ rad/s^2, where θ is in radians. If initially the blades have an angular velocity of 5 rad/s, determine the speed of point P, located at the tip of one of the blades, just after the blade has turned two revolutions.



Angular Motion: The angular velocity of the blade can be obtained by applying Eq. 16–4.

$$\omega d\omega = \alpha d\theta$$

$$\int_{5\text{ rad/s}}^{\omega} \omega d\omega = \int_0^{4\pi} 0.2\theta d\theta$$

$$\omega = 7.522 \text{ rad/s}$$

Motion of P: The speed of point P can be determined using Eq. 16–8.

$v_P = \omega r_P = 7.522(2.5) = 18.8$ ft/s **Ans.**

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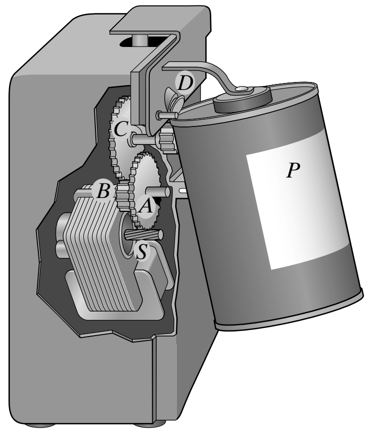
16–11. The can opener operates such that the can is driven by the drive wheel D. If the armature shaft S on the motor turns with a constant angular velocity of 40 rad/s, determine the angular velocity of the can. The radii of S, can P, drive wheel D, gears A, B, and C, are $r_S = 5$ mm, $r_P = 40$ mm, $r_D = 7.5$ mm, $r_A = 20$ mm, $r_B = 10$ mm, and $r_C = 25$ mm, respectively.



Gears A and B will have the same angular velocity since they are mounted on the same axle. Thus,

$$\omega_A r_A = \omega_s r_s$$

$$\omega_B = \omega_A = \left(\frac{r_s}{r_A}\right)\omega_s = \left(\frac{5}{20}\right)(40) = 10 \text{ rad/s}$$

Wheel D is mounted on the same axle as gear C, which in turn is in mesh with gear B.

$$\omega_C r_C = \omega_B r_B$$

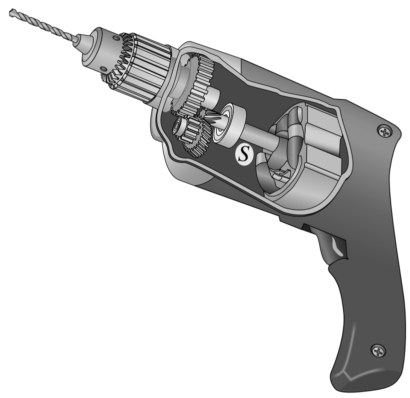
$$\omega_D = \omega_C = \left(\frac{r_B}{r_C}\right)\omega_B = \left(\frac{10}{25}\right)(10) = 4 \text{ rads/s}$$

Finally, the rim of can P is in mesh with wheel D.

$$\omega_P r_P = \omega_D r_D$$

$$\omega_P = \left(\frac{r_D}{r_P}\right)\omega_D = \left(\frac{7.5}{40}\right)(4) = 0.75 \text{ rad/s} \qquad \textbf{Ans.}$$

***16–12.** If the motor of the electric drill turns the armature shaft S with a constant angular acceleration of $\alpha_S = 30 \text{ rad/s}^2$, determine the angular velocity of the shaft after it has turned 200 rev, starting from rest.



Motion of Pulley A: Here, $\theta_s = (200 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 400\pi$ rad. Since the angular acceleration of shaft s is constant, its angular velocity can be determined from

$$\omega_s^{\,2} = (\omega_s)_0^{\,2} + 2\alpha_C\left[\theta_s - (\theta_s)_0\right]$$

$$\omega_s^{\,2} = 0^2 + 2(30)(400\pi - 0)$$

$$\omega_s = 274.6 \text{ rad/s} \qquad \textbf{Ans.}$$

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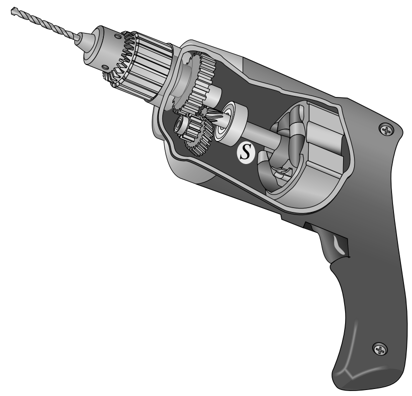
•16–13. If the motor of the electric drill turns the armature shaft S with an angular velocity of $\omega_S = (100t^{1/2})$ rad/s, determine the angular velocity and angular acceleration of the shaft at the instant it has turned 200 rev, starting from rest.



Motion of Armature Shaft *S*: Here, $\theta_s = (200\text{ rev})\left(\frac{2\pi\text{rad}}{1\text{ rev}}\right) = 400\pi$. The angular velocity of A can be determined from

$$\int d\theta_s = \int \omega_s dt$$

$$\int_0^{\theta_s} \theta_s = \int_0^t 100t^{1/2}dt$$

$$\theta_s\Big|_0^{\theta_s} = 66.67t^{3/2}\Big|_0^t$$

$$\theta_s = \left(66.67t^{3/2}\right)\text{rad}$$

When $\theta_s = 400\pi$ rad,

$$400\pi = 66.67t^{3/2}$$

$$t = 7.083\text{ s}$$

Thus, the angular velocity of the shaft after it turns 200 rev ($t = 7.083$ s) is

$$\omega_s = 100(7.083)^{1/2} = 266\text{ rad/s} \qquad \textbf{Ans.}$$

The angular acceleration of the shaft is

$$\alpha_s = \frac{d\omega_s}{dt} = 100\left(\frac{1}{2}t^{-1/2}\right) = \left(\frac{50}{t^{1/2}}\right)\text{rad/s}^2$$

When $t = 7.083$ s,

$$\alpha_s = \frac{50}{7.083^{1/2}} = 18.8\text{ rad/s}^2 \qquad \textbf{Ans.}$$

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16–14. A disk having a radius of 6 in. rotates about a fixed axis with an angular velocity of $\omega = (2t + 3)$ rad/s, where t is in seconds. Determine the tangential and normal components of acceleration of a point located on the rim of the disk at the instant the angular displacement is $\theta = 40$ rad.

Motion of the Disk: We have

$$\int d\theta = \int \omega dt$$

$$\int_0^\theta d\theta = \int_0^t (2t + 3)dt$$

$$\theta\Big|_0^\theta = \left(t^2 + 3t\right)\Big|_0^t$$

$$\theta = \left(t^2 + 3t\right) \text{ rad}$$

When $\theta = 40$ rad,

$$40 = t^2 + 3t$$

$$t^2 + 3t - 40 = 0$$

Solving for the positive root,

$$t = 5 \text{ s}$$

Also,

$$\alpha = \frac{d\omega}{dt} = 2 \text{ rad/s}^2$$

When $t = 5$ s($\theta = 40$ rad),

$$\omega = 2(5) + 3 = 13 \text{ rad/s}$$

Motion of Point *P*: Using the result for ω and α, the tangential and normal components of the acceleration of point P are

$$a_t = \alpha r_p = 2\left(\frac{6}{12}\right) = 1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_n = \omega^2 r_p = (13)^2\left(\frac{6}{12}\right) = 84.5 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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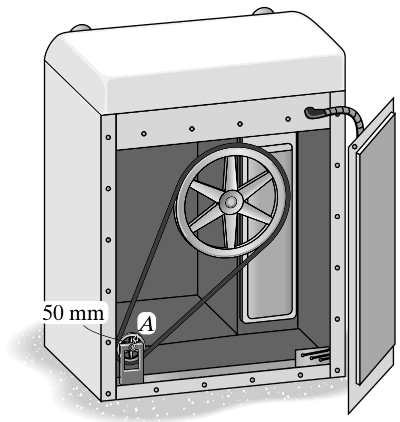
16–15. The 50-mm-radius pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (27\theta_A^{1/2})$ rad/s^2, where θ_A is in radians. Determine its angular acceleration when $t = 1$ s, starting from rest.



Motion of Pulley *A*: The angular velocity of pulley A can be determined from

$$\int \omega_A\, d\omega_A = \int \alpha_A\, d\theta_A$$

$$\int_0^{\omega_A} \omega_A d\omega_A = \int_0^{\theta_A} 27\theta_A{}^{1/2} d\theta_A$$

$$\left.\frac{\omega_A{}^2}{2}\right|_0^{\omega_A} = 18\theta_A{}^{3/2}\Big|_0^{\theta_A}$$

$$\omega_A = \left(6\theta_A{}^{3/4}\right) \text{ rad/s}$$

Using this result, the angular displacement of A as a function of t can be determined from

$$\int dt = \int \frac{d\theta_A}{\omega_A}$$

$$\int_0^t dt = \int_0^{\theta_A} \frac{d\theta_A}{6\theta_A{}^{3/4}}$$

$$t\Big|_0^t = \left.\frac{2}{3}\theta_A{}^{1/4}\right|_0^{\theta_A}$$

$$t = \left(\frac{2}{3}\theta_A{}^{1/4}\right) \text{s}$$

$$\theta_A = \left(\frac{3}{2}t\right)^4 \text{ rad}$$

When $t = 1$ s

$$\theta_A = \left[\frac{3}{2}(1)\right]^4 = 5.0625 \text{ rad}$$

Thus, when $t = 1$ s, α_A is

$$\alpha_A = 27\left(5.0625^{1/2}\right) = 60.8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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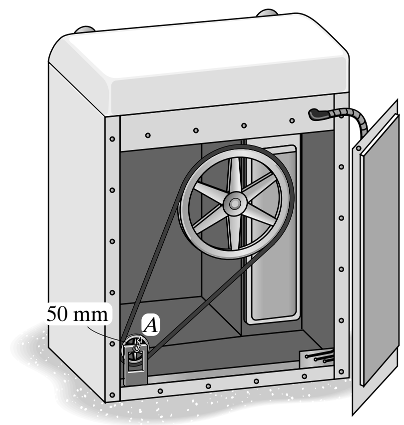
***16–16.** If the 50-mm-radius motor pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (10 + 50t)\ \text{rad/s}^2$, where t is in seconds, determine its angular velocity when $t = 3$ s, starting from rest.



Motion of Pulley A: The angular velocity of pulley A can be determined from

$$\int d\omega_A = \int \alpha_A dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t (10 + 50t)dt$$

$$\omega_A\Big|_0^{\omega_A} = \left(10t + 25t^2\right)\Big|_0^t$$

$$\omega_A = \left(10t + 25t^2\right)\text{rad/s}$$

When $t = 3$ s

$$\omega_A = 10(3) + 25\left(3^2\right) = 225\ \text{rad/s} \qquad \textbf{Ans.}$$

•16–17. The vacuum cleaner's armature shaft S rotates with an angular acceleration of $\alpha = 4\omega^{3/4}\ \text{rad/s}^2$, where ω is in rad/s. Determine the brush's angular velocity when $t = 4$ s, starting from rest. The radii of the shaft and the brush are 0.25 in. and 1 in., respectively. Neglect the thickness of the drive belt.

Motion of the Shaft: The angular velocity of the shaft can be determined from

$$\int dt = \int \frac{d\omega_S}{\alpha_S}$$

$$\int_0^t dt = \int_0^{\omega_s} \frac{d\omega_S}{4\omega_S{}^{3/4}}$$

$$t\Big|_0^t = \omega_S{}^{1/4}\Big|_0^{\omega_s}$$

$$t = \omega_S{}^{1/4}$$

$$\omega_S = \left(t^4\right)\ \text{rad/s}$$

When $t = 4$ s

$$\omega_s = 4^4 = 256\ \text{rad/s}$$

Motion of the Beater Brush: Since the brush is connected to the shaft by a non-slip belt, then

$$\omega_B r_B = \omega_s r_s$$

$$\omega_B = \left(\frac{r_s}{r_B}\right)\omega_s = \left(\frac{0.25}{1}\right)(256) = 64\ \text{rad/s} \qquad \textbf{Ans.}$$

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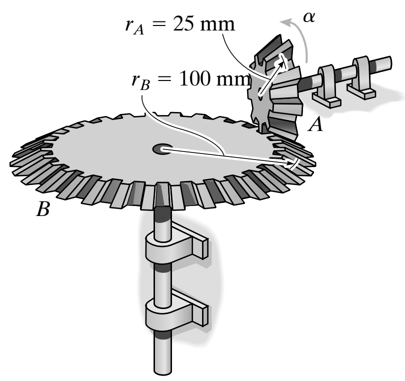
16–18. Gear A is in mesh with gear B as shown. If A starts from rest and has a constant angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$, determine the time needed for B to attain an angular velocity of $\omega_B = 50\ \text{rad/s}$.



Angular Motion: The angular acceleration of gear B must be determined first. Here, $\alpha_A r_A = \alpha_B r_B$. Then,

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{25}{100}\right)(2) = 0.5\ \text{rad/s}^2$$

The time for gear B to attain an angular velocity of $\omega_B = 50\ \text{rad/s}$ can be obtained by applying Eq. 16–5.

$$\omega_B = (\omega_0)_B + \alpha_B t$$

$$50 = 0 + 0.5t$$

$$t = 100\ \text{s} \qquad \textbf{Ans.}$$

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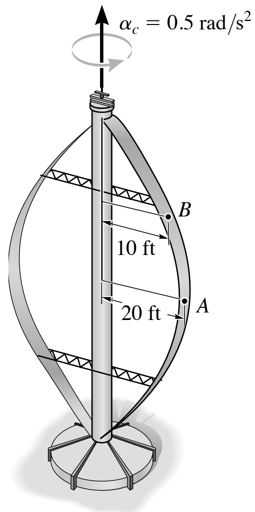
16–19. The vertical-axis windmill consists of two blades that have a parabolic shape. If the blades are originally at rest and begin to turn with a constant angular acceleration of $\alpha_c = 0.5\ \text{rad/s}^2$, determine the magnitude of the velocity and acceleration of points A and B on the blade after the blade has rotated through two revolutions.



Angular Motion: The angular velocity of the blade after the blade has rotated $2(2\pi) = 4\pi$ rad can be obtained by applying Eq. 16–7.

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = 0^2 + 2(0.5)(4\pi - 0)$$

$$\omega = 3.545\ \text{rad/s}$$

Motion of A and B: The magnitude of the velocity of point A and B on the blade can be determined using Eq. 16–8.

$$v_A = \omega r_A = 3.545(20) = 70.9\ \text{ft/s} \qquad \textbf{Ans.}$$

$$\omega_B = \omega r_B = 3.545(10) = 35.4\ \text{ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point A and B can be determined using Eqs. 16–11 and 16–12 respectively.

$$(a_t)_A = \alpha r_A = 0.5(20) = 10.0\ \text{ft/s}^2$$

$$(a_n)_A = \omega^2\, r_A = \left(3.545^2\right)(20) = 251.33\ \text{ft/s}^2$$

$$(a_t)_B = \alpha r_B = 0.5(10) = 5.00\ \text{ft/s}^2$$

$$(a_n)_B = \omega^2\, r_B = \left(3.545^2\right)(10) = 125.66\ \text{ft/s}^2$$

The magnitude of the acceleration of points A and B are

$$(a)_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{10.0^2 + 251.33^2} = 252\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$(a)_B = \ = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{5.00^2 + 125.66^2} = 126\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

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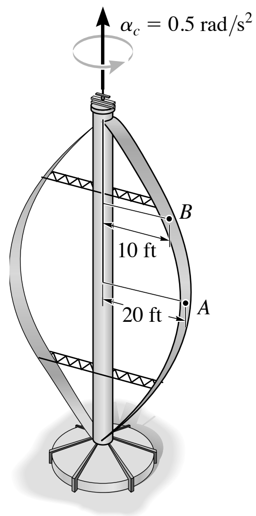
***16–20.** The vertical-axis windmill consists of two blades that have a parabolic shape. If the blades are originally at rest and begin to turn with a constant angular acceleration of $\alpha_c = 0.5\ \text{rad/s}^2$, determine the magnitude of the velocity and acceleration of points A and B on the blade when $t = 4$ s.



Angular Motion: The angular velocity of the blade at $t = 4$ s can be obtained by applying Eq. 16–5.

$$\omega = \omega_0 + \alpha_c t = 0 + 0.5(4) = 2.00\ \text{rad/s}$$

Motion of A and B: The magnitude of the velocity of points A and B on the blade can be determined using Eq. 16–8.

$$v_A = \omega r_A = 2.00(20) = 40.0\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_B = \omega r_B = 2.00(10) = 20.0\ \text{ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of points A and B can be determined using Eqs. 16–11 and 16–12 respectively.

$$(a_t)_A = \alpha r_A = 0.5(20) = 10.0\ \text{ft/s}^2$$

$$(a_n)_A = \omega^2 r_A = \left(2.00^2\right)(20) = 80.0\ \text{ft/s}^2$$

$$(a_t)_B = \alpha r_B = 0.5(10) = 5.00\ \text{ft/s}^2$$

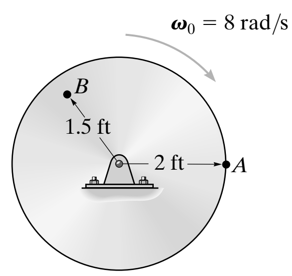
$$(a_n)_B = \omega^2 r_B = \left(2.00^2\right)(10) = 40.0\ \text{ft/s}^2$$

The magnitude of the acceleration of points A and B are

$$(a)_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{10.0^2 + 80.0^2} = 80.6\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$(a)_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{5.00^2 + 40.0^2} = 40.3\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

16.21. The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6\ \text{rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 0.5$ s.



$$\omega = \omega_0 + \alpha_c t$$

$$\omega = 8 + 6(0.5) = 11\ \text{rad/s}$$

$v = r\omega; \qquad v_A = 2(11) = 22\ \text{ft/s}$ **Ans.**

$a_t = r\alpha; \qquad (a_A)_t = 2(6) = 12.0\ \text{ft/s}^2$ **Ans.**

$a_n = \omega^2 r; \qquad (a_A)_n = (11)^2(2) = 242\ \text{ft/s}^2$ **Ans.**

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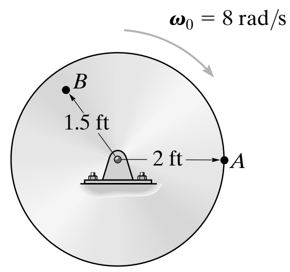
16–22. The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point B just after the wheel undergoes 2 revolutions.



$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = (8)^2 + 2(6)[2(2\pi) - 0]$$

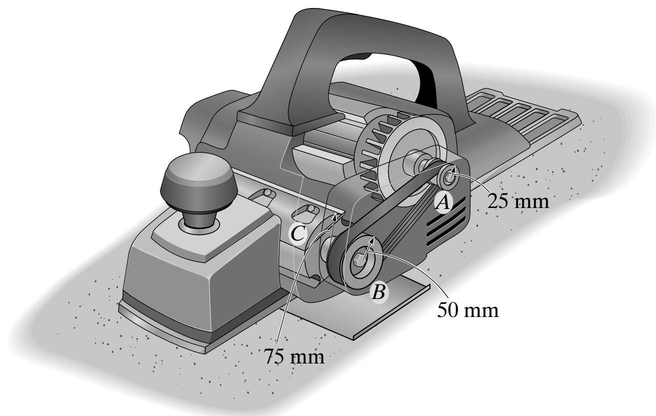
$$\omega = 14.66 \text{ rad/s}$$

$$v_B = \omega r = 14.66(1.5) = 22.0 \text{ ft/s}$$ **Ans.**

$$(a_B)_t = \alpha r = 6(1.5) = 9.00 \text{ ft/s}^2$$ **Ans.**

$$(a_B)_n = \omega^2 r = (14.66)^2(1.5) = 322 \text{ ft/s}^2$$ **Ans.**

16–23. The blade C of the power plane is driven by pulley A mounted on the armature shaft of the motor. If the constant angular acceleration of pulley A is $\alpha_A = 40$ rad/s^2, determine the angular velocity of the blade at the instant A has turned 400 rev, starting from rest.



Motion of Pulley *A*: Here, $\theta_A = (400 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 800\pi$ rad. Since the angular velocity can be determined from

$$\omega_A{}^2 = (\omega_A)_0{}^2 + 2\alpha_C\left[\theta_A - (\theta_A)_0\right]$$

$$\omega_A{}^2 = 0^2 + 2(40)(800\pi - 0)$$

$$\omega_A = 448.39 \text{ rad/s}$$

Motion of Pulley *B*: Since blade C and pulley B are on the same axle, both will have the same angular velocity. Pulley B is connected to pulley A by a nonslip belt. Thus,

$$\omega_B r_B = \omega_A r_A$$

$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{50}\right)(448.39) = 224 \text{ rad/s}$$ **Ans.**

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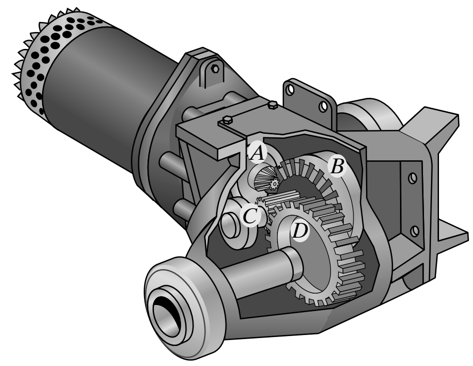
***16–24.** For a short time the motor turns gear A with an angular acceleration of $\alpha_A = (30t^{1/2})$ rad/s^2, where t is in seconds. Determine the angular velocity of gear D when $t = 5$ s, starting from rest. Gear A is initially at rest. The radii of gears A, B, C, and D are $r_A = 25$ mm, $r_B = 100$ mm, $r_C = 40$ mm, and $r_D = 100$ mm, respectively.



Motion of the Gear *A*: The angular velocity of gear A can be determined from

$$\int d\omega_A = \int \alpha dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t 30t^{1/2} dt$$

$$\omega_A\Big|_0^{\omega_A} = 20t^{3/2}\Big|_0^t$$

$$\omega_A = \left(20t^{3/2}\right) \text{ rad/s}$$

When $t = 5$ s

$$\omega_A = 20\left(5^{3/2}\right) = 223.61 \text{ rad/s}$$

Motion of Gears *B*, *C*, and *D*: Gears B and C which are mounted on the same axle will have the same angular velocity. Since gear B is in mesh with gear A, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{100}\right)(223.61) = 55.90 \text{ rad/s}$$

Also, gear D is in mesh with gear C. Then

$$\omega_D r_D = \omega_C r_C$$

$$\omega_D = \left(\frac{r_C}{r_D}\right)\omega_C = \left(\frac{40}{100}\right)(55.90) = 22.4 \text{ rad/s}$$ **Ans.**

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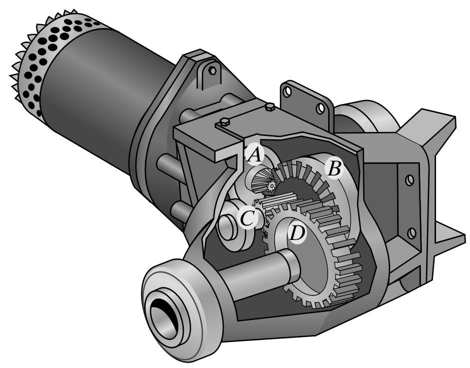
•16–25. The motor turns gear A so that its angular velocity increases uniformly from zero to 3000 rev/min after the shaft turns 200 rev. Determine the angular velocity of gear D when $t = 3$ s. The radii of gears A, B, C, and D are $r_A = 25$ mm, $r_B = 100$ mm, $r_C = 40$ mm, and $r_D = 100$ mm, respectively.



Motion of Wheel *A*: Here, $\omega_A = \left(3000\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{rev}}\right) = 100\pi$ rad/s when $\theta_A = (200\text{ rev})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = 400\pi$ rad. Since the angular acceleration of gear A is constant, it can be determined from

$$\omega_A{}^2 = (\omega_A)_0{}^2 + 2\alpha_A\left[\theta_A - (\theta_A)_0\right]$$

$$(100\pi)^2 = 0^2 + 2\alpha_A\,(400\pi - 0)$$

$$\alpha_A = 39.27\text{ rad/s}^2$$

Thus, the angular velocity of gear A when $t = 3$ s is

$$\omega_A = \left(\omega_A\right)_0 + \alpha_A\,t$$

$$= 0 + 39.27(3)$$

$$= 117.81\text{ rad/s}$$

Motion of Gears *B, C,* and *D*: Gears B and C which are mounted on the same axle will have the same angular velocity. Since gear B is in mesh with gear A, then

$$\omega_B\,r_B = \omega_B\,r_A$$

$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{100}\right)(117.81) = 29.45\text{ rad/s}$$

Also, gear D is in mesh with gear C. Then

$$\omega_D\,r_D = \omega_C\,r_C$$

$$\omega_D = \left(\frac{r_C}{r_D}\right)\omega_C = \left(\frac{40}{100}\right)(29.45) = 11.8\text{ rad/s}$$ **Ans.**

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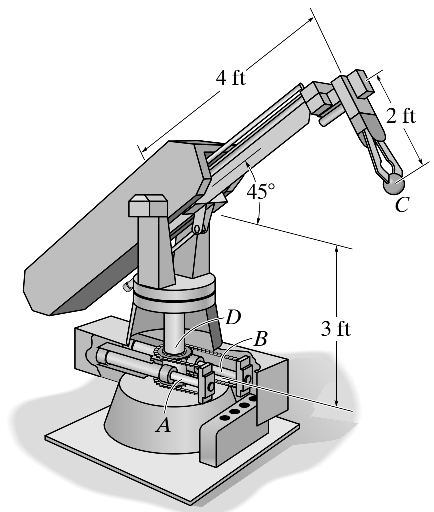
16–26. Rotation of the robotic arm occurs due to linear movement of the hydraulic cylinders A and B. If this motion causes the gear at D to rotate clockwise at 5 rad/s, determine the magnitude of velocity and acceleration of the part C held by the grips of the arm.



Motion of Part *C*: Since the shaft that turns the robot's arm is attached to gear D, then the angular velocity of the robot's arm $\omega_R = \omega_D = 5.00$ rad/s. The distance of part C from the rotating shaft is $r_C = 4 \cos 45° + 2 \sin 45° = 4.243$ ft. The magnitude of the velocity of part C can be determined using Eq. 16–8.

$$v_C = \omega_R\, r_C = 5.00(4.243) = 21.2 \text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of part C can be determined using Eqs. 16–11 and 16–12 respectively.

$$a_t = \alpha r_C = 0$$

$$a_n = \omega_R^2\, r_C = \left(5.00^2\right)(4.243) = 106.07 \text{ ft/s}^2$$

The magnitude of the acceleration of point C is

$$a_C = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 106.07^2} = 106 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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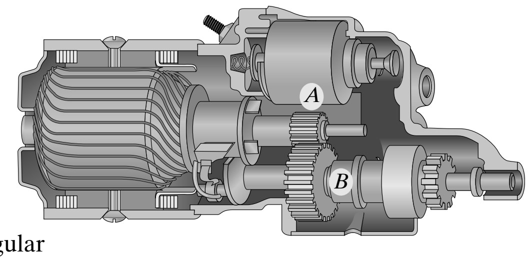
16–27. For a short time, gear A of the automobile starter rotates with an angular acceleration of $\alpha_A = (450t^2 + 60)\ \text{rad/s}^2$, where t is in seconds. Determine the angular velocity and angular displacement of gear B when $t = 2$ s, starting from rest. The radii of gears A and B are 10 mm and 25 mm, respectively.



Motion of Gear *A*: Applying the kinematic equation of variable angular acceleration,

$$\int d\omega_A \int \alpha_A dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t \left(450t^2 + 60\right)dt$$

$$\omega_A\Big|_0^{\omega_A} = 150t^3 + 60t\Big|_0^t$$

$$\omega_A = \left(150t^3 + 60t\right)\ \text{rad/s}$$

When $t = 2$ s,

$$\omega_A = 150(2)^3 + 60(2) = 1320\ \text{rad/s}$$

$$\int d\theta_A = \int \omega_A\, dt$$

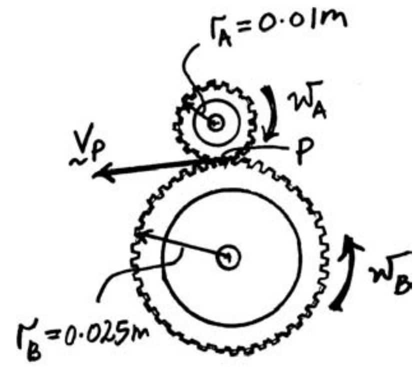
$$\int_0^{\theta_A} d\theta_A = \int_0^t \left(150t^3 + 60t\right)dt$$

$$\theta_A\Big|_0^{\theta_A} = 37.5t^4 + 30t^2\Big|_0^t$$

$$\theta_A = \left(37.5t^4 + 30t^2\right)\ \text{rad}$$

When $t = 2$ s

$$\theta_A = 37.5(2)^4 + 30(2)^2 = 720\ \text{rad}$$



(a)

Motion of Gear *B*: Since gear B is meshed with gear A, Fig. a, then

$$v_p = \omega_A r_A = \omega_B r_B$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right)$$

$$= (1320)\left(\frac{0.01}{0.025}\right)$$

$$= 528\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\theta_B = \theta_A\left(\frac{r_A}{r_B}\right)$$

$$= 720\left(\frac{0.01}{0.025}\right)$$

$$= 288\ \text{rad} \qquad \textbf{Ans.}$$

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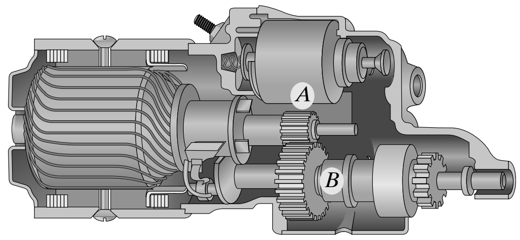
***16–28.** For a short time, gear A of the automobile starter rotates with an angular acceleration of $\alpha_A = (50\omega^{1/2})$ rad/s^2, where ω is in rad/s. Determine the angular velocity of gear B after gear A has rotated 50 rev, starting from rest. The radii of gears A and B are 10 mm and 25 mm, respectively.



Motion of Gear *A*: We have

$$\int dt = \int \frac{d\omega_A}{\alpha_A}$$

$$\int_0^t dt = \int_0^{\omega_A} \frac{d\omega_A}{50\omega_A{}^{1/2}}$$

$$t\Big|_0^t = \frac{1}{25}\omega_A{}^{1/2}\Big|_0^{\omega_A}$$

$$\omega_A = \left(625t^2\right) \text{ rad/s}$$

The angular displacement of gear A can be determined using this result.

$$\int d\theta_A = \int \omega_A\, dt$$

$$\int_0^{\theta_A} d\theta_A = \int_0^t \left(625t^2\right) dt$$

$$\theta_A\Big|_0^{\theta_A} = 208.33t^3\Big|_0^t$$

$$\theta_A = \left(208.33t^3\right) \text{ rad}$$

When $\theta_A = 50 \text{ rev}\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 100\pi \text{ rad}$,

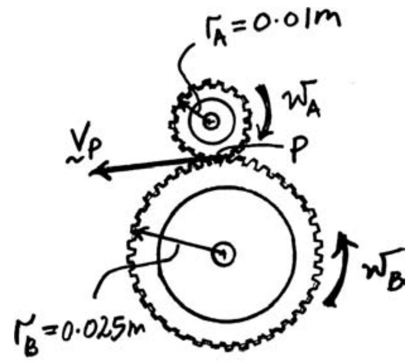
$$100\pi = 208.33t^3$$

$$t = 1.147 \text{ s}$$

Thus, the angular velocity of gear A at $t = 1.147$ s$(\theta_A = 100\pi$ rad) is

$$\omega_A = 625(1.147^2) = 821.88 \text{ rad/s}$$

Motion of Gear *B*: Since gear B is meshed with gear A, Fig. a, then



(a)

$$v_p = \omega_A r_A = \omega_B r_B$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right)$$

$$= 821.88\left(\frac{0.01}{0.025}\right)$$

$$= 329 \text{ rad/s} \qquad \textbf{Ans.}$$

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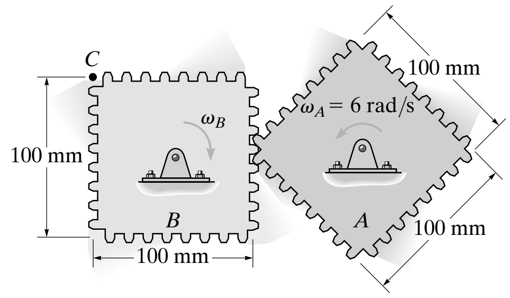
•16–29. Gear A rotates with a constant angular velocity of $\omega_A = 6$ rad/s. Determine the largest angular velocity of gear B and the speed of point C.



$$(r_B)_{\max} = (r_A)_{\max} = 50\sqrt{2}\text{ mm}$$

$$(r_B)_{\min} = (r_A)_{\min} = 50\text{ mm}$$

When r_A is max., r_B is min.

$$\omega_B\,(r_B) = \omega_A\,r_A$$

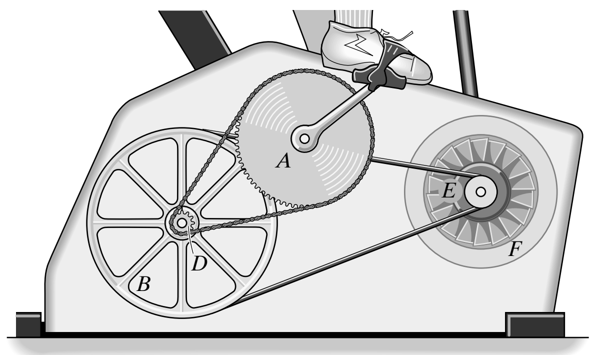
$$(\omega_B)_{\max} = 6\left(\frac{r_A}{r_B}\right) = 6\left(\frac{50\sqrt{2}}{50}\right)$$

$(\omega_B)_{\max} = 8.49$ rad/s **Ans.**

$$v_C = (\omega_B)_{\max}\,r_C = 8.49\left(0.05\sqrt{2}\right)$$

$v_C = 0.6$ m/s **Ans.**

16–30. If the operator initially drives the pedals at 20 rev/min, and then begins an angular acceleration of 30 rev/min^2, determine the angular velocity of the flywheel F when $t = 3$ s. Note that the pedal arm is fixed connected to the chain wheel A, which in turn drives the sheave B using the fixed connected clutch gear D. The belt wraps around the sheave then drives the pulley E and fixed-connected flywheel.



$r_A = 125$ mm $\quad r_B = 175$ mm
$r_D = 20$ mm $\quad r_E = 30$ mm

$$\omega = \omega_0 + \alpha_c t$$

$$\omega_A = 20 + 30\left(\frac{3}{60}\right) = 21.5\text{ rev/min}$$

$$\omega_A\,r_A = \omega_D\,r_D$$

$$21.5(125) = \omega_D\,(20)$$

$$\omega_D = \omega_B = 134.375$$

$$\omega_B\,r_B = \omega_E\,r_E$$

$$134.375(175) = \omega_E(30)$$

$$\omega_E = 783.9\text{ rev/min}$$

$\omega_F = 784$ rev/min **Ans.**

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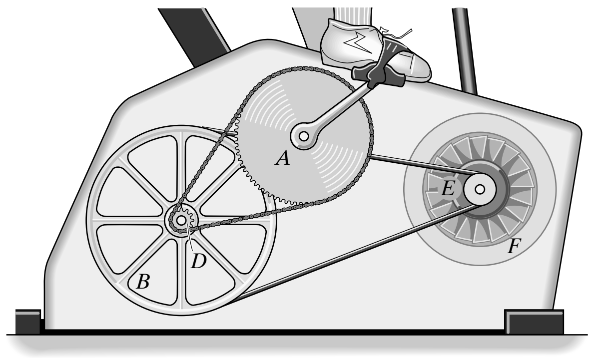
16–31. If the operator initially drives the pedals at 12 rev/min, and then begins an angular acceleration of 8 rev/min^2, determine the angular velocity of the flywheel F after the pedal arm has rotated 2 revolutions. Note that the pedal arm is fixed connected to the chain wheel A, which in turn drives the sheave B using the fixed-connected clutch gear D. The belt wraps around the sheave then drives the pulley E and fixed-connected flywheel.



$r_A = 125$ mm $\quad r_B = 175$ mm
$r_D = 20$ mm $\quad r_E = 30$ mm

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = (12)^2 + 2(8)(2 - 0)$$

$$\omega = 13\,266 \text{ rev/min}$$

$$\omega_A r_A = \omega_D r_D$$

$$13\,266(125) = \omega_D(20)$$

$$\omega_D = \omega_D = 82.916$$

$$\omega_B r_B = \omega_E r_E$$

$$82.916(175) = \omega_E(30)$$

$$\omega_E = 483.67$$

$$\omega_F = 484 \text{ rev/min}$$ **Ans.**

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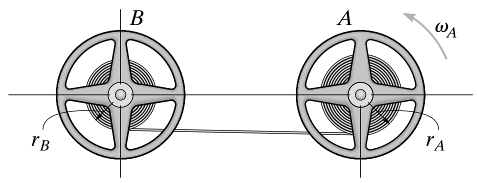
***16–32.** The drive wheel A has a constant angular velocity of ω_A. At a particular instant, the radius of rope wound on each wheel is as shown. If the rope has a thickness T, determine the angular acceleration of wheel B.



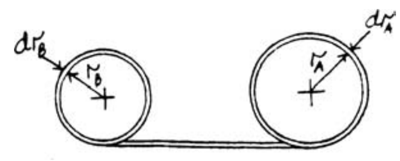
Angular Motion: The angular velocity between wheels A and B can be related by

$$\omega_A r_A = \omega_B r_B \text{ or } \omega_B = \frac{r_A}{r_B}\omega_A$$

During time dt, the volume of the tape exchange between the wheel is

$$-2\pi r_B\, dr_B = 2\pi r_A\, dr_A$$

$$dr_B = -\left(\frac{r_A}{r_B}\right) dr_A \qquad \textbf{[1]}$$

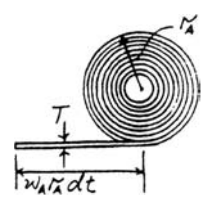


Applying Eq. 16–2 with $\omega_B = \dfrac{r_A}{r_B}\omega_A$, we have

$$\alpha_B = \frac{d\omega_B}{dt} = \frac{d}{dt}\left[\frac{r_A}{r_B}\omega_A\right] = \omega_A\left(\frac{1}{r_B}\frac{dr_A}{dt} - \frac{r_A}{r_B^2}\frac{dr_B}{dt}\right) \qquad \textbf{[2]}$$

Substituting Eq.[1] into [2] yields

$$\alpha_B = \omega_A\left(\frac{r_A^2 + r_B^2}{r_B^3}\right)\frac{dr_A}{dt} \qquad \textbf{[3]}$$



The volume of tape coming out from wheel A in time dt is

$$2\pi r_A\, dr_A = (\omega_A r_A\, dt)\, T$$

$$\frac{dr_A}{dt} = \frac{\omega_A T}{2\pi} \qquad \textbf{[4]}$$

Substitute Eq.[4] into [3] gives

$$\alpha_B = \frac{\omega_A^2 T}{2\pi r_B^3}\left(r_A^2 + r_B^2\right) \qquad \textbf{Ans.}$$

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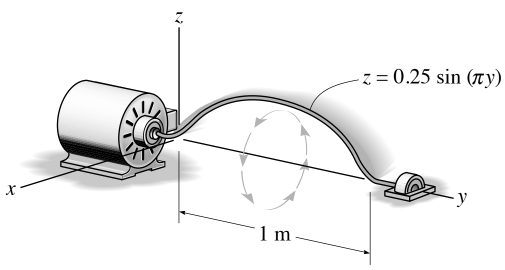
•16–33. If the rod starts from rest in the position shown and a motor drives it for a short time with an angular acceleration of $\alpha = (1.5e^t)$ rad/s^2, where t is in seconds, determine the magnitude of the angular velocity and the angular displacement of the rod when $t = 3$ s. Locate the point on the rod which has the greatest velocity and acceleration, and compute the magnitudes of the velocity and acceleration of this point when $t = 3$ s. The rod is defined by $z = 0.25 \sin(\pi y)$ m, where the argument for the sine is given in radians and y is in meters.



$$d\omega = \alpha\, dt$$

$$\int_0^\omega d\omega = \int_0^t 1.5e^t\, dt$$

$$\omega = 1.5e^t\Big|_0^t = 1.5\left[e^t - 1\right]$$

$$d\theta = \omega\, dt$$

$$\int_0^\theta d\theta = 1.5\int_0^t \left[e^t - 1\right] dt$$

$$\theta = 1.5\left[e^t - t\right]_0^t = 1.5\left[e^t - t - 1\right]$$

When $t = 3$ s

$$\omega = 1.5\left[e^3 - 1\right] = 28.63 = 28.6 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\theta = 1.5\left[e^3 - 3 - 1\right] = 24.1 \text{ rad} \qquad \textbf{Ans.}$$

The point having the greatest velocity and acceleration is located furthest from the axis of rotation. This is at $y = 0.5$ m, where $z = 0.25 \sin(\pi 0.5) = 0.25$ m.

Hence,

$$v_P = \omega(z) = 28.63(0.25) = 7.16 \text{ m/s} \qquad \textbf{Ans.}$$

$$(a_t)_P = \alpha(z) = \left(1.5e^3\right)(0.25) = 7.532 \text{ m/s}^2$$

$$(a_n)_P = \omega^2(z) = (28.63)^2(0.25) = 204.89 \text{ m/s}^2$$

$$a_P = \sqrt{(a_t)_P^2 + (a_n)_P^2} = \sqrt{(7.532)^2 + (204.89)^2}$$

$$a_P = 205 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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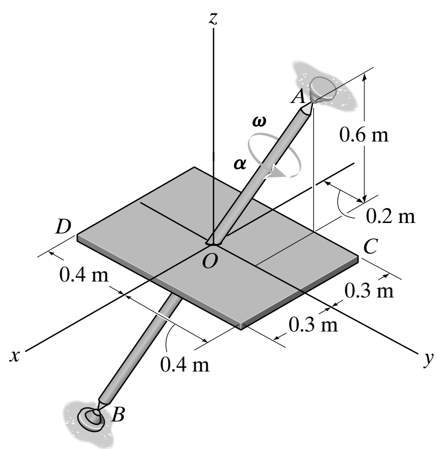
16–34. If the shaft and plate rotate with a constant angular velocity of $\omega = 14$ rad/s, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.



We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

Since ω is constant

$$\alpha = 0$$

For convenience, $\mathbf{r}_C = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point C can be determined from

$$\mathbf{v}_C = \omega \times \mathbf{r}_C$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})$$

$$= [-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \alpha \times \mathbf{r}_C$$

$$= 0 + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}$$

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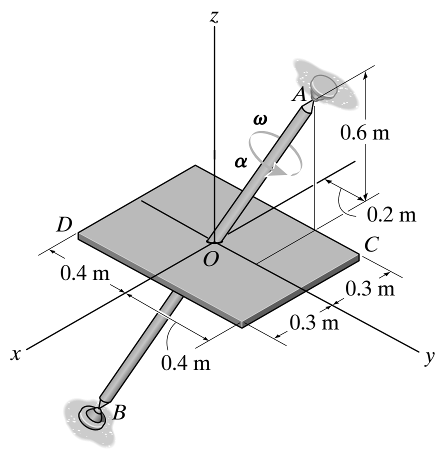
16–35. At the instant shown, the shaft and plate rotates with an angular velocity of $\omega = 14$ rad/s and angular acceleration of $\alpha = 7$ rad/s^2. Determine the velocity and acceleration of point D located on the corner of the plate at this instant. Express the result in Cartesian vector form.



We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω and α is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

$$\alpha = \alpha\mathbf{u}_{OA} = 7\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}] \text{ rad/s}$$

For convenience, $\mathbf{r}_D = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point D can be determined from

$$\mathbf{v}_D = \omega \times r_D$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})$$

$$= [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}]\text{m/s}$$ **Ans.**

and

$$\mathbf{a}_D = \alpha \times \mathbf{r}_D - \omega^2\,\mathbf{r}_D$$

$$= (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j}) + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [-36.0\mathbf{i} + 66.6\mathbf{j} + 40.2\mathbf{k}]\text{m/s}^2$$ **Ans.**

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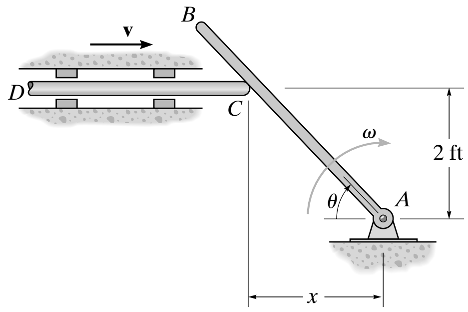
***16–36.** Rod CD presses against AB, giving it an angular velocity. If the angular velocity of AB is maintained at $\omega = 5$ rad/s, determine the required magnitude of the velocity **v** of CD as a function of the angle θ of rod AB.

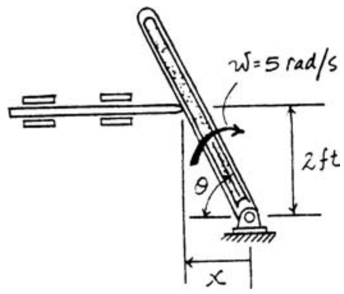


Position Coordinate Equation: From the geometry,

$$x = \frac{2}{\tan\theta} = 2\cot\theta \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

$$\frac{dx}{dt} = -2\csc^2\theta\frac{d\theta}{dt} \qquad \textbf{[2]}$$

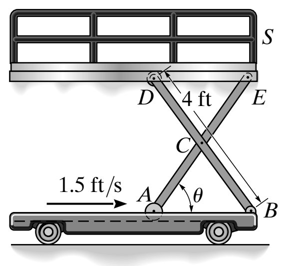


However, $\dfrac{dx}{dt} = v$ and $\dfrac{d\theta}{dt} = \omega = 5$ rad/s, then from Eq. [2]

$$v = -2\csc^2\theta(5) = \left(-10\csc^2\theta\right) \qquad \textbf{Ans.}$$

Note: Negative sign indicates that v is directed in the opposite direction to that of positive x.

•16–37. The scaffold S is raised by moving the roller at A toward the pin at B. If A is approaching B with a speed of 1.5 ft/s, determine the speed at which the platform rises as a function of θ. The 4-ft links are pin connected at their midpoint.



Position Coordinate Equation:

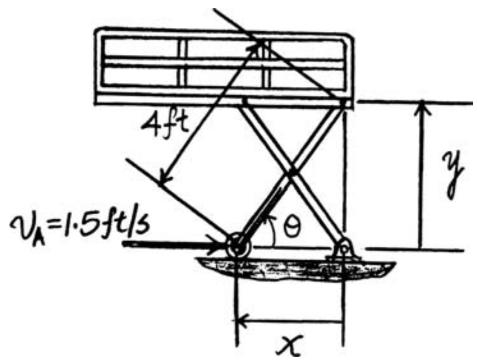
$$x = 4\cos\theta \qquad y = 4\sin\theta$$

Time Derivatives:

$$\dot{x} = -4\sin\theta\dot{\theta} \qquad \text{However,} \quad \dot{x} = -v_A = -1.5 \text{ ft/s}$$

$$-1.5 = -4\sin\theta\dot{\theta} \qquad \dot{\theta} = \frac{0.375}{\sin\theta}$$

$$\dot{y} = v_y = 4\cos\theta\dot{\theta} = 4\cos\theta\left(\frac{0.375}{\sin\theta}\right) = 1.5\cot\theta \qquad \textbf{Ans.}$$



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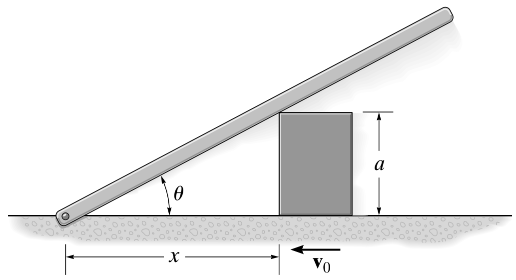
16–38. The block moves to the left with a constant velocity $\mathbf{v}_0$. Determine the angular velocity and angular acceleration of the bar as a function of θ.



Position Coordinate Equation: From the geometry,

$$x = \frac{a}{\tan\theta} = a\cot\theta \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

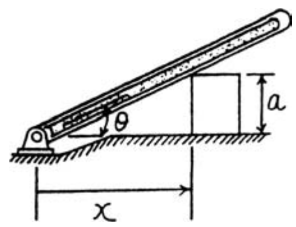
$$\frac{dx}{dt} = -a\csc^2\theta\frac{d\theta}{dt} \qquad \textbf{[2]}$$

Since v_0 is directed toward negative x, then $\frac{dx}{dt} = -v_0$. Also, $\frac{d\theta}{dt} = \omega$.

From Eq.[2],

$$-v_0 = -a\csc^2\theta(\omega)$$

$$\omega = \frac{v_0}{a\csc^2\theta} = \frac{v_0}{a}\sin^2\theta \qquad \textbf{Ans.}$$



Here, $\alpha = \frac{d\omega}{dt}$. Then from the above expression

$$\alpha = \frac{v_0}{a}(2\sin\theta\cos\theta)\frac{d\theta}{dt} \qquad \textbf{[3]}$$

However, $2\sin\theta\cos\theta = \sin 2\theta$ and $\omega = \frac{d\theta}{dt} = \frac{v_0}{a}\sin^2\theta$. Substitute these values into Eq.[3] yields

$$\alpha = \frac{v_0}{a}\sin 2\theta\left(\frac{v_0}{a}\sin^2\theta\right) = \left(\frac{v_0}{a}\right)^2\sin 2\theta\sin^2\theta \qquad \textbf{Ans.}$$

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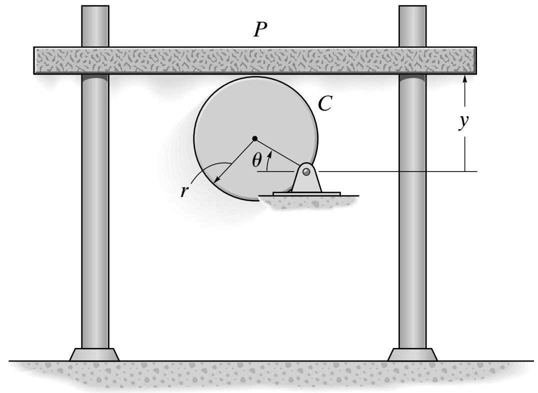
16–39. Determine the velocity and acceleration of platform P as a function of the angle θ of cam C if the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection does not cause interference with the motion of P on C. The platform is constrained to move vertically by the smooth vertical guides.

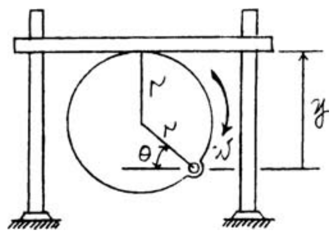


Position Coordinate Equation: From the geometry.

$$y = r\sin\theta + r \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

$$\frac{dy}{dt} = r\cos\theta\frac{d\theta}{dt} \qquad \textbf{[2]}$$



However $v = \dfrac{dy}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. From Eq.[2],

$$v = \omega r\cos\theta \qquad \textbf{Ans.}$$

Taking the time derivative of the above expression, we have

$$\frac{dv}{dt} = r\left[\omega(-\sin\theta)\frac{d\theta}{dt} + \cos\theta\frac{d\omega}{dt}\right]$$

$$= r\left(\cos\theta\frac{d\omega}{dt} - \omega^2\sin\theta\right) \qquad \textbf{[4]}$$

However $a = \dfrac{dv}{dt}$ and $\alpha = \dfrac{d\omega}{dt} = 0$. From Eq.[4],

$$a = -\omega^2 r\sin\theta \qquad \textbf{Ans.}$$

Note: Negative sign indicates that a is directed in the opposite direction to that of positive y.

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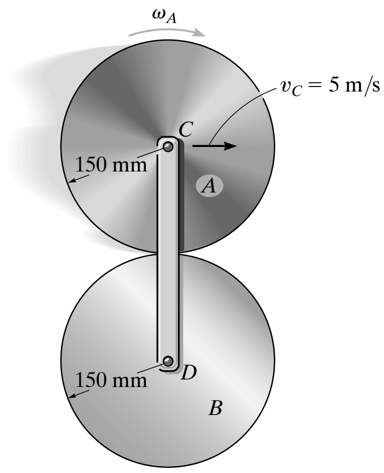
***16–40.** Disk A rolls without slipping over the surface of the *fixed* cylinder B. Determine the angular velocity of A if its center C has a speed $v_C = 5$ m/s. How many revolutions will A rotate about its center just after link DC completes one revolution?



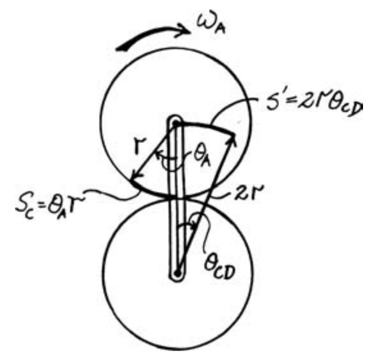
As shown by the construction, as A rolls through the arc $s = \theta_A r$, the center of the disk moves through the same distance $s' = s$. Hence,

$$s = \theta_A r$$

$$\dot{s} = \dot{\theta}_A r$$

$$5 = \omega_A (0.15)$$

$$\omega_A = 33.3 \text{ rad/s}$$ **Ans.**



Link

$$s' = 2r\theta_{CD} = s = \theta_A r$$

$$2\theta_{CD} = \theta_A$$

Thus, A makes 2 revolutions for each revolution of CD. **Ans.**

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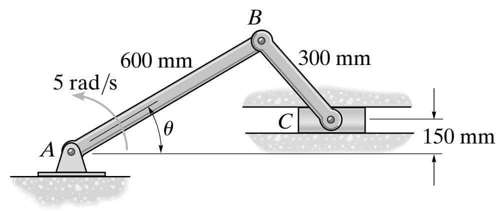
•16–41. Crank AB rotates with a constant angular velocity of 5 rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 30°$.



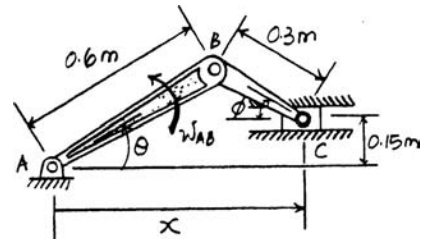
Position Coordinate Equation: From the geometry,

$$x = 0.6\cos\theta + 0.3\cos\phi \qquad \textbf{[1]}$$

$$0.6\sin\theta = 0.15 + 0.3\sin\phi \qquad \textbf{[2]}$$

Eliminate ϕ from Eqs. [1] and [2] yields

$$x = 0.6\cos\theta + 0.3\sqrt{2\sin\theta - 4\sin^2\theta + 0.75} \qquad \textbf{[3]}$$



Time Derivatives: Taking the time derivative of Eq. [3], we have

$$\frac{dx}{dt} = \left[-0.6\sin\theta + \frac{0.15(2\cos\theta - 4\sin 2\theta)}{\sqrt{2\sin\theta - 4\sin^2\theta + 0.75}}\right]\frac{d\theta}{dt} \qquad \textbf{[4]}$$

However, $\frac{dx}{dt} = v_C$ and $\frac{d\theta}{dt} = \omega_{AB}$, then from Eq.[4]

$$v_C = \left[-0.6\sin\theta + \frac{0.15(2\cos\theta - 4\sin 2\theta)}{\sqrt{2\sin\theta - 4\sin^2\theta + 0.75}}\right]\omega_{AB} \qquad \textbf{[5]}$$

At the instant $\theta = 30°$, $\omega_{AB} = 5$ rad/s. Substitute into Eq.[5] yields

$$v_C = \left[-0.6\sin 30° + \frac{0.15(2\cos 30° - 4\sin 60°)}{\sqrt{2\sin 30° - 4\sin^2 30° + 0.75}}\right](5) = -3.00\text{ m/s} \qquad \textbf{Ans.}$$

Taking the time derivative of Eq. [2], we have

$$0.6\cos\theta\,\frac{d\theta}{dt} = 0.3\cos\phi\,\frac{d\phi}{dt} \qquad \textbf{[6]}$$

However, $\frac{d\phi}{dt} = \omega_{BC}$ and $\frac{d\theta}{dt} = \omega_{AB}$, then from Eq.[6]

$$\omega_{BC} = \left(\frac{2\cos\theta}{\cos\phi}\right)\omega_{AB} \qquad \textbf{[7]}$$

At the instant $\theta = 30°$, from Eq.[2], $\phi = 30.0°$. From Eq.[7]

$$\omega_{BC} = \left(\frac{2\cos 30°}{\cos 30.0°}\right)(5) = 10.0\text{ rad/s} \qquad \textbf{Ans.}$$

Note: Negative sign indicates that v_C is directed in the opposite direction to that of positive x.

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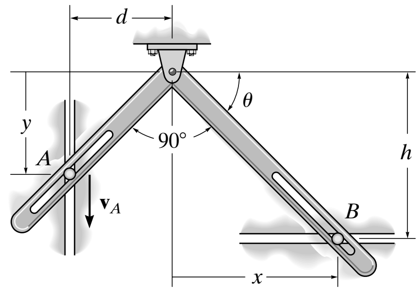
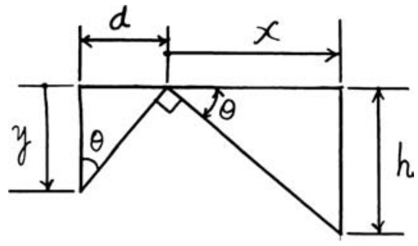
16–42. The pins at A and B are constrained to move in the vertical and horizontal tracks. If the slotted arm is causing A to move downward at $\mathbf{v}_A$, determine the velocity of B as a function of θ.



Position Coordinate Equation:

$$\tan\theta = \frac{h}{x} = \frac{d}{y}$$

$$x = \left(\frac{h}{d}\right)y$$



Time Derivatives:

$$\dot{x} = \left(\frac{h}{d}\right)\dot{y}$$

$$v_B = \left(\frac{h}{d}\right)v_A \qquad \textbf{Ans.}$$

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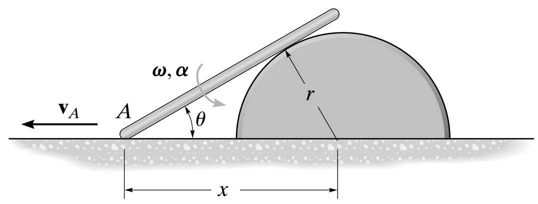
16–43. End A of the bar moves to the left with a constant velocity $\mathbf{v}_A$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar as a function of its position x.

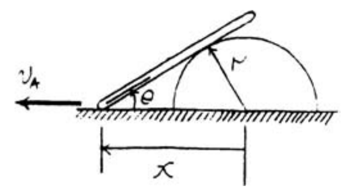


Position Coordinate Equation: From the geometry.

$$x = \frac{r}{\sin\theta} \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq.[1], we have

$$\frac{dx}{dt} = -\frac{r\cos\theta\, d\theta}{r\sin^2\theta\, dt} \qquad \textbf{[2]}$$



Since v_0 is directed toward positive x, then $\dfrac{dx}{dt} = v_A$. Also, $\dfrac{d\theta}{dt} = \omega$. From the geometry, $\sin\theta = \dfrac{r}{x}$ and $\cos\theta = \dfrac{\sqrt{x^2 - r^2}}{x}$. Substitute these values into Eq.[2], we have

$$v_A = -\left(\frac{r\left(\sqrt{x^2 - r^2}/x\right)}{(r/x)^2}\right)\omega$$

$$\omega = -\left(\frac{r}{x\sqrt{x^2 - r^2}}\right)v_A \qquad \textbf{Ans.}$$

Taking the time derivative of Eq. [2], we have

$$\frac{d^2x}{dt^2} = \frac{r}{\sin^2\theta} = \left[\left(\frac{1 + \cos^2\theta}{\sin\theta}\right)\left(\frac{d\theta}{dt}\right)^2 - \cos\theta\frac{d^2\theta}{dt^2}\right] \qquad \textbf{[3]}$$

Here, $\dfrac{d^2x}{dt^2} = a = 0$ and $\dfrac{d^2\theta}{dt^2} = \alpha$. Substitute into Eq.[3], we have

$$0 = \frac{r}{\sin^2\theta}\left[\left(\frac{1 + \cos^2\theta}{\sin\theta}\right)\omega^2 - \alpha\cos\theta\right]$$

$$\alpha = \left(\frac{1 + \cos^2\theta}{\sin\theta\cos\theta}\right)\omega^2 \qquad \textbf{[4]}$$

However, $\sin\theta = \dfrac{r}{x}$, $\cos\theta = \dfrac{\sqrt{x^2 - r^2}}{x}$ and $\omega = -\left(\dfrac{r}{x\sqrt{x^2 - r^2}}\right)v_A$. Substitute these values into Eq.[4] yields

$$\alpha = \left[\frac{r(2x^2 - r^2)}{x^2(x^2 - r^2)^{3/2}}\right]v_A^2 \qquad \textbf{Ans.}$$

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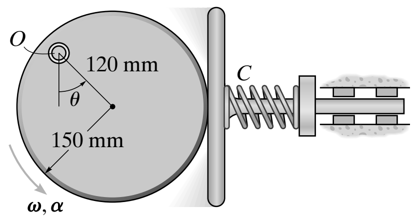
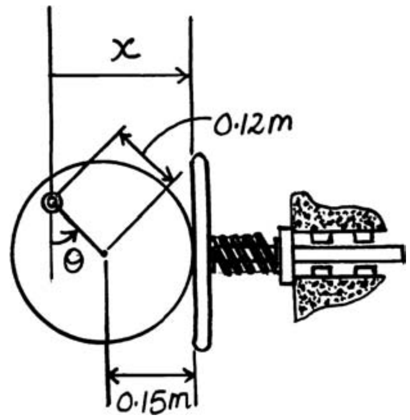
***16–44.** Determine the velocity and acceleration of the plate at the instant $\theta = 30°$, if at this instant the circular cam is rotating about the fixed point O with an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 2$ rad/s^2.



Position Coordinate Equation: From the geometry,

$$x = 0.12 \sin \theta + 0.15 \qquad \textbf{[1]}$$



Time Derivatives: Taking the time derivative of Eq. [1], we have

$$\frac{dx}{dt} = 0.12 \cos \theta \frac{d\theta}{dt} \qquad \textbf{[2]}$$

However $v = \frac{dx}{dt}$ and $\omega = \frac{d\theta}{dt}$. From Eq.[2],

$$v = 0.12\omega \cos \theta \qquad \textbf{[3]}$$

At the instant $\theta = 30°$, $\omega = 4$ rad/s, then substitute these values into Eq.[3] yields

$$v = 0.12(4) \cos 30° = 0.416 \text{ m/s} \qquad \textbf{Ans.}$$

Taking the time derivative of Eq. [3], we have

$$\frac{dv}{dt} = 0.12\left[\omega(-\sin \theta)\frac{d\theta}{dt} + \cos \theta \frac{d\omega}{dt}\right]$$

$$= 0.12\left(\cos \theta \frac{d\omega}{dt} - \omega^2 \sin \theta\right) \qquad \textbf{[4]}$$

However $a = \frac{dv}{dt}$ and $\alpha = \frac{d\omega}{dt}$. From Eq.[4],

$$a = 0.12\left(\alpha \cos \theta - \omega^2 \sin \theta\right) \qquad \textbf{[5]}$$

At the instant $\theta = 30°$, $\omega = 4$ rad/s and $\alpha = 2$ rad/s^2, then substitute these values into Eq.[5] yields

$$a = 0.12\left(2 \cos 30° - 4^2 \sin 30°\right) = -0.752 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Note: Negative sign indicates that a is directed in the opposite direction to that of positive x.

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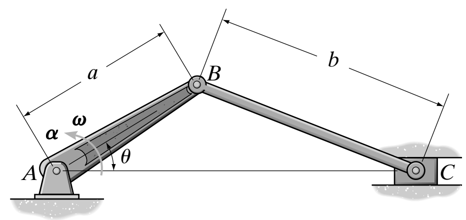
•16–45. At the instant $\theta = 30°$, crank AB rotates with an angular velocity and angular acceleration of $\omega = 10$ rad/s and $\alpha = 2$ rad/s^2, respectively. Determine the velocity and acceleration of the slider block C at this instant. Take $a = b = 0.3$ m.



Position Coordinates: Due to symmetry, $\phi = \theta$. Thus, from the geometry shown in Fig. a,

$$x_C = 2[0.3\cos\theta]\text{m} = 0.6\cos\theta\text{ m}$$

Time Derivative: Taking the time derivative,

$$v_C = \dot{x}_C = \left(-0.6\sin\theta\dot{\theta}\right)\text{m/s} \qquad \textbf{(1)}$$

When $\theta = 30°, \dot{\theta} = \omega = 10$ rad/s Thus,

$$v_C = -0.6\sin 30°(10) = -3\text{ m/s} = 3\text{ m/s}\leftarrow \qquad \textbf{Ans.}$$

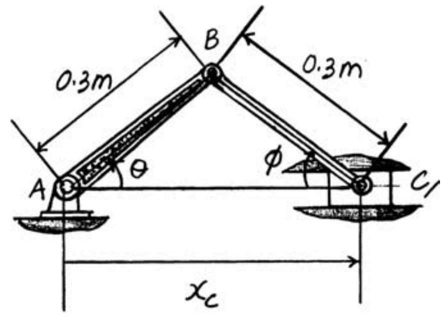
The time derivative of Eq. (1) gives

$$a_C = \ddot{x}_C = -0.6\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right)\text{ m/s}^2$$

When $\theta = 30°, \ddot{\theta} = \alpha = 2$ rad/s^2, and $\dot{\theta} = 10$ rad/s. Thus,

$$a_C = -0.6\left[\sin 30°(2) + \cos 30°(10^2)\right]$$
$$= -52.6\text{ m/s}^2 = 52.6\text{ m/s}^2\leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_C$ and $\mathbf{a}_C$ are in the negative sense of x_C.



(a)

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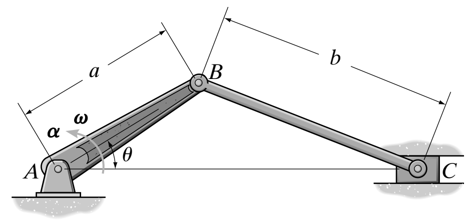
16–46. At the instant $\theta = 30°$, crank AB rotates with an angular velocity and angular acceleration of $\omega = 10$ rad/s and $\alpha = 2$ rad/s^2, respectively. Determine the angular velocity and angular acceleration of the connecting rod BC at this instant. Take $a = 0.3$ m and $b = 0.5$ m.



Position Coordinates: The angles θ and ϕ can be related using the law of sines and referring to the geometry shown in Fig. a.

$$\frac{\sin\phi}{0.3} = \frac{\sin\theta}{0.5}$$

$$\sin\phi = 0.6\sin\theta \qquad \textbf{(1)}$$

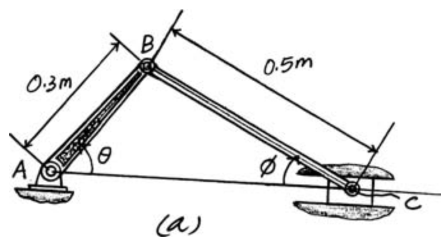
When $\theta = 30°$,

$$\phi = \sin^{-1}(0.6\sin 30°) = 17.46°$$

Time Derivative: Taking the time derivative of Eq. (1),

$$\cos\phi\dot{\phi} = 0.6\cos\theta\dot{\theta} \qquad \textbf{(2)}$$

$$\omega_{BC} = \dot{\phi} = \frac{0.6\cos\theta}{\cos\phi}\dot{\theta}$$



When $\theta = 30°$, $\phi = 17.46°$ and $\dot{\theta} = 10$ rad/s,

$$\omega_{BC} = \dot{\phi} = \frac{0.6\cos 30°}{\cos 17.46°}(10) = 5.447 \text{ rad/s} = 5.45 \text{ rad/s} \qquad \textbf{Ans.}$$

The time derivative of Eq. (2) gives

$$\cos\phi\ddot{\phi} - \sin\phi\dot{\phi}^2 = 0.6\left(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2\right)$$

$$\alpha_{BC} = \ddot{\phi} = \frac{0.6\left(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2\right) + \sin\phi\dot{\phi}^2}{\cos\phi}$$

When $\theta = 30°$, $\phi = 17.46°$, $\dot{\theta} = 10$ rad/s, $\dot{\phi} = 5.447$ rad/s and $\ddot{\theta} = \alpha = 2$ rad/s^2,

$$\alpha_{BC} = \frac{0.6\left[\cos 30°(2) - \sin 30°(10^2)\right] + \sin 17.46°(5.447^2)}{\cos 17.46°}$$

$$= -21.01 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

The negative sign indicates that α_{BC} acts counterclockwise.

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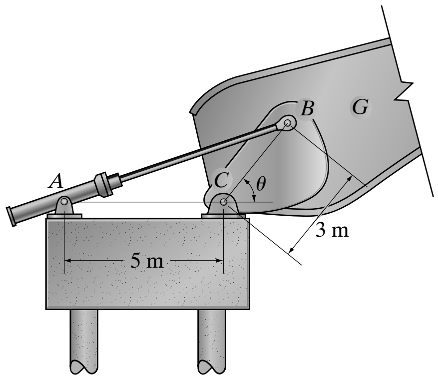
16–47. The bridge girder G of a bascule bridge is raised and lowered using the drive mechanism shown. If the hydraulic cylinder AB shortens at a constant rate of $0.15\,\text{m/s}$, determine the angular velocity of the bridge girder at the instant $\theta = 60°$.



Position Coordinates: Applying the law of cosines to the geometry shown in Fig. a,

$$s^2 = 3^2 + 5^2 - 2(3)(5)\cos\left(180°-\theta\right)$$

$$s^2 = 34 - 30\cos\left(180°-\theta\right)$$

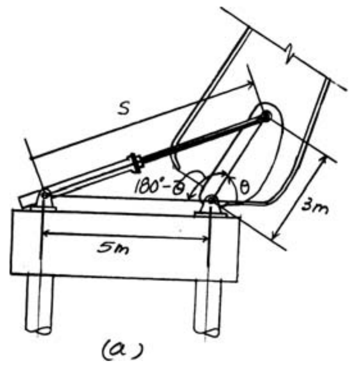
However, $\cos\left(180°-\theta\right) = -\cos\theta$. Thus,

$$s^2 = 34 + 30\cos\theta$$

Time Derivatives: Taking the time derivative,

$$2s\dot{s} = 0 + 30\left(-\sin\theta\dot{\theta}\right)$$

$$s\dot{s} = -15\sin\theta\dot{\theta} \qquad \textbf{(1)}$$



When $\theta = 60°$, $s = \sqrt{34 + 30\cos 60°} = 7$ m. Also, $\dot{s} = -0.15$ m/s since $\dot{s}$ is directed towards the negative sense of s. Thus, Eq. (1) gives

$$7\left(-0.15\right) = -15\sin 60°\dot{\theta}$$

$$\omega = \dot{\theta} = 0.0808\ \text{rad/s} \qquad \textbf{Ans.}$$

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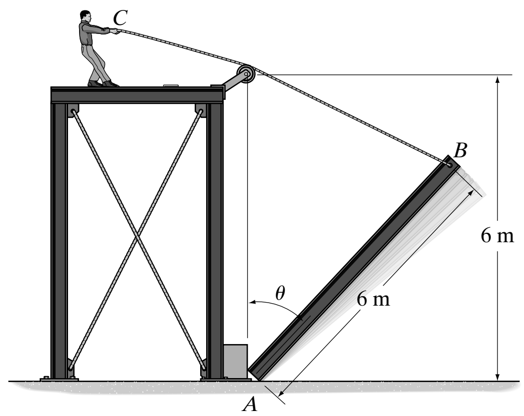
***16–48.** The man pulls on the rope at a constant rate of 0.5 m/s. Determine the angular velocity and angular acceleration of beam AB when $\theta = 60°$. The beam rotates about A. Neglect the thickness of the beam and the size of the pulley.



Position Coordinates: Applying the law of cosines to the geometry,

$$s^2 = 6^2 + 6^2 - 2(6)(6)\cos\theta$$

$$s^2 = \left(72 - 72\cos\theta\right)\text{m}^2$$

Time Derivatives: Taking the time derivative,

$$2s\dot{s} = 0 - 72\left(-\sin\theta\dot{\theta}\right)$$

$$s\dot{s} = 36\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

Here, $\dot{s} = -0.5\text{ m/s}$ since $\dot{s}$ acts in the negative sense of s. When $\theta = 60°$, $s = \sqrt{72 - 72\cos 60°} = 6\text{ m}$. Thus, Eq. (1) gives

$$6\left(-0.5\right) = 36\sin 60°\dot{\theta}$$

$$\omega = \dot{\theta} = -0.09623\text{ rad/s} - 0.0962\text{ rad/s} \qquad \textbf{Ans.}$$

The negative sign indicates that ω acts in the negative rotational sense of θ. The time derivative of Eq.(1) gives

$$s\ddot{s} + \dot{s}^2 = 36\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right) \qquad \textbf{(2)}$$

Since $\dot{s}$ is constant, $\ddot{s} = 0$. When $\theta = 60°$.

$$6(0) + (-0.5)^2 = 36\left[\sin 60°\,\ddot{\theta} + \cos 60°\left(-0.09623\right)^2\right]$$

$$\alpha = \ddot{\theta} = 0.00267\text{ rad/s}^2 \qquad \textbf{Ans.}$$

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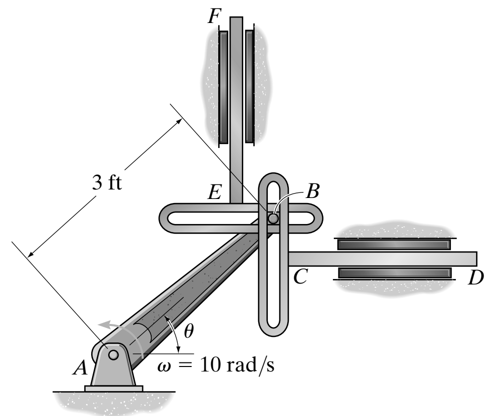
•16–49. Peg B attached to the crank AB slides in the slots mounted on follower rods, which move along the vertical and horizontal guides. If the crank rotates with a constant angular velocity of $\omega = 10$ rad/s, determine the velocity and acceleration of rod CD at the instant $\theta = 30°$.



Position Coordinates: From the geometry shown in Fig.*a*,

$$x_B = 3 \cos \theta \text{ ft}$$

Time Derivative: Taking the time derivative,

$$v_{CD} = \dot{x}_B = -3 \sin \theta\dot{\theta} \text{ ft/s} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = \omega = 10$ rad/s since ω acts in the positive rotational sense of θ. When $\theta = 30°$,

$$v_{CD} = -3 \sin 30° \left(10\right) = -15 \text{ ft/s} = 15 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

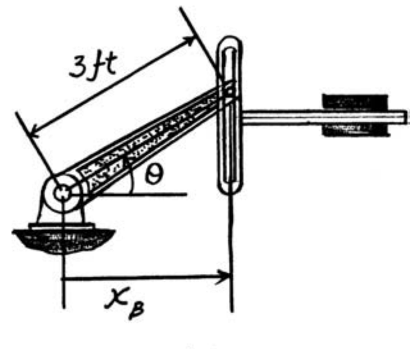
Taking the time derivative of Eq.(1) gives

$$a_{CD} = \ddot{x}_B = -3\left(\sin \theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right)$$

Since ω is constant, $\ddot{\theta} = \alpha = 0$. When $\theta = 30°$,

$$a_{CD} = -3\left[\sin 30°(0) + \cos 30°(10^2)\right]$$

$$= -259.80 \text{ ft/s} = 260 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$



(a)

The negative signs indicates that $\mathbf{v}_{CD}$ and $\mathbf{a}_{CD}$ act towards the negative sense of x_B.

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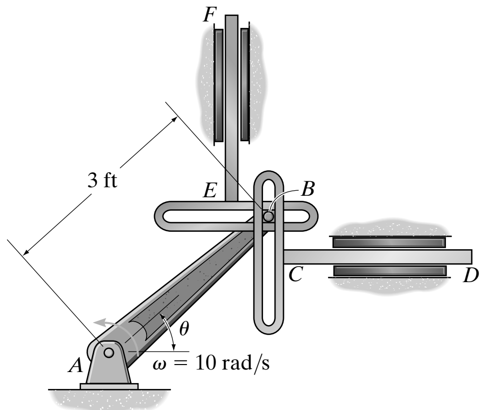
16–50. Peg B attached to the crank AB slides in the slots mounted on follower rods, which move along the vertical and horizontal guides. If the crank rotates with a constant angular velocity of $\omega = 10$ rad/s, determine the velocity and acceleration of rod EF at the instant $\theta = 30°$.



Position Coordinates: From the geometry shown in Fig.*a*,

$$y_B = 3\sin\theta \text{ ft}$$

Time Derivatives: Taking the time derivative,

$$v_{EF} = \dot{y}_B = 3\cos\theta\dot{\theta}\text{ ft/s} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = \omega = 10$ rad/s since ω acts in the positive rotational sense of θ. When $\theta = 30°$,

$$v_{EF} = 3\cos 30°(10) = 25.98\text{ ft/s} = 26\text{ ft/s}\uparrow \qquad \textbf{Ans.}$$

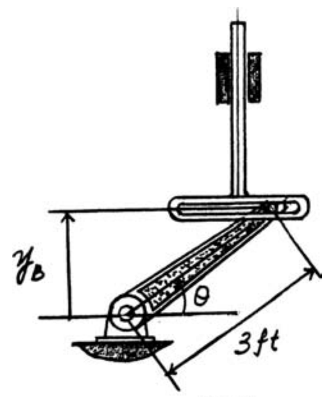
The time derivative of Eq.(1) gives

$$a_{EF} = \ddot{y}_B = 3\left[\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2\right]\text{ ft/s}^2$$

Since ω is constant, $\ddot{\theta} = \alpha = 0$. When $\theta = 30°$,

$$a_{EF} = 3\left[\cos 30°(0) - \sin 30°(10^2)\right]$$

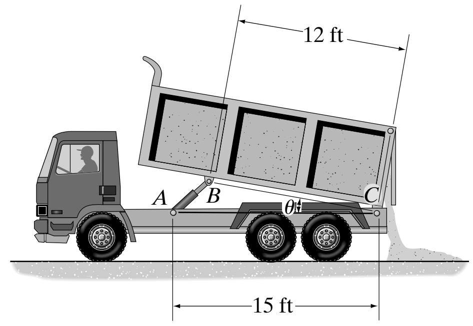
$$= -150\text{ ft/s}^2 = 150\text{ ft/s}^2\downarrow \qquad \textbf{Ans.}$$



(a)

The negative signs indicates that $\mathbf{a}_{EF}$ acts towards the negative sense of y_B.

16–51. If the hydraulic cylinder AB is extending at a constant rate of 1 ft/s, determine the dumpster's angular velocity at the instant $\theta = 30°$.



Position Coordinates: Applying the law of cosines to the geometry shown in Fig. *a*,

$$s^2 = 15^2 + 12^2 - 2(15)(12)\cos\theta$$

$$s^2 = (369 - 360\cos\theta)\text{ ft}^2 \qquad \textbf{(1)}$$

Time Derivatives: Taking the time derivative,

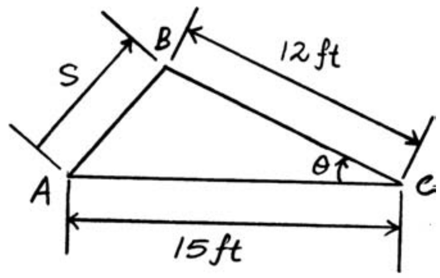
$$2s\dot{s} = 360\sin\theta\dot{\theta}$$

$$s\dot{s} = 180\sin\theta\dot{\theta} \qquad \textbf{(2)}$$

$\dot{s} = +1$ ft/s since the hydraulic cylinder is extending towards the positive sense of s. When $\theta = 30°$, from Eq. (1), $s = \sqrt{369 - 360\cos 30°} = 7.565$ ft. Thus, Eq.(2) gives

$$7.565(1) = 180\sin 30°\dot{\theta}$$

$$\dot{\theta} = 0.0841\text{ rad/s} \qquad \textbf{Ans.}$$



(a)

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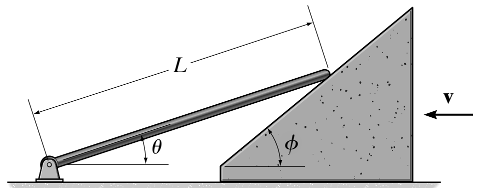
***16–52.** If the wedge moves to the left with a constant velocity **v**, determine the angular velocity of the rod as a function of θ.



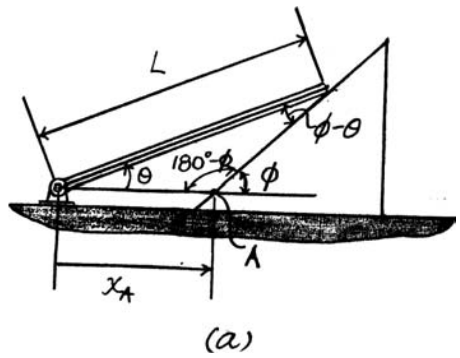
Position Coordinates: Applying the law of sines to the geometry shown in Fig. a,

$$\frac{x_A}{\sin(\phi - \theta)} = \frac{L}{\sin\left(180^\circ - \phi\right)}$$

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\left(180^\circ - \phi\right)}$$

However, $\sin\left(180^\circ - \phi\right) = \sin\phi$. Therefore,

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\phi}$$



Time Derivative: Taking the time derivative,

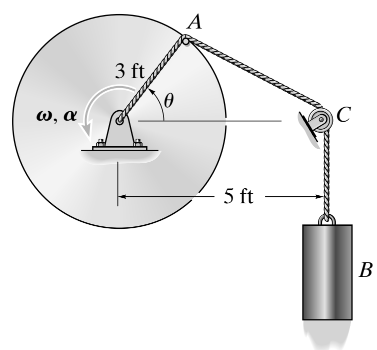
$$\dot{x}_A = \frac{L\cos(\phi - \theta)(-\dot{\theta})}{\sin\phi}$$

$$v_A = \dot{x}_A = -\frac{L\cos(\phi - \theta)\dot{\theta}}{\sin\phi} \qquad \textbf{(1)}$$

Since point A is on the wedge, its velocity is $v_A = -v$. The negative sign indicates that $\mathbf{v}_A$ is directed towards the negative sense of x_A. Thus, Eq. (1) gives

$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi - \theta)} \qquad \textbf{Ans.}$$

•16–53. At the instant shown, the disk is rotating with an angular velocity of $\boldsymbol{\omega}$ and has an angular acceleration of $\boldsymbol{\alpha}$. Determine the velocity and acceleration of cylinder B at this instant. Neglect the size of the pulley at C.



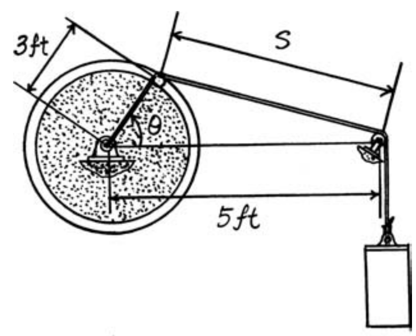
$$s = \sqrt{3^2 + 5^2 - 2(3)(5)\cos\theta}$$

$$v_B = \dot{s} = \frac{1}{2}(34 - 30\cos\theta)^{-\frac{1}{2}}(30\sin\theta)\dot{\theta}$$

$$v_B = \frac{15\,\omega\sin\theta}{(34 - 30\cos\theta)^{\frac{1}{2}}} \qquad \textbf{Ans.}$$

$$a_B = \dot{s} = \frac{15\,\omega\cos\theta\dot{\theta} + 15\dot{\omega}\sin\theta}{\sqrt{34 - 30\cos\theta}} + \frac{\left(-\frac{1}{2}\right)(15\omega\sin\theta)\left(30\sin\theta\dot{\theta}\right)}{(34 - 30\cos\theta)^{\frac{3}{2}}}$$

$$= \frac{15\left(\omega^2\cos\theta + \alpha\sin\theta\right)}{(34 - 30\cos\theta)^{\frac{1}{2}}} - \frac{225\,\omega^2\sin^2\theta}{(34 - 30\cos\theta)^{\frac{3}{2}}} \qquad \textbf{Ans.}$$



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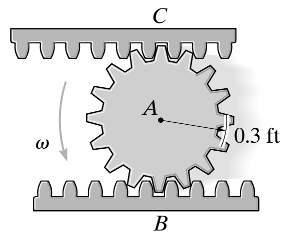
16–54. Pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 4$ rad/s. Determine the velocity of the gear rack C.

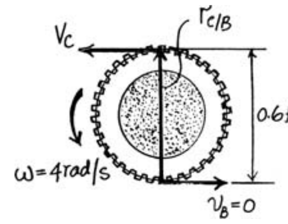


$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$(\overset{+}{\leftarrow})$ $\quad v_C = 0 + 4(0.6)$

$$v_C = 2.40 \text{ ft/s}$$ **Ans.**

Also:

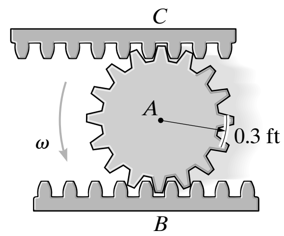


$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$-v_C\,\mathbf{i} = 0 + (4\mathbf{k}) \times (0.6\mathbf{j})$$

$$v_C = 2.40 \text{ ft/s}$$ **Ans.**

16–55. Pinion gear A rolls on the gear racks B and C. If B is moving to the right at 8 ft/s and C is moving to the left at 4 ft/s, determine the angular velocity of the pinion gear and the velocity of its center A.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$(\overset{+}{\rightarrow})$ $\quad -4 = 8 - 0.6(\omega)$

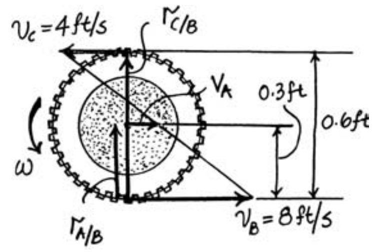
$$\omega = 20 \text{ rad/s}$$ **Ans.**

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$(\overset{+}{\rightarrow})$ $\quad v_A = 8 - 20(0.3)$

$$v_A = 2 \text{ ft/s} \rightarrow$$ **Ans.**

Also,



$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$-4\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (0.6\mathbf{j})$$

$$-4 = 8 - 0.6\omega$$

$$\omega = 20 \text{ rad/s}$$ **Ans.**

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$v_A\,\mathbf{i} = 8\mathbf{i} + 20\mathbf{k} \times (0.3\mathbf{j})$$

$$v_A = 2 \text{ ft/s} \rightarrow$$ **Ans.**

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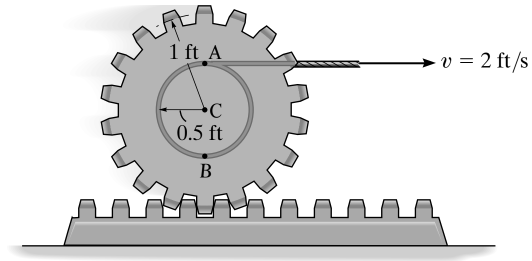
***16–56.** The gear rests in a fixed horizontal rack. A cord is wrapped around the inner core of the gear so that it remains horizontally tangent to the inner core at A. If the cord is pulled to the right with a constant speed of 2 ft/s, determine the velocity of the center of the gear, C.



$$\mathbf{v}_A = \mathbf{v}_D + \mathbf{v}_{A/D}$$

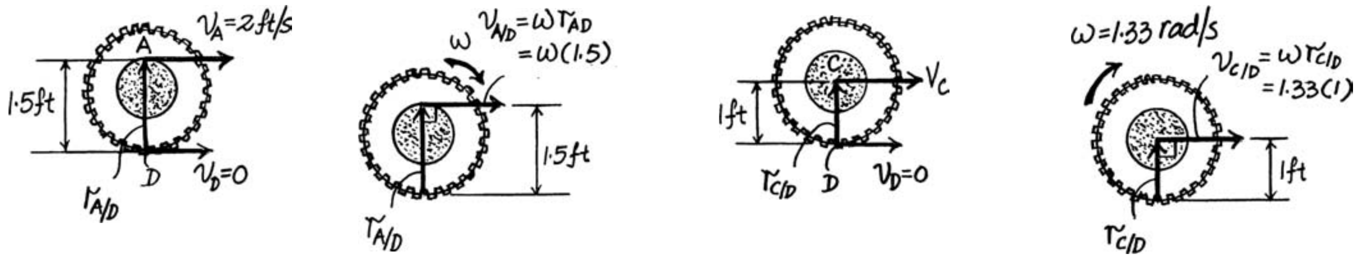
$$\left[\underset{\rightarrow}{2}\right] = 0 + \left[\omega(\underset{\rightarrow}{1.5})\right]$$

$(\overset{+}{\rightarrow})$ $\quad 2 = 1.5\omega \qquad \omega = 1.33 \text{ rad/s}$

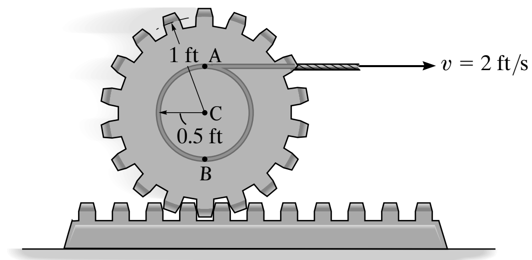
$$\mathbf{v}_C = \mathbf{v}_D + \mathbf{v}_{C/D}$$

$$\left[\underset{\rightarrow}{v_C}\right] = 0 + \left[\underset{\rightarrow}{1.33\,(1)}\right]$$

$(\overset{+}{\rightarrow})$ $\quad v_C = 1.33 \text{ ft/s} \rightarrow$ **Ans.**



•16–57. Solve Prob. 16–56 assuming that the cord is wrapped around the gear in the opposite sense, so that the end of the cord remains horizontally tangent to the inner core at B and is pulled to the right at 2 ft/s.



$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D}$$

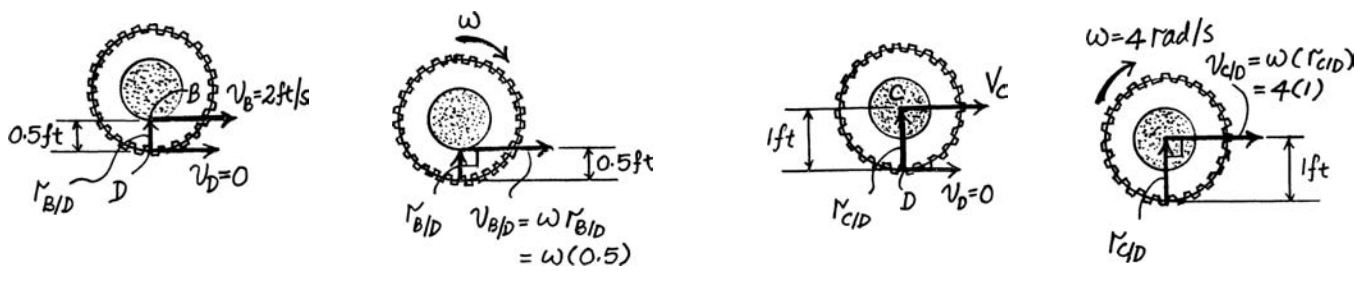
$$\left[\underset{\rightarrow}{2}\right] = 0 + \left[\omega(\underset{\rightarrow}{0.5})\right]$$

$(\overset{+}{\rightarrow})$ $\quad 2 = 0.5\omega \qquad \omega = 4 \text{ rad/s}$

$$\mathbf{v}_C = \mathbf{v}_D + \mathbf{v}_{C/D}$$

$$\left[\underset{\rightarrow}{v_C}\right] = 0 + \left[4\,(\underset{\rightarrow}{1})\right]$$

$(\overset{+}{\rightarrow})$ $\quad v_C = 4 \text{ ft/s} \rightarrow$ **Ans.**



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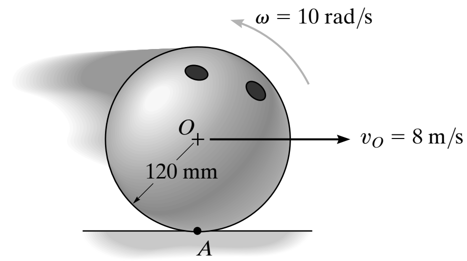
16–58. A bowling ball is cast on the "alley" with a backspin of $\omega = 10$ rad/s while its center O has a forward velocity of $v_O = 8$ m/s. Determine the velocity of the contact point A in contact with the alley.



$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

$(\overset{+}{\rightarrow})$ $$v_A = 8 + 10(0.12)$$

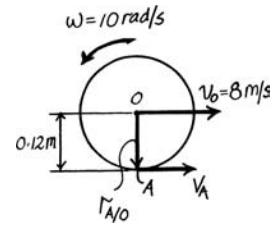
$$v_A = 9.20 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

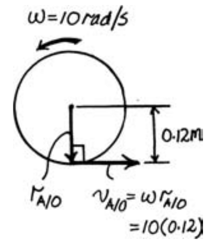
Also,

$$\mathbf{v}_A = \mathbf{v}_O + \omega \times \mathbf{r}_{A/O}$$

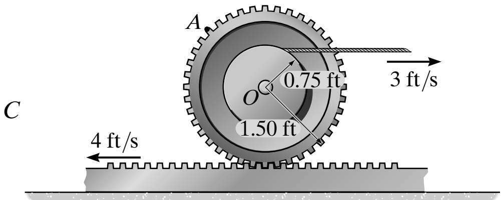
$$v_A\,\mathbf{i} = 8\mathbf{i} + (10\mathbf{k}) \times (-0.12\mathbf{j})$$

$(\overset{+}{\rightarrow})$ $$v_A = 9.20 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$





16–59. Determine the angular velocity of the gear and the velocity of its center O at the instant shown.



General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

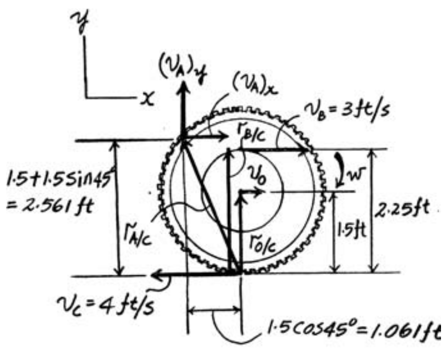
$$3\mathbf{i} = -4\mathbf{i} + \left(-\omega\mathbf{k}\right) \times \left(2.25\mathbf{j}\right)$$

$$3\mathbf{i} = \left(2.25\omega - 4\right)\mathbf{i}$$

Equating the **i** components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{Ans. (2)}$$



(a)

For points O and C,

$$\mathbf{v}_O = \mathbf{v}_C + \omega \times \mathbf{r}_{O/C}$$

$$= -4\mathbf{i} + \left(-3.111\mathbf{k}\right) \times \left(1.5\mathbf{j}\right)$$

$$= [0.6667\mathbf{i}] \text{ ft/s}$$

Thus,

$$v_O = 0.667 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

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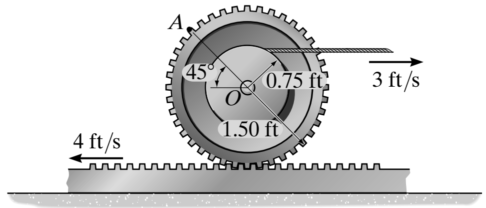
***16–60.** Determine the velocity of point A on the rim of the gear at the instant shown.

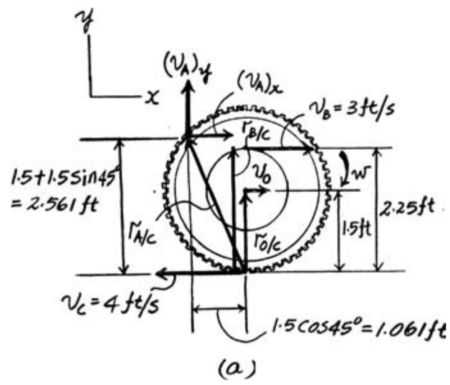


General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{B/C}$$

$$3\mathbf{i} = -4\mathbf{i} + \left(-\omega\mathbf{k}\right) \times \left(2.25\mathbf{j}\right)$$

$$3\mathbf{i} = \left(2.25\omega - 4\right)\mathbf{i}$$



Equating the $\mathbf{i}$ components yields

$$3 = 2.25\omega - 4 \tag{1}$$

$$\omega = 3.111 \text{ rad/s} \tag{2}$$

For points A and C,

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C}$$

$$\left(v_A\right)_x \mathbf{i} + \left(v_A\right)_y \mathbf{j} = -4\mathbf{i} + \left(-3.111\mathbf{k}\right) \times \left(-1.061\mathbf{i} + 2.561\mathbf{j}\right)$$

$$\left(v_A\right)_x \mathbf{i} + \left(v_A\right)_y \mathbf{j} = 3.9665\mathbf{i} + 3.2998\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$\left(v_A\right)_x = 3.9665 \text{ ft/s} \qquad \left(v_A\right)_y = 3.2998 \text{ ft/s}$$

Thus, the magnitude of v_A is

$$v_A = \sqrt{\left(v_A\right)_x^{\,2} + \left(v_A\right)_y^{\,2}} = \sqrt{3.9665^2 + 3.2998^2} = 5.16 \text{ ft/s} \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{\left(v_A\right)_y}{\left(v_A\right)_x}\right] = \tan^{-1}\left(\frac{3.2998}{3.9665}\right) = 39.8° \qquad \textbf{Ans.}$$

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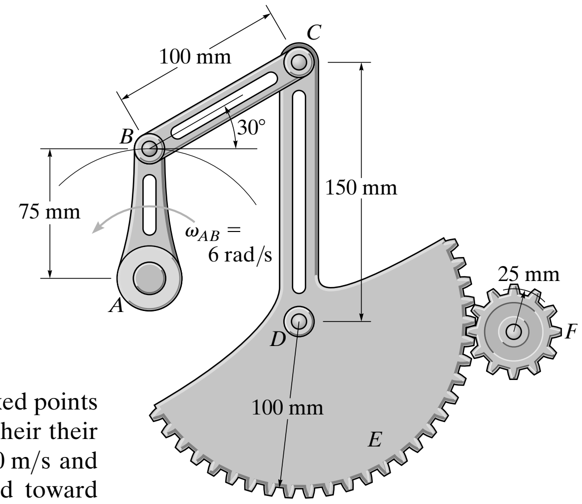
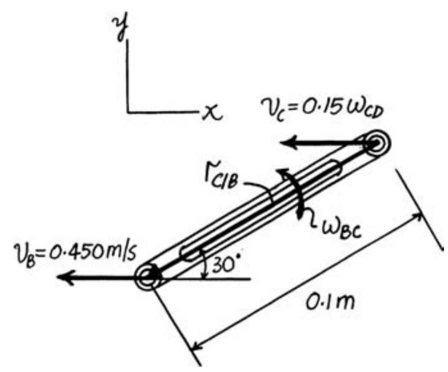
•16–61. The rotation of link AB creates an oscillating movement of gear F. If AB has an angular velocity of $\omega_{AB} = 6$ rad/s, determine the angular velocity of gear F at the instant shown. Gear E is rigidly attached to arm CD and pinned at D to a fixed point.



Kinematic Diagram: Since link AB and arm CD are rotating about the fixed points A and D respectively, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular their their respective arms with the magnitude of $v_B = \omega_{AB}\, r_{AB} = 6(0.075) = 0.450$ m/s and $v_C = \omega_{CD}\, r_{CD} = 0.15\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ and $\mathbf{v}_C$ are directed toward negative x axis.

Velocity Equation: Here, $\mathbf{r}_{B/C} = \{-0.1 \cos 30°\mathbf{i} + 0.1 \sin 30°\mathbf{j}\}$ m $= \{-0.08660\mathbf{i} + 0.05\mathbf{j}\}$ m. Applying Eq. 16–16, we have



$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

$$-0.450\mathbf{i} = -0.15\omega_{CD}\,\mathbf{i} + (\omega_{BC}\mathbf{k}) \times (0.08660\mathbf{i} + 0.05\mathbf{j})$$

$$-0.450\mathbf{i} = -(0.05\omega_{BC} + 0.15\omega_{CD})\mathbf{i} + 0.08660\omega_{BC}\mathbf{j}$$

Equating **i** and **j** components gives

$$0 = 0.08660\omega_{BC} \qquad \omega_{BC} = 0$$

$$-0.450 = -[0.05(0) + 0.15\omega_{CD}] \qquad \omega_{CD} = 3.00 \text{ rad/s}$$

Angular Motion About a Fixed Point: The angular velocity of gear E is the same with arm CD since they are attached together. Then, $\omega_E = \omega_{CD} = 3.00$ rad/s. Here, $\omega_E\, r_E = \omega_F\, r_F$ where ω_F is the angular velocity of gear F.

$$\omega_F = \frac{r_E}{r_F}\omega_E = \left(\frac{100}{25}\right)(3.00) = 12.0 \text{ rad/s} \qquad \textbf{Ans.}$$

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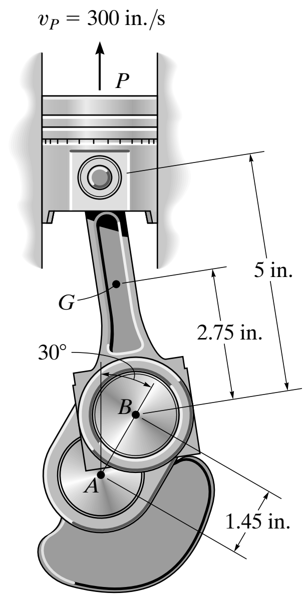
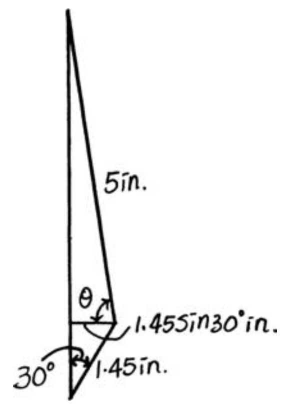
16–62. Piston P moves upward with a velocity of 300 in./s at the instant shown. Determine the angular velocity of the crankshaft AB at this instant.



From the geometry:

$$\cos\theta = \frac{1.45\sin 30^\circ}{5} \qquad \theta = 81.66^\circ$$



For link BP

$\mathbf{v}_P = \{300\mathbf{j}\}$ in/s $\qquad \mathbf{v}_B = -v_B\cos 30^\circ\mathbf{i} + v_B\sin 30^\circ\mathbf{j} \qquad \omega = -\omega_{BP}\mathbf{k}$

$\mathbf{r}_{P/B} = \{-5\cos 81.66^\circ\mathbf{i} + 5\sin 81.66^\circ\mathbf{j}\}$ in.

$\mathbf{v}_P = \mathbf{v}_B + \omega \times \mathbf{r}_{P/B}$

$300\mathbf{j} = (-v_B\cos 30^\circ\mathbf{i} + v_B\sin 30^\circ\mathbf{j}) + (-\omega_{BP}\mathbf{k}) \times (-5\cos 81.66^\circ\mathbf{i} + 5\sin 81.66^\circ\mathbf{j})$

$300\mathbf{j} = (-v_B\cos 30^\circ\mathbf{i} + 5\sin 81.66^\circ\omega_{BP})\mathbf{i} + (v_B\sin 30^\circ + 5\cos 81.66^\circ\,\omega_{BP})\mathbf{j}$

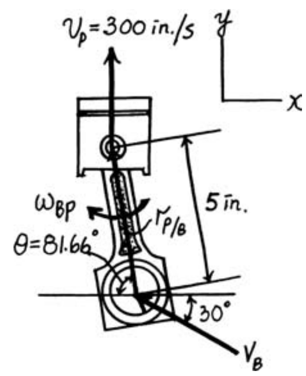
Equating the **i** and **j** components yields:

$$0 = -v_B\cos 30^\circ + 5\sin 81.66^\circ\,\omega_{BP} \qquad \textbf{(1)}$$

$$300 = v_B\sin 30^\circ + 5\cos 81.66^\circ\,\omega_{BP} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$\omega_{BP} = 83.77\text{ rad/s} \qquad v_B = 478.53\text{ in./s}$$



For crankshaft AB: Crankshaft AB rotates about the fixed point A. Hence

$$v_B = \omega_{AB}\,r_{AB}$$

$$478.53 = \omega_{AB}(1.45) \qquad \omega_{AB} = 330\text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

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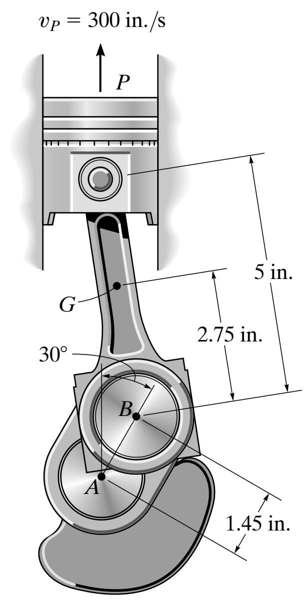
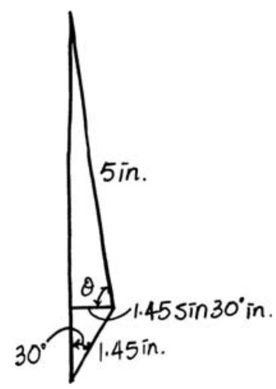
16–63. Determine the velocity of the center of gravity G of the connecting rod at the instant shown. Piston P is moving upward with a velocity of 300 in./s.



From the geometry:

$$\cos\theta = \frac{1.45 \sin 30^\circ}{5} \qquad \theta = 81.66^\circ$$



For link BP

$\mathbf{v}_P = \{300\mathbf{j}\}$ in/s $\qquad \mathbf{v}_B = -v_B \cos 30^\circ \mathbf{i} + v_B \sin 30^\circ \mathbf{j} \qquad \omega = -\omega_{BP}\mathbf{k}$

$\mathbf{r}_{P/B} = \{-5 \cos 81.66^\circ \mathbf{i} + 5 \sin 81.66^\circ \mathbf{j}\}$ in.

$\mathbf{v}_P = \mathbf{v}_B + \omega \times \mathbf{r}_{P/B}$

$300\mathbf{j} = (-v_B \cos 30^\circ \mathbf{i} + v_B \sin 30^\circ \mathbf{j}) + (-\omega_{BP}\mathbf{k}) \times (-5 \cos 81.66^\circ \mathbf{i} + 5 \sin 81.66^\circ \mathbf{j})$

$300\mathbf{j} = (-v_B \cos 30^\circ + 5 \sin 81.66^\circ\, \omega_{BP})\mathbf{i} + (v_B \sin 30^\circ + 5 \cos 81.66^\circ\, \omega_{BP})\mathbf{j}$

Equating the **i** and **j** components yields:

$$0 = -v_B \cos 30^\circ + 5 \sin 81.66^\circ\, \omega_{BP} \qquad \textbf{(1)}$$

$$300 = v_B \sin 30^\circ + 5 \cos 81.66^\circ\, \omega_{BP} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$\omega_{BP} = 83.77$ rad/s $\qquad v_B = 478.53$ in./s

$\mathbf{v}_P = \{300\mathbf{j}\}$ in/s $\qquad \omega = \{-83.77\mathbf{k}\}$ rad/s

$\mathbf{r}_{G/P} = \{2.25 \cos 81.66^\circ \mathbf{i} - 2.25 \sin 81.66^\circ \mathbf{j}\}$ in.

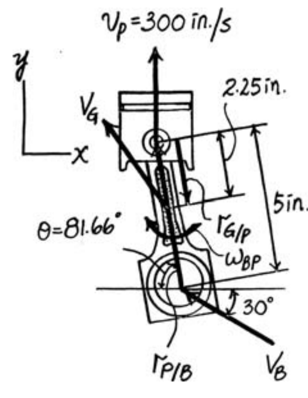
$\mathbf{v}_G = \mathbf{v}_P + \omega \times \mathbf{r}_{G/P}$

$\quad = 300\mathbf{j} + (-83.77\mathbf{k}) \times (2.25 \cos 81.66^\circ \mathbf{i} - 2.25 \sin 81.66^\circ \mathbf{j})$

$\quad = \{-186.49\mathbf{i} + 272.67\mathbf{j}\}$ in./s

$v_G = \sqrt{(-186.49)^2 + 272.67^2} = 330$ in./s **Ans.**

$\theta = \tan^{-1}\left(\dfrac{272.67}{186.49}\right) = 55.6^\circ$ ◺ **Ans.**



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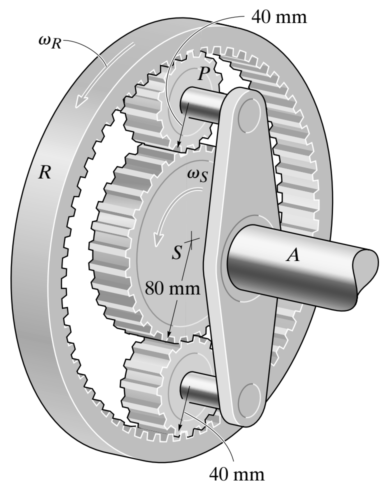
***16–64.** The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



$$v_A = 5(80) = 400 \text{ mm/s} \leftarrow$$

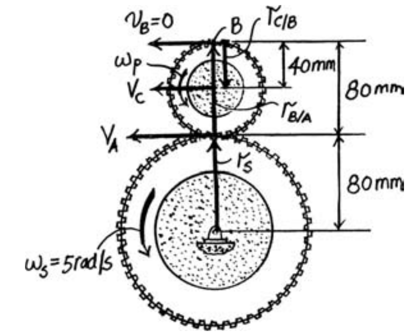
$$v_B = 0$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$0 = -400\mathbf{i} + (\omega_p\,\mathbf{k}) \times (80\mathbf{j})$$

$$0 = -400\mathbf{i} - 80\omega_p\,\mathbf{i}$$

$$\omega_P = -5 \text{ rad/s} = 5 \text{ rad/s}$$ **Ans.**

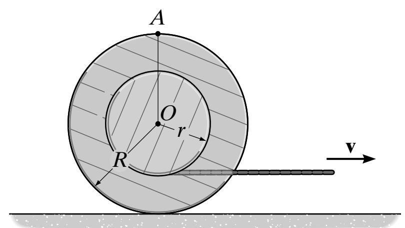


$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$$

$$\omega_A = \frac{200}{120} = 1.67 \text{ rad/s}$$ **Ans.**

•16–65. Determine the velocity of the center O of the spool when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. *a*.

General Plane Motion: Applying the relative velocity equation and referring to Fig. *a*,

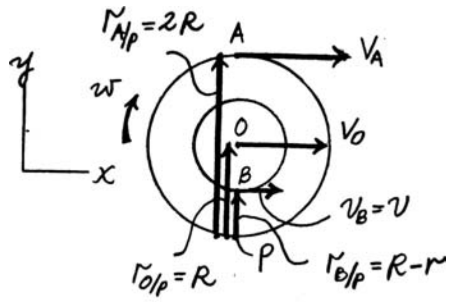
$$\mathbf{v}_B = \mathbf{v}_P + \omega \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad\qquad \omega = \frac{v}{R - r}$$



(a)

Using this result,

$$\mathbf{v}_O = \mathbf{v}_P + \omega \times \mathbf{r}_{O/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times R\mathbf{j}$$

$$\mathbf{v}_O = \left(\frac{R}{R - r}\right)v \rightarrow$$ **Ans.**

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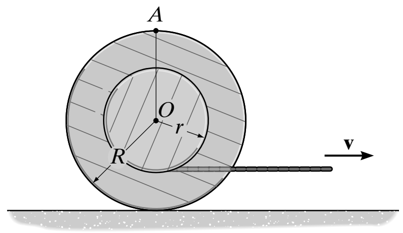
16–66. Determine the velocity of point A on the outer rim of the spool at the instant shown when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. *a*.

General Plane Motion: Applying the relative velocity equation and referring to Fig. *a*,

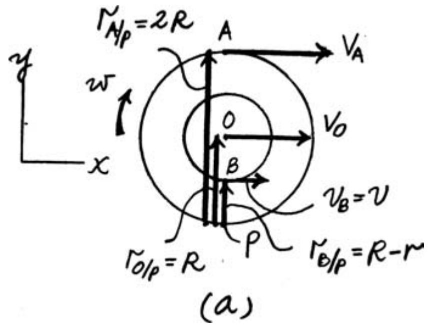
$$\mathbf{v}_B = \mathbf{v}_P + \omega \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the **i** components, yields

$$v = \omega(R - r) \qquad\qquad \omega = \frac{v}{R - r}$$



Using this result,

$$\mathbf{v}_A = \mathbf{v}_P + \omega \times \mathbf{r}_{A/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times 2R\mathbf{j}$$

$$= \left[\left(\frac{2R}{R - r}\right)v\right]\mathbf{i}$$

Thus,

$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow \qquad \textbf{Ans.}$$

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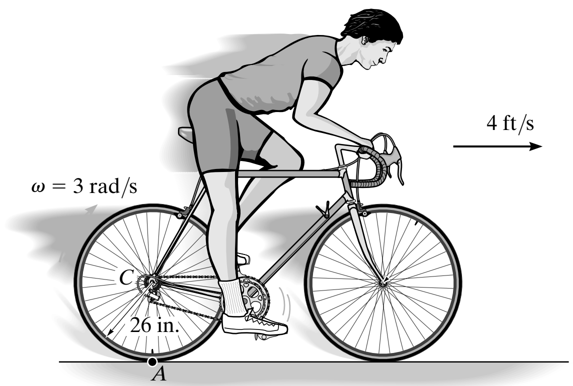
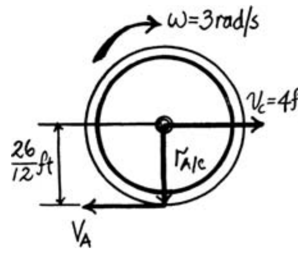
16–67. The bicycle has a velocity $v = 4$ ft/s, and at the same instant the rear wheel has a clockwise angular velocity $\omega = 3$ rad/s, which causes it to slip at its contact point A. Determine the velocity of point A.



$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$\left[\underset{\leftarrow}{v_A}\right] = \left[\underset{\rightarrow}{4}\right] + \left[\underset{\leftarrow}{\left(\frac{26}{12}\right)}(3)\right]$$

$v_A = 2.5$ ft/s $\leftarrow$ **Ans.**



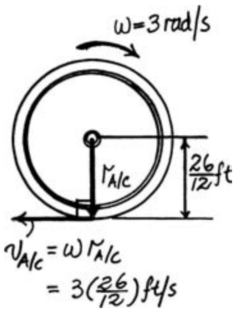
Also,

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C}$$

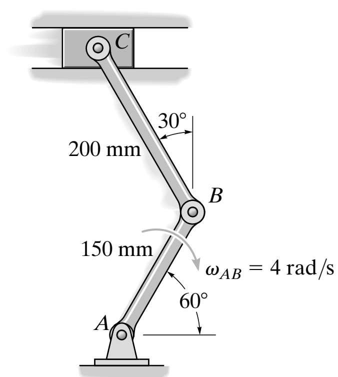
$$\mathbf{v}_A = 4\mathbf{i} + (-3\mathbf{k}) \times \left(-\frac{26}{12}\mathbf{j}\right)$$

$$\mathbf{v}_A = 4\mathbf{i} - 6.5\mathbf{i} = -2.5\mathbf{i}$$

$v_A = 2.5$ ft/s $\leftarrow$ **Ans.**



***16–68.** If bar AB has an angular velocity $\omega_{AB} = 4$ rad/s, determine the velocity of the slider block C at the instant shown.



For link AB: Link AB rotates about a fixed point A. Hence

$$v_B = \omega_{AB}\, r_{AB} = 4(0.15) = 0.6 \text{ m/s}$$

For link BC

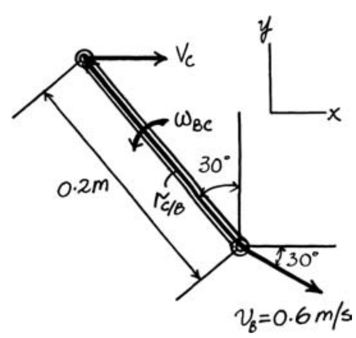
$$\mathbf{v}_B = \{0.6 \cos 30°\mathbf{i} - 0.6 \sin 30°\mathbf{j}\}\text{m/s} \qquad \mathbf{v}_C = v_C\mathbf{i} \qquad \boldsymbol{\omega} = \omega_{BC}\,\mathbf{k}$$

$$\mathbf{r}_{C/B} = \{-0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j}\} \text{ m}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$v_C\,\mathbf{i} = (0.6 \cos 30°\mathbf{i} - 0.6 \sin 30°\mathbf{j}) + (\omega_{BC}\,\mathbf{k}) \times (-0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j})$$

$$v_C\mathbf{i} = (0.5196 - 0.1732\omega_{BC})\mathbf{i} - (0.3 + 0.1\omega_{BC})\mathbf{j}$$



Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields:

$$0 = 0.3 + 0.1\omega_{BC} \qquad \omega_{BC} = -3 \text{ rad/s}$$

$v_C = 0.5196 - 0.1732(-3) = 1.04$ m/s $\rightarrow$ **Ans.**

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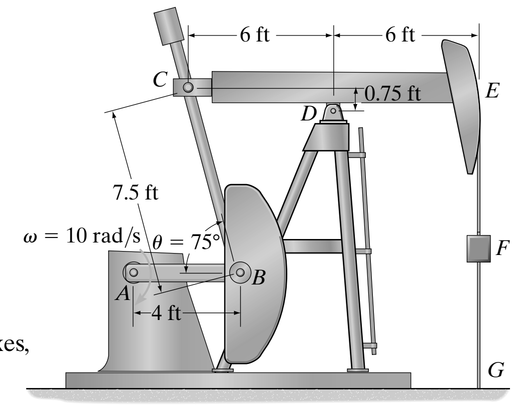
•16–69. The pumping unit consists of the crank pitman AB, connecting rod BC, walking beam CDE and pull rod F. If the crank is rotating with an angular velocity of $\omega = 10$ rad/s, determine the angular velocity of the walking beam and the velocity of the pull rod EFG at the instant shown.

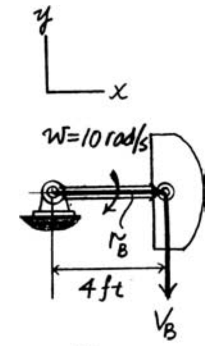


Rotation About a Fixed Axis: The crank and walking beam rotate about fixed axes, Figs. *a* and *b*. Thus, the velocity of points B, C, and E can be determined from

$$v_B = \omega \times r_B = \left(-10\mathbf{k}\right) \times \left(4\mathbf{i}\right) = \left[-40\mathbf{j}\right]\text{ft/s}$$

$$v_C = \omega_{CDE} \times r_{DC} = \left(\omega_{CDE}\mathbf{k}\right) \times \left(-6\mathbf{i} + 0.75\mathbf{j}\right) = -0.75\omega_{CDE}\mathbf{i} - 6\omega_{CDE}\mathbf{j}$$

$$v_E = \omega_{CDE} \times r_{DE} = \left(\omega_{CDE}\mathbf{k}\right) \times \left(6\mathbf{i}\right) = 6\omega_{CDE}\mathbf{j} \qquad \textbf{(1)}$$



(a)

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of link BC shown in Fig. *c*,

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

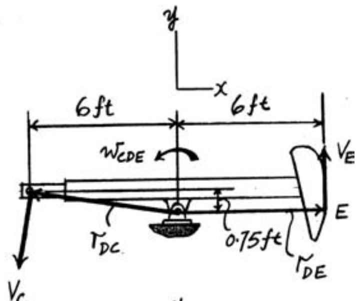
$$-0.75\omega_{CDE}\mathbf{i} - 6\omega_{CDE}\mathbf{j} = -40\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (-7.5\cos 75°\,\mathbf{i} + 7.5\sin 75°\,\mathbf{j})$$

$$-0.75\omega_{CDE}\mathbf{i} - 6\omega_{CDE}\mathbf{j} = -7.244\omega_{BC}\,\mathbf{i} - (1.9411\omega_{BC} + 40)\mathbf{j}$$

Equating the **i** and **j** components

$$-0.75\omega_{CDE} = -7.244\omega_{BC} \qquad \textbf{(2)}$$

$$-6\omega_{CDE} = -(1.9411\omega_{BC} + 40) \qquad \textbf{(3)}$$



(b)

Solving Eqs. (1) and (2) yields

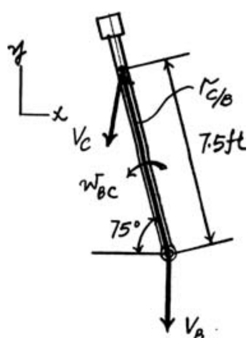
$\omega_{BC} = 0.714$ rad/s $\qquad \omega_{CDE} = 6.898$ rad/s $= 6.90$ rad/s **Ans.**

Substituting the result for ω_{CDE} into Eq. (1),

$$\mathbf{v}_E = \theta(6.898) = [41.39\mathbf{j}]\text{ ft/s}$$

Thus,

$$v_E = 41.4\text{ ft/s} \uparrow \qquad \textbf{Ans.}$$



(C)

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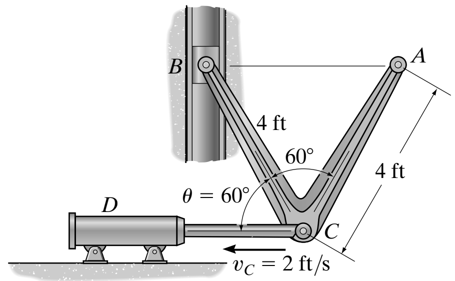
16–70. If the hydraulic cylinder shortens at a constant rate of $v_C = 2$ ft/s, determine the angular velocity of link ACB and the velocity of block B at the instant shown.



General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of link ABC shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

$$v_B\,\mathbf{j} = -2\mathbf{i} + (-\omega\mathbf{k}) \times (-4\cos 60°\mathbf{i} + 4\sin 60°\,\mathbf{j})$$

$$v_B\,\mathbf{j} = (3.464\omega - 2)\mathbf{i} + 2\omega\mathbf{j}$$

Equating the **i** and **j** components yields

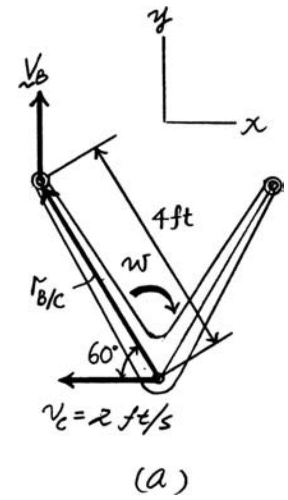
$$0 = 3.464\omega - 2$$

$$v_B = 2\omega$$

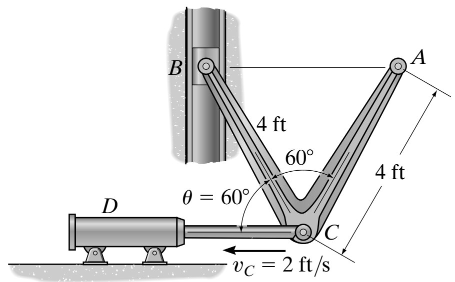
Solving,

$$\omega = 0.577 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_B = 1.15\text{ft/s} \uparrow \qquad \textbf{Ans.}$$



16–71. If the hydraulic cylinder shortens at a constant rate of $v_C = 2$ ft/s, determine the velocity of end A of link ACB at the instant shown.



General Plane Motion: First, applying the relative velocity equation to points B and C and referring to the kinematic diagram of link ABC shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

$$v_B\,\mathbf{j} + -2\mathbf{i} + (-\omega\mathbf{k}) \times (-4\cos 60°\,\mathbf{i} + 4\sin 60°\,\mathbf{j})$$

$$v_B\,\mathbf{j} = (3.464\omega - 2)\mathbf{i} + 2\omega\mathbf{j}$$

Equating the **i** components yields

$$0 = 3.464\omega - 2 \qquad \omega = 0.5774 \text{ rad/s}$$

Then, for points A and C using the result of ω,

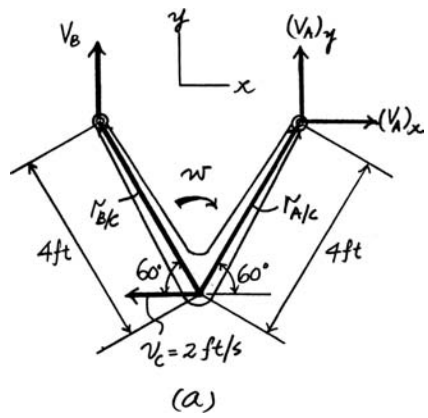
$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C}$$

$$(v_A)_x\,\mathbf{i} + (v_A)_y\,\mathbf{j} = -2\mathbf{i} + (-0.5774\mathbf{k}) \times (4\cos 60°\,\mathbf{i} + 4\sin 60°\,\mathbf{j})$$

$$(v_A)_x\,\mathbf{i} + (v_A)_y\,\mathbf{j} = -1.1547\mathbf{j}$$

Equating the **i** and **j** components yields

$$(v_A)_x = 0 \qquad (v_A)_y = -1.1547 \text{ ft/s} = 1.1547 \text{ ft/s} \downarrow$$



Thus,

$$v_A = (v_A)_y = 1.15 \text{ ft/s}\downarrow \qquad \textbf{Ans.}$$

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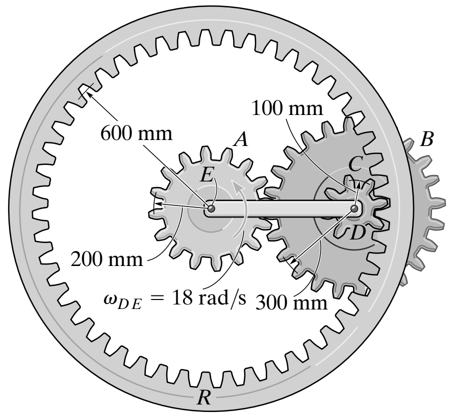
***16–72.** The epicyclic gear train consists of the sun gear A which is in mesh with the planet gear B. This gear has an inner hub C which is fixed to B and in mesh with the fixed ring gear R. If the connecting link DE pinned to B and C is rotating at $\omega_{DE} = 18$ rad/s about the pin at E, determine the angular velocities of the planet and sun gears.



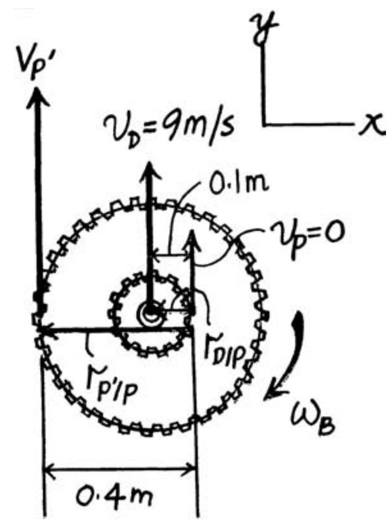
$v_D = r_{DE}\,\omega_{DE} = (0.5)(18) = 9$ m/s $\uparrow$

The velocity of the contact point P with the ring is zero.

$\mathbf{v}_D = \mathbf{v}_P + \omega \times \mathbf{r}_{D/P}$

$9\mathbf{j} = 0 + (-\omega_B\,\mathbf{k}) \times (-0.1\mathbf{i})$

$\omega_B = 90$ rad/s $\circlearrowright$ **Ans.**



Let P' be the contact point between A and B.

$\mathbf{v}_{P'} = \mathbf{v}_P + \omega \times \mathbf{r}_{P'/P}$

$v_{P'}\,\mathbf{j} = \mathbf{0} + (-90\mathbf{k}) \times (-0.4\mathbf{i})$

$v_{P'} = 36$ m/s $\uparrow$

$\omega_A = \dfrac{v_{P'}}{r_A} = \dfrac{36}{0.2} = 180$ rad/s $\circlearrowleft$ **Ans.**

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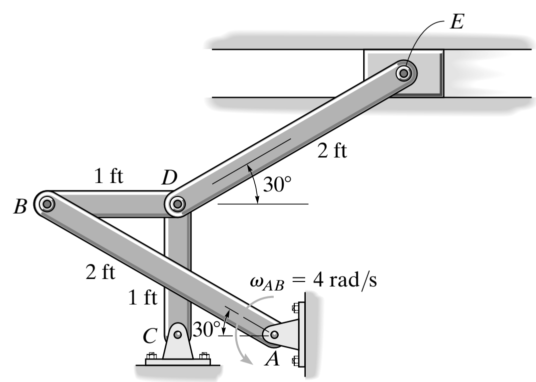
•16–73. If link AB has an angular velocity of $\omega_{AB} = 4$ rad/s at the instant shown, determine the velocity of the slider block E at this instant. Also, identify the type of motion of each of the four links.

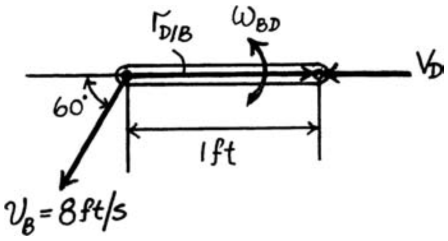


Link AB rotates about the fixed point A. Hence

$$v_B = \omega_{AB}\, r_{AB} = 4(2) = 8 \text{ ft/s}$$

For link BD

$$\mathbf{v}_B = \{-8\cos 60°\mathbf{i} - 8\sin 60°\,\mathbf{j}\} \text{ ft/s} \qquad \mathbf{v}_D = -v_D\mathbf{i} \qquad \boldsymbol{\omega}_{BD} = \omega_{BD}\,\mathbf{k}$$



$$\mathbf{r}_{D/B} = \{1\mathbf{i}\} \text{ ft}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

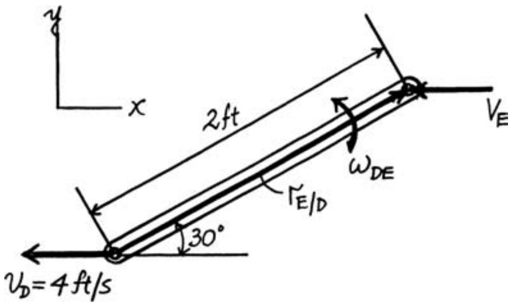
$$-v_D\mathbf{i} = (-8\cos 60°\mathbf{i} - 8\sin 60°\mathbf{j}) + (\omega_{BD}\mathbf{k}) \times (1\mathbf{i})$$

$$-v_D\,\mathbf{i} = -8\cos 60°\mathbf{i} + (\omega_{BD} - 8\sin 60°)\mathbf{j}$$

$(\overset{+}{\rightarrow})$ $\quad -v_D = -8\cos 60° \qquad v_D = 4$ ft/s

$(+\uparrow)$ $\quad 0 = \omega_{BD} - 8\sin 60° \qquad \omega_{BD} = 6.928$ rad/s

For Link DE



$$\mathbf{v}_D = \{-4\mathbf{i}\} \text{ ft/s} \qquad \boldsymbol{\omega}_{DE} = \omega_{DE}\,\mathbf{k} \qquad \mathbf{v}_E = -v_E\mathbf{i}$$

$$\mathbf{r}_{E/D} = \{2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\} \text{ ft}$$

$$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$$

$$-v_E\mathbf{i} = -4\mathbf{i} + (\omega_{DE}\mathbf{k}) \times (2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$-v_E\mathbf{i} = (-4 - 2\sin 30°\,\omega_{DE})\mathbf{i} + 2\cos 30°\omega_{DE}\mathbf{j}$$

$(\overset{+}{\rightarrow})$ $\quad 0 = 2\cos 30°\,\omega_{DE} \qquad \omega_{DE} = 0$

$(+\uparrow)$ $\quad -v_E = -4 - 2\sin 30°(0) \qquad v_E = 4\text{ft/s} \quad \leftarrow$ **Ans.**

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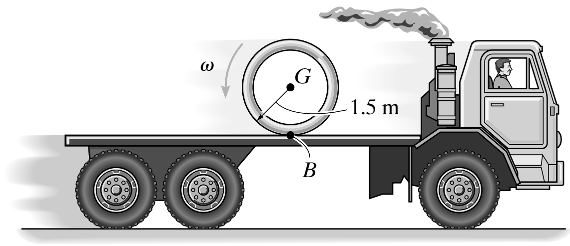
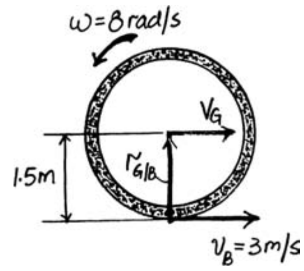
16–74. At the instant shown, the truck travels to the right at 3 m/s, while the pipe rolls counterclockwise at $\omega = 8$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



$$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$$

$$\left[\underset{\rightarrow}{\mathbf{v}_G}\right] = \left[\underset{\rightarrow}{3\cdot}\right] + \left[\underset{\leftarrow}{1.5(8)}\right]$$

$v_G = 9\text{ m/s} \leftarrow$ **Ans.**



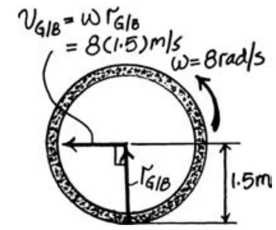
Also:

$$\mathbf{v}_G = \mathbf{v}_B + \omega \times \mathbf{r}_{G/B}$$

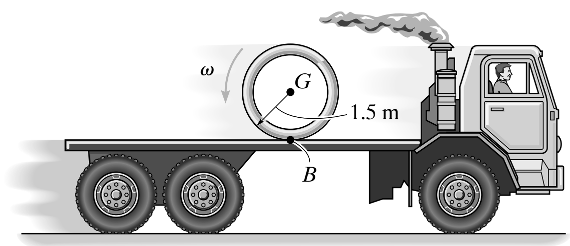
$$v_G\mathbf{i} = 3\mathbf{i} + (8\mathbf{k}) \times (1.5\mathbf{j})$$

$$v_G = 3 - 12$$

$v_G = -9\text{ m/s} = 9\text{ m/s} \leftarrow$ **Ans.**



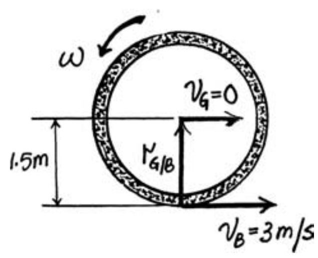
16–75. At the instant shown, the truck travels to the right at 8 m/s. If the pipe does not slip at B, determine its angular velocity if its mass center G appears to remain stationary to an observer on the ground.



$$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$$

$$0 = \left[\underset{\rightarrow}{8}\right] + \left[\underset{\leftarrow}{1.5\omega}\right]$$

$\omega = \dfrac{8}{1.5} = 5.33\text{ rad/s} \circlearrowleft$ **Ans.**



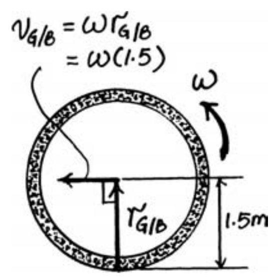
Also:

$$\mathbf{v}_G = \mathbf{v}_B + \omega \times \mathbf{r}_{G/B}$$

$$0\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (1.5\mathbf{j})$$

$$0 = 8 - 1.5\omega$$

$\omega = \dfrac{8}{1.5} = 5.33\text{ rad/s} \circlearrowleft$ **Ans.**



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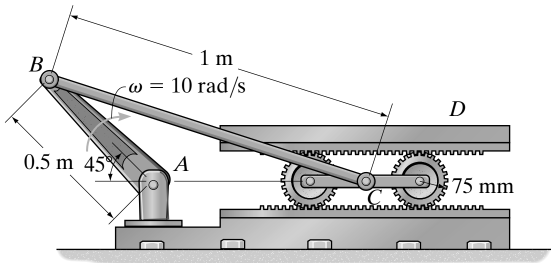
***16–76.** The mechanism of a reciprocating printing table is driven by the crank AB. If the crank rotates with an angular velocity of $\omega = 10$ rad/s, determine the velocity of point C at the instant shown.

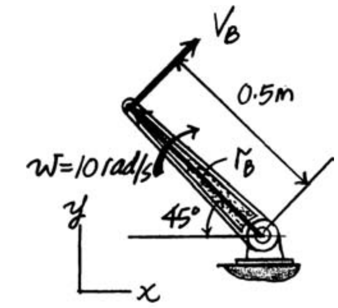


Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega \times \mathbf{r}_B$$

$$= (-10\mathbf{k}) \times (-0.5\cos 45°\,\mathbf{i} + 0.5\sin 45°\mathbf{j})$$

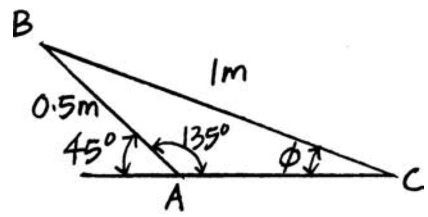
$$= [3.536\mathbf{i} + 3.536\mathbf{j}]\text{ m}$$



(a)

General Plane Motion: Applying the law of sines to the geometry shown in Fig. *b*,

$$\frac{\sin\phi}{0.5} = \frac{\sin 135°}{1} \qquad \phi = 20.70°$$



(b)

Applying the relative velocity equation to the kinematic diagram of link BC shown in Fig. *c*,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$3.536\mathbf{i} + 3.536\mathbf{j} = v_C\,\mathbf{i} + (-\omega_{BC}\,\mathbf{k}) \times (-1\cos 20.70°\,\mathbf{i} + 1\sin 20.70°\,\mathbf{j})$$

$$3.536\mathbf{i} + 3.536\mathbf{j} = (v_C + 0.3536\omega_{BC})\mathbf{i} + 0.9354\omega_{BC}\,\mathbf{j}$$

Equating the **i** and **j** components yields,

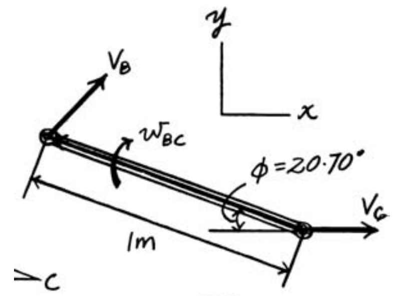
$$3.536 = v_C + 0.3536\omega_{BC}$$

$$3.536 = 0.9354\omega_{BC}$$

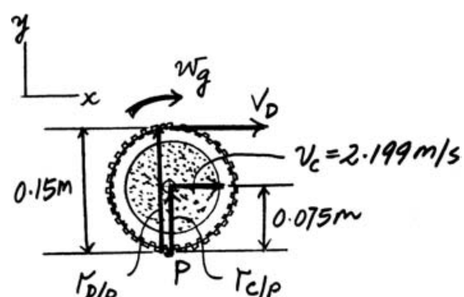
Solving,

$$\omega_{BC} = 3.780\text{ rad/s} \qquad \textbf{Ans.}$$

$$v_C = 2.199\text{ m/s} \qquad \textbf{Ans.}$$



(c)



(d)

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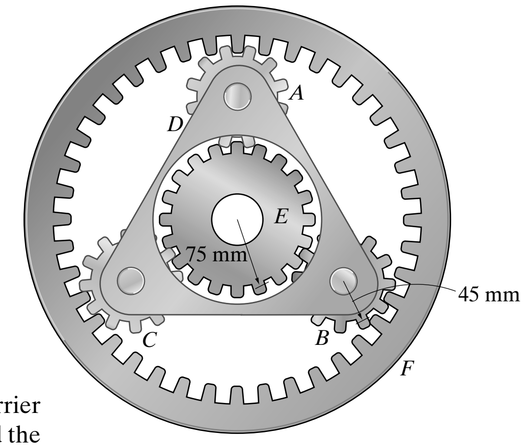
•16–77. The planetary gear set of an automatic transmission consists of three planet gears A, B, and C, mounted on carrier D, and meshed with the sun gear E and ring gear F. By controlling which gear of the planetary set rotates and which gear receives the engine's power, the automatic transmission can alter a car's speed and direction. If the carrier is rotating with a counterclockwise angular velocity of $\omega_D = 20$ rad/s while the ring gear is rotating with a clockwise angular velocity of $\omega_F = 10$ rad/s, determine the angular velocity of the planet gears and the sun gear. The radii of the planet gears and the sun gear are 45 mm and 75 mm, respectively.



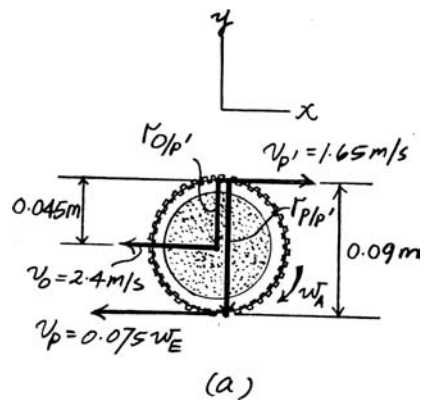
Rotation About a Fixed Axis: Here, the ring gear, the sun gear, and the carrier rotate about a fixed axis. Thus, the velocity of the center O of the planet gear and the contact points P' and P with the ring and sun gear can be determined from

$$v_O = \omega_D r_O = 20(0.045 + 0.075) = 2.4 \text{ m/s} \leftarrow$$

$$v_{P'} = \omega_F r_F = 10(0.045 + 0.045 + 0.075) = 1.65 \text{ m/s} \rightarrow$$

$$v_P = \omega_E r_E = \omega_E(0.075) = 0.075\omega_E$$

General Plane Motion: First, applying the relative velocity equation for O and P' and referring to the kinematic diagram of planet gear A shown in Fig. a,



$$\mathbf{v}_O = \mathbf{v}_{P'} + \omega_A \times \mathbf{r}_{O/P'}$$

$$-2.4\mathbf{i} = 1.65\mathbf{i} + (-\omega_A\,\mathbf{k}) \times (-0.045\mathbf{j})$$

$$-2.4\mathbf{i} = (1.65 - 0.045\omega_A)\mathbf{i}$$

Thus,

$$-2.4 = 1.65 - 0.045\omega_A$$

$$\omega_A = 90 \text{ rad/s} \qquad \textbf{Ans.}$$

Using this result to apply the relative velocity equation for P' and P,

$$\mathbf{v}_P = \mathbf{v}_{P'} + \omega_A \times \mathbf{r}_{P/P'}$$

$$-0.075\omega_E\mathbf{i} = 1.65\mathbf{i} + (-90\mathbf{j}) \times (-0.09\mathbf{j})$$

$$-0.075\omega_E\mathbf{i} = -6.45\mathbf{i}$$

Thus,

$$-0.075\omega_E = -6.45 \qquad \omega_E = 86 \text{ rad/s} \qquad \textbf{Ans.}$$

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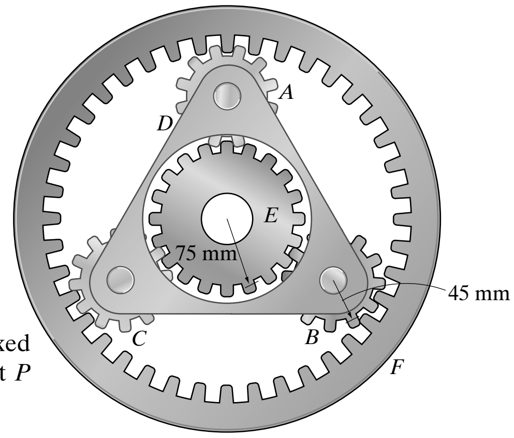
16–78. The planetary gear set of an automatic transmission consists of three planet gears A, B, and C, mounted on carrier D, and meshed with sun gear E and ring gear F. By controlling which gear of the planetary set rotates and which gear receives the engine's power, the automatic transmission can alter a car's speed and direction. If the ring gear is held stationary and the carrier is rotating with a clockwise angular velocity of $\omega_D = 20$ rad/s, determine the angular velocity of the planet gears and the sun gear. The radii of the planet gears and the sun gear are 45 mm and 75 mm, respectively.

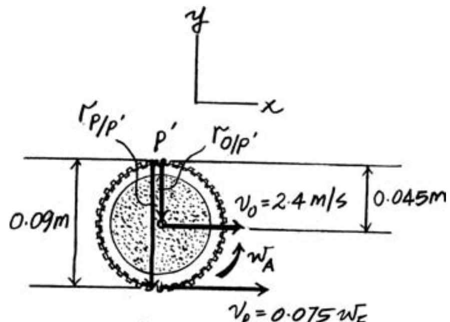


Rotation About a Fixed Axis: Here, the carrier and the sun gear rotate about a fixed axis. Thus, the velocity of the center O of the planet gear and the contact point P with the sun gear can be determined from

$$v_O = \omega_D r_D = 20(0.045 + 0.075) = 2.4 \text{ m/s}$$

$$v_P = \omega_E r_E = \omega_E (0.075) = 0.075\omega_E$$

General Plane Motion: Since the ring gear is held stationary, the velocity of the contact point P' with the planet gear A is zero. Applying the relative velocity equation for O and P' and referring to the kinematic diagram of planet gear A shown in Fig. *a*,



(a)

$$\mathbf{v}_O = \mathbf{v}_{P'} + \boldsymbol{\omega}_A \times \mathbf{r}_{O/P'}$$

$$2.4\mathbf{i} = \mathbf{0} + (\omega_A \mathbf{k}) \times (-0.045\mathbf{j})$$

$$2.4\mathbf{i} = 0.045\omega_A\, \mathbf{i}$$

Thus,

$$2.4 = 0.045\omega_A$$

$$\omega_A = 53.33 \text{ rad/s} = 53.3 \text{ rad/s} \qquad \textbf{Ans.}$$

Using this result to apply the relative velocity equation for points P' and P,

$$\mathbf{v}_P = \mathbf{v}_{P'} + \boldsymbol{\omega}_A \times \mathbf{r}_{P/P'}$$

$$0.075\omega_E\, \mathbf{i} = \mathbf{0} + (53.33\mathbf{k}) \times (-0.09\mathbf{j})$$

$$0.075\omega_E\, \mathbf{i} = 4.8\mathbf{i}$$

Thus,

$$0.075\omega_E = 4.8 \qquad \textbf{Ans.}$$

$$\omega_E = 64 \text{ rad/s} \qquad \textbf{Ans.}$$

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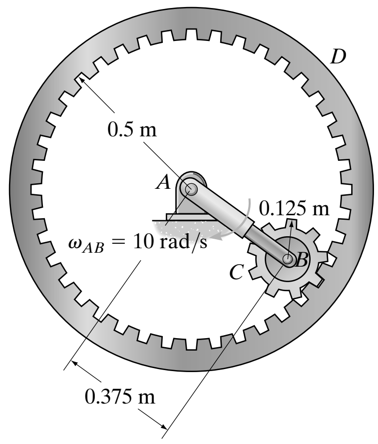
16–79. If the ring gear D is held fixed and link AB rotates with an angular velocity of $\omega_{AB} = 10$ rad/s, determine the angular velocity of gear C.



Rotation About a Fixed Axis: Since link AB rotates about a fixed axis, Fig. a, the velocity of the center B of gear C is

$$v_B = \omega_{AB}\, r_{AB} = 10(0.375) = 3.75 \text{ m/s}$$

General Plane Motion: Since gear D is fixed, the velocity of the contact point P between the gears is zero. Applying the relative velocity equation and referring to the kinematic diagram of gear C shown in Fig. b,

$$\mathbf{v}_B = \mathbf{v}_P + \omega_C \times \mathbf{r}_{B/P}$$

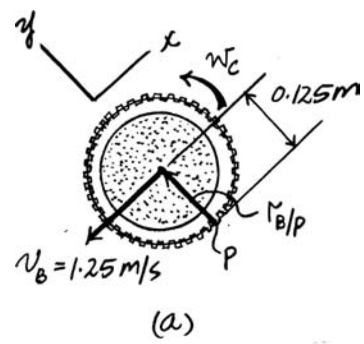
$$-3.75\mathbf{i} = \mathbf{0} + (\omega_C\,\mathbf{k}) \times (0.125\mathbf{j})$$

$$-3.75\mathbf{i} = -0.125\omega_C\mathbf{i}$$

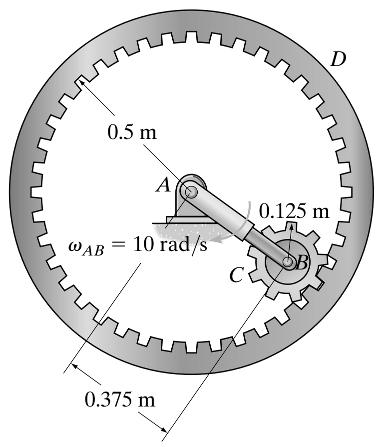
Thus,

$$-3.75 = -0.125\omega_C$$

$$\omega_C = 30 \text{ rad/s}$$ **Ans.**



***16–80.** If the ring gear D rotates counterclockwise with an angular velocity of $\omega_D = 5$ rad/s while link AB rotates clockwise with an angular velocity of $\omega_{AB} = 10$ rad/s, determine the angular velocity of gear C.



Rotation About a Fixed Axis: Since link AB and gear D rotate about a fixed axis, Fig. a, the velocity of the center B and the contact point of gears D and C is

$$v_B = \omega_{AB}\, r_B = 10(0.375) = 3.75 \text{ m/s}$$

$$v_P = \omega_D\, r_P = 5(0.5) = 2.5 \text{ m/s}$$

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of gear C shown in Fig. b,

$$\mathbf{v}_B = \mathbf{v}_P + \omega_C \times r_{B/P}$$

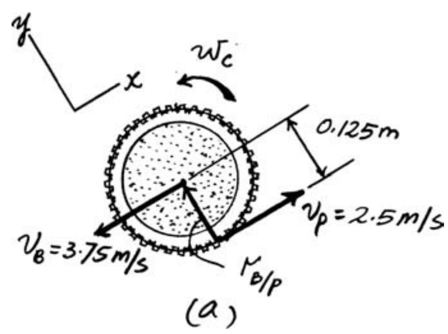
$$-3.75\mathbf{i} = 2.5\mathbf{i} + (\omega_C\,\mathbf{k}) \times (0.125\mathbf{j})$$

$$-3.75\mathbf{i} = (2.5 - 0.125\omega_C)\mathbf{i}$$

Thus,

$$-3.75 = 2.5 - 0.125\omega_C$$

$$\omega_C = 50 \text{ rad/s}$$ **Ans.**



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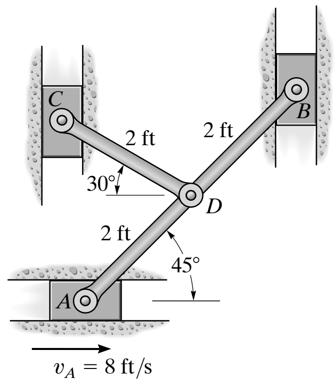
•16–81. If the slider block A is moving to the right at $v_A = 8$ ft/s, determine the velocity of blocks B and C at the instant shown. Member CD is pin connected to member ADB.



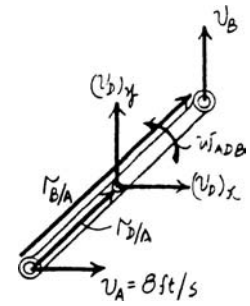
Kinematic Diagram: Block B and C are moving along the guide and directed towards the *positive* y axis and *negative* y axis, respectively. Then, $\mathbf{v}_B = v_B\,\mathbf{j}$ and $\mathbf{v}_C = -v_C\,\mathbf{j}$. Since the direction of the velocity of point D is unknown, we can assume that its x and y components are directed in the *positive direction* of their respective axis.

Velocity Equation: Here, $\mathbf{r}_{B/A} = \{4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j}\}$ ft $= \{2.828\mathbf{i} + 2.828\mathbf{j}\}$ ft and $\mathbf{r}_{D/A} = \{2\cos 45°\mathbf{i} + 2\sin 45°\mathbf{j}\}$ ft $= \{1.414\mathbf{i} + 1.414\mathbf{j}\}$ ft. Applying Eq. 16–16 to link ADB, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{ADB} \times \mathbf{r}_{B/A}$$

$$v_B\,\mathbf{j} = 8\mathbf{i} + (\omega_{ADB}\,\mathbf{k}) \times (2.828\mathbf{i} + 2.828\mathbf{j})$$

$$v_B\,\mathbf{j} = (8 - 2.828\omega_{ADB})\,\mathbf{i} + 2.828\omega_{ADB}\,\mathbf{j}$$



Equating **i** and **j** components gives

$$0 = 8 - 2.828\omega_{ADB} \qquad \textbf{[1]}$$

$$v_B = 2.828\omega_{ADB} \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

$$\omega_{ADB} = 2.828 \text{ rad/s}$$

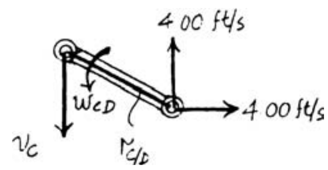
$$v_B = 8.00 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

The x and y component of velocity of v_D are given by

$$\mathbf{v}_D = \mathbf{v}_A + \omega_{ADB} \times \mathbf{r}_{D/A}$$

$$(v_D)_x\,\mathbf{i} + (v_D)_y\mathbf{j} = 8\mathbf{i} + (2.828\mathbf{k}) \times (1.414\mathbf{i} + 1.414\mathbf{j})$$

$$(v_D)_x\,\mathbf{i} + (v_D)_y\,\mathbf{j} = 4.00\mathbf{i} + 4.00\mathbf{j}$$



Equating **i** and **j** components gives

$$(v_D)_x = 4.00 \text{ ft/s} \qquad (v_D)_y = 4.00 \text{ ft/s}$$

Here, $r_{C/D} = \{-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\}$ ft $= \{-1.732\mathbf{i} + 1\mathbf{j}\}$ ft. Applying Eq. 16–16 to link CD, we have

$$\mathbf{v}_C = \mathbf{v}_D + \omega_{CD} \times \mathbf{r}_{C/D}$$

$$-v_C\,\mathbf{j} = 4.00\mathbf{i} + 4.00\mathbf{j} + (\omega_{CD}\mathbf{k}) \times (-1.732\mathbf{i} + 1\mathbf{j})$$

$$-v_C\,\mathbf{j} = (4.00 - \omega_{CD})\,\mathbf{i} + (4 - 1.732\omega_{CD})\,\mathbf{j}$$

Equating **i** and **j** components gives

$$0 = 4.00 - \omega_{CD} \qquad \textbf{[3]}$$

$$-v_C = 4 - 1.732\omega_{CD} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

$$\omega_{CD} = 4.00 \text{ rad/s}$$

$$v_C = 2.93 \text{ ft/s} \downarrow \qquad \textbf{Ans.}$$

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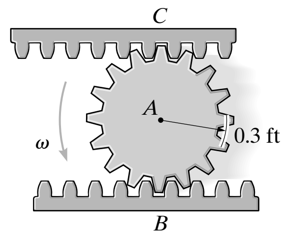
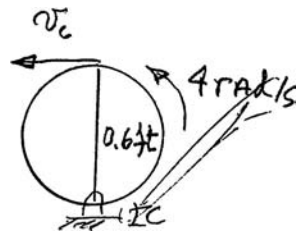
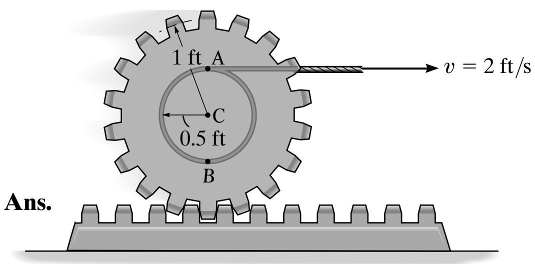
16–82. Solve Prob. 16–54 using the method of instantaneous center of zero velocity.



$$v_C = 4\text{ rad/s}(0.6\text{ ft}) = 2.40\text{ ft/s}$$ **Ans.**

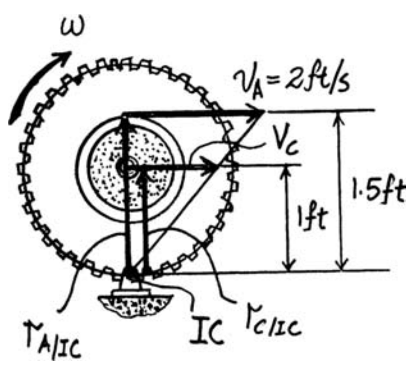


16–83. Solve Prob. 16–56 using the method of instantaneous center of zero velocity.



$$\omega = \frac{2}{1.5} = 1.33\text{ rad/s}$$

$$v_C = 1(1.33) = 1.33\text{ ft/s} \rightarrow$$ **Ans.**



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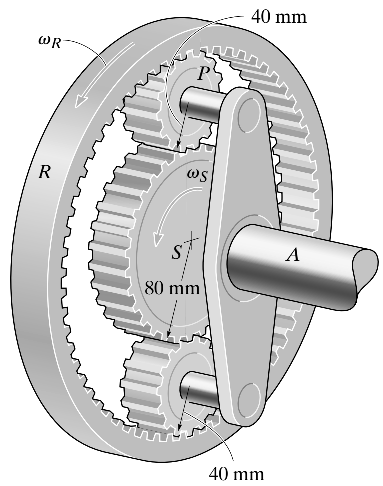
***16–84.** Solve Prob. 16–64 using the method of instantaneous center of zero velocity.

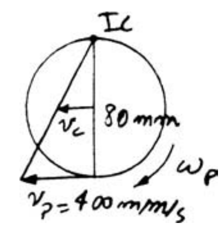


$$v_P = (80)(5) = 400 \text{ mm/s}$$

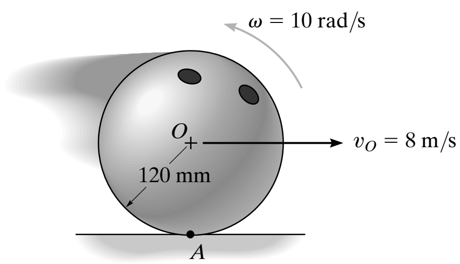
$$\omega_P = \frac{400}{80} = 5 \text{ rad/s} \quad \circlearrowright \qquad \textbf{Ans.}$$

$$v_C = (5)(40) = 200 \text{ mm/s}$$

$$\omega_A = \frac{200}{(80 + 40)} = 1.67 \text{ rad/s} \quad \circlearrowleft \qquad \textbf{Ans.}$$

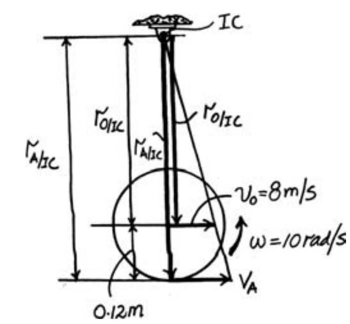


•16–85. Solve Prob. 16–58 using the method of instantaneous center of zero velocity.



$$r_{OI/C} = \frac{8}{10} = 0.8 \text{ m}$$

$$v_A = 10(0.8 + 0.120) = 9.20 \text{ m/s} \qquad \textbf{Ans.}$$



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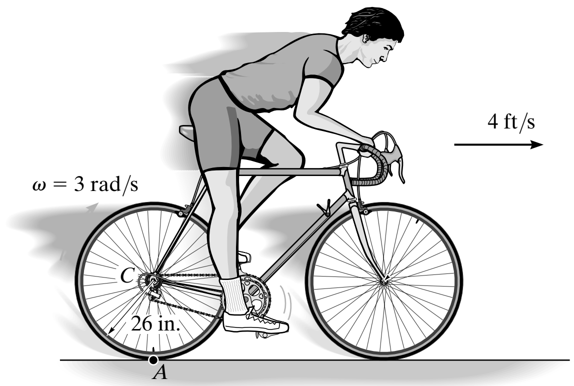
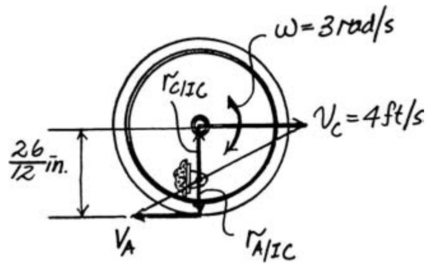
16–86. Solve Prob. 16–67 using the method of instantaneous center of zero velocity.



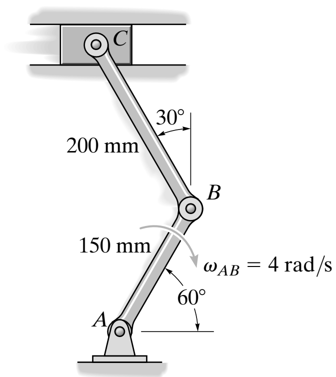
$$r_{C/IC} = \ = \frac{4}{3} = 1.33 \text{ ft}$$

$$r_{A/IC} = \ = \frac{26}{12} - 1.33 \text{ ft} = 0.833 \text{ ft}$$

$$v_A = 3(0.833) = 2.5 \text{ ft/s} \quad \leftarrow$$ **Ans.**



16–87. Solve Prob. 16–68 using the method of instantaneous center of zero velocity.



$$v_B = 4(0.150) = 0.6 \text{ m/s}$$

$$\frac{r_{C/IC}}{\sin 120^\circ} = \frac{0.2}{\sin 30^\circ}$$

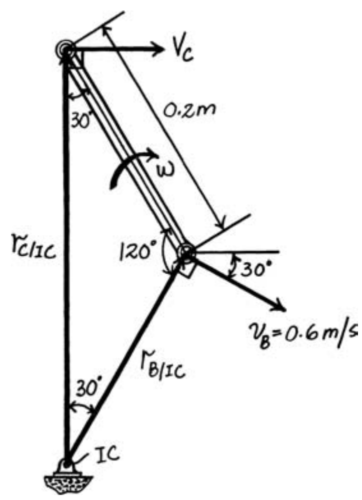
$$r_{C/IC} = 0.34641 \text{ m}$$

$$\frac{r_{B/IC}}{\sin 30^\circ} = \frac{0.2}{\sin 30^\circ}$$

$$r_{B/IC} = 0.2 \text{ m}$$

$$\omega = \frac{0.6}{0.2} = 3 \text{ rad/s}$$

$$v_C = 0.34641(3) = 1.04 \text{ m/s} \quad \rightarrow$$ **Ans.**



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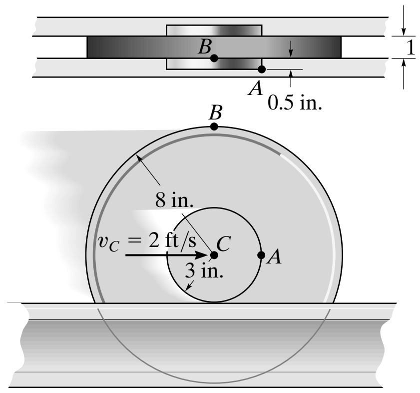
***16–88.** The wheel rolls on its hub without slipping on the horizontal surface. If the velocity of the center of the wheel is $v_C = 2$ ft/s to the right, determine the velocities of points A and B at the instant shown.



$$v_C = \omega r_{C/IC}$$

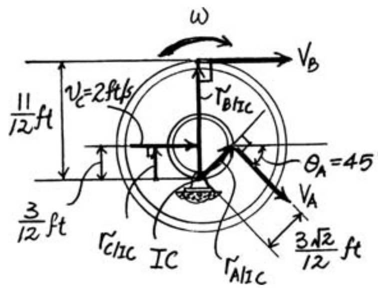
$$2 = \omega\left(\frac{3}{12}\right)$$

$$\omega = 8 \text{ rad/s}$$

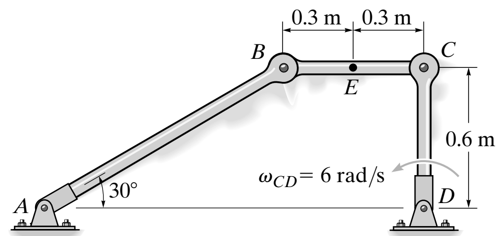
$$v_B = \omega r_{B/IC} = 8\left(\frac{11}{12}\right) = 7.33 \text{ ft/s} \quad \rightarrow$$ **Ans.**

$$v_A = \omega r_{A/IC} = 8\left(\frac{3\sqrt{2}}{12}\right) = 2.83 \text{ ft/s}$$ **Ans.**

$$\theta_A = \tan^{-1}\left(\frac{3}{3}\right) = 45° \searrow$$ **Ans.**



•16–89. If link CD has an angular velocity of $\omega_{CD} = 6$ rad/s, determine the velocity of point E on link BC and the angular velocity of link AB at the instant shown.



$$v_C = \omega_{CD}\,(r_{CD}) = (6)(0.6) = 3.60 \text{ m/s}$$

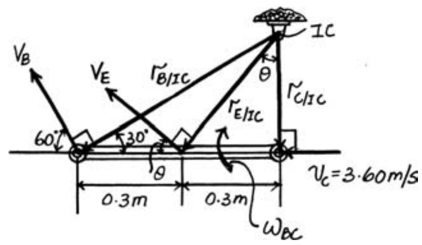
$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3.60}{0.6 \tan 30°} = 10.39 \text{ rad/s}$$

$$v_B = \omega_{BC}\, r_{B/IC} = (10.39)\left(\frac{0.6}{\cos 30°}\right) = 7.20 \text{ m/s}$$

$$\omega_{AB} = \frac{v_B}{r_{AB}} = \frac{7.20}{\left(\dfrac{0.6}{\sin 30°}\right)} = 6 \text{ rad/s} \quad \circlearrowleft$$ **Ans.**

$$v_E = \omega_{BC}\, r_{E/IC} = 10.39\sqrt{(0.6 \tan 30°)^2 + (0.3)^2} = 4.76 \text{ m/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{0.3}{0.6 \tan 30°}\right) = 40.9° \nwarrow$$ **Ans.**



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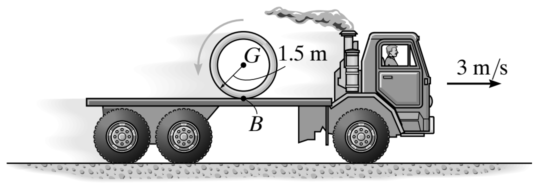
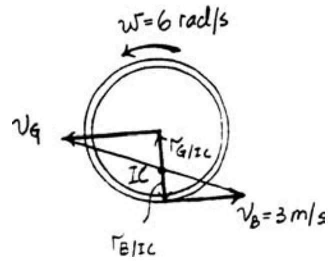
16–90. At the instant shown, the truck travels to the right at 3 m/s, while the pipe rolls counterclockwise at $\omega = 6$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



Kinematic Diagram: Since the pipe rolls without slipping, then the velocity of point B must be the same as that of the truck, i.e; $v_B = 3$ m/s.

Instantaneous Center: $r_{B/IC}$ must be determined first in order to locate the the instantaneous center of zero velocity of the pipe.



$$v_B = \omega r_{B/IC}$$

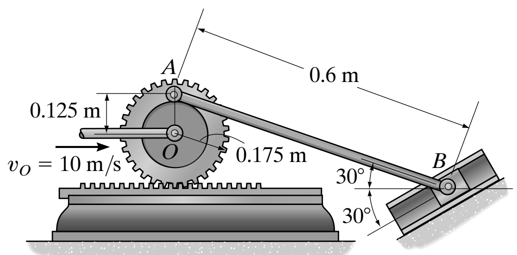
$$3 = 6(r_{B/IC})$$

$$r_{B/IC} = 0.5 \text{ m}$$

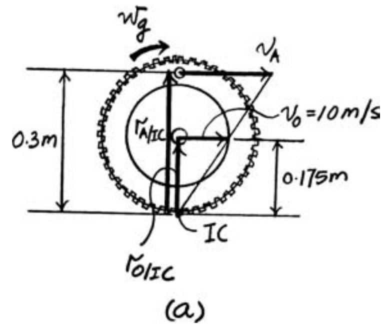
Thus, $r_{G/IC} = 1.5 - r_{B/IC} = 1.5 - 0.5 = 1.00$ m. Then

$$v_G = \omega r_{G/IC} = 6(1.00) = 6.00 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

16–91. If the center O of the gear is given a velocity of $v_O = 10$ m/s, determine the velocity of the slider block B at the instant shown.



General Plane Motion: Since the gear rack is stationary, the IC of the gear is located at the contact point between the gear and the rack, Fig. a. Here, $r_{O/IC} = 0.175$ m and $r_{A/IC} = 0.6$ m. Thus, the velocity of point A can be determined using the similar triangles shown in Fig. a,

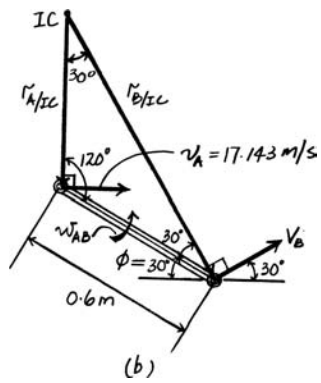


$$\omega_g = \frac{v_A}{r_{A/IC}} = \frac{v_O}{r_{O/IC}}$$

$$\frac{v_A}{0.3} = \frac{10}{0.175}$$

$$v_A = 17.143 \text{ m/s} \rightarrow$$

The location of the IC for rod AB is indicated in Fig. b. From the geometry shown in Fig. b,



$$r_{A/IC} = 0.6 \text{ m}$$

$$r_{B/IC} = 2(0.6 \cos 30°) = 1.039 \text{ m}$$

Thus, the angular velocity of the gear can be determined from

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{17.143}{0.6} = 28.57 \text{ rad/s}$$

Then,

$$v_B = \omega_{AB}\, r_{B/IC} = 28.57(1.039) = 29.7 \text{ m/s} \qquad \textbf{Ans.}$$

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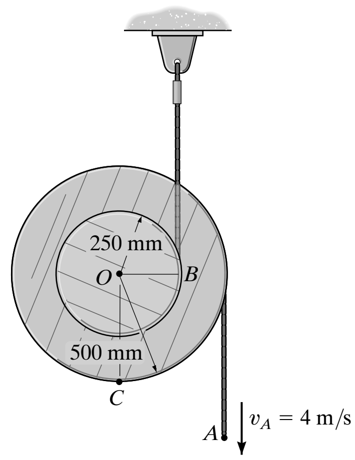
***16–92.** If end A of the cord is pulled down with a velocity of $v_A = 4$ m/s, determine the angular velocity of the spool and the velocity of point C located on the outer rim of the spool.



General Plane Motion: Since the contact point B between the rope and the spool is at rest, the IC is located at point B, Fig. a. From the geometry of Fig. a,

$$r_{A/IC} = 0.25 \text{ m}$$

$$r_{C/IC} = \sqrt{0.25^2 + 0.5^2} = 0.5590 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{0.25}{0.5}\right) = 26.57°$$

Thus, the angular velocity of the spool can be determined from

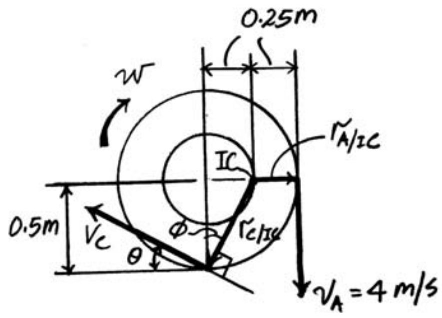
$$\omega = \frac{v_A}{r_{A/IC}} = \frac{4}{0.25} = 16\text{rad/s} \qquad \textbf{Ans.}$$

Then,

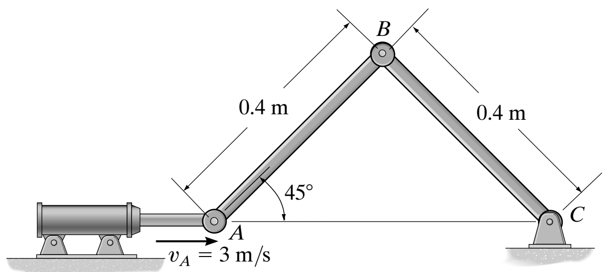
$$v_C = \omega r_{C/IC} = 16(0.5590) = 8.94\text{m/s} \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \phi = 26.6° \qquad \textbf{Ans.}$$



•16–93. If end A of the hydraulic cylinder is moving with a velocity of $v_A = 3$ m/s, determine the angular velocity of rod BC at the instant shown.



Rotation About a Fixed Axis: Referring to Fig. a,

$$v_B = \omega_{BC}\, r_B = \omega_{BC}\,(0.4)$$

General Plane Motion: The location of the IC for rod AB is indicated in Fig. b. From the geometry shown in this figure, we obtain

$$r_{A/IC} = \frac{0.4}{\cos 45°} \qquad r_{A/IC} = 0.5657 \text{ m}$$

$$r_{B/IC} = 0.4 \tan 45° = 0.4 \text{ m}$$

Thus, the angular velocity of rod AB can be determined from

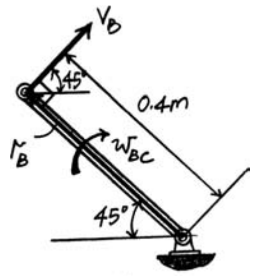
$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{3}{0.5657} = 5.303 \text{ rad/s}$$

Then,

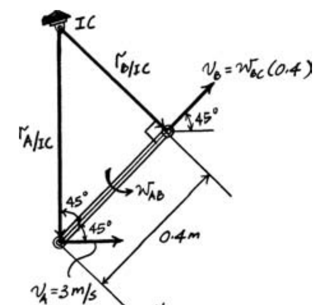
$$v_B = \omega_{AB}\, r_{B/IC}$$

$$\omega_{BC}\,(0.4) = 5.303(0.4)$$

$$\omega_{BC} = 5.30 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)



(b)

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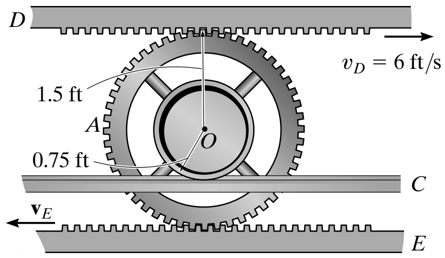
16–94. The wheel is rigidly attached to gear A, which is in mesh with gear racks D and E. If D has a velocity of $v_D = 6$ ft/s to the right and wheel rolls on track C without slipping, determine the velocity of gear rack E.



General Plane Motion: Since the wheel rolls without slipping on track C, the IC is located there, Fig. a. Here,

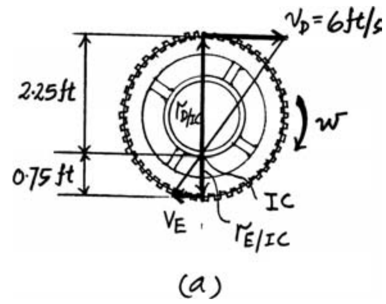
$$r_{D/IC} = 2.25 \text{ ft} \qquad r_{E/IC} = 0.75 \text{ ft}$$

Thus, the angular velocity of the gear can be determined from

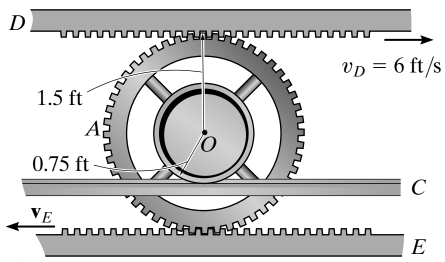
$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{2.25} = 2.667 \text{ rad/s}$$

Then,

$$v_E = \omega r_{E/IC} = 2.667(0.75) = 2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$



16–95. The wheel is rigidly attached to gear A, which is in mesh with gear racks D and E. If the racks have a velocity of $v_D = 6$ ft/s and $v_E = 10$ ft/s, show that it is necessary for the wheel to slip on the fixed track C. Also find the angular velocity of the gear and the velocity of its center O.



General Plane Motion: The location of the IC can be found using the similar triangles shown in Fig. a,

$$\frac{r_{D/IC}}{6} = \frac{3 - r_{D/IC}}{10} \qquad r_{D/IC} = 1.125 \text{ ft}$$

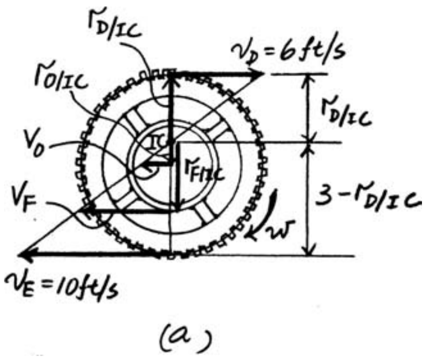
Thus,

$$r_{O/IC} = 1.5 - r_{D/IC} = 1.5 - 1.125 = 0.375\text{ft}$$

$$r_{F/IC} = 2.25 - r_{D/IC} = 2.25 - 1.125 = 1.125\text{ft}$$

Thus, the angular velocity of the gear is

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{1.125} = 5.333 \text{ rad/s} = 5.33 \text{ rad/s} \qquad \textbf{Ans.}$$



The velocity of the contact point F between the wheel and the track is

$$v_F = \omega r_{F/IC} = 5.333(1.125) = 6 \text{ ft/s} \leftarrow$$

Since $v_F \neq 0$, **the wheel slips on the track** **(Q.E.D.)**

The velocity of center O of the gear is

$$v_O = \omega r_{O/IC} = 5.333(0.375) = 2\text{ft/s} \leftarrow \qquad \textbf{Ans.}$$

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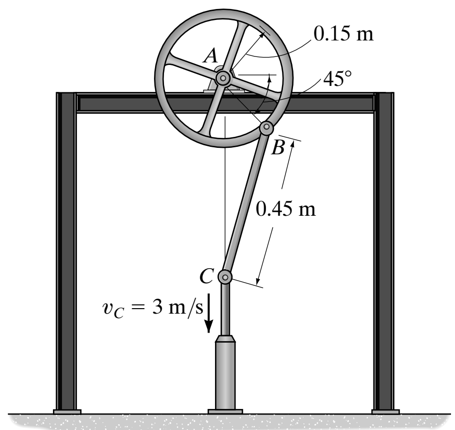
***16–96.** If C has a velocity of $v_C = 3$ m/s, determine the angular velocity of the wheel at the instant shown.



Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega_W r_B = \omega_W(0.15)$$

General Plane Motion: Applying the law of sines to the geometry shown in Fig. *b*,

$$\frac{\sin\phi}{0.15} = \frac{\sin 45°}{0.45} \qquad \phi = 13.63°$$

The location of the *IC* for rod *BC* is indicated in Fig. *c*. Applying the law of sines to the geometry of Fig. *c*,

$$\frac{r_{C/IC}}{\sin 58.63°} = \frac{0.45}{\sin 45°} \qquad r_{C/IC} = 0.5434 \text{ m}$$

$$\frac{r_{B/IC}}{\sin 76.37°} = \frac{0.45}{\sin 45°} \qquad r_{B/IC} = 0.6185 \text{ m}$$

Thus, the angular velocity of rod *BC* is

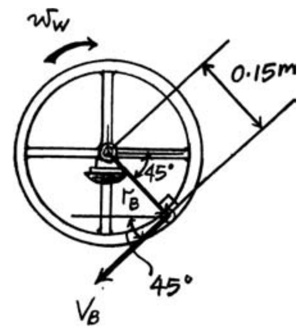
$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3}{0.5434} = 5.521 \text{ rad/s}$$

and

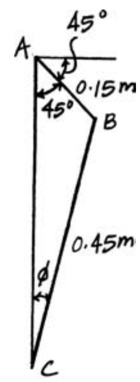
$$v_B = \omega_{BC}\, r_{B/IC}$$

$$\omega_W(0.15) = 5.521(0.6185)$$

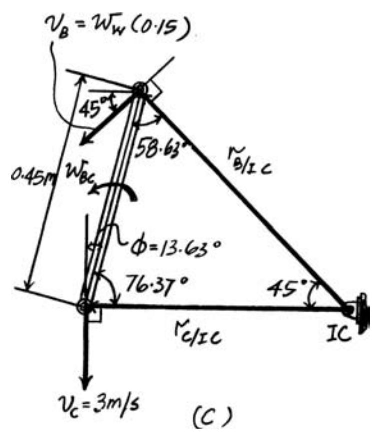
$$\omega_W = 22.8 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

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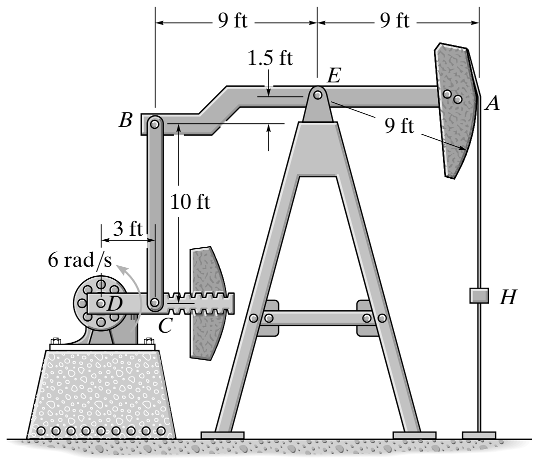
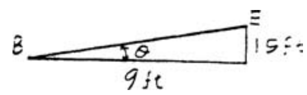
•**16–97.** The oil pumping unit consists of a walking beam AB, connecting rod BC, and crank CD. If the crank rotates at a constant rate of 6 rad/s, determine the speed of the rod hanger H at the instant shown. *Hint:* Point B follows a circular path about point E and therefore the velocity of B is *not* vertical.



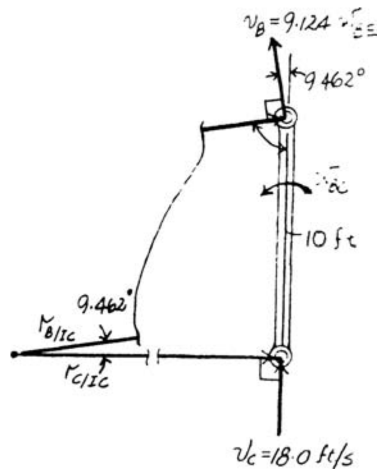
Kinematic Diagram: From the geometry, $\theta = \tan^{-1}\left(\dfrac{1.5}{9}\right) = 9.462°$ and $r_{BE} = \sqrt{9^2 + 1.5^2} = 9.124$ ft. Since crank CD and beam BE are rotating about fixed points D and E, then $\mathbf{v}_C$ and $\mathbf{v}_B$ are always directed perpendicular to crank CD and beam BE, respectively. The magnitude of $\mathbf{v}_C$ and $\mathbf{v}_B$ are $v_C = \omega_{CD} r_{CD} = 6(3) = 18.0$ ft/s and $v_B = \omega_{BE} r_{BE} = 9.124\omega_{BE}$. At the instant shown, $\mathbf{v}_C$ is directed vertically while $\mathbf{v}_B$ is directed with an angle 9.462° with the vertical.



Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. From the geometry

$$r_{B/IC} = \frac{10}{\sin 9.462°} = 60.83 \text{ ft}$$

$$r_{C/IC} = \frac{10}{\tan 9.462°} = 60.0 \text{ ft}$$



The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{18.0}{60.0} = 0.300 \text{ rad/s}$$

Thus, the angular velocity of beam BE is given by

$$v_B = \omega_{BC} r_{B/IC}$$

$$9.124\omega_{BE} = 0.300(60.83)$$

$$\omega_{BE} = 2.00 \text{ rad/s}$$

The speed of rod hanger H is given by

$$v_H = \omega_{BE} r_{EA} = 2.00(9) = 18.0 \text{ ft/s}$$ **Ans.**

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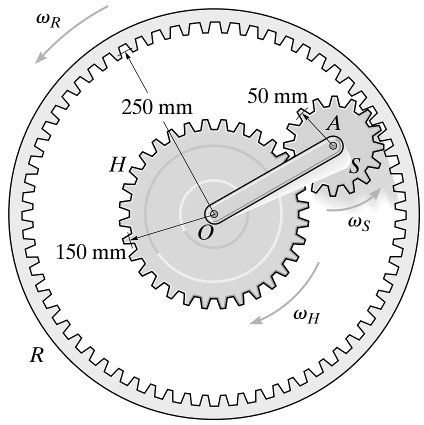
16–98. If the hub gear H and ring gear R have angular velocities $\omega_H = 5$ rad/s and $\omega_R = 20$ rad/s, respectively, determine the angular velocity ω_S of the spur gear S and the angular velocity of arm OA.



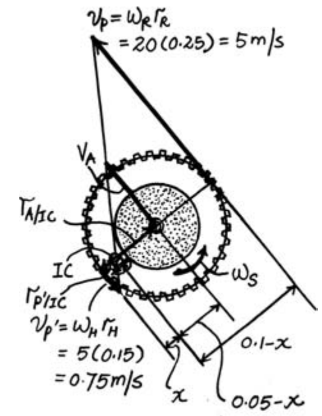
$$\frac{5}{0.1 - x} = \frac{0.75}{x}$$

$$x = 0.01304 \text{ m}$$

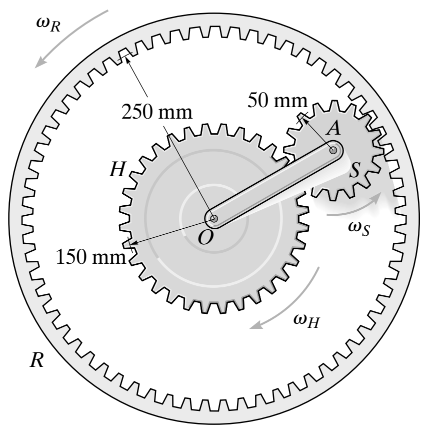
$$\omega_s = \frac{0.75}{0.01304} = 57.5 \text{ rad/s} \quad \circlearrowleft \qquad \textbf{Ans.}$$

$$v_A = 57.5(0.05 - 0.01304) = 2.125 \text{ m/s}$$

$$\omega_{OA} = \frac{2.125}{0.2} = 10.6 \text{ rad/s} \quad \circlearrowleft \qquad \textbf{Ans.}$$



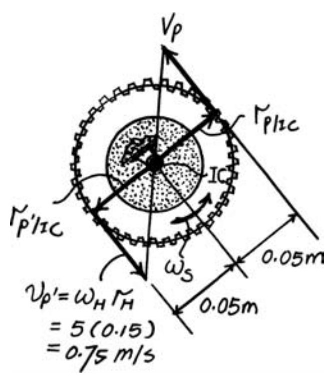
16–99. If the hub gear H has an angular velocity $\omega_H = 5$ rad/s, determine the angular velocity of the ring gear R so that the arm OA which is pinned to the spur gear S remains stationary ($\omega_{OA} = 0$). What is the angular velocity of the spur gear?



The *IC* is at *A*.

$$\omega_S = \frac{0.75}{0.05} = 15.0 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\omega_R = \frac{0.75}{0.250} = 3.00 \text{ rad/s} \qquad \textbf{Ans.}$$



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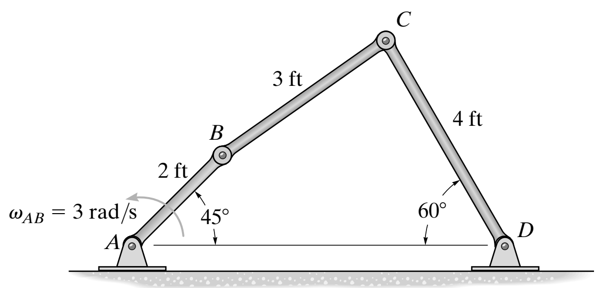
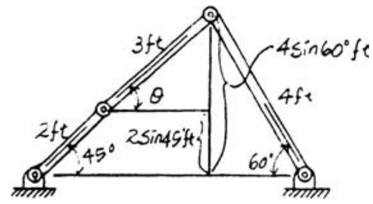
***16–100.** If rod AB is rotating with an angular velocity $\omega_{AB} = 3$ rad/s, determine the angular velocity of rod BC at the instant shown.



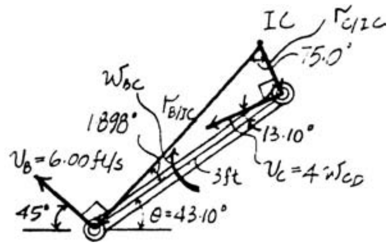
Kinematic Diagram: From the geometry, $\theta = \sin^{-1}\left(\frac{4 \sin 60° - 2 \sin 45°}{3}\right) = 43.10°$. Since links AB and CD is rotating about fixed points A and D, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular to links AB and CD, respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_C$ are $v_B = \omega_{AB} r_{AB} = 3(2) = 6.00$ ft/s and $v_C = \omega_{CD} r_{CD} = 4\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle of 45° while $\mathbf{v}_C$ is directed at 30°



Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 103.1°} = \frac{3}{\sin 75°} \qquad r_{B/IC} = 3.025 \text{ ft}$$

$$\frac{r_{C/IC}}{\sin 1.898°} = \frac{3}{\sin 75°} \qquad r_{C/IC} = 0.1029 \text{ ft}$$



The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s} = 1.98 \text{ rad/s} \qquad \textbf{Ans.}$$

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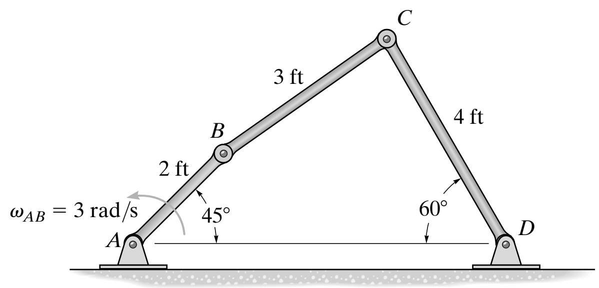
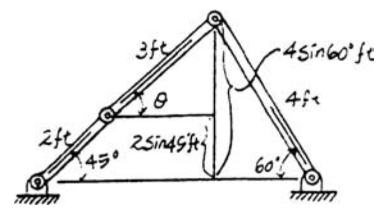
•16–101. If rod AB is rotating with an angular velocity $\omega_{AB} = 3$ rad/s, determine the angular velocity of rod CD at the instant shown.



Kinematic Diagram: From the geometry. $\theta = \sin^{-1}\left(\frac{4\sin 60° - 2\sin 45°}{3}\right) = 43.10°$. Since links AB and CD is rotating about fixed points A and D, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular to links AB and CD, respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_C$ are $v_B = \omega_{AB} r_{AB} = 3(2) = 6.00$ ft/s and $v_C = \omega_{CD} r_{CD} = 4\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle of 45° while $\mathbf{v}_C$ is directed at 30°.



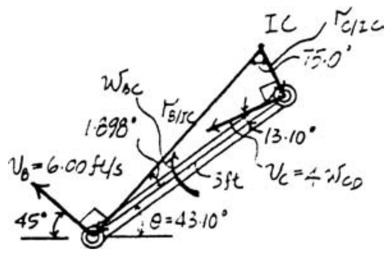
Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 103.1°} = \frac{3}{\sin 75°} \qquad r_{B/IC} = 3.025 \text{ ft}$$

$$\frac{r_{C/IC}}{\sin 1.898°} = \frac{3}{\sin 75°} \qquad r_{C/IC} = 0.1029 \text{ ft}$$

The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s}$$



Thus, the angular velocity of link CD is given by

$$v_C = \omega_{BC} r_{C/IC}$$

$$4\omega_{CD} = 1.983(0.1029)$$

$$\omega_{CD} = 0.0510 \text{ rad/s} \qquad \textbf{Ans.}$$

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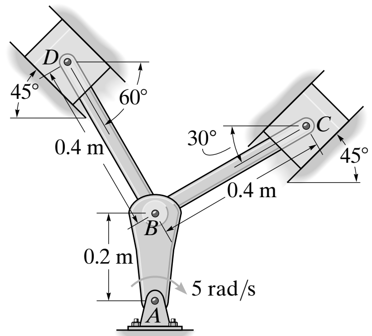
16–102. The mechanism used in a marine engine consists of a crank AB and two connecting rods BC and BD. Determine the velocity of the piston at C the instant the crank is in the position shown and has an angular velocity of 5 rad/s.



$v_B = 0.2(5) = 1 \text{ m/s} \rightarrow$

Member BC:

$$\frac{r_{C/IC}}{\sin 60°} = \frac{0.4}{\sin 45°}$$

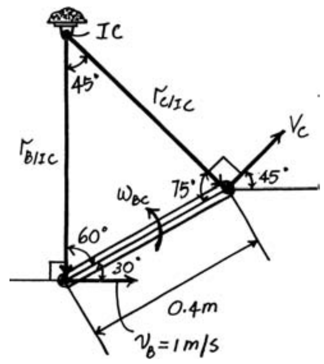
$r_{C/IC} = 0.4899 \text{ m}$

$$\frac{r_{B/IC}}{\sin 75°} = \frac{0.4}{\sin 45°}$$

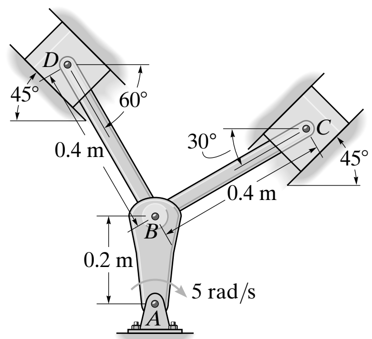
$r_{B/IC} = 0.5464 \text{ m}$

$$\omega_{BC} = \frac{1}{0.5464} = 1.830 \text{ rad/s}$$

$v_C = 0.4899(1.830) = 0.897 \text{ m/s} \nearrow$ **Ans.**



16–103. The mechanism used in a marine engine consists of a crank AB and two connecting rods BC and BD. Determine the velocity of the piston at D the instant the crank is in the position shown and has an angular velocity of 5 rad/s.



$v_B = 0.2(5) = 1 \text{ m/s} \rightarrow$

Member BD:

$$\frac{r_{B/IC}}{\sin 105°} = \frac{0.4}{\sin 45°}$$

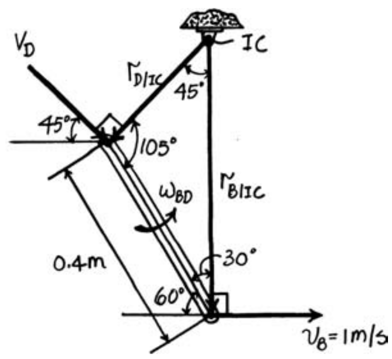
$r_{B/IC} = 0.54641 \text{ m}$

$$\frac{r_{D/IC}}{\sin 30°} = \frac{0.4}{\sin 45°}$$

$r_{D/IC} = 0.28284 \text{ m}$

$$\omega_{BD} = \frac{1}{0.54641} = 1.830 \text{ rad/s}$$

$v_D = 1.830(0.28284) = 0.518 \text{ m/s} \searrow$ **Ans.**



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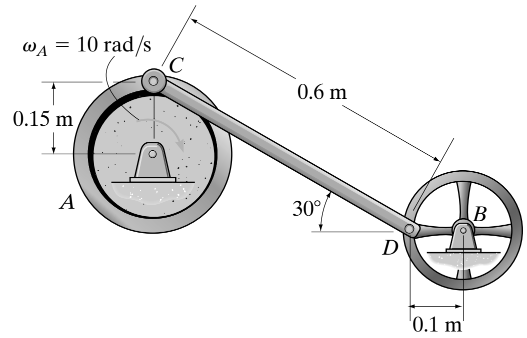
***16–104.** If flywheel A is rotating with an angular velocity of $\omega_A = 10$ rad/s, determine the angular velocity of wheel B at the instant shown.



Rotation About a Fixed Axis: Referring to Figs. *a* and *b*,

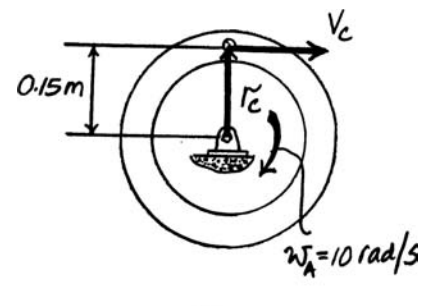
$$v_C = \omega_A r_C = 10(0.15) = 1.5 \text{ m/s} \rightarrow$$

$$v_D = \omega_B r_D = \omega_B(0.1) \downarrow$$

General Plane Motion: The location of the *IC* for rod *CD* is indicated in Fig. *c*. From the geometry of this figure, we obtain

$$r_{C/IC} = 0.6 \sin 30° = 0.3 \text{ m}$$

$$r_{D/IC} = 0.6 \cos 30° = 0.5196 \text{ m}$$



(a)

Thus, the angular velocity of rod *CD* can be determined from

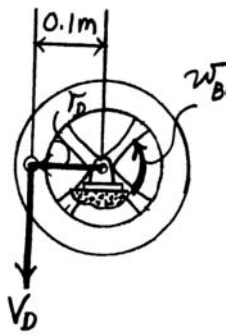
$$\omega_{CD} = \frac{v_D}{r_{C/IC}} = \frac{1.5}{0.3} = 5 \text{ rad/s}$$

Then,

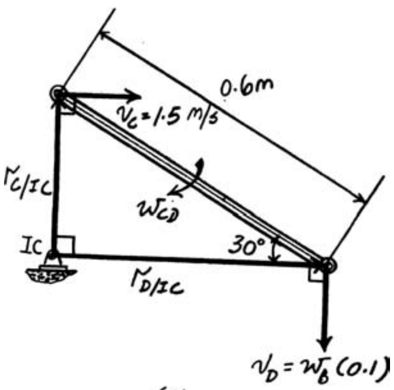
$$v_D = \omega_{CD}\, r_{D/IC}$$

$$\omega_B(0.1) = 5(0.5196)$$

$$\omega_B = 26.0 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)



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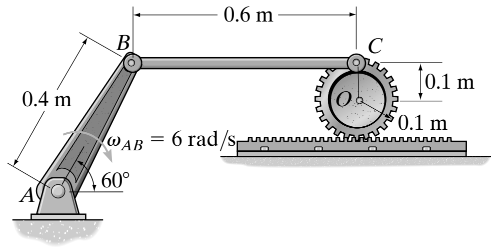
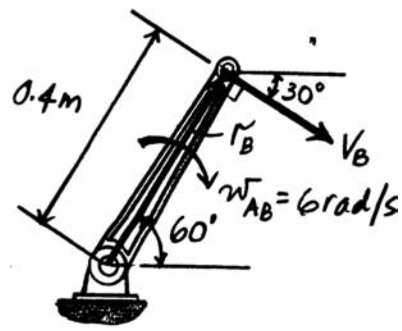
•16–105. If crank AB is rotating with an angular velocity of $\omega_{AB} = 6$ rad/s, determine the velocity of the center O of the gear at the instant shown.



Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega_{AB}\, r_B = 6(0.4) = 2.4 \text{ m/s}$$

General Plane Motion: Since the gear rack is stationary, the *IC* of the gear is located at the contact point between the gear and the rack, Fig. *b*. Thus, $\mathbf{v}_O$ and $\mathbf{v}_C$ can be related using the similar triangles shown in Fig. *b*,



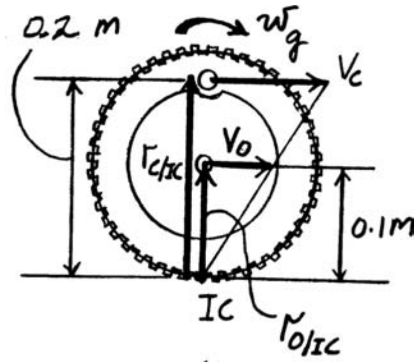
(a)

$$\omega_g = \frac{v_C}{r_{C/IC}} = \frac{v_O}{r_{O/IC}}$$

$$\frac{v_C}{0.2} = \frac{v_O}{0.1}$$

$$v_C = 2v_O$$

The location of the *IC* for rod *BC* is indicated in Fig. *c*. From the geometry shown,



(b)

$$r_{B/IC} = \frac{0.6}{\cos 60°} = 1.2 \text{ m}$$

$$r_{C/IC} = 0.6 \tan 60° = 1.039 \text{ m}$$

Thus, the angular velocity of rod BC can be determined from

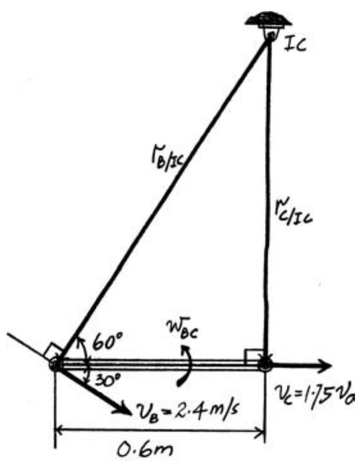
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{2.4}{1.2} = 2 \text{ rad/s}$$

Then,

$$v_C = \omega_{BC}\, r_{C/IC}$$

$$2v_O = 2(1.039)$$

$$v_O = 1.04 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



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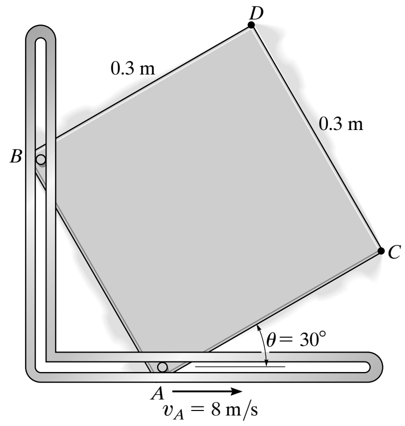
16–106. The square plate is constrained within the slots at A and B. When $\theta = 30^\circ$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point C at this instant.



$r_{A/IC} = 0.3 \cos 30^\circ = 0.2598 \text{ m}$

$$\omega = \frac{8}{0.2598} = 30.792 \text{ rad/s}$$

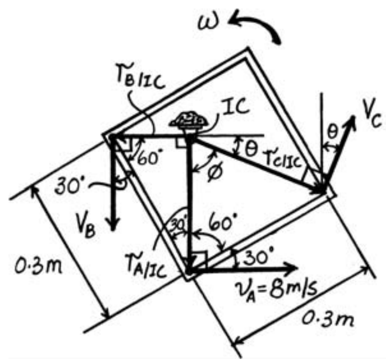
$r_{C/IC} = \sqrt{(0.2598)^2 + (0.3)^2 - 2(0.2598)(0.3)\cos 60^\circ} = 0.2821 \text{ m}$

$v_C = (0.2821)(30.792) = 8.69 \text{ m/s}$ **Ans.**

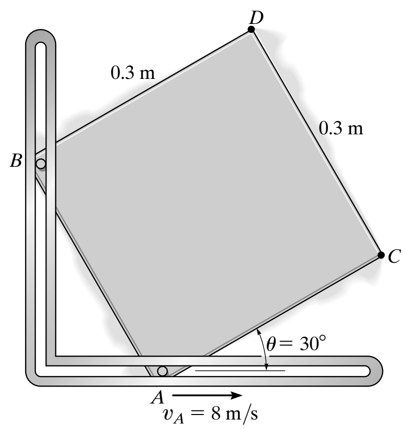
$$\frac{\sin \phi}{0.3} = \frac{\sin 60^\circ}{0.2821}$$

$\phi = 67.09^\circ$

$\theta = 90^\circ - 67.09^\circ = 22.9^\circ$ ∡ **Ans.**



16–107. The square plate is constrained within the slots at A and B. When $\theta = 30^\circ$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point D at this instant.



$r_{A/IC} = 0.3 \cos 30^\circ = 0.2598 \text{ m}$

$$\omega = \frac{8}{0.2598} = 30.792 \text{ rad/s}$$

$r_{B/IC} = 0.3 \sin 30^\circ = 0.15 \text{ m}$

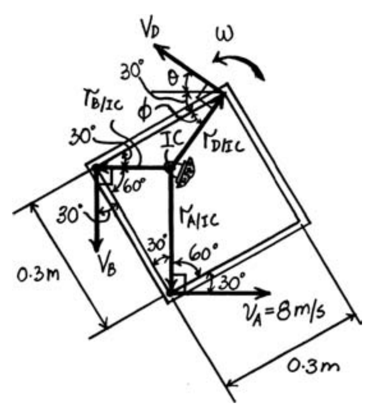
$r_{D/IC} = \sqrt{(0.3)^2 + (0.15)^2 - 2(0.3)(0.15)\cos 30^\circ} = 0.1859 \text{ m}$

$v_D = (30.792)(0.1859) = 5.72 \text{ m/s}$ **Ans.**

$$\frac{\sin \phi}{0.15} = \frac{\sin 30^\circ}{0.1859}$$

$\phi = 23.794^\circ$

$\theta = 90^\circ - 30^\circ - 23.794^\circ = 36.2^\circ$ ∡ **Ans.**



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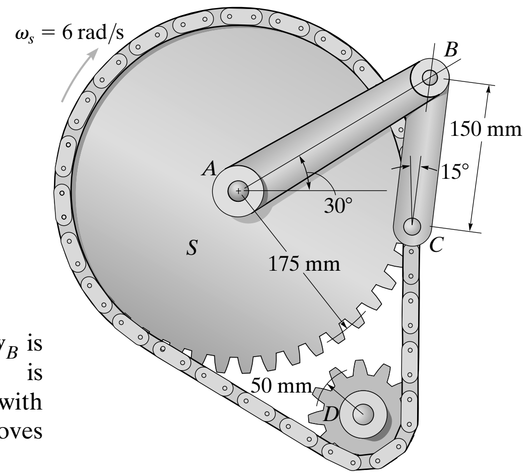
***16–108.** The mechanism produces intermittent motion of link AB. If the sprocket S is turning with an angular velocity of $\omega_S = 6$ rad/s, determine the angular velocity of link BC at this instant. The sprocket S is mounted on a shaft which is separate from a collinear shaft attached to AB at A. The pin at C is attached to one of the chain links.

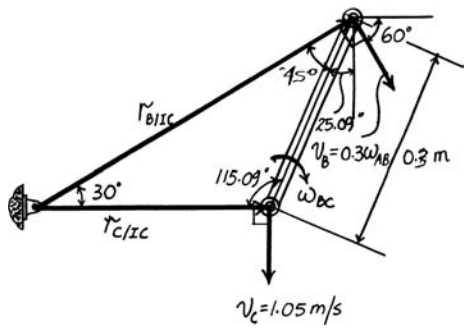


Kinematic Diagram: Since link AB is rotating about the fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB}\, r_{AB} = 0.2\omega_{AB}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle 60° with the horizontal. Since point C is attached to the chain, at the instant shown, it moves vertically with a speed of $v_C = \omega_S\, r_S = 6(0.175) = 1.05$ m/s.

Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 105^\circ} = \frac{0.15}{\sin 30^\circ} \qquad r_{B/IC} = 0.2898 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 45^\circ} = \frac{0.15}{\sin 30^\circ} \qquad r_{C/IC} = 0.2121 \text{ m}$$



The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{1.05}{0.2121} = 4.950 \text{ rad/s} \qquad \textbf{Ans.}$$

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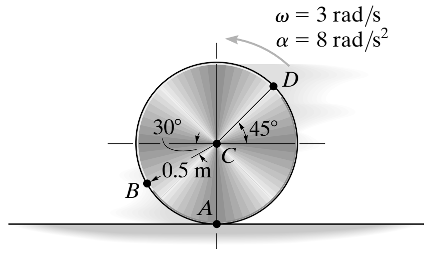
•16–109. The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point B.



$$a_C = 0.5(8) = 4\ \text{m/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

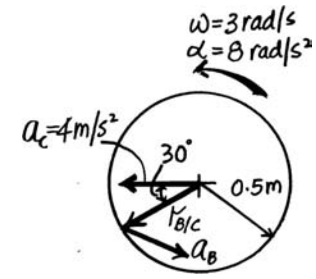
$$\mathbf{a}_B = \left[\underset{\leftarrow}{4}\right] + \left[\underset{\measuredangle 30^\circ}{(3)^2(0.5)}\right] + \left[\underset{30^\circ}{(0.5)(8)}\right]$$

$(\overset{+}{\rightarrow})$ $\quad (a_B)_x = -4 + 4.5\cos 30^\circ + 4\sin 30^\circ = 1.897\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_B)_y = 0 + 4.5\sin 30^\circ - 4\cos 30^\circ = -1.214\ \text{m/s}^2$

$$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{1.214}{1.897}\right) = 32.6^\circ \qquad \textbf{Ans.}$$



Also,

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2\,\mathbf{r}_{B/C}$$

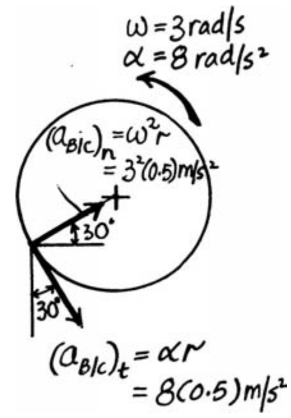
$$(a_B)_x\,\mathbf{i} + (a_B)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (-0.5\cos 30^\circ\mathbf{i} - 0.5\sin 30^\circ\mathbf{j}) - (3)^2\,(-0.5\cos 30^\circ\mathbf{i} - 0.5\sin 30^\circ\mathbf{j})$$

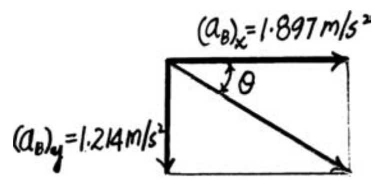
$(\overset{+}{\rightarrow})$ $\quad (a_B)_x = -4 + 8(0.5\sin 30^\circ) + (3)^2(0.5\cos 30^\circ) = 1.897\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_B)_y = 0 - 8(0.5\cos 30^\circ) + (3)^2\,(0.5\sin 30^\circ) = -1.214\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{1.214}{1.897}\right) = 32.6^\circ \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2 \qquad \textbf{Ans.}$$





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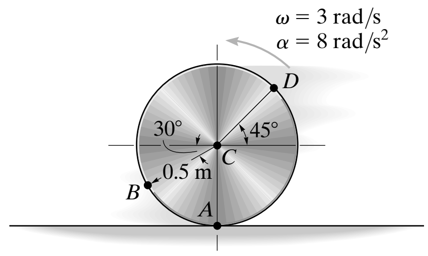
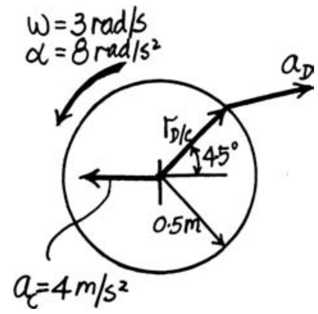
16–110. The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point D.



$$a_C = 0.5(8) = 4\ \text{m/s}^2$$

$$\mathbf{a}_D = \mathbf{a}_C + \mathbf{a}_{D/C}$$

$$\mathbf{a}_D = \begin{bmatrix} 4 \\ \leftarrow \end{bmatrix} + \begin{bmatrix} (3)^2(0.5) \\ \swarrow 45^\circ \end{bmatrix} + \begin{bmatrix} 8\,(0.5) \\ 45^\circ \searrow \end{bmatrix}$$



$(\overset{+}{\rightarrow})$ $\quad (a_D)_x = -4 - 4.5 \sin 45^\circ - 4 \cos 45^\circ = -10.01\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_D)_y = 0 - 4.5 \cos 45^\circ + 4 \sin 45^\circ = -0.3536\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \quad \textbf{Ans.}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \quad \textbf{Ans.}$$

Also,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha \times \mathbf{r}_{D/C} - \omega^2\,\mathbf{r}_{D/C}$$

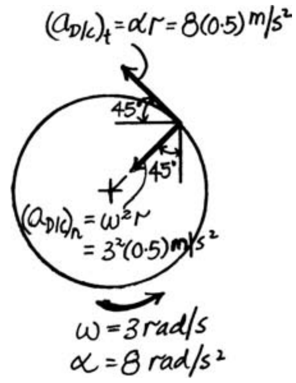
$$(a_D)_x\,\mathbf{i} + (a_D)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (0.5 \cos 45^\circ \mathbf{i} + 0.5 \sin 45^\circ \mathbf{j}) - (3)^2\,(0.5 \cos 45^\circ \mathbf{i} + 0.5 \sin 45^\circ \mathbf{j})$$

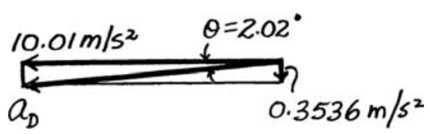
$(\overset{+}{\rightarrow})$ $\quad (a_D)_x = -4 - 8(0.5 \sin 45^\circ) - (3)^2(0.5 \cos 45^\circ) = -10.01\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_D)_y = +8(0.5 \cos 45^\circ) - (3)^2\,(0.5 \sin 45^\circ) = -0.3536\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \quad \textbf{Ans.}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \quad \textbf{Ans.}$$





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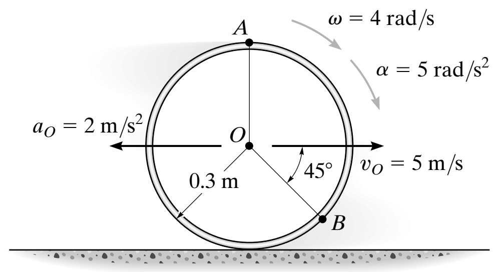
16–111. The hoop is cast on the rough surface such that it has an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s^2. Also, its center has a velocity $v_O = 5$ m/s and a deceleration $a_O = 2$ m/s^2. Determine the acceleration of point A at this instant.



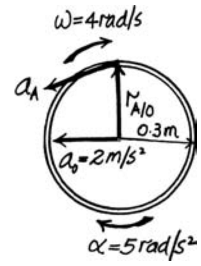
$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O}$$

$$\mathbf{a}_A = \left[\underset{\leftarrow}{2}\right] + \left[\underset{\downarrow}{(4)^2(0.3)}\right] + \left[\underset{\rightarrow}{5(0.3)}\right]$$

$$\mathbf{a}_A = \left[\underset{\leftarrow}{0.5}\right] + \left[\underset{\downarrow}{4.8}\right]$$

$a_A = 4.83$ m/s^2 **Ans.**

$\theta = \tan^{-1}\left(\dfrac{4.8}{0.5}\right) = 84.1°$ ⦣ **Ans.**



Also,

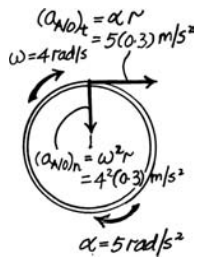
$$\mathbf{a}_A = \mathbf{a}_O - \omega^2\mathbf{r}_{A/O} + \alpha \times \mathbf{r}_{A/O}$$

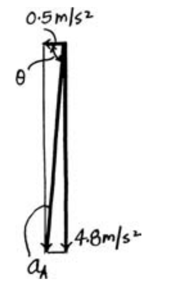
$$\mathbf{a}_A = -2\mathbf{i} - (4)^2(0.3\mathbf{j}) + (-5\mathbf{k}) \times (0.3\mathbf{j})$$

$$\mathbf{a}_A = \{-0.5\mathbf{i} - 4.8\mathbf{j}\} \text{ m/s}^2$$

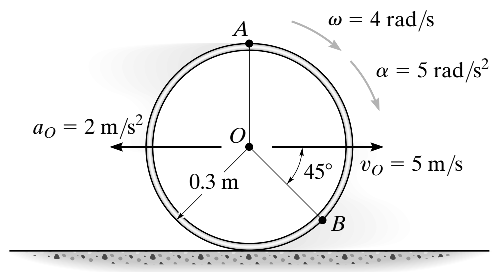
$a_A = 4.83$ m/s^2 **Ans.**

$\theta = \tan^{-1}\left(\dfrac{4.8}{0.5}\right) = 84.1°$ ⦣ **Ans.**





***16–112.** The hoop is cast on the rough surface such that it has an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s^2. Also, its center has a velocity of $v_O = 5$ m/s and a deceleration $a_O = 2$ m/s^2. Determine the acceleration of point B at this instant.



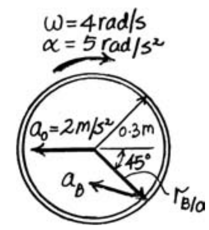
$$\mathbf{a}_B = \mathbf{a}_O + \mathbf{a}_{B/O}$$

$$\mathbf{a}_B = \left[\underset{\leftarrow}{2}\right] + \left[\underset{⦣}{5(0.3)}\right] + \left[\underset{⊾}{(4)^2(0.3)}\right]$$

$$\mathbf{a}_B = \left[\underset{\leftarrow}{6.4548}\right] + \left[\underset{\uparrow}{2.333}\right]$$

$a_B = 6.86$ m/s^2 **Ans.**

$\theta = \tan^{-1}\left(\dfrac{2.333}{6.4548}\right) = 19.9°$ ⊾



Also:

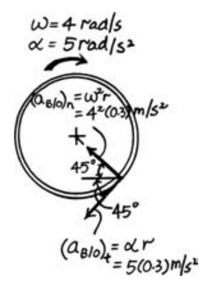
$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2\,\mathbf{r}_{B/O}$$

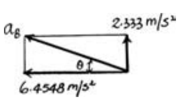
$$\mathbf{a}_B = -2\mathbf{i} + (-5\mathbf{k}) \times (0.3\cos 45°\mathbf{i} - 0.3\sin 45°\mathbf{j}) - (4)^2(0.3\cos 45°\mathbf{i} - 0.3\sin 45°\mathbf{j})$$

$$\mathbf{a}_B = \{-6.4548\mathbf{i} + 2.333\mathbf{j}\} \text{ m/s}^2$$

$a_B = 6.86$ m/s^2 **Ans.**

$\theta = \tan^{-1}\left(\dfrac{2.333}{6.4548}\right) = 19.9°$ ⊾





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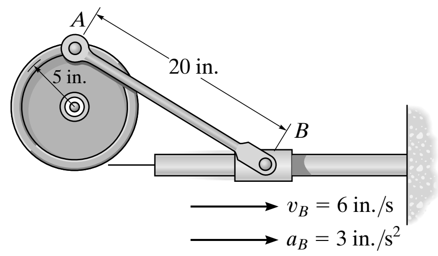
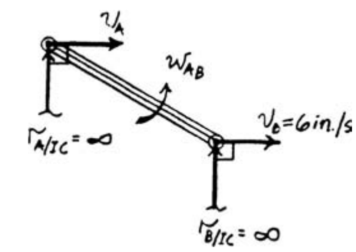
•**16–113.** At the instant shown, the slider block B is traveling to the right with the velocity and acceleration shown. Determine the angular acceleration of the wheel at this instant.



Velocity Analysis: The angular velocity of link AB can be obtained by using the method of instantaneous center of zero velocity. Since $\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel, $r_{A/IC} = r_{B/IC} = \infty$. Thus, $\omega_{AB} = 0$. Since $\omega_{AB} = 0$, $v_A = v_B = 6$ in./s. Thus, the angular velocity of the wheel is $\omega_W = \dfrac{v_A}{r_{OA}} = \dfrac{6}{5} = 1.20$ rad/s.



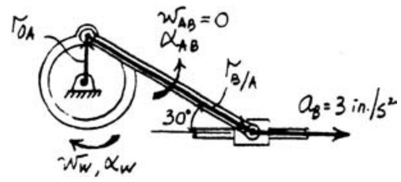
Acceleration Equation: The acceleration of point A can be obtained by analyzing the angular motion of link OA about point O. Here, $\mathbf{r}_{OA} = \{5\mathbf{j}\}$ in..

$$\mathbf{a}_A = \alpha_W \times \mathbf{r}_{OA} - \omega_W^2\, \mathbf{r}_{OA}$$

$$= (-\alpha_W\, \mathbf{k}) \times (5\mathbf{j}) - 1.20^2\,(5\mathbf{j})$$

$$= \{5\alpha_W\, \mathbf{i} - 7.20\mathbf{j}\} \text{ in./s}^2$$

Link AB is subjected to general plane motion. Applying Eq. 16–18 with $\mathbf{r}_{B/A} = \{20\cos 30°\mathbf{i} - 20\sin 30°\mathbf{j}\}$ in. $= \{17.32\mathbf{i} - 10.0\mathbf{j}\}$ in., we have



$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$3\mathbf{i} = 5\alpha_W\, \mathbf{i} - 7.20\mathbf{j} + \alpha_{AB}\,\mathbf{k} \times (17.32\mathbf{i} - 10.0\mathbf{j}) - \mathbf{0}$$

$$3\mathbf{i} = (10.0\alpha_{AB} + 5\alpha_W)\,\mathbf{i} + (17.32\alpha_{AB} - 7.20)\,\mathbf{j}$$

Equating **i** and **j** components, we have

$$3 = 10.0\alpha_{AB} + 5\alpha_W \qquad \textbf{[1]}$$

$$0 = 17.32\alpha_{AB} - 7.20 \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

$$\alpha_{AB} = 0.4157 \text{ rad/s}^2$$

$$\alpha_W = -0.2314 \text{ rad/s}^2 = 0.231 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

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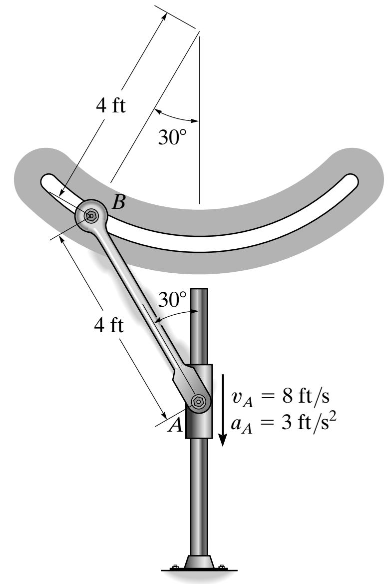
16–114. The ends of bar AB are confined to move along the paths shown. At a given instant, A has a velocity of 8 ft/s and an acceleration of 3 ft/s^2. Determine the angular velocity and angular acceleration of AB at this instant.



$\omega = \dfrac{8}{4} = 2\text{ rad/s}$ ↻ **Ans.**

$v_B = 4(2) = 8\text{ ft/s}$

$(a_B)_n = \dfrac{(8)^2}{4} = 16\text{ ft/s}^2$

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

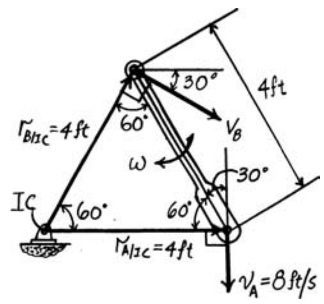
$$\begin{bmatrix} 16 \\ \nearrow 30^\circ \end{bmatrix} + \begin{bmatrix} (a_B)_t \\ \searrow 30^\circ \end{bmatrix} = \begin{bmatrix} 3 \\ \downarrow \end{bmatrix} + \begin{bmatrix} \alpha(4) \\ \nearrow 60^\circ \end{bmatrix} + \begin{bmatrix} (2)^2(4) \\ \searrow 60^\circ \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $16\sin 30^\circ + (a_B)_t \cos 30^\circ = 0 + \alpha(4)\sin 60^\circ + 16\cos 60^\circ$

$(+\uparrow)$ $16\cos 30^\circ - (a_B)_t \sin 30^\circ = -3 + \alpha(4)\cos 60^\circ - 16\sin 60^\circ$

$\alpha = 7.68\text{ rad/s}^2$ ↻ **Ans.**

$(a_B)_t = 30.7\text{ ft/s}^2$



Also,

$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

$(a_B)_t \cos 30^\circ\mathbf{i} - (a_B)_t \sin 30^\circ\mathbf{j} + (\dfrac{(8)^2}{4})\sin 30^\circ\mathbf{i} + (\dfrac{(8)^2}{4})\cos 30^\circ\mathbf{j} = -3\mathbf{j}$

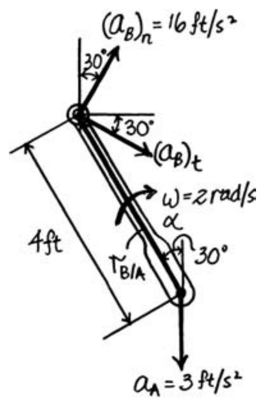
$-(\alpha\mathbf{k}) \times (-4\sin 30^\circ\mathbf{i} + 4\cos 30^\circ\mathbf{j}) - (2)^2(-4\sin 30^\circ\mathbf{i} + 4\cos 30^\circ\mathbf{j})$

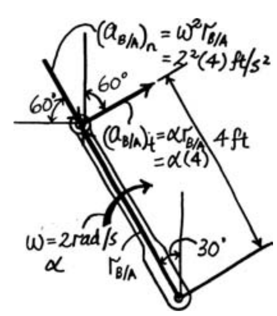
$(\overset{+}{\rightarrow})$ $(a_B)_t \cos 30^\circ + 8 = -3.464\alpha + 8$

$(+\uparrow)$ $-(a_B)_t \sin 30^\circ + 13.8564 = -3 + 2\alpha - 13.8564$

$\alpha = 7.68\text{ rad/s}^2$ ↻ **Ans.**

$(a_B)_t = 30.7\text{ ft/s}^2$





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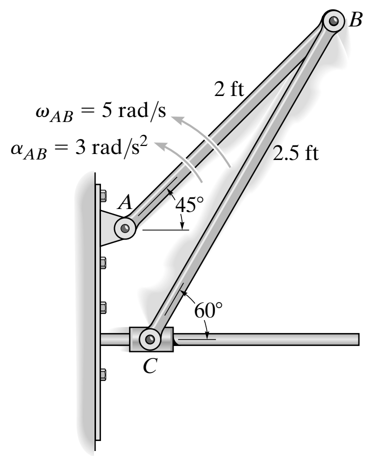
16–115. Rod AB has the angular motion shown. Determine the acceleration of the collar C at this instant.



$$\frac{r_{B/IC}}{\sin 30^\circ} = \frac{2.5}{\sin 135^\circ}$$

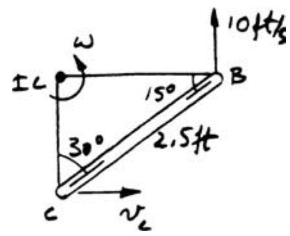
$$r_{B/IC} = 1.7678 \text{ ft}$$

$$\omega = \frac{10}{1.7678} = 5.66 \text{ rad/s} \quad \circlearrowright$$

$$(a_B)_n = 25(2) = 50 \text{ ft/s}^2$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

$$\left[\underset{\rightarrow}{a_C}\right] = \left[\underset{45^\circ \searrow}{6}\right] + \left[\underset{45^\circ \swarrow}{50}\right] + \left[\underset{\swarrow 60^\circ}{(5.66)^2(2.5)}\right] + \left[\underset{\searrow 30^\circ}{\alpha(2.5)}\right]$$



$(\overset{+}{\rightarrow})$ $\quad a_C = -6 \cos 45^\circ - 50 \cos 45^\circ + 80 \cos 60^\circ + \alpha(2.5) \cos 30^\circ$

$(+\uparrow)$ $\quad 0 = 6 \sin 45^\circ - 50 \sin 45^\circ + 80 \sin 60^\circ - \alpha(2.5)\sin 30^\circ$

$$\alpha = 30.5 \text{ rad/s}^2 \quad \circlearrowright$$

$$a_C = 66.5 \text{ ft/s}^2 \rightarrow$$ **Ans.**

Also,

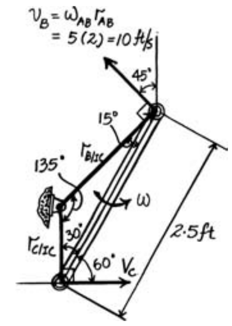
$$v_B = 5(2) = 10 \text{ ft/s}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$-v_C\,\mathbf{i} = -10 \cos 45^\circ \mathbf{i} + 10 \sin 45^\circ \mathbf{j} + \omega\mathbf{k} \times (-2.5 \sin 30^\circ \mathbf{i} - 2.5 \cos 30^\circ \mathbf{j})$$

$(+\uparrow)$ $\quad 0 = 10 \sin 45^\circ - 2.5\,\omega \sin 30^\circ$

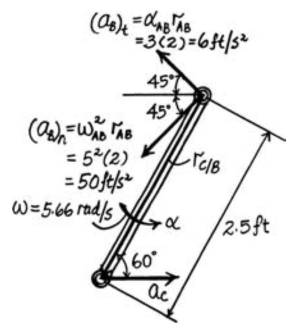
$$\omega = 5.66 \text{ rad/s}$$



$$\mathbf{a}_C = \mathbf{a}_B + \alpha \times \mathbf{r}_{C/B} - \omega^2 \mathbf{r}_{C/B}$$

$$a_C\,\mathbf{i} = -\frac{(10)^2}{2} \cos 45^\circ \mathbf{i} - \frac{(10)^2}{2} \sin 45^\circ \mathbf{j} - 6 \cos 45^\circ \mathbf{i} + 6 \sin 45^\circ \mathbf{j}$$

$$+ (\alpha\mathbf{k}) \times (-2.5 \cos 60^\circ \mathbf{i} - 2.5 \sin 60^\circ \mathbf{j}) - (5.66)^2 (-2.5 \cos 60^\circ \mathbf{i} - 2.5 \sin 60^\circ \mathbf{j})$$

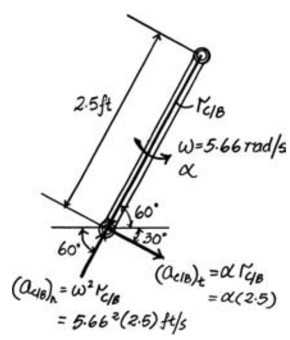


$(\overset{+}{\rightarrow})$ $\quad a_C = -35.355 - 4.243 + 2.165\alpha + 40$

$(+\uparrow)$ $\quad 0 - -35.355 + 4.243 - 1.25\alpha + 69.282$

$$\alpha = 30.5 \text{ rad/s}^2 \quad \circlearrowright$$

$$a_C = 66.5 \text{ ft/s}^2 \quad \rightarrow$$ **Ans.**



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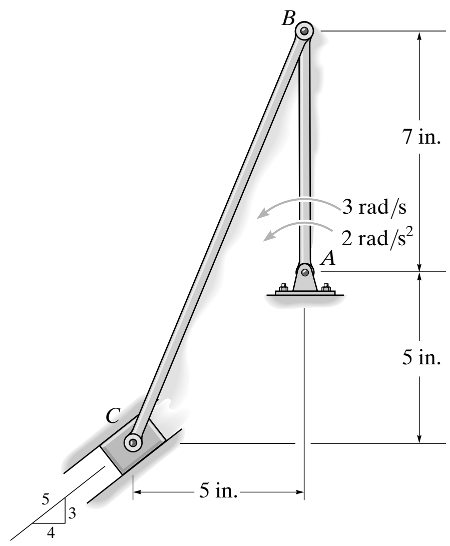
***16–116.** At the given instant member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



$v_B = 3(7) = 21 \text{ in./s} \leftarrow$

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$-v_C\left(\frac{4}{5}\right)\mathbf{i} - v_C\left(\frac{3}{5}\right)\mathbf{j} = -21\mathbf{i} + \omega\mathbf{k} \times (-5\mathbf{i} - 12\mathbf{j})$

$(\overset{+}{\rightarrow}) \qquad -0.8v_C = -21 + 12\omega$

$(+\uparrow) \qquad -0.6v_C = -5\omega$

Solving:

$\omega = 1.125 \text{ rad/s}$

$v_C = 9.375 \text{ in./s} = 9.38 \text{ in./s}$ **Ans.**

$(a_B)_n = (3)^2(7) = 63 \text{ in./s}^2 \downarrow$

$(a_B)_t = (2)(7) = 14 \text{ in./s}^2 \leftarrow$

$\mathbf{a}_C = a_B + \alpha \times \mathbf{r}_{C/B} - \omega^2 \mathbf{r}_{C/B}$

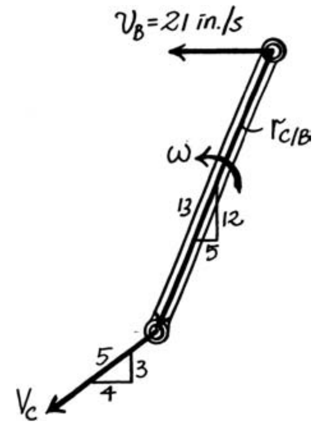
$-a_C\left(\frac{4}{5}\right)\mathbf{i} - a_C\left(\frac{3}{5}\right)\mathbf{j} = -14\mathbf{i} - 63\mathbf{j} + (\alpha\mathbf{k}) \times (-5\mathbf{i} - 12\mathbf{j}) - (1.125)^2(-5\mathbf{i} - 12\mathbf{j})$

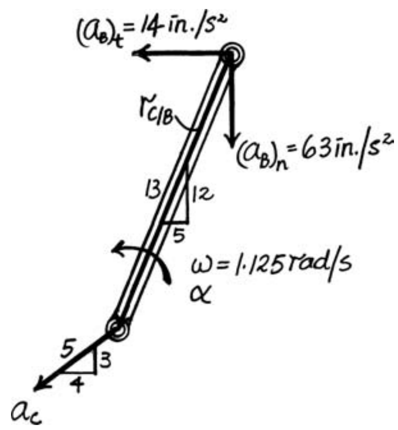
$(\overset{+}{\rightarrow}) \qquad -0.8a_C = -14 + 12\alpha + 6.328$

$(+\uparrow) \qquad -0.6a_C = -63 - 5\alpha + 15.1875$

$a_C = 54.7 \text{ in./s}^2$ **Ans.**

$\alpha = -3.00 \text{ rad/s}^2$





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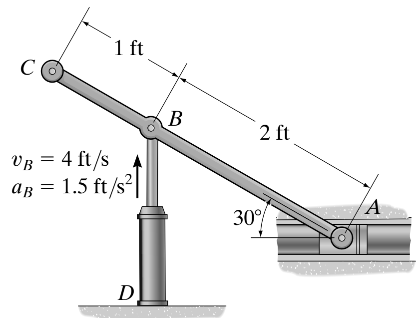
•16–117. The hydraulic cylinder D extends with a velocity of $v_B = 4\text{ ft/s}$ and an acceleration of $a_B = 1.5\text{ ft/s}^2$. Determine the acceleration of A at the instant shown.

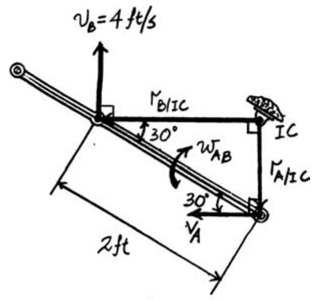


Angular Velocity: The location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,

$$r_{B/IC} = 2\cos 30° = 1.732\text{ ft}$$

Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{4}{1.732} = 2.309\text{ rad/s}$$



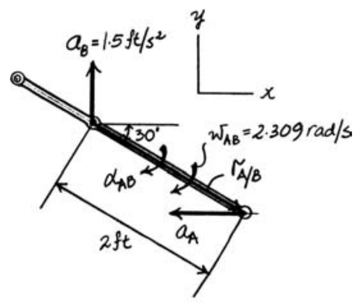
(a)

Acceleration and Angular Acceleration: Here, $\mathbf{r}_{A/B} = 2\cos 30°\mathbf{i} - 2\sin 30°\,\mathbf{j} = [1.732\mathbf{i} - 1\mathbf{j}]$ ft. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = 1.5\mathbf{j} + (-\alpha_{AB}\mathbf{k})\times(1.732\mathbf{i} - 1\mathbf{j}) - 2.309^2(1.732\mathbf{i} - 1\mathbf{j})$$

$$-a_A\mathbf{i} = -(\alpha_{AB} + 9.238)\mathbf{i} + (6.833 - 1.732\alpha_{AB})\mathbf{j}$$



(b)

Equating the **i** and **j** components,

$$-a_A = -(\alpha_{AB} + 9.238) \qquad \textbf{(1)}$$

$$0 = 6.833 - 1.732\alpha_{AB} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\alpha_{AB} = 3.945\text{ rad/s}^2$$

$$a_A = 13.2\text{ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

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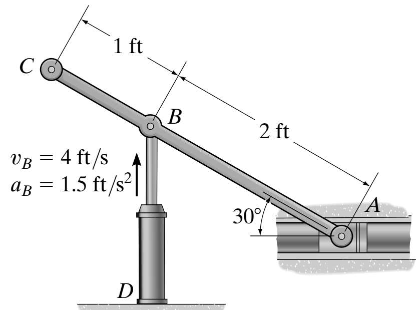
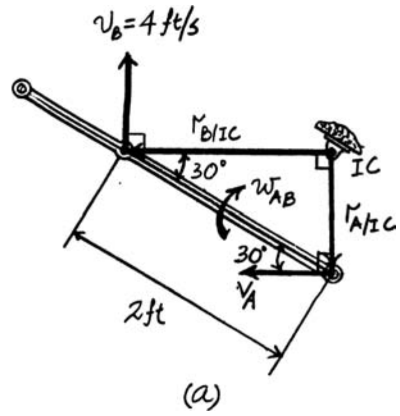
16–118. The hydraulic cylinder D extends with a velocity of $v_B = 4\text{ ft/s}$ and an acceleration of $a_B = 1.5\text{ ft/s}^2$. Determine the acceleration of C at the instant shown.



Angular Velocity: The location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure, $r_{B/IC} = 2\cos 30° = 1.732$ ft

Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{4}{1.732} = 2.309 \text{ rad/s}$$



Acceleration and Angular Acceleration: Here, $\mathbf{r}_{A/B} = 2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j} = [1.732\mathbf{i} - 1\mathbf{j}]$ ft. Applying the relative acceleration equation to points A and B and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\,\mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = 1.5\mathbf{j} + (-\alpha_{AB}\mathbf{k}) \times (1.732\mathbf{i} - 1\mathbf{j}) - 2.309^2(1.732\mathbf{i} - 1\mathbf{j})$$

$$-a_A\,\mathbf{i} = -(\alpha_{AB} + 9.2376)\mathbf{i} + (6.833 - 1.732\alpha_{AB})\mathbf{j}$$

Equating the **i** and **j** components, we obtain

$$0 = 6.833 - 1.732\alpha_{AB} \qquad \alpha_{AB} = 3.945 \text{ rad/s}^2$$

Using this result and $\mathbf{r}_{C/B} = -1\cos 30°\,\mathbf{i} + 1\sin 30°\mathbf{j} = [-0.8660\mathbf{i} + 0.5\mathbf{j}]$ ft, the relative acceleration equation is applied at points B and C, Fig. b, which gives

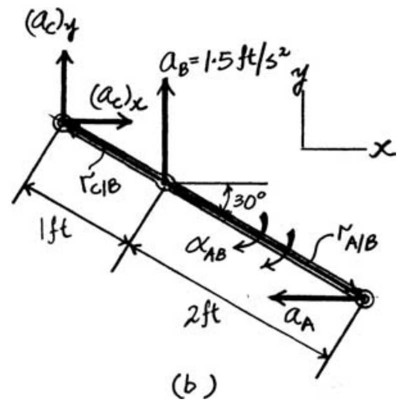
$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{C/B} - \omega_{AB}{}^2\mathbf{r}_{C/B}$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\mathbf{j} = 1.5\mathbf{j} + (-3.945\mathbf{k}) \times (-0.8660\mathbf{i} + 0.5\mathbf{j}) - (2.309)^2(-0.8660\mathbf{i} + 0.5\mathbf{j})$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\,\mathbf{j} = 6.591\mathbf{i} + 2.25\mathbf{j}$$

Equating the **i** and **j** components,

$$(a_C)_x = 6.591 \text{ ft/s}^2 \rightarrow \qquad (a_C)_y = 2.25 \text{ ft/s}^2 \uparrow$$



Thus, the magnitude of a_C is

$$a_C = \sqrt{(a_C)_x{}^2 + (a_C)_y{}^2} = \sqrt{6.591^2 + 2.25^2} = 6.96 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_C)_y}{(a_C)_x}\right] = \tan^{-1}\left(\frac{2.25}{6.591}\right) = 18.8° \measuredangle \qquad \textbf{Ans.}$$

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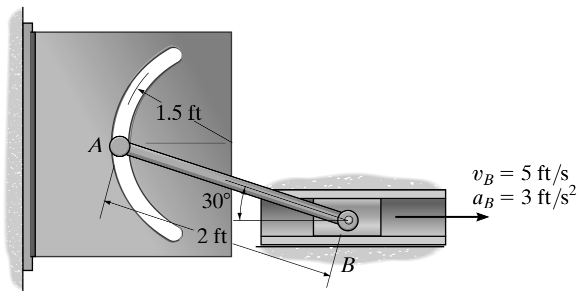
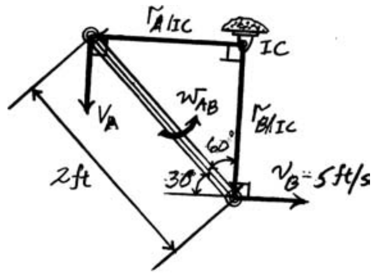
16–119. The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the angular acceleration of rod AB at the instant shown.



Angular Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,

$$r_{B/IC} = 2 \sin 30° = 1 \text{ ft} \qquad\qquad r_{A/IC} = 2 \cos 30° = 1.732 \text{ ft}$$



(a)

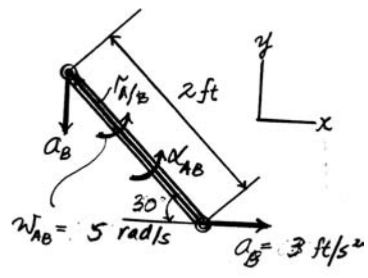
Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5 \text{ rad/s}$$

Then

$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660 \text{ ft/s}$$

Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. b,



(b)

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\, \mathbf{r}_{A/B}$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\,\mathbf{k}) \times (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) - 5^2(-2\cos 30°\,\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} + (1.732\alpha_{AB} + 25)\mathbf{j}$$

Equating the **i** components,

$$50 = 46.30 - \alpha_{AB}$$

$$\alpha_{AB} = -3.70 \text{ rad/s}^2 = 3.70 \text{ rad/s}^2 \; \circlearrowright \qquad \textbf{Ans.}$$

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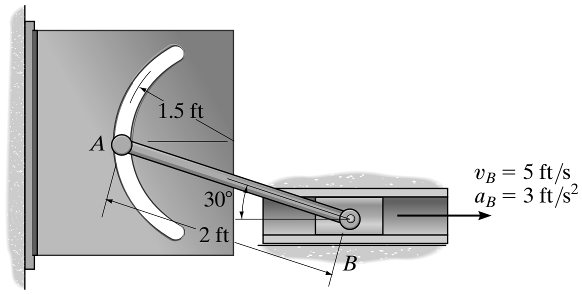
***16–120.** The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the acceleration of A at the instant shown.



Angualr Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,

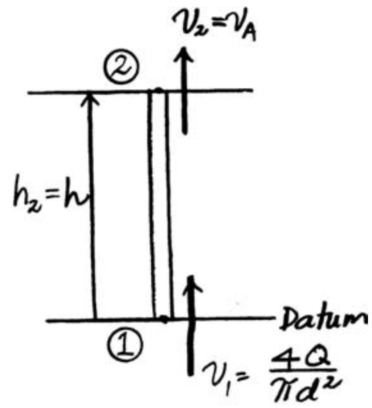
$$r_{B/IC} = 2 \sin 30° = 1 \text{ ft} \qquad\qquad r_{A/IC} = 2 \cos 30° = 1.732 \text{ ft}$$

Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5 \text{ rad/s}$$

Then

$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660 \text{ ft/s}$$



(a)

Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \frac{v_A{}^2}{\rho} = \frac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\, \mathbf{r}_{A/B}$$

$$50\mathbf{i} - (a_A)_t\, \mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\, \mathbf{k}) \times (-2\cos 30°\mathbf{i} + 2 \sin 30°\mathbf{j}) - 5^2(-2 \cos 30°\mathbf{i} + 2 \sin 30°\mathbf{j})$$

$$50\mathbf{i} - (a_A)_t\, \mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} - (1.732\alpha_{AB} + 25)\mathbf{j}$$

Equating the **i** and **j** components,

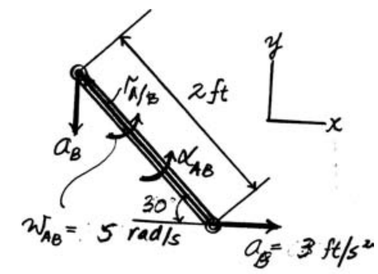
$$50 = 46.30 - \alpha_{AB}$$

$$-(a_A)_t = -(1.732\alpha_{AB} + 25)$$

Solving,

$$\alpha_{AB} = -3.70 \text{ rad/s}^2$$

$$(a_A)_t = 18.59 \text{ ft/s}^2 \downarrow$$



(b)

Thus, the magnitude of $\mathbf{a}_A$ is

$$a_A = \sqrt{(a_A)_t{}^2 + (a_A)_n{}^2} = \sqrt{18.59^2 + 50^2} = 53.3 \text{ft/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_A)_t}{(a_A)_n}\right] = \tan^{-1}\left(\frac{18.59}{50}\right) = 20.4° \qquad \textbf{Ans.}$$

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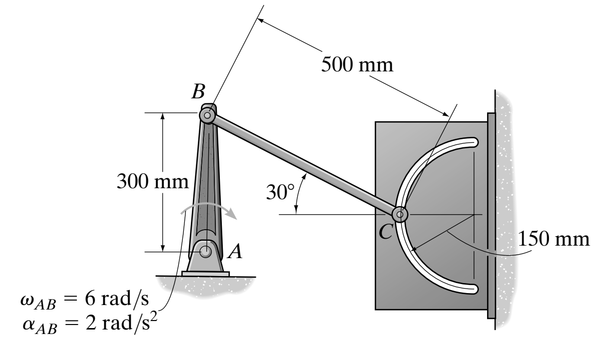
•16–121. Crank AB rotates with an angular velocity of $\omega_{AB} = 6\text{ rad/s}$ and an angular acceleration of $\alpha_{AB} = 2\text{ rad/s}^2$. Determine the acceleration of C and the angular acceleration of BC at the instant shown.



Angular Velocity: Since crank AB rotates about a fixed axis, then

$$v_B = \omega_{AB}\, r_B = 6(0.3) = 1.8\text{ m/s} \rightarrow$$

The location of the IC for rod BC is

$r_{B/IC} = 0.5 \sin 30° = 0.25\text{ m}$ $\qquad$ $r_{C/IC} = 0.5 \cos 30° = 0.4330\text{ m}$

Then,

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.8}{0.25} = 7.2\text{ rad/s}$$

and

$$v_C = \omega_{BC}\, r_{C/IC} = 7.2(0.4330) = 3.118\text{ ft/s}$$

Acceleration and Angular Acceleration: Since crank AB rotates about a fixed axis, then

$$\begin{aligned}\mathbf{a}_B &= \alpha'_{AB} \times \mathbf{r}_B - \omega_{AB}{}^2\mathbf{r}_B \\ &= (-2\mathbf{k}) \times (0.3\mathbf{j}) - 6^2(0.3\mathbf{j}) \\ &= \{0.6\mathbf{i} - 10.8\mathbf{j}\}\text{ m/s}^2\end{aligned}$$

Since point C travels along a circular slot, the normal component of its acceleration has a magnitude of $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{3.118^2}{0.15} = 64.8\text{ m/s}^2$ and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation,

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B}$$

$$64.8\mathbf{i} - (a_C)_t\,\mathbf{j} = (0.6\mathbf{i} - 10.8\mathbf{j}) + (\alpha_{BC}\mathbf{k}) \times (0.5 \cos 30°\mathbf{i} - 0.5 \sin 30°\,\mathbf{j}) - 7.2^2(0.5 \cos 30°\mathbf{i} - 0.5 \sin 30°\,\mathbf{j})$$

$$64.8\mathbf{i} - (a_C)_t\,\mathbf{j} = -(0.25\alpha_{BC} - 21.85)\mathbf{i} + (2.16 + 0.4330\alpha_{BC})\mathbf{j}$$

Equating the **i** and **j** components,

$$64.8 = -(0.25\alpha_{BC} - 21.85)$$

$$-(a_C)_t = 2.16 + 0.4330\alpha_{BC}$$

Solving,

$$\alpha_{BC} = -346.59\text{ rad/s}^2 = 347\text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$(a_C)_t = -152.24\text{ m/s}^2 = 152.24\text{ m/s}^2 \uparrow$$

Thus, the magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{(a_C)_t{}^2 + (a_C)_n{}^2} = \sqrt{152.24^2 + 64.7^2} = 165\text{m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_C)_t}{(a_C)_n}\right] = \tan^{-1}\left(\frac{152.24}{64.8}\right) = 66.9° \measuredangle \qquad \textbf{Ans.}$$

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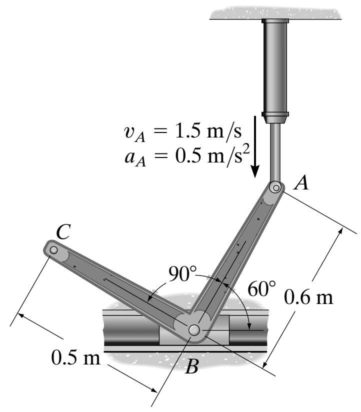
16–122. The hydraulic cylinder extends with a velocity of $v_A = 1.5\text{ m/s}$ and an acceleration of $a_A = 0.5\text{ m/s}^2$. Determine the angular acceleration of link ABC and the acceleration of end C at the instant shown. Point B is pin connected to the slider block.



Angular Velocity: The location of the IC for link ABC is indicated in Fig. a. From the geometry of this figure,

$$r_{A/IC} = 0.6\cos 60° = 0.3\text{ m}$$

Then

$$\omega_{ABC} = \frac{v_A}{r_{A/IC}} = \frac{1.5}{0.3} = 5\text{ rad/s}$$

Acceleration and Angular Acceleration: Applying the relative acceleration equation to points A and B,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{ABC} \times \mathbf{r}_{B/A} - \omega_{ABC}{}^2 r_{B/A}$$

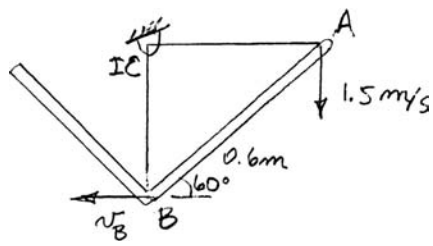
$$-a_B\,\mathbf{i} = -0.5\mathbf{j} + (-\alpha_{ABC}\,\mathbf{k}) \times (-0.6\cos 60°\mathbf{i} - 0.6\sin 60°\,\mathbf{j}) - 5^2(-0.6\cos 60°\mathbf{i} - 0.6\sin 60°\mathbf{j})$$

$$-a_B\,\mathbf{i} = (7.5 - 0.5196\alpha_{ABC})\mathbf{i} + (0.3\alpha_{ABC} + 12.490)\mathbf{j}$$

Equating the **i** and **j** components,

$$-a_B = 7.5 - 0.5196\alpha_{ABC} \qquad \textbf{(1)}$$

$$0 = 0.3\alpha_{ABC} + 12.490 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2),

$$\alpha_{ABC} = -41.63\text{ rad/s}^2 = 41.6\text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = -29.13\text{ m/s}^2$$

From points B and C,

$$a_C = \mathbf{a}_B + \alpha_{ABC} \times \mathbf{r}_{C/B} - \omega_{ABC}{}^2 r_{C/B}$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\mathbf{j} = [-(-29.13)\mathbf{i}] + [-(-41.63)\mathbf{k}] \times (-0.5\cos 30°\,\mathbf{i} + 0.5\sin 30°\mathbf{j}) - 5^2(-0.5\cos 30°\mathbf{i} + 0.5\sin 30°\mathbf{j})$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\,\mathbf{j} = 29.55\mathbf{i} - 24.28\,\mathbf{j}$$

Equating the **i** and **j** components,

$$(a_C)_x = 29.55\text{ m/s}^2 \qquad (a_C)_y = -24.28\text{ m/s}^2 = 24.28\text{ m/s}^2 \downarrow$$

Thus, the magnitude of a_C is

$$a_C = \sqrt{(a_C)_x{}^2 + (a_C)_y{}^2} = \sqrt{29.55^2 + 24.28^2} = 38.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_C)_y}{(a_C)_x}\right] = \tan^{-1}\left(\frac{24.28}{29.55}\right) = 39.4° \qquad \textbf{Ans.}$$

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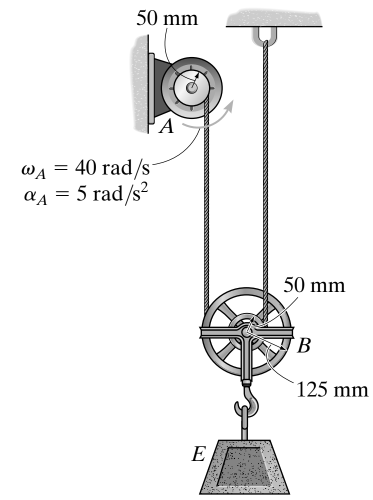
16–123. Pulley A rotates with the angular velocity and angular acceleration shown. Determine the angular acceleration of pulley B at the instant shown.

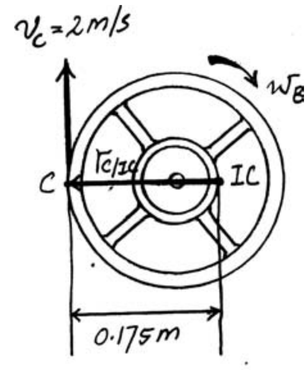


Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

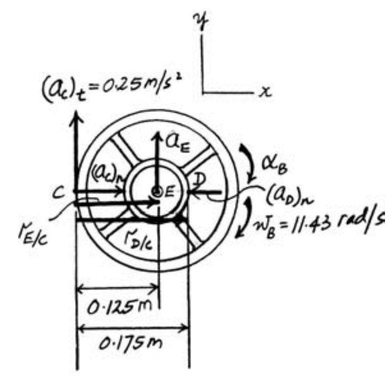
$$(a_D)_n\,\mathbf{i} = (a_C)_n\,\mathbf{i} + 0.25\mathbf{j} + (-\alpha_B\,\mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\,\mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.43 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(b)

91962_06_s16_p0513-0640 6/8/09 2:52 PM Page 605

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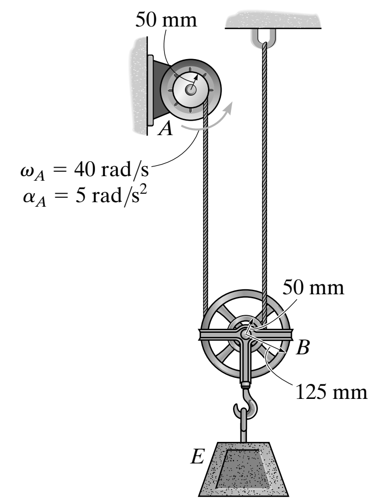
***16–124.** Pulley A rotates with the angular velocity and angular acceleration shown. Determine the acceleration of block E at the instant shown.

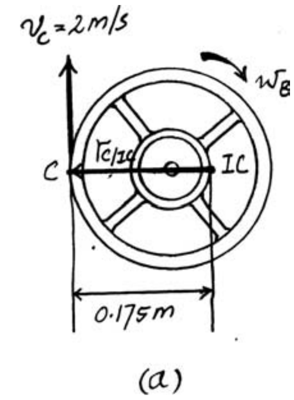


Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

$$(a_D)_n\,\mathbf{i} = (a_C)_n\,\mathbf{i} + 0.25\mathbf{j} + (-\alpha_B\mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\,\mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.429 \text{ rad/s} = 1.43 \text{ rad/s}^2$$

Using this result, the relative acceleration equation applied to points C and E, Fig. b, gives

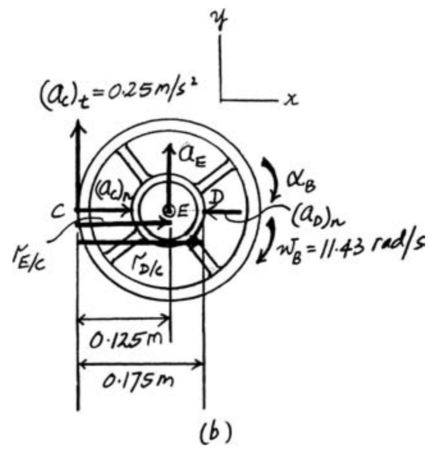
$$\mathbf{a}_E = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{E/C} - \omega_B{}^2 r_{E/C}$$

$$a_E\,\mathbf{j} = [(a_C)_n\,\mathbf{i} + 0.25\mathbf{j}] + (-1.429\mathbf{k}) \times (0.125\mathbf{i}) - 11.43^2(0.125\mathbf{i})$$

$$a_E\,\mathbf{j} = [(a_C)_n - 16.33]\mathbf{i} + 0.0714\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$a_E = 0.0714 \text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$



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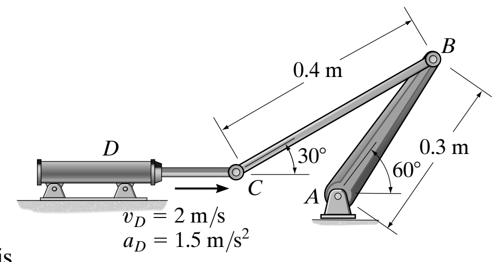
•16–125. The hydraulic cylinder is extending with the velocity and acceleration shown. Determine the angular acceleration of crank AB and link BC at the instant shown.

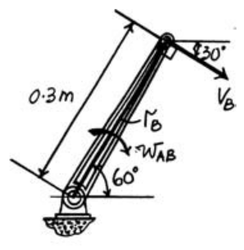


Angular Velocity: Crank AB rotates about a fixed axis. Thus,

$$v_B = \omega_{AB}\, r_B = \omega_{AB}\,(0.3)$$

The location of the IC for link BC is indicated in Fig. b. From the geometry of this figure,

$$r_{C/IC} = 0.4 \text{ m} \qquad\qquad r_{B/IC} = 2(0.4\cos 30°) = 0.6928 \text{ m}$$



(a)

Then

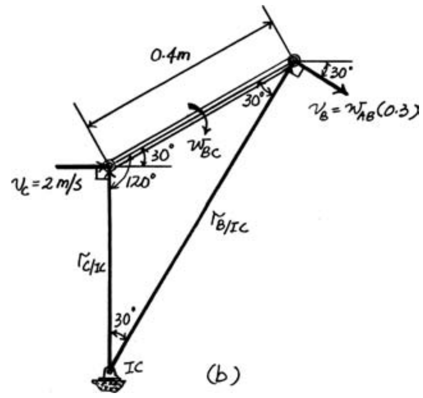
$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{2}{0.4} = 5 \text{ rad/s}$$

and

$$v_B = \omega_{BC}\, r_{B/IC}$$

$$\omega_{AB}(0.3) = 5(0.6928)$$

$$\omega_{AB} = 11.55 \text{ rad/s}$$



(b)

Acceleration and Angular Acceleration: Since crank AB rotates about a fixed axis, Fig. c,

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_B - \omega_{AB}{}^2\mathbf{r}_B$$

$$= (-\alpha_{AB}\,\mathbf{k}) \times (0.3\cos 60°\mathbf{i} + 0.3\sin 60°\mathbf{j}) - 11.55^2(0.3\cos 60°\mathbf{i} + 0.3\sin 60°\mathbf{j})$$

$$= (0.2598\alpha_{AB})\mathbf{i} - (0.15\alpha_{AB} + 34.64)\mathbf{j}$$

Using these results and applying the relative acceleration equation to points B and C of link BC, Fig. d,

$$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}{}^2 r_{B/C}$$

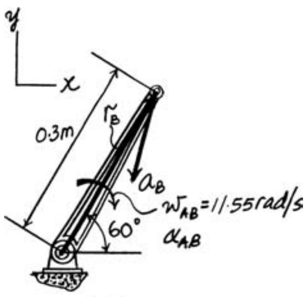
$$(0.2598\alpha_{AB} - 20)\mathbf{i} - (0.15\alpha_{AB} + 34.64)\mathbf{j} = 1.5\mathbf{i} + (\alpha_{BC}\mathbf{k}) \times (0.4\cos 30°\mathbf{i} + 0.4\sin 30°\mathbf{j}) - 5^2(0.4\cos 30°\mathbf{i} + 0.4\sin 30°\mathbf{j})$$

$$(0.2598\alpha_{AB} - 20)\mathbf{i} - (0.15\alpha_{AB} + 34.64)\mathbf{j} = -(0.2\alpha_{BC} + 7.160)\mathbf{i} + (0.3464\alpha_{BC} - 5)\mathbf{j}$$

Equating the **i** and **j** components,

$$0.2598\alpha_{AB} - 20 = -(0.2\alpha_{BC} + 7.160)$$

$$-(0.15\alpha_{AB} + 34.64) = 0.3464\alpha_{BC} - 5$$

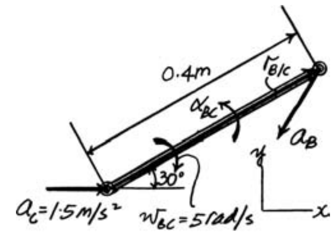


(c)

Solving,

$$\alpha_{BC} = -160.44 \text{ rad/s}^2 = 160 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$\alpha_{AB} = 172.93 \text{ rad/s}^2 = 173 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(d)

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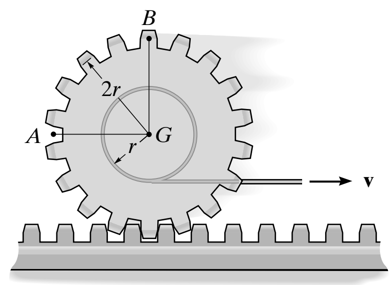
16–126. A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity **v**, determine the velocities and accelerations of points A and B. The gear rolls on the fixed gear rack.

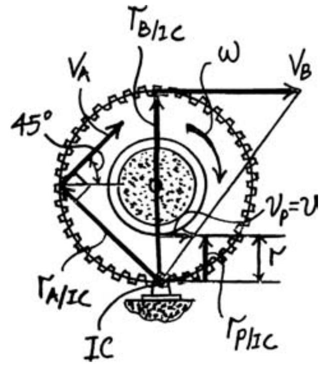


Velocity Analysis:

$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \rightarrow$$ **Ans.**

$$v_A = \omega r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \quad \measuredangle 45^\circ$$ **Ans.**



Acceleration Equation: From Example 16–3, Since $a_G = 0, \alpha = 0$

$$\mathbf{r}_{B/G} = 2\,r\,\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\,\mathbf{i}$$

$$\mathbf{a}_B = \mathbf{a}_G + \alpha \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

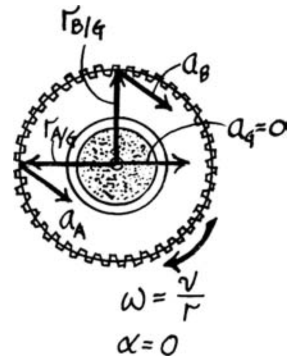
$$= \mathbf{0} + \mathbf{0} - \left(\frac{v}{r}\right)^2 (2\,r\,\mathbf{j}) = -\frac{2v^2}{r}\mathbf{j}$$

$$a_B = \frac{2\,v^2}{r} \downarrow$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2 (-2r\mathbf{i}) = \frac{2v^2}{r}\,\mathbf{i}$$

$$a_A = \frac{2v^2}{r} \rightarrow$$ **Ans.**



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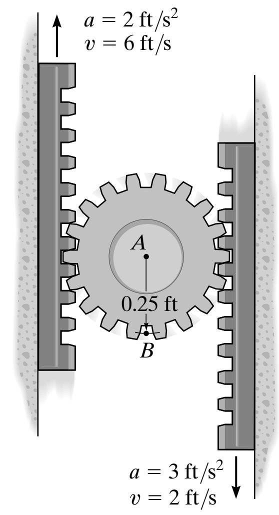
16–127. At a given instant, the gear racks have the velocities and accelerations shown. Determine the acceleration of points A and B.



Velocity Analysis: The angular velocity of the gear can be obtained by using the method of instantaneous center of zero velocity. From similar triangles,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{v_C}{r_{C/IC}}$$

$$\frac{6}{r_{D/IC}} = \frac{2}{r_{C/IC}} \qquad \textbf{[1]}$$

Where

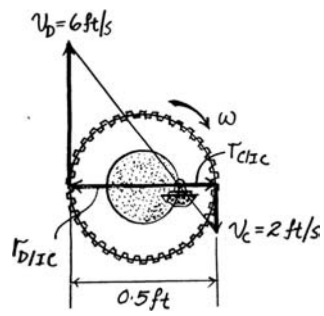
$$r_{D/IC} + r_{C/IC} = 0.5 \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

$$r_{D/IC} = 0.375\ \text{ft} \qquad r_{C/IC} = 0.125\ \text{ft}$$

Thus,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{0.375} = 16.0\ \text{rad/s}$$



Acceleration Equation: The angular acceleration of the gear can be obtained by analyzing the angular motion point C and D. Applying Eq. 16–18 with $\mathbf{r}_{D/C} = \{-0.5\mathbf{i}\}$ ft, we have

$$\mathbf{a}_D = \mathbf{a}_C + \alpha \times \mathbf{r}_{D/C} - \omega^2\, \mathbf{r}_{D/C}$$

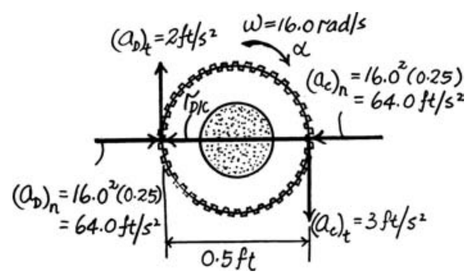
$$64.0\mathbf{i} + 2\mathbf{j} = -64.0\mathbf{i} - 3\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.5\mathbf{i}) - 16.0^2(-0.5\mathbf{i})$$

$$64.0\mathbf{i} + 2\mathbf{j} = 64.0\mathbf{i} + (0.5\alpha - 3)\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$64.0 = 64.0\ (\textbf{\textit{Check!}})$$

$$2 = 0.5\,\alpha - 3 \qquad \alpha = 10.0\ \text{rad/s}^2$$



The acceleration of point A can be obtained by analyzing the angular motion point A and C. Applying Eq. 16–18 with $\mathbf{r}_{A/C} = \{-0.25\mathbf{i}\}$ ft, we have

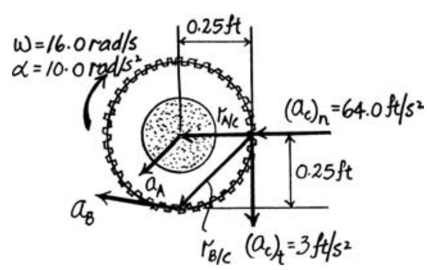
$$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2\, \mathbf{r}_{A/C}$$

$$= -64.0\mathbf{i} - 3\mathbf{j} + (-10.0\mathbf{k}) \times (-0.25\mathbf{i}) - 16.0^2(-0.25\mathbf{i})$$

$$= \{0.500\mathbf{j}\}\ \text{ft/s}^2$$

Thus,

$$a_A = 0.500\ \text{ft/s}^2 \downarrow \qquad \textbf{Ans.}$$



The acceleration of point B can be obtained by analyzing the angular motion point B and C. Applying Eq. 16–18 with $\mathbf{r}_{B/C} = \{-0.25\mathbf{i} - 0.25\mathbf{j}\}$ ft, we have

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$= -64.0\mathbf{i} - 3\mathbf{j} + (-10.0\mathbf{k}) \times (-0.25\mathbf{i} - 0.25\mathbf{j}) - 16.0^2(-0.25\mathbf{i} - 0.25\mathbf{j})$$

$$= \{-2.50\mathbf{i} + 63.5\mathbf{j}\}\ \text{ft/s}^2$$

The magnitude and direction of the acceleration of point B are given by

$$a_C = \sqrt{(-2.50)^2 + 63.5^2} = 63.5\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{63.5}{2.50} = 87.7° \qquad \textbf{Ans.}$$

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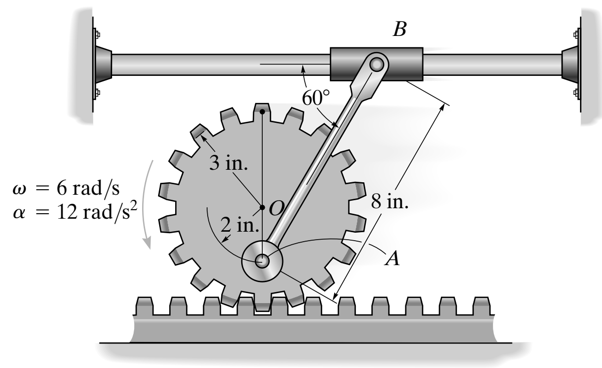
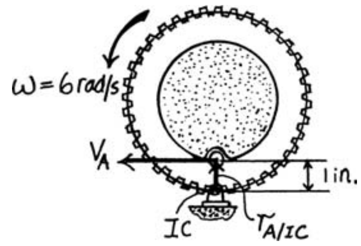
***16–128.** At a given instant, the gear has the angular motion shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant.



For the gear

$v_A = \omega r_{A/IC} = 6(1) = 6$ in./s

$\mathbf{a}_O = -12(3)\mathbf{i} = \{-36\mathbf{i}\}$ in./s^2 $\qquad \mathbf{r}_{A/O} = \{-2\mathbf{j}\}$ in. $\qquad \alpha = \{12\mathbf{k}\}$ rad/s^2



$\mathbf{a}_A = \mathbf{a}_0 + \alpha \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$

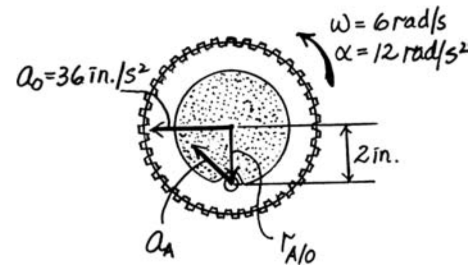
$= -36\mathbf{i} + (12\mathbf{k}) \times (-2\mathbf{j}) - (6)^2(-2\mathbf{j})$

$= \{-12\mathbf{i} + 72\mathbf{j}\}$ in./s^2

$a_A = \sqrt{(-12)^2 + 72^2} = 73.0$ in./s^2 **Ans.**

$\theta = \tan^{-1}\left(\dfrac{72}{12}\right) = 80.5°$ **Ans.**

For link AB



The IC is at ∞, so $\omega_{AB} = 0$, i.e.,

$\omega_{AB} = \dfrac{v_A}{r_{A/IC}} = \dfrac{6}{\infty} = 0$

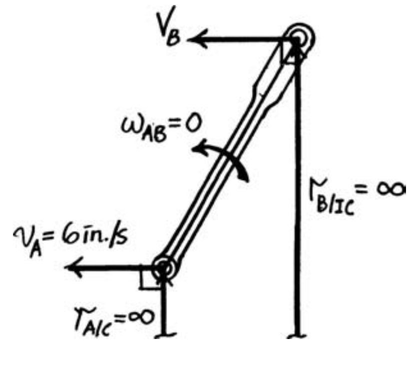
$\mathbf{a}_B = a_B\mathbf{i} \qquad \alpha_{AB} = -\alpha_{AB}\,\mathbf{k} \qquad \mathbf{r}_{B/A} = \{8 \cos 60°\mathbf{i} + 8 \sin 60°\mathbf{j}\}$ in.

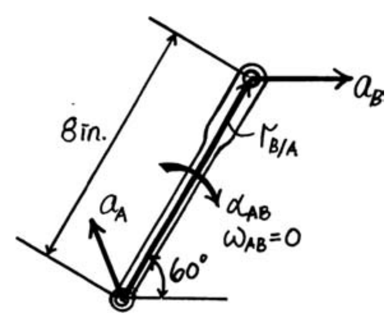
$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$

$a_B\,\mathbf{i} = (-12\mathbf{i} + 72\mathbf{j}) + (-\alpha_{AB}\,\mathbf{k}) \times (8 \cos 60°\mathbf{i} + 8 \sin 60°\mathbf{j}) - \mathbf{0}$

$(\overset{+}{\rightarrow}) \qquad a_B = -12 + 8 \sin 60°(18) = 113$ in./s$^2 \rightarrow$ **Ans.**

$(+\uparrow) \qquad 0 = 72 - 8 \cos 60°\alpha_{AB} \qquad\qquad \alpha_{AB} = 18$ rad/s^2 ↻ **Ans.**





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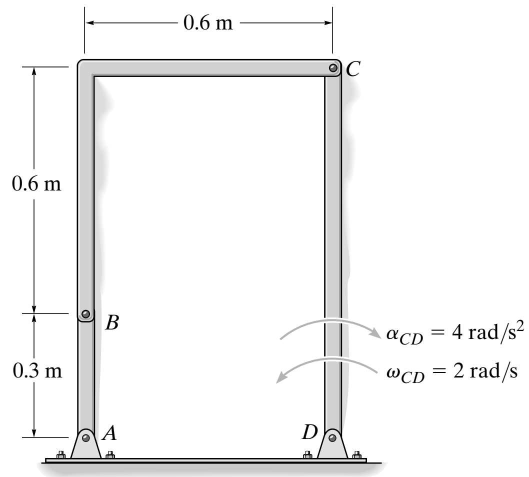
•16–129. Determine the angular acceleration of link AB if link CD has the angular velocity and angular deceleration shown.



IC is at ∞, thus

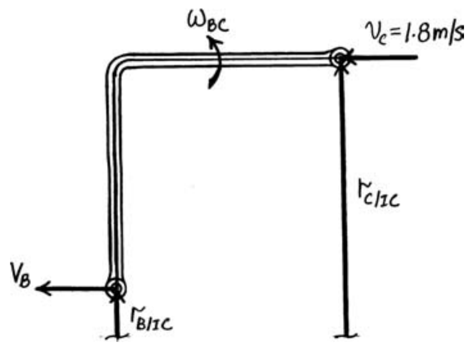
$\omega_{BC} = 0$

$v_B = v_C = (0.9)(2) = 1.8\ \text{m/s}$

$(a_C)_n = (2)^2(0.9) = 3.6\ \text{m/s}^2 \downarrow$

$(a_C)_t = 4(0.9) = 3.6\ \text{m/s}^2 \rightarrow$

$(a_B)_n = \dfrac{(1.8)^2}{0.3} = 10.8\ \text{m/s}^2 \downarrow$



$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\,\mathbf{r}_{B/C}$

$(a_B)_t\,\mathbf{i} - 10.8\mathbf{j} = 3.6\mathbf{i} - 3.6\mathbf{j} + (\alpha_{BC}\mathbf{k}) \times (-0.6\mathbf{i} - 0.6\mathbf{j}) - \mathbf{0}$

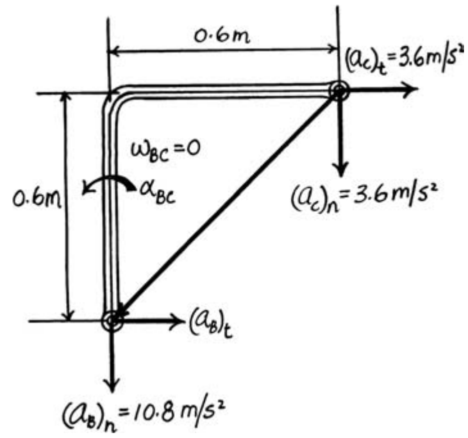
$(\overset{+}{\rightarrow})$ $\quad (a_B)_t = 3.6 + 0.6\,\alpha_{BC}$

$(+\uparrow)$ $\quad -10.8 = -3.6 - 0.6\,\alpha_{BC}$

$\alpha_{BC} = 12\ \text{rad/s}^2$

$(a_B)_t = 10.8\ \text{m/s}^2$

$\alpha_{AB} = \dfrac{10.8}{0.3} = 36\ \text{rad/s}^2 \circlearrowright$ **Ans.**



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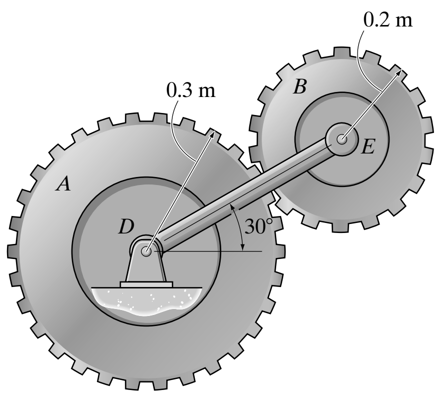
16–130. Gear A is held fixed, and arm DE rotates clockwise with an angular velocity of $\omega_{DE} = 6$ rad/s and an angular acceleration of $\alpha_{DE} = 3$ rad/s^2. Determine the angular acceleration of gear B at the instant shown.

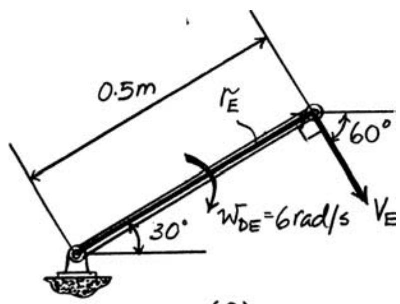


Angular Velocity: Arm DE rotates about a fixed axis, Fig. *a*. Thus,

$$v_E = \omega_{DE}\, r_E = 6(0.5) = 3 \text{ m/s}$$

The IC for gear B is located at the point where gears A and B are meshed, Fig. *b*. Thus,

$$\omega_B = \frac{v_E}{r_{E/IC}} = \frac{3}{0.2} = 15 \text{ rad/s}$$



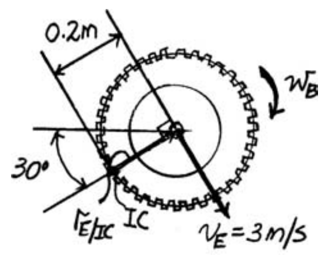
(a)

Acceleration and Angular Acceleration: Since arm DE rotates about a fixed axis, Fig. *c*,

$$\mathbf{a}_E = \alpha_{DE} \times \mathbf{r}_E - \omega_{DE}^{\;2}\, \mathbf{r}_E$$

$$= (-3\mathbf{k}) \times (0.5 \cos 30°\mathbf{i} + 0.5 \sin 30°\,\mathbf{j}) - 6^2\,(0.5 \cos 30°\,\mathbf{i} + 0.5 \sin 30°\,\mathbf{j})$$

$$= [-14.84\mathbf{i} - 10.30\mathbf{j}] \text{ m/s}^2$$



(b)

Using these results to apply the relative acceleration equation to points E and F of gear B, Fig. *d*, we have

$$\mathbf{a}_F = \mathbf{a}_E + \alpha_B \times \mathbf{r}_{F/E} - \omega_B^{\;2} r_{F/E}$$

$$a_F \cos 30°\mathbf{i} + a_F \sin 30°\mathbf{j} = (-14.84\mathbf{i} - 10.30\mathbf{j}) + (-\alpha_B\,\mathbf{k}) \times$$

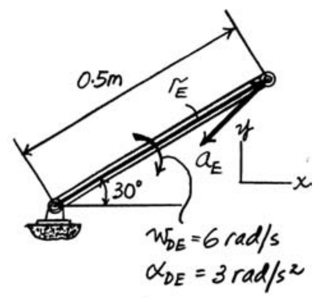
$$(-0.2 \cos 30°\,\mathbf{i} - 0.2 \sin 30°\mathbf{j}) - 15^2(-0.2 \cos 30°\mathbf{i} - 0.2 \sin 30°\mathbf{j})$$

$$a_F \cos 30°\,\mathbf{i} + a_F \sin 30°\mathbf{j} = (24.13 - 0.1\alpha_B)\mathbf{i} + (0.1732\alpha_B + 12.20)\mathbf{j}$$

Equating the **i** and **j** components yields

$$0.8660 a_F = 24.13 - 0.1\alpha_B$$

$$0.5 a_F = 0.1732\alpha_B + 12.20$$

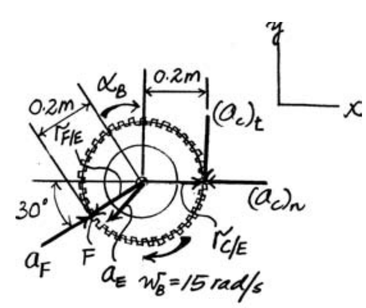


(c)

Solving,

$$a_F = 27 \text{ m/s}^2$$

$$\alpha_B = 7.5 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(d)

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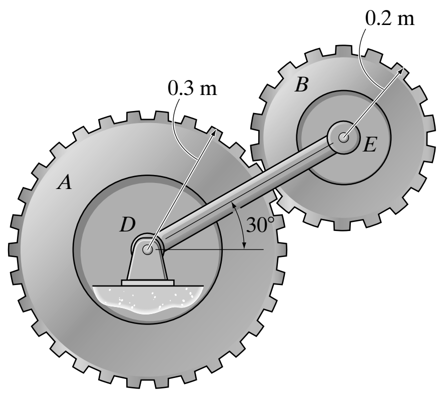
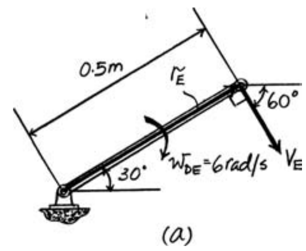
16–131. Gear A rotates counterclockwise with a constant angular velocity of $\omega_A = 10$ rad/s, while arm DE rotates clockwise with an angular velocity of $\omega_{DE} = 6$ rad/s and an angular acceleration of $\alpha_{DE} = 3\text{ rad/s}^2$. Determine the angular acceleration of gear B at the instant shown.



Angular Velocity: Arm DE and gear A rotate about a fixed axis, Figs. a and b. Thus,

$$v_E = \omega_{DE}\, r_E = 6(0.5) = 3\text{ m/s}$$

$$v_F = \omega_A\, r_F = 10(0.3) = 3\text{ m/s}$$

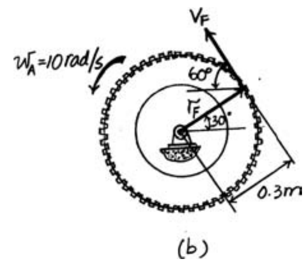


The location of the IC for gear B is indicated in Fig. c. Thus,

$$r_{E/IC} = r_{F/IC} = 0.1\text{ m}$$

Then,

$$\omega_B = \frac{v_E}{r_{E/IC}} = \frac{3}{0.1} = 30\text{ rad/s}$$

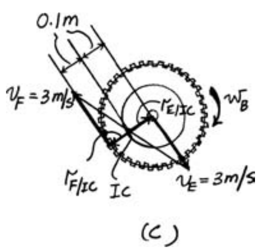


Acceleration and Angular Acceleration: Since arm DE rotates about a fixed axis, Fig. c, then

$$\mathbf{a}_E = \alpha_{DE} \times \mathbf{r}_E - \omega_{DE}^{\,2}\,\mathbf{r}_E$$

$$= (-3\mathbf{k}) \times (0.5\cos 30°\mathbf{i} + 0.5\sin 30°\,\mathbf{j}) - 6^2\,(0.5\cos 30°\,\mathbf{i} + 0.5\sin 30°\,\mathbf{j})$$

$$= [-14.84\mathbf{i} - 10.30\mathbf{j}]\text{ m/s}^2$$



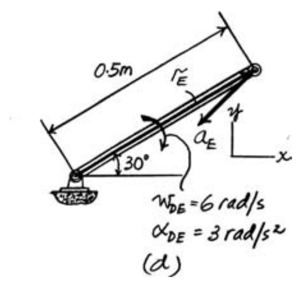
Using these results and applying the acceleration equation to points E and F of gear B, Fig. e,

$$\mathbf{a}_F = \mathbf{a}_E + \alpha_B \times \mathbf{r}_{F/E} - \omega_B^{\,2}\mathbf{r}_{F/E}$$

$$a_F\cos 30°\mathbf{i} + a_F\sin 30°\mathbf{j} = (-14.84\mathbf{i} - 10.30\mathbf{j}) + (-\alpha_B\,\mathbf{k}) \times$$

$$(-0.2\cos 30°\,\mathbf{i} - 0.2\sin 30°\mathbf{j}) - 30^2(-0.2\cos 30°\mathbf{i} - 0.2\sin 30°\mathbf{j})$$

$$0.8660a_F\,\mathbf{i} + 0.5a_F\,\mathbf{j} = (141.05 - 0.1\alpha_B)\mathbf{i} + (79.70 + 0.1732\alpha_B)\mathbf{j}$$



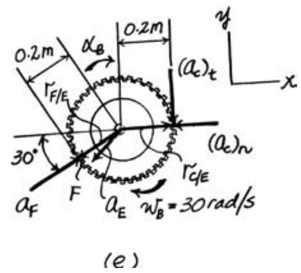
Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$0.8660a_F = 141.05 - 0.1\alpha_B$$

$$0.5a_F = 79.70 + 0.1732\alpha_B$$

$$a_F = 162\text{ m/s}^2$$

$$\alpha_B = 7.5\text{ rad/s}^2 \qquad \textbf{Ans.}$$



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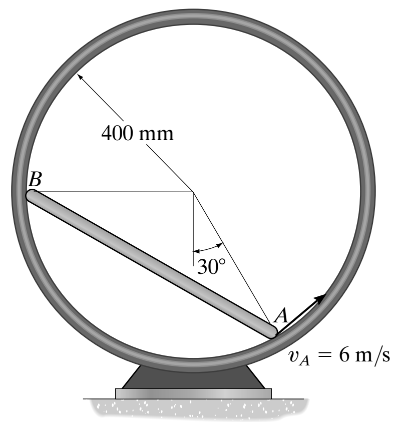
***16–132.** If end A of the rod moves with a constant velocity of $v_A = 6$ m/s, determine the angular velocity and angular acceleration of the rod and the acceleration of end B at the instant shown.



Angular Velocity: The location of the *IC* is indicated in Fig. *a*. Thus,

$$r_{A/IC} = r_{B/IC} = 0.4 \text{ m}$$

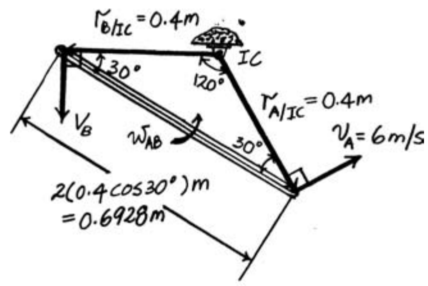
Then,

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{6}{0.4} = 15 \text{ rad/s}$$

and

$$v_B = \omega_{AB}\, r_{B/IC} = 15(0.4) = 6 \text{ m/s}$$

Acceleration and Angular Acceleration: The magnitude of the normal component of its acceleration of points A and B are $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{6^2}{0.4} = 90 \text{ m/s}^2$ and $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{6^2}{0.4} = 90 \text{ m/s}^2$ and both are directed towards the center of the circular track. Since $\mathbf{v}_A$ is constant, $(a_A)_t = 0$. Thus, $a_A = 90 \text{ m/s}^2$. Applying the relative acceleration equation to points A and B, Fig. b,

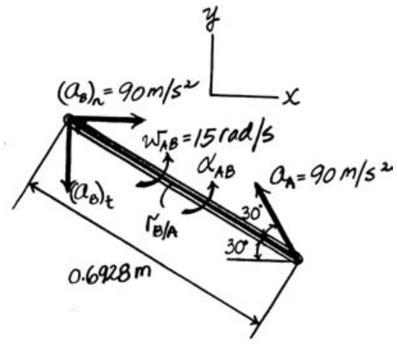


(a)

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$90\mathbf{i} - (a_B)_t\,\mathbf{j} = (-90\cos 60°\mathbf{i} + 90\sin 60°\mathbf{j}) + (\alpha_{AB}\,\mathbf{k}) \times$$
$$(-0.6928\cos 30°\mathbf{i} + 0.6928\sin 30°\mathbf{j}) - 15^2(-0.6928\cos 30°\mathbf{i} + 0.6928\sin 30°\mathbf{j})$$

$$90\mathbf{i} - (a_B)_t\,\mathbf{j} = (-0.3464\alpha_{AB} + 90)\mathbf{i} - (0.6\alpha_{AB})\mathbf{j}$$



(b)

Equating the **i** and **j** components yields

$$90 = -0.3464\alpha_{AB} + 90$$

$$-(a_B) = -0.6\alpha_{AB}$$

$$\alpha_{AB} = 0 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$(a_B)_t = 0 \text{ m/s}^2$$

Thus, the magnitude of a_B is

$$a_B = \sqrt{(a_B)_t{}^2 + (a_B)_n{}^2} = \sqrt{0^2 + 90^2} = 90 \text{ m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_B)_t}{(a_B)_n}\right] = \tan^{-1}\left(\frac{0}{90}\right) = 0° \rightarrow \qquad \textbf{Ans.}$$

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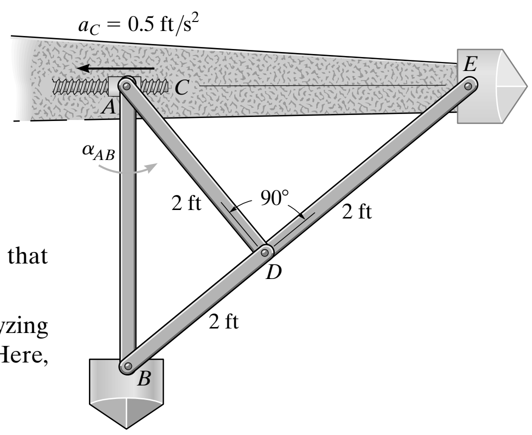
•16–133. The retractable wing-tip float is used on an airplane able to land on water. Determine the angular accelerations α_{CD}, α_{BD}, and α_{AB} at the instant shown if the trunnion C travels along the horizontal rotating screw with an acceleration of $a_C = 0.5$ ft/s^2. In the position shown, $v_C = 0$. Also, points A and E are pin connected to the wing and points A and C are coincident at the instant shown.



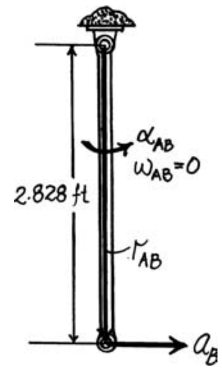
Velocity Analysis: Since $v_C = 0$, then $\omega_{CD} = 0$. Also, one can then show that $\omega_{BD} = \omega_{AB} = \omega_{ED} = 0$.

Acceleration Equation: The acceleration of point D can be obtained by analyzing the angular motion of link ED about point E. Here, $\mathbf{r}_{ED} = \{-2\cos 45°\mathbf{i} - 2\sin 45°\mathbf{j}\}$ ft $= \{-1.414\mathbf{i} - 1.414\mathbf{j}\}$ ft.

$$\mathbf{a}_D = \alpha_{ED} \times \mathbf{r}_{ED} - \omega_{ED}^2 \mathbf{r}_{ED}$$

$$= (\alpha_{ED}\mathbf{k}) \times (-1.414\mathbf{i} - 1.414\mathbf{j}) - \mathbf{0}$$

$$= \{1.414\,\alpha_{ED}\,\mathbf{i} - 1.414\,\alpha_{ED}\,\mathbf{j}\}\ \text{ft/s}^2$$



The acceleration of point B can be obtained by analyzing the angular motion of links AB about point A. Here, $\mathbf{r}_{AB} = \{-2.828\mathbf{j}\}$ ft

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= (\alpha_{AB}\mathbf{k}) \times (-2.828\mathbf{j}) - \mathbf{0}$$

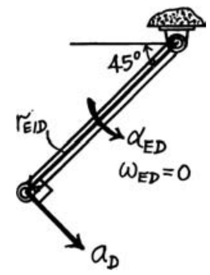
$$= \{2.828\,\alpha_{AB}\,\mathbf{i}\}\ \text{ft/s}^2$$

Link CD is subjected to general plane motion. Applying Eq. 16–18 with $\mathbf{r}_{D/C} = \{2\cos 44°\mathbf{i} - 2\sin 45°\mathbf{j}\}$ ft $= \{1.414\mathbf{i} - 1.414\mathbf{j}\}$ ft, we have

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}^2\mathbf{r}_{D/C}$$

$$1.414\,\alpha_{ED}\,\mathbf{i} - 1.414\,\alpha_{ED}\,\mathbf{j} = -0.5\mathbf{i} + \alpha_{CD}\mathbf{k} \times (1.414\mathbf{i} - 1.414\mathbf{j}) - \mathbf{0}$$

$$1.414\,\alpha_{ED}\,\mathbf{i} - 1.414\,\alpha_{ED}\,\mathbf{j} = (1.414\,\alpha_{CD} - 0.5)\,\mathbf{i} + 1.414\,\alpha_{CD}\mathbf{j}$$



Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

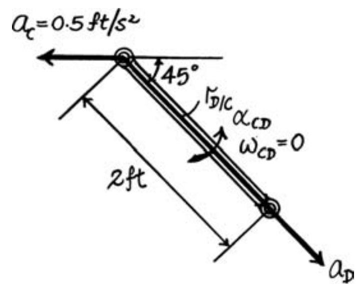
$$1.414\,\alpha_{ED} = 1.414\alpha_{CD} - 0.5 \qquad \textbf{[1]}$$

$$-1.414\alpha_{ED} = 1.414\alpha_{CD} \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

$$\alpha_{ED} = -0.1768\ \text{rad/s}^2$$

$$\alpha_{CD} = 0.177\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



Link BD is subjected to general plane motion. Applying Eq. 16–18 with $\mathbf{r}_{B/D} = \{-2\cos 45°\mathbf{i} - 2\sin 45°\mathbf{j}\}$ ft $= \{-1.414\mathbf{i} - 1.414\mathbf{j}\}$ ft and $\mathbf{a}_D = [1.414\,(-0.1768)\,\mathbf{i} - 1.414\,(-0.1768)\mathbf{j} = \{-0.25\mathbf{i} + 0.25\mathbf{j}\}$ rad/s^2, we have

$$\mathbf{a}_B = \mathbf{a}_D + \alpha_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2\mathbf{r}_{B/D}$$

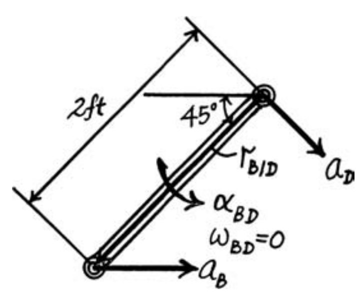
$$2.828\,\alpha_{AB}\,\mathbf{i} = -0.25\mathbf{i} + 0.25\mathbf{j} + \alpha_{BD}\mathbf{k} \times (-1.414\mathbf{i} - 1.414\mathbf{j}) - \mathbf{0}$$

$$2.828\,\alpha_{AB}\,\mathbf{i} = (1.414\,\alpha_{BD} - 0.25)\,\mathbf{i} + (0.25 - 1.414\,\alpha_{BD})\,\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$2.828\,\alpha_{AB} = 1.414\,\alpha_{BD} - 0.25 \qquad \textbf{[3]}$$

$$0 = 0.25 - 1.414\,\alpha_{BD} \qquad \textbf{[4]}$$



Solving Eqs. [3] and [4] yields

$$\alpha_{BD} = 0.177\ \text{rad/s}^2 \qquad \alpha_{AB} = 0 \qquad \textbf{Ans.}$$

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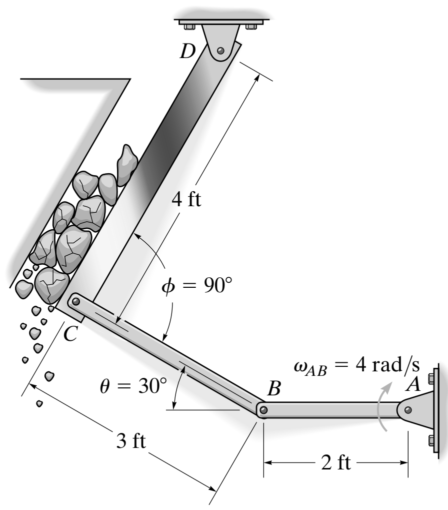
16–134. Determine the angular velocity and the angular acceleration of the plate CD of the stone-crushing mechanism at the instant AB is horizontal. At this instant $\theta = 30^\circ$ and $\phi = 90^\circ$. Driving link AB is turning with a constant angular velocity of $\omega_{AB} = 4$ rad/s.

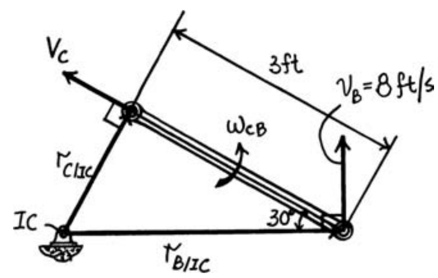


$$v_B = \omega_{AB}\, r_{BA} = (4)(2) = 8 \text{ ft/s} \uparrow$$

$$\omega_{CB} = \frac{v_B}{r_{B/IC}} = \frac{8}{3/\cos 30^\circ} = 2.309 \text{ rad/s}$$

$$v_C = \omega_{BC}\, r_{C/IC} = (2.309)(3 \tan 30^\circ) = 4 \text{ ft/s}$$

$$\omega_{CD} = \frac{v_C}{r_{CD}} = \frac{4}{4} = 1 \text{ rad/s} \quad \curvearrowright$$ **Ans.**



$$a_B = (a_B)_n = (4)^2(2) = 32 \text{ ft/s}^2 \rightarrow$$

$$(\mathbf{a}_C)_t + (\mathbf{a}_C)_n = \mathbf{a}_B + \alpha_{CB} \times \mathbf{r}_{C/B} - \omega^2 \mathbf{r}_{C/B}$$

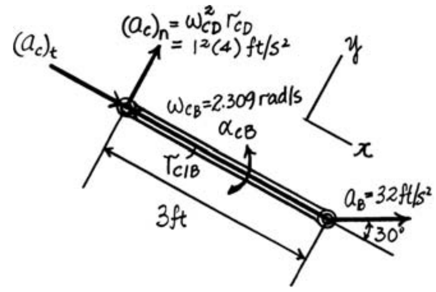
$$(a_C)_t\,\mathbf{i} + (1)^2(4)\mathbf{j} = 32\cos 30^\circ\mathbf{i} + 32\sin 30^\circ\mathbf{j} + (\alpha_{CB}\,\mathbf{k}) \times (-3\mathbf{i}) - (2.309)^2(-3\mathbf{i})$$

$$(a_C)_t = 32\cos 30^\circ - (2.309)^2(-3) = 43.71 \text{ ft/s}^2$$

$$4 = 32\sin 30^\circ - \alpha_{CB}(3)$$

$$\alpha_{CB} = 4 \text{ rad/s}^2 \quad \circlearrowleft$$

$$\alpha_{CD} = \frac{43.71}{4} = 10.9 \text{ rad/s}^2 \quad \circlearrowleft$$ **Ans.**



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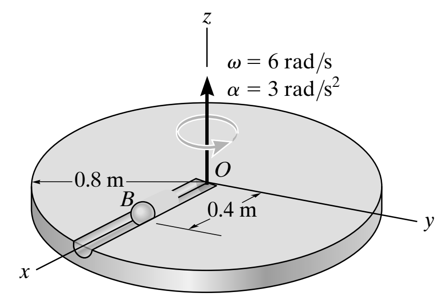
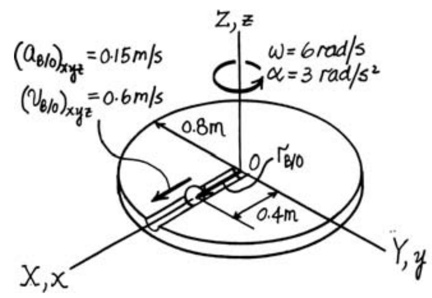
16–135. At the instant shown, ball B is rolling along the slot in the disk with a velocity of 600 mm/s and an acceleration of 150 mm/s^2, both measured relative to the disk and directed away from O. If at the same instant the disk has the angular velocity and angular acceleration shown, determine the velocity and acceleration of the ball at this instant.



Kinematic Equations:

$$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad \textbf{(2)}$$



$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

$\Omega = \{6\mathbf{k}\}$ rad/s

$\Omega = \{3\mathbf{k}\}$ rad/s^2

$\mathbf{r}_{B/O} = \{0.4\,\mathbf{i}\}$ m

$(\mathbf{v}_{B/O})_{xyz} = \{0.6\mathbf{i}\}$ m/s

$(\mathbf{a}_{B/O})_{xyz} = \{0.15\mathbf{i}\}$ m/s^2

Substitute the data into Eqs.(1) and (2) yields:

$$\mathbf{v}_B = \mathbf{0} + (6\mathbf{k}) \times (0.4\mathbf{i}) + (0.6\mathbf{i}) = \{0.6\mathbf{i} + 2.4\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{0} + (3\mathbf{k}) \times (0.4\mathbf{i}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.4\mathbf{i})] + 2\,(6\mathbf{k}) \times (0.6\mathbf{i}) + (0.15\mathbf{i})$$

$$= \{-14.2\mathbf{i} + 8.40\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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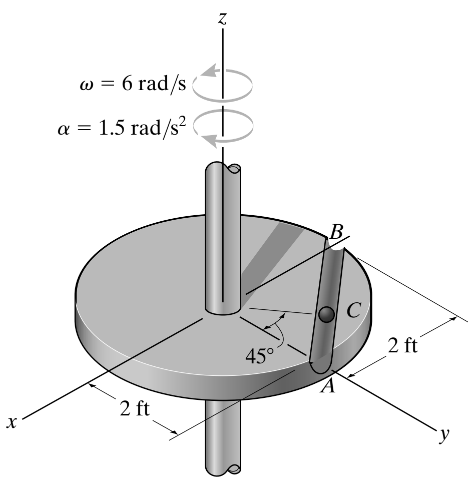
***16–136.** Ball C moves along the slot from A to B with a speed of 3 ft/s, which is increasing at 1.5 ft/s^2, both measured relative to the circular plate. At this same instant the plate rotates with the angular velocity and angular deceleration shown. Determine the velocity and acceleration of the ball at this instant.



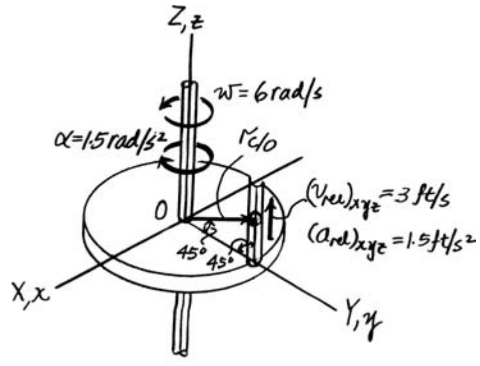
Reference Frames: The xyz rotating reference frame is attached to the plate and coincides with the fixed reference frame XYZ at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [6\mathbf{k}]\text{ rad/s} \qquad \dot{\omega} = \alpha = [-1.5\mathbf{k}]\text{ rad/s}^2$$

For the motion of ball C with respect to the xyz frame,

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-3\sin 45°\mathbf{i} - 3\cos 45°\mathbf{j})\text{ ft/s} = [-2.121\mathbf{i} - 2.121\mathbf{j}]\text{ ft/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-1.5\sin 45°\mathbf{i} - 1.5\cos 45°\mathbf{j})\text{ ft/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ ft/s}^2$$



(a)

From the geometry shown in Fig. b, $r_{C/O} = 2\cos 45° = 1.414$ ft. Thus,

$$\mathbf{r}_{C/O} = (-1.414\sin 45°\mathbf{i} + 1.414\cos 45°\mathbf{j})\text{ft} = [-1\mathbf{i} + 1\mathbf{j}]\text{ ft}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_O + \omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j}) + (-2.121\mathbf{i} - 2.121\mathbf{j})$$

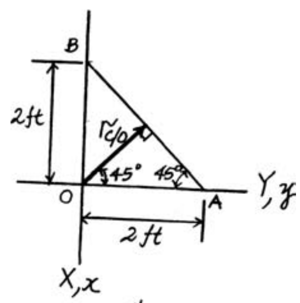
$$= [-8.12\mathbf{i} - 8.12\mathbf{j}]\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation, we have

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{C/O} + \omega \times (\omega \times \mathbf{r}_{C/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (a_{\text{rel}})_{xyz}$$

$$= 0 + (1.5\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j})] + 2(6\mathbf{k}) \times (-2.121\mathbf{i} - 2.121\mathbf{j}) + (-1.061\mathbf{i} - 1.061\mathbf{j})$$

$$= [61.9\mathbf{i} - 61.0\mathbf{j}]\text{ft/s}^2 \qquad \textbf{Ans.}$$



(b)

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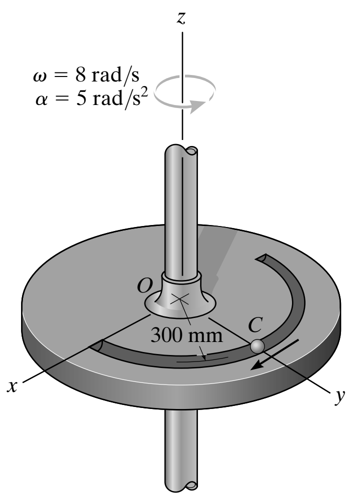
•16–137. Ball C moves with a speed of 3 m/s, which is increasing at a constant rate of 1.5 m/s^2, both measured relative to the circular plate and directed as shown. At the same instant the plate rotates with the angular velocity and angular acceleration shown. Determine the velocity and acceleration of the ball at this instant.



Reference Frames: The xyz rotating reference frame is attached to the plate and coincides with the fixed reference frame XYZ at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [8\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \alpha = [5\mathbf{k}] \text{ rad/s}^2$$

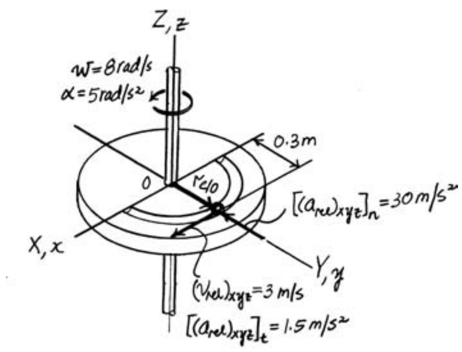
For the motion of ball C with respect to the xyz frame, we have

$$\mathbf{r}_{C/O} = [0.3\mathbf{j}] \text{ m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [3\mathbf{i}] \text{ m/s}$$

The normal component of $\left(\mathbf{a}_{\text{rel}}\right)_{xyz}$ is $\left[\left(a_{\text{rel}}\right)_{xyz}\right]_n = \dfrac{\left(v_{\text{rel}}\right)_{xyz}{}^2}{\rho} = \dfrac{3^2}{0.3} = 30 \text{ m/s}^2$. Thus,

$$\left(\mathbf{a}_{\text{rel}}\right)_{xyz} = [1.5\mathbf{i} - 30\mathbf{j}] \text{ m/s}$$



(a)

Velocity: Applying the relative velocity equation,

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_O + \omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{\text{rel}})_{xyz} \\ &= \mathbf{0} + (8\mathbf{k}) \times (0.3\mathbf{j}) + (3\mathbf{i}) \\ &= [0.6\mathbf{i}] \text{ m/s} \end{aligned} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation.

$$\begin{aligned}\mathbf{a}_C &= \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{C/O} + \omega \times (\omega \times \mathbf{r}_{C/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz} \\ &= \mathbf{0} + (5\mathbf{k}) \times (0.3\mathbf{j}) + (8\mathbf{k}) \times [(8\mathbf{k}) \times (0.3\mathbf{j})] + \mathbf{2}(8\mathbf{k}) \times (3\mathbf{i}) + (1.5\mathbf{i} - 30\mathbf{j}) \\ &= [-1.2\mathbf{j}] \text{ m/s}^2 \end{aligned} \qquad \textbf{Ans.}$$

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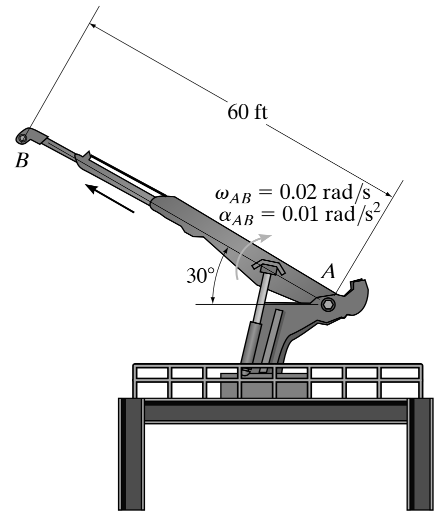
16–138. The crane's telescopic boom rotates with the angular velocity and angular acceleration shown. At the same instant, the boom is extending with a constant speed of 0.5 ft/s, measured relative to the boom. Determine the magnitudes of the velocity and acceleration of point B at this instant.

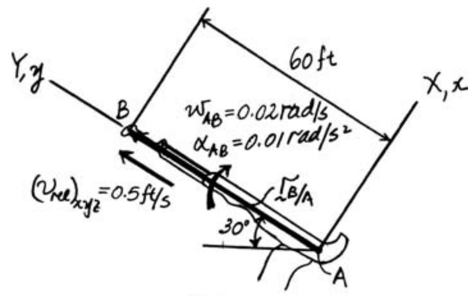


Reference Frames: The xyz rotating reference frame is attached to boom AB and coincides with the XY fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xy frame with respect to the XY frame is

$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0}$ $\qquad \omega_{AB} = [-0.02\mathbf{k}]$ rad/s $\qquad \dot{\omega}_{AB} = \alpha = [-0.01\mathbf{k}]$ rad/s^2

For the motion of point B with respect to the xyz frame, we have

$\mathbf{r}_{B/A} = [60\mathbf{j}]$ ft $\qquad (\mathbf{v}_{\text{rel}})_{xyz} = [0.5\mathbf{j}]$ ft/s $\qquad (\mathbf{a}_{\text{rel}})_{xyz} = \mathbf{0}$



(a)

Velocity: Applying the relative velocity equation,

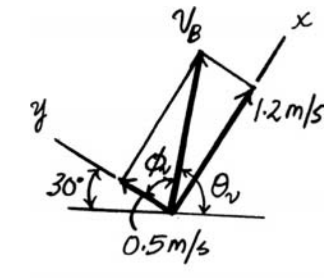
$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.02\mathbf{k}) \times (60\mathbf{j}) + 0.5\mathbf{j}$$

$$= [1.2\mathbf{i} + 0.5\mathbf{j}] \text{ ft / s}$$

Thus, the magnitude of $\mathbf{v}_B$, Fig. b, is

$$v_B = \sqrt{1.2^2 + 0.5^2} = 1.30 \text{ ft/s} \qquad \textbf{Ans.}$$



(b)

Acceleration: Applying the relative acceleration equation,

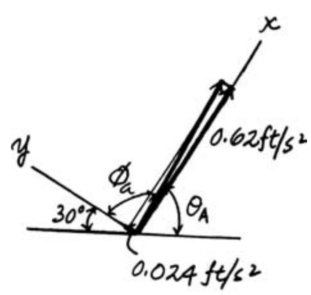
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.01\mathbf{k}) \times (60\mathbf{j}) + (-0.02\mathbf{k}) \times [(-0.02\mathbf{k}) \times (60\mathbf{j})] + 2(-0.02\mathbf{k}) \times (0.5\mathbf{j}) + \mathbf{0}$$

$$= [0.62\mathbf{i} - 0.024\,\mathbf{j}] \text{ ft/s}^2$$

Thus, the magnitude of a_B, Fig. c, is

$$a_B = \sqrt{0.62^2 + (-0.024)^2} = 0.6204 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



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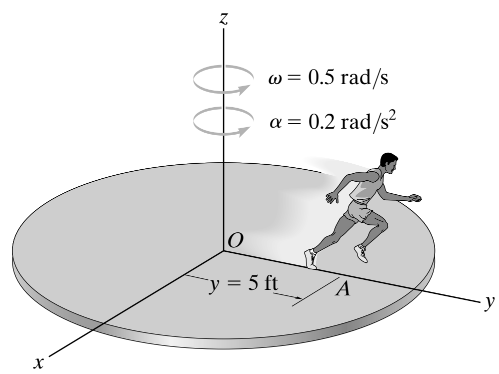
16–139. The man stands on the platform at O and runs out toward the edge such that when he is at A, $y = 5$ ft, his mass center has a velocity of 2 ft/s and an acceleration of 3 ft/s^2, both measured relative to the platform and directed along the positive y axis. If the platform has the angular motions shown, determine the velocity and acceleration of his mass center at this instant.



$$\mathbf{v}_A = \mathbf{v}_O + \Omega \times \mathbf{r}_{A/O} + (\mathbf{v}_{A/O})_{xyz}$$

$$\mathbf{v}_A = \mathbf{0} + (0.5\mathbf{k}) \times (5\mathbf{j}) + 2\mathbf{j}$$

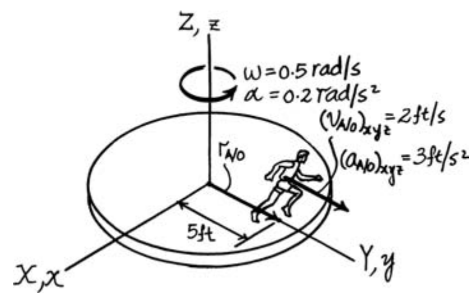
$$\mathbf{v}_A = \{-2.50\mathbf{i} + 2.00\mathbf{j}\} \text{ ft/s}$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{A/O} + \Omega \times (\Omega \times \mathbf{r}_{A/O}) + 2\Omega \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$

$$\mathbf{a}_A = \mathbf{0} + (0.2\mathbf{k}) \times (5\mathbf{j}) + (0.5\mathbf{k}) \times (0.5\mathbf{k} \times 5\mathbf{j}) + 2(0.5\mathbf{k}) \times (2\mathbf{j}) + 3\mathbf{j}$$

$$\mathbf{a}_A = -1\mathbf{i} - 1.25\mathbf{j} - 2\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{a}_A = \{-3.00\mathbf{i} + 1.75\mathbf{j}\} \text{ ft/s}^2$$ **Ans.**



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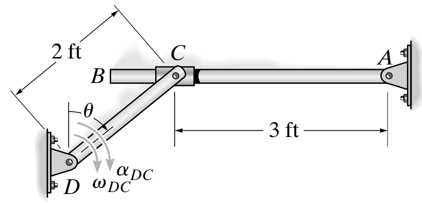
***16–140.** At the instant $\theta = 45°$, link DC has an angular velocity of $\omega_{DC} = 4$ rad/s and an angular acceleration of $\alpha_{DC} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod AB at this instant. The collar at C is pin connected to DC and slides freely along AB.



$\mathbf{v}_A = 0$

$\mathbf{a}_A = \mathbf{0}$

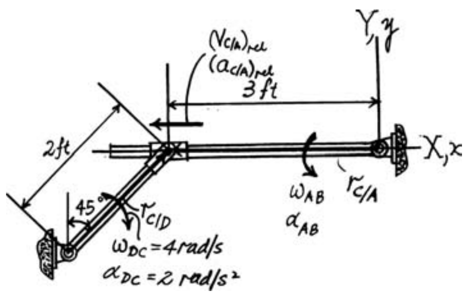
$\Omega = \omega_{AB}\,\mathbf{k}$

$\dot{\Omega} = \alpha_{AB}\,\mathbf{k}$

$\mathbf{r}_{C/A} = \{-3\mathbf{i}\}$ ft

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{rel}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{rel}\,\mathbf{i}$



$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = (-4\mathbf{k}) \times (2\sin 45°\mathbf{i} + 2\cos 45°\mathbf{j}) = \{5.6569\mathbf{i} - 5.6569\mathbf{j}\}$ ft/s

$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\,\mathbf{r}_{C/D}$

$= (-2\mathbf{k}) \times (2\sin 45°\mathbf{i} + 2\cos 45°\mathbf{j}) - (4)^2\,(2\sin 45°\mathbf{i} + 2\cos 45°\mathbf{j})$

$= \{-19.7990\mathbf{i} - 25.4558\mathbf{j}\}$ ft/s^2

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$5.6569\mathbf{i} - 5.6569\mathbf{j} = \mathbf{0} + (\omega_{AB}\mathbf{k}) \times (-3\mathbf{i}) + (v_{C/A})_{xyz}\,\mathbf{i}$

$5.6569\mathbf{i} - 5.6569\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} - 3\omega_{AB}\,\mathbf{j}$

Solving:

$(v_{C/A})_{xyz} = 5.6569$ ft/s

$\omega_{AB} = 1.89$ rad/s ↺ **Ans.**

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$-19.7990\mathbf{i} - 25.4558\mathbf{j} = \mathbf{0} + (\alpha_{AB}\,\mathbf{k}) \times (-3\mathbf{i}) + (1.89\mathbf{k}) \times [(1.89\mathbf{k}) \times (-3\mathbf{i})] + 2(1.89\mathbf{k}) \times (5.6569\mathbf{i}) + (a_{C/A})_{xyz}\,\mathbf{i}$

$-19.7990\mathbf{i} - 25.4558\mathbf{j} = [10.6667 + (a_{C/A})_{xyz}]\mathbf{i} + (21.334 - 3\alpha_{AB})\mathbf{j}$

Solving:

$(a_{C/A})_{xyz} = -30.47$ ft/s^2

$\alpha_{AB} = 15.6$ rad/s^2 ↺ **Ans.**

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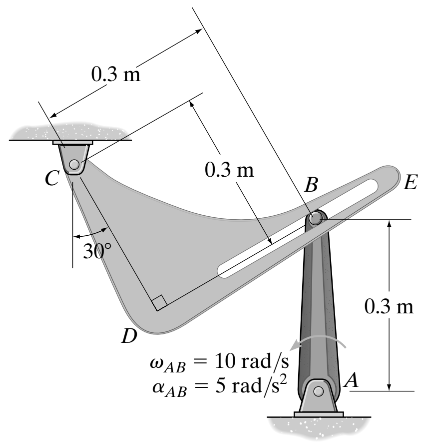
•**16–141.** Peg B fixed to crank AB slides freely along the slot in member CDE. If AB rotates with the motion shown, determine the angular velocity of CDE at the instant shown.



Reference Frame: The xyz rotating reference frame is attached to member CDE and coincides with the XYZ fixed reference frame at the instant considered. Thus, the motion of the xyz reference frame with respect to the XYZ frame is

$$\mathbf{v}_C = \mathbf{0} \qquad \omega_{CDE} = \omega_{CDE}\mathbf{k}$$

For the motion of point B with respect to the xyz frame,

$$\mathbf{r}_{B/C} = \left[\sqrt{0.3^2 + 0.3^2}\mathbf{i}\right] \text{m} = 0.3\sqrt{2}\mathbf{i}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz} \cos 45°\mathbf{i} + (v_{\text{rel}})_{xyz} \sin 45°\mathbf{j} = 0.7071(v_{\text{rel}})_{xyz}\,\mathbf{i} + 0.7071\,(v_{\text{rel}})_{xyz}\,\mathbf{j}$$

Since crank AB rotates about a fixed axis, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ reference frame can be determined from

$$\begin{aligned}\mathbf{v}_B &= \omega_{AB} \times \mathbf{r}_B \\ &= (10\mathbf{k}) \times (-0.3 \cos 75°\mathbf{i} + 0.3 \sin 75°\mathbf{j}) \\ &= [-2.898\mathbf{i} - 0.7765\mathbf{j}] \text{ m/s}\end{aligned}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{CDE} \times \mathbf{r}_{B/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(-2.898\mathbf{i} - 0.7765\mathbf{j}) = \mathbf{0} + (\omega_{CDE}\,\mathbf{k}) \times \left(0.3\sqrt{2}\mathbf{i}\right) + 0.7071(v_{\text{rel}})_{xyz}\,\mathbf{i} + 0.7071(v_{rel})_{xyz}\,\mathbf{j}$$

$$-2.898\mathbf{i} - 0.7765\mathbf{j} = 0.7071(v_{\text{rel}})_{xyz}\mathbf{i} + \left[0.3\sqrt{2}\omega_{CDE} + 0.7071(v_{\text{rel}})_{xyz}\right]\mathbf{j}$$

Equating the **i** and **j** components yields

$$-2.898 = 0.7071(v_{\text{rel}})_{xyz} \qquad \textbf{(1)}$$

$$-0.7765 = 0.3\sqrt{2}\omega_{CDE} + 0.7071(v_{\text{rel}})_{xyz} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_{\text{rel}})_{xyz} = -4.098 \text{ m/s}$$

$$\omega_{CDE} = 5 \text{ rad/s} \qquad \textbf{Ans.}$$

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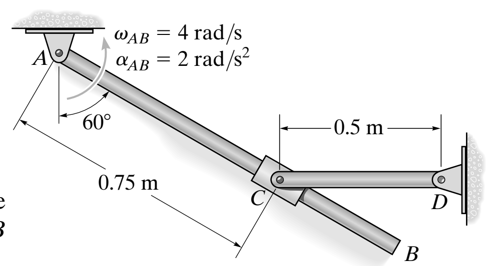
16–142. At the instant shown rod AB has an angular velocity $\omega_{AB} = 4$ rad/s and an angular acceleration $\alpha_{AB} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin connected to CD and slides freely along AB.



Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point A. The x, y, z moving frame is attached to and rotate with rod AB since collar C slides along rod AB.

Kinematic Equation: Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \qquad \textbf{[1]}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \qquad \textbf{[2]}$$

Motion of moving reference

$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = 4\mathbf{k}$ rad/s

$\dot{\Omega} = 2\mathbf{k}$ rad/s^2

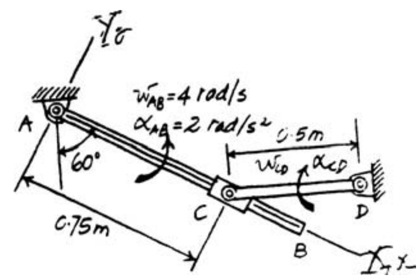
Motion of C with respect to moving reference

$r_{C/A} = \{0.75\mathbf{i}\}$m

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

The velocity and acceleration of collar C can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{C/D} = \{-0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j}\}$m $= \{-0.4330\mathbf{i} - 0.250\mathbf{j}\}$ m.



$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = -\omega_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j})$$
$$= -0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$
$$= -\alpha_{CD}\,\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j}) - \omega_{CD}^2(-0.4330\mathbf{i} - 0.250\mathbf{j})$$
$$= \left(0.4330\omega_{CD}^2 - 0.250\,\alpha_{CD}\right)\mathbf{i} + \left(0.4330\alpha_{CD} + 0.250\omega_{CD}^2\right)\mathbf{j}$$

Substitute the above data into Eq.[1] yields

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$-0.250\,\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = \mathbf{0} + 4\mathbf{k} \times 0.75\mathbf{i} + (v_{C/A})_{xyz}\,\mathbf{i}$$

$$-0.250\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\,\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} + 3.00\mathbf{j}$$

Equating **i** and **j** components and solve, we have

$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$

$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \qquad \textbf{Ans.}$$

Substitute the above data into Eq.[2] yields

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$\left[0.4330\left(6.928^2\right) - 0.250\,\alpha_{CD}\right]\mathbf{i} + \left[0.4330\alpha_{CD} + 0.250\left(6.928^2\right)\right]\mathbf{j}$$
$$= \mathbf{0} + 2\mathbf{k} \times 0.75\mathbf{i} + 4\mathbf{k} \times (4\mathbf{k} \times 0.75\mathbf{i}) + \mathbf{2}\,(4\mathbf{k}) \times (-1.732\mathbf{i}) + (a_{C/A})_{xyz}\,\mathbf{i}$$

$$(20.78 - 0.250\alpha_{CD})\mathbf{i} + (0.4330\,\alpha_{CD} + 12)\mathbf{j} = \left[(a_{C/A})_{xyz} - 12.0\right]\mathbf{i} - 12.36\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_{C/A})_{xyz} = 46.85 \text{ m/s}^2$$

$$\alpha_{CD} = -56.2 \text{ rad/s}^2 = 56.2 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

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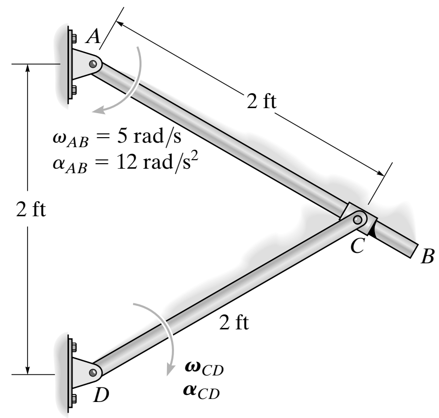
16–143. At a given instant, rod AB has the angular motions shown. Determine the angular velocity and angular acceleration of rod CD at this instant. There is a collar at C.



$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = \{-5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \{-12\mathbf{k}\}$ rad/s^2

$\mathbf{r}_{C/A} = \{2\mathbf{i}\}$ ft

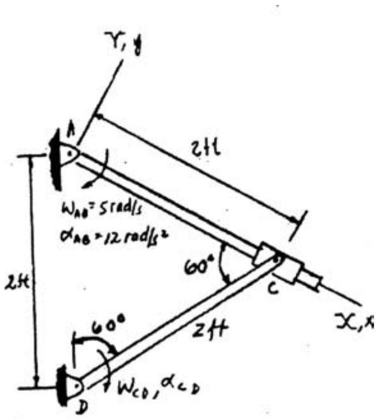
$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$\mathbf{v}_C = \mathbf{0} + (-5\mathbf{k}) \times (2\mathbf{i}) + (v_{C/A})_{xyz}\mathbf{i}$

$\quad = (v_{C/A})_{xyz}\,\mathbf{i} - 10\mathbf{j}$



$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{CD}$$

$(v_{C/A})_{xyz}\,\mathbf{i} - 10\mathbf{j} = (-\omega_{CD}\,\mathbf{k}) \times (2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j})$

$(v_{C/A})_{xyz}\,\mathbf{i} - 10\mathbf{j} = 1.732\omega_{CD}\,\mathbf{i} - \omega_{CD}\mathbf{j}$

Solving:

$$\omega_{CD} = 10 \text{ rad/s} \quad \circlearrowright \qquad \textbf{Ans.}$$

$(v_{C/A})_{xyz} = 1.732(10) = 17.32$ ft/s

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$\mathbf{a}_C = \mathbf{0} + (-12\mathbf{k}) \times (2\mathbf{i}) + (-5\mathbf{k}) \times \left[(-5\mathbf{k}) \times (2\mathbf{i})\right] + 2(-5\mathbf{k}) \times \left[(v_{C/A})_{xyz}\mathbf{i}\right] + (a_{C/A})_{xyz}\,\mathbf{i}$

$\quad = \left[(a_{C/A})_{xyz} - 50\right]\mathbf{i} - \left[10(v_{C/A})_{xyz} + 24\right]\mathbf{j}$

$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\,\mathbf{r}_{CD}$

$\left[(a_{C/A})_{xyz} - 50\right]\mathbf{i} - \left[10(17.32) + 24\right]\mathbf{j} = (-\alpha_{CD}\,\mathbf{k}) \times (2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j}) - (10)^2(2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j})$

$\left[(a_{C/A})_{xyz} - 50\right]\mathbf{i} - \left(10(17.32) + 24\right]\mathbf{j} = (1.732\,\alpha_{CD} - 100)\mathbf{i} - (\alpha_{CD} + 173.2)\mathbf{j}$

Solving:

$-\left[10(17.32) + 24\right] = -(\alpha_{CD} + 173.2) \qquad \alpha_{CD} = 24 \text{ rad/s}^2 \quad \circlearrowright \qquad$ **Ans.**

$(a_{C/A})_{xyz} - 50 = 1.732(24) - 100 \qquad (a_{C/A})_{xyz} = -8.43 \text{ ft/s}^2$

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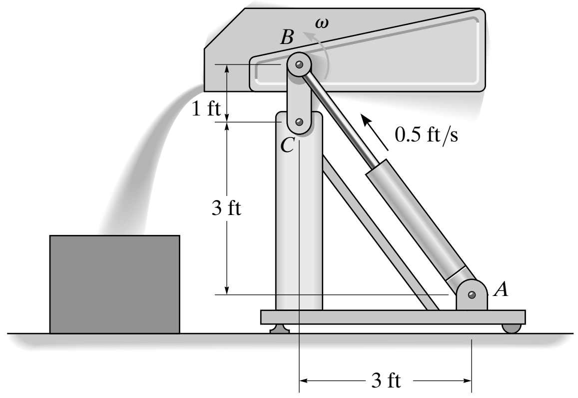
***16–144.** The dumpster pivots about C and is operated by the hydraulic cylinder AB. If the cylinder is extending at a constant rate of 0.5 ft/s, determine the angular velocity $\boldsymbol{\omega}$ of the container at the instant it is in the horizontal position shown.



$\mathbf{r}_{B/A} = 5\,\mathbf{j}$

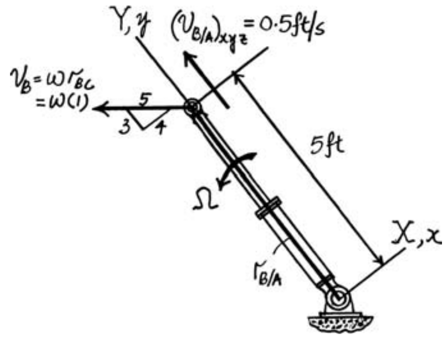
$\mathbf{v}_{B/A} = 0.5\mathbf{j}$

$\mathbf{v}_B = -\frac{4}{5}\,\omega(1)\mathbf{i} + \frac{3}{5}\,\omega(1)\mathbf{j}$

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$-\frac{4}{5}\,\omega(1)\mathbf{i} + \frac{3}{5}\,\omega(1)\mathbf{j} = \mathbf{0} + (\Omega\mathbf{k}) \times (5\mathbf{j}) + 0.5\mathbf{j}$

$-\frac{4}{5}\,\omega(1)\mathbf{i} + \frac{3}{5}\omega(1)\mathbf{j} = -\Omega(5)\mathbf{i} + 0.5\mathbf{j}$



Thus,

$$\omega = 0.833 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\Omega = 0.133 \text{ rad/s}$$

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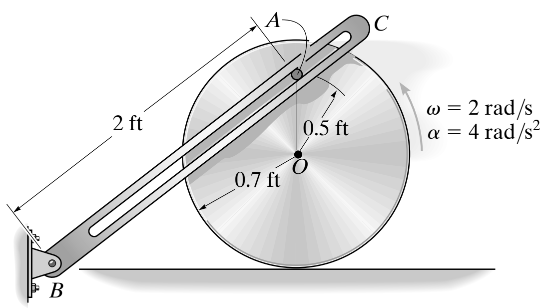
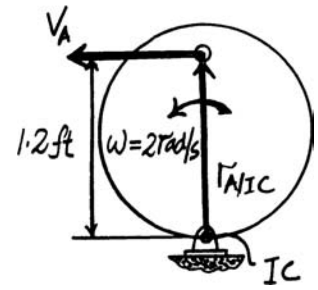
•16–145. The disk rolls without slipping and at a given instant has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link BC at this instant. The peg at A is fixed to the disk.



$\mathbf{v}_A = -(1.2)(2)\mathbf{i} = -2.4\mathbf{i}$ ft/s



$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$

$\mathbf{a}_A = -4(0.7)\mathbf{i} + (4\mathbf{k}) \times (0.5\mathbf{j}) - (2)^2(0.5\mathbf{j})$

$\mathbf{a}_A = -4.8\,\mathbf{i} - 2\mathbf{j}$

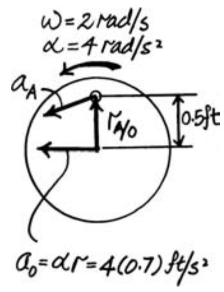
$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$

$-2.4\mathbf{i} = \mathbf{0} + (\omega_{BC}\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + v_{A/B}\left(\frac{4}{5}\right)\mathbf{i} + v_{A/B}\left(\frac{3}{5}\right)\mathbf{j}$

$-2.4\mathbf{i} = 1.6\,\omega_{BC}\,\mathbf{j} - 1.2\,\omega_{BC}\,\mathbf{i} + 0.8v_{A/B}\mathbf{i} + 0.6v_{A/B}\,\mathbf{j}$

$-2.4 = -1.2\,\omega_{BC} + 0.8\,v_{A/B}$

$0 = 1.6\omega_{BC} + 0.6v_{A/B}$



Solving,

$\omega_{BC} = 0.720$ rad/s ↺ **Ans.**

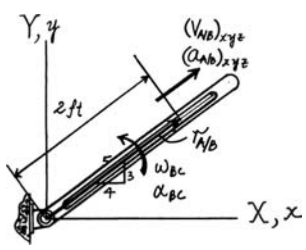
$v_{A/B} = -1.92$ ft/s

$\mathbf{a}_A = \mathbf{a}_B + \Omega \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$

$-4.8\mathbf{i} - 2\mathbf{j} = \mathbf{0} + (\alpha_{BC}\,\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + (0.72\mathbf{k}) \times (0.72\mathbf{k} \times (1.6\mathbf{i} + 1.2\mathbf{j}))$

$+2(0.72\mathbf{k}) \times \left[-(0.8)(1.92)\mathbf{i} - 0.6(1.92)\mathbf{j}\right] + 0.8\,a_{B/A}\mathbf{i} + 0.6a_{B/A}\,\mathbf{j}$

$-4.8\mathbf{i} - 2\mathbf{j} = 1.6\alpha_{BC}\mathbf{j} - 1.2\alpha_{BC}\,\mathbf{i} - 0.8294\mathbf{i} - 0.6221\mathbf{j} - 2.2118\mathbf{j} + 1.6589\mathbf{i} + 0.8a_{B/A}\,\mathbf{i} + 0.6a_{B/A}\,\mathbf{j}$



$-4.8 = -1.2\alpha_{BC} - 0.8294 + 1.6589 + 0.8a_{B/A}$

$-2 = 1.6\alpha_{BC} - 0.6221 - 2.2118 + 0.6a_{B/A}$

Solving,

$\alpha_{BC} = 2.02$ rad/s² ↺ **Ans.**

$a_{B/A} = -4.00$ ft/s²

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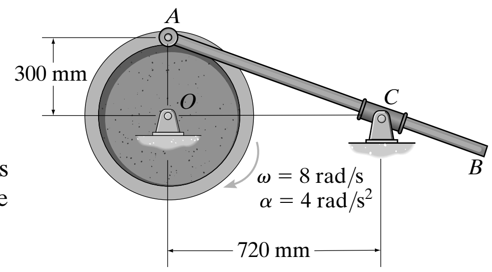
16–146. The wheel is rotating with the angular velocity and angular acceleration at the instant shown. Determine the angular velocity and angular acceleration of the rod at this instant. The rod slides freely through the smooth collar.



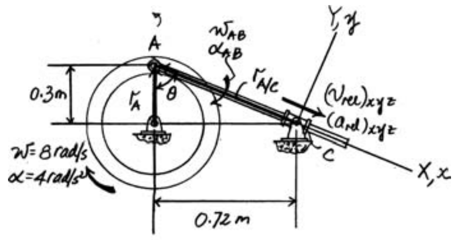
Reference Frame: The xyz rotating reference frame is attached to C and coincides with the XYZ fixed reference frame at the instant considered, Fig. *a*. Thus, the motion of the xyz reference frame with respect to the XYZ frame is

$$\mathbf{v}_C = \mathbf{a}_C = 0 \qquad \omega_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

From the geometry shown in Fig. *a*,

$$r_{A/C} = \sqrt{0.3^2 + 0.72^2} = 0.78 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{0.72}{0.3}\right) = 67.38°$$



(a)

For the motion of point A with respect to the xyz frame,

$$\mathbf{r}_{A/C} = [-0.78\mathbf{i}] \text{ m} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\,\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\,\mathbf{i}$$

Since the wheel A rotates about a fixed axis, $\mathbf{v}_A$ and $\mathbf{a}_A$ with respect to the XYZ reference frame can be determined from

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$
$$= (-8\mathbf{k}) \times (-0.3\cos 67.38°\mathbf{i} + 0.3\sin 67.38°\mathbf{j})$$
$$= [2.215\mathbf{i} + 0.9231\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A - \omega^2\mathbf{r}_A$$
$$= (-4\mathbf{k}) \times (-0.3\cos 67.38°\mathbf{i} + 0.3\sin 67.38°\,\mathbf{j}) - 8^2(-0.3\cos 67.38°\mathbf{i} + 0.3\sin 67.38°\mathbf{j})$$
$$= [8.492\mathbf{i} - 17.262\mathbf{j}] \text{ m/s}^2$$

Velocity: Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \omega_{AB} \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$2.215\mathbf{i} + 0.9231\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (-0.78\mathbf{i}) + (v_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$2.215\mathbf{i} + 0.9231\mathbf{j} = (v_{\text{rel}})_{xyz}\,\mathbf{i} + 0.78\omega_{AB}\,\mathbf{j}$$

Equating the **i** and **j** components yields

$$(v_{\text{rel}})_{xyz} = 2.215 \text{ m/s}$$

$$0.78\omega_{AB} = 0.9231 \qquad \omega_{AB} = 1.183 \text{ rad/s} = 1.18 \text{ rad/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\omega}_{AB} \times \mathbf{r}_{A/C} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{A/C}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyx} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$8.492\mathbf{i} - 17.262\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (-0.78\mathbf{i}) + (-1.183\mathbf{k}) \times [(-1.183\mathbf{k}) \times (-0.78\mathbf{i})] + 2(-1.183\mathbf{k}) \times (2.215\mathbf{i}) + (a_{\text{rel}})_{xyz}\mathbf{i}$$

$$8.492\mathbf{i} - 17.262\mathbf{j} = \left[\left(a_{\text{rel}}\right)_{xyz} + 1.092\right]\mathbf{i} + \left(0.78\alpha_{AB} - 5.244\right)\mathbf{j}$$

Equating the **j** components yields

$$-17.262 = 0.78\alpha_{AB} - 5.244$$

$$\alpha_{AB} = -15.41 \text{ rad/s}^2 = 15.4 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

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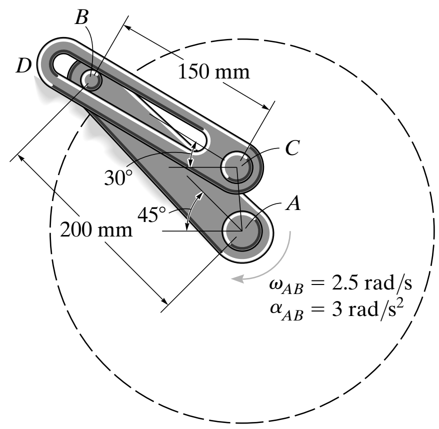
16–147. The two-link mechanism serves to amplify angular motion. Link AB has a pin at B which is confined to move within the slot of link CD. If at the instant shown, AB (input) has an angular velocity of $\omega_{AB} = 2.5$ rad/s and an angular acceleration of $\alpha_{AB} = 3\text{ rad/s}^2$, determine the angular velocity and angular acceleration of CD (output) at this instant.



$\mathbf{v}_C = \mathbf{0}$

$\mathbf{a}_C = \mathbf{0}$

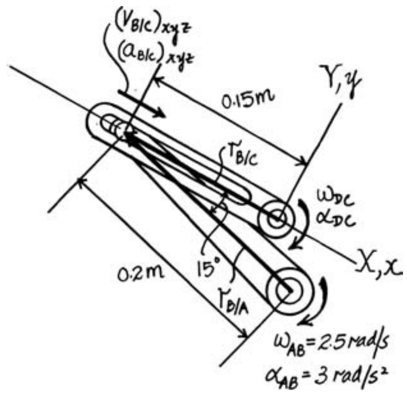
$\Omega = -\omega_{DC}\,\mathbf{k}$

$\dot{\Omega} = -\alpha_{DC}\,\mathbf{k}$

$\mathbf{r}_{B/C} = \{-0.15\,\mathbf{i}\}$ m

$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\,\mathbf{i}$



$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = (-2.5\mathbf{k}) \times (-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j})$

$= \{0.1294\mathbf{i} + 0.4830\mathbf{j}\}$ m/s

$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$

$= (-3\mathbf{k}) \times (-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j}) - (2.5)^2(-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j})$

$= \{1.3627\mathbf{i} + 0.2560\mathbf{j}\}\text{ m/s}^2$

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.1294\mathbf{i} + 0.4830\mathbf{j} = \mathbf{0} + (-\omega_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (v_{B/C})_{xyz}\,\mathbf{i}$

$0.1294\mathbf{i} + 0.4830\mathbf{j} = (v_{B/C})_{xyz}\,\mathbf{i} + 0.15\omega_{DC}\,\mathbf{j}$

Solving:

$(v_{B/C})_{xyz} = 0.1294$ m/s

$\omega_{DC} = 3.22$ rad/s ↻ **Ans.**

$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

$1.3627\mathbf{i} + 0.2560\mathbf{j} = \mathbf{0} + (-\alpha_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (-3.22\mathbf{k}) \times \left[(-3.22\mathbf{k}) \times (-0.15\mathbf{i})\right] + 2(-3.22\mathbf{k}) \times (0.1294\mathbf{i}) + (a_{B/C})_{xyz}\,\mathbf{i}$

$1.3627\mathbf{i} + 0.2560\mathbf{j} = \left[1.5550 + (a_{B/C})_{xyz}\right]\mathbf{i} + (0.15\,\alpha_{DC} - 0.8333)\mathbf{j}$

Solving:

$(a_{B/C})_{xyz} = -0.1923\text{ m/s}^2$

$\alpha_{DC} = 7.26\text{ rad/s}^2$ ↻ **Ans.**

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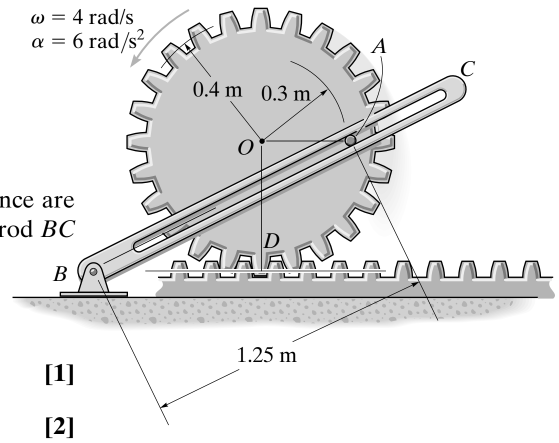
***16–148.** The gear has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link BC at this instant. The peg A is fixed to the gear.



Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point B. The x, y, z moving frame is attached to and rotates with rod BC since peg A slides along slot in member BC.

Kinematic Equation: Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} \qquad \textbf{[1]}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz} \qquad \textbf{[2]}$$

Motion of moving reference

$\mathbf{v}_B = \mathbf{0}$

$\mathbf{a}_B = \mathbf{0}$

$\Omega = \omega_{BC}\,\mathbf{k}$

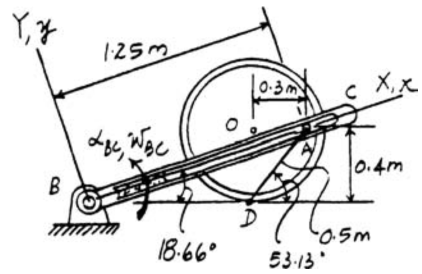
$\dot{\Omega} = \alpha_{BC}\,\mathbf{k}$

Motion of C with respect to moving reference

$\mathbf{r}_{A/B} = \{1.25\mathbf{i}\}$ m

$(\mathbf{v}_{A/B})_{xyz} = (v_{A/B})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{A/B})_{xyz} = (a_{A/B})_{xyz}\,\mathbf{i}$



The velocity and acceleration of peg A can be determined using Eqs. 16–16 and 16–18 with $\mathbf{r}_{A/D} = \{0.5\cos 34.47°\mathbf{i} + 0.5\sin 34.47°\mathbf{j}\}$ m $= \{0.4122\mathbf{i} + 0.2830\mathbf{j}\}$ m.

$$\mathbf{v}_A = \mathbf{v}_D + \omega \times \mathbf{r}_{A/D} = \mathbf{0} + 4\mathbf{k} \times (0.4122\mathbf{i} + 0.2830\mathbf{j})$$

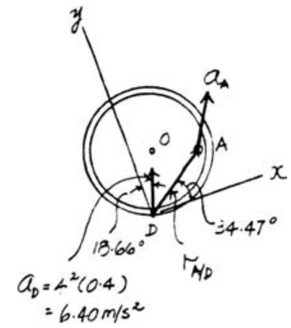
$$= \{-1.1319\mathbf{i} + 1.6489\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_A = \mathbf{a}_D + \alpha \times \mathbf{r}_{A/D} - \omega^2\mathbf{r}_{A/D}$$

$$= 6.40\sin 18.66°\mathbf{i} + 6.40\cos 18.66°\mathbf{j} + 6\mathbf{k} \times (0.4122\mathbf{i} + 0.2830\mathbf{j}) - 4^2(0.4122\mathbf{i} + 0.2830\mathbf{j})$$

$$= \{-6.2454\mathbf{i} + 4.0094\mathbf{j}\}\text{ m/s}^2$$

Substitute the above data into Eq.[1] yields



$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$-1.1319\mathbf{i} + 1.6489\mathbf{j} = \mathbf{0} + \omega_{BC}\mathbf{k} \times 1.25\mathbf{i} + (v_{A/B})_{xyz}\,\mathbf{i}$$

$$-1.1319\mathbf{i} + 1.6489\mathbf{j} = (v_{A/B})_{xyz}\,\mathbf{i} + 1.25\,\omega_{BC}\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components and solving, we have

$$(v_{A/B})_{xyz} = -1.1319\text{ m/s}$$

$$\omega_{BC} = 1.3191\text{ rad/s} = 1.32\text{ rad/s} \qquad \textbf{Ans.}$$

Substitute the above data into Eq.[2] yields

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$-6.2454\mathbf{i} + 4.0094\mathbf{j} = \mathbf{0} + \alpha_{BC}\,\mathbf{k} \times 1.25\mathbf{i} + 1.3191\mathbf{k} \times (1.3191\mathbf{k} \times 1.25\mathbf{i}) + 2(1.3191\mathbf{k}) \times (-1.1319\mathbf{i}) + (a_{A/B})_{xyz}\,\mathbf{i}$$

$$-6.2454\mathbf{i} + 4.0094\mathbf{j} = \left[(a_{A/B})_{xyz} - 2.1751\right]\mathbf{i} + (1.25\alpha_{BC} - 2.9861)\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$(a_{A/B})_{xyz} = -4.070\text{ m/s}^2$$

$$\alpha_{BC} = 5.60\text{ rad/s}^2 \qquad \textbf{Ans.}$$

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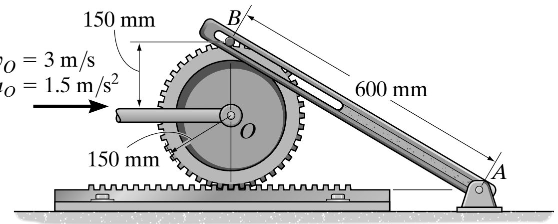
•16–149. Peg B on the gear slides freely along the slot in link AB. If the gear's center O moves with the velocity and acceleration shown, determine the angular velocity and angular acceleration of the link at this instant.

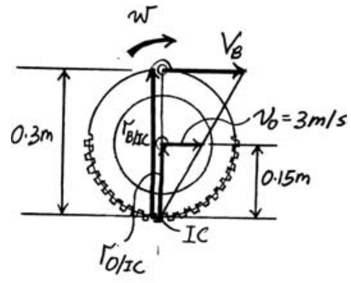


Gear Motion: The IC of the gear is located at the point where the gear and the gear rack mesh, Fig. a. Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.15} = 20 \text{ rad/s}$$

Then,

$$v_B = \omega r_{B/IC} = 20(0.3) = 6 \text{ m/s} \rightarrow$$



(a)

Since the gear rolls on the gear rack, $\alpha = \dfrac{a_O}{r} = \dfrac{1.5}{0.15} = 10$ rad/s. By referring to Fig. b,

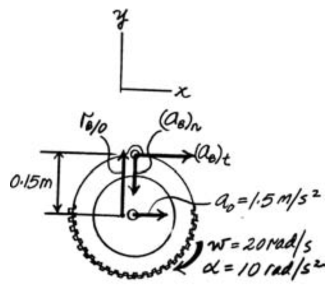
$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$(a_B)_t\,\mathbf{i} - (a_B)_n\,\mathbf{j} = 1.5\mathbf{i} + (-10\mathbf{k}) \times 0.15\mathbf{j} - 20^2(0.15\mathbf{j})$$

$$(a_B)_t\,\mathbf{i} - (a_B)_n\,\mathbf{j} = 3\mathbf{i} - 60\mathbf{j}$$

Thus,

$$(a_B)_t = 3 \text{ m/s}^2 \qquad\qquad (a_B)_n = 60 \text{ m/s}^2$$



(b)

Reference Frame: The $x'y'z'$ rotating reference frame is attached to link AB and coincides with the XYZ fixed reference frame, Figs. c and d. Thus, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ frame is

$$\mathbf{v}_B = [6 \sin 30°\mathbf{i} - 6 \cos 30°\,\mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_B = (3 \sin 30° - 60 \cos 30°)\mathbf{i} + (-3 \cos 30° - 60 \sin 30°)\mathbf{j}$$

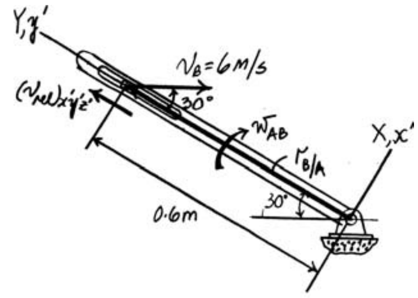
$$= [-50.46\mathbf{i} - 32.60\mathbf{j}] \text{ m/s}^2$$

For motion of the $x'y'z'$ frame with reference to the XYZ reference frame,

$$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0} \qquad \omega_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

For the motion of point B with respect to the $x'y'z'$ frame is

$$\mathbf{r}_{B/A} = [0.6\mathbf{j}]\text{m} \qquad (\mathbf{v}_{\text{rel}})_{x'y'z'} = (v_{\text{rel}})_{x'y'z'}\,\mathbf{j} \qquad (\mathbf{a}_{\text{rel}})_{x'y'z'} = (a_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$



(c)

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{x'y'z'}$$

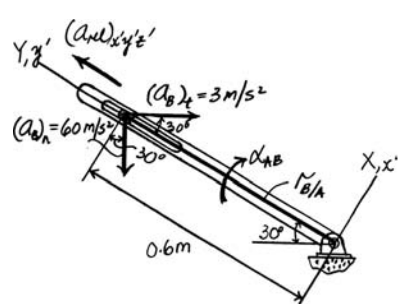
$$3\mathbf{i} - 5.196\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (v_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

$$3\mathbf{i} - 5.196\mathbf{j} = 0.6\omega_{AB}\,\mathbf{i} + (v_{\text{rel}})_{x'y'z'}\mathbf{j}$$

Equating the **i** and **j** components yields

$$3 = 0.6\omega_{AB} \qquad\qquad \omega_{AB} = 5 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(v_{\text{rel}})_{x'y'z'} = -5.196 \text{ m/s}$$



(d)

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{x'y'z'} + (\mathbf{a}_{\text{ rel}})_{x'y'z'}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (-5\mathbf{k}) \times [(-5\mathbf{k}) \times (0.6\mathbf{j})] + 2(-5\mathbf{k}) \times (-5.196\mathbf{j}) + (a_{\text{rel}})_{x'y'z'}\mathbf{j}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = (0.6\alpha_{AB} - 51.96)\mathbf{i} + \left[(a_{\text{rel}})_{x'y'z'} - 15\right]\mathbf{j}$$

Equating the **i** components,

$$-50.46 = 0.6\alpha_{AB} - 51.96$$

$$\alpha_{AB} = 2.5 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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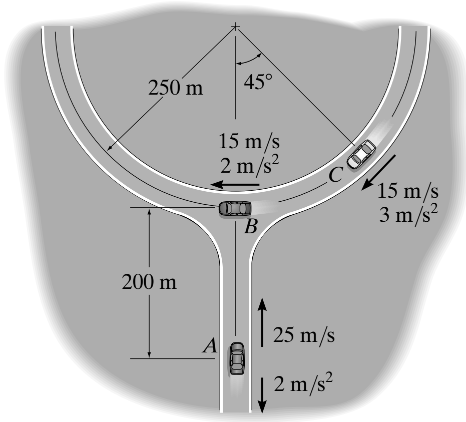
16–150. At the instant shown, car A travels with a speed of 25 m/s, which is decreasing at a constant rate of $2\,\text{m/s}^2$, while car B travels with a speed of 15 m/s, which is increasing at a constant rate of $2\,\text{m/s}^2$. Determine the velocity and acceleration of car A with respect to car B.



Reference Frames: The xyz rotating reference frame is attached to car B and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since car B moves along the circular road, its normal component of acceleration is $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$. Thus, the motion of car B with respect to the XYZ frame is

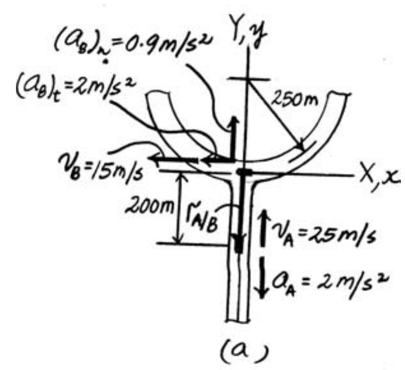
$$\mathbf{v}_B = [-15\mathbf{i}]\ \text{m/s}$$

$$\mathbf{a}_B = [-2\mathbf{i} + 0.9\mathbf{j}]\ \text{m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz frame with respect to the XYZ frame is

$$\omega = \frac{v_B}{\rho} = \frac{15}{250} = 0.06\ \text{rad/s} \qquad \omega = [-0.06\mathbf{k}]\ \text{rad/s}$$

$$\dot{\omega} = \frac{(a_B)_t}{\rho} = \frac{2}{250} = 0.008\ \text{rad/s}^2 \qquad \dot{\omega} = [-0.008\mathbf{k}]\ \text{rad/s}^2$$



The velocity of car A with respect to the XYZ reference frame is

$$\mathbf{v}_A = [25\mathbf{j}]\ \text{m/s} \qquad \mathbf{a}_A = [-2\mathbf{j}]\ \text{m/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{A/B} = [-200\mathbf{j}]\ \text{m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -15\mathbf{i} + (-0.06\mathbf{k}) \times (-200\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -27\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r}_{A/B} + \omega \times (\omega \times \mathbf{r}_{A/B}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = (-2\mathbf{i} + 0.9\mathbf{j}) + (-0.008\mathbf{k}) \times (-200\mathbf{j}) + (-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-200\mathbf{j})] + 2(-0.06\mathbf{k}) \times (27\mathbf{i} + 25\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = -0.6\mathbf{i} - 1.62\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [0.6\mathbf{i} - 0.38\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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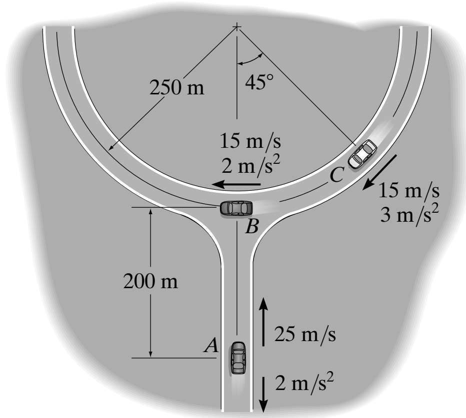
16–151. At the instant shown, car A travels with a speed of 25 m/s, which is decreasing at a constant rate of 2 m/s^2, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of 3 m/s^2. Determine the velocity and acceleration of car A with respect to car C.



Reference Frame: The xyz rotating reference frame is attached to car C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since car C moves along the circular road, its normal component of acceleration is $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{15^2}{250} = 0.9$ m/s^2. Thus, the motion of car C with respect to the XYZ frame is

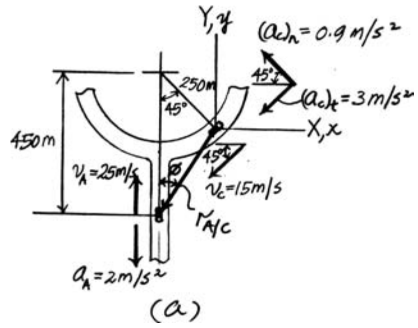
$$\mathbf{v}_C = -15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j} = [-10.607\mathbf{i} - 10.607\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\mathbf{j}] \text{ m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame is

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06 \text{ rad/s} \qquad \omega = [-0.06\mathbf{k}] \text{ rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012 \text{ rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}] \text{ rad/s}^2$$



The velocity and accdeleration of car A with respect to the XYZ frame is

$$\mathbf{v}_A = [25\mathbf{j}] \text{ m/s} \qquad \mathbf{a}_A = [-2\mathbf{j}] \text{ m/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{A/C} = -250\sin 45°\mathbf{i} - (450 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 273.22\mathbf{j}] \text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -27\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}] \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\omega} \times r_{A/C} + \omega \times (\omega \times \mathbf{r}_{A/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})$$
$$+ (-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times (27\mathbf{i} + 25\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = -2.4\mathbf{i} - 1.62\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

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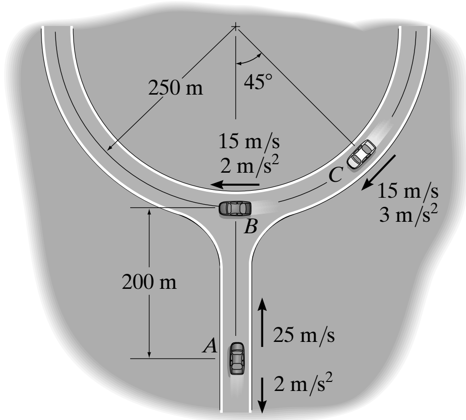
***16–152.** At the instant shown, car B travels with a speed of 15 m/s, which is increasing at a constant rate of $2\,\text{m/s}^2$, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of $3\,\text{m/s}^2$. Determine the velocity and acceleration of car B with respect to car C.



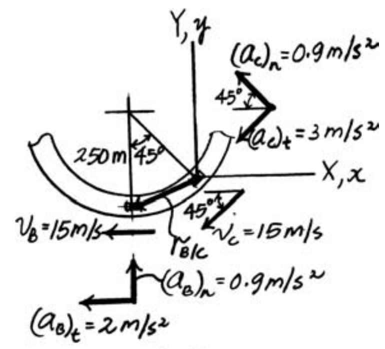
Reference Frame: The xyz rotating reference frame is attached to C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since B and C move along the circular road, their normal components of acceleration are $(a_B)_n = \dfrac{v_B^{\,2}}{\rho} = \dfrac{15^2}{250} = 0.9\text{ m/s}^2$ and $(a_C)_n = \dfrac{v_C^{\,2}}{\rho} = \dfrac{15^2}{250} = 0.9\text{ m/s}^2$. Thus, the motion of cars B and C with respect to the XYZ frame are

$$\mathbf{v}_B = [-15\mathbf{i}]\text{ m/s}$$

$$\mathbf{v}_C = [-15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j}] = [-10.607\mathbf{i} - 10.607\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_B = [-2\mathbf{i} + 0.9\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\,\mathbf{j}]\text{ m/s}^2$$



(a)

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\text{ rad/s} \qquad \omega = [-0.06\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\text{ rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}]\text{ rad/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{B/C} = -250\sin 45°\mathbf{i} - (250 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 73.22\,\mathbf{j}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times r_{B/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = -15\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = \mathbf{0} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \dot{\omega} \times \mathbf{r}_{B/C} + \omega \times (\omega \times \mathbf{r}_{B/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})$$

$$+(-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times \mathbf{0} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = -3\mathbf{i} + 0.9\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(a_{\text{rel}})_{xyz} = [1\mathbf{i}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

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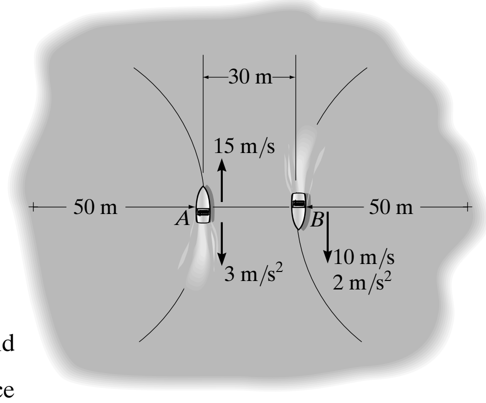
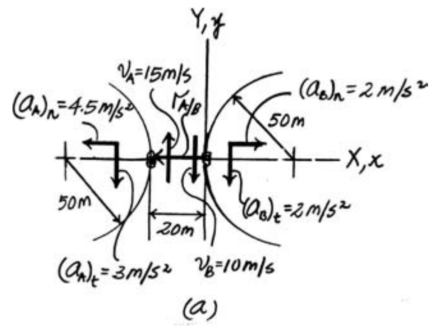
•16–153. At the instant shown, boat A travels with a speed of 15 m/s, which is decreasing at 3 m/s^2, while boat B travels with a speed of 10 m/s, which is increasing at 2 m/s^2. Determine the velocity and acceleration of boat A with respect to boat B at this instant.



Reference Frame: The xyz rotating reference frame is attached to boat B and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since boats A and B move along the circular paths, their normal components of acceleration are $(a_A)_n = \frac{v_A{}^2}{\rho} = \frac{15^2}{50} = 4.5\text{ m/s}^2$ and $(a_B)_n = \frac{v_B{}^2}{\rho} = \frac{10^2}{50} = 2\text{ m/s}^2$. Thus, the motion of boats A and B with respect to the XYZ frame are

$$\mathbf{v}_A = [15\mathbf{j}]\text{ m/s} \qquad \mathbf{v}_B = [-10\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_A = [-4.5\mathbf{i} - 3\mathbf{j}]\text{ m/s}^2 \qquad \mathbf{a}_B = [2\mathbf{i} - 2\mathbf{j}]\text{ m/s}^2$$



Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_B}{\rho} = \frac{10}{50} = 0.2\text{ rad/s} \qquad \omega = [0.2\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_B)_t}{\rho} = \frac{2}{50} = 0.04\text{ rad/s}^2 \qquad \dot{\omega} = [0.04\mathbf{k}]\text{ rad/s}^2$$

And the position of boat A with respect to B is

$$\mathbf{r}_{A/B} = [-20\mathbf{i}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$15\mathbf{j} = -10\mathbf{j} + (0.2\mathbf{k}) \times (-20\mathbf{i}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$15\mathbf{j} = -14\mathbf{j} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [29\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r}_{A/B} + \omega \times (\omega \times \mathbf{r}_{A/B}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(-4.5\mathbf{i} - 3\mathbf{j}) = (2\mathbf{i} - 2\mathbf{j}) + (0.04\mathbf{k}) \times (-20\mathbf{i}) + (0.2\mathbf{k}) \times \left[(0.2\mathbf{k}) \times (-20\mathbf{i})\right] + 2(0.2\mathbf{k}) \times (29\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-4.5\mathbf{i} - 3\mathbf{j} = -8.8\mathbf{i} - 2.8\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [4.3\mathbf{i} - 0.2\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

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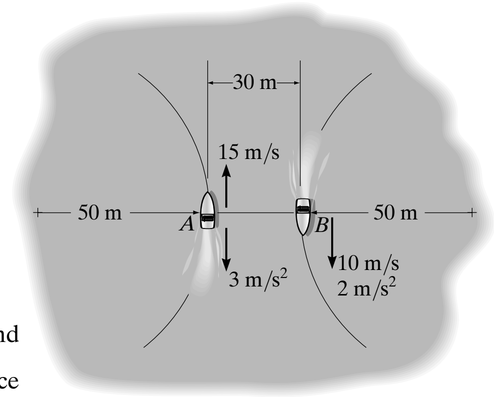
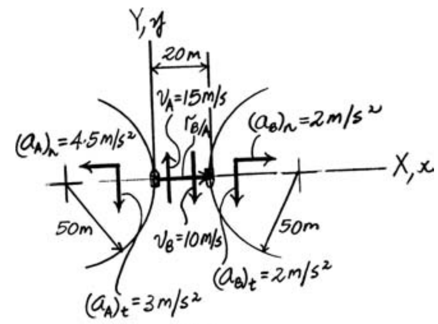
16–154. At the instant shown, boat A travels with a speed of $15\,\text{m/s}$, which is decreasing at $3\,\text{m/s}^2$, while boat B travels with a speed of $10\,\text{m/s}$, which is increasing at $2\,\text{m/s}^2$. Determine the velocity and acceleration of boat B with respect to boat A at this instant.



Reference Frame: The xyz rotating reference frame is attached to boat A and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since boats A and B move along the circular paths, their normal components of acceleration are $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{15^2}{50} = 4.5\text{ m/s}^2$ and $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{10^2}{50} = 2\text{ m/s}^2$. Thus, the motion of boats A and B with respect to the XYZ frame are

$$\mathbf{v}_A = [15\mathbf{j}]\text{ m/s} \qquad \mathbf{v}_B = [-10\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_A = [-4.5\mathbf{i} - 3\mathbf{j}]\text{ m/s}^2 \qquad \mathbf{a}_B = [2\mathbf{i} - 2\mathbf{j}]\text{ m/s}^2$$



(a)

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_A}{\rho} = \frac{15}{50} = 0.3\text{ rad/s} \qquad \boldsymbol{\omega} = [0.3\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_A)_t}{\rho} = \frac{3}{50} = 0.06\text{ rad/s}^2 \qquad \dot{\boldsymbol{\omega}} = [-0.06\mathbf{k}]\text{ rad/s}^2$$

And the position of boat B with respect to boat A is

$$\mathbf{r}_{B/A} = [20\mathbf{i}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-10\mathbf{j} = 15\mathbf{j} + (0.3\mathbf{k}) \times (20\mathbf{i}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-10\mathbf{j} = 21\mathbf{j} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [-31\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}(\boldsymbol{\omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\omega} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(2\mathbf{i} - 2\mathbf{j}) = (-4.5\mathbf{i} - 3\mathbf{j}) + (-0.06\mathbf{k}) \times (20\mathbf{i}) + (0.3\mathbf{k}) \times \left[(0.3\mathbf{k}) \times (20\mathbf{i})\right] + 2(0.3\mathbf{k}) \times (-31\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$2\mathbf{i} - 2\mathbf{j} = 12.3\mathbf{i} - 4.2\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [-10.3\mathbf{i} + 2.2\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

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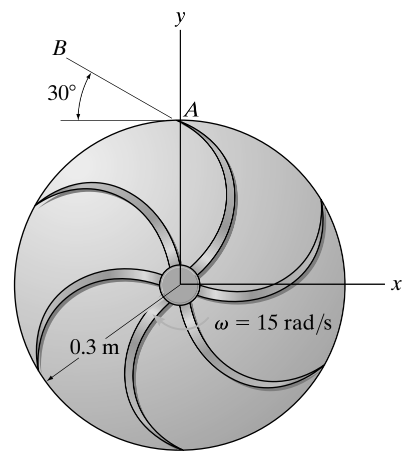
16–155. Water leaves the impeller of the centrifugal pump with a velocity of 25 m/s and acceleration of 30 m/s^2, both measured relative to the impeller along the blade line AB. Determine the velocity and acceleration of a water particle at A as it leaves the impeller at the instant shown. The impeller rotates with a constant angular velocity of $\omega = 15$ rad/s.



Reference Frame: The xyz rotating reference frame is attached to the impeller and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

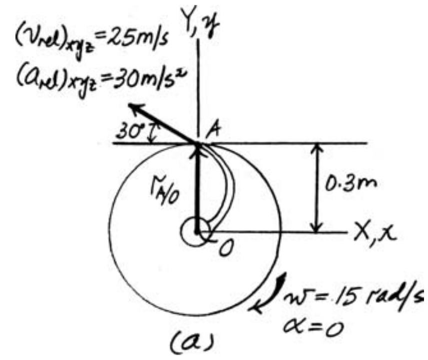
$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [-15\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \mathbf{0}$$

The motion of point A with respect to the xyz frame is

$$\mathbf{r}_{A/O} = [0.3\mathbf{j}]\ \text{m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-25\cos 30°\,\mathbf{i} + 25\sin 30°\,\mathbf{j}) = [-21.65\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-30\cos 30°\,\mathbf{i} + 30\sin 30°\,\mathbf{j}) = [-25.98\mathbf{i} + 15\mathbf{j}]\ \text{m/s}^2$$



Velocity: Applying the relative velocity equation.

$$\mathbf{v}_A = \mathbf{v}_O + \omega \times \mathbf{r}_{A/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-15\mathbf{k}) \times (0.3\mathbf{j}) + (-21.65\mathbf{i} + 12.5\mathbf{j})$$

$$= [-17.2\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{A/O} + \omega \times (\omega \times \mathbf{r}_{A/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-15\mathbf{k}) \times [(-15\mathbf{k}) \times (0.3\mathbf{j})] + 2(-15\mathbf{k}) \times (-21.65\mathbf{i} + 12.5\mathbf{j}) + (-25.98\mathbf{i} + 15\mathbf{j})$$

$$= [349\mathbf{i} + 597\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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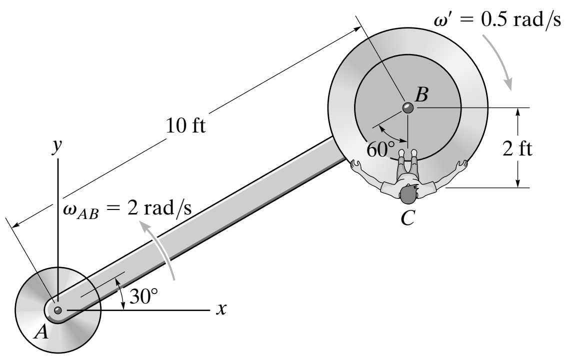
***16–156.** A ride in an amusement park consists of a rotating arm AB having a constant angular velocity $\omega_{AB} = 2$ rad/s about point A and a car mounted at the end of the arm which has a constant angular velocity $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at C.



$$\mathbf{r}_{B/A} = (10\cos 30°\,\mathbf{i} + 10\sin 30°\,\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}\text{ ft}$$

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}\text{ ft/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$= 0 - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-34.64\mathbf{i} - 20\mathbf{j}\}\text{ ft/s}^2$$

$$\Omega = (2 - 0.5)\mathbf{k} = 1.5\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$$

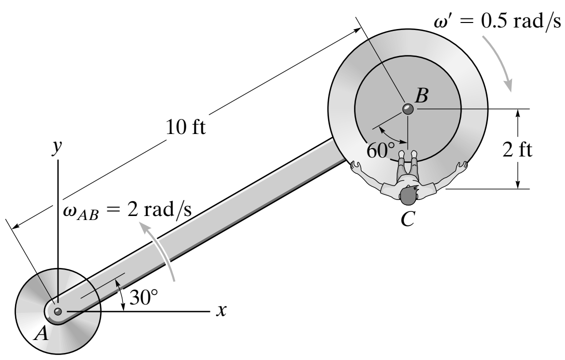
$$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}\text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= -34.64\mathbf{i} - 20\mathbf{j} + 0 + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$$

$$= \{-34.6\mathbf{i} - 15.5\mathbf{j}\}\text{ ft/s}^2 \qquad \textbf{Ans.}$$

•16–157. A ride in an amusement park consists of a rotating arm AB that has an angular acceleration of $\alpha_{AB} = 1$ rad/s^2 when $\omega_{AB} = 2$ rad/s at the instant shown. Also at this instant the car mounted at the end of the arm has an angular acceleration of $\boldsymbol{\alpha}' = \{-0.6\mathbf{k}\}$ rad/s^2 and angular velocity of $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. Determine the velocity and acceleration of the passenger C at this instant.



$$\mathbf{r}_{B/A} = (10\cos 30°\mathbf{i} + 10\sin 30°\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}\text{ ft}$$

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}\text{ ft/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$= (1\mathbf{k}) \times (8.66\mathbf{i} + 5\mathbf{j}) - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-39.64\mathbf{i} - 11.34\mathbf{j}\}\text{ ft/s}^2$$

$$\Omega = (2-0.5)\mathbf{k} = 1.5\mathbf{k}$$

$$\dot{\Omega} = (1 - 0.6)\mathbf{k} = 0.4\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$$

$$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}\text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= -39.64\mathbf{i} - 11.34\mathbf{j} + (0.4\mathbf{k}) \times (-2\mathbf{j}) + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$$

$$= \{-38.8\mathbf{i} - 6.84\mathbf{j}\}\text{ ft/s}^2 \qquad \textbf{Ans.}$$

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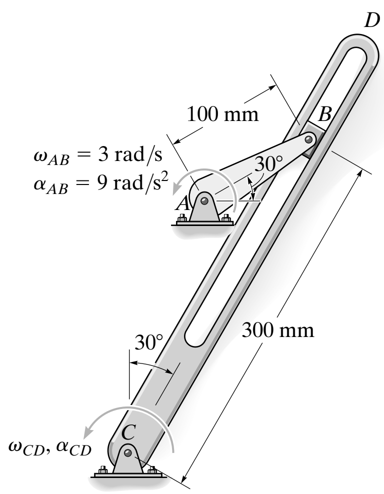
16–158. The "quick-return" mechanism consists of a crank AB, slider block B, and slotted link CD. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



$v_B = 3(0.1) = 0.3$ m/s

$(a_B)_t = 9(0.1) = 0.9$ m/s^2

$(a_B)_n = (3)^2\,(0.1) = 0.9$ m/s^2

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.3\cos 60°\mathbf{i} + 0.3\sin 60°\mathbf{j} = \mathbf{0} + (\omega_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + v_{B/C}\,\mathbf{i}$

$v_{B/C} = 0.15$ m/s

$\omega_{CD} = 0.866$ rad/s ↺ **Ans.**

$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

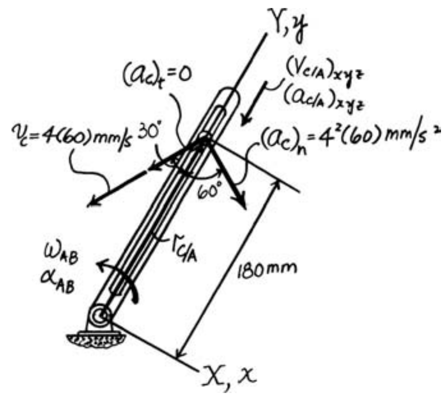
$0.9\cos 60°\mathbf{i} - 0.9\cos 30°\mathbf{i} + 0.9\sin 60°\mathbf{j} + 0.9\sin 30°\mathbf{j} = \mathbf{0} + (\alpha_{CD}\,\mathbf{k}) \times (0.3\mathbf{i})$

$+(0.866\mathbf{k}) \times (0.866\mathbf{k} \times 0.3\mathbf{i}) + 2(0.866\mathbf{k} \times 0.15\mathbf{i}) + a_{B/C}\,\mathbf{i}$

$-0.3294\mathbf{i} + 1.2294\mathbf{j} = 0.3\alpha_{CD}\mathbf{j} - 0.225\mathbf{i} + 0.2598\mathbf{j} + a_{B/C}\,\mathbf{i}$

$a_{B/C} = -0.104$ m/s^2

$\alpha_{CD} = 3.23$ rad/s^2 ↺ **Ans.**



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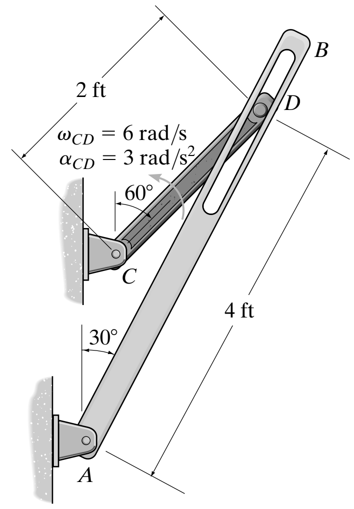
16–159. The quick return mechanism consists of the crank CD and the slotted arm AB. If the crank rotates with the angular velocity and angular acceleration at the instant shown, determine the angular velocity and angular acceleration of AB at this instant.



Reference Frame: The xyz rotating reference frame is attached to slotted arm AB and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz reference frame with respect to the XYZ frame is

$$\mathbf{v}_A = a_A = \mathbf{0} \qquad \omega_{AB} = \omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = \alpha_{AB}\,\mathbf{k}$$

For the motion of point D with respect to the xyz frame, we have

$$\mathbf{r}_{D/A} = [4\mathbf{i}] \text{ ft} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\,\mathbf{i}$$

Since the crank CD rotates about a fixed axis, $\mathbf{v}_D$ and $\mathbf{a}_D$ with respect to the XYZ reference frame can be determined from

$$\mathbf{v}_D = \omega_{CD} \times \mathbf{r}_D$$

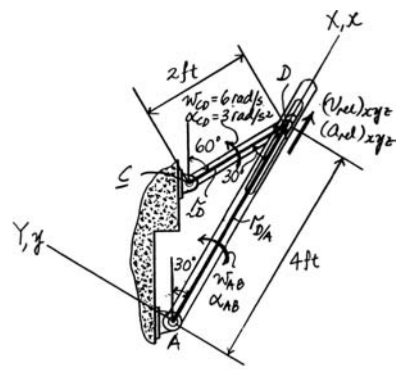
$$= (6\mathbf{k}) \times (2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j})$$

$$= [6\mathbf{i} + 10.39\mathbf{j}] \text{ ft/s}$$

$$\mathbf{a}_D = \alpha_{CD} \times \mathbf{r}_D - \omega_{CD}{}^2\mathbf{r}_D$$

$$= (3\mathbf{k}) \times (2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j}) - 6^2(2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j})$$

$$= [-59.35\mathbf{i} + 41.20\mathbf{j}] \text{ ft/s}^2$$



(a)

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_D = \mathbf{v}_A + \omega_{AB} \times r_{D/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$6\mathbf{i} + 10.39\,\mathbf{j} = \mathbf{0} + (\omega_{AB}\mathbf{k}) \times (4\mathbf{i}) + (v_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$6\mathbf{i} + 10.39\mathbf{j} = (v_{\text{rel}})_{xyz}\,\mathbf{i} + 4\omega_{AB}\,\mathbf{j}$$

Equating the **i** and **j** components yields

$$(v_{\text{rel}})_{xyz} = 6 \text{ ft/s}$$

$$10.39 = 4\omega_{AB} \qquad \omega_{AB} = 2.598 \text{ rad/s} = 2.60 \text{ rad/s} \quad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{D/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{AB}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-59.35\mathbf{i} + 41.20\,\mathbf{j} = \mathbf{0} + (\alpha_{AB}\mathbf{k}) \times 4\mathbf{i} + 2.598\mathbf{k} \times [(2.598\mathbf{k}) \times (4\mathbf{i})] + 2(2.598\mathbf{k}) \times (6\mathbf{i}) + (\mathbf{a}_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$-59.35\mathbf{i} + 41.20\,\mathbf{j} = \left[(a_{\text{rel}})_{xyz} - 27\right]\mathbf{i} + (4\alpha_{AB} + 31.18)\mathbf{j}$$

Equating the **i** and **j** components yields

$$41.20 = 4\alpha_{AB} + 31.18$$

$$\alpha_{AB} = 2.50 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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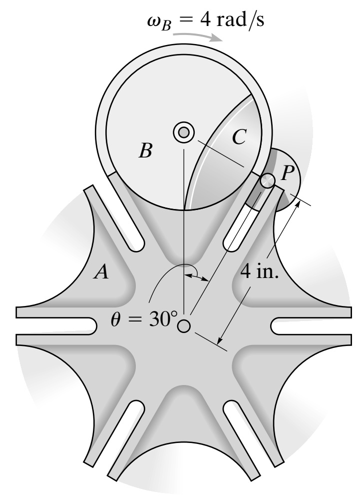
***16–160.** The Geneva mechanism is used in a packaging system to convert constant angular motion into intermittent angular motion. The star wheel A makes one sixth of a revolution for each full revolution of the driving wheel B and the attached guide C. To do this, pin P, which is attached to B, slides into one of the radial slots of A, thereby turning wheel A, and then exits the slot. If B has a constant angular velocity of $\omega_B = 4$ rad/s, determine $\boldsymbol{\omega}_A$ and $\boldsymbol{\alpha}_A$ of wheel A at the instant shown.



The circular path of motion of P has a radius of

$$r_P = 4 \tan 30° = 2.309 \text{ in.}$$

Thus,

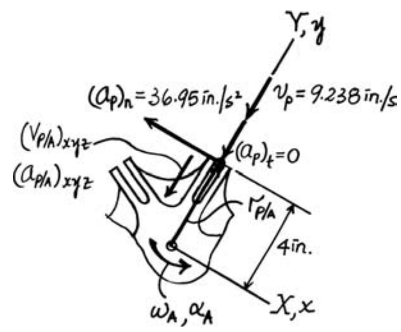
$$\mathbf{v}_P = -4(2.309)\mathbf{j} = -9.238\mathbf{j}$$

$$\mathbf{a}_P = -(4)^2(2.309)\mathbf{i} = -36.95\mathbf{i}$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$-9.238\mathbf{j} = \mathbf{0} + (\omega_A\,\mathbf{k}) \times (4\mathbf{j}) - v_{P/A}\,\mathbf{j}$$



Solving,

$$\omega_A = 0 \qquad \textbf{Ans.}$$

$$v_{P/A} = 9.238 \text{ in./s}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

$$-36.95\mathbf{i} = \mathbf{0} + (\alpha_A\mathbf{k}) \times (4\mathbf{j}) + \mathbf{0} + \mathbf{0} - a_{P/A}\,\mathbf{j}$$

Solving,

$$-36.95 = -4\alpha_A$$

$$\alpha_A = 9.24 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

$$a_{P/A} = 0$$

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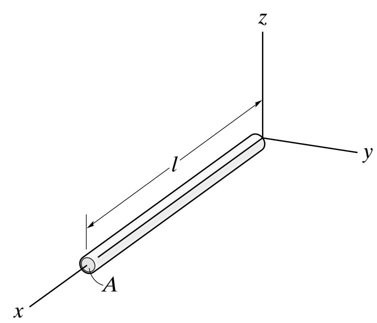
•17–1. Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.

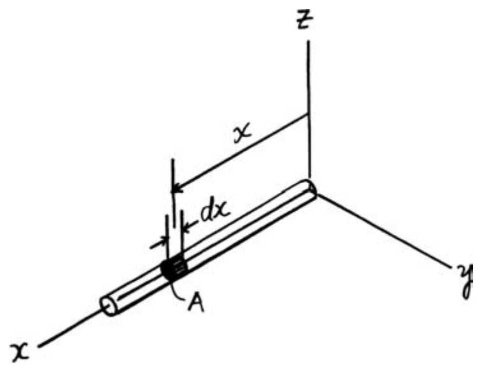


$$I_y = \int_M x^2\, dm$$

$$= \int_0^l x^2\,(\rho\, A\, dx)$$

$$= \frac{1}{3}\rho\, A\, l^3$$

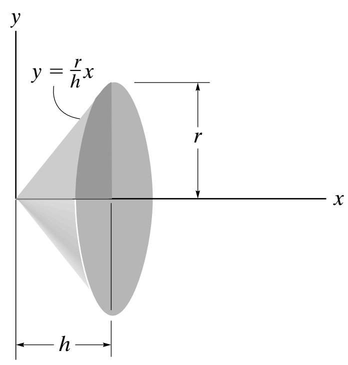
$$m = \rho\, A\, l$$



Thus,

$$I_y = \frac{1}{3} m\, l^2$$ **Ans.**

17–2. The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



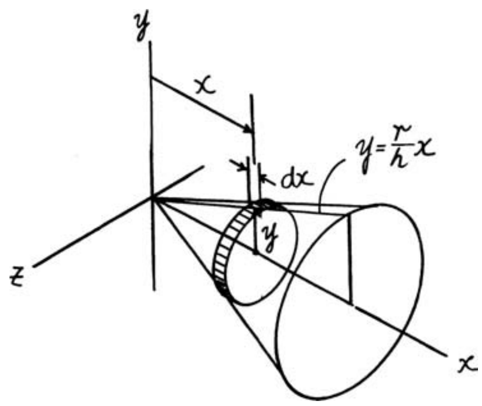
$$dm = \rho\, dV = \rho(\pi\, y^2\, dx)$$

$$m = \int_0^h \rho(\pi)\left(\frac{r^2}{h^2}\right)x^2\, dx = \rho\pi\left(\frac{r^2}{h^2}\right)\left(\frac{1}{3}\right)h^3 = \frac{1}{3}\rho\pi\, r^2 h$$

$$dI_x = \frac{1}{2}\, y^2\, dm$$

$$= \frac{1}{2}\, y^2\,(\rho\pi\, y^2\, dx)$$

$$= \frac{1}{2}\,\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\, dx$$



$$I_x = \int_0^h \frac{1}{2}\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\, dx = \frac{1}{10}\rho\pi\, r^4\, h$$

Thus,

$$I_x = \frac{3}{10}\, m\, r^2$$ **Ans.**

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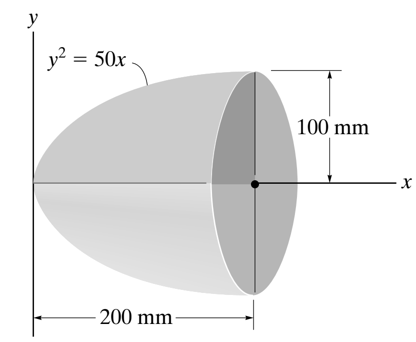
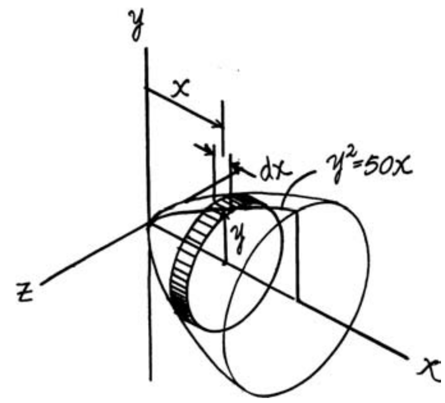
17–3. The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration k_x. The density of the material is $\rho = 5\ \mathrm{Mg/m^3}$.





$$dm = \rho\,\pi\,y^2\,dx = \rho\,\pi\,(50x)\,dx$$

$$I_x = \int \frac{1}{2}\,y^2\,dm = \frac{1}{2}\int_0^{200} 50\,x\,\{\pi\,\rho\,(50x)\}\,dx$$

$$= \rho\,\pi\left(\frac{50^2}{2}\right)\left[\frac{1}{3}\,x^3\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50^2}{6}\right)(200)^3$$

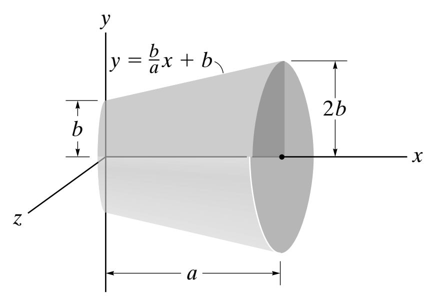
$$m = \int dm = \int_0^{200} \pi\,\rho\,(50x)\,dx$$

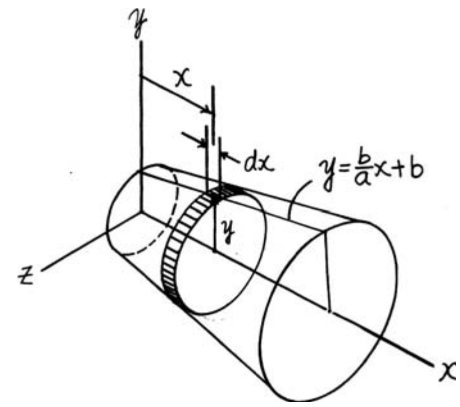
$$= \rho\,\pi\,(50)\left[\frac{1}{2}\,x^2\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50}{2}\right)(200)^2$$

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{50}{3}\,(200)} = 57.7\ \mathrm{mm}$$ **Ans.**

***17–4.** The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The frustum has a constant density ρ.





$$dm = \rho\,dV = \rho\pi y^2\,dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \frac{1}{2}dm y^2 = \frac{1}{2}\rho\pi y^4\,dx$$

$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4\,b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

$$I_x = \int dI_x = \frac{1}{2}\rho\pi\int_0^a\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4\,b^4}{a}x + b^4\right)dx$$

$$= \frac{31}{10}\rho\pi a b^4$$

$$m = \int_m dm = \rho\pi\int_0^a\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi a b^2$$

$$I_x = \frac{93}{70}mb^2$$ **Ans.**

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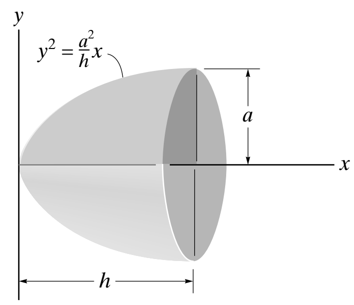
•**17–5.** The paraboloid is formed by revolving the shaded area around the x axis. Determine the moment of inertia about the x axis and express the result in terms of the total mass m of the paraboloid. The material has a constant density ρ.

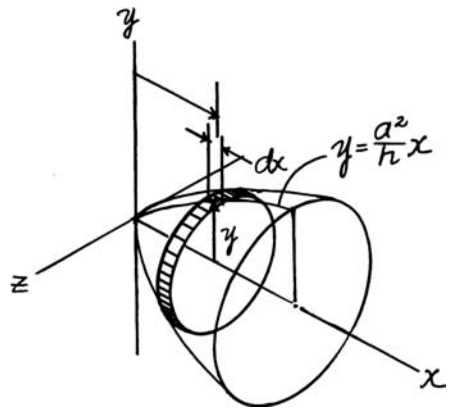


$$dm = \rho\, dV = \rho\,(\pi\, y^2\, dx)$$

$$d\,I_x = \frac{1}{2}\,dm\,y^2 = \frac{1}{2}\rho\,\pi\,y^4\,dx$$

$$I_x = \int_0^h \frac{1}{2}\rho\,\pi\left(\frac{a^4}{h^2}\right)x^2\,dx$$

$$= \frac{1}{6}\,\pi\,\rho a^4\,h$$

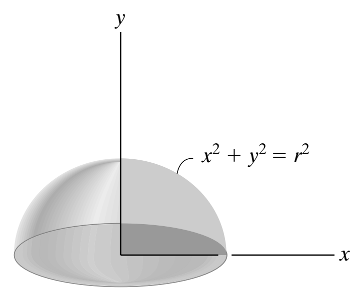


$$m = \int_0^h \frac{1}{2}\rho\,\pi\left(\frac{a^2}{h}\right)x\,dx$$

$$= \frac{1}{2}\,\rho\,\pi\,a^2\,h$$

$$I_x = \frac{1}{3}\,ma^2 \qquad \textbf{Ans.}$$

17–6. The hemisphere is formed by rotating the shaded area around the y axis. Determine the moment of inertia I_y and express the result in terms of the total mass m of the hemisphere. The material has a constant density ρ.



$$m = \int_V \rho\,dV = \rho\int_0^r \pi\,x^2\,dy = \rho\pi\int_0^r (r^2 - y^2)dy$$

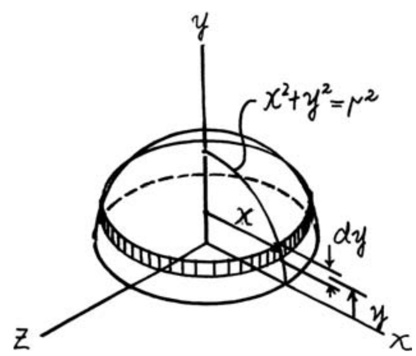
$$= \rho\pi\left[r^2\,y - \frac{1}{3}\,y^3\right]_0^r = \frac{2}{3}\rho\pi\,r^3$$

$$I_y = \int_m \frac{1}{2}\,(dm)\,x^2 = \frac{\rho}{2}\int_0^r \pi x^4\,dy = \frac{\rho\pi}{2}\int_0^r (r^2 - y^2)^2\,dy$$

$$= \frac{\rho\pi}{2}\left[r^4 y - \frac{2}{3}\,r^2\,y^3 + \frac{y^5}{5}\right]_0^r = \frac{4\rho\pi}{15}\,r^5$$

Thus,

$$I_y = \frac{2}{5}\,m\,r^2 \qquad \textbf{Ans.}$$



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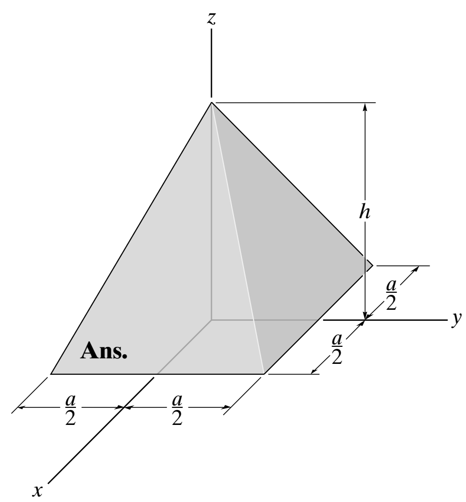
17–7. Determine the moment of inertia of the homogeneous pyramid of mass m about the z axis. The density of the material is ρ. *Suggestion:* Use a rectangular plate element having a volume of $dV = (2x)(2y)dz$.



$$dI_z = \frac{dm}{12}\left[(2y)^2 + (2y)^2\right] = \frac{2}{3}y^2\,dm \qquad I_z = \frac{m}{10}a^2 \qquad \textbf{Ans.}$$

$$dm = 4\rho y^2\,dz$$

$$dI_z = \frac{8}{3}\rho y^4\,dz = \frac{8}{3}\rho(h - z)^4\left(\frac{a^4}{16h^4}\right)dz$$

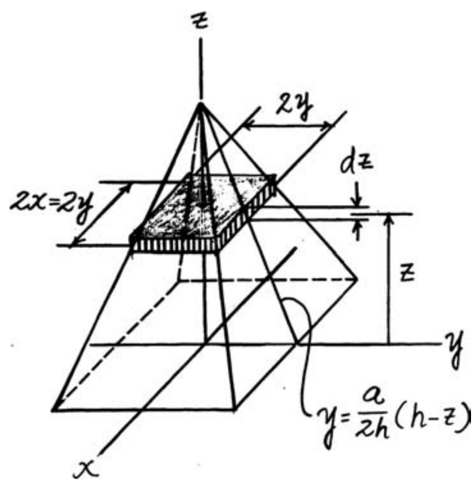
$$I_z = \frac{\rho}{6}\left(\frac{a^4}{h^4}\right)\int_0^h (h^4 - 4h^3z + 6h^2z^2 - 4hz^3 + z^4)dz = \frac{\rho}{6}\left(\frac{a^4}{h^4}\right)\left[h^5 - 2h^5 + 2h^5 - h^5 + \frac{1}{5}h^5\right]$$

$$= \frac{\rho a^4 h}{30}$$

$$m = \int_0^h 4\rho(h - z)^2\left(\frac{a^2}{4h^2}\right)dz = \frac{\rho a^2}{h^2}\int_0^h (h^2 - 2hz + z^2)dz$$

$$= \frac{\rho a^2}{h^2}\left[h^3 - h^3 + \frac{1}{3}h^3\right]$$

$$= \frac{\rho a^2 h}{3}$$



Thus,

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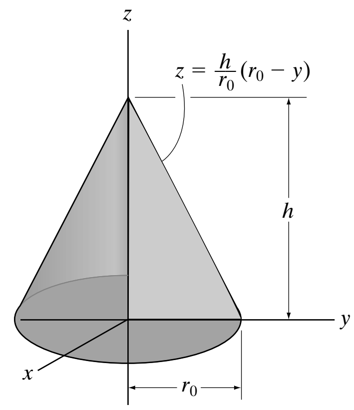
***17–8.** Determine the mass moment of inertia I_z of the cone formed by revolving the shaded area around the z axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho\, dV = \rho\pi r^2 dz$. Here, $r = y = r_o - \frac{r_o}{h}z$. Thus, $dm = \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$. The mass moment of inertia of this element about the z axis is

$$dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2 = \frac{1}{2}\rho\pi r^4 dz = \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

Mass: The mass of the cone can be determined by integrating dm. Thus,

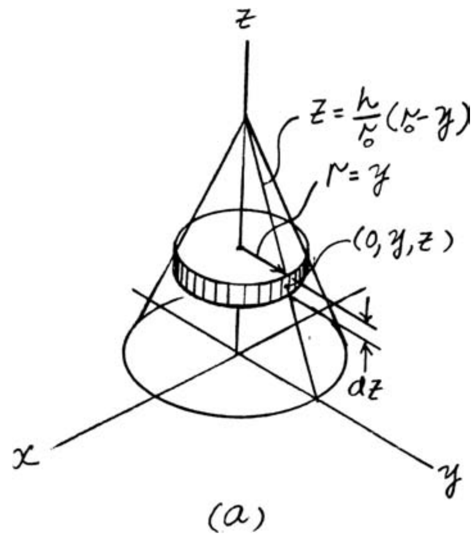
$$m = \int dm = \int_0^h \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$$

$$= \rho\pi\left[\frac{1}{3}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{3}\rho\pi r_o^2 h$$

Mass Moment of Inertia: Integrating dI_z, we obtain

$$I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_o - \frac{r_o}{h}z\right)^5\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{10}\rho\pi r_o^4 h$$



From the result of the mass, we obtain $\rho\pi r_o^2 h = 3m$. Thus, I_z can be written as

$$I_z = \frac{1}{10}\left(\rho\pi r_o^2 h\right)r_o^2 = \frac{1}{10}(3m)r_o^2 = \frac{3}{10}mr_o^2 \qquad \textbf{Ans.}$$

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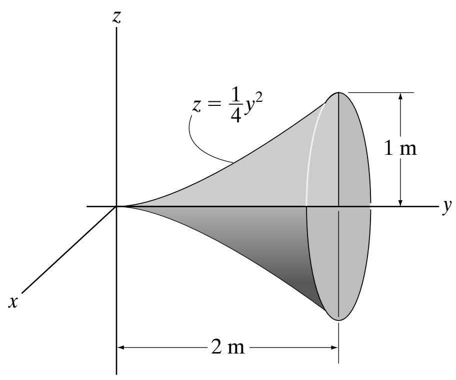
•17–9. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the solid.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho\, dV = \rho\pi r^2 dy$. Here, $r = z = \frac{1}{4}y^2$. Thus, $dm = \rho\pi\left(\frac{1}{4}y^2\right)^2 dy = \frac{\rho\pi}{16}y^4\,dy$.

The mass moment of inertia of this element about the y axis is

$$dI_y = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(\frac{1}{4}y^2\right)^4 dy = \frac{\rho\pi}{512}y^8 dy$$

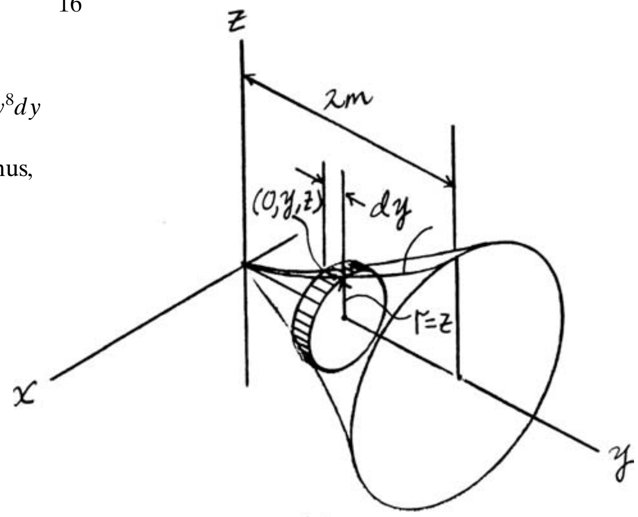
Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{2\text{ m}} \frac{\rho\pi}{16}y^4 dy = \frac{\rho\pi}{16}\left(\frac{y^5}{5}\right)\Bigg|_0^{2\text{ m}} = \frac{2}{5}\rho\pi$$

Mass Moment of Inertia: Integrating dI_y, we obtain

$$I_y = \int dI_y = \int_0^{2\text{ m}} \frac{\rho\pi}{572}y^8 dy$$

$$= \frac{\rho\pi}{512}\left(\frac{y^9}{9}\right)\Bigg|_0^{2\text{ m}} = \frac{\pi\rho}{9}$$



(a)

From the result of the mass, we obtain $\pi\rho = \frac{5m}{2}$. Thus, I_y can be written as

$$I_y = \frac{1}{9}\left(\frac{5m}{2}\right) = \frac{5}{18}m$$ **Ans.**

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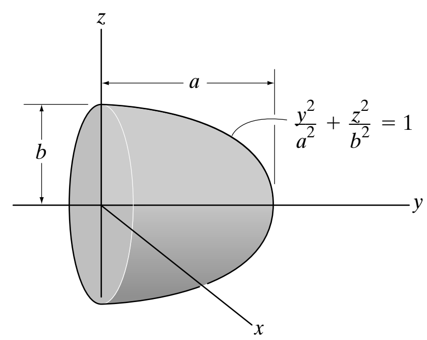
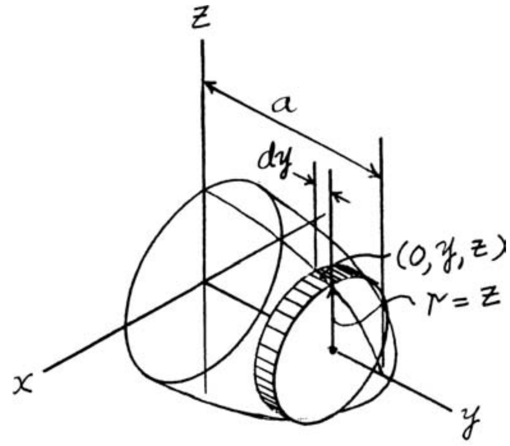
17–10. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the semi-ellipsoid.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho\, dV = \rho\pi r^2 dy$. Here, $r = z = b\sqrt{1 - \frac{y^2}{a^2}}$. Thus, $dm = \rho\pi\left(b\sqrt{1 - \frac{y^2}{a^2}}\right)^2 dz$ $= \rho\pi b^2\left(1 - \frac{y^2}{a^2}\right)dy$. The mass moment of inertia of this element about the y axis is

$$dI_y = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(b\sqrt{1 - \frac{y^2}{a^2}}\right)^4 dy$$

$$= \frac{1}{2}\rho\pi b^4\left(1 - \frac{y^2}{a^2}\right)^2 dy = \frac{1}{2}\rho\pi b^4\left(1 + \frac{y^4}{a^4} - \frac{2y^2}{a^2}\right)dy$$



(a)

Mass: The mass of the semi-ellipsoid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^a \rho\pi b^2\left(1 - \frac{y^2}{a^2}\right)dy = \rho\pi b^2\left(y - \frac{y^3}{3a^2}\right)\Bigg|_0^a = \frac{2}{3}\rho\,\pi\,ab^2$$

Mass Moment of Inertia: Integrating dI_y, we obtain

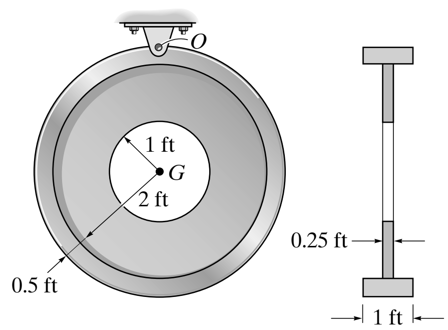
$$I_y = \int dI_y = \int_0^a \frac{1}{2}\rho\pi b^4\left(H\frac{y^4}{a^4} - \frac{2y^2}{a^2}\right)dy$$

$$= \frac{1}{2}\rho\pi b^4\left(y + \frac{y^5}{5a^4} - \frac{2y^3}{3a^2}\right)\Bigg|_0^a = \frac{4}{15}\rho\,\pi\,ab^4$$

From the result of the mass, we obtain $\rho\pi ab^2 = \frac{3m}{2}$. Thus, I_y can be written as

$$I_y = \frac{4}{15}\left(\rho\pi ab^2\right)b^2 = \frac{4}{15}\left(\frac{3m}{2}\right)b^2 = \frac{2}{5}mb^2 \qquad \textbf{Ans.}$$

17–11. Determine the moment of inertia of the assembly about an axis that is perpendicular to the page and passes through the center of mass G. The material has a specific weight of $\gamma = 90\ \text{lb/ft}^3$.



$$I_G = \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2.5)^2(1)\right](2.5)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(1)\right](2)^2$$

$$+ \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(0.25)\right](2)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(1)^2(0.25)\right](1)^2$$

$$= 118\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

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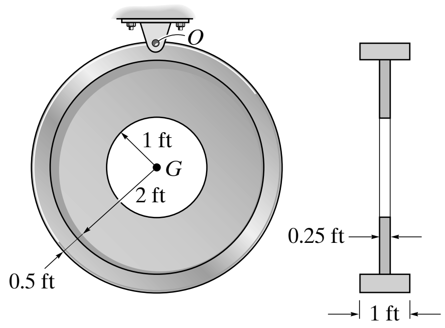
***17–12.** Determine the moment of inertia of the assembly about an axis that is perpendicular to the page and passes through point O. The material has a specific weight of $\gamma = 90 \text{ lb/ft}^3$.



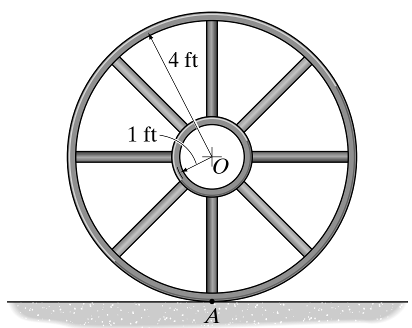
$$I_G = \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2.5)^2(1)\right](2.5)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(1)\right](2)^2$$
$$+ \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(0.25)\right](2)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(1)^2(0.25)\right](1)^2$$
$$= 117.72 \text{ slug}\cdot\text{ft}^2$$

$$I_O = I_G + md^2$$

$$m = \left(\frac{90}{32.2}\right)\pi(2^2 - 1^2)(0.25) + \left(\frac{90}{32.2}\right)\pi(2.5^2 - 2^2)(1) = 26.343 \text{ slug}$$

$$I_O = 117.72 + 26.343(2.5)^2 = 282 \text{ slug}\cdot\text{ft}^2$$ **Ans.**

•17–13. If the large ring, small ring and each of the spokes weigh 100 lb, 15 lb, and 20 lb, respectively, determine the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A.



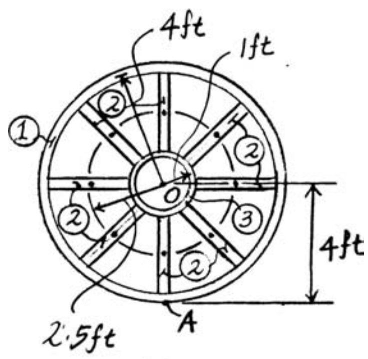
Composite Parts: The wheel can be subdivided into the segments shown in Fig. *a*. The spokes which have a length of $(4 - 1) = 3$ ft and a center of mass located at a distance of $\left(1 + \frac{3}{2}\right)$ ft $= 2.5$ ft from point O can be grouped as segment (2).

Mass Moment of Inertia: First, we will compute the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O.

$$I_O = \left(\frac{100}{32.2}\right)(4^2) + 8\left[\frac{1}{12}\left(\frac{20}{32.2}\right)(3^2) + \left(\frac{20}{32.2}\right)(2.5^2)\right] + \left(\frac{15}{32.2}\right)(1^2)$$
$$= 84.94 \text{ slug}\cdot\text{ft}^2$$

The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A can be found using the parallel-axis theorem $I_A = I_O + md^2$, where $m = \frac{100}{32.2} + 8\left(\frac{20}{32.2}\right) + \frac{15}{32.2} = 8.5404$ slug and $d = 4$ ft. Thus,

$$I_A = 84.94 + 8.5404(4^2) = 221.58 \text{ slug}\cdot\text{ft}^2 = 222 \text{ slug}\cdot\text{ft}^2$$ **Ans.**



(a)

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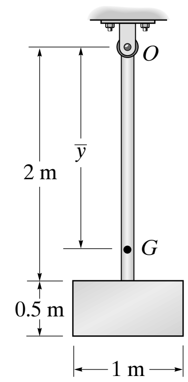
17–14. The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



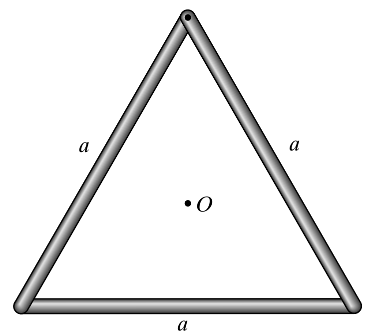
$$\bar{y} = \frac{\Sigma \bar{y}m}{\Sigma m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781 \text{ m} = 1.78 \text{ m}$$ **Ans.**

$$I_G = \Sigma \bar{I}_G + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

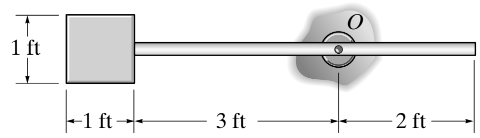
$$= 4.45 \text{ kg} \cdot \text{m}^2$$ **Ans.**

17–15. Each of the three slender rods has a mass m. Determine the moment of inertia of the assembly about an axis that is perpendicular to the page and passes through the center point O.



$$I_O = 3\left[\frac{1}{12}ma^2 + m\left(\frac{a \sin 60^\circ}{3}\right)^2\right] = \frac{1}{2}ma^2$$ **Ans.**

***17–16.** The pendulum consists of a plate having a weight of 12 lb and a slender rod having a weight of 4 lb. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



$$I_O = \Sigma I_G + md^2$$

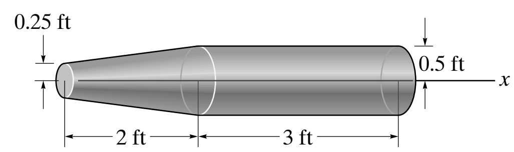
$$= \frac{1}{12}\left(\frac{4}{32.2}\right)(5)^2 + \left(\frac{4}{32.2}\right)(0.5)^2 + \frac{1}{12}\left(\frac{12}{32.2}\right)(1^2 + 1^2) + \left(\frac{12}{32.2}\right)(3.5)^2$$

$$= 4.917 \text{ slug} \cdot \text{ft}^2$$

$$m = \left(\frac{4}{32.2}\right) + \left(\frac{12}{32.2}\right) = 0.4969 \text{ slug}$$

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{4.917}{0.4969}} = 3.15 \text{ ft}$$ **Ans.**

•**17–17.** Determine the moment of inertia of the solid steel assembly about the x axis. Steel has a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$.



$$I_x = \frac{1}{2}m_1(0.5)^2 + \frac{3}{10}m_2(0.5)^2 - \frac{3}{10}m_3(0.25)^2$$

$$= \left[\frac{1}{2}\pi(0.5)^2(3)(0.5)^2 + \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.5)^2(4)(0.5)^2 - \frac{3}{10}\left(\frac{1}{2}\right)\pi(0.25)^2(2)(0.25)^2\right]\left(\frac{490}{32.2}\right)$$

$$= 5.64 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

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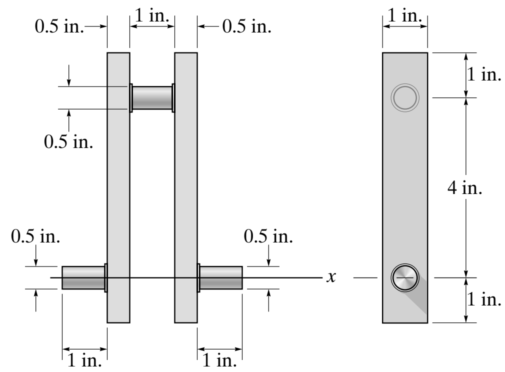
17–18. Determine the moment of inertia of the center crank about the x axis. The material is steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$.



$$m_s = \frac{490}{32.2}\left(\frac{\pi\,(0.25)^2(1)}{(12)^3}\right) = 0.0017291\ \text{slug}$$

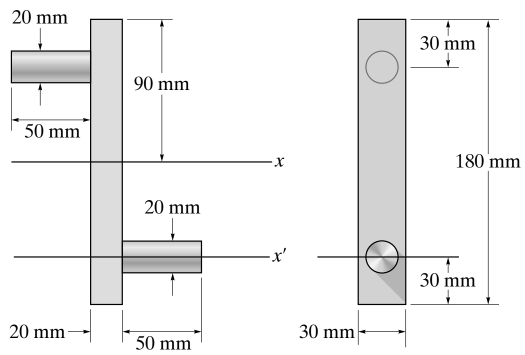
$$m_p = \frac{490}{32.2}\left(\frac{(6)(1)(0.5)}{(12)^3}\right) = 0.02642\ \text{slug}$$

$$I_x = 2\left[\frac{1}{12}(0.02642)\big((1)^2 + (6)^2\big) + (0.02642)(2)^2\right]$$

$$+\ 2\left[\frac{1}{2}(0.0017291)(0.25)^2\right] + \frac{1}{2}(0.0017291)(0.25)^2 + (0.0017291)(4)^2\Big]$$

$$= 0.402\ \text{slug}\cdot\text{in}^2$$ **Ans.**

17–19. Determine the moment of inertia of the overhung crank about the x axis. The material is steel for which the density is $\rho = 7.85\ \text{Mg/m}^3$.



$$m_c = 7.85\big(10^3\big)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

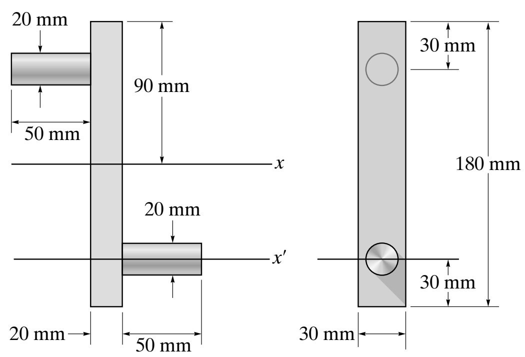
$$m_p = 7.85\big(10^3\big)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$

$$+\left[\frac{1}{12}(0.8478)\big((0.03)^2 + (0.180)^2\big)\right]$$

$$= 0.00325\ \text{kg}\cdot\text{m}^2 = 3.25\ \text{g}\cdot\text{m}^2$$ **Ans.**

***17–20.** Determine the moment of inertia of the overhung crank about the x' axis. The material is steel for which the density is $\rho = 7.85\ \text{Mg/m}^3$.



$$m_c = 7.85\big(10^3\big)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

$$m_p = 7.85\big(10^3\big)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

$$+\left[\frac{1}{12}(0.8478)\big((0.03)^2 + (0.180)^2\big) + (0.8478)(0.06)^2\right]$$

$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2$$ **Ans.**

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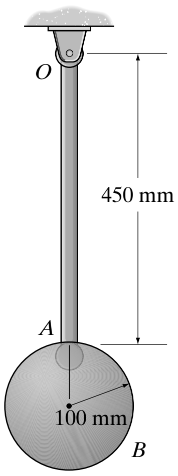
•17–21. Determine the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through point O. The slender rod has a mass of 10 kg and the sphere has a mass of 15 kg.



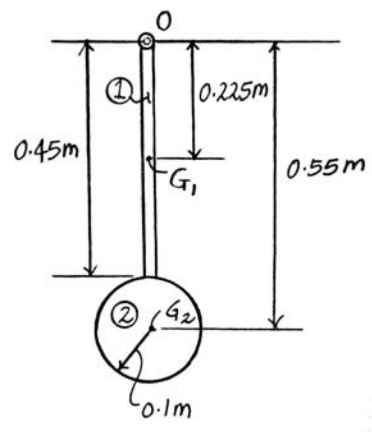
Composite Parts: The pendulum can be subdivided into two segments as shown in Fig. *a*. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Moment of Inertia: The moment of inertia of the slender rod segment (1) and the sphere segment (2) about the axis passing through their center of mass can be computed from $(I_G)_1 = \frac{1}{12}ml^2$ and $(I_G)_2 = \frac{2}{5}mr^2$. The mass moment of inertia of each segment about an axis passing through point O can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{12}(10)(0.45^2) + 10(0.225^2)\right] + \left[\frac{2}{5}(15)(0.1^2) + 15(0.55^2)\right]$$

$$= 5.27 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$



(a)

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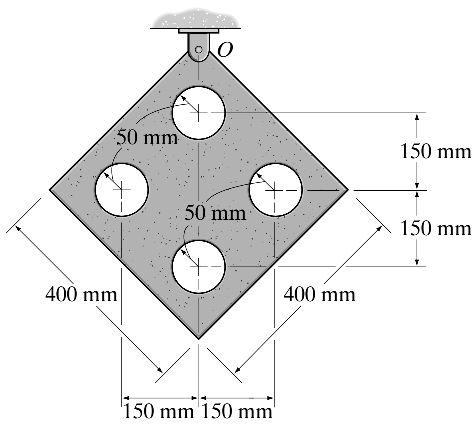
17–22. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of 20 kg/m^2.



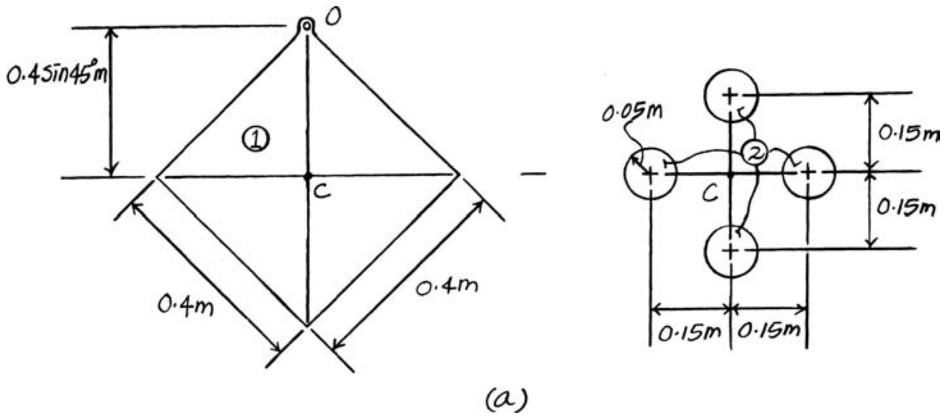
Composite Parts: The plate can be subdivided into the segments shown in Fig. *a*. Here, the four similar holes of which the perpendicular distances measured from their centers of mass to point C are the same and can be grouped as segment (2). This segment should be considered as a negative part.

Mass Moment of Inertia: The mass of segments (1) and (2) are $m_1 = (0.4)(0.4)(20) = 3.2$ kg and $m_2 = \pi(0.05^2)(20) = 0.05\pi$ kg, respectively. The mass moment of inertia of the plate about an axis perpendicular to the page and passing through point C is

$$I_C = \frac{1}{12}(3.2)(0.4^2 + 0.4^2) - 4\left[\frac{1}{2}(0.05\pi)(0.05^2) + 0.05\pi(0.15^2)\right]$$
$$= 0.07041 \text{ kg} \cdot \text{m}^2$$

The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O can be determined using the parallel-axis theorem $I_O = I_C + md^2$, where $m = m_1 - m_2 = 3.2 - 4(0.05\pi) = 2.5717$ kg and $d = 0.4 \sin 45° m$. Thus,

$$I_O = 0.07041 + 2.5717(0.4 \sin 45°)^2 = 0.276 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$



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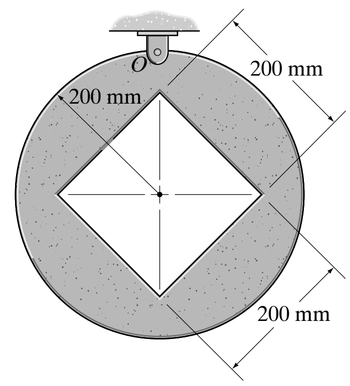
17–23. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



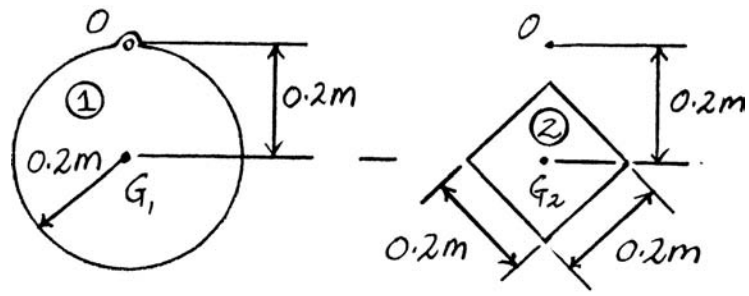
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass Moment of Inertia: The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the plate about an axis perpendicular to the page and passing through point O for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113\ \text{kg}\cdot\text{m}^2$$ **Ans.**



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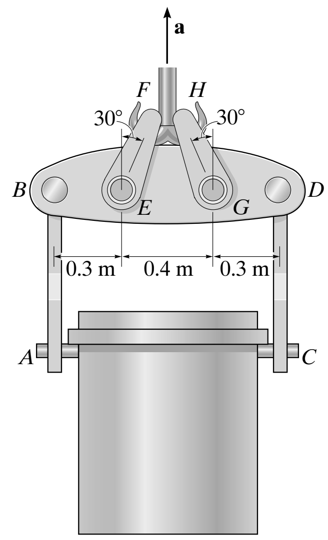
***17–24.** The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the force in each of the links AB, CD, EF, and GH when the system is lifted with an acceleration of $a = 2\ \text{m/s}^2$ for a short period of time.



Canister:

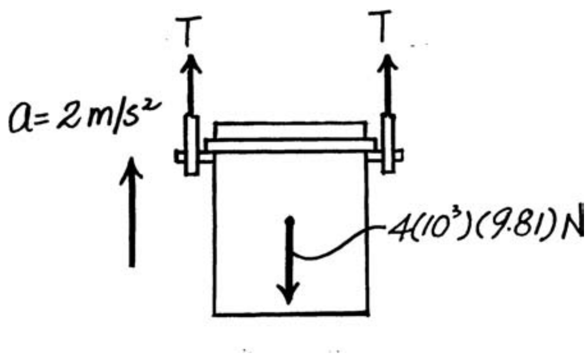
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T - 4\left(10^3\right)(9.81) = 4\left(10^3\right)(2)$$

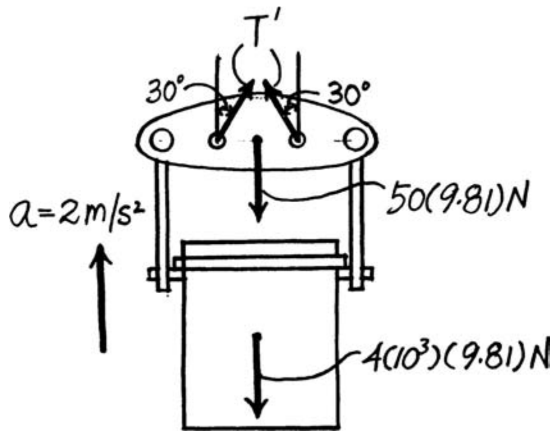
$$T_{AB} = T_{CD} = T = 23.6\ \text{kN} \qquad \textbf{Ans.}$$

System:

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T' \cos 30° - 4050(9.81) = 4050(2)$$

$$T_{EF} = T_{GH} = T' = 27.6\ \text{kN} \qquad \textbf{Ans.}$$





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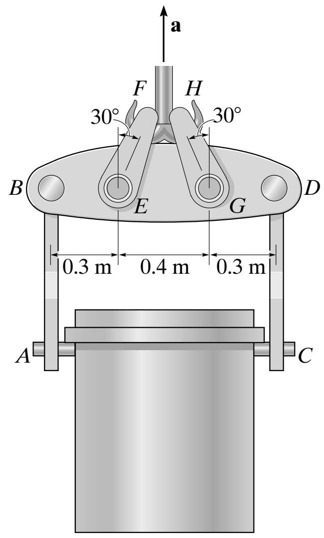
•17–25. The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the largest vertical acceleration **a** of the system so that each of the links AB and CD are not subjected to a force greater than 30 kN and links EF and GH are not subjected to a force greater than 34 kN.



Canister:

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2(30)\left(10^3\right) - 4\left(10^3\right)(9.81) = 4\left(10^3\right)a$

$a = 5.19 \text{ m/s}^2$

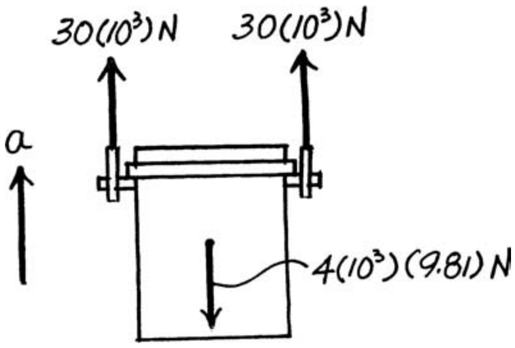
System:

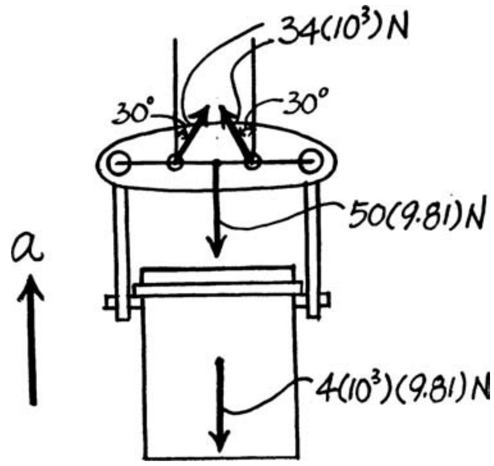
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2\left[34\left(10^3\right)\cos 30°\right] - 4050(9.81) = 4050a$

$a = 4.73 \text{ m/s}^2$

Thus,

$a_{\max} = 4.73 \text{ m/s}^2$ **Ans.**





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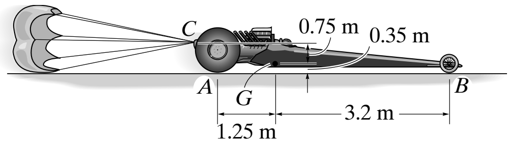
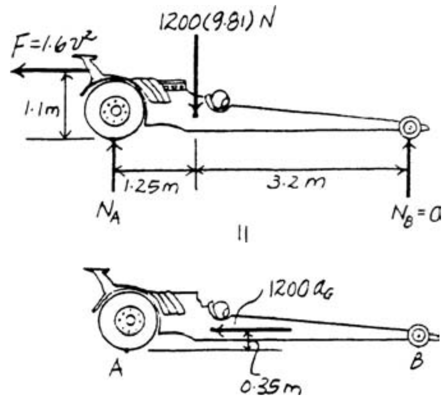
17–26. The dragster has a mass of 1200 kg and a center of mass at G. If a braking parachute is attached at C and provides a horizontal braking force of $F = (1.6v^2)$ N, where v is in meters per second, determine the critical speed the dragster can have upon releasing the parachute, such that the wheels at B are on the verge of leaving the ground; i.e., the normal reaction at B is zero. If such a condition occurs, determine the dragster's initial deceleration. Neglect the mass of the wheels and assume the engine is disengaged so that the wheels are free to roll.





If the front wheels are on the verge of lifting off the ground, then $N_B = 0$.

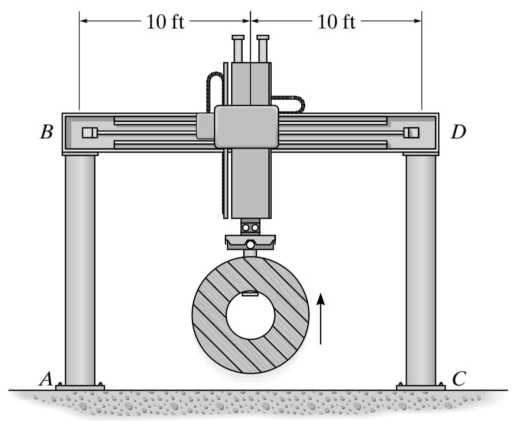
$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 1.6\,v^2\,(1.1) - 1200(9.81)(1.25) = 1200a_G(0.35) \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 1.6v^2 = 1200a_G \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

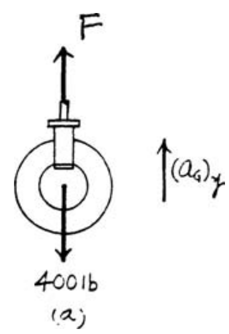
$$a_G = 16.35 \text{ m/s}^2 \qquad v = 111 \text{ m/s} \qquad \textbf{Ans.}$$

17–27. When the lifting mechanism is operating, the 400-lb load is given an upward acceleration of 5 ft/s^2. Determine the compressive force the load creates in each of the columns, AB and CD. What is the compressive force in each of these columns if the load is moving upward at a constant velocity of 3 ft/s? Assume the columns only support an axial load.



Equations of Motion: Applying Eq. 17–12 to FBD(a), we have

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F - 400 = \left(\frac{400}{32.2}\right)(a_G)_y \qquad \textbf{(1)}$$



Equation of Equilibrium: Due to symmetry $F_{CD} = F_{AB}$. From FBD(b).

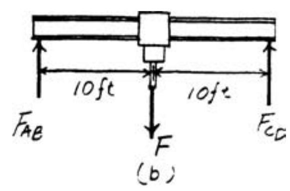
$$+\uparrow \Sigma F_y = 0; \qquad 2F_{AB} - F = 0 \qquad \textbf{(2)}$$

If $(a_G)_y = 5$ ft/s^2, from Eq. (1), $F = 462.11$ lb. Substitute into Eq. (2) yields

$$F_{AB} = F_{CD} = 231 \text{ lb} \qquad \textbf{Ans.}$$

If the load travels with a constant speed, $(a_G)_y = 0$. From Eq. (1), $F = 400$ lb. Substitute into Eq. (2) yields

$$F_{AB} = F_{CD} = 200 \text{ lb} \qquad \textbf{Ans.}$$



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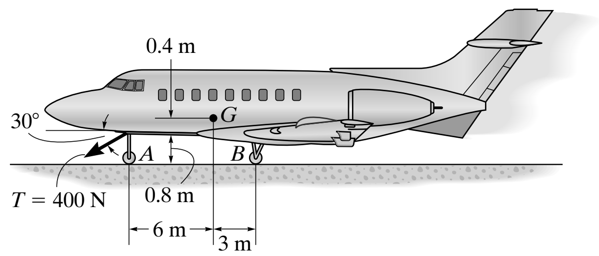
***17–28.** The jet aircraft has a mass of 22 Mg and a center of mass at G. If a towing cable is attached to the upper portion of the nose wheel and exerts a force of $T = 400$ N as shown, determine the acceleration of the plane and the normal reactions on the nose wheel and each of the two wing wheels located at B. Neglect the lifting force of the wings and the mass of the wheels.



$$\leftarrow \Sigma F_x = m(a_G)_x; \qquad 400 \cos 30° = 22\left(10^3\right) a_G$$

$$a_G = 0.01575 \text{ m/s}^2 = 0.0157 \text{ m/s}^2 \qquad \textbf{Ans.}$$

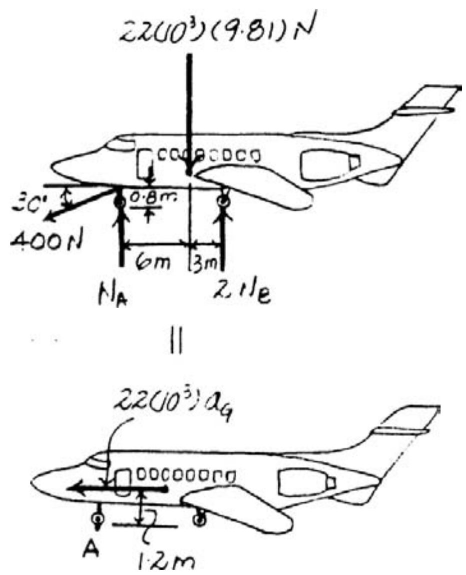
$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 400 \cos 30° \,(0.8) + 2N_B\,(9) - 22\left(10^3\right)(9.81)(6)$$

$$= 22\left(10^3\right)(0.01575)(1.2)$$

$$N_B = 71\,947.70 \text{ N} = 71.9 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + 2(71\,947.70) - 22\left(10^3\right)(9.81) - 400 \sin 30° = 0$$

$$N_A = 72\,124.60 \text{ N} = 72.1 \text{ kN} \qquad \textbf{Ans.}$$



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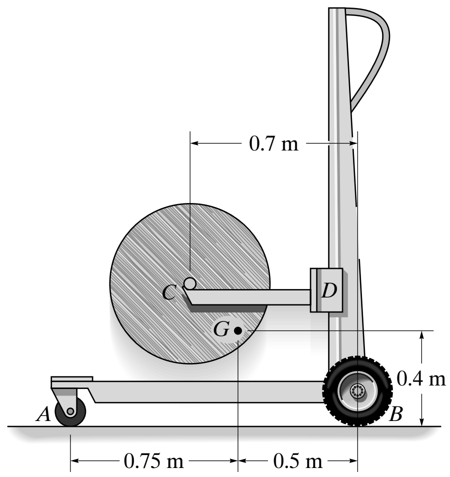
•17–29. The lift truck has a mass of 70 kg and mass center at G. If it lifts the 120-kg spool with an acceleration of $3\ \text{m/s}^2$, determine the reactions on each of the four wheels. The loading is symmetric. Neglect the mass of the movable arm CD.



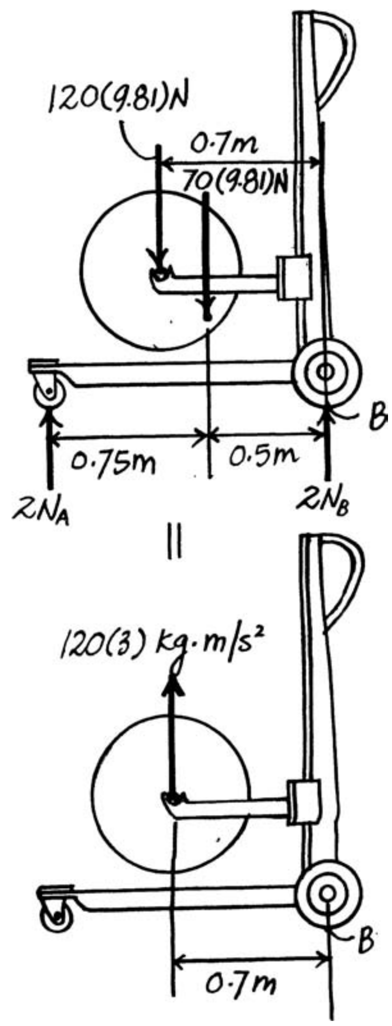
$$\zeta + \Sigma M_B = \Sigma (M_k)_B; \qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2N_A(1.25)$$

$$= -120(3)(0.7)$$

$$N_A = 567.76\ \text{N} = 568\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2(567.76) + 2N_B - 120(9.81) - 70(9.81) = 120(3)$$

$$N_B = 544\ \text{N} \qquad \textbf{Ans.}$$



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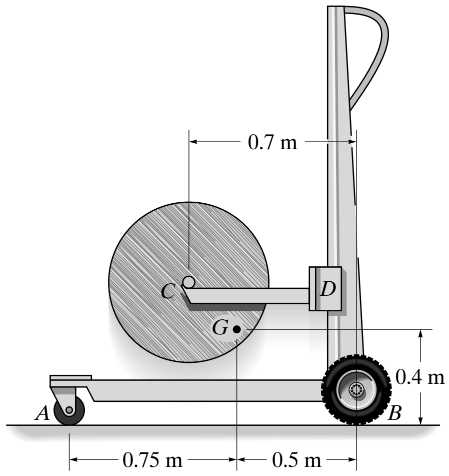
17–30. The lift truck has a mass of 70 kg and mass center at G. Determine the largest upward acceleration of the 120-kg spool so that no reaction on the wheels exceeds 600 N.



Assume $N_A = 600$ N.

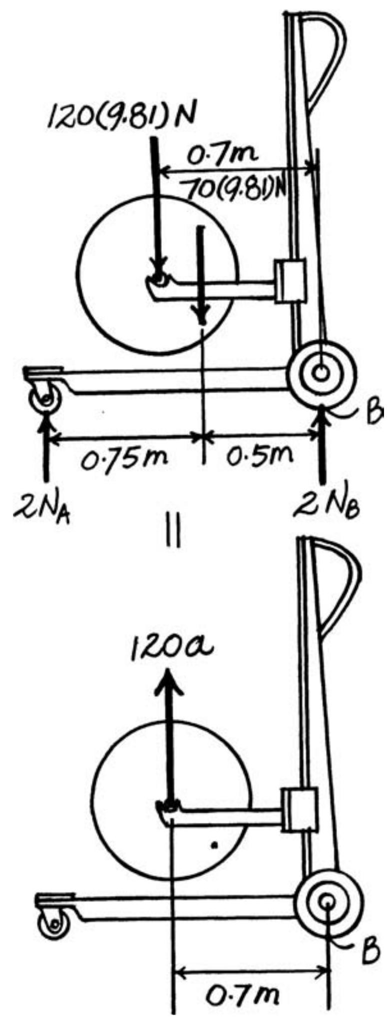
$\zeta + \Sigma M_B = \Sigma(M_k)_B; \qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2(600)(1.25) = -120a(0.7)$

$a = 3.960 \text{ m/s}^2$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2(600) + 2N_B - 120(9.81) - 70(9.81) = 120(3.960)$

$N_B = 570 \text{ N} < 600 \text{ N}$ **OK**

Thus $a = 3.96 \text{ m/s}^2$ **Ans.**



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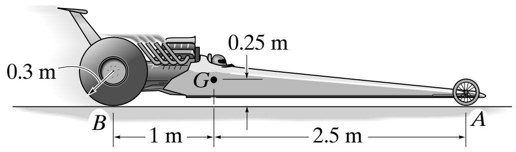
17–31. The dragster has a mass of 1500 kg and a center of mass at G. If the coefficient of kinetic friction between the rear wheels and the pavement is $\mu_k = 0.6$, determine if it is possible for the driver to lift the front wheels, A, off the ground while the rear drive wheels are slipping. Neglect the mass of the wheels and assume that the front wheels are free to roll.



If the front wheels A lift off the ground, then $N_A = 0$.

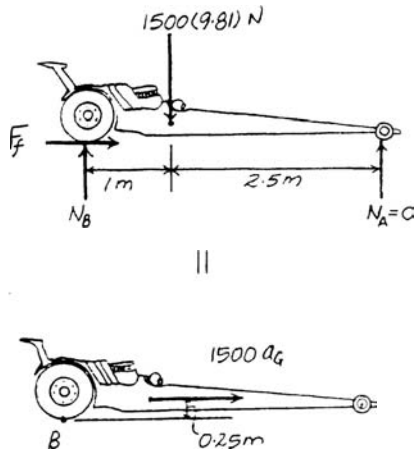
$\zeta + \Sigma M_B = \Sigma(M_k)_B;$ $\quad -1500(9.81)(1) = -1500a_G(0.25)$

$$a_G = 39.24 \text{ m/s}^2$$

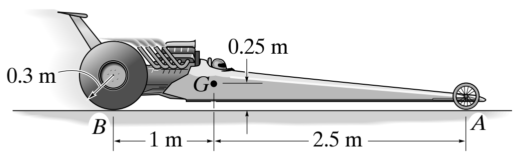
$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\quad F_f = 1500(39.24) = 58860 \text{ N}$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_B - 1500(9.81) = 0 \qquad N_B = 14715 \text{ N}$

Since the required friction $F_f > (F_f)_{max} = \mu_k N_B = 0.6(14715) = 8829$ N,
it is not possible to lift the front wheels off the ground. **Ans.**



***17–32.** The dragster has a mass of 1500 kg and a center of mass at G. If no slipping occurs, determine the frictional force $\mathbf{F}_B$ which must be developed at each of the rear drive wheels B in order to create an acceleration of $a = 6 \text{ m/s}^2$. What are the normal reactions of each wheel on the ground? Neglect the mass of the wheels and assume that the front wheels are free to roll.



$\zeta + \Sigma M_B = \Sigma(M_k)_B;$ $\quad 2N_A(3.5) - 1500(9.81)(1) = -1500(6)(0.25)$

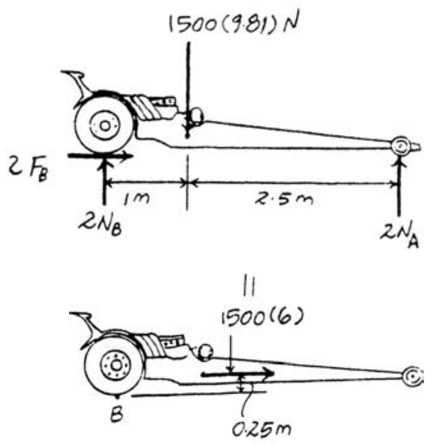
$$N_A = 1780.71 \text{ N} = 1.78 \text{ kN} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 2N_B + 2(1780.71) - 1500(9.81) = 0$

$$N_B = 5576.79 \text{ N} = 5.58 \text{ kN} \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\quad 2F_B = 1500(6)$

$$F_B = 4500 \text{ N} = 4.50 \text{ kN} \qquad \textbf{Ans.}$$



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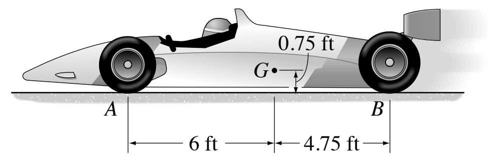
•17–33. At the start of a race, the rear drive wheels B of the 1550-lb car slip on the track. Determine the car's acceleration and the normal reaction the track exerts on the front pair of wheels A and rear pair of wheels B. The coefficient of kinetic friction is $\mu_k = 0.7$, and the mass center of the car is at G. The front wheels are free to roll. Neglect the mass of all the wheels.



Equations of Motion: Since the rear wheels B are required to slip, the frictional force developed is $F_B = \mu_s N_B = 0.7N_B$.

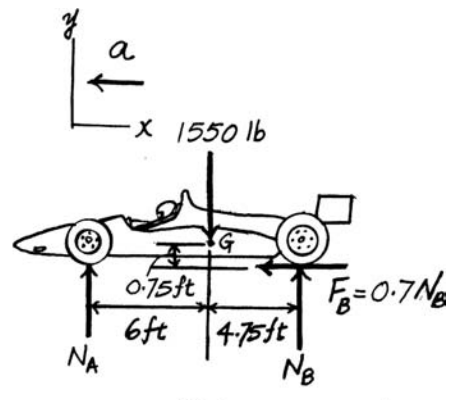
$$\xleftarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.7N_B = \frac{1550}{32.2}a \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(4.75) - 0.7N_B(0.75) - N_A(6) = 0 \qquad \textbf{(3)}$$

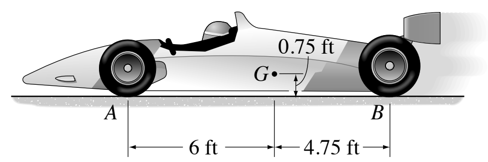
Solving Eqs. (1), (2), and (3) yields

$N_A = 640.46\text{ lb} = 640\text{ lb}$ $\qquad N_B = 909.54\text{ lb} = 910\text{ lb}$ $\qquad a = 13.2\text{ ft/s}^2$ **Ans.**



(a)

17–34. Determine the maximum acceleration that can be achieved by the car without having the front wheels A leave the track or the rear drive wheels B slip on the track. The coefficient of static friction is $\mu_s = 0.9$. The car's mass center is at G, and the front wheels are free to roll. Neglect the mass of all the wheels.



Equations of Motion:

$$\xleftarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_B = \frac{1550}{32.2}a \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(4.75) - F_B(0.75) - N_A(6) = 0 \qquad \textbf{(3)}$$

If we assume that the front wheels are about to leave the track, $N_A = 0$. Substituting this value into Eqs. (2) and (3) and solving Eqs. (1), (2), (3),

$N_B = 1550\text{ lb}$ $\qquad F_B = 9816.67\text{ lb}$ $\qquad a = 203.93\text{ ft/s}^2$

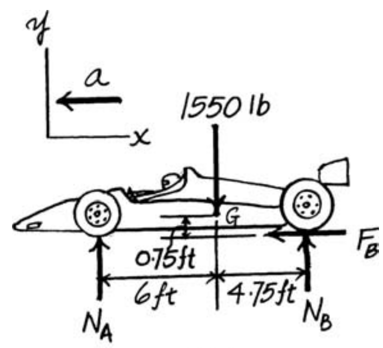
Since $F_B > (F_B)_{max} = \mu_s N_B = 0.9(1550)\text{ lb} = 1395\text{ lb}$, the rear wheels will slip. Thus, the solution must be reworked so that the rear wheels are about to slip.

$$F_B = \mu_s N_B = 0.9N_B \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), (3), and (4) yields

$N_A = 626.92\text{ lb}$ $\qquad N_B = 923.08\text{ lb}$

$a = 17.26\text{ ft/s}^2 = 17.3\text{ ft/s}^2$ **Ans.**



(a)

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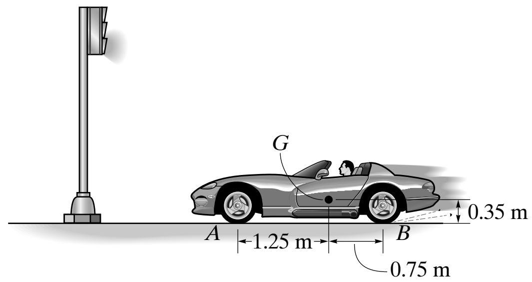
17–35. The sports car has a mass of 1.5 Mg and a center of mass at G. Determine the shortest time it takes for it to reach a speed of 80 km/h, starting from rest, if the engine only drives the rear wheels, whereas the front wheels are free rolling. The coefficient of static friction between the wheels and the road is $\mu_s = 0.2$. Neglect the mass of the wheels for the calculation. If driving power could be supplied to all four wheels, what would be the shortest time for the car to reach a speed of 80 km/h?



$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x\,; \qquad 0.2N_A + 0.2N_B = 1500a_G \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y\,; \qquad N_A + N_B - 1500(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad -N_A(1.25) + N_B(0.75) - (0.2N_A + 0.2N_B)(0.35) = 0 \qquad \textbf{(3)}$$

For Rear-Wheel Drive:

Set the friction force $0.2N_A = 0$ in Eqs. (1) and (3).

Solving yields:

$N_A = 5.18 \text{ kN} > 0$ **(OK)** $N_B = 9.53 \text{ kN}$ $a_G = 1.271 \text{m/s}^2$

Since $v = 80 \text{ km/h} = 22.22 \text{ m/s}$, then

$$\left(\overset{+}{\leftarrow}\right) \qquad v = v_0 + a_G t$$

$$22.22 = 0 + 1.271t$$

$$t = 17.5 \text{ s} \qquad \textbf{Ans.}$$

For 4-Wheel Drive:

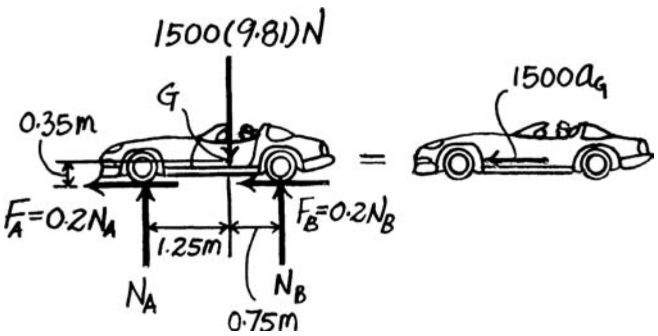
$N_A = 5.00 \text{ kN} > 0$ **(OK)** $N_B = 9.71 \text{ kN}$ $a_G = 1.962 \text{m/s}^2$

Since $v_2 = 80 \text{ km/h} = 22.22 \text{ m/s}$, then

$$v_2 = v_1 + a_G t$$

$$22.22 = 0 + 1.962t$$

$$t = 11.3 \text{ s} \qquad \textbf{Ans.}$$



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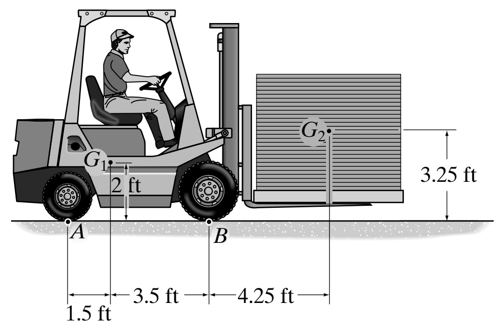
***17–36.** The forklift travels forward with a constant speed of 9 ft/s. Determine the shortest stopping distance without causing any of the wheels to leave the ground. The forklift has a weight of 2000 lb with center of gravity at G_1, and the load weighs 900 lb with center of gravity at G_2. Neglect the weight of the wheels.



Equations of Motion: Since it is required that the rear wheels are about to leave the ground, $N_A = 0$. Applying the moment equation of motion of about point B,

$$\zeta + \Sigma M_B = (M_k)_B; \qquad 2000(3.5) - 900(4.25) = \left(\frac{2000}{32.2}a\right)(2) + \left(\frac{900}{32.2}a\right)(3.25)$$

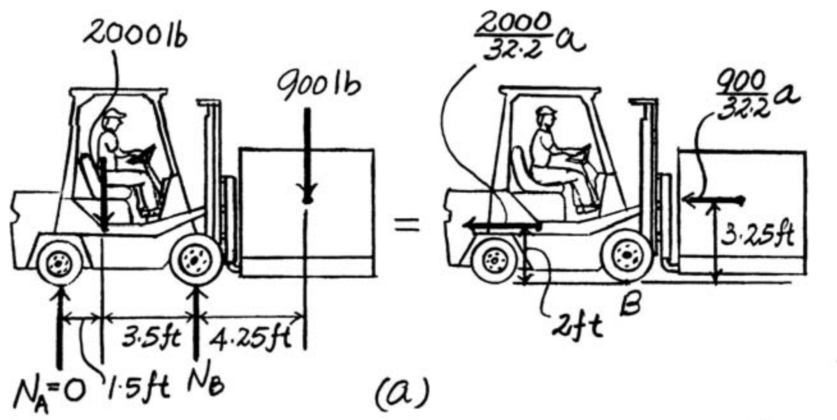
$$a = 14.76 \text{ ft/s}^2 \leftarrow$$

Kinematics: Since the acceleration of the forklift is constant,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 9^2 + 2(-14.76)(s - 0)$$

$$s = 2.743 \text{ ft} = 2.74 \text{ ft} \qquad \textbf{Ans.}$$



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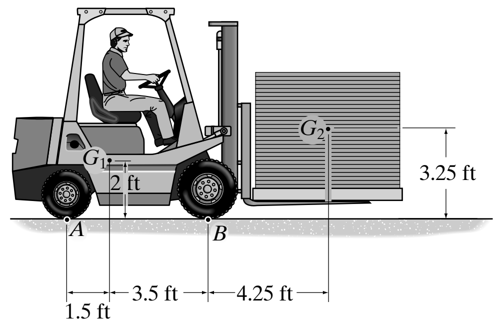
•17–37. If the forklift's rear wheels supply a combined traction force of $F_A = 300$ lb, determine its acceleration and the normal reactions on the pairs of rear wheels and front wheels. The forklift has a weight of 2000 lb, with center of gravity at G_1, and the load weighs 900 lb, with center of gravity at G_2. The front wheels are free to roll. Neglect the weight of the wheels.



Equations of Motion: The acceleration of the forklift can be obtained directly by writing the force equation of motion along the x axis.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 300 = \frac{2000}{32.2}a + \frac{900}{32.2}a$$

$$a = 3.331 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Using this result and writing the moment equation of motion about point A,

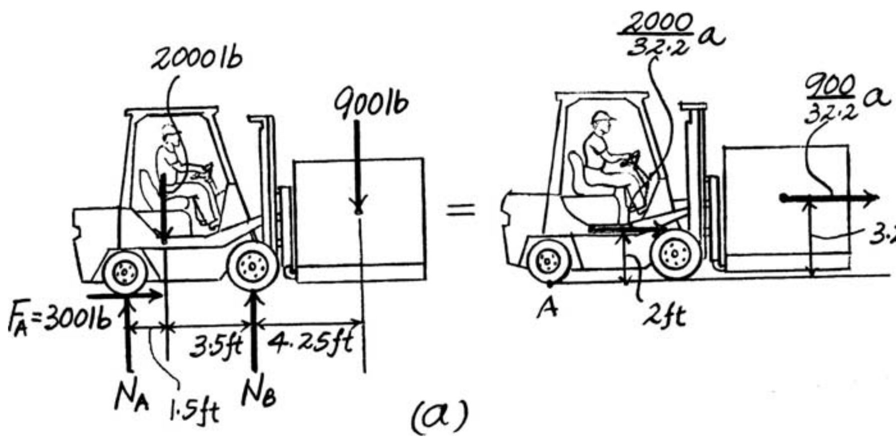
$$\circlearrowleft + \Sigma M_A = (\mathcal{M}_k)_A; \qquad N_B(5) - 2000(1.5) - 900(9.25) = -\left(\frac{2000}{32.2}\right)(3.331)(2) - \left(\frac{900}{32.2}\right)(3.331)(3.25)$$

$$N_B = 2121.72 \text{ lb} = 2122 \text{ lb} \qquad \textbf{Ans.}$$

Finally, writing the force equation of motion along the y axis and using this result,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 2121.72 - 2000 - 900 = 0$$

$$N_A = 778.28 \text{ lb} = 778 \text{ lb} \qquad \textbf{Ans.}$$



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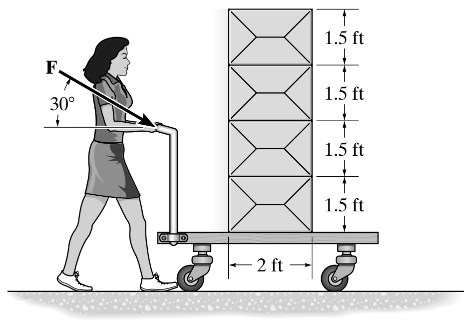
17–38. Each uniform box on the stack of four boxes has a weight of 8 lb. The stack is being transported on the dolly, which has a weight of 30 lb. Determine the maximum force **F** which the woman can exert on the handle in the direction shown so that no box on the stack will tip or slip. The coefficient of the static friction at all points of contact is $\mu_s = 0.5$. The dolly wheels are free to roll. Neglect their mass.



Assume that the boxes up, then $x = 1$ ft. Applying Eq. 17–12 to FBD(a). we have

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad -32(1) = -\left[\left(\frac{32}{32.2}\right)a_G\right](3) \qquad a_G = 10.73\ \text{ft/s}^2$$

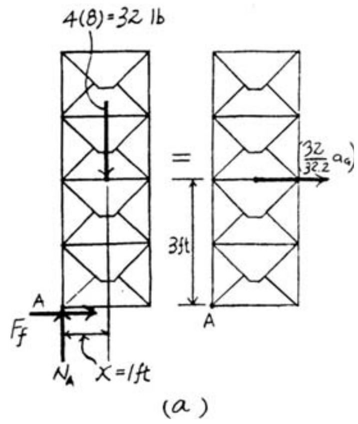
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 32 = 0 \qquad N_A = 32.0\ \text{lb}$$

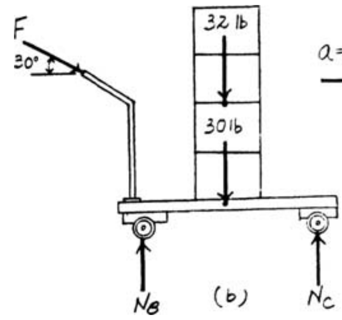
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_f = \left(\frac{32}{32.2}\right)(10.73) = 10.67\ \text{lb}$$

Since $F_f < (F_f)_{\max} = \mu_s N_A = 0.5(32.0) = 16.0$ lb. slipping will not occur. Hence, the boxes and the dolly moves as a unit. From FBD(b),

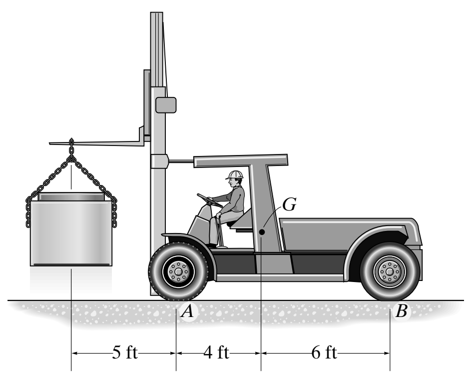
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F\cos 30° = \left(\frac{32 + 30}{32.2}\right)(10.73)$$

$$F = 23.9\ \text{lb} \qquad \textbf{Ans.}$$





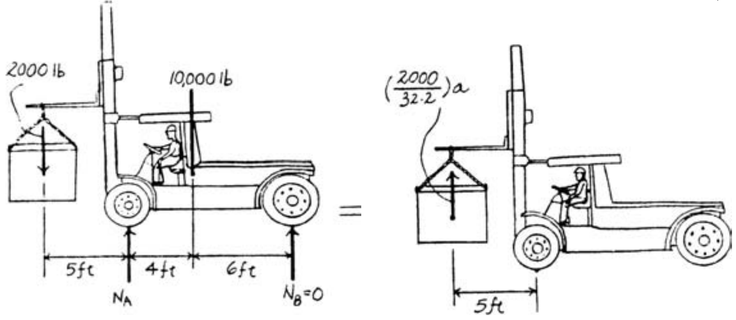
17–39. The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the maximum vertical acceleration it can give to the pipe so that it does not tip forward on its front wheels.



It is required that $N_B = 0$.

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad 2000(5) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)a\right](5)$$

$$a = 96.6\ \text{ft/s}^2 \qquad \textbf{Ans.}$$



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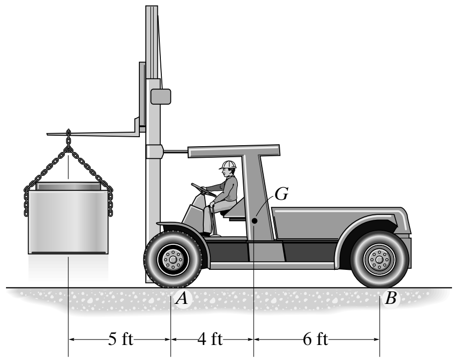
***17–40.** The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the normal reactions on each of its four wheels if the pipe is given an upward acceleration of 4 ft/s^2.

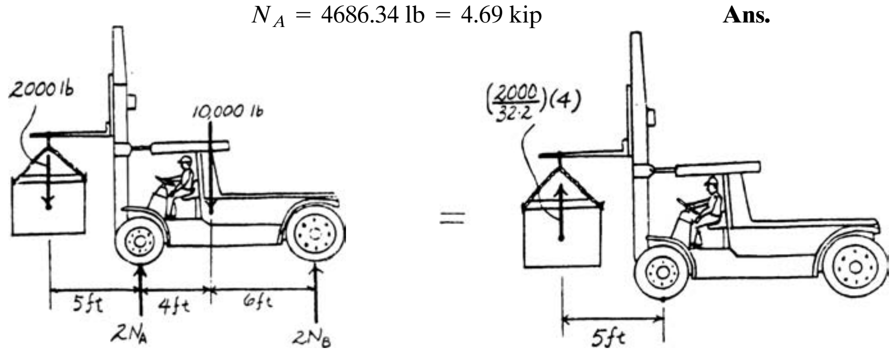


$$\zeta+\Sigma M_A = \Sigma(M_k)_A; \qquad 2000(5) + 2N_B(10) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)(4)\right](5)$$

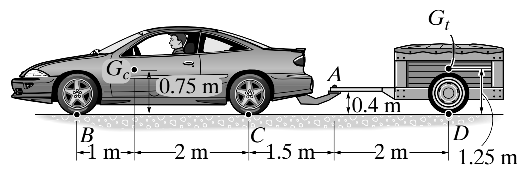
$$N_B = 1437.89 \text{ lb} = 1.44 \text{ kip} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad 2N_A + 2(1437.89) - 2000 - 10000 = \left(\frac{2000}{32.2}\right)(4)$$

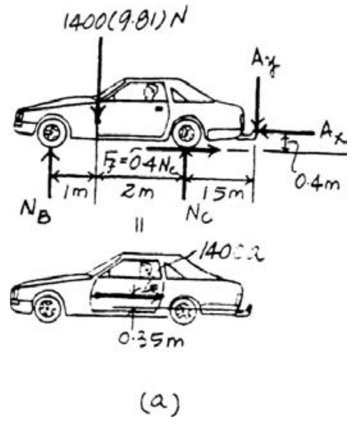
$$N_A = 4686.34 \text{ lb} = 4.69 \text{ kip} \qquad \textbf{Ans.}$$



•17–41. The car, having a mass of 1.40 Mg and mass center at G_c, pulls a loaded trailer having a mass of 0.8 Mg and mass center at G_t. Determine the normal reactions on both the car's front and rear wheels and the trailer's wheels if the driver applies the car's rear brakes C and causes the car to skid. Take $\mu_C = 0.4$ and assume the hitch at A is a pin or ball-and-socket joint. The wheels at B and D are free to roll. Neglect their mass and the mass of the driver.



Equations of Motion: Since the car skids, then $F_f = \mu_C N_C = 0.4N_C$. Applying Eq. 17–12 to FBD(a), we have

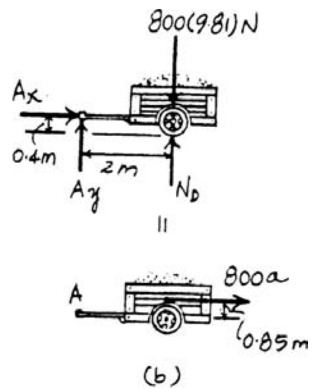


$$\zeta+\Sigma M_A = \Sigma(M_k)_A; \qquad 1400(9.81)(3.5) + 0.4N_C(0.4) - N_B(4.5) - N_C(1.5) = -1400a(0.35) \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad N_B + N_C - 1400(9.81) - A_y = 0 \qquad \textbf{(2)}$$

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 0.4N_C - A_x = 1400a \qquad \textbf{(3)}$$

From FBD(b),



$$\zeta+\Sigma M_A = \Sigma(M_k)_A; \qquad N_D(2) - 800(9.81)(2) = -800a(0.85) \qquad \textbf{(4)}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad N_D + A_y - 800(9.81) = 0 \qquad \textbf{(5)}$$

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad A_x = 800a \qquad \textbf{(6)}$$

Solving Eqs. (1), (2), (3), (4), (5), and (6) yields

$N_B = 9396.95 \text{ N} = 9.40 \text{ kN}$ $\qquad N_C = 4622.83 \text{ N} = 4.62 \text{ kN}$ **Ans.**

$N_D = 7562.23 \text{ N} = 7.56 \text{ kN}$ **Ans.**

$a = 0.8405 \text{ m/s}^2 \qquad A_x = 672.41 \text{ N} \qquad A_y = 285.77 \text{ N}$

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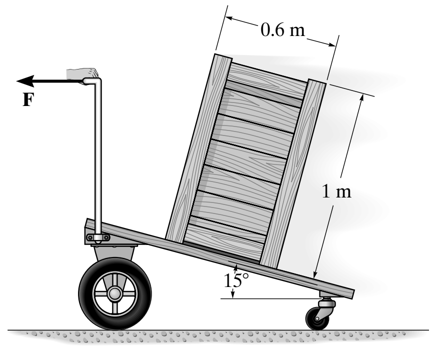
17–42. The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



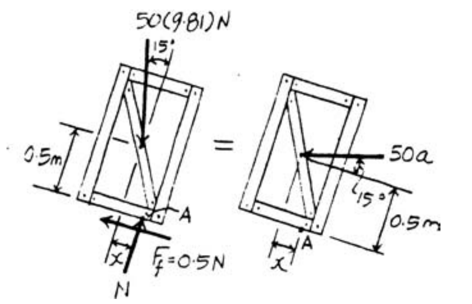
Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$$\zeta + \Sigma M_A = \Sigma (M_k)_A; \quad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5)$$

$$= 50a\cos 15°(0.5) + 50a\sin 15°(x) \quad \textbf{(1)}$$

$$+\nearrow \Sigma F_{y'} = m(a_G)_{y'}; \quad N - 50(9.81)\cos 15° = -50a\sin 15° \quad \textbf{(2)}$$

$$\searrow + \Sigma F_{x'} = m(a_G)_{x'}; \quad 50(9.81)\sin 15° - 0.5N = -50a\cos 15° \quad \textbf{(3)}$$



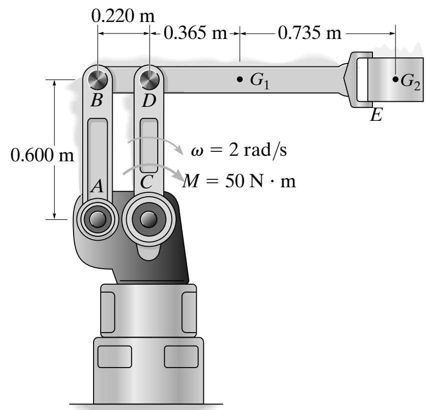
Solving Eqs. (1), (2), and (3) yields

$$N = 447.81 \text{ N} \qquad x = 0.250 \text{ m}$$

$$a = 2.01 \text{ m/s}^2 \quad \textbf{Ans.}$$

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips**. **Ans.**

17–43. Arm BDE of the industrial robot is activated by applying the torque of $M = 50$ N·m to link CD. Determine the reactions at pins B and D when the links are in the position shown and have an angular velocity of 2 rad/s. Arm BDE has a mass of 10 kg with center of mass at G_1. The container held in its grip at E has a mass of 12 kg with center of mass at G_2. Neglect the mass of links AB and CD.



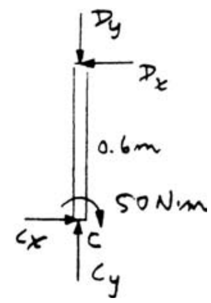
Curvilinear translation:

$$(a_D)_n = (a_G)_n = (2)^2(0.6) = 2.4 \text{ m/s}^2$$

Member DC:

$$\zeta + \Sigma M_C = 0; \quad -D_x(0.6) + 50 = 0$$

$$D_x = 83.33 \text{ N} = 83.3 \text{ N} \quad \textbf{Ans.}$$



Member BDE:

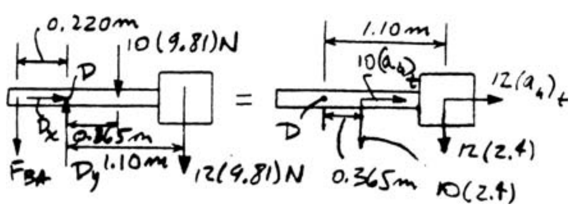
$$\zeta + \Sigma M_D = \Sigma (M_k)_D; \quad -F_{BA}(0.220) + 10(9.81)(0.365) + 12(9.81)(1.10)$$

$$= 10(2.4)(0.365) + 12(2.4)(1.10)$$

$$F_{BA} = 567.54 \text{ N} = 568 \text{ N} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad -567.54 + D_y - 10(9.81) - 12(9.81) = -10(2.4) - 12(2.4)$$

$$D_y = 731 \text{ N} \quad \textbf{Ans.}$$



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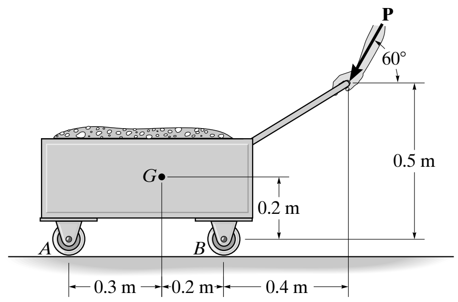
***17–44.** The handcart has a mass of 200 kg and center of mass at G. Determine the normal reactions at each of the two wheels at A and at B if a force of $P = 50$ N is applied to the handle. Neglect the mass of the wheels.

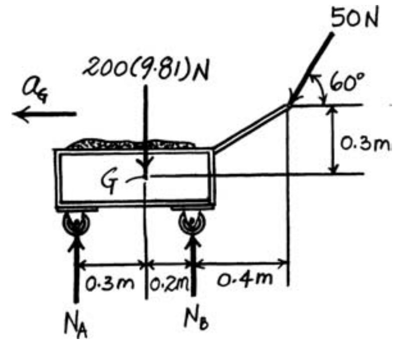


$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad 50 \cos 60° = 200 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + N_B - 200(9.81) - 50 \sin 60° = 0$

$\circlearrowleft + \Sigma M_G = 0;\qquad -N_A(0.3) + N_B(0.2) + 50 \cos 60°(0.3) - 50 \sin 60°(0.6) = 0$

$a_G = 0.125 \text{ m/s}^2 \qquad N_A = 765.2 \text{ N} \qquad N_B = 1240 \text{ N}$

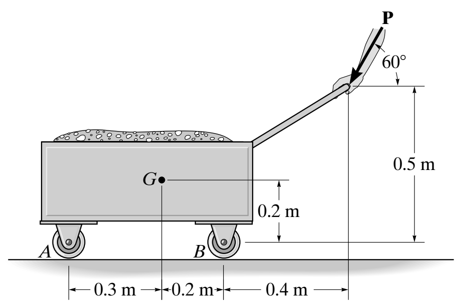


At each wheel,

$$N_A' = \frac{N_A}{2} = 383 \text{ N} \qquad \textbf{Ans.}$$

$$N_B' = \frac{N_B}{2} = 620 \text{ N} \qquad \textbf{Ans.}$$

•17–45. The handcart has a mass of 200 kg and center of mass at G. Determine the largest magnitude of force **P** that can be applied to the handle so that the wheels at A or B continue to maintain contact with the ground. Neglect the mass of the wheels.



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad P \cos 60° = 200 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + N_B - 200(9.81) - P \sin 60° = 0$

$\circlearrowleft + \Sigma M_G = 0;\qquad -N_A(0.3) + N_B(0.2) + P \cos 60°(0.3) - P \sin 60°(0.6) = 0$

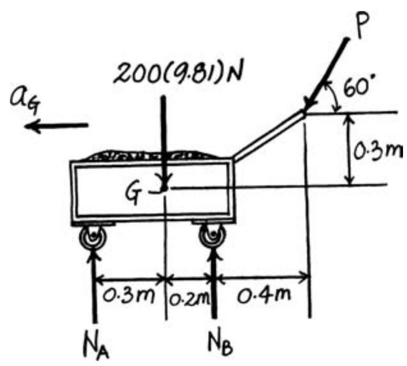
For P_{max}, require

$$N_A = 0$$

$$P = 1998 \text{ N} = 2.00 \text{ kN} \qquad \textbf{Ans.}$$

$$N_B = 3692 \text{ N}$$

$$a_G = 4.99 \text{ m/s}^2$$



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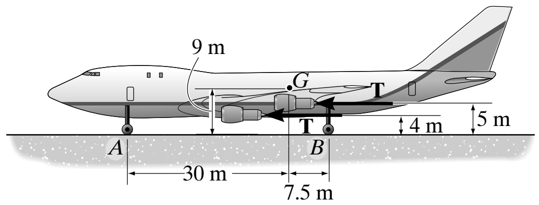
17–46. The jet aircraft is propelled by four engines to increase its speed uniformly from rest to 100 m/s in a distance of 500 m. Determine the thrust **T** developed by each engine and the normal reaction on the nose wheel A. The aircraft's total mass is 150 Mg and the mass center is at point G. Neglect air and rolling resistance and the effect of lift.



Kinematics: The acceleration of the aircraft can be determined from

$$v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$100^2 = 0^2 + 2a(500 - 0)$$

$$a = 10 \text{ m/s}^2$$

Equations of Motion: The thrust **T** can be determined directly by writing the force equation of motion along the x axis.

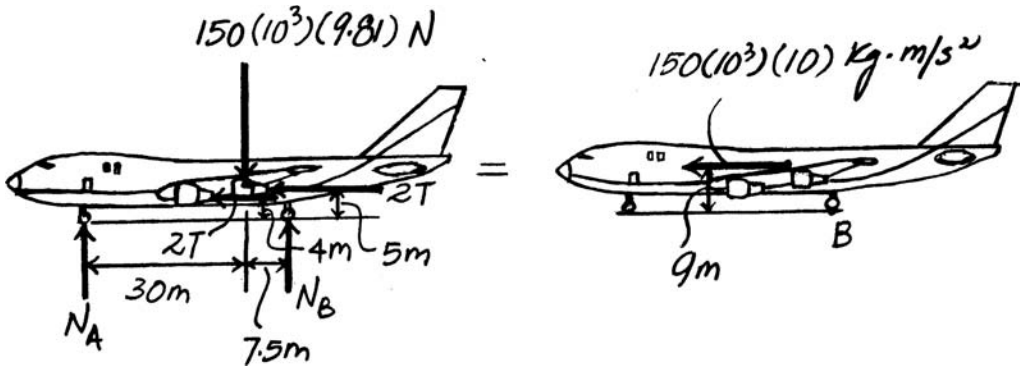
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 4T = 150\left(10^3\right)(10)$$

$$T = 375\left(10^3\right)\text{N} = 375 \text{ kN} \qquad \textbf{Ans.}$$

Writing the moment equation of equilibrium about point B and using the result of **T**,

$$\circlearrowleft + \Sigma M_B = (M_k)_B; \qquad 150\left(10^3\right)(9.81)(7.5) + 2\left[375\left(10^3\right)\right](5) + 2\left[375\left(10^3\right)\right](4) - N_A(37.5) = 150\left(10^3\right)(10)(9)$$

$$N_A = 114.3\left(10^3\right)\text{N} = 114 \text{ kN} \qquad \textbf{Ans.}$$



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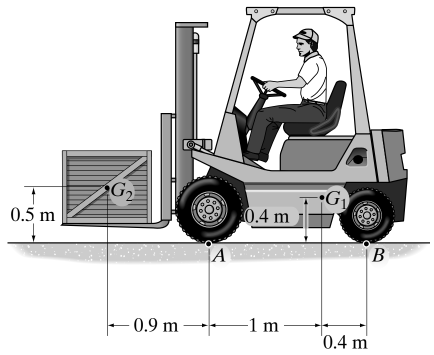
17–47. The 1-Mg forklift is used to raise the 750-kg crate with a constant acceleration of $2\,\text{m/s}^2$. Determine the reaction exerted by the ground on the pairs of wheels at A and at B. The centers of mass for the forklift and the crate are located at G_1 and G_2, respectively.



Equations of Motion: $\mathbf{N}_B$ can be obtained directly by writing the moment equation of motion about point A.

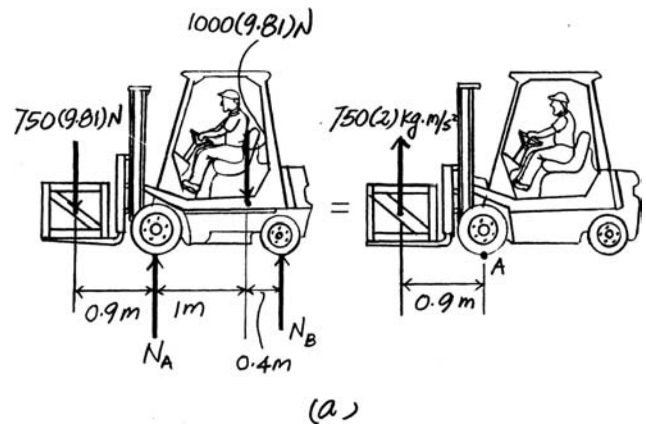
$$\circlearrowleft + \Sigma M_A = (\mathcal{M}_k)_A; \quad N_B\,(1.4) + 750(9.81)(0.9) - 1000(9.81)(1) = -750(2)(0.9)$$

$$N_B = 1313.03\text{ N} = 1.31\text{ kN} \qquad \textbf{Ans.}$$

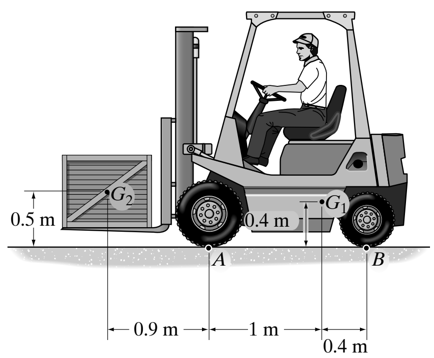
Using this result to write the force equation of motion along the y axis,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 1313.03 - 750(9.81) - 1000(9.81) = 750(2)$$

$$N_A = 17354.46\text{ N} = 17.4\text{ kN} \qquad \textbf{Ans.}$$



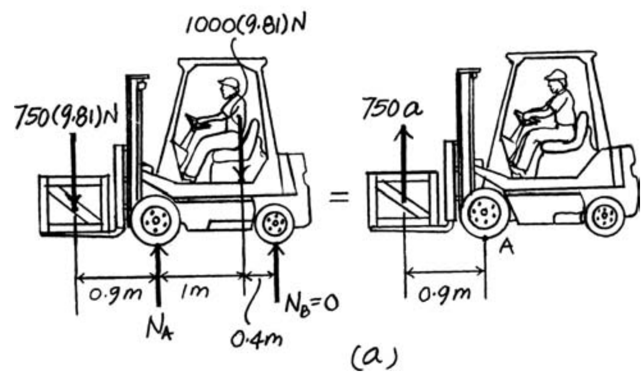
***17–48.** Determine the greatest acceleration with which the 1-Mg forklift can raise the 750-kg crate, without causing the wheels at B to leave the ground. The centers of mass for the forklift and the crate are located at G_1 and G_2, respectively.



Equations of Motion: Since the wheels at B are required to just lose contact with the ground, $N_B = 0$. The direct solution for **a** can be obtained by writing the moment equation of motion about point A.

$$\circlearrowleft + \Sigma M_A = (\mathcal{M}_k)_A; \quad 750(9.81)(0.9) - 1000(9.81)(1) = -750a(0.9)$$

$$a = 4.72\text{ m/s}^2 \qquad \textbf{Ans.}$$



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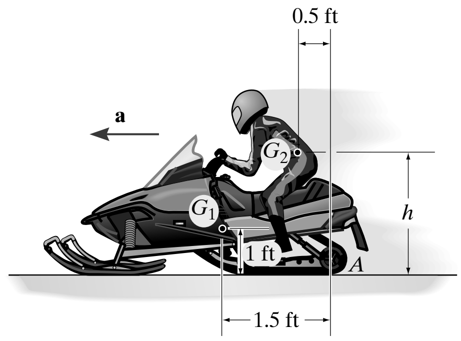
•17–49. The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If the acceleration is $a = 20 \text{ ft/s}^2$, determine the maximum height h of G_2 of the rider so that the snowmobile's front skid does not lift off the ground. Also, what are the traction (horizontal) force and normal reaction under the rear tracks at A?



Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 250(1.5) + 150(0.5) = \frac{150}{32.2}(20)(h_{\max}) + \frac{250}{32.2}(20)(1)$$

$$h_{\max} = 3.163 \text{ ft} = 3.16 \text{ ft} \qquad \textbf{Ans.}$$

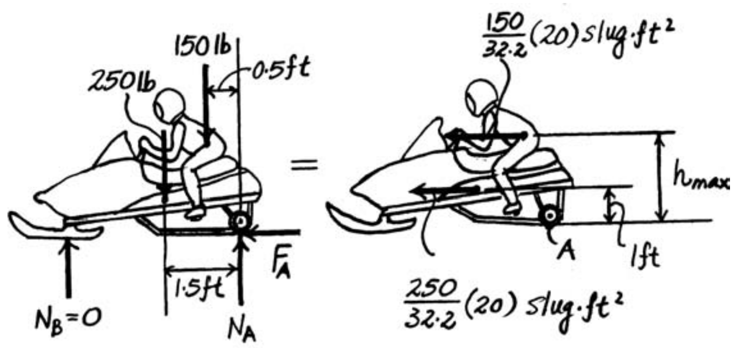
Writing the force equations of motion along the x and y axes,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20) + \frac{250}{32.2}(20)$$

$$F_A = 248.45 \text{ lb} = 248 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 250 - 150 = 0$$

$$N_A = 400 \text{ lb} \qquad \textbf{Ans.}$$



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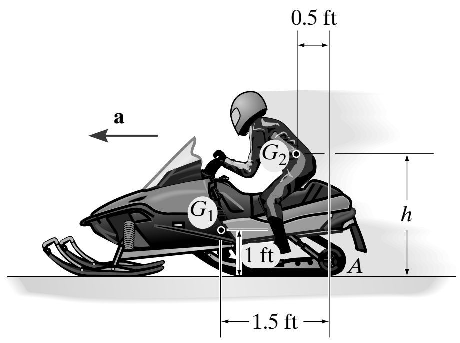
17–50. The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If $h = 3$ ft, determine the snowmobile's maximum permissible acceleration **a** so that its front skid does not lift off the ground. Also, find the traction (horizontal) force and the normal reaction under the rear tracks at A.



Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 250(1.5) + 150(0.5) = \left(\frac{150}{32.2}a_{\text{max}}\right)(3) + \left(\frac{250}{32.2}a_{\text{max}}\right)(1)$$

$$a_{\text{max}} = 20.7 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

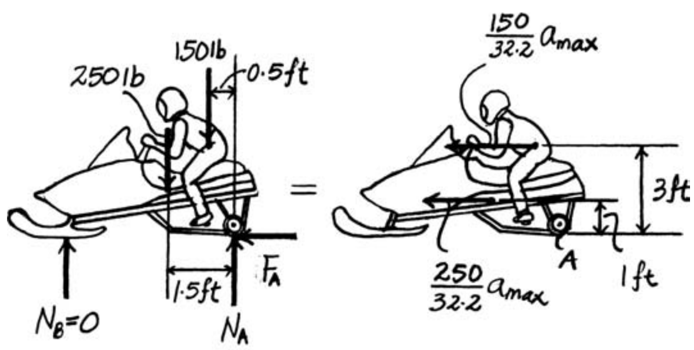
Writing the force equations of motion along the x and y axes and using this result, we have

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20.7) + \frac{250}{32.2}(20.7)$$

$$F_A = 257.14 \text{ lb} = 257 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 150 - 250 = 0$$

$$N_A = 400 \text{ lb} \qquad \textbf{Ans.}$$



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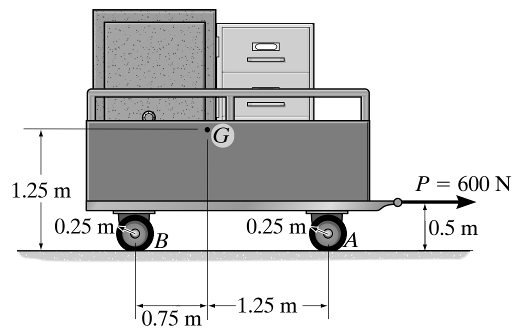
17–51. The trailer with its load has a mass of 150 kg and a center of mass at G. If it is subjected to a horizontal force of $P = 600$ N, determine the trailer's acceleration and the normal force on the pair of wheels at A and at B. The wheels are free to roll and have negligible mass.



Equations of Motion: Writing the force equation of motion along the x axis,

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 600 = 150a \qquad a = 4 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans.}$$

Using this result to write the moment equation about point A,

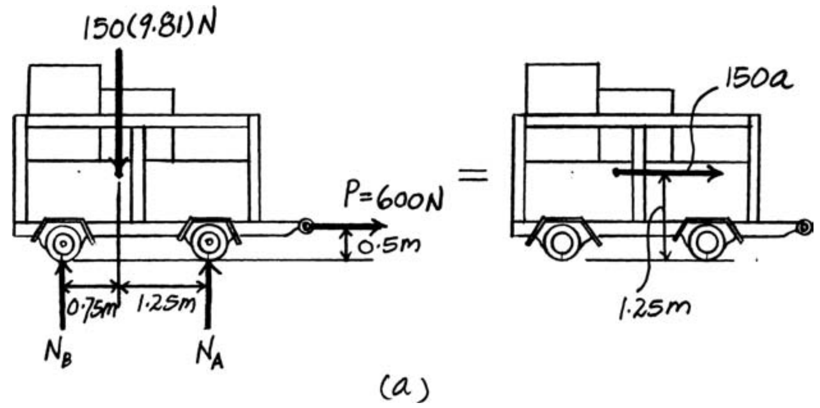
$$\circlearrowleft + \Sigma M_A = (M_k)_A; \quad 150(9.81)(1.25) - 600(0.5) - N_B(2) = -150(4)(1.25)$$

$$N_B = 1144.69 \text{ N} = 1.14 \text{ kN} \qquad \textbf{Ans.}$$

Using this result to write the force equation of motion along the y axis,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 1144.69 - 150(9.81) = 150(0)$$

$$N_A = 326.81 \text{ N} = 327 \text{ N} \qquad \textbf{Ans.}$$



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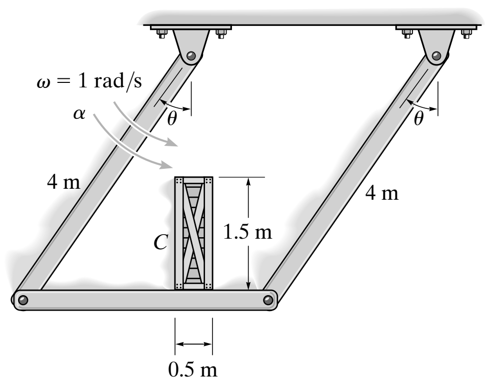
***17–52.** The 50-kg uniform crate rests on the platform for which the coefficient of static friction is $\mu_s = 0.5$. If the supporting links have an angular velocity $\omega = 1$ rad/s, determine the greatest angular acceleration α they can have so that the crate does not slip or tip at the instant $\theta = 30°$.



Curvilinear Translation:

$(a_G)_n = (1)^2(4) = 4 \text{ m/s}^2$

$(a_G)_t = \alpha(4) \text{ m/s}^2$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_C = 50(4)\sin 30° + 50(\alpha)(4)\cos 30°$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_C - 50(9.81) = 50(4)\cos 30° - 50(\alpha)(4)\sin 30°$

$\circlearrowleft + \Sigma M_G = \Sigma(M_k)_G; \qquad N_C(x) - F_C(0.75) = 0$

Assume crate is about to slip. $F_C = 0.5N_C$

Thus,

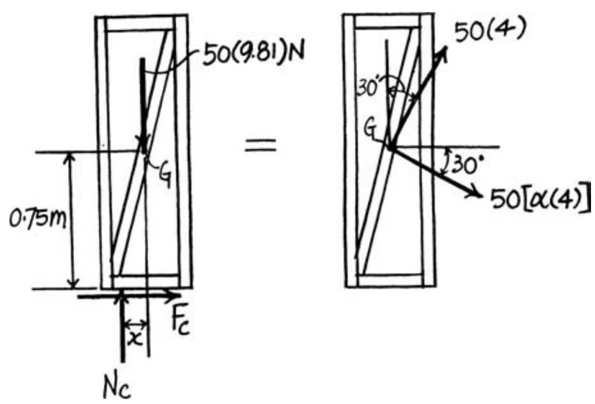
$$x = 0.375 \text{ m} > 0.25 \text{ m}$$

Crate must tip. Set $x = 0.25$ m.

$$N_C = 605 \text{ N} \qquad F_C = 202 \text{ N}$$

$$\alpha = 0.587 \text{ rad/s}^2$$ **Ans.**

Note: $(F_C)_{max} = 0.5(605) = 303 \text{ N} > 202 \text{ N}$ **O.K.**



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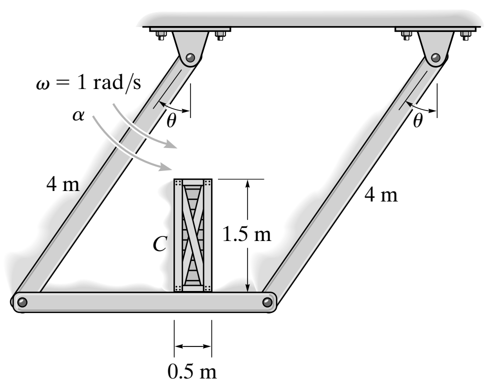
•**17–53.** The 50-kg uniform crate rests on the platform for which the coefficient of static friction is $\mu_s = 0.5$. If at the instant $\theta = 30°$ the supporting links have an angular velocity $\omega = 1$ rad/s and angular acceleration $\alpha = 0.5 \text{ rad/s}^2$, determine the frictional force on the crate.



Curvilinear Translation:

$$(a_G)_n = (1)^2(4) = 4 \text{ m/s}^2$$

$$(a_G)_t = 0.5(4) \text{ m/s}^2 = 2 \text{ m/s}^2$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_C = 50(4)\sin 30° + 50(2)\cos 30°$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_C - 50(9.81) = 50(4)\cos 30° - 50(2)\sin 30°$$

Solving,

$$F_C = 186.6 \text{ N}$$

$$N_C = 613.7 \text{ N}$$

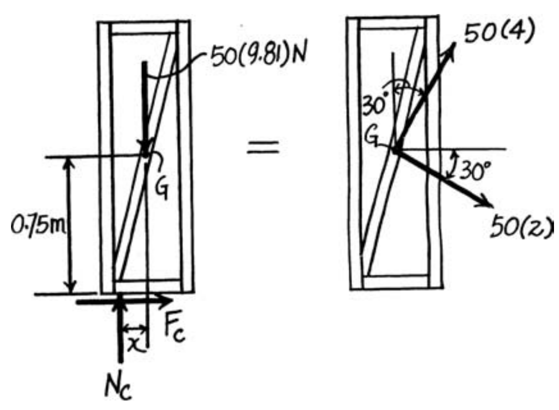
$$(F_C)_{max} = 0.5(613.7) = 306.9 \text{ N} > 186.6 \text{ N} \qquad \textbf{OK}$$

$$\circlearrowleft + \Sigma M_G = \Sigma(M_k)_G; \qquad N_C(x) - F_C(0.75) = 0$$

$$613.7(x) - 186.6(0.75) = 0$$

$$x = 0.228 \text{ m} < 0.25 \text{ m} \qquad \textbf{OK}$$

Thus, $F_C = 187$ N **Ans.**



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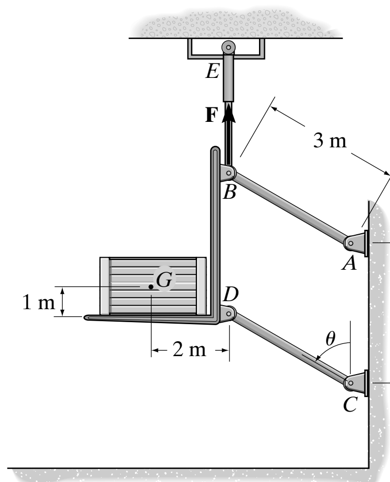
17–54. If the hydraulic cylinder BE exerts a vertical force of $F = 1.5$ kN on the platform, determine the force developed in links AB and CD at the instant $\theta = 90°$. The platform is at rest when $\theta = 45°$. Neglect the mass of the links and the platform. The 200-kg crate does not slip on the platform.



Equations of Motion: The free-body diagram of the crate and platform at the general position is shown in Fig. *a*. Here, $(a_G)_t = \alpha r = \alpha(3)$ and $(a_G)_n = \omega^2 r = \omega^2(3)$, where ω and α are the angular velocity and acceleration of the links. Writing the force equation of motion along the t axis by referring to Fig. *a*, we have

$$+\nearrow \Sigma F_t = m(a_G)_t; \qquad 200(9.81)\sin\theta - 1500\sin\theta = 200\left[\alpha(3)\right]$$

$$\alpha = 0.77\sin\theta$$

Kinematics: Using this result, the angular velocity of the links can be obtained by integrating

$$\int \omega\, d\omega = \int \alpha\, d\theta$$

$$\int_0^\omega \omega\, d\omega = \int_{45°}^{\theta} 0.77\sin\theta\, d\theta$$

$$\omega = \sqrt{1.54(0.7071 - \cos\theta)}$$

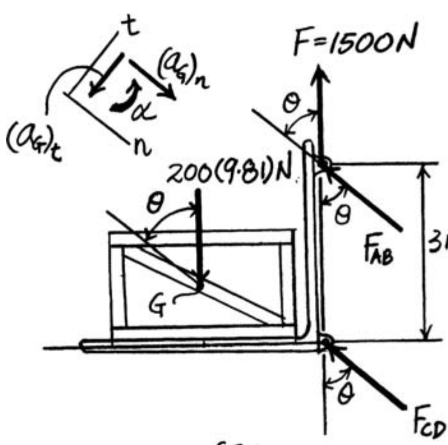
When $\theta = 90°, \omega = 1.044$ rad/s. Referring to the free-body diagram of the crate and platform when $\theta = 90°$, Fig. *b*,

$$\xrightarrow{+} \Sigma F_n = m(a_G)_n; \qquad F_{AB} - F_{CD} = 200\left[1.044^2(3)\right] \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad 1500(2) - F_{AB}(2) - F_{CD}(1) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$F_{AB} = 1217.79 \text{ N} = 1.22 \text{ kN}$ $\qquad F_{CD} = 564.42 \text{ N} = 564\text{N}$ **Ans.**



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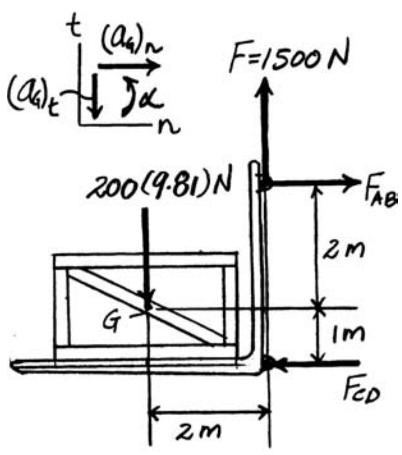

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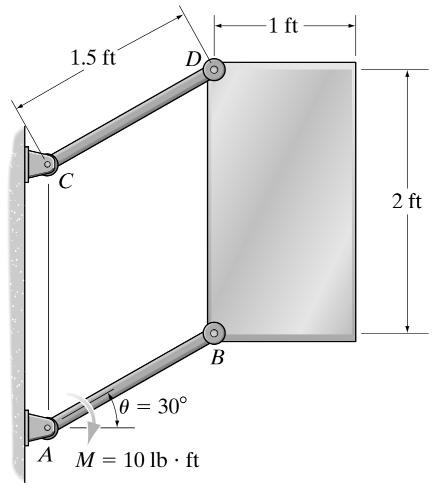
17–55. A uniform plate has a weight of 50 lb. Link AB is subjected to a couple moment of $M = 10\text{ lb}\cdot\text{ft}$ and has a clockwise angular velocity of 2 rad/s at the instant $\theta = 30°$. Determine the force developed in link CD and the tangential component of the acceleration of the plate's mass center at this instant. Neglect the mass of links AB and CD.



Equations of Motion: Since the plate undergoes the cantilever translation, $(a_G)_n = (2^2)(1.5) = 6\text{ ft/s}^2$. Referring to the free-body diagram of the plate shown in Fig. *a*,

$$\Sigma F_n = m(a_G)_n; \quad -F_{CD} - B_x \cos 30° - B_y \sin 30° + 50 \sin 30° = \left(\frac{50}{32.2}\right)(6) \quad \textbf{(1)}$$

$$\Sigma F_t = m(a_G)_t; \quad B_x \sin 30° - B_y \cos 30° + 50 \cos 30° = \frac{50}{32.2}(a_G)_t \quad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = 0; \quad B_x(1) - B_y(0.5) - F_{CD}\cos 30°(1) - F_{CD}\sin 30°(0.5) = 0 \quad \textbf{(3)}$$

Since the mass of link AB can be neglected, we can apply the moment equation of equilibrium to link AB. Referring to its free-body diagram, Fig. *b*,

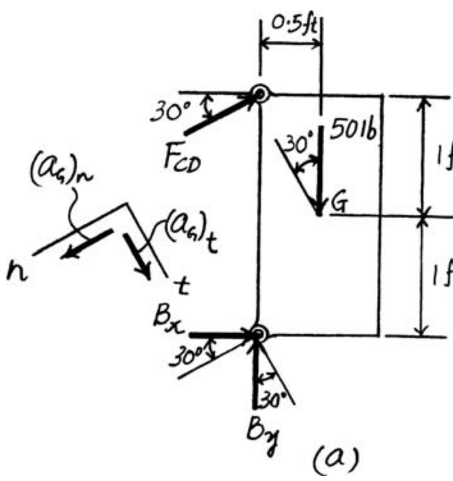
$$\circlearrowleft + \Sigma M_A = 0; \quad B_x(1.5 \sin 30°) - B_y(1.5 \cos 30°) - 10 = 0 \quad \textbf{(4)}$$

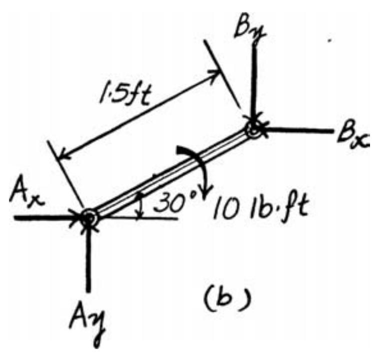
Solving Eqs. (1) through (4) yields

$B_x = 8.975\text{ lb}$ $\quad B_y = -2.516\text{ lb}$

$F_{CD} = 9.169\text{ lb} = 9.17\text{ lb}$ **Ans.**

$(a_G)_t = 32.18\text{ ft/s}^2 = 32.2\text{ ft/s}^2$ **Ans.**





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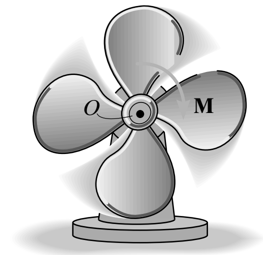
***17–56.** The four fan blades have a total mass of 2 kg and moment of inertia $I_O = 0.18\ \text{kg}\cdot\text{m}^2$ about an axis passing through the fan's center O. If the fan is subjected to a moment of $M = 3(1 - e^{-0.2t})\ \text{N}\cdot\text{m}$, where t is in seconds, determine its angular velocity when $t = 4$ s starting from rest.



$$\circlearrowleft +\Sigma M_O = I_O\alpha; \qquad 3\left(1 - e^{-0.2t}\right) = 0.18\alpha$$

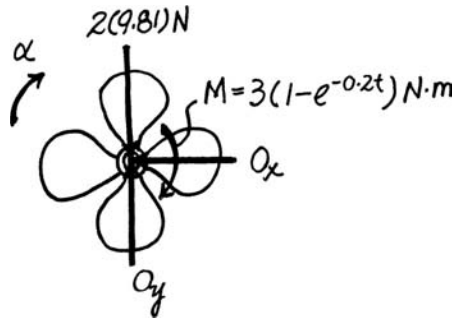
$$\alpha = 16.67\left(1 - e^{-0.2t}\right)$$

$$d\omega = \alpha\, dt$$

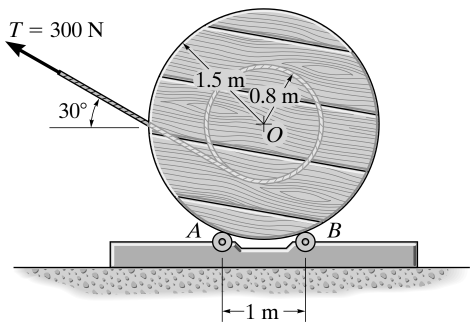
$$\int_0^{\omega} d\omega = \int_0^4 16.67\left(1 - e^{-0.2t}\right) dt$$

$$\omega = 16.67\left[t + \frac{1}{0.2}e^{-0.2t}\right]_0^4$$

$$\omega = 20.8\ \text{rad/s}$$ **Ans.**



•17–57. Cable is unwound from a spool supported on small rollers at A and B by exerting a force of $T = 300$ N on the cable in the direction shown. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a centroidal radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of the rollers at A and B. The rollers turn with no friction.



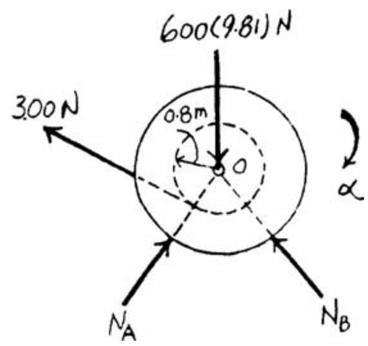
Equations of Motion: The mass moment of inertia of the spool about point O is given by $I_O = mk_O^2 = 600\left(1.2^2\right) = 864\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$$\circlearrowright +\Sigma M_O = I_O\,\alpha; \qquad -300(0.8) = -864\alpha \qquad \alpha = 0.2778\ \text{rad/s}^2$$

Kinematics: Here, the angular displacement $\theta = \dfrac{s}{r} = \dfrac{5}{0.8} = 6.25$ rad. Applying equation $\theta = \theta_0 + \omega_0 t + \dfrac{1}{2}\alpha t^2$, we have

$$(\circlearrowleft +) \qquad 6.25 = 0 + 0 + \frac{1}{2}(0.2778)t^2$$

$$t = 6.71\ \text{s}$$ **Ans.**



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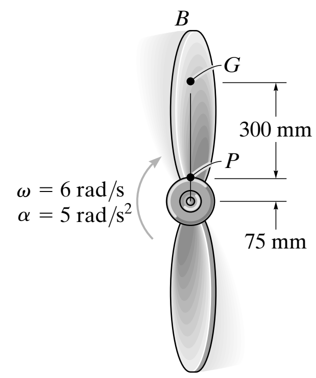
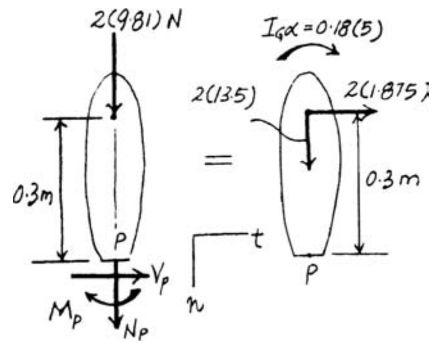
17–58. The single blade PB of the fan has a mass of 2 kg and a moment of inertia $I_G = 0.18\ \text{kg}\cdot\text{m}^2$ about an axis passing through its center of mass G. If the blade is subjected to an angular acceleration $\alpha = 5\ \text{rad/s}^2$, and has an angular velocity $\omega = 6\ \text{rad/s}$ when it is in the vertical position shown, determine the internal normal force N, shear force V, and bending moment M, which the hub exerts on the blade at point P.



Equations of Motion: Here, $(a_G)_t = \alpha r_G = 5(0.375) = 1.875\ \text{m/s}^2$ and $(a_G)_n = \omega^2 r_G = 6^2(0.375) = 13.5\ \text{m/s}^2$.



$\circlearrowleft + \Sigma M_P = \Sigma(M_k)_P;\qquad -M_P = -0.18(5) - 2(1.875)(0.3)$

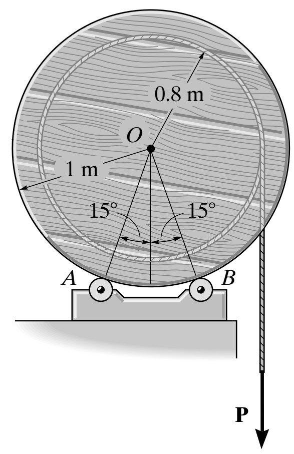
$M_P = 2.025\ \text{N}\cdot\text{m}$ **Ans.**

$\Sigma F_n = m(a_G)_n;\qquad N_P + 2(9.81) = 2(13.5)$

$N_P = 7.38\ \text{N}$ **Ans.**

$\Sigma F_t = m(a_G)_t;\qquad V_P = 2(1.875) = 3.75\ \text{N}$ **Ans.**

17–59. The uniform spool is supported on small rollers at A and B. Determine the constant force **P** that must be applied to the cable in order to unwind 8 m of cable in 4 s starting from rest. Also calculate the normal forces on the spool at A and B during this time. The spool has a mass of 60 kg and a radius of gyration about O of $k_O = 0.65$ m. For the calculation neglect the mass of the cable and the mass of the rollers at A and B.

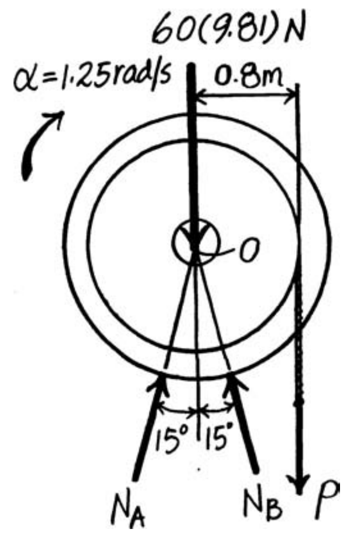


$(\downarrow +)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$8 = 0 + 0 + \frac{1}{2} a_c (4)^2$

$a_c = 1\ \text{m/s}^2$

$\alpha = \frac{1}{0.8} = 1.25\ \text{rad/s}^2$



$\circlearrowright + \Sigma M_O = I_O\,\alpha;\qquad P(0.8) = 60(0.65)^2(1.25)$

$P = 39.6\ \text{N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = m a_x;\qquad N_A \sin 15° - N_B \sin 15° = 0$

$+\uparrow \Sigma F_y = m a_y;\qquad N_A \cos 15° + N_B \cos 15° - 39.6 - 588.6 = 0$

$N_A = N_B = 325\ \text{N}$ **Ans.**

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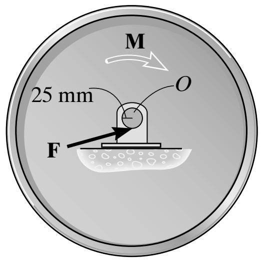
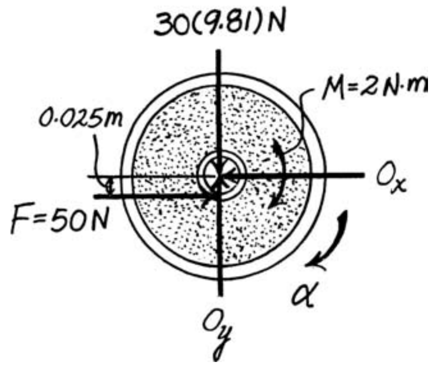
***17–60.** A motor supplies a constant torque $M = 2\ \text{N}\cdot\text{m}$ to a 50-mm-diameter shaft O connected to the center of the 30-kg flywheel. The resultant bearing friction $\mathbf{F}$, which the bearing exerts on the shaft, acts tangent to the shaft and has a magnitude of 50 N. Determine how long the torque must be applied to the shaft to increase the flywheel's angular velocity from 4 rad/s to 15 rad/s. The flywheel has a radius of gyration $k_O = 0.15$ m about its center O.





$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad 2 - 50(0.025) = 30(0.15)^2\alpha$

$$\alpha = 1.11\ \text{rad/s}^2$$

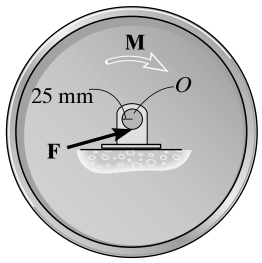
$\circlearrowleft +$

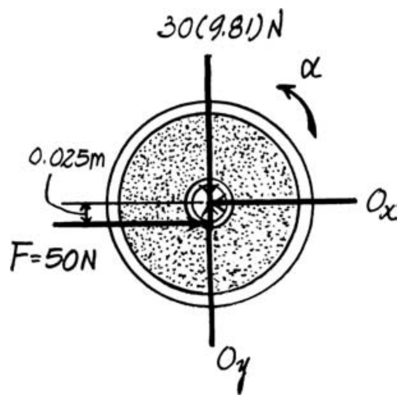
$$\omega = \omega_0 + \alpha_c\,t$$

$$15 = 4 + (1.11)t$$

$$t = 9.90\ \text{s}$$ **Ans.**

•17–61. If the motor in Prob. 17–60 is disengaged from the shaft once the flywheel is rotating at 15 rad/s, so that $M = 0$, determine how long it will take before the resultant bearing frictional force $F = 50$ N stops the flywheel from rotating.





$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad 50(0.025) = 30(0.15)^2\alpha$

$$\alpha = 1.852\ \text{rad/s}^2$$

$\circlearrowleft +$

$$\omega = \omega_0 + \alpha_c\,t$$

$$0 = -15 + (1.852)t$$

$$t = 8.10\ \text{s}$$ **Ans.**

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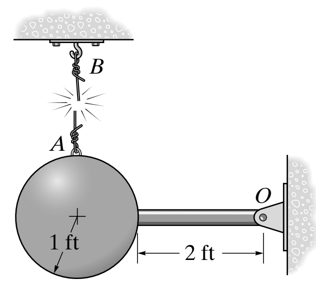
17–62. The pendulum consists of a 30-lb sphere and a 10-lb slender rod. Compute the reaction at the pin O just after the cord AB is cut.

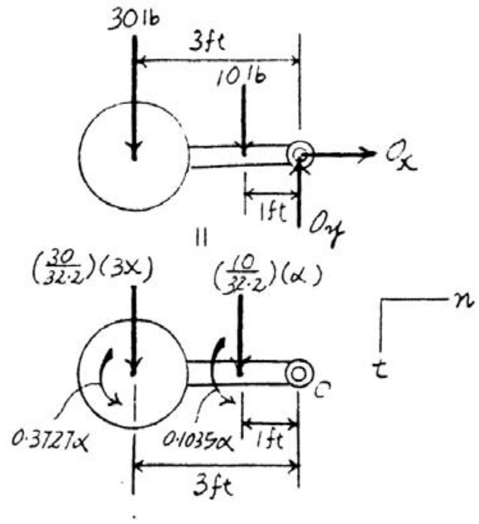


Mass Moment Inertia: From the inside back cover of the text.

$$(I_G)_S = \frac{2}{5}mr^2 = \frac{2}{5}\left(\frac{30}{32.2}\right)(1^2) = 0.3727 \text{ slug} \cdot \text{ft}^2$$

$$(I_G)_R = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug} \cdot \text{ft}^2$$

Equations of Motion: At the instant shown, the normal component of acceleration of the mass center for the sphere and the rod are $\left[(a_G)_n\right]_S = \left[(a_G)_n\right]_R = 0$ since the angular velocity of the pendulum $\omega = 0$ at that instant. The tangential component of acceleration of the mass center for the sphere and the rod are $\left[(a_G)_t\right]_S = \alpha r_S = 3\alpha$ and $\left[(a_G)_t\right]_R = \alpha r_R = \alpha$.



$$\circlearrowleft + \Sigma M_O = \Sigma(M_k)_O; \qquad 30(3) + 10(1) = 0.3727\alpha + 0.1035\alpha + \left(\frac{30}{32.2}\right)(3\alpha)(3) + \left(\frac{10}{32.2}\right)(\alpha)(1)$$

$$\alpha = 10.90 \text{ rad/s}^2$$

$$\Sigma F_n = m(a_G)_n; \qquad O_x = 0$$

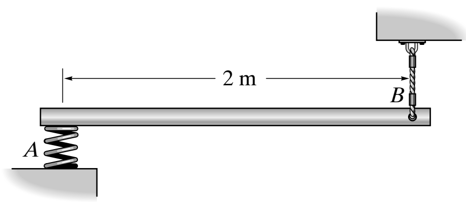
$$\Sigma F_t = m(a_G)_t; \qquad 30 + 10 - O_y = \left(\frac{30}{32.2}\right)[3(10.90)] + \left(\frac{10}{32.2}\right)(10.90)$$

$$O_y = 6.140 \text{ lb}$$

Thus,

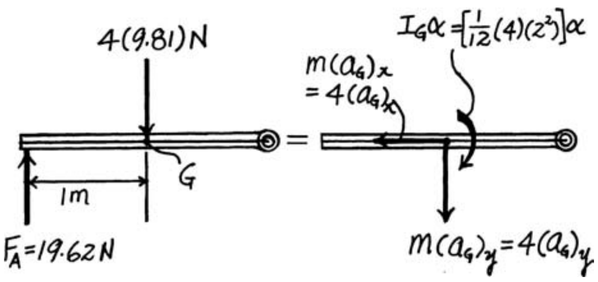
$$F_O = \sqrt{O_x^2 + O_y^2} = \sqrt{0^2 + 6.140^2} = 6.14 \text{ lb} \qquad \textbf{Ans.}$$

17–63. The 4-kg slender rod is supported horizontally by a spring at A and a cord at B. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the cord at B is cut. *Hint:* The stiffness of the spring is not needed for the calculation.



Since the deflection of the spring is unchanged at the instant the cord is cut, the reaction at A is

$$F_A = \frac{4}{2}(9.81) = 19.62 \text{ N}$$



$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0 = 4(a_G)_x$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 4(9.81) - 19.62 = 4(a_G)_y$$

$$\circlearrowright + \Sigma M_G = I_G\alpha; \qquad (19.62)(1) = \left[\frac{1}{12}(4)(2)^2\right]\alpha$$

Solving:

$$(a_G)_x = 0$$

$$(a_G)_y = 4.905 \text{ m/s}^2$$

$$\alpha = 14.7 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Thus,

$$(a_G) = 4.90 \text{ m/s}^2 \qquad \textbf{Ans.}$$

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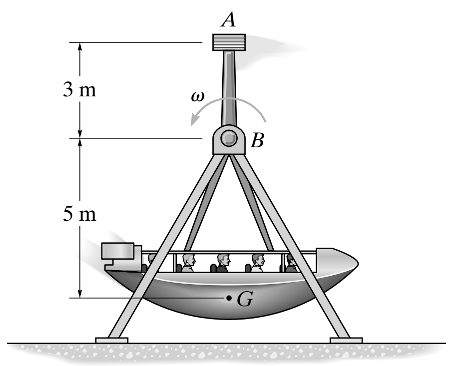
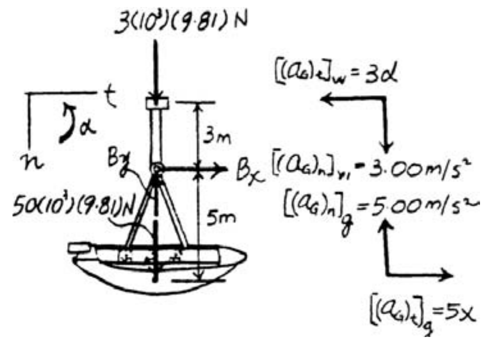
***17–64.** The passengers, the gondola, and its swing frame have a total mass of 50 Mg, a mass center at G, and a radius of gyration $k_B = 3.5$ m. Additionally, the 3-Mg steel block at A can be considered as a point of concentrated mass. Determine the horizontal and vertical components of reaction at pin B if the gondola swings freely at $\omega = 1$ rad/s when it reaches its lowest point as shown. Also, what is the gondola's angular acceleration at this instant?



Equations of Motion: The mass moment of inertia of the gondola and the counter weight about point B is given by $I_B = m_g k_B^2 + m_W r_W^2 = 50\left(10^3\right)\left(3.5^2\right) + 3\left(10^3\right)\left(3^2\right) = 639.5\left(10^3\right)\ \text{kg}\cdot\text{m}^2$. At the instant shown, the normal component of acceleration of the mass center for the gondola and the counter weight are $\left[(a_G)_n\right]_g = \omega^2 r_g = 1^2(5) = 5.00\ \text{m/s}^2$ and $\left[(a_G)_n\right]_W = \omega^2 r_W = 1^2(3) = 3.00\ \text{m/s}^2$. The tangential component of acceleration of the mass center for the gondola and the counter weight are $\left[(a_G)_t\right]_g = \alpha r_g = 5\alpha$ and $\left[(a_G)_t\right]_W = \alpha r_W = 3\alpha$. Applying Eq. 17–16, we have



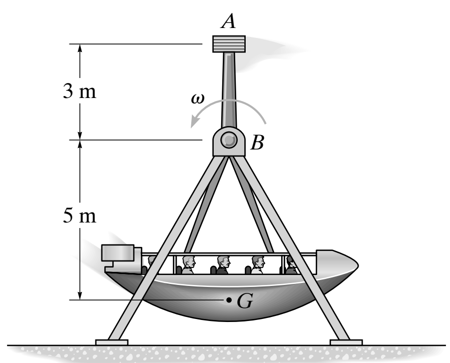
$\circlearrowleft + \Sigma M_B = I_B\,\alpha;\qquad 0 = 639.5\left(10^3\right)\alpha \qquad \alpha = 0$ **Ans.**

$\Sigma F_t = m(a_G)_t;\qquad B_x = 0$ **Ans.**

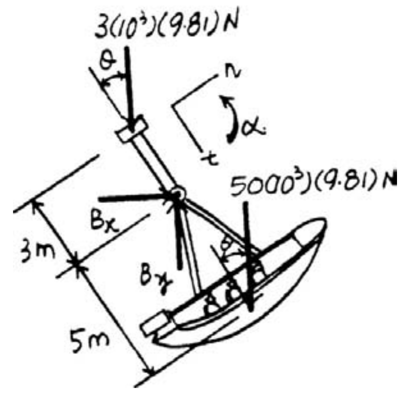
$$\Sigma F_n = m(a_G)_n;\qquad 3\left(10^3\right)(9.81) + 50\left(10^3\right)(9.81) - B_y = 3\left(10^3\right)(3.00) - 50\left(10^3\right)(5.00)$$

$B_y = 760.93\left(10^3\right)\ \text{N} = 761\ \text{kN}$ **Ans.**

•17–65. The passengers, the gondola, and its swing frame have a total mass of 50 Mg, a mass center at G, and a radius of gyration $k_B = 3.5$ m. Additionally, the 3-Mg steel block at A can be considered as a point of concentrated mass. Determine the angle θ to which the gondola will swing before it stops momentarily, if it has an angular velocity of $\omega = 1$ rad/s at its lowest point.



Equations of Motion: The mass moment of inertia of the gondola and the counter weight about point B is given by $I_B = m_g k_B^2 + m_W r_W^2 = 50\left(10^3\right)\left(3.5^2\right) + 3\left(10^3\right)\left(3^2\right) = 639.5\left(10^3\right)\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_B = I_B\,\alpha;\qquad 3\left(10^3\right)(9.81)\sin\theta(3) - 50\left(10^3\right)(9.81)\sin\theta(5) = 639.5\left(10^3\right)\alpha$$

$$\alpha = -3.6970\sin\theta$$

Kinematics: Applying equation $\omega\, d\omega = \alpha\, d\theta$, we have

$$\int_{1\,\text{rad/s}}^{0} \omega\, d\omega = \int_{0^\circ}^{\theta} -3.6970\sin\theta\, d\theta$$

$\theta = 30.1^\circ$ **Ans.**

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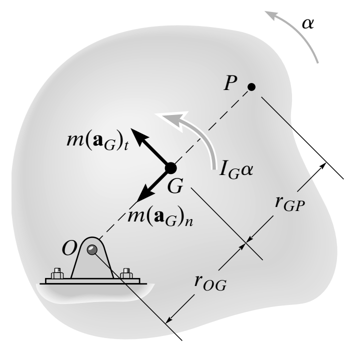
17–66. The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis passing through O is shown in the figure. Show that $I_G\boldsymbol{\alpha}$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about an axis passing through G. The point P is called the *center of percussion* of the body.



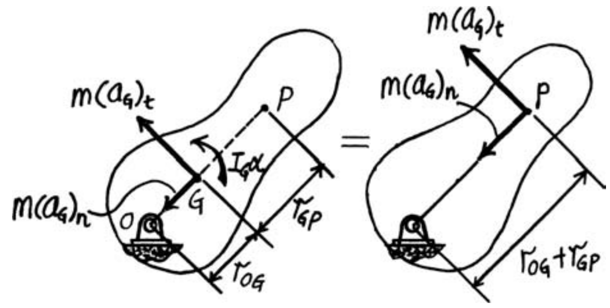
$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + \left(mk_G^2\right)\alpha$$

However,

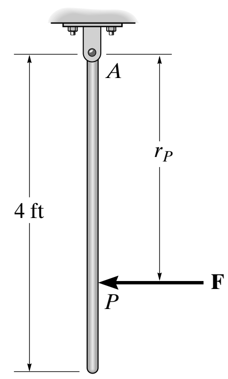
$$k_G^2 = r_{OG}\, r_{GP} \text{ and } \alpha = \frac{(a_G)_t}{r_{OG}}$$

$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + (mr_{OG}\, r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$$

$$= m(a_G)_t(r_{OG} + r_{GP}) \qquad \textbf{Q.E.D.}$$



17–67. Determine the position r_P of the center of percussion P of the 10-lb slender bar. (See Prob. 17–66.) What is the horizontal component of force that the pin at A exerts on the bar when it is struck at P with a force of $F = 20$ lb?



Using the result of Prob 17–66,

$$r_{GP} = \frac{k_G^2}{r_{AG}} = \frac{\left[\sqrt{\frac{1}{12}\left(\frac{ml^2}{m}\right)}\right]^2}{\frac{l}{2}} = \frac{1}{6}\,l$$

Thus,

$$r_P = \frac{1}{6}\,l + \frac{1}{2}\,l = \frac{2}{3}\,l = \frac{2}{3}\,(4) = 2.67 \text{ ft} \qquad \textbf{Ans.}$$

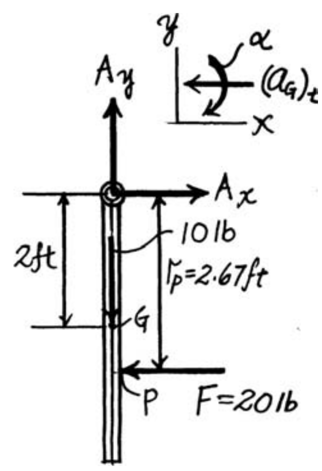
$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 20(2.667) = \left[\frac{1}{3}\left(\frac{10}{32.2}\right)(4)^2\right]\alpha$$

$$\alpha = 32.2 \text{ rad/s}^2$$

$$(a_G)_t = 2(32.2) = 64.4 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x\,; \qquad -A_x + 20 = \left(\frac{10}{32.2}\right)(64.4)$$

$$A_x = 0 \qquad \textbf{Ans.}$$



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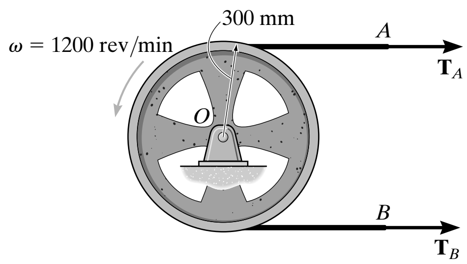
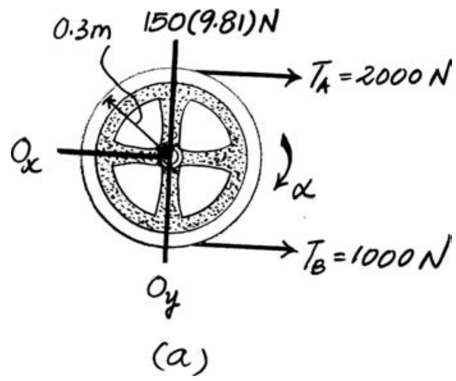
***17–68.** The 150-kg wheel has a radius of gyration about its center of mass O of $k_O = 250$ mm. If it rotates counterclockwise with an angular velocity of $\omega = 1200$ rev/min at the instant the tensile forces $T_A = 2000$ N and $T_B = 1000$ N are applied to the brake band at A and B, determine the time needed to stop the wheel.



Equations of Motion: Here, the mass moment of inertia of the flywheel about its mass center O is $I_O = mk_O{}^2 = 150\left(0.25^2\right) = 9.375 \text{ kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel in Fig. b, we have

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 1000(0.3) - 2000(0.3) = -9.375\alpha \qquad \alpha = 32 \text{ rad/s}^2$$



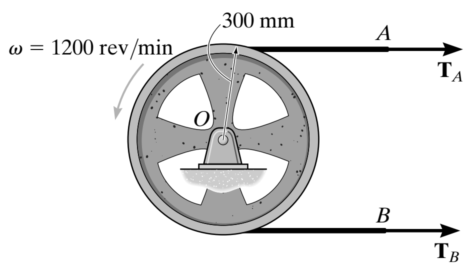
Kinematics: Here, $\omega_0 = \left(1200\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = 40\pi$ rad. Since the angular acceleration is constant, we can apply

$$\circlearrowleft + \qquad \omega = \omega_0 + \alpha_c t$$

$$0 = 40\pi + (-32)t$$

$$t = 3.93 \text{ s} \qquad \textbf{Ans.}$$

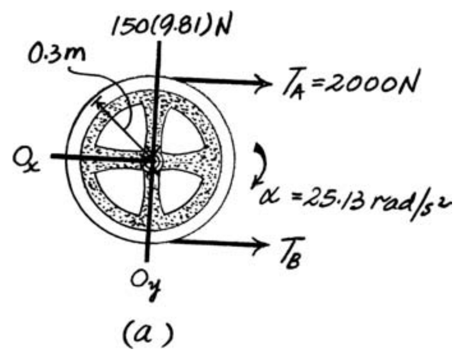
•17–69. The 150-kg wheel has a radius of gyration about its center of mass O of $k_O = 250$ mm. If it rotates counterclockwise with an angular velocity of $\omega = 1200$ rev/min and the tensile force applied to the brake band at A is $T_A = 2000$ N, determine the tensile force $\mathbf{T}_B$ in the band at B so that the wheel stops in 50 revolutions after $\mathbf{T}_A$ and $\mathbf{T}_B$ are applied.



Kinematics: Here, $\omega_0 = \left(1200\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = 40\pi$ rad and

$$\theta = (50 \text{ rev})$$

$$\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 100\pi\text{rad}$$



Since the angular acceleration is constant,

$$\circlearrowleft + \qquad \omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$

$$0 = (40\pi)^2 + 2\alpha(100\pi - 0)$$

$$\alpha = -25.13 \text{ rad/s}^2 = 25.13 \text{ rad/s}^2$$

Equations of Motion: Here, the mass moment of inertia of the flywheel about its mass center O is $I_O = mk_O{}^2 = 150\left(0.25^2\right) = 9.375 \text{ kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel,

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad T_B(0.3) - 2000(0.3) = -9.375(25.13)$$

$$T_B = 1214.60 \text{ N} = 1.21 \text{ kN} \qquad \textbf{Ans.}$$

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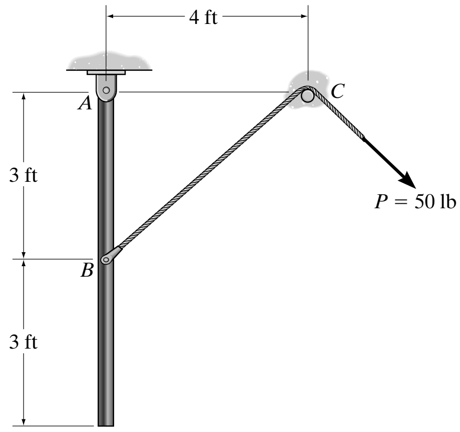
17–70. The 100-lb uniform rod is at rest in a vertical position when the cord attached to it at B is subjected to a force of $P = 50$ lb. Determine the rod's initial angular acceleration and the magnitude of the reactive force that pin A exerts on the rod. Neglect the size of the smooth peg at C.



Equations of Motion: Since the rod rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(3)$ and $(a_G)_n = \omega^2 r_G = 0$. The mass moment of inertia of the rod about G is $I_G = \frac{1}{12}\left(\frac{100}{32.2}\right)(6^2) = 9.317 \text{ slug}\cdot\text{ft}^2$. Writing the moment equation of motion about point A,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 50\left(\frac{4}{5}\right)(3) = \frac{100}{32.2}\big[\alpha(3)\big](3) + 9.317\alpha$$

$$\alpha = 3.220 \text{ rad/s}^2 = 3.22 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

This result can also be obtained by applying $\Sigma M_A = I_A\alpha$, where

$$I_A = 9.317 + \left(\frac{100}{32.2}\right)(3^2) = 37.267 \text{ slug}\cdot\text{ft}^2$$

Thus,

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 50\left(\frac{4}{5}\right)(3) = 37.267\alpha$$

$$\alpha = 3.220 \text{ rad/s}^2 = 3.22 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

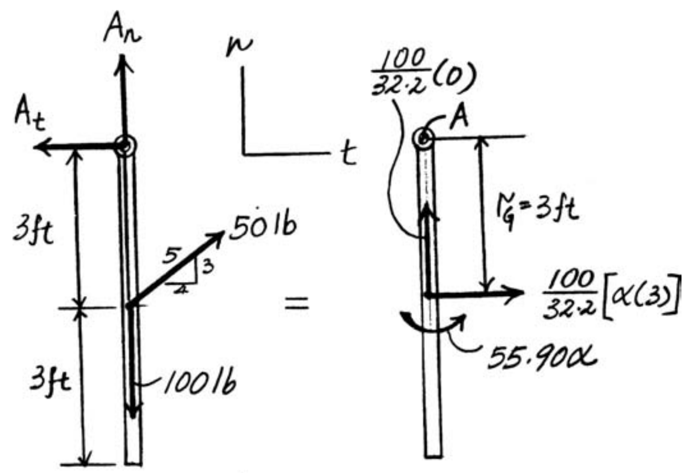
Using this result to write the force equation of motion along the n and t axes,

$$+\uparrow \Sigma F_n = m(a_G)_n; \qquad A_n + 50\left(\frac{3}{5}\right) - 100 = \frac{100}{32.2}(0) \qquad A_n = 70 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_t = m(a_G)_t; \qquad 50\left(\frac{4}{5}\right) - A_t = \frac{100}{32.2}\big[3.220(3)\big] \qquad A_t = 10.0 \text{ lb}$$

Thus,

$$F_A = \sqrt{A_t^{\,2} + A_n^{\,2}} = \sqrt{10^2 + 70^2} = 70.7 \text{ lb} \qquad \textbf{Ans.}$$



(a)

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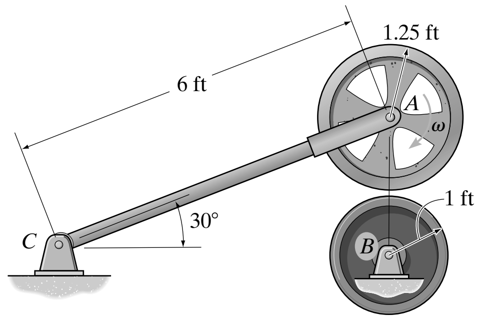
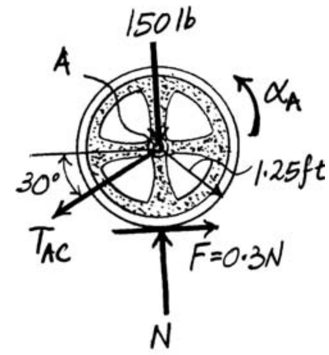
17–71. Wheels A and B have weights of 150 lb and 100 lb, respectively. Initially, wheel A rotates clockwise with a constant angular velocity of $\omega = 100$ rad/s and wheel B is at rest. If A is brought into contact with B, determine the time required for both wheels to attain the same angular velocity. The coefficient of kinetic friction between the two wheels is $\mu_k = 0.3$ and the radii of gyration of A and B about their respective centers of mass are $k_A = 1$ ft and $k_B = 0.75$ ft. Neglect the weight of link AC.



Equations of Motion: Wheel A will slip on wheel B until both wheels attain the same angular velocity. The frictional force developed at the contact point is $F = \mu_k N = 0.3N$. The mass moment of inertia of wheel A about its mass center is $I_A = m_A k_A{}^2 = \frac{150}{32.2}\left(1^2\right)$ slug·ft^2. Referring to the free-body diagram of wheel A shown in Fig. *a*.



(a)

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 0.3N - T_{AC}\cos 30^\circ = 0$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N - T_{AC}\sin 30^\circ - 150 = 0$$

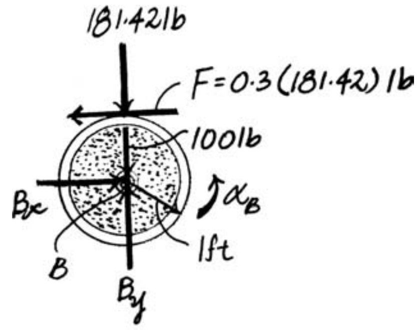
$$\circlearrowleft+\Sigma M_A = I_A\alpha_A; \qquad 0.3N(1.25) = \left[\frac{150}{32.2}\left(1^2\right)\right]\alpha_A$$

Solving,

$$N = 181.42 \text{ lb} \qquad T_{AC} = 62.85 \text{ lb} \qquad \alpha_A = 14.60 \text{ rad/s}$$

The mass moment of inertia of wheel B about its mass center is

$$I_B = m_B k_B{}^2 = \frac{100}{32.2}\left(0.75^2\right) \text{ slug}\cdot\text{ft}^2$$



(b)

Writing the moment equation of motion about point B using the free-body diagram of wheel B shown in Fig. *b*,

$$+\Sigma M_B = I_B\alpha_B; \qquad 0.3(181.42)(1) = \frac{100}{32.2}\left(0.75^2\right)\alpha_B$$

$$\alpha_B = 31.16 \text{ rad/s}^2$$

Kinematics: Since the angular acceleration of both wheels is constant,

$$\circlearrowleft+ \qquad \omega_A = (\omega_A)_0 + \alpha_A t$$

$$\omega_A = 100 + (-14.60)t$$

and

$$\circlearrowleft+ \qquad \omega_B = (\omega_B)_0 + \alpha_B t$$

$$\omega_B = 0 + 31.16t$$

Since ω_A is required to be equal to ω_B, we obtain

$$100 + (-14.60)t = 31.16t$$

$$t = 2.185 \text{ s} = 2.19 \text{ s} \qquad \textbf{Ans.}$$

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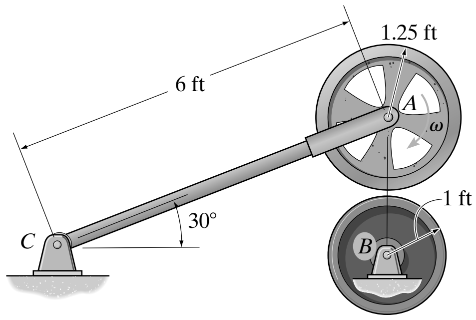
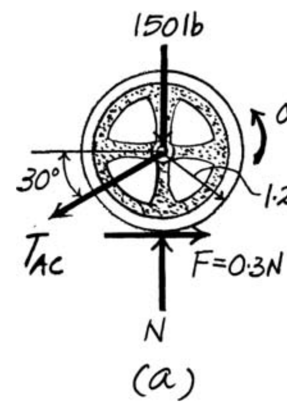
***17–72.** Initially, wheel A rotates clockwise with a constant angular velocity of $\omega = 100$ rad/s. If A is brought into contact with B, which is held fixed, determine the number of revolutions before wheel A is brought to a stop. The coefficient of kinetic friction between the two wheels is $\mu_k = 0.3$, and the radius of gyration of A about its mass center is $k_A = 1$ ft. Neglect the weight of link AC.



Equations of Motion: Since wheel B is fixed, wheel A will slip on wheel B. The frictional force developed at the contact point is $F = \mu_k N = 0.3N$. The mass moment of inertia of wheel A about its mass center is $I_A = m_A k_A^2 = \frac{150}{32.2}(1^2)\text{ slug}\cdot\text{ft}^2$. Referring to the free-body diagram of wheel A shown in Fig. a,



$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 0.3N - T_{AC}\cos 30^\circ = 0$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N - T_{AC}\sin 30^\circ - 150 = 0$$

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha_A; \qquad 0.3N(1.25) = \left[\frac{150}{32.2}(1^2)\right]\alpha_A$$

Solving,

$$N = 181.42\text{ lb} \qquad T_{AC} = 62.85\text{ lb} \qquad \alpha_A = 14.60\text{ rad/s}$$

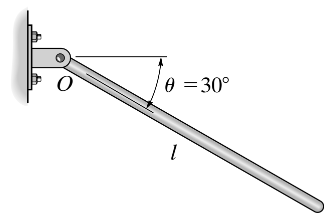
Kinematics: Since the angular acceleration is constant,

$$\circlearrowleft + \qquad \omega_A{}^2 = (\omega_A)^2{}_0 + 2\alpha_A(\theta - \theta_0)$$

$$0^2 = 100^2 + 2(-14.60)(\theta - 0)$$

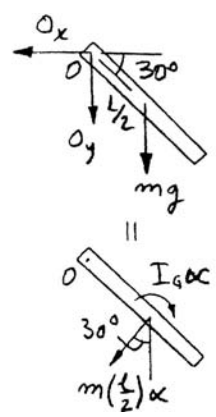
$$\theta = 342.36\text{ rad}\left(\frac{1\text{ rad}}{2\pi\text{ rad}}\right) = 54.49\text{ rev} = 54.5\text{ rev} \qquad \textbf{Ans.}$$

•17–73. The bar has a mass m and length l. If it is released from rest from the position $\theta = 30^\circ$, determine its angular acceleration and the horizontal and vertical components of reaction at the pin O.



$$\circlearrowright + \Sigma M_O = I_O\,\alpha; \qquad (mg)\left(\frac{l}{2}\right)\cos 30^\circ = \frac{1}{3}ml^2\,\alpha$$

$$\alpha = \frac{1.299g}{l} = \frac{1.30g}{l} \qquad \textbf{Ans.}$$



$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \qquad O_x = m\left(\frac{l}{2}\right)\left(\frac{1.299g}{l}\right)\sin 30^\circ$$

$$O_x = 0.325mg \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad O_y - mg = -m\left(\frac{l}{2}\right)\left(\frac{1.299g}{l}\right)\cos 30^\circ$$

$$O_y = 0.438mg \qquad \textbf{Ans.}$$

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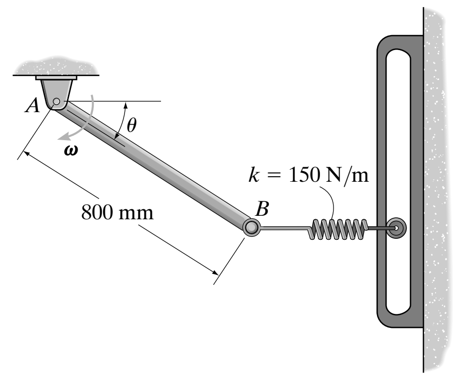
17–74. The uniform slender rod has a mass of 9 kg. If the spring is unstretched when $\theta = 0°$, determine the magnitude of the reactive force exerted on the rod by pin A when $\theta = 45°$, if at this instant $\omega = 6$ rad/s. The spring has a stiffness of $k = 150$ N/m and always remains in the horizontal position.



Equations of Motion: The stretch of the spring when $\theta = 45°$ is $s = 0.8 - 0.8\cos 45° = 0.2343$ m. Thus, $F_{sp} = ks = 150(0.2343) = 35.15$ N. Since the rod rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(0.4)$ and $(a_G)_n = \omega^2 r_G = 6^2(0.4) = 14.4\text{ m/s}^2$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(9)\left(0.8^2\right) = 0.48\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A, Fig. a,

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 35.15\cos 45°(0.8) - 9(9.81)\cos 45°(0.4)$$

$$= -9[\alpha(0.4)](0.4) - 0.48\alpha$$

$$\alpha = 2.651\text{ rad/s}^2$$

The above result can also be obtained by applying $\Sigma M_A = I_A\,\alpha$, where

$$I_A = I_G + md^2 = \frac{1}{12}(9)\left(0.8^2\right) + 9\left(0.4^2\right) = 1.92\text{ kg}\cdot\text{m}^2$$

Thus,

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 35.15\cos 45°(0.8) - 9(9.81)\cos 45°(0.4) = -1.92\alpha$$

$$\alpha = 2.651\text{ rad/s}^2$$

Using this result and writing the force equation of motion along the n and t axes,

$$+\swarrow \Sigma F_t = m(a_G)_t; \qquad 9(9.81)\cos 45° - 35.15\cos 45° - A_t = 9[2.651(0.4)]$$

$$A_t = 28.03\text{ N}$$

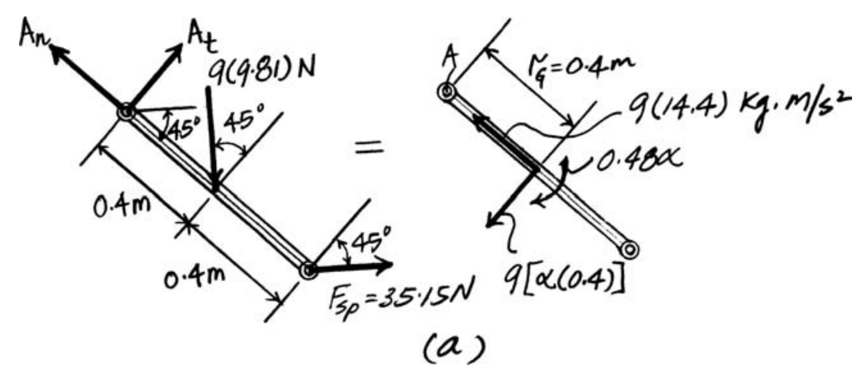
$$+\nwarrow \Sigma F_n = m(a_G)_n; \qquad A_n - 9(9.81)\sin 45° - 35.15\sin 45° = 9(14.4)$$

$$A_n = 216.88\text{ N}$$

Thus,

$$F_A = \sqrt{{A_t}^2 + {A_n}^2} = \sqrt{28.03^2 + 216.88^2}$$

$$= 218.69\text{ N} = 219\text{ N} \qquad \textbf{Ans.}$$



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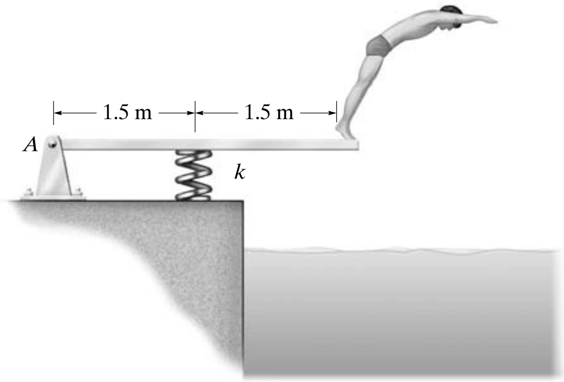
17–75. Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



$$\circlearrowleft + \sum M_A = I_A\alpha; \qquad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t; \qquad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

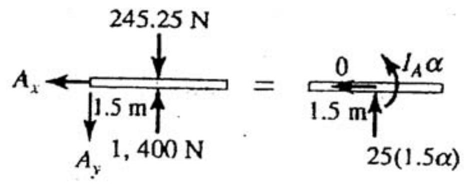
$$\overset{+}{\leftarrow} \sum F_n = m(a_G)_n; \qquad A_x = 0$$

Solving,

$$A_x = 0 \qquad \textbf{Ans.}$$

$$A_y = 289 \text{ N} \qquad \textbf{Ans.}$$

$$\alpha = 23.1 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



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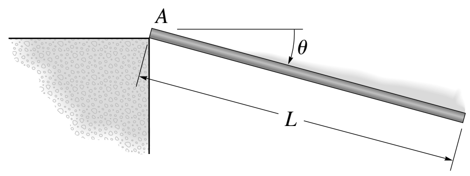
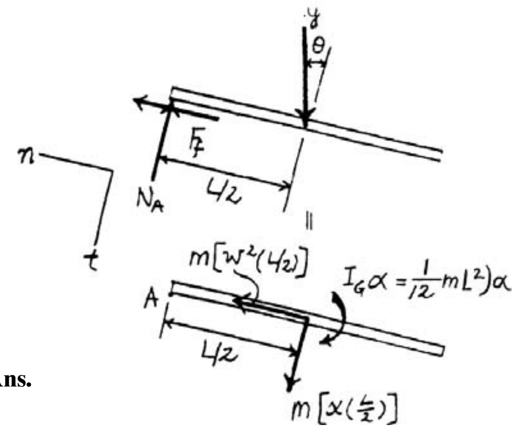
***17–76.** The slender rod of length L and mass m is released from rest when $\theta = 0°$. Determine as a function of θ the normal and the frictional forces which are exerted by the ledge on the rod at A as it falls downward. At what angle θ does the rod begin to slip if the coefficient of static friction at A is μ?



Equations of Motion: The mass moment inertia of the rod about its mass center is given by $I_G = \frac{1}{12}mL^2$. At the instant shown, the normal component of acceleration of the mass center for the rod is $(a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{2}\right)$. The tangential component of acceleration of the mass center for the rod is $(a_G)_t = \alpha r_s = \alpha\left(\frac{L}{2}\right)$.



$$\zeta + \Sigma M_A = \Sigma(M_k)_O; \qquad -mg\cos\theta\left(\frac{L}{2}\right) = -\left(\frac{1}{12}mL^2\right)\alpha - m\left[\alpha\left(\frac{L}{2}\right)\right]\left(\frac{L}{2}\right)$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

$$+\swarrow \Sigma F_t = m(a_G)_t; \qquad mg\cos\theta - N_A = m\left[\frac{3g}{2L}\cos\theta\left(\frac{L}{2}\right)\right]$$

$$N_A = \frac{mg}{4}\cos\theta \qquad \textbf{Ans.}$$

$$\nwarrow + \Sigma F_n = m(a_G)_n; \qquad F_f - mg\sin\theta = m\left[\omega^2\left(\frac{L}{2}\right)\right] \qquad \textbf{(1)}$$

Kinematics: Applying equation $\omega\, d\omega = a\, d\theta$, we have

$$\int_0^\omega \omega\, d\omega = \int_{0°}^{\theta} \frac{3g}{2L}\cos\theta\, d\theta$$

$$\omega^2 = \frac{3g}{L}\sin\theta$$

Substitute $\omega^2 = \frac{3g}{L}\sin\theta$ into Eq. (1) gives

$$F_f = \frac{5mg}{2}\sin\theta \qquad \textbf{Ans.}$$

If the rod is on the verge of slipping at A, $F_f = \mu N_A$. Substitute the data obtained above, we have

$$\frac{5mg}{2}\sin\theta = \mu\left(\frac{mg}{4}\cos\theta\right)$$

$$\theta = \tan^{-1}\left(\frac{\mu}{10}\right) \qquad \textbf{Ans.}$$

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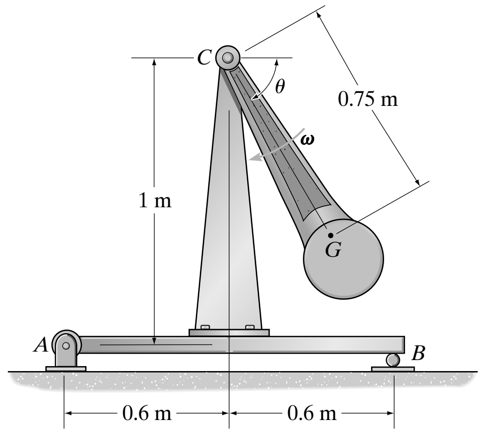
•**17–77.** The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 90°$ when the pendulum is rotating at $\omega = 8$ rad/s. Neglect the weight of the beam and the support.



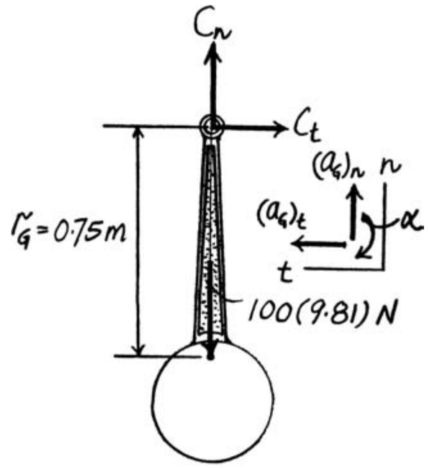
Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 8^2(0.75) = 48\ \text{m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25)^2 + 100(0.75^2) = 62.5\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram of the pendulum, Fig. a, we have

$$\circlearrowleft + \Sigma M_C = I_C\alpha; \qquad 0 = 62.5\alpha \qquad \alpha = 0$$

Using this result to write the force equations of motion along the n and t axes,

$$\overset{+}{\leftarrow} \Sigma F_t = m(a_G)_t; \qquad -C_t = 100[0(0.75)] \qquad C_t = 0$$

$$+\uparrow \Sigma F_n = m(a_G)_n; \qquad C_n - 100(9.81) = 100(48) \qquad C_n = 5781\ \text{N}$$



(a)

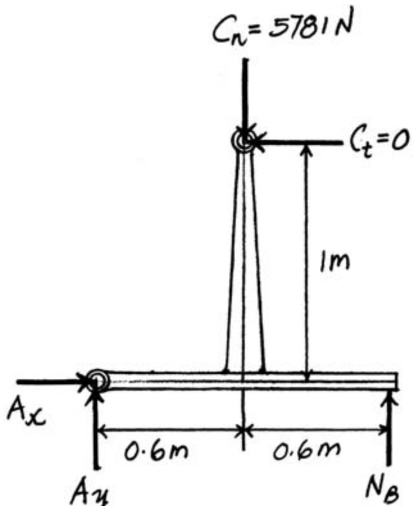
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. b, we have

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 5781(0.6) = 0 \qquad N_B = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$

Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 2890.5 - 5781 = 0 \qquad A_y = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$



(b)

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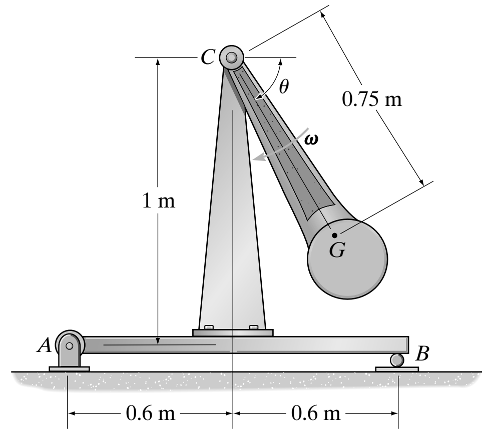
17–78. The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 0°$ when the pendulum is rotating at $\omega = 4$ rad/s. Neglect the weight of the beam and the support.



Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 4^2(0.75) = 12\text{ m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25^2) + 100(0.75)^2 = 62.5\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_C = I_C\alpha; \qquad -100(9.81)(0.75) = -62.5\alpha \qquad \alpha = 11.772\text{ rad/s}^2$$

Using this result to write the force equations of motion along the n and t axes, we have

$$+\uparrow \Sigma F_t = m(a_G)_t; \qquad C_t - 100(9.81) = -100[11.772(0.75)] \qquad C_t = 98.1\text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad C_n = 100(12) \qquad C_n = 1200\text{ N}$$

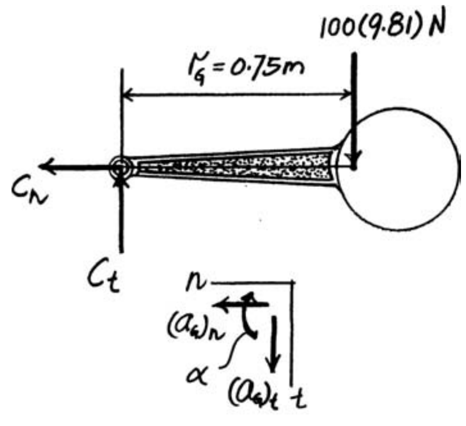
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. *b*,

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 98.1(0.6) - 1200(1) = 0 \qquad N_B = 1049.05\text{ N} = 1.05\text{ kN} \quad \textbf{Ans.}$$

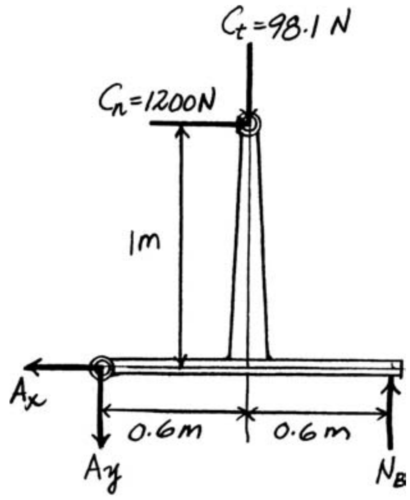
Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1200 - A_x = 0 \qquad A_x = 1200\text{ N} = 1.20\text{ kN} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 1049.05 - 98.1 - A_y = 0 \qquad A_y = 950.95\text{ N} = 951\text{ N} \quad \textbf{Ans.}$$



(a)



(b)

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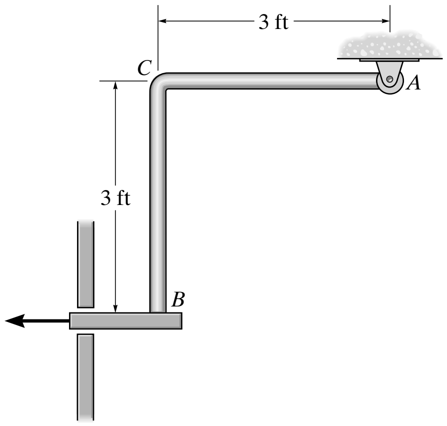
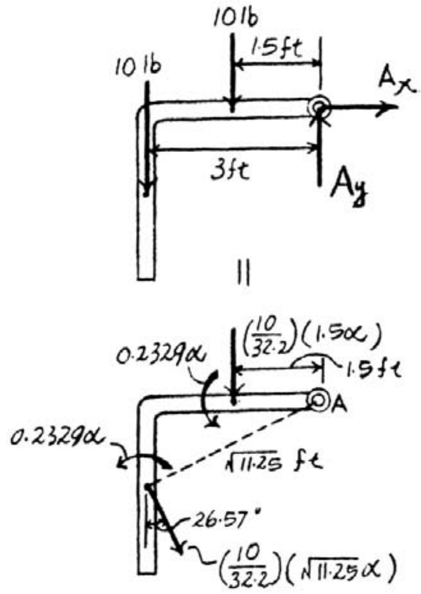
17–79. If the support at B is suddenly removed, determine the initial horizontal and vertical components of reaction that the pin A exerts on the rod ACB. Segments AC and CB each have a weight of 10 lb.



Equations of Motion: The mass moment inertia of the rod segment AC and BC about their respective mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(3^2) = 0.2329 \text{ slug}\cdot\text{ft}^2$. At the instant shown, the normal component of acceleration of the mass center for rod segment AB and BC are $[(a_G)_n]_{AB} = [(a_G)_n]_{BC} = 0$ since the angular velocity of the assembly $\omega = 0$ at that instant. The tangential component of acceleration of the mass center for rod segment AC and BC are $[(a_G)_t]_{AB} = 1.5\,\alpha$ and $[(a_G)_t]_{BC} = \sqrt{11.25}\alpha$.



$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 10(1.5) + 10(3) = 0.2329\alpha + \left(\frac{10}{32.2}\right)(1.5\alpha)(1.5)$$

$$+ 0.2329\,\alpha + \left(\frac{10}{32.2}\right)(\sqrt{11.25}\alpha)\,(\sqrt{11.25})$$

$$\alpha = 9.660 \text{ rad/s}^2$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad A_x = \left(\frac{10}{32.2}\right)[\,\sqrt{11.25}\,(9.660)\,] \sin 26.57°$$

$$A_x = 4.50 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 20 = -\left(\frac{10}{32.2}\right)[\,1.5(9.660)]$$

$$-\left(\frac{10}{32.2}\right)[\,\sqrt{11.25}\,(9.660)\,] \cos 26.57°$$

$$A_y = 6.50 \text{ lb} \qquad \textbf{Ans.}$$

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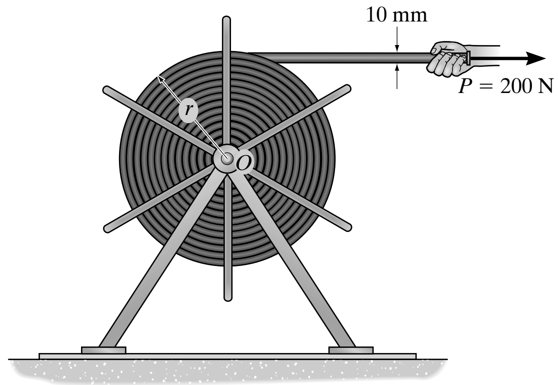
***17–80.** The hose is wrapped in a spiral on the reel and is pulled off the reel by a horizontal force of $P = 200$ N. Determine the angular acceleration of the reel after it has turned 2 revolutions. Initially, the radius is $r = 500$ mm. The hose is 15 m long and has a mass per unit length of 10 kg/m. Treat the wound-up hose as a disk.



Equations of Motion: The mass of the hose on the reel when it rotates through an angle θ is $m = 15(10) - r\theta(10) = (150 - 10r\theta)$ kg. Then, the mass moment of inertia of the reel about point O at any instant is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(150 - 10r\theta)r^2$. Also, the acceleration of the unwound hose is $a = \alpha r$. Writing the moment equation of motion about point O,

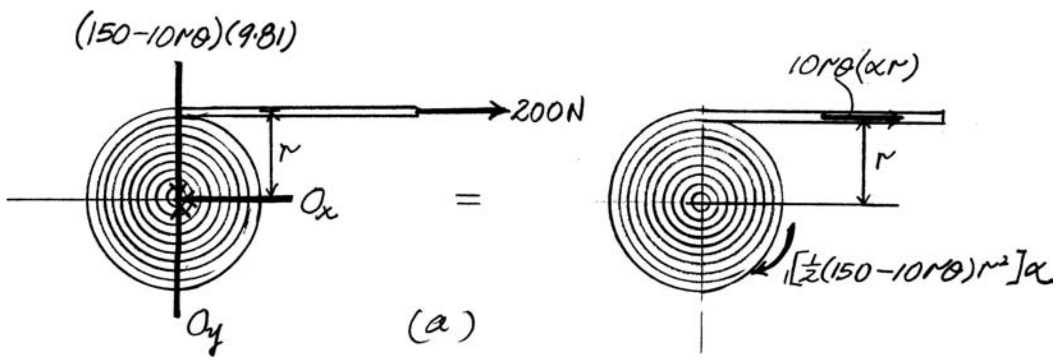
$$\circlearrowleft +\Sigma M_O = \Sigma(M_k)_O; \qquad -200(r) = -\left[\frac{1}{2}(150 - 10r\theta)r^2\right]\alpha - 10r\theta(\alpha r)r$$

$$\alpha = \frac{200}{75r + 5r^2\theta}$$

However, $r = 0.5 - \frac{\theta}{2\pi}(0.01) = 0.5 - \frac{0.005}{\pi}\theta$. Thus, when $\theta = 2\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 4\pi$ rad, $r = 0.48$ m. Then

$$\alpha = \frac{200}{75(0.48) + 5(0.48^2)(4\pi)}$$

$$= 3.96\text{ rad/s}^2 \qquad \textbf{Ans.}$$



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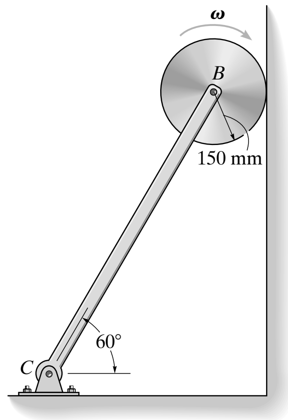
•17–81. The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, where the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_{CB} \sin 30° - N_A = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F_{CB} \cos 30° - 20(9.81) + 0.3N_A = 0$

$\circlearrowleft + \Sigma M_B = I_B\,\alpha; \qquad 0.3N_A(0.15) = \left[\frac{1}{2}(20)(0.15)^2\right]\alpha$

$N_A = 96.6$ N

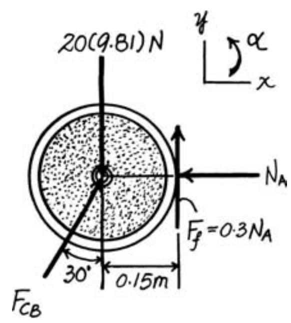
$F_{CB} = 193$ N **Ans.**

$\alpha = 19.3$ rad/s^2

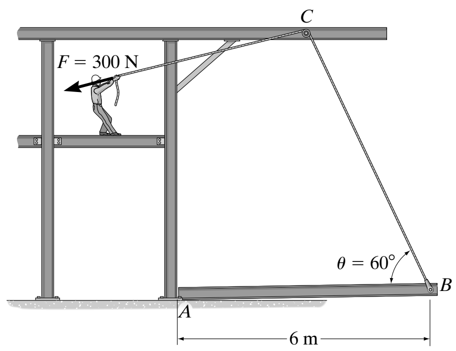
$\circlearrowleft + \qquad \omega = \omega_0 + \alpha_c t$

$0 = 60 + (-19.3)\,t$

$t = 3.11$ s **Ans.**



17–82. The 50-kg uniform beam (slender rod) is lying on the floor when the man exerts a force of $F = 300$ N on the rope, which passes over a small smooth peg at C. Determine the initial angular acceleration of the beam. Also find the horizontal and vertical reactions on the beam at A (considered to be a pin) at this instant.



Equations of Motion: Since the beam rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(3)$ and $(a_G)_n = \omega^2 r_G = \omega^2(3)$. However, the beam is initially at rest, so $\omega = 0$. Thus, $(a_G)_n = 0$. Here, the mass moment of inertia of the beam about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(50)(6^2) = 150$ kg·m^2. Writing the moment equation of motion about point A, Fig. a,

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 300 \sin 60°(6) - 50(9.81)(3) = 50[\alpha(3)](3) + 150\alpha$

$\alpha = 0.1456$ rad/s$^2 = 0.146$ rad/s^2 **Ans.**

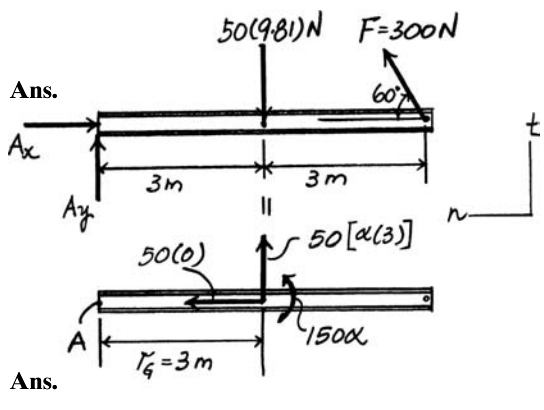
This result can also be obtained by applying $\Sigma M_A = I_A\,\alpha$, where

$$I_A = \frac{1}{12}(50)(6^2) + 50(3^2) = 600 \text{ kg}\cdot\text{m}^2$$

Thus,

$\circlearrowleft + \Sigma M_A = \Sigma(\mu_k)_A; \qquad 300 \sin 60°(6) - 50(9.81)(3) = 600\alpha$

$\alpha = 0.1456$ rad/s$^2 = 0.146$ rad/s^2 **Ans.**



(a)

Using this result to write the force equations of motion along the n and t axes,

$\xleftarrow{+} \Sigma F_t = m(a_G)_t; \qquad 300 \cos 60° - A_x = 50(0) \qquad A_x = 150$ N **Ans.**

$+\uparrow \Sigma F_n = m(a_G)_n; \quad A_y + 300 \sin 60° - 50(9.81) = 50[0.1456(3)]$

$A_y = 252.53$ N $= 253$ N **Ans.**

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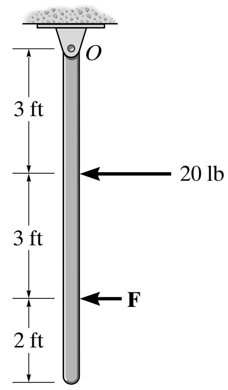
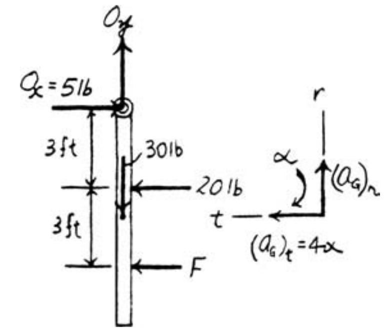
17–83. At the instant shown, two forces act on the 30-lb slender rod which is pinned at O. Determine the magnitude of force **F** and the initial angular acceleration of the rod so that the horizontal reaction which the *pin exerts on the rod* is 5 lb directed to the right.



Equations of Motion: The mass moment of inertia of the rod about point O is given by $I_O = I_G = mr_G^2 = \frac{1}{12}\left(\frac{30}{32.2}\right)(8^2) + \left(\frac{30}{32.2}\right)(4^2) = 19.88 \text{ slug}\cdot\text{ft}^2$. At the instant shown, the tangential component of acceleration of the mass center for the rod is $(a_G)_t = \alpha r_g = 4\alpha$. Applying Eq. 17–16, we have

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad -20(3) - F(6) = -19.88\alpha \qquad \textbf{(1)}$$

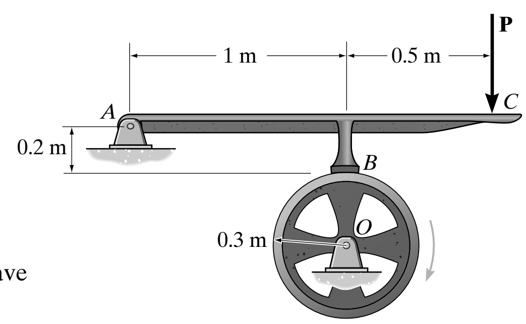
$$\Sigma F_t = m(a_G)_t\,; \qquad 20 + F - 5 = \left(\frac{30}{32.2}\right)(4\alpha) \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields:

$$\alpha = 12.1 \text{ rad/s}^2 \qquad F = 30.0 \text{ lb} \qquad \textbf{Ans.}$$

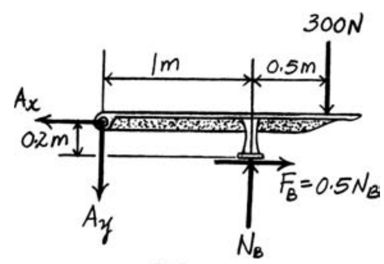
***17–84.** The 50-kg flywheel has a radius of gyration about its center of mass of $k_O = 250$ mm. It rotates with a constant angular velocity of 1200 rev/min before the brake is applied. If the coefficient of kinetic friction between the brake pad B and the wheel's rim is $\mu_k = 0.5$, and a force of $P = 300$ N is applied to the braking mechanism's handle, determine the time required to stop the wheel.



Equilibrium: Writing the moment equation of equilibrium about point A, we have

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B\,(1) + 0.5N_B\,(0.2) - 300(1.5) = 0$$

$$N_B = 409.09 \text{ N}$$



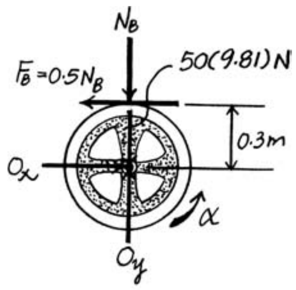
(a)

Equations of Motion: The mass moment of inertia of the flywheel about its center is $I_O = 50\left(0.25^2\right) = 3.125 \text{ kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel shown in Fig. b, we have

$$+\Sigma M_O = I_O\,\alpha; \qquad 0.5(409.09)(0.3) = 3.125\alpha$$

$$\alpha = 19.64 \text{ rad/s}^2$$

Kinematics: Here, $\omega_0 = \left(1200\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 40\pi$ rad/s. Since the angular acceleration is constant,



(b)

$$\circlearrowleft + \qquad \omega = \omega_0 + \alpha t$$

$$0 = 40\pi + (-19.64)t$$

$$t = 6.40 \text{ s} \qquad \textbf{Ans.}$$

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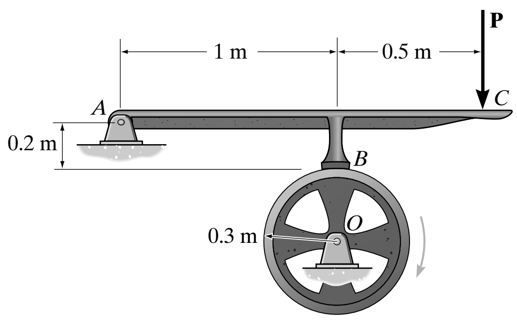
•17–85. The 50-kg flywheel has a radius of gyration about its center of mass of $k_O = 250$ mm. It rotates with a constant angular velocity of 1200 rev/min before the brake is applied. If the coefficient of kinetic friction between the brake pad B and the wheel's rim is $\mu_k = 0.5$, determine the constant force $\mathbf{P}$ that must be applied to the braking mechanism's handle in order to stop the wheel in 100 revolutions.



Kinematics: Here,

$$\omega_0 = \left(1200\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 40\pi\text{ rad/s}$$

and

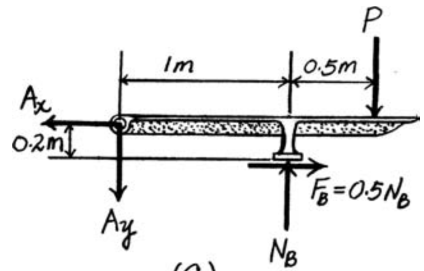
$$\theta = (100\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 200\pi\text{ rad}$$

Since the angular acceleration is constant,

$$\circlearrowleft+\qquad \omega^2 = \omega_0{}^2 + \alpha(\theta - \theta_0)$$

$$0^2 = (40\pi)^2 + 2\alpha(200\pi - 0)$$

$$\alpha = -12.57\text{ rad/s}^2 = 12.57\text{ rad/s}^2$$

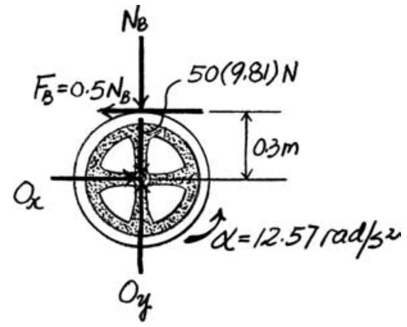


(a)

Equilibrium: Writing the moment equation of equilibrium about point A using the free-body diagram of the brake shown in Fig. a,

$$\circlearrowleft+\Sigma M_A = 0;\qquad N_B\,(1) + 0.5N_B\,(0.2) - P(1.5) = 0$$

$$N_B = 1.3636P$$



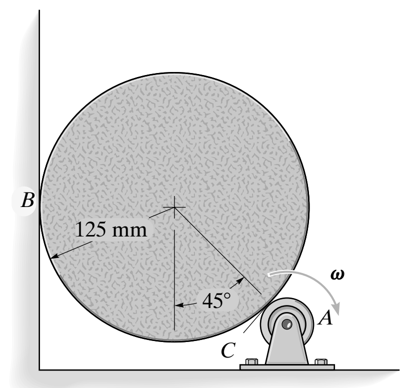
(b)

Equations of Motion: The mass moment of inertia of the flywheel about its center is $I_O = mk_O{}^2 = 50(0.25^2) = 3.125\text{ kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel shown in Fig. b,

$$+\Sigma M_O = I_O\alpha;\qquad 0.5(1.3636\,P)(0.3) = 3.125(12.57)$$

$$P = 191.98\text{ N} = 192\text{ N}\qquad\textbf{Ans.}$$

17–86. The 5-kg cylinder is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.

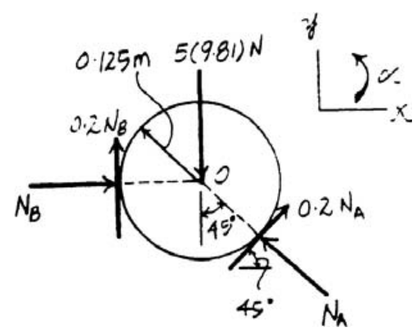


Equations of Motion: The mass moment of inertia of the cylinder about point O is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.125^2) = 0.0390625\text{ kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;\qquad N_B + 0.2N_A\cos 45^\circ - N_A\sin 45^\circ = 0\qquad\textbf{(1)}$$

$$+\uparrow\Sigma F_y = m(a_G)_y;\qquad 0.2N_B + 0.2N_A\sin 45^\circ + N_A\cos 45^\circ - 5(9.81) = 0\qquad\textbf{(2)}$$

$$\circlearrowleft+\Sigma M_O = I_O\,\alpha;\qquad 0.2N_A\,(0.125) - 0.2N_B\,(0.125) = 0.0390625\alpha\qquad\textbf{(3)}$$



Solving Eqs. (1), (2), and (3) yields;

$$N_A = 51.01\text{ N}\qquad N_B = 28.85\text{ N}$$

$$\alpha = 14.2\text{ rad/s}^2\qquad\textbf{Ans.}$$

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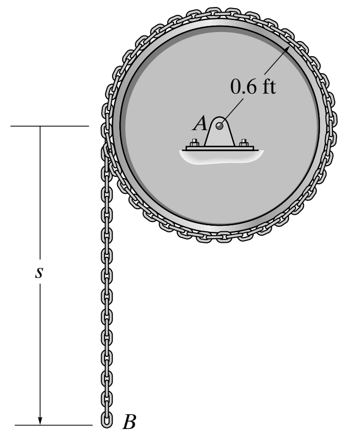
17–87. The drum has a weight of 50 lb and a radius of gyration $k_A = 0.4$ ft. A 35-ft-long chain having a weight of 2 lb/ft is wrapped around the outer surface of the drum so that a chain length of $s = 3$ ft is suspended as shown. If the drum is originally at rest, determine its angular velocity after the end B has descended $s = 13$ ft. Neglect the thickness of the chain.



$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 2s(0.6) = \left(\frac{2s}{32.2}\right)[(\alpha)(0.6)](0.6) + \left[\left(\frac{50}{32.2}\right)(0.4)^2 + \frac{2(35 - s)}{32.2}(0.6)^2\right]\alpha$$

$$1.2s = 0.02236s\alpha + (0.24845 + 0.7826 - 0.02236s)\alpha$$

$$1.164s = \alpha$$

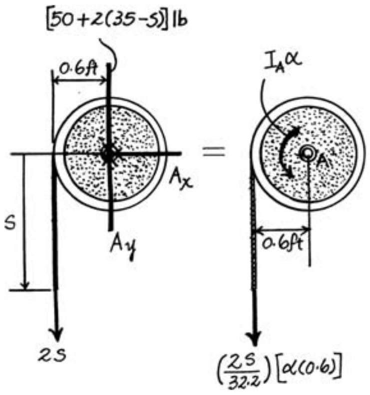
$$\alpha\, d\theta = \alpha\left(\frac{ds}{0.6}\right) = \omega\, d\omega$$

$$1.164s\left(\frac{ds}{0.6}\right) = \omega\, d\omega$$

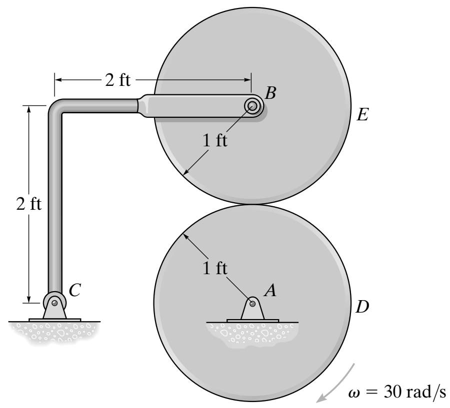
$$1.9398\int_3^{13} s\, ds = \int_0^{\omega} \omega\, d\omega$$

$$1.9398\left[\frac{(13)^2}{2} - \frac{(3)^2}{2}\right] = \frac{1}{2}\omega^2$$

$$\omega = 17.6 \text{ rad/s} \qquad \textbf{Ans.}$$



***17–88.** Disk D turns with a constant clockwise angular velocity of 30 rad/s. Disk E has a weight of 60 lb and is initially at rest when it is brought into contact with D. Determine the time required for disk E to attain the same angular velocity as disk D. The coefficient of kinetic friction between the two disks is $\mu_k = 0.3$. Neglect the weight of bar BC.



Equations of Motion: The mass moment of inertia of disk E about point B is given by $I_B = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{60}{32.2}\right)(1^2) = 0.9317 \text{ slug}\cdot\text{ft}^2$. Applying Eq. 17–16, we have

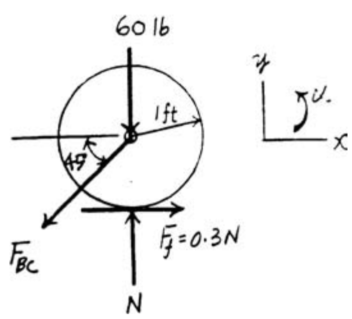
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.3N - F_{BC}\cos 45^\circ = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - F_{BC}\sin 45^\circ - 60 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad 0.3N(1) = 0.9317\alpha \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields:

$$F_{BC} = 36.37 \text{ lb} \qquad N = 85.71 \text{ lb} \qquad \alpha = 27.60 \text{ rad/s}^2$$



Kinematics: Applying equation $\omega = \omega_0 + \alpha_t$, we have

$$(\circlearrowleft +) \qquad 30 = 0 + 27.60t$$

$$t = 1.09 \text{ s} \qquad \textbf{Ans.}$$

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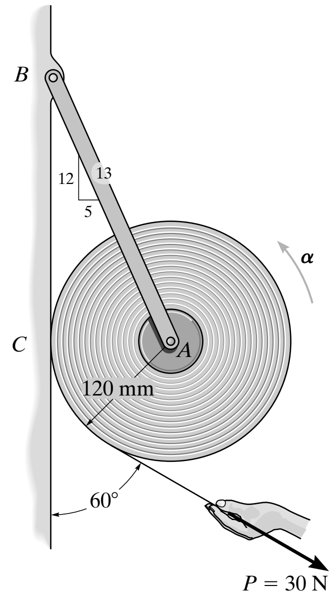
•17–89. A 17-kg roll of paper, originally at rest, is supported by bracket AB. If the roll rests against a wall where the coefficient of kinetic friction is $\mu_C = 0.3$, and a constant force of 30 N is applied to the end of the sheet, determine the tension in the bracket as the paper unwraps, and the angular acceleration of the roll. For the calculation, treat the roll as a cylinder.

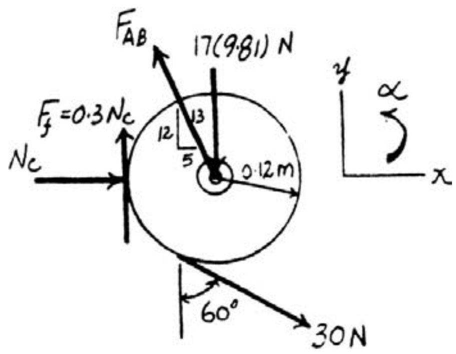


Equations of Motion: The mass moment of inertia of the paper roll about point A is given by $I_A = \frac{1}{2}mr^2 = \frac{1}{2}(17)\left(0.12^2\right) = 0.1224\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad N_C - F_{AB}\left(\frac{5}{13}\right) + 30\sin 60° = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad 0.3N_C + F_{AB}\left(\frac{12}{13}\right) - 30\cos 60° - 17(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft +\Sigma M_A = I_A\,\alpha; \qquad 30(0.12) - 0.3N_C(0.12) = 0.1224\alpha \qquad \textbf{(3)}$$



Solving Eqs. (1), (2), and (3) yields:

$$F_{AB} = 183\ \text{N} \qquad \alpha = 16.4\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$N_C = 44.23\ \text{N}$$

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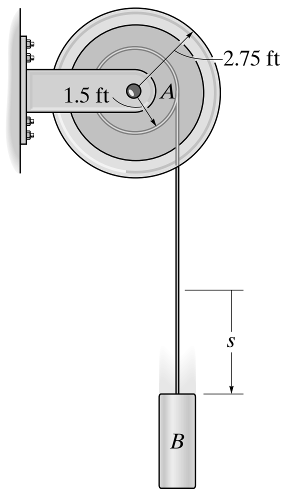
17–90. The cord is wrapped around the inner core of the spool. If a 5-lb block B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = 3$ s. Neglect the mass of the cord. The spool has a weight of 180 lb and the radius of gyration about the axle A is $k_A = 1.25$ ft. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.



System:

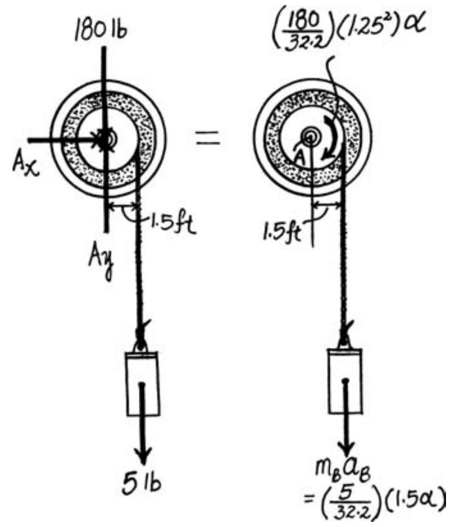
$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 5(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\alpha + \left(\frac{5}{32.2}\right)(1.5\alpha)(1.5)$$

$$\alpha = 0.8256 \text{ rad/s}^2$$

$$(\circlearrowleft +) \qquad \omega = \omega_0 + a_c t$$

$$\omega = 0 + (0.8256)(3)$$

$$\omega = 2.48 \text{ rad/s} \qquad \textbf{Ans.}$$



Also,

Spool:

$$\circlearrowleft +\Sigma M_A = I_A\alpha; \qquad T(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\,\alpha$$

Weight:

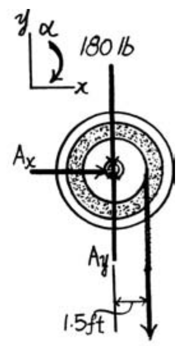
$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 5 - T = \left(\frac{5}{32.2}\right)(1.5\alpha)$$

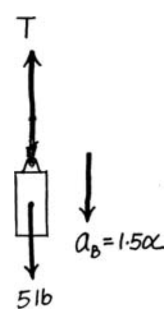
$$\alpha = 0.8256 \text{ rad/s}^2$$

$$(\circlearrowleft +) \qquad \omega = \omega_0 + a_c t$$

$$\omega = 0 + (0.8256)(3)$$

$$\omega = 2.48 \text{ rad/s} \qquad \textbf{Ans.}$$





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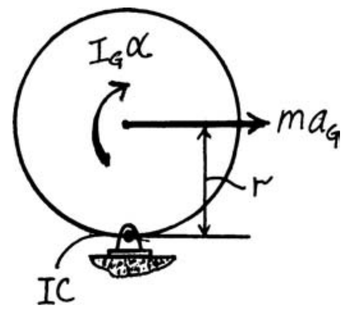
17–91. If a disk *rolls without slipping* on a horizontal surface, show that when moments are summed about the instantaneous center of zero velocity, IC, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.



$\circlearrowleft + \Sigma M_{IC} = \Sigma (M_k)_{IC}; \qquad \Sigma M_{IC} = I_G\,\alpha + (ma_G)r$

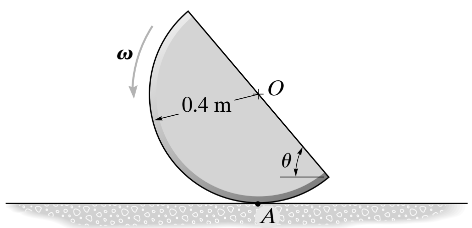
Since there is no slipping, $a_G = \alpha r$.

Thus, $\Sigma M_{IC} = (I_G + mr^2)\,\alpha$.

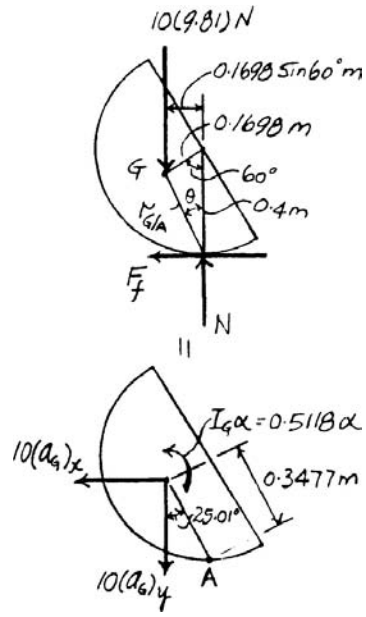
By the parallel-axis theorem, the term in parenthesis represents I_{IC}. Thus,

$$\Sigma M_{IC} = I_{IC}\,\alpha \qquad \textbf{Q.E.D.}$$

***17–92.** The 10-kg semicircular disk is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. Determine the normal and frictional forces it exerts on the ground at A at this instant. Assume the disk does not slip as it rolls.



Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\left(0.1698^2\right) = 0.5118 \text{ kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4)\cos 60°} = 0.3477$ m. Also, using the law of sines, $\dfrac{\sin\theta}{0.1698} = \dfrac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad 10(9.81)(0.1698 \sin 60°) = 0.5118\alpha + 10(a_G)_x \cos 25.01°\,(0.3477) + 10(a_G)_y \sin 25.01°(0.3477) \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m\,(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$$

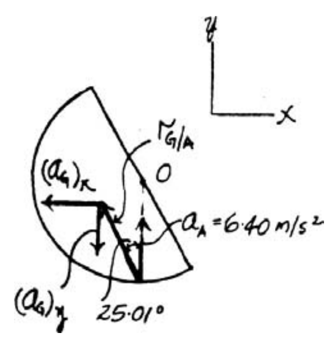
$$+\uparrow F_y = m(a_G)_y; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$$

Kinematics: Since the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477 \sin 25.01°\mathbf{i} + 0.3477 \cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$



Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields:

$\alpha = 13.85 \text{ rad/s}^2 \qquad (a_G)_x = 2.012 \text{ m/s}^2 \qquad (a_G)_y = 0.6779 \text{ m/s}^2$

$F_f = 20.1$ N $\qquad N = 91.3$ N $\qquad$ **Ans.**

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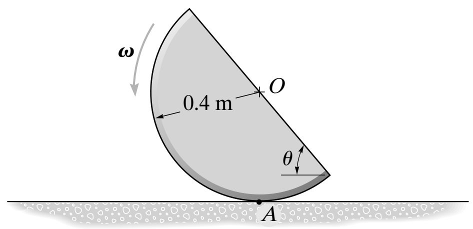
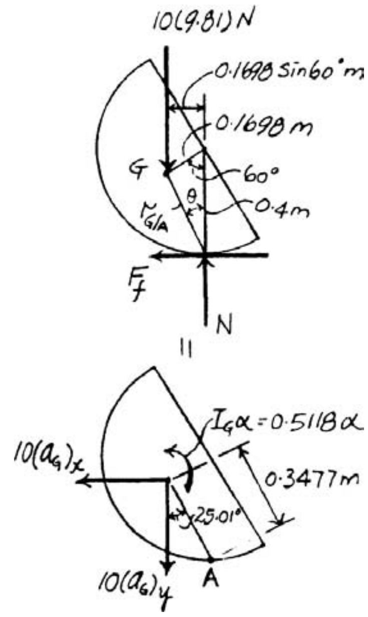
•17–93. The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60^\circ$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.



Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\,(0.1698^2) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)\,(0.4)\cos 60^\circ} = 0.3477$ m Also, using law of sines, $\frac{\sin\theta}{0.1698} = \frac{\sin 60^\circ}{0.3477}$, $\theta = 25.01^\circ$. Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 10(9.81)(0.1698 \sin 60^\circ) = 0.5118\alpha$$
$$+ 10(a_G)_x \cos 25.01^\circ(0.3477)$$
$$+ 10(a_G)_y \sin 25.01^\circ(0.3477) \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$$

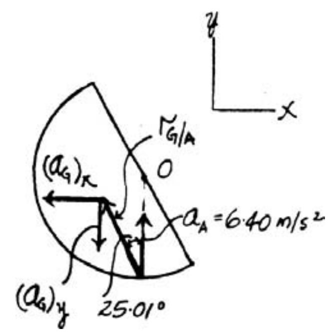
$$+\uparrow F_y = m(a_G)_y; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$$

Kinematics: Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477 \sin 25.01^\circ\mathbf{i} + 0.3477 \cos 25.01^\circ\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\,\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$



Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields:

$$\alpha = 13.85\ \text{rad/s}^2 \qquad (a_G)_x = 2.012\ \text{m/s}^2 \qquad (a_G)_y = 0.6779\ \text{m/s}^2$$

$$F_f = 20.12\ \text{N} \qquad N = 91.32\ \text{N}$$

Since $F_f < (F_f)_{\text{max}} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**. **Ans.**

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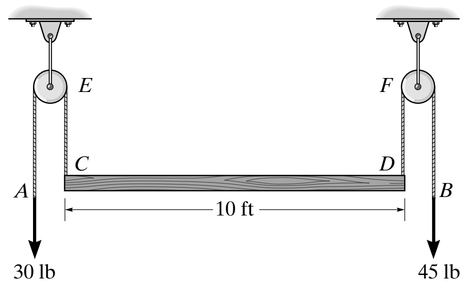
17–94. The uniform 50-lb board is suspended from cords at C and D. If these cords are subjected to constant forces of 30 lb and 45 lb, respectively, determine the initial acceleration of the board's center and the board's angular acceleration. Assume the board is a thin plate. Neglect the mass of the pulleys at E and F.



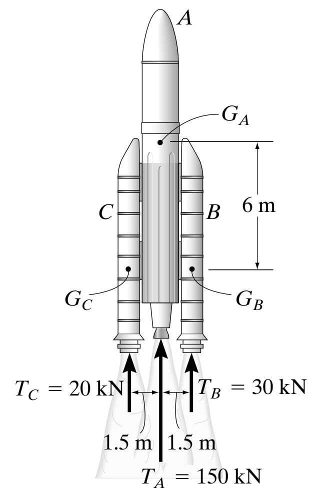
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 45 + 30 - 50 = \left(\frac{50}{32.2}\right) a_G$$

$$a_G = 16.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_G = I_G \alpha; \qquad -30(5) + 45(5) = \left[\frac{1}{12}\left(\frac{50}{32.2}\right)(10)^2\right]\alpha$$

$$\alpha = 5.80 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

17–95. The rocket consists of the main section A having a mass of 10 Mg and a center of mass at G_A. The two identical booster rockets B and C each have a mass of 2 Mg with centers of mass at G_B and G_C, respectively. At the instant shown, the rocket is traveling vertically and is at an altitude where the acceleration due to gravity is $g = 8.75 \text{ m/s}^2$. If the booster rockets B and C suddenly supply a thrust of $T_B = 30$ kN and $T_C = 20$ kN, respectively, determine the angular acceleration of the rocket. The radius of gyration of A about G_A is $k_A = 2$ m and the radii of gyration of B and C about G_B and G_C are $k_B = k_C = 0.75$ m.



Equations of Motion: The mass moment of inertia of the main section and booster rockets about G is

$$(I_G)_A = 10(10^3)(2^2) + 2\left(2(10^3)(0.75^2) + 2(10^3)(1.5^2 + 6^2)\right)$$

$$= 195.25(10^3) \text{ kg}\cdot\text{m}^2$$

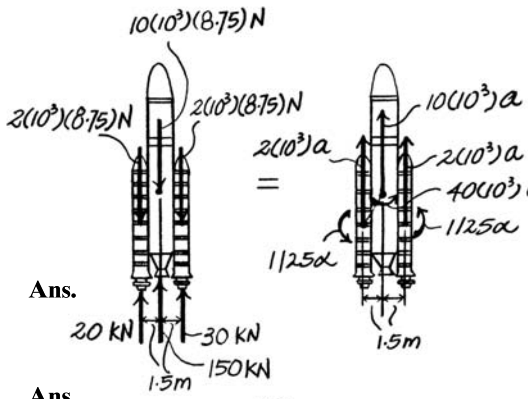
$$+\uparrow \Sigma F_y = m(a_G)_y;$$

$$150(10^3) + 20(10^3) + 30(10^3)$$

$$-\left[2(10^3) + 2(10^3) + 10(10^3)\right](8.75)$$

$$= \left[2(10^3) + 2(10^3) + 10(10^3)\right]a$$

$$a = 5.536 \text{ m/s}^2 = 5.54 \text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$



(a)

$$+\Sigma(M_G)_A = \Sigma(I_G)_A\,\alpha; \qquad 30(10^3)(1.5) - 20(10^3)(1.5) = 195.25(10^3)\alpha$$

$$\alpha = 0.0768 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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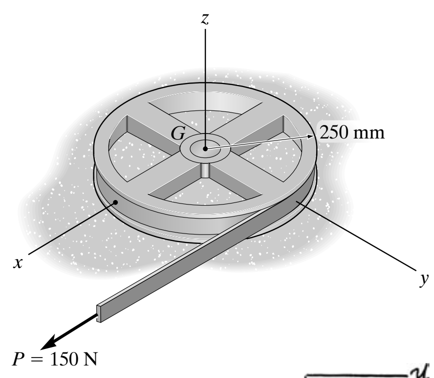
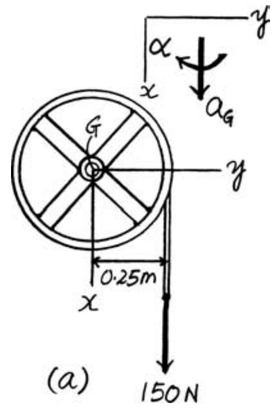
***17–96.** The 75-kg wheel has a radius of gyration about the z axis of $k_z = 150$ mm. If the belt of negligible mass is subjected to a force of $P = 150$ N, determine the acceleration of the mass center and the angular acceleration of the wheel. The surface is smooth and the wheel is free to slide.



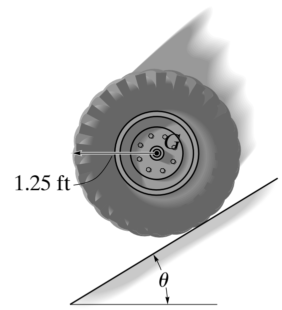
Equations of Motion: The mass moment of inertia of the wheel about the z axis is $(I_G)_z = mk_z^{\,2} = 75\left(0.15^2\right) = 1.6875\ \text{kg}\cdot\text{m}^2$. Referring to the free-body diagram of the wheel shown in Fig. a, we have

$+\downarrow \Sigma F_x = m(a_G)_x;\qquad 150 = 75a_G \qquad a_G = 2\ \text{m/s}^2$ **Ans.**

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad -150(0.25) = -1.6875\alpha \qquad \alpha = 22.22\ \text{rad/s}^2$ **Ans.**



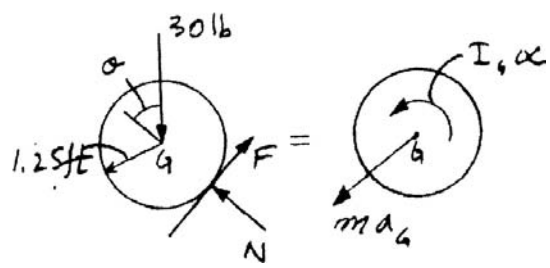
•17–97. The wheel has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the wheel's angular acceleration as it rolls down the incline. Set $\theta = 12°$.



$+\swarrow \Sigma F_x = m(a_G)_x;\qquad 30 \sin 12° - F = \left(\dfrac{30}{32.2}\right)a_G$

$+\nwarrow \Sigma F_y = m(a_G)_y;\qquad N - 30 \cos 12° = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad F(1.25) = \left[\left(\dfrac{30}{32.2}\right)(0.6)^2\right]\alpha$



Assume the wheel does not slip.

$$a_G = (1.25)\alpha$$

Solving:

$F = 1.17$ lb

$N = 29.34$ lb

$a_G = 5.44\ \text{ft/s}^2$

$\alpha = 4.35\ \text{rad/s}^2$ **Ans.**

$F_{max} = 0.2(29.34) = 5.87\ \text{lb} > 1.17\ \text{lb}$ **OK**

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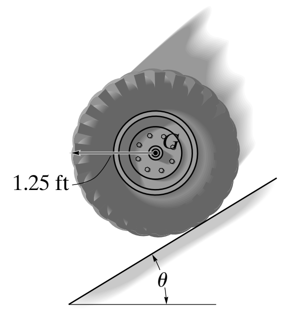
17–98. The wheel has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the maximum angle θ of the inclined plane so that the wheel rolls without slipping.

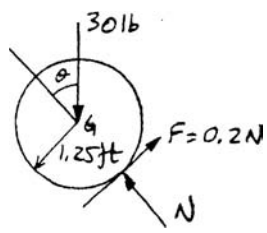


Since wheel is on the verge of slipping:

$+\swarrow\Sigma F_x = m(a_G)_x\,; \qquad 30\sin\theta - 0.2N = \left(\dfrac{30}{32.2}\right)(1.25\alpha)$ **(1)**

$+\nwarrow\Sigma F_y = m(a_G)_y\,; \qquad N - 30\cos\theta = 0$ **(2)**

$\circlearrowleft +\Sigma M_C = I_G\,\alpha; \qquad 0.2N(1.25) = \left[\left(\dfrac{30}{32.2}\right)(0.6)^2\right]\alpha$ **(3)**



Substituting Eqs.(2) and (3) into Eq. (1),

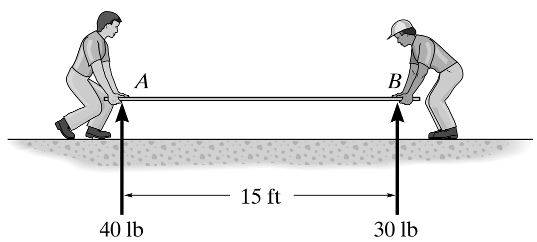
$$30\sin\theta - 6\cos\theta = 26.042\cos\theta$$

$$30\sin\theta = 32.042\cos\theta$$

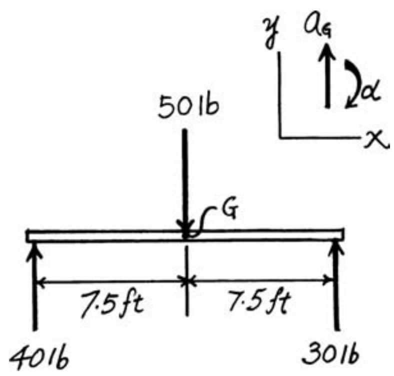
$$\tan\theta = 1.068$$

$$\theta = 46.9^\circ$$ **Ans.**

17–99. Two men exert constant vertical forces of 40 lb and 30 lb at ends A and B of a uniform plank which has a weight of 50 lb. If the plank is originally at rest in the horizontal position, determine the initial acceleration of its center and its angular acceleration. Assume the plank to be a slender rod.



Equations of Motion: The mass moment of inertia of the plank about its mass center is given by $I_G = \dfrac{1}{12}ml^2 = \dfrac{1}{12}\left(\dfrac{50}{32.2}\right)\left(15^2\right) = 29.115\ \text{slug}\cdot\text{ft}^2$ Applying Eq. 17–14, we have



$+\uparrow\Sigma F_y = m(a_G)_y; \qquad 40 + 30 - 50 = \left(\dfrac{50}{32.2}\right)a_G$

$$a_G = 12.9\ \text{ft/s}^2$$ **Ans.**

$\circlearrowleft +\Sigma M_G = I_G\,\alpha; \qquad 30(7.5) - 40(7.5) = -29.115\,\alpha$

$$\alpha = 2.58\ \text{rad/s}^2$$ **Ans.**

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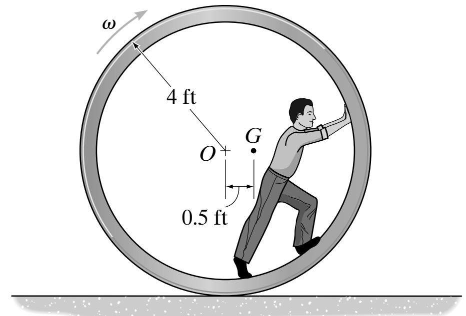
***17–100.** The circular concrete culvert rolls with an angular velocity of $\omega = 0.5$ rad/s when the man is at the position shown. At this instant the center of gravity of the culvert and the man is located at point G, and the radius of gyration about G is $k_G = 3.5$ ft. Determine the angular acceleration of the culvert. The combined weight of the culvert and the man is 500 lb. Assume that the culvert rolls without slipping, and the man does not move within the culvert.



Equations of Motion: The mass moment of inertia of the system about its mass center is $I_G = mk_G{}^2 = \frac{500}{32.2}\left(3.5^2\right) = 190.22 \text{ slug}\cdot\text{ft}^2$. Writing the moment equation of motion about point A, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A; \quad -500(0.5) = -\frac{500}{32.2}(a_G)_x(4) - \frac{500}{32.2}(a_G)_y(0.5) - 190.22\alpha \quad \textbf{(1)}$$

Kinematics: Since the culvert rolls without slipping,

$$a_0 = \alpha r = \alpha(4) \rightarrow$$

Applying the relative acceleration equation and referring to Fig. b,

$$a_G = a_O + \alpha \times r_{G/O} - \omega^2 \mathbf{r}_{G/A}$$

$$(a_G)_x \mathbf{i} - (a_G)_y \mathbf{j} = 4\alpha\mathbf{i} + (-\alpha\mathbf{k}) \times (0.5\mathbf{i}) - 0.5^2(0.5\mathbf{i})$$

$$(a_G)_x \mathbf{i} - (a_G)_y \mathbf{j} = (4\alpha - 0.125)\mathbf{i} - 0.5\alpha\mathbf{j}$$

Equating the **i** and **j** components,

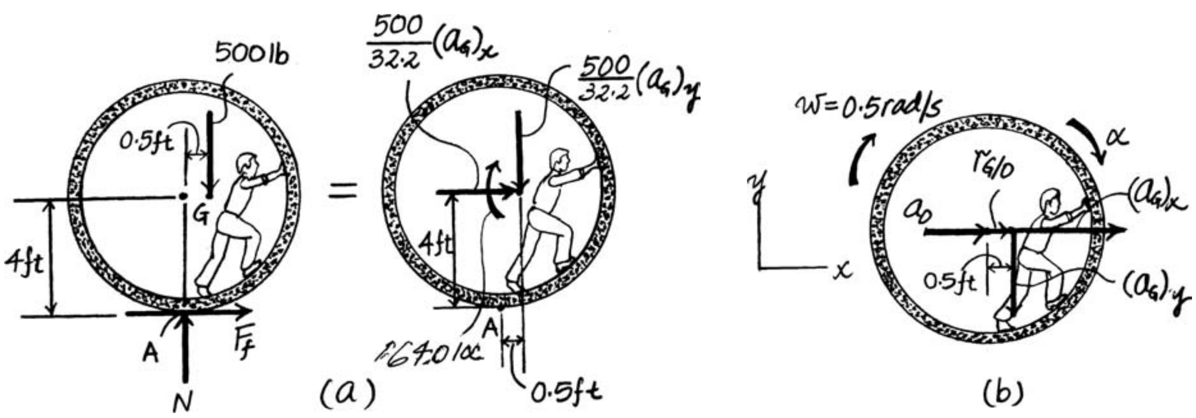
$$(a_G)_x = 4\alpha - 0.125 \quad \textbf{(2)}$$

$$(a_G)_y = 0.5\alpha \quad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$-500(0.5) = -\frac{500}{32.2}(4\alpha - 0.125)(4) - \frac{500}{32.2}(0.5\alpha)(0.5) - 190.22\alpha$$

$$\alpha = 0.582 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



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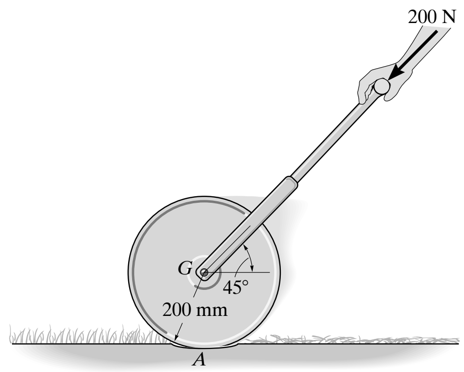
•17–101. The lawn roller has a mass of 80 kg and a radius of gyration $k_G = 0.175$ m. If it is pushed forward with a force of 200 N when the handle is at 45°, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are $\mu_s = 0.12$ and $\mu_k = 0.1$, respectively.



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $\quad 200 \cos 45° - F_A = 80 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A - 80(9.81) - 200 \sin 45° = 0$

$\zeta + \Sigma M_G = I_G \alpha;$ $\quad F_A(0.2) = 80(0.175)^2 \alpha$

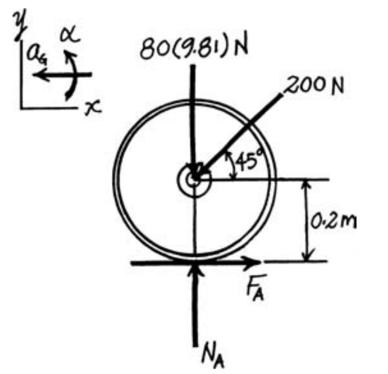
Assume no slipping: $a_G = 0.2\alpha$

$F_A = 61.32$ N

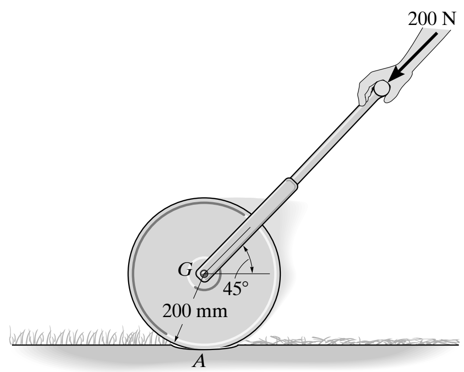
$N_A = 926.2$ N

$\alpha = 5.01 \text{ rad/s}^2$ **Ans.**

$(F_A)_{max} = \mu_s N_A = 0.12(926.2) = 111.1 \text{ N} > 61.32 \text{ N}$ **OK**



17–102. Solve Prob. 17–101 if $\mu_s = 0.6$ and $\mu_k = 0.45$.



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $\quad 200 \cos 45° - F_A = 80 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A - 80(9.81) - 200 \sin 45° = 0$

$\zeta + \Sigma M_G = I_G \alpha;$ $\quad F_A(0.2) = 80(0.175)^2 \alpha$

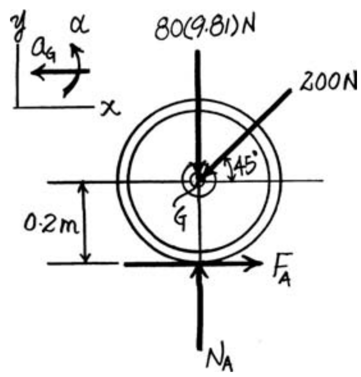
Assume no slipping: $a_G = 0.2\,\alpha$

$F_A = 61.32$ N

$N_A = 926.2$ N

$\alpha = 5.01 \text{ rad/s}^2$ **Ans.**

$(F_A)_{max} = \mu_s N_A = 0.6(926.2 \text{ N}) = 555.7 \text{ N} > 61.32 \text{ N}$ **OK**



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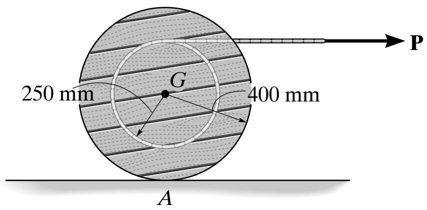
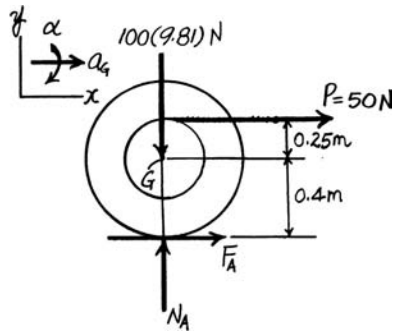
17–103. The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x ; \qquad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y ; \qquad N_A - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$



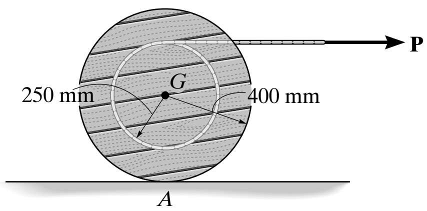
Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 1.30 \text{ rad/s}^2$ **Ans.**

$a_G = 0.520 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 2.00 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$ **OK**

***17–104.** Solve Prob. 17–103 if the cord and force $P = 50$ N are directed vertically upwards.

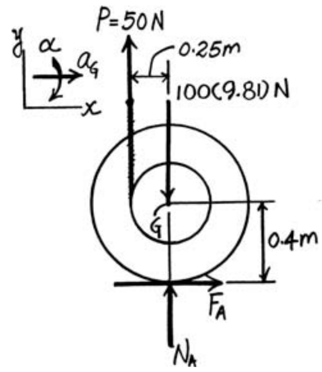


$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x ; \qquad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y ; \qquad N_A + 50 - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\,\alpha$



$\alpha = 0.500 \text{ rad/s}^2$ **Ans.**

$a_G = 0.2 \text{ m/s}^2 \qquad N_A = 931 \text{ N} \qquad F_A = 20 \text{ N}$

Since $(F_A)_{max} = 0.2(931) = 186.2 \text{ N} > 20 \text{ N}$ **OK**

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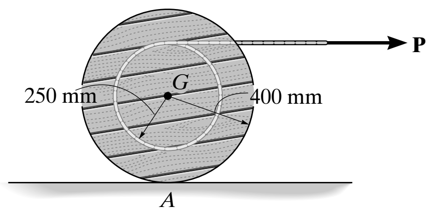
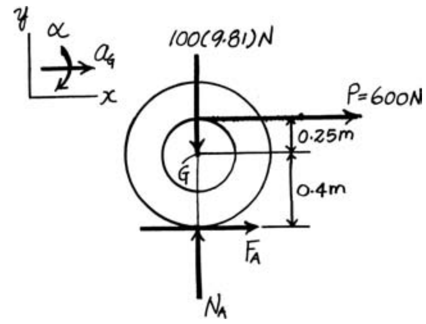
•17–105. The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.





$\stackrel{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad 600 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad 600(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

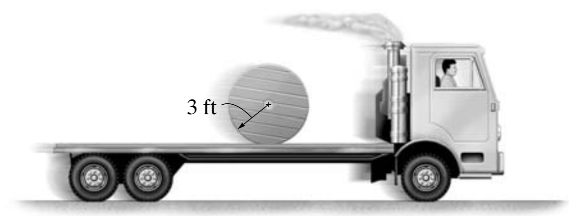
Assume no slipping: $a_G = 0.4\alpha$

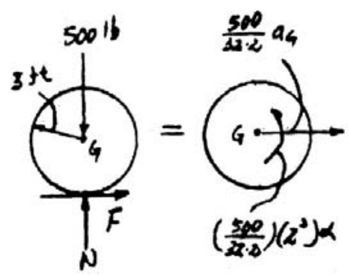
$\alpha = 15.6 \text{ rad/s}^2$ **Ans.**

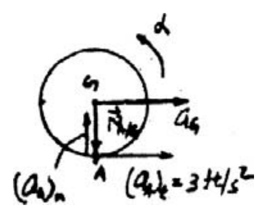
$a_G = 6.24 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 24.0 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 24.0 \text{ N}$ **OK**

17–106. The truck carries the spool which has a weight of 500 lb and a radius of gyration of $k_G = 2$ ft. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at 3 ft/s². Assume the spool does not slip on the bed of the truck.







$\stackrel{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad F = \left(\frac{500}{32.2}\right)a_G$ **(1)**

$\circlearrowleft + \ \Sigma M_G = I_G\,\alpha;\qquad F(3) = \left(\frac{500}{32.2}\right)(2)^2\,\alpha$ **(2)**

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$\left[\underset{\rightarrow}{(a_A)_t}\right] + \left[\underset{\uparrow}{(a_A)_n}\right] = \left[\underset{\rightarrow}{a_G}\right] + \left[\underset{\rightarrow}{3\alpha}\right] + \left[\underset{\uparrow}{(a_{A/G})_n}\right]$$

$\left(\stackrel{+}{\rightarrow}\right)\quad 3 = a_G + 3\alpha$ **(3)**

Solving Eqs. (1), (2), and (3) yields:

$F = 14.33 \text{ lb} \qquad a_G = 0.923 \text{ ft/s}^2$

$\alpha = 0.692 \text{ rad/s}^2.$ **Ans.**

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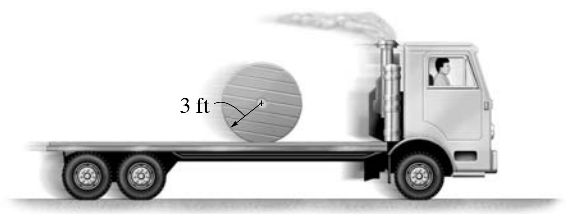
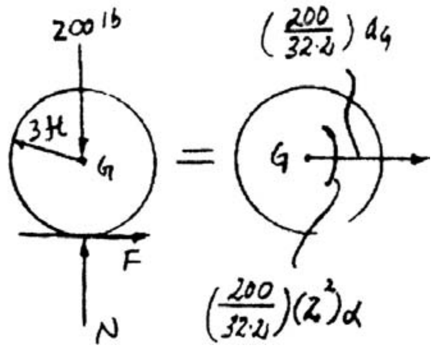
17–107. The truck carries the spool which has a weight of 200 lb and a radius of gyration of $k_G = 2$ ft. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at 5 ft/s^2. The coefficients of static and kinetic friction between the spool and the truck bed are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.



$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 200 = 0 \qquad N = 200 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F = \left(\frac{200}{32.2}\right)a_G \qquad$ **(1)**

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad F(3) = \left(\frac{200}{32.2}\right)(2)^2\alpha \qquad$ **(2)**



Assume no slipping occurs at the point of contact. Hence, $(a_A)_t = 5$ ft/s^2.

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

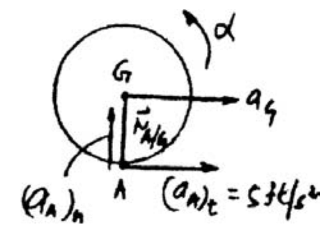
$$\left[\underset{\rightarrow}{(a_A)_t}\right] + \left[\underset{\uparrow}{(a_A)_n}\right] = \left[\underset{\rightarrow}{a_G}\right] + \left[\underset{\rightarrow}{3\alpha}\right] + \left[\underset{\uparrow}{(a_{A/G})_n}\right]$$

$\left(\xrightarrow{+}\right) \qquad 5 = a_G + 3\alpha \qquad$ **(3)**

Solving Eqs. (1), (2), and (3) yields:

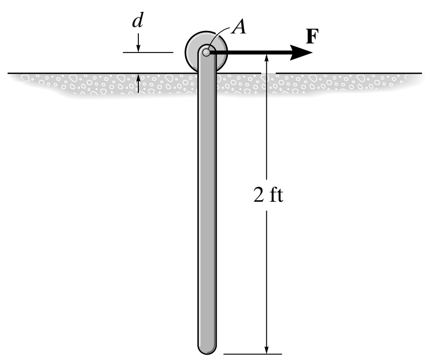
$$F = 9.556 \text{ lb} \qquad a_G = 1.538 \text{ ft/s}^2$$

$$\alpha = 1.15 \text{ rad/s}^2$$ **Ans.**



Since $F_{max} = (200 \text{ lb})(0.15) = 30 \text{ lb} > 9.556$ lb **OK**

***17–108.** A uniform rod having a weight of 10 lb is pin supported at A from a roller which rides on a horizontal track. If the rod is originally at rest, and a horizontal force of $F = 15$ lb is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size d in the computations.

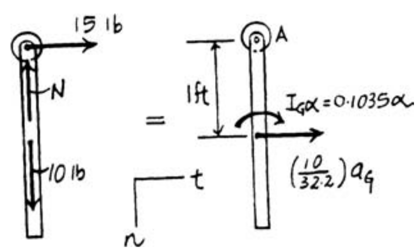


Equations of Motion: The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug}\cdot\text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17–16, we have

$\Sigma F_t = m(a_G)_t; \qquad 15 = \left(\frac{10}{32.2}\right)a_G \quad a_G = 48.3 \text{ ft/s}^2$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 0 = \left(\frac{10}{32.2}\right)(48.3)(1) - 0.1035\,\alpha$

$$\alpha = 144.9 \text{ rad/s}^2$$



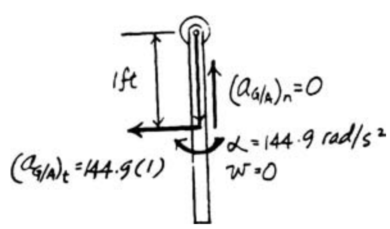
Kinematics: Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of roller A can be obtain by analyzing the motion of points A and G. Applying Eq. 16–17, we have

$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\left[\underset{\rightarrow}{48.3}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{144.9(1)}\right] + \left[0\right]$$

$(\xrightarrow{+}) \qquad 48.3 = a_A - 144.9$

$$a_A = 193 \text{ ft/s}^2$$ **Ans.**



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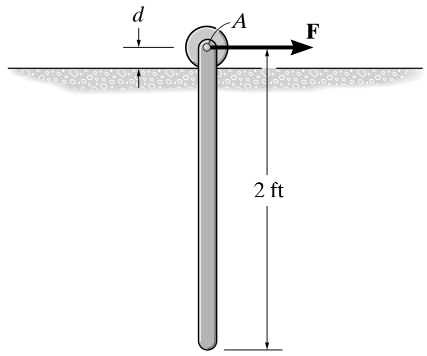
•17–109. Solve Prob. 17–108 assuming that the roller at A is replaced by a slider block having a negligible mass. The coefficient of kinetic friction between the block and the track is $\mu_k = 0.2$. Neglect the dimension d and the size of the block in the computations.



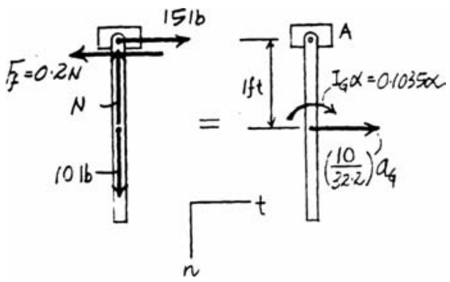
Equations of Motion: The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug} \cdot \text{ft}^2$. At the instant force $\mathbf{F}$ is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17–16, we have

$$\Sigma F_n = m(a_G)_n; \qquad 10 - N = 0 \qquad N = 10.0 \text{ lb}$$

$$\Sigma F_t = m(a_G)_t; \qquad 15 - 0.2(10.0) = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 41.86 \text{ ft/s}^2$$

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = \left(\frac{10}{32.2}\right)(41.86)(1) - 0.1035\alpha$$

$$\alpha = 125.58 \text{ rad/s}^2$$



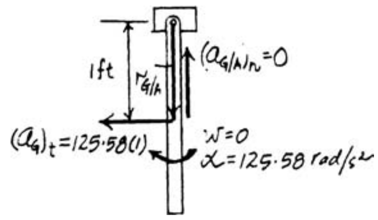
Kinematics: Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of block A can be obtain by analyzing the motion of points A and G. Applying Eq. 16–17, we have

$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

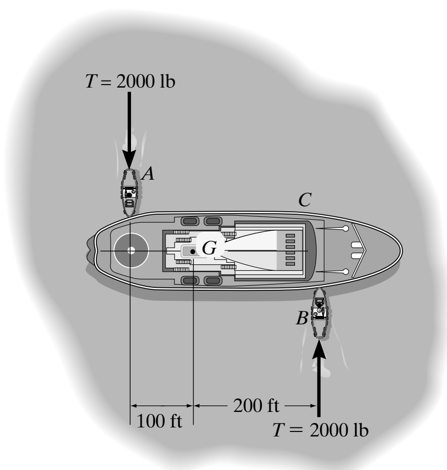
$$\left[\underset{\rightarrow}{41.86}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{125.58(1)}\right] + [0]$$

$$(\overset{+}{\rightarrow}) \qquad 41.86 = a_A - 125.58$$

$$a_A = 167 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



17–110. The ship has a weight of $4(10^6)$ lb and center of gravity at G. Two tugboats of negligible weight are used to turn it. If each tugboat pushes on it with a force of $T = 2000$ lb, determine the initial acceleration of its center of gravity G and its angular acceleration. Its radius of gyration about its center of gravity is $k_G = 125$ ft. Neglect water resistance.



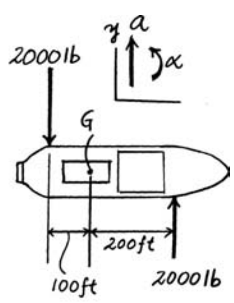
Equations of Motion: Here, the mass moment of inertia of the ship about its mass center is $I_G = mk_G{}^2 = \frac{4(10^6)}{32.2}(125^2) = 1.941(10^9) \text{ slug} \cdot \text{ft}^2$. Referring to the free-body diagrams of the ship shown in Fig. a,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2000 - 2000 = \frac{4(10^6)}{32.2}a$$

$$a = 0 \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_G = I_G\alpha; \qquad 2000(100) + 2000(200) = 1.941(10^9)\alpha$$

$$\alpha = 0.30912(10^{-3}) \text{ rad/s}^2 = 0.309(10^{-3}) \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(a)

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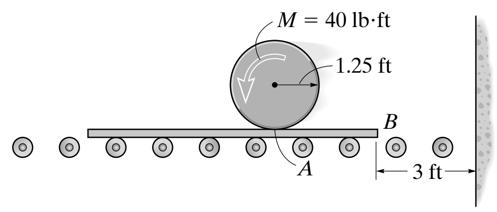
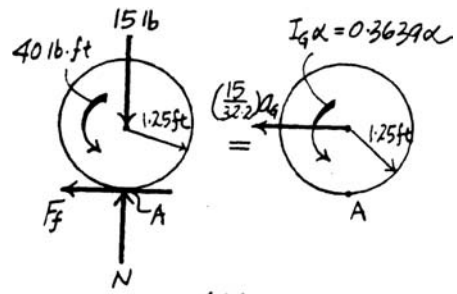
17–111. The 15-lb cylinder is initially at rest on a 5-lb plate. If a couple moment $M = 40 \text{ lb} \cdot \text{ft}$ is applied to the cylinder, determine the angular acceleration of the cylinder and the time needed for the end B of the plate to travel 3 ft to the right and strike the wall. Assume the cylinder does not slip on the plate, and neglect the mass of the rollers under the plate.



Equation of Motions: The mass moment of inertia of the cylinder about its mass center is given by $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{15}{32.2}\right)(1.25^2) = 0.3639 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 17–16 to the cylinder [FBD(a)], we have

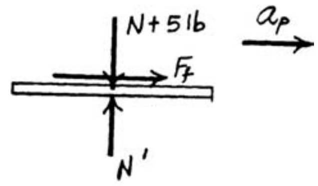


(a)

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad -40 = -\left(\frac{15}{32.2}\right)a_G(1.25) - 0.3639\alpha \qquad \textbf{(1)}$$

$$\xleftarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_f = \left(\frac{15}{32.2}\right)a_G \qquad \textbf{(2)}$$

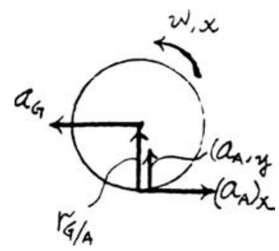
Applying the equation of motion to the place [FBD(b)], we have



(b)

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_f = \left(\frac{5}{32.2}\right)a_P \qquad \textbf{(3)}$$

Kinematics: Analyzing the motion of points G and A by applying Eq. 16–18 with $\mathbf{r}_{G/A} = \{1.25\mathbf{j}\}$ ft, we have



$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-a_G\,\mathbf{i} = (a_A)_x\,\mathbf{i} + (a_A)_y\,\mathbf{j} + \alpha\mathbf{k} \times (1.25\mathbf{j}) - \omega^2(1.25\mathbf{j})$$

$$-a_G\,\mathbf{i} = \left[(a_A)_x - 1.25\alpha\right]\mathbf{i} + \left[(a_A)_y - 1.25\,\omega^2\right]\mathbf{j}$$

Equating **i** components, we have

$$a_G = 1.25\alpha - (a_A)_x \qquad \textbf{(4)}$$

Since the cylinder rolls without slipping on the plate, then $a_P = (a_A)_x$. Substitute into Eq. (4) yields

$$a_G = 1.25\,\alpha - a_P \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), and (5) yields:

$$\alpha = 73.27 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_G = 22.90 \text{ ft/s}^2 \qquad a_P = 68.69 \text{ ft/s}^2 \qquad F_f = 10.67 \text{ lb}$$

The time required for the plate to travel 3 ft is given by

$$s = s_o + v_o\,t + \frac{1}{2}\,a_P\,t^2$$

$$3 = 0 + 0 + \frac{1}{2}\,(68.69)t^2$$

$$t = 0.296 \text{ s} \qquad \textbf{Ans.}$$

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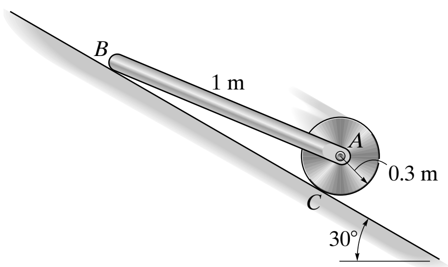
***17–112.** The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. Neglect friction at B.



Equation of Motions:

Disk:

$+\searrow \Sigma F_x = m(a_G)_x; \qquad A_x - F_C + 8(9.81)\sin 30° = 8a_G \qquad$ **(1)**

$+\nearrow \Sigma F_y = m(a_G)_y; \qquad N_C - A_y - 8(9.81)\cos 30° = 0 \qquad$ **(2)**

$\circlearrowleft + \Sigma M_A = I_A \alpha; \qquad F_C(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha \qquad$ **(3)**

Bar:

$+\searrow \Sigma F_x = m(a_G)_x; \qquad 10(9.81)\sin 30° - A_x = 10a_G \qquad$ **(4)**

$+\nearrow \Sigma F_y = m(a_G)_y; \qquad N_B + A_y - 10(9.81)\cos 30° = 0 \qquad$ **(5)**

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad -N_B(0.5\cos 17.46°) + A_x(0.5\sin 17.46°)$

$+ A_y(0.5\cos 17.46°) = 0 \qquad$ **(6)**

Kinematics: Assume no slipping of the disk:

$$a_G = 0.3\alpha \qquad \textbf{(7)}$$

Solving Eqs. (1) through (7):

$A_x = 8.92\text{ N} \qquad A_y = 41.1\text{ N} \qquad N_B = 43.9\text{ N}$

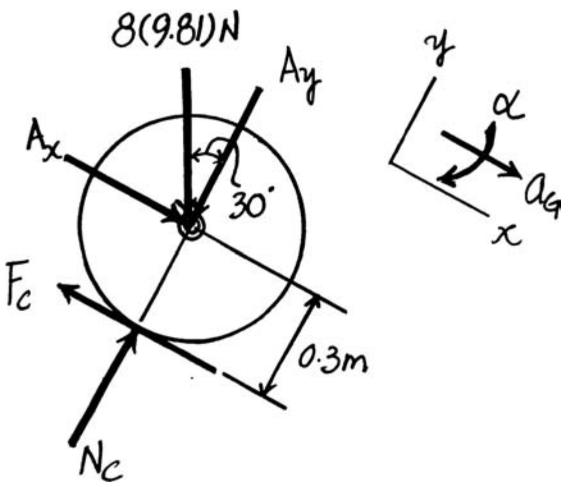
$a_G = 4.01\text{ m/s}^2$

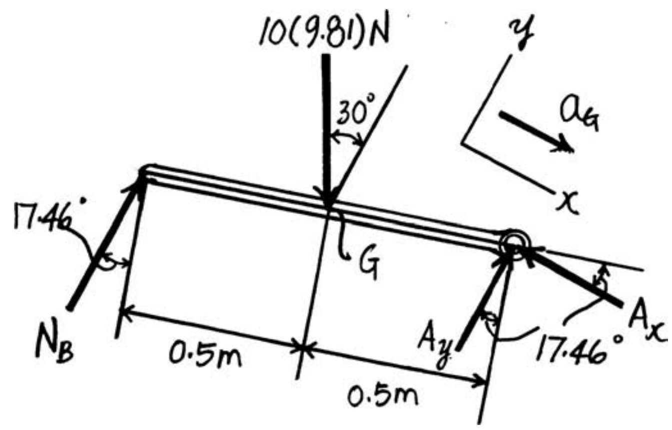
$\alpha = 13.4\text{ rad/s}^2$ **Ans.**

$N_C = 109\text{ N}$

$F_C = 16.1\text{ N}$

$(F_C)_{max} = 0.6(109) = 65.4\text{ N} > 16.1\text{ N}$ **OK**





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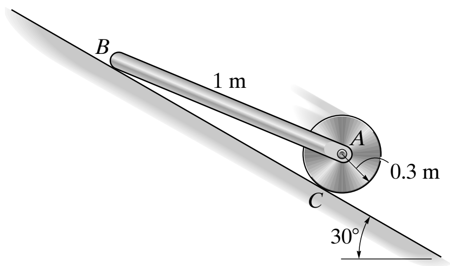
•**17–113.** Solve Prob. 17–112 if the bar is removed. The coefficients of static and kinetic friction between the disk and inclined plane are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.



Equation of Motions:

$+\searrow\Sigma F_x = m(a_G)_x;\quad 8(9.81)\sin 30^\circ - F_C = 8a_G$ **(1)**

$+\nearrow\Sigma F_y = m(a_G)_y;\quad -8(9.81)\cos 30^\circ + N_C = 0$ **(2)**

$\circlearrowleft+\Sigma M_G = I_G\alpha;\quad F_C\,(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha$ **(3)**

Kinematics: Assume no slipping: $a_G = 0.3\alpha$

Solving Eqs. (1)–(3):

$N_C = 67.97$ N

$a_G = 3.27$ m/s^2

$\alpha = 10.9$ rad/s^2

$F_C = 13.08$ N

$(F_C)_{max} = 0.15(67.97) = 10.2\text{ N} < 13.08\text{ N}$ **NG**

Slipping occurs:

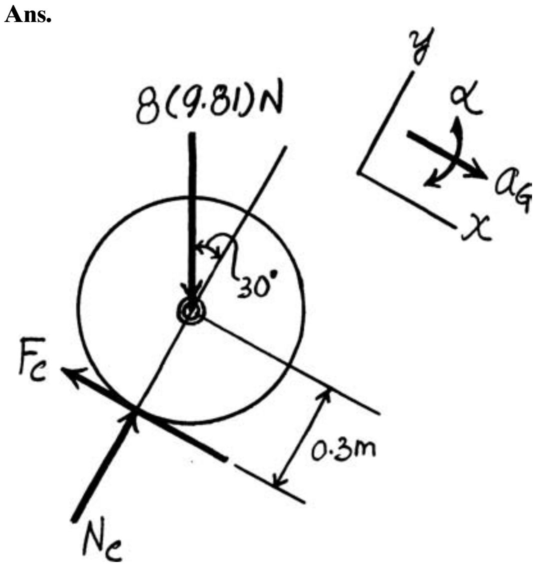
$$F_C = 0.1N_C$$

Solving Eqs. (1) through (3):

$N_C = 67.97$ N

$\alpha = 5.66$ rad/s^2 **Ans.**

$a_G = 4.06$ m/s^2



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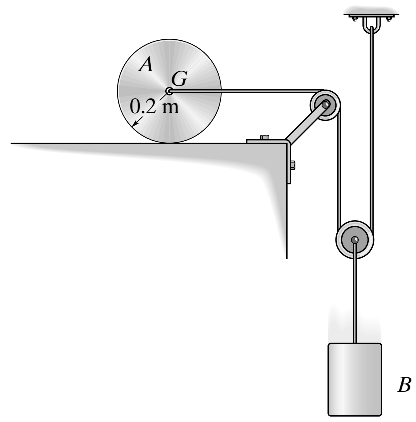
17–114. The 20-kg disk A is attached to the 10-kg block B using the cable and pulley system shown. If the disk rolls without slipping, determine its angular acceleration and the acceleration of the block when they are released. Also, what is the tension in the cable? Neglect the mass of the pulleys.



Equation of Motions:

Disk:

$$\circlearrowleft + \Sigma M_{IC} = \Sigma(M_k)_{IC}; \qquad T(0.2) = -\left[\frac{1}{2}(20)(0.2)^2 + 20(0.2)^2\right]\alpha \qquad \textbf{(1)}$$

Block:

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 10(9.81) - 2T = 10a_B \qquad \textbf{(2)}$$

Kinematics:

$$2s_B + s_A = l$$

$$2a_B = -a_A$$

Also,

$$a_A = 0.2\alpha$$

Thus,

$$a_B = -0.1\alpha \qquad \textbf{(3)}$$

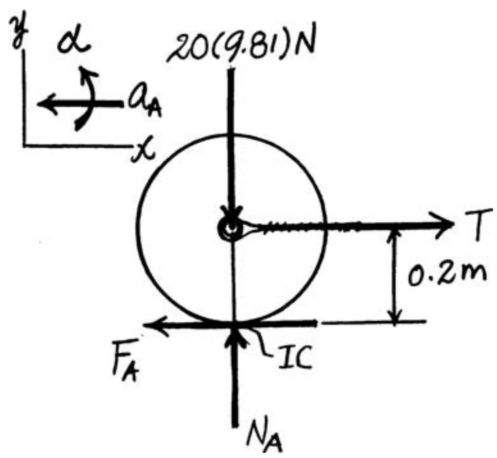
Note the direction for α and a_B are the same for all equations.

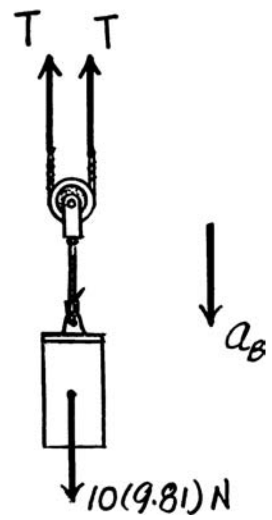
Solving Eqs. (1) through (3):

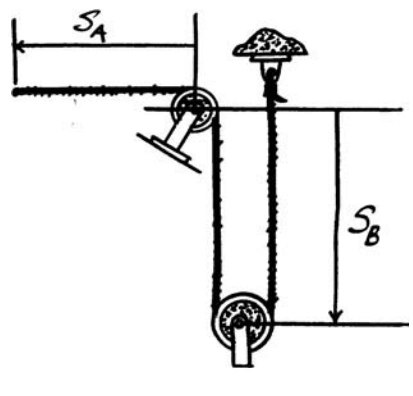
$$a_B = 0.755 \text{ m/s}^2 = 0.755 \text{ m/s}^2 \downarrow \qquad \textbf{Ans.}$$

$$\alpha = -7.55 \text{ rad/s}^2 = 7.55 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$T = 45.3 \text{ N} \qquad \textbf{Ans.}$$







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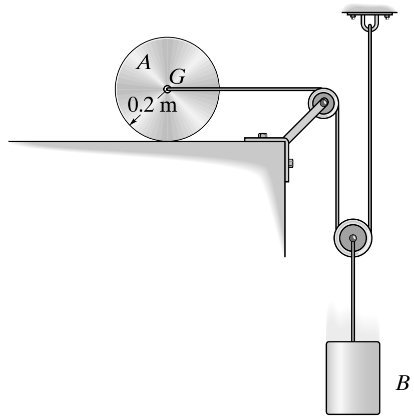
17–115. Determine the minimum coefficient of static friction between the disk and the surface in Prob. 17–114 so that the disk will roll without slipping. Neglect the mass of the pulleys.



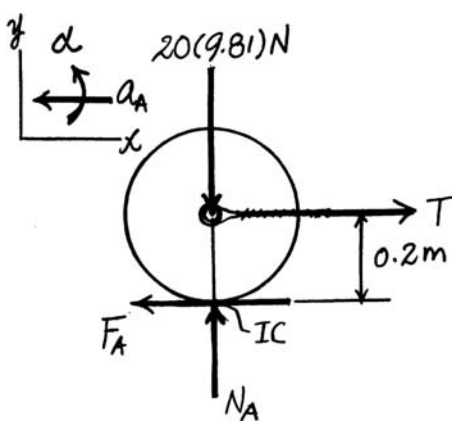
Equation of Motions:

Disk:

$\circlearrowleft + \Sigma M_{IC} = \Sigma (M_k)_{IC}; \quad T(0.2) = -\left[\frac{1}{2}(20)(0.2)^2 + 20(0.2)^2\right]\alpha$ **(1)**

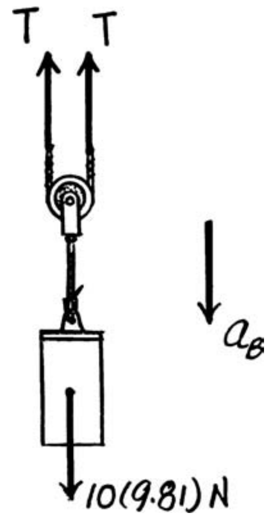
$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad -T + F_A = 20a_A$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 20(9.81) = 0$



Block:

$+\downarrow \Sigma F_y = m(a_G)_y; \quad 10(9.81) - 2T = 10a_B$ **(2)**



Kinematics:

$$2s_B + s_A = l$$

$$2a_B = -a_A$$

Also,

$$a_A = 0.2\alpha$$

Thus,

$$a_B = -0.1\alpha \quad \textbf{(3)}$$

Note the direction for α and a_B are the same for all equations. Solving Eqs. (1) through (3):

$$a_B = 0.755 \text{ m/s}^2$$

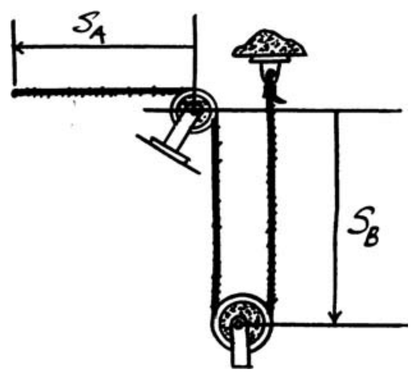
$$\alpha = -7.55 \text{ rad/s}^2$$

$$T = 45.3 \text{ N}$$

Also,

$a_A = 0.2(-7.55) = -1.509 \text{ m/s}^2, \qquad N_A = 196.2 \text{ N}, \qquad F_A = 15.09 \text{ N}$

$\mu_{min} = \frac{15.09}{196.2} = 0.0769$ **Ans.**



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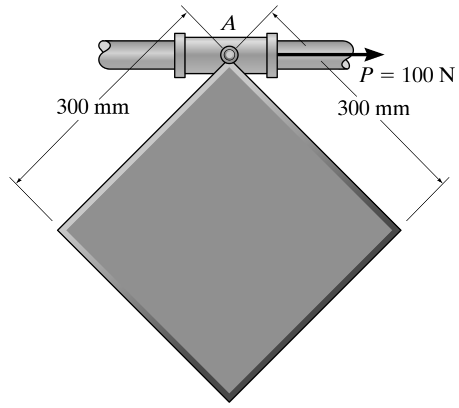
***17–116.** The 20-kg square plate is pinned to the 5-kg smooth collar. Determine the initial angular acceleration of the plate when $P = 100$ N is applied to the collar. The plate is originally at rest.



Equations of Motion: The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(20)\left(0.3^2 + 0.3^2\right) = 0.3\ \text{kg}\cdot\text{m}^2$.

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 100 = 5a_A + 20(a_G)_x \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = \Sigma(\mu_k)_A; \qquad 0 = 20(a_G)_x(0.3\sin 45°) - 0.3\alpha \qquad \textbf{(2)}$$

Kinematics: Applying the relative acceleration equation and referring to Fig. *b*,

$$a_G = a_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\,\mathbf{r}_{G/A}$$

$$(a_G)_x\,\mathbf{i} + (a_G)_y\,\mathbf{j} = a_A\mathbf{i} + (-\alpha\mathbf{k}) \times (-0.3\sin 45°\,\mathbf{j}) - 0$$

$$(a_G)_x\,\mathbf{i} + (a_G)_y\,\mathbf{j} = (a_A - 0.2121\alpha)\mathbf{i}$$

Equating the **i** and **j** components,

$$(a_G)_x = a_A - 0.2121\alpha \qquad \textbf{(3)}$$

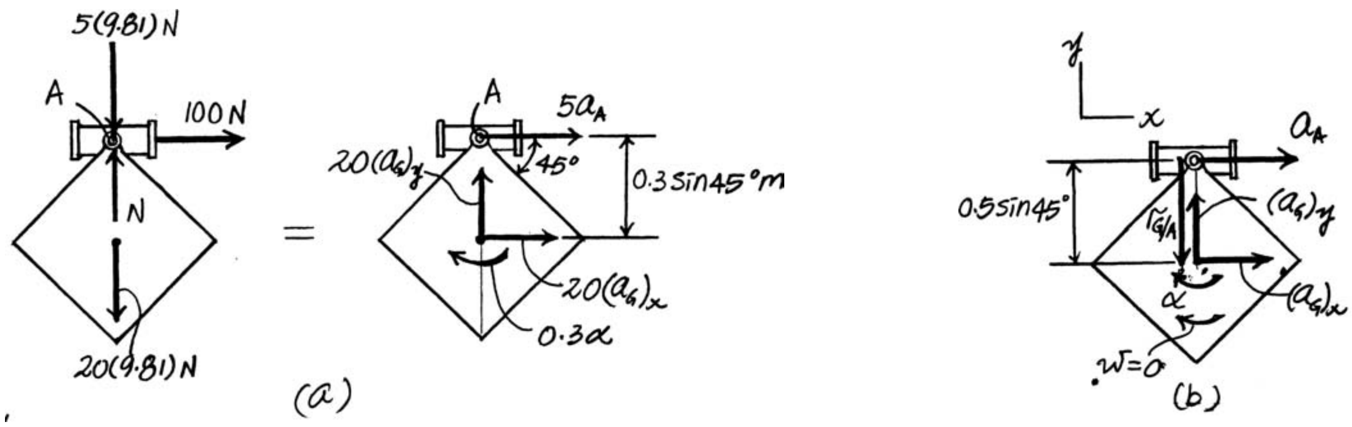
$$(a_G)_y = 0$$

Solving Eqs. (1) through (3) yields:

$$a_A = 10\ \text{m/s}^2 \rightarrow$$

$$(a_G)_x = 2.5\ \text{m/s}^2 \rightarrow$$

$$\alpha = 35.4\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



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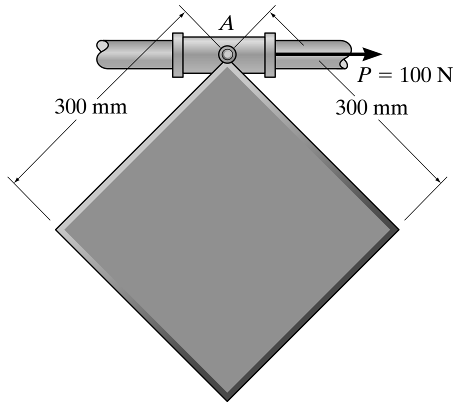
•17–117. The 20-kg square plate is pinned to the 5-kg smooth collar. Determine the initial acceleration of the collar when $P = 100$ N is applied to the collar. The plate is originally at rest.



Equations of Motion: The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(20)\left(0.3^2 + 0.3^2\right) = 0.3 \text{ kg} \cdot \text{m}^2$.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 100 = 5a_x + 20(a_G)_x \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = \Sigma(\mu_k)_A; \qquad 0 = 20(a_G)_x(0.3\sin 45°) - 0.3\alpha \qquad \textbf{(2)}$$

Kinematics: Applying the relative acceleration equation and referring to Fig. *b*,

$$a_G = a_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

$$(a_G)_x\,\mathbf{i} + (a_G)_y\,\mathbf{j} = a_A\mathbf{i} + (-\alpha\mathbf{k}) \times (-0.3 \sin 45°\mathbf{j}) - 0$$

$$(a_G)_x\,\mathbf{i} + (a_G)_y\,\mathbf{j} = (a_A - 0.2121\alpha)\mathbf{i}$$

Equating the **i** and **j** components,

$$(a_G)_x = a_A - 0.2121\alpha \qquad \textbf{(3)}$$

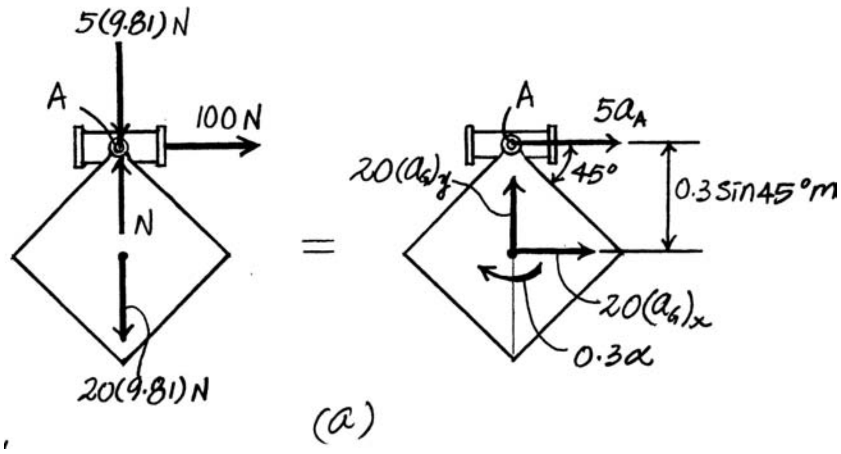
$$(a_G)_y = 0$$

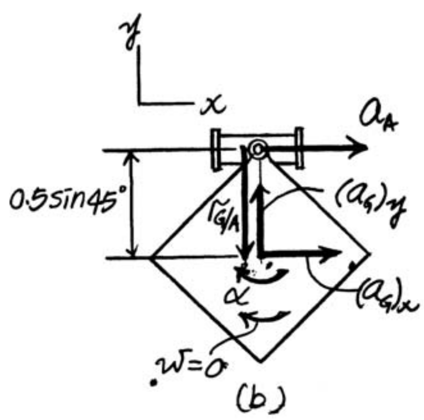
Solving Eqs. (1) through (3) yields

$$a_A = 10 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans.}$$

$$(a_G)_x = 2.5 \text{ m/s}^2 \rightarrow$$

$$\alpha = 35.4 \text{ rad/s}^2$$





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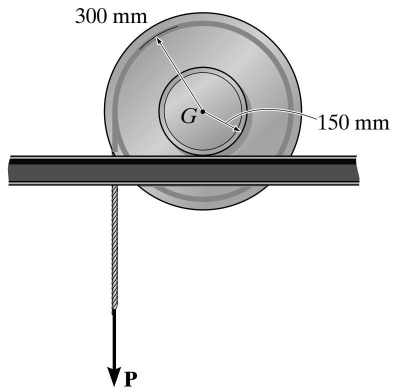
17–118. The spool has a mass of 100 kg and a radius of gyration of $k_G = 200$ mm about its center of mass G. If a vertical force of $P = 200$ N is applied to the cable, determine the acceleration of G and the angular acceleration of the spool. The coefficients of static and kinetic friction between the rail and the spool are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively.



Equations of Motion: The mass moment of inertia of the spool about its mass center is $I_G = mk_G{}^2 = 100\left(0.2^2\right) = 4\ \text{kg}\cdot\text{m}^2$.

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 100a_G \qquad$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 100(9.81) - 200 = 0 \qquad N = 1181\ \text{N}$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 200(0.3) - F_f(0.15) = 4\alpha \qquad$ **(2)**

Kinematics: Assuming that the spool rolls without slipping on the rail,

$$a_G = \alpha r_G = \alpha(0.15) \qquad \textbf{(3)}$$

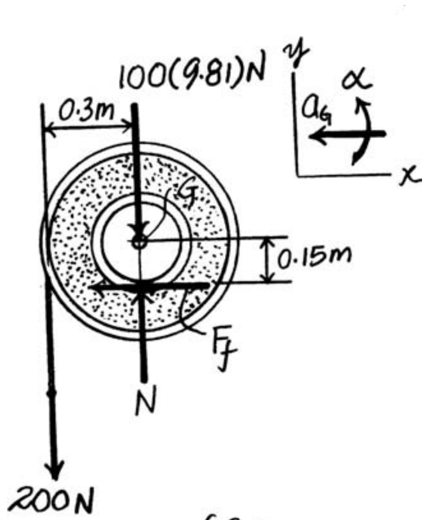
Solving Eqs. (1) through (3) yields:

$$\alpha = 9.60\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$a_G = 1.44\ \text{m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$F_f = 144\ \text{N}$$

Since $F_f < \mu_k N = 0.3(1181) = 354.3$ N, the spool does not slip as assumed.



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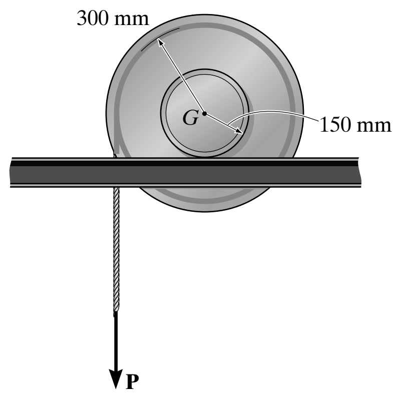
17–119. The spool has a mass of 100 kg and a radius of gyration of $k_G = 200$ mm about its center of mass G. If a vertical force of $P = 500$ N is applied to the cable, determine the acceleration of G and the angular acceleration of the spool. The coefficients of static and kinetic friction between the rail and the spool are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively.



Equations of Motion: The mass moment of inertia of the spool about its mass center is $I_G = mk_G{}^2 = 100\left(0.2^2\right) = 4\ \text{kg}\cdot\text{m}^2$.

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 100a_G \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 100(9.81) - 500 = 0 \qquad N = 1481\ \text{N}$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 500(0.3) - F_f(0.15) = 4\alpha \qquad \textbf{(2)}$$

Kinematics: Assuming that the spool rolls without slipping on the rail,

$$a_G = \alpha r_G = \alpha(0.15) \qquad \textbf{(3)}$$

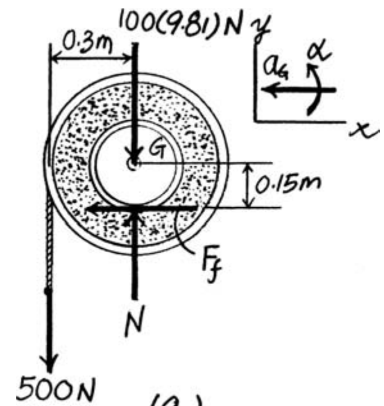
Solving Eqs. (1) through (3) yields:

$$\alpha = 24\ \text{rad/s}^2 \qquad a_G = 3.6\ \text{m/s}^2 \qquad F_f = 360\ \text{N}$$

Since $F_f > \mu_k N = 0.2(1481) = 296.2$ N, the spool slips. Thus, the solution must be reworked using $F_f = \mu_k N = 0.15(1481) = 222.15$ N. Substituting this result into Eqs. (1) and (2),

$$222.15 = 100a_G \qquad a_G = 2.22\ \text{m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$500(0.3) - 222.15(0.15) = 4\alpha \qquad \alpha = 29.17\ \text{rad/s}^2 = 29.2\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



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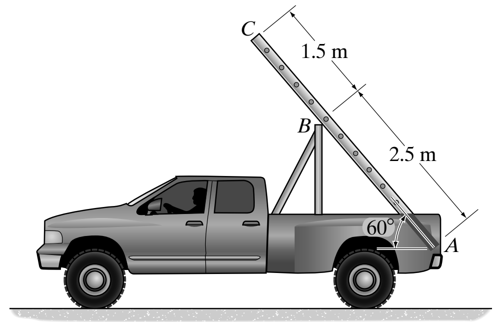
***17–120.** If the truck accelerates at a constant rate of $6\,\text{m/s}^2$, starting from rest, determine the initial angular acceleration of the 20-kg ladder. The ladder can be considered as a uniform slender rod. The support at B is smooth.



Equations of Motion: We must first show that the ladder will rotate when the acceleration of the truck is $6\ \text{m/s}^2$. This can be done by determining the minimum acceleration of the truck that will cause the ladder to lose contact at B, $N_B = 0$. Writing the moment equation of motion about point A using Fig. *a*,

$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 20(9.81)\cos 60°(2) = 20a_{\text{min}}\,(2\sin 60°)$$

$$a_{\text{min}} = 5.664\ \text{m/s}^2$$

Since $a_{\text{min}} < 6\ \text{m/s}^2$, the ladder will in the fact rotate. The mass moment of inertia about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(20)\left(4^2\right) = 26.67\ \text{kg}\cdot\text{m}^2$. Referring to Fig. *b*,

$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 20(9.81)\cos 60°(2) = -20(a_G)_x(2\sin 60°)$$
$$- 20(a_G)_y(2\cos 60°) - 26.67\alpha \qquad \textbf{(1)}$$

Kinematics: The acceleration of A is equal to that of the truck. Thus, $a_A = 6\ \text{m/s}^2 \leftarrow$. Applying the relative acceleration equation and referring to Fig. *c*,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -6\mathbf{i} + (-\alpha\mathbf{k}) \times (-2\cos 60°\,\mathbf{i} + 2\sin 60°\,\mathbf{j}) - \mathbf{0}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = (2\sin 60°\,\alpha - 6)\mathbf{i} + \alpha\mathbf{j}$$

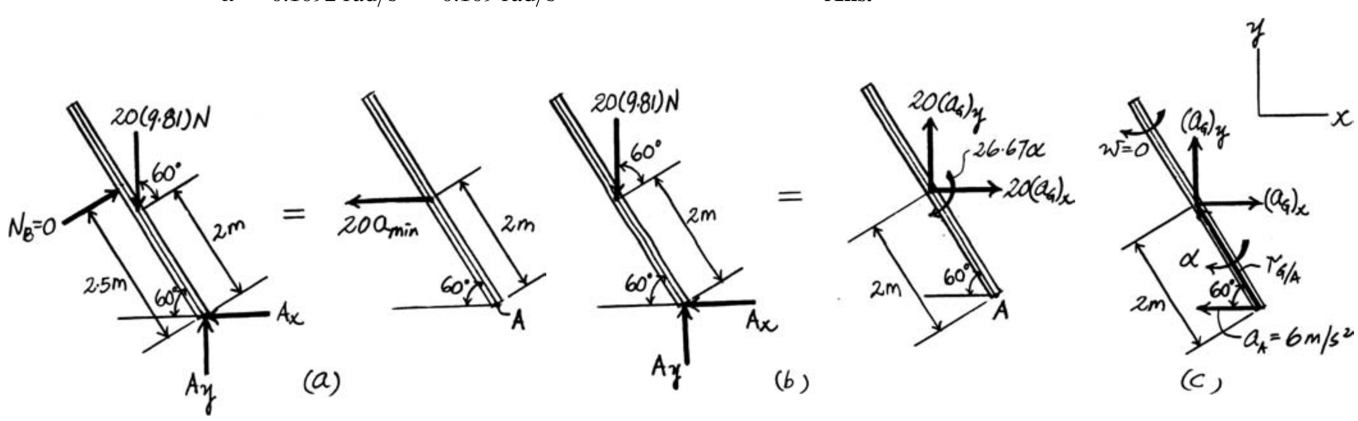
Equating the **i** and **j** components,

$$(a_G)_x = 2\sin 60°\,\alpha - 6 \qquad \textbf{(2)}$$

$$(a_G)_y = \alpha \qquad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$\alpha = 0.1092\ \text{rad/s}^2 = 0.109\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



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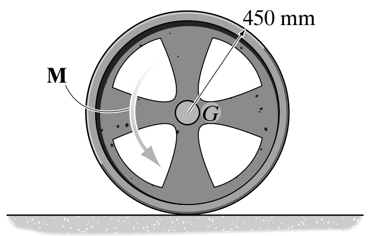
•17–121. The 75-kg wheel has a radius of gyration about its mass center of $k_G = 375$ mm. If it is subjected to a torque of $M = 100\ \text{N}\cdot\text{m}$, determine its angular acceleration. The coefficients of static and kinetic friction between the wheel and the ground are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively.



Equations of Motion: The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 75\left(0.375^2\right) = 10.55\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A,

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \quad 100 = 75a_G(0.45) + 10.55\alpha \qquad \textbf{(1)}$$

Assuming that the wheel rolls without slipping.

$$a_G = \alpha r_G = \alpha(0.45) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$\alpha = 3.886\ \text{rad/s}^2 = 3.89\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

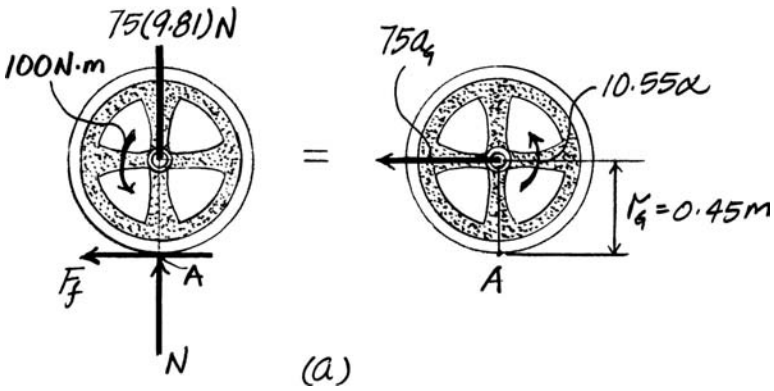
$$a_G = 1.749\ \text{m/s}^2$$

Writing the force equation of motion along the x and y axes,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - 75(9.81) = 0 \qquad N = 735.75\ \text{N}$$

$$\xleftarrow{+} \Sigma F_x = m(a_G)_x; \quad F_f = 75(1.749) = 131.15\text{N}$$

Since $F_f < \mu_k N = 0.2(735.75) = 147.15\ \text{N}$, the wheel does not slip as assumed.



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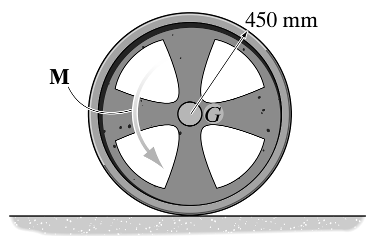
17–122. The 75-kg wheel has a radius of gyration about its mass center of $k_G = 375$ mm. If it is subjected to a torque of $M = 150\ \text{N}\cdot\text{m}$, determine its angular acceleration. The coefficients of static and kinetic friction between the wheel and the ground are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively.



Equations of Motion: The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 75\left(0.375^2\right) = 10.55\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A, we have

$$\circlearrowleft + \Sigma M_A = \Sigma(\mu_k)_A; \quad 150 = 75a_G(0.45) + 10.55\alpha \qquad \textbf{(1)}$$

Assuming that the wheel rolls without slipping,

$$a_G = \alpha r_G = \alpha(0.45) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a_G = 2.623\ \text{m/s}^2$$

$$\alpha = 5.829\ \text{rad/s}^2$$

Writing the force equations of motion along the x and y axes,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 75(9.81) = 0 \qquad N = 735.75\ \text{N}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 75(2.623) = 196.72\ \text{N}$$

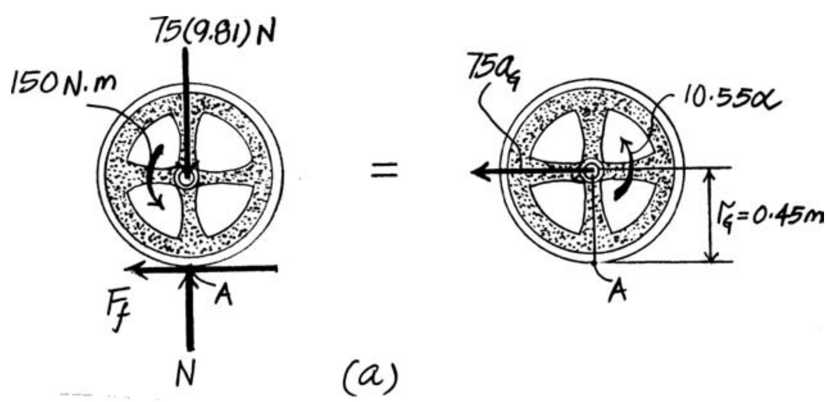
Since $F_f > \mu_k N = 0.2(735.75) = 147.15$ N, the wheel slips. The solution must be reworked using $F_f = \mu_k N = 0.15(735.75) = 110.36$ N. Thus,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 110.36 = 75a_G \qquad a_G = 1.4715\ \text{m/s}^2$$

Substituting this result into Eq. (1), we obtain

$$150 = 75(1.4715)(0.45) + 10.55\alpha$$

$$\alpha = 9.513\ \text{rad/s}^2 = 9.51\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



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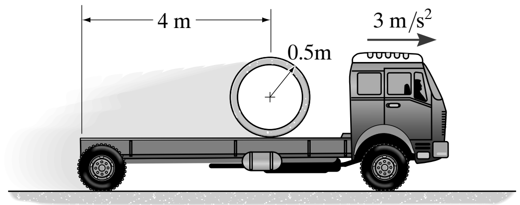
17–123. The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of $3\,\text{m/s}^2$, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500\left(0.5^2\right) = 125\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A using Fig. a,

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3\ \text{m/s}^2$. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} - 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

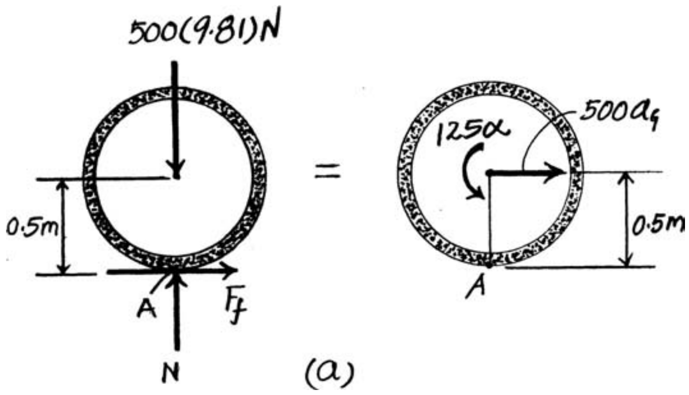
Equating the **i** components,

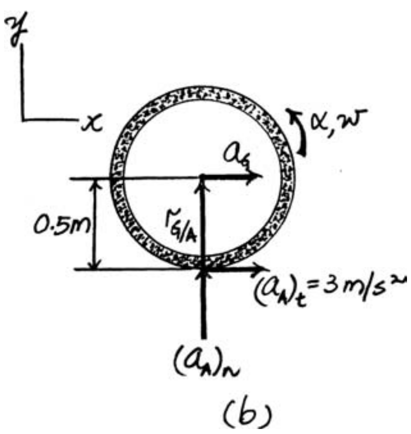
$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a_G = 1.5\ \text{m/s}^2 \rightarrow$$

$$\alpha = 3\ \text{rad/s}^2 \qquad \textbf{Ans.}$$





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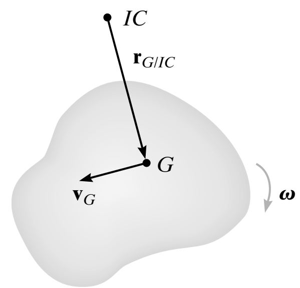
•18–1. At a given instant the body of mass m has an angular velocity $\boldsymbol{\omega}$ and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = \frac{1}{2}I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.



$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G\,\omega^2 \qquad \text{where } v_G = \omega r_{G/IC}$$

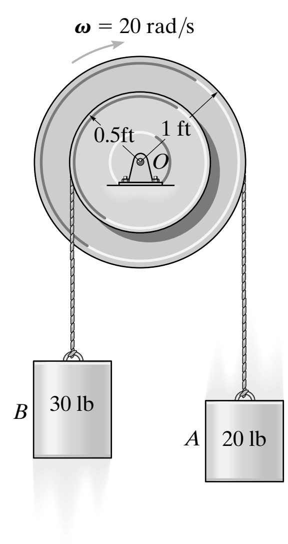
$$= \frac{1}{2} m(\omega r_{G/IC})^2 + \frac{1}{2} I_G\,\omega^2$$

$$= \frac{1}{2}\left(m r_{G/IC}^2 + I_G\right)\omega^2 \qquad \text{However } m r_{G/IC}^2 + I_G = I_{IC}$$

$$= \frac{1}{2} I_{IC}\,\omega^2$$

Q.E.D.

18–2. The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a radius of gyration about its center of $k_O = 0.6$ ft. If it rotates with an angular velocity of 20 rad/s clockwise, determine the kinetic energy of the system. Assume that neither cable slips on the pulley.



$$T = \frac{1}{2} I_O\,\omega_O^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left[(20)(1)\right]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)\left[(20)(0.5)\right]^2$$

$$= 283 \text{ ft}\cdot\text{lb}$$

Ans.

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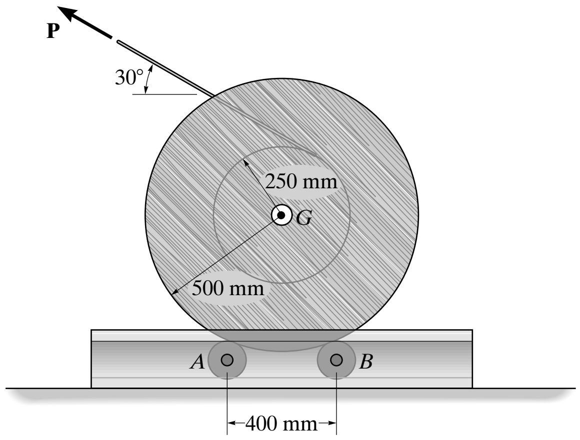
18–3. A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn without slipping on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has rotated two revolutions starting from rest. Neglect the mass of the cable. Each roller can be considered as an 18-kg cylinder, having a radius of 0.1 m. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.



System:

$$T_1 + \Sigma U_{1-2} = T_2$$

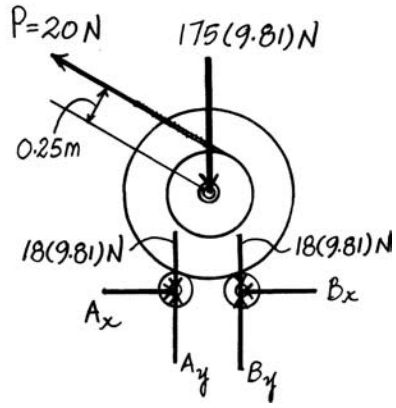
$$[0 + 0 + 0] + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42^2)\right]\omega^2 + \frac{2}{2}\left[\frac{1}{2}(18)(0.1)^2\right]\omega_r^2$$

$$v = \omega_r\,(0.1) = \omega(0.5)$$

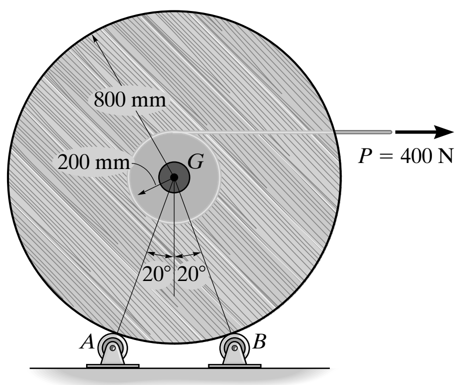
$$\omega_r = 5\omega$$

Solving:

$$\omega = 1.88 \text{ rad/s}$$ **Ans.**



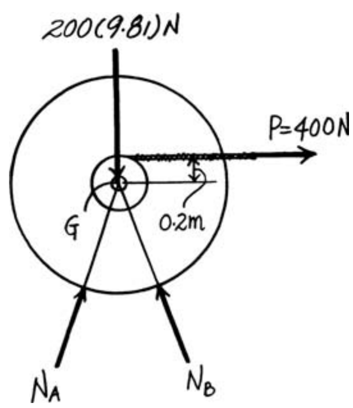
***18–4.** The spool of cable, originally at rest, has a mass of 200 kg and a radius of gyration of $k_G = 325$ mm. If the spool rests on two small rollers A and B and a constant horizontal force of $P = 400$ N is applied to the end of the cable, determine the angular velocity of the spool when 8 m of cable has been unwound. Neglect friction and the mass of the rollers and unwound cable.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (400)(8) = \frac{1}{2}\left[200(0.325)^2\right]\omega_2^2$$

$$\omega_2 = 17.4 \text{ rad/s}$$ **Ans.**



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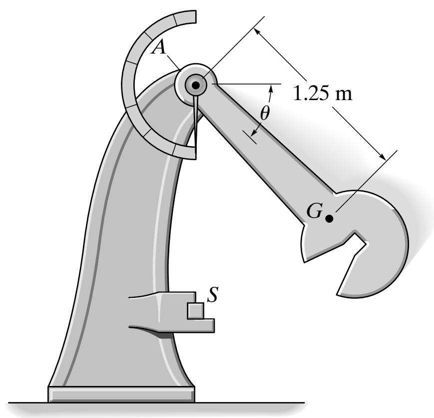
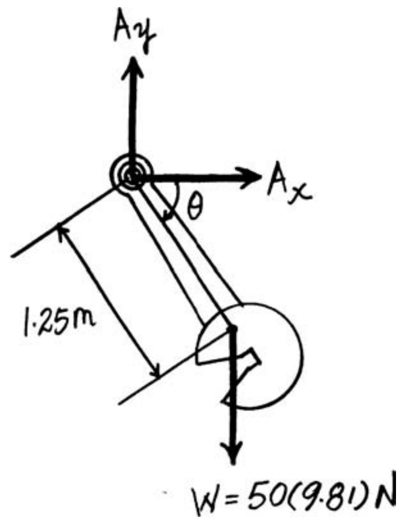
•18–5. The pendulum of the Charpy impact machine has a mass of 50 kg and a radius of gyration of $k_A = 1.75$ m. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.



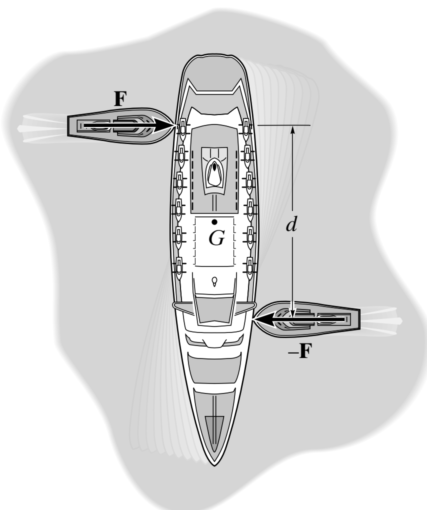
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (50)(9.81)(1.25) = \frac{1}{2}\left[(50)(1.75)^2\right]\omega_2^2$$

$$\omega_2 = 2.83 \text{ rad/s} \qquad \textbf{Ans.}$$



18–6. The two tugboats each exert a constant force **F** on the ship. These forces are always directed perpendicular to the ship's centerline. If the ship has a mass m and a radius of gyration about its center of mass G of k_G, determine the angular velocity of the ship after it turns 90°. The ship is originally at rest.



Principle of Work and Energy: The two tugboats create a couple moment of $M = Fd$ to rotate the ship through an angular displacement of $\theta = \frac{\pi}{2}$ rad. The mass moment of inertia about its mass center is $I_G = mk_G^2$. Applying Eq. 18–14, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + M\theta = \frac{1}{2} I_G\,\omega^2$$

$$0 + Fd\left(\frac{\pi}{2}\right) = \frac{1}{2}\left(mk_G^2\right)\omega^2$$

$$\omega = \frac{1}{k_G}\sqrt{\frac{\pi F d}{m}} \qquad \textbf{Ans.}$$

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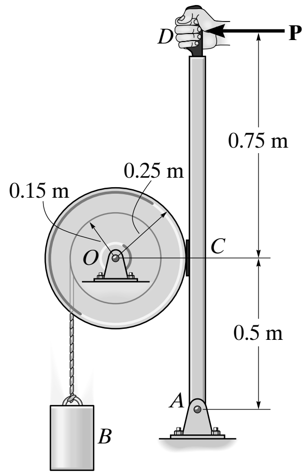
18–7. The drum has a mass of 50 kg and a radius of gyration about the pin at O of $k_O = 0.23$ m. Starting from rest, the suspended 15-kg block B is allowed to fall 3 m without applying the brake ACD. Determine the speed of the block at this instant. If the coefficient of kinetic friction at the brake pad C is $\mu_k = 0.5$, determine the force **P** that must be applied at the brake handle which will then stop the block after it descends *another* 3 m. Neglect the thickness of the handle.



Before braking:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(9.81)(3) = \frac{1}{2}(15)v_B^2 + \frac{1}{2}\left[50(0.23)^2\right]\left(\frac{v_B}{0.15}\right)^2$$

$$v_B = 2.58 \text{ m/s} \qquad \textbf{Ans.}$$

$$\frac{s_B}{0.15} = \frac{s_C}{0.25}$$

Set $s_B = 3$ m, then $s_C = 5$ m.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 - F(5) + 15(9.81)(6) = 0$$

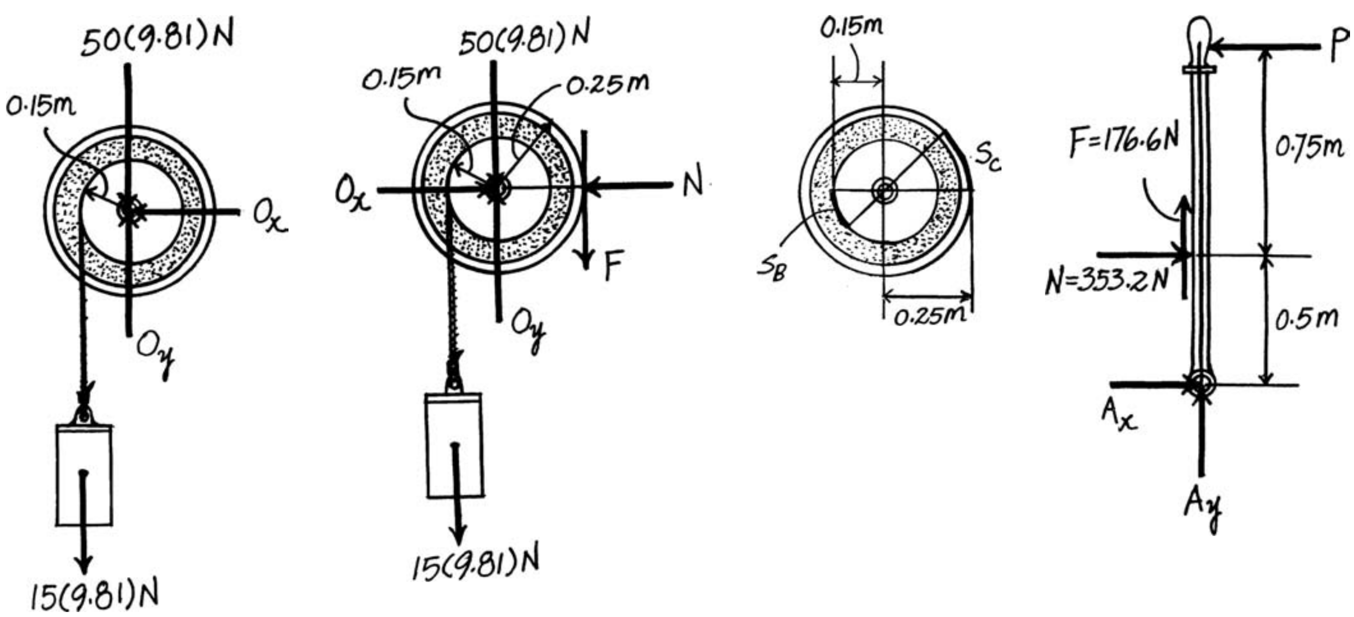
$$F = 176.6 \text{ N}$$

$$N = \frac{176.6}{0.5} = 353.2 \text{ N}$$

Brake arm:

$$\circlearrowleft + \Sigma M_A = 0; \qquad -353.2(0.5) + P(1.25) = 0$$

$$P = 141 \text{ N} \qquad \textbf{Ans.}$$



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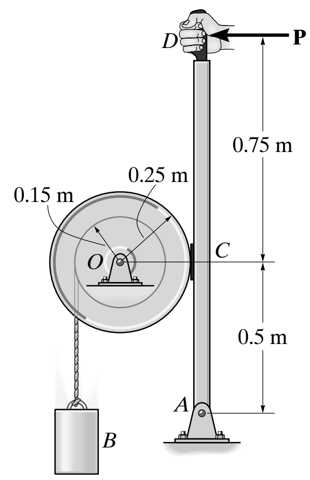
***18–8.** The drum has a mass of 50 kg and a radius of gyration about the pin at O of $k_O = 0.23$ m. If the 15-kg block is moving downward at 3 m/s, and a force of $P = 100$ N is applied to the brake arm, determine how far the block descends from the instant the brake is applied until it stops. Neglect the thickness of the handle. The coefficient of kinetic friction at the brake pad is $\mu_k = 0.5$.



Brake arm:

$$\circlearrowleft + \Sigma M_A = 0; \qquad -N(0.5) + 100(1.25) = 0$$

$$N = 250 \text{ N}$$

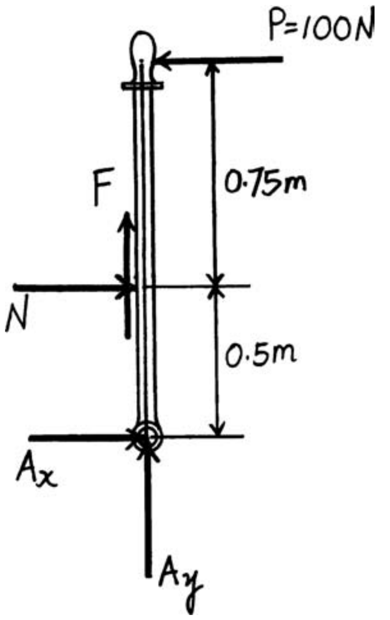
$$F = 0.5(250) = 125 \text{ N}$$

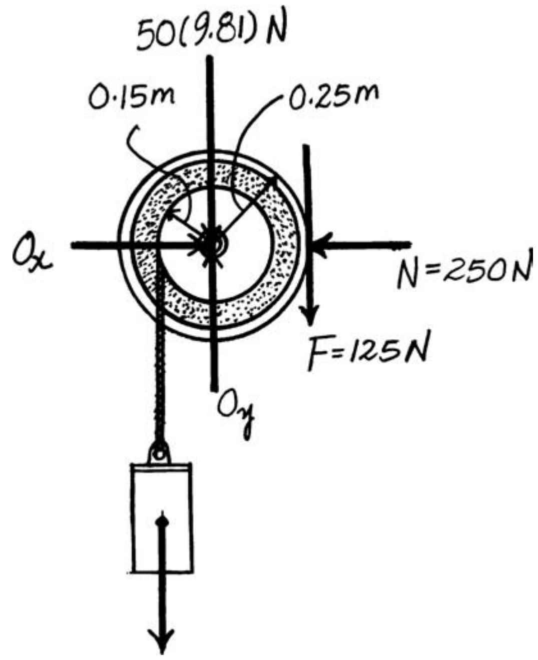
If block descends s, then F acts through a distance $s' = s\left(\frac{0.25}{0.15}\right)$.

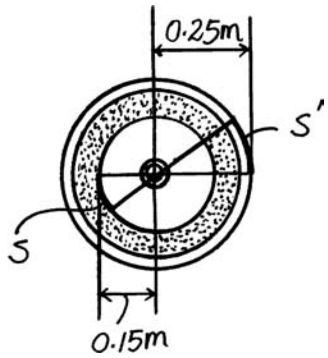
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left[(50)(0.23)^2\right]\left(\frac{3}{0.15}\right)^2 + \frac{1}{2}(15)(3)^2 + 15(9.81)(s) - 125(s)\left(\frac{0.25}{0.15}\right) = 0$$

$$s = 9.75 \text{ m}$$ **Ans.**







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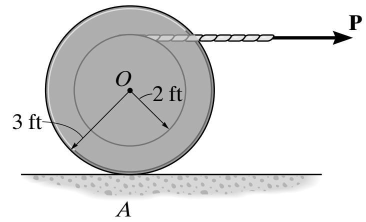
•18–9. The spool has a weight of 150 lb and a radius of gyration $k_O = 2.25$ ft. If a cord is wrapped around its inner core and the end is pulled with a horizontal force of $P = 40$ lb, determine the angular velocity of the spool after the center O has moved 10 ft to the right. The spool starts from rest and does not slip at A as it rolls. Neglect the mass of the cord.



Kinematics: Since the spool rolls without slipping, the instantaneous center of zero velocity is located at point A. Thus,

$$v_O = \omega r_{O/IC} = \omega(3)$$

Also, using similar triangles

$$\frac{s_P}{5} = \frac{10}{3} \qquad s_P = 16.67 \text{ ft}$$

Free-Body Diagram: The 40 lb force does *positive* work since it acts in the same direction of its displacement s_P. The normal reaction N and the weight of the spool do no work since they do not displace. Also, since the spool does not slip, friction does no work.

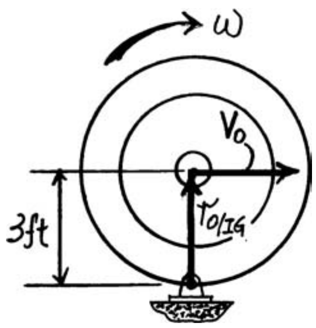
Principle of Work and Energy: The mass moment of inertia of the spool about point O is $I_O = mk_O^2 = \left(\frac{150}{32.2}\right)\left(2.25^2\right) = 23.58 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 18–14, we have

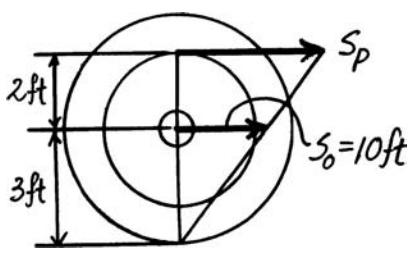
$$T_1 + \sum U_{1-2} = T_2$$

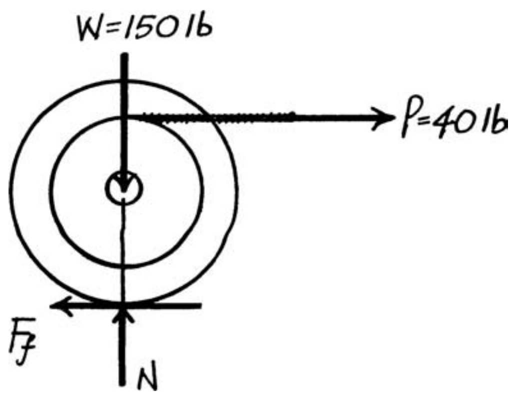
$$0 + P(s_P) = \frac{1}{2} m v_O^2 + \frac{1}{2} I_O \, \omega^2$$

$$0 + 40(16.67) = \frac{1}{2}\left(\frac{150}{32.2}\right)[\omega(3)]^2 + \frac{1}{2}(23.58)\,\omega^2$$

$$\omega = 4.51 \text{ rad/s}$$ **Ans.**







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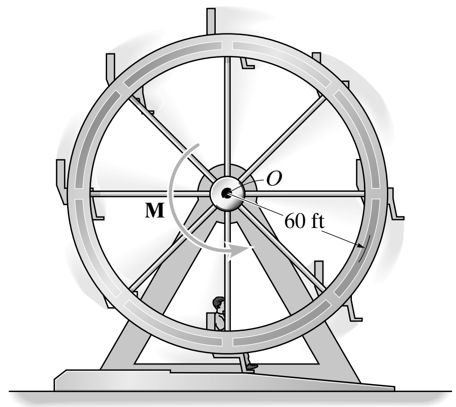
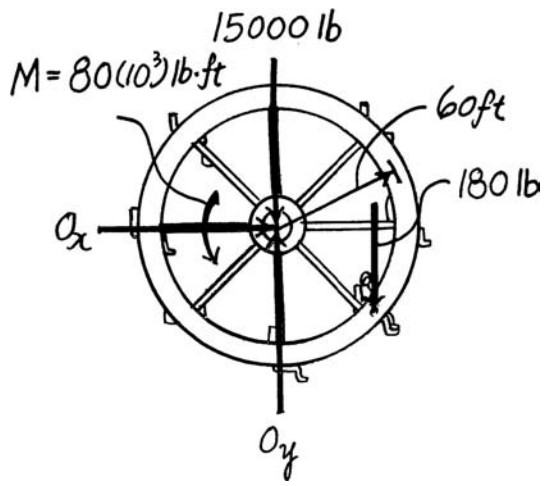
18–10. A man having a weight of 180 lb sits in a chair of the Ferris wheel, which, excluding the man, has a weight of 15 000 lb and a radius of gyration $k_O = 37$ ft. If a torque $M = 80(10^3)$ lb·ft is applied about O, determine the angular velocity of the wheel after it has rotated 180°. Neglect the weight of the chairs and note that the man remains in an upright position as the wheel rotates. The wheel starts from rest in the position shown.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 80\left(10^3\right)(\pi) - (180)(120) = \frac{1}{2}\left[\left(\frac{15\,000}{32.2}\right)(37)^2\right]\omega^2 + \frac{1}{2}\left(\frac{180}{32.2}\right)(60\omega)^2$$

$$\omega = 0.836 \text{ rad/s}$$ **Ans.**



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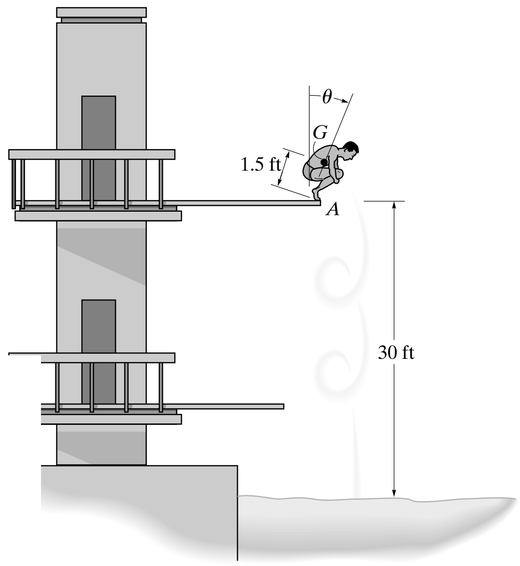
18–11. A man having a weight of 150 lb crouches down on the end of a diving board as shown. In this position the radius of gyration about his center of gravity is $k_G = 1.2$ ft. While holding this position at $\theta = 0°$, he rotates about his toes at A until he loses contact with the board when $\theta = 90°$. If he remains rigid, determine approximately how many revolutions he makes before striking the water after falling 30 ft.



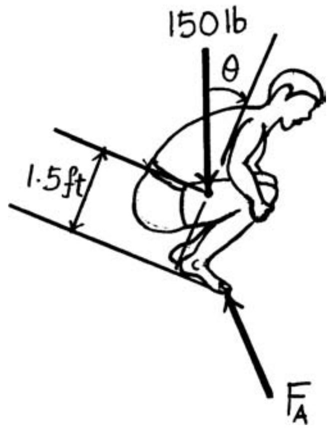
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150(1.5) = \frac{1}{2}\left(\frac{150}{32.2}\right)(1.5\omega)^2 + \frac{1}{2}\left[\left(\frac{150}{32.2}\right)(1.2)^2\right]\omega^2$$

$$\omega = 5.117 \text{ rad/s}$$

$$v_G = (1.5)(5.117) = 7.675 \text{ ft/s}$$

During the fall no forces act on the man to cause an angular acceleration, so $\alpha = 0$.



$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$30 = 0 + 7.675t + \frac{1}{2}(32.2)t^2$$

Choosing the positive root,

$$t = 1.147 \text{ s}$$

$(\circlearrowright +)$ $\quad \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$

$$\theta = 0 + 5.117(1.147) + 0$$

$$\theta = 5.870 \text{ rad} = 0.934 \text{ rev.}$$ **Ans.**

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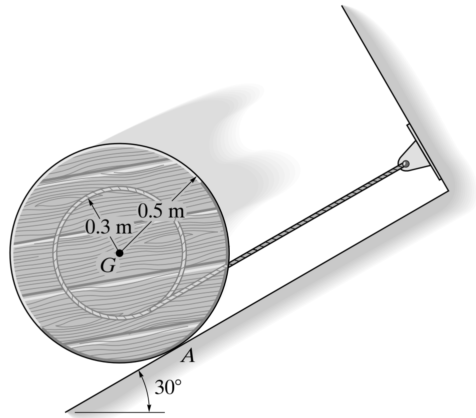
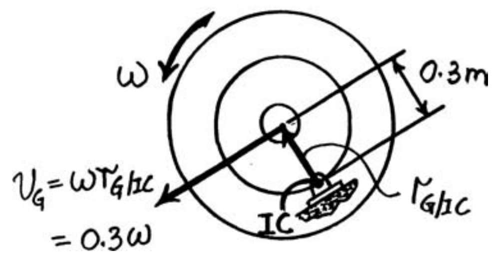
***18–12.** The spool has a mass of 60 kg and a radius of gyration $k_G = 0.3$ m. If it is released from rest, determine how far its center descends down the smooth plane before it attains an angular velocity of $\omega = 6$ rad/s. Neglect friction and the mass of the cord which is wound around the central core.

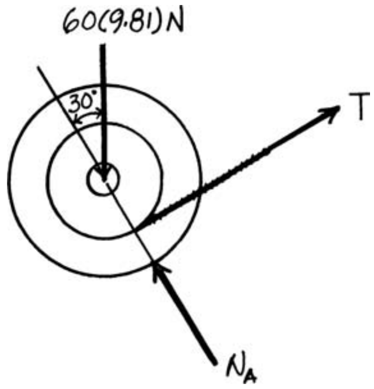


$$T_1 + \Sigma U_{1-2} = T_2$$

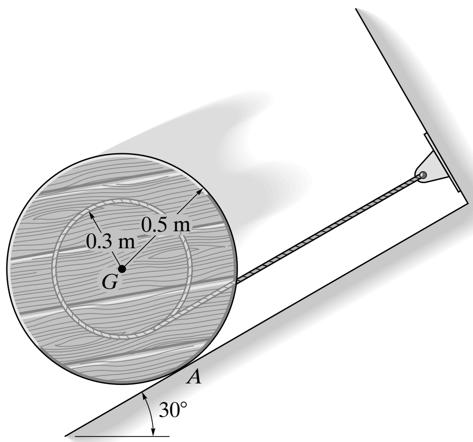
$$0 + 60(9.81)\sin 30°(s) = \frac{1}{2}\left[60(0.3)^2\right](6)^2 + \frac{1}{2}(60)\left[0.3(6)\right]^2$$

$$s = 0.661 \text{ m} \qquad \textbf{Ans.}$$





•18–13. Solve Prob. 18–12 if the coefficient of kinetic friction between the spool and plane at A is $\mu_k = 0.2$.



$$\frac{s_G}{0.3} = \frac{s_A}{(0.5 - 0.3)}$$

$$s_A = 0.6667 s_G$$

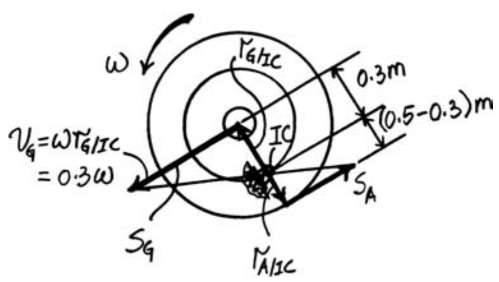
$+\nwarrow \Sigma F_y = 0; \qquad N_A - 60(9.81)\cos 30° = 0$

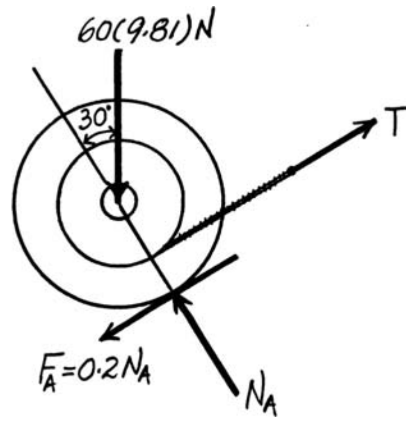
$$N_A = 509.7 \text{ N}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60(9.81)\sin 30°(s_G) - 0.2(509.7)(0.6667 s_G) = \frac{1}{2}\left[60(0.3)^2\right](6)^2 + \frac{1}{2}(60)\left[(0.3)(6)\right]^2$$

$$s_G = 0.859 \text{ m} \qquad \textbf{Ans.}$$





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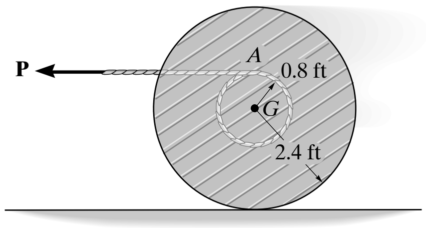
18–14. The spool has a weight of 500 lb and a radius of gyration of $k_G = 1.75$ ft. A horizontal force of $P = 15$ lb is applied to the cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center G has moved 6 ft to the left. The spool rolls without slipping. Neglect the mass of the cable.



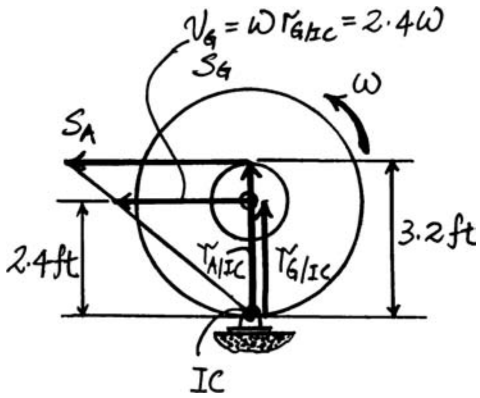
$$\frac{s_G}{2.4} = \frac{s_A}{3.2}$$

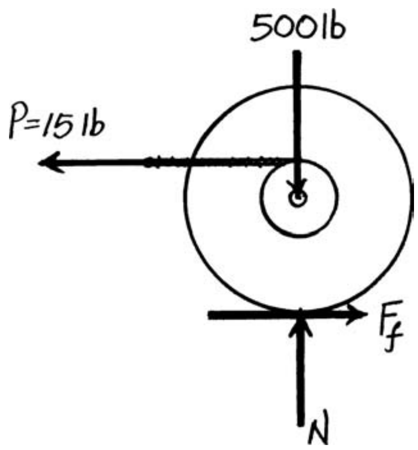
For $s_G = 6$ ft, then $s_A = 8$ ft.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(8) = \frac{1}{2}\left[\left(\frac{500}{32.2}\right)(1.75)^2\right]\omega^2 + \frac{1}{2}\left(\frac{500}{32.2}\right)(2.4\omega)^2$$

$$\omega = 1.32 \text{ rad/s} \qquad \textbf{Ans.}$$





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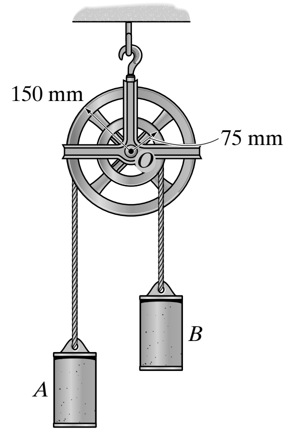
18–15. If the system is released from rest, determine the speed of the 20-kg cylinders A and B after A has moved downward a distance of 2 m. The differential pulley has a mass of 15 kg with a radius of gyration about its center of mass of $k_O = 100$ mm.



Kinetic Energy and Work: The kinetic energy of the pulley and cylinders A and B is

$$T_P = \frac{1}{2} I_O \omega^2 = \frac{1}{2}\left[15\left(0.1^2\right)\right]\omega^2 = 0.075\omega^2$$

$$T_A = \frac{1}{2} m_A {v_A}^2 = \frac{1}{2}(20){v_A}^2 = 10{v_A}^2$$

$$T_B = \frac{1}{2} m_B {v_B}^2 = \frac{1}{2}(20){v_B}^2 = 10{v_B}^2$$

Thus, the kinetic energy of the system is

$$T = T_P + T_A + T_B$$

$$T = 0.075\omega^2 + 10{v_A}^2 + 10{v_B}^2 \qquad \textbf{(1)}$$

However, since the pulley rotates about a fixed axis,

$$\omega = \frac{v_A}{r_A} = \frac{v_A}{0.15} = 6.667 v_A$$

then

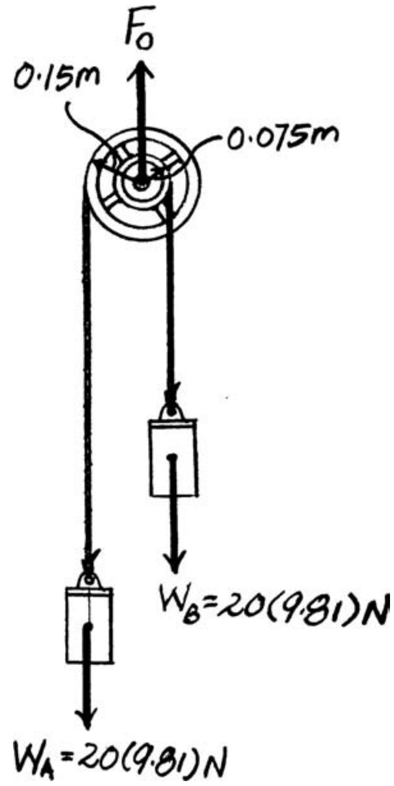
$$v_B = \omega r_B = 6.667 v_A\,(0.075) = 0.5 v_A$$

Substituting these results into Eq. (1), we obtain

$$T = 15.833{v_A}^2$$

Since the system is initially at rest,

$$T_1 = 0$$



Referring to Fig. *a*, $\mathbf{F}_O$ does no work, while W_A does positive work, and W_B does negative work. Thus,

$$U_A = W_A s_A \qquad\qquad U_B = -W_B s_B$$

Here, $s_A = 2$ m. Thus, the pulley rotates through an angle of $\theta = \frac{s_A}{r_A} = \frac{2}{0.15} = 13.33$ rad. Then, $s_B = r_B\theta = 0.075(13.33) = 1$ m. Thus,

$$U_A = 20(9.81)(2) = 392.4 \text{ J}$$

$$U_B = -20(9.81)(1) = -196.2 \text{ J}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[392.4 + (-196.2)\right] = 15.833{v_A}^2$$

$$v_A = 3.520 \text{ m/s} = 3.52 \text{ m/s}\downarrow \qquad \textbf{Ans.}$$

Then

$$v_B = 0.5(3.520) = 1.76 \text{ m/s}\uparrow \qquad \textbf{Ans.}$$

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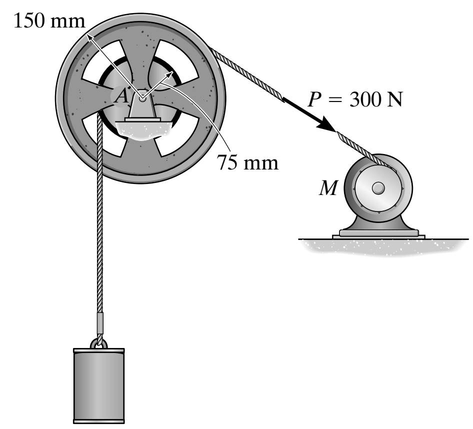
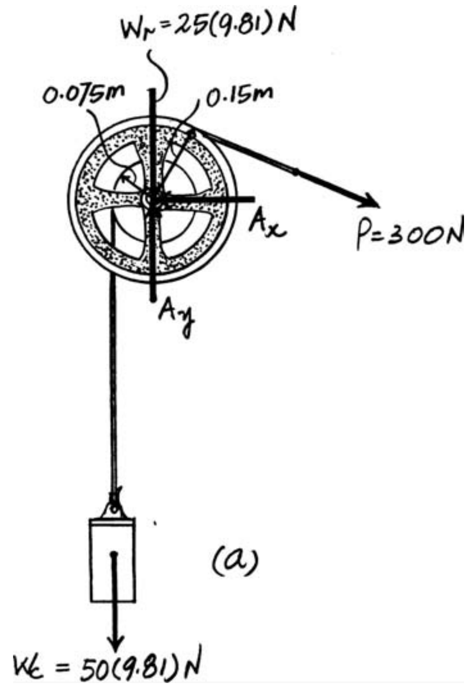
***18–16.** If the motor M exerts a constant force of $P = 300$ N on the cable wrapped around the reel's outer rim, determine the velocity of the 50-kg cylinder after it has traveled a distance of 2 m. Initially, the system is at rest. The reel has a mass of 25 kg, and the radius of gyration about its center of mass A is $k_A = 125$ mm.



Kinetic Energy and Work: Since the reel rotates about a fixed axis, $v_C = \omega_r r_C$ or $\omega_r = \dfrac{v_C}{r_C} = \dfrac{v_C}{0.075} = 13.33v_C$. The mass moment of inertia of the reel about its mass centers is $I_A = m_r k_A{}^2 = 25\left(0.125^2\right) = 0.390625\ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is

$$
\begin{aligned}
T &= T_r + T_C \\
&= \frac{1}{2} I_A \,\omega_r{}^2 + \frac{1}{2} m_C \,v_C{}^2 \\
&= \frac{1}{2}(0.390625)(13.33v_C)^2 + \frac{1}{2}(50)v_C{}^2 \\
&= 59.72v_C{}^2
\end{aligned}
$$



Since the system is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{A}_y$, $\mathbf{A}_x$, and $\mathbf{W}_r$ do no work, while $\mathbf{P}$ does positive work, and $\mathbf{W}_C$ does negative work. When the cylinder displaces upwards through a distance of $s_C = 2$ m, the wheel rotates $\theta = \dfrac{s_C}{r_C} = \dfrac{2}{0.075} = 26.67$ rad. Thus, $\mathbf{P}$ displaces a distance of $s_P = r_P\theta = 0.15(26.67) = 4$ m. The work done by $\mathbf{P}$ and $\mathbf{W}_C$ is therefore

$$U_P = Ps_P = 300(4) = 1200\ \text{J}$$

$$U_{W_C} = -W_C\, s_C = -50(9.81)(2) = -981\ \text{J}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[1200 + (-981)\right] = 59.72v_C{}^2$$

$$v_C = 1.91\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

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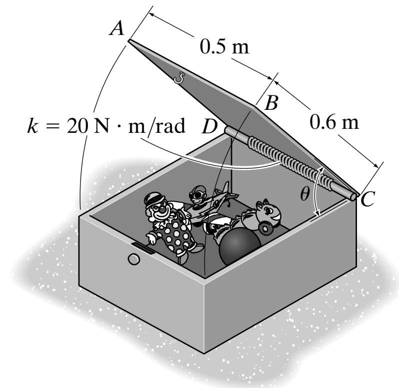
•18–17. The 6-kg lid on the box is held in equilibrium by the torsional spring at $\theta = 60°$. If the lid is forced closed, $\theta = 0°$, and then released, determine its angular velocity at the instant it opens to $\theta = 45°$.



Equilibrium: Here, $M = k\theta_0 = 20\theta_0$, where θ_0 is the initial angle of twist for the torsional spring. Referring to Fig. *a*, we have

$+\Sigma M_C = 0;$ $\quad 6(9.81)\cos 60°(0.3) - 20\theta_0 = 0 \quad$ $\theta_0 = 0.44145$ rad

Kinetic Energy and Work: Since the cover rotates about a fixed axis passing through point C, the kinetic energy of the cover can be obtained by applying $T = \frac{1}{2} I_C\,\omega^2$, where $I_C = \frac{1}{3} mb^2 = \frac{1}{3}(6)\left(0.6^2\right) = 0.72\ \text{kg} \cdot \text{m}^2$. Thus,

$$T = \frac{1}{2} I_C\omega^2 = \frac{1}{2}(0.72)\omega^2 = 0.36\omega^2$$

Since the cover is initially at rest ($\theta = 0°$), $T_1 = 0$. Referring to Fig. *b*, C_x and C_y do no work. **M** does positive work, and W does negative work. When $\theta = 0°$ and 45°, the angles of twist for the torsional spring are $\theta_1 = 1.489$ rad and $\theta_2 = 1.489 - \frac{\pi}{4} = 0.703$ rad, respectively. Also, when $\theta = 45°$, **W** displaces vertically upward through a distance of $h = 0.3 \sin 45° = 0.2121$ m. Thus, the work done by **M** and **W** are

$$U_M = \int M\,d\theta = \int_{\theta_2}^{\theta_1} 20\theta\,d\theta = 10\theta^2\Big|_{0.7032\text{ rad}}^{1.4886\text{ rad}} = 17.22\text{ J}$$

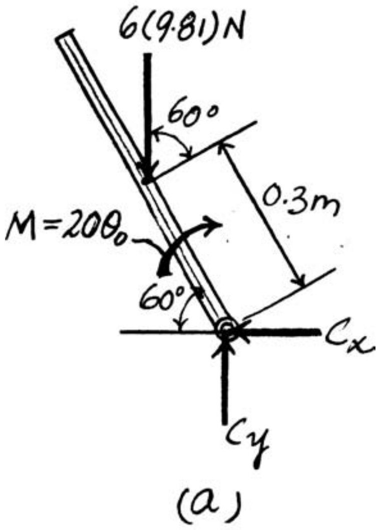
$$U_W = -Wh = -6(9.81)(0.2121) = -12.49\text{ J}$$

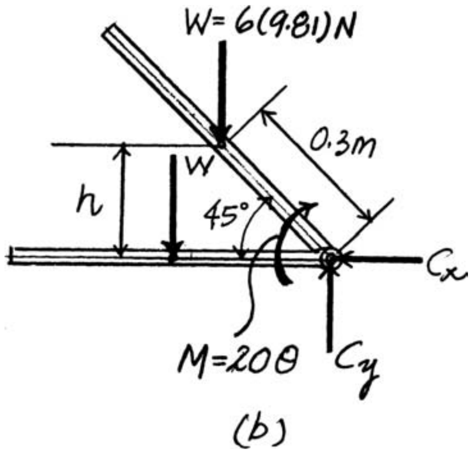
Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[17.22 + (-12.49)\right] = 0.36\omega^2$$

$$\omega = 3.62\text{ rad/s}$$ **Ans.**





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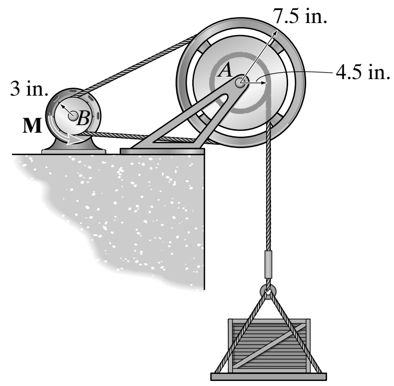
18–18. The wheel and the attached reel have a combined weight of 50 lb and a radius of gyration about their center of $k_A = 6$ in. If pulley B attached to the motor is subjected to a torque of $M = 40(2 - e^{-0.1\theta})$ lb · ft, where θ is in radians, determine the velocity of the 200-lb crate after it has moved upwards a distance of 5 ft, starting from rest. Neglect the mass of pulley B.



Kinetic Energy and Work: Since the wheel rotates about a fixed axis, $v_C = \omega r_C = \omega(0.375)$. The mass moment of inertia of A about its mass center is $I_A = m k_A{}^2 = \left(\dfrac{50}{32.2}\right)(0.5^2) = 0.3882 \text{ slug} \cdot \text{ft}^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A \omega^2 + \frac{1}{2} m_C v_C{}^2$$

$$= \frac{1}{2}(0.3882)\omega^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left[\omega(0.375)\right]^2$$

$$= 0.6308\omega^2$$

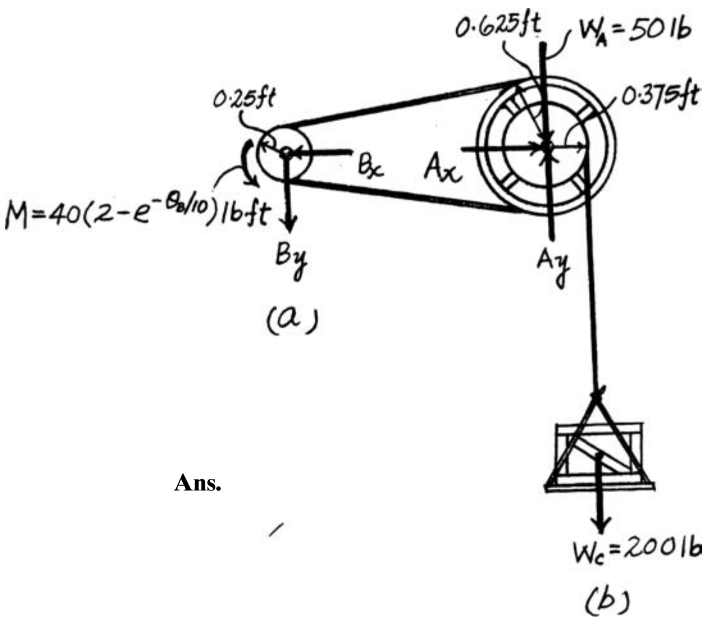
Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When crate C moves 5 ft upward, wheel A rotates through an angle of $\theta_A = \dfrac{s_C}{r} = \dfrac{5}{0.375} = 13.333$ rad. Then, pulley B rotates through an angle of $\theta_B = \dfrac{r_A}{r_B}\theta_A = \left(\dfrac{0.625}{0.25}\right)(13.333) = 33.33$ rad. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$$U_M = \int M d\theta_B = \int_0^{33.33 \text{ rad}} 40\left(2 - e^{-0.1\theta}\right) d\theta$$

$$= \left[40\left(2\theta + 10e^{-0.1\theta}\right)\right]\Big|_0^{33.33 \text{ rad}}$$

$$= 2280.93 \text{ ft} \cdot \text{lb}$$

$$U_{W_C} = -W_C s_C = -200(5) = -1000 \text{ ft} \cdot \text{lb}$$



Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [2280.93 - 1000] = 0.6308\omega^2$$

$$\omega = 45.06 \text{ rad/s}$$

Thus,

$$v_C = 45.06(0.375) = 16.9 \text{ ft/s} \uparrow$$ **Ans.**

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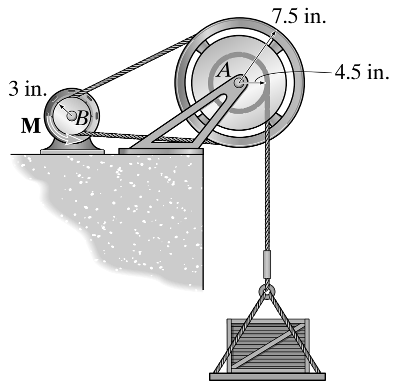
18–19. The wheel and the attached reel have a combined weight of 50 lb and a radius of gyration about their center of $k_A = 6$ in. If pulley B that is attached to the motor is subjected to a torque of $M = 50\text{ lb}\cdot\text{ft}$, determine the velocity of the 200-lb crate after the pulley has turned 5 revolutions. Neglect the mass of the pulley.



Kinetic Energy and Work: Since the wheel at A rotates about a fixed axis, $v_C = \omega r_C = \omega(0.375)$. The mass moment of inertia of wheel A about its mass center is $I_A = mk_A{}^2 = \left(\frac{50}{32.2}\right)(0.5^2) = 0.3882\text{ slug}\cdot\text{ft}^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A\,\omega^2 + \frac{1}{2} m_C\, v_C{}^2$$

$$= \frac{1}{2}(0.3882)\omega^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left[\omega(0.375)\right]^2$$

$$= 0.6308\omega^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When pulley B rotates $\theta_B = (5\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 10\pi\text{ rad}$, the wheel rotates through an angle of $\theta_A = \frac{r_B}{r_A}\theta_B = \left(\frac{0.25}{0.625}\right)(10\pi) = 4\pi$. Thus, the crate displaces upwards through a distance of $s_C = r_C\,\theta_A = 0.375(4\pi) = 1.5\pi$ ft. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$$U_M = M\theta_B = 50(10\pi) = 500\pi\text{ ft}\cdot\text{lb}$$

$$U_{W_C} = -W_C\, s_C = -200(1.5\pi) = -300\pi\text{ ft}\cdot\text{lb}$$

Principle of Work and Energy:

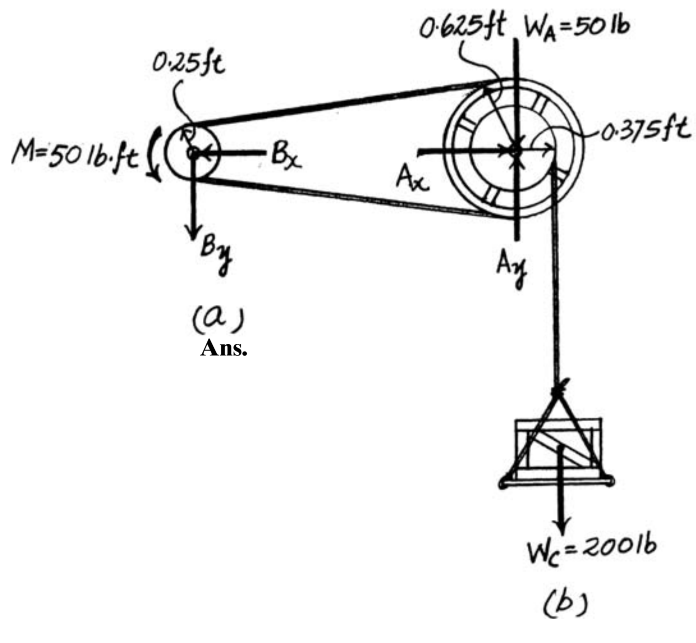
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [500\pi - 300\pi] = 0.6308\omega^2$$

$$\omega = 31.56\text{ rad/s}$$

Thus,

$$v_C = 31.56(0.375) = 11.8\text{ ft/s}\uparrow$$ **Ans.**



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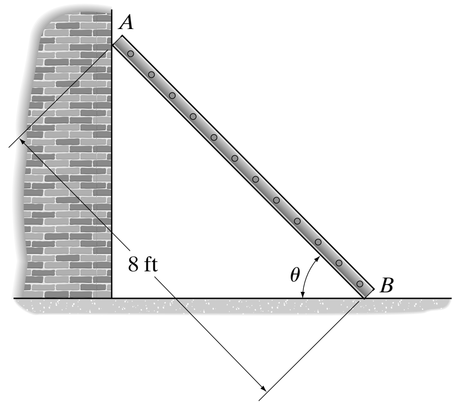
***18–20.** The 30-lb ladder is placed against the wall at an angle of $\theta = 45°$ as shown. If it is released from rest, determine its angular velocity at the instant just before $\theta = 0°$. Neglect friction and assume the ladder is a uniform slender rod.



Kinetic Energy and Work: Referring to Fig. *a*,

$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(4)$$

The mass moment of inertia of the ladder about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{30}{32.2}\right)(8^2) = 4.969 \text{ slug}\cdot\text{ft}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2$$

$$= \frac{1}{2}\left(\frac{30}{32.2}\right)\left[\omega_2(4)\right]^2 + \frac{1}{2}(4.969)\omega_2{}^2$$

$$= 9.938\omega_2{}^2$$

Since the ladder is initially at rest, $T_1 = 0$. Referring to Fig. *b*, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while $\mathbf{W}$ does positive work. When $\theta = 0°$, $\mathbf{W}$ displaces vertically through a distance of $h = 4 \sin 45° \text{ ft} = 2.828 \text{ ft}$. Thus, the work done by $\mathbf{W}$ is

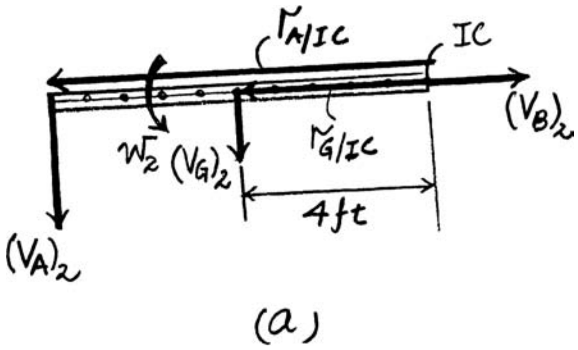
$$U_W = Wh = 30(2.828) = 84.85 \text{ ft}\cdot\text{lb}$$

Principle of Work and Energy:

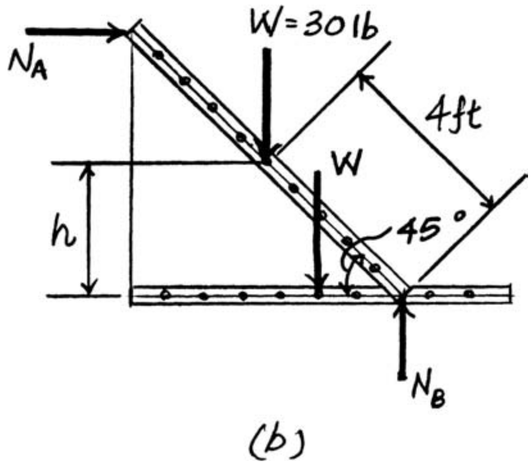
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 84.85 = 9.938\omega_2{}^2$$

$$\omega_2 = 2.92 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)



(b)

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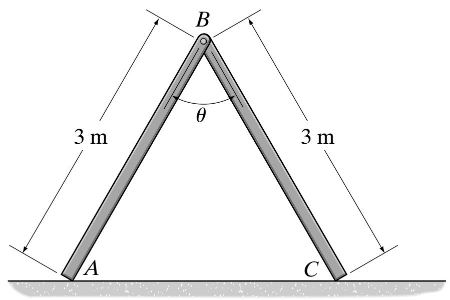
•18–21. Determine the angular velocity of the two 10-kg rods when $\theta = 180°$ if they are released from rest in the position $\theta = 60°$. Neglect friction.



Kinetic Energy and Work: Due to symmetry, the velocity of point B is directed along the vertical, as shown in Fig. *a*. Also, $(\omega_{AB})_2 = (\omega_{BC})_2 = \omega_2$ and $\left(v_{G_{AB}}\right)_2 = \left(v_{G_{BC}}\right)_2 = (v_G)_2$. Here, $(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(1.5)$. The mass moment of inertia of the rods about their respective mass centers is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(10)\left(3^2\right) = 7.5\ \text{kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = (T_{AB})_2 + (T_{BC})_2$$

$$= 2\left[\frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2\right]$$

$$= 2\left[\frac{1}{2}(10)\left[\omega_2(1.5)\right]^2 + \frac{1}{2}(7.5)\omega_2{}^2\right]$$

$$= 30\omega_2{}^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. *b*, **N** does no work, while **W** does positive work. When $\theta = 180°$, **W** displaces vertically downward through a distance of $h = 1.5\cos 30° = 1.2990$ m. Thus, the work done by **W** is

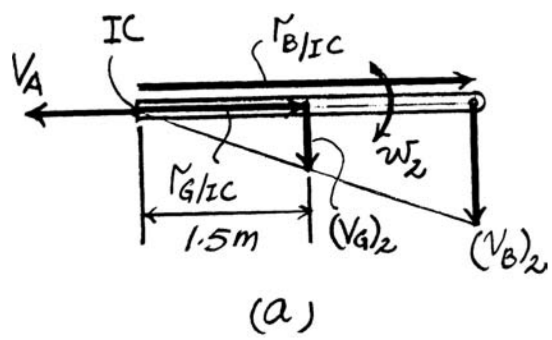
$$U_W = Wh = 10(9.81)(1.2990) = 127.44\ \text{J}$$

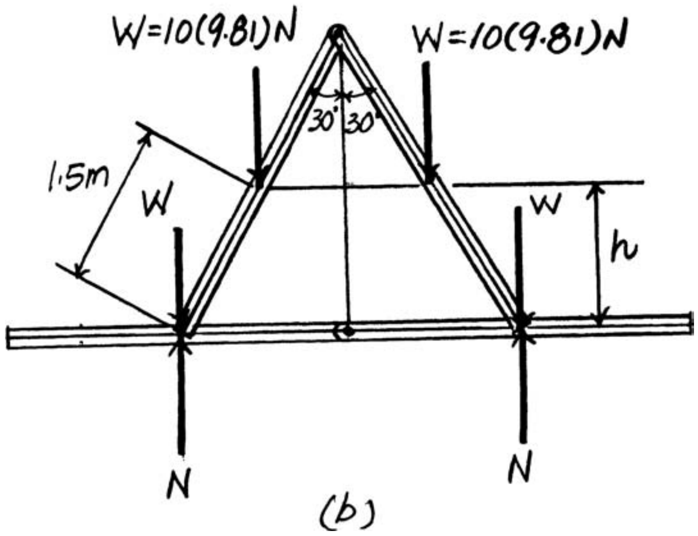
Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2(127.44) = 30\omega_2{}^2$$

$$\omega_2 = 2.91\ \text{rad/s} \qquad \textbf{Ans.}$$





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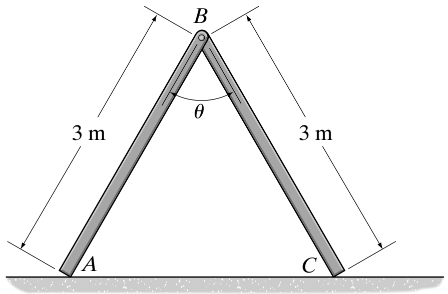
18–22. Determine the angular velocity of the two 10-kg rods when $\theta = 90°$ if they are released from rest in the position $\theta = 60°$. Neglect friction.



Kinetic Energy and Work: Due to symmetry, the velocity of point B is directed along the vertical, as shown in Fig. *a*. Also, $(\omega_{AB})_2 = (\omega_{BC})_2 = \omega_2$ and $\left(v_{G_{AB}}\right)_2 = \left(v_{G_{BC}}\right)_2 = (v_G)_2$. From the geometry of this diagram, $r_{G/IC} = 1.5$ m. Thus, $(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(1.5)$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}(10)\left(3^2\right) = 7.5\ \text{kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = (T_{AB})_2 + (T_{BC})_2$$

$$= 2\left[\frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2\right]$$

$$= 2\left[\frac{1}{2}(10)[\omega_2(1.5)]^2 + \frac{1}{2}(7.5)\omega_2{}^2\right]$$

$$= 30\omega_2{}^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. *b*, **N** does no work, while **W** does positive work. When $\theta = 90°$, **W** displaces vertically downward through a distance of $h = 1.5\cos 30° - 1.5\cos 45° = 0.2384$ m. Thus, the work done by **W** is

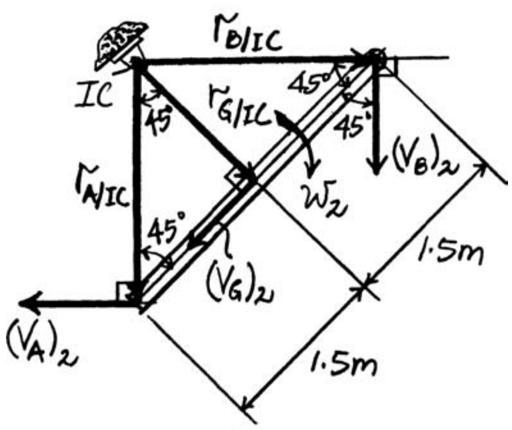
$$U_W = Wh = 10(9.81)(0.2384) = 23.38\ \text{J}$$

Principle of Work and Energy:

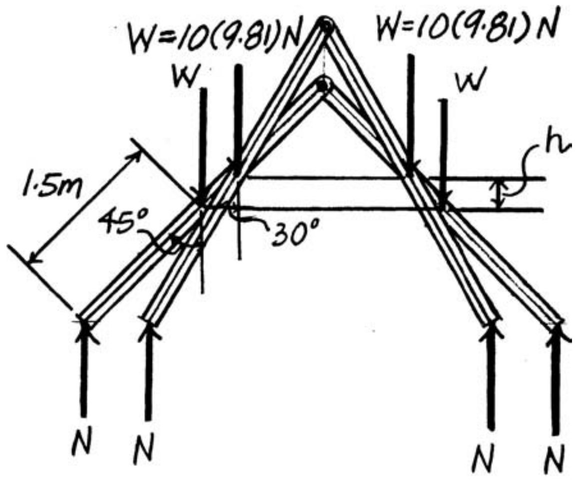
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2(23.38) = 30\omega_2{}^2$$

$$\omega_2 = 1.25\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)



(b)

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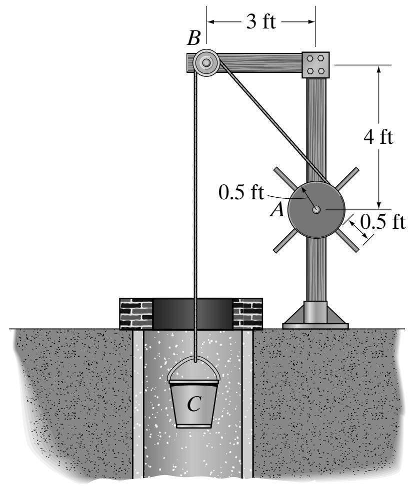
18–23. If the 50-lb bucket is released from rest, determine its velocity after it has fallen a distance of 10 ft. The windlass A can be considered as a 30-lb cylinder, while the spokes are slender rods, each having a weight of 2 lb. Neglect the pulley's weight.



Kinetic Energy and Work: Since the windlass rotates about a fixed axis, $v_C = \omega_A r_A$ or $\omega_A = \dfrac{v_C}{r_A} = \dfrac{v_C}{0.5} = 2v_C$. The mass moment of inertia of the windlass about its mass center is

$$I_A = \frac{1}{2}\left(\frac{30}{32.2}\right)(0.5^2) + 4\left[\frac{1}{12}\left(\frac{2}{32.2}\right)(0.5^2) + \frac{2}{32.2}(0.75^2)\right] = 0.2614 \text{ slug} \cdot \text{ft}^2$$

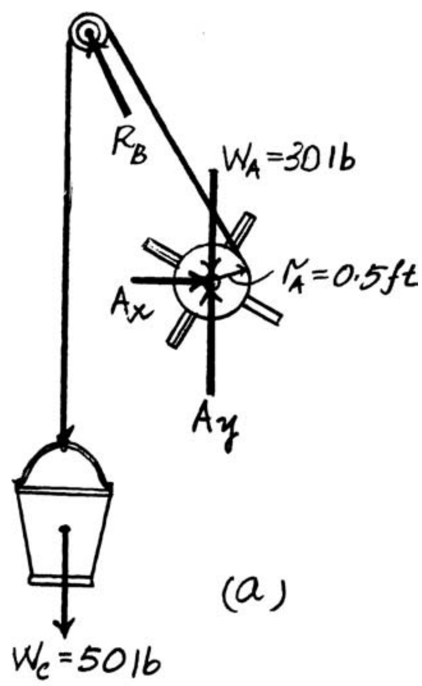
Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2}I_A\omega^2 + \frac{1}{2}m_C v_C{}^2$$

$$= \frac{1}{2}(0.2614)(2v_C)^2 + \frac{1}{2}\left(\frac{50}{32.2}\right)v_C{}^2$$

$$= 1.2992 v_C{}^2$$



Since the system is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{W}_A$, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{R}_B$ do no work, while $\mathbf{W}_C$ does positive work. Thus, the work done by $\mathbf{W}_C$, when it displaces vertically downward through a distance of $s_C = 10$ ft, is

$$U_{W_C} = W_C s_C = 50(10) = 500 \text{ ft} \cdot \text{lb}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 500 = 1.2992 v_C{}^2$$

$$v_C = 19.6 \text{ ft/s} \qquad \textbf{Ans.}$$

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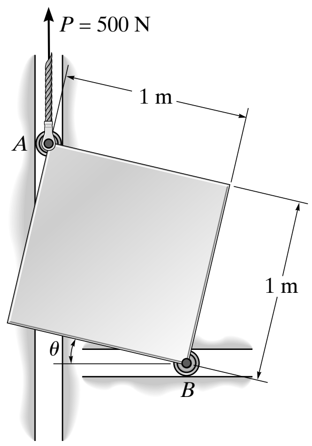
***18–24.** If corner A of the 60-kg plate is subjected to a vertical force of $P = 500$ N, and the plate is released from rest when $\theta = 0°$, determine the angular velocity of the plate when $\theta = 45°$.



Kinetic Energy and Work: Since the plate is initially at rest, $T_1 = 0$. Referring to Fig. *a*,

$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(1\cos 45°) = 0.7071\omega_2$$

The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(60)\left(1^2 + 1^2\right) = 10\text{ kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2$$

$$= \frac{1}{2}m(60)(0.7071\omega_2)^2 + \frac{1}{2}(10)\omega_2{}^2$$

$$= 20\omega_2{}^2$$

Referring to Fig. *b*, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while **P** does positive work, and **W** does negative work. When $\theta = 45°$, **W** and **P** displace upwards through a distance of $h = 1\cos 45° - 0.5 = 0.2071$ m and $s_P = 2(1\cos 45°) - 1 = 0.4142$ m. Thus, the work done by **P** and **W** is

$$U_P = Ps_P = 500(0.4142) = 207.11\text{ J}$$

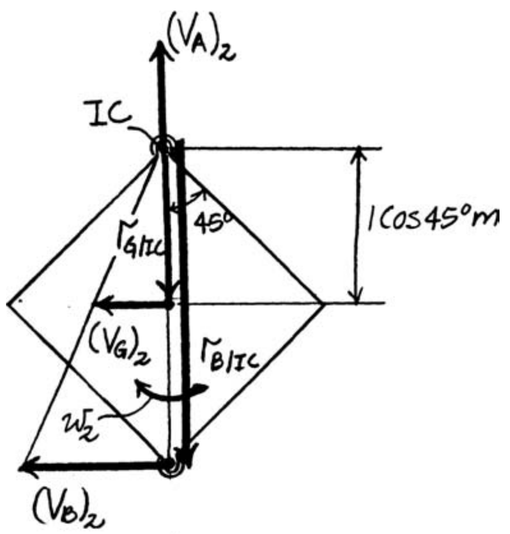
$$U_W = -Wh = -60(9.81)(0.2071) = -121.90\text{ J}$$

Principle of Work and Energy:

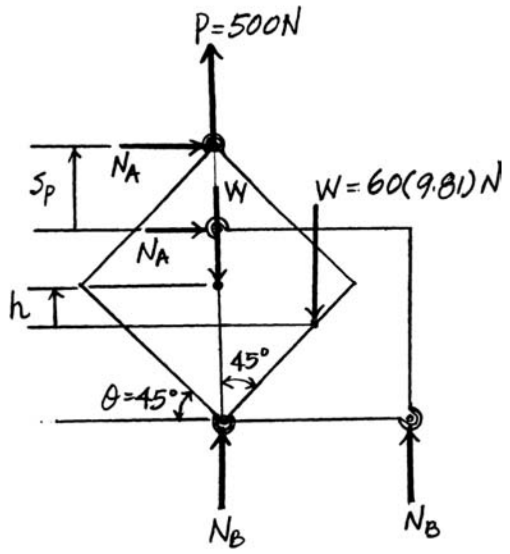
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [207.11 - 121.90] = 20\omega_2{}^2$$

$$\omega_2 = 2.06\text{ rad/s}$$ **Ans.**



(a)



(b)

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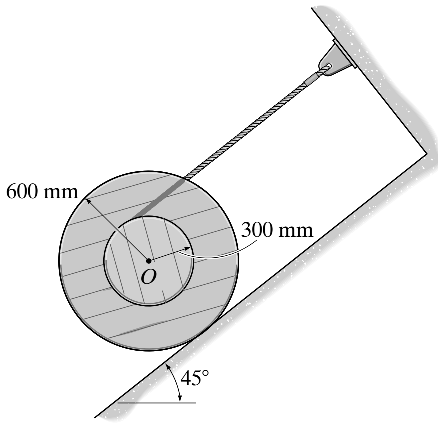
•18–25. The spool has a mass of 100 kg and a radius of gyration of 400 mm about its center of mass O. If it is released from rest, determine its angular velocity after its center O has moved down the plane a distance of 2 m. The contact surface between the spool and the inclined plane is smooth.



Kinetic Energy and Work: Referring to Fig. *a*,

$$v_O = \omega r_{O/IC} = \omega(0.3)$$

The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = 100\left(0.4^2\right) = 16 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy of the spool is

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\omega^2$$

$$= \frac{1}{2}(100)[\omega(0.3)]^2 + \frac{1}{2}(16)\omega^2$$

$$= 12.5\omega^2$$

Since the spool is initially at rest, $T_1 = 0$. Referring to Fig. *b*, **T** and **N** do no work, while **W** does positive work. When the center of the spool moves down the plane through a distance of $s_O = 2$ m, **W** displaces vertically downward $h = s_O \cos 45° = 2 \cos 45° = 1.4142$ m. Thus, the work done by **W** is

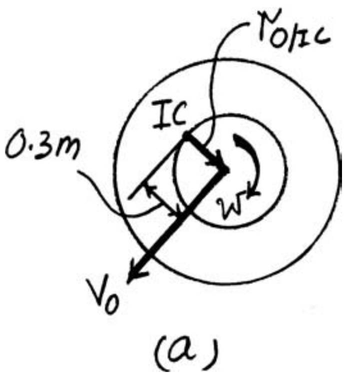
$$U_W = Wh = 100(9.81)(1.4142) = 1387.34 \text{ N}$$

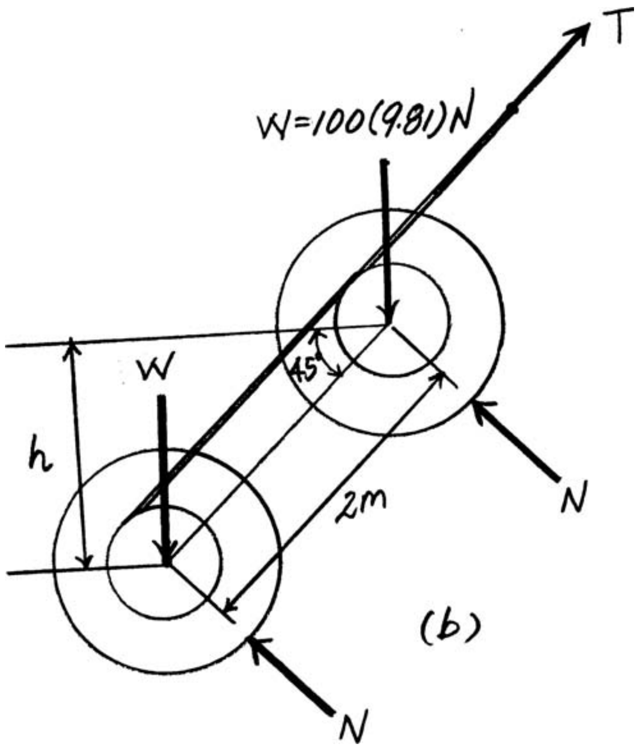
Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1387.34 = 12.5\omega^2$$

$$\omega = 10.5 \text{ rad/s} \qquad \textbf{Ans.}$$





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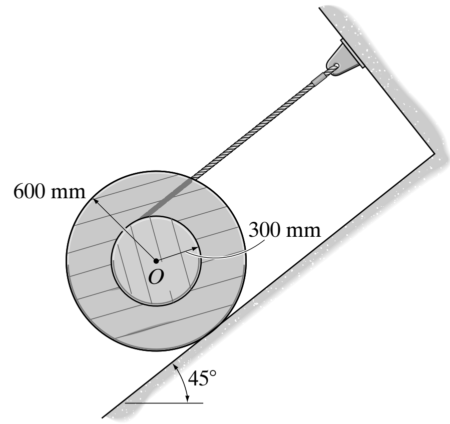
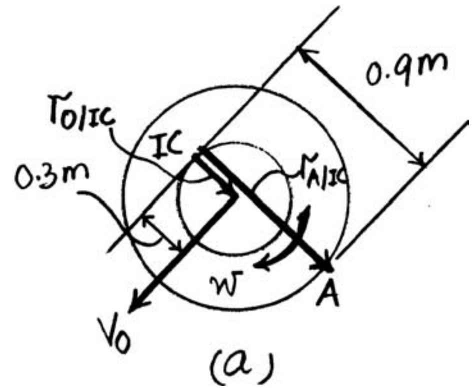
18–26. The spool has a mass of 100 kg and a radius of gyration of 400 mm about its center of mass O. If it is released from rest, determine its angular velocity after its center O has moved down the plane a distance of 2 m. The coefficient of kinetic friction between the spool and the inclined plane is $\mu_k = 0.15$.



Kinetic Energy and Work: Referring to Fig. a,

$$v_O = \omega r_{O/IC} = \omega(0.3)$$



The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = 100\left(0.4^2\right) = 16\ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the spool is

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\omega^2$$

$$= \frac{1}{2}(100)\left[\omega(0.3)\right]^2 + \frac{1}{2}(16)\omega^2$$

$$= 12.5\omega^2$$

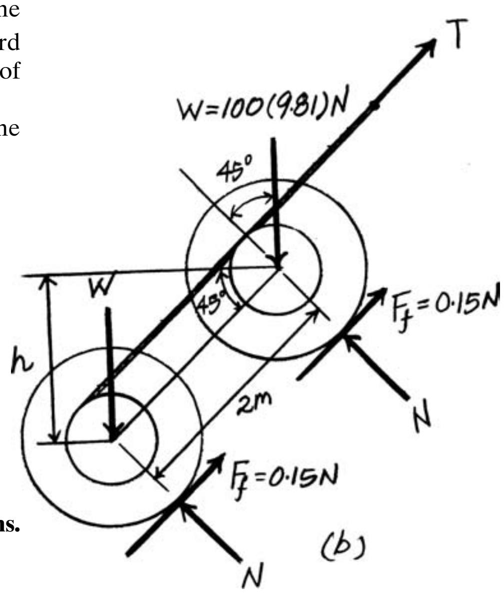
Since the spool is initially at rest, $T_1 = 0$. Referring to Fig. b, **T** and **N** do no work, while **W** does positive work, and $\mathbf{F}_f$ does negative work. Since the spool slips at the contact point on the inclined plane, $F_f = \mu_k N = 0.15N$, where **N** can be obtained using the equation of motion,

$$\Sigma F_{y'} = m(a_a)_{y'}; \qquad N - 100(9.81)\cos 45° = 0 \qquad\qquad N = 693.67\ \text{N}$$

Thus, $F_f = 0.15(693.67) = 104.05$ N. When the center of the spool moves down the inclined plane through a distance of $s_O = 2$ m, **W** displaces vertically downward $h = s_O \sin 45° = 2 \sin 45° = 1.4142$ m. Also, the contact point A on the outer rim of the spool travels a distance of $s_A = \left(\dfrac{r_{A/IC}}{r_{O/IC}}\right)s_O = \dfrac{0.9}{0.3}(2) = 6$ m, Fig. a. Thus, the work done by **W** and $\mathbf{F}_f$ is

$$U_W = Wh = 100(9.81)(1.4142) = 1387.34\ \text{J}$$

$$U_{F_f} = -F_f s_A = -104.05(6) = -624.30\ \text{J}$$



Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [1387.34 - 624.30] = 12.5\omega^2$$

$$\omega = 7.81\ \text{rad/s} \qquad\qquad \textbf{Ans.}$$

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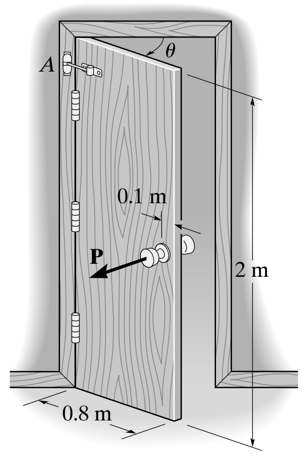
18–27. The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has a stiffness of $k = 80\ \text{N}\cdot\text{m/rad}$, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, when k is the stiffness and θ is the angle of twist.

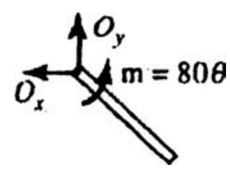


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_{\theta_O}^{\theta_O + \frac{\pi}{2}} 80\theta\, d\theta = \frac{1}{2}\left[\frac{1}{3}(20)(0.8)^2\right](12)^2$$

$$40\left[\left(\theta_O + \tfrac{\pi}{2}\right)^2 - \theta_0^2\right] = 307.2$$

$$\theta_O = 1.66 \text{ rad}$$ **Ans.**



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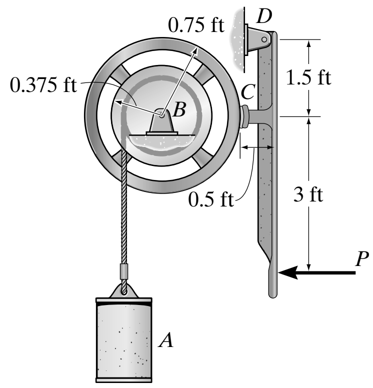
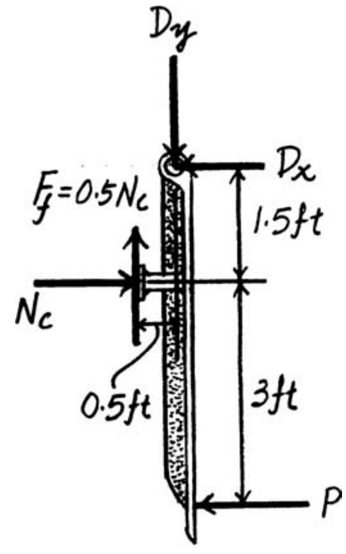
***18–28.** The 50-lb cylinder A is descending with a speed of 20 ft/s when the brake is applied. If wheel B must be brought to a stop after it has rotated 5 revolutions, determine the constant force **P** that must be applied to the brake arm. The coefficient of kinetic friction between the brake pad C and the wheel is $\mu_k = 0.5$. The wheel's weight is 25 lb, and the radius of gyration about its center of mass is $k = 0.6$ ft.



Equilibrium: Referring to Fig. a, we have

$$\circlearrowleft + \Sigma M_D = 0; \qquad N_C(1.5) - 0.5N_C(0.5) - P(4.5) = 0$$

$$N_C = 3.6\,P$$



(a)

Kinetic Energy and Work: Since the wheel rotates about a fixed axis, $(\omega_B)_1 = \dfrac{(v_A)_1}{r_A} = \dfrac{20}{0.375} = 53.33$ rad/s. The mass moment of inertia of the wheel about its mass center is $I_B = m_B k^2 = \dfrac{25}{32.2}\left(0.6^2\right) = 0.2795\ \text{slug}\cdot\text{ft}^2$. Thus, the initial kinetic energy of the system is

$$T_1 = (T_A)_1 + (T_B)_1$$

$$= \frac{1}{2} m_A (v_A)_1^2 + \frac{1}{2} I_B (\omega_B)_1^2$$

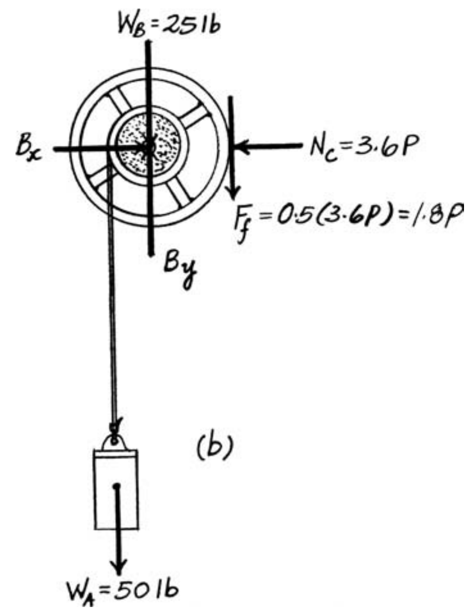
$$= \frac{1}{2}\left(\frac{50}{32.2}\right)\left(20^2\right) + \frac{1}{2}(0.2795)\left(53.33^2\right)$$

$$= 708.07\ \text{ft}\cdot\text{lb}$$

Since the system is brought to rest, $T_2 = 0$. Referring to Fig. b, $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{W}_B$, and $\mathbf{N}_C$ do no work, while $\mathbf{W}_A$ does positive work, and $\mathbf{F}_f$ does negative work. When wheel B rotates through the angle $\theta = (5\text{ rev})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = 10\pi$ rad, $\mathbf{W}_A$ displaces $s_A = r_A\theta = 0.375(10\pi) = 3.75\pi$ ft vertically downward, and the contact point C on the outer rim of the wheel travels a distance of $s_C = r_B\theta = 0.75(10\pi) = 7.5\pi$. Thus, the work done by $\mathbf{W}_A$ and $\mathbf{F}_f$ is

$$U_{W_A} = W_A s_A = 50(3.75\pi) = 187.5\pi\ \text{ft}\cdot\text{lb}$$

$$U_{F_f} = -F_f s_C = -1.8P(7.5\pi) = -13.5\pi P$$



(b)

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$708.07 + [187.5\pi - 13.5\pi P] = 0$$

$$P = 30.6\ \text{lb} \qquad \textbf{Ans.}$$

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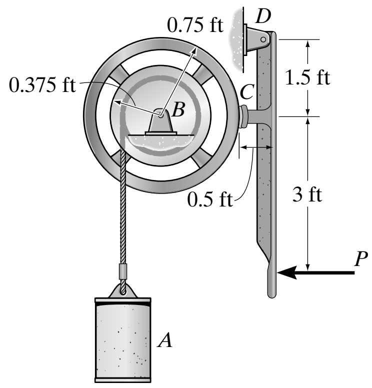
•18–29. When a force of $P = 30$ lb is applied to the brake arm, the 50-lb cylinder A is descending with a speed of 20 ft/s. Determine the number of revolutions wheel B will rotate before it is brought to a stop. The coefficient of kinetic friction between the brake pad C and the wheel is $\mu_k = 0.5$. The wheel's weight is 25 lb, and the radius of gyration about its center of mass is $k = 0.6$ ft.



Equilibrium: Referring to Fig. a,

$$\zeta + \Sigma M_D = 0; \qquad N_C(1.5) - 0.5N_C(0.5) - 30(4.5) = 0$$

$$N_C = 108 \text{ lb}$$

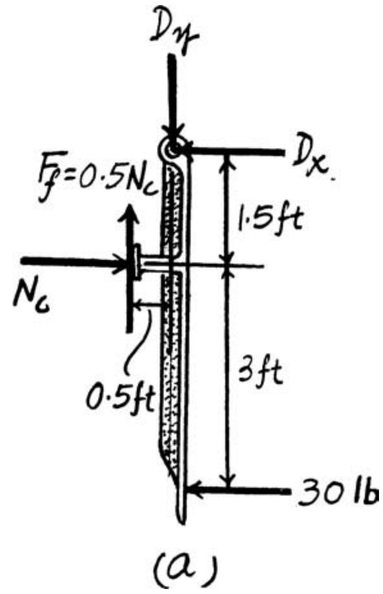
Kinetic Energy and Work: Since the wheel rotates about a fixed axis, $(\omega_B)_1 = \dfrac{(v_A)_1}{r_A} = \dfrac{20}{0.375} = 53.33$ rad/s. The mass moment of inertia of the wheel about its mass center is $I_B = m_B k^2 = \dfrac{25}{32.2}\left(0.6^2\right) = 0.2795 \text{ slug} \cdot \text{ft}^2$. Thus, the initial kinetic energy of the system is

$$T_1 = (T_A)_1 + (T_B)_1$$

$$= \frac{1}{2} m_A (v_A)_1^2 + \frac{1}{2} I_B (\omega_B)_1^2$$

$$= \frac{1}{2}\left(\frac{50}{32.2}\right)\left(20^2\right) + \frac{1}{2}(0.2795)\left(53.33^2\right)$$

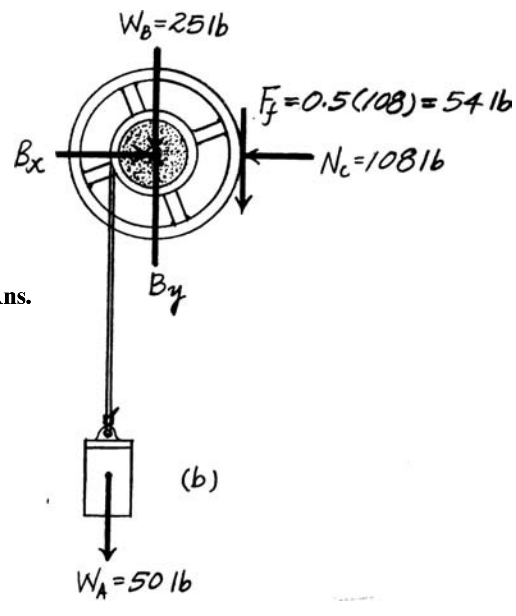
$$= 708.07 \text{ ft} \cdot \text{lb}$$



Since the system is brought to rest, $T_2 = 0$. Referring to Fig. b, $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{W}_B$, and N_C do no work, while $\mathbf{W}_A$ does positive work, and $\mathbf{F}_f$ does negative work. When wheel B rotates through the angle θ, $\mathbf{W}_A$ displaces $s_A = r_A\theta = 0.375\theta$ and the contact point on the outer rim of the wheel travels a distance of $s_C = r_B\theta = 0.75\theta$. Thus, the work done by $\mathbf{W}_A$ and $\mathbf{F}_f$ are

$$U_{W_A} = W_A s_A = 50(0.375\theta) = 18.75\theta$$

$$U_{F_f} = -F_f s_C = -0.5(108)(0.75\theta) = -40.5\theta$$



Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$708.07 + [18.75\theta - 40.5\theta] = 0$$

$$\theta = 32.55 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi}\right) = 5.18 \text{ rev} \qquad \textbf{Ans.}$$

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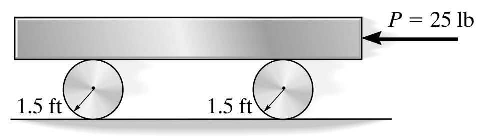
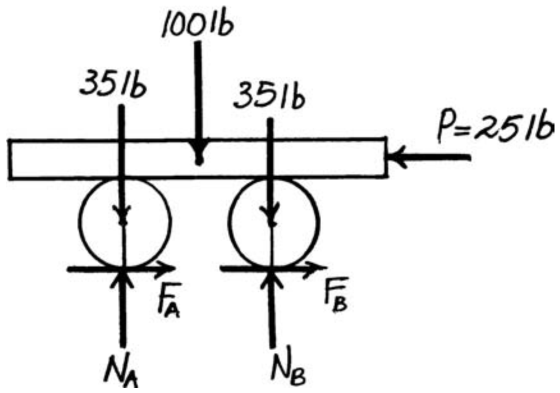
18–30. The 100-lb block is transported a short distance by using two cylindrical rollers, each having a weight of 35 lb. If a horizontal force $P = 25$ lb is applied to the block, determine the block's speed after it has been displaced 2 ft to the left. Originally the block is at rest. No slipping occurs.

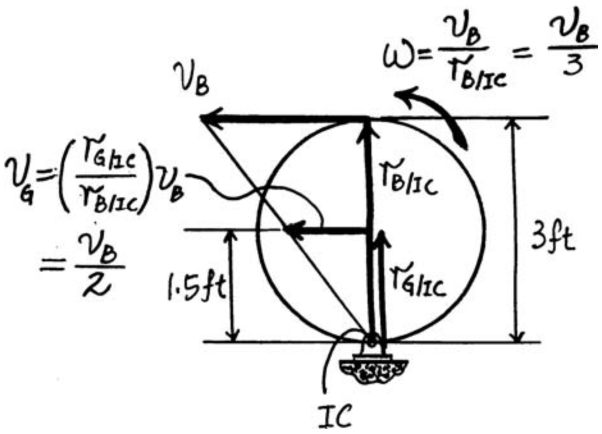


$$T_1 + \Sigma U_{1-2} = T_2$$

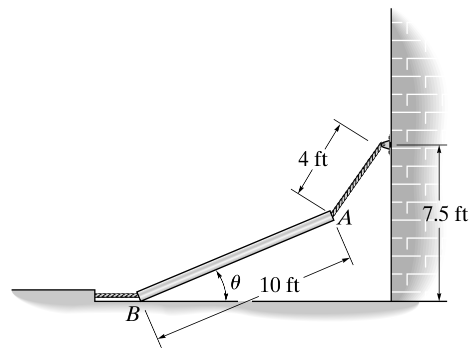
$$0 + 25(2) = \frac{1}{2}\left(\frac{100}{32.2}\right)(v_B)^2 + 2\left[\frac{1}{2}\left(\frac{35}{32.2}\right)\left(\frac{v_B}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}\left(\frac{35}{32.2}\right)(1.5)^2\right)\left(\frac{v_B}{3}\right)^2\right]$$

$$v_B = 5.05 \text{ ft/s} \qquad \textbf{Ans.}$$





18–31. The slender beam having a weight of 150 lb is supported by two cables. If the cable at end B is cut so that the beam is released from rest when $\theta = 30°$, determine the speed at which end A strikes the wall. Neglect friction at B.

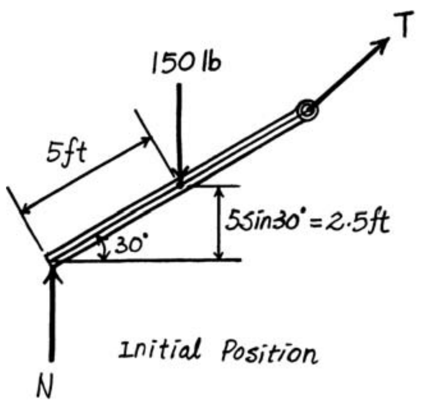


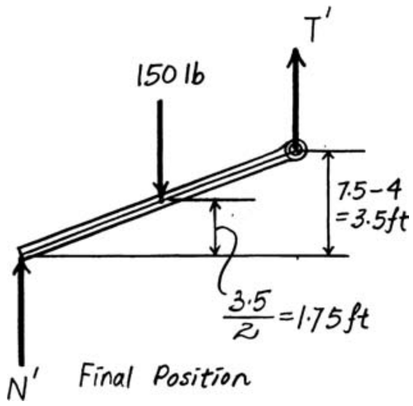
In the final position, the rod is in translation since the *IC* is at infinity.

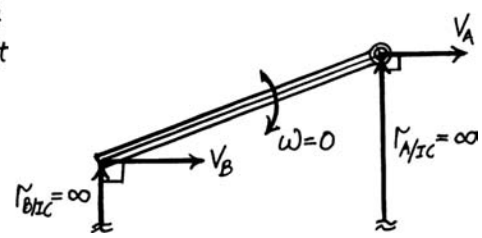
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150(2.5 - 1.75) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_G^2$$

$$v_G = v_A = 6.95 \text{ ft/s} \qquad \textbf{Ans.}$$







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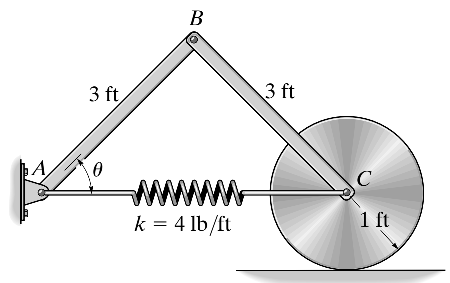
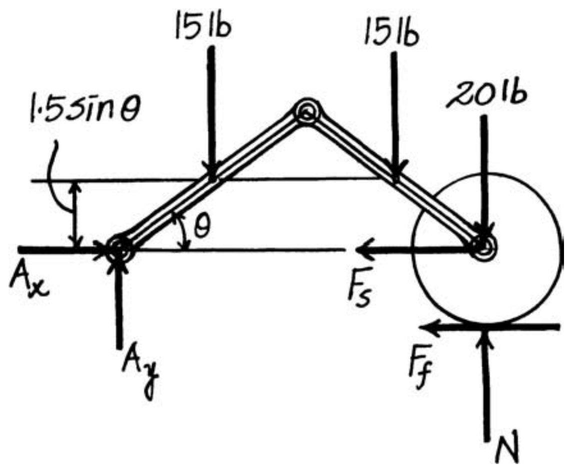
***18–32.** The assembly consists of two 15-lb slender rods and a 20-lb disk. If the spring is unstretched when $\theta = 45^\circ$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0^\circ$. The disk rolls without slipping.

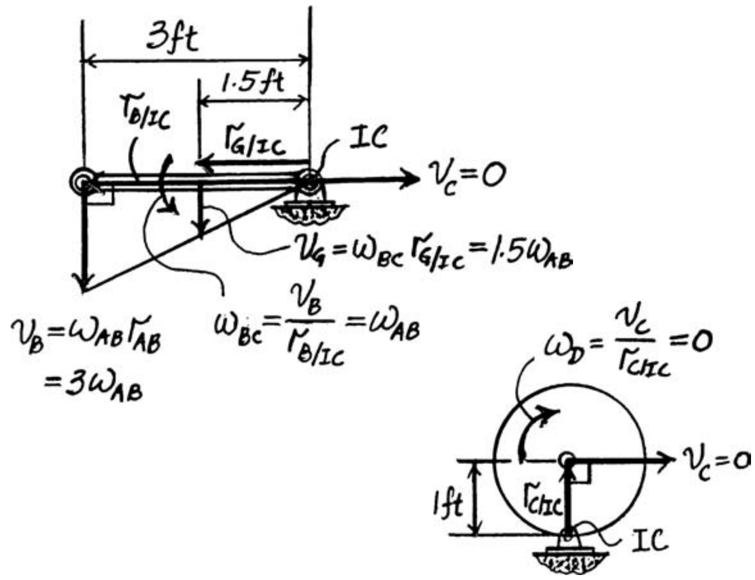


$$T_1 + \Sigma U_{1-2} = T_2$$

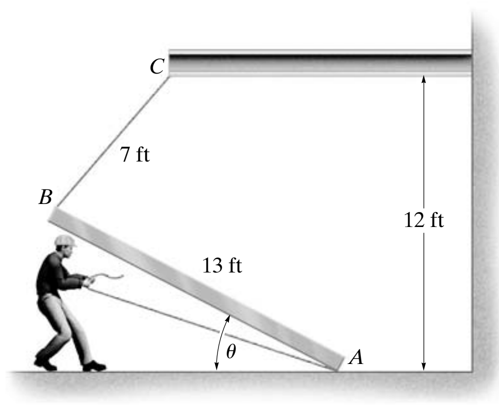
$$[0 + 0] + 2(15)(1.5)\sin 45^\circ - \frac{1}{2}(4)\left[6 - 2(3)\cos 45^\circ\right]^2 = 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right]$$

$$\omega_{AB} = 4.28 \text{ rad/s}$$ **Ans.**





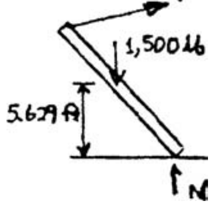
18–33. The beam has a weight of 1500 lb and is being raised to a vertical position by pulling very slowly on its bottom end A. If the cord fails when $\theta = 60^\circ$ and the beam is essentially at rest, determine the speed of A at the instant cord BC becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.

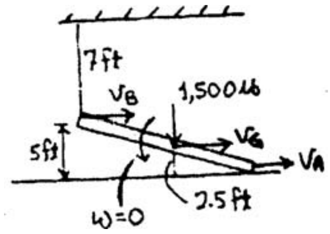


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1500(5.629) - 1500(2.5) = \frac{1}{2}\left(\frac{1500}{32.2}\right)(v_G)^2$$

$$v_G = v_A = 14.2 \text{ ft/s}$$ **Ans.**





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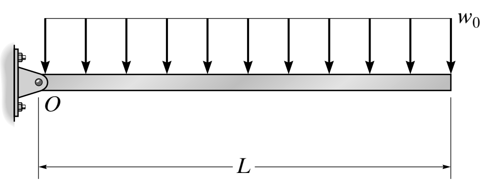
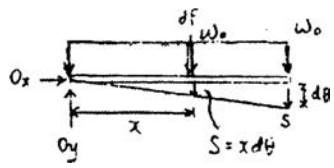
18–34. The uniform slender bar that has a mass m and a length L is subjected to a uniform distributed load w_0, which is always directed perpendicular to the axis of the bar. If the bar is released from rest from the position shown, determine its angular velocity at the instant it has rotated 90°. Solve the problem for rotation in (a) the horizontal plane, and (b) the vertical plane.



a)

$$T_1 + \Sigma U_{1-2} = T_2$$

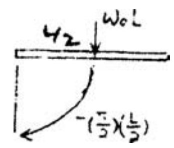
$$[0] + \int_0^{\frac{\pi}{2}} \int_0^L (w_0\,dx)(x\,d\theta) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$



$$\int_0^{\frac{\pi}{2}} \frac{w_0 L^2}{2} d\theta = \frac{1}{6}mL^2\omega^2$$

$$\frac{w_0\,L^2}{2}\left(\frac{\pi}{2}\right) = \frac{1}{6}mL^2\omega^2$$

$$\omega = \sqrt{\frac{3\pi}{2}\left(\frac{w_0}{m}\right)}$$ **Ans.**



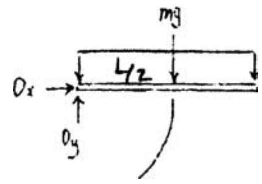
Note: The work of the distributed load can also be determined from its resultant.

$$U_{1-2} = w_0 L\left(\frac{\pi}{2}\right)\left(\frac{L}{2}\right) = \frac{w_0}{4}\pi L^2$$

b)

$$T_1 + \Sigma U_{1-2} = T_2$$

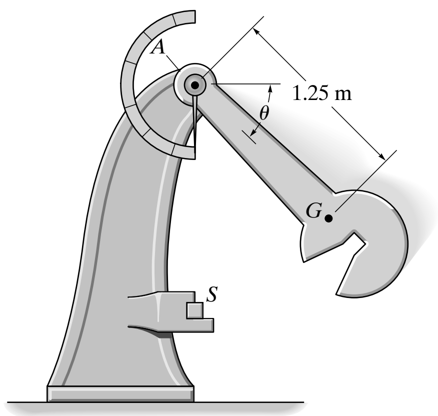
$$[0] + \frac{w_0}{4}\pi L^2 + mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{1}{3}\,mL^2\right)\omega^2$$



$$\omega^2 = \frac{3}{2}\frac{w_0\,\pi L}{mL} + \frac{mg(6)}{2mL}$$

$$\omega = \sqrt{\frac{3\pi}{2}\frac{w_0}{m} + \frac{3g}{L}}$$ **Ans.**

18–35. Solve Prob. 18–5 using the conservation of energy equation.



Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 50(9.81)(1.25) = \frac{1}{2}\left[50(1.75)^2\right]\omega^2 + 0$$

$$\omega = 2.83 \text{ rad/s}$$ **Ans.**

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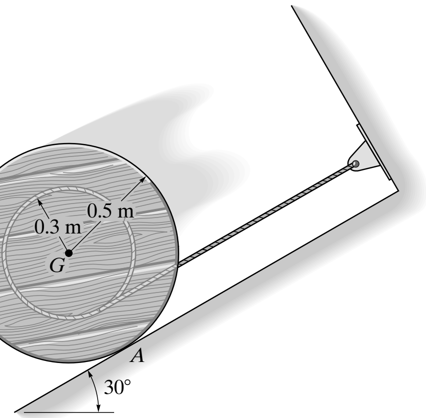
***18–36.** Solve Prob. 18–12 using the conservation of energy equation.

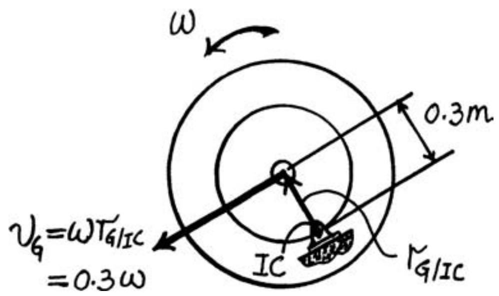


Datum at lowest point through G.

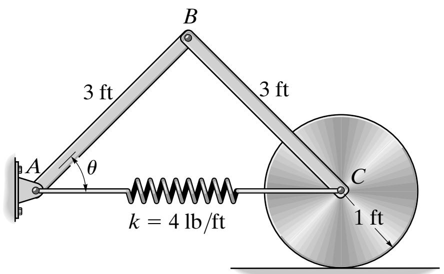
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 60(9.81)(s \sin 30°) = \frac{1}{2}\left[60(0.3)^2\right](6)^2 + \frac{1}{2}(60)\left[(0.3)(6)\right]^2 + 0$$

$$s = 0.661 \text{ m} \qquad \textbf{Ans.}$$



•18–37. Solve Prob. 18–32 using the conservation of energy equation.

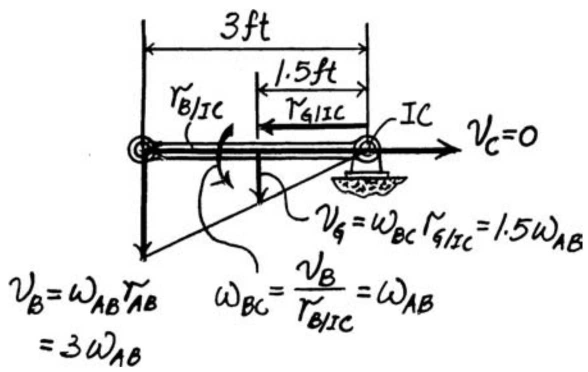


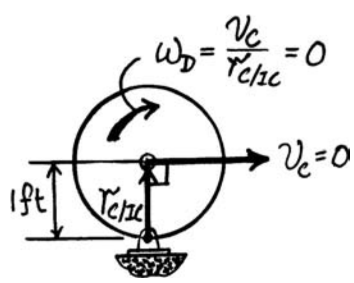
Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[15(1.5 \sin 45°)\right] = 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right] + \frac{1}{2}(4)\left[6 - 2(3 \cos 45°)\right]^2 + 0$$

$$\omega_{AB} = 4.28 \text{ rad/s} \qquad \textbf{Ans.}$$





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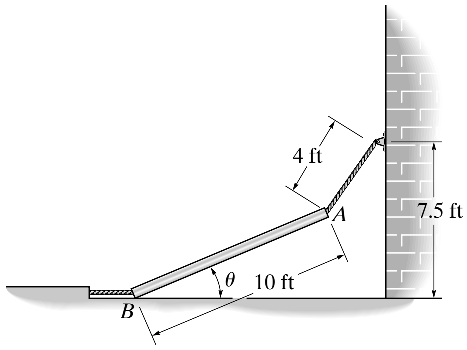
18–38. Solve Prob. 18–31 using the conservation of energy equation.



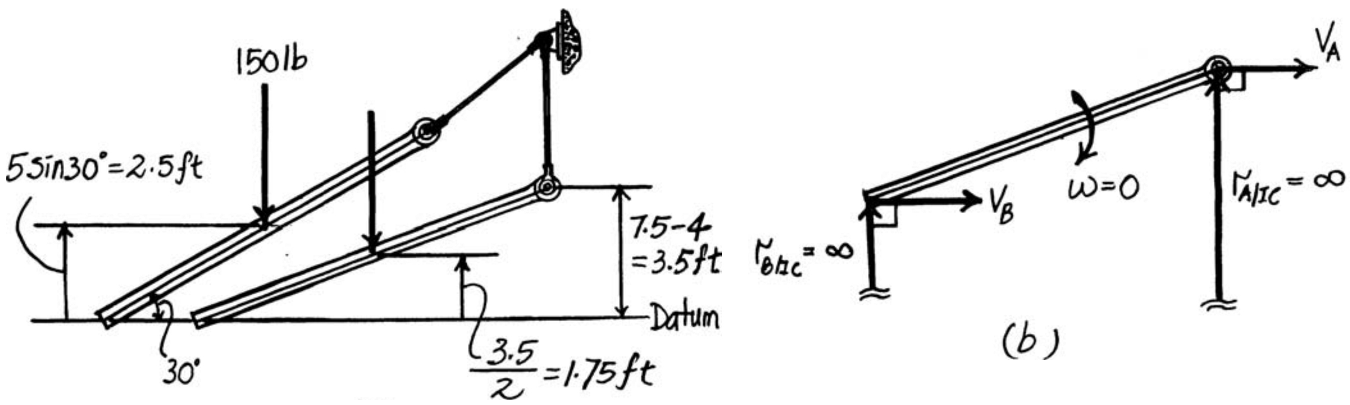
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 150(2.5) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_G^2 + 150(1.75)$$

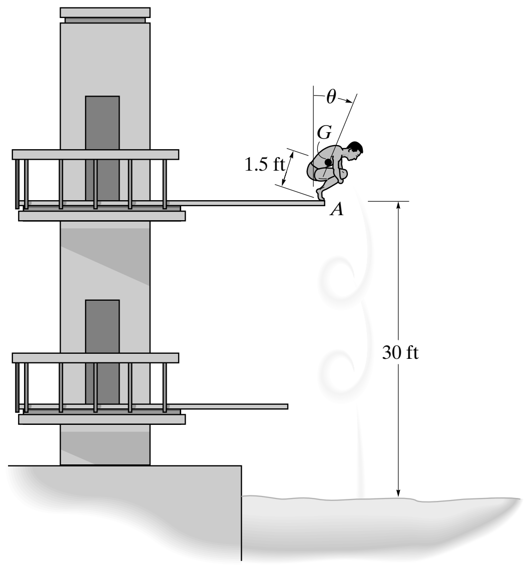
$$v_G = 6.95 \text{ ft/s}$$

Since the rod is in translation at the final instant, then

$$v_A = 6.95 \text{ ft/s}$$ **Ans.**



18–39. Solve Prob. 18–11 using the conservation of energy equation.



Datum at A.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 150(1.5) = \frac{1}{2}\left[\left(\frac{150}{32.2}\right)(1.2)^2\right]\omega^2 + \frac{1}{2}\left(\frac{150}{32.2}\right)(1.5\omega)^2 + 0$$

$$\omega = 5.117 \text{ rad/s}$$

Time to fall:

$$s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2$$

$$30 = 0 + 1.5(5.117)t + \frac{1}{2}(32.2)t^2$$

Choosing the positive root: $t = 1.147$ s

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\theta = 0 + 5.117(1.147) + 0 = 5.870 \text{ rad} = 0.934 \text{ rev.}$$ **Ans.**

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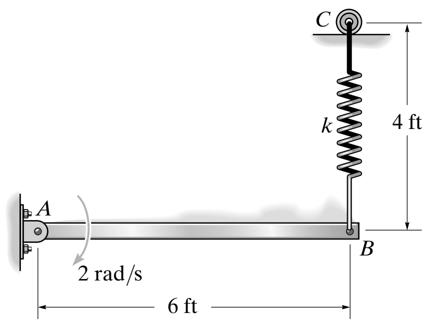
***18–40.** At the instant shown, the 50-lb bar rotates clockwise at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 2 ft and a stiffness of $k = 6$ lb/ft, determine the angular velocity of the bar the instant it has rotated 30° clockwise.

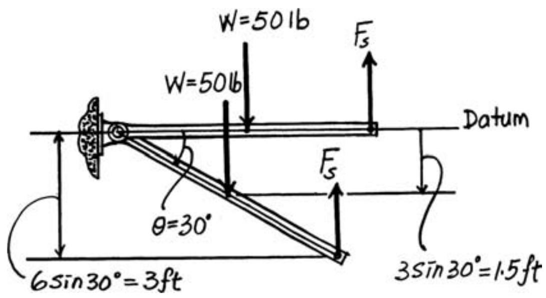


Datum through A.

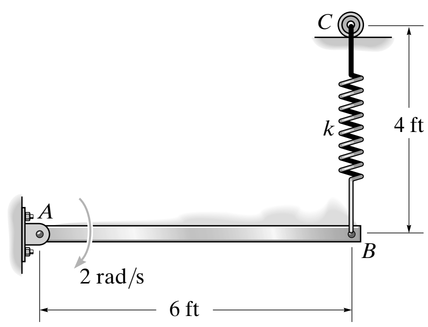
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(6)(4-2)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right]\omega^2 + \frac{1}{2}(6)(7-2)^2 - 50(1.5)$$

$$\omega = 2.30 \text{ rad/s} \qquad \textbf{Ans.}$$



•18–41. At the instant shown, the 50-lb bar rotates clockwise at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 2 ft and a stiffness of $k = 12$ lb/ft, determine the angle θ, measured from the horizontal, to which the bar rotates before it momentarily stops.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(12)(4-2)^2 = 0 + \frac{1}{2}(12)(4 + 6\sin\theta - 2)^2 - 50(3\sin\theta)$$

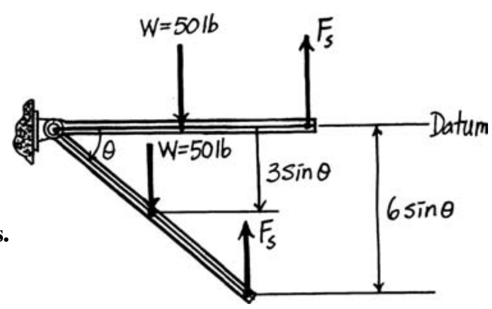
$$61.2671 = 24(1 + 3\sin\theta)^2 - 150\sin\theta$$

$$37.2671 = -6\sin\theta + 216\sin^2\theta$$

Set $x = \sin\theta$, and solve the quadratic equation for the positive root:

$$\sin\theta = 0.4295$$

$$\theta = 25.4° \qquad \textbf{Ans.}$$



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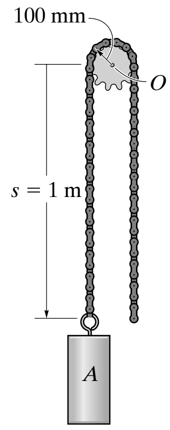
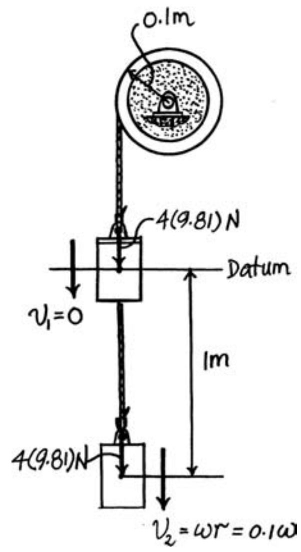
18–42. A chain that has a negligible mass is draped over the sprocket which has a mass of 2 kg and a radius of gyration of $k_O = 50$ mm. If the 4-kg block A is released from rest from the position $s = 1$ m, determine the angular velocity of the sprocket at the instant $s = 2$ m.



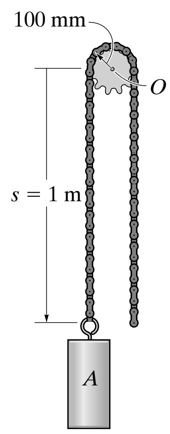
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 + 0 = \frac{1}{2}(4)(0.1\,\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 - 4(9.81)(1)$$

$$\omega = 41.8 \text{ rad/s} \qquad \textbf{Ans.}$$



18–43. Solve Prob. 18–42 if the chain has a mass per unit length of 0.8 kg/m. For the calculation neglect the portion of the chain that wraps over the sprocket.

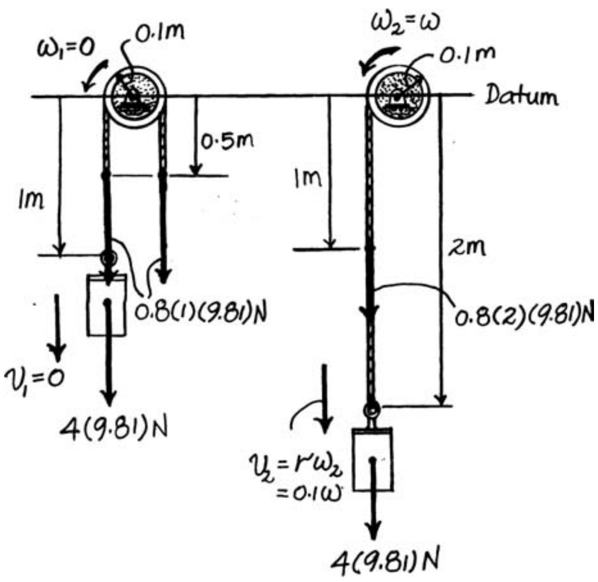


$$T_1 + V_1 = T_2 + V_2$$

$$0 - 4(9.81)(1) - 2\left[0.8(1)(9.81)(0.5)\right] = \frac{1}{2}(4)(0.1\,\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2$$

$$+ \frac{1}{2}(0.8)(2)(0.1\,\omega)^2 - 4(9.81)(2) - 0.8(2)(9.81)(1)$$

$$\omega = 39.3 \text{ rad/s} \qquad \textbf{Ans.}$$



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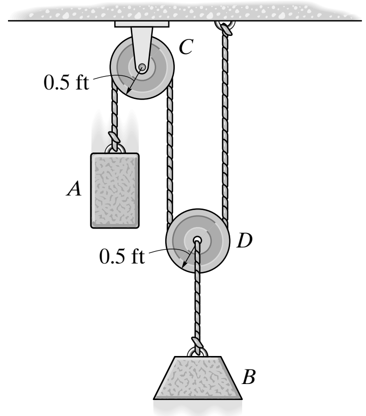
***18–44.** The system consists of 60-lb and 20-lb blocks A and B, respectively, and 5-lb pulleys C and D that can be treated as thin disks. Determine the speed of block A after block B has risen 5 ft, starting from rest. Assume that the cord does not slip on the pulleys, and neglect the mass of the cord.



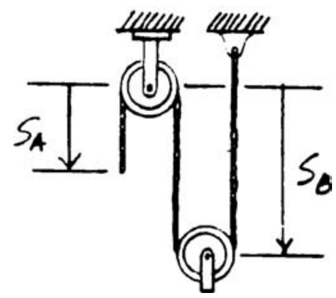
Kinematics: The speed of block A and B can be related using the position coordinate equation.

$$s_A + 2s_B = l \qquad \textbf{(1)}$$

$$\Delta s_A + 2\Delta s_B = 0 \qquad \Delta s_A + 2(5) = 0 \qquad \Delta s_A = -10\text{ ft} = 10\text{ ft}\downarrow$$

Taking time derivative of Eq. (1), we have

$$v_A + 2v_B = 0 \qquad v_B = -0.5v_A$$



Potential Energy: Datum is set at fixed pulley C. When blocks A and B (pulley D) are at their initial position, their centers of gravity are located at s_A and s_B. Their initial gravitational potential energies are $-60s_A$, $-20s_B$, and $-5s_B$. When block B (pulley D) rises 5 ft, block A decends 10 ft. Thus, the final position of blocks A and B (pulley D) are $(s_A + 10)$ ft and $(s_B - 5)$ ft *below* datum. Hence, their respective final gravitational potential energy are $-60(s_A + 10)$, $-20(s_B - 5)$, and $-5(s_B - 5)$. Thus, the initial and final potential energy are

$$V_1 = -60s_A - 20s_B - 5s_B = -60s_A - 25s_B$$

$$V_2 = -60(s_A + 10) - 20(s_B - 5) - 5(s_B - 5) = -60s_A - 25s_B - 475$$

Kinetic Energy: The mass moment inertia of the pulley about its mass center is $I_G = \frac{1}{2}\left(\frac{5}{32.2}\right)(0.5^2) = 0.01941\text{ slug}\cdot\text{ft}^2$. Since pulley D rolls without slipping, $\omega_D = \frac{v_B}{r_D} = \frac{v_B}{0.5} = 2v_B = 2(-0.5v_A) = -v_A$. Pulley C rotates about the fixed point hence $\omega_C = \frac{v_A}{r_C} = \frac{v_A}{0.5} = 2v_A$. Since the system is at initially rest, the initial kinectic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}m_D v_B^2 + \frac{1}{2}I_G\omega_D^2 + \frac{1}{2}I_G\,\omega_C^2$$

$$= \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(-0.5v_A)^2 + \frac{1}{2}\left(\frac{5}{32.2}\right)(-0.5v_A)^2$$

$$+ \frac{1}{2}(0.01941)(-v_A)^2 + \frac{1}{2}(0.01941)(2v_A)^2$$

$$= 1.0773v_A^2$$

Conservation of Energy: Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-60s_A - 25s_B) = 1.0773v_A^2 + (-60s_A - 25s_B - 475)$$

$$v_4 = 21.0\text{ ft/s} \qquad \textbf{Ans.}$$

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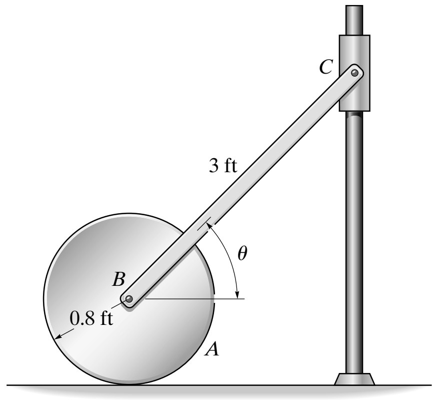
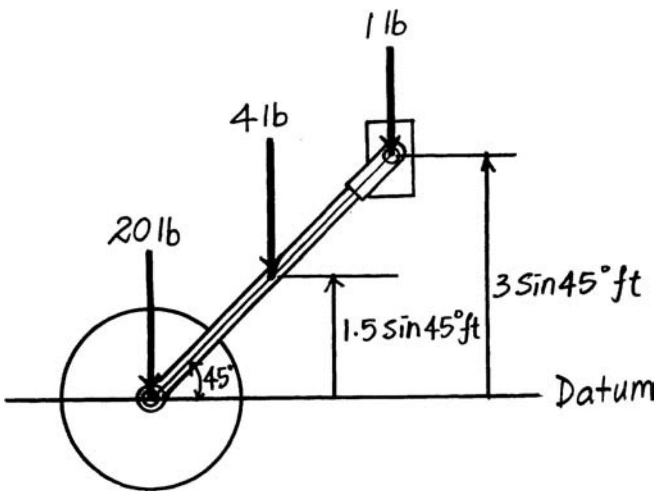
•18–45. The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., $\theta = 0°$. The system is released from rest when $\theta = 45°$.

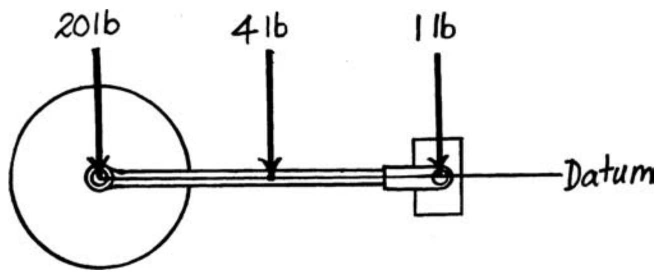


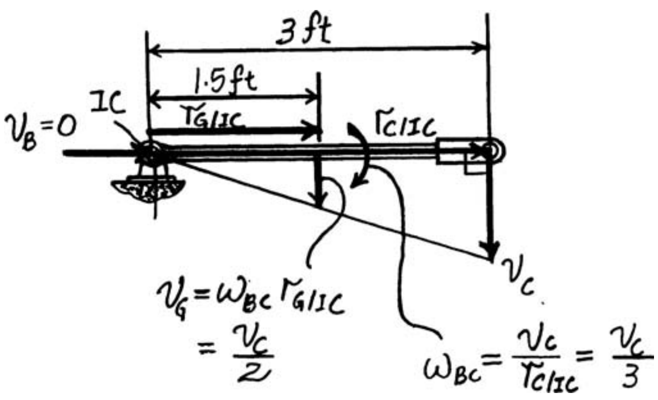
$$T_1 + V_1 = T_2 + V_2$$

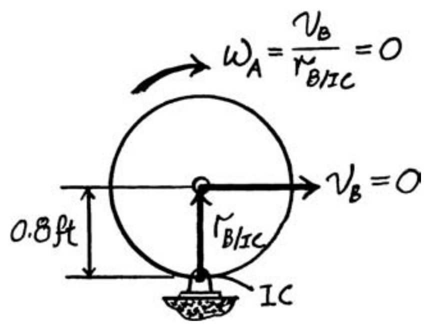
$$0 + 4(1.5 \sin 45°) + 1(3 \sin 45°) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{4}{32.2}\right)(3)^2\right]\left(\frac{v_C}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{32.2}\right)(v_C)^2 + 0$$

$$v_C = 13.3 \text{ ft/s}$$ **Ans.**









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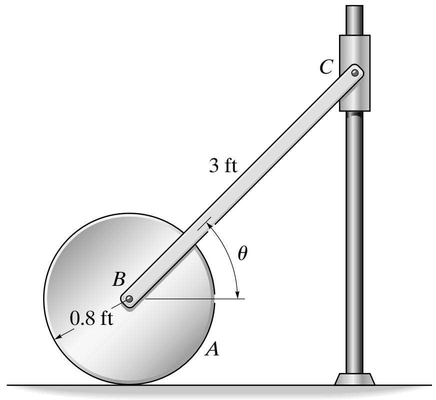
18–46. The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$. The system is released from rest when $\theta = 45°$.



$$v_B = 0.8\omega_A$$

$$\omega_{BC} = \frac{v_B}{1.5} = \frac{v_C}{2.598} = \frac{v_G}{1.5}$$

Thus,

$$v_B = v_G = 1.5\omega_{BC} \qquad v_C = 2.598\omega_{BC}$$

$$\omega_A = 1.875\,\omega_{BC}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1.5 \sin 45°) + 1(3 \sin 45°)$$

$$= \frac{1}{2}\left[\frac{1}{2}\left(\frac{20}{32.2}\right)(0.8)^2\right](1.875\omega_{BC})^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(1.5\,\omega_{BC})^2$$

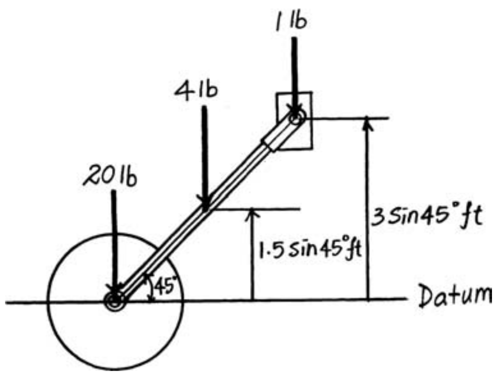
$$+\frac{1}{2}\left[\frac{1}{12}\left(\frac{4}{32.2}\right)(3)^2\right]\omega_{BC}^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)(1.5\omega_{BC})^2$$

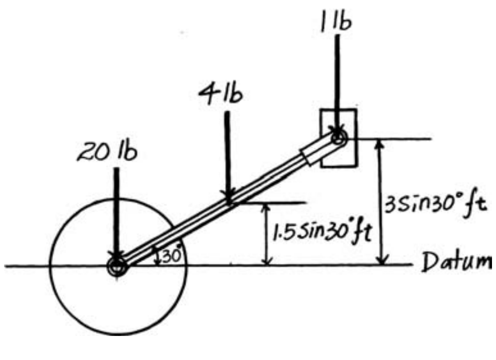
$$+\frac{1}{2}\left(\frac{1}{32.2}\right)(2.598\omega_{BC})^2 + 4(1.5 \sin 30°) + 1(3 \sin 30°)$$

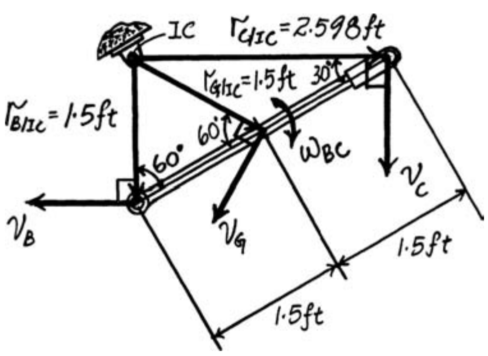
$$\omega_{BC} = 1.180 \text{ rad/s}$$

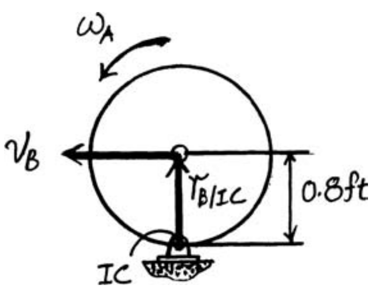
Thus,

$$v_C = 2.598(1.180) = 3.07 \text{ ft/s}$$ **Ans.**









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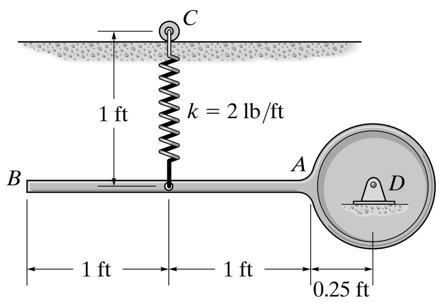
18–47. The pendulum consists of a 2-lb rod *BA* and a 6-lb disk. The spring is stretched 0.3 ft when the rod is horizontal as shown. If the pendulum is released from rest and rotates about point *D*, determine its angular velocity at the instant the rod becomes vertical. The roller at *C* allows the spring to remain vertical as the rod falls.



Potential Energy: Datum is set at point O. When rod AB is at vertical position, its center of gravity is located 1.25 ft *below* the datum. Its gravitational potential energy at this position is $-2(1.25)$ ft · lb. The initial and final stretch of the spring are 0.3 ft and $(1.25 + 0.3)$ ft $= 1.55$ ft, respectively. Hence, the initial and final elastic potential energy are $\frac{1}{2}(2)(0.3^2) = 0.09$ lb · ft and $\frac{1}{2}(2)(1.55^2) = 2.4025$ lb · ft. Thus,

$$V_1 = 0.09 \text{ lb}\cdot\text{ft} \qquad V_2 = 2.4025 + [-2(1.25)] = -0.0975 \text{ lb}\cdot\text{ft}$$

Kinetic Energy: The mass moment inertia for rod AB and the disk about point O are

$$(I_{AB})_O = \frac{1}{12}\left(\frac{2}{32.2}\right)(2^2) + \left(\frac{2}{32.2}\right)(1.25^2) = 0.1178 \text{ slug}\cdot\text{ft}^2$$

and

$$(I_D)_O = \frac{1}{2}\left(\frac{6}{32.2}\right)(0.25^2) = 0.005823 \text{ slug}\cdot\text{ft}^2$$

Since rod AB and the disk are initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}(I_{AB})_O\,\omega^2 + \frac{1}{2}(I_D)_O\,\omega^2$$

$$= \frac{1}{2}(0.1178)\,\omega^2 + \frac{1}{2}(0.005823)\,\omega^2$$

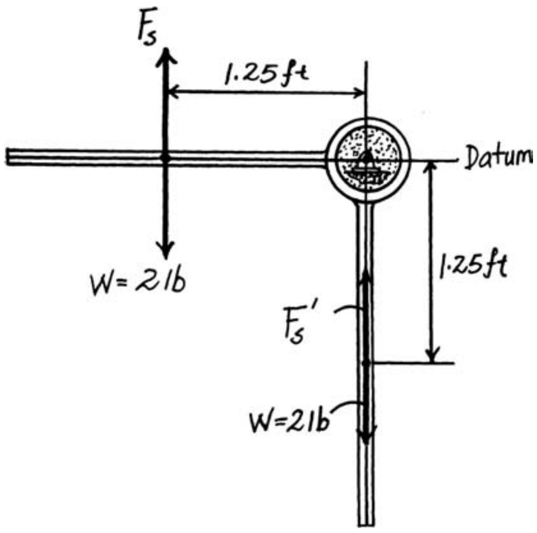
$$= 0.06179\,\omega^2$$

Conservation of Energy: Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0.09 = 0.06179\,\omega^2 + (-0.0975)$$

$$\omega = 1.74 \text{ rad/s} \qquad \textbf{Ans.}$$



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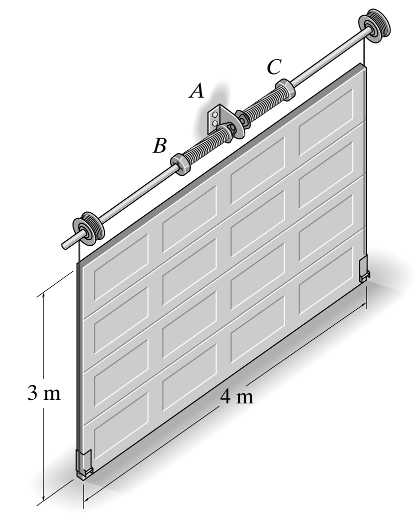
***18–48.** The uniform garage door has a mass of 150 kg and is guided along smooth tracks at its ends. Lifting is done using the two springs, each of which is attached to the anchor bracket at A and to the counterbalance shaft at B and C. As the door is raised, the springs begin to unwind from the shaft, thereby assisting the lift. If each spring provides a torsional moment of $M = (0.7\theta)\ \text{N}\cdot\text{m}$, where θ is in radians, determine the angle θ_0 at which both the left-wound and right-wound spring should be attached so that the door is completely balanced by the springs, i.e., when the door is in the vertical position and is given a slight force upwards, the springs will lift the door along the side tracks to the horizontal plane with no final angular velocity. *Note:* The elastic potential energy of a torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and in this case $k = 0.7\ \text{N}\cdot\text{m/rad}$.

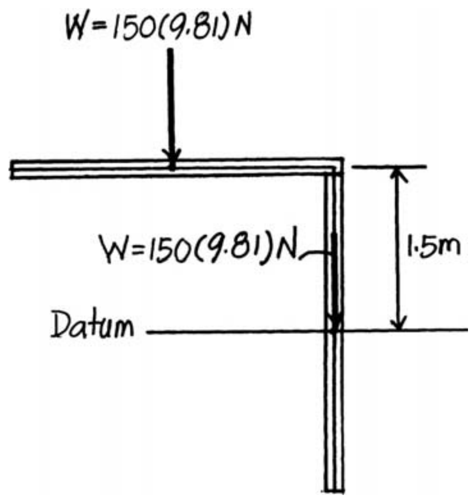


Datum at initial position.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(0.7)\theta_0^2\right] + 0 = 0 + 150(9.81)(1.5)$$

$$\theta_0 = 56.15\ \text{rad} = 8.94\ \text{rev.}$$ **Ans.**



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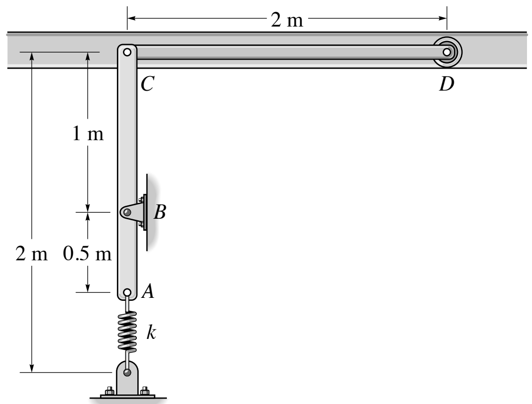
•18–49. The garage door *CD* has a mass of 50 kg and can be treated as a thin plate. Determine the required unstretched length of each of the two side springs when the door is in the open position, so that when the door falls freely from the open position it comes to rest when it reaches the fully closed position, i.e., when *AC* rotates 180°. Each of the two side springs has a stiffness of $k = 350$ N/m. Neglect the mass of the side bars *AC*.



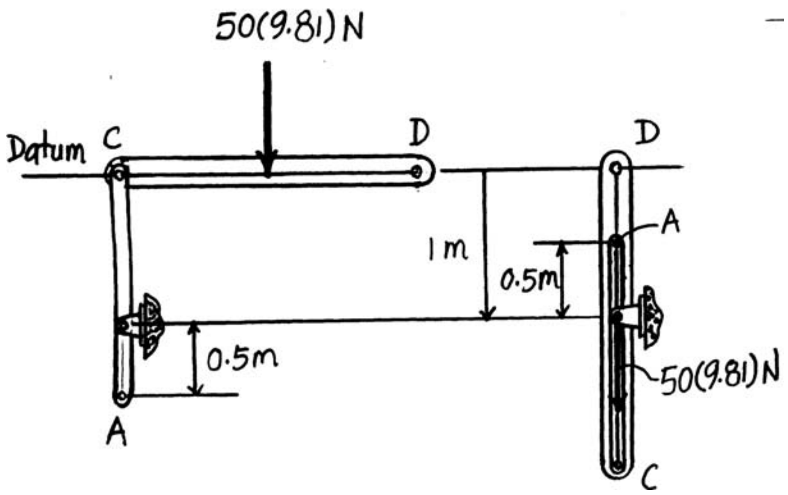
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(350)(x_1)^2\right] = 0 + 2\left[\frac{1}{2}(350)(x_1 + 1)^2\right] - 50(9.81)(1)$$

$$x_1 = 0.201 \text{ m}$$

Thus,

$$I_0 = 0.5 \text{ m} - 0.201 \text{ m} = 299 \text{ mm} \qquad \textbf{Ans.}$$



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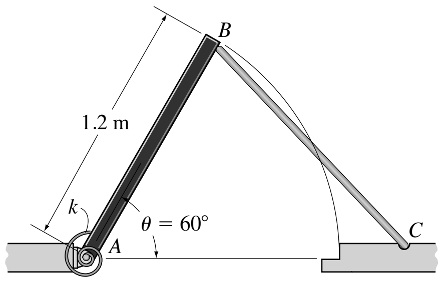
18–50. The uniform rectangular door panel has a mass of 25 kg and is held in equilibrium above the horizontal at the position $\theta = 60°$ by rod BC. Determine the required stiffness of the torsional spring at A, so that the door's angular velocity becomes zero when the door reaches the closed position ($\theta = 0°$) once the supporting rod BC is removed. The spring is undeformed when $\theta = 60°$.



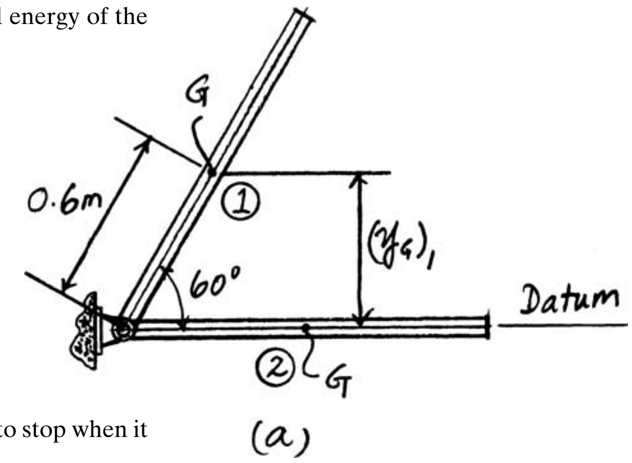
Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the panel at positions (1) and (2) is

$$(V_g)_1 = W(y_G)_1 = 25(9.81)(0.6 \sin 60°) = 127.44 \text{ J}$$

$$(V_g)_2 = W(y_G)_2 = 25(9.81)(0) = 0$$

Since the spring is initially untwisted, $(V_e)_1 = 0$. The elastic potential energy of the spring when $\theta = 60° = \frac{\pi}{3}$ rad is

$$(V_e)_2 = \frac{1}{2}k\theta^2 = \frac{1}{2}(k)\left(\frac{\pi}{3}\right)^2 = \frac{\pi^2}{18}k$$



Thus, the potential energy of the panel is

$$V_1 = (V_g)_1 + (V_e)_1 = 127.44 + 0 = 127.44 \text{ J}$$

$$V_2 = (V_g)_2 + (V_e)_2 = 0 + \frac{\pi^2}{18}k = \frac{\pi^2}{18}k$$

Kinetic Energy. Since the rod is at rest at position (1) and is required to stop when it is at position (2), $T_1 = T_2 = 0$.

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 127.44 = 0 + \frac{\pi^2}{18}k$$

$$k = 232 \text{ N}\cdot\text{m/rad}$$ **Ans.**

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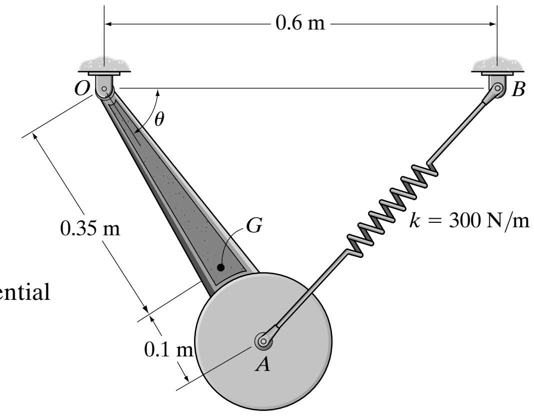
18–51. The 30 kg pendulum has its mass center at G and a radius of gyration about point G of $k_G = 300$ mm. If it is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 90°$. Spring AB has a stiffness of $k = 300$ N/m and is unstretched when $\theta = 0°$.



Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the pendulum at positions (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 30(9.81)(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -30(9.81)(0.35) = -103.005 \text{ J}$$

Since the spring is unstretched initially, $(V_e)_1 = 0$. When $\theta = 90°$, the spring stretches $s = AB - A'B = \sqrt{0.45^2 + 0.6^2} - 0.15 = 0.6$ m. Thus,

$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}(300)\left(0.6^2\right) = 54 \text{ J}$$

and

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 0$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = -103.005 + 54 = -49.005 \text{ J}$$

Kinetic Energy: Since the pendulum rotates about a fixed axis, $v_G = \omega r_G = \omega(0.35)$. The mass moment of inertia of the pendulum about its mass center is $I_G = mk_G{}^2 = 30\left(0.3^2\right) = 2.7 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the pendulum is

$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2$$

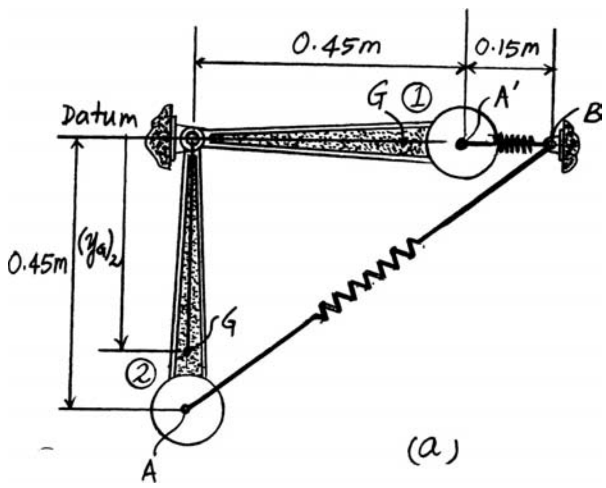
$$= \frac{1}{2}(30)\left[\omega(0.35)\right]^2 + \frac{1}{2}(2.7)\omega^2 = 3.1875\omega^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 3.1875\omega^2 - 49.005$$

$$\omega = 3.92 \text{ rad/s}$$ **Ans.**



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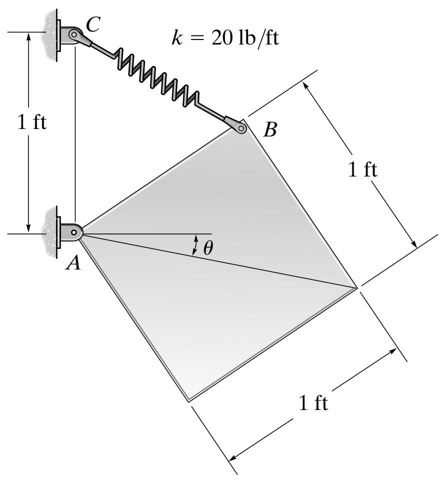
***18–52.** The 50-lb square plate is pinned at corner A and attached to a spring having a stiffness of $k = 20$ lb/ft. If the plate is released from rest when $\theta = 0°$, determine its angular velocity when $\theta = 90°$. The spring is unstretched when $\theta = 0°$.



Potential Energy: With reference to the datum shown in Fig. *a*, the gravitational potential energy of the plate at position (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 50(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -50(1 \cos 45°) = -35.36 \text{ lb} \cdot \text{ft}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. When the plate is at position (2), the spring stetches $s = BC - B'C = 2[1 \cos 22.5°] - 2(1 \cos 67.5°) = 1.082$ ft. Therefore,

$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}(20)\left(1.082^2\right) = 11.72 \text{ lb} \cdot \text{ft}$$

Thus,

$$V_1 = \left(V_g\right)_1 + \left(V_e\right)_1 = 0 + 0 = 0$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = -35.36 + 11.72 = -23.64 \text{ ft} \cdot \text{lb}$$

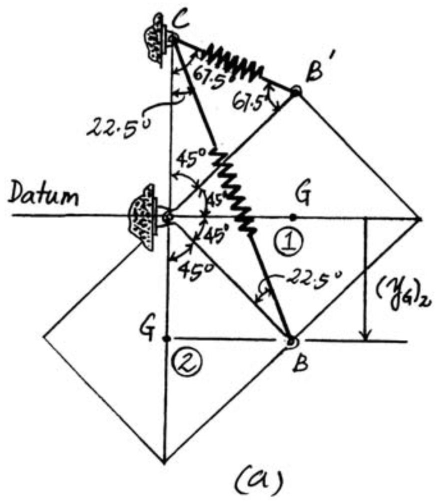
Kinetic Energy: Since the plate rotates about a fixed axis passing through point A, its kinetic energy can be determined from $T = \frac{1}{2}I_A\,\omega^2$, where

$$I_A = \frac{1}{12}\left(\frac{50}{32.2}\right)\left(1^2 + 1^2\right) + \frac{50}{32.2}(1 \cos 45°)^2 = 1.035 \text{ slug} \cdot \text{ft}^2$$

Thus,

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(1.035)\omega^2 = 0.5176\omega^2$$

Since the plate is initially at rest $T_1 = 0$.



Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0.5176\omega^2 - 23.64$$

$$\omega = 6.76 \text{ rad/s} \qquad \textbf{Ans.}$$

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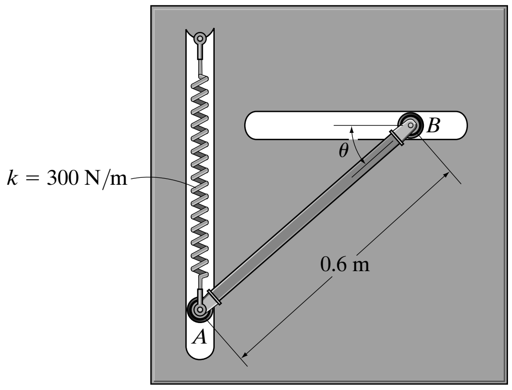
•18–53. A spring having a stiffness of $k = 300$ N/m is attached to the end of the 15-kg rod, and it is unstretched when $\theta = 0°$. If the rod is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 30°$. The motion is in the vertical plane.



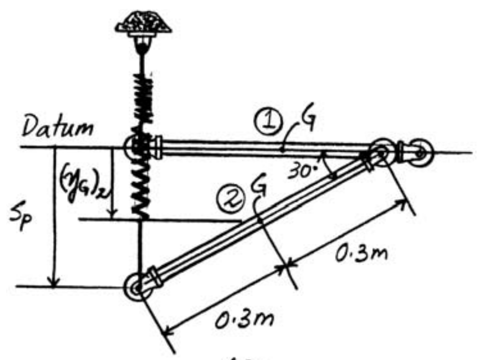
Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the rod at positions (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 15(9.81)(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -15(9.81)(0.3 \sin 30°) = -22.0725 \text{ J}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. When $\theta = 30°$, the stretch of the spring is $s_P = 0.6 \sin 30° = 0.3$ m. Thus, the final elastic potential energy of the spring is

$$\left(V_e\right)_2 = \frac{1}{2} k s_P{}^2 = \frac{1}{2}(300)\left(0.3^2\right) = 13.5 \text{ J}$$



(a)

Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = 0 + 0 = 0$$

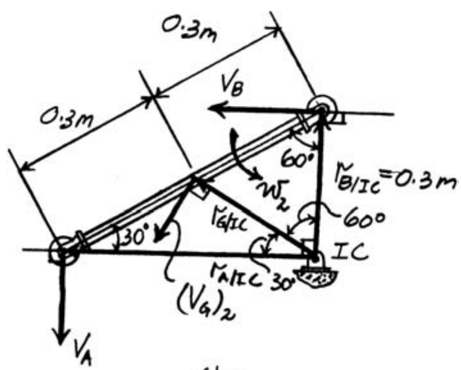
$$V_2 = (V_g)_2 + (V_e)_2 = -22.0725 + 13.5 = -8.5725 \text{ J}$$

Kinetic Energy: Since the rod is initially at rest, $T_1 = 0$. From the geometry shown in Fig. *b*, $r_{G/IC} = 0.3$ m. Thus, $(V_G)_2 = \omega_2 r_{G/IC} = \omega_2\,(0.3)$. The mass moment of inertia of the rod about its mass center is $I_G = \dfrac{1}{12} ml^2 = \dfrac{1}{12}(15)\left(0.6^2\right) = 0.45 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy of the rod is

$$T_2 = \frac{1}{2} m(v_G)_2{}^2 + \frac{1}{2} I_G \omega_2{}^2$$

$$= \frac{1}{2}(15)\left[\omega_2\left(0.3\right)\right]^2 + \frac{1}{2}\left(0.45\right)\omega_2{}^2$$

$$= 0.9\omega_2{}^2$$



(b)

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0.9\omega_2{}^2 - 8.5725$$

$$\omega_2 = 3.09 \text{ rad/s} \qquad \textbf{Ans.}$$

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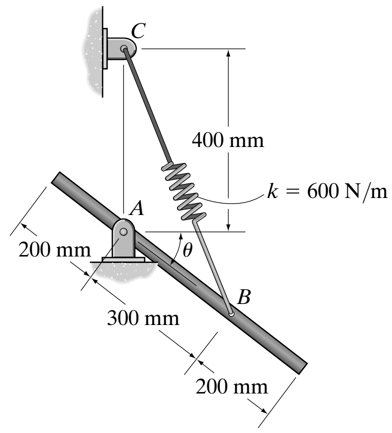
18–54. If the 6-kg rod is released from rest at $\theta = 30°$, determine the angular velocity of the rod at the instant $\theta = 0°$. The attached spring has a stiffness of $k = 600$ N/m, with an unstretched length of 300 mm.

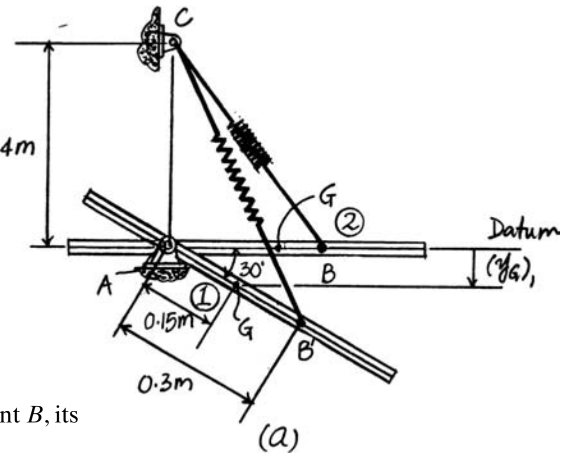


Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the rod at positions (1) and (2) is

$$\left(V_g\right)_1 = -W(y_G)_1 = -6(9.81)(0.15 \sin 30°) = -4.4145 \text{ J}$$

$$\left(V_g\right)_2 = W(y_G)_2 = 6(9.81)(0) = 0$$

The stretch of the spring when the rod is in positions (1) and (2) is $s_1 = B'C - l_0 = \sqrt{0.3^2 + 0.4^2 - 2(0.3)(0.4)\cos 120°} - 0.3 = 0.3083$ m and $s_2 = BC - l_0 = \sqrt{0.3^2 + 0.4^2} - 0.3 = 0.2$ m. Thus, the initial and final elastic potential energy of the spring is



$$\left(V_e\right)_1 = \frac{1}{2}ks_1{}^2 = \frac{1}{2}(600)\left(0.3083^2\right) = 28.510 \text{ J}$$

$$\left(V_e\right)_2 = \frac{1}{2}ks_2{}^2 = \frac{1}{2}(600)\left(0.2^2\right) = 12 \text{ J}$$

Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = -4.4145 + 28.510 = 24.096 \text{ J}$$

$$V_2 = (V_g)_2 + (V_e)_2 = 0 + 12 = 12 \text{ J}$$

Kinetic Energy: Since the rod rotates about a fixed axis passing through point *B*, its kinetic energy can be determined from $T = \frac{1}{2} I_B\omega^2$, where

$$I_B = \frac{1}{12}(6)\left(0.7^2\right) + 6\left(0.15^2\right) = 0.38 \text{ kg} \cdot \text{m}^2$$

Thus,

$$T = \frac{1}{2} I_B\omega^2 = \frac{1}{2}(0.38)\omega^2 = 0.19\omega^2$$

Since the rod is initially at rest, $T_1 = 0$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 24.096 = 0.19\omega^2 + 12$$

$$\omega = 7.98 \text{ rad/s} \qquad \textbf{Ans.}$$

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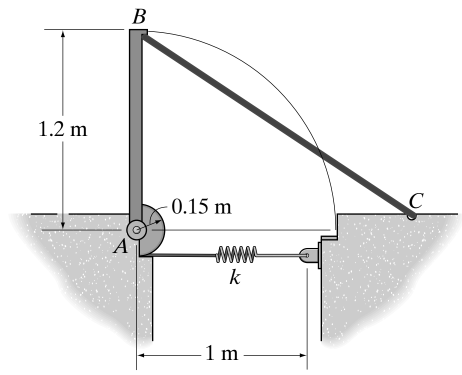
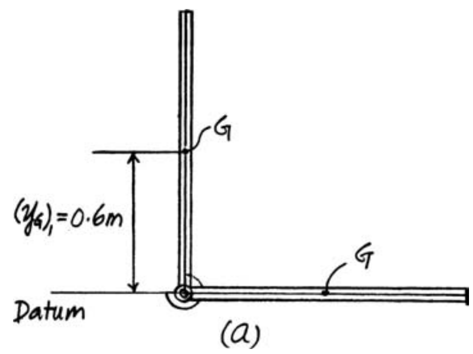
18–55. The 50-kg rectangular door panel is held in the vertical position by rod CB. When the rod is removed, the panel closes due to its own weight. The motion of the panel is controlled by a spring attached to a cable that wraps around the half pulley. To reduce excessive slamming, the door panel's angular velocity is limited to 0.5 rad/s at the instant of closure. Determine the minimum stiffness k of the spring if the spring is unstretched when the panel is in the vertical position. Neglect the half pulley's mass.



Potential Energy: With reference to the datum in Fig. a, the gravitational potential energy of the door panel at its open and closed positions is

$$(V_g)_1 = W(y_G)_1 = 50(9.81)(0.6) = 294.3 \text{ J}$$

$$(V_g)_2 = W(y_G)_2 = 50(9.81)(0) = 0$$



Since the spring is unstretched when the door panel is at the open position, $(V_e)_1 = 0$. When the door is closed, the half pulley rotates through and angle of $\theta = \dfrac{\pi}{2}$ rad. Thus, the spring stretches $s = r\theta = 0.15\left(\dfrac{\pi}{2}\right) = 0.075\pi$ m. Then,

$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}k(0.075\pi)^2 = 0.0028125\pi^2\, k$$

Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = 294.3 + 0 = 294.3 \text{ J}$$

$$V_2 = (V_g)_2 + (V_e)_2 = 0 + 0.0028125\pi^2 k = 0.0028125\pi^2 k$$

Kinetic Energy: Since the door panel rotates about a fixed axis passing through point A, its kinetic energy can be determined from $T = \dfrac{1}{2} I_A\, \omega^2$, where

$$I_A = \frac{1}{12}(50)\left(1.2^2\right) + 50\left(0.6^2\right) = 24 \text{ kg} \cdot \text{m}^2$$

Thus,

$$T = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(24)\omega^2 = 12\omega^2$$

Since the door panel is at rest in the open position and required to have an angular velocity of $\omega = 0.5$ rad/s at closure, then

$$T_1 = 0 \qquad\qquad T_2 = 12\left(0.5^2\right) = 3 \text{ J}$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 294.3 = 3 + 0.0028125\pi^2 k$$

$$k = 10494.17 \text{ N/m} = 10.5 \text{ kN/m} \qquad \textbf{Ans.}$$

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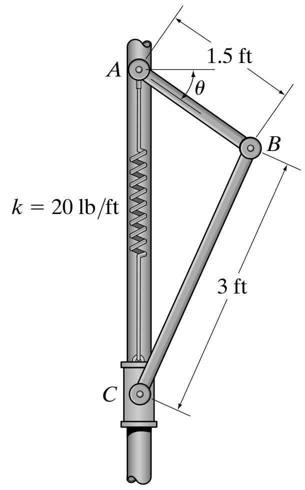
***18–56.** Rods AB and BC have weights of 15 lb and 30 lb, respectively. Collar C, which slides freely along the smooth vertical guide, has a weight of 5 lb. If the system is released from rest when $\theta = 0°$, determine the angular velocity of the rods when $\theta = 90°$. The attached spring is unstretched when $\theta = 0°$.



Potential Energy: From the geometry in Fig. a, $\theta = \sin^{-1}\left(\frac{1.5}{3}\right) = 30°$. With reference to the datum, the initial and final gravitational potential energy of the system is

$$\left(V_g\right)_1 = W_{AB}\,(y_{G1})_1 - W_{BC}\,(y_{G2})_1 - W_C\,(y_{G3})_1$$

$$= 15(0) - 30(1.5\cos 30°) - 5(3\cos 30°)$$

$$= -51.96\text{ ft}\cdot\text{lb}$$

$$\left(V_g\right)_2 = -W_{AB}\,(y_{G1})_2 - W_{BC}\,(y_{G2})_2 - W_C\,(y_{G3})_2$$

$$= -15(0.75) - 30(3) - 5(4.5)$$

$$= -123.75\text{ ft}\cdot\text{lb}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. When $\theta = 90°$, the spring stretches $s = 4.5 - 3\cos 30° = 1.902$ ft. Thus,

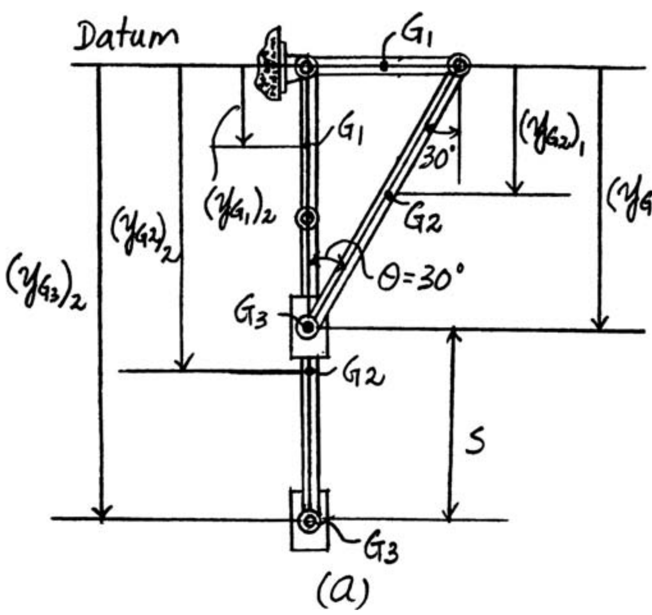
$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}(20)\left(1.902^2\right) = 36.17\text{ ft}\cdot\text{lb}$$

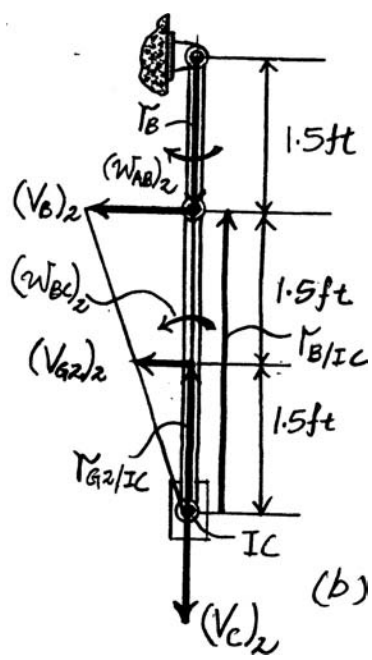
And,

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = -51.96 + 0 = -51.96\text{ ft}\cdot\text{lb}$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = -123.75 + 36.17 = -87.58\text{ ft}\cdot\text{lb}$$

Kinetic Energy: Since the system is initially at rest $T_1 = 0$. Referring to Fig. b, $(v_B)_2 = (\omega_{AB})_2\, r_B = (\omega_{AB})_2(1.5)$. Then $(\omega_{BC})_2 = \dfrac{(v_B)_2}{r_{B/IC}} = \dfrac{(\omega_{AB})_2(1.5)}{3} = 0.5(\omega_{AB})_2$.





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***18–56. Continued**

Subsequently, $(v_{G2})_2 = (\omega_{BC})_2\, r_{G2/IC} = 0.5(\omega_{AB})_2(1.5) = 0.75(\omega_{AB})_2$. Since point C is located at the IC, $v_C = 0$. The mass moments of inertia of AB about point A and BC about its mass center are $(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}\left(\frac{15}{32.2}\right)(1.5^2) = 0.3494\ \text{slug/ft}^2$ and $(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{30}{32.2}\right)(3^2) = 0.6988\ \text{slug/ft}^2$. Thus, the final kinetic energy of the system is

$$T_2 = T_{AB} + T_{BC} + T_C$$

$$= \frac{1}{2}(I_{AB})_A(\omega_{AB})_2{}^2 + \left[\frac{1}{2}m_{BC}(v_{G2})^2 + \frac{1}{2}(I_{BC})_{G2}(\omega_{BC})_2{}^2\right] + \frac{1}{2}m_C v_C{}^2$$

$$= \frac{1}{2}(0.3494)(\omega_{AB})_2{}^2 + \left[\frac{1}{2}\left(\frac{30}{32.2}\right)\left[0.75(\omega_{AB})_2\right]^2 + \frac{1}{2}(0.6988)\left[0.5(\omega_{AB})_2\right]^2\right] + 0$$

$$= 0.5241(\omega_{AB})_2{}^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

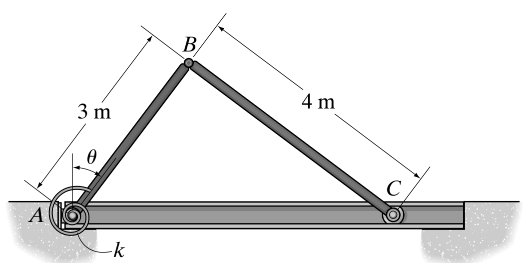
$$0 - 51.96 = 0.5241(\omega_{AB})_2{}^2 - 87.58$$

$$(\omega_{AB})_2 = 8.244\ \text{rad/s} = 8.24\ \text{rad/s} \qquad \textbf{Ans.}$$

Thus,

$$(\omega_{BC})_2 = 0.5(8.244) = 4.12\ \text{rad/s} \qquad \textbf{Ans.}$$

•18–57. Determine the stiffness k of the torsional spring at A, so that if the bars are released from rest when $\theta = 0°$, bar AB has an angular velocity of 0.5 rad/s at the closed position, $\theta = 90°$. The spring is uncoiled when $\theta = 0°$. The bars have a mass per unit length of 10 kg/m.



Potential Energy: With reference to the datum in Fig. a, the gravitational potential energy of the system at its open and closed positions is

$$\left(V_g\right)_1 = W_{AB}(y_{G1})_1 + W_{BC}(y_{G2})_1$$

$$= 10(3)(9.81)(1.5) + 10(4)(9.81)(1.5) = 1030.5\ \text{J}$$

$$\left(V_g\right)_2 = W_{AB}(y_{G1})_2 + W_{BC}(y_{G2})_2$$

$$= 10(3)(9.81)(0) + 10(4)(9.81)(0) = 0$$

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•18–57. Continued

Since the spring is initially uncoiled, $(V_e)_1 = 0$. When the panels are in the closed position, the coiled angle of the spring is $\theta = \frac{\pi}{2}$ rad. Thus,

$$(V_e)_2 = \frac{1}{2}k\theta^2 = \frac{1}{2}k\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{8}k$$

And so,

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 1030.5 + 0 = 1030.5 \text{ J}$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = 0 + \frac{\pi^2}{8}k = \frac{\pi^2}{8}k$$

Kinetic Energy: Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $(v_B)_2 = (\omega_{AB})_2 r_B = 0.5(3) = 1.5$ m/s. Then, $(\omega_{BC})_2 = \frac{(v_B)_2}{r_{B/IC}} = \frac{1.5}{4} = 0.375$ rad/s. Subsequently, $(v_G)_2 = (\omega_{BC})_2 r_{G2/IC} = 0.375(2) = 0.75$ m/s. The mass moments of inertia of AB about point A and BC about its mass center are

$$(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}[10(3)]\left(3^2\right) = 90 \text{ kg}\cdot\text{m}^2$$

and

$$(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}[10(4)]\left(4^2\right) = 53.33 \text{ kg}\cdot\text{m}^2$$

Thus,

$$T_2 = \frac{1}{2}(I_{AB})_A\,(\omega_{AB})_2{}^2 + \left[\frac{1}{2}m_{BC}(v_{G2})^2 + \frac{1}{2}(I_{BC})_{G2}\,(\omega_{BC})_2{}^2\right]$$

$$= \frac{1}{2}(90)\left(0.5^2\right) + \left[\frac{1}{2}[10(4)]\left(0.75^2\right) + \frac{1}{2}(53.33)\left(0.375^2\right)\right]$$

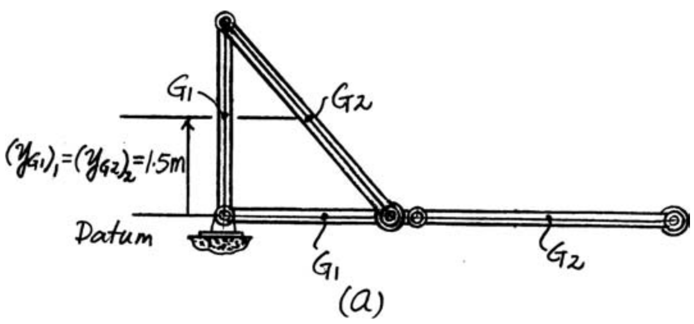
$$= 26.25 \text{ J}$$

Conservation of Energy:

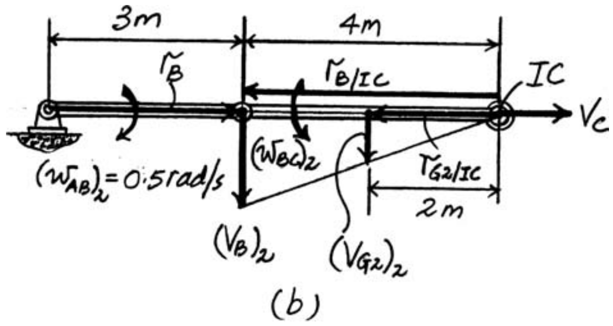
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 1030.5 = 26.25 + \frac{\pi^2}{8}k$$

$$k = 814 \text{ N}\cdot\text{m/rad} \qquad \textbf{Ans.}$$



(a)



(b)

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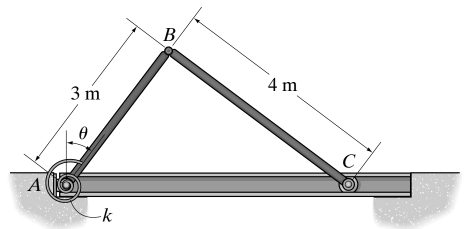
18–58. The torsional spring at A has a stiffness of $k = 900\ \text{N}\cdot\text{m/rad}$ and is uncoiled when $\theta = 0°$. Determine the angular velocity of the bars, AB and BC, when $\theta = 0°$, if they are released from rest at the closed position, $\theta = 90°$. The bars have a mass per unit length of 10 kg/m.



Potential Energy: With reference to the datum in Fig. a, the gravitational potential energy of the system at its open and closed positions is

$$\left(V_g\right)_1 = W_{AB}\,(y_{G1})_1 + W_{BC}\,(y_{G2})_1$$

$$= 10(3)(9.81)(0) + 10(4)(9.81)(0) = 0$$

$$\left(V_g\right)_2 = W_{AB}\,(y_{G1})_2 + W_{BC}\,(y_{G2})_2$$

$$= 10(3)(9.81)(1.5) + 10(4)(9.81)(1.5) = 1030.05\ \text{J}$$

When the panel is in the closed position, the coiled angle of the spring is $\theta = \dfrac{\pi}{2}$ rad. Thus,

$$(V_e)_1 = \frac{1}{2}k\theta^2 = \frac{1}{2}(900)\left(\frac{\pi}{2}\right)^2 = 112.5\pi^2\ \text{J}$$

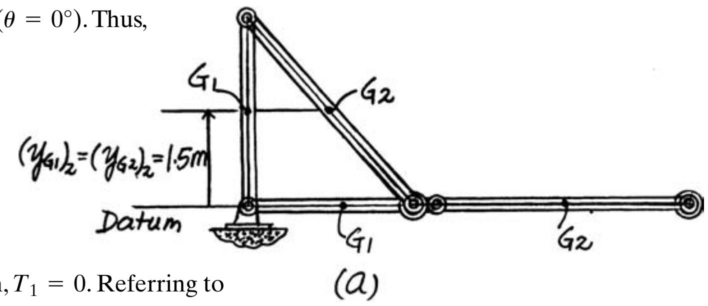
The spring is uncoiled when the panel is in the open position ($\theta = 0°$). Thus,

$$(V_e)_2 = 0$$

And so,

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 0 + 112.5\pi^2 = 112.5\pi^2\ \text{J}$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = 1030.05 + 0 = 1030.05\ \text{J}$$



Kinetic Energy: Since the panel is at rest in the closed position, $T_1 = 0$. Referring to Fig. b, the IC for BC is located at infinity. Thus,

$$(\omega_{BC})_2 = 0 \qquad \textbf{Ans.}$$

Then,

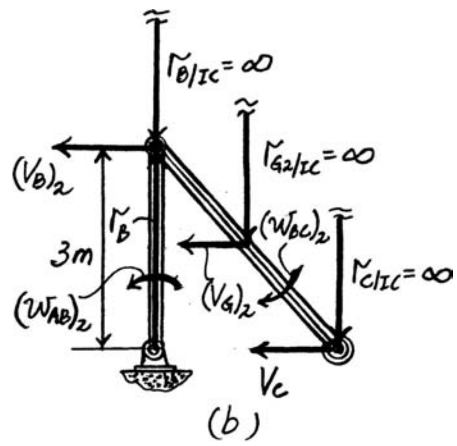
$$(v_G)_2 = (v_B)_2 = (\omega_{AB})_2\, r_B = (\omega_{AB})_2(3)$$

The mass moments of inertia of AB about point A and BC about its mass center are

$$(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}[10(3)]\left(3^2\right) = 90\ \text{kg}\cdot\text{m}^2$$

and

$$(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}[10(4)]\left(4^2\right) = 53.33\ \text{kg}\cdot\text{m}^2$$



Thus,

$$T_2 = \frac{1}{2}(I_{AB})_A(\omega_{AB})_2{}^2 + \frac{1}{2}m_{BC}(v_{G2})^2$$

$$= \frac{1}{2}(90)(\omega_{AB})_2{}^2 + \frac{1}{2}[10(4)]\left[(\omega_{AB})_2(3)\right]^2$$

$$= 225(\omega_{AB})_2{}^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 112.5\pi^2 = 225(\omega_{AB})_2{}^2 + 1030.05$$

$$(\omega_{AB})_2 = 0.597\ \text{rad/s} \qquad \textbf{Ans.}$$

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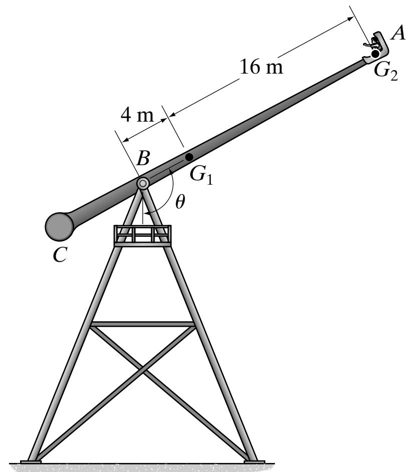
18–59. The arm and seat of the amusement-park ride have a mass of 1.5 Mg, with the center of mass located at point G_1. The passenger seated at A has a mass of 125 kg, with the center of mass located at G_2 If the arm is raised to a position where $\theta = 150°$ and released from rest, determine the speed of the passenger at the instant $\theta = 0°$. The arm has a radius of gyration of $k_{G1} = 12$ m about its center of mass G_1. Neglect the size of the passenger.



Potential Energy: With reference to the datum shown in Fig. *a*, the gravitational potential energy of the system at position (1) and (2) is

$$V_1 = \left(V_g\right)_1 = W_1(y_{G1})_1 + W_2(y_{G2})_1$$

$$= 1500(9.81)(4 \sin 60°) + 125(9.81)(20 \sin 60°)$$

$$= 72\,213.53 \text{ J}$$

$$V_2 = \left(V_g\right)_2 = -W_1(y_{G1})_2 - W_2(y_{G2})_2$$

$$= -1500(9.81)(4) - 125(9.81)(20)$$

$$= -83\,385 \text{ J}$$

Kinetic Energy: Since the arm rotates about a fixed axis passing through B, $v_{G1} = \omega r_{G1} = \omega(4)$ and $v_{G2} = \omega r_{G2} = \omega(20)$. The mass moment of inertia of the arm about its mass center is $I_{G1} = m_1 k_{G1}^2 = 1500\left(12^2\right) = 216\,000 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the system is

$$T = \left[\frac{1}{2} m_1 (v_G)_1{}^2 + \frac{1}{2} I_{G1}\omega^2\right] + \frac{1}{2} m_2 (v_{G2})^2$$

$$= \left[\frac{1}{2}(1500)[\omega(4)]^2 + \frac{1}{2}(216\,000)\omega^2\right] + \frac{1}{2}(125)[\omega(20)]^2$$

$$= 145\,000\omega^2$$

Since the system is initially at rest, $T_1 = 0$.

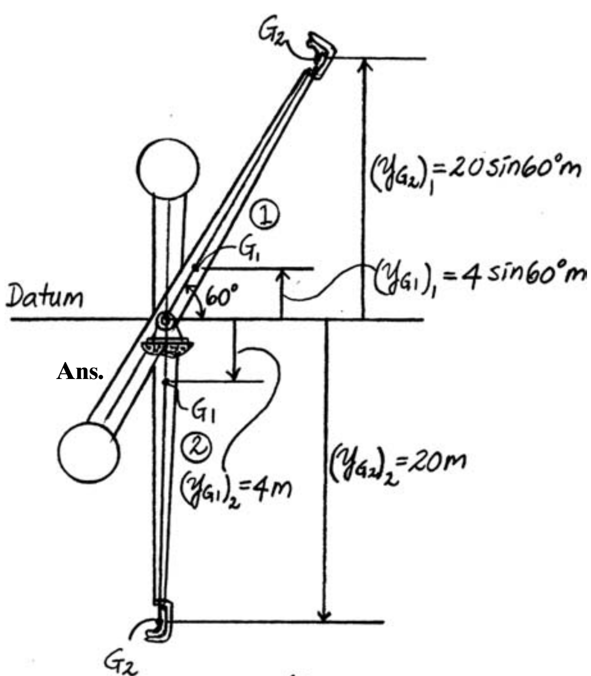
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 72\,213.53 = 145\,000\omega^2 - 83\,385$$

$$\omega = 1.0359 \text{ rad/s}$$

$$v = \omega r = (1.0359 \text{ rad/s})(20 \text{ m}) = 20.7 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

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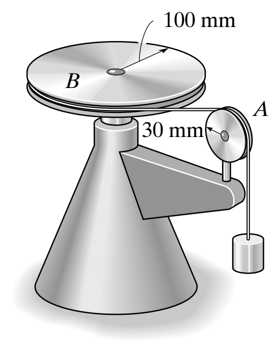
18–60. The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0 + 0] + [0] = \frac{1}{2}\left[\frac{1}{2}(3)(0.03)^2\right]\omega_A^2 + \frac{1}{2}\left[\frac{1}{2}(10)(0.1)^2\right]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$$

$$v_C = \omega_B(0.1) = 0.03\,\omega_A$$

Thus,

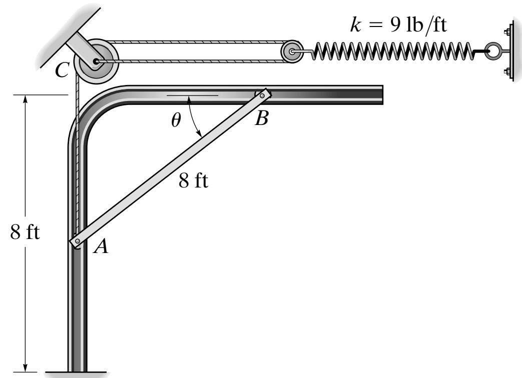
$$\omega_B = 10\,v_C$$

$$\omega = 33.33 v_C$$

Substituting and solving yields,

$$v_C = 1.52 \text{ m/s}$$ **Ans.**

•18–61. The motion of the uniform 80-lb garage door is guided at its ends by the track. Determine the required initial stretch in the spring when the door is open, $\theta = 0°$, so that when it falls freely it comes to rest when it just reaches the fully closed position, $\theta = 90°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.



$$s_A + 2s_s = l$$

$$\Delta s_A = -2\Delta s_s$$

$$8 \text{ ft} = -2\Delta s_s$$

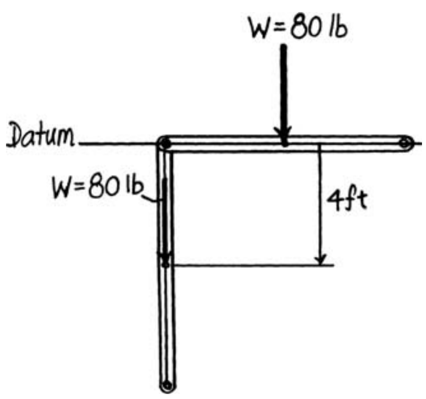
$$\Delta s_s = -4 \text{ ft}$$

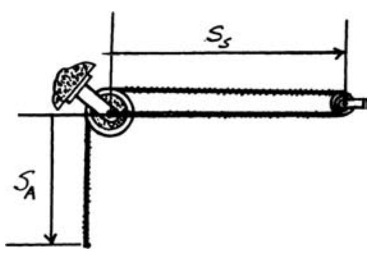
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(9)s^2\right] = 0 - 80(4) + 2\left[\frac{1}{2}(9)(4 + s)^2\right]$$

$$9s^2 = -320 + 9(16 + 8s + s^2)$$

$$s = 2.44 \text{ ft}$$ **Ans.**





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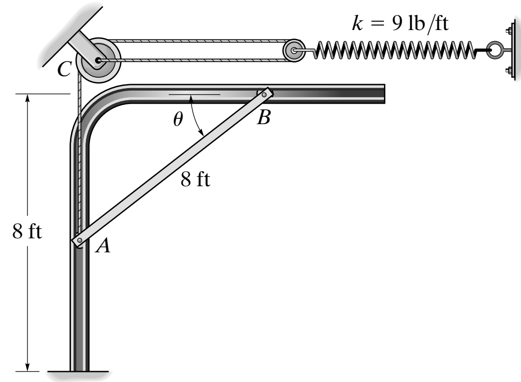
18–62. The motion of the uniform 80-lb garage door is guided at its ends by the track. If it is released from rest at $\theta = 0°$, determine the door's angular velocity at the instant $\theta = 30°$. The spring is originally stretched 1 ft when the door is held open, $\theta = 0°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.



$$v_G = 4\omega$$

$$s_A + 2s_s = l$$

$$\Delta s_A = -2\Delta s_s$$

$$4 \text{ ft} = -2\Delta s_s$$

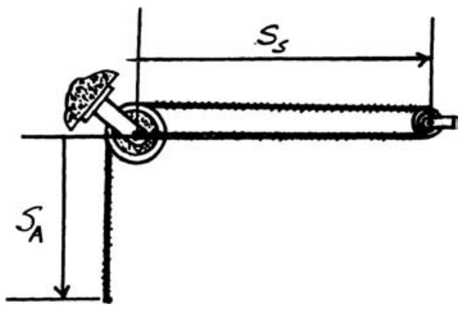
$$\Delta s_s = -2 \text{ ft}$$

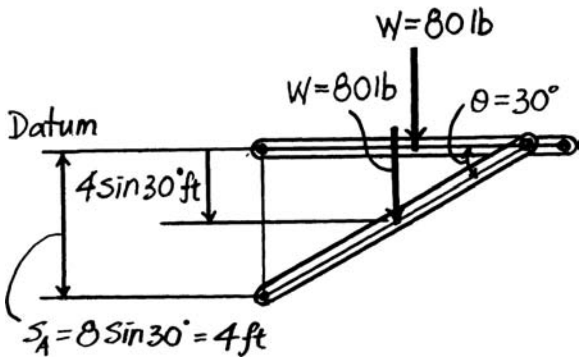
$$T_1 + V_1 = T_2 + V_2$$

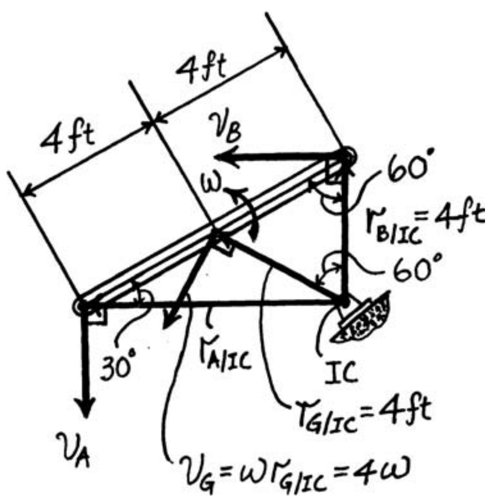
$$0 + 2\left[\frac{1}{2}(9)(1)^2\right] = \frac{1}{2}\left(\frac{80}{32.2}\right)(4\omega)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{80}{32.2}\right)(8)^2\right]\omega^2 - 80(4\sin 30°)$$

$$+ 2\left[\frac{1}{2}(9)(2+1)^2\right]$$

$$\omega = 1.82 \text{ rad/s} \qquad \textbf{Ans.}$$







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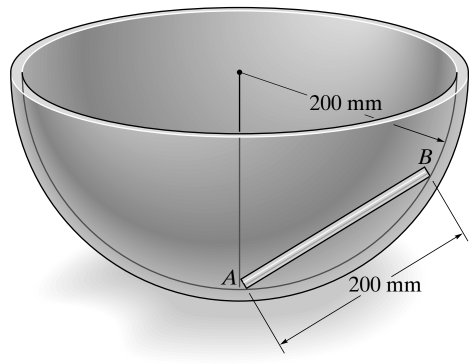
18–63. The 500-g rod AB rests along the smooth inner surface of a hemispherical bowl. If the rod is released from rest from the position shown, determine its angular velocity at the instant it swings downward and becomes horizontal.



Select datum at the bottom of the bowl.

$$\theta = \sin^{-1}\left(\frac{0.1}{0.2}\right) = 30^\circ$$

$$h = 0.1 \sin 30^\circ = 0.05$$

$$CE = \sqrt{(0.2)^2 - (0.1)^2} = 0.1732 \text{ m}$$

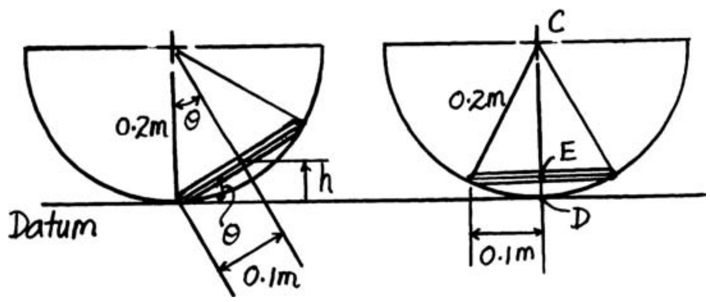
$$ED = 0.2 - 0.1732 = 0.02679$$

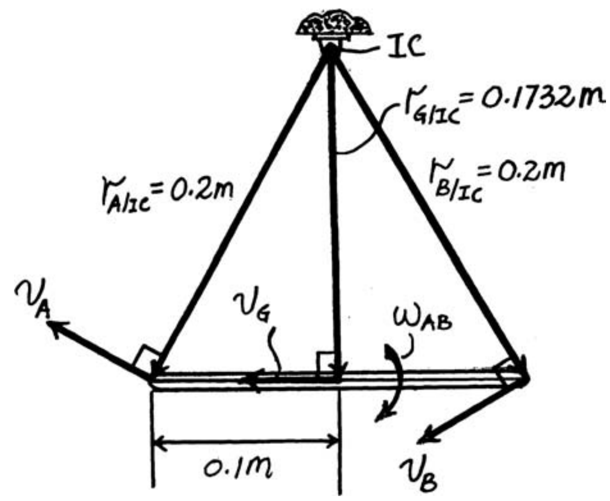
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(9.81)(0.05) = \frac{1}{2}\left[\frac{1}{12}(0.5)(0.2)^2\right]\omega_{AB}^2 + \frac{1}{2}(0.5)(v_G)^2 + (0.5)(9.81)(0.02679)$$

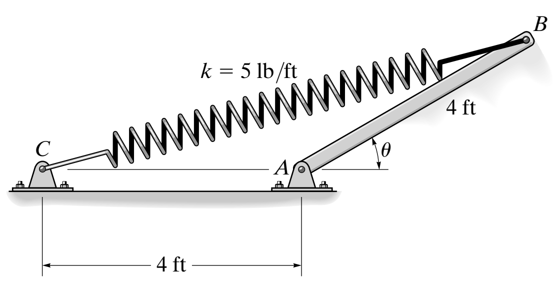
Since $v_G = 0.1732\omega_{AB}$

$$\omega_{AB} = 3.70 \text{ rad/s} \qquad \textbf{Ans.}$$





***18–64.** The 25-lb slender rod AB is attached to spring BC which has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30^\circ$, determine its angular velocity at the instant $\theta = 90^\circ$.

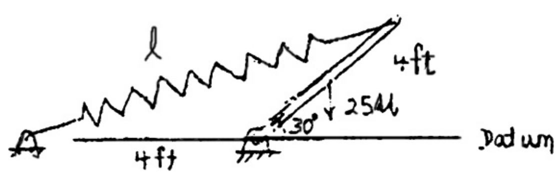


$$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150^\circ} = 7.727 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 25(2)\sin 30^\circ + \frac{1}{2}(5)(7.727 - 4)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{25}{32.2}\right)(4)^2\right]\omega^2 + 25(2)$$

$$+ \frac{1}{2}(5)(4\sqrt{2} - 4)^2$$



$$\omega = 1.18 \text{ rad/s} \qquad \textbf{Ans.}$$

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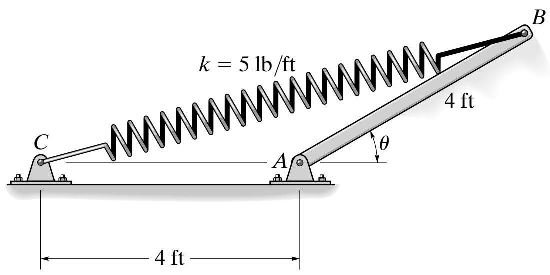
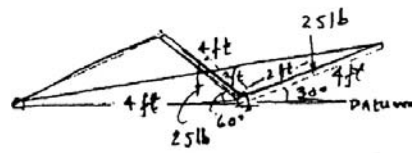
•18–65. The 25-lb slender rod AB is attached to spring BC which has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.



$$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$$

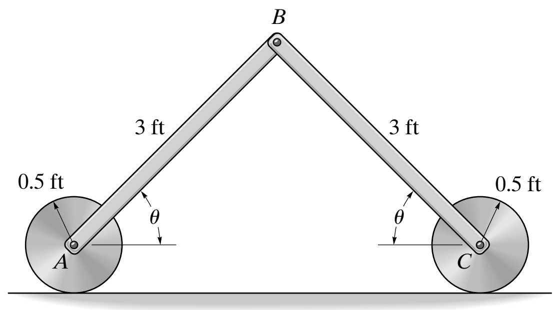
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727 - 4)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{25}{32.2}\right)(4)^2\right]\omega^2 + 25(2)(\sin 60°) + 0$$



$$\omega = 2.82 \text{ rad/s} \qquad \textbf{Ans.}$$

18–66. The assembly consists of two 8-lb bars which are pin connected to the two 10-lb disks. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 0°$. Assume the disks roll without slipping.

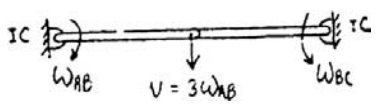


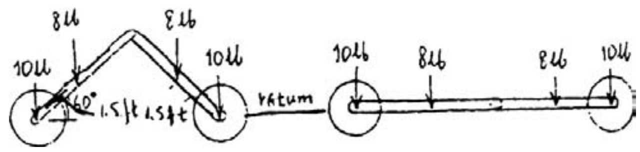
$$\omega_{AB} = \omega_{BC}$$

$$T_1 + V_1 = T_2 + V_2$$

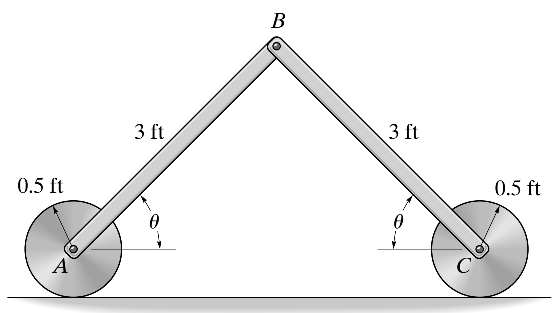
$$[0] + 2(8)(1.5 \sin 60°) = 2\left[\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{8}{32.2}\right)(3)^2\omega^2\right] + [0]$$

$$\omega = 5.28 \text{ rad/s} \qquad \textbf{Ans.}$$





18–67. The assembly consists of two 8-lb bars which are pin connected to the two 10-lb disks. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 30°$. Assume the disks roll without slipping.



$$\omega_D = \frac{v_A}{0.5} \qquad v_A = \omega_{AB}(1.5)$$

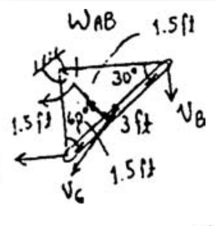
$$\omega_D = 3\omega_{AB} \qquad v_G = 1.5\omega_{AB}$$

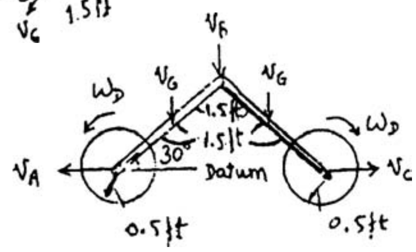
$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0] + 2[8(1.5 \sin 60°)]$$

$$= 2\left[\frac{1}{2}\left\{\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right\}(3\omega_{AB})^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)\{\omega_{AB}(1.5)\}^2\right.$$

$$\left. + \frac{1}{2}\left(\frac{8}{32.2}\right)(1.5\omega_{AB})^2 + \frac{1}{2}\left\{\frac{1}{12}\left(\frac{8}{32.2}\right)(3)^2\right\}(\omega_{AB})^2\right] + 2[8(1.5 \sin 30°)]$$





$$\omega_{AB} = 2.21 \text{ rad/s} \qquad \textbf{Ans.}$$

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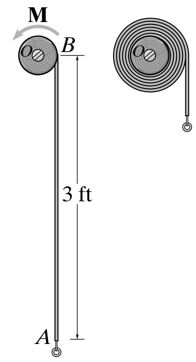
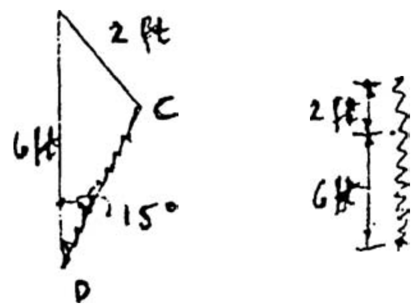
***18–68.** The uniform window shade AB has a total weight of 0.4 lb. When it is released, it winds up around the spring-loaded core O. Motion is caused by a spring within the core, which is coiled so that it exerts a torque $M = 0.3(10^{-3})\theta$ lb·ft, where θ is in radians, on the core. If the shade is released from rest, determine the angular velocity of the core at the instant the shade is completely rolled up, i.e., after 12 revolutions. When this occurs, the spring becomes uncoiled and the radius of gyration of the shade about the axle at O is $k_O = 0.9$ in. *Note:* The elastic potential energy of the torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and $k = 0.3(10^{-3})$ lb·ft/rad.



$$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15°$$

$$CD^2 - 11.591CD + 32 = 0$$



Selecting the smaller root:

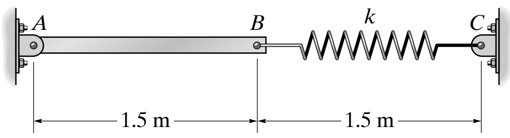
$$CD = 4.5352 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + 2\left[\frac{1}{2}(k)(8 - 4.5352)^2\right] - 200(6)$$

$$k = 100 \text{ lb/ft} \qquad \textbf{Ans.}$$

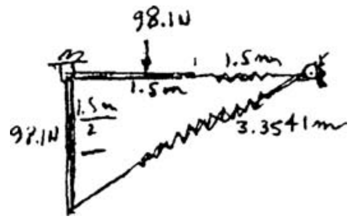
18–69. When the slender 10-kg bar AB is horizontal it is at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 90°.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \frac{1}{2}(k)(3.3541 - 1.5)^2 - 98.1\left(\frac{1.5}{2}\right)$$

$$k = 42.8 \text{ N/m} \qquad \textbf{Ans.}$$



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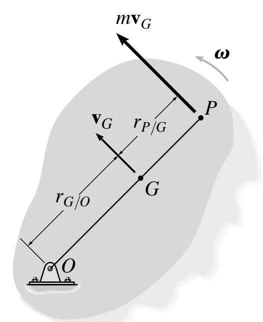
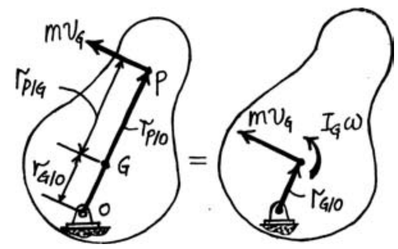
•19–1. The rigid body (slab) has a mass m and rotates with an angular velocity $\boldsymbol{\omega}$ about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the *center of percussion,* which lies at a distance $r_{P/G} = k_G^2/r_{G/O}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.



$$H_O = (r_{G/O} + r_{P/G})\, mv_G = r_{G/O}\,(mv_G) + I_G\omega, \qquad \text{where} \quad I_G = mk_G^2$$

$$r_{G/O}\,(mv_G) + r_{P/G}\,(mv_G) = r_{G/O}\,(mv_G) + (mk_G^2)\,\omega$$

$$r_{P/G} = \frac{k_G^2}{v_G/\omega}$$

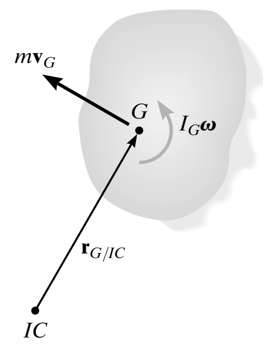


However, $\quad v_G = \omega r_{G/O} \quad$ or $\quad r_{G/O} = \dfrac{v_G}{\omega}$

$$r_{P/G} = \frac{k_G^2}{r_{G/O}}$$

Q.E.D.

19–2. At a given instant, the body has a linear momentum $\mathbf{L} = m\mathbf{v}_G$ and an angular momentum $\mathbf{H}_G = I_G\boldsymbol{\omega}$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $\mathbf{H}_{IC} = I_{IC}\boldsymbol{\omega}$, where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance $r_{G/IC}$ away from the mass center G.



$$H_{IC} = r_{G/IC}\,(mv_G) + I_G\,\omega, \qquad \text{where} \quad v_G = \omega r_{G/IC}$$

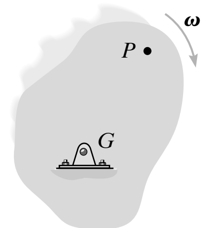
$$= r_{G/IC}\,(m\omega r_{G/IC}) + I_G\,\omega$$

$$= (I_G + mr_{G/IC}^2)\,\omega$$

$$= I_{IC}\,\omega$$

Q.E.D.

19–3. Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P.



Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\,\omega$$

Since ω is a free vector, so is $\mathbf{H}_P$. **Q.E.D.**

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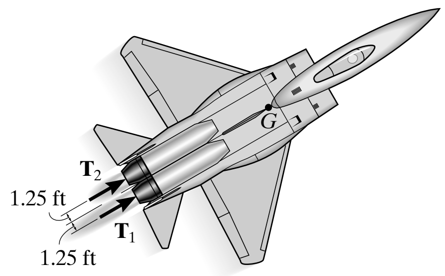
***19–4.** The pilot of a crippled jet was able to control his plane by throttling the two engines. If the plane has a weight of 17 000 lb and a radius of gyration of $k_G = 4.7$ ft about the mass center G, determine the angular velocity of the plane and the velocity of its mass center G in $t = 5$ s if the thrust in each engine is altered to $T_1 = 5000$ lb and $T_2 = 800$ lb as shown. Originally the plane is flying straight at 1200 ft/s. Neglect the effects of drag and the loss of fuel.



$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M_G\,dt = (H_G)_2$$

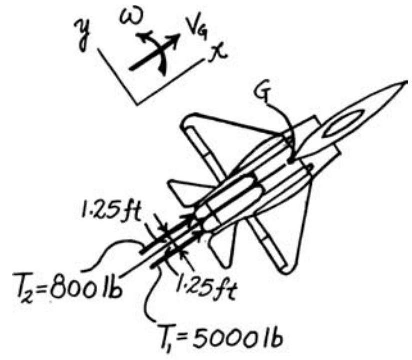
$$0 + 5000(5)(1.25) - 800(5)(1.25) = \left[\left(\frac{17\,000}{32.2}\right)(4.7)^2\right]\omega$$

$$\omega = 2.25 \text{ rad/s} \qquad \textbf{Ans.}$$

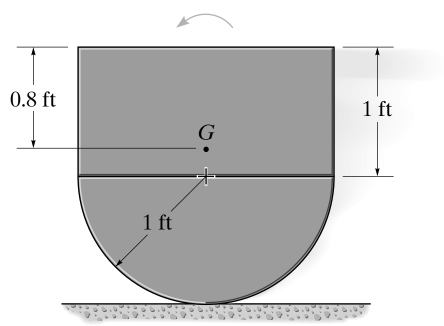
$$\left(\xrightarrow{+}\right) \qquad m(v_{Gx})_1 + \Sigma \int F_x\,dt = m(v_{Gx})_2$$

$$\left(\frac{17\,000}{32.2}\right)(1200) + 5800(5) = \left(\frac{17\,000}{32.2}\right)(v_G)_2$$

$$(v_G)_2 = 1.25\left(10^3\right) \text{ ft/s} \qquad \textbf{Ans.}$$



•19–5. The assembly weighs 10 lb and has a radius of gyration $k_G = 0.6$ ft about its center of mass G. The kinetic energy of the assembly is 31 ft·lb when it is in the position shown. If it rolls counterclockwise on the surface without slipping, determine its linear momentum at this instant.

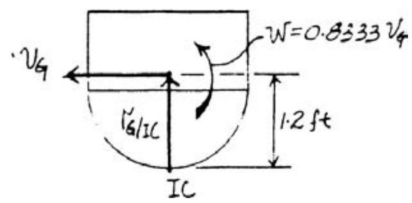


Kinetic Energy: Since the assembly rolls without slipping, then $\omega = \frac{v_G}{r_{G/IC}} = \frac{v_G}{1.2} = 0.8333 v_G$.

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$31 = \frac{1}{2}\left(\frac{10}{32.2}\right) v_G^2 + \frac{1}{2}\left[\frac{10}{32.2}\left(0.6^2\right)\right](0.8333 v_G)^2$$

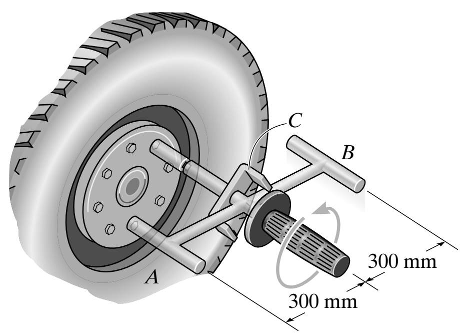
$$v_G = 12.64 \text{ ft/s}$$



Linear Momentum: Applying Eq. 19–7, we have

$$L = m v_G = \frac{10}{32.2}(12.64) = 3.92 \text{ slug} \cdot \text{ft/s} \qquad \textbf{Ans.}$$

19–6. The impact wrench consists of a slender 1-kg rod AB which is 580 mm long, and cylindrical end weights at A and B that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to rotate about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod AB is given an angular velocity of 4 rad/s and it strikes the bracket C on the handle without rebounding, determine the angular impulse imparted to the lug nut.



$$I_{\text{axle}} = \frac{1}{12}(1)(0.6 - 0.02)^2 + 2\left[\frac{1}{2}(1)(0.01)^2 + 1(0.3)^2\right] = 0.2081 \text{ kg} \cdot \text{m}^2$$

$$\int M\,dt = I_{\text{axle}}\,\omega = 0.2081(4) = 0.833 \text{ kg} \cdot \text{m}^2/\text{s} \qquad \textbf{Ans.}$$

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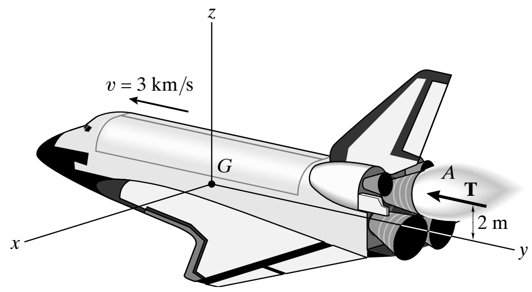
19–7. The space shuttle is located in "deep space," where the effects of gravity can be neglected. It has a mass of 120 Mg, a center of mass at G, and a radius of gyration $(k_G)_x = 14$ m about the x axis. It is originally traveling forward at $v = 3$ km/s when the pilot turns on the engine at A, creating a thrust $T = 600(1 - e^{-0.3t})$ kN, where t is in seconds. Determine the shuttle's angular velocity 2 s later.

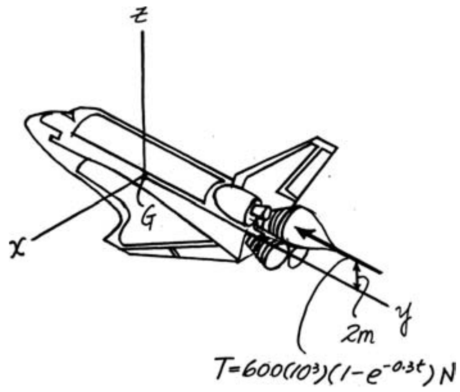


$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$$

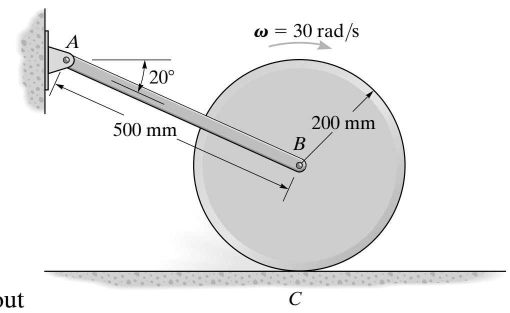
$$0 + \int_0^2 600\left(10^3\right)\left(1 - e^{-0.3\,t}\right)(2)\, dt = \left[120\left(10^3\right)(14)^2\right]\omega$$

$$1200\left(10^3\right)\left[t + \frac{1}{0.3}e^{-0.3\,t}\right]_0^2 = 120\left(10^3\right)(14)^2\,\omega$$

$$\omega = 0.0253 \text{ rad/s} \qquad \textbf{Ans.}$$



***19–8.** The 50-kg cylinder has an angular velocity of 30 rad/s when it is brought into contact with the horizontal surface at C. If the coefficient of kinetic friction is $\mu_C = 0.2$, determine how long it will take for the cylinder to stop spinning. What force is developed in link AB during this time? The axle through the cylinder is connected to two symmetrical links. (Only AB is shown.) For the computation, neglect the weight of the links.



Principle of Impulse and Momentum: The mass moment inertia of the cylinder about its mass center is $I_G = \frac{1}{2}(50)\left(0.2^2\right) = 1.00 \text{ kg}\cdot\text{m}^2$. Applying Eq. 19–14, we have

$$m\left(v_{G_y}\right)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m\left(v_{G_y}\right)_2$$

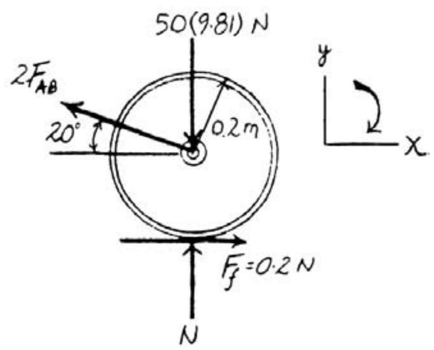
$$(+\uparrow) \qquad 0 + N(t) + 2F_{AB}\sin 20^\circ\,(t) - 50(9.81)(t) = 0 \qquad \textbf{(1)}$$

$$m\left(v_{G_x}\right)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m\left(v_{G_x}\right)_2$$

$$(\xrightarrow{+}) \qquad 0 + 0.2N(t) - 2F_{AB}\cos 20^\circ(t) = 0 \qquad \textbf{(2)}$$

$$I_G\omega_1 + \Sigma \int_{t_1}^{t_2} M_G\, dt = I_G\,\omega_2$$

$$(\circlearrowleft +) \qquad -1.00(30) + [0.2N(t)](0.2) = 0 \qquad \textbf{(3)}$$



Solving Eqs. (1), (2), and (3) yields

$$F_{AB} = 48.7 \text{ N} \qquad t = 1.64 \text{ s} \qquad \textbf{Ans.}$$

$$N = 457.22 \text{ N}$$

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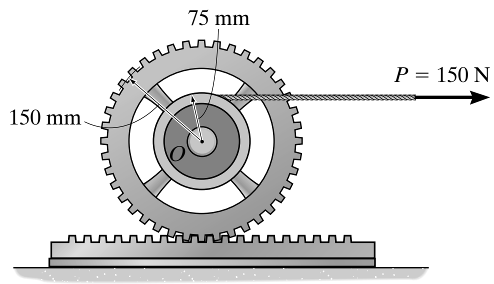
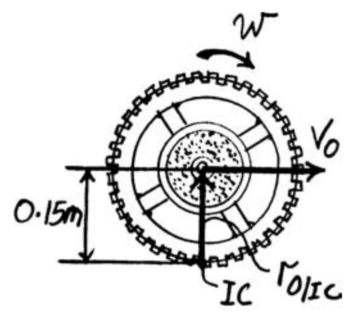
•19–9. If the cord is subjected to a horizontal force of $P = 150$ N, and the gear rack is fixed to the horizontal plane, determine the angular velocity of the gear in 4 s, starting from rest. The mass of the gear is 50 kg, and it has a radius of gyration about its center of mass O of $k_O = 125$ mm.



Kinematics: Referring to Fig. *a*,

$$v_O = \omega r_{O/IC} = \omega(0.15)$$



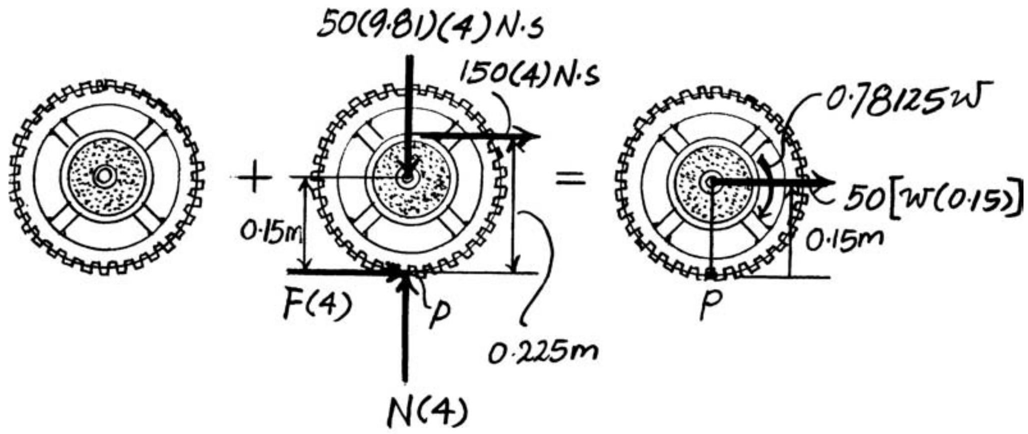
(a)

Principle of Angular Impulse and Momentum: The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = 50\left(0.125^2\right) = 0.78125\ \text{kg}\cdot\text{m}^2$. From Fig. *b*,

$$\circlearrowleft + I_P\omega_1 + \Sigma\int_{t_1}^{t_2} M_P\, dt = I_P\omega_2$$

$$0 + 150(4)(0.225) = 0.78125\omega + 50[\omega(0.15)](0.15)$$

$$\omega = 70.8 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

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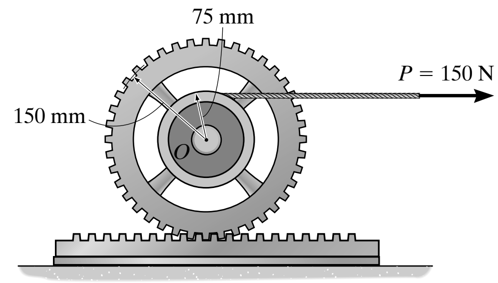
19–10. If the cord is subjected to a horizontal force of $P = 150$ N, and gear is supported by a fixed pin at O, determine the angular velocity of the gear and the velocity of the 20-kg gear rack in 4 s, starting from rest. The mass of the gear is 50 kg and it has a radius of gyration of $k_O = 125$ mm. Assume that the contact surface between the gear rack and the horizontal plane is smooth.



Principle of Impulse and Momentum: The mass moment of inertia of the gear about its mass center is $I_O = mk_O^2 = 50\left(0.125^2\right) = 0.78125\ \text{kg}\cdot\text{m}^2$. Referring to the free-body diagram of the gear shown in Fig. *a*,

$$\circlearrowleft + \quad I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

$$0 + F(4)(0.15) - 150(4)(0.075) = -0.78125\omega_A$$

$$F = 75 - 1.302\omega_A \qquad \textbf{(1)}$$

Since the gear rotates about the fixed axis, $v_P = \omega_A r_P = \omega_A(0.15)$. Referring to the free-body diagram of the gear rack shown in Fig. *b*,

$$(\overset{+}{\leftarrow}) \qquad mv_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = mv_2$$

$$0 + F(4) = 20[\omega_A(0.15)]$$

$$F = 0.75\omega_A \qquad \textbf{(2)}$$

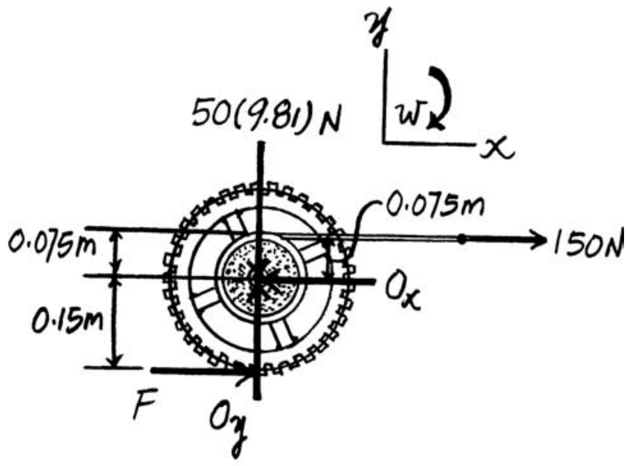
Equating Eqs. (1) and (2),

$$0.75\omega_A = 75 - 1.302\omega_A$$

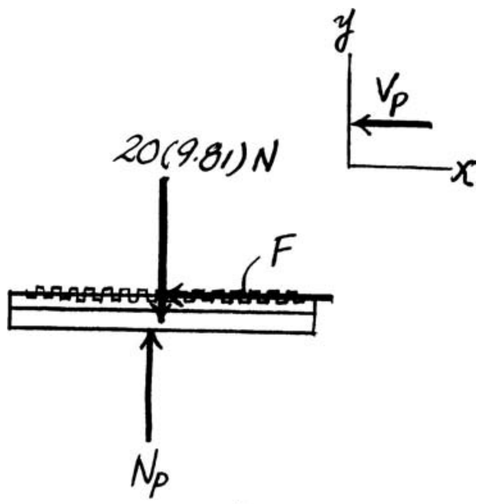
$$\omega_A = 36.548\ \text{rad/s} = 36.5\ \text{rad/s} \qquad \textbf{Ans.}$$

Then,

$$v = 36.548(0.15) = 5.48\ \text{m/s} \qquad \textbf{Ans.}$$



(a)



(b)

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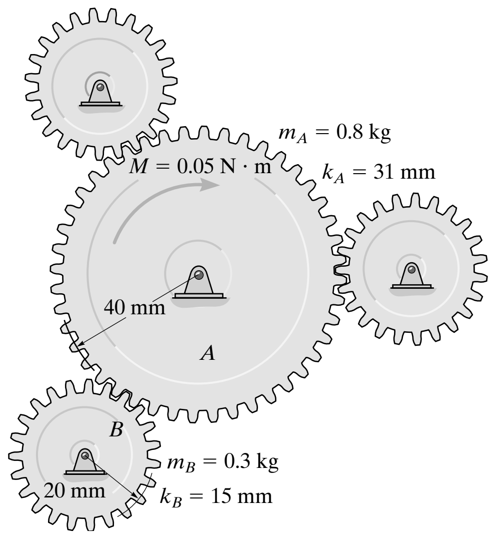
19–11. A motor transmits a torque of $M = 0.05\ \text{N}\cdot\text{m}$ to the center of gear A. Determine the angular velocity of each of the three (equal) smaller gears in 2 s starting from rest. The smaller gears (B) are pinned at their centers, and the masses and centroidal radii of gyration of the gears are given in the figure.



Gear A:

$(\circlearrowleft +)$ $$(H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$$

$$0 - 3(F)(2)(0.04) + 0.05(2) = [0.8(0.031)^2]\,\omega_A$$

Gear B:

$(\circlearrowright +)$ $$(H_B)_1 + \Sigma \int M_B\, dt = (H_B)_2$$

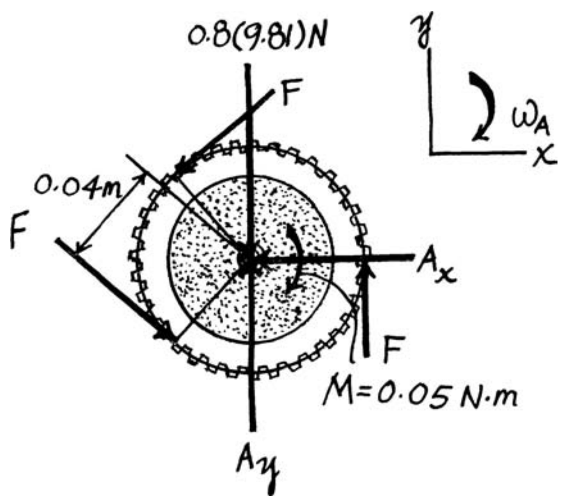
$$0 + (F)(2)(0.02) = [0.3(0.015)^2]\,\omega_B$$

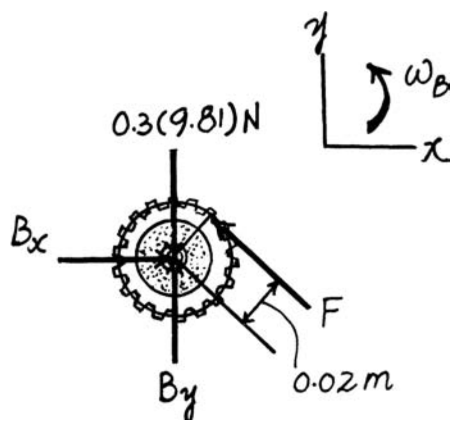
Since $0.04\omega_A = 0.02\,\omega_B$, or $\omega_B = 2\,\omega_A$, then solving,

$F = 0.214$ N

$\omega_A = 63.3$ rad/s

$\omega_B = 127$ rad/s **Ans.**





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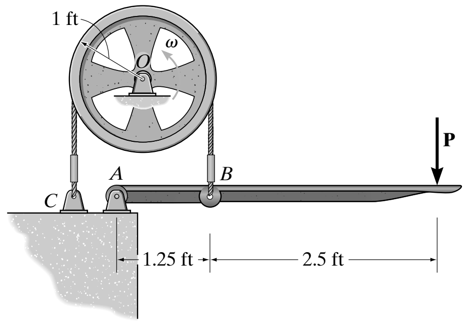
***19–12.** The 200-lb flywheel has a radius of gyration about its center of gravity O of $k_O = 0.75$ ft. If it rotates counterclockwise with an angular velocity of 1200 rev/min before the brake is applied, determine the time required for the wheel to come to rest when a force of $P = 200$ lb is applied to the handle. The coefficient of kinetic friction between the belt and the wheel rim is $\mu_k = 0.3$. (*Hint*: Recall from the statics text that the relation of the tension in the belt is given by $T_B = T_C e^{\mu\beta}$, where β is the angle of contact in radians.)



Equilibrium: Writing the moment equation of equilibrium about point A and referring to the free-body diagram of the arm brake shown in Fig. *a*,

$$\zeta + \Sigma M_A = 0; \qquad T_B(1.25) - 200(3.75) = 0 \qquad T_B = 600 \text{ lb}$$

Using the belt friction formula,

$$T_B = T_C e^{\mu\beta}$$

$$600 = T_C e^{0.3(\pi)}$$

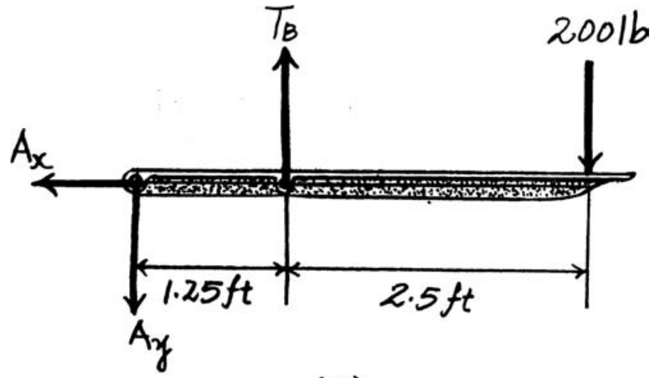
$$T_C = 233.80 \text{ lb}$$

Principle of Angular Impulse and Momentum: The mass moment of inertia of the wheel about its mass center is $I_O = mk_O{}^2 = \left(\frac{200}{32.2}\right)(0.75^2) = 3.494 \text{ slug} \cdot \text{ft}^2$, and the initial angular velocity of the wheel is $\omega_1 = \left(1200 \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$ $= 40\pi$ rad/s. Applying the angular impulse and momentum equation about point O using the free-body diagram of the wheel shown in Fig. *b*,

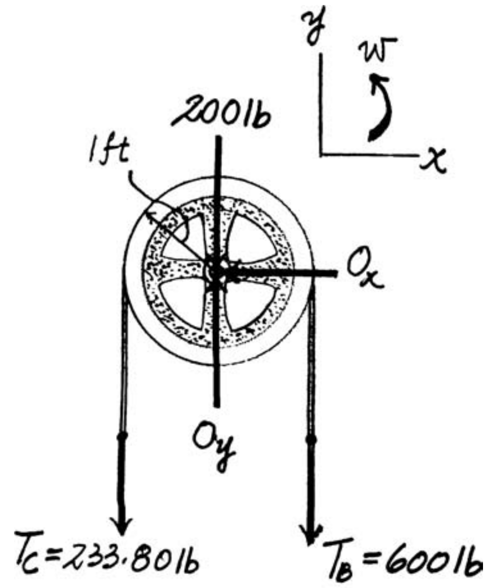
$$\zeta + \quad I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_O \, dt = I_O\omega_2$$

$$3.494(40\pi) + 233.80(t)(1) - 600(t)(1) = 0$$

$$t = 1.20 \text{ s} \qquad \textbf{Ans.}$$



(a)



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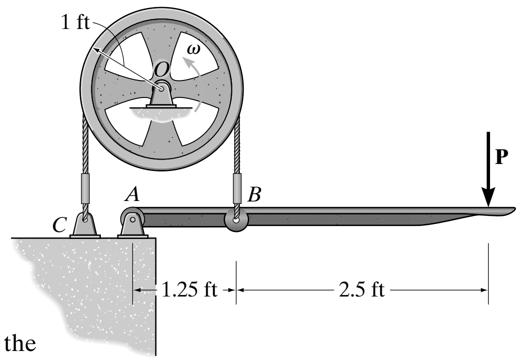
•19–13. The 200-lb flywheel has a radius of gyration about its center of gravity O of $k_O = 0.75$ ft. If it rotates counterclockwise with a constant angular velocity of 1200 rev/min before the brake is applied, determine the required force **P** that must be applied to the handle to stop the wheel in 2 s. The coefficient of kinetic friction between the belt and the wheel rim is $\mu_k = 0.3$. (*Hint*: Recall from the statics text that the relation of the tension in the belt is given by $T_B = T_C e^{\mu\beta}$, where β is the angle of contact in radians.)



Principle of Angular Impulse and Momentum: The mass moment of inertia of the wheel about its mass center is $I_O = mk_O{}^2 = \left(\frac{200}{32.2}\right)(0.75^2) = 3.494 \text{ slug}\cdot\text{ft}^2$, and the initial angular velocity of the wheel is $\omega_1 = \left(1200\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 40\pi$ rad/s. Applying the angular impulse and momentum equation about point O using the free-body diagram shown in Fig. *a*,

$$\circlearrowleft + \quad I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

$$3.494(40\pi) + T_C(2)(1) - T_B(2)(1) = 0$$

$$T_B - T_C = 219.52 \qquad \textbf{(1)}$$

Using the belt friction formula,

$$T_B = T_C e^{\mu\beta}$$

$$T_B = T_C e^{0.3(\pi)} \qquad \textbf{(2)}$$

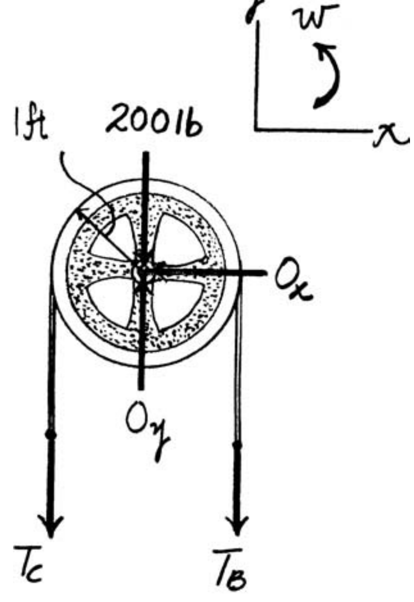
Solving Eqs. (1) and (2),

$$T_C = 140.15 \text{ lb} \qquad T_B = 359.67 \text{ lb}$$

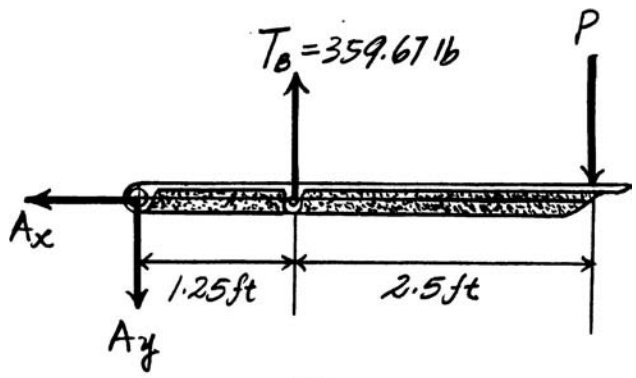
Equilibrium: Using this result and writing the moment equation of equilibrium about point A using the free-body diagram of the brake arm shown in Fig. *b*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 359.67(1.25) - P(3.75) = 0$$

$$P = 120 \text{ lb} \qquad \textbf{Ans.}$$



(a)



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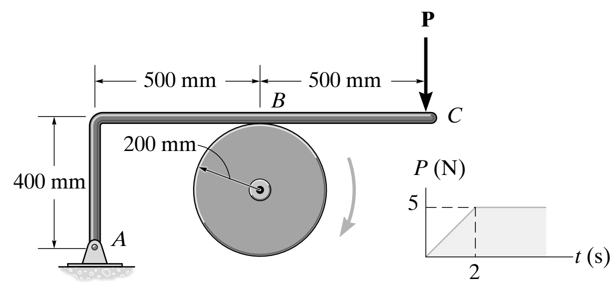
19–14. The 12-kg disk has an angular velocity of $\omega = 20$ rad/s. If the brake ABC is applied such that the magnitude of force **P** varies with time as shown, determine the time needed to stop the disk. The coefficient of kinetic friction at B is $\mu_k = 0.4$. Neglect the thickness of the brake.

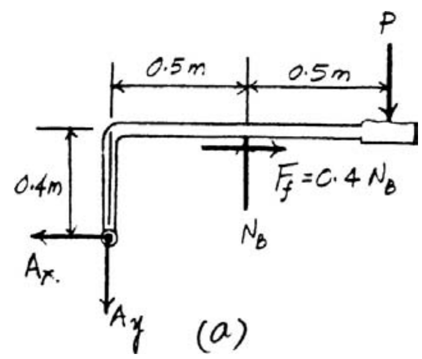


Equation of Equilibrium: Since slipping occurs at B, the friction $F_f = \mu_k N_B = 0.4N_B$. From FBD(a), the normal reaction N_B can be obtained directed by summing moments about point A.

$$\zeta + \Sigma M_A = 0; \qquad N_B(0.5) - 0.4N_B(0.4) - P(1) = 0$$

$$N_B = 2.941P$$

Thus, the friction $F_f = 0.4N_B = 0.4(2.941P) = 1.176P$.



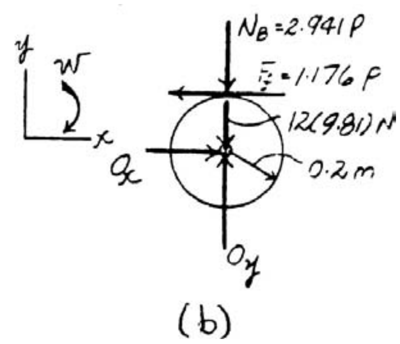
Principle of Impulse and Momentum: The mass moment inertia of the cylinder about its mass center is $I_O = \frac{1}{2}(12)\left(0.2^2\right) = 0.240 \text{ kg} \cdot \text{m}^2$. Applying Eq. 19–14, we have

$$I_O \omega_1 + \Sigma \int_{t_1}^{t_2} M_O \, dt = I_O \omega_2$$

$$(\zeta +) \qquad -0.240(20) + \left[-\left(1.176 \int_0^t P dt\right)(0.2)\right] = 0 \qquad \textbf{(1)}$$

However, $\int_0^t P dt$ is the area under the $P-t$ graph. Assuming $t > 2$ s, then

$$\int_0^t P dt = \frac{1}{2}(5)(2) + 5(t-2) = (5t - 5) \text{ N} \cdot \text{s}$$



Substitute into Eq. (1) yields

$$-0.240(20) + [-1.176(5t - 5)(0.2)] = 0$$

$$t = 5.08 \text{ s} \qquad \textbf{Ans.}$$

Since $t = 5.08 \text{ s} > 2$ s, the above assumption is correct.

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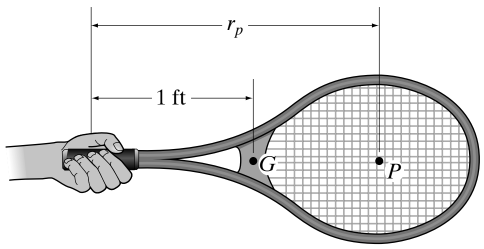
19–15. The 1.25-lb tennis racket has a center of gravity at G and a radius of gyration about G of $k_G = 0.625$ ft. Determine the position P where the ball must be hit so that 'no sting' is felt by the hand holding the racket, i.e., the horizontal force exerted by the racket on the hand is zero.



Principle of Impulse and Momentum: Here, we will assume that the tennis racket is initially at rest and rotates about point A with an angular velocity of ω immediately after it is hit by the ball, which exerts an impulse of $\int F dt$ on the racket, Fig. a. The mass moment of inertia of the racket about its mass center is $I_G = \left(\frac{1.25}{32.2}\right)(0.625^2) = 0.01516 \text{ slug} \cdot \text{ft}^2$. Since the racket about point A, $(v_G) = \omega r_G = \omega(1)$. Referring to Fig. b,

$$\overset{+}{\leftarrow} \qquad m(v_G)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_G)_2$$

$$0 + \int F dt = \left(\frac{1.25}{32.2}\right)\left[\omega(1)\right]$$

$$\int F dt = 0.03882\omega \qquad \textbf{(1)}$$

and

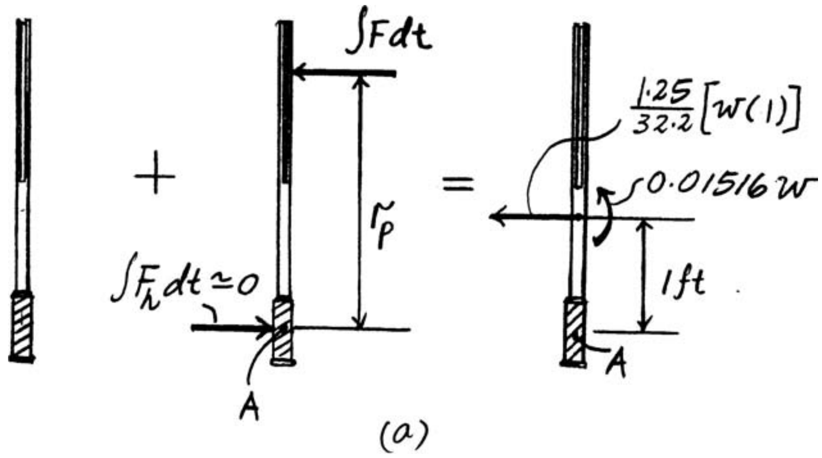
$$\circlearrowleft + \quad (H_A)_1 + \Sigma \int_{t_1}^{t_2} M_A \, dt = (H_A)_2$$

$$0 + \left(\int F dt\right) r_P = 0.01516\omega + \frac{1.25}{32.2}\left[\omega(1)\right](1)$$

$$\int F dt = \frac{0.05398\omega}{r_P} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) yields

$$0.03882\omega = \frac{0.05398\omega}{r_P} \qquad r_P = 1.39 \text{ ft} \qquad \textbf{Ans.}$$



(a)

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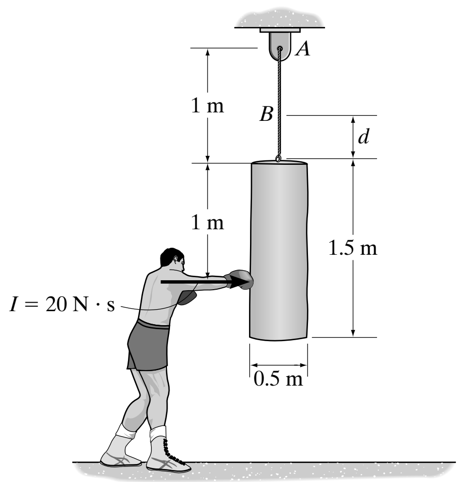
***19–16.** If the boxer hits the 75-kg punching bag with an impulse of $I = 20\ \text{N} \cdot \text{s}$, determine the angular velocity of the bag immediately after it has been hit. Also, find the location d of point B, about which the bag appears to rotate. Treat the bag as a uniform cylinder.



Principle of Impulse and Momentum: The mass moment of inertia of the bag about its mass center is $I_G = \frac{1}{12}m\left(3r^2 + h^2\right) = \frac{1}{12}(75)\left[3\left(0.25^2\right) + 1.5^2\right] = 15.23\ \text{kg} \cdot \text{m}^2$. Referring to the impulse and momentum diagrams of the bag shown in Fig. a,

$$(\xrightarrow{+}) \qquad m(v_G)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m\,(v_G)_2$$

$$0 + 20 = 75v_G \qquad\qquad v_G = 0.2667\ \text{m/s}$$

and

$$\circlearrowleft + \quad I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\omega_2$$

$$0 + 20(0.25) = 15.23\omega$$

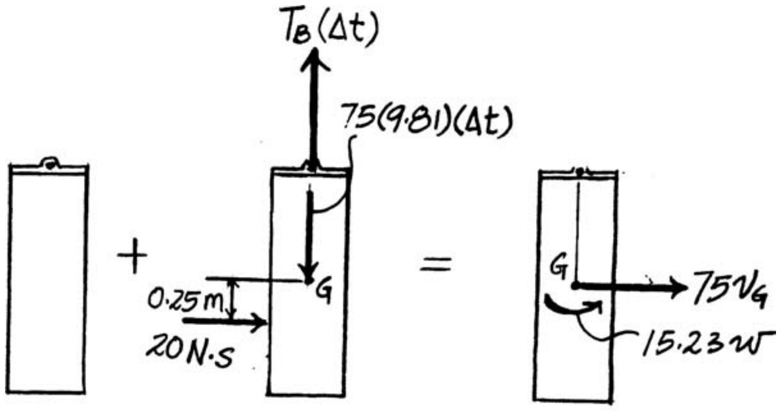
$$\omega = 0.3282\ \text{rad/s} = 0.328\ \text{rad/s} \qquad \textbf{Ans.}$$

Kinematics: Referring to Fig. b,

$$v_G = \omega r_{G/IC}$$

$$0.2667 = 0.3282(0.75 + d)$$

$$d = 0.0625\ \text{m} \qquad \textbf{Ans.}$$



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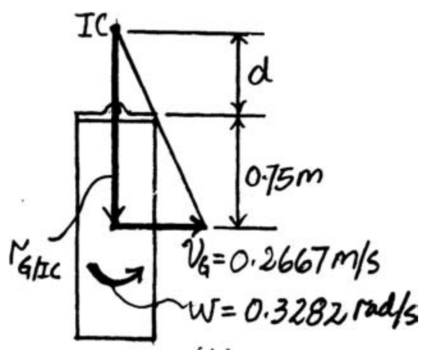

(b)

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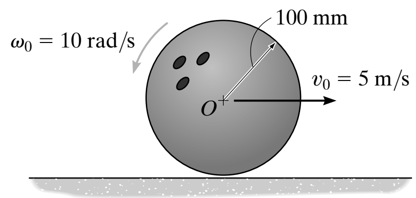
•19–17. The 5-kg ball is cast on the alley with a backspin of $\omega_0 = 10$ rad/s, and the velocity of its center of mass O is $v_0 = 5$ m/s. Determine the time for the ball to stop back spinning, and the velocity of its center of mass at this instant. The coefficient of kinetic friction between the ball and the alley is $\mu_k = 0.08$.



Principle of Impulse and Momentum: Since the ball slips, $F_f = \mu_k N = 0.08N$. The mass moment of inertia of the ball about its mass center is

$$I_O = \frac{2}{5}mr^2 = \frac{2}{5}(5)\left(0.1^2\right) = 0.02\ \text{kg}\cdot\text{m}^2$$

Referring to Fig. a,

$$(+\uparrow) \qquad m\left[(v_O)_y\right]_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m\left[(v_O)_y\right]_2$$

$$0 + N(t) - 5(9.81)t = 0 \qquad\qquad N = 49.05\ \text{N}$$

$$\circlearrowleft + (H_A)_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = (H_A)_2$$

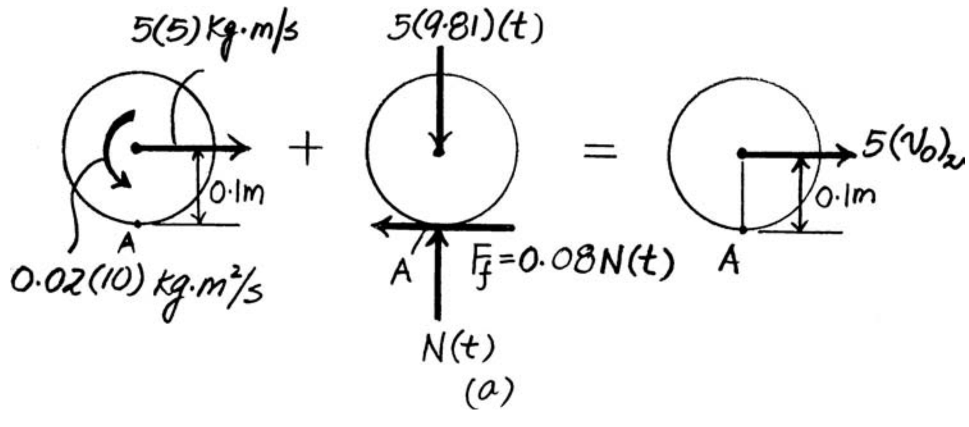
$$0.02(10) - 5(5)(0.1) + 0 = -5(v_O)_2\,(0.1)$$

$$(v_O)_2 = 4.6\ \text{m/s} \qquad\qquad \textbf{Ans.}$$

$$(\overset{+}{\rightarrow}) \qquad m\left[(v_O)_x\right]_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m\left[(v_O)_x\right]_2$$

$$5(5) - 0.08(49.05)(t) = 5(4.6)$$

$$t = 0.510\ \text{s} \qquad\qquad \textbf{Ans.}$$



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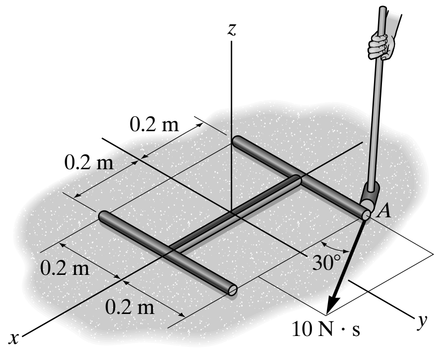
19–18. The smooth rod assembly shown is at rest when it is struck by a hammer at A with an impulse of 10 N·s. Determine the angular velocity of the assembly and the magnitude of velocity of its mass center immediately after it has been struck. The rods have a mass per unit length of 6 kg/m.



Principle of Impulse and Momentum: The total mass of the assembly is $m = 3[6(0.4)] = 7.2$ kg. The mass moment of inertia of the assembly about its mass center is

$$I_G = \frac{1}{12}[6(0.4)](0.4^2) + 2\left[\frac{1}{12}[6(0.4)](0.4^2) + 6(0.4)(0.2^2)\right] = 0.288\ \text{kg}\cdot\text{m}^2$$

Referring to Fig. b,

$$(+\downarrow) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + 10\cos 30^\circ = 7.2(v_G)_x \qquad (v_G)_x = 1.203\ \text{m/s}$$

$$(\overset{+}{\rightarrow}) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + 10\sin 30^\circ = 7.2(v_G)_y \qquad (v_G)_y = 0.6944\ \text{m/s}$$

Thus, the magnitude of $\mathbf{v}_G$ is

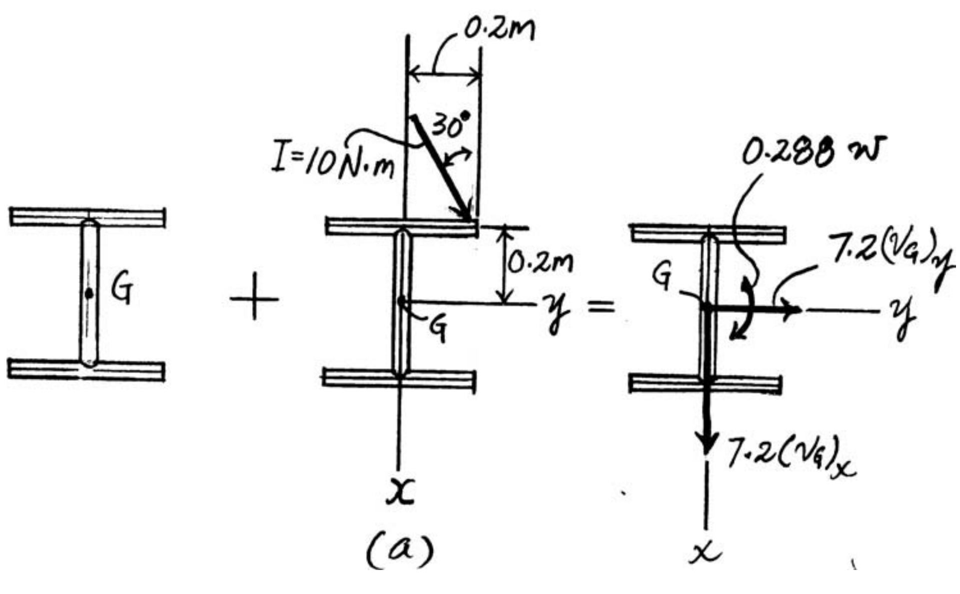
$$v_G = \sqrt{(v_G)_x^2 + (v_G)_y^2} = \sqrt{1.203^2 + 0.6944^2} = 1.39\ \text{m/s} \qquad \textbf{Ans.}$$

Also

$$\circlearrowleft + \qquad I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\omega_2$$

$$0 + [-10\cos 30^\circ(0.2) - 10\sin 30^\circ(0.2)] = -0.288\omega$$

$$\omega = 9.49\ \text{rad/s} \qquad \textbf{Ans.}$$



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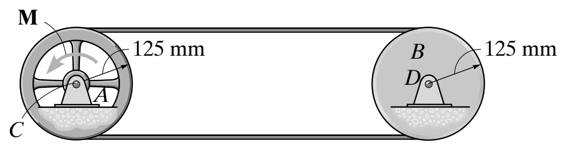
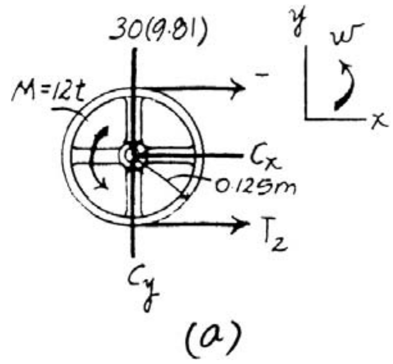
19–19. The flywheel A has a mass of 30 kg and a radius of gyration of $k_C = 95$ mm. Disk B has a mass of 25 kg, is pinned at D, and is coupled to the flywheel using a belt which is subjected to a tension such that it does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque or twist to the flywheel, having a magnitude of $M = (12t)$ N · m, where t is in seconds, determine the angular velocity of the disk 3 s after the motor is turned on. Initially, the flywheel is at rest.



Principle of Impulse and Momentum: The mass moment inertia of the flywheel about point C is $I_C = 30(0.095^2) = 0.27075\ \text{kg}\cdot\text{m}^2$. Applying Eq. 19–14 to the flywheel [FBD(a)], we have

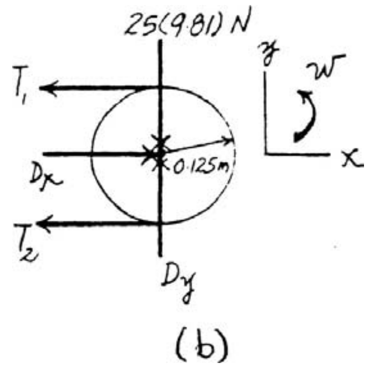


$$I_C\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$$(\circlearrowleft +)\quad 0 + \int_0^{3\text{ s}} 12t\,dt + [T_2(3)](0.125) - T_1(3)](0.125) = 0.27075\omega$$

$$54.0 + 0.375T_2 - 0.375T_1 = 0.27075\omega \qquad \textbf{(1)}$$

The mass moment inertia of the disk about point D is $I_D = \frac{1}{2}(25)\left(0.125^2\right) = 0.1953125\ \text{kg}\cdot\text{m}^2$. Applying Eq. 19–14 to the disk [FBD(b)], we have



$$I_D\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_D\,dt = I_D\,\omega_2$$

$$(\circlearrowleft +)\qquad 0 + \left[T_1(3)\right](0.125) - \left[T_2(3)\right](0.125) = 0.1953125\omega$$

$$0.375T_2 - 0.375T_1 = -0.1953125\omega \qquad \textbf{(2)}$$

Substitute Eq. (2) into Eq. (1) and solving yields

$$\omega = 116\ \text{rad/s} \qquad \textbf{Ans.}$$

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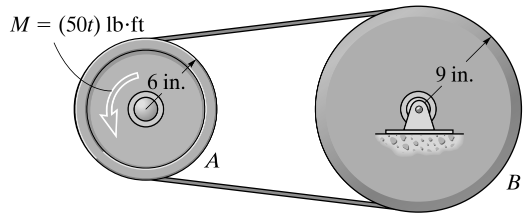
***19–20.** The 30-lb flywheel A has a radius of gyration about its center of 4 in. Disk B weighs 50 lb and is coupled to the flywheel by means of a belt which does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque to the flywheel of $M = (50t)\ \text{lb}\cdot\text{ft}$, where t is in seconds, determine the time required for the disk to attain an angular velocity of 60 rad/s starting from rest.

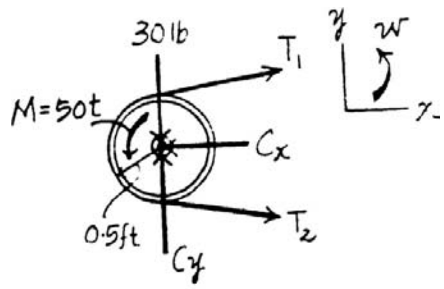


Principle of Impulse and Momentum: The mass moment inertia of the flywheel about point C is $I_C = \dfrac{30}{32.2}\left(\dfrac{4}{12}\right)^2 = 0.1035\ \text{slug}\cdot\text{ft}^2$. The angular velocity of the flywheel is $\omega_A = \dfrac{r_B}{r_A}\omega_B = \dfrac{0.75}{0.5}(60) = 90.0\ \text{rad/s}$. Applying Eq. 19–14 to the flywheel [FBD(a)], we have

$$I_C\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$$(\circlearrowleft +)\quad 0 + \int_0^t 50t\,dt + \left[\int T_2\,(dt)\right](0.5) - \left[\int T_1(dt)\right](0.5) = 0.1035(90)$$

$$25t^2 + 0.5\int (T_2 - T_1)dt = 9.317 \qquad \textbf{(1)}$$



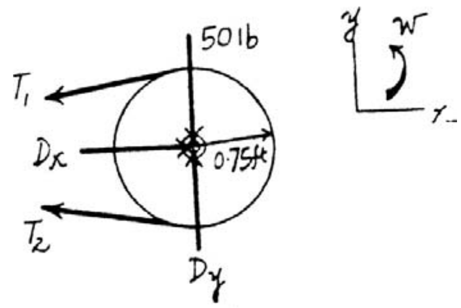
(a)

The mass moment inertia of the disk about point D is $I_D = \dfrac{1}{2}\left(\dfrac{50}{32.2}\right)(0.75^2) = 0.4367\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 19–14 to the disk [FBD(b)], we have

$$I_D\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_D\,dt = I_D\,\omega_2$$

$$(\circlearrowleft +)\qquad 0 + \left[\int T_1\,(dt)\right](0.75) - \left[\int T_2\,(dt)\right](0.75) = 0.4367(60)$$

$$\int (T_2 - T_1)dt = -34.94 \qquad \textbf{(2)}$$



(b)

Substitute Eq. (2) into Eq. (1) and solving yields

$$t = 1.04\ \text{s} \qquad \textbf{Ans.}$$

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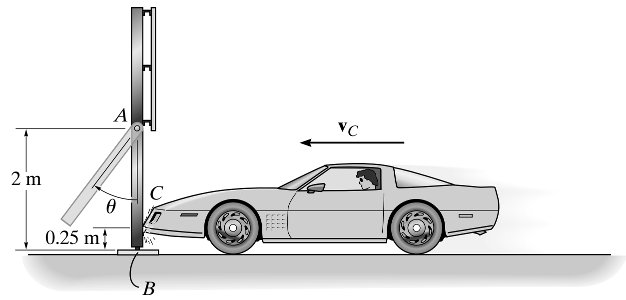
•19–21. For safety reasons, the 20-kg supporting leg of a sign is designed to break away with negligible resistance at B when the leg is subjected to the impact of a car. Assuming that the leg is pinned at A and approximates a thin rod, determine the impulse the car bumper exerts on it, if after the impact the leg appears to rotate clockwise to a maximum angle of $\theta_{\max} = 150°$.



$$(+\circlearrowright) \qquad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

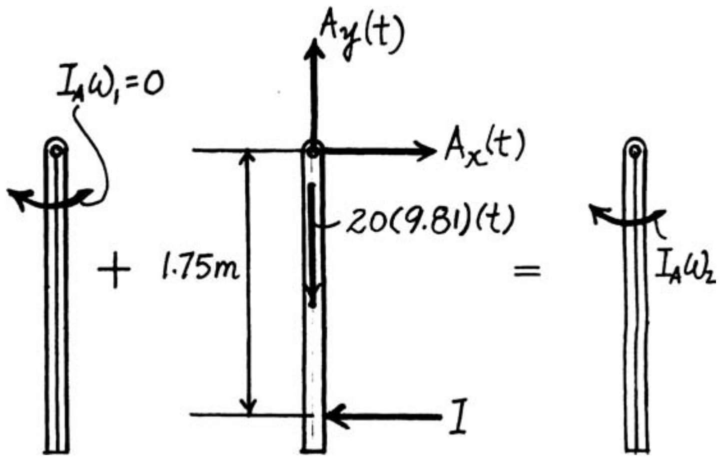
$$0 + I(1.75) = \left[\frac{1}{3}(20)(2)^2\right]\omega_2$$

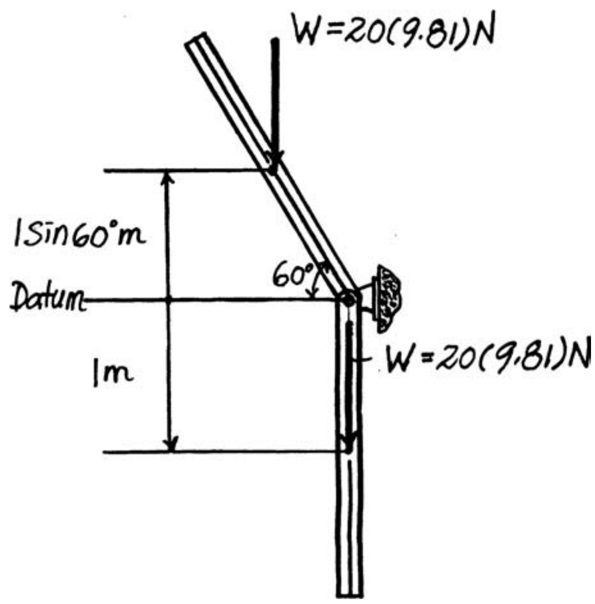
$$\omega_2 = 0.065625I$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{1}{3}(20)(2)^2\right](0.065625I)^2 + 20(9.81)(-1) = 0 + 20(9.81)(1\sin 60°)$$

$I = 79.8\ \text{N}\cdot\text{s}$ **Ans.**





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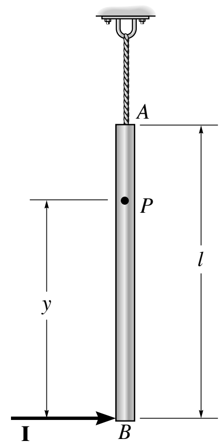
19–22. The slender rod has a mass m and is suspended at its end A by a cord. If the rod receives a horizontal blow giving it an impulse $\mathbf{I}$ at its bottom B, determine the location y of the point P about which the rod appears to rotate during the impact.



Principle of Impulse and Momentum:

$(\circlearrowleft +)$
$$I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$0 + I\left(\frac{l}{2}\right) = \left[\frac{1}{12}ml^2\right]\omega \qquad I = \frac{1}{6}ml\omega$$

$\left(\xrightarrow{+}\right)$
$$m(v_{Ax})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Ax})_2$$

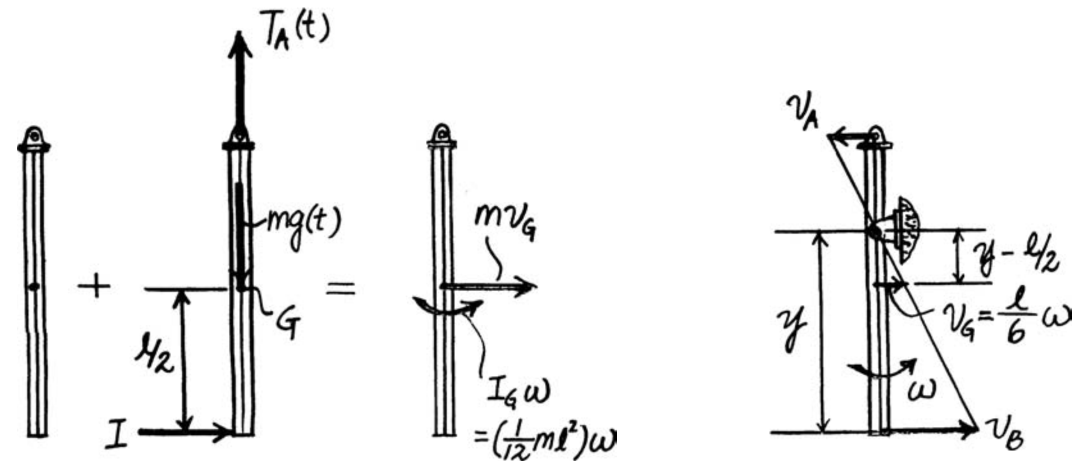
$$0 + \frac{1}{6}ml\omega = mv_G \qquad v_G = \frac{l}{6}\omega$$

Kinematics: Point P is the IC.

$$v_B = \omega\,y$$

Using similar triangles,

$$\frac{\omega\,y}{y} = \frac{\frac{l}{6}\omega}{y - \frac{l}{2}} \qquad y = \frac{2}{3}l \qquad \textbf{Ans.}$$



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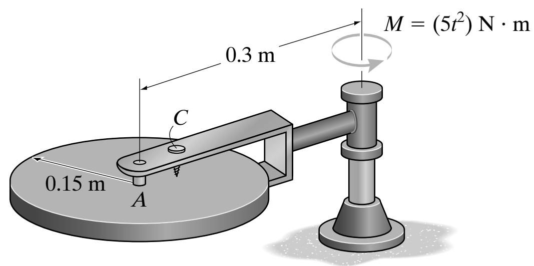
19–23. The 25-kg circular disk is attached to the yoke by means of a smooth axle A. Screw C is used to lock the disk to the yoke. If the yoke is subjected to a torque of $M = (5t^2)\ \text{N}\cdot\text{m}$, where t is in seconds, and the disk is unlocked, determine the angular velocity of the yoke when $t = 3$ s, starting from rest. Neglect the mass of the yoke.



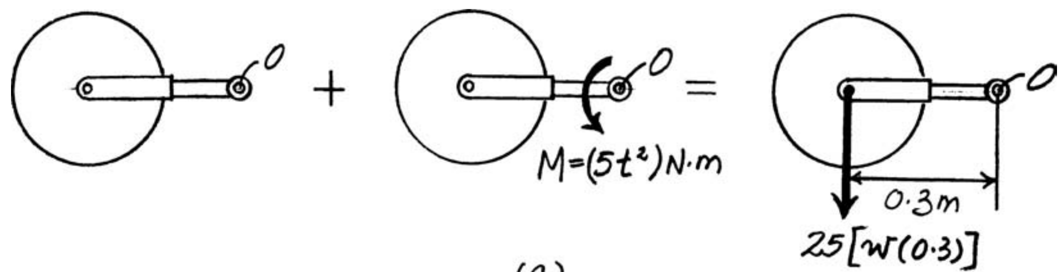
Principle of Angular Momentum: Since the disk is not rigidly attached to the yoke, only the linear momentum of its mass center contributes to the angular momentum about point O. Here, the yoke rotates about the fixed axis, thus $v_A = \omega r_{OA} = \omega(0.3)$. Referring to Fig. a,

$$\circlearrowleft + \qquad (H_O)_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = (H_O)_2$$

$$0 + \int_0^{3\,\text{s}} 5t^2\,dt = 25\big[\omega(0.3)\big](0.3)$$

$$\left.\frac{5t^3}{3}\right|_0^{3\,\text{s}} = 2.25\omega$$

$$\omega = 20 \text{ rad/s} \qquad \textbf{Ans.}$$



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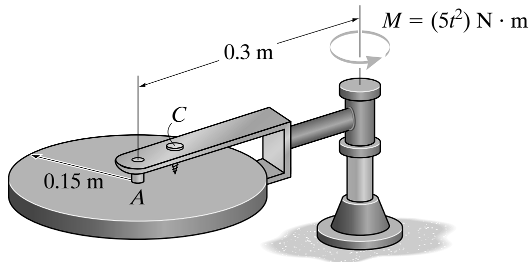
***19–24.** The 25-kg circular disk is attached to the yoke by means of a smooth axle A. Screw C is used to lock the disk to the yoke. If the yoke is subjected to a torque of $M = (5t^2)\ \text{N}\cdot\text{m}$, where t is in seconds, and the disk is locked, determine the angular velocity of the yoke when $t = 3$ s, starting from rest. Neglect the mass of the yoke.



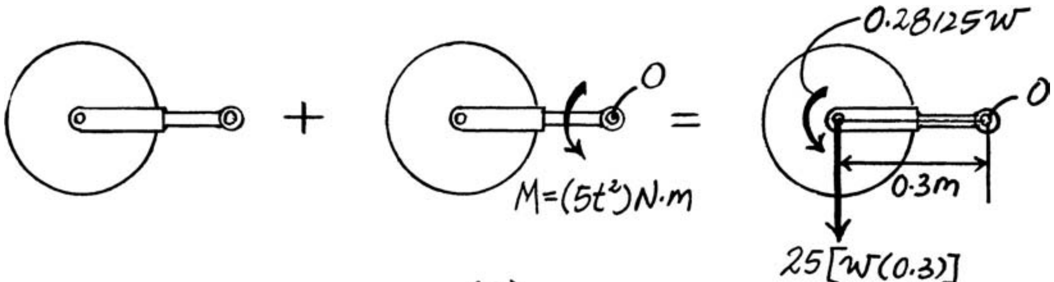
Principle of Angular Momentum: The mass moment of inertia of the disk about its mass center is $I_A = \frac{1}{2}mr^2 = \frac{1}{2}(25)\left(0.15^2\right) = 0.28125\ \text{kg}\cdot\text{m}^2$. Since the yoke rotates about a fixed axis, $v_A = \omega r_{OA} = \omega(0.3)$. Referring to Fig. *a*,

$$\zeta+ \qquad (H_O)_1 + \Sigma\int_{t_1}^{t_2} M_O\, dt = (H_O)_2$$

$$0 + \int_0^{3\text{ s}} 5t^2 dt = 0.28125\omega + 25\left[\omega(0.3)\right](0.3)$$

$$\left.\frac{5t^3}{3}\right|_0^{3\text{ s}} = 2.53125\omega$$

$$\omega = 17.8\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)

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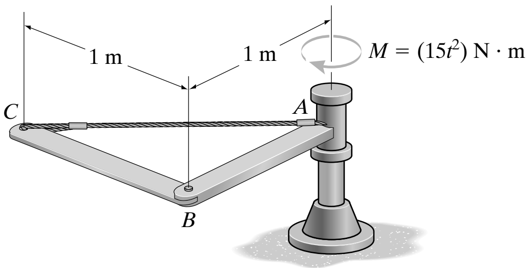
•19–25. If the shaft is subjected to a torque of $M = (15t^2)$ N·m, where t is in seconds, determine the angular velocity of the assembly when $t = 3$ s, starting from rest. Rods AB and BC each have a mass of 9 kg.



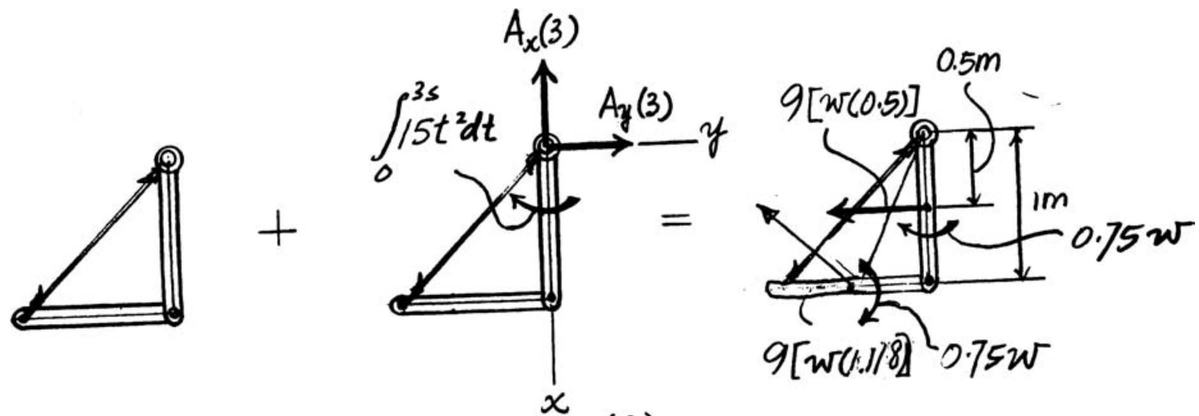
Principle of Impulse and Momentum: The mass moment of inertia of the rods about their mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(9)\left(1^2\right) = 0.75\ \text{kg}\cdot\text{m}^2$. Since the assembly rotates about the fixed axis, $(v_G)_{AB} = \omega(r_G)_{AB} = \omega(0.5)$ and $(v_G)_{BC} = \omega(r_G)_{BC} = \omega\left(\sqrt{1^2 + (0.5)^2}\right) = \omega(1.118)$. Referring to Fig. a,

$$\circlearrowleft + \qquad (H_z)_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

$$0 + \int_0^{3\text{s}} 15t^2 dt = 9\left[\omega(0.5)\right](0.5) + 0.75\omega + 9\left[\omega(1.118)\right](1.118) + 0.75\omega$$

$$5t^3\Big|_0^{3\text{ s}} = 15\omega$$

$$\omega = 9 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$



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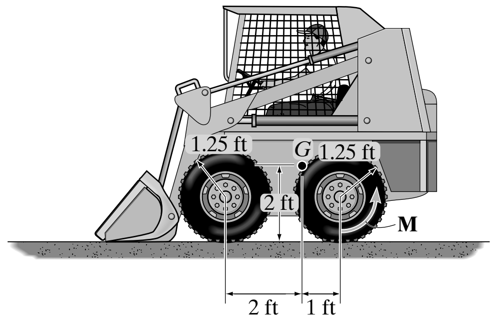
19–26. The body and bucket of a skid steer loader has a weight of 2000 lb, and its center of gravity is located at G. Each of the four wheels has a weight of 100 lb and a radius of gyration about its center of gravity of 1 ft. If the engine supplies a torque of $M = 100\ \text{lb}\cdot\text{ft}$ to each of the rear drive wheels, determine the speed of the loader in $t = 10$ s, starting from rest. The wheels roll without slipping.

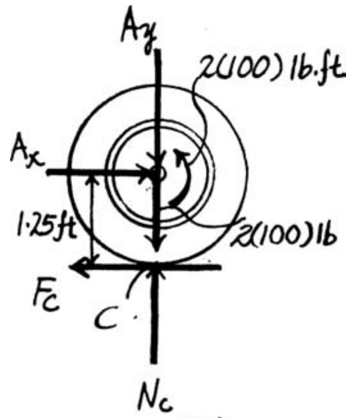


Principle of Impulse and Momentum. The mass moment of inertia of the wheels about their mass center are $I_A = I_B = 2mk^2 = 2\left(\frac{100}{32.2}\right)(1^2) = 6.211\ \text{slug}\cdot\text{ft}^2$. Since the wheels roll without slipping, $\omega = \frac{v}{r} = \frac{v}{1.25} = 0.8v$. From Figs. *a, b,* and *c,*

$$\zeta+ \qquad (H_C)_1 + \Sigma\int_{t_1}^{t_2} M_C\,dt = (H_C)_2$$

$$0 + 2(100)(10) - A_x(10)(1.25) = 6.211(0.8v) + 2\left[\left(\frac{100}{32.2}\right)v\right](1.25)$$

$$A_x = 160 - 1.019v \qquad \textbf{(1)}$$



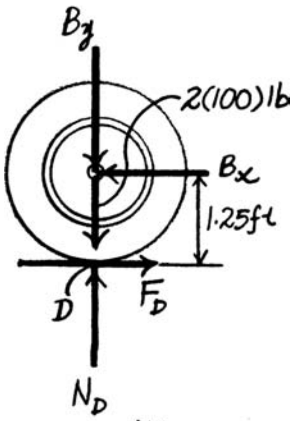
(a)

and

$$\zeta+ \qquad (H_D)_1 + \Sigma\int_{t_1}^{t_2} M_D\,dt = (H_D)_2$$

$$0 + B_x(10)(1.25) = 6.211(0.8v) + 2\left[\left(\frac{100}{32.2}\right)v\right](1.25)$$

$$B_x = 1.019v \qquad \textbf{(2)}$$



(b)

From Fig. *d,*

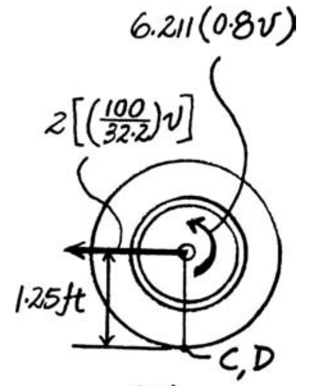
$$\left(\overset{+}{\leftarrow}\right) \qquad m\left[(v_G)_x\right]_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m\left[(v_G)_x\right]_2$$

$$0 + A_x(10) - B_x(10) = \left(\frac{2000}{32.2}\right)v \qquad \textbf{(3)}$$

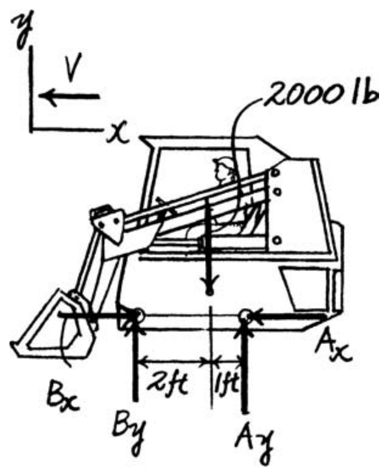
Substituting Eqs. (1) and (2) into Eq. (3),

$$(160 - 1.019v)(10) - 1.019v(10) = \left(\frac{2000}{32.2}\right)v$$

$$v = 19.4\ \text{ft/s} \qquad \textbf{Ans.}$$



(c)



(d)

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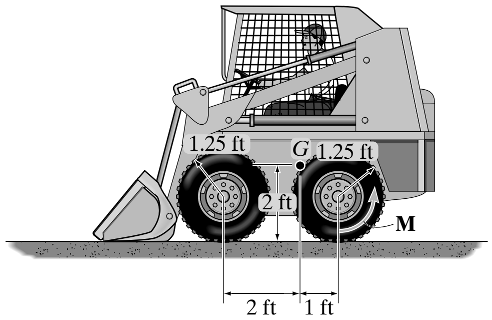
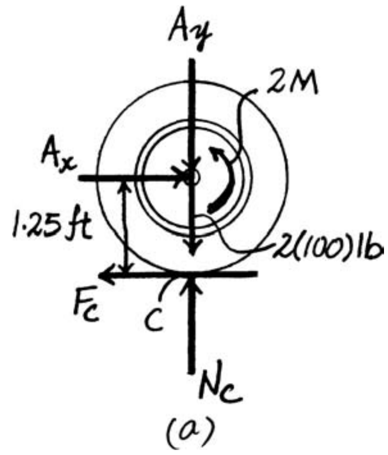
19–27. The body and bucket of a skid steer loader has a weight of 2000 lb, and its center of gravity is located at G. Each of the four wheels has a weight of 100 lb and a radius of gyration about its center of gravity of 1 ft. If the loader attains a speed of 20 ft/s in 10 s, starting from rest, determine the torque **M** supplied to each of the rear drive wheels. The wheels roll without slipping.



Principle of Impulse and Momentum: The mass momentum of inertia of the wheels about their mass centers are $I_A = I_B = 2mk^2 = 2\left(\frac{100}{32.2}\right)(1^2) = 6.211 \text{ slug} \cdot \text{ft}^2$. Since the wheels roll without slipping, $\omega = \frac{v}{r} = \frac{20}{1.25} = 16$ rad/s. From Figs. *a, b,* and *c*,



$$\circlearrowleft + \qquad (H_C)_1 + \Sigma \int_{t_1}^{t_2} M_C \, dt = (H_C)_2$$

$$0 + 2M(10) - A_x(10)(1.25) = 6.211(16) + 2\left[\frac{100}{32.2}(20)\right](1.25)$$

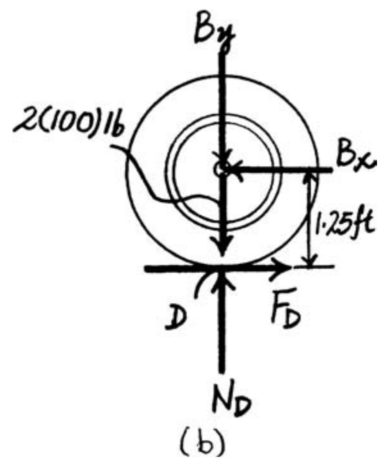
$$A_x = 1.6M - 20.37 \qquad \textbf{(1)}$$

and

$$\circlearrowright + \qquad (H_D)_1 + \Sigma \int_{t_1}^{t_2} M_D \, dt = (H_D)_2$$

$$0 + B_x(10)(1.25) = 6.211(16) + 2\left[\frac{100}{32.2}(20)\right](1.25)$$

$$B_x = 20.37 \text{ lb} \qquad \textbf{(2)}$$



From Fig. *d,*

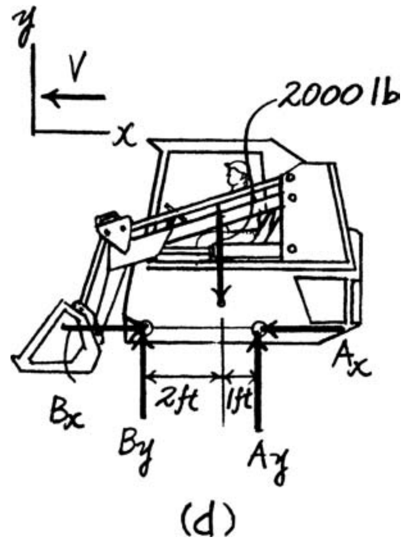
$$\left(\overset{+}{\leftarrow}\right) \qquad m\left[(v_G)_x\right]_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m\left[(v_G)_x\right]_2$$

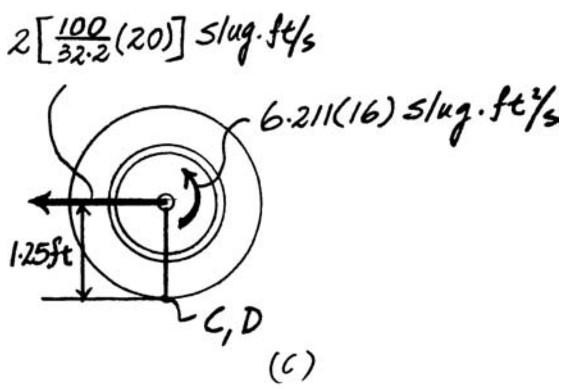
$$0 + A_x(10) - B_x(10) = \frac{2000}{32.2}(20) \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (2) into Eq. (3),

$$(1.6M - 20.37)(10) - 20.37(10) = \frac{2000}{32.2}(20)$$

$$M = 103 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$





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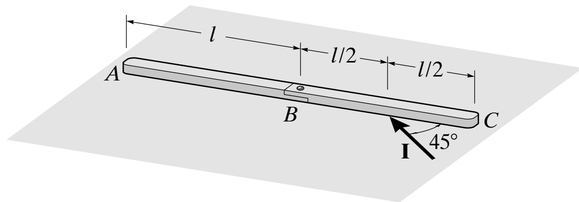
***19–28.** The two rods each have a mass m and a length l, and lie on the smooth horizontal plane. If an impulse **I** is applied at an angle of 45° to one of the rods at midlength as shown, determine the angular velocity of each rod just after the impact. The rods are pin connected at B.



Bar BC:

$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$$

$$0 + \int B_y\, dt \left(\frac{l}{2}\right) = I_G\, \omega_{BC} \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad m(v_{Gy})_1 + \Sigma \int F_y\, dt = m(v_{Gy})_2$$

$$0 - \int B_y\, dt + I \sin 45^\circ = m(v_G)_y \qquad \textbf{(2)}$$

Bar AB:

$$(\circlearrowleft +) \qquad (H_{G'})_1 + \Sigma \int M_{G'}\, dt = (H_{G'})_2$$

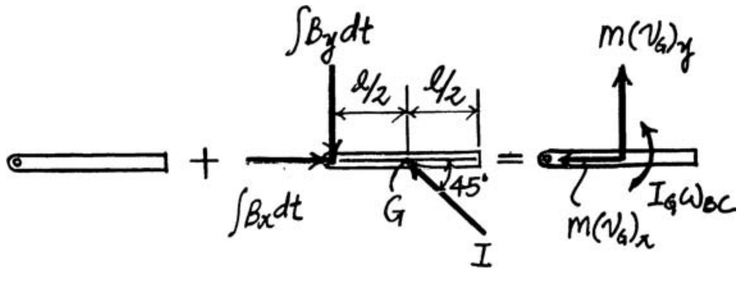
$$0 + \int B_y\, dt \left(\frac{l}{2}\right) = I_G\, \omega_{AB} \qquad \textbf{(3)}$$

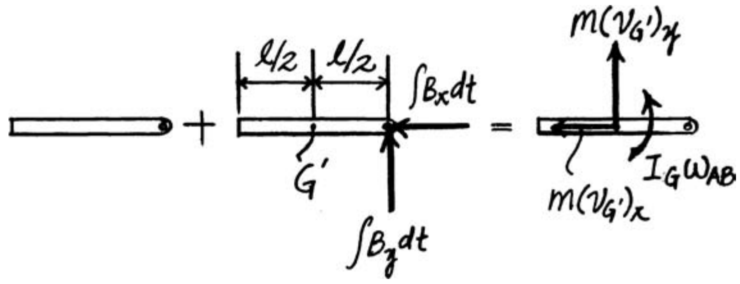
$$(+\uparrow) \qquad m(v_{Gy})_1 + \Sigma \int F_y\, dt = m(v_{Gy})_2$$

$$0 + \int B_y\, dt = m(v_{G'})_y \qquad \textbf{(4)}$$

$$\mathbf{v}_B = \mathbf{v}_{G'} + \mathbf{v}_{B/G'} = \mathbf{v}_G + \mathbf{v}_{B/G}$$

$$(+\uparrow) \qquad v_{By} = (v_{G'})_y + \omega_{AB}\left(\frac{l}{2}\right) = (v_G)_y - \omega_{BC}\left(\frac{l}{2}\right) \qquad \textbf{(5)}$$





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***19–28. Continued**

Eliminate $\int B_y\,dt$ from Eqs. (1) and (2), from Eqs. (3) and (4), and between Eqs. (1) and (3). This yields

$$I_G\,\omega_{BC} = \frac{l}{2}\left(I \sin 45^\circ - m(v_G)_y\right)$$

$$m(v_G)_y\left(\frac{l}{2}\right) = I_G\,\omega_{AB}$$

$$\omega_{BC} = \omega_{AB}$$

Substituting into Eq. (5),

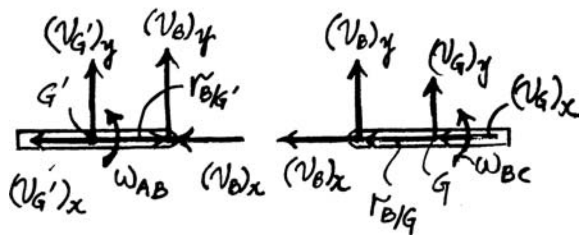
$$\frac{1}{m}\left(\frac{2}{l}\right)I_G\,\omega_{AB} + \omega_{AB}\left(\frac{l}{2}\right) = -\left[I_G\left(\frac{\omega_{AB}}{m}\right)\left(\frac{2}{l}\right)\right] + \frac{I}{m}\sin 45^\circ - \omega_{AB}\left(\frac{l}{2}\right)$$

$$\left(\frac{4}{ml}\right)I_G\omega_{AB} + \omega_{AB}\,l = \frac{I}{m}\sin 45^\circ$$

$$\left(\frac{4}{ml}\right)\left(\frac{1}{12}\,ml^2\right)\omega_{AB} + \omega_{AB}\,l = \frac{I}{m}\sin 45^\circ$$

$$\frac{4}{3}\,\omega_{AB}\,l = \frac{I}{m}\sin 45^\circ$$

$$\omega_{AB} = \omega_{BC} = \frac{3}{4\sqrt{2}}\left(\frac{I}{ml}\right) \qquad \textbf{Ans.}$$



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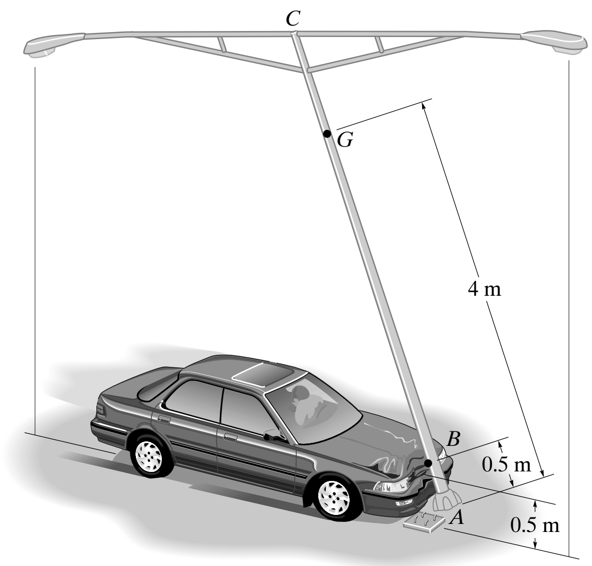
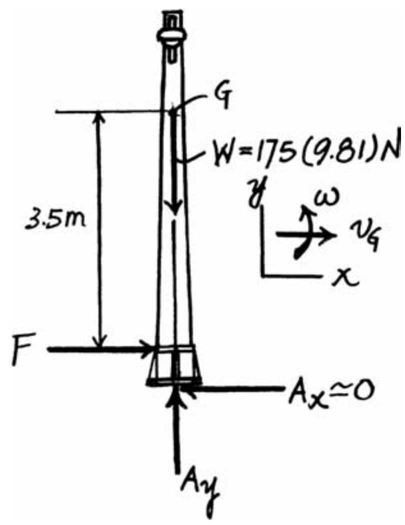
•19–29. The car strikes the side of a light pole, which is designed to break away from its base with negligible resistance. From a video taken of the collision it is observed that the pole was given an angular velocity of 60 rad/s when AC was vertical. The pole has a mass of 175 kg, a center of mass at G, and a radius of gyration about an axis perpendicular to the plane of the pole assembly and passing through G of $k_G = 2.25$ m. Determine the horizontal impulse which the car exerts on the pole at the instant AC is essentially vertical.



$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$$

$$0 + \left[\int F\, dt\right](3.5) = 175(2.25)^2\,(60)$$

$$\int F\, dt = 15.2 \text{ kN}\cdot\text{s}$$ **Ans.**



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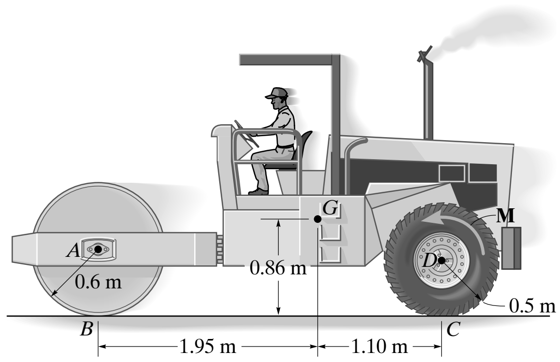
19–30. The frame of the roller has a mass of 5.5 Mg and a center of mass at G. The roller has a mass of 2 Mg and a radius of gyration about its mass center of $k_A = 0.45$ m. If a torque of $M = 600\ \text{N}\cdot\text{m}$ is applied to the rear wheels, determine the speed of the compactor in $t = 4$ s, starting from rest. No slipping occurs. Neglect the mass of the driving wheels.



Driving Wheels: (mass is neglected)

$$\zeta + \Sigma M_D = 0; \qquad 600 - F_C(0.5) = 0$$

$$F_C = 1200\ \text{N}$$

Frame and driving wheels:

$$(\overset{+}{\leftarrow}) \qquad m(v_{Gx})_1 + \Sigma \int F_x\,dt = m(v_{Gx})_2$$

$$0 + 1200(4) - A_x(4) = 5500 v_G$$

$$A_x = 12\,00 - 1375 v_G \qquad \textbf{(1)}$$

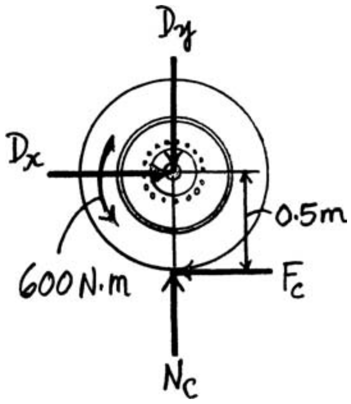
Roller:

$$v_G = v_A = 0.6\omega$$

$$(\zeta +) \qquad (H_B)_1 + \Sigma \int M_B\,dt = (H_B)_2$$

$$0 + A_x(4)(0.6) = \left[2000(0.45)^2\right]\left(\frac{v_G}{0.6}\right) + \left[2000(v_G)\right](0.6)$$

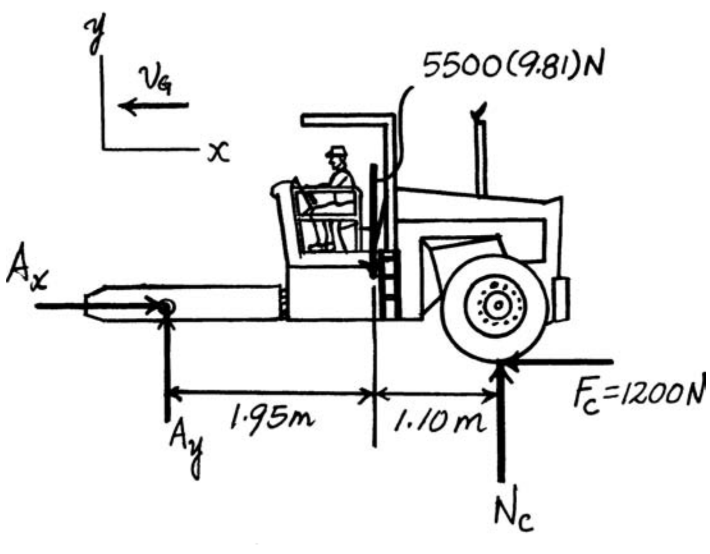
$$A_x = 781.25 v_G \qquad \textbf{(2)}$$

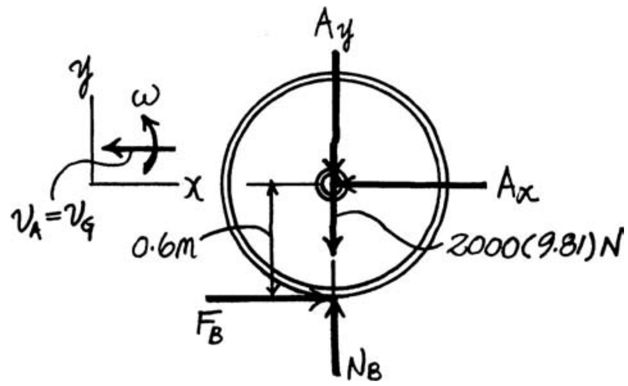


Solving Eqs. (1) and (2):

$$A_x = 435\ \text{N}$$

$$v_G = 0.557\ \text{m/s} \qquad \textbf{Ans.}$$





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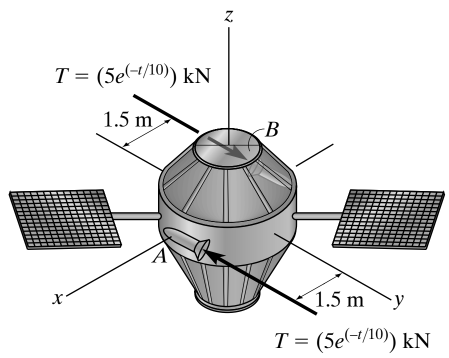
19–31. The 200-kg satellite has a radius of gyration about the centroidal z axis of $k_z = 1.25$ m. Initially it is rotating with a constant angular velocity of $\boldsymbol{\omega}_0 = \{1500\,\mathbf{k}\}$ rev/min. If the two jets A and B are fired simultaneously and produce a thrust of $T = (5e^{-0.1t})$ kN, where t is in seconds, determine the angular velocity of the satellite, five seconds after firing.



Principle of Angular Impulse and Momentum: The mass moment of inertia of the satellite about its centroidal z axis is $I_z = mk_z^2 = 200\left(1.25^2\right) = 312.5\ \text{kg}\cdot\text{m}^2$. The initial angular velocity of the satellite is $\omega_1 = \left(1500\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 50\pi$ rad/s. Applying the angular impulse and momentum equation about the z axis,

$$I_z\omega_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = I_z\omega_2$$

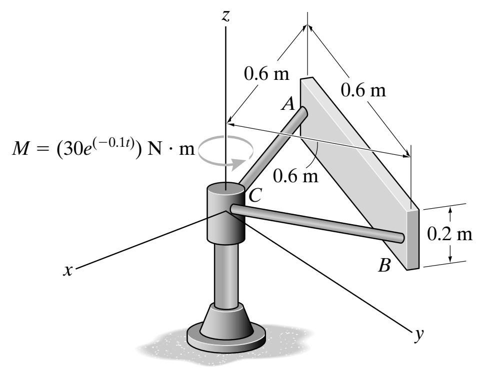
$$312.5(50\pi) - \left[2\int_0^{5\text{ s}} 5000e^{-0.1t}(1.5)\,dt\right] = 312.5\omega_2$$

$$15625\pi + \left(150\,000e^{-0.1t}\right)\Big|_0^{5\text{ s}} = 312.5\omega_2$$

Thus,

$$\omega_2 = [-31.8\mathbf{k}]\ \text{rad/s}$$ **Ans.**

***19–32.** If the shaft is subjected to a torque of $M = (30e^{-0.1t})$ N·m, where t is in seconds, determine the angular velocity of the assembly when $t = 5$ s, starting from rest. The rectangular plate has a mass of 25 kg. Rods AC and BC have the same mass of 5 kg.



Principle of Angular Impulse and Momentum: The mass moment of inertia of the assembly about the z axis is $I_z = 2\left[\frac{1}{3}(5)\left(0.6^2\right)\right] + \left[\frac{1}{12}(25)\left(0.6^2\right) + 25(0.6\sin 60°)^2\right] = 8.70\ \text{kg}\cdot\text{m}^2$. Using the free-body diagram of the assembly shown in Fig. *a*,

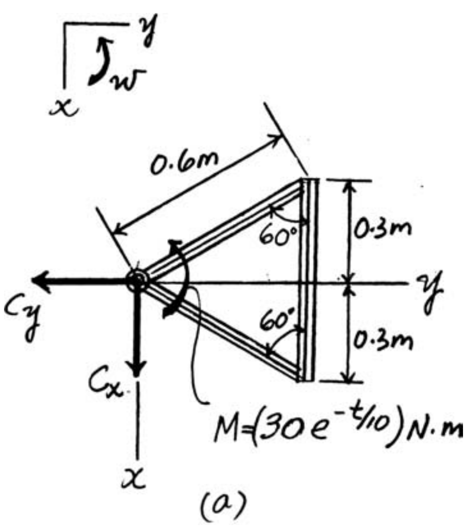
$$\circlearrowleft + \ I_z\omega_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = I_z\omega_2$$

$$0 + \int_0^{5\text{s}} 30e^{-0.1t}\,dt = 8.70\omega_2$$

$$\left(-300e^{-0.1t}\right)\Big|_0^{5\text{ s}} = 8.70\omega_2$$

Thus,

$$\omega_2 = 13.6\ \text{rad/s}$$ **Ans.**



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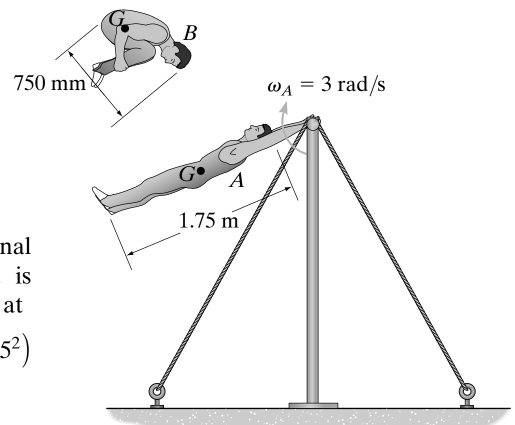
•19–33. The 75-kg gymnast lets go of the horizontal bar in a fully stretched position A, rotating with an angular velocity of $\omega_A = 3$ rad/s. Estimate his angular velocity when he assumes a tucked position B. Assume the gymnast at positions A and B as a uniform slender rod and a uniform circular disk, respectively.



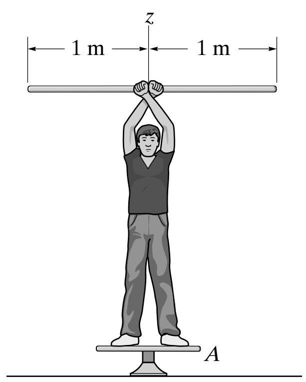
Conservation of Angular Momentum: Other than the weight, there is no external impulse during the motion. Thus, the angular momentum of the gymnast is conserved about his mass center G. The mass moments of inertia of the gymnast at the fully-stretched and tucked positions are $(I_A)_G = \frac{1}{12}ml^2 = \frac{1}{12}(75)\left(1.75^2\right)$ $= 19.14\ \text{kg}\cdot\text{m}^2$ and $(I_B)_G = \frac{1}{2}mr^2 = \frac{1}{2}(75)\left(0.375^2\right) = 5.273\ \text{kg}\cdot\text{m}^2$. Thus,

$$(H_A)_G = (H_B)_G$$

$$19.14(3) = 5.273\omega_B$$

$$\omega_B = 10.9\ \text{rad/s}$$ **Ans.**

19–34. A 75-kg man stands on the turntable A and rotates a 6-kg slender rod over his head. If the angular velocity of the rod is $\omega_r = 5$ rad/s measured relative to the man and the turntable is observed to be rotating in the opposite direction with an angular velocity of $\omega_t = 3$ rad/s, determine the radius of gyration of the man about the z axis. Consider the turntable as a thin circular disk of 300-mm radius and 5-kg mass.



Conservation of Angular Momentum: The mass moment of inertia of the rod about the z axis is $(I_r)_z = \frac{1}{12}ml^2 = \frac{1}{12}(6)\left(2^2\right) = 2\ \text{kg}\cdot\text{m}^2$ and the mass moment of inertia of the man and the turntable about the z axis is $(I_m)_z = \frac{1}{2}(5)\left(0.3^2\right) + 75k_z^2$ $= 0.225 + 75k_z^2$. Since no external angular impulse acts on the system, the angular momentum of the system is conserverved about the z axis.

$$\left(H_z\right)_1 = \left(H_z\right)_2$$

$$0 = 2(\omega_r) - \left(0.225 + 75k_z^2\right)(3)$$

$$\omega_r = \omega_m + lm$$

$$\omega_r = -3 + 5 = 2\ \text{rad/s}$$

$$2(2) = \left(0.225 + 75k_z^2\right)3$$

$$k_z = 0.122\ \text{m}$$ **Ans.**

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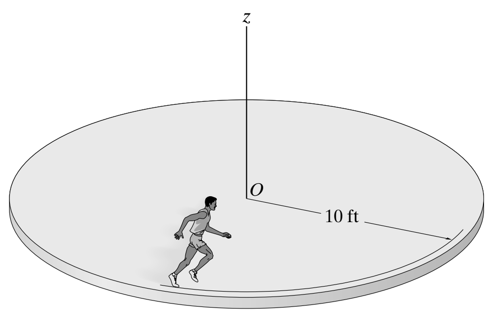
19–35. A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb begins to run along the edge in a circular path of radius 10 ft. If he maintains a speed of 4 ft/s relative to the platform, determine the angular velocity of the platform. Neglect friction.



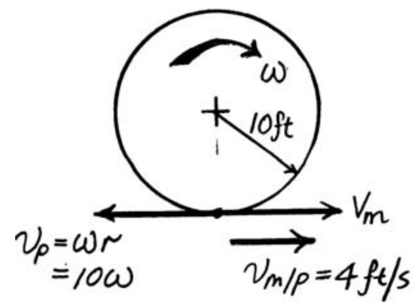
$$\mathbf{v}_m = \mathbf{v}_p + \mathbf{v}_{m/p}$$

$\left(\overset{+}{\rightarrow}\right)$ $\quad v_m = -10\omega + 4$

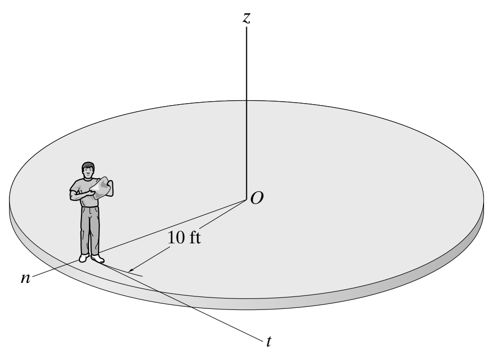
$(\circlearrowleft +)$ $\quad (H_z)_1 = (H_z)_2$

$$0 = -\left(\frac{300}{32.2}\right)(8)^2\,\omega + \left(\frac{150}{32.2}\right)(-10\omega + 4)(10)$$

$\omega = 0.175$ rad/s **Ans.**



***19–36.** A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb throws a 15-lb block off the edge of the platform with a horizontal velocity of 5 ft/s, *measured relative to the platform*. Determine the angular velocity of the platform if the block is thrown (a) tangent to the platform, along the $+t$ axis, and (b) outward along a radial line, or $+n$ axis. Neglect the size of the man.



a) $\quad (H_z)_1 = (H_z)_2$

$$0 + 0 = \left(\frac{15}{32.2}\right)(v_b)(10) - \left(\frac{300}{32.2}\right)(8)^2\,\omega - \left(\frac{150}{32.2}\right)(10\omega)(10)$$

$$v_b = 228\omega$$

$$\mathbf{v}_b = \mathbf{v}_m + \mathbf{v}_{b/m}$$

$\left(\overset{+}{\rightarrow}\right)$ $\quad v_b = -10\omega + 5$

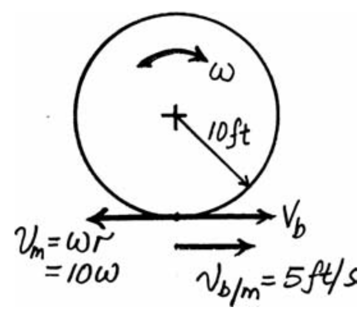
$$228\omega = -10\omega + 5$$

$\omega = 0.0210$ rad/s **Ans.**

b) $\quad (H_z)_1 = (H_z)_2$

$$0 + 0 = 0 - \left(\frac{300}{32.2}\right)(8)^2\,\omega - \left(\frac{150}{32.2}\right)(10\omega)(10)$$

$\omega = 0$ **Ans.**



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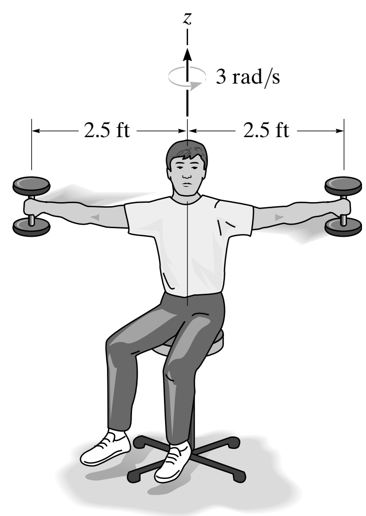
•19–37. The man sits on the swivel chair holding two 5-lb weights with his arms outstretched. If he is rotating at 3 rad/s in this position, determine his angular velocity when the weights are drawn in and held 0.3 ft from the axis of rotation. Assume he weighs 160 lb and has a radius of gyration $k_z = 0.55$ ft about the z axis. Neglect the mass of his arms and the size of the weights for the calculation.



Mass Moment of Inertia: The mass moment inertia of the man and the weights about z axis when the man arms are fully stretched is

$$(I_z)_1 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(2.5^2)\right] = 3.444 \text{ slug} \cdot \text{ft}^2$$

The mass moment inertia of the man and the weights about z axis when the weights are drawn in to a distance 0.3 ft from z axis

$$(I_z)_2 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(0.3^2)\right] = 1.531 \text{ slug} \cdot \text{ft}^2$$

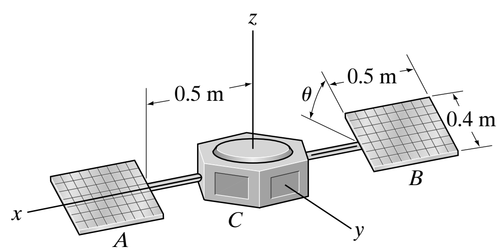
Conservation of Angular Momentum: Applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$3.444(3) = 1.531(\omega_z)_2$$

$$(\omega_z)_2 = 6.75 \text{ rad/s} \qquad \textbf{Ans.}$$

19–38. The satellite's body C has a mass of 200 kg and a radius of gyration about the z axis of $k_z = 0.2$ m. If the satellite rotates about the z axis with an angular velocity of 5 rev/s, when the solar panels A and B are in a position of $\theta = 0°$, determine the angular velocity of the satellite when the solar panels are rotated to a position of $\theta = 90°$. Consider each solar panel to be a thin plate having a mass of 30 kg. Neglect the mass of the rods.



Conservation of Angular Momentum: When $\theta = 0°$ and $\theta = 90°$, the mass momentum of inertia of the satellite are

$$(I_z)_1 = 200(0.2^2) + 2\left[\frac{1}{12}(30)(0.5^2 + 0.4^2) + 30(0.75^2)\right]$$
$$= 43.8 \text{ kg} \cdot \text{m}^2$$

$$(I_z)_2 = 200(0.2^2) + 2\left[\frac{1}{12}(30)(0.5^2) + 30(0.75^2)\right]$$
$$= 43 \text{ kg} \cdot \text{m}^2$$

Thus,

$$(H_z)_1 = (H_z)_2$$

$$(I_z)_1\,\omega_1 = (I_z)_2\,\omega_2$$

$$43.8(5) = 43\omega_2$$

$$\omega_2 = 5.09 \text{ rev/s} \qquad \textbf{Ans.}$$

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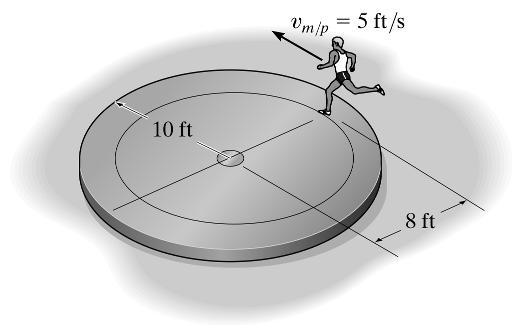
19–39. A 150-lb man leaps off the circular platform with a velocity of $v_{m/p} = 5\text{ ft/s}$, relative to the platform. Determine the angular velocity of the platform afterwards. Initially the man and platform are at rest. The platform weighs 300 lb and can be treated as a uniform circular disk.



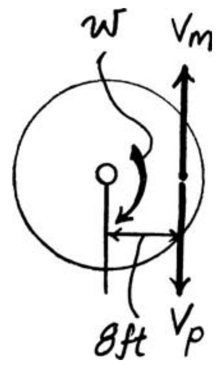
Kinematics: Since the platform rotates about a fixed axis, the speed of point P on the platform to which the man leaps is $v_P = \omega r = \omega(8)$. Applying the relative velocity equation,

$$v_m = v_P + v_{m/P}$$

$$(+\uparrow) \qquad v_m = -\omega(8) + 5 \qquad \textbf{(1)}$$

Conservation of Angular Momentum: As shown in Fig. b, the impulse $\int F dt$ generated during the leap is internal to the system. Thus, angular momentum of the system is conserved about the axis perpendicular to the page passing through point O. The mass moment of inertia of the platform about this axis is

$$I_O = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{300}{32.2}\right)\left(10^2\right) = 465.84\text{ slug}\cdot\text{ft}^2$$



Then

$$(H_O)_1 = (H_O)_2$$

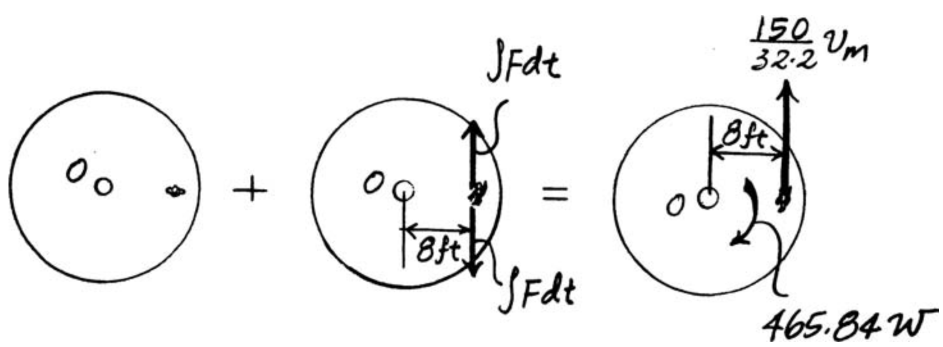
$$0 = \left(\frac{150}{32.2}v_m\right)(8) - 465.84\omega$$

$$v_m = 12.5\omega \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\omega = 0.244\text{ rad/s} \qquad \textbf{Ans.}$$

$$v_m = 3.05\text{ ft/s}$$



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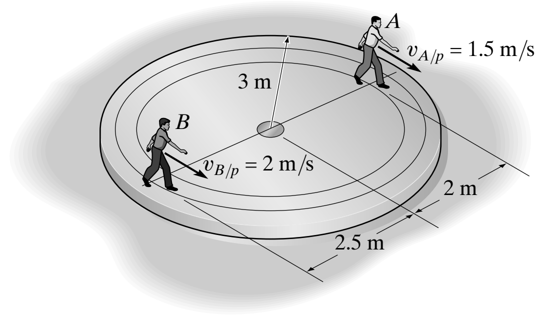
***19–40.** The 150-kg platform can be considered as a circular disk. Two men, A and B, of 60-kg and 75-kg mass, respectively, stand on the platform when it is at rest. If they start to walk around the circular paths with speeds of $v_{A/p} = 1.5$ m/s and $v_{B/p} = 2$ m/s, measured relative to the platform, determine the angular velocity of the platform.



Kinematics: Since the platform rotates about a fixed axis, the speed of points P and P' on the platform at which men B and A are located is $v_P = \omega r_P = \omega(2.5)$ and $v_{P'} = \omega r_{P'} = \omega(2)$. Applying the relative velocity equation,

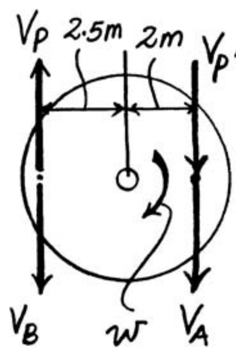
$$\mathbf{v}_B = \mathbf{v}_P + v_{B/P}$$

$$(+\downarrow) \qquad v_B = -\omega(2.5) + 2 \qquad \textbf{(1)}$$

and

$$\mathbf{v}_A = \mathbf{v}_{P'} + \mathbf{v}_{A/P'}$$

$$(+\downarrow) \qquad v_B = \omega(2) + 1.5 \qquad \textbf{(2)}$$



(a)

Conservation of Angular Momentum: As shown in Fig. b, the impulses $\int F_A\,dt$ and $\int F_B\,dt$ are internal to the system. Thus, angular momentum of the system is conserved about the axis perpendicular to the page passing through point O. The mass moment of inertia of the platform about this axis is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(150)(3^2) = 675\ \text{kg}\cdot\text{m}^2$. Then

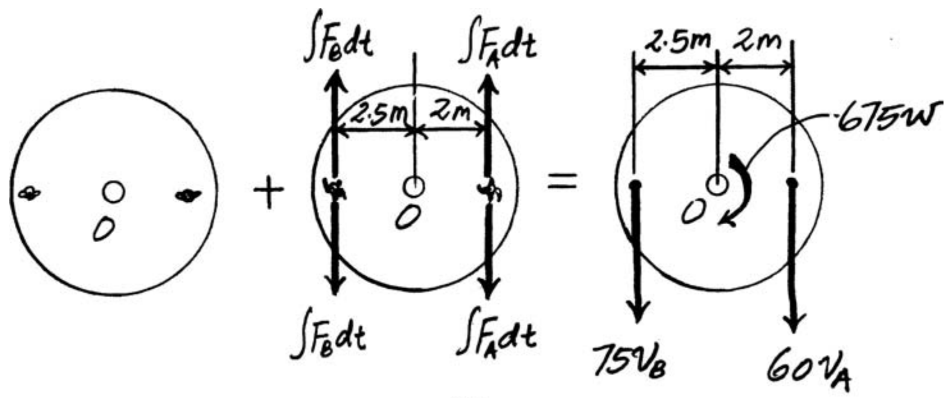
$$(H_O)_1 = (H_O)_2$$

$$0 = 75v_B(2.5) - 60v_A(2) - 675\omega \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (2) into Eq. (3),

$$0 = 75(-2.5\omega + 2)(2.5) - 60(2\omega + 1.5)(2) - 675\omega$$

$$\omega = 0.141 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

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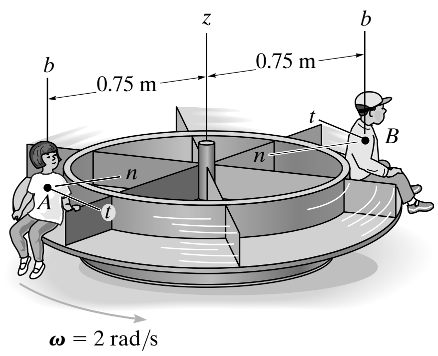
•19–41. Two children A and B, each having a mass of 30 kg, sit at the edge of the merry-go-round which rotates at $\omega = 2$ rad/s. Excluding the children, the merry-go-round has a mass of 180 kg and a radius of gyration $k_z = 0.6$ m. Determine the angular velocity of the merry-go-round if A jumps off horizontally in the $-n$ direction with a speed of 2 m/s, measured relative to the merry-go-round. What is the merry-go-round's angular velocity if B then jumps off horizontally in the $-t$ direction with a speed of 2 m/s, measured relative to the merry-go-round? Neglect friction and the size of each child.



Mass Moment of Inertia: The mass moment inertia of the merry-go-round about the z axis when both children are still on it is

$$(I_z)_1 = 180\left(0.6^2\right) + 2\left[30\left(0.75^2\right)\right] = 98.55 \text{ kg} \cdot \text{m}^2$$

The mass moment inertia of the merry-go-round about z axis when child A jumps off

$$(I_z)_2 = 180\left(0.6^2\right) + 30\left(0.75^2\right) = 81.675 \text{ kg} \cdot \text{m}^2$$

The mass moment inertia of the merry-go-round about z axis when both children jump off

$$(I_z)_3 = 180\left(0.6^2\right) + 0 = 64.80 \text{ kg} \cdot \text{m}^2$$

Conservation of Angular Momentum: When child A jumps off in the $-n$ direction, applying Eq. 19–17, we have

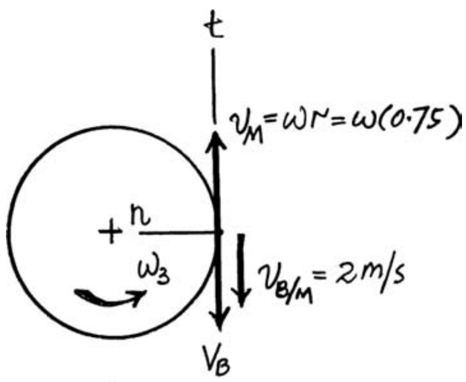
$$(H_z)_1 = (H_z)_2$$

$$(I_z)_1\,\omega_1 = (I_z)_2\,\omega_2$$

$$98.55(2) = 81.675\omega_2$$

$$\omega_2 = 2.413 \text{ rad/s} = 2.41 \text{ rad/s} \qquad \textbf{Ans.}$$

Subsequently, when child B jumps off from the merry-go-round in the $-t$ direction, applying Eq. 19–17, we have



$$(H_z)_2 = (H_z)_3$$

$$(I_z)_2\,\omega_2 = (I_z)_3\,\omega_3 - (m_B\,v_B)(0.75)$$

$$81.675(2.413) = 64.80\omega_3 - 30v_B\,(0.75) \qquad \textbf{(1)}$$

Relative Velocity: The speed of a point located on the edge of the merry-go-round at the instant child B jumps off is $v_M = \omega_3\,(0.75)$.

$$v_B = -v_M + v_{B/M} = -\omega_3\,(0.75) + 2 \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1) and solving yields

$$\omega_3 = 2.96 \text{ rad/s} \qquad \textbf{Ans.}$$

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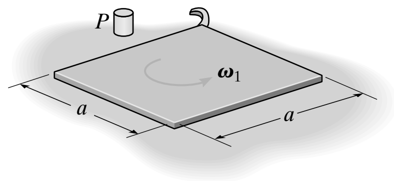
19–42. A thin square plate of mass m rotates on the smooth surface with an angular velocity ω_1. Determine its new angular velocity just after the hook at its corner strikes the peg P and the plate starts to rotate about P without rebounding.



Mass Moment of Inertia: The mass moment inertia of the thin plate about the z axis passing through its mass center is

$$(I_z)_G = \frac{1}{12}(m)\left(a^2 + a^2\right) = \frac{1}{6}ma^2$$

The mass moment inertia of the thin plate about z axis passing through peg P is

$$(I_z)_P = \frac{1}{12}(m)\left(a^2 + a^2\right) + m\left[\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}\right]^2 = \frac{2}{3}ma^2$$

Conservation of Angular Momentum: Applying Eq. 19–17, we have

$$H_G = H_P$$

$$\left(\frac{1}{6}ma^2\right)\omega_1 = \left(\frac{2}{3}ma^2\right)\omega_2$$

$$\omega_2 = \frac{1}{4}\omega_1 \qquad \textbf{Ans.}$$

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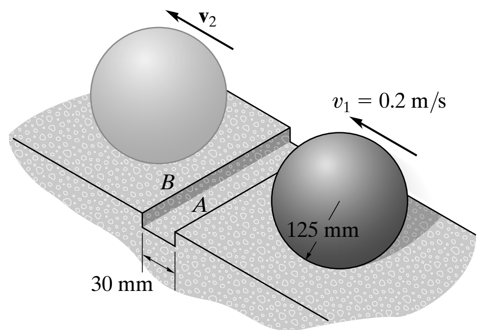
19–43. A ball having a mass of 8 kg and initial speed of $v_1 = 0.2$ m/s rolls over a 30-mm-long depression. Assuming that the ball rolls off the edges of contact first A, then B, without slipping, determine its final velocity $\mathbf{v}_2$ when it reaches the other side.



$$\omega_1 = \frac{0.2}{0.125} = 1.6 \text{ rad/s} \qquad \omega_2 = \frac{v_2}{0.125} = 8v_2$$

$$\theta = \sin^{-1}\left(\frac{15}{125}\right) = 6.8921°$$

$$h = 125 - 125 \cos 6.8921° = 0.90326 \text{ mm}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(8)(0.2)^2 + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](1.6)^2 + 0$$

$$= -(0.90326)(10^{-3})8(9.81) + \frac{1}{2}(8)\omega^2(0.125)^2 + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](\omega)^2$$

$$\omega = 1.836 \text{ rad/s}$$

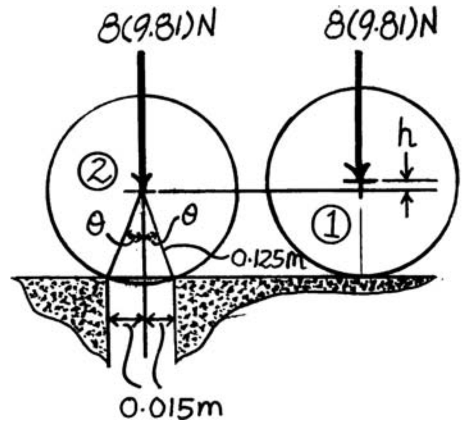
$$(H_B)_2 = (H_B)_3$$

$$\left[\frac{2}{5}(8)(0.125)^2\right](1.836) + 8(1.836)(0.125)\cos 6.892°(0.125 \cos 6.892°)$$

$$- 8(0.22948 \sin 6.892°)(0.125 \sin 6.892°)$$

$$= \left[\frac{2}{5}(8)(0.125)^2\right]\omega_3 + 8(0.125)\omega_3(0.125)$$

$$\omega_3 = 1.7980 \text{ rad/s}$$



$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](1.7980)^2 + \frac{1}{2}(8)(1.7980)^2(0.125)^2 + 0$$

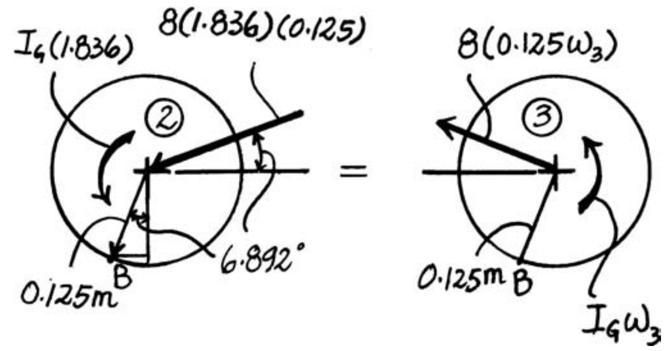
$$= 8(9.81)(0.90326(10^{-3})) + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](\omega_4)^2$$

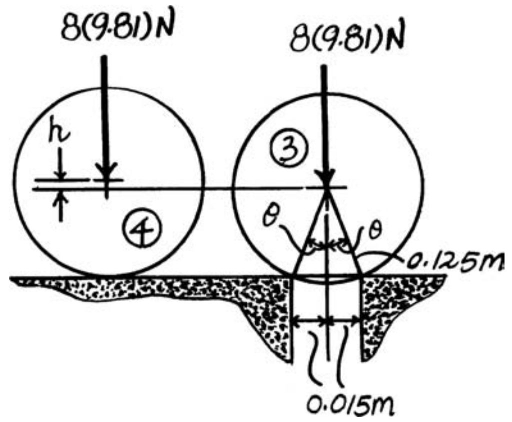
$$+ \frac{1}{2}(8)(\omega_4)^2(0.125)^2$$

$$\omega_4 = 1.56 \text{ rad/s}$$

So that

$$v_2 = 1.56(0.125) = 0.195 \text{ m/s}$$ **Ans.**





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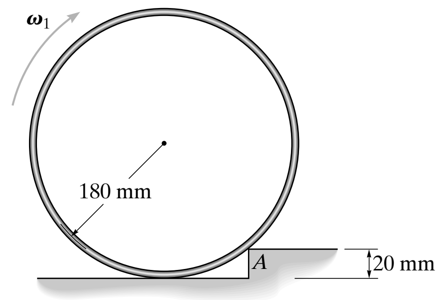
***19–44.** The 15-kg thin ring strikes the 20-mm-high step. Determine the smallest angular velocity ω_1 the ring can have so that it will just roll over the step at A without slipping



The weight is non-impulsive.

$$(H_A)_1 = (H_A)_2$$

$$15(\omega_1)(0.18)(0.18 - 0.02) + \left[15(0.18)^2\right](\omega_1) = \left[15(0.18)^2 + 15(0.18)^2\right]\omega_2$$

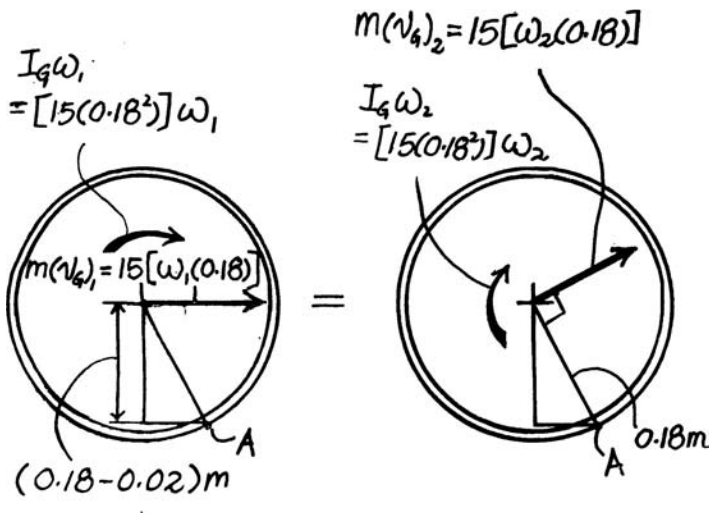
$$\omega_2 = 0.9444\omega_1$$

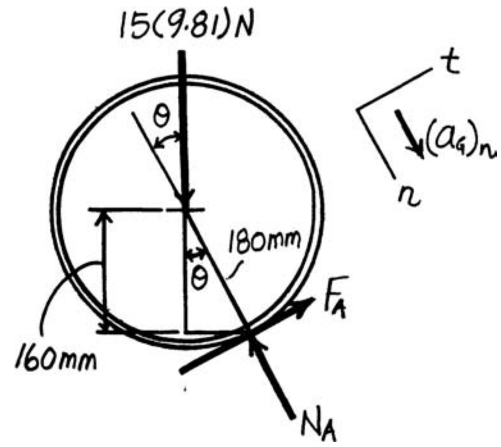
$$+\searrow \Sigma F_n = m(a_G)_n; \qquad (15)(9.81)\cos\theta - N_A = 15\omega_2^2\,(0.18)$$

When hoop is about to rebound, $N_A - 0$. Also, $\cos\theta = \dfrac{160}{180}$, and so

$$\omega_2 = 6.9602 \text{ rad/s}$$

$$\omega_1 = \frac{6.9602}{0.9444} = 7.37 \text{ rad/s} \qquad \textbf{Ans.}$$





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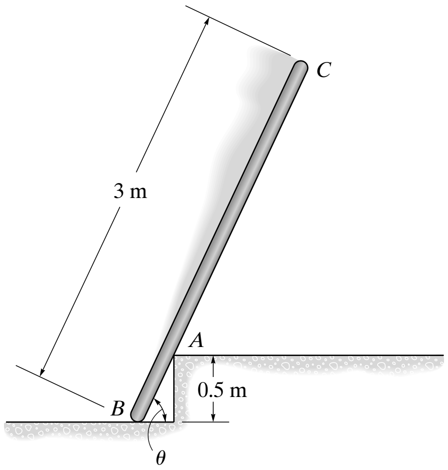
•19–45. The uniform pole has a mass of 15 kg and falls from rest when $\theta = 90^\circ$. It strikes the edge at A when $\theta = 60^\circ$. If the pole then begins to pivot about this point after contact, determine the pole's angular velocity just after the impact. Assume that the pole does not slip at B as it falls until it strikes A.

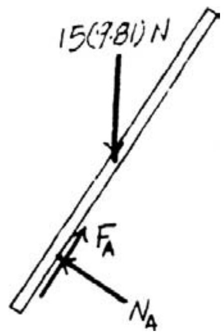


Conservation of Energy: Datum is set at point B. When the pole is at its initial and final position, its center of gravity is located 1.5 m and 1.5 sin 60° m = 1.299 m *above* the datum. Its initial and final potential energy are $15(9.81)(1.5) = 220.725\ \text{N}\cdot\text{m}$ and $15(9.81)(1.299) = 191.15\ \text{N}\cdot\text{m}$. The mass moment of inertia about point B is $I_B = \frac{1}{12}(15)(3^2) + 15(1.5^2) = 45.0\ \text{kg}\cdot\text{m}^2$. The kinetic energy of the pole before the impact is $\frac{1}{2}I_B\,\omega_1^2 = \frac{1}{2}(45.0)\omega_1^2 = 22.5\omega_1^2$. Applying Eq. 18–18, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 220.725 = 22.5\omega_1^2 + 191.15$$

$$\omega_1 = 1.146\ \text{rad/s}$$



Conservation of Angular Momentum: Since the weight of the pole is *nonimpulsive* force, the angular momentum is conserved about point A. The velocity of its mass center before impact is $v_G = \omega_1 r_{GB} = 1.146(1.5) = 1.720\ \text{m/s}$. The mass moment of inertia of the pole about its mass center and point A are

$$I_G = \frac{1}{12}(15)(3^2) = 11.25\ \text{kg}\cdot\text{m}^2$$

and

$$I_A = \frac{1}{12}(15)(3^2) + 15\left(1.5 - \frac{0.5}{\sin 60^\circ}\right)^2 = 24.02\ \text{kg}\cdot\text{m}^2$$

Applying Eq. 19–17, we have

$$(H_A)_1 = (H_A)_2$$

$$(mv_G)(r_{GA}) + I_G\,\omega_1 = I_A\,\omega_2$$

$$[15(1.720)]\left(1.5 - \frac{0.5}{\sin 60^\circ}\right) + 11.25(1.146) = 24.02\omega_2$$

$$\omega_2 = 1.53\ \text{rad/s} \qquad \textbf{Ans.}$$

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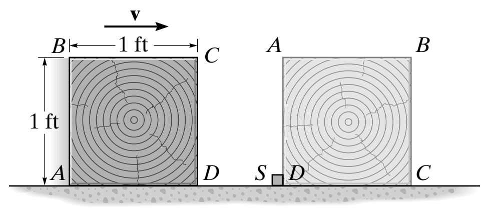
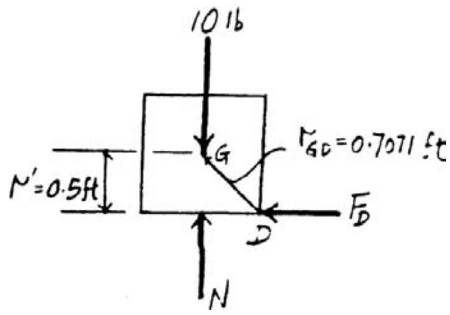
19–46. The 10-lb block slides on the smooth surface when the corner D hits a stop block S. Determine the minimum velocity **v** the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of S. *Hint:* During impact consider the weight of the block to be nonimpulsive.



Conservation of Energy: If the block tips over about point D, it must at least achieve the dash position shown. Datum is set at point D. When the block is at its initial and final position, its center of gravity is located 0.5 ft and 0.7071 ft *above* the datum. Its initial and final potential energy are $10(0.5) = 5.00$ ft · lb and $10(0.7071) = 7.071$ ft · lb. The mass moment of inertia of the block about point D is



$$I_D = \frac{1}{12}\left(\frac{10}{32.2}\right)\left(1^2 + 1^2\right) + \left(\frac{10}{32.2}\right)\left(\sqrt{0.5^2 + 0.5^2}\right)^2 = 0.2070 \text{ slug} \cdot \text{ft}^2$$

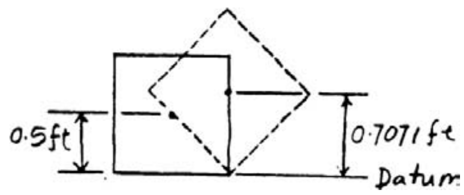
The initial kinetic energy of the block (after the impact) is $\frac{1}{2} I_D \omega_2^2 = \frac{1}{2}(0.2070)\,\omega_2^2$. Applying Eq. 18–18, we have

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(0.2070)\,\omega_2^2 + 5.00 = 0 + 7.071$$

$$\omega_2 = 4.472 \text{ rad/s}$$

Conservation of Angular Momentum: Since the weight of the block and the normal reaction N are *nonimpulsive* forces, the angular momentum is conserves about point D. Applying Eq. 19–17, we have



$$(H_D)_1 = (H_D)_2$$

$$(mv_G)(r') = I_D\,\omega_2$$

$$\left[\left(\frac{10}{32.2}\right)v\right](0.5) = 0.2070(4.472)$$

$$v = 5.96 \text{ ft/s} \qquad \textbf{Ans.}$$

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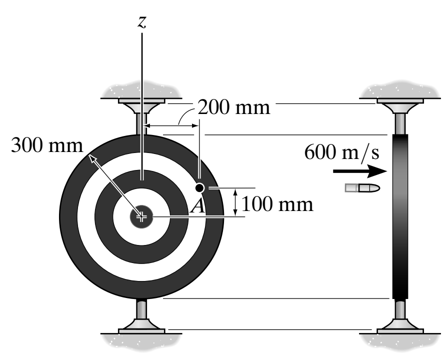
19–47. The target is a thin 5-kg circular disk that can rotate freely about the z axis. A 25-g bullet, traveling at 600 m/s, strikes the target at A and becomes embedded in it. Determine the angular velocity of the target after the impact. Initially, it is at rest.

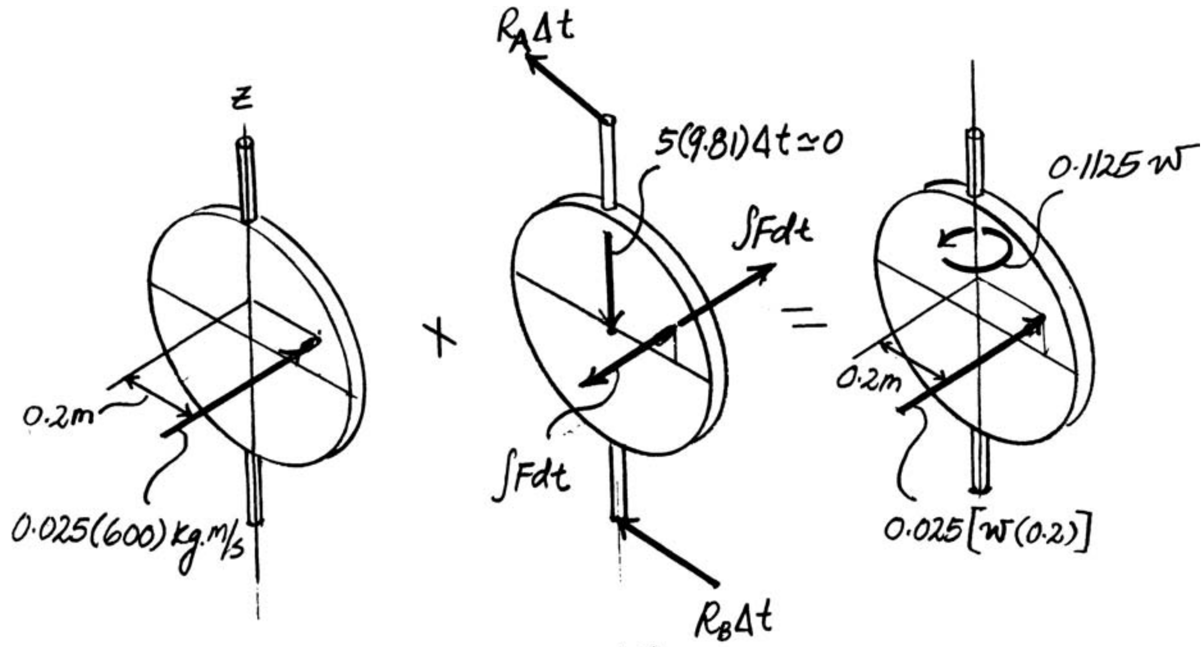


Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulse of the system about the z axis is zero. Thus, the angular impulse of the system is conserved about the z axis. The mass moment of inertia of the target about the z axis is $I_z = \frac{1}{4}mr^2 = \frac{1}{4}(5)\left(0.3^2\right) = 0.1125\ \text{kg}\cdot\text{m}^2$. Since the target rotates about the z axis when the bullet is embedded in the target, the bullet's velocity is $(v_b)_2 = \omega(0.2)$. Then,

$$(H_z)_1 = (H_z)_2$$

$$0.025(600)(0.2) = 0.1125\omega + 0.025\left[\omega(0.2)\right](0.2)$$

$$\omega = 26.4\ \text{rad/s}$$ **Ans.**



(a)

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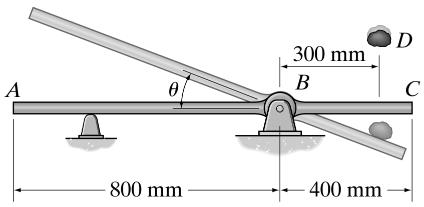
***19–48.** A 2-kg mass of putty D strikes the uniform 10-kg plank ABC with a velocity of 10 m/s. If the putty remains attached to the plank, determine the maximum angle θ of swing before the plank momentarily stops. Neglect the size of the putty.



Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulses about point B is zero. Thus, angular impulse of the system is conserved about this point. Since rod AC rotates about point B, $(v_{G_{AC}})_2 = \omega_2 r_{G_{AC}} = \omega_2(0.2)$ and $(v_D)_2 = \omega_2 r_{G_D} = \omega_2(0.3)$. The mass moment of inertia of rod AC about its mass center is $I_{G_{AC}} = \frac{1}{12}ml^2 = \frac{1}{12}(10)\left(1.2^2\right) = 1.2\ \text{kg}\cdot\text{m}^2$. Then,

$$(H_B)_1 = (H_B)_2$$

$$2(10)(0.3) = 1.2\omega_2 + 10\left[\omega_2(0.2)\right](0.2) + 2\left[\omega_2(0.3)\right](0.3)$$

$$\omega_2 = 3.371\ \text{rad/s}$$ **Ans.**

Conservation of Energy: With reference to the datum in Fig. a,

$$V_2 = \left(V_g\right)_2 = W_{AC}\,(y_{G_{AC}})_2 + W_D(y_{G_D})_2 = 0$$

and

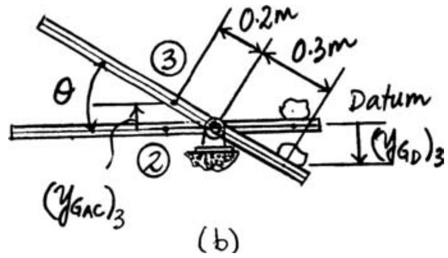
$$V_3 = \left(V_g\right)_3 = W_{AC}\,(y_{G_{AC}})_3 - W_D(y_{G_D})_3$$

$$= 10(9.81)(0.2\sin\theta) - 2(9.81)(0.3\sin\theta) = 13.734\sin\theta$$

The initial kinetic energy of the system is

$$T_2 = \frac{1}{2}I_{G_{AC}}\,\omega_2{}^2 + \frac{1}{2}m_{AC}\,(v_{G_{AC}})_2{}^2 + \frac{1}{2}m_D(v_{G_D})_2{}^2$$

$$= \frac{1}{2}(1.2)\left(3.371^2\right) + \frac{1}{2}(10)\left[3.371(0.2)\right]^2 + \frac{1}{2}(2)\left[3.371(0.3)\right]^2 = 10.11\ \text{J}$$

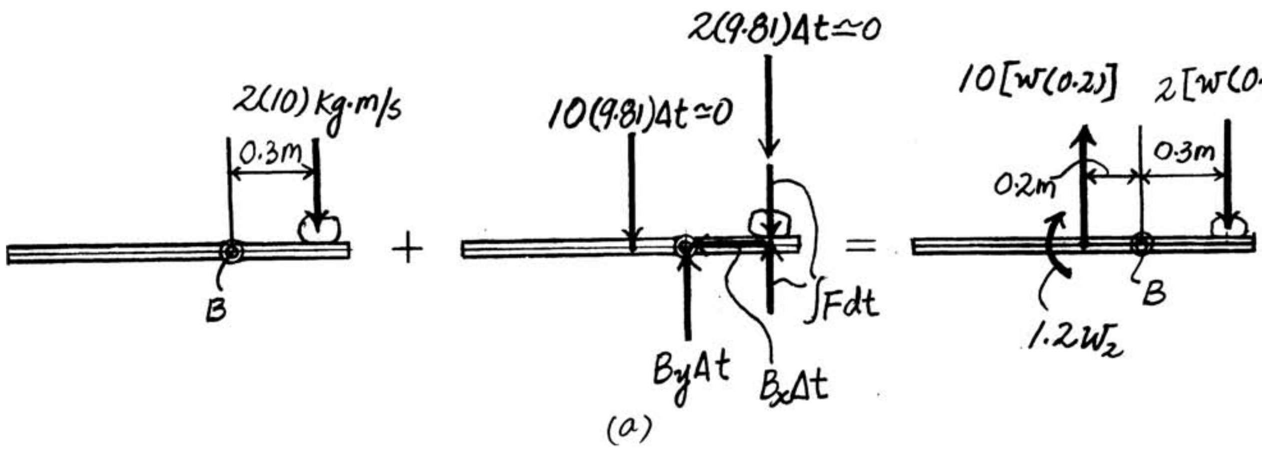


Since the system is required to be at rest in the final position, $T_3 = 0$. Then,

$$T_2 + V_2 = T_3 + V_3$$

$$10.11 + 0 = 0 + 13.734\sin\theta$$

$$\theta = 47.4^\circ$$ **Ans.**



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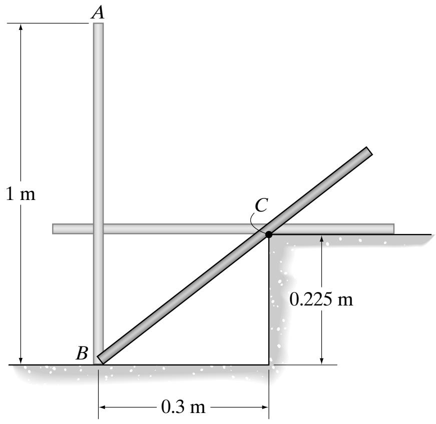
•19–49. The uniform 6-kg slender rod AB is given a slight horizontal disturbance when it is in the vertical position and rotates about B without slipping. Subsequently, it strikes the step at C. The impact is perfectly plastic and so the rod rotates about C without slipping after the impact. Determine the angular velocity of the rod when it is in the horizontal position shown.



Conservation of Energy: From the geometry of Fig. a, $\theta = \tan^{-1}\left(\frac{0.225}{0.3}\right) = 36.87°$ and $BC = \sqrt{0.3^2 + 0.225^2} = 0.375$ m. Thus, $r_{CG} = 0.5 - 0.375 = 0.125$ m. With reference to the datum, $V_1 = W(y_G)_1 = 6(9.81)(0.5) = 29.43$ J, $V_2 = V_3 = W(y_G)_3 = 6(9.81)(0.5 \sin 36.87°) = 17.658$ J, and $V_4 = W(y_G)_4 = 6(9.81)(0.225) = 13.2435$ J. Since the rod is initially at rest, $T_1 = 0$. The rod rotates about point B before impact. Thus, $(v_G)_2 = \omega_2 r_{BG} = \omega_2\,(0.5)$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(6)\left(1^2\right) = 0.5\ \text{kg}\cdot\text{m}^2$. Then, $T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(6)\left[\omega_2(0.5)\right]^2 + \frac{1}{2}(0.5)\omega_2{}^2 = 1\omega_2{}^2$. Therefore,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 29.43 = 1\omega_2{}^2 + 17.658$$

$$\omega_2 = 3.431 \text{ rad/s}$$

The rod rotates about point C after impact. Thus, $v_G = \omega r_{CG} = \omega(0.125)$. Then,

$$T = \frac{1}{2}m(v_G)^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(6)\left[\omega(0.125)\right]^2 + \frac{1}{2}(0.5)\omega^2 = 0.296875\omega^2$$

so that

$$T_3 = 0.296875\omega_3{}^2 \text{ and } T_4 = 0.296875\omega_4{}^2$$

$$T_3 + V_3 = T_4 + V_4$$

$$0.296875\omega_3{}^2 + 17.658 = 0.296875\omega_4{}^2 + 13.2435$$

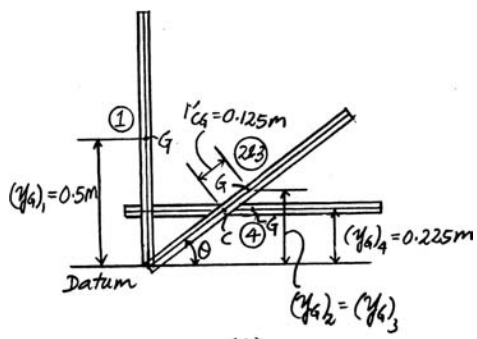
$$\omega_4{}^2 - \omega_3{}^2 = 14.87 \qquad \textbf{(1)}$$

Conservation of Angular Momentum: Referring to Fig. b, the sum of the angular impulses about point C is zero. Thus, angular momentum of the rod is conserved about this point during the impact. Then,

$$(H_C)_1 = (H_C)_2$$

$$6\left[3.431(0.5)\right](0.125) + 0.5(3.431) = 6\left[\omega_3(0.125)\right](0.125) + 0.5\omega_3$$

$$\omega_3 = 5.056 \text{ rad/s}$$

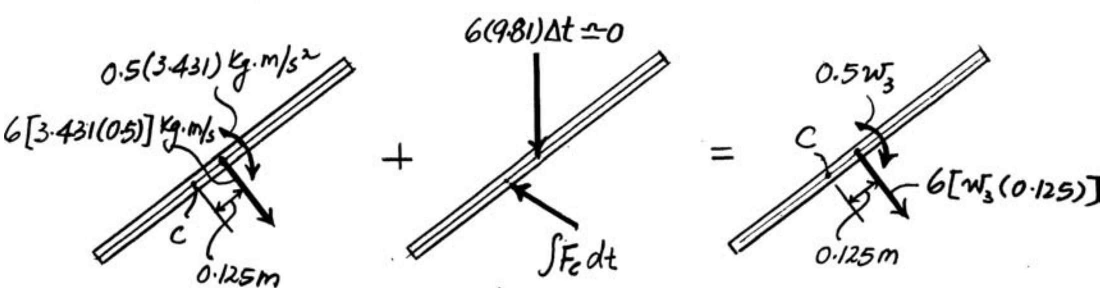


(a)

Substituting this result into Eq. (1), we obtain

$$\omega_4{}^2 - (5.056)^2 = 14.87$$

$$\omega_4 = 6.36 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

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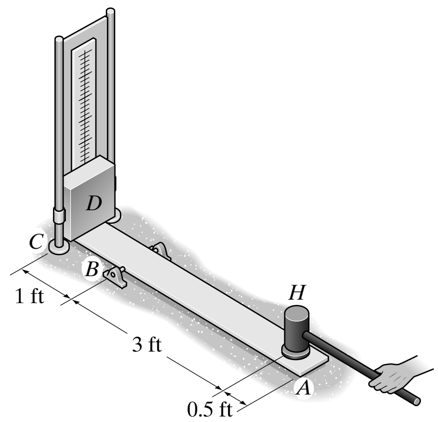
19–50. The rigid 30-lb plank is struck by the 15-lb hammer head H. Just before the impact the hammer is gripped loosely and has a vertical velocity of 75 ft/s. If the coefficient of restitution between the hammer head and the plank is $e = 0.5$, determine the maximum height attained by the 50-lb block D. The block can slide freely along the two vertical guide rods. The plank is initially in a horizontal position.



Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulses about point B is zero. Thus, angular momentum of the system is conserved about this point during the impact. Since the plank rotates about point B, $(v_D)_2 = \omega_2(1)$ and $(v_G)_2 = \omega_2(1.25)$. The mass moment of inertia of the plank about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{30}{32.2}\right)(4.5^2) = 1.572\ \text{slug}\cdot\text{ft}^2$. Thus,

$$(H_B)_1 = (H_B)_2$$

$$\frac{15}{32.2}(75)(3) = \frac{50}{32.2}\left[\omega_2(1)\right](1) + \frac{30}{32.2}\left[\omega_2(1.25)\right](1.25) + 1.572\omega_2 - \frac{15}{32.2}(v_H)_2(3)$$

$$4.581\omega_2 - 1398(v_H)_2 = 104.81 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_A)_2 = \omega_2(3)\downarrow$. Thus,

$$(+\uparrow) \qquad e = \frac{(v_A)_2 - (v_H)_2}{(v_H)_1 - (v_A)_1}$$

$$0.5 = \frac{-\omega_2(3) - (v_H)_2}{-75 - 0}$$

$$3\omega_2 + (v_H)_2 = 37.5 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$\omega_2 = 17.92\ \text{rad/s} \qquad (v_H)_2 = -16.26\ \text{ft/s} = 16.26\ \text{ft/s}\downarrow$$

Conservation of Energy: With reference to the datum in Fig. b, $V_2 = \left(V_g\right)_2 = W_D(y_G)_2 = 0$ and $V_3 = \left(V_g\right)_3 = W_D(y_G)_3 = 50h$.

$$\left(v_D\right)_2 = \omega_2(1) = 17.92(1) = 17.92\ \text{ft/s and } (v_D)_3 = 0$$

Thus,

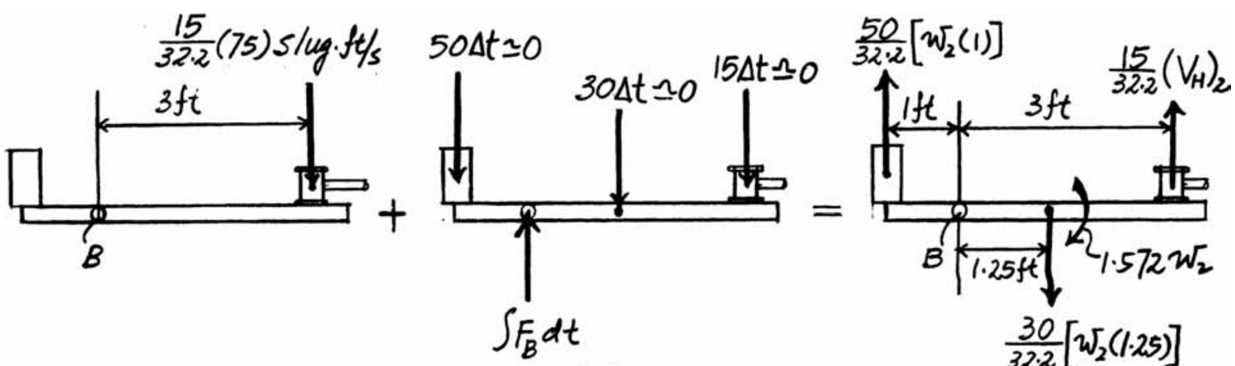
$$T_2 = \frac{1}{2}m_D(v_D)_2{}^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)\left(17.92^2\right) = 249.33\ \text{ft}\cdot\text{lb and } T_3 = 0$$

Then

$$T_2 + V_2 = T_3 + V_3$$

$$249.33 + 0 = 0 + 50h$$

$$h = 4.99\ \text{ft} \qquad \textbf{Ans.}$$



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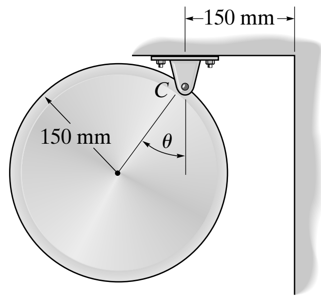
19–51. The disk has a mass of 15 kg. If it is released from rest when $\theta = 30°$, determine the maximum angle θ of rebound after it collides with the wall. The coefficient of restitution between the disk and the wall is $e = 0.6$. When $\theta = 0°$, the disk hangs such that it just touches the wall. Neglect friction at the pin C.



Datum at lower position of G.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (15)(9.81)(0.15)(1 - \cos 30°) = \frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right]\omega^2 + 0$$

$$\omega = 3.418 \text{ rad/s}$$

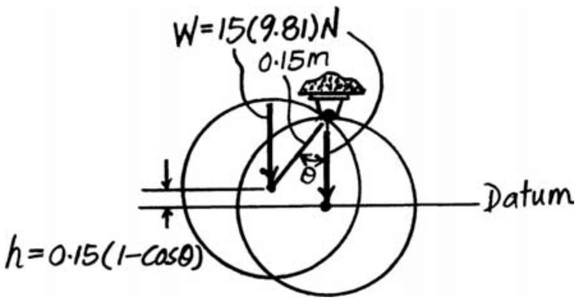
$$\left(\overset{+}{\rightarrow}\right) \qquad e = 0.6 = \frac{0 - (-0.15\omega')}{3.418(0.15) - 0}$$

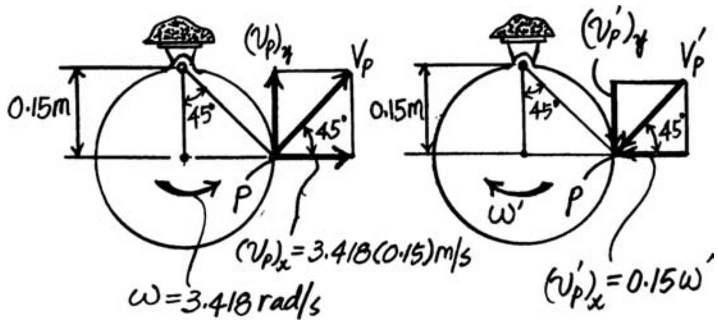
$$\omega' = 2.0508 \text{ rad/s}$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right](2.0508)^2 + 0 = 0 + 15(9.81)(0.15)(1 - \cos\theta)$$

$$\theta = 17.9° \qquad \textbf{Ans.}$$





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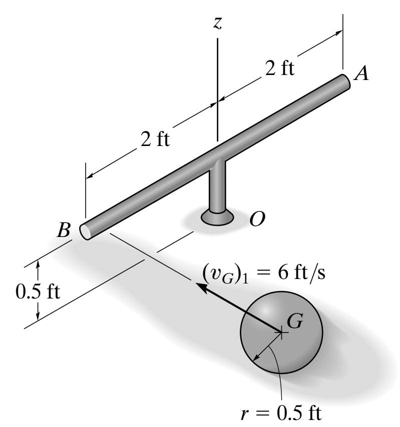
***19–52.** The mass center of the 3-lb ball has a velocity of $(v_G)_1 = 6$ ft/s when it strikes the end of the smooth 5-lb slender bar which is at rest. Determine the angular velocity of the bar about the z axis just after impact if $e = 0.8$.



Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender bar and the ball, it will cancel out. Thus, angular momentum is conserved about the z axis. The mass moment of inertia of the slender bar about the z axis is $I_z = \frac{1}{12}\left(\frac{5}{32.2}\right)(4^2) = 0.2070$ slug · ft^2. Here, $\omega_2 = \frac{(v_B)_2}{2}$. Applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$\left[m_b\,(v_G)_1\right](r_b) = I_z\,\omega_2 + \left[m_b\,(v_G)_2\right](r_b)$$

$$\left(\frac{3}{32.2}\right)(6)(2) = 0.2070\left[\frac{(v_B)_2}{2}\right] + \left(\frac{3}{32.2}\right)(v_G)_2(2) \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{(v_B)_2 - (v_G)_2}{(v_G)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_G)_2}{6 - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_G)_2 = 2.143 \text{ ft/s} \qquad (v_B)_2 = 6.943 \text{ ft/s}$$

Thus, the angular velocity of the slender rod is given by

$$\omega_2 = \frac{(v_B)_2}{2} = \frac{6.943}{2} = 3.47 \text{ rad/s} \qquad \textbf{Ans.}$$

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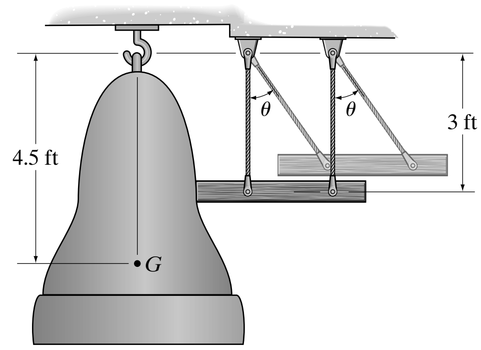
•19–53. The 300-lb bell is at rest in the vertical position before it is struck by a 75-lb wooden post suspended from two equal-length ropes. If the post is released from rest at $\theta = 45°$, determine the angular velocity of the bell and the velocity of the post immediately after the impact. The coefficient of restitution between the bell and the post is $e = 0.6$. The center of gravity of the bell is located at point G and its radius of gyration about G is $k_G = 1.5$ ft.



Conservation of Energy: With reference to the datum in Fig. *a*, $V_1 = \left(V_g\right)_1 = -W(y_G)_1 = -75(3\cos 45°) = -159.10$ ft · lb and $V_2 = \left(V_g\right)_2 = -W(y_G)_2 = -75(3) = -225$ ft · lb. Since the post is initially at rest, $T_1 = 0$. The post undergoes curvilinear translation, $T_2 = \frac{1}{2}m(v_P)_2{}^2 = \frac{1}{2}\left[\frac{75}{32.2}\right](v_P)_2{}^2$. Thus,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-159.10) = \frac{1}{2}\left[\frac{75}{32.2}\right](v_G)_2{}^2 + (-225)$$

$$(v_P)_2 = 7.522 \text{ ft/s}$$

Conservation of Angular Momentum: The sum of the angular impulses about point O is zero. Thus, angular momentum of the system is conserved about this point during the impact. Since the bell rotates about point O, $(v_G)_3 = \omega_3 r_{OG} = \omega_3(4.5)$. The mass moment of inertia of the bell about its mass center is $IG = \frac{1}{12}mk_G{}^2 = \frac{300}{32.2}\left(1.5^2\right) = 20.96$ slug · ft². Thus,

$$(H_O)_2 = (H_O)_3$$

$$\frac{75}{32.2}(7.522)(3) = \frac{300}{32.2}\left[\omega_3(4.5)\right](4.5) + 20.96\omega_3 - \frac{75}{32.2}(v_P)_3(3)$$

$$209.63\omega_3 - 6.988(v_P)_3 = 52.56 \qquad \textbf{(1)}$$

Coefficient of Restitution: The impact point A on the bell along the line of impact (x axis) is $\left[(v_A)_3\right]_x = \omega_3(3)$. Thus,

$$e = \frac{\left[(v_A)_3\right]_x - (v_P)_3}{(v_P)_2 - \left[(v_A)_2\right]_x}$$

$$\left(\overset{+}{\rightarrow}\right) \qquad 0.6 = \frac{-\omega_3(3) - (v_P)_3}{-7.522 - 0}$$

$$3\omega_3 + (v_P)_3 = 4.513 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$\omega_3 = 0.365 \text{ rad/s} \qquad (v_P)_3 = 3.42 \text{ ft/s} \qquad \textbf{Ans.}$$

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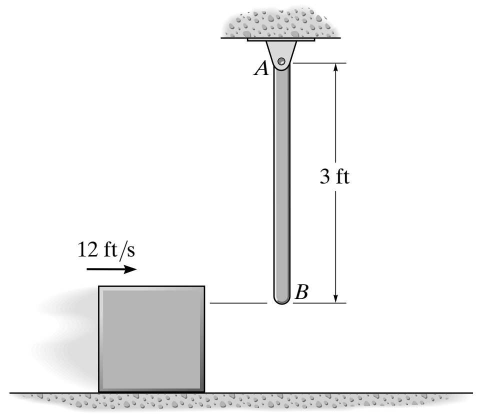
19–54. The 4-lb rod AB hangs in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end B. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at B is $e = 0.8$.

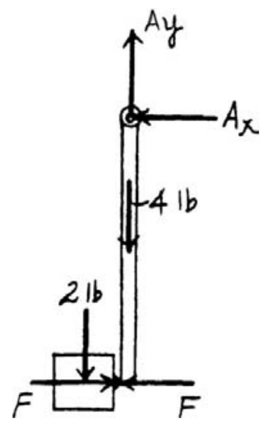


Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point A. The mass moment of inertia of the slender rod about point A is $I_A = \frac{1}{12}\left(\frac{4}{32.2}\right)(3^2) + \frac{4}{32.2}(1.5^2) = 0.3727 \text{ slug}\cdot\text{ft}^2$. Here, $\omega_2 = \frac{(v_B)_2}{3}$. Applying Eq. 19–17, we have

$$(H_A)_1 = (H_A)_2$$

$$\left[m_b\,(v_b)_1\right](r_b) = I_A\,\omega_2 + \left[m_b\,(v_b)_2\right](r_b)$$

$$\left(\frac{2}{32.2}\right)(12)(3) = 0.3727\left[\frac{(v_B)_2}{3}\right] + \left(\frac{2}{32.2}\right)(v_b)_2(3) \qquad \textbf{(1)}$$



Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_b)_2}{12 - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

$$(v_B)_2 = 12.96 \text{ ft/s} \rightarrow$$

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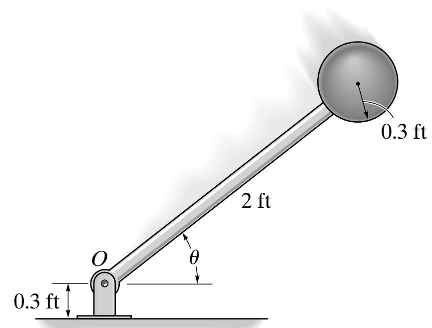
19–55. The pendulum consists of a 10-lb sphere and 4-lb rod. If it is released from rest when $\theta = 90°$, determine the angle θ of rebound after the sphere strikes the floor. Take $e = 0.8$.



$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \left(\frac{10}{32.2}\right)(2.3)^2 = 1.8197 \text{ slug}\cdot\text{ft}^2$$

Just before impact:

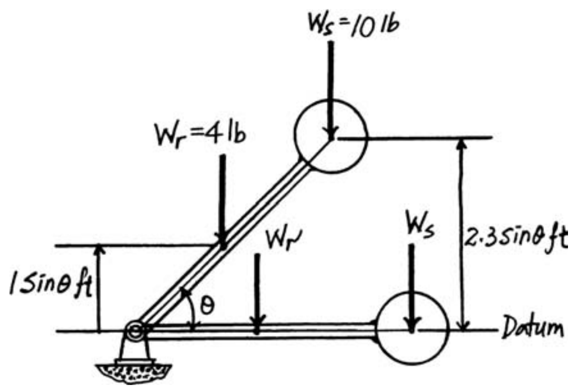
Datum through O.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1) + 10(2.3) = \frac{1}{2}(1.8197)\omega^2 + 0$$

$$\omega_2 = 5.4475 \text{ rad/s}$$

$$v = 2.3(5.4475) = 12.529 \text{ ft/s}$$



Since the floor does not move,

$$(+\uparrow) \qquad e = 0.8 = \frac{(v_P) - 0}{0 - (-12.529)}$$

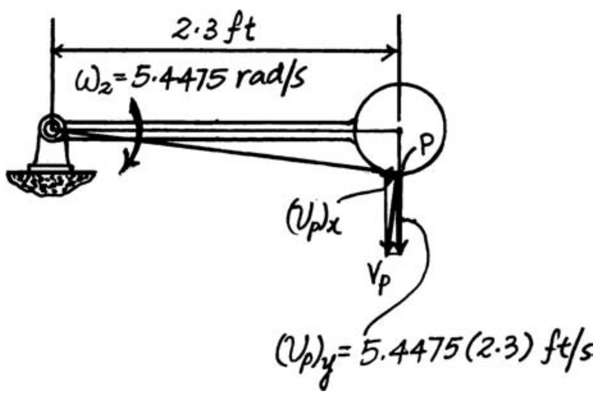
$$(v_P)_3 = 10.023 \text{ ft/s}$$

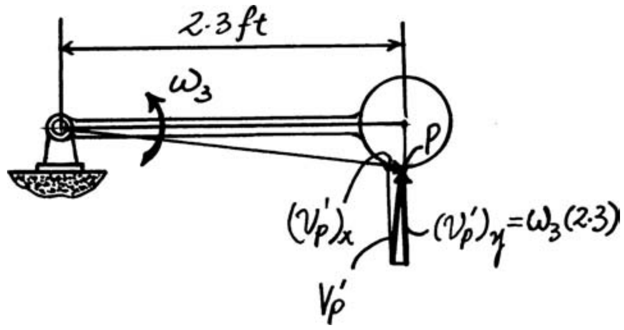
$$\omega_3 = \frac{10.023}{2.3} = 4.358 \text{ rad/s}$$

$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}(1.8197)(4.358)^2 + 0 = 4(1 \sin\theta_1) + 10(2.3 \sin\theta_1)$$

$$\theta_1 = 39.8° \qquad \textbf{Ans.}$$





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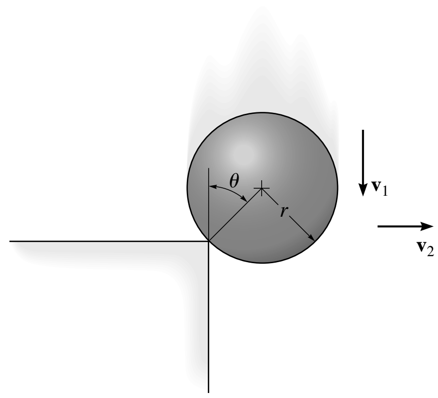
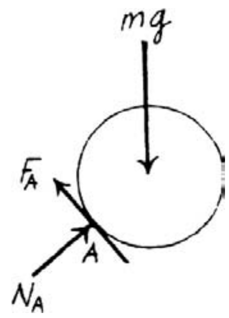
***19–56.** The solid ball of mass m is dropped with a velocity $\mathbf{v}_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $\mathbf{v}_2$, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.



Conservation of Angular Momentum: Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A. The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$. Here, $\omega_2 = \frac{v_2 \cos\theta}{r}$. Applying Eq. 19–17, we have



$$(H_A)_1 = (H_A)_2$$

$$\left[m_b\,(v_b)_1\right](r') = I_G\,\omega_2 + \left[m_b\,(v_b)_2\right](r'')$$

$$(mv_1)(r\sin\theta) = \left(\frac{2}{5}mr^2\right)\left(\frac{v_2\cos\theta}{r}\right) + (mv_2)(r\cos\theta)$$

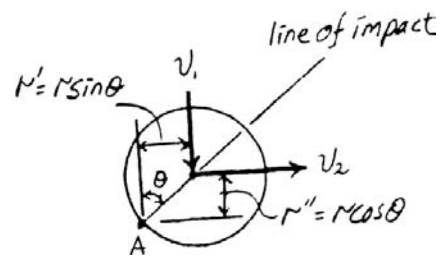
$$\frac{v_2}{v_1} = \frac{5}{7}\tan\theta \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{0 - (v_b)_2}{(v_b)_1 - 0}$$

$$e = \frac{-(v_2\sin\theta)}{-v_1\cos\theta}$$

$$\frac{v_2}{v_1} = \frac{e\cos\theta}{\sin\theta} \qquad \textbf{(2)}$$



Equating Eqs. (1) and (2) yields

$$\frac{5}{7}\tan\theta = \frac{e\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{7}{5}e$$

$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right) \qquad \textbf{Ans.}$$

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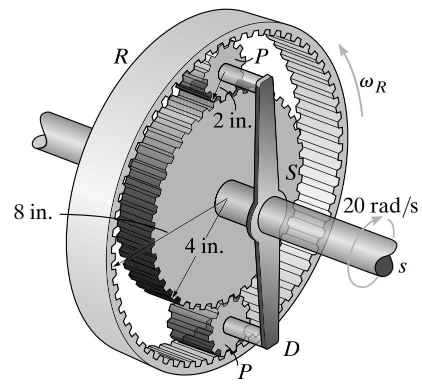
R2–1. An automobile transmission consists of the planetary gear system shown. If the ring gear R is held fixed so that $\omega_R = 0$, and the shaft s and sun gear S, rotates at 20 rad/s, determine the angular velocity of each planet gear P and the angular velocity of the connecting rack D, which is free to rotate about the center shaft s.



For planet gear P: The velocity of point A is $v_A = \omega_s r_s = 20\left(\frac{4}{12}\right) = 6.667$ ft/s.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$0 = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{\omega_P\left(\frac{4}{12}\right)}\right]$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 6.667 - \omega_P\left(\frac{4}{12}\right) \qquad \omega_P = 20 \text{ rad/s} \qquad \textbf{Ans.}$$

For connecting rack D:

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$$

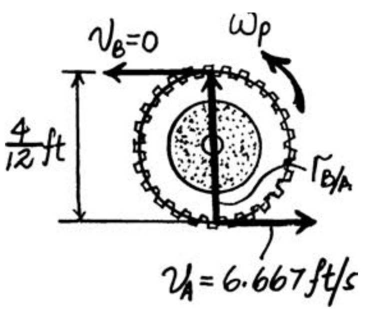
$$\left[\underset{\rightarrow}{v_C}\right] = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{20\left(\frac{2}{12}\right)}\right]$$

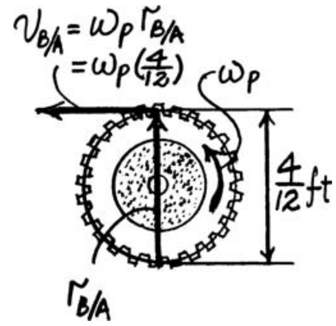
$$(\overset{+}{\rightarrow}) \qquad v_C = 6.667 - 20\left(\frac{2}{12}\right) \qquad v_C = 3.333 \text{ ft/s}$$

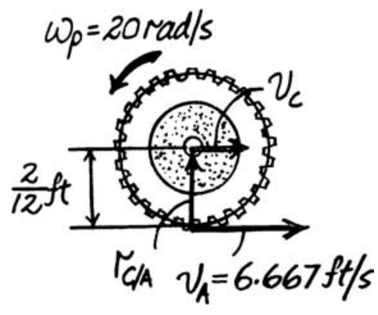
The rack is rotating about a fixed axis (shaft s). Hence,

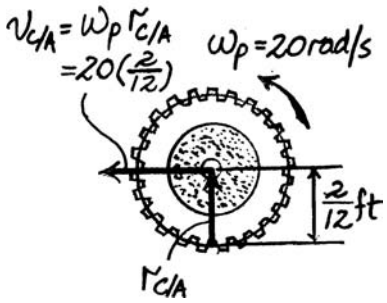
$$v_C = \omega_D r_D$$

$$3.333 = \omega_D\left(\frac{6}{12}\right) \qquad \omega_D = 6.67 \text{ rad/s} \qquad \textbf{Ans.}$$









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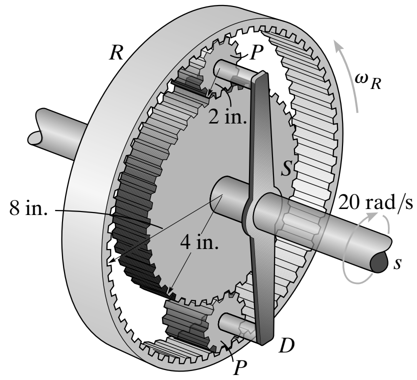
R2–2. An automobile transmission consists of the planetary gear system shown. If the ring gear R rotates at $\omega_R = 2$ rad/s, and the shaft s and sun gear S, rotates at 20 rad/s, determine the angular velocity of each planet gear P and the angular velocity of the connecting rack D, which is free to rotate about the center shaft s.



For planet gear P: The velocity of points A and B are $v_A = \omega_S\, r_S = 20\left(\frac{4}{12}\right) = 6.667$ ft/s and $v_B = \omega_B\, r_B = 2\left(\frac{8}{12}\right) = 1.333$ ft/s.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\left[\underset{\leftarrow}{1.333}\right] = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{\omega_P\left(\frac{4}{12}\right)}\right]$$

$$\left(\xrightarrow{+}\right) \qquad -1.333 = 6.667 - \omega_P\left(\frac{4}{12}\right) \qquad \omega_P = 24 \text{ rad/s} \qquad \textbf{Ans.}$$

For connecting rack D:

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$$

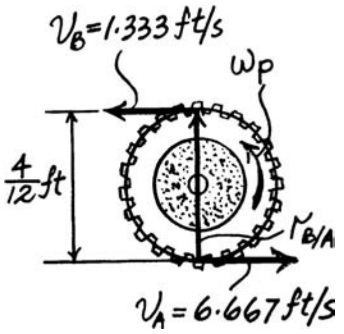
$$\left[\underset{\rightarrow}{v_C}\right] = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{24\left(\frac{2}{12}\right)}\right]$$

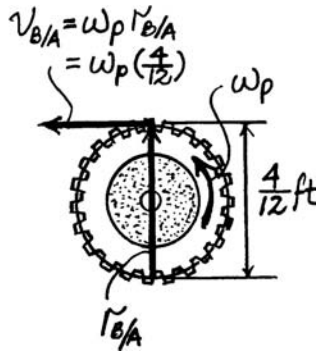
$$\left(\xrightarrow{+}\right) \qquad v_C = 6.667 - 24\left(\frac{2}{12}\right) \qquad v_C = 2.667 \text{ ft/s}$$

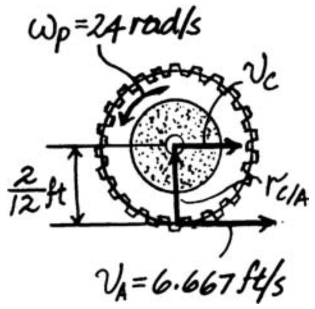
The rack is rotating about a fixed axis (shaft s). Hence,

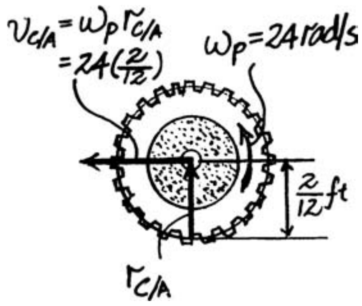
$$v_C = \omega_D\, r_D$$

$$2.667 = \omega_D\left(\frac{6}{12}\right) \qquad \omega_D = 5.33 \text{ rad/s} \qquad \textbf{Ans.}$$









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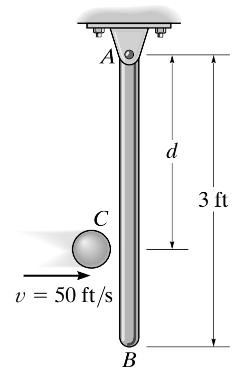
R2–3. The 6-lb slender rod AB is released from rest when it is in the *horizontal position* so that it begins to rotate clockwise. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s. The ball strikes the rod at C at the instant the rod is in the vertical position as shown. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.



Datum at A:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega^2 - 6(1.5)$$

$$\omega = 5.675 \text{ rad/s}$$

$$\circlearrowright + \qquad (H_A)_1 = (H_A)_2$$

$$\frac{1}{32.2}(50)(2) - \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right](5.675) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

$$e = 0.7 = \frac{v_C - v_{BL}}{50 - [-5.675(2)]}$$

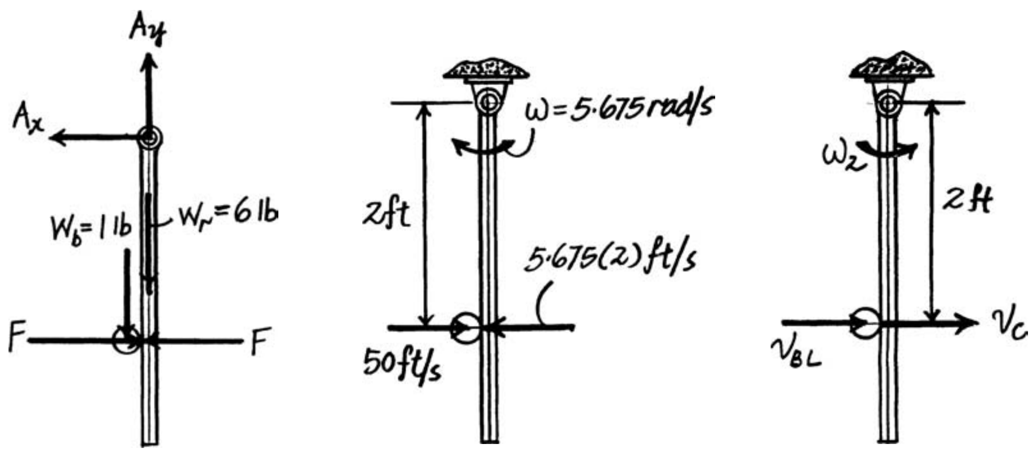
$$v_C = 2\omega_2$$

Solving,

$$\omega_2 = 3.81 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_{BL} = -35.3 \text{ ft/s}$$

$$v_C = 7.61 \text{ ft/s}$$



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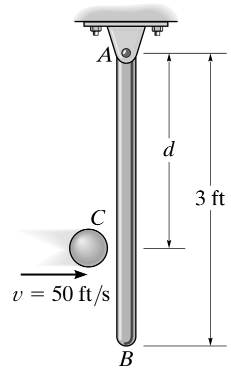
***R2–4.** The 6-lb slender rod AB is originally at rest, suspended in the vertical position. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s and strikes the rod at C. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.



$$\circlearrowleft + \quad (H_A)_1 = (H_A)_2$$

$$\left(\frac{1}{32.2}\right)(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

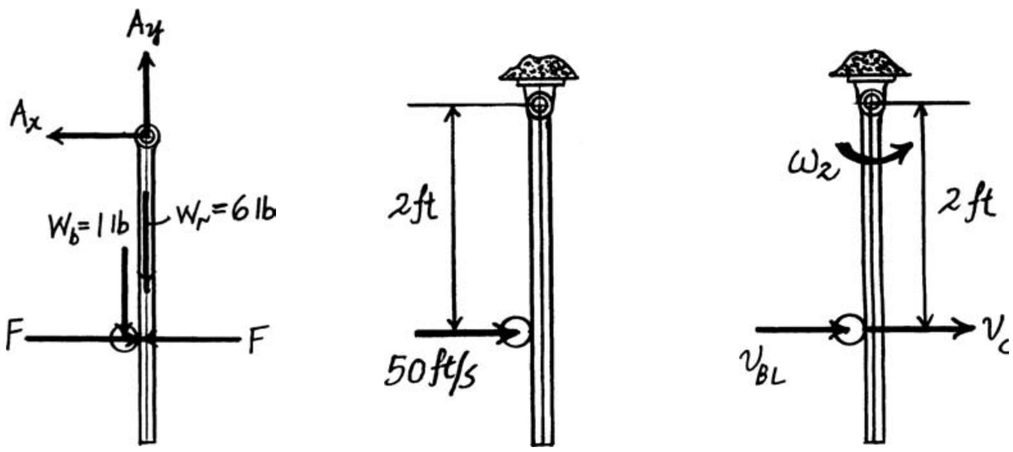
$$e = 0.7 = \frac{v_C - v_{BL}}{50 - 0}$$

$$v_C = 2\omega_2$$

Thus,

$$\omega_2 = 7.73 \text{ rad/s}$$ **Ans.**

$$v_{BL} = -19.5 \text{ ft/s}$$



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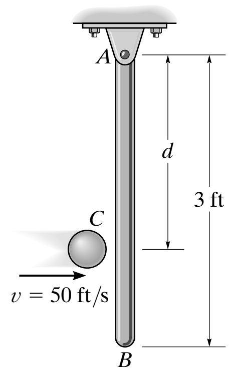
R2–5. The 6-lb slender rod is originally at rest, suspended in the vertical position. Determine the distance d where the 1-lb ball, traveling at $v = 50\text{ ft/s}$, should strike the rod so that it does not create a horizontal impulse at A. What is the rod's angular velocity just after the impact? Take $e = 0.5$.



Rod:

$$\zeta+ \quad (H_G)_1 + \Sigma \int M_G\,dt = (H_G)_2$$

$$0 + \int F\,dt\,(d - 1.5) = \left(\frac{1}{12}(m)(3)^2\right)\omega$$

$$m(v_G)_1 + \Sigma \int F\,dt = m(v_G)_2$$

$$0 + \int F\,dt = m(1.5\omega)$$

Thus,

$$m(1.5\omega)(d - 1.5) = \frac{1}{12}(m)(3)^2\,\omega$$

$$d = 2\text{ ft} \qquad \textbf{Ans.}$$

This is called the center of percussion. See Example 19–5.

$$\zeta+ \quad (H_A)_1 = (H_A)_2$$

$$\frac{1}{32.2}(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

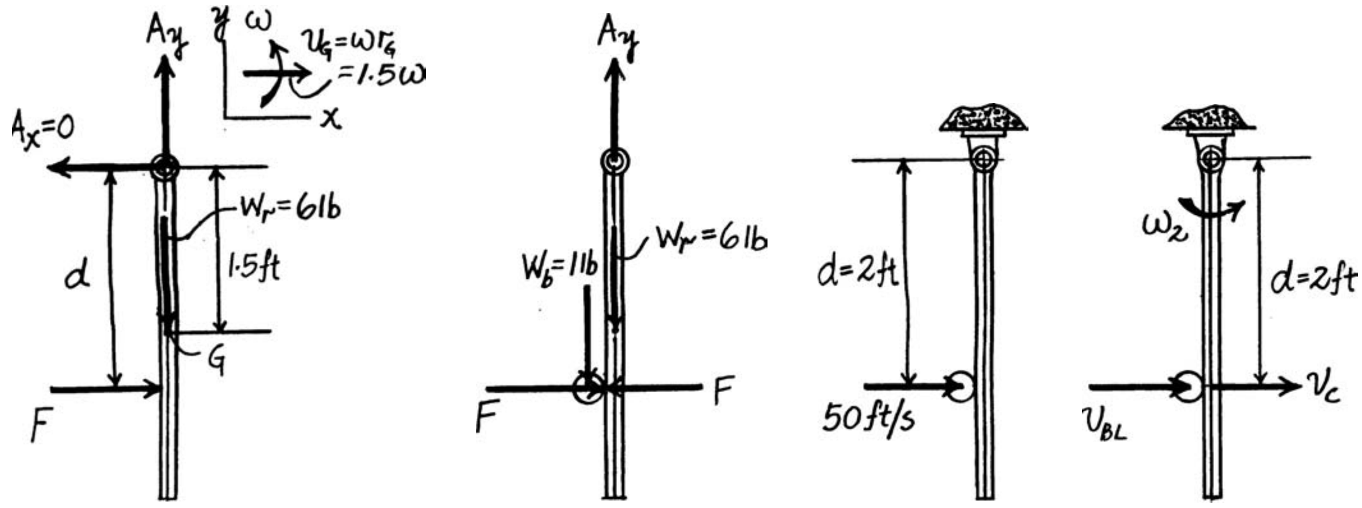
$$e = 0.5 = \frac{v_C - v_{BL}}{50 - 0}$$

$$v_C = 2\omega_2$$

Thus,

$$\omega_2 = 6.82\text{ rad/s} \qquad \textbf{Ans.}$$

$$v_{BL} = -11.4\text{ ft/s}$$



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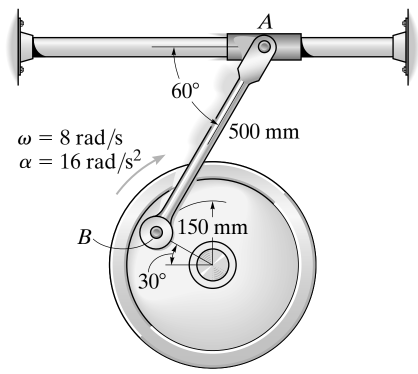
R2–6. At a given instant, the wheel rotates with the angular motions shown. Determine the acceleration of the collar at A at this instant.



Using instantaneous center method:

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{8(0.15)}{0.5\tan 30^\circ} = 4.157 \text{ rad/s}$$

$$\mathbf{a}_B = 16(0.15)\mathbf{i} - 8^2\,(0.15)\mathbf{j} = \{2.4\mathbf{i} - 9.6\mathbf{j}\}\text{ m/s}^2$$

$$\mathbf{a}_A = -a_A\cos 60^\circ\mathbf{i} + a_A\sin 60^\circ\mathbf{j} \qquad \alpha = \alpha\mathbf{k} \qquad \mathbf{r}_{B/A} = \{-0.5\mathbf{i}\}\text{ m}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$$

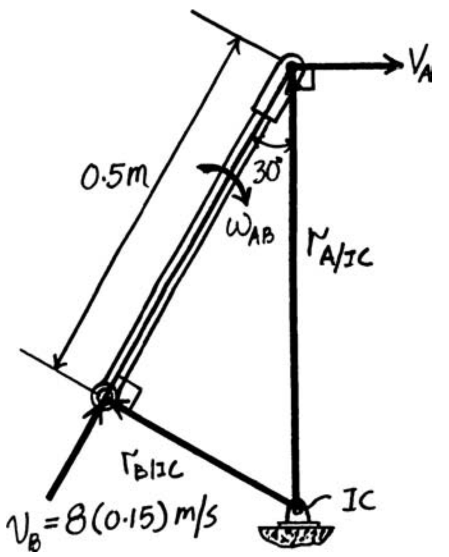
$$2.4\mathbf{i} - 9.6\mathbf{j} = (-a_A\cos 60^\circ\mathbf{i} + a_A\sin 60^\circ\mathbf{j}) + (\alpha\mathbf{k}) \times (-0.5\mathbf{i}) - (4.157)^2(-0.5\mathbf{i})$$

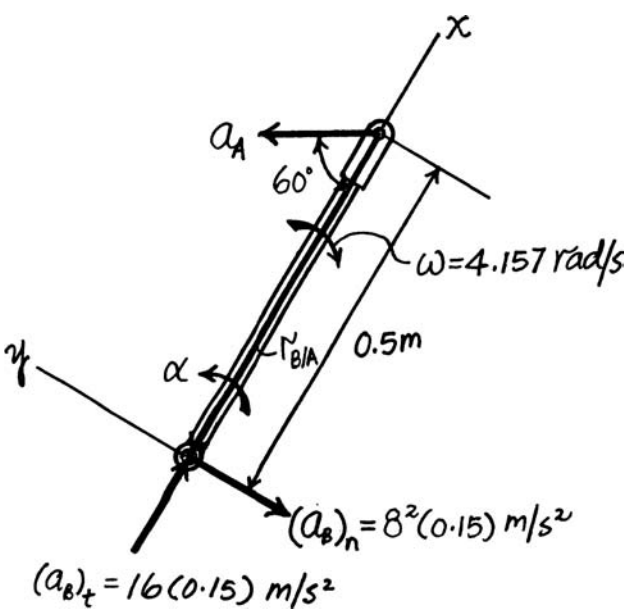
$$2.4\mathbf{i} - 9.6\mathbf{j} = (-a_A\cos 60^\circ + 8.64)\mathbf{i} + (-0.5\alpha + a_A\sin 60^\circ)\mathbf{j}$$

Equating the **i** and **j** components yields:

$$2.4 = -a_A\cos 60^\circ + 8.64 \qquad a_A = 12.5\text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$-9.6 = -0.5\alpha + (12.5)\sin 60^\circ \qquad \alpha = 40.8\text{ rad/s}^2 \circlearrowleft$$





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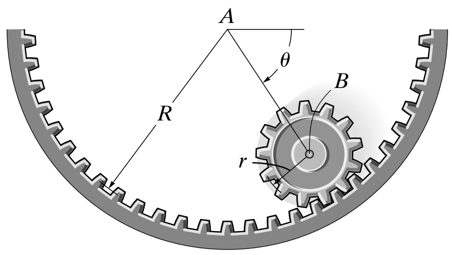
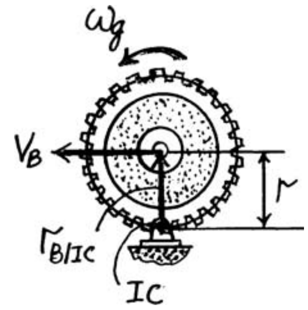
R2–7. The small gear which has a mass m can be treated as a uniform disk. If it is released from rest at $\theta = 0°$, and rolls along the fixed circular gear rack, determine the angular velocity of the radial line AB at the instant $\theta = 90°$.



Potential Energy: Datum is set at point A. When the gear is at its final position ($\theta = 90°$), its center of gravity is located $(R - r)$ *below* the datum. Its gravitational potential energy at this position is $-mg(R - r)$. Thus, the initial and final potential energies are

$$V_1 = 0 \qquad V_2 = -mg(R - r)$$



Kinetic Energy: When gear B is at its final position ($\theta = 90°$), the velocity of its mass center is $v_B = \omega_g r$ or $\omega_g = \dfrac{v_B}{r}$ since the gear rolls without slipping on the fixed circular gear track. The mass moment of inertia of the gear about its mass center is $I_B = \dfrac{1}{2}mr^2$. Since the gear is at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}mv_B^2 + \frac{1}{2}I_B\,\omega_g^2 = \frac{1}{2}mv_B^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_B}{r}\right)^2 = \frac{3}{4}mv_B^2$$

Conservation of Energy: Applying Eq. 18–18, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{3}{4}mv_B^2 + [-mg(R - r)]$$

$$v_B = \sqrt{\frac{4g(R - r)}{3}}$$

Thus, the angular velocity of the radical line AB is given by

$$\omega_{AB} = \frac{v_B}{R - r} = \sqrt{\frac{4g}{3(R - r)}} \qquad \textbf{Ans.}$$

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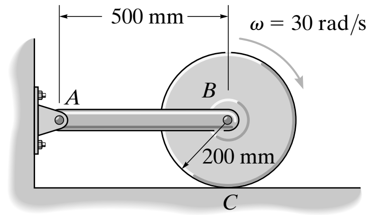
***R2–8.** The 50-kg cylinder has an angular velocity of 30 rad/s when it is brought into contact with the surface at C. If the coefficient of kinetic friction is $\mu_k = 0.2$, determine how long it will take for the cylinder to stop spinning. What force is developed in link AB during this time? The axis of the cylinder is connected to *two* symmetrical links. (Only AB is shown.) For the computation, neglect the weight of the links.



$(+\uparrow)\qquad m(v_{Ay})_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_{Ay})_2$

$0 + N_C(t) - 50(9.81)(t) = 0 \qquad N_C = 490.5\text{ N}$

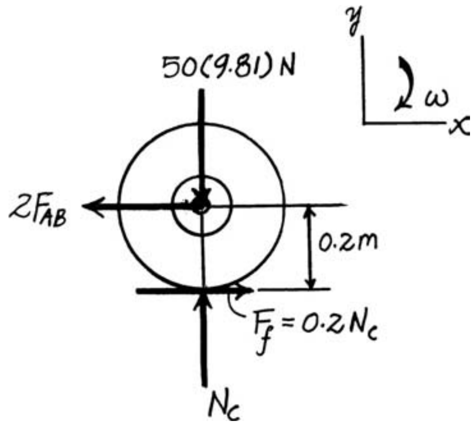
$(\xrightarrow{+})\qquad m(v_{Ax})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Ax})_2$

$0 + 0.2(490.5)(t) - 2F_{AB}(t) = 0 \qquad F_{AB} = 49.0\text{ N}$ **Ans.**

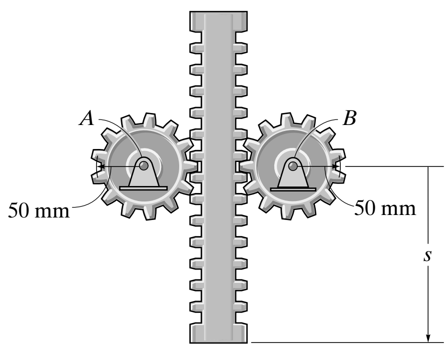
$(\circlearrowleft +)\qquad I_B\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_B\,dt = I_B\,\omega_2$

$-\left[\frac{1}{2}(50)(0.2)^2\right](30) + 0.2(490.5)(0.2)(t) = 0$

$t = 1.53\text{ s}$ **Ans.**



R2–9. The gear rack has a mass of 6 kg, and the gears each have a mass of 4 kg and a radius of gyration of $k = 30$ mm about their center. If the rack is originally moving downward at 2 m/s, when $s = 0$, determine the speed of the rack when $s = 600$ mm. The gears are free to rotate about their centers, A and B.



Originally, both gears rotate with an angular velocity of $\omega_t = \dfrac{2}{0.05} = 40$ rad/s. After the rack has traveled $s = 600$ mm, both gears rotate with an angular velocity of $\omega_2 = \dfrac{v_2}{0.05}$, where v_2 is the speed of the rack at that moment.

Put datum through points A and B.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(6)(2)^2 + \left\{\frac{1}{2}\left[4(0.03)^2\right](40)^2\right\} + 0 = \frac{1}{2}(6)v_2^2 + 2\left\{\frac{1}{2}\left[4(0.03)^2\right]\left(\frac{v_2}{0.05}\right)\right\} - 6(9.81)(0.6)$$

$v_2 = 3.46\text{ m/s}$ **Ans.**

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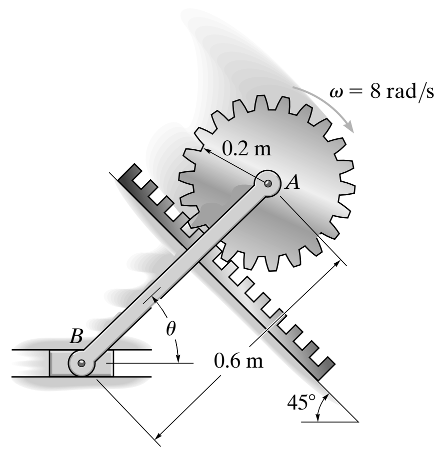
R2–10. The gear has a mass of 2 kg and a radius of gyration $k_A = 0.15$ m. The connecting link AB (slender rod) and slider block at B have a mass of 4 kg and 1 kg, respectively. If the gear has an angular velocity $\omega = 8$ rad/s at the instant $\theta = 45°$, determine the gear's angular velocity when $\theta = 0°$.



At position 1:

$$(\omega_{AB})_1 = \frac{(v_A)_1}{r_{A/IC}} = \frac{1.6}{0.6} = 2.6667 \text{ rad/s} \qquad (v_B)_1 = 0$$

$$(v_{AB})_1 = (\omega_{AB})_1\, r_{G/IC} = 2.6667(0.3) = 0.8 \text{ m/s}$$

At position 2:

$$(\omega_{AB})_2 = \frac{(v_A)_2}{r_{A/IC}} = \frac{\omega_2\,(0.2)}{\dfrac{0.6}{\cos 45°}} = 0.2357\omega_2$$

$$(v_B)_2 = (\omega_{AB})_2\, r_{B/IC} = 0.2357\,\omega_2(0.6) = 0.1414\omega_2$$

$$(v_{AB})_2 = (\omega_{AB})_2\, r_{G/IC} = 0.2357\,\omega_2(0.6708) = 0.1581\omega_2$$

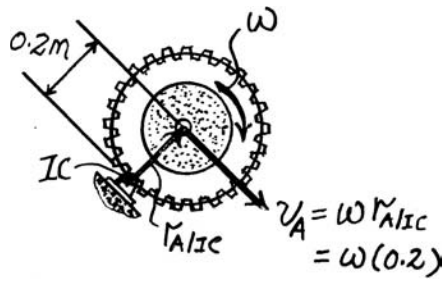
$$T_1 = \frac{1}{2}\left[(2)(0.15)^2\right](8)^2 + \frac{1}{2}(2)(1.6)^2 + \frac{1}{2}(4)(0.8)^2 + \frac{1}{2}\left[\frac{1}{12}(4)(0.6)^2\right](2.6667)^2$$

$$= 5.7067 \text{ J}$$

$$T_2 = \frac{1}{2}\left[(2)(0.15)^2\right](\omega_2)^2 + \frac{1}{2}(2)(0.2\,\omega_2)^2 + \frac{1}{2}(4)(0.1581\omega_2)^2$$

$$+ \frac{1}{2}\left[\frac{1}{12}(4)(0.6)^2\right](0.2357\omega_2)^2 + \frac{1}{2}(1)(0.1414\omega_2)^2$$

$$T_2 = 0.1258\,\omega_2^2$$



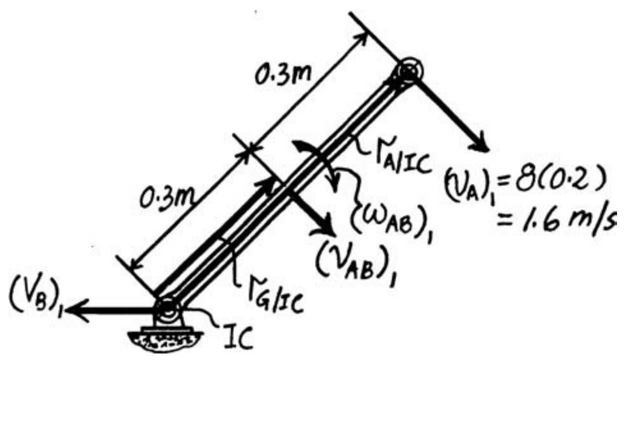
Put datum through bar in position 2.

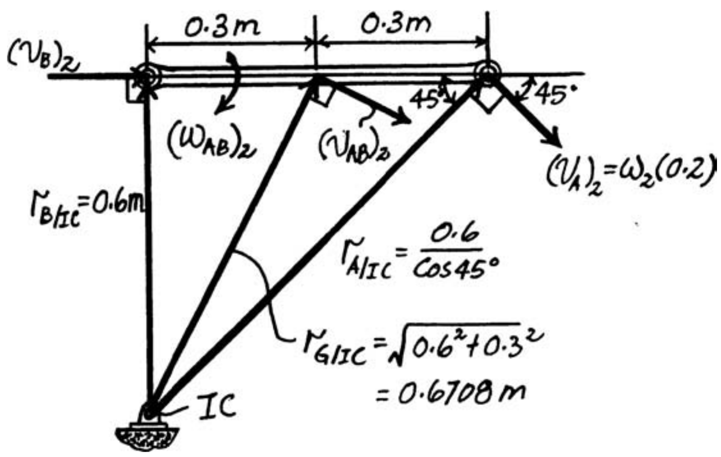
$$V_1 = 2(9.81)(0.6 \sin 45°) + 4(9.81)(0.3 \sin 45°) = 16.6481 \text{ J} \qquad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$5.7067 + 16.6481 = 0.1258\omega_2^2 + 0$$

$$\omega_2 = 13.3 \text{ rad/s}$$ **Ans.**





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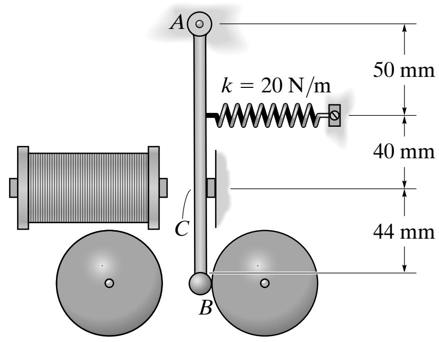
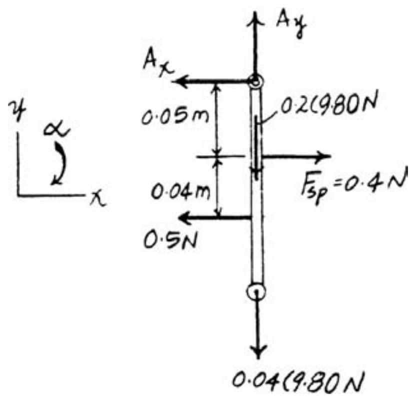
***R2–11.** The operation of a doorbell requires the use of an electromagnet, that attracts the iron clapper AB that is pinned at end A and consists of a 0.2-kg slender rod to which is attached a 0.04-kg steel ball having a radius of 6 mm. If the attractive force of the magnet at C is 0.5 N when the switch is on, determine the initial angular acceleration of the clapper. The spring is originally stretched 20 mm.



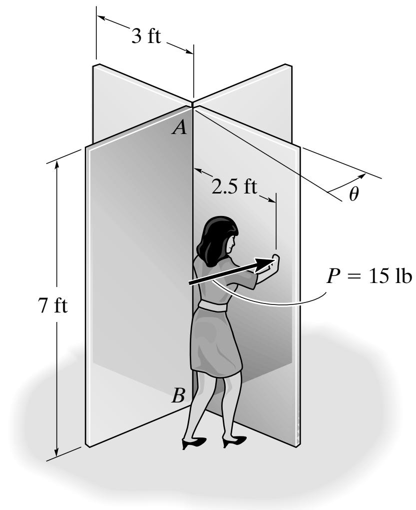
Equation of Motion: The spring force is given by $F_{sp} = kx = 20(0.02) = 0.4$ N. The mass moment of inertia for the clapper AB is $(I_{AB})_A = \frac{1}{12}(0.2)\left(0.134^2\right) + 0.2\left(0.067^2\right) + \frac{2}{5}(0.04)\left(0.006^2\right) + 0.04\left(0.14^2\right) = 1.9816\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–12, we have

$$+\Sigma M_A = I_A\,\alpha; \qquad 0.4(0.05) - 0.5(0.09) = -1.9816\left(10^{-3}\right)\alpha$$

$$\alpha = 12.6\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



***R2–12.** The revolving door consists of four doors which are attached to an axle AB. Each door can be assumed to be a 50-lb thin plate. Friction at the axle contributes a moment of 2 lb·ft which resists the rotation of the doors. If a woman passes through one door by always pushing with a force $P = 15$ lb perpendicular to the plane of the door as shown, determine the door's angular velocity after it has rotated 90°. The doors are originally at rest.



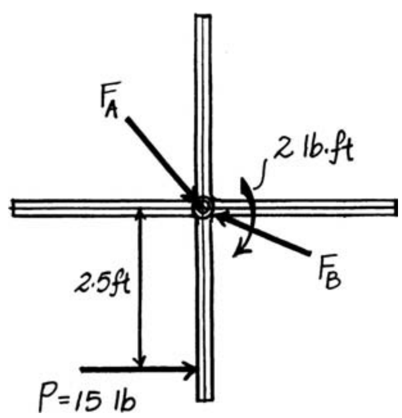
Moment of inertia of the door about axle AB:

$$I_{AB} = 2\left[\frac{1}{12}\left(\frac{100}{32.2}\right)(6)^2\right] = 18.6335\ \text{slug}\cdot\text{ft}^2$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left\{15(2.5)\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{2}\right)\right\} = \frac{1}{2}(18.6335)\,\omega^2$$

$$\omega = 2.45\ \text{rad/s} \qquad \textbf{Ans.}$$



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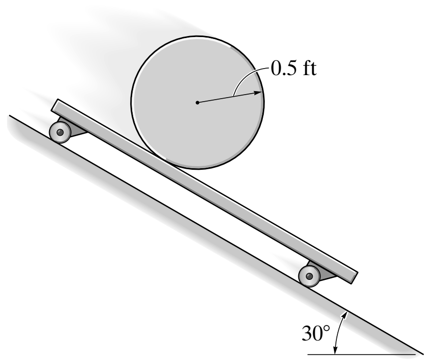
R2–13. The 10-lb cylinder rests on the 20-lb dolly. If the system is released from rest, determine the angular velocity of the cylinder in 2 s. The cylinder does not slip on the dolly. Neglect the mass of the wheels on the dolly.



For the cylinder,

$$(+\searrow) \quad m(v_{Cx'})_1 + \Sigma \int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Cx'})_2$$

$$0 + 10 \sin 30°(2) - F(2) = \left(\frac{10}{32.2}\right) v_C \qquad \textbf{(1)}$$

$$(\circlearrowleft +) \quad I_C\,\omega_1 + \Sigma \int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$$0 + F(0.5)(2) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega \qquad \textbf{(2)}$$

For the dolly,

$$(+\searrow) \quad m(v_{Dx'})_1 + \Sigma \int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Dx'})_2$$

$$0 + F(2) + 20 \sin 30°(2) = \left(\frac{20}{32.2}\right) v_D \qquad \textbf{(3)}$$

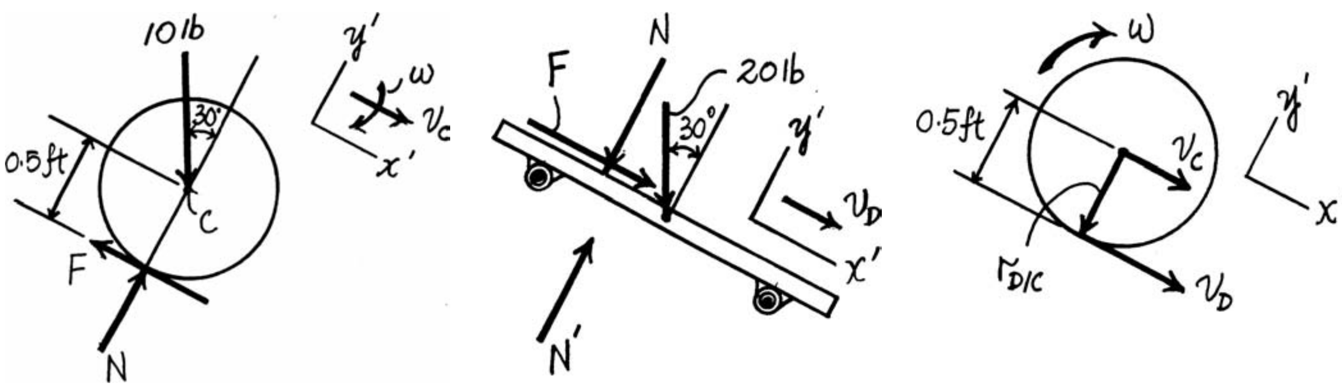
$$(+\searrow) \quad \mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

$$v_D = v_C - 0.5\omega \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$$\omega = 0 \qquad \textbf{Ans.}$$

$$v_C = 32.2 \text{ ft/s} \qquad v_D = 32.2 \text{ ft/s} \qquad F = 0$$



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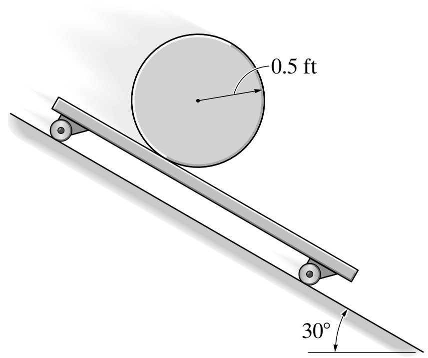
R2–14. Solve Prob. R2–13 if the coefficients of static and kinetic friction between the cylinder and the dolly are $\mu_s = 0.3$ and $\mu = 0.2$, respectively.



For the cylinder,

$$(+\searrow) \quad m(v_{Cx'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Cx'})_2$$

$$0 + 10\sin 30°(2) - F(2) = \left(\frac{10}{32.2}\right)v_C \qquad \textbf{(1)}$$

$$(\circlearrowleft +) \quad I_C\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$$0 + F(0.5)(2) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega \qquad \textbf{(2)}$$

For the dolly,

$$(+\searrow) \quad m(v_{Dx'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Dx'})_2$$

$$0 + F(2) + 20\sin 30°(2) = \left(\frac{20}{32.2}\right)v_D \qquad \textbf{(3)}$$

$$(+\searrow) \quad \mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

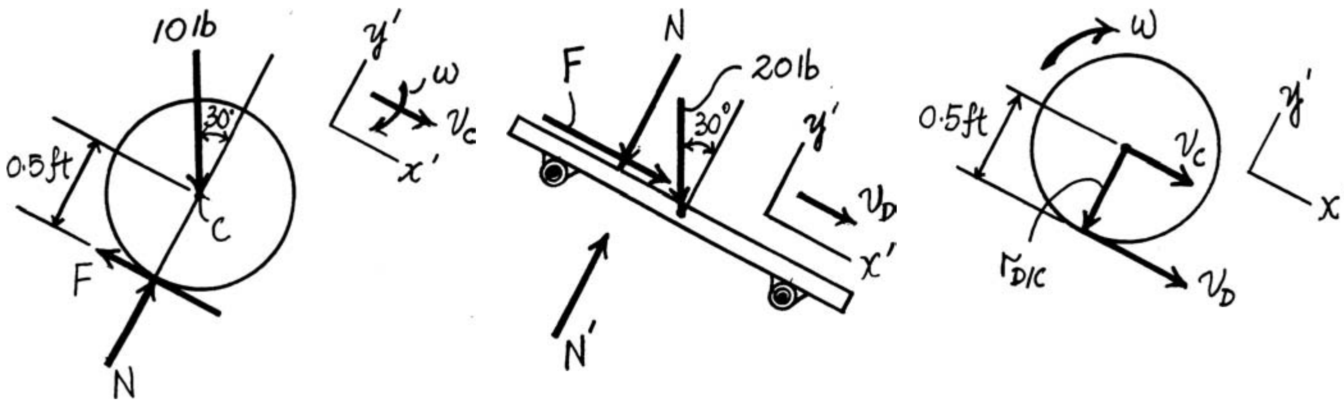
$$v_D = v_C - 0.5\omega \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$$\omega = 0 \qquad \textbf{Ans.}$$

$$v_C = 32.2 \text{ ft/s} \qquad v_D = 32.2 \text{ ft/s} \qquad F = 0$$

Note: No friction force develops.



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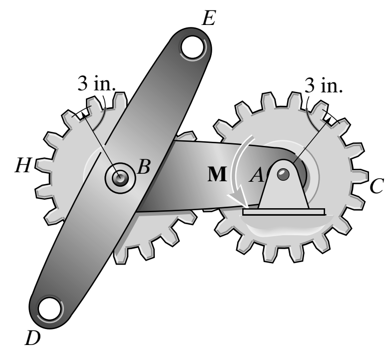
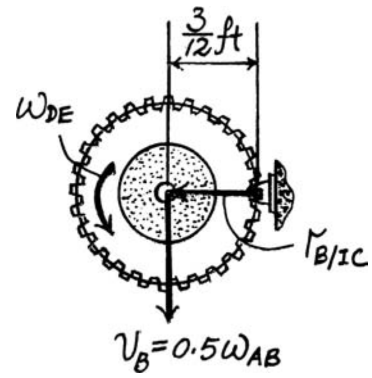
R2–15. Gears H and C each have a weight of 0.4 lb and a radius of gyration about their mass center of $(k_H)_B = (k_C)_A = 2$ in. Link AB has a weight of 0.2 lb and a radius of gyration of $(k_{AB})_A = 3$ in., whereas link DE has a weight of 0.15 lb and a radius of gyration of $(k_{DE})_B = 4.5$ in. If a couple moment of $M = 3\ \text{lb}\cdot\text{ft}$ is applied to link AB and the assembly is originally at rest, determine the angular velocity of link DE when link AB has rotated 360°. Gear C is prevented from rotating, and motion occurs in the horizontal plane. Also, gear H and link DE rotate together about the same axle at B.



For link AB,

$$v_B = \omega_{AB}\, r_{AB} = \omega_{AB}\left(\frac{6}{12}\right) = 0.5\omega_{AB}$$

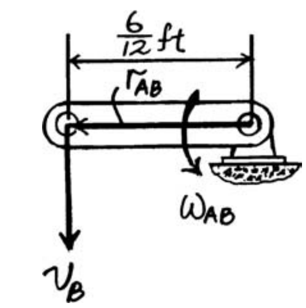


For gear H,

$$\omega_{DE} = \frac{v_B}{r_{B/IC}} = \frac{0.5\omega_{AB}}{3/12} = 2\omega_{AB}$$

$$\omega_{AB} = \frac{1}{2}\omega_{DE}$$

$$v_B = \left(\frac{1}{2}\omega_{DE}\right)\frac{6}{12} = 0.25\omega_{DE}$$



Principle of Work and Energy: For the system,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 3(2\pi) = \frac{1}{2}\left[\left(\frac{0.2}{32.2}\right)\left(\frac{3}{12}\right)^2\right]\left(\frac{1}{2}\,\omega_{DE}\right)^2 + \frac{1}{2}\left[\left(\frac{0.4}{32.2}\right)\left(\frac{2}{12}\right)^2\right]\omega_{DE}^2$$

$$+ \frac{1}{2}\left(\frac{0.4}{32.2}\right)(0.25\omega_{DE})^2 + \frac{1}{2}\left[\left(\frac{0.15}{32.2}\right)\left(\frac{4.5}{12}\right)^2\right]\omega_{DE}^2 + \frac{1}{2}\left(\frac{0.15}{32.2}\right)(0.25\omega_{DE})^2$$

$$\omega_{DE} = 132\ \text{rad/s} \qquad \textbf{Ans.}$$

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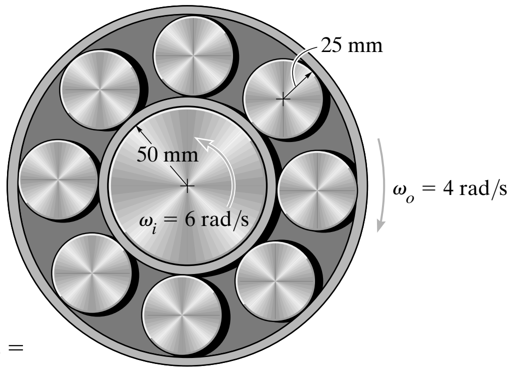
***R2–16.** The inner hub of the roller bearing rotates with an angular velocity of $\omega_i = 6$ rad/s, while the outer hub rotates in the opposite direction at $\omega_o = 4$ rad/s. Determine the angular velocity of each of the rollers if they roll on the hubs without slipping.

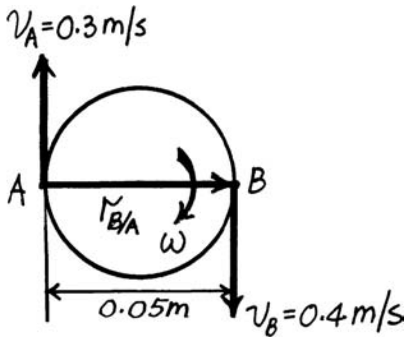


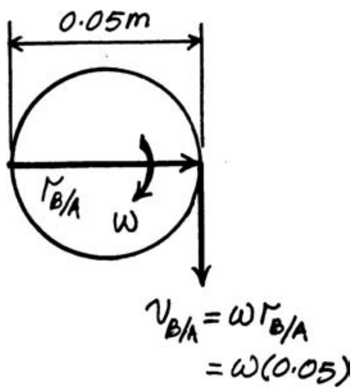
Since the hub does not slip, $v_A = \omega_i r_i = 6(0.05) = 0.3$ m/s and $v_B = \omega_O r_O = 4(0.1) = 0.4$ m/s.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\left[\underset{\downarrow}{0.4}\right] = \left[\underset{\uparrow}{0.3}\right] + \left[\underset{\downarrow}{\omega(0.05)}\right]$$

$(+\downarrow)$ $\quad 0.4 = -0.3 + 0.05\omega \qquad \omega = 14$ rad/s ↻ **Ans.**





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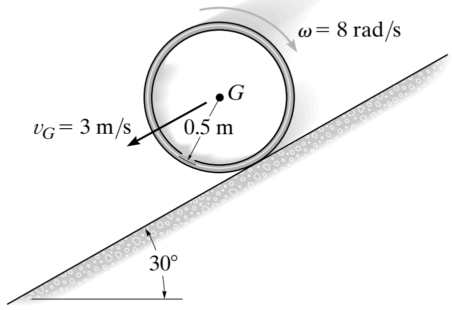
R2–17. The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin $\omega = 8$ rad/s and its center has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$, determine how long the hoop rolls before it stops slipping.

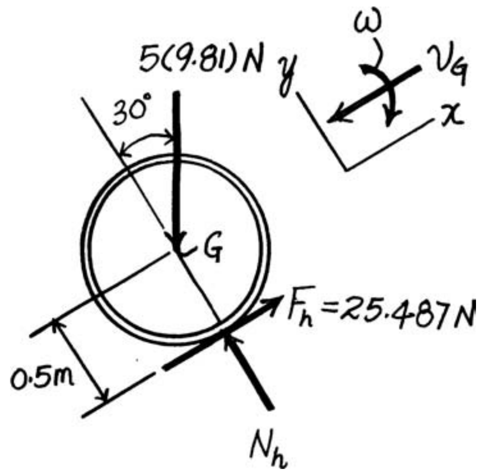


$(+\nwarrow)\qquad mv_{y1} + \Sigma\int F_y\,dt = mv_{y2}$

$$0 + N_h(1) - 5(9.81)t\cos 30^\circ = 0$$

$$N_h = 42.479 \text{ N}$$

$$F_h = 0.6N_h = 0.6(42.479 \text{ N}) = 25.487 \text{ N}$$



$(\swarrow +)\qquad mv_{x1} + \Sigma\int F_x\,dt = mv_{x2}$

$$5(3) + 5(9.181)\sin 30^\circ(t) - 25.487t = 5v_G$$

$(\circlearrowleft +)\qquad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

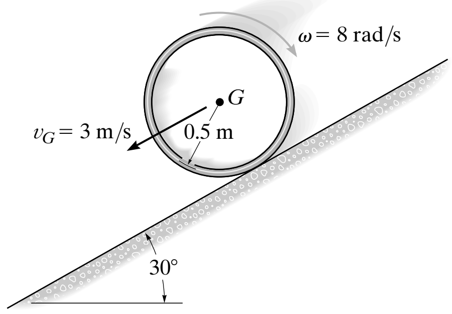
$$-5(0.5)^2(8) + 25.487(0.5)(t) = 5(0.5)^2\left(\frac{v_G}{0.5}\right)$$

Solving,

$$v_G = 2.75 \text{ m/s}$$

$$t = 1.32 \text{ s} \qquad \textbf{Ans.}$$

R2–18. The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin $\omega = 8$ rad/s and its center has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$, determine the hoop's angular velocity 1 s after it is released.



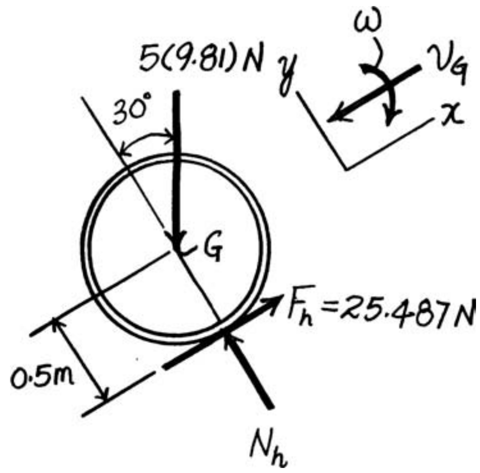
See solution to Prob. R2–17. Since backspin will not stop in $t = 1\text{ s} < 1.32\text{ s}$, then

$(+\nwarrow)\qquad mv_{y1} + \Sigma\int F_y\,dt = mv_{y2}$

$$0 + N_h(t) - 5(9.81)t\cos 30^\circ = 0$$

$$N_h = 42.479 \text{ N}$$

$$F_h = 0.6N_h = 0.6(42.479 \text{ N}) = 25.487 \text{ N}$$



$\circlearrowleft +\qquad (H_G)_1 + \Sigma\int M\,dt = (H_G)_2$

$$-5(0.5)^2(8) + 25.487(0.5)(1) = -5(0.5)^2\,\omega$$

$$\omega = 2.19 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

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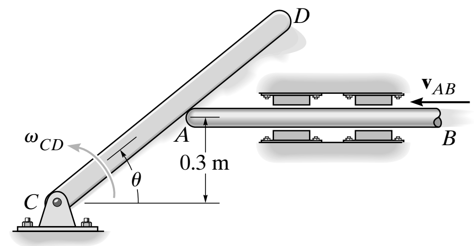
R2–19. Determine the angular velocity of rod CD at the instant $\theta = 30°$. Rod AB moves to the left at a constant speed of $v_{AB} = 5$ m/s.

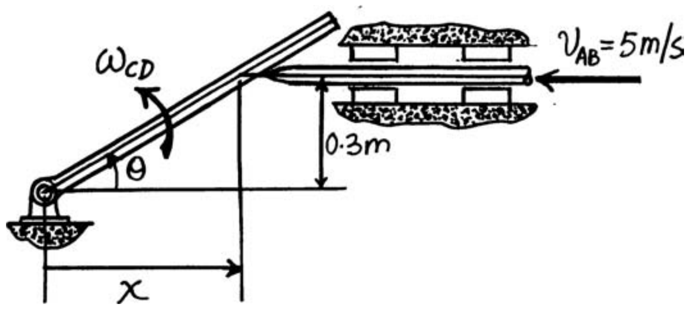


$$x = \frac{0.3}{\tan\theta} = 0.3\cot\theta$$

$$\dot{x} = v_{AB} = -0.3\csc^2\theta\dot{\theta}$$

Here $\dot{\theta} = \omega_{CD}, v_{AB} = -5$ m/s and $\theta = 30°$.

$-5 = -0.03\csc^2 30°(\omega_{CD})$ $\qquad$ $\omega_{CD} = 4.17$ rad/s $\qquad$ **Ans.**



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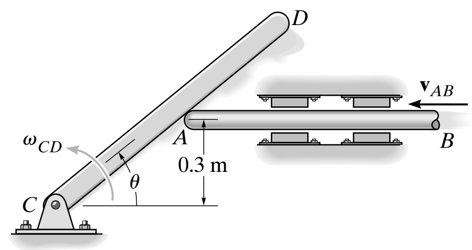
***R2–20.** Determine the angular acceleration of rod CD at the instant $\theta = 30°$. Rod AB has zero velocity, i.e., $v_{AB} = 0$, and an acceleration of $a_{AB} = 2\ \text{m/s}^2$ to the right when $\theta = 30°$.



$$x = \frac{0.3}{\tan\theta} = 0.3\cot\theta$$

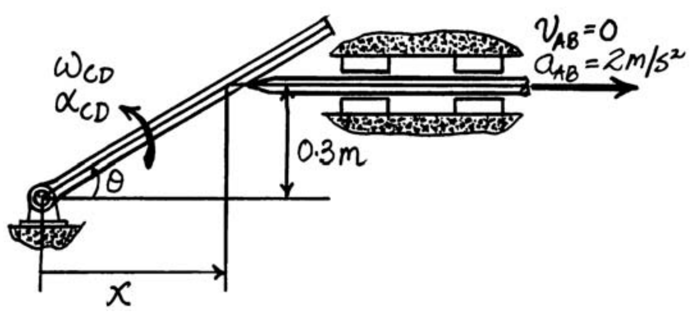
$$\dot{x} = v_{AB} = -0.3\csc^2\theta\dot{\theta}$$

$$\ddot{x} = a_{AB} = -0.3\left[\csc^2\theta\ddot{\theta} - 2\csc^2\theta\cot\theta\dot{\theta}^2\right] = 0.3\csc^2\theta\left(2\cot\theta\dot{\theta}^2 - \ddot{\theta}\right)$$

Here $\dot{\theta} = \omega_{CD}$, $v_{AB} = 0$, $a_{AB} = 2\ \text{m/s}^2$, $\ddot{\theta} = \alpha_{CD}$, and $\theta = 30°$.

$$0 = -0.3\csc^2 30°(\omega_{CD}) \qquad \omega_{CD} = 0$$

$$2 = 0.3\csc^2 30°\left[2\cot 30°(0)^2 - \alpha_{CD}\right] \qquad \alpha_{CD} = -1.67\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



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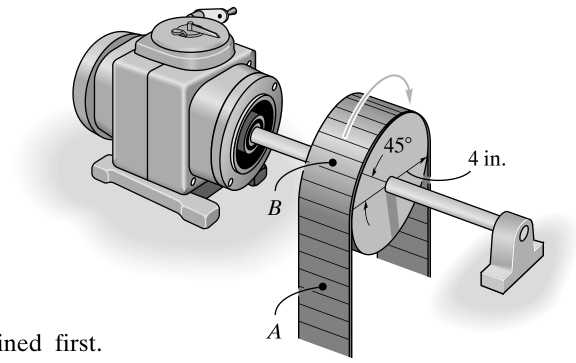
R2–21. If the angular velocity of the drum is increased uniformly from 6 rad/s when $t = 0$ to 12 rad/s when $t = 5$ s, determine the magnitudes of the velocity and acceleration of points A and B on the belt when $t = 1$ s. At this instant the points are located as shown.



Angular Motion: The angular acceleration of drum must be determined first. Applying Eq. 16–5, we have

$$\omega = \omega_0 + \alpha_c t$$

$$12 = 6 + \alpha_c (5)$$

$$\alpha_c = 1.20 \text{ rad/s}^2$$

The angular velocity of the drum at $t = 1$ s is given by

$$\omega = \omega_0 + \alpha_c t = 6 + 1.20(1) = 7.20 \text{ rad/s}$$

Motion of *P*: The magnitude of the velocity of points A and B can be determined using Eq. 16–8.

$$v_A = v_B = \omega r = 7.20\left(\frac{4}{12}\right) = 2.40 \text{ ft/s} \qquad \textbf{Ans.}$$

Also,

$$a_A = (a_t)_A = \alpha_c r = 1.20\left(\frac{4}{12}\right) = 0.400 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of points B can be determined using Eqs. 16–11 and 16–12, respectively.

$$(a_t)_B = \alpha_c r = 1.20\left(\frac{4}{12}\right) = 0.400 \text{ ft/s}^2$$

$$(a_n)_B = \omega^2 r = \left(7.20^2\right)\left(\frac{4}{12}\right) = 17.28 \text{ ft/s}^2$$

The magnitude of the acceleration of points B is

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0.400^2 + 17.28^2} = 17.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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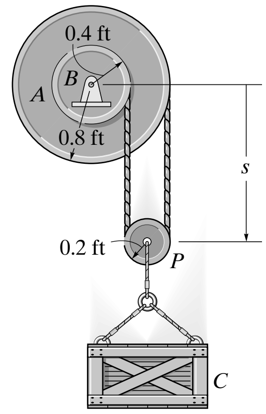
R2–22. Pulley A and the attached drum B have a weight of 20 lb and a radius of gyration of $k_B = 0.6$ ft. If pulley P "rolls" downward on the cord without slipping, determine the speed of the 20-lb crate C at the instant $s = 10$ ft. Initially, the crate is released from rest when $s = 5$ ft. For the calculation, neglect the mass of pulley P and the cord.

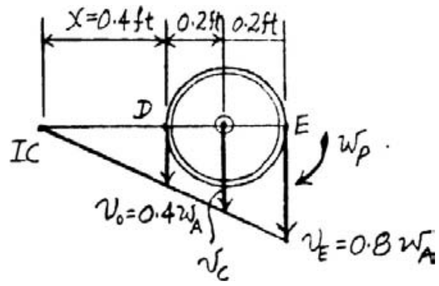


Kinematics: Since pulley A is rotating about a fixed point B and pulley P rolls down without slipping, the velocity of points D and E on the pulley P are given by $v_D = 0.4\omega_A$ and $v_E = 0.8\omega_A$ where ω_A is the angular velocity of pulley A. Thus, the instantaneous center of zero velocity can be located using similar triangles.

$$\frac{x}{0.4\omega_A} = \frac{x + 0.4}{0.8\omega_A} \qquad x = 0.4 \text{ ft}$$

Thus, the velocity of block C is given by

$$\frac{v_C}{0.6} = \frac{0.4\omega_A}{0.4} \qquad v_C = 0.6\omega_A$$



Potential Energy: Datumn is set at point B. When block C is at its initial and final position, its locations are 5 ft and 10 ft *below* the datum. Its initial and final gravitational potential energies are $20(-5) = -100$ ft · lb and $20(-10) = -200$ ft · lb, respectively. Thus, the initial and final potential energy are

$$V_1 = -100 \text{ ft} \cdot \text{lb} \qquad V_2 = -200 \text{ ft} \cdot \text{lb}$$

Kinetic Energy: The mass moment of inertia of pulley A about point B is $I_B = mk_B^2 = \frac{20}{32.2}\left(0.6^2\right) = 0.2236 \text{ slug} \cdot \text{ft}^2$. Since the system is initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2} m_C v_C^2 + \frac{1}{2} I_B \omega_A^2$$

$$= \frac{1}{2}\left(\frac{20}{32.2}\right)(0.6\omega_A)^2 + \frac{1}{2}(0.2236)\,\omega_A^2$$

$$= 0.2236\omega_A^2$$

Conservation of Energy: Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + -100 = 0.2236\omega_A^2 + (-200)$$

$$\omega_A = 21.15 \text{ rad/s}$$

Thus, the speed of block C at the instant $s = 10$ ft is

$$v_C = 0.6\omega_A = 0.6(21.15) = 12.7 \text{ ft/s} \qquad \textbf{Ans.}$$

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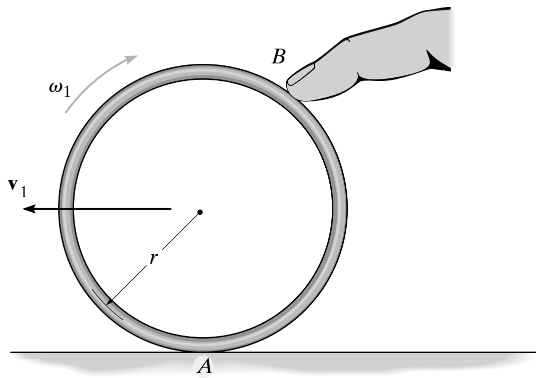
R2–23. By pressing down with the finger at B, a thin ring having a mass m is given an initial velocity v_1 and a backspin ω_1 when the finger is released. If the coefficient of kinetic friction between the table and the ring is μ, determine the distance the ring travels forward before the backspin stops.

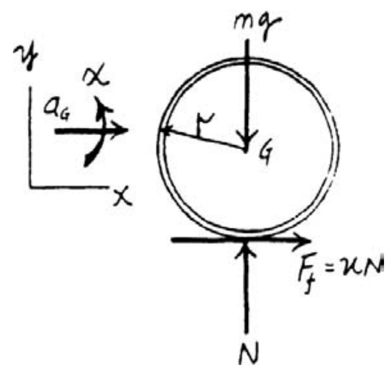


Equations of Motion: The mass moment of inertia of the ring about its mass center is given by $I_G = mr^2$. Applying Eq. 17–16, we have

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - mg = 0 \qquad N = mg$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad \mu mg = ma_G \qquad a_G = \mu g$$

$$\circlearrowleft + \Sigma M_G = I_G\,\alpha; \qquad \mu mgr = mr^2\,\alpha \qquad \alpha = \frac{\mu g}{r}$$



Kinematics: The time required for the ring to stop back spinning can be determined by applying Eq. 16–5.

$$\omega = \omega_0 + \alpha_c\,t$$

$$(\circlearrowright +) \qquad 0 = \omega_1 + \left(-\frac{\mu g}{r}\right)t$$

$$t = \frac{\omega_1\,r}{\mu g}$$

The distance traveled by the ring just before back spinning stops can be determine by applying Eq. 12–5.

$$\left(\overset{+}{\leftarrow}\right) \qquad s = s_0 + v_0\,t + \frac{1}{2}a_c\,t^2$$

$$= 0 + v_1\left(\frac{\omega_1\,r}{\mu g}\right) + \frac{1}{2}(-\mu g)\left(\frac{\omega_1 r}{\mu g}\right)^2$$

$$= \frac{\omega_1\,r}{2\mu g}(2v_1 - \omega_1 r) \qquad \textbf{Ans.}$$

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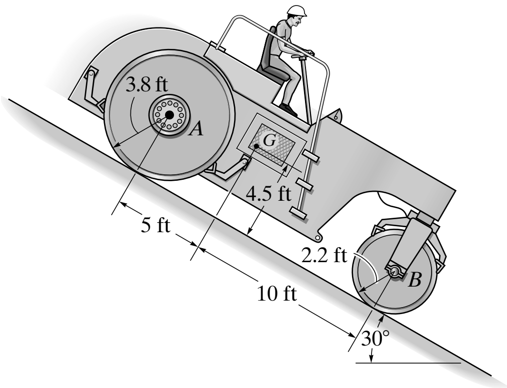
***R2–24.** The pavement roller is traveling down the incline at $v_1 = 5$ ft/s when the motor is disengaged. Determine the speed of the roller when it has traveled 20 ft down the plane. The body of the roller, excluding the rollers, has a weight of 8000 lb and a center of gravity at G. Each of the two rear rollers weighs 400 lb and has a radius of gyration of $k_A = 3.3$ ft. The front roller has a weight of 800 lb and a radius of gyration of $k_B = 1.8$ ft. The rollers do not slip as they turn.



The wheels roll without slipping, hence $\omega = \dfrac{v_G}{r}$.

$$T_1 = \frac{1}{2}\left(\frac{8000 + 800 + 800}{32.2}\right)(5)^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(3.3)^2\right]\left(\frac{5}{3.8}\right)^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(1.8)^2\right]\left(\frac{5}{2.2}\right)^2$$

$$= 4168.81 \text{ ft} \cdot \text{lb}$$

$$T_2 = \frac{1}{2}\left(\frac{8000 + 800 + 800}{32.2}\right)v^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(3.3)^2\right]\left(\frac{v}{3.8}\right)^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(1.8)^2\right]\left(\frac{v}{2.2}\right)^2$$

$$= 166.753\, v^2$$

Put datum through the mass center of the wheels and body of the roller when it is in the initial position.

$$V_1 = 0$$

$$V_2 = -800(20 \sin 30°) - 8000(20 \sin 30°) - 800(20 \sin 30°)$$

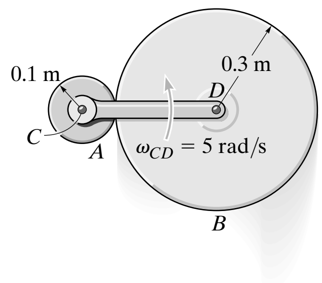
$$= -96000 \text{ ft} \cdot \text{lb}$$

$$T_1 + V_1 = T_2 + V_2$$

$$4168.81 + 0 = 166.753v^2 - 96000$$

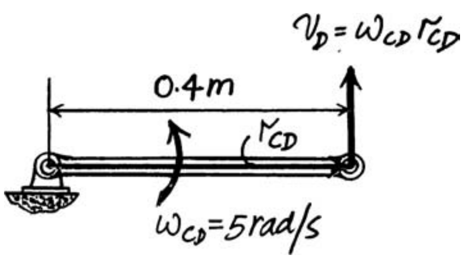
$$v = 24.5 \text{ ft/s}$$ **Ans.**

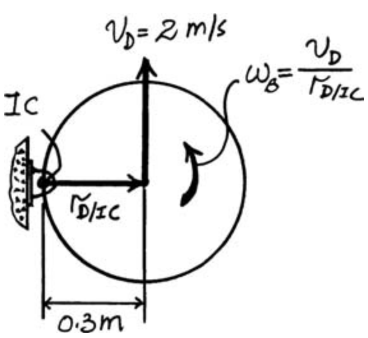
R2–25. The cylinder B rolls on the fixed cylinder A without slipping. If bar CD rotates with an angular velocity $\omega_{CD} = 5$ rad/s, determine the angular velocity of cylinder B. Point C is a fixed point.



$$v_D = 5(0.4) = 2 \text{ m/s}$$

$$\omega_B = \frac{2}{0.3} = 6.67 \text{ rad/s}$$ **Ans.**





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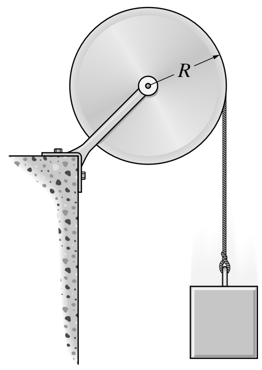
R2–26. The disk has a mass M and a radius R. If a block of mass m is attached to the cord, determine the angular acceleration of the disk when the block is released from rest. Also, what is the distance the block falls from rest in the time t?



$$I_0 = \frac{1}{2} MR^2$$

$\circlearrowleft + \Sigma M_O = \Sigma t M_k)_0;$ $\quad mgR = \frac{1}{2} MR^2 (\alpha) + m(\alpha R)R$

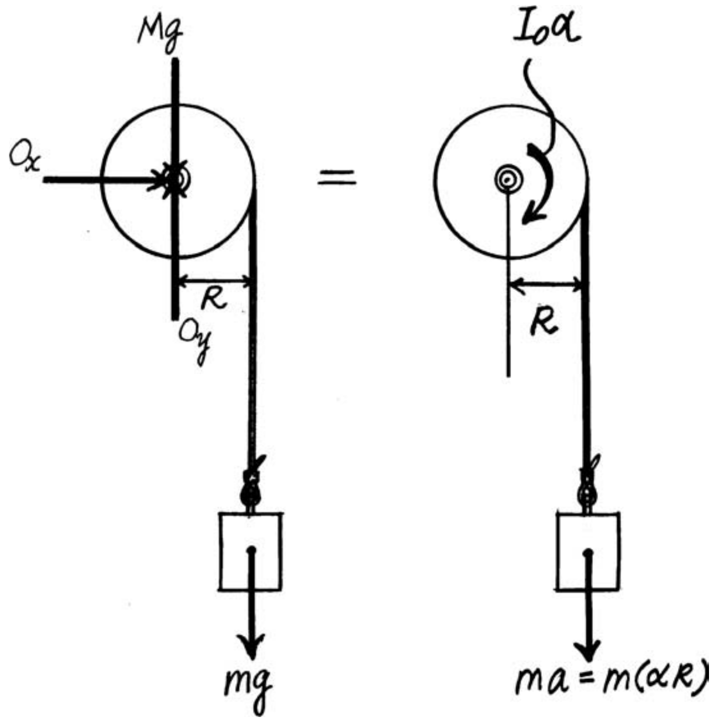
$$\alpha = \frac{2mg}{R(M + 2m)}$$ **Ans.**

The displacement $h = R\theta$, hence $\theta = \dfrac{h}{R}$

$$\theta - \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$$

$$\frac{h}{R} = 0 + 0 + \frac{1}{2}\left(\frac{2mg}{R(M + 2m)}\right) t^2$$

$$h = \frac{mg}{M + 2m} t^2$$ **Ans.**



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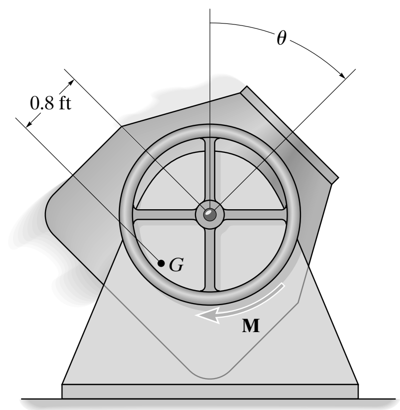
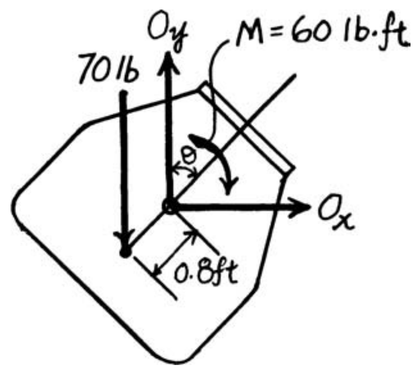
R2–27. The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity G. If a constant torque $M = 60$ lb · ft is applied to the dumping wheel, determine the angular velocity of the tub when it has rotated $\theta = 90°$. Originally the tub is at rest when $\theta = 0°$. Neglect the mass of the wheel.



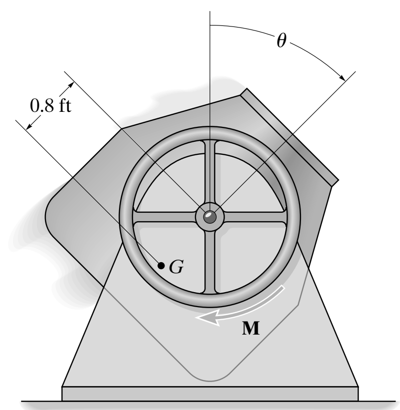
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60\left(\frac{\pi}{2}\right) - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right](\omega)^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$

$$\omega = 3.89 \text{ rad/s} \qquad \textbf{Ans.}$$



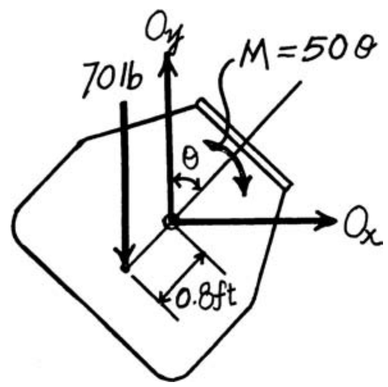
***R2–28.** Solve Prob. R2–27 if the applied torque is $M = (50\theta)$ lb · ft, where θ is in radians.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{\pi/2} 50\theta\, d\theta - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right]\omega^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$

$$\omega = 1.50 \text{ rad/s} \qquad \textbf{Ans.}$$



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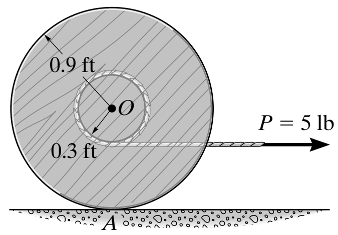
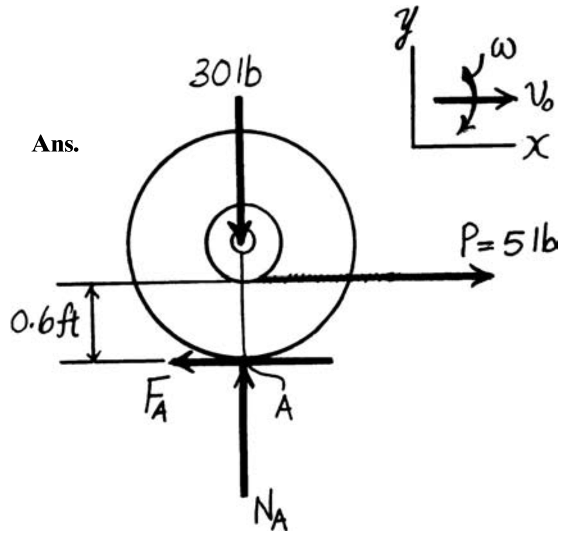
R2–29. The spool has a weight of 30 lb and a radius of gyration $k_O = 0.45$ ft. A cord is wrapped around the spool's inner hub and its end subjected to a horizontal force $P = 5$ lb. Determine the spool's angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.



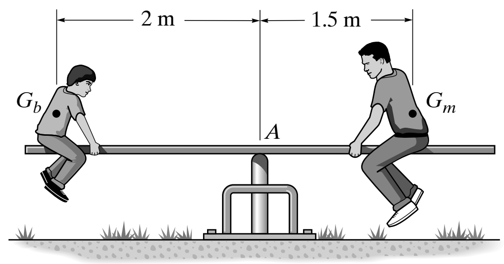
$$(+\circlearrowright) \qquad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + 5(0.6)(4) = \left[\left(\frac{30}{32.2}\right)(0.45)^2 + \left(\frac{30}{32.2}\right)(0.9)^2\right]\omega_2$$

$$\omega_2 = 12.7 \text{ rad/s}$$ **Ans.**



R2–30. The 75-kg man and 40-kg boy sit on the horizontal seesaw, which has negligible mass. At the instant the man lifts his feet from the ground, determine their accelerations if each sits upright, i.e., they do not rotate. The centers of mass of the man and boy are at G_m and G_b, respectively.



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 40(9.81)(2) - 75(9.81)(1.5)$$

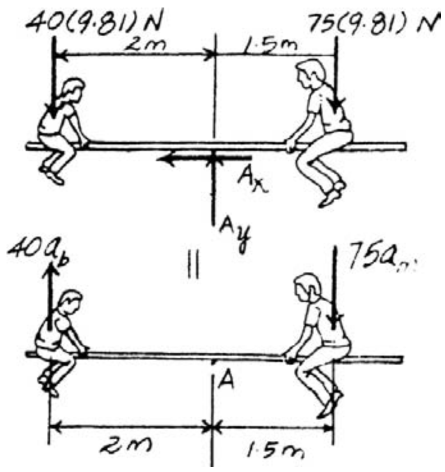
$$= -40a_b\,(2) - 75a_m\,(1.5) \qquad \textbf{[1]}$$

Since the seesaw is rotating about point A, then

$$\alpha = \frac{a_b}{2} = \frac{a_m}{1.5} \qquad \text{or} \qquad a_m = 0.75a_b \qquad \textbf{[2]}$$

Solving Eqs. (1) and (2) yields:

$$a_m = 1.45 \text{ m/s}^2 \qquad a_b = 1.94 \text{ m/s}^2$$ **Ans.**



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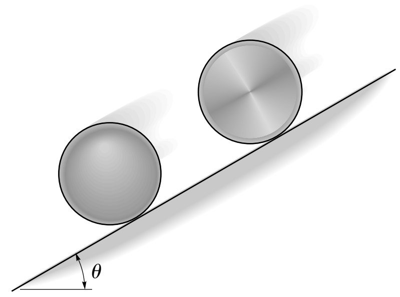
R2–31. A sphere and cylinder are released from rest on the ramp at $t = 0$. If each has a mass m and a radius r, determine their angular velocities at time t. Assume no slipping occurs.



Principle of Impulse and Momentum: For the sphere,

$$(+\circlearrowleft) \qquad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + mg\sin\theta(r)(t) = \left[\frac{2}{5}mr^2 + mr^2\right](\omega_S)_2$$

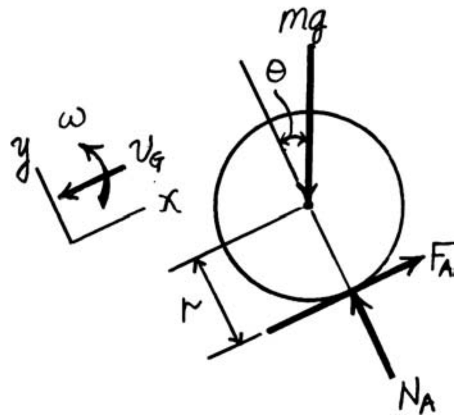
$$(\omega_S)_2 = \frac{5g\sin\theta}{7r}t \qquad \textbf{Ans.}$$

Principle of Impulse and Momentum: For the cyclinder,

$$(+\circlearrowleft) \qquad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + mg\sin\theta(r)(t) = \left[\frac{1}{2}mr^2 + mr^2\right](\omega_C)_2$$

$$(\omega_C)_2 = \frac{2g\sin\theta}{3r}t \qquad \textbf{Ans.}$$



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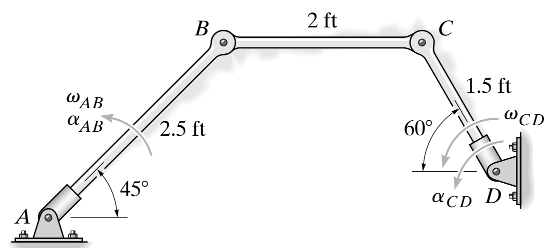
***R2–32.** At a given instant, link AB has an angular acceleration $\alpha_{AB} = 12\text{ rad/s}^2$ and an angular velocity $\omega_{AB} = 4\text{ rad/s}$. Determine the angular velocity and angular acceleration of link CD at this instant.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$\begin{bmatrix} v_C \\ \scriptstyle 30^\circ \end{bmatrix} = \begin{bmatrix} 10 \\ \scriptstyle 45^\circ \end{bmatrix} + \begin{bmatrix} 2\omega_{BC} \\ \downarrow \end{bmatrix}$$

$(\overset{+}{\leftarrow})$ $\quad v_C \cos 30^\circ = 10 \cos 45^\circ + 0$

$(+\downarrow)$ $\quad v_C \sin 30^\circ = -10 \sin 45^\circ + 2\omega_{BC}$

$$\omega_{BC} = 5.58\text{ rad/s}$$

$$v_C = 8.16\text{ ft/s}$$

$$\omega_{CD} = \frac{8.16}{1.5} = 5.44\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

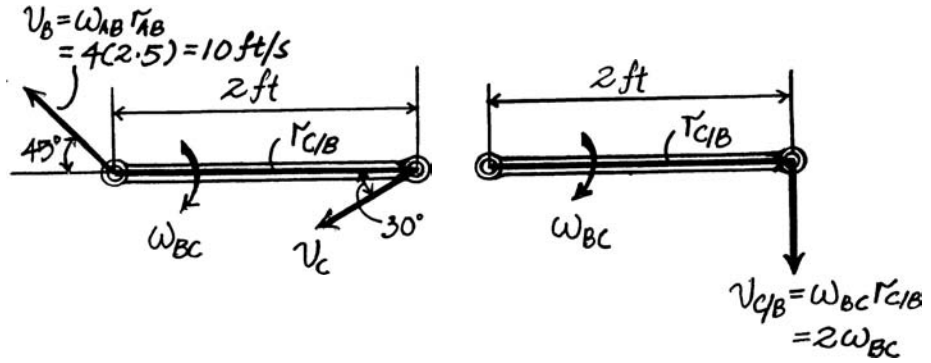
$$\begin{bmatrix} 44.44 \\ \scriptstyle 60^\circ \end{bmatrix} + \begin{bmatrix} (a_C)_t \\ \scriptstyle 30^\circ \end{bmatrix} = \begin{bmatrix} 30 \\ \scriptstyle 45^\circ \end{bmatrix} + \begin{bmatrix} 40 \\ \scriptstyle 45^\circ \end{bmatrix} + \begin{bmatrix} 2(5.58)^2 \\ \leftarrow \end{bmatrix} + \begin{bmatrix} 2\alpha_{BC} \\ \downarrow \end{bmatrix}$$

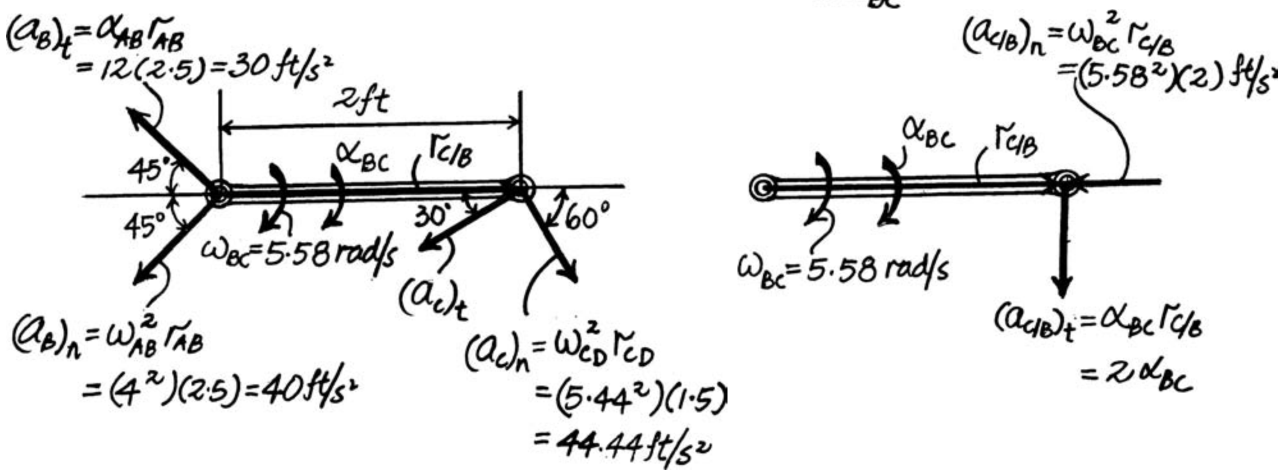
$(\overset{+}{\leftarrow})$ $\quad -44.44 \cos 60^\circ + (a_C)_t \cos 30^\circ = 30 \cos 45^\circ + 40 \cos 45^\circ + 62.21$

$(+\downarrow)$ $\quad 44.44 \sin 60^\circ + (a_C)_t \sin 30^\circ = -30 \sin 45^\circ + 40 \sin 45^\circ + 2\alpha_{BC}$

$$(a_C)_t = 155\text{ ft/s}^2 \qquad \alpha_{BC} = 54.4\text{ rad/s}^2$$

$$\alpha_{CD} = \frac{155}{1.5} = 103\text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$





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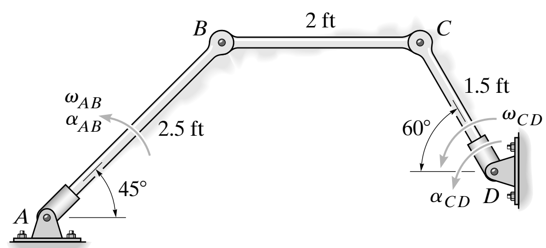
R2–33. At a given instant, link CD has an angular acceleration $\alpha_{CD} = 5\ \text{rad/s}^2$ and an angular velocity $\omega_{CD} = 2\ \text{rad/s}$. Determine the angular velocity and angular acceleration of link AB at this instant.



$$\frac{r_{IC-C}}{\sin 45^\circ} = \frac{2}{\sin 75^\circ} \qquad r_{IC-C} = 1.464\ \text{ft}$$

$$\frac{r_{IC-B}}{\sin 60^\circ} = \frac{2}{\sin 75^\circ} \qquad r_{IC-B} = 1.793\ \text{ft}$$

$$\omega_{BC} = \frac{3}{1.464} = 2.0490\ \text{rad/s}$$

$$v_B = 2.0490(1.793) = 3.6742\ \text{ft/s}$$

$$\omega_{AB} = \frac{3.6742}{2.5} = 1.47\ \text{rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$$(a_B)_n = \frac{v_B^2}{r_{BA}} = \frac{(3.6742)^2}{2.5} = 5.4000\ \text{ft/s}^2$$

$$(a_C)_n = \frac{v_C^2}{r_{CD}} = \frac{(3)^2}{1.5} = 6\ \text{ft/s}^2$$

$$(a_C)_t = \alpha_{CD}(r_{CD}) = 5(1.5) = 7.5\ \text{ft/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2\,\mathbf{r}_{B/C}$$

$$-5.400\cos 45^\circ\mathbf{i} - 5.400\sin 45^\circ\mathbf{j} - (a_B)_t\cos 45^\circ\mathbf{i} + (a_B)_t\sin 45^\circ\mathbf{j} = 6\sin 30^\circ\mathbf{i} - 6\cos 30^\circ\mathbf{j}$$

$$-7.5\cos 30^\circ\mathbf{i} - 7.5\sin 30^\circ\mathbf{j} + (\alpha\mathbf{k}) \times (-2\mathbf{i}) - (2.0490)^2(-2\mathbf{i})$$

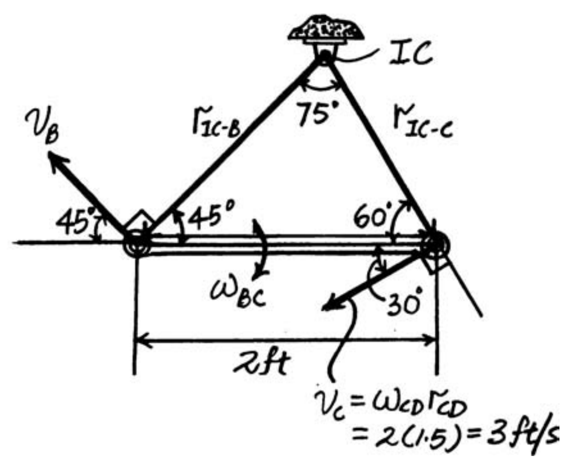
$$-3.818 - (a_B)_t(0.7071) = 3 - 6.495 + 8.3971$$

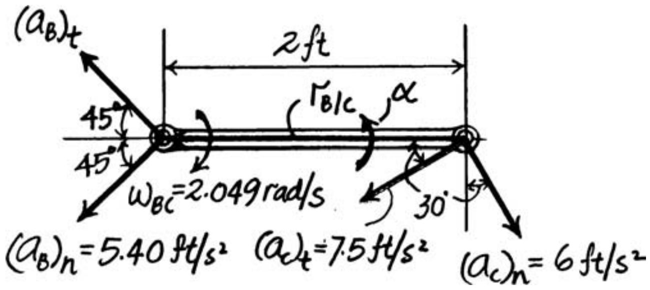
$$-3.818 + (a_B)_t(0.7071) = -5.1962 - 3.75 - 2\alpha$$

$$(a_B)_t = -12.332\ \text{ft/s}^2$$

$$\alpha = 1.80\ \text{rad/s}^2$$

$$\alpha_{AB} = \frac{12.332}{2.5} = 4.93\ \text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$





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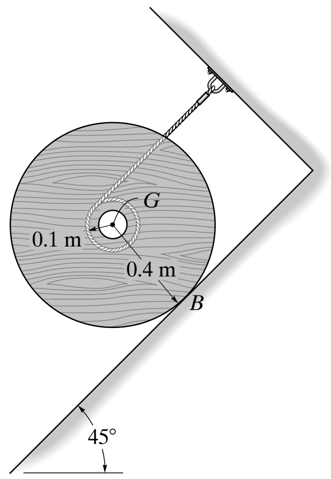
R2–34. The spool and the wire wrapped around its core have a mass of 50 kg and a centroidal radius of gyration of $k_G = 235$ mm. If the coefficient of kinetic friction at the surface is $\mu_k = 0.15$, determine the angular acceleration of the spool after it is released from rest.



$$I_G = mk_G^2 = 500(0.235)^2 = 2.76125 \text{ kg} \cdot \text{m}^2$$

$+\swarrow \Sigma F_{x'} = m(a_G)_{x'}$; $\quad 50(9.81) \sin 45° - T - 0.15N_B = 50a_G$ **(1)**

$+\nwarrow \Sigma F_{y'} = m(a_G)_{y'}$; $\quad N_B - 50(9.81) \cos 45° = 0$ **(2)**

$\circlearrowleft + \Sigma M_G = I_G \alpha$; $\quad T(0.1) - 0.15N_B(0.4) = 2.76125\alpha$ **(3)**

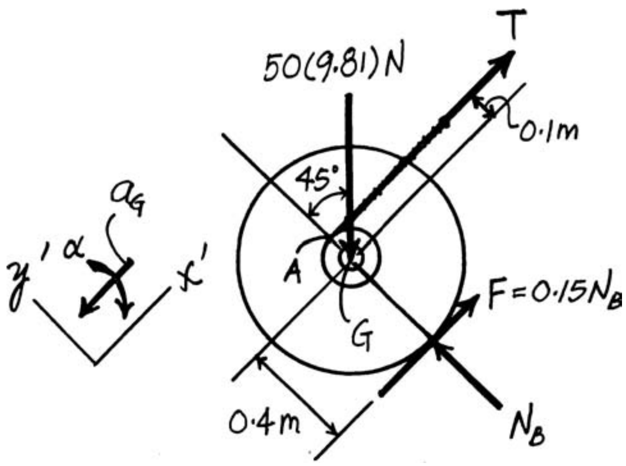
The spool does not slip at point A, therefore

$$a_G = 0.1\alpha \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$$N_B = 346.8 \text{ N} \qquad T = 281.5 \text{ N} \qquad a_G = 0.2659 \text{ m/s}^2$$

$$\alpha = 2.66 \text{ rad/s}^2 \circlearrowright$$ **Ans.**



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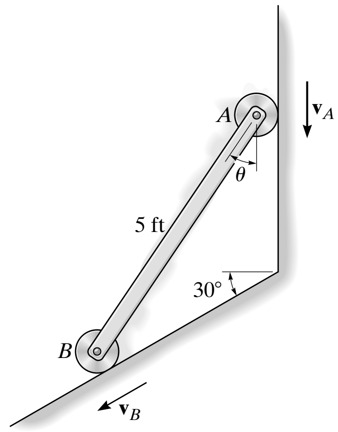
R2–35. The bar is confined to move along the vertical and inclined planes. If the velocity of the roller at A is $v_A = 6$ ft/s when $\theta = 45°$, determine the bar's angular velocity and the velocity of B at this instant.



$$s_B \cos 30° = 5 \sin \theta$$

$$\dot{s}_B = 5.774 \sin \theta$$

$$\ddot{s}_B = 5.774 \cos \theta\dot{\theta} \qquad \textbf{(1)}$$

$$5 \cos \theta = s_A + s_B \sin 30°$$

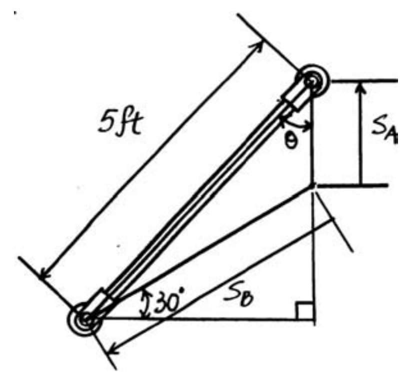
$$-5 \sin \theta\, \ddot{\theta} = \dot{s}_A + \dot{s}_B \sin 30° \qquad \textbf{(2)}$$

Combine Eqs. (1) and (2):

$$-5 \sin \theta\, \dot{\theta} = -6 + 5.774 \cos \theta(\dot{\theta})(\sin 30°)$$

$$-3.536\dot{\theta} = -6 + 2.041\dot{\theta}$$

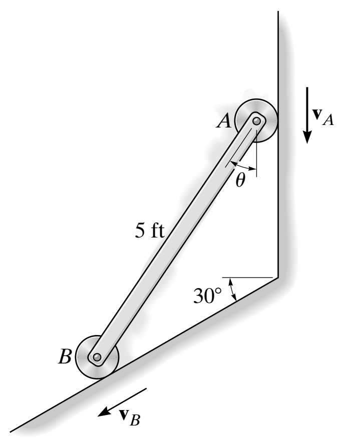
$$\omega = \theta = 1.08 \text{ rad/s} \qquad \textbf{Ans.}$$



From Eq. (1),

$$v_B = s_B = 5.774 \cos 45°(1.076) = 4.39 \text{ ft/s} \qquad \textbf{Ans.}$$

***R2–36.** The bar is confined to move along the vertical and inclined planes. If the roller at A has a constant velocity of $v_A = 6$ ft/s, determine the bar's angular acceleration and the acceleration of B when $\theta = 45°$.



See solution to Prob. R2–35.

Taking the time derivatives of Eqs. (1) and (2) yields:

$$a_B = \ddot{s}_B = -5.774 \sin \theta(\dot{\theta})^2 + 5.774 \cos \theta(\ddot{\theta})$$

$$-5 \cos \theta\, \dot{\theta}^2 - 5 \sin \theta(\ddot{\theta}) = \ddot{s}_A + \ddot{s}_B \sin 30°$$

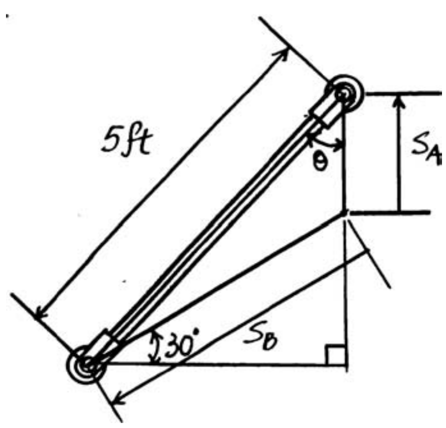
Substitute the data:

$$a_B = -5.774 \sin 45°(1.076)^2 + 5.774 \cos 45°(\ddot{\theta})$$

$$-5 \cos 45°(1.076)^2 - 5 \sin 45°(\ddot{\theta}) = 0 + a_B \sin 30°$$

$$a_B = -4.726 + 4.083\, \ddot{\theta}$$

$$a_B = -8.185 - 7.071\, \ddot{\theta}$$



Solving:

$$\dot{\theta} = -0.310 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = -5.99 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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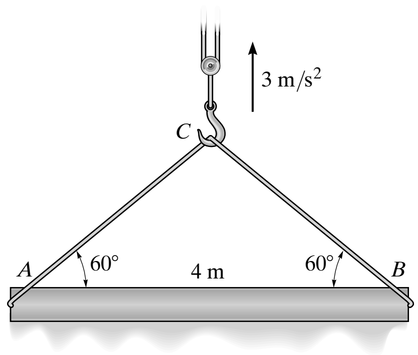
R2–37. The uniform girder AB has a mass of 8 Mg. Determine the internal axial force, shear, and bending moment at the center of the girder if a crane gives it an upward acceleration of 3 m/s^2.



Equations of Motion: By considering the entire beam [FBD(a)], we have

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T \sin 60^\circ - 8000(9.81) = 8000(3)$$

$$T = 59166.86 \text{ N}$$

From the FBD(b) (beam segment),

$$\circlearrowleft + \Sigma M_O = \Sigma (M_k)_O; \qquad M + 4000(9.81)(1)$$

$$-59166.86 \sin 60^\circ(2) = -4000(3)(1)$$

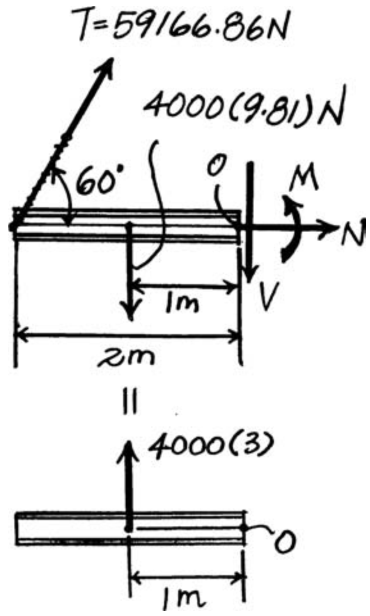
$$M = 51240 \text{ N} \cdot \text{m} = 51.2 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 59166.86 \cos 60^\circ + N = 0$$

$$N = -29583.43 \text{ N} = -29.6 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 59166.86 \sin 60^\circ - 4000(9.81) - V = 4000(3)$$

$$V = 0 \qquad \textbf{Ans.}$$



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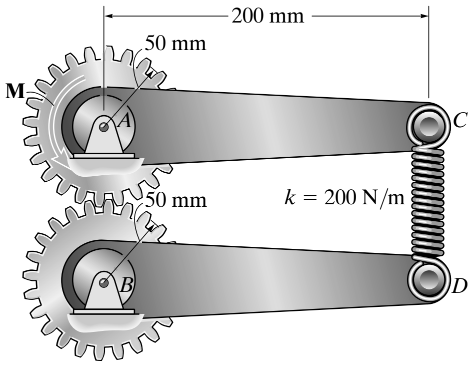
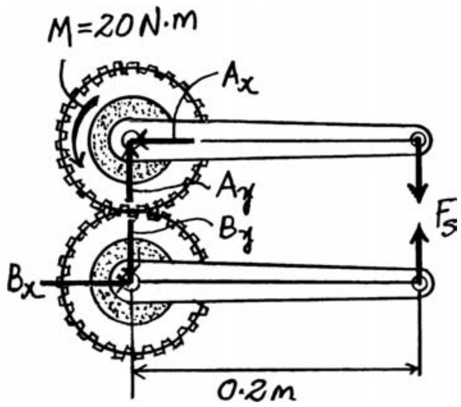
R2–38. Each gear has a mass of 2 kg and a radius of gyration about its pinned mass centers A and B of $k_g = 40$ mm. Each link has a mass of 2 kg and a radius of gyration about its pinned ends A and B of $k_l = 50$ mm. If originally the spring is unstretched when the couple moment $M = 20$ N·m is applied to link AC, determine the angular velocities of the links at the instant link AC rotates $\theta = 45°$. Each gear and link is connected together and rotates in the horizontal plane about the fixed pins A and B.



Consider the system of both gears and the links.

The spring stretches $s = 2(0.2 \sin 45°) = 0.2828$ m.



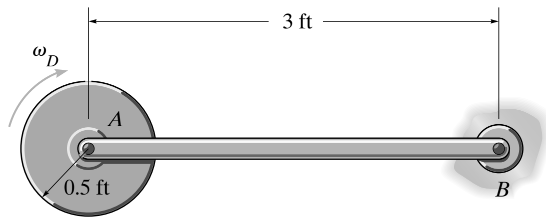
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left\{20\left(\frac{\pi}{4}\right) - \frac{1}{2}(200)(0.2828)^2\right\} = 2\left\{\frac{1}{2}\left[(2)(0.05)^2 + (2)(0.04)^2\right]\omega^2\right\}$$

$$\omega = 30.7 \text{ rad/s}$$ **Ans.**

Note that work is done by the tangential force between the gears since each move. For the system, though, this force is equal but opposite and the work cancels.

R2–39. The 5-lb rod AB supports the 3-lb disk at its end A. If the disk is given an angular velocity $\omega_D = 8$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing A. Motion is in the *horizontal plane*. Neglect friction at the fixed bearing B.



Conservation of Momentum:

$$\circlearrowleft + \Sigma(H_B)_1 = \Sigma(H_B)_2$$

$$\left[\frac{1}{2}\left(\frac{3}{32.2}\right)(0.5)^2\right](8) + 0 = \left[\frac{1}{3}\left(\frac{5}{32.2}\right)(3)^2\right]\omega$$

$$+\left[\frac{1}{2}\left(\frac{3}{32.2}\right)(0.5)^2\right]\omega + \left(\frac{3}{32.2}\right)(3\omega)(3)$$

$$\omega = 0.0708 \text{ rad/s}$$ **Ans.**

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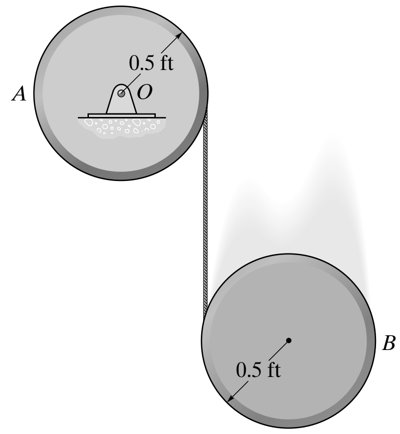
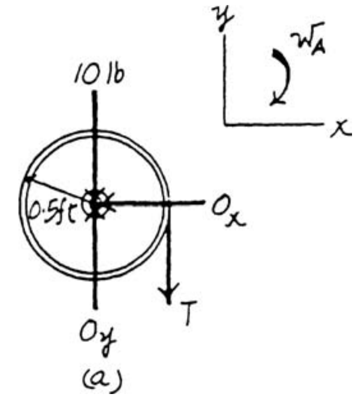
***R2–40.** A cord is wrapped around the rim of each 10-lb disk. If disk B is released from rest, determine the angular velocity of disk A in 2 s. Neglect the mass of the cord.



Principle of Impulse and Momentum: The mass moment inertia of disk A about point O is $I_O = \frac{1}{2}\left(\frac{10}{32.2}\right)(0.5^2) = 0.03882\ \text{slug} \cdot \text{ft}^2$. Applying Eq. 19–14 to disk A [FBD(a)], we have

$$I_O\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\,\omega_2$$

$$(\;+) \qquad 0 - [T(2)](0.5) = -0.03882\omega_A \qquad \textbf{(1)}$$



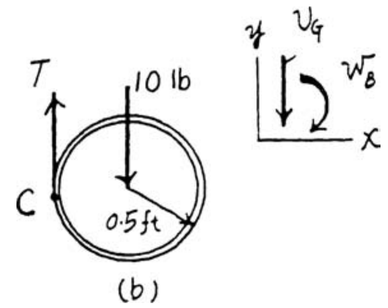
The mass moment inertia of disk B about its mass center is $I_G = \frac{1}{2}\left(\frac{10}{32.2}\right)(0.5^2) = 0.03882\ \text{slug} \cdot \text{ft}^2$. Applying Eq. 19–14 to disk B [FBD(b)], we have

$$m\left(v_{G_y}\right)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m\left(v_{G_y}\right)_1$$

$$(+\uparrow) \qquad 0 + T(2) - 10(2) = -\left(\frac{10}{32.2}\right)v_G \qquad \textbf{(2)}$$

$$I_G\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$(\circlearrowleft +) \qquad 0 - [T(2)](0.5) = -0.03882\omega_B \qquad \textbf{(3)}$$

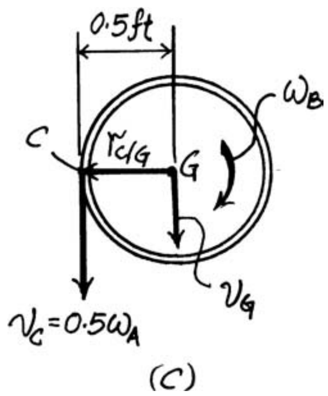


Kinematics: The speed of point C on disk B is $v_C = \omega_A r_A = 0.5\omega_A$. Here, $v_{C/G} = \omega_B r_{C/G} = 0.5\omega_B$ which is directed vertically upward. Applying Eq. 16–15, we have

$$\mathbf{v}_C = \mathbf{v}_G + \mathbf{v}_{C/G}$$

$$\begin{bmatrix} 0.5\omega_A \\ \downarrow \end{bmatrix} = \begin{bmatrix} v_G \\ \downarrow \end{bmatrix} + \begin{bmatrix} 0.5\omega_B \\ \uparrow \end{bmatrix}$$

$$(+\uparrow) \qquad -0.5\omega_A = -v_G + 0.5\omega_B \qquad \textbf{(4)}$$



Solving Eqs. (1), (2), (3), and (4) yields:

$$\omega_A = 51.5\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\omega_B = 51.52\ \text{rad/s} \qquad v_G = 51.52\ \text{ft/s} \qquad T = 2.00\ \text{lb}$$

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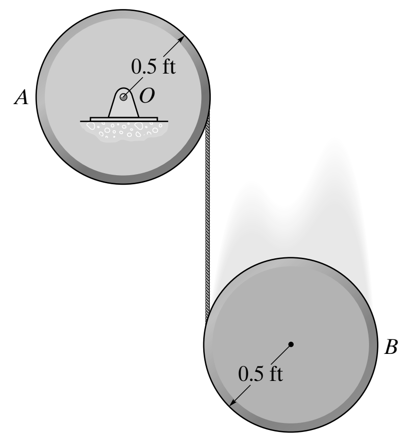
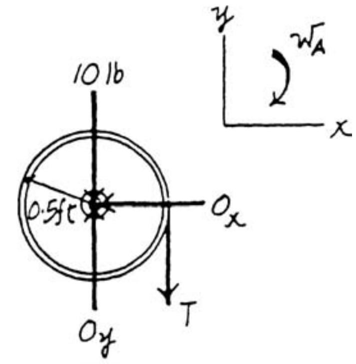
R2–41. A cord is wrapped around the rim of each 10-lb disk. If disk B is released from rest, determine how much time t is required before A attains an angular velocity $\omega_A = 5$ rad/s.



Principle of Impulse and Momentum: The mass moment inertia of disk A about point O is $I_O = \frac{1}{2}\left(\frac{10}{32.2}\right)\left(0.5^2\right) = 0.03882\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 19–14 to disk A [FBD(a)], we have

$$I_O\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\,\omega_2$$

$$(+)\qquad 0 - [T(t)](0.5) = -0.03882(5) \qquad \textbf{(1)}$$



(a)

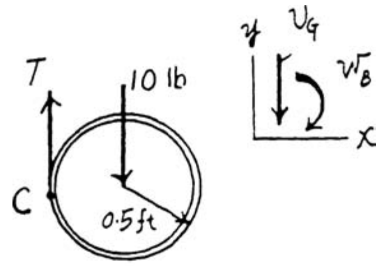
The mass moment inertia of disk B about its mass center is $I_G = \frac{1}{2}\left(\frac{10}{32.2}\right)\left(0.5^2\right) = 0.03882\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 19–14 to disk B [FBD(b)], we have

$$m\left(v_{G_y}\right)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m\left(v_{G_y}\right)_1$$

$$(+\uparrow)\qquad 0 + T(t) - 10(t) = -\left(\frac{10}{32.2}\right)v_G \qquad \textbf{(2)}$$

$$I_G\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$(\circlearrowleft +)\qquad 0 - [T(t)](0.5) = -0.03882\omega_B \qquad \textbf{(3)}$$



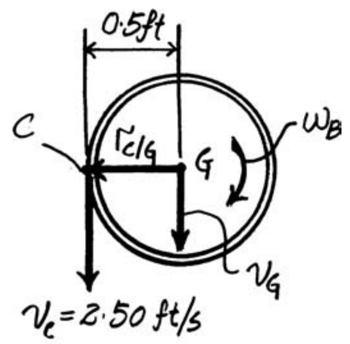
(b)

Kinematics: The speed of point C on disk B is $v_C = \omega_A\,r_A = 0.5(5) = 2.50$ ft/s. Here, $v_{C/G} = \omega_B\,r_{C/G} = 0.5\,\omega_B$ which is directed vertically upward. Applying Eq. 16–15, we have

$$\mathbf{v}_C = \mathbf{v}_G + \mathbf{v}_{C/G}$$

$$\begin{bmatrix} 2.50 \\ \downarrow \end{bmatrix} = \begin{bmatrix} v_G \\ \downarrow \end{bmatrix} + \begin{bmatrix} 0.5\,\omega_B \\ \uparrow \end{bmatrix}$$

$$(+\uparrow)\qquad -2.50 = -v_G + 0.5\,\omega_B \qquad \textbf{[4]}$$



(c)

Solving Eqs. (1), (2), (3), and (4) yields:

$$t = 0.194\ \text{s} \qquad \textbf{Ans.}$$

$$\omega_B = 5.00\ \text{rad/s} \qquad v_G = 5.00\ \text{ft/s} \qquad T = 2.00\ \text{lb}$$

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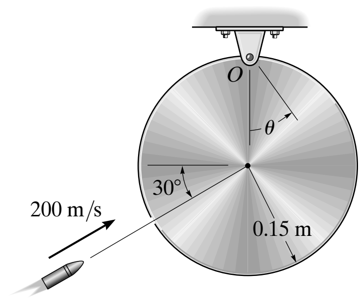
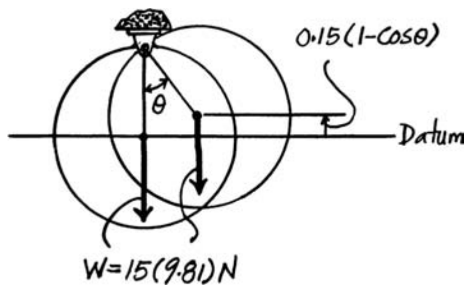
R2–42. The 15-kg disk is pinned at O and is initially at rest. If a 10-g bullet is fired into the disk with a velocity of 200 m/s, as shown, determine the maximum angle θ to which the disk swings. The bullet becomes embedded in the disk.





$$\circlearrowleft + (H_O)_1 = (H_O)_2$$

$$0.01(200 \cos 30°)(0.15) = \left[\left[\frac{1}{2}(15)(0.15)^2 + 15(0.15)^2\right]\omega\right.$$

$$\omega = 0.5132 \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{2}(15)(0.15)^2 + 15(0.15)^2\right](0.5132)^2 + 0 = 0 + 15(9.81)(0.15)(1 - \cos\theta)$$

$$\theta = 4.45° \qquad \textbf{Ans.}$$

Note that the calculation neglects the small mass of the bullet after it becomes embedded in the plate, since its position in the plate is not specified.

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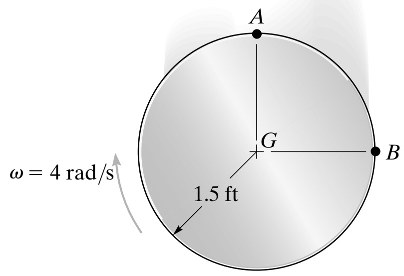
R2–43. The disk rotates at a constant rate of 4 rad/s as it falls freely so that its center G has an acceleration of 32.2 ft/s^2. Determine the accelerations of points A and B on the rim of the disk at the instant shown.



$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$\left[\underset{\rightarrow}{(a_A)_x}\right] + \left[\underset{\uparrow}{(a_A)_y}\right] = \left[\underset{\downarrow}{32.2}\right] + 0 + \left[\underset{\downarrow}{(4)^2(1.5)}\right]$$

$$\left(\stackrel{+}{\rightarrow}\right) \qquad (a_A)_x = 0$$

$$(+\uparrow) \qquad (a_A)_y = -32.2 - (4)^2(1.5) = -56.2 \text{ ft/s}^2 = 56.2 \text{ ft/s}^2 \downarrow$$

$$a_A = (a_A)_y = 56.2 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

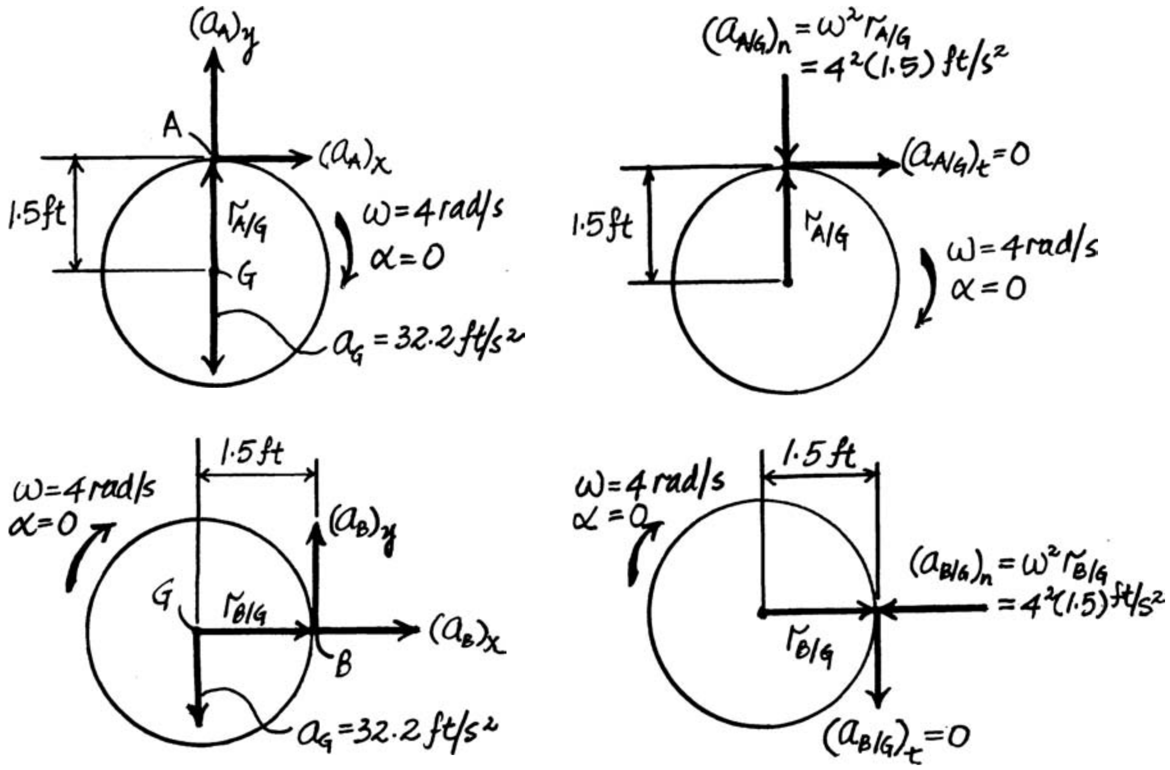
$$\left[\underset{\rightarrow}{(a_B)_x}\right] + \left[\underset{\uparrow}{(a_B)_y}\right] = \left[\underset{\downarrow}{32.2}\right] + 0 + \left[\underset{\leftarrow}{(4)^2(1.5)}\right]$$

$$\left(\stackrel{+}{\rightarrow}\right) \qquad (a_B)_x = -(4)^2(1.5) = -24 \text{ ft/s}^2 = 24 \text{ ft/s}^2 \leftarrow$$

$$(+\uparrow) \qquad (a_B)_y = -32.2 \text{ ft/s}^2 = 32.2 \text{ ft/s}^2 \downarrow$$

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = \sqrt{24^2 + 32.2^2} = 40.2 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{(a_B)_y}{(a_B)_x}\right) = \tan^{-1}\left(\frac{32.2}{24}\right) = 53.3° \qquad \textbf{Ans.}$$



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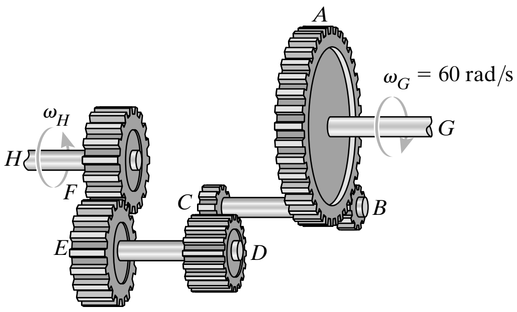
***R2–44.** The operation of "reverse" for a three-speed automotive transmission is illustrated schematically in the figure. If the shaft G is turning with an angular velocity of $\omega_G = 60$ rad/s, determine the angular velocity of the drive shaft H. Each of the gears rotates about a fixed axis. Note that gears A and B, C and D, E and F are in mesh. The radius of each of these gears is reported in the figure.



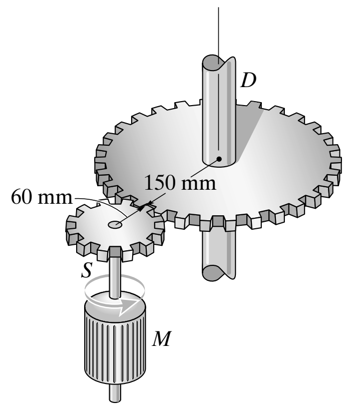
$r_A = 90$ mm
$r_B = r_C = 30$ mm
$r_D = 50$ mm
$r_E = 70$ mm
$r_F = 60$ mm

$$\omega_C = \omega_B = \frac{r_A}{r_B}\omega_G = \frac{90}{30}(60) = 180 \text{ rad/s}$$

$$\omega_E = \omega_D = \frac{r_C}{r_D}\omega_C = \frac{30}{50}(180) = 108 \text{ rad/s}$$

$$\omega_H = \frac{r_E}{r_F}\omega_E = \frac{70}{60}(108) = 126 \text{ rad/s}$$ **Ans.**

R2–45. Shown is the internal gearing of a "spinner" used for drilling wells. With constant angular acceleration, the motor M rotates the shaft S to 100 rev/min in $t = 2$ s starting from rest. Determine the angular acceleration of the drill-pipe connection D and the number of revolutions it makes during the 2-s startup.



For shaft S,

$$\omega = \omega_0 + \alpha_c t$$

$$\frac{100(2\pi)}{60} = 0 + \alpha_S(2) \qquad \alpha_S = 5.236 \text{ rad/s}^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\theta_S = 0 + 0 + \frac{1}{2}(5.236)(2)^2 = 10.472 \text{ rad}$$

For connection D,

$$\alpha_D = \frac{r_S}{r_D}\alpha_S = \frac{60}{150}(5.236) = 2.09 \text{ rad/s}^2$$ **Ans.**

$$\theta_D = \frac{r_S}{r_D}\theta_S = \frac{60}{150}(10.472) = 4.19 \text{ rad} = 0.667 \text{ rev}$$ **Ans.**

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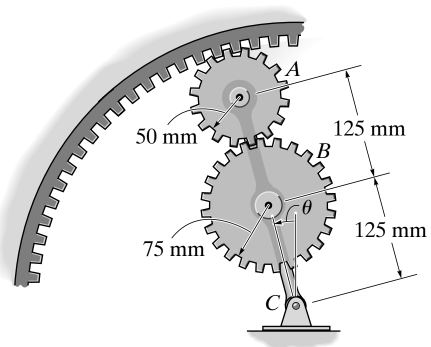
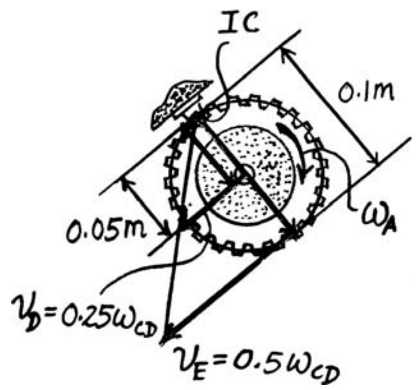
R2–46. Gear A has a mass of 0.5 kg and a radius of gyration of $k_A = 40$ mm, and gear B has a mass of 0.8 kg and a radius of gyration of $k_B = 55$ mm. The link is pinned at C and has a mass of 0.35 kg. If the link can be treated as a slender rod, determine the angular velocity of the link after the assembly is released from rest when $\theta = 0°$ and falls to $\theta = 90°$.



Kinematics: The velocity of the mass center of gear A is $v_D = 0.25\,\omega_{CD}$, and since is rolls without slipping on the fixed circular gear track, the location of the instantaneous center of zero velocity is as shown. Thus,

$$\omega_A = \frac{v_D}{r_{D/IC}} = \frac{0.25\,\omega_{CD}}{0.05} = 5\omega_{CD} \qquad v_E = \omega_A\, r_{E/IC} = 5\omega_{CD}(0.1) = 0.5\,\omega_{CD}$$



The velocity of the mass center of gear B is $v_F = 0.125\omega_{CD}$. The location of the instantaneous center of zero velocity is as shown. Thus,

$$\omega_B = \frac{v_E}{r_{E/(IC)1}} = \frac{0.5\,\omega_{CD}}{0.1} = 5\omega_{CD}$$

Potential Energy: Datum is set at point C. When gears A, B and link AC are at their initial position ($\theta = 0°$), their centers of gravity are located 0.25 m, 0.125 m, and 0.125 m *above* the datum, respectively. The total gravitational potential energy when they are at these positions is $0.5(9.81)(0.25) + 0.8(9.81)(0.125) + 0.35(9.81)(0.125) = 2.636$ N·m. Thus, the initial and final potential energy is

$$V_1 = 2.636\ \text{N}\cdot\text{m} \qquad V_2 = 0$$

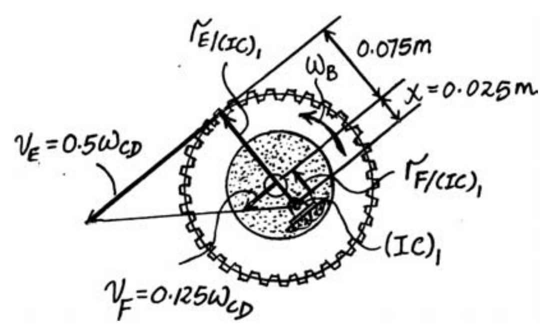
Kinetic Energy: The mass moment of inertia of gears A and B about their mass center is $I_D = 0.5\left(0.04^2\right) = 0.8\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$ and $I_F = 0.8\left(0.055^2\right) = 2.42\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$. The mass moment of inertia of link CD about point C is $(I_{CD})_C = \frac{1}{12}(0.35)\left(0.25^2\right) + 0.35\left(0.125^2\right) = 7.292\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$. Since the system is at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}m_A v_D^2 + \frac{1}{2}I_D\,\omega_A^2 + \frac{1}{2}m_B v_F^2 + \frac{1}{2}I_F\omega_B^2 + \frac{1}{2}(I_{CD})_C\,\omega_{CD}^2$$

$$= \frac{1}{2}(0.5)(0.25\,\omega_{CD})^2 + \frac{1}{2}\left[0.8\left(10^{-3}\right)\right](5\omega_{CD})^2 + \frac{1}{2}(0.8)(0.125\,\omega_{CD})^2$$

$$+ \frac{1}{2}\left[2.42\left(10^{-3}\right)\right](5\,\omega_{CD})^2 + \frac{1}{2}\left[7.292\left(10^{-3}\right)\right]\left(\omega_{CD}^2\right)$$

$$= 0.06577\,\omega_{CD}^2$$



Conservation of Energy: Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.636 = 0.06577\,\omega_{CD}^2$$

$$\omega_{CD} = 6.33\ \text{rad/s} \qquad \textbf{Ans.}$$

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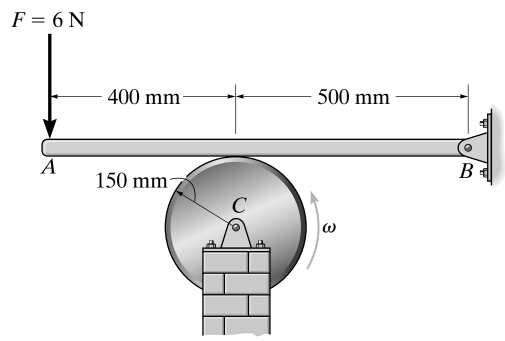
R2–47. The 15-kg cylinder rotates with an angular velocity of $\omega = 40$ rad/s. If a force $F = 6$ N is applied to bar AB, as shown, determine the time needed to stop the rotation. The coefficient of kinetic friction between AB and the cylinder is $\mu_k = 0.4$. Neglect the thickness of the bar.



For link AB,

$\circlearrowleft + \Sigma M_B = 0; \qquad 6(0.9) - N_E(0.5) = 0 \qquad N_E = 10.8 \text{ N}$

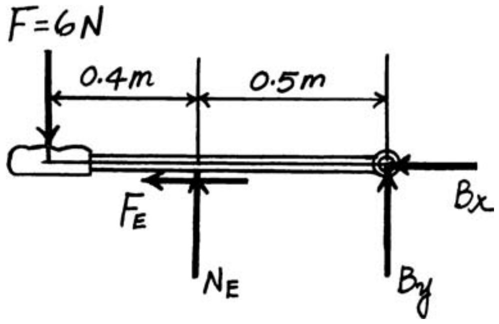
$$I_C = \frac{1}{2}mr^2 = \frac{1}{2}(15)(0.15)^2 = 0.16875 \text{ kg} \cdot \text{m}^2$$

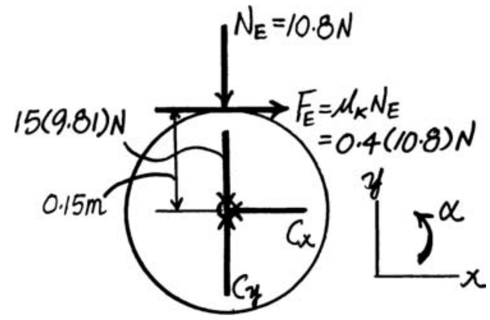
$\circlearrowleft + \Sigma M_C = I_C\,\alpha; \qquad -0.4(10.8)(0.15) = 0.16875(\alpha) \qquad \alpha = -3.84 \text{ rad/s}^2$

$\circlearrowleft + \omega = \omega_0 + \alpha t$

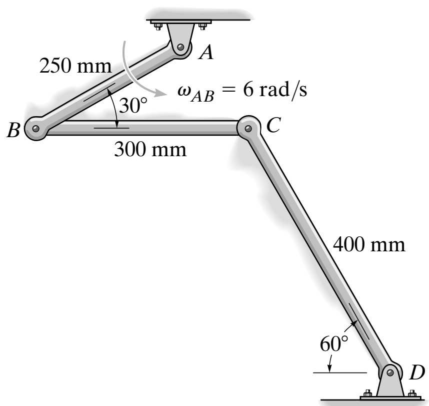
$0 = 40 + (-3.84)\,t$

$t = 10.4$ s **Ans.**





***R2–48.** If link AB rotates at $\omega_{AB} = 6$ rad/s, determine the angular velocities of links BC and CD at the instant shown.



Link AB rotates about the fixed point A. Hence,

$$v_B = \omega_{AB}\, r_{AB} = 6(0.25) = 1.5 \text{ m/s}$$

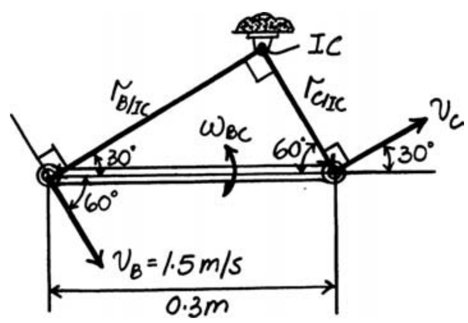
For link BC,

$r_{B/IC} = 0.3 \cos 30° = 0.2598$ m $\qquad r_{C/IC} = 0.3 \cos 60° = 0.15$ m

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.5}{0.2598} = 5.77 \text{ rad/s}$$ **Ans.**

$$v_C = \omega_{BC}\, r_{C/IC} = 5.77(0.15) = 0.8660 \text{ m/s}$$

Link CD rotates about the fixed point D. Hence,



$$v_C = \omega_{CD}\, r_{CD}$$

$0.8660 = \omega_{CD}\,(0.4) \qquad \omega_{CD} = 2.17$ rad/s **Ans.**

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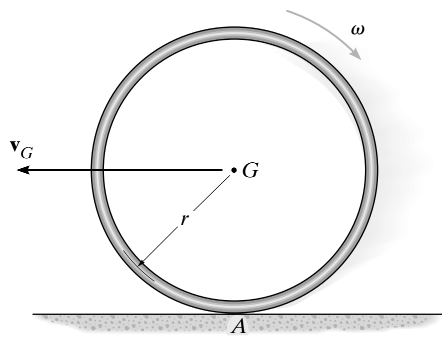
R2–49. If the thin hoop has a weight W and radius r and is thrown onto a *rough surface* with a velocity $\mathbf{v}_G$ parallel to the surface, determine the backspin, $\boldsymbol{\omega}$, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at A for the calculation.

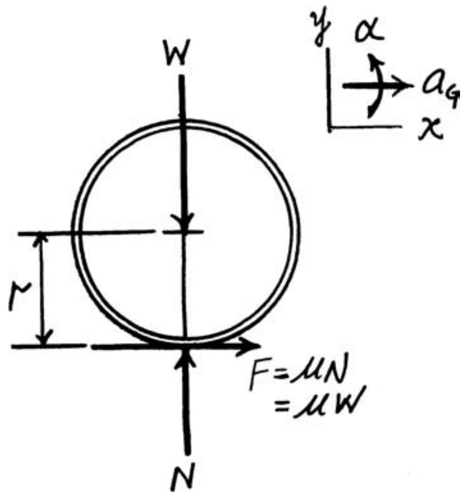


Equations of Motion: The mass moment of inertia of the hoop about its mass center is given by $I_G = mr^2 = \frac{W}{g}r^2$. Applying Eq. 17–16, we have

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - W = 0 \qquad N = W$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad \mu W = \frac{W}{g}a_G \qquad a_G = \mu g$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad \mu W r = \frac{W}{g}r^2\alpha \qquad \alpha = \frac{\mu g}{r}$$



Kinematics: The time required for the hoop to stop back spinning can be determined by applying Eq. 16–5.

$$\omega = \omega_0 + \alpha t_1$$

$$(\circlearrowleft +) \qquad 0 = \omega + \left(-\frac{\mu g}{r}\right)t_1$$

$$t_1 = \frac{\omega r}{\mu g}$$

The time required for the hoop to stop can be determined by applying Eq. 12–4.

$$\left(\overset{+}{\leftarrow}\right) \qquad v = v_0 + at_2$$

$$0 = v_G + (-\mu g)t_2$$

$$t_2 = \frac{v_G}{\mu g}$$

It is required that $t_1 = t_2$. Thus,

$$\frac{\omega r}{\mu g} = \frac{v_G}{\mu g}$$

$$\omega = \frac{v_G}{r} \qquad \textbf{Ans.}$$

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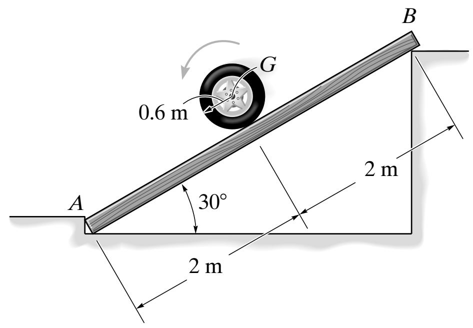
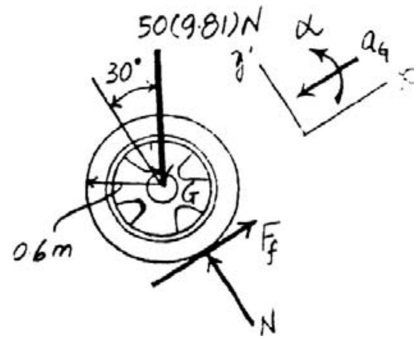
R2–50. The wheel has a mass of 50 kg and a radius of gyration $k_G = 0.4$ m. If it rolls without slipping down the inclined plank, determine the horizontal and vertical components of reaction at A, and the normal reaction at the smooth support B at the instant the wheel is located at the midpoint of the plank. The plank has negligible thickness and has a mass of 20 kg.



Equations of Motion: Since the tire rolls down the plane without slipping, then $a_G = \alpha r = 0.6\alpha$. The mass moment of inertia of the tire about its mass center is given by $I_G = mk_G^2 = 50\left(0.4^2\right) = 8.00\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16 to [FBD(a)], we have



$\nwarrow + \Sigma F_{y'} = m(a_G)_{y'}; \qquad N - 50(9.81)\cos 30° = 0 \qquad N = 424.79\text{ N}$

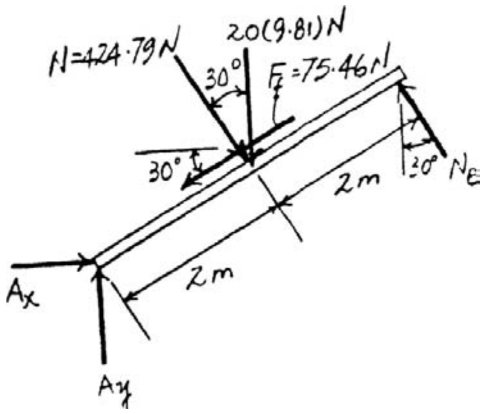
$\swarrow + \Sigma F_{x'} = m(a_G)_{x'}; \qquad 50(9.81)\sin 30° - Ff = 50(0.6\alpha) \qquad$ **(1)**

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad F_f(0.6) = 8.00\alpha \qquad$ **(2)**

Solving Eqs. [1] and [2] yields

$$F_f = 75.46\text{ N} \qquad \alpha = 5.660\text{ rad/s}^2$$

Equations of Equilibrium: From FBD(b).



$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(4) - 20(9.81)\cos 30°(2) - 424.79(2) = 0$

$$N_B = 297.35\text{ N} = 297\text{ N} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0; \qquad A_y + 297.35\cos 30° - 20(9.81)$

$$- 424.79\cos 30° - 75.46\sin 30° = 0$$

$$A_y = 344\text{ N} \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x + 424.79\sin 30° - 75.46\cos 30° - 297.35\sin 30° = 0$

$$A_x = 1.63\text{ N} \qquad \textbf{Ans.}$$

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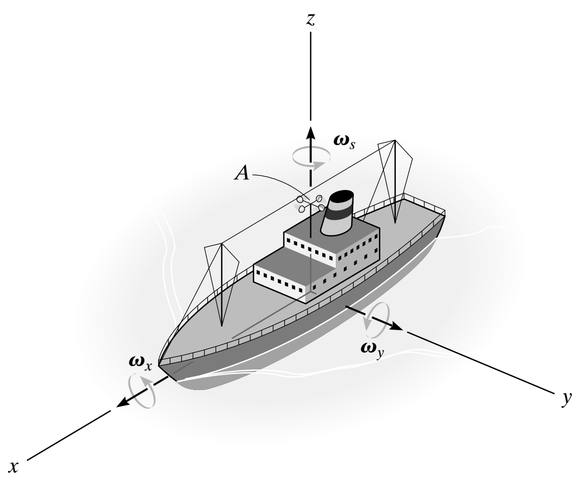
•20–1. The anemometer located on the ship at A spins about its own axis at a rate ω_s, while the ship rolls about the x axis at the rate ω_x and about the y axis at the rate ω_y. Determine the angular velocity and angular acceleration of the anemometer at the instant the ship is level as shown. Assume that the magnitudes of all components of angular velocity are constant and that the rolling motion caused by the sea is independent in the x and y directions.



$$\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k} \qquad \textbf{Ans.}$$

$$\text{Let } \Omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j}.$$

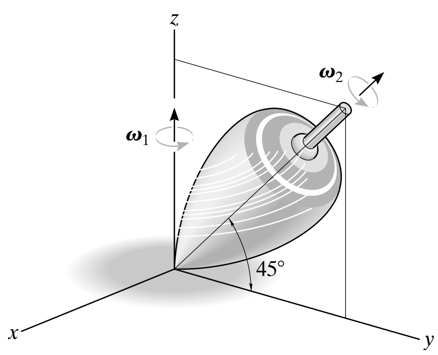
Since ω_x and ω_y are independent of one another, they do not change their direction or magnitude. Thus,

$$\alpha = \dot{\omega} = \left(\dot{\omega}\right)_{xyz} + (\omega_x + \omega_y) \times \omega_z$$

$$\alpha = \mathbf{0} + (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j}) \times (\omega_y\,\mathbf{k})$$

$$\alpha = \omega_y\,\omega_x\,\mathbf{i} - \omega_x\,\omega_z\,\mathbf{j} \qquad \textbf{Ans.}$$

20–2. The motion of the top is such that at the instant shown it rotates about the z axis at $\omega_1 = 0.6$ rad/s, while it spins at $\omega_2 = 8$ rad/s. Determine the angular velocity and angular acceleration of the top at this instant. Express the result as a Cartesian vector.



$$\omega = \omega_1 + \omega_2$$

$$\omega = 0.6\mathbf{k} + 8\cos 45°\,\mathbf{j} + 8\sin 45°\mathbf{k}$$

$$\omega = \{5.66\mathbf{j} + 6.26\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

Let x, y, z axes have angular velocity of $\Omega = \omega_1$, thus

$$\dot{\omega}_1 = \mathbf{0}$$

$$\omega_2 = \left(\omega_2\right)_{xyz} + (\omega_1 \times \omega_2) = \mathbf{0} + (0.6\mathbf{k}) \times (8\cos 45°\mathbf{j} + 8\sin 45°\mathbf{k}) = -3.394\mathbf{i}$$

$$\alpha = \omega = \{-3.39\mathbf{i}\}\text{ rad/s}^2 \qquad \textbf{Ans.}$$

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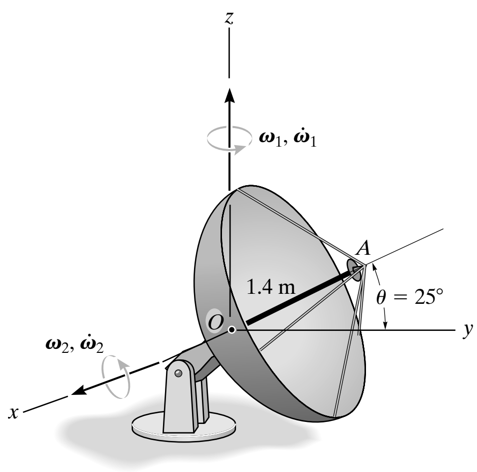
20–3. At a given instant, the satellite dish has an angular motion $\omega_1 = 6$ rad/s and $\dot{\omega}_1 = 3$ rad/s^2 about the z axis. At this same instant $\theta = 25°$, the angular motion about the x axis is $\omega_2 = 2$ rad/s, and $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant.



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the satellite at this instant (with reference to X, Y, Z) can be expressed in terms of **i, j, k** components.

$$\omega = \omega_1 + \omega_2 = \{2\mathbf{i} + 6\mathbf{k}\} \text{ rad/s}$$

Angular Acceleration: The angular acceleration α will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame. ω_2 is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s. Applying Eq. 20–6 with $(\dot{\omega}_2)_{xyz} = \{1.5\mathbf{i}\}$ rad/s^2. we have

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{i} + 6\mathbf{k} \times 2\mathbf{i} = \{1.5\mathbf{i} + 12\mathbf{j}\} \text{ rad/s}^2$$

Since ω_1 is always directed along the Z axis ($\Omega = \mathbf{0}$), then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \mathbf{0} \times \omega_1 = \{3\mathbf{k}\} \text{ rad/s}^2$$

Thus, the angular acceleration of the satellite is

$$\alpha = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{1.4\cos 25°\mathbf{j} + 1.4\sin 25°\mathbf{k}\}$ m $= \{1.2688\mathbf{j} + 0.5917\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})$$

$$= \{-7.61\mathbf{i} - 1.18\mathbf{j} + 2.54\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (1.3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})$$

$$+ (2\mathbf{i} + 6\mathbf{k}) \times [(2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})]$$

$$= \{10.4\mathbf{i} - 51.6\mathbf{j} - 0.463\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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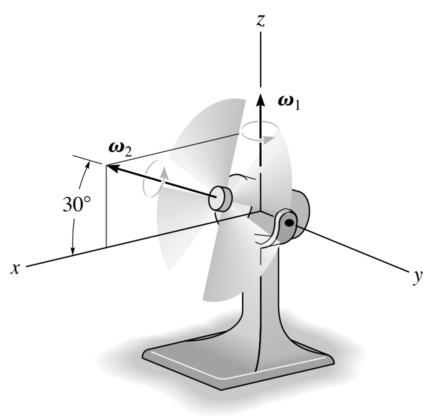
***20–4.** The fan is mounted on a swivel support such that at the instant shown it is rotating about the z axis at $\omega_1 = 0.8$ rad/s, which is increasing at $12\ \text{rad/s}^2$. The blade is spinning at $\omega_2 = 16$ rad/s, which is decreasing at $2\ \text{rad/s}^2$. Determine the angular velocity and angular acceleration of the blade at this instant.



$$\omega = \omega_1 + \omega_2$$

$$= 0.8\mathbf{k} + (16\cos 30°\mathbf{i} + 16\sin 30°\mathbf{k})$$

$$= \{13.9\mathbf{i} + 8.80\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

For ω_2, $\quad \Omega = \omega_1 = \{0.8\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2$$

$$= (-2\cos 30°\mathbf{i} - 2\sin 30°\mathbf{k}) + (0.8\mathbf{k}) \times (16\cos 30°\mathbf{i} + 16\sin 30°\mathbf{k})$$

$$= \{-1.7320\mathbf{i} + 11.0851\mathbf{j} - 1\mathbf{k}\}\ \text{rad/s}^2$$

For ω_1, $\quad \Omega = \mathbf{0}$.

$$(\omega_1)_{XYZ} = (\omega_1)_{xyz} + \Omega \times \omega_1$$

$$= (12\mathbf{k}) + 0$$

$$= \{12\mathbf{k}\}\ \text{rad/s}^2$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = 12\mathbf{k} + (-1.7320\mathbf{i} + 11.0851\mathbf{j} - 1\mathbf{k})$$

$$= \{-1.73\mathbf{i} + 11.1\mathbf{j} + 11.0\mathbf{k}\}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

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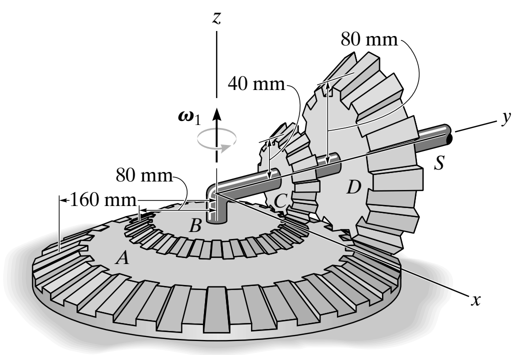
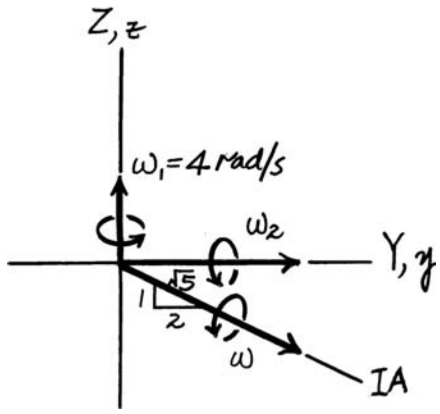
•20–5. Gears A and B are fixed, while gears C and D are free to rotate about the shaft S. If the shaft turns about the z axis at a constant rate of $\omega_1 = 4$ rad/s, determine the angular velocity and angular acceleration of gear C.



The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.

$$\omega = \omega_1 + \omega_2$$

$$\frac{2}{\sqrt{5}}\omega\mathbf{j} - \frac{1}{\sqrt{5}}\omega\mathbf{k} = 4\mathbf{k} + \omega_2\mathbf{j}$$



Equating **j** and **k** components

$$-\frac{1}{\sqrt{5}}\omega = 4 \qquad \omega = -8.944 \text{ rad/s}$$

$$\omega_2 = \frac{2}{\sqrt{5}}(-8.944) = -8.0 \text{ rad/s}$$

Hence $\quad \omega = \frac{2}{\sqrt{5}}(-8.944)\mathbf{j} - \frac{1}{\sqrt{5}}(-8.944)\mathbf{k} = \{-8.0\mathbf{j} + 4.0\mathbf{k}\}$ rad/s **Ans.**

For ω_2, $\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2$$

$$= 0 + (4\mathbf{k}) \times (-8\mathbf{j})$$

$$= \{32\mathbf{i}\} \text{ rad/s}^2$$

For ω_1, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \Omega \times \omega_1 = 0 + 0 = 0$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = \mathbf{0} + (32\mathbf{i}) = \{32\mathbf{i}\} \text{ rad/s}^2$$ **Ans.**

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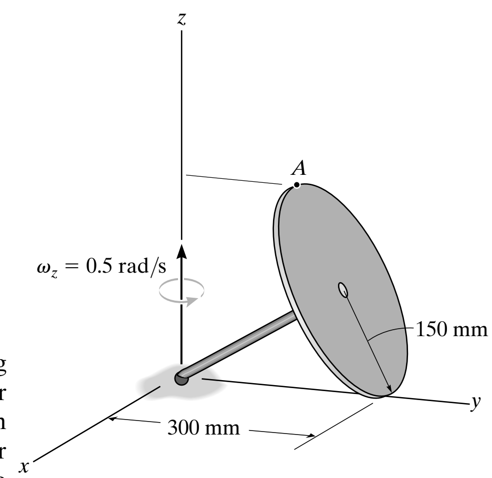
20–6. The disk rotates about the z axis $\omega_z = 0.5$ rad/s without slipping on the horizontal plane. If at this same instant ω_z is increasing at $\dot{\omega}_z = 0.3$ rad/s^2, determine the velocity and acceleration of point A on the disk.



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant (with reference to X, Y, Z) can be expressed in terms of **i, j, k** components. Since the disk rolls without slipping, then its angular velocity $\omega = \omega_s + \omega_z$ is always directed along the instantaneous axis of zero velocity (y axis). Thus,

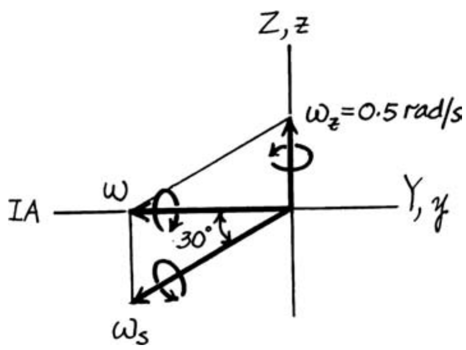
$$\omega = \omega_s + \omega_z$$

$$-\omega\mathbf{j} = -\omega_s \cos 30°\mathbf{j} - \omega_s \sin 30°\mathbf{k} + 0.5\mathbf{k}$$

Equating **k** and **j** components, we have

$$0 = -\omega_s \sin 30° + 0.5 \qquad \omega_s = 1.00 \text{ rad/s}$$

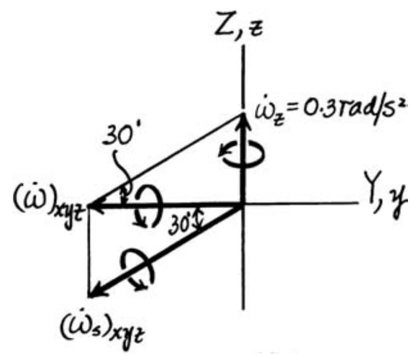
$$-\omega = -1.00 \cos 30° \qquad \omega = 0.8660 \text{ rad/s}$$



Angular Acceleration: The angular acceleration α will be determined by investigating the time rate of change of *angular velocity* with respect to the fixed XYZ frame. Since ω always lies in the fixed X–Y plane, then $\omega = \{-0.8660\mathbf{j}\}$ rad/s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{0.5\mathbf{k}\}$ rad/s. $(\dot{\omega}_s)_{xyz}$

$$= \left\{-\frac{0.3}{\sin 30°}(\cos 30°)\,\mathbf{j} - \frac{0.3}{\sin 30°}(\sin 30°)\,\mathbf{k}\right\} \text{rad/s}^2 = \{-0.5196\mathbf{j} - 0.3\mathbf{k}\} \text{ rad/s}^2.$$

Thus, $(\dot{\omega})_{xyz} = \dot{\omega}_z + (\dot{\omega}_x)_{xyz} = \{-0.5196\mathbf{j}\}$ rad/s^2. Applying Eq. 20–6, we have



$$\alpha = \dot{\omega} = (\dot{\omega})_{xyz} + \omega_z \times \omega$$

$$= -0.5196\mathbf{j} + 0.5\mathbf{k} \times (-0.8660\mathbf{j})$$

$$= \{0.4330\mathbf{i} - 0.5196\mathbf{j}\} \text{ rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{(0.3 - 0.3\cos 60°)\mathbf{j} + 0.3\sin 60°\mathbf{k}\}$ m $= \{0.15\mathbf{j} + 0.2598\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k}) = \{-0.225\mathbf{i}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (0.4330\mathbf{i} - 0.5196\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})$$

$$+ (-0.8660\mathbf{j}) \times [(-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})]$$

$$= \{-0.135\mathbf{i} - 0.1125\mathbf{j} - 0.130\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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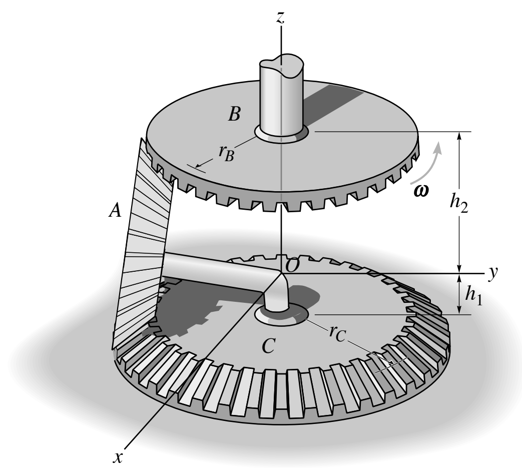
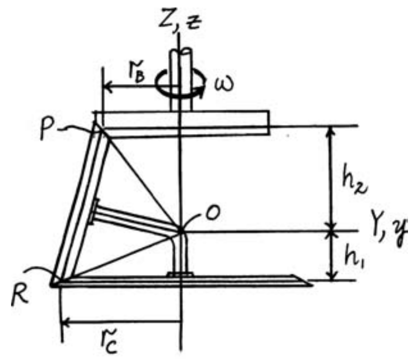
20–7. If the top gear B rotates at a constant rate of $\boldsymbol{\omega}$, determine the angular velocity of gear A, which is free to rotate about the shaft and rolls on the bottom fixed gear C.



$$\mathbf{v}_P = \omega\mathbf{k} \times (-r_B\,\mathbf{j}) = \omega r_B\,\mathbf{i}$$

Also,



$$\mathbf{v}_P = \omega_A \times (-r_B\,\mathbf{j} + h_2\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_{Ax} & \omega_{Ay} & \omega_{Az} \\ 0 & -r_B & h_2 \end{vmatrix}$$

$$= (\omega_{Ay}\,h_2 + \omega_{Az}\,r_B)\mathbf{i} - (\omega_{Ax}\,h_2)\mathbf{j} - \omega_{Ax}\,r_B\,\mathbf{k}$$

Thus,

$$\omega r_B = \omega_{Ay}\,h_2 + \omega_{Az}\,r_B \qquad \textbf{(1)}$$

$$0 = \omega_{Ax}\,h_2$$

$$0 = \omega_{Ax}\,r_B$$

$$\omega_{Ax} = 0$$

$$\mathbf{v}_R = \mathbf{0} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_{Ay} & \omega_{Az} \\ 0 & -r_C & -h_1 \end{vmatrix} = (-\omega_{Ay}\,h_1 + \omega_{Az}\,r_C)\mathbf{i}$$

$$\omega_{Ay} = \omega_{Az}\left(\frac{r_C}{h_1}\right)$$

From Eq. (1)

$$\omega r_B = \omega_{Az}\left[\left(\frac{r_C\,h_2}{h_1}\right) + r_B\right]$$

$$\omega_{Az} = \frac{r_B\,h_1\,\omega}{r_C\,h_2 + r_B\,h_1}; \qquad \omega_{Ay} = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B\,h_1\omega}{r_C\,h_2 + r_B\,h_1}\right)$$

$$\omega_A = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B\,h_1\,\omega}{r_C\,h_2 + r_B\,h_1}\right)\mathbf{j} + \left(\frac{r_B\,h_1\omega}{r_C\,h_2 + r_B\,h_1}\right)\mathbf{k} \qquad \textbf{Ans.}$$

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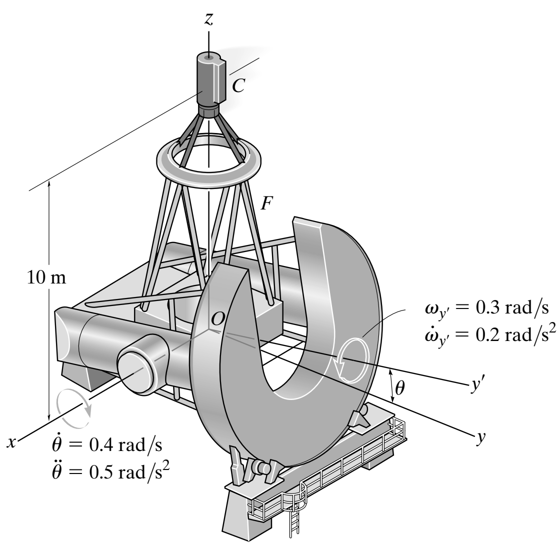
***20–8.** The telescope is mounted on the frame F that allows it to be directed to any point in the sky. At the instant $\theta = 30°$, the frame has an angular acceleration of $\alpha_{y'} = 0.2\ \text{rad/s}^2$ and an angular velocity of $\omega_{y'} = 0.3$ rad/s about the y' axis, and $\ddot{\theta} = 0.5\ \text{rad/s}^2$ while $\dot{\theta} = 0.4$ rad/s. Determine the velocity and acceleration of the observing capsule at C at this instant.



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are coincident. Thus, the angular velocity of the frame at this instant is

$$\omega = \dot{\theta} + \omega_{y'} = -0.4\mathbf{i} + (0.3\cos 30°\mathbf{j} + 0.3\sin 30°\mathbf{k})$$

$$= [-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}]\ \text{rad/s}$$

Angular Acceleration: $\omega_{y'}$ is observed to have a constant direction relative to the rotating xyz frame which rotates at $\Omega = \dot{\theta} = [-0.4\mathbf{i}]$ rad/s. With $\left(\dot{\omega}_{y'}\right)_{xyz} = \alpha_{y'} = 0.2\cos 30°\mathbf{j} + 0.2\sin 30°\mathbf{k} = [0.1732\mathbf{j} + 0.1\mathbf{k}]\ \text{rad/s}^2$, we obtain

$$\dot{\omega}_{y'} = \left(\dot{\omega}_{y'}\right)_{xyz} + \Omega \times \omega_{y'}$$

$$= (0.1732\mathbf{j} + 0.1\mathbf{k}) + (-0.4\mathbf{i}) \times (0.3\cos 30°\mathbf{j} + 0.3\sin 30°\mathbf{k})$$

$$= [0.2332\mathbf{j} - 0.003923\mathbf{k}]\ \text{rad/s}^2$$

Since $\dot{\theta}$ is always directed along the X axis $(\Omega = 0)$, then

$$\ddot{\theta} = (\ddot{\theta})_{xyz} + 0 \times \dot{\theta} = [-0.5\mathbf{i}]\ \text{rad/s}^2$$

Thus, the angular acceleration of the frame is

$$\alpha = \dot{\omega}_{y'} + \ddot{\theta} = [-0.5\mathbf{i} + 0.2332\mathbf{j} - 0.003923\mathbf{k}]\ \text{rad/s}^2$$

Velocity and Acceleration:

$$\mathbf{v}_c = \omega \times \mathbf{r}_{oc} = (-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}) \times (10\mathbf{k})$$

$$= [2.598\mathbf{i} + 4.00\mathbf{j}]\ \text{m/s} = [2.60\mathbf{i} + 4.00\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_c = \alpha \times \mathbf{r}_{oc} + \omega \times (\omega \times \mathbf{r}_{oc})$$

$$= (-0.5\mathbf{i} + 0.2332\mathbf{j} - 0.003923\mathbf{k}) \times (10\mathbf{k}) + (-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}) \times [(-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}) \times (10\mathbf{k})]$$

$$= [1.732\mathbf{i} + 5.390\mathbf{j} - 2.275\mathbf{k}]\ \text{m/s}^2$$

$$= [1.73\mathbf{i} + 5.39\mathbf{j} - 2.275\mathbf{k}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

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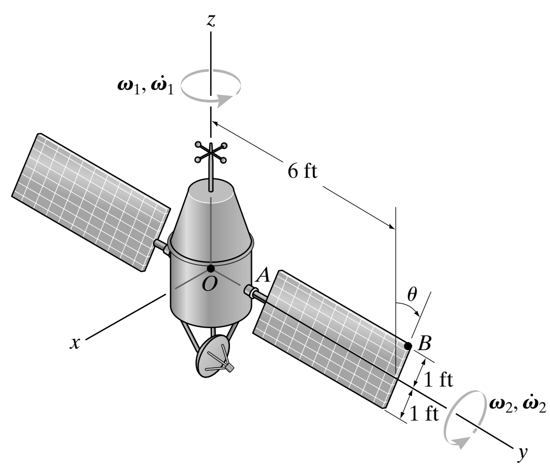
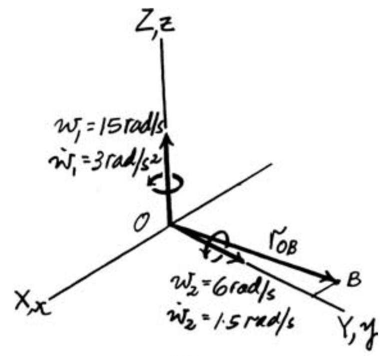
•20–9. At the instant when $\theta = 90°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 15$ rad/s and angular acceleration of $\dot{\omega}_1 = 3$ rad/s^2. Simultaneously, the solar panels rotate with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of point B on the solar panel at this instant.



Here, the solar panel rotates about a fixed point O. The XYZ fixed reference frame is set to coincide with the xyz rotating frame at the instant considered. Thus, the angular velocity of the solar panel can be obtained by vector addition of ω_1 and ω_2.

$$\omega = \omega_1 + \omega_2 = [6\mathbf{j} + 15\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

The angular acceleration of the solar panel can be determined from

$$\alpha = \dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

If we set the xyz frame to have an angular velocity of $\Omega = \omega_1 = [15\mathbf{k}]$ rad/s, then the direction of ω_2 will remain constant with respect to the xyz frame, which is along the y axis. Thus,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{j} + (15\mathbf{k} \times 6\mathbf{j}) = [-90\mathbf{i} + 1.5\mathbf{j}] \text{ rad/s}^2$$

Since ω_1 is always directed along the Z axis when $\Omega = \omega_1$, then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 = [3\mathbf{k}] \text{ rad/s}^2$$

Thus,

$$\alpha = 3\mathbf{k} + (-90\mathbf{i} + 1.5\mathbf{j})$$

$$= [-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}] \text{ rad/s}^2$$

When $\theta = 90°$, $\mathbf{r}_{OB} = [-1\mathbf{i} + 6\mathbf{j}]$ ft. Thus,

$$\mathbf{v}_B = \omega \times r_{OB} = (6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})$$

$$= [-90\mathbf{i} - 15\mathbf{j} + 6\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{OB} + \omega \times (\omega \times \mathbf{r}_{OB})$$

$$= (-90\mathbf{i} + 15\mathbf{j} + 3\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j}) + (6\mathbf{j} + 15\mathbf{k}) \times [(6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})]$$

$$= [243\mathbf{i} - 1353\mathbf{j} + 1.5\mathbf{k}] \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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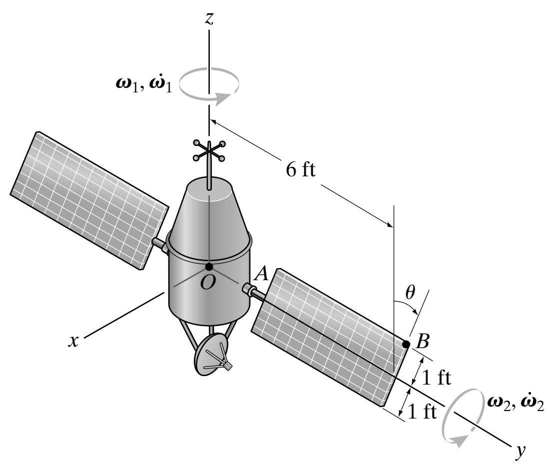
20–10. At the instant when $\theta = 90°$, the satellite's body travels in the x direction with a velocity of $\mathbf{v}_O = \{500\mathbf{i}\}$ m/s and acceleration of $\mathbf{a}_O = \{50\mathbf{i}\}$ m/s^2. Simultaneously, the body also rotates with an angular velocity of $\omega_1 = 15$ rad/s and angular acceleration of $\dot{\omega}_1 = 3$ rad/s^2. At the same time, the solar panels rotate with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 1.5$ rad/s^2 Determine the velocity and acceleration of point B on the solar panel.

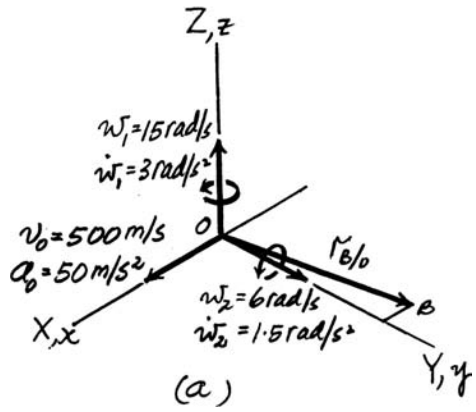


The XYZ translating reference frame is set to coincide with the xyz rotating frame at the instant considered. Thus, the angular velocity of the solar panel at this instant can be obtained by vector addition of ω_1 and ω_2.

$$\omega = \omega_1 + \omega_2 = [6\mathbf{j} + 15\mathbf{k}]\text{ rad/s}$$

The angular acceleration of the solar panel can be determined from

$$\alpha = \dot{\omega} = \omega_1 + \omega_2$$



If we set the xyz frame to have an angular velocity of $\Omega = \omega_1 = [15\mathbf{k}]$ rad/s, then the direction of ω_2 will remain constant with respect to the xyz frame, which is along the y axis. Thus,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{j} + (15\mathbf{k} \times 6\mathbf{j}) = [-90\mathbf{i} + 15\mathbf{j}]\text{ rad/s}^2$$

Since ω_1 is always directed along the Z axis when $\Omega = \omega_1$, then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 = [3\mathbf{k}]\text{ rad/s}^2$$

Thus,

$$\alpha = 3\mathbf{k} + (-90\mathbf{i} + 1.5\mathbf{j})$$
$$= [-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}]\text{ rad/s}^2$$

When $\theta = 90°$, $\mathbf{r}_{B/O} = [-1\mathbf{i} + 6\mathbf{j}]$ ft. Since the satellite undergoes general motion, then

$$\mathbf{v}_B = \mathbf{v}_O + \omega \times r_{B/O} = (500\mathbf{i}) + (6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})$$
$$= [410\mathbf{i} - 15\mathbf{j} + 6\mathbf{k}]\text{ ft/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} + \omega \times (\omega \times \mathbf{r}_{B/O})$$
$$= 50\mathbf{i} + (-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j}) + (6\mathbf{j} + 15\mathbf{k}) \times [(6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})]$$
$$= [293\mathbf{i} - 1353\mathbf{j} + 1.5\mathbf{k}]\text{ ft/s}^2 \qquad \textbf{Ans.}$$

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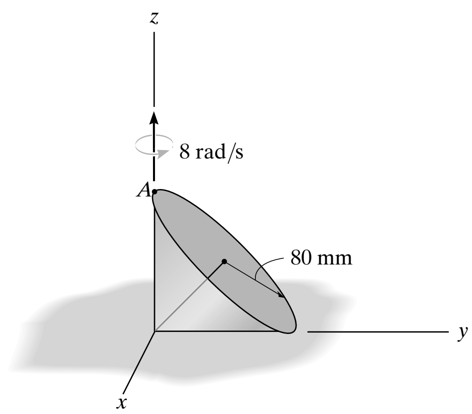
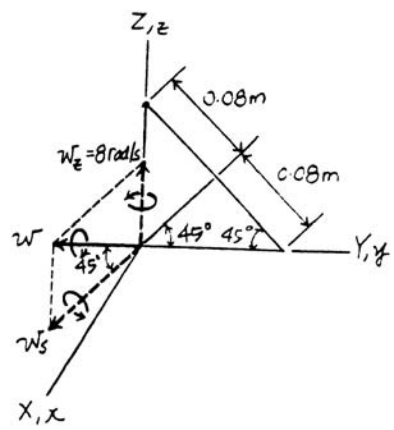
20–11. The cone rolls in a circle and rotates about the z axis at a constant rate $\omega_z = 8$ rad/s. Determine the angular velocity and angular acceleration of the cone if it rolls without slipping. Also, what are the velocity and acceleration of point A?



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant (with reference to X, Y, Z) can be expressed in terms of **i, j, k** components. Since the disk rolls without slipping, then its angular velocity $\omega = \omega_s + \omega_z$ is always directed along the instantaneuos axis of zero velocity (y axis). Thus,



$$\omega = \omega_s + \omega_z$$

$$-\omega\mathbf{j} = -\omega_s \cos 45°\mathbf{j} - \omega_s \sin 45°\mathbf{k} + 8\mathbf{k}$$

Equating **k** and **j** components, we have

$$0 = -\omega_s \sin 45° + 8 \qquad \omega_s = 11.31 \text{ rad/s}$$

$$-\omega = -11.13 \cos 45° \qquad \omega = 8.00 \text{ rad/s}$$

Thus, $\qquad \omega = \{-8.00\mathbf{j}\}$ rad/s **Ans.**

Angular Acceleration: The angular acceleration α will be determined by investigating the time rate of change of *angular velocity* with respect to the fixed XYZ frame. Since ω always lies in the fixed X–Y plane, then $\omega = \{-8.00\mathbf{j}\}$ rad/s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{8\mathbf{k}\}$ rad/s. Applying Eq. 20–6 with $(\dot{\omega})_{xyz} = \mathbf{0}$, we have

$$\alpha = \dot{\omega} = (\dot{\omega})_{xyz} + \omega_z \times \omega = \mathbf{0} + 8\mathbf{k} \times (-8.00\mathbf{j}) = \{64.0\mathbf{i}\} \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{0.16 \cos 45°\mathbf{k}\}$ m $= \{0.1131\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-8.00\mathbf{j}) \times (0.1131\mathbf{k}) = \{-0.905\mathbf{i}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (64.0\mathbf{i}) \times (0.1131\mathbf{k}) + (-8.00\mathbf{j}) \times [(-8.00\mathbf{j}) \times (0.1131\mathbf{k})]$$

$$= \{-7.24\mathbf{j} - 7.24\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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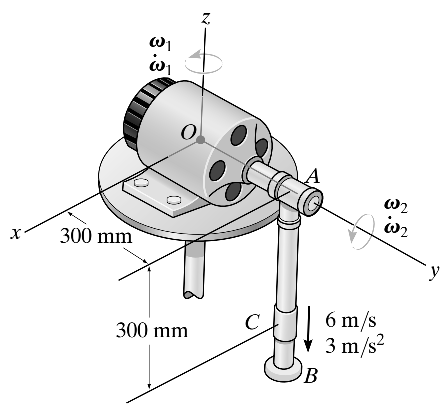
***20–12.** At the instant shown, the motor rotates about the z axis with an angular velocity of $\omega_1 = 3$ rad/s and angular acceleration of $\dot{\omega}_1 = 1.5$ rad/s^2. Simultaneously, shaft OA rotates with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 3$ rad/s^2, and collar C slides along rod AB with a velocity and acceleration of 6 m/s and 3 m/s^2. Determine the velocity and acceleration of collar C at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [3\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = [1.5\mathbf{k}] \text{ rad/s}^2$$

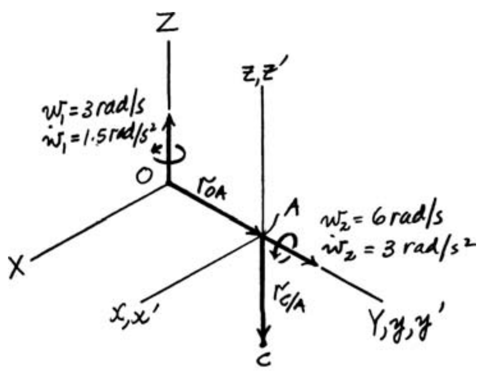
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (3\mathbf{k}) \times (0.3\mathbf{j}) = [-0.9\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega \times \mathbf{r}_{OA})$$

$$= (1.5\mathbf{k}) \times (0.3\mathbf{j}) + (3\mathbf{k}) \times (3\mathbf{k} \times 0.3\mathbf{j})$$

$$= [-0.45\mathbf{i} - 2.7\mathbf{j}] \text{ m/s}^2$$



(a)

In order to determine the motion of point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [6\mathbf{j}]$ rad/s, the direction of $(\mathbf{r}_{C/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{C/A})_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{C/A})_{xyz}\right]$$

$$= (-6\mathbf{k}) + 6\mathbf{j} \times (-0.3\mathbf{k})$$

$$= [-1.8\mathbf{i} - 6\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [3\mathbf{j}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A})_{xyz} = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{C/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{xyz}$$

$$= [(-3\mathbf{k}) + 6\mathbf{j} \times (-6\mathbf{k})] + (3\mathbf{j}) \times (-0.3\mathbf{k}) + 6\mathbf{j} \times (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-72.9\mathbf{i} + 7.8\mathbf{k}] \text{ m/s}$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$= (-0.9\mathbf{i}) + 3\mathbf{k} \times (-0.3\mathbf{k}) + (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-2.7\mathbf{i} - 6\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (-0.45\mathbf{i} - 2.7\mathbf{j}) + 1.5\mathbf{k} \times (-0.3\mathbf{k}) + (3\mathbf{k}) \times [(3\mathbf{k}) \times (-0.3\mathbf{k})] + 2(3\mathbf{k}) \times (-1.8\mathbf{i} - 6\mathbf{k}) + (-72.9\mathbf{i} + 7.8\mathbf{k})$$

$$= [-73.35\mathbf{i} - 13.5\mathbf{j} + 7.8\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

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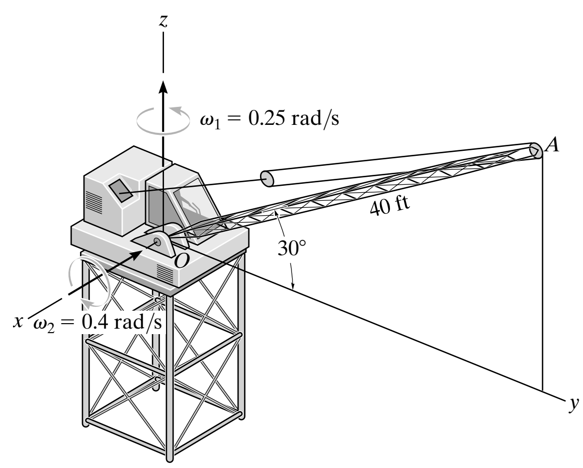
•20–13. At the instant shown, the tower crane rotates about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA rotates downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the end of the boom at this instant.



$\omega = \omega_1 \cdot \omega_2 = \{-0.4\,\mathbf{i} + 0.25\mathbf{k}\}$ rad/s

$\Omega = \{0.25\,\mathbf{k}\}$ rad/s

$\omega = \omega_{1\cdot 2} + \Omega \times \omega = (-0.8\,\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\,\mathbf{i} + 0.25\mathbf{k})$

$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\}\text{ rad/s}^2$

$\mathbf{r}_A = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\}$ ft

$\mathbf{v}_A = \omega \times \mathbf{r}_A = 1 - 0.4\,\mathbf{i} + 0.25\,\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$

$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}$ft/s **Ans.**

$\mathbf{a}_A = \alpha \cdot \mathbf{r}_A + \omega \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$

$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}\text{ft/s}^2$ **Ans.**

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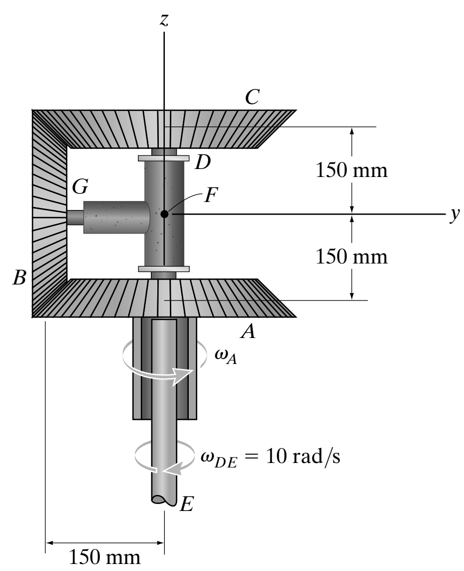
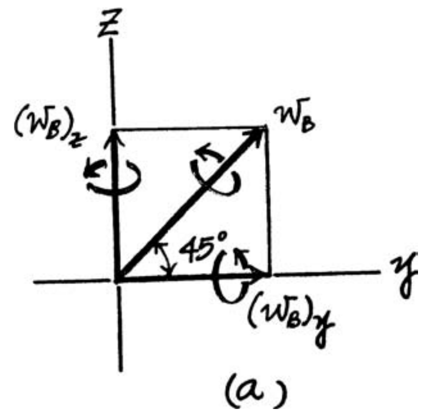
20–14. Gear C is driven by shaft DE, while gear B spins freely about its axle GF, which precesses freely about shaft DE. If gear A is held fixed ($\omega_A = 0$), and shaft DE rotates with a constant angular velocity of $\omega_{DE} = 10$ rad/s, determine the angular velocity of gear B.



Since gear C rotates about the fixed axis ($z$axis), the velocity of the contact point P between gears B and C is

$$\mathbf{v}_P = \omega_{DE} \times \mathbf{r}_C = (10\mathbf{k}) \times (-0.15\mathbf{j}) = [1.5\mathbf{i}] \text{ m/s}$$



Here, gear B spins about its axle with an angular velocity of $(\omega_B)_y$ and precesses about shaft DE with an angular velocity of $(\omega_B)_z$. Thus, the angular velocity of gear B is

$$\omega_B = (\omega_B)_y\,\mathbf{j} + (\omega_B)_z\,\mathbf{k}$$

Here, $\mathbf{r}_{FP} = [-0.15\mathbf{j} + 0.15\mathbf{k}]$ m. Thus,

$$\mathbf{v}_P = \omega_B \times \mathbf{r}_{FP}$$

$$1.5\mathbf{i} = \left[(\omega_B)_y\,\mathbf{j} + (\omega_B)_z\,\mathbf{k}\right] \times (-0.15\mathbf{j} + 0.15\mathbf{k})$$

$$1.5\mathbf{i} = \left[0.15(\omega_B)_y - (-0.15)(\omega_B)_z\right]\mathbf{i}$$

$$1.5 = 0.15(\omega_B)_y + 0.15(\omega_B)_z$$

$$(\omega_B)_y + (\omega_B)_z = 10 \qquad \textbf{(1)}$$

Since gear A is held fixed, ω_B will be directed along the instantaneous axis of zero velocity, which is along the line where gears A and B mesh. From the geometry of Fig. a,

$$\frac{(\omega_B)_z}{(\omega_B)_y} = \tan 45^\circ \qquad (\omega_B)_z = (\omega_B)_y \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$(\omega_B)_y = (\omega_B)_z = 5 \text{ rad/s}$$

Thus,

$$\omega_B = [5\mathbf{j} + 5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

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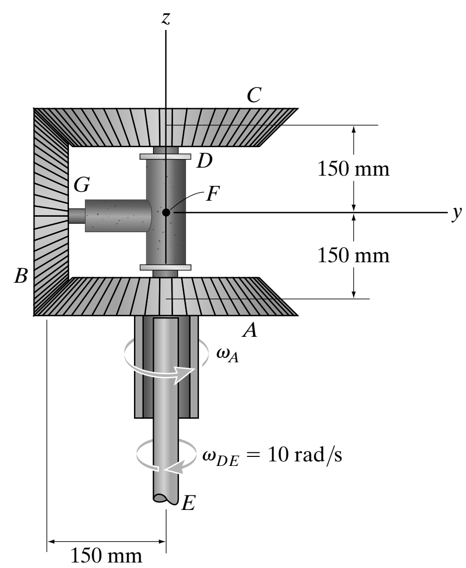
20–15. Gear C is driven by shaft DE, while gear B spins freely about its axle GF, which precesses freely about shaft DE. If gear A is driven with a constant angular velocity of $\omega_A = 5$ rad/s and shaft DE rotates with a constant angular velocity of $\omega_{DE} = 10$ rad/s, determine the angular velocity of gear B.



Since gears A and C rotate about the fixed axis (z axis), the velocity of the contact point P between gears B and C and point P' between gears A and B are

$$\mathbf{v}_P = \omega_{DE} \times \mathbf{r}_C = (10\mathbf{k}) \times (-0.15\mathbf{j}) = [1.5\mathbf{i}] \text{ m/s}$$

and

$$\mathbf{v}_{P'} = \omega_A \times \mathbf{r}_A = (-5\mathbf{k}) \times (-0.15\mathbf{j}) = [-0.75\mathbf{i}] \text{ m/s}$$

Gear B spins about its axle with an angular velocity of $(\omega_B)_y$ and precesses about shaft DE with an angular velocity of $(\omega_B)_z$. Thus, the angular velocity of gear B is

$$\omega_B = (\omega_B)_y\mathbf{j} + (\omega_B)_z\,\mathbf{k}$$

Here, $r_{FP} = [-0.15\mathbf{j} + 0.15\mathbf{k}]$ m and $r_{FP'} = [-0.15\mathbf{j} - 0.15\mathbf{k}]$. Thus,

$$\mathbf{v}_P = \omega_B \times \mathbf{r}_{FP}$$

$$1.5\mathbf{i} = \left[(\omega_B)_y\mathbf{j} + (\omega_B)_z\,\mathbf{k}\right] \times (-0.15\mathbf{j} + 0.15\mathbf{k})$$

$$1.5\mathbf{i} = \left[0.15(\omega_B)_y + 0.15(\omega_B)_z\right]\mathbf{i}$$

so that

$$1.5 = 0.15(\omega_B)_y + 0.15(\omega_B)_z$$

$$(\omega_B)_y + (\omega_B)_z = 10 \qquad \textbf{(1)}$$

and

$$\mathbf{v}_{P'} = \omega_B \times \mathbf{r}_{FP'}$$

$$-0.75\mathbf{i} = \left[(\omega_B)_y\mathbf{j} + (\omega_B)_z\mathbf{k}\right] \times (-0.15\mathbf{j} - 0.15\mathbf{k})$$

$$-0.75\mathbf{i} = \left[0.15(\omega_B)_z - 0.15(\omega_B)_y\right]\mathbf{i}$$

Thus,

$$-0.75 = 0.15(\omega_B)_z - 0.15(\omega_B)_y$$

$$(\omega_B)_y - (\omega_B)_z = 5 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), we obtain

$$(\omega_B)_y = 7.5 \text{ rad/s}$$

$$(\omega_B)_z = 2.5 \text{ rad/s}$$

Thus,

$$\omega_B = [7.5\mathbf{j} + 2.5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

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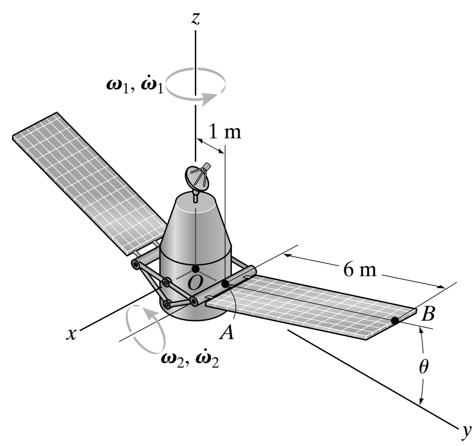
***20–16.** At the instant $\theta = 0°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 20$ rad/s, and it has an angular acceleration of $\dot{\omega}_1 = 5$ rad/s^2. Simultaneously, the solar panels rotate with an angular velocity of $\omega_2 = 5$ rad/s and angular acceleration of $\dot{\omega}_2 = 3$ rad/s^2. Determine the velocity and acceleration of point B located at the end of one of the solar panels at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [20\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_1 = [5\mathbf{k}] \text{ rad/s}^2$$

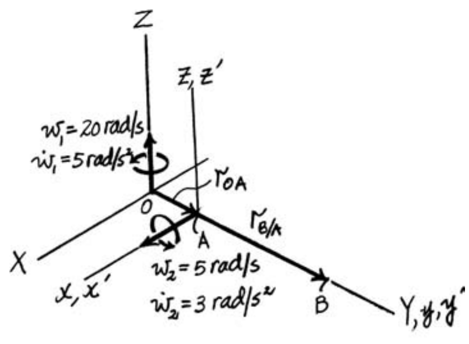
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (20\mathbf{k}) \times (1\mathbf{j}) = [-20\mathbf{i}] \text{ m/s}$$

and

$$\begin{aligned} \mathbf{a}_A &= \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA}) \\ &= (5\mathbf{k}) \times (1\mathbf{j}) + (20\mathbf{i}) \times \left[(20\mathbf{i}) \times (1\mathbf{j})\right] \\ &= [-5\mathbf{i} - 400\mathbf{j}] \text{ m/s}^2 \end{aligned}$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,



(a)

$$\begin{aligned} (\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} &= \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right] \\ &= 0 + (5\mathbf{i}) \times (6\mathbf{j}) \\ &= [30\mathbf{k}] \text{ m/s} \end{aligned}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [3\mathbf{i}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$\begin{aligned} (\mathbf{a}_{B/A})_{xyz} = (\ddot{\mathbf{r}}_{B/A})_{xyz} &= \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz} \\ &= [\mathbf{0} + \mathbf{0}] + (3\mathbf{i}) \times (6\mathbf{j}) + (5\mathbf{i}) \times (30\mathbf{k}) \\ &= [-150\mathbf{j} + 18\mathbf{k}] \text{ m/s}^2 \end{aligned}$$

Thus,

$$\begin{aligned} \mathbf{v}_B &= \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz} \\ &= (-20\mathbf{i}) + (20\mathbf{k}) \times (6\mathbf{j}) + (30\mathbf{k}) \\ &= [-140\mathbf{i} + 30\mathbf{k}] \text{ m/s} \end{aligned}$$ **Ans.**

and

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times r_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz} \\ &= (-5\mathbf{i} - 400\mathbf{j}) + (5\mathbf{k}) \times (6\mathbf{j}) + (20\mathbf{k}) \times \left[(20\mathbf{k}) \times (6\mathbf{j})\right] + 2(20\mathbf{k}) \times 30\mathbf{k} + (-150\mathbf{j} + 18\mathbf{k}) \\ &= [-35\mathbf{i} - 2950\mathbf{j} + 18\mathbf{k}] \text{ m/s}^2 \end{aligned}$$ **Ans.**

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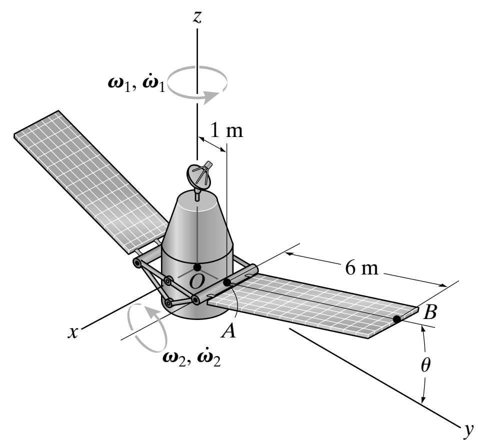
•**20–17.** At the instant $\theta = 30°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 20$ rad/s, and it has an angular acceleration of $\dot{\omega}_1 = 5$ rad/s^2. Simultaneously, the solar panels rotate with a constant angular velocity of $\omega_2 = 5$ rad/s. Determine the velocity and acceleration of point B located at the end of one of the solar panels at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [20\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_1 = [5\mathbf{k}] \text{ rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

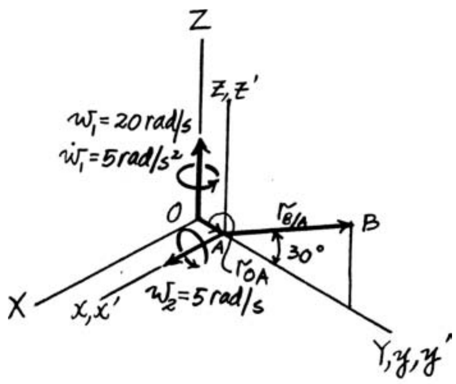
$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (20\mathbf{k}) \times (1\mathbf{j}) = [-20\mathbf{i}] \text{ m/s}$$

and

$$a_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (5\mathbf{k}) \times (1\mathbf{j}) + (20\mathbf{k}) \times [(20\mathbf{k}) \times (1\mathbf{j})]$$

$$= [-5\mathbf{i} - 400\mathbf{j}] \text{ m/s}^2$$



(a)

In order to determined the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [5\mathbf{k}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (5\mathbf{i}) \times (6\cos 30°\,\mathbf{j} + 6\sin 30°\,\mathbf{k})$$

$$= [-15\mathbf{j} + 25.98\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A})_{xyz} = \left(\ddot{\mathbf{r}}_{B/A}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{B/A}\right)_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [\mathbf{0} + \mathbf{0}] + \mathbf{0} + (5\mathbf{i}) \times (-15\mathbf{j} + 25.98\mathbf{k})$$

$$= [-129.90\mathbf{j} - 75\mathbf{k}] \text{ m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-20\mathbf{i}) + (20\mathbf{k}) \times (6\cos 30°\,\mathbf{j} + 6\sin 30°\mathbf{k}) + (-15\mathbf{j} + 25.98\mathbf{k})$$

$$= [-124\mathbf{i} - 15\mathbf{j} + 26.0\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-5\mathbf{i} - 400\mathbf{j}) + (5\mathbf{k}) \times (6\cos 30°\mathbf{j} + 6\sin 30°\,\mathbf{k}) + (20\mathbf{k}) \times \left[(20\mathbf{k}) \times (6\cos 30°\,\mathbf{j} + 6\sin 30°\mathbf{k})\right]$$

$$+2(20\mathbf{k}) \times (-15\mathbf{j} + 25.98\mathbf{k}) + (-129.90\mathbf{j} - 75\mathbf{k})$$

$$= [569\mathbf{i} - 2608\mathbf{j} - 75\mathbf{k}] \text{m/s}^2 \qquad \textbf{Ans.}$$

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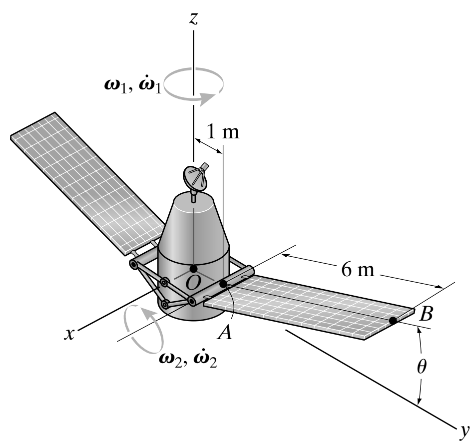
20–18. At the instant $\theta = 30^\circ$, the satellite's body is rotating with an angular velocity of $\omega_1 = 20$ rad/s, and it has an angular acceleration of $\dot{\omega}_1 = 5$ rad/s^2. At the same instant, the satellite travels in the x direction with a velocity of $\mathbf{v}_O = \{5000\mathbf{i}\}$ m/s, and it has an acceleration of $\mathbf{a}_O = \{500\mathbf{i}\}$ m/s^2. Simultaneously, the solar panels rotate with a constant angular speed of $\omega_2 = 5$ rad/s. Determine the velocity and acceleration of point B located at the end of one of the solar panels at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [20\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_1 = [5\mathbf{k}] \text{ rad/s}^2$$

Since the body of the satellite undergoes general motion, the motion of points O and A can be related using

$$\mathbf{v}_A = \mathbf{v}_O + \omega_1 \times \mathbf{r}_{A/O} = 5000\mathbf{i} + (20\mathbf{k}) \times (1\mathbf{j}) = [4980\mathbf{i}] \text{ m/s}$$

and

$$\begin{aligned}
\mathbf{a}_A &= \mathbf{a}_O + \dot{\omega}_1 \times \mathbf{r}_{A/O} + \omega_1 \times (\omega_1 \times \mathbf{r}_{O/A}) \\
&= (500\mathbf{i}) + (5\mathbf{k}) \times (1\mathbf{j}) + (20\mathbf{k}) \times [(20\mathbf{k}) \times (1\mathbf{j})] \\
&= [495\mathbf{i} - 400\mathbf{j}] \text{ m/s}^2
\end{aligned}$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = [5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$\begin{aligned}
(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} &= \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right] \\
&= \mathbf{0} + (5\mathbf{i}) \times (6\cos 30^\circ\,\mathbf{j} + 6\sin 30^\circ\,\mathbf{k}) \\
&= [-15\mathbf{j} + 25.98\mathbf{k}] \text{ m/s}
\end{aligned}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

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20–18. Continued

$$(\mathbf{a}_{B/A})_{xyz} = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [0 + 0] + 0 + (5\mathbf{i}) \times (-15\mathbf{j} + 25.98\mathbf{k})$$

$$= [-129.90\mathbf{j} - 75\mathbf{k}]\ \text{m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (4980\mathbf{i}) + (20\mathbf{k}) \times (6\cos 30°\,\mathbf{j} + 6\sin 30°\mathbf{k}) + (-15\mathbf{j} + 25.98\mathbf{k})$$

$$= [4876\mathbf{i} - 15\mathbf{j} + 26.0\mathbf{k}]\text{m/s}$$ **Ans.**

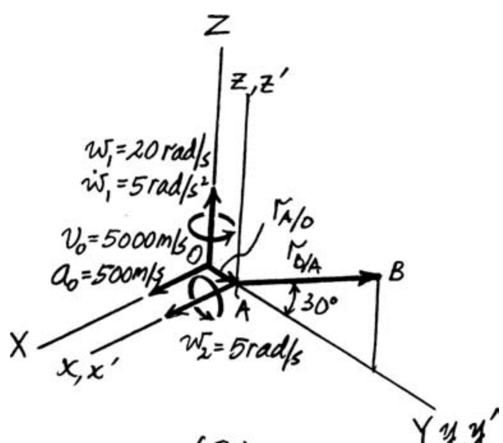
and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (495\mathbf{i} - 400\mathbf{j}) + (5\mathbf{k})\times(6\cos 30°\mathbf{j} + 6\sin 30°\,\mathbf{k}) + (20\mathbf{k})\times\left[(20\mathbf{k})\times(6\cos 30°\,\mathbf{j} + 6\sin 30°\mathbf{k})\right]$$

$$+ 2(20\mathbf{k})\times(-15\mathbf{j} + 25.98\mathbf{k}) + (-129.90\mathbf{j} - 75\mathbf{k})$$

$$= [1069\mathbf{i} - 2608\mathbf{j} - 75\mathbf{k}]\ \text{m/s}^2$$ **Ans.**



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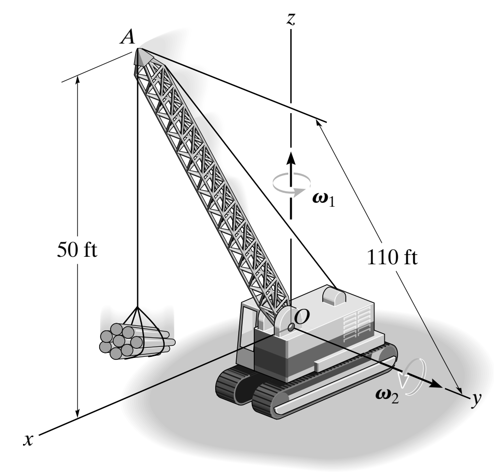
20–19. The crane boom OA rotates about the z axis with a constant angular velocity of $\omega_1 = 0.15$ rad/s, while it is rotating downward with a constant angular velocity of $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



$$\dot{\omega} = \omega_1 + \omega_2 = \{0.2\mathbf{j} + 0.15\mathbf{k}\} \text{ rad/s}$$

$$\omega = \omega_1 - \omega_2$$

Let the x, y, z axes rotate at $\Omega = \omega_1$, then

$$\omega = \omega = |\omega| + \omega_1 \times \omega_2$$

$$\omega = \mathbf{0} + 0.15\mathbf{k} \times \mathbf{0.2j} = \{-0.03\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = \left[\sqrt{(110)^2 - (50)^2}\right]\mathbf{i} + 50\mathbf{k} = \{97.98\mathbf{i} + 50\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_A = \omega_A\,\mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 97.98 & 0 & 50 \end{vmatrix}$$

$$\mathbf{v}_A = \{10\mathbf{i} + 117\mathbf{j} - 19.6\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha + \mathbf{r}_A + \omega + \mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.03 & 0 & 0 \\ 97.98 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 10 & 14.7 & -19.6 \end{vmatrix}$$

$$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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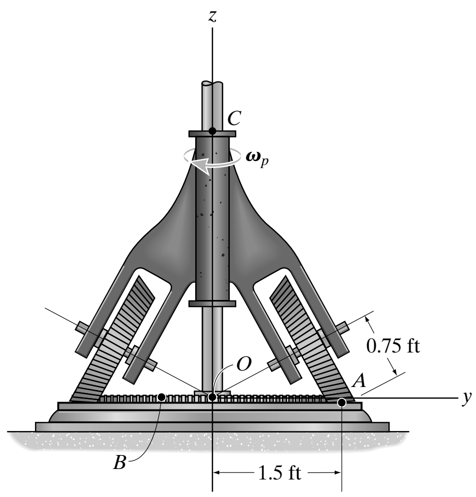
***20–20.** If the frame rotates with a constant angular velocity of $\omega_p = \{-10\mathbf{k}\}$ rad/s and the horizontal gear B rotates with a constant angular velocity of $\omega_B = \{5\mathbf{k}\}$ rad/s, determine the angular velocity and angular acceleration of the bevel gear A.



If the bevel gear A spins about its axle with an angular velocity of ω_S, then its angular velocity is

$$\omega = \omega_s + \omega_p$$

$$= (\omega_s \cos 30^\circ\,\mathbf{j} + \omega_s \sin 30^\circ\,\mathbf{k}) - 10\,\mathbf{k}$$

$$= 0.8660\omega_s\,\mathbf{j} + (0.5\omega_s - 10)\mathbf{k}$$

Since gear B rotates about the fixed axis ($z$axis), the velocity of the contact point P between gears A and B is

$$\mathbf{v}_p = \omega_B \times \mathbf{r}_B = (5\mathbf{k}) \times (1.5\mathbf{j}) = [-7.5\mathbf{i}]\text{ft/s}$$

Since gear A rotates about a fixed point O then $r_{OP} = [1.5\mathbf{j}]$ ft. Also,

$$\mathbf{v}_p = \omega \times \mathbf{r}_{OP}$$

$$-7.5\mathbf{i} = [0.8660\omega_s\,\mathbf{j} + (0.5\omega_s - 10)\mathbf{k}] \times (1.5\mathbf{j})$$

$$-7.5\mathbf{i} = -1.5(0.5\omega_s - 10)\mathbf{i}$$

$$-7.5 = -1.5(0.5\omega_s - 10)$$

$$\omega_s = 30 \text{ rad/s}$$

Thus,

$$\omega_s = 30\cos 30^\circ\,\mathbf{j} + 30\sin 30^\circ\mathbf{k} = [25.98\mathbf{j} + 15\mathbf{k}] \text{ rad/s}$$

$$\omega = 0.8660(30)\mathbf{j} + [0.5(30) - 10]\mathbf{k} = [26.0\mathbf{j} + 5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

We will set the XYZ fixed reference frame to coincide with the xyz rotating reference frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_p = [-10\mathbf{k}]$ rad/s, the direction of ω_s will remain constant with respect to the xyz frame. Thus,

$$\dot{\omega}_s = (\dot{\omega}_S)_{xyz} + \omega_p \times \omega_s$$

$$= \mathbf{0} + (-10\mathbf{k}) \times (25.98\mathbf{j} + 15\mathbf{k})$$

$$= [259.81\mathbf{i}] \text{ rad/s}^2$$

If $\Omega = \omega_p$, then ω_p is always directed along the z axis. Thus,

$$\dot{\omega}_p = \left(\dot{\omega}_p\right)_{xyz} + \omega_p \times \omega_p = \mathbf{0} + \mathbf{0} = 0$$

Thus,

$$\alpha = \dot{\omega} = \dot{\omega}_s + \dot{\omega}_p = (259.81\mathbf{i}) + \mathbf{0} = [260\mathbf{i}] \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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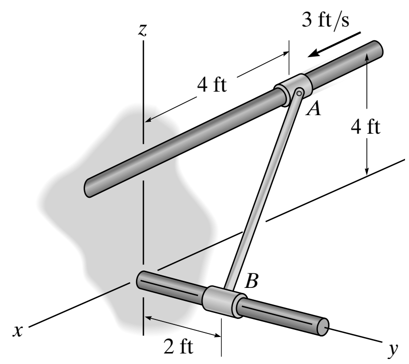
•**20–21.** Rod AB is attached to collars at its ends by ball-and-socket joints. If the collar A has a velocity of $v_A = 3$ ft/s, determine the angular velocity of the rod and the velocity of collar B at the instant shown. Assume the angular velocity of the rod is directed perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{[0 - (-4)]\,\mathbf{i} + (2 - 0)\,\mathbf{j} + (0 - 4)\mathbf{k}\}$ ft $= \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}$ ft, $\mathbf{v}_A = \{3\mathbf{i}\}$ ft/s, $\mathbf{v}_B = v_B\,\mathbf{j}$ and $\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$v_B\,\mathbf{j} = 3\mathbf{i} + \left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}\right) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$v_B\,\mathbf{j} = \left(3 - 4\omega_y - 2\omega_z\right)\mathbf{i} + (4\omega_x + 4\omega_z)\,\mathbf{j} + \left(2\omega_x - 4\omega_y\right)\mathbf{k}$$

Equating **i**, **j** and **k** components, we have

$$3 - 4\omega_y - 2\omega_z = 0 \qquad \textbf{[1]}$$

$$v_B = 4\omega_x + 4\omega_z \qquad \textbf{[2]}$$

$$2\omega_x - 4\omega_y = 0 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of the rod AB. then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}\right) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) = 0$$

$$4\omega_x + 2\omega_y - 4\omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 6.00 \text{ ft/s} \qquad \omega_x = 0.6667 \text{ rad/s}$$

$$\omega_y = 0.3333 \text{ rad/s} \qquad \omega_z = 0.8333 \text{ rad/s}$$

Thus,

$$\mathbf{v}_B = \{6.00\mathbf{j}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\omega = \{0.667\mathbf{i} + 0.333\mathbf{j} + 0.833\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

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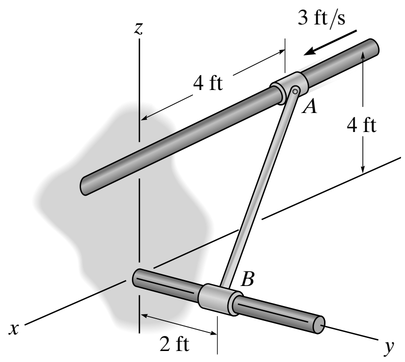
20–22. The rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has an acceleration of $\mathbf{a}_A = \{8\mathbf{i}\}\ \text{ft/s}^2$ and a velocity $\mathbf{v}_A = \{3\mathbf{i}\}\ \text{ft/s}$, determine the angular acceleration of the rod and the acceleration of collar B at the instant shown. Assume the angular acceleration of the rod is directed perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{[0-(-4)]\,\mathbf{i} + (2-0)\,\mathbf{j} + (0-4)\mathbf{k}\}$ ft $= \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}$ ft, $\mathbf{v}_A = \{3\mathbf{i}\}$ ft/s, $\mathbf{v}_B = v_B\,\mathbf{j}$ and $\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7. we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$v_B\,\mathbf{j} = 3\mathbf{i} + \left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}\right) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$v_B\,\mathbf{j} = \left(3 - 4\omega_y - 2\omega_z\right)\mathbf{i} + (4\omega_x + 4\omega_z)\,\mathbf{j} + \left(2\omega_x - 4\omega_y\right)\mathbf{k}$$

Equating $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components, we have

$$3 - 4\omega_y - 2\omega_z = 0 \qquad \textbf{[1]}$$

$$v_B = 4\omega_x + 4\omega_z \qquad \textbf{[2]}$$

$$2\omega_x - 4\omega_y = 0 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}\right) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) = 0$$

$$4\omega_x + 2\omega_y - 4\omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 6.00\ \text{ft/s} \qquad \omega_x = 0.6667\ \text{rad/s}$$

$$\omega_y = 0.3333\ \text{rad/s} \qquad \omega_z = 0.8333\ \text{rad/s}$$

Thus, $\omega = \{0.6667\mathbf{i} + 0.3333\mathbf{j} + 0.8333\mathbf{k}\}$ rad/s

Acceleration Equation: With $\alpha = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$ and the result obtained above, applying Eq. 20–8, we have

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A})$$

$$a_B\,\mathbf{j} = 8\,\mathbf{i} + \left(\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}\right) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

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20–22. Continued

$$+(0.6667\mathbf{i} + 0.3333\mathbf{j} + 0.8333\mathbf{k}) \times [(0.6667\mathbf{i} + 0.3333\mathbf{j} + 0.8333\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$a_B\,\mathbf{j} = \left(3 - 4\alpha_y - 2\alpha_z\right)\mathbf{i} + (-2.50 + 4\alpha_x + 4\alpha_z)\mathbf{j} + \left(5 + 2\alpha_x - 4\alpha_y\right)\mathbf{k}$$

Equating **i**, **j**, **k** components, we have

$$3 - 4\alpha_y - 2\alpha_z = 0 \qquad \textbf{[5]}$$

$$a_B = -2.50 + 4\alpha_x + 4\alpha_z \qquad \textbf{[6]}$$

$$5 + 2\alpha_x - 4\alpha_y = 0 \qquad \textbf{[7]}$$

If α is specified acting *perpendicular* to the axis of rod AB, then

$$\alpha \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_2\,\mathbf{k}\right)\cdot(4\mathbf{i} + 2 - 4\mathbf{k}) = 0$$

$$4\alpha_x + 2\alpha_y - 4\alpha_z = 0 \qquad \textbf{[8]}$$

Solving Eqs. [5], [6], [7] and [8] yields

$$a_B = -6.50 \text{ ft/s}^2 \qquad \alpha_x = -0.7222 \text{ rad/s}^2$$

$$\alpha_y = 0.8889 \text{ rad/s} \qquad \alpha_z = -0.2778 \text{ rad/s}^2$$

Thus,

$$\mathbf{a}_B = \{-6.50\mathbf{j}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$\alpha = \{-0.722\mathbf{i} + 0.889\mathbf{j} - 0.278\mathbf{k}\}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

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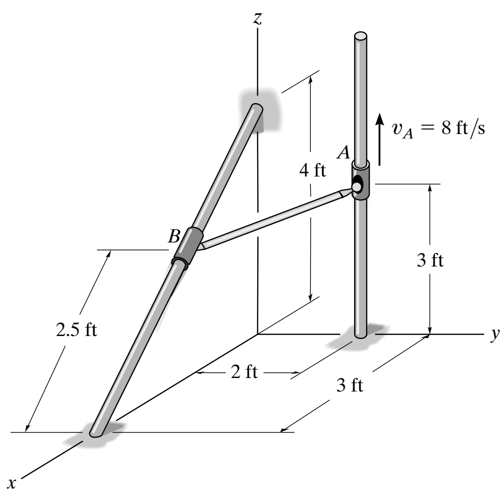
20–23. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with a velocity of $\mathbf{v}_A = \{8\mathbf{k}\}$ ft/s, determine the angular velocity of the rod and the speed of collar B at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the rod.



$\mathbf{v}_B = \{8\mathbf{k}\}$ ft/s $\qquad \mathbf{v}_B = -\frac{3}{5}v_B\mathbf{i} + \frac{4}{5}v_B\,\mathbf{k} \qquad \omega_{AB} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$-\frac{3}{5}v_B\,\mathbf{i} + \frac{4}{5}v_B\mathbf{k} = 8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.5 & -2 & -1 \end{vmatrix}$$

Equating **i**, **j**, and **k**

$$-\omega_z + 2\omega_z = -\frac{3}{5}v_B \qquad \textbf{(1)}$$

$$\omega_z + 1.5\omega_z = 0 \qquad \textbf{(2)}$$

$$\omega - 2\omega_x - 1.5\omega_x = \frac{4}{5}v_B \qquad \textbf{(3)}$$

Since ω_{AB} is perpendicular to the axis of the rod,

$$\omega_{AB} \cdot \mathbf{r}_{B/A} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$$

$$1.5\omega_x - 2\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs.(1) to (4) yields:

$\omega_x = 1.1684$ rad/s $\qquad \omega_y = 1.2657$rad/s $\qquad \omega_z = -0.7789$ rad/s

$$v_B = 4.71\text{ ft/s} \qquad \textbf{Ans.}$$

Then $\omega_{AB} = \{1.17\mathbf{i} + 1.27\mathbf{j} - 0.779\mathbf{k}\}$ rad/s **Ans.**

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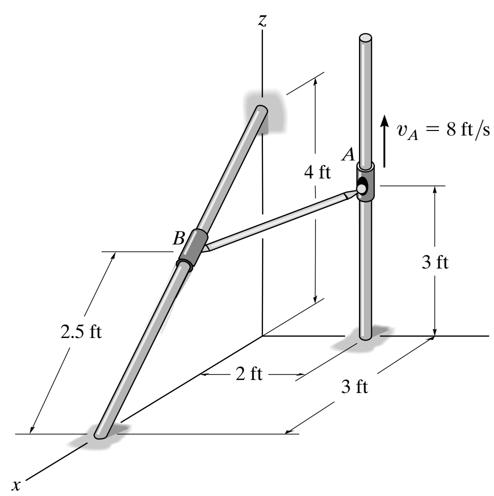
***20–24.** Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with an acceleration of $\mathbf{a}_A = \{4\mathbf{k}\}\text{ft/s}^2$, determine the angular acceleration of rod AB and the magnitude of acceleration of collar B. Assume that the rod's angular acceleration is directed perpendicular to the rod.



From Prob. 20–23

$\omega_{AB} = \{1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}\}$ rad/s

$\mathbf{r}_{B\cdot A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}$ ft

$\alpha_{AB} = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$

$\mathbf{a}_A = \{4\mathbf{k}\}\ \text{ft/s}^2 \qquad \mathbf{a}_B = -\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{k}$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

$$-\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{k} = 4\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})$$

$$+(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k})$$

$$\times\left[(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})\right]$$

Equating **i**, **j**, and **k** components

$$-\alpha_y + 2\alpha_z - 5.3607 = -\frac{3}{5}a_B \qquad \textbf{(1)}$$

$$\alpha_x + 1.5\alpha_z + 7.1479 = 0 \qquad \textbf{(2)}$$

$$7.5737 - 2\alpha_x - 1.5\alpha_y = \frac{4}{5}a \qquad \textbf{(3)}$$

Since α_{AB} is perpendicular to the axis of the rod,

$\alpha_{AB} \cdot \mathbf{r}_{B/A} = (\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$

$$1.5\alpha_x - 2\alpha_y - \alpha_z = 0 \qquad \textbf{(4)}$$

Solving Eqs.(1) to (4) yields:

$\alpha_x = -2.7794\ \text{rad/s}^2 \qquad \alpha_y = -0.6285\text{rad/s}^2 \qquad \alpha_z = -2.91213\ \text{rad/s}^2$

$a_B = 17.6\ \text{ft/s}^2$ **Ans.**

Then $\alpha_{AB} = \{-2.78\mathbf{i} - 0.628\mathbf{j} - 2.91\mathbf{k}\}\ \text{rad/s}^2$ **Ans.**

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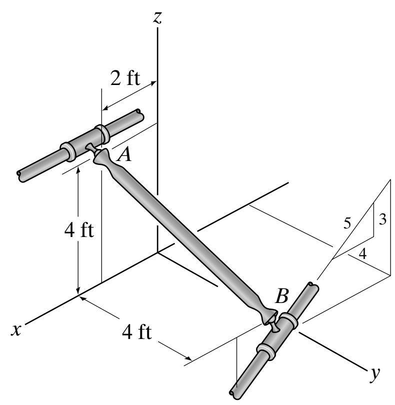
•20–25. If collar A moves with a constant velocity of $\mathbf{v}_A = \{10\mathbf{i}\}$ ft/s, determine the velocity of collar B when rod AB is in the position shown. Assume the angular velocity of AB is perpendicular to the rod.



Since rod AB undergoes general motion $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

Assume $\mathbf{v}_B = \frac{4}{5}v_B\,\mathbf{i} - \frac{3}{5}v_B\,\mathbf{k}$ and $\omega_{AB} = (\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}$. Also,

$\mathbf{r}_{B/A} = [-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}]$ ft. Thus,

$$\frac{4}{5}v_B\mathbf{i} - \frac{3}{5}v_B\mathbf{k} = 10\mathbf{i} + \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \times (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})$$

$$\frac{4}{5}v_B\mathbf{i} - \frac{3}{5}v_B\,\mathbf{k} = \left[10 - 4(\omega_{AB})_y - 4(\omega_{AB})_z\right]\mathbf{i} + \left[4(\omega_{AB})_x - 2(\omega_{AB})_z\right]\mathbf{j} + \left[4(\omega_{AB})_x + 2(\omega_{AB})_y\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$\frac{4}{5}v_B = 10 - 4(\omega_{AB})_y - 4(\omega_{AB})_z \qquad \textbf{(1)}$$

$$0 = 4(\omega_{AB})_x - 2(\omega_{AB})_z \qquad \textbf{(2)}$$

$$-\frac{3}{5}v_B = 4(\omega_{AB})_x + 2(\omega_{AB})_y \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\omega_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \cdot (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}) = 0$$

$$-2(\omega_{AB})_x + 4(\omega_{AB})_y - 4(\omega_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$(\omega_{AB})_x = 1.667$ rad/s $\qquad (\omega_{AB})_y = 4.167$ rad/s $\qquad (\omega_{AB})_z = 3.333$ rad/s

$$v_B = -25 \text{ ft/s}$$

Then,

$$\mathbf{v}_B = \frac{4}{5}(-25)\mathbf{i} - \frac{3}{5}(-25)\mathbf{k} = [-20\mathbf{i} + 15\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

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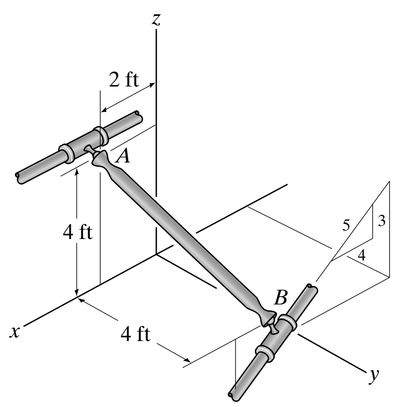
20–26. When rod AB is in the position shown, collar A moves with a velocity of $\mathbf{v}_A = \{10\mathbf{i}\}$ ft/s and acceleration of $\mathbf{a}_A = \{2\mathbf{i}\}$ ft/s^2. Determine the acceleration of collar B at this instant. Assume the angular velocity and angular acceleration of AB are perpendicular to the rod.



For general motion, $\mathbf{a}_A$ and $\mathbf{a}_B$ can be related using the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

Using the result of Prob. 19-17, $\omega_{AB} = [1.667\mathbf{i} + 4.166\mathbf{j} + 3.333\mathbf{k}]$ rad/s.

Assume $\mathbf{a}_B = \frac{4}{5}a_B\,\mathbf{i} - \frac{3}{5}a_B\,\mathbf{k}$ and $\alpha_{AB} = (\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}$. Also,

$\mathbf{r}_{B/A} = [-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}]$ ft. Thus,

$$\frac{4}{5}a_B\,\mathbf{i} - \frac{3}{5}a_B\mathbf{k} = 2\mathbf{i} + \left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \times (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})$$

$$+(1.667\mathbf{i} + 4.166\mathbf{j} + 3.333\mathbf{k}) \times [(1.667\mathbf{i} + 4.166\mathbf{j} + 3.333\mathbf{k}) \times (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})]$$

$$\frac{4}{5}a_B\,\mathbf{i} - \frac{3}{5}a_B\mathbf{k} = \left[64.5 - 4(\alpha_{AB})_y - 4(\alpha_{AB})_z\right]\mathbf{i} + \left[4(\alpha_{AB})_x - 2(\alpha_{AB})_z - 125\right]\mathbf{j} + \left[4(\alpha_{AB})_x + 2(\alpha_{AB})_y + 125\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$\frac{4}{5}a_B = 64.5 - 4(\alpha_{AB})_y - 4(\alpha_{AB})_z \qquad \textbf{(1)}$$

$$0 = 4(\alpha_{AB})_x - 2(\alpha_{AB})_z - 125 \qquad \textbf{(2)}$$

$$-\frac{3}{5}a_B = 4(\alpha_{AB})_x + 2(\alpha_{AB})_y + 125 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\mathbf{k}\right] \cdot (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}) = 0$$

$$-2(\alpha_{AB})_x + 4(\alpha_{AB})_y - 4(\alpha_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$(\alpha_{AB})_x = 94.08$ rad/s^2 $\qquad (\alpha_{AB})_y = 172.71$ rad/s^2 $\qquad (\alpha_{AB})_z = 125.67$ rad/s^2

$$a_B = -1411.25 \text{ ft/s}^2$$

Then,

$$\mathbf{a}_B = \frac{4}{5}(-1411.25)\mathbf{i} - \frac{3}{5}(-1411.25)\mathbf{k} = [-1129\mathbf{i} + 846.75\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

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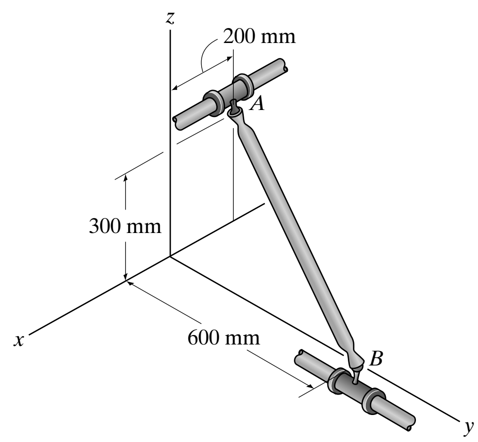
20–27. If collar A moves with a constant velocity of $\mathbf{v}_A = \{3\mathbf{i}\}$ m/s, determine the velocity of collar B when rod AB is in the position shown. Assume the angular velocity of AB is perpendicular to the rod.



Since rod AB undergoes general motion $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

Assume $\mathbf{v}_B = v_B\,\mathbf{j}$, $\omega_{AB} = \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right]$, and

$$\mathbf{r}_{B/A} = [0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}] \text{ ft. Thus,}$$

$$v_B\,\mathbf{j} = 3\mathbf{i} + \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \times (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k})$$

$$v_B\,\mathbf{j} = \left[3 - 0.3(\omega_{AB})_y - 0.6(\omega_{AB})_z\right]\mathbf{i} + \left[0.3(\omega_{AB})_x + 0.2(\omega_{AB})_z\right]\mathbf{j} + \left[0.6(\omega_{AB})_x - 0.2(\omega_{AB})_y\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = 3 - 0.3(\omega_{AB})_y - 0.6(\omega_{AB})_z \qquad \textbf{(1)}$$

$$v_B = 0.3(\omega_{AB})_x + 0.2(\omega_{AB})_z \qquad \textbf{(2)}$$

$$0 = 0.6(\omega_{AB})_x - 0.2(\omega_{AB})_y \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\omega_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \cdot (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}) = 0$$

$$0.2(\omega_{AB})_x + 0.6(\omega_{AB})_y - 0.3(\omega_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$(\omega_{AB})_x = 0.6122$ rad/s $\quad (\omega_{AB})_y = 1.837$ rad/s $\quad (\omega_{AB})_z = 4.082$ rad/s

$$v_B = 1 \text{ m/s}$$

Then,

$$\mathbf{v}_B = [1\mathbf{j}]\text{m/s} \qquad \textbf{Ans.}$$

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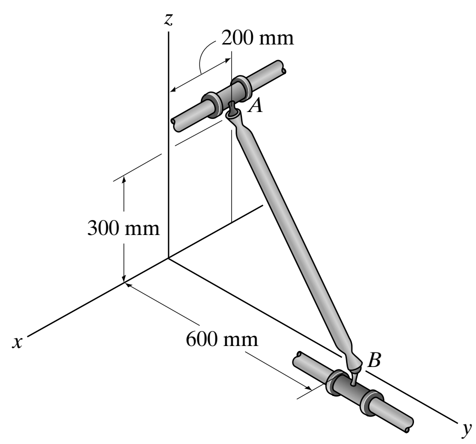
***20–28.** When rod AB is in the position shown, collar A moves with a velocity of $\mathbf{v}_A = \{3\mathbf{i}\}$ m/s and acceleration of $\mathbf{a}_A = \{0.5\mathbf{i}\}$ m/s^2. Determine the acceleration of collar B at this instant. Assume the angular velocity and angular acceleration of AB are perpendicular to the rod.



For general motion, $\mathbf{a}_A$ and $\mathbf{a}_B$ can be related using the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

Using the result of Prob. 19-22, $\omega_{AB} = [0.6122\mathbf{i} + 1837\mathbf{j} + 4.082\mathbf{k}]$ rad/s

Also, $\mathbf{a}_B = a_B\,\mathbf{j}$, $\alpha_{AB} = (\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}$, and
$\mathbf{r}_{B/A} = [0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}]$ m. Thus,

$$\mathbf{a}_B\,\mathbf{j} = (0.5\mathbf{i}) + \left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \times (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k})$$
$$+(0.6122\mathbf{i} + 1837\mathbf{j} + 4.082\mathbf{k}) \times [(0.6122\mathbf{i} + 1837\mathbf{j} + 4.082\mathbf{k}) \times (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k})]$$

$$a_B\mathbf{j} = -\left[0.3(\alpha_{AB})_y + 0.6(\alpha_{AB})_z + 3.5816\right]\mathbf{i} + \left[0.3(\alpha_{AB})_x + 0.2(\alpha_{AB})_z - 12.2449\right]\mathbf{j} + \left[0.6(\alpha_{AB})_x - 0.2(\alpha_{AB})_y + 6.1224\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = -\left[0.3(\alpha_{AB})_y + 0.6(\alpha_{AB})_z + 3.5816\right] \qquad \textbf{(1)}$$

$$a_B = 0.3(\alpha_{AB})_x + 0.2(\alpha_{AB})_z - 12.2449 \qquad \textbf{(2)}$$

$$0 = 0.6(\alpha_{AB})_x - 0.2(\alpha_{AB})_y + 6.1224 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \cdot (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}) = 0$$

$$0.2(\alpha_{AB})_x + 0.6(\alpha_{AB})_y - 0.3(\alpha_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$(\alpha_{AB})_x = -10.1020$ rad/s^2 $\quad (\alpha_{AB})_y = 0.3061$rad/s^2 $\quad (\alpha_{AB})_z = -6.1224$ rad/s^2

$$a_B = -16.5 \text{ m/s}^2$$

Then,

$$\mathbf{a}_B = [-16.5\mathbf{j}]\text{m/s}^2 \qquad \textbf{Ans.}$$

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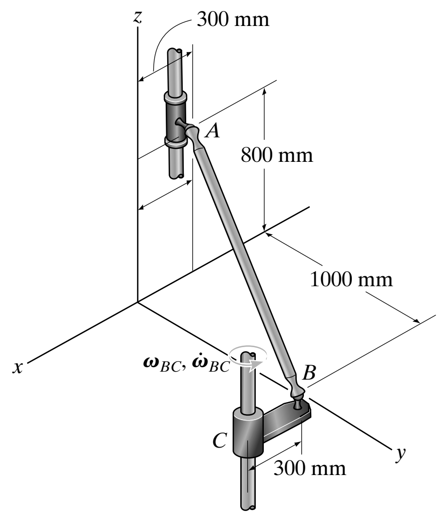
•20–29. If crank BC rotates with a constant angular velocity of $\omega_{BC} = 6$ rad/s, determine the velocity of the collar at A. Assume the angular velocity of AB is perpendicular to the rod.



Here, $\mathbf{r}_{C/B} = [-0.3\mathbf{i}]$ m and $\omega_{BC} = [6\mathbf{k}]$ rad/s. Since crank BC rotates about a fixed axis, then

$$\mathbf{v}_B = \omega_{AB} \times r_{B/C} = \left(6\mathbf{k}\right) \times \left(-0.3\mathbf{i}\right) = [-1.8\mathbf{j}] \text{ m/s}$$

Since rod AB undergoes general motion $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_A = \mathbf{v}_B + \omega_{AB} \times \mathbf{r}_{A/B}$$

Here, $\mathbf{v}_B = v_A\,\mathbf{k}$, $\omega_{AB} = \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right]$, and

$$\mathbf{r}_{A/B} = [-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}] \text{ m. Thus,}$$

$$v_A\,\mathbf{k} = -1.8\mathbf{j} + \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \times (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k})$$

$$v_A\,\mathbf{k} = \left[0.8(\omega_{AB})_y + (\omega_{AB})_z\right]\mathbf{i} - \left[0.8(\omega_{AB})_x + 0.3(\omega_{AB})_z + 1.8\right]\mathbf{j} + \left[0.3(\omega_{AB})_y - (\omega_{AB})_x\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = 0.8(\omega_{AB})_y + (\omega_{AB})_z \qquad \textbf{(1)}$$

$$0 = -\left[0.8(\omega_{AB})_x + 0.3(\omega_{AB})_z + 1.8\right] \qquad \textbf{(2)}$$

$$v_A = 0.3(\omega_{AB})_y - (\omega_{AB})_x \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\omega_{AB} \cdot \mathbf{r}_{A/B} = 0$$

$$\left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \cdot (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}) = 0$$

$$-0.3(\omega_{AB})_x - (\omega_{AB})_y + 0.8(\omega_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$(\omega_{AB})_x = -2.133$ rad/s $\qquad (\omega_{AB})_y = 0.3902$rad/s $\qquad (\omega_{AB})_z = -0.3121$ rad/s^2

$v_A = 2.25$ m/s

Then,

$$\mathbf{v}_A = [2.25\mathbf{k}]\text{m/s} \qquad \textbf{Ans.}$$

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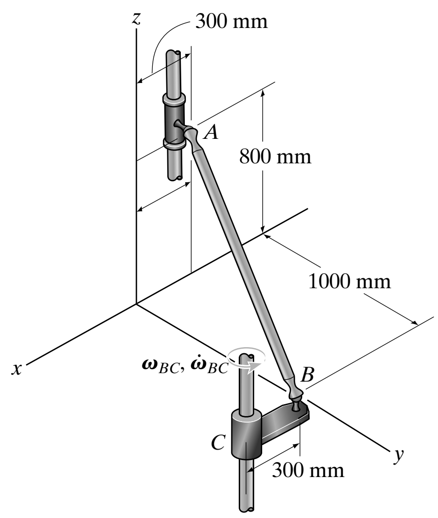
20–30. If crank BC is rotating with an angular velocity of $\omega_{BC} = 6$ rad/s and an angular acceleration of $\dot{\omega}_{BC} = 1.5$ rad/s^2, determine the acceleration of collar A at this instant. Assume the angular velocity and angular acceleration of AB are perpendicular to the rod.



Here, $\mathbf{r}_{CB} = [-0.3\mathbf{i}]$ m and $\alpha_{BC} = [1.5\mathbf{k}]$ rad/s^2. Since crank BC rotates about a fixed axis, then

$$\mathbf{a}_B = \alpha_{BC} \times \mathbf{r}_{CB} + \omega_{BC} \times (\omega_{BC} \times r_{CB}) = (1.5\mathbf{k}) \times (-0.3\mathbf{i}) + 6\mathbf{k} \times [(6\mathbf{k}) \times (-0.3\mathbf{i})]$$

$$= [10.8\mathbf{i} - 0.45\mathbf{j}] \text{ m/s}^2$$

For general motion, $\mathbf{a}_A$ and $\mathbf{a}_B$ can be related using the acceleration equation.

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{A/B})$$

Using the result of Prob. 20-29, $\omega_{AB} = [-2.133\mathbf{i} + 0.3902\mathbf{j} - 0.3121\mathbf{k}]$ rad/s.

Also, $\mathbf{a}_A = a_A\,\mathbf{k}$, $\alpha_{AB} = (\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}$, and

$$\mathbf{r}_{A/B} = [-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}] \text{ m. Thus,}$$

$$a_A\mathbf{k} = (10.8\mathbf{i} - 0.45\mathbf{j}) + \left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \times (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k})$$
$$+(-2.133\mathbf{i} + 0.3902\mathbf{j} - 0.3121\mathbf{k}) \times [(-2.133\mathbf{i} + 0.3902\mathbf{j} - 0.3121\mathbf{k}) \times (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k})]$$

$$a_A\mathbf{k} = \left[0.8(\alpha_{AB})_y + (\alpha_{AB})_z + 12.24\right]\mathbf{i} + \left[4.349 - 0.8(\alpha_{AB})_x - 0.3(\alpha_{AB})_z\right]\mathbf{j} + \left[0.3(\alpha_{AB})_y - (\alpha_{AB})_x - 3.839\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = 0.8(\alpha_{AB})_y + (\alpha_{AB})_z + 12.24 \qquad \textbf{(1)}$$

$$0 = 4.349 - 0.8(\alpha_{AB})_x - 0.3(\alpha_{AB})_z \qquad \textbf{(2)}$$

$$a_A = 0.3(\alpha_{AB})_y - (\alpha_{AB})_x - 3.839 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\alpha_{AB} \cdot \mathbf{r}_{A/B} = 0$$

$$\left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \cdot (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}) = 0$$

$$-0.3(\alpha_{AB})_x - (\alpha_{AB})_y + 0.8(\alpha_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4).

$(\alpha_{AB})_x = 7.807$ rad/s^2 $\qquad (\alpha_{AB})_y = -7.399$rad/s^2 $\qquad (\alpha_{AB})_z = -6.321$ rad/s^2

$$a_A = -13.9 \text{ m/s}^2$$

Then,

$$\mathbf{a}_A = [-13.9\mathbf{k}] \text{m/s}^2 \qquad \textbf{Ans.}$$

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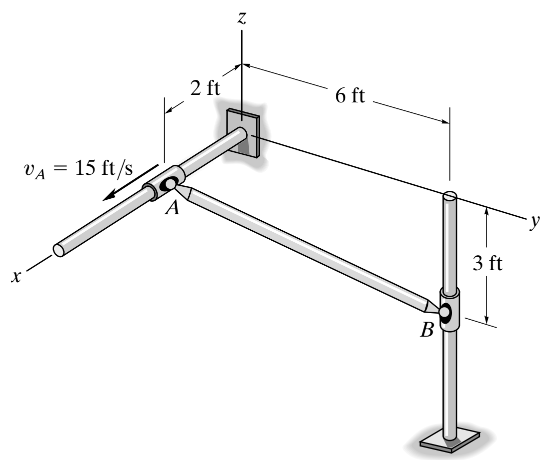
20–31. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a velocity $v_A = 15$ ft/s at the instant shown, determine the velocity of collar B. Assume the angular velocity is perpendicular to the rod.



$\mathbf{v}_A = \{15\mathbf{i}\}$ ft/s $\quad \mathbf{v}_B = v_B\mathbf{k} \quad \omega_{AB} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$\mathbf{r}_{B/A} = \{-2\,\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{k} = 15\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -2 & 6 & -3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components yields:

$$15 - 3\,\omega_y - 6\omega_z = 0 \qquad \textbf{(1)}$$

$$3\,\omega_x - 2\,\omega_z = 0 \qquad \textbf{(2)}$$

$$6\omega_x + 2\omega_y = v_B \qquad \textbf{(3)}$$

If ω_{AB} is perpendicular to the axis of the rod,

$$\omega_{AB} \cdot \mathbf{r}_{B/A} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \cdot (-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$$-2\omega_x + 6\omega_y - 3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 1.2245$ rad/s $\quad \omega_y = 1.3265$ rad/s $\quad \omega_z = 1.8367$ rad/s $\quad v_B = 10$ ft/s

Note: v_B can be obtained by solving Eqs. (1)-(3) without knowing the direction of ω

Hence $\omega_{AB} = \{1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k}\}$ rad/s

$$\mathbf{v}_B = \{10\mathbf{k}\}\text{ ft/s} \qquad \textbf{Ans.}$$

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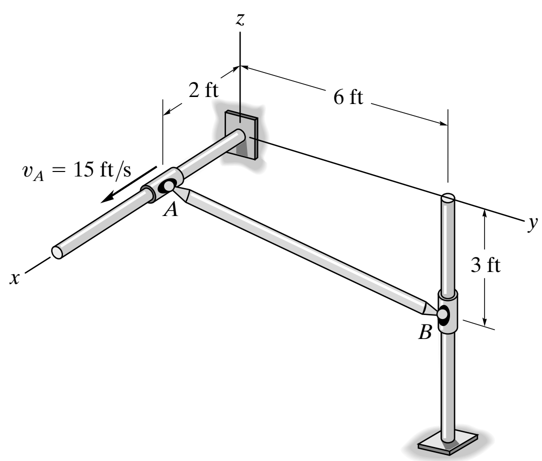
***20–32.** Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a velocity of $\mathbf{v}_A = \{15\mathbf{i}\}$ ft/s and an acceleration of $\mathbf{a}_A = \{2\mathbf{i}\}$ ft/s^2 at the instant shown, determine the acceleration of collar B. Assume the angular velocity and angular acceleration are perpendicular to the rod.



From Prob. 20–31

$$\omega_{AB} = \{1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k}\} \text{ rad/s}$$

$$\mathbf{r}_{B/A} = \{-2\,\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\} \text{ ft}$$

$$\alpha_{AB} = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$$

$$\mathbf{a}_A = \{2\mathbf{i}\} \quad \text{ft/s}^2 \qquad\qquad \mathbf{a}_B = a_B\,\mathbf{k}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

$$a_B\,\mathbf{k} = 2\mathbf{i} + (\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}) \times (-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$$
$$+(1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k})$$
$$\times\left[(1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k}) \times (-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})\right]$$

Equating **i**, **j**, and **k** components yields:

$$15.2653 - 3\,\alpha_y - 6\alpha_z = 0 \qquad \textbf{(1)}$$

$$3\alpha_x - 2\alpha_z - 39.7955 = 0 \qquad \textbf{(2)}$$

$$6\alpha_x + 2\alpha_y + 19.8975 = a_B \qquad \textbf{(3)}$$

If α_{AB} is perpendicular to the axis of the rod,

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = (\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}) \cdot (-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$$-2\alpha_x + 6\alpha_y - 3\alpha_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\alpha_x = 13.43 \text{ rad/s}^2 \qquad \alpha_y = 4.599 \text{ rad/s}^2 \qquad \alpha_z = 0.2449 \text{ rad/s}^2 \qquad a_B = 109.7 \text{ ft/s}^2$

Note: $\mathbf{a}_B$ can be obtained by solving Eqs. (1)-(3) without knowing the direction of α

Hence $\qquad \mathbf{a}_B = \{110\mathbf{k}\} \text{ ft/s}^2 \qquad$ **Ans.**

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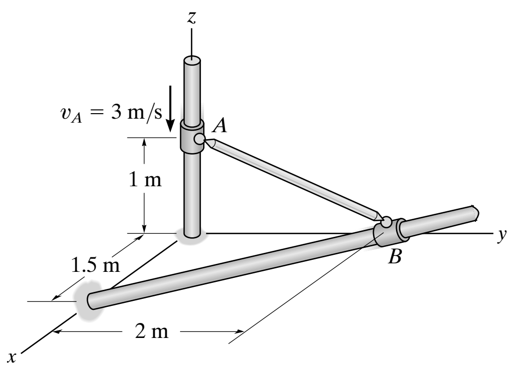
•20–33. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a speed $v_A = 3$ m/s, determine the speed of collar B at the instant shown. Assume the angular velocity is perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{(2-0)\,\mathbf{j} + (0-1\,\mathbf{k})\text{ m} = \{2\mathbf{j} - 1\mathbf{k}\}$ m,

$\mathbf{v}_A = \{-3\mathbf{k}\}$ m/s, $\mathbf{v}_B = v_B\left[\dfrac{(0-1.5)\,\mathbf{i} + (2-0)\,\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right] = -0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j}$ and

$\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$-0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$-0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = (-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\,\mathbf{j} + (2\omega_x - 3)\,\mathbf{k}$$

Equating **i**, **j** and **k** components, we have.

$$-0.6\,v_B = -\omega_y - 2\omega_z \qquad \textbf{[1]}$$

$$0.8\,v_B = \omega_x \qquad \textbf{[2]}$$

$$0 = 2\,\omega_x - 3 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 1.875 \text{ m/s} \qquad \textbf{Ans.}$$

$$\omega_x = 1.50 \text{ rad/s} \qquad \omega_y = 0.225 \text{ rad/s} \qquad \omega_z = 0.450 \text{ rad/s}$$

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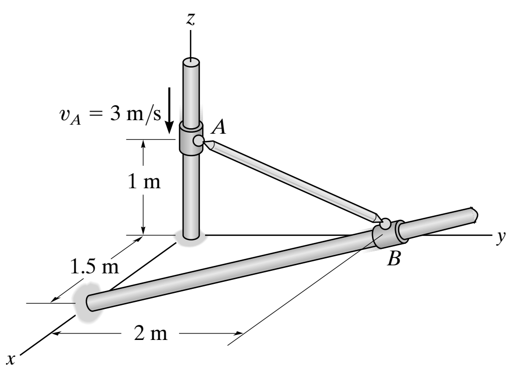
20–34. If the collar at A in Prob 20–33 has an acceleration of $\mathbf{a}_A = \{-2\mathbf{k}\}\ \text{m/s}^2$ at the instant its velocity is $\mathbf{v}_A = \{-3\mathbf{k}\}\ \text{m/s}$, determine the magnitude of the acceleration of the collar at B at this instant. Assume the angular velocity and angular acceleration are perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{(2-0)\mathbf{j} + (0-1)\,\mathbf{k}\}\ \text{m} = \{2\mathbf{j} - 1\mathbf{k})\ \text{m}$,

$\mathbf{v}_A = \{-3\mathbf{k}\}\ \text{m/s}, \mathbf{v}_B = v_B\left[\dfrac{(0-1.5)\mathbf{i} + (2-0)\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right] = -0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j}$ and

$\omega = \omega\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$-0.6v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$-0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = (-3-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\,\mathbf{j} + (2\,\omega_x - 3)\,\mathbf{k}$$

Equating **i**, **j**, and **k** components, we have

$$-0.6\,v_B = -3-\omega_y - 2\omega_z \qquad \textbf{[1]}$$

$$0.8\,v_B = \omega_x \qquad \textbf{[2]}$$

$$0 = 2\omega_x - 3 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}\right)\cdot(2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 1.875\ \text{m/s} \qquad \omega_x = 1.50\ \text{rad/s}$$

$$\omega_y = 0.225\ \text{rad/s} \qquad \omega_z = 0.450\ \text{rad/s}$$

Thus,

$$\omega = \{1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}\}\ \text{rad/s}$$

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20–34. Continued

Acceleration Equation: With $\alpha = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$ and the result obtained above, Applying Eq. 20–8, we have

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A})$$

$$-0.6a_B\,\mathbf{i} + 0.8a_B\,\mathbf{j} = -2\mathbf{k} + \left(\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}\right) \times (2\mathbf{j} - 1\mathbf{k})$$

$$+ (1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times [(1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})]$$

$$-0.6a_B\,\mathbf{i} + 0.8a_B\,\mathbf{j} = \left(-\alpha_y - 2\alpha_z\right)\mathbf{i} + (\alpha_x - 5.00625)\,\mathbf{j} + (2\alpha_x + 0.503125)\,\mathbf{k}$$

Equating **i**, **j**, and **k** components. we have

$$-0.6a_B = -\alpha_y - 2\alpha_z \qquad \textbf{[5]}$$

$$0.8a_B = \alpha_y - 5.00625 \qquad \textbf{[6]}$$

$$0 = 2\alpha_y + 0.503125 \qquad \textbf{[7]}$$

Solving Eqs. [6] and [7] yields

$$\alpha_x = -0.2515\ \text{rad/s}^2$$

$$a_B = -6.57\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Negative sign indicates that $\mathbf{a}_B$ is directed in the opposite direction to that of the above assumed direction

Note: In order to determine α_y and α_z. one should obtain another equation by pacifying the direction of α which acts *perpendicular* to the axis of rod AB.

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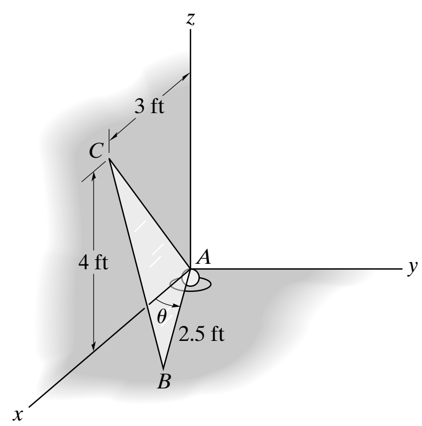
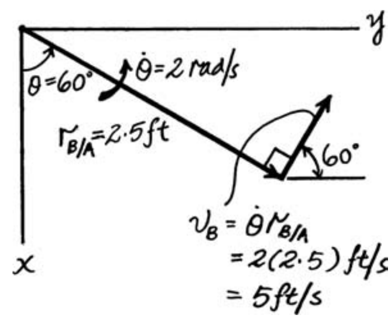
20–35. The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the $x-z$ plane. The side AB lies in the $x-y$ plane. At the instant $\theta = 60^\circ$, $\dot{\theta} = 2$ rad/s and point C has the coordinates shown. Determine the angular velocity of the plate and the velocity of point C at this instant.





$$\mathbf{v}_B = -5\sin 60^\circ\mathbf{i} + 5\cos 60^\circ\mathbf{j}$$

$$= \{-4.33\mathbf{i} + 2.5\mathbf{j}\}\text{ ft/s}$$

$$\mathbf{v}_C = (v_C)_x\,\mathbf{i} + (v_C)_z\,\mathbf{k}$$

$$\mathbf{r}_{C/A} = \{3\mathbf{i} + 4\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{B/A} = \{1.25\mathbf{i} + 2.165\mathbf{j}\}\text{ ft}$$

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A}$$

$$-4.33\mathbf{i} + 2.5\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.25 & 2.165 & 0 \end{vmatrix}$$

$$-2.165\omega_z = -4.33\,; \qquad \omega_z = 2\text{ rad/s}$$

$$2.165\omega_x - 1.25\omega_y = 0; \qquad \omega_y = 1.732\omega_x$$

$$\mathbf{v}_C = \omega \times \mathbf{r}_{C/A}$$

$$(v_C)_x\,\mathbf{i} + (v_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & 2 \\ 3 & 0 & 4 \end{vmatrix}$$

$$(v_C)_x = 4\omega_y$$

$$0 = 4\omega_x - 6; \qquad \omega_x = 1.5\text{ rad/s}$$

$$(v_C)_z = -3\omega_y$$

Solving,

$$\omega_y = 2.5981\text{ rad/s}$$

$$(v_C)_x = 10.392\text{ ft/s}$$

$$(v_C)_z = -7.7942\text{ ft/s}$$

Thus,

$$\omega = \{1.50\mathbf{i} + 2.60\mathbf{j} + 2.00\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_C = \{10.4\mathbf{i} - 7.79\mathbf{k}\}\text{ ft/s} \qquad \textbf{Ans.}$$

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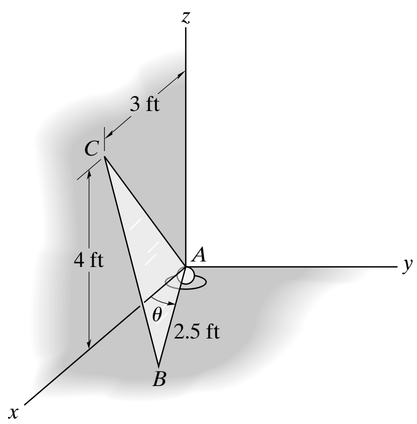
***20–36.** The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the $x-z$ plane. The side AB lies in the $x-y$ plane. At the instant $\theta = 60°$, $\dot{\theta} = 2$ rad/s, $\ddot{\theta} = 3$ rad/s^2 and point C has the coordinates shown. Determine the angular acceleration of the plate and the acceleration of point C at this instant.



From Prob. 20–35.

$$\omega = 1.5\mathbf{i} + 2.5981\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 1.25\mathbf{i} + 2.165\mathbf{j}$$

$$\mathbf{v}_B = -4.33\mathbf{i} + 2.5\mathbf{j}$$

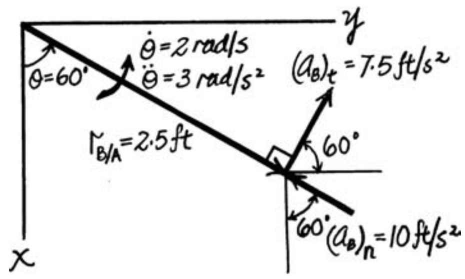
$$(a_B)_t = 3(2.5) = 7.5 \text{ ft/s}^2$$

$$(a_B)_n = (2)^2(2.5) = 10 \text{ ft/s}^2$$

$$\mathbf{a}_B = -7.5 \sin 60°\mathbf{i} + 7.5 \cos 60°\mathbf{j} - 10 \cos 60°\mathbf{i} - 10 \sin 60°\mathbf{j}$$

$$\mathbf{a}_B = -11.4952\mathbf{i} - 4.91025\mathbf{j}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} + \omega \times \mathbf{v}_{B/A}$$



$$-11.4952\mathbf{i} - 4.91025\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 1.25 & 2.165 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2.5981 & 2 \\ -4.33 & 2.5 & 0 \end{vmatrix}$$

$$-11.4952 = -2.165\alpha_z - 5$$

$$-4.91025 = 1.25\alpha_z - 8.66$$

$$\alpha_z = 3 \text{ rad/s}^2$$

$$0 = 2.165\alpha_x - 1.25\alpha_y + 15 \qquad \textbf{(1)}$$

$$\mathbf{a}_C = \alpha \times \mathbf{r}_{C/A} + \omega \times \mathbf{v}_{C/A}$$

$$\mathbf{v}_{C/A} = 10.39\mathbf{i} - 7.794\mathbf{k}$$

$$\mathbf{a}_C = (a_C)_x\,\mathbf{i} + (a_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 3 & 0 & 4 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2.5981 & 2 \\ 10.39 & 0 & -7.794 \end{vmatrix}$$

$$(a_C)_x = 4\alpha_y - 20.25 \qquad \textbf{(2)}$$

$$0 = 3\alpha_z - 4\alpha_x + 32.4760 \qquad \textbf{(3)}$$

$$(a_C)_z = -3\alpha_y - 27 \qquad \textbf{(4)}$$

Solving Eqs. (1)-(4),

$$\alpha_x = 10.369 \text{ rad/s}^2$$

$$\alpha_y = 29.96 \text{ rad/s}^2$$

$$(a_C)_x = 99.6 \text{ ft/s}^2$$

$$(a_C)_z = -117 \text{ ft/s}^2$$

$$\mathbf{a}_C = \{99.6\mathbf{i} - 117\mathbf{k}\}\text{ft/s}^2 \qquad \textbf{Ans.}$$

$$\alpha = \{10.4\mathbf{i} + 30.0\mathbf{j} + 3\mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans.}$$

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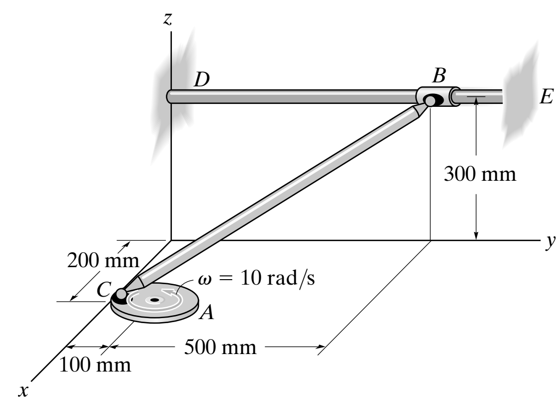
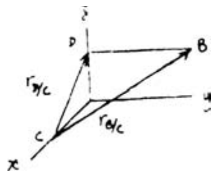
•20–37. Disk A rotates at a constant angular velocity of 10 rad/s. If rod BC is joined to the disk and a collar by ball-and-socket joints, determine the velocity of collar B at the instant shown. Also, what is the rod's angular velocity $\boldsymbol{\omega}_{BC}$ if it is directed perpendicular to the axis of the rod?



$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\qquad \mathbf{v}_B = -v_B\mathbf{j} \qquad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$r_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$



$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-\mathbf{v}_B = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components

$$1 - 0.3\omega_y - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_y = 0 \qquad \textbf{(3)}$$

Since ω_{BC} is perpendicular to the axis of the rod,

$$\omega_{BC} \quad \mathbf{r}_{B/C} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k})\cdot(-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}) = 0$$

$$-0.2\omega_x + 0.6\omega_y + 0.3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 0.204$ rad/s $\qquad \omega_y = -0.612$ rad/s $\qquad \omega_z = 1.36$ rad/s $\qquad v_B = 0.333$ m/s

Then

$$\omega_{BC} = \{0.204\mathbf{i} - 0.612\mathbf{j} + 1.36\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

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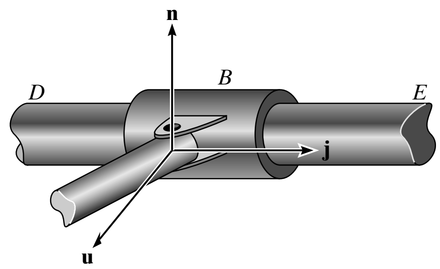
20–38. Solve Prob. 20–37 if the connection at B consists of a pin as shown in the figure below, rather than a ball-and-socket joint. *Hint:* The constraint allows rotation of the rod both about bar DE ($\mathbf{j}$ direction) and about the axis of the pin ($\mathbf{n}$ direction). Since there is no rotational component in the $\mathbf{u}$ direction, i.e., perpendicular to $\mathbf{n}$ and $\mathbf{j}$ where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector $\mathbf{n}$ is in the same direction as $\mathbf{r}_{B/C} \times \mathbf{r}_{D/C}$.



$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\quad \mathbf{v}_B = -v_B\mathbf{j} \quad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-v_B\mathbf{j} = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components

$$1 + 0.3\omega_x - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_z = 0 \qquad \textbf{(3)}$$

Also,

$$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{D/C} = \{-0.2\mathbf{i} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{B/C} \times \mathbf{r}_{D/C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.2 & 0.6 & 0.3 \\ -0.2 & 0 & 0.3 \end{vmatrix} = \{0.18\mathbf{i} + 0.12\mathbf{k}\}\text{ m}^2$$

$$\mathbf{n} = \frac{0.18\mathbf{i} + 0.12\mathbf{k}}{\sqrt{0.18^2 + 0.12^2}} = 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$$\mathbf{u} = \mathbf{j} \times \mathbf{n} = \mathbf{j} \times (0.8321\mathbf{i} + 0.5547\mathbf{k}) = 0.5547\mathbf{i} - 0.8321\mathbf{k}$$

$$\omega_{BC} \cdot \mathbf{u} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}) \cdot (0.5547\mathbf{i} - 0.8321\mathbf{k}) = 0$$

$$0.5547\omega_x - 0.8321\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 0.769$ rad/s $\quad \omega_y = -2.31$ rad/s $\quad \omega_z = 0.513$ rad/s $\quad v_B = 0.333$ m/s

Then

$$\omega_{BC} = \{0.769\mathbf{i} - 2.31\mathbf{j} + 0.513\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

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20–39. Solve Example 20–5 such that the x, y, z axes move with curvilinear translation, $\mathbf{\Omega} = \mathbf{0}$ in which case the collar appears to have both an angular velocity $\mathbf{\Omega}_{xyz} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$ and radial motion.

Relative to XYZ, let xyz have

$$\Omega = 0 \qquad \dot{\Omega} = 0$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \{2\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_B = \{0.75\mathbf{j} + 8\mathbf{k}\}\text{ m/s}^2$$

Relative to xyz, let $x'\ y'\ z'$ be coincident with xyz and be fixed to BD. Then

$$\Omega_{xyz} = \omega_1 + \omega_2 = \{4\mathbf{i} + 5\mathbf{k}\}\text{ rad/s} \qquad \dot{\omega}_{xyz} = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} - 6\mathbf{k}\}\text{ rad/s}^2$$

$$(\mathbf{r}_{C/B})_{xyz} = \{0.2\mathbf{j}\}\text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\mathbf{r}_{C/B})_{xyz}$$

$$= 3\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})$$

$$= \{-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = (\ddot{\mathbf{r}}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{x'y'z'}\right]$$

$$+\left[(\dot{\omega}_1 + \dot{\omega}_2) \times (\mathbf{r}_{C/B})_{xyz}\right] + \left[(\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right]$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[2\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times 3\mathbf{j}\right] + \left[(1.5\mathbf{i} - 6\mathbf{k}) \times 0.2\mathbf{j}\right] + \left[(4\mathbf{i} + 5\mathbf{k}) \times (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})\right]$$

$$= \{-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k}\}\text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + 0 + (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})$$

$$= \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \mathbf{0} + \mathbf{0} + \mathbf{0} + (-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k})$$

$$= \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

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***20–40.** Solve Example 20–5 by fixing x, y, z axes to rod BD so that $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$. In this case the collar appears only to move radially outward along BD; hence $\boldsymbol{\Omega}_{xyz} = \mathbf{0}$.

Relative to XYZ, let $x'\ y'\ z'$ be concident with XYZ and have $\Omega' = \omega_1$ and $\dot{\Omega}' = \dot{\omega}_1$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \{4\mathbf{i} + 5\mathbf{k}\}\text{ rad/s}$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \left[\left(\dot{\omega}_1\right)_{x'y'z'} + \omega_1 \times \omega_1\right] + \left[\left(\omega_2\right)_{x'y'z'} + \omega_1 \times \omega_2\right]$$

$$= (1.5\mathbf{i} + \mathbf{0}) + \left[-6\mathbf{k} + (4\mathbf{i}) \times (5\mathbf{k})\right] = \{1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = \left(\dot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \mathbf{r}_B = \mathbf{0} + (4\mathbf{i}) \times (-0.5\mathbf{k}) = \{2\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_B = \dot{\mathbf{r}}_B = \left[\left(\ddot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \left(\dot{\mathbf{r}}_\mathbf{B}\right)_{x'y'z'}\right] + \dot{\omega}_1 \times r_B + \omega_1 \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + \left[(1.5\mathbf{i}) \times (-0.5\mathbf{k})\right] + (4\mathbf{i} \times 2\mathbf{j}) = \{0.75\mathbf{j} + 8\mathbf{k}\}\text{ m/s}^2$$

Relative to $x'y'z'$, let xyz have

$$\Omega_{x'y'z'} = \mathbf{0}; \qquad \dot{\Omega}_{x'y'z'} = \mathbf{0};$$

$$\left(r_{C/B}\right)_{xyz} = \{0.2\mathbf{j}\}\text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \{3\mathbf{j}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \{2\mathbf{j}\}\text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 3\mathbf{j}$$

$$= \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}\text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \left[(1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}) \times (0.2\mathbf{j})\right] + (4\mathbf{i} + 5\mathbf{k}) \times \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 2\left[(4\mathbf{i} + 5\mathbf{k}) \times (3\mathbf{j})\right] + 2\mathbf{j}$$

$$\mathbf{a}_C = \{-28.2\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

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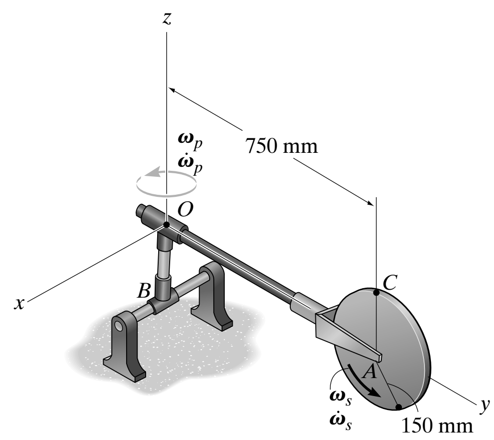
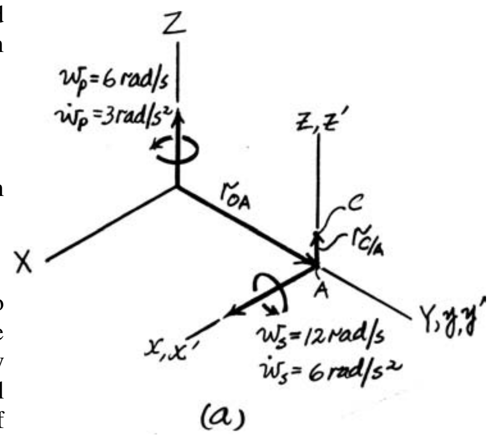
•**20–41.** At the instant shown, the shaft rotates with an angular velocity of $\omega_p = 6$ rad/s and has an angular acceleration of $\dot{\omega}_p = 3$ rad/s^2. At the same instant, the disk spins about its axle with an angular velocity of $\omega_s = 12$ rad/s, increasing at a constant rate of $\dot{\omega}_s = 6$ rad/s^2. Determine the velocity of point C located on the rim of the disk at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_p = [6\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_p = [3\mathbf{k}] \text{ rad/s}^2$$



Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_P \times \mathbf{r}_{OA} = (6\mathbf{k}) \times (0.75\mathbf{j}) = [-4.5\mathbf{i}] \text{ m/s}$$

In order to determine the motion of point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_S = [12\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{C/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{C/A})_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_s \times (\mathbf{r}_{C/A})_{xyz}\right]$$
$$= 0 + (12\mathbf{i}) \times (0.15\mathbf{k})$$
$$= [-1.8\mathbf{j}] \text{ m/s}$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$
$$= (-4.5\mathbf{i}) + 6\mathbf{k} \times 0.15\mathbf{k} + (-1.8\mathbf{j})$$
$$= [-4.5\mathbf{i} - 1.8\mathbf{j}] \text{ m/s} \qquad \textbf{Ans.}$$

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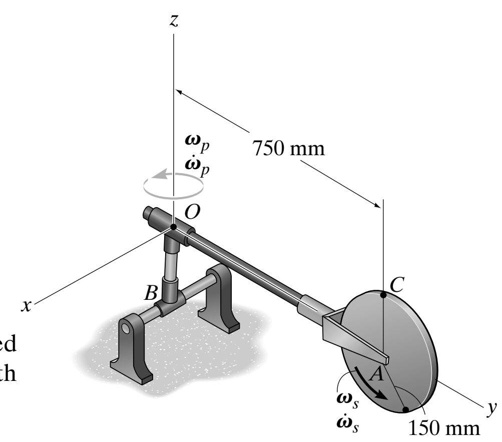
20–42. At the instant shown, the shaft rotates with an angular velocity of $\omega_p = 6$ rad/s and has an angular acceleration of $\dot{\omega}_p = 3\ \text{rad/s}^2$. At the same instant, the disk spins about its axle with an angular velocity of $\omega_s = 12$ rad/s, increasing at a constant rate of $\dot{\omega}_s = 6\ \text{rad/s}^2$. Determine the acceleration of point C located on the rim of the disk at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_p = [6\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \dot{\Omega}_p = [3\mathbf{k}]\ \text{rad/s}^2$$

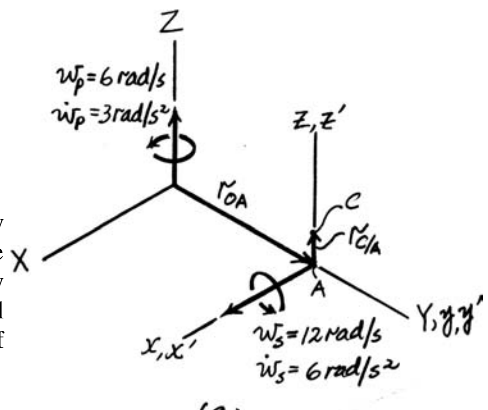
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_p \times \mathbf{r}_{OA} = (6\mathbf{k}) \times (0.75\mathbf{j}) = [-4.5\mathbf{i}]\ \text{m/s}$$

$$a_A = \dot{\omega}_p \times \mathbf{r}_{OA} + \omega_p \times \left(\omega_p \times \mathbf{r}_{OA}\right)$$

$$= (3\mathbf{k}) \times (0.75\mathbf{j}) + 6\mathbf{k} \times [6\mathbf{k} \times 0.75\mathbf{j}]$$

$$= [-2.25\mathbf{i} - 27\mathbf{j}]\ \text{m/s}^2$$



(a)

In order to determine the motion of the point C relative to point A, it is neccessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_s = [12\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{C/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{C/A})_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_S \times (\mathbf{r}_{C/A})_{xyz}\right]$$

$$= 0 + (12\mathbf{i}) \times (0.15\mathbf{k})$$

$$= [-1.8\mathbf{j}]\ \text{m/s}$$

Since $\Omega' = \omega_s$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_s = [6\mathbf{i}]\ \text{rad/s}^2$. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A}) = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_s \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_s \times (\mathbf{r}_{C/A})_{xyz} + \omega_s \times (\dot{r}_{C/A})_{xyz}$$

$$= [0 + 0] + (6\mathbf{i}) \times (0.15\mathbf{k}) + (12\mathbf{i}) \times (-1.8\mathbf{j})$$

$$= [-0.9\mathbf{j} - 21.6\mathbf{k}]\ \text{m/s}^2$$

Thus,

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (-2.25\mathbf{i} - 27\mathbf{j}) + 3\mathbf{k} \times 0.15\mathbf{k} + 6\mathbf{k} \times (6\mathbf{k} \times 0.15\mathbf{k}) + 2(6\mathbf{k}) \times (-1.8\mathbf{j}) + (-0.9\mathbf{j} - 21.6\mathbf{k})$$

$$= [19.35\mathbf{i} - 27.9\mathbf{j} - 21.6\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}$$

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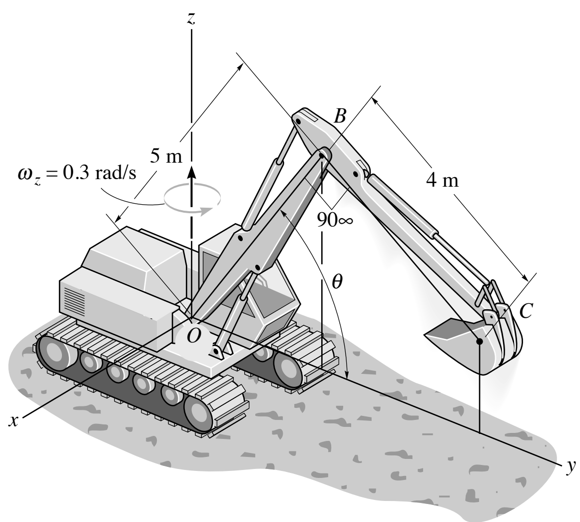
20–43. At the instant shown, the cab of the excavator rotates about the z axis with a constant angular velocity of $\omega_z = 0.3$ rad/s. At the same instant $\theta = 60°$, and the boom OBC has an angular velocity of $\dot{\theta} = 0.6$ rad/s, which is increasing at $\ddot{\theta} = 0.2$ rad/s^2, both measured relative to the cab. Determine the velocity and acceleration of point C on the grapple at this instant.



Relative to XYZ, let xyz have

$\Omega = \{0.3\mathbf{k}\}$ rad/s, $\quad \dot{\omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$$\mathbf{r}_O = \mathbf{0}, \quad \mathbf{v}_O = \mathbf{0}, \quad \mathbf{a}_O = \mathbf{0}$$

Relative to xyz, let $x'y'z'$ be coincident with xyz at O and have

$\Omega_{xyz} = \{0.6\mathbf{i}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \{0.2\mathbf{i}\}$ rad/s^2 (Ω_{xyz} does not change direction relative to XYZ.)

$$(\mathbf{r}_{C/O})_{xyz} = (5\cos 60° + 4\cos 30°)\mathbf{j} + (5\sin 60° - 4\sin 30°)\mathbf{k} = \{5.9641\mathbf{j} + 2.3301\mathbf{k}\}\text{ m}$$

$(\mathbf{r}_{C/O})_{xyz}$ change direction relative to XYZ.)

$$(\mathbf{v}_{C/O})_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/O}\right)_{xyz}$$

$$= \mathbf{0} + (0.6\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) = \{-1.3981\mathbf{j} + 3.5785\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{C/O})_{xyz} = \left(\ddot{\mathbf{r}}_{C/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \dot{\mathbf{r}}_{C/O} + \Omega_{xyz} \times \dot{\mathbf{r}}_{C/O}$$

$$= \left[\mathbf{0} + \mathbf{0}\right] + (0.2\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) + (0.6\mathbf{i}) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k})$$

$$= \{-2.61310\mathbf{j} + 0.35397\mathbf{k}\}\text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_O + \Omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz} = \mathbf{0} + (0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) - 1.3981\mathbf{j} + 3.5785\mathbf{k}$$

$$= \{-1.79\mathbf{i} - 1.40\mathbf{j} + 3.58\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{C/O} + \Omega \times \left(\Omega \times \mathbf{r}_{C/O}\right) + 2\Omega \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (0.3\mathbf{k}) \times \left[(0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k})\right]$$

$$+ 2(0.3k) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k}) - 2.61310\mathbf{j} + 0.35397\mathbf{k}$$

$$= \{0.839\mathbf{i} - 3.15\mathbf{j} + 0.354\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

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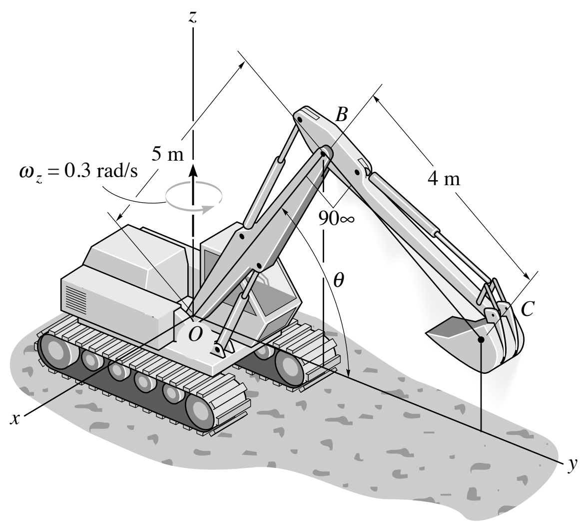
***20–44.** At the instant shown, the frame of the excavator travels forward in the y direction with a velocity of 2 m/s and an acceleration of 1 m/s^2, while the cab rotates about the z axis with an angular velocity of $\omega_z = 0.3$ rad/s, which is increasing at $\alpha_z = 0.4$ rad/s^2. At the same instant $\theta = 60°$, and the boom OBC has an angular velocity of $\dot{\theta} = 0.6$ rad/s, which is increasing at $\ddot{\theta} = 0.2$ rad/s^2, both measured relative to the cab. Determine the velocity and acceleration of point C on the grapple at this instant.



Relative to XYZ, let xyz have

$\Omega = \{0.3\mathbf{k}\}$ rad/s, $\dot{\omega} = \{0.4\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_O = \mathbf{0}$ (r_o does not change direction relative to XYZ.)

$$\mathbf{v}_O = \{2\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_O = \{1\mathbf{j}\} \text{ m/s}^2$$

Relative to xyz, let $x'y'z'$ have

$\Omega_{xyz} = \{0.6\mathbf{i}\}$ rad/s, $\dot{\omega}_{xyz} = \{0.2\mathbf{i}\}$ rad/s^2 (Ω_{xyz} does not change direction relative to xyz.)

$$(\mathbf{r}_{C/O})_{xyz} = (5\cos 60° + 4\cos 30°)\mathbf{j} + (5\sin 60° - 4\sin 30°)\mathbf{k} = \{5.9641\mathbf{j} + 2.3301\mathbf{k}\} \text{ m}$$

($(\mathbf{r}_{C/O})_{xyz}$ change direction relative to xyz.)

$$(\mathbf{v}_{C/O})_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/O}\right)_{xyz}$$

$$= 0 + (0.6\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) = \{-1.3981\mathbf{j} + 3.5785\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/O})_{xyz} = \left(\ddot{\mathbf{r}}_{C/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'}\right] + \Omega_{xyz} \times \left(\mathbf{r}_{C/O}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/O}\right)_{xyz}$$

$$= \left[\mathbf{0} + \mathbf{0}\right] + (0.2\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) + (0.6\mathbf{i}) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k})$$

$$= \{-2.61310\mathbf{j} + 0.35397\mathbf{k}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_O + \Omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz} = 2\mathbf{j} + (0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) - 1.3981\mathbf{j} + 3.5785\mathbf{k}$$

$$= \{-1.79\mathbf{i} + 0.602\mathbf{j} + 3.58\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{C/O} + \Omega \times \left(\Omega \times \mathbf{r}_{C/O}\right) + 2\Omega \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz}$$

$$= 1\mathbf{j} + 0.4\mathbf{k} \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) + (0.3\mathbf{k}) \times \left[(0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k})\right]$$

$$+ 2(0.3\mathbf{k}) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k}) - 2.61310\mathbf{j} + 0.35397\mathbf{k}$$

$$= \{-1.55\mathbf{i} - 2.15\mathbf{j} + 0.354\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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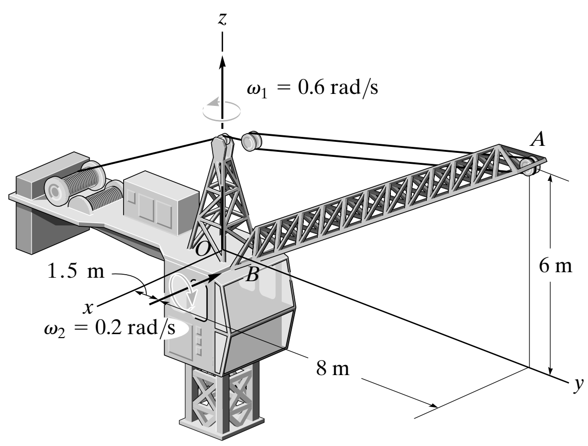
•20–45. The crane rotates about the z axis with a constant rate $\omega_1 = 0.6$ rad/s, while the boom rotates downward with a constant rate $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



Coordinate Axes: The rotating x, y, z frame and fixed X, Y, Z frame are set with the origin at point B and O respectively.

Motion of B: Here, $\mathbf{r}_B$ changes direction with respect to X, Y, Z frame. The time derivatives of $\mathbf{r}_B$ can be found by setting another set of coordinate axis x', y', z', coincident with X, Y, Z rotating at $\Omega = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\Omega' = \dot{\omega}_1 = \mathbf{0}$. Here, $\mathbf{r}_B = \{1.5\mathbf{j}\}$ m

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{x'y'z'} + \Omega' \times \mathbf{r}_B = \mathbf{0} + 0.6\mathbf{k} \times 1.5\mathbf{j} = \{-0.9\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{x'y'z'} + \Omega' \times (\dot{\mathbf{r}}_B)_{x'y'z'}\right] + \dot{\Omega}' \times \mathbf{r}_B + \Omega' \times \dot{\mathbf{r}}_B$$

$$= (\mathbf{0} + \mathbf{0}) + \mathbf{0} + 0.6\mathbf{k} \times (-0.9\mathbf{i}) = \{-0.540\mathbf{j}\} \text{ m/s}^2$$

Motion of A with Respect to B: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{-0.2\mathbf{i}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = \mathbf{0}$. Here, $\mathbf{r}_{A/B} = \{8\mathbf{j} + 6\mathbf{k}\}$ m.

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B} = \mathbf{0} + (-0.2\mathbf{i}) \times (8\mathbf{j} + 6\mathbf{k}) = \{1.20\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{A/B} + \Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}$$

$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (-0.2\mathbf{i}) \times (1.20\mathbf{j} - 1.60\mathbf{k})$$

$$= \{-0.320\mathbf{j} - 0.240\mathbf{k}\} \text{ m/s}^2$$

Motion of Point A: Here, $\Omega = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\dot{\omega} = \dot{\omega}_1 = \mathbf{0}$. Applying Eqs. 20–11 and 20-12, we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} = (-0.9\mathbf{i}) + 0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k}) + (1.20\mathbf{j} - 1.60\mathbf{k})$$

$$= \{-5.70\mathbf{i} + 1.20\mathbf{j} - 1.60\mathbf{k}\}\text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{AB} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= (-0.540\mathbf{j}) + \mathbf{0} + 0.6\mathbf{k} \times [0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k})] + 2(0.6\mathbf{k}) \times (1.20\mathbf{j} - 1.60\mathbf{k}) + (-0.320\mathbf{j} - 0.240\mathbf{k})$$

$$= \{-1.44\mathbf{i} - 3.74\mathbf{j} - 0.240\mathbf{k}\}\text{m/s}^2 \qquad \textbf{Ans.}$$

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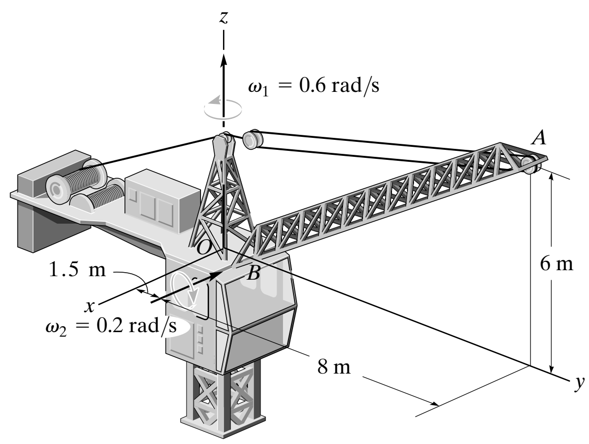
20–46. The crane rotates about the z axis with a rate of $\omega_1 = 0.6$ rad/s, which is increasing at $\dot{\omega}_1 = 0.6$ rad/s^2. Also, the boom rotates downward at $\omega_2 = 0.2$ rad/s, which is increasing at $\dot{\omega}_2 = 0.3$ rad/s^2. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



Coordinate Axes: The rotating x, y, z frame and fixed and fixed X, Y, Z frame are set with the origin at point B and O respectively.

Motion of B: Here, $\mathbf{r}_B$ change direction with respect to X, Y, Z frame. The time derivatives of $\mathbf{r}_B$ can be found by seeting another set of coordinate axis x', y', z' coincident with X, Y, Z rotating at $\Omega' = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{0.6\mathbf{k}\}$ rad/s^2. Here, $\mathbf{r}_B = \{1.5\mathbf{j}\}$ m

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{'y'z'} + \Omega' \times \mathbf{r}_B = \mathbf{0} + 0.6\mathbf{k} \times 1.5\mathbf{j} = \{-0.9\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{x'y'z'} + \Omega' \times (\dot{\mathbf{r}}_B)_{x'y'z'}\right] + \dot{\Omega}' \times \mathbf{r}_B + \Omega' \times \dot{\mathbf{r}}_B$$

$$= (\mathbf{0} + \mathbf{0}) + 0.6\mathbf{k} \times 1.5\mathbf{j} + 0.6\mathbf{k} \times (-0.9\mathbf{i}) = \{-0.9\mathbf{i} - 0.540\mathbf{j}\} \text{ m/s}^2$$

Motion of A with Respect to B: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{-0.2\mathbf{i}\}$ rad/s and $\Omega_{xyz} = \dot{\omega}_2 = \{-0.3\mathbf{i}\}$ rad/s^2. Here, $\mathbf{r}_{A/B} = \{8\mathbf{j} + 6\mathbf{k}\}$ m

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B} = \mathbf{0} + (-0.2\mathbf{i}) \times (8\mathbf{j} + 6\mathbf{k}) = \{1.20\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{A/B} + \Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}$$

$$= \mathbf{0} + \mathbf{0} + (-0.3\mathbf{i}) \times (8\mathbf{i} + 6\mathbf{k}) + (-0.2\mathbf{i}) \times (1.20\mathbf{j} - 1.60\mathbf{k})$$

$$= \{1.48\mathbf{j} - 2.64\mathbf{k}\} \text{ m/s}^2$$

Motion of Point A: Here, $\Omega = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{0.6\mathbf{k}\}$ rad/s^2. Applying Eqs. 20–11 and 20-12, we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} = (-0.9\mathbf{i}) + 0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k}) + (1.20\mathbf{j} - 1.60\mathbf{k})$$

$$= \{-5.70\mathbf{i} + 1.20\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= (-0.9\mathbf{i} - 0.540\mathbf{j}) + 0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k}) + 0.6\mathbf{k} \times [0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k})]$$

$$+ 2(0.6\mathbf{k}) \times (1.20\mathbf{j} - 1.60\mathbf{k}) + (1.48\mathbf{j} - 2.64\mathbf{k})$$

$$= \{-7.14\mathbf{i} - 1.94\mathbf{j} - 2.64\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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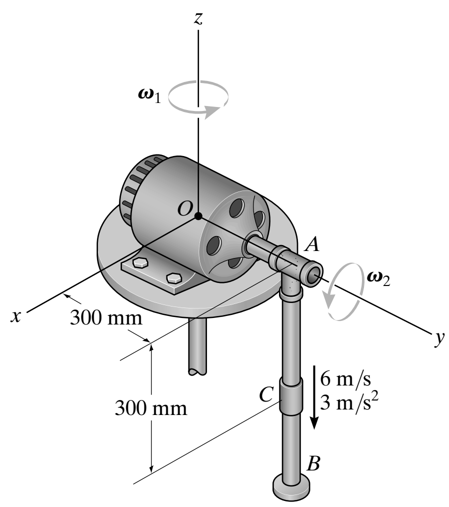
20–47. The motor rotates about the z axis with a constant angular velocity of $\omega_1 = 3$ rad/s. Simultaneously, shaft OA rotates with a constant angular velocity of $\omega_2 = 6$ rad/s. Also, collar C slides along rod AB with a velocity and acceleration of 6 m/s and 3 m/s^2. Determine the velocity and acceleration of collar C at the instant shown.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. *a*. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [3\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$

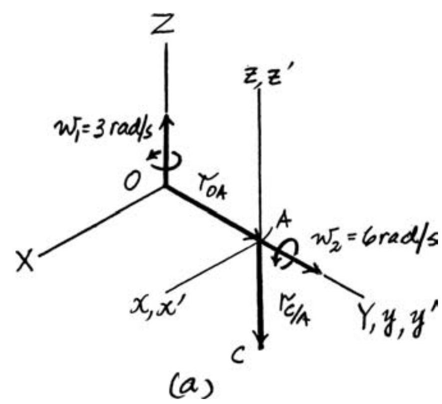
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (3\mathbf{k}) \times (0.3\mathbf{j}) = [-0.9\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= \mathbf{0} + (3\mathbf{k}) \times \left[(3\mathbf{k}) \times (0.3\mathbf{j})\right]$$

$$= [-2.7\mathbf{j}] \text{ m/s}^2$$



In order to determine the motion of the point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. *a*. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [6\mathbf{j}]$ rad/s, the direction of $(\mathbf{r}_{C/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{C/A})_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{C/A})_{xyz}\right]$$

$$= (-6\mathbf{k}) + 6\mathbf{j} \times (-0.3\mathbf{k})$$

$$= [-1.8\mathbf{i} - 6\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A})_{xyz} = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{C/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{xyz}$$

$$= [(-3\mathbf{k}) + 6\mathbf{j} \times (-6\mathbf{k})] + \mathbf{0} + [6\mathbf{j} \times (-18\mathbf{j} - 6\mathbf{k})]$$

$$= [-72\mathbf{i} + 7.8\mathbf{k}] \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$= (-0.9\mathbf{i}) + 3\mathbf{k} \times (-0.3\mathbf{k}) + (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-2.7\mathbf{i} - 6\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (-2.7\mathbf{j}) + \mathbf{0} + 3\mathbf{k} \times [(3\mathbf{k}) \times (-0.3\mathbf{k})] + 2(3\mathbf{k}) \times (-1.8\mathbf{i} - 6\mathbf{k}) + (-72\mathbf{i} + 7.8\mathbf{k})$$

$$= [-72\mathbf{i} - 13.5\mathbf{j} + 7.8\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

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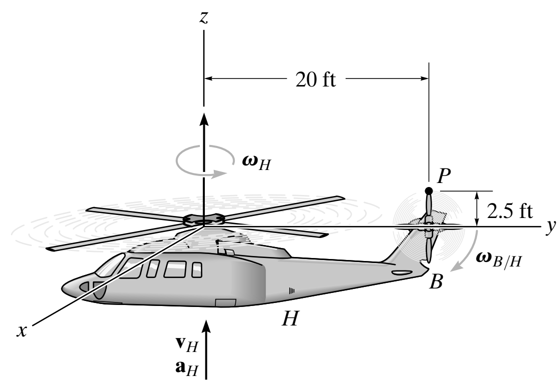
***20–48.** At the instant shown, the helicopter is moving upwards with a velocity $v_H = 4$ ft/s and has an acceleration $a_H = 2$ ft/s^2. At the same instant the frame H, *not* the horizontal blade, rotates about a vertical axis with a constant angular velocity $\omega_H = 0.9$ rad/s. If the tail blade B rotates with a constant angular velocity $\omega_{B/H} = 180$ rad/s, measured relative to H, determine the velocity and acceleration of point P, located on the end of the blade, at the instant the blade is in the vertical position.



Relative to XYZ, let xyz have

$\Omega = \{0.9\mathbf{k}\}$ rad/s $\quad \dot{\omega} = \mathbf{0}$ (Ω does note change direction relative to XYZ.)

$\mathbf{r}_B = \{20\mathbf{j}\}$ ft ($\mathbf{r}_B$ changes direction relative to XYZ.)

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = 4\mathbf{k} + (0.9\mathbf{k}) \times (20\mathbf{j}) = \{-18\mathbf{i} + 4\mathbf{k}\} \text{ ft/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times \left(\dot{\mathbf{r}}_B\right)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= \left[2\mathbf{k} + \mathbf{0}\right] + \mathbf{0} + \left[(0.9\mathbf{k}) \times (-18\mathbf{i} + 4\mathbf{k})\right]$$

$$= \{-16.2\mathbf{j} + 2\mathbf{k}\} \text{ ft/s}^2$$

Relative to xyz, let $x'y'z'$ have

$\Omega_{xyz} = \{-180\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{P/B})_{xyz} = \{2.5\mathbf{k}\}$ ft ($(\mathbf{r}_{P/B})_{xyz}$ change direction relative to xyz.)

$$(\mathbf{v}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\mathbf{r}_{P/B})_{xyz} = \mathbf{0} + (-180\mathbf{i}) \times (2.5\mathbf{k}) = \{450\mathbf{j}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/B})_{xyz} = (\ddot{\mathbf{r}}_{P/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times (\mathbf{r}_{P/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{xyz}$$

$$(\mathbf{a}_{P/B})_{xyz} = \left[\mathbf{0} + \mathbf{0}\right] + \mathbf{0} + (-180\mathbf{i}) \times (450\mathbf{j}) = \{-81\,000\mathbf{k}\} \text{ ft/s}^2$$

Thus,

$$\mathbf{v}_F = \mathbf{v}_B + \Omega \times \mathbf{r}_{P/B} + (\mathbf{v}_{P/B})_{xyz}$$

$$= (-18\mathbf{i} + 4\mathbf{k}) + \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + (450\mathbf{j})$$

$$= \{-18\mathbf{i} + 450\mathbf{j} + 4\mathbf{k}\}\text{ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_P = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{P/B} + \Omega \times (\Omega \times \mathbf{r}_{P/B}) + 2\Omega \times (\mathbf{v}_{P/B})_{xyz} + (\mathbf{a}_{P/B})_{xyz}$$

$$= (-16.2\mathbf{j} + 2\mathbf{k}) + \mathbf{0} + (0.9\mathbf{k}) \times \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + \left[2(0.9\mathbf{k}) \times (450\mathbf{j})\right] + (-81000\mathbf{k})$$

$$= \{-810\mathbf{i} - 16.2\mathbf{j} - 81\,000\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

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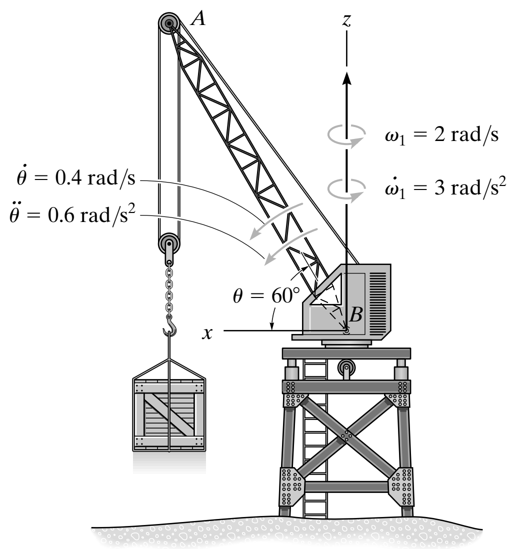
•20–49. At a given instant the boom AB of the tower crane rotates about the z axis with the motion shown. At this same instant, $\theta = 60°$ and the boom is rotating downward such that $\dot{\theta} = 0.4$ rad/s and $\ddot{\theta} = 0.6$ rad/s^2. Determine the velocity and acceleration of the end of the boom A at this instant. The boom has a length of $l_{AB} = 40$ m.



Coordinate Axis: The rotating x, y, z frame is set to be coincident with the fixed X, Y, Z frame with origin at point B.

Motion of *B*: Since point B does not move, then

$$\mathbf{a}_B = \mathbf{v}_B = \mathbf{0}$$

Motion of *A* with Respect to *B*: Let xyz axis rotate at $\Omega_{xyz} = \dot{\theta} = \{0.4\mathbf{j}\}$ rad/s and $\dot{\Omega}_{xyz} = \ddot{\theta} = \{0.6\mathbf{j}\}$ rad/s^2. Here.
$\mathbf{r}_{A/B} = \{40\cos 60°\mathbf{i} + 40\sin 60°\mathbf{k}\}$ m $= \{20.0\mathbf{i} + 34.64\mathbf{k}\}$ m.

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B} = \mathbf{0} + 0.4\mathbf{j} \times (20.0\mathbf{i} + 34.640\mathbf{k}) = \{13.86\mathbf{i} - 8.00\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{A/B} + \Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}$$

$$= \mathbf{0} + \mathbf{0} + 0.6\mathbf{j} \times (20.0\mathbf{i} + 34.64\mathbf{k}) + 0.4\mathbf{j} \times (13.86\mathbf{i} - 8.00\mathbf{k})$$

$$= \{17.58\mathbf{i} - 17.54\mathbf{k}\}\text{ m/s}^2$$

Motion of Point *A*: Here, $\Omega = \omega_1 = \{2\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{3\mathbf{k}\}$ rad/s^2. Applying Eqs. 20–11 and 20-12. we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} = \mathbf{0} + 2\mathbf{k} \times (20.0\mathbf{i} + 34.64\mathbf{k}) + (13.86\mathbf{i} - 8.00\mathbf{k})$$

$$= \{13.9\mathbf{i} - 40.0\mathbf{j} - 8.00\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= \mathbf{0} + 3\mathbf{k} \times (20.0\mathbf{i} + 34.64\mathbf{k}) + 2\mathbf{k} \times [2\mathbf{k} \times (20.0\mathbf{i} + 34.64\mathbf{k})] + 2(2\mathbf{k}) \times (13.86\mathbf{i} - 8.00\mathbf{k}) + 17.58\mathbf{i} - 17.54\mathbf{k}$$

$$= \{-62.4\mathbf{i} + 115\mathbf{j} - 17.5\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

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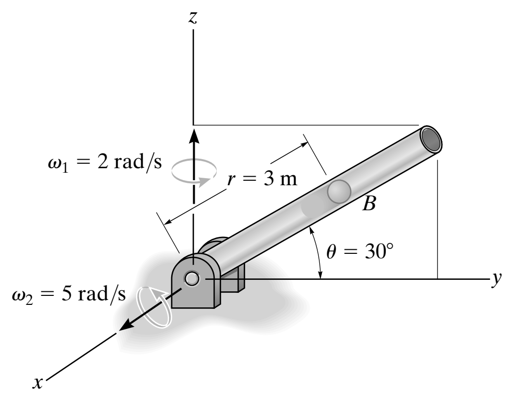
20–50. At the instant shown, the tube rotates about the z axis with a constant angular velocity $\omega_1 = 2$ rad/s, while at the same instant the tube rotates upward at a constant rate $\omega_2 = 5$ rad/s. If the ball B is blown through the tube at a rate $\dot{r} = 7$ m/s, which is increasing at $\ddot{r} = 2$ m/s^2, determine the velocity and acceleration of the ball at the instant shown. Neglect the size of the ball.



Coordinate Axis: The rotating x, y, z frame is set to be coincident with the fixed X, Y, Z frame with origin at point A.

Motion of A: Since point A does not move, then

$$\mathbf{a}_A = \mathbf{v}_A = \mathbf{0}$$

Motion of B with Respect to A: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{5\mathbf{i}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = \mathbf{0}$. Here, $\mathbf{r}_{B/A} = \{3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k}\}$ m $= \{2.5981\mathbf{j} + 1.50\mathbf{k}\}$ m, $(\dot{\mathbf{r}}_{B/A})_{xyz} = \{7\cos 30°\mathbf{j} + 7\sin 30°\mathbf{k}\}$ m/s $= \{6.0621\mathbf{j} + 3.50\mathbf{k}\}$ m/s and $(\ddot{\mathbf{r}}_{B/A})_{xyz} = \{2\cos 30\mathbf{j} + 2\sin 30°\mathbf{k}\}$ m/s^2 $= \{1.7321\mathbf{j} + 1\mathbf{k}\}$ m/s^2.

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{B/A}$$
$$= 6.0621\mathbf{j} + 3.50\mathbf{k} + 5\mathbf{i} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) = \{-1.4378\mathbf{j} + 16.4903\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = \left[(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{B/A} + \Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}$$
$$= 1.7321\mathbf{j} + 1\mathbf{k} + 5\mathbf{i} \times (6.0621\mathbf{j} + 3.50\mathbf{k}) + \mathbf{0} + 5\mathbf{i} \times (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$
$$= \{-98.2199\mathbf{j} - 24.1218\mathbf{k}\} \text{ m/s}^2$$

Motion of Point B: Here, $\Omega = \omega_1 = \{2\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$. Applying Eqs. 20–11 and 20–12, we have

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz} = \mathbf{0} + 2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) + (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$
$$= \{-5.20\mathbf{i} - 1.44\mathbf{j} + 16.5\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{a}_A + \ + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$
$$= \mathbf{0} + \mathbf{0} + 2\mathbf{k} \times [2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k})] + 2(2\mathbf{k}) \times (-1.4378\mathbf{j} + 16.4903\mathbf{k}) + (-98.2199\mathbf{j} + 24.1218\mathbf{k})$$
$$= \{5.75\mathbf{i} - 109\mathbf{j} + 24.1\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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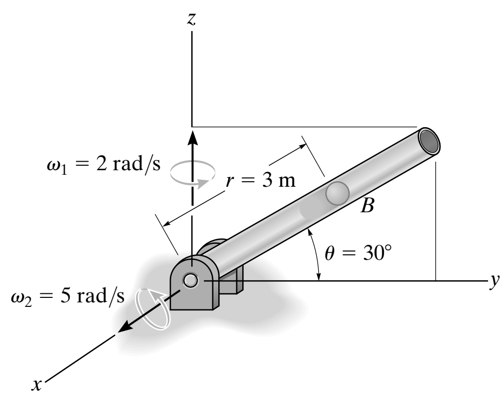
20–51. At the instant shown, the tube rotates about the z axis with a constant angular velocity $\omega_1 = 2$ rad/s, while at the same instant the tube rotates upward at a constant rate $\omega_2 = 5$ rad/s. If the ball B is blown through the tube at a constant rate $\dot{r} = 7$ m/s, determine the velocity and acceleration of the ball at the instant shown. Neglect the size of the ball.



Coordinate Axis: The rotating x, y, z frame is set to be coincident with the fixed X, Y, Z frame with origin at point A.

Motion of A: Since point A does not move, then

$$\mathbf{a}_A = \mathbf{v}_A = \mathbf{0}$$

Motion of B with Respect to A: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{5\mathbf{i}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = \mathbf{0}$. Here, $\mathbf{r}_{B/A} = \{3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k}\}$ m $= \{2.5981\mathbf{j} + 1.50\mathbf{k}\}$ m and $(\dot{\mathbf{r}}_{B/A})_{xyz} = \{7\cos 30\mathbf{j} + 7\sin 30°\mathbf{k}\}$ m/s $= \{6.0621\mathbf{j} + 3.50\mathbf{k}\}$ m/s.

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{B/A}$$

$$= 6.0621\mathbf{j} + 3.50\mathbf{k} + 5\mathbf{i} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) = \{-1.4378\mathbf{j} + 16.4903\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = \left[(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{B/A} + \Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}$$

$$= \mathbf{0} + 5\mathbf{i} \times (6.0621\mathbf{j} + 3.50\mathbf{k}) + \mathbf{0} + 5\mathbf{i} \times (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$

$$= \{-99.9519\mathbf{j} + 23.1218\mathbf{k}\} \text{ m/s}^2$$

Motion of Point B: Here, $\Omega = \omega_1 = \{2\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$. Applying Eqs. 20–11 and 20–12, we have

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz} = \mathbf{0} + 2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) + (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$

$$= \{-5.20\mathbf{i} - 1.44\mathbf{j} + 16.5\mathbf{k}\}\text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{a}_A + \ + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + 2\mathbf{k} \times [2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k})] + 2(2\mathbf{k}) \times (-1.4378\mathbf{j} + 16.4903\mathbf{k}) + (-99.9519\mathbf{j} + 23.1218\mathbf{k})$$

$$= \{5.75\mathbf{i} - 110\mathbf{j} + 23.1\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

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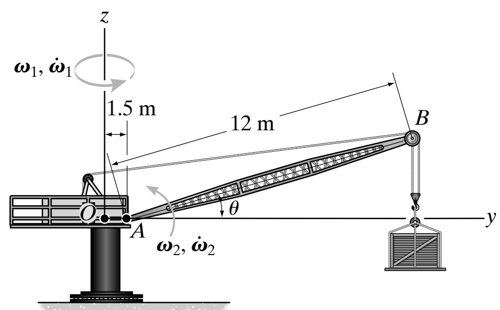
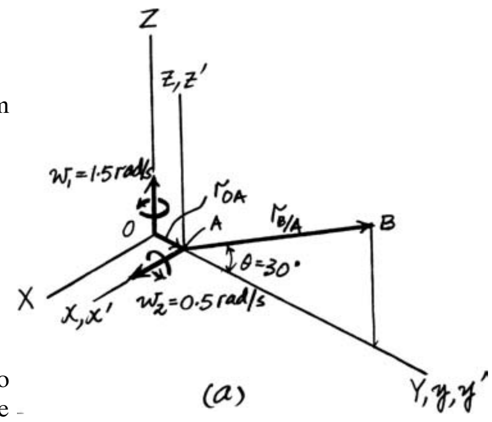
***20–52.** At the instant $\theta = 30°$, the frame of the crane and the boom AB rotate with a constant angular velocity of $\omega_1 = 1.5$ rad/s and $\omega_2 = 0.5$ rad/s, respectively. Determine the velocity and acceleration of point B at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$



Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= \mathbf{0} + (1.5\mathbf{k}) \times [(1.5\mathbf{k}) \times (1.5\mathbf{j})]$$

$$= [-3.375\mathbf{j}] \text{ m/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30°\,\mathbf{j} + 12 \sin 30°\mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{A/B})_{xyz}$,

$$(\mathbf{a}_{A/B})_{xyz} = (\ddot{\mathbf{r}}_{A/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/B})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{A/B})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{xyz}$$

$$= [0 + 0] + 0 + (0.5\mathbf{i}) \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-2.598\mathbf{j} - 1.5\mathbf{k}] \text{ m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30°\,\mathbf{j} + 12 \sin 30°\,\mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{AB})_{xyz} + (\mathbf{a}_{AB})_{xyz}$$

$$= (-3.375\mathbf{j}) + 0 + 1.5\mathbf{k} \times \left[(1.5\mathbf{k}) \times (12 \cos 30°\,\mathbf{j} + 12 \sin 30°\,\mathbf{k})\right] + 2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-2.598\mathbf{j} - 1.5\mathbf{k})$$

$$= [9\mathbf{i} - 29.4\mathbf{j} - 1.5\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

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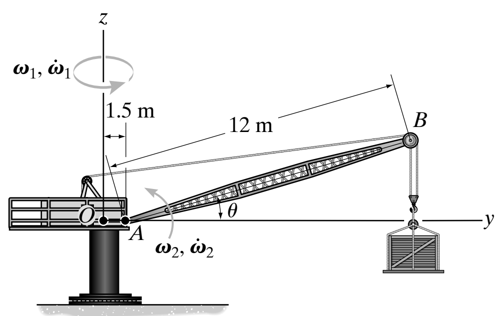
•20–53. At the instant $\theta = 30°$, the frame of the crane is rotating with an angular velocity of $\omega_1 = 1.5$ rad/s and angular acceleration of $\dot{\omega}_1 = 0.5$ rad/s^2, while the boom AB rotates with an angular velocity of $\omega_2 = 0.5$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.25$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = [0.5\mathbf{k}] \text{ rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.5\mathbf{k}) \times (1.5\mathbf{j}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (1.5\mathbf{j})\right]$$

$$= [-0.75\mathbf{i} - 3.375\mathbf{j}] \text{ m/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30° \, \mathbf{j} + 12 \sin 30° \, \mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [0.25\mathbf{i}]$ m/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

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•20–53. Continued

$$(\mathbf{a}_{B/A}) = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\Omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [0 + 0] + (0.25\mathbf{i}) \times (12 \cos 30^\circ\, \mathbf{j} + 12 \sin 30^\circ \mathbf{k}) + 0.5\mathbf{i} \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-4.098\mathbf{j} + 1.098\mathbf{k}] \text{ m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30^\circ\, \mathbf{j} + 12 \sin 30^\circ \mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

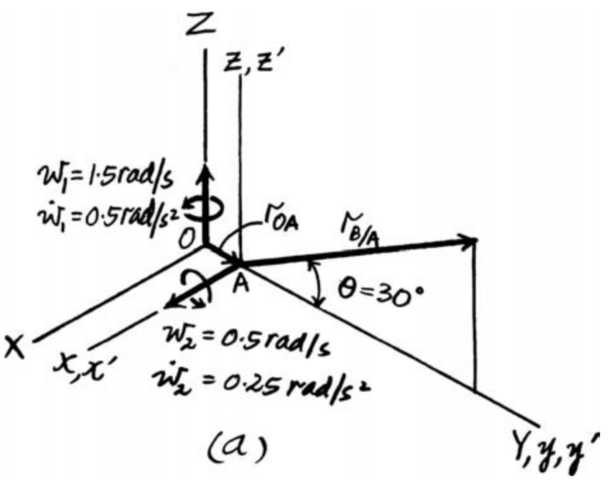
and

$$\mathbf{a}_B = \mathbf{a}_A = \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-0.75\mathbf{i} - 3.375\mathbf{j}) + 0.5\mathbf{k} \times (12 \cos 30^\circ \mathbf{j} + 12 \sin 30^\circ\, \mathbf{k}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (12 \cos 30^\circ\, \mathbf{j} + 12 \sin 30^\circ\, \mathbf{k})\right]$$

$$+2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-4.098\mathbf{j} + 1.098\mathbf{k})$$

$$= [3.05\mathbf{i} - 30.9\mathbf{j} + 1.10\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$



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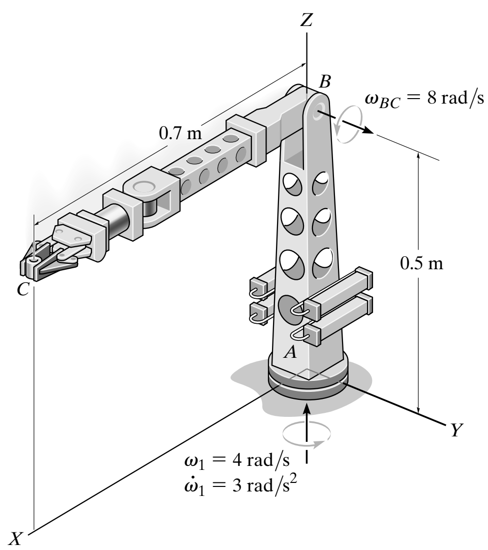
20–54. At the instant shown, the base of the robotic arm rotates about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom BC rotates at a constant rate of $\omega_{BC} = 8$ rad/s. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz have origin at B and have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\quad \Omega = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m ($\mathbf{r}_B$ does not change direction relative to XYZ.)

$\mathbf{v}_B = 0$

$\mathbf{a}_B = 0$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\quad \Omega_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m ($(\mathbf{r}_{C/B})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = \mathbf{0} + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = \mathbf{0} + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= \mathbf{0} + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$= 2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j}\} \text{ m/s}^2$$ **Ans.**

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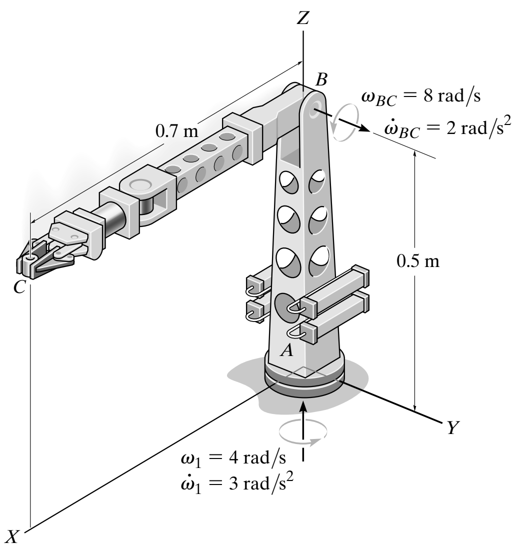
20–55. At the instant shown, the base of the robotic arm rotates about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom BC rotates at $\omega_{BC} = 8$ rad/s, which is increasing at $\dot{\omega}_{BC} = 2$ rad/s^2. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz with origin at B have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\Omega = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m (r_B does not change direction relative to XYZ.)

$\mathbf{v}_B = \mathbf{0}$

$\mathbf{a}_B = \mathbf{0}$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\Omega_{xyz} = \{2\mathbf{j}\}$ rad/s^2 (Ω does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m (Ω does not change direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left(\mathbf{r}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = \mathbf{0} + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (2\mathbf{j}) \times (0.7\mathbf{i}) + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i} - 1.40\mathbf{k}\}\text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = \mathbf{0} + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\}\text{ m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= \mathbf{0} + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$= 2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i} - 1.40\mathbf{k}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j} - 1.40\mathbf{k}\}\text{ m/s}^2$$ **Ans.**

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•21–1. Show that the sum of the moments of inertia of a body, $I_{xx} + I_{yy} + I_{zz}$, is independent of the orientation of the x, y, z axes and thus depends only on the location of its origin.

$$I_{xx} + I_{yy} + I_{zz} = \int_m (y^2 + z^2)dm + \int_m (x^2 + z^2)dm + \int_m (x^2 + y^2)dm$$

$$= 2\int_m (x^2 + y^2 + z^2)dm$$

However, $x^2 + y^2 + z^2 = r^2$, where r is the distance from the origin O to dm. Since $|r|$ is constant, it does not depend on the orientation of the x, y, z axis. Consequently, $I_{xx} + I_{yy} + I_{zz}$ is also indepenent of the orientation of the x, y, z axis. **Q.E.D.**

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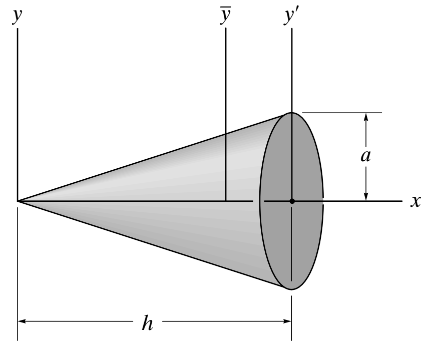
21–2. Determine the moment of inertia of the cone with respect to a vertical $\bar{y}$ axis that passes through the cone's center of mass. What is the moment of inertia about a parallel axis y' that passes through the diameter of the base of the cone? The cone has a mass m.



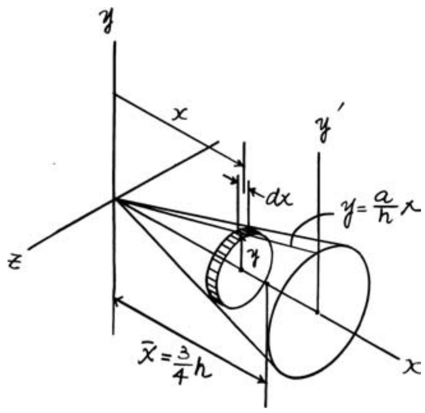
The mass of the differential element is $dm = \rho dV = \rho(\pi y^2)\, dx = \dfrac{\rho\pi a^2}{h^2} x^2 dx$.

$$dI_v = \frac{1}{4} dmy^2 + dmx^2$$

$$= \frac{1}{4}\left[\frac{\rho\pi a^2}{h^2} x^2\, dx\right]\left(\frac{a}{h}x\right)^2 + \left(\frac{\rho\pi a^2}{h^2} x^2\right)x^2\, dx$$

$$= \frac{\rho\,\pi a^2}{4h^4}(4h^2 + a^2)\, x^4\, dx$$

$$I_v = \int dI_v = \frac{\rho\pi a^2}{4h^4}(4h^2 + a^2)\int_0^h x^4 dx = \frac{\rho\,\pi a^2 h}{20}(4h^2 + a^2)$$



However,

$$m = \int_m dm = \frac{\rho\pi a^2}{h^2}\int_0^h x^2\, dx = \frac{\rho\,\pi a^2 h}{3}$$

Hence,

$$I_y = \frac{3m}{20}(4h^2 + a^2)$$

Using the parallel axis theorem:

$$I_v = I_v + md^2$$

$$\frac{3m}{20}(4h^2 + a^2) = I_v + m\left(\frac{3h}{4}\right)^2$$

$$I_v = \frac{3m}{80}(h^2 + 4a^2) \qquad \textbf{Ans.}$$

$$I_v = I_y + md^2$$

$$= \frac{3m}{80}(h^2 + 4a^2) + m\left(\frac{h}{4}\right)^2$$

$$= \frac{m}{20}(2h^2 + 3a^2) \qquad \textbf{Ans.}$$

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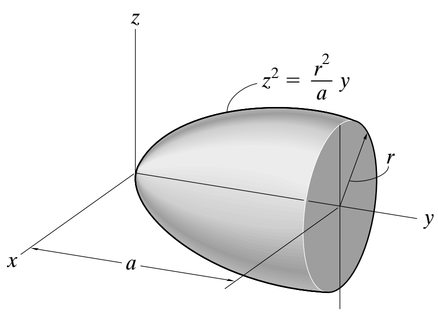
21–3. Determine the moments of inertia I_x and I_y of the paraboloid of revolution. The mass of the paraboloid is m.

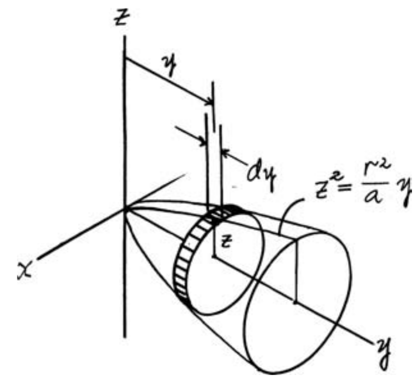


$$m = \rho \int_0^a \pi z^2\, dy = \rho\pi \int_0^a \left(\frac{r^2}{a}\right) y\,dy = \rho r\left(\frac{r^2}{2}\right)a$$

$$I_v = \int_m \frac{1}{2}\,dm\, z^2 = \frac{1}{2}\rho\pi \int_0^a z^4 dy = \frac{1}{2}\rho\pi\left(\frac{r^4}{a^2}\right)\int_0^a v^2 dy = \rho\pi\left(\frac{r^4}{6}\right)a$$

Thus,

$$I_x = \frac{1}{3}mr^2 \qquad \textbf{Ans.}$$

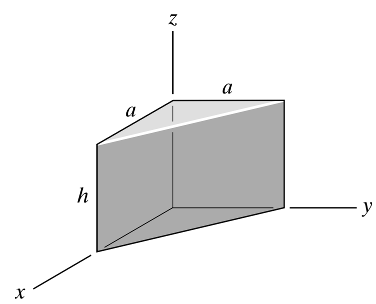


$$I_x = \int_m \left(\frac{1}{4}\,dm\, z^2 + dm\, y^2\right) = \frac{1}{4}\rho\pi \int_0^a z^4\, dy + \rho \int_0^a \pi z^2 y^2\, dy$$

$$= \frac{1}{4}\rho\pi\left(\frac{r^4}{a^2}\right)\int_0^a y^2 dy + \rho\pi\left(\frac{r^2}{a}\right)\int_0^a y^3 dy = \frac{\rho\pi r^4 a}{12} + \frac{\rho\pi r^2 a^3}{4} = \frac{1}{6}mr^2 + \frac{1}{2}ma^2$$

$$I_x = \frac{m}{6}(r^2 + 3a^2) \qquad \textbf{Ans.}$$

***21–4.** Determine by direct integration the product of inertia I_{yz} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the total mass m of the prism.



The mass of the differential element is $dm = \rho dV = \rho hx dy = \rho h(a - y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a - y)dy = \frac{\rho a^2 h}{2}$$

Using the parallel axis theorem:

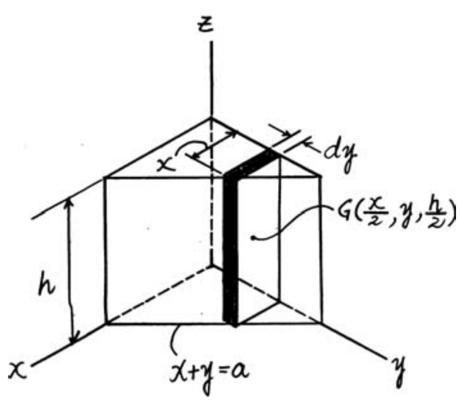
$$dI_{yz} = (dI_{y'z'})_G + dm y_G z_G$$

$$= 0 + (\rho hx dy)(y)\left(\frac{h}{2}\right)$$

$$= \frac{\rho h^2}{2} xy dy$$

$$= \frac{\rho h^2}{2}(ay - y^2)\, dy$$

$$I_{yz} = \frac{\rho h^2}{2}\int_0^a (ay - y^2)\, dy = \frac{\rho a^3 h^2}{12} = \frac{1}{6}\left(\frac{\rho a^2 h}{2}\right)(ah) = \frac{m}{6}ah \qquad \textbf{Ans.}$$



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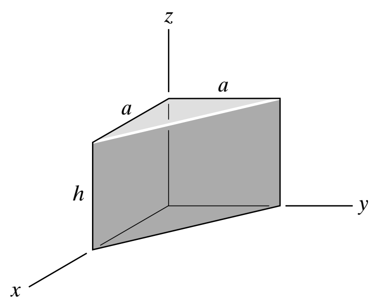
•21–5. Determine by direct integration the product of inertia I_{xy} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the total mass m of the prism.



The mass of the differential element is $dm = \rho dV = \rho hxdy = \rho h(a - y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a - y)dy = \frac{\rho a^2 h}{2}$$

Using the parallel axis theorem:

$$dI_{xy} = (dI_{x'y'})_G + dm x_G y_G$$

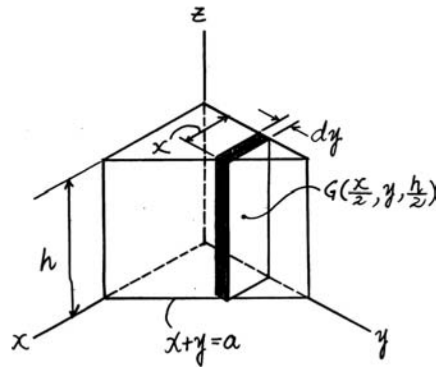
$$= 0 + (\rho hxdy)\left(\frac{x}{2}\right)(y)$$

$$= \frac{\rho h^2}{2} x^2 y dy$$

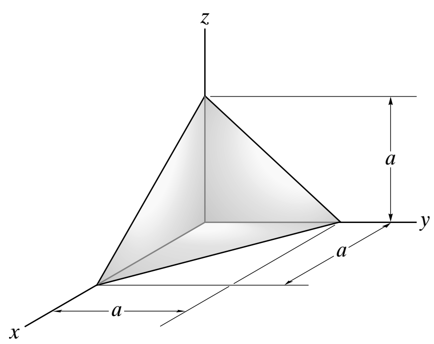
$$= \frac{\rho h^2}{2} (y^3 - 2ay^2 + a^2 y)\, dy$$

$$I_{xy} = \frac{\rho h}{2} \int_0^a (y^3 - 2ay^2 + a^2 y)\, dy$$

$$= \frac{\rho a^4 h}{24} = \frac{1}{12}\left(\frac{\rho a^2 h}{2}\right)a^2 = \frac{m}{12}a^2 \qquad \textbf{Ans.}$$



21–6. Determine the product of inertia I_{xy} for the homogeneous tetrahedron. The density of the material is ρ. Express the result in terms of the total mass m of the solid. *Suggestion:* Use a triangular element of thickness dz and then express dI_{xy} in terms of the size and mass of the element using the result of Prob. 21–5.



$$dm = \rho\, dV = \rho\left[\frac{1}{2}(a - z)(a - z)\right]dz = \frac{\rho}{2}(a - z)^2\, dz$$

$$m = \frac{\rho}{2}\int_0^a (a^2 - 2az + z^2)dz = \frac{\rho a^3}{6}$$

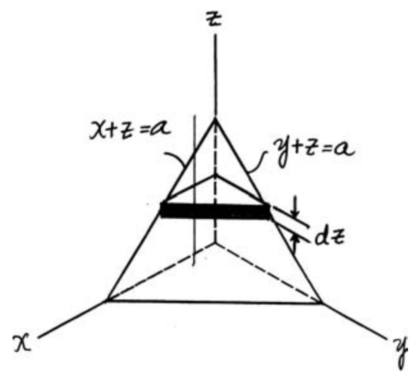
From Prob. 21–5 the product of inertia of a triangular prism with respect to the xz and yz planes is $I_{xy} = \frac{\rho a^4 h}{24}$. For the element above, $dI_{xy} = \frac{\rho dz}{24}(a - z)^4$. Hence,

$$I_{xy} = \frac{\rho}{24}\int_0^a (a^4 - 4a^3 z + 6z^2 a^2 - 4az^3 + z^4)dz$$

$$I_{xy} = \frac{\rho a^5}{120}$$

or,

$$I_{xy} = \frac{ma^2}{20} \qquad \textbf{Ans.}$$



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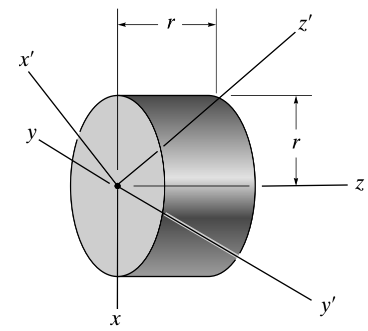
21–7. Determine the moments of inertia for the homogeneous cylinder of mass m about the x', y', z' axes.



Due to symmetry

$$I_{xy} = I_{yz} = I_{zx} = 0$$

$$I_y = I_x = \frac{1}{12}m(3r^2 + r^2) + m\left(\frac{r}{2}\right)^2 = \frac{7mr^2}{12} \qquad I_z = \frac{1}{2}mr^2$$

For x',

$$u_x = \cos 135^\circ = -\frac{1}{\sqrt{2}}, \qquad u_y = \cos 90^\circ = 0, \qquad u_z = \cos 135^\circ = -\frac{1}{\sqrt{2}}$$

$$I_x = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy}u_x u_y - 2I_{yz}u_y u_z - 2I_{zx}u_z u_x$$

$$= \frac{7mr^2}{12}\left(-\frac{1}{\sqrt{2}}\right)^2 + 0 + \frac{1}{2}mr^2\left(-\frac{1}{\sqrt{2}}\right)^2 - 0 - 0 - 0$$

$$= \frac{13}{24}mr^2$$ **Ans.**

For y',

$$I_{y'} = I_y = \frac{7mr^2}{12}$$ **Ans.**

For z',

$$u_x = \cos 135^\circ = -\frac{1}{\sqrt{2}}, \qquad u_y = \cos 90^\circ = 0, \qquad u_z = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy}u_x u_y - 2I_{yz}u_y u_z - 2I_{zx}u_z u_x$$

$$= \frac{7mr^2}{12}\left(-\frac{1}{\sqrt{2}}\right)^2 + 0 + \frac{1}{2}mr^2\left(-\frac{1}{\sqrt{2}}\right)^2 - 0 - 0 - 0$$

$$= \frac{13}{24}mr^2$$ **Ans.**

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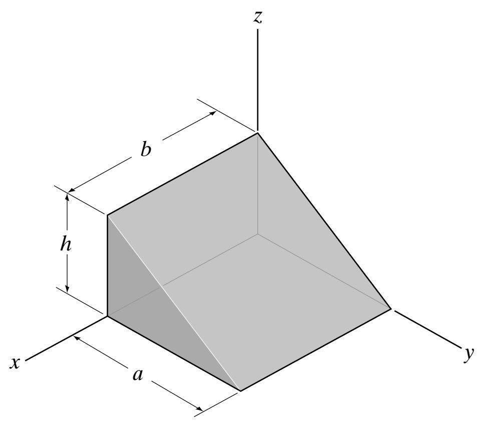
***21–8.** Determine the product of inertia I_{xy} of the homogeneous triangular block. The material has a density of ρ. Express the result in terms of the total mass m of the block.

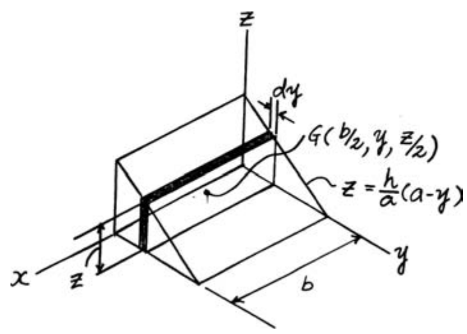


The mass of the differential rectangular volume element shown in Fig. a is $dm = \rho dV = \rho bzdy$. Using the parallel - plane theorem,

$$dI_{xy} = dI_{x'y'} + dm x_G y_G$$

$$= 0 + [\rho bzdy]\left(\frac{b}{2}\right)y$$

$$= \frac{\rho b^2}{2} zydy$$



However, $z = \frac{h}{a}(a - y)$. Then

$$dI_{xy} = \frac{\rho b^2}{2}\left[\frac{h}{a}(a - y)y\right]dy = \frac{\rho b^2 h}{2a}\left(ay - y^2\right) dy$$

Thus,

$$I_{xy} = \int dI_{xy} = \frac{\rho b^2 h}{2a}\int_0^a \left(ay - y^2\right)dy$$

$$= \frac{\rho b^2 h}{2a}\left(\frac{ay^2}{2} - \frac{y^3}{3}\right)\Big|_0^a$$

$$= \frac{1}{12}\rho a^2 b^2 h$$

However, $m = \rho V = \left(\frac{1}{2}ahb\right) = \frac{1}{2}\rho abh$. Then

$$I_{xy} = \frac{1}{12}\rho a^2 b^2 h\left(\frac{m}{\frac{1}{2}\rho abh}\right) = \frac{1}{6}mab \qquad \textbf{Ans.}$$

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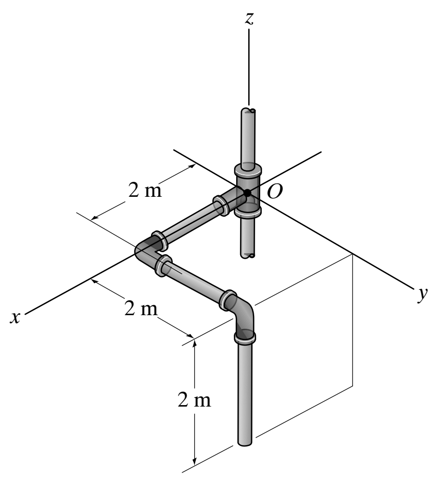
•21–9. The slender rod has a mass per unit length of 6 kg/m. Determine its moments and products of inertia with respect to the x, y, z axes.



The mass of segments (1), (2), and (3) shown in Fig. a is $m_1 = m_2 = m_3 = 6(2) = 12$ kg. The mass moments of inertia of the bent rod about the x, y, and z axes are

$$I_x = \Sigma \bar{I}_{x'} + m\left(y_G{}^2 + z_G{}^2\right)$$

$$= (0 + 0) + \left[\frac{1}{12}(12)\left(2^2\right) + 12\left(1^2 + 0^2\right)\right] + \left[\frac{1}{12}(12)\left(2^2\right) + 12\left[2^2 + (-1)^2\right]\right]$$

$$= 80 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_y = \Sigma \bar{I}_{y'} + m\left(x_G{}^2 + z_G{}^2\right)$$

$$= \left[\frac{1}{12}(12)\left(2^2\right) + 12\left(1^2 + 0^2\right)\right] + \left[0 + 12\left(2^2 + 0^2\right)\right] + \left[\frac{1}{12}(12)\left(2^2\right) + 12\left[2^2 + (-1)^2\right]\right]$$

$$= 128 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_z = \Sigma \bar{I}_{z'} + m\left(x_G{}^2 + y_G{}^2\right)$$

$$= \left[\frac{1}{12}(12)\left(2^2\right) + 12\left(1^2 + 0^2\right)\right] + \left[\frac{1}{2}(12)\left(2^2\right) + 12\left(2^2 + 1^2\right)\right] + \left[0 + 12\left(2^2 + 2^2\right)\right]$$

$$= 176 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Due to symmetry, the products of inertia of segments (1), (2), and (3) with respect to their centroidal planes are equal to zero. Thus,

$$I_{xy} = \Sigma \bar{I}_{x'y'} + m x_G y_G$$

$$= [0 + 12(1)(0)] + [0 + 12(2)(1)] + [0 + 12(2)(2)]$$

$$= 72 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{yz} = \Sigma \bar{I}_{y'z'} + m y_G z_G$$

$$= [0 + 12(0)(0)] + [0 + 12(1)(0)] + [0 + 12(2)(-1)]$$

$$= -24 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{xz} = \Sigma \bar{I}_{x'z'} + m x_G z_G$$

$$= [0 + 12(1)(0)] + [0 + 12(2)(0)] + [0 + 12(2)(-1)]$$

$$= -24 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

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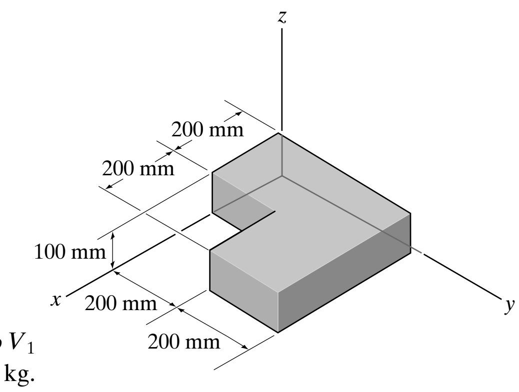
21–10. Determine the products of inertia I_{xy}, I_{yz}, and I_{xz} of the homogeneous solid. The material has a density of 7.85 Mg/m^3.



The masses of segments (1) and (2) shown in Fig. *a* are $m_1 = \rho V_1 = 7850(0.4)(0.4)(0.1) = 125.6$ kg and $m_2 = \rho V_2 = 7850(0.2)(0.2)(0.1) = 31.4$ kg. Due to symmetry $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{x'z'} = 0$ for segment (1) and $\bar{I}_{x''y''} = \bar{I}_{y''z''} = \bar{I}_{x''z''} = 0$ for segment (2). Since segment (2) is a hole, it should be considered as a negative segment. Thus

$$I_{xy} = \Sigma \bar{I}_{x'y'} + m x_G y_G$$

$$= \left[0 + 125.6(0.2)(0.2)\right] - \left[0 + 31.4(0.3)(0.1)\right]$$

$$= 4.08 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{yz} = \Sigma \bar{I}_{y'z'} + m y_G z_G$$

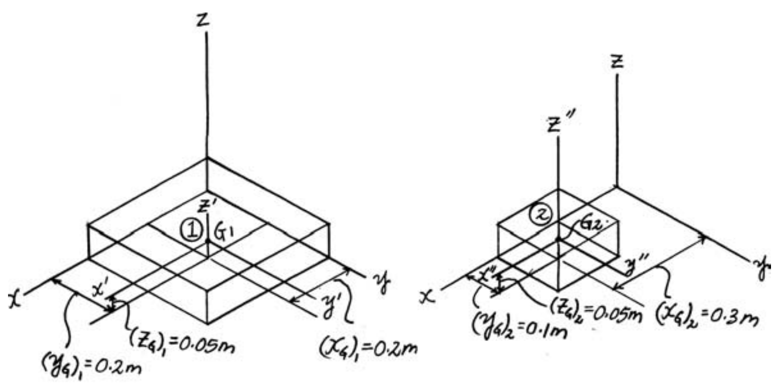
$$= \left[0 + 125.6(0.2)(0.05)\right] - \left[0 + 31.4(0.1)(0.05)\right]$$

$$= 1.10 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{xz} = \Sigma \bar{I}_{x'z'} + m x_G z_G$$

$$= \left[0 + 125.6(0.2)(0.05)\right] - \left[0 + 31.4(0.3)(0.05)\right]$$

$$= 0.785 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

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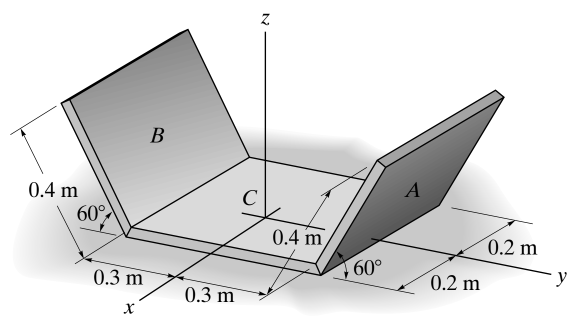
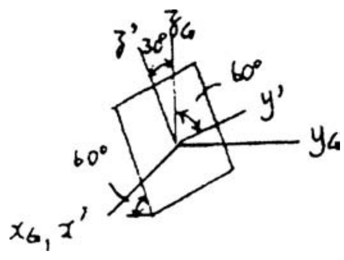
21–11. The assembly consists of two thin plates A and B which have a mass of 3 kg each and a thin plate C which has a mass of 4.5 kg. Determine the moments of inertia I_x, I_y and I_z.



$$I_{x'} = I_{z'} = \frac{1}{12}(3)(0.4)^2 = 0.04\ \text{kg}\cdot\text{m}^2$$

$$I_{y'} = \frac{1}{12}(3)[(0.4)^2 + (0.4)^2] = 0.08\ \text{kg}\cdot\text{m}^2$$

$$I_{x'y'} = I_{z'y'} = I_{z'x'} = 0$$



For z_G,

$$u_{x'} = 0$$

$$u_{y'} = \cos 60° = 0.50$$

$$u_{z'} = \cos 30° = 0.8660$$

$$I_{z_G} = 0 + 0.08(0.5)^2 + 0.04(0.866)^2 - 0 - 0 - 0$$
$$= 0.05\ \text{kg}\cdot\text{m}^2$$

$$I_{x_G} = I_{x'} = 0.04\ \text{kg}\cdot\text{m}^2$$

For y_G,

$$u_{x'} = 0$$

$$u_{y'} = \cos 30° = 0.866$$

$$u_{z'} = \cos 120° = -0.50$$

$$I_{y_G} = 0 + 0.08(0.866)^2 + 0.04(-0.5)^2 - 0 - 0 - 0$$
$$= 0.07\ \text{kg}\cdot\text{m}^2$$

$$I_x = \frac{1}{12}(4.5)(0.6)^2 + 2[0.04 + 3\{(0.3 + 0.1)^2 + (0.1732)^2\}]$$

$I_x = 1.36\ \text{kg}\cdot\text{m}^2$ **Ans.**

$$I_y = \frac{1}{12}(4.5)(0.4)^2 + 2[0.07 + 3(0.1732)^2]$$

$I_y = 0.380\ \text{kg}\cdot\text{m}^2$ **Ans.**

$$I_z = \frac{1}{12}(4.5)[(0.6)^2 + (0.4)^2] + 2[0.05 + 3(0.3 + 0.1)^2]$$

$I_z = 1.26\ \text{kg}\cdot\text{m}^2$ **Ans.**

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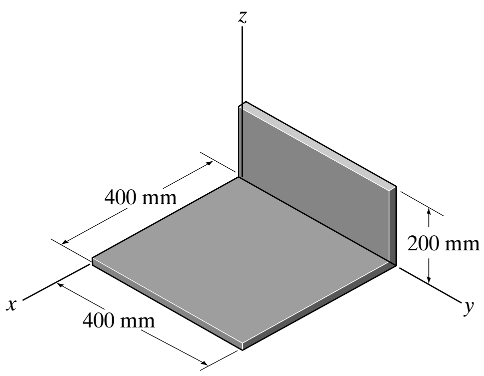
***21–12.** Determine the products of inertia I_{xy}, I_{yz}, and I_{xz}, of the thin plate. The material has a density per unit area of 50 kg/m^2.



The masses of segments (1) and (2) shown in Fig. *a* are $m_1 = 50(0.4)(0.4) = 8$ kg and $m_2 = 50(0.4)(0.2) = 4$ kg. Due to symmetry $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{x'z'} = 0$ for segment (1) and $\bar{I}_{x''y''} = \bar{I}_{y''z''} = \bar{I}_{x''z''} = 0$ for segment (2).

$$I_{xy} = \Sigma \bar{I}_{x'y'} + m x_G y_G$$

$$= \left[0 + 8(0.2)(0.2)\right] + \left[0 + 4(0)(0.2)\right]$$

$$= 0.32 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

$$I_{yz} = \Sigma \bar{I}_{y'z'} + m y_G z_G$$

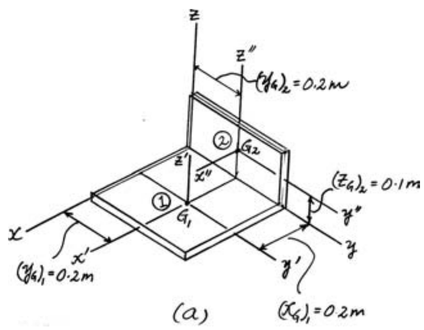
$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 4(0.2)(0.1)\right]$$

$$= 0.08 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

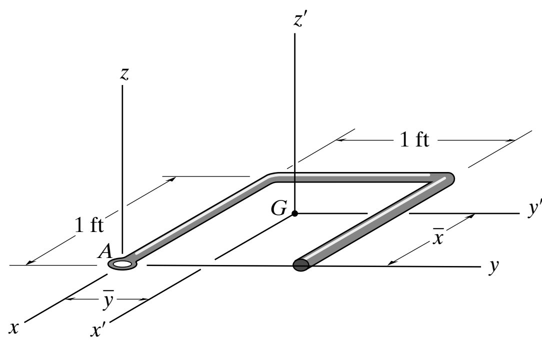
$$I_{xz} = \Sigma \bar{I}_{x'z'} + m x_G z_G$$

$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 4(0)(0.1)\right]$$

$$= 0 \qquad \textbf{Ans.}$$



•21–13. The bent rod has a weight of 1.5 lb/ft. Locate the center of gravity $G(\bar{x}, \bar{y})$ and determine the principal moments of inertia $I_{x'}$, $I_{y'}$, and $I_{z'}$ of the rod with respect to the x', y', z' axes.



Due to symmetry

$$\bar{y} = 0.5 \text{ ft} \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma w} = \frac{(-1)(1.5)(1) + 2\left[(-0.5)(1.5)(1)\right]}{3\left[1.5(1)\right]} = -0.667 \text{ ft} \qquad \textbf{Ans.}$$

$$I_{x'} = 2\left[\left(\frac{1.5}{32.2}\right)(0.5)^2\right] + \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2$$

$$= 0.0272 \text{ slug} \cdot \text{ft}^2 \qquad \textbf{Ans.}$$

$$I_{y'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.667 - 0.5)^2\right] + \left(\frac{1.5}{32.2}\right)(1 - 0.667)^2$$

$$= 0.0155 \text{ slug} \cdot \text{ft}^2 \qquad \textbf{Ans.}$$

$$I_{z'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.5^2 + 0.1667^2)\right]$$

$$+ \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.3333)^2$$

$$= 0.0427 \text{ slug} \cdot \text{ft}^2 \qquad \textbf{Ans.}$$

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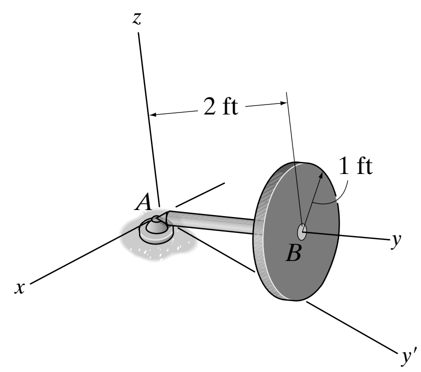
21–14. The assembly consists of a 10-lb slender rod and a 30-lb thin circular disk. Determine its moment of inertia about the y' axis.



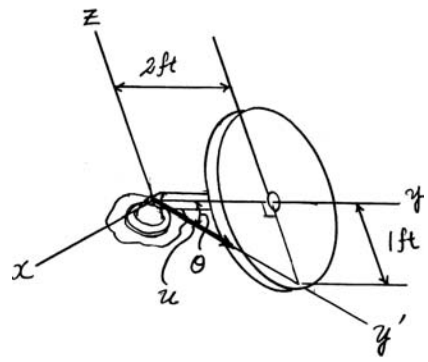
The mass of moment inertia of the assembly about the x, y, and z axes are

$$I_x = I_z = \left[\frac{1}{4}\left(\frac{30}{32.2}\right)(1^2) + \frac{30}{32.2}(2^2)\right] + \left[\frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) + \frac{10}{32.2}(1^2)\right]$$

$$= 4.3737 \text{ slug} \cdot \text{ft}^2$$

$$I_y = \frac{1}{2}\left(\frac{30}{32.2}\right)(1^2) + 0 = 0.4658 \text{ slug} \cdot \text{ft}^2$$

Due to symmetry, $I_{xy} = I_{yz} = I_{xz} = 0$. From the geometry shown in Fig. *a*, $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$. Thus, the direction of the y' axis is defined by the unit vector



(a)

$$\mathbf{u} = \cos 26.57°\mathbf{j} - \sin 26.57°\mathbf{k} = 0.8944\mathbf{j} - 0.4472\mathbf{k}$$

Thus,

$$u_x = 0 \qquad u_y = 0.8944 \qquad u_z = -0.4472$$

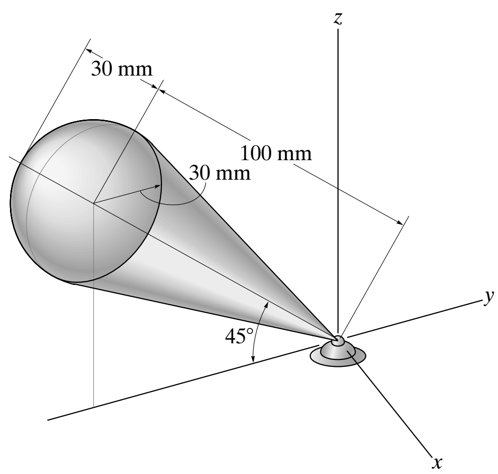
Then

$$I_{y'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy}u_x u_y - 2I_{yz}u_y u_z - 2I_{xz}u_x u_z$$

$$= 4.3737(0) + 0.4658(0.8944)^2 + 4.3737(-0.4472)^2 - 0 - 0 - 0$$

$$= 1.25 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

21–15. The top consists of a cone having a mass of 0.7 kg and a hemisphere of mass 0.2 kg. Determine the moment of inertia I_z when the top is in the position shown.



$$I_{x'} = I_{y'} = \frac{3}{80}(0.7)\left[(4)(0.3)^2 + (0.1)^2\right] + (0.7)\left[\frac{3}{4}(0.1)\right]^2$$

$$+ \left(\frac{83}{320}\right)(0.2)(0.03)^2 + (0.2)\left[\frac{3}{8}(0.03) + (0.1)\right]^2 = 6.816\left(10^{-3}\right)\text{kg} \cdot \text{m}^2$$

$$I_{z'} = \left(\frac{3}{10}\right)(0.7)(0.03)^2 + \left(\frac{2}{5}\right)(0.2)(0.03)^2$$

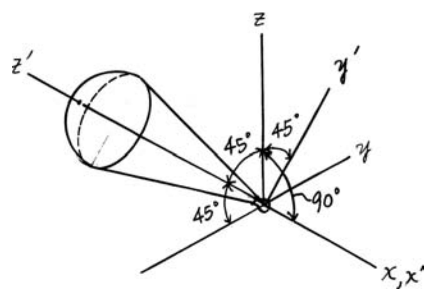
$$I_z = 0.261\left(10^{-3}\right) \text{kg} \cdot \text{m}^2$$

$$u_x = \cos 90° = 0, \qquad u_{y'} = \cos 45° = 0.7071, \qquad u_{z'} = \cos 45° = 0.7071$$

$$I_z = I_{x'}u_{x'}^2 + I_{y'}u_{y'}^2 + I_{z'}u_{z'}^2 - 2I_{x'y'}u_{x'}u_{y'} - 2I_{y'z'}u_{y'}u_{z'} - 2I_{x'z'}u_{x'}u_{z'}$$

$$= 0 + 6.816\left(10^{-3}\right)(0.7071)^2 + (0.261)\left(10^{-3}\right)(0.7071)^2 - 0 - 0 - 0$$

$$I_z = 3.54\left(10^{-3}\right) \text{kg} \cdot \text{m}^2$$ **Ans.**



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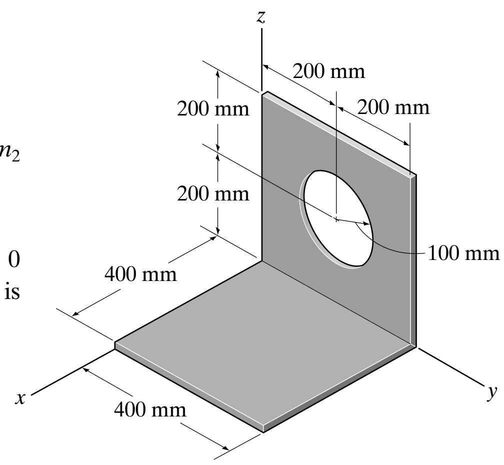
***21–16.** Determine the products of inertia I_{xy}, I_{yz}, and I_{xz} of the thin plate. The material has a mass per unit area of 50 kg/m^2.



The masses of segments (1), (2), and (3) shown in Fig. *a* are $m_1 = m_2 = 50(0.4)(0.4) = 8$ kg and $m_3 = 50\left[\pi(0.1)^2\right] = 0.5\pi$ kg.

Due to symmetry $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{x'z'} = 0$ for segment (1), $\bar{I}_{x''y''} = \bar{I}_{y''z''} = \bar{I}_{x''z''} = 0$ for segment (2), and $\bar{I}_{x'''y'''} = \bar{I}_{y'''z'''} = \bar{I}_{x'''z'''} = 0$ for segment (3). Since segment (3) is a hole, it should be considered as a negative segment. Thus

$$I_{xy} = \Sigma \bar{I}_{x'y'} + m x_G y_G$$

$$= \left[0 + 8(0.2)(0.2)\right] + \left[0 + 8(0)(0.2)\right] - \left[0 + 0.5\pi(0)(0.2)\right]$$

$$= 0.32 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{yz} = \Sigma \bar{I}_{y'z'} + m y_G z_G$$

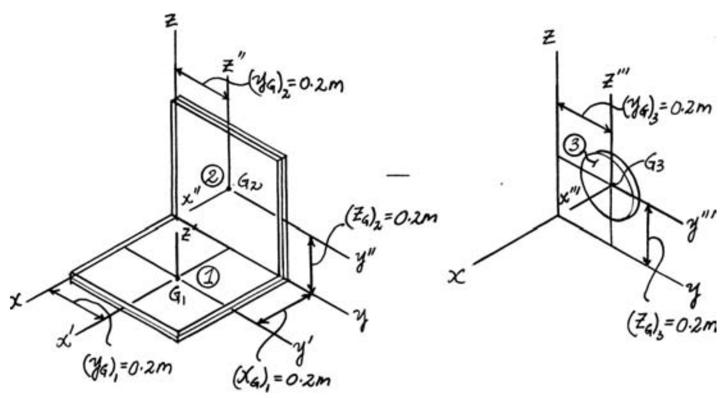
$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 8(0.2)(0.2)\right] - \left[0 + 0.5\pi(0.2)(0.2)\right]$$

$$= 0.257 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{xz} = \Sigma \bar{I}_{x'z'} + m x_G z_G$$

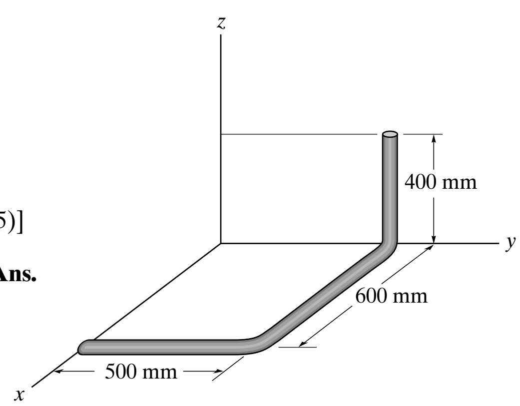
$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 8(0)(0.2)\right] - \left[0 + 0.5\pi(0)(0.2)\right]$$

$$= 0 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

•21–17. Determine the product of inertia I_{xy} for the bent rod. The rod has a mass per unit length of 2 kg/m.



Product of Inertia: Applying Eq. 21–4. we have

$$I_{xy} = \Sigma\left(I_{x'y'}\right)_G + m x_G\, y_G$$

$$= [0 + 0.4\,(2)\,(0)\,(0.5)] + [0 + 0.6\,(2)\,(0.3)\,(0.5)] + [0 + 0.5\,(2)\,(0.6)\,(0.25)]$$

$$= 0.330 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

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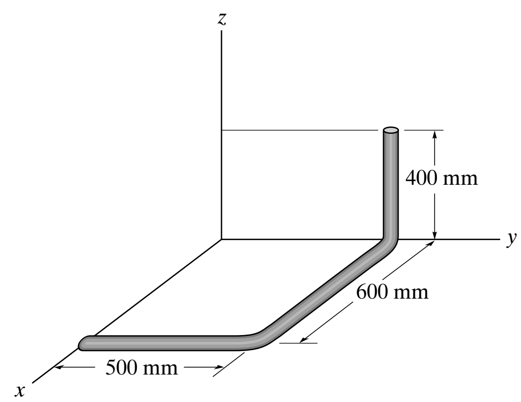
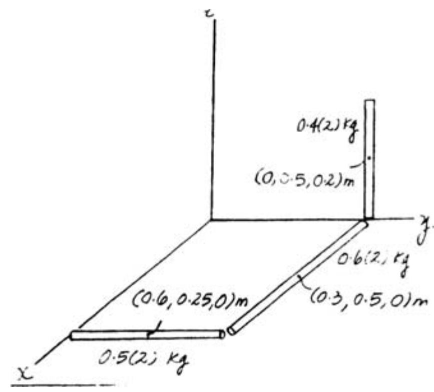
21–18. Determine the moments of inertia I_{xx}, I_{yy}, I_{zz} for the bent rod. The rod has a mass per unit length of 2 kg/m.



Moments of Inertia: Applying Eq. 21–3, we have



$$I_{xx} = \Sigma(I_{x'x'})_G + m(y_G^2 + z_G^2)$$

$$= \left[\frac{1}{12}(0.4)(2)\left(0.4^2\right) + 0.4(2)\left(0.5^2 + 0.2^2\right)\right]$$

$$+ \left[0 + 0.6(2)\left(0.5^2 + 0^2\right)\right]$$

$$+ \left[\frac{1}{12}(0.5)(2)\left(0.5^2\right) + 0.5(2)\left(0.25^2 + 0^2\right)\right]$$

$$= 0.626 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

$$I_{xy} = \Sigma(I_{y'y'})_G + m(x_G^2 + z_G^2)$$

$$= \left[\frac{1}{12}(0.4)(2)\left(0.4^2\right) + 0.4(2)\left(0^2 + 0.2^2\right)\right]$$

$$+ \left[\frac{1}{12}(0.6)(2)\left(0.6^2\right) + 0.6(2)\left(0.3^2 + 0^2\right)\right]$$

$$+ \left[0 + 0.5(2)\left(0.6^2 + 0^2\right)\right]$$

$$= 0.547 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

$$I_{zz} = \Sigma(I_{z'z'})_G + m(x_G^2 + y_G^2)$$

$$= \left[0 + 0.4(2)\left(0^2 + 0.5^2\right)\right]$$

$$+ \left[\frac{1}{12}(0.6)(2)\left(0.6^2\right) + 0.6(2)\left(0.3^2 + 0.5^2\right)\right]$$

$$+ \left[\frac{1}{12}(0.5)(2)\left(0.5^2\right) + 0.5(2)\left(0.6^2 + 0.25^2\right)\right]$$

$$= 1.09 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

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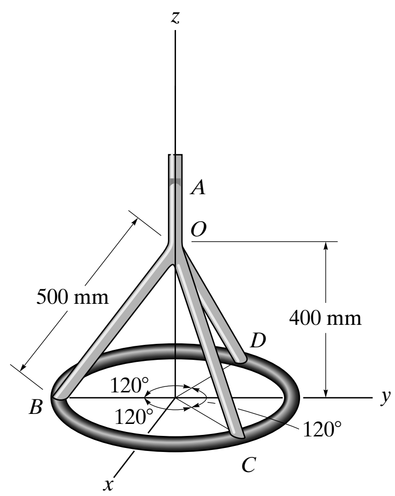
21–19. Determine the moment of inertia of the rod-and-thin-ring assembly about the z axis. The rods and ring have a mass per unit length of 2 kg/m.



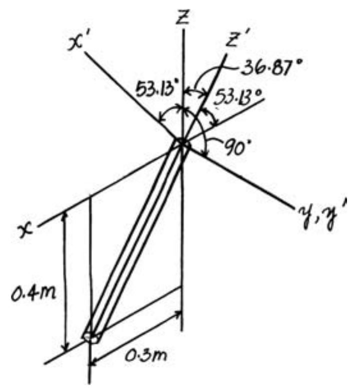
For the rod,

$$u_{x'} = 0.6, \qquad u_{y'} = 0, \qquad u_{z'} = 0.8$$

$$I_x = I_y = \frac{1}{3}[(0.5)(2)](0.5)^2 = 0.08333 \text{ kg} \cdot \text{m}^2$$

$$I_{x'} = 0$$

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$



From Eq. 21–5,

$$I_z = 0.08333(0.6)^2 + 0 + 0 - 0 - 0 - 0$$

$$I_z = 0.03 \text{ kg} \cdot \text{m}^2$$

For the ring,

The radius is $r = 0.3$ m

Thus,

$$I_z = mR^2 = [2\,(2\pi)(0.3)](0.3)^2 = 0.3393 \text{ kg} \cdot \text{m}^2$$

Thus the moment of inertia of the assembly is

$$I_z = 3(0.03) + 0.339 = 0.429 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

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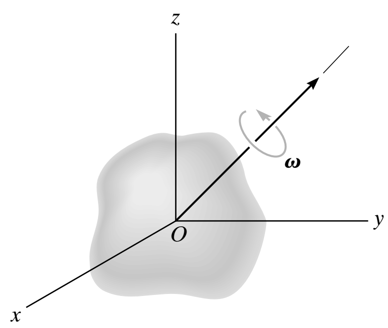
***21–20.** If a body contains *no planes of symmetry,* the principal moments of inertia can be determined mathematically. To show how this is done, consider the rigid body which is spinning with an angular velocity $\boldsymbol{\omega}$, directed along one of its principal axes of inertia. If the principal moment of inertia about this axis is I, the angular momentum can be expressed as $\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$. The components of $\mathbf{H}$ may also be expressed by Eqs. 21–10, where the inertia tensor is assumed to be known. Equate the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components of both expressions for $\mathbf{H}$ and consider ω_x, ω_y, and ω_z to be unknown. The solution of these three equations is obtained provided the determinant of the coefficients is zero. Show that this determinant, when expanded, yields the cubic equation

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0$$

The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.



$$\mathbf{H} = I\omega = I\omega_x\,\mathbf{i} + I\omega_y\,\mathbf{j} + I\omega_z\,\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components to the scalar equations (Eq. 21–10) yields

$$(I_{xx} - I)\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z = 0$$

$$-I_{xx}\,\omega_x + (I_{xy} - I)\,\omega_y - I_{yz}\,\omega_z = 0$$

$$-I_{zx}\,\omega_z - I_{zy}\,\omega_y + (I_{zz} - I)\,\omega_z = 0$$

Solution for ω_x, ω_y, and ω_z requires

$$\begin{vmatrix} (I_{xx} - I) & -I_{xy} & -I_{xz} \\ -I_{yx} & (I_{yy} - I) & -I_{yz} \\ -I_{zx} & -I_{zy} & (I_{zz} - I) \end{vmatrix} = 0$$

Expanding

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + \left(I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\right)I - \left(I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2\right) = 0 \quad \textbf{Q.E.D.}$$

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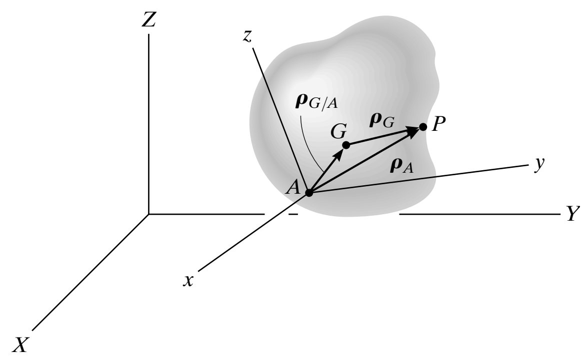
•21–21. Show that if the angular momentum of a body is determined with respect to an arbitrary point A, then $\mathbf{H}_A$ can be expressed by Eq. 21–9. This requires substituting $\boldsymbol{\rho}_A = \boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A}$ into Eq. 21–6 and expanding, noting that $\int \boldsymbol{\rho}_G\, dm = \mathbf{0}$ by definition of the mass center and $\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}$.



$$\mathbf{H}_A = \left(\int_m \rho_A\, dm\right) \times \mathbf{v}_A + \int_m \rho_A \times (\omega \times \rho_A) dm$$

$$= \left(\int_m (\rho_G + \rho_{G/A})\, dm\right) \times \mathbf{v}_A + \int_m (\rho_G + \rho_{G/A}) \times \left[\omega \times \rho_G + \rho_{G/A})\right] dm$$

$$= \left(\int_m \rho_G\, dm\right) \times \mathbf{v}_A + (\rho_{G/A} \times \mathbf{v}_A) \int_m dm + \int_m \rho_G \times (\omega \times \rho_G)\, dm$$

$$+ \left(\int_m \rho_G dm\right) \times (\omega \times \rho_{G/A}) + \rho_{G/A} \times \left(\omega \times \int_m \rho_G\, dm\right) + \rho_{G/A} \times (\omega \times \rho_{G/A}) \int_m dm$$

Since $\int_m \rho_G\, dm = 0$ and from Eq. 21–8 $\mathbf{H}_G = \int_m \rho_G \times (\omega \times \rho_G) dm$

$$\mathbf{H}_A = (\rho_{G/A} \times \mathbf{v}_A) m + \mathbf{H}_G + \rho_{G/A} \times (\omega \times \rho_{G/A}) m$$

$$= \rho_{G/A} \times (\mathbf{v}_A + (\omega \times \rho_{G/A})) m + \mathbf{H}_G$$

$$= (\rho_{G/A} \times m\mathbf{v}_G) + \mathbf{H}_G \qquad \textbf{Q.E.D.}$$

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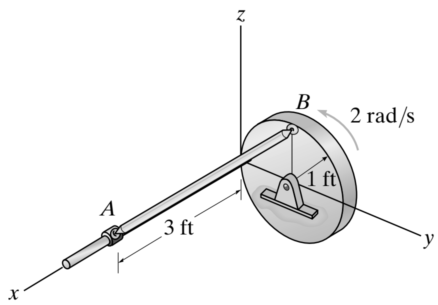
21–22. The 4-lb rod AB is attached to the disk and collar using ball-and-socket joints. If the disk has a constant angular velocity of 2 rad/s, determine the kinetic energy of the rod when it is in the position shown. Assume the angular velocity of the rod is directed perpendicular to the axis of the rod.



$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{i} = -(1)(2)\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 3 & -1 & -1 \end{vmatrix}$$

Expand and equate components:

$$v_A = -\omega_y + \omega_z \qquad \textbf{(1)}$$

$$2 = \omega_x + 3\,\omega_z \qquad \textbf{(2)}$$

$$0 = -\omega_x - 3\,\omega_y \qquad \textbf{(3)}$$

Also:

$$\omega \cdot \mathbf{r}_{A/B} = 0$$

$$3\omega_x - \omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1)–(4):

$$\omega_x = 0.1818 \text{ rad/s}$$

$$\omega_y = -0.06061 \text{ rad/s}$$

$$\omega_z = 0.6061 \text{ rad/s}$$

$$v_A = 0.667 \text{ ft/s}$$

ω is perpendicular to the rod.

$$\omega_z = 0.1818 \text{ rad/s}, \qquad \omega_y = -0.06061 \text{ rad/s}, \qquad \omega_z = 0.6061 \text{ rad/s}$$

$$\mathbf{v}_B = \{-2\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{r}_{A/B} = \{3\mathbf{i} - 1\mathbf{j} - 1\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_G = \mathbf{v}_B + \omega \times \frac{\mathbf{r}_{A/B}}{2}$$

$$\mathbf{v}_G = -2\mathbf{j} + \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1818 & -0.06061 & 0.6061 \\ 3 & -1 & -1 \end{vmatrix}$$

$$\mathbf{v}_G = \{0.333\mathbf{i} - 1\mathbf{j}\} \text{ ft/s}$$

$$v_G = \sqrt{(0.333)^2 + (-1)^2} = 1.054 \text{ ft/s}$$

$$\omega = \sqrt{(0.1818)^2 + (-0.06061)^2 + (0.6061)^2} = 0.6356 \text{ rad/s}$$

$$T = \left(\frac{1}{2}\right)\left(\frac{4}{32.2}\right)(1.054)^2 + \left(\frac{1}{2}\right)\left[\frac{1}{12}\left(\frac{4}{32.2}\right)(3.3166)^2\right](0.6356)^2$$

$$T = 0.0920 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans.}$$

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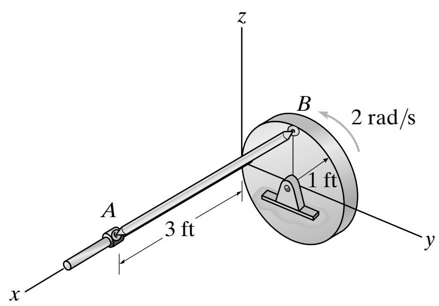
21–23. Determine the angular momentum of rod AB in Prob. 21–22 about its mass center at the instant shown. Assume the angular velocity of the rod is directed perpendicular to the axis of the rod.



$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{i} = -(1)(2)\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 3 & -1 & -1 \end{vmatrix}$$

Expand and equate components:

$$v_A = -\omega_y + \omega_z \qquad \textbf{(1)}$$

$$2 = \omega_x + 3\,\omega_z \qquad \textbf{(2)}$$

$$0 = -\omega_x - 3\,\omega_y \qquad \textbf{(3)}$$

Also:

$$\omega \cdot \mathbf{r}_{A/B} = 0$$

$$3\omega_x - \omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1)–(4):

$$\omega_x = 0.1818 \text{ rad/s}$$

$$\omega_y = -0.06061 \text{ rad/s}$$

$$\omega_z = 0.6061 \text{ rad/s}$$

$$v_A = 0.667 \text{ ft/s}$$

ω is perpendicular to the rod.

$$r_{A/B} = \sqrt{(3)^2 + (-1)^2 + (-1)^2} = 3.3166 \text{ ft}$$

$$I_G = \left(\frac{1}{12}\right)\left(\frac{4}{32.2}\right)(3.3166)^2 = 0.1139 \text{ slug} \cdot \text{ft}^2$$

$$\mathbf{H}_G = I_G\,\omega = 0.1139\,(0.1818\mathbf{i} - 0.06061\mathbf{j} + 0.6061\mathbf{k})$$

$$\mathbf{H}_G = \{0.0207\mathbf{i} - 0.00690\mathbf{j} + 0.0690\mathbf{k})\ \text{slug} \cdot \text{ft}^2/\text{s} \qquad \textbf{Ans.}$$

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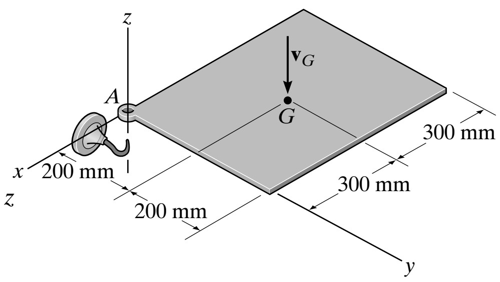
***21–24.** The uniform thin plate has a mass of 15 kg. Just before its corner A strikes the hook, it is falling with a velocity of $\mathbf{v}_G = \{-5\mathbf{k}\}$ m/s with no rotational motion. Determine its angular velocity immediately after corner A strikes the hook without rebounding.



Referring to Fig. a, the mass moments of inertia of the plate about the x, y, and z axes are

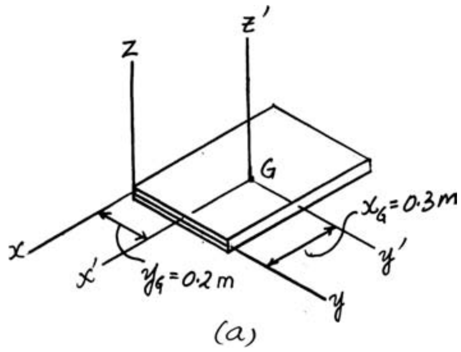
$$I_x = I_{x'} + m\left(y_G{}^2 + z_G{}^2\right) = \frac{1}{12}(15)\left(0.4^2\right) + 15\left(0.2^2 + 0^2\right) = 0.8\ \text{kg}\cdot\text{m}^2$$

$$I_y = I_{y'} + m\left(x_G{}^2 + z_G{}^2\right) = \frac{1}{12}(15)\left(0.6^2\right) + 15\left[(-0.3)^2 + 0^2\right] = 1.8\ \text{kg}\cdot\text{m}^2$$

$$I_z = I_{z'} + m\left(x_G{}^2 + y_G{}^2\right) = \frac{1}{12}(15)\left(0.4^2 + 0.6^2\right) + 15\left[(-0.3)^2 + 0.2^2\right] = 2.6\ \text{kg}\cdot\text{m}^2$$

Due to symmetry, $I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$. Thus,

$$I_{xy} = I_{x'y'} + mx_Gy_G = 0 + 15(-0.3)(0.2) = -0.9\ \text{kg}\cdot\text{m}^2$$
$$I_{yz} = I_{y'z'} + my_Gz_G = 0 + 15(0.2)(0) = 0$$
$$I_{xz} = I_{x'z'} + mx_Gz_G = 0 + 15(-0.3)(0) = 0$$



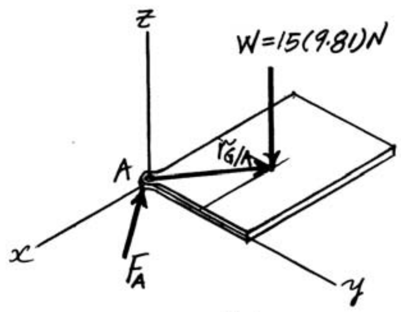
(a)

Since the plate falls without rotational motion just before the impact, its angular momentum about point A is

$$(\mathbf{H}_A)_1 = \mathbf{r}_{G/A} \times m\mathbf{v}_G = (-0.3\mathbf{i} + 0.2\mathbf{j}) \times 15(-5\mathbf{k})$$
$$= [-15\mathbf{i} - 22.5\mathbf{j}]\ \text{kg}\cdot\text{m}^2/\text{s}$$

Since the plate rotates about point A just after impact, the components of its angular momentum at this instant can be determined from

$$\left[(H_A)_2\right]_x = I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z$$
$$= 0.8\omega_x - (-0.9)\omega_y - 0(\omega_z)$$
$$= 0.8\omega_x + 0.9\omega_y$$
$$\left[(H_A)_2\right]_y = -I_{xy}\omega_x + I_y\omega_y - I_{yz}\omega_z$$
$$= -(-0.9)\omega_x + 1.8\omega_y - 0(\omega_z)$$
$$= 0.9\omega_x + 1.8\omega_y$$
$$\left[(H_A)_2\right]_z = -I_{xz}\omega_x + I_{yz}\omega_y - I_z\omega_z$$
$$= 0(\omega_x) - 0(\omega_y) + 2.6\omega_z$$
$$= 2.6\omega_z$$



(b)

Thus,

$$(\mathbf{H}_A)_2 = (0.8\omega_x + 0.9\omega_y)\mathbf{i} + (0.9\omega_x + 1.8\omega_y)\mathbf{j} + 2.6\omega_z\,\mathbf{k}$$

Referring to the free-body diagram of the plate shown in Fig. b, the weight $\mathbf{W}$ is a nonimpulsive force and the impulsive force $\mathbf{F}_A$ acts through point A. Therefore, angular momentum of the plate is conserved about point A. Thus,

$$(\mathbf{H}_A)_1 = (\mathbf{H}_A)_2$$
$$-15\mathbf{i} - 22.5\mathbf{j} = (0.8\omega_x + 0.9\omega_y)\mathbf{i} + (0.9\omega_x + 1.8\omega_y)\mathbf{j} + 2.6\omega_z\,\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$-15 = 0.8\omega_x + 0.9\omega_y \qquad \textbf{(1)}$$
$$-22.5 = 0.9\omega_x + 1.8\omega_y \qquad \textbf{(2)}$$
$$0 = 2.6\omega_z \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3),

$\omega_x = -10.71$ rad/s $\qquad \omega_y = -7.143$ rad/s $\qquad \omega_z = 0$

Thus,

$$\omega = [-10.7\mathbf{i} - 7.14\mathbf{j}]\ \text{rad/s} \qquad \textbf{Ans.}$$

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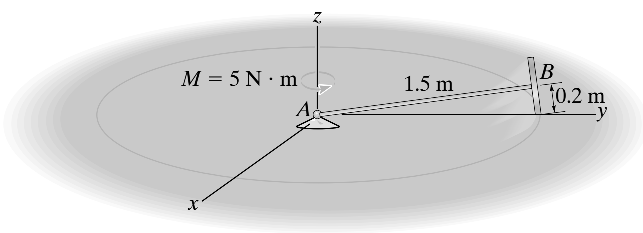
•21–25. The 5-kg disk is connected to the 3-kg slender rod. If the assembly is attached to a ball-and-socket joint at A and the 5-N · m couple moment is applied, determine the angular velocity of the rod about the z axis after the assembly has made two revolutions about the z axis starting from rest. The disk rolls without slipping.

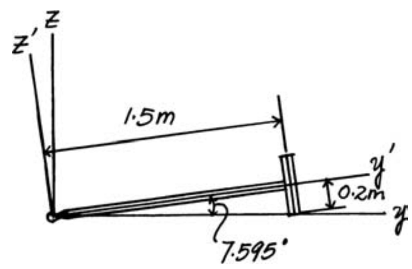


$$I_x = I_z = \frac{1}{4}(5)(0.2)^2 + 5(1.5)^2 + \frac{1}{3}(3)(1.5)^2 = 13.55$$

$$I_v = \frac{1}{2}(5)(0.2)^2 = 0.100$$

$$\omega = -\omega_y \mathbf{j}' + \omega_z \mathbf{k} = -\omega_{y'} \mathbf{j}' + \omega_z \sin 7.595° \mathbf{j}' + \omega_z \cos 7.595° \mathbf{k}'$$

$$= (0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\, \omega_z \mathbf{k}'$$



Since points A and C have zero velocity,

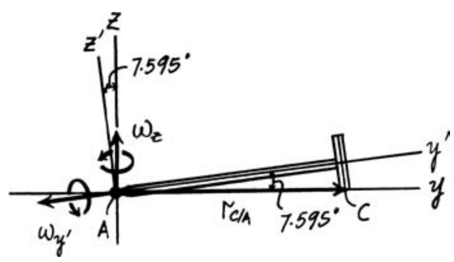
$$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$$

$$\mathbf{0} = \mathbf{0} + \left[(0.13216\,\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z \mathbf{k}'\right] \times (1.5\mathbf{j}' - 0.2\mathbf{k}')$$

$$0 = -1.48684\omega_z - 0.026433\,\omega_z + 0.2\,\omega_{y'}$$

$$\omega_{y'} = 7.5664\,\omega_z$$

Thus,

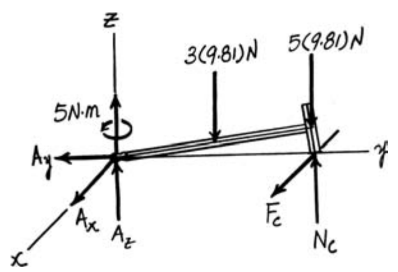


$$\omega = -7.4342\,\omega_z \mathbf{j}' + 0.99123\,\omega_z \mathbf{k}'$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 5(2\pi)(2) = 0 + \frac{1}{2}(0.100)(-7.4342\,\omega_z)^2 + \frac{1}{2}(13.55)(0.99123\,\omega_z)^2$$

$$\omega_z = 2\ 58 \text{ rad/s}$$ **Ans.**



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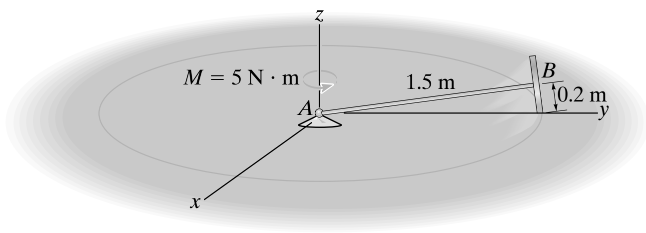
21–26. The 5-kg disk is connected to the 3-kg slender rod. If the assembly is attached to a ball-and-socket joint at A and the 5-N · m couple moment gives it an angular velocity about the z axis of $\omega_z = 2$ rad/s, determine the magnitude of the angular momentum of the assembly about A.

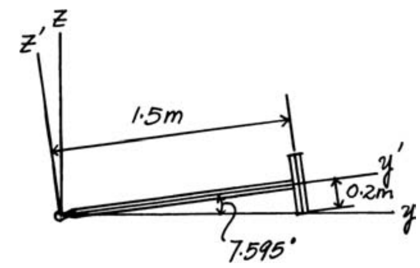


$$I_x = I_z = \frac{1}{4}(5)(0.2)^2 + 5(1.5)^2 + \frac{1}{3}(3)(1.5)^2 = 13.55$$

$$I_x = \frac{1}{2}(5)(0.2)^2 = 0.100$$

$$\omega = -\omega_y \mathbf{j}' + \omega_z \mathbf{k} = -\omega_{y'} \mathbf{j}' + \omega_z \sin 7.595° \mathbf{j}' + \omega_z \cos 7.595° \mathbf{k}'$$

$$= (0.13216\omega_z - \omega_y)\mathbf{j}' + 0.99123\,\omega_z\,\mathbf{k}'$$



Since points A and C have zero velocity,

$$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$$

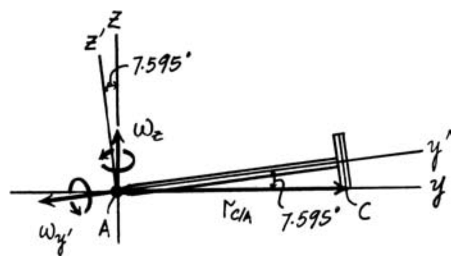
$$\mathbf{0} = \mathbf{0} + \left[(0.13216\,\omega_z - \omega_y\,)\,\mathbf{j}' + 0.99123\,\omega_z\,\mathbf{k}'\right] \times (1.5\mathbf{j}' - 0.2\mathbf{k}')$$

$$0 = -1\;\;48684\omega_z - 0.26433\omega_z + 0.2\omega_y$$

$$\omega_{y'} = -7.5664\,\omega_z$$

Thus,

$$\omega = -7.4342\,\omega_z\,\mathbf{j}' + 0.99123\,\omega_z\,\mathbf{k}'$$



Since $\omega_z = 2$ rad/s

$$\omega = -14.868\mathbf{j}' + 1\;\;9825\mathbf{k}'$$

So that,

$$\mathbf{H}_A = I_{x'}\,\omega_x\,\mathbf{i}' + I_{y'}\,\omega_{y'}\,\mathbf{j} + I_{z'}\,\omega_{z'}\,\mathbf{k}' = \mathbf{0} + 0.100(-14.868)\mathbf{j}' + 13.55(1.9825)\,\mathbf{k}'$$

$$= -1.4868\mathbf{j}' + 26.862\mathbf{k}'$$

$$H_A = \sqrt{(-1.4868)^2 + (26.862)^2} = 26.9 \text{ kg} \cdot \text{m}^2/\text{s}$$ **Ans.**

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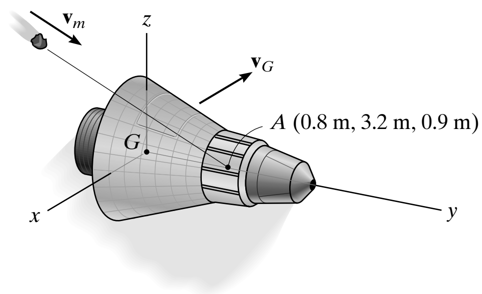
21–27. The space capsule has a mass of 5 Mg and the radii of gyration are $k_x = k_z = 1.30$ m and $k_y = 0.45$ m. If it travels with a velocity $\mathbf{v}_G = \{400\mathbf{j} + 200\mathbf{k}\}$ m/s, compute its angular velocity just after it is struck by a meteoroid having a mass of 0.80 kg and a velocity $\mathbf{v}_m = \{-300\mathbf{i} + 200\mathbf{j} - 150\mathbf{k}\}$ m/s. Assume that the meteoroid embeds itself into the capsule at point A and that the capsule initially has no angular velocity.



Conservation of Angular Momentum: The angular momentum is conserved about the center of mass of the space capsule G. Neglect the mass of the meteroid after the impact.

$$(H_G)_1 = (H_G)_2$$

$$\mathbf{r}_{GA} \times m_m \mathbf{v}_m = I_G \omega$$

$$(0.8\mathbf{i} + 3.2\mathbf{j} + 0.9\mathbf{k}) \times 0.8(-300\mathbf{i} + 200\mathbf{j} - 150\mathbf{k})$$
$$= 5000\left(1.30^2\right)\omega_x\,\mathbf{i} + 5000\left(0.45^2\right)\omega_y\,\mathbf{j} + 5000\left(1.30^2\right)\omega_z\,\mathbf{k}$$

$$-528\mathbf{i} - 120\mathbf{j} + 896\mathbf{k} = 8450\omega_x\,\mathbf{i} + 1012.5\omega_y\,\mathbf{j} + 8450\,\omega_z\,\mathbf{k}$$

Equating **i**, **j** and **k** components, we have

$$-528 = 8450\omega_x \qquad \omega_x = -0.06249 \text{ rad/s}$$

$$-120 = 1012.5\omega_y \qquad \omega_y = -0.11852 \text{ rad/s}$$

$$896 = 8450\omega_z \qquad \omega_z = 0.1060 \text{ rad/s}$$

Thus,

$$\omega = \{-0.0625\mathbf{i} - 0.119\mathbf{j} + 0.106\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

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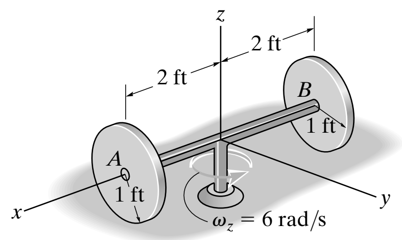
***21–28.** Each of the two disks has a weight of 10 lb. The axle AB weighs 3 lb. If the assembly rotates about the z axis at $\omega_z = 6$ rad/s, determine its angular momentum about the z axis and its kinetic energy. The disks roll without slipping.

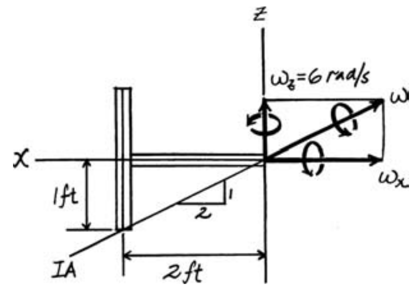


$$\frac{6}{\omega_x} = \frac{1}{2} \qquad \omega_x = 12 \text{ rad/s}$$

$$\omega_A = \{-12\mathbf{i}\} \text{ rad/s} \qquad \omega_B = \{12\mathbf{i}\} \text{ rad/s}$$

$$\mathbf{H}_z = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(1)^2\right](12\mathbf{i}) + \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(1)^2\right](-12\mathbf{i})$$

$$+\ \mathbf{0} + \left\{2\left[\frac{1}{4}\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}(2)^2\right](6) + \frac{1}{12}\left(\frac{3}{32.2}\right)(4)^2(6)\right\}\mathbf{k}$$



$$\mathbf{H}_z = \{16.6\mathbf{k}\} \text{ slug}\cdot\text{ft}^2/\text{s}$$ **Ans.**

$$T = \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2$$

$$= \frac{1}{2}\left[2\left(\frac{1}{2}\left(\frac{10}{32.2}\right)(1)^2\right)\right](12)^2 + 0$$

$$+ \frac{1}{2}\left\{2\left[\frac{1}{4}\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}(2)^2\right] + \frac{1}{12}\left(\frac{3}{32.2}\right)(4)^2\right\}(6)^2$$

$$= 72.1 \text{ lb}\cdot\text{ft}$$ **Ans.**

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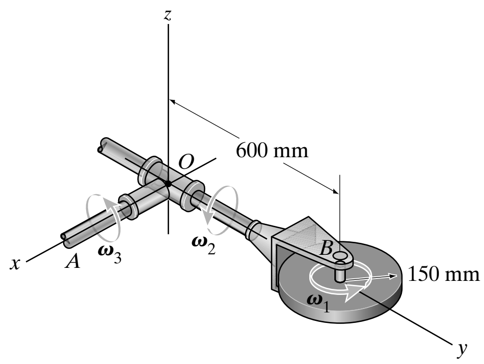
•21–29. The 10-kg circular disk spins about its axle with a constant angular velocity of $\omega_1 = 15$ rad/s. Simultaneously, arm OB and shaft OA rotate about their axes with constant angular velocities of $\omega_2 = 0$ and $\omega_3 = 6$ rad/s, respectively. Determine the angular momentum of the disk about point O, and its kinetic energy.



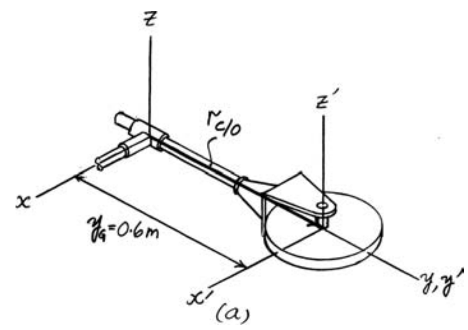
The mass moments of inertia of the disk about the centroidal x', y', and z' axes, Fig. a, are

$$I_{x'} = I_{y'} = \frac{1}{4}mr^2 = \frac{1}{4}(10)\left(0.15^2\right) = 0.05625\ \text{kg}\cdot\text{m}^2$$

$$I_{z'} = \frac{1}{2}mr^2 = \frac{1}{2}(10)\left(0.15^2\right) = 0.1125\ \text{kg}\cdot\text{m}^2$$

Due to symmetry, the products of inertia of the disk with respect to its centroidal planes are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$



Here, the angular velocity of the disk can be determined from the vector addition of ω_1 and ω_3. Thus,

$$\omega = \omega_1 + \omega_2 = [6\mathbf{i} + 15\mathbf{k}]\ \text{rad/s}$$

The angular momentum of the disk about its mass center G can be obtained by applying

$$H_x = I_{x'}\omega_x = 0.05625(6) = 0.3375\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$H_y = I_{y'}\omega_y = 0.05625(0) = 0$$

$$H_z = I_{z'}\omega_z = 0.1125(15) = 1.6875\ \text{kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$\mathbf{H}_G = [0.3375\mathbf{i} + 1.6875\mathbf{k}]\ \text{kg}\cdot\text{m}^2/\text{s}$$

Since the mass center G rotates about the x axis with a constant angular velocity of $\omega_3 = [6\mathbf{i}]$ rad/s, its velocity is

$$\mathbf{v}_G = \omega_3 \times \mathbf{r}_{C/O} = (6\mathbf{i}) \times (0.6\mathbf{j}) = [3.6\mathbf{k}]\ \text{m/s}$$

Since the disk does not rotate about a fixed point O, its angular momentum must be determined from

$$\mathbf{H}_O = \mathbf{r}_{C/O} \times m\mathbf{v}_C + \mathbf{H}_G$$

$$= (0.6\mathbf{j}) \times 10(3.6\mathbf{k}) + (0.3375\mathbf{i} + 1.6875\mathbf{k})$$

$$= [21.9375\mathbf{i} + 1.6875\mathbf{k}]\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$= [21.9\mathbf{i} + 1.69\mathbf{k}]\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

The kinetic energy of the disk is therefore

$$T = \frac{1}{2}\omega\cdot\mathbf{H}_O$$

$$= \frac{1}{2}(6\mathbf{i} + 15\mathbf{k})\cdot(21.9375\mathbf{i} + 1.6875\mathbf{k})$$

$$= 78.5\ \text{J} \qquad \textbf{Ans.}$$

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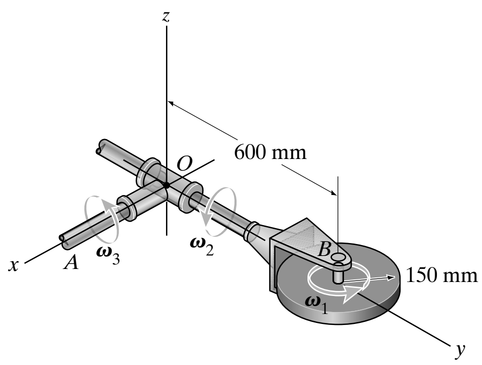
21–30. The 10-kg circular disk spins about its axle with a constant angular velocity of $\omega_1 = 15$ rad/s. Simultaneously, arm OB and shaft OA rotate about their axes with constant angular velocities of $\omega_2 = 10$ rad/s and $\omega_3 = 6$ rad/s, respectively. Determine the angular momentum of the disk about point O, and its kinetic energy.



The mass moments of inertia of the disk about the centroidal x', y', and z' axes. Fig. a, are

$$I_{x'} = I_{y'} = \frac{1}{4}mr^2 = \frac{1}{4}(10)\left(0.15^2\right) = 0.05625 \text{ kg}\cdot\text{m}^2$$

$$I_{z'} = \frac{1}{2}mr^2 = \frac{1}{2}(10)\left(0.15^2\right) = 0.1125 \text{ kg}\cdot\text{m}^2$$

Due to symmetry, the products of inertia of the disk with respect to its centroidal planes are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

Here, the angular velocity of the disk can be determined from the vector addition of ω_1, ω_2, and ω_3. Thus,

$$\omega = \omega_1 + \omega_2 + \omega_3 = [6\mathbf{i} + 10\mathbf{j} + 15\mathbf{k}] \text{ rad/s}$$

The angular momentum of the disk about its mass center G can be obtained by applying

$$H_x = I_{x'}\omega_x = 0.05625(6) = 0.3375 \text{ kg}\cdot\text{m}^2$$

$$H_y = I_{y'}\omega_y = 0.05625(10) = 0.5625 \text{ kg}\cdot\text{m}^2$$

$$H_z = I_{z'}\omega_z = 0.1125(15) = 1.6875 \text{ kg}\cdot\text{m}^2$$

Thus,

$$\mathbf{H}_G = [0.3375\mathbf{i} + 0.5625\mathbf{j} + 1.6875\mathbf{k}] \text{ kg}\cdot\text{m}^2$$

Since the mass center G rotates about the fixed point O with an angular velocity of $\Omega = \omega_2 + \omega_3 = [6\mathbf{i} + 10\mathbf{j}]$, its velocity is

$$\mathbf{v}_G = \Omega \times \mathbf{r}_{G/O} = (6\mathbf{i} + 10\mathbf{j}) \times (0.6\mathbf{j}) = [3.6\mathbf{k}] \text{ m/s}$$

Since the disk does not rotate about a fixed point O, its angular momentum must be determined from

$$\mathbf{H}_O = \mathbf{r}_{C/O} \times m\mathbf{v}_G + \mathbf{H}_G$$

$$= (0.6\mathbf{j}) \times 10(3.6\mathbf{k}) + (0.3375\mathbf{i} + 0.5625\mathbf{j} + 1.6875\mathbf{k})$$

$$= [21.9375\mathbf{i} + 0.5625\mathbf{j} + 1.6875\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$

$$= [21.9\mathbf{i} + 0.5625\mathbf{j} + 1.69\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

The kinetic energy of the disk is therefore

$$T = \frac{1}{2}\omega\cdot\mathbf{H}_O$$

$$= \frac{1}{2}(6\mathbf{i} + 10\mathbf{j} + 15\mathbf{k})\cdot(21.9375\mathbf{i} + 0.5625\mathbf{j} + 1.6875\mathbf{k})$$

$$= 81.3\text{J} \qquad \textbf{Ans.}$$

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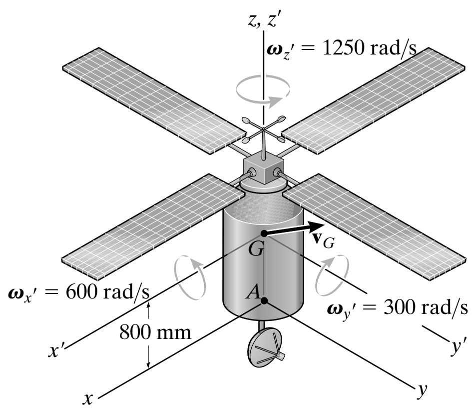
21–31. The 200-kg satellite has its center of mass at point G. Its radii of gyration about the z', x', y' axes are $k_{z'} = 300$ mm, $k_{x'} = k_{y'} = 500$ mm, respectively. At the instant shown, the satellite rotates about the x', y', and z' axes with the angular velocity shown, and its center of mass G has a velocity of $\mathbf{v}_G = \{-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}\}$ m/s. Determine the angular momentum of the satellite about point A at this instant.



The mass moments of inertia of the satellite about the x', y', and z' axes are

$$I_{x'} = I_{y'} = 200\left(0.5^2\right) = 50 \text{ kg}\cdot\text{m}^2$$

$$I_{z'} = 200\left(0.3^2\right) = 18 \text{ kg}\cdot\text{m}^2$$

Due to symmetry, the products of inertia of the satellite with respect to the x', y', and z' coordinate system are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

The angular velocity of the satellite is

$$\omega = [600\mathbf{i} + 300\mathbf{j} + 1250\mathbf{k}] \text{ rad/s}$$

Thus,

$$\omega_{x'} = 600 \text{ rad/s} \qquad \omega_{y'} = -300 \text{ rad/s} \qquad \omega_{z'} = 1250 \text{ rad/s}$$

Then, the components of the angular momentum of the satellite about its mass center G are

$$(H_G)_{x'} = I_{x'}\omega_{x'} = 50(600) = 30\,000 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_G)_{y'} = I_{y'}\omega_{y'} = 50(-300) = 15\,000 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_G)_{z'} = I_{z'}\omega_{z'} = 18(1250) = 22\,500 \text{ kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$\mathbf{H}_G = [30\,000\mathbf{i} + 15\,000\mathbf{j} + 22\,500\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$

The angular momentum of the satellite about point A can be determined from

$$\mathbf{H}_A = \mathbf{r}_{G/A} \times m\mathbf{v}_G + \mathbf{H}_G$$

$$= (0.8\mathbf{k}) \times 200(-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}) + (30\,000\mathbf{i} + 15\,000\mathbf{j} + 22\,500\mathbf{k})$$

$$= [-2000\mathbf{i} - 25\,000\mathbf{j} + 22\,500\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

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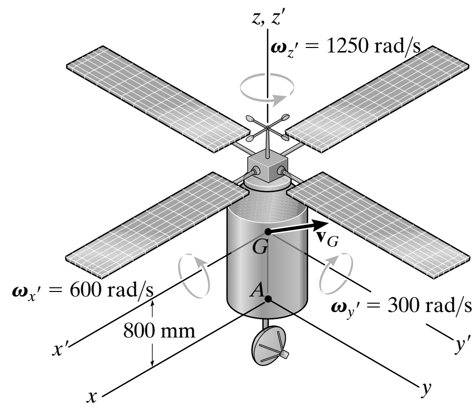
***21–32.** The 200-kg satellite has its center of mass at point G. Its radii of gyration about the z', x', y' axes are $k_{z'} = 300$ mm, $k_{x'} = k_{y'} = 500$ mm, respectively. At the instant shown, the satellite rotates about the x', y', and z' axes with the angular velocity shown, and its center of mass G has a velocity of $\mathbf{v}_G = \{-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}\}$ m/s. Determine the kinetic energy of the satellite at this instant.



The mass moments of inertia of the satellite about the x', y', and z' axes are

$$I_{x'} = I_{y'} = 200\left(0.5^2\right) = 50 \text{ kg}\cdot\text{m}^2$$

$$I_{z'} = 200\left(0.3^2\right) = 18 \text{ kg}\cdot\text{m}^2$$

Due to symmetry, the products of inertia of the satellite with respect to the x', y', and z' coordinate system are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

The angular velocity of the satellite is

$$\omega = [600\mathbf{i} - 300\mathbf{j} + 1250\mathbf{k}] \text{ rad/s}$$

Thus,

$$\omega_{x'} = 600 \text{ rad/s} \qquad \omega_{y'} = -300 \text{ rad/s} \qquad \omega_{z'} = 1250 \text{ rad/s}$$

Since $v_G{}^2 = (-250)^2 + 200^2 + 120^2 = 116\,900 \text{ m}^2/\text{s}^2$, the kinetic energy of the satellite can be determined from

$$T = \frac{1}{2} m v_G{}^2 + \frac{1}{2} I_{x'}\omega_{x'}{}^2 + \frac{1}{2} I_{y'}\omega_{y'}{}^2 + \frac{1}{2} I_{z'}\omega_{z'}{}^2$$

$$= \frac{1}{2}(200)(116\,900) + \frac{1}{2}(50)\left(600^2\right) + \frac{1}{2}(50)(-300)^2 + \frac{1}{2}(18)\left(1250^2\right)$$

$$= 37.0025\left(10^6\right)\text{J} = 37.0\text{MJ}$$ **Ans.**

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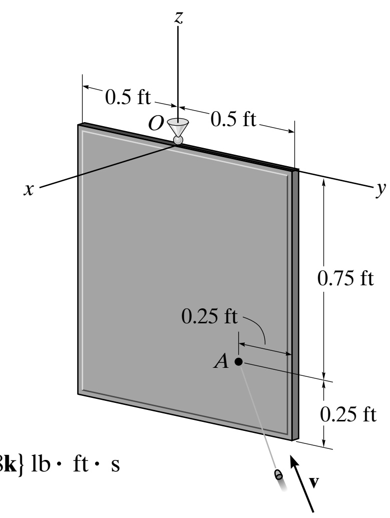
•21–33. The 25-lb thin plate is suspended from a ball-and-socket joint at O. A 0.2-lb projectile is fired with a velocity of $\mathbf{v} = \{-300\mathbf{i} - 250\mathbf{j} + 300\mathbf{k}\}$ ft/s into the plate and becomes embedded in the plate at point A. Determine the angular velocity of the plate just after impact and the axis about which it begins to rotate. Neglect the mass of the projectile after it embeds into the plate.



Angular momentum about point O is conserved.

$$(\mathbf{H}_O)_2 = (\mathbf{H}_O)_1 = \mathbf{r}_{OA} \times m_p \mathbf{v}_p$$

$$(\mathbf{H}_O)_1 = (0.25\mathbf{j} - 0.75\mathbf{k}) \times \left(\frac{0.2}{32.2}\right)(-300\mathbf{i} - 250\mathbf{j} + 300\mathbf{k}) = \{-0.6988\mathbf{i} + 1.3975\mathbf{j} + 0.4658\mathbf{k}\}\ \text{lb}\cdot\text{ft}\cdot\text{s}$$

$$I_x = \left(\frac{1}{12}\right)\left(\frac{25}{32.2}\right)\left[(1)^2 + (1)^2\right] + \left(\frac{25}{32.2}\right)(0.5)^2 = 0.3235\ \text{slug}\cdot\text{ft}^2$$

$$I_y = \left(\frac{1}{12}\right)\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(0.5)^2 = 0.2588\ \text{slug}\cdot\text{ft}^2$$

$$I_z = \left(\frac{1}{12}\right)\left(\frac{25}{32.2}\right)(1)^2 = 0.06470\ \text{slug}\cdot\text{ft}^2$$

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$-0.6988\mathbf{i} + 1.3975\mathbf{j} + 0.4658\mathbf{k} = 0.3235\omega_x\,\mathbf{i} + 0.2588\omega_y\,\mathbf{j} + 0.06470\omega_z\,\mathbf{k}$$

$$\omega_x = \frac{-0.6988}{0.3235} = -2.160\ \text{rad/s}$$

$$\omega_y = \frac{1.3975}{0.2588} = 5.400\ \text{rad/s}$$

$$\omega_z = \frac{0.4658}{0.06470} = 7.200\ \text{rad/s}$$

$\omega = \{-2.16\mathbf{i} + 5.40\mathbf{j} + 7.20\mathbf{k}\}$ rad/s **Ans.**

Axis of rotation line is along ω:

$$\mathbf{u}_O = \frac{-2.160\mathbf{i} + 5.400\mathbf{j} + 7.200\mathbf{k}}{\sqrt{(-2.160)^2 + (5.400)^2 + (7.200)^2}}$$

$= -0.233\mathbf{i} + 0.583\mathbf{j} + 0.778\mathbf{k}$ **Ans.**

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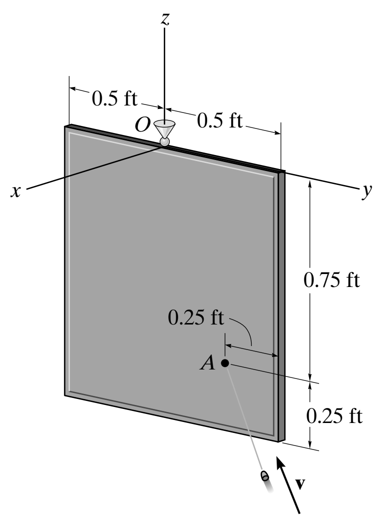
21–34. Solve Prob. 21–33 if the projectile emerges from the plate with a velocity of 275 ft/s in the same direction.



$$\mathbf{u_v} = \left(\frac{-300}{492.4}\right)\mathbf{i} - \left(\frac{250}{492.4}\right)\mathbf{j} + \left(\frac{300}{492.4}\right)\mathbf{k} = -0.6092\mathbf{i} - 0.5077\mathbf{j} + 0.6092\mathbf{k}$$

$$I_x = \left(\frac{1}{12}\right)\left(\frac{25}{32.2}\right)\left[(1)^2 + (1)^2\right] + \left(\frac{25}{32.2}\right)(0.5)^2 = 0.32350 \text{ slug}\cdot\text{ft}^2$$

$$I_y = \left(\frac{1}{12}\right)\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(0.5)^2 = 0.25880 \text{ slug}\cdot\text{ft}^2$$

$$I_z = \left(\frac{1}{12}\right)\left(\frac{25}{32.2}\right)(1)^2 = 0.06470 \text{ slug}\cdot\text{ft}^2$$

$$\mathbf{H}_1 + \Sigma\int \mathbf{M}_O\, dt = \mathbf{H}_2$$

$$(0.25\mathbf{j} - 0.75\mathbf{k}) \times \left(\frac{0.2}{32.2}\right)(-300\mathbf{i} - 250\mathbf{j} + 300\mathbf{k}) + \mathbf{0} = 0.32350\omega_x\mathbf{i} + 0.25880\omega_y\mathbf{j} + 0.06470\omega_z\mathbf{k}$$

$$+(0.25\mathbf{j} - 0.75\mathbf{k}) \times \left(\frac{0.2}{32.2}\right)(275)(-0.6092\mathbf{i} - 0.5077\mathbf{j} + 0.6092\mathbf{k})$$

Expanding, the **i**, **j**, **k**, components are:

$$-0.6988 = 0.32350\omega_x - 0.390215$$

$$1.3975 = 0.25880\omega_y + 0.78043$$

$$0.4658 = 0.06470\omega_z + 0.26014$$

$$\omega_x = -0.9538, \qquad \omega_y = 2.3844, \qquad \omega_z = 3.179$$

$$\omega = \{-0.954\mathbf{i} + 2.38\mathbf{j} + 3.18\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

Axis of rotation is along ω:

$$\mathbf{u}_A = \frac{-0.954\mathbf{i} + 2.38\mathbf{j} + 3.18\mathbf{k}}{\sqrt{(-0.954)^2 + (2.38)^2 + (3.18)^2}}$$

$$\mathbf{u}_A = -0.233\mathbf{i} + 0.583\mathbf{j} + 0.778\mathbf{k} \qquad \textbf{Ans.}$$

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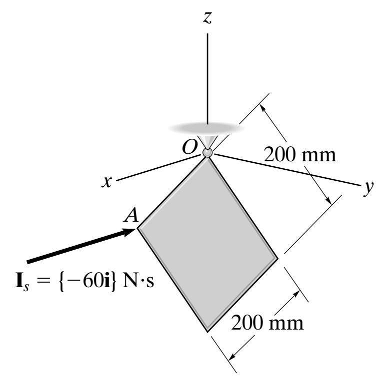
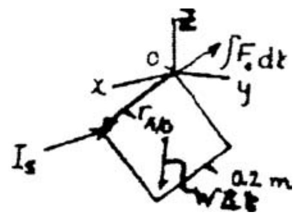
21–35. A thin plate, having a mass of 4 kg, is suspended from one of its corners by a ball-and-socket joint O. If a stone strikes the plate perpendicular to its surface at an adjacent corner A with an impulse of $\mathbf{I}_s = \{-60\mathbf{i}\}\ \text{N}\cdot\text{s}$, determine the instantaneous axis of rotation for the plate and the impulse created at O.



$$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$$

$$\mathbf{0} + \mathbf{r}_{A/O} \times \mathbf{I}_S = (\mathbf{H}_O)_2$$

$$\mathbf{0} + (-0.2(0.7071)\mathbf{j} - 0.2(0.7071)\mathbf{k}) \times (-60\mathbf{i}) = (I_O)_x\,\omega_x\,\mathbf{i} + (I_O)_y\,\omega_y\,\mathbf{j} + (I_O)_z\,\omega_z\,\mathbf{k}$$



Expand and equate components:

$$0 = (I_O)_x\,\omega_x \qquad \textbf{(1)}$$

$$8.4853 = (I_O)_y\,\omega_y \qquad \textbf{(2)}$$

$$-8.4853 = (I_O)_z\,\omega_z \qquad \textbf{(3)}$$

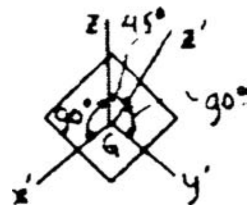
$$I_{x'y'} = 0, \qquad I_{y'z'} = 0, \qquad I_{x'z'} = 0$$

$$I_{y'} = \left(\frac{1}{12}\right)(4)(0.2)^2 = 0.01333, \qquad I_{z'} = \left(\frac{1}{12}\right)(4)(0.2)^2 = 0.01333$$

$$u_{x'} = \cos 90^\circ = 0, \qquad u_{y'} = \cos 135^\circ = -0.7071, \qquad u_{z'} = \cos 45^\circ = 0.7071$$

$$(I_G)_z = I_{x'}u_{x'}^2 + I_{y'}u_{y'}^2 + I_{z'}u_{z'}^2 - 2I_{x'y'}u_{x'}u_{y'} - 2I_{y'z'}u_{y'}u_{z'} - 2I_{z'x'}u_{z'}u_{x'}$$

$$= 0 + (0.01333)(-0.7071)^2 + (0.01333)(0.7071)^2 - 0 - 0 - 0$$



$$(I_G)_z = (I_O)_z = 0.01333$$

For $(I_O)_y$, use the parallel axis theorem.

$$(I_O)_y = 0.01333 + 4\left[0.7071(0.2)\right]^2, \qquad (I_O)_y = 0.09333$$

Hence, from Eqs. (1) and (2):

$$\omega_x = 0, \qquad \omega_y = 90.914, \qquad \omega_z = -636.340$$

The instantaneous axis of rotation is thus,

$$\mathbf{u}_{IA} = \frac{90.914\mathbf{j} - 636.340\mathbf{k}}{\sqrt{(90.914)^2 + (-636.340)^2}} = 0.141\mathbf{j} - 0.990\mathbf{k} \qquad \textbf{Ans.}$$

The velocity of G just after the plate is hit is

$$\mathbf{v}_G = \omega \times \mathbf{r}_{G/O}$$

$$\mathbf{v}_G = (90.914\mathbf{j} - 636.340\mathbf{k}) \times (-0.2(0.7071)\mathbf{k}) = -12.857\mathbf{i}$$

$$m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$$

$$\mathbf{0} - 60\mathbf{i} + \int \mathbf{F}_O\,dt = -4(12.857)\mathbf{i}$$

$$\int \mathbf{F}_O\,dt = \{8.57\mathbf{i}\}\ \text{N}\cdot\text{s} \qquad \textbf{Ans.}$$

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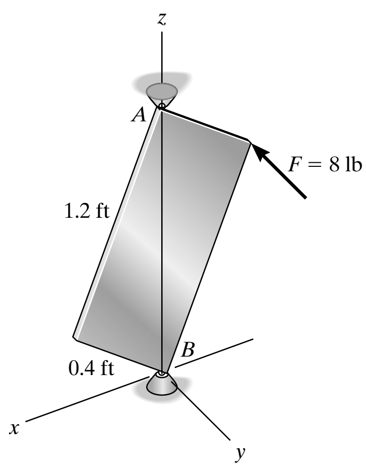
***21–36.** The 15-lb plate is subjected to a force $F = 8$ lb which is always directed perpendicular to the face of the plate. If the plate is originally at rest, determine its angular velocity after it has rotated one revolution (360°). The plate is supported by ball-and-socket joints at A and B.



Due to symmetry

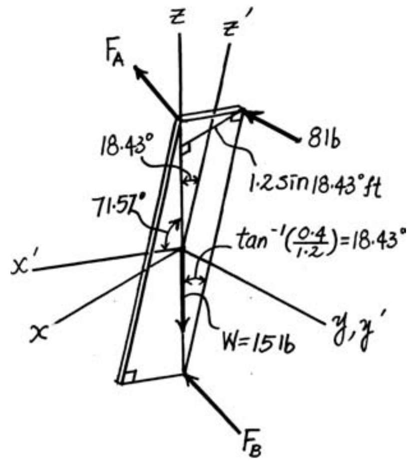
$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

$$I_{x'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(1.2)^2 = 0.05590 \text{ slug} \cdot \text{ft}^2$$

$$I_{y'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(1.2^2 + 0.4^2) = 0.06211 \text{ slug} \cdot \text{ft}^2$$

$$I_{z'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(0.4)^2 = 0.006211 \text{ slug} \cdot \text{ft}^2$$

For z axis



$$u_{x'} = \cos 71.57° = 0.3162 \qquad u_{y'} = \cos 90° = 0$$

$$u_{z'} = \cos 18.43° = 0.9487$$

$$I_z = I_{x'}u_{x'}^2 + I_{y'}u_{y'}^2 + I_{z'}u_{z'}^2 - 2I_{x'y'}u_{x'}u_{y'} - 2I_{y'z'}u_{y'}u_{z'} - 2I_{z'x'}u_{z'}u_{x'}$$

$$= 0.05590(0.3162)^2 + 0 + 0.006211(0.9487)^2 - 0 - 0 - 0$$

$$= 0.01118 \text{ slug} \cdot \text{ft}^2$$

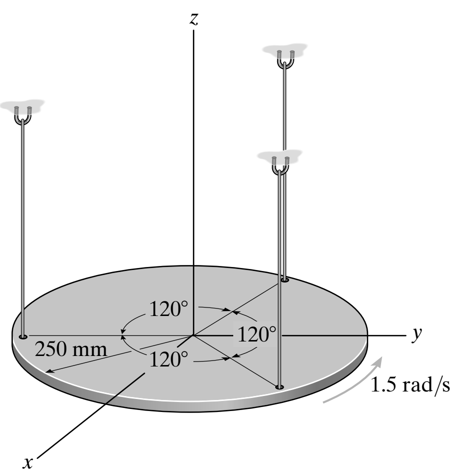
Principle of work and energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 8(1.2 \sin 18.43°)(2\pi) = \frac{1}{2}(0.01118)\omega^2$$

$$\omega = 58.4 \text{ rad/s} \qquad \textbf{Ans.}$$

•21–37. The plate has a mass of 10 kg and is suspended from parallel cords. If the plate has an angular velocity of 1.5 rad/s about the z axis at the instant shown, determine how high the center of the plate rises at the instant the plate momentarily stops swinging.



Consevation Energy: Datum is set at the initial position of the plate. When the plate is at its final position and its mass center is located h *above* the datum. Thus, its gravitational potential energy at this position is $10(9.81)h = 98.1h$. Since the plate momentarily stops swinging, its final kinetic energy $T_2 = 0$. Its initial kinetic energy i

$$T_1 = \frac{1}{2}I_G\,\omega^2 = \frac{1}{2}\left[\frac{1}{2}(10)\left(0.25^2\right)\right]\left(1.5^2\right) = 0.3516 \text{ J}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0.3516 + 0 = 0 + 98.1h$$

$$h = 0.00358 \text{ m} = 3.58 \text{ mm} \qquad \textbf{Ans.}$$

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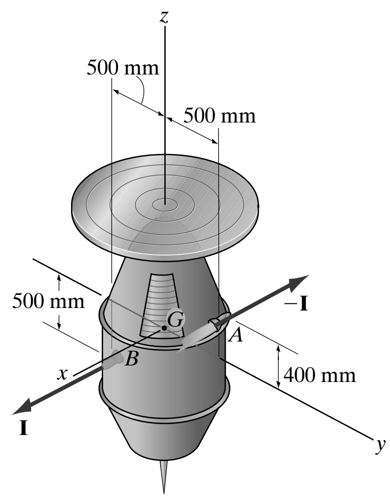
21–38. The satellite has a mass of 200 kg and radii of gyration of $k_x = k_y = 400$ mm and $k_z = 250$ mm. When it is not rotating, the two small jets A and B are ignited simultaneously, and each jet provides an impulse of $I = 1000\ \text{N} \cdot \text{s}$ on the satellite. Determine the satellite's angular velocity immediately after the ignition.



The mass moments of inertia of the satellite about the x, y, and z axes are

$$I_x = I_y = 200\left(0.4^2\right) = 32\ \text{kg} \cdot \text{m}^2$$

$$I_z = 200\left(0.25^2\right) = 12.5\ \text{kg} \cdot \text{m}^2$$

Due to symmetry,

$$I_{xy} = I_{yz} = I_{xz} = 0$$

Thus, the angular momentum of the satellite about its mass center G is

$$H_x = I_x\omega_x = 32\omega_x \qquad H_y = I_y\omega_y = 32\omega_y \qquad H_z = I_z\omega_z = 12.5\omega_z$$

Applying the principle of angular impulse and momentum about the x, y, and z axes,

$$(H_x)_1 + \Sigma\int_{t_1}^{t_2} M_x\,dt = (H_x)_2$$

$$0 + 0 = 32\omega_x$$

$$\omega_x = 0$$

$$\left(H_y\right)_1 + \Sigma\int_{t_1}^{t_2} M_y\,dt = \left(H_y\right)_2$$

$$0 - 1000(0.4) - 1000(0.5) = 32\omega_y$$

$$\omega_y = -28.125\ \text{rad/s}$$

$$(H_z)_1 + \Sigma\int_{t_1}^{t_2} M_y\,dt = (H_z)_2$$

$$0 + 1000(0.5) + 1000(0.5) = 12.5\omega_z$$

$$\omega_z = 80\ \text{rad/s}$$

Thus

$$\omega = \{-28.1\mathbf{j} + 80\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

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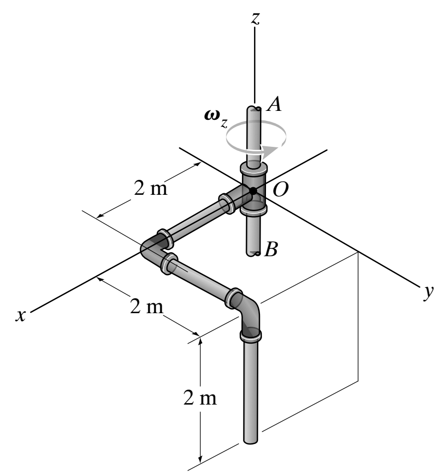
21–39. The bent rod has a mass per unit length of 6 kg/m, and its moments and products of inertia have been calculated in Prob. 21–9. If shaft AB rotates with a constant angular velocity of $\omega_z = 6$ rad/s, determine the angular momentum of the rod about point O, and the kinetic energy of the rod.



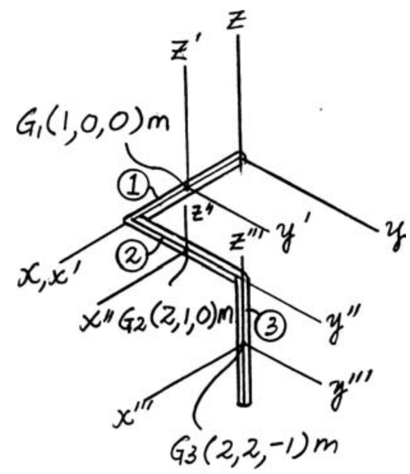
Here, the angular velocity of the rod is

$$\omega = [6\mathbf{k}] \text{ rad/s}$$

Thus,

$$\omega_x = \omega_y = 0 \qquad \omega_z = 6 \text{ rad/s}$$

The rod rotates about a fixed point O. Using the results of Prob. 20–91



(a)

$$H_x = I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z$$

$$= 80(0) - 72(0) - (-24)(6) = 144 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$H_y = -I_{xy}\omega_x + I_y\omega_y - I_{yz}\omega_z$$

$$= -72(0) + 128(0) - (-24)(6) = 144 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$H_z = -I_{xz}\omega_x - I_{yz}\omega_y + I_z\omega_z$$

$$= -(-24)(0) - (-24)(0) + 176(6) = 1056 \text{ kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$\mathbf{H}_O = [144\mathbf{i} + 144\mathbf{j} + 1056\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

The kinetic energy of the rod can be determined from

$$T = \frac{1}{2}\omega\cdot\mathbf{H}_O$$

$$= \frac{1}{2}(6\mathbf{k})\cdot(144\mathbf{i} + 144\mathbf{j} + 1056\mathbf{k})$$

$$= 3168 \text{ J} = 3.17 \text{ kJ} \qquad \textbf{Ans.}$$

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***21–40.** Derive the scalar form of the rotational equation of motion about the x axis if $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *not constant* with respect to time.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Subsitute H_x, H_y and H_z using Eq. 21–10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y) \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

•21–41. Derive the scalar form of the rotational equation of motion about the x axis if $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *constant* with respect to time.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_{\mathbf{y}}\mathbf{j} + \dot{H}_{\mathbf{z}}\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Subsitute H_x, H_y and H_z using Eq. 21–10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y)$$

For constant inertia, expanding the time derivative of the above equation yields

$$\Sigma M_x = (I_x\,\dot{\omega}_x - I_{xy}\,\dot{\omega}_y - I_{xz}\dot{\omega}_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\,\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\omega_x - I_{zy}\,\omega_y) \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

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21–42. Derive the Euler equations of motion for $\mathbf{\Omega} \neq \boldsymbol{\omega}$, i.e., Eqs. 21–26.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Substitute H_x, H_y and H_z using Eq. 21–10. For the **i** component

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y)$$

Set $I_{xy} = I_{yz} = I_{zx} = 0$ and require I_x, I_y, I_z to be constant. This yields

$$\Sigma M_x = I_x\,\dot{\omega}_x - I_y\,\Omega_z\,\omega_y + I_z\,\Omega_y\omega_z \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

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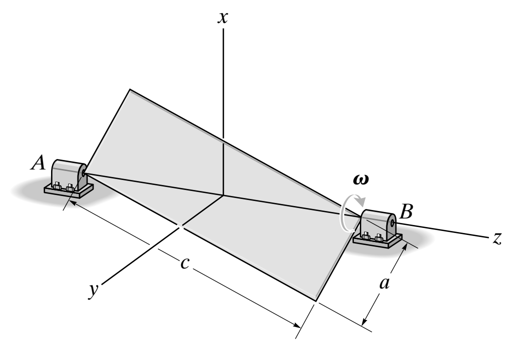
21–43. The uniform rectangular plate has a mass of $m = 2$ kg and is given a rotation of $\omega = 4$ rad/s about its bearings at A and B. If $a = 0.2$ m and $c = 0.3$ m, determine the vertical reactions at A and B at the instant the plate is vertical as shown. Use the x, y, z axes shown and note that $I_{zx} = -\left(\dfrac{mac}{12}\right)\left(\dfrac{c^2 - a^2}{c^2 + a^2}\right)$.



$$\omega_x = 0, \quad \omega_y = 0, \quad \omega_z = -4$$

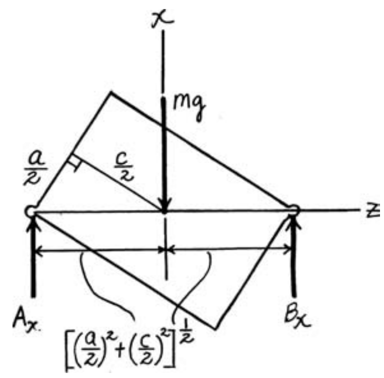
$$\dot{\omega}_x = 0, \quad \dot{\omega}_y = 0, \quad \dot{\omega}_z = 0$$

$$\Sigma M_y = I_{yy}\,\dot{\omega}_y - (I_{zz} - I_{xx})\omega_z\,\omega_x - I_{yz}\left(\dot{\omega}_z - \omega_x\,\omega_y\right) - I_{zx}\left(\omega_z^2 - \omega_x^2\right) - I_{xy}\left(\dot{\omega}_x + \omega_y\,\omega_z\right)$$

$$B_x\left[\left(\frac{a}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]^{\frac{1}{2}} - A_x\left[\left(\frac{a}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]^{\frac{1}{2}} = -I_{zx}\,(\omega)^2$$

$$B_x - A_x = \left(\frac{mac}{6}\right)\left(\frac{c^2 - a^2}{\left[a^2 + c^2\right]^{\frac{3}{2}}}\right)\omega^2$$

$$\Sigma F_x = m(a_G)_x; \qquad A_x + B_x - mg = 0$$



Substitute the data,

$$B_x - A_x = \frac{2(0.2)(0.3)}{6}\left[\frac{(0.3)^2 - (0.2)^2}{\left[(0.3)^2 + (0.2)^2\right]^{\frac{3}{2}}}\right](-4)^2 = 0.34135$$

$$A_x + B_x = 2(9.81)$$

Solving:

$A_x = 9.64$ N **Ans.**

$B_x = 9.98$ N **Ans.**

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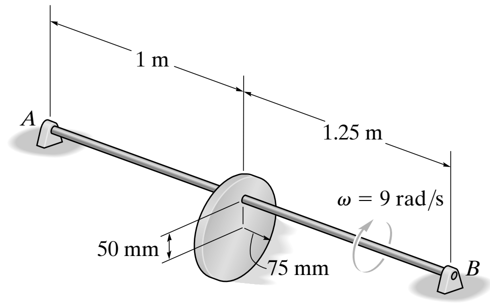
***21–44.** The disk, having a mass of 3 kg, is mounted eccentrically on shaft AB. If the shaft is rotating at a constant rate of 9 rad/s, determine the reactions at the journal bearing supports when the disk is in the position shown.



$\omega_x = 0, \qquad \omega_y = -9, \qquad \omega_z = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z$

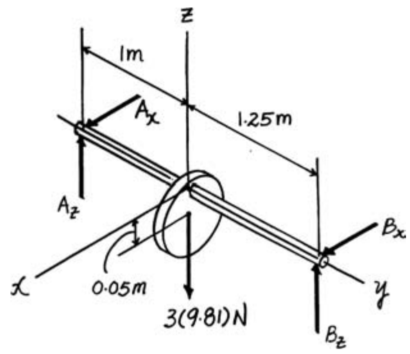
$B_z(1.25) - A_z(1) = 0 - 0$

$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y$

$A_x(1) - B_x(1.25) = 0 - 0$

$\Sigma F_x = ma_x; \qquad A_x + B_x = 0$

$\Sigma F_z = ma_z; \qquad A_z + B_z - 3(9.81) = 3(9)^2(0.05)$



Solving,

$A_x = B_x = 0$ **Ans.**

$A_z = 23.1 \text{ N}$ **Ans.**

$B_z = 18.5 \text{ N}$ **Ans.**

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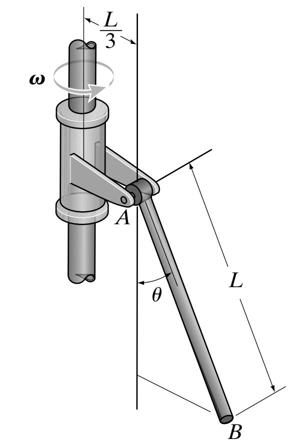
•21–45. The slender rod AB has a mass m and it is connected to the bracket by a smooth pin at A. The bracket is rigidly attached to the shaft. Determine the required constant angular velocity of $\boldsymbol{\omega}$ of the shaft, in order for the rod to make an angle of θ with the vertical.



The rotating xyz frame is set with its origin at the rod's mass center, Fig. *a*. This frame will be attached to the rod so that its angular velocity is $\Omega = \omega$ and the x, y, z axes will always be the principal axes of inertia. Referring to Fig. *b*,

$$\omega = -\omega\cos\theta\mathbf{j} + \omega\sin\theta\mathbf{k}$$

Thus,

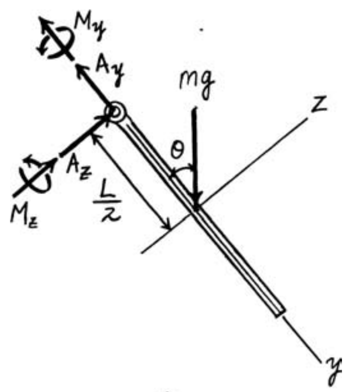
$$\omega_x = 0 \qquad \omega_y = -\omega\cos\theta \qquad \omega_z = \omega\sin\theta$$

Since both the direction and the magnitude is constant $\dot{\omega} = \mathbf{0}$. Also, since $\Omega = \omega$, $\left(\dot{\omega}_{xyz}\right) = \dot{\omega} = 0$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$

The mass moments of inertia of the rod about the x, y, z axes are

$$I_x = I_z = \frac{1}{12}mL^2 \qquad I_y = 0$$

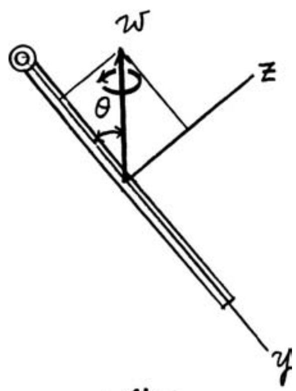


(a)

Applying the equation of motion and referring to the free-body diagram of the rod, Fig. *a*,

$$\Sigma M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z; \quad -A_z\left(\frac{L}{2}\right) = 0 - \left(0 - \frac{1}{12}mL^2\right)(-\omega\cos\theta)(\omega\sin\theta)$$

$$A_z = \frac{m\omega^2 L}{6}\sin\theta\cos\theta \qquad \textbf{(1)}$$



(b)

The acceleration of the mass center of the rod can be determined from $a_G = \omega^2 r = \omega^2\left(\frac{L}{2}\sin\theta + \frac{L}{3}\right) = \frac{\omega^2 L}{6}(3\sin\theta + 2)$ and is directed as shown in Fig. *c*. Thus,

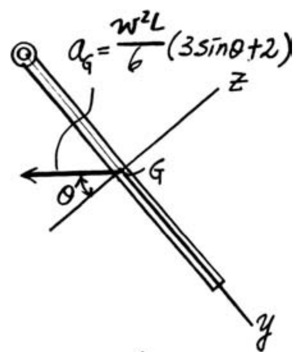
$$\Sigma F_z = m(a_G)_z; \qquad A_z - mg\sin\theta = -\frac{m\omega^2 L}{6}(3\sin\theta + 2)\cos\theta$$

$$A_z = mg\sin\theta = -\frac{m\omega^2 L}{6}(3\sin\theta + 2)\cos\theta \quad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{m\omega^2 L}{6}\sin\theta\cos\theta = mg\sin\theta - \frac{m\omega^2 L}{6}(3\sin\theta + 2)\cos\theta$$

$$\omega = \sqrt{\frac{3g\tan\theta}{L(2\sin\theta + 1)}} \qquad \textbf{Ans.}$$



(c)

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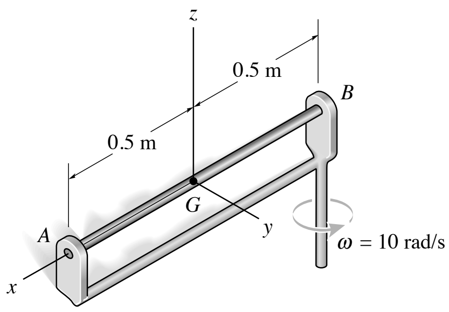
21–46. The 5-kg rod AB is supported by a rotating arm. The support at A is a journal bearing, which develops reactions normal to the rod. The support at B is a thrust bearing, which develops reactions both normal to the rod and along the axis of the rod. Neglecting friction, determine the x, y, z components of reaction at these supports when the frame rotates with a constant angular velocity of $\omega = 10$ rad/s.



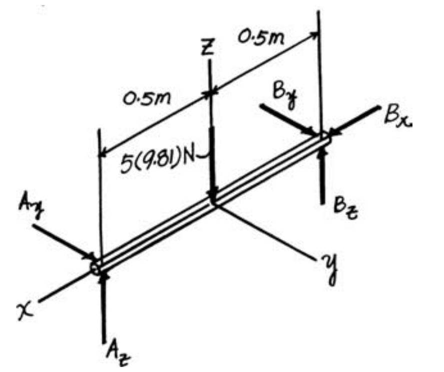
$$I_y = I_z = \frac{1}{12}(5)(1)^2 = 0.4167\ \text{kg}\cdot\text{m}^2 \qquad I_x = 0$$

Applying Eq. 21–25 with $\omega_x = \omega_y = 0$ $\omega_z = 10$ rad/s $\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;\qquad 0 = 0$

$\Sigma M_y = I_y \dot{\omega}_y - (I_z - I_x)\omega_z \omega_x;\qquad B_z(0.5) - A_z(0.5) = 0$ **(1)**

$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y;\qquad A_y(0.5) - B_y(0.5) = 0$ **(2)**



Also,

$\Sigma F_x = m(a_G)_x;\qquad B_x = -5(10)^2(0.5)\qquad B_x = -250\text{N}$ **Ans.**

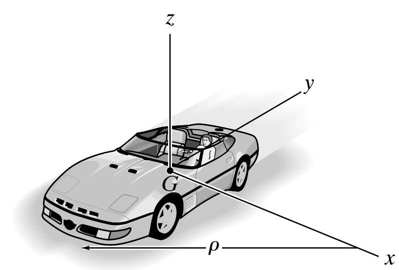
$\Sigma F_y = m(a_G)_y;\qquad A_y + B_y = 0$ **(3)**

$\Sigma F_z = m(a_G)_z;\qquad A_z + B_z - 5(9.81) = 0$ **(4)**

Solving Eqs. (1) to (4) yields:

$$A_y = B_y = 0 \qquad A_z = B_z = 24.5\ \text{N}$$ **Ans.**

21–47. The car travels around the curved road of radius ρ such that its mass center has a constant speed v_G. Write the equations of rotational motion with respect to the x, y, z axes. Assume that the car's six moments and products of inertia with respect to these axes are known.



Applying Eq. 21–24 with $\omega_x = 0,\quad \omega_y = 0,\quad \omega_z = \dfrac{v_G}{\rho},$

$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$$\Sigma M_x = -I_{yz}\left[0 - \left(\frac{v_G}{\rho}\right)^2\right] = \frac{I_{yz}}{\rho^2}v_G^2$$ **Ans.**

$$\Sigma M_y = -I_{zx}\left[\left(\frac{v_G}{\rho}\right)^2 - 0\right] = -\frac{I_{zx}}{\rho^2}v_G^2$$ **Ans.**

$\Sigma M_z = 0$ **Ans.**

Note: This result indicates the normal reactions of the tires on the ground are not all necessarily equal. Instead, they depend upon the speed of the car, radius of curvature, and the products of inertia I_{yz} and I_{zx}. (See Example 13–6.)

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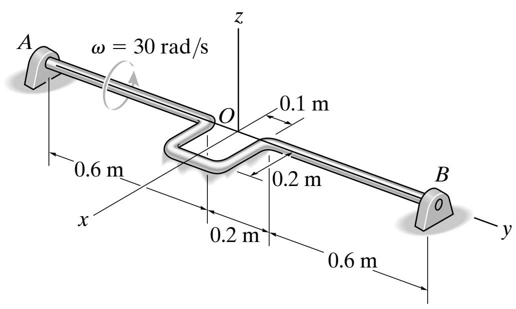
***21–48.** The shaft is constructed from a rod which has a mass per unit length of 2 kg/m. Determine the x, y, z components of reaction at the bearings A and B if at the instant shown the shaft spins freely and has an angular velocity of $\omega = 30$ rad/s. What is the angular acceleration of the shaft at this instant? Bearing A can support a component of force in the y direction, whereas bearing B cannot.

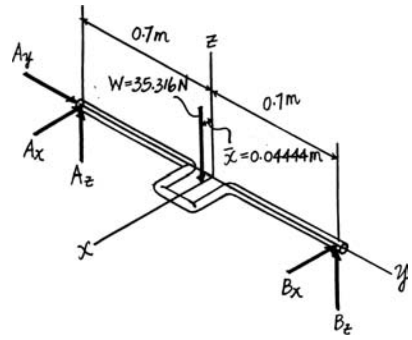


$$\Sigma W = \left[3(0.2) + 1.2\right](2)(9.81) = 35.316\text{ N}$$

$$\Sigma \bar{x}W = 0\left[1.2(2)(9.81)\right] + 0.1\left[0.4(2)(9.81)\right] + 0.2\left[0.2(2)(9.81)\right] = 1.5696\text{ N}\cdot\text{m}$$

$$\bar{x} = \frac{\Sigma\,\bar{x}W}{\Sigma\,W} = \frac{1.5696}{35.316} = 0.04444\text{ m}$$

$$I_x = 2\left[\frac{1}{3}\left[0.2(2)\right](0.2)^2\right] + \left[0.2(2)\right](0.2)^2 = 0.02667\text{ kg}\cdot\text{m}^2$$



Applying Eq. 21–25 with $\omega_x = \omega_z = 0$ $\omega_y = 30$ rad/s $\dot{\omega}_x = \dot{\omega}_z = 0$

$\Sigma M_x = I_x\,\dot{\omega}_x - (I_y - I_z)\omega_y\,\omega_z;\qquad B_z(0.7) - A_z(0.7) = 0$ **(1)**

$\Sigma M_y = I_y\,\dot{\omega}_y - (I_z - I_x)\omega_z\,\omega_x;\qquad 35.316(0.04444) = 0.02667\dot{\omega}_y$

$\dot{\omega}_y = 58.9\text{ rad/s}^2$ **Ans.**

$\Sigma M_z = I_z\,\dot{\omega}_z - (I_x - I_y)\omega_x\,\omega_y;\qquad B_x(0.7) - A_x(0.7) = 0$ **(2)**

Also,

$\Sigma F_x = m(a_G)_x;\qquad -A_x - B_x = -1.8(2)(0.04444)(30)^2$ **(3)**

$\Sigma F_y = m(a_G)_y;\qquad A_y = 0$ **Ans.**

$\Sigma F_z = m(a_G)_z;\qquad A_z + B_z - 35.316 = -1.8(2)(0.04444)(58.9)$ **(4)**

Solving Eqs. (1) to (4) yields:

$A_x = B_x = 72.0\text{ N}\qquad A_z = B_z = 12.9\text{ N}$ **Ans.**

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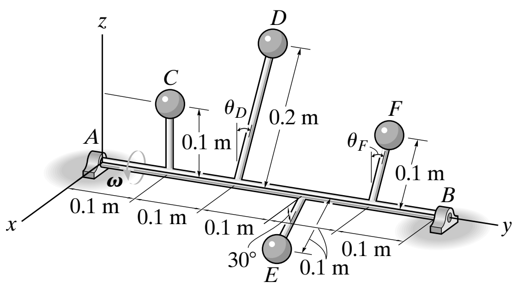
•21–49. Four spheres are connected to shaft AB. If $m_C = 1$ kg and $m_E = 2$ kg, determine the mass of spheres D and F and the angles of the rods, θ_D and θ_F, so that the shaft is dynamically balanced, that is, so that the bearings at A and B exert only vertical reactions on the shaft as it rotates. Neglect the mass of the rods.



For $\bar{x} = 0$; $\Sigma \bar{x}_1 m_1 = 0$

$$(0.1 \cos 30°)(2) - (0.1 \sin \theta_F)m_F - (0.2 \sin \theta_D)m_D = 0 \qquad \textbf{(1)}$$

For $\bar{z} = 0$; $\Sigma \bar{z}_1 m_1 = 0$

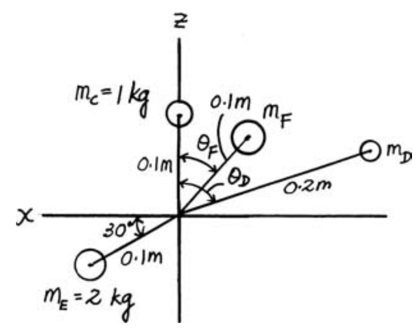
$$(0.1)(1) - (0.1 \sin 30°)(2) + (0.2 \cos \theta_D)m_D + (0.1 \cos \theta_F)m_F = 0 \qquad \textbf{(2)}$$

For $I_{xz} = 0$; $\Sigma \bar{x}_1 \bar{z}_1 m_1 = 0$

$$-(0.2)(0.2 \sin \theta_D)m_D + (0.3)(0.1 \cos 30°)(2) - (0.4)(0.1 \sin \theta_F)m_F = 0 \qquad \textbf{(3)}$$

For $I_{xy} = 0$; $\Sigma \bar{x}_1 \bar{y}_1 m_1 = 0$

$$(0.1)(0.1)(1) + (0.2)(0.2 \cos \theta_D)m_D - (0.3)(0.1 \sin 30°)(2)$$
$$+ (0.1 \cos \theta_F)(0.4)(m_F) = 0 \qquad \textbf{(4)}$$



Solving,

$\theta_D = 139°$ **Ans.**

$m_D = 0.661$ kg **Ans.**

$\theta_F = 40.9°$ **Ans.**

$m_F = 1.32$ kg **Ans.**

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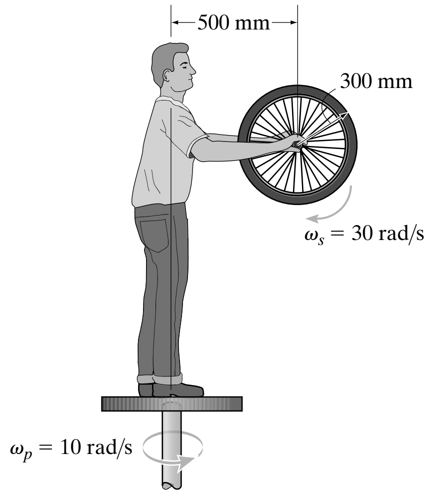
21–50. A man stands on a turntable that rotates about a vertical axis with a constant angular velocity of $\omega_p = 10$ rad/s. If the wheel that he holds spins with a constant angular speed of $\omega_s = 30$ rad/s, determine the magnitude of moment that he must exert on the wheel to hold it in the position shown. Consider the wheel as a thin circular hoop (ring) having a mass of 3 kg and a mean radius of 300 mm.



The rotating xyz frame will be set with an angular velocity of $\Omega = \omega_P = [10\mathbf{k}]$ rad/s. Since the wheel is symmetric about its spinning axis, the x, y, and z axes will remain as the principle axes of inertia. Thus,

$$I_y = I_z = \frac{1}{2}mr^2 = \frac{1}{2}(3)\left(0.3^2\right) = 0.135 \text{ kg}\cdot\text{m}^2$$

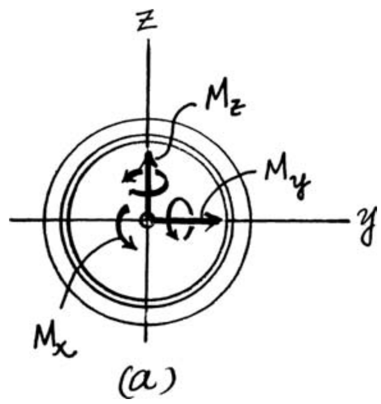
$$I_x = mr^2 = 3\left(0.3^2\right) = 0.27 \text{ kg}\cdot\text{m}^2$$

The angular velocity of the wheel is $\omega = \omega_s + \omega_P = [-30\mathbf{i} + 10\mathbf{k}]$ rad/s. Thus,

$$\omega_x = -30 \text{ rad/s} \qquad \omega_y = 0 \qquad \omega_z = 10 \text{ rad/s}$$

Since the directions of ω_s and ω_p do not change with respect to the xyz frame and their magnitudes are constant, $\dot{\omega}_{xyz} = 0$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$



Applying the equations of motion and referring to the free-body diagram shown in Fig. a,

$$\Sigma M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z; \qquad M_x = 0$$

$$\Sigma M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x; \qquad M_y = 0 - 0 + 0.27(10)(-30) = -81.0 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y; \qquad M_z = 0$$

Thus,

$$M = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{0^2 + (-81.0)^2 + 0^2} = 81.0 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

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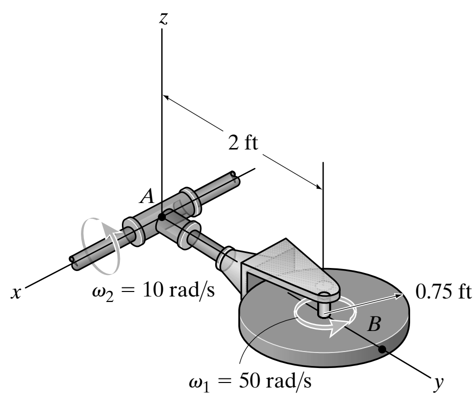
21–51. The 50-lb disk spins with a constant angular rate of $\omega_1 = 50$ rad/s about its axle. Simultaneously, the shaft rotates with a constant angular rate of $\omega_2 = 10$ rad/s. Determine the x, y, z components of the moment developed in the arm at A at the instant shown. Neglect the weight of arm AB.



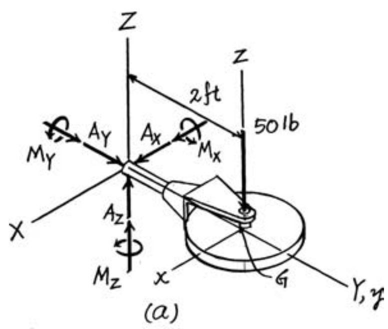
The rotating xyz frame is established as shown in Fig. a. This frame will be set to have an angular velocity of $\Omega = \omega_2 = [10\mathbf{i}]$ rad/s. Since the disk is symmetric about its spinning axis, the x, y, and z axes will remain as the principle axes of inertia. Thus,

$$I_x = I_y = \frac{1}{4}\left(\frac{50}{32.2}\right)\left(0.75^2\right) = 0.2184 \text{ slug}\cdot\text{ft}^2$$

$$I_z = \frac{1}{2}\left(\frac{50}{32.2}\right)\left(0.75^2\right) = 0.4367 \text{ slug}\cdot\text{ft}^2$$

The angular velocity of the disk is $\omega = \omega_s + \omega_p = [10\mathbf{i} + 50\mathbf{k}]$ rad/s. Thus,

$$\omega_x = 10 \text{ rad/s} \qquad \omega_y = 0 \qquad \omega_z = 50 \text{ rad/s}$$



Since the directions of ω_1 and ω_2 do not change with respect to the xyz frame and their magnitudes are constant, $\dot{\omega}_{xyz} = \mathbf{0}$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$

Applying the equations of motion and referring to the free-body diagram shown in Fig. a,

$\Sigma M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z;\qquad M_X - A_Z(2) = 0 \qquad$ **(1)**

$\Sigma M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x;\qquad M_Y = 0 - 0.4367(10)(50) + 0$

$M_Y = -218.36 \text{ lb}\cdot\text{ft} = -218 \text{ lb}\cdot\text{ft}$ **Ans.**

$\Sigma M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_{zx} + I_y\Omega_x\omega_y;\qquad M_Z = 0 - 0 + 0$

$M_Z = 0$ **Ans.**

Since the mass center of the disk rotates about the X axis with a constant angular velocity of $\omega_1 = [10\mathbf{i}]$ rad/s, its acceleration is $\mathbf{a}_G = \dot{\omega}_2 \times \mathbf{r}_G - \omega^2\mathbf{r}_G = \mathbf{0} - 10^2(2\mathbf{j}) = [-200\mathbf{j}]$ ft/s². Thus,

$$\Sigma F_Z = m(a_G)_Z;\qquad A_Z - 50 = \frac{50}{32.2}(0)\qquad A_Z = 50 \text{ lb}$$

Substituting this result into Eq. (1), we have

$M_X = 100 \text{ lb}\cdot\text{ft}$ **Ans.**

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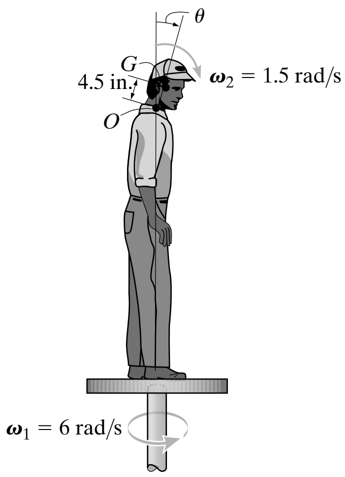
***21–52.** The man stands on a turntable that rotates about a vertical axis with a constant angular velocity of $\omega_1 = 6$ rad/s. If he tilts his head forward at a constant angular velocity of $\omega_2 = 1.5$ rad/s about point O, determine the magnitude of the moment that must be resisted by his neck at O at the instant $\theta = 30°$. Assume that his head can be considered as a uniform 10-lb sphere, having a radius of 4.5 in. and center of gravity located at G, and point O is on the surface of the sphere.

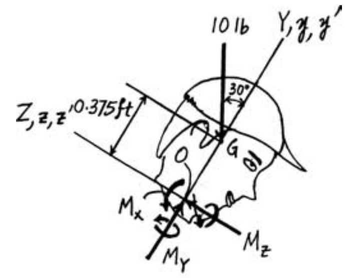


The rotating xyz frame shown in Fig. a will be attached to the head so that it rotates with an angular velocity of $\Omega = \omega$, where $\omega = \omega_1 + \omega_2$. Referring to Fig. b, $\omega_1 = [6\cos 30°\,\mathbf{j} + 6\sin 30°\,\mathbf{k}]$ rad/s $= [5.196\mathbf{j} + 3\mathbf{k}]$ rad/s. Thus, $\omega = [-1.5\mathbf{i} + 5.196\mathbf{j} + 3\mathbf{k}]$ rad/s. Then

$$\omega_x = -1.5 \text{ rad/s} \qquad \omega_y = 5.196 \text{ rad/s} \qquad \omega_z = 3 \text{ rad/s}$$

The angular acceleration of the head $\dot{\omega}$ with respect to the XYZ frame can be obtained by setting another $x'y'z'$ frame having an angular velocity of $\Omega' = \omega_1 = [5.196\mathbf{j} + 3\mathbf{k}]$ rad/s. Thus

$$\begin{aligned}\dot{\omega} &= \left(\dot{\omega}_{x'y'z'}\right) + \Omega' \times \omega \\ &= (\dot{\omega}_1)_{x'y'z'} + (\dot{\omega}_2)_{x'y'z'} + \Omega' \times \omega_1 + \Omega' \times \omega_2 \\ &= 0 + 0 + 0 + (5.196\mathbf{j} + 3\mathbf{k}) \times (-1.5\mathbf{i}) \\ &= [-4.5\mathbf{j} + 7.794\mathbf{k}] \text{ rad/s}^2\end{aligned}$$



(a)

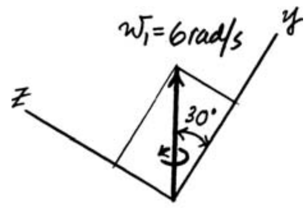
Since $\Omega = \omega$, $\dot{\omega}_{x'y'z'} = \dot{\omega} = [-4.5\mathbf{j} + 7.794\mathbf{k}]$ rad/s². Thus,

$$\dot{\omega}_x = 0 \qquad \dot{\omega}_y = -4.5 \text{ rad/s}^2 \qquad \dot{\omega}_z = 7.794 \text{ rad/s}^2$$

Also, the x, y, z axes will remain as principal axes of inertia. Thus,

$$I_x = I_z = \frac{2}{5}\frac{10}{32.2}\left(0.375^2\right) + \left(\frac{10}{32.2}\right)\left(0.375^2\right) = 0.06114 \text{ slug} \cdot \text{ft}^2$$

$$I_y = \frac{2}{5}\left(\frac{10}{32.2}\right)\left(0.375^2\right) = 0.01747 \text{ slug} \cdot \text{ft}^2$$



(b)

Applying the moment equations of motion and referring to the free-body diagram shown in Fig. a,

$$\Sigma M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z; \qquad M_x - 10\sin 30°(0.375) = 0 - (0.01747 - 0.06114)(5.196)(3)$$

$$M_x = 2.556 \text{ lb} \cdot \text{ft}$$

$$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \qquad M_y = 0.01747(-4.5) - 0 \qquad M_y = -0.07861 \text{ lb} \cdot \text{ft}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y; \qquad M_z = 0.06114(7.794) - (0.06114 - 0.01747)(-1.5)(5.196)$$

$$= 0.8161 \text{ lb} \cdot \text{ft}$$

Thus,

$$M_A = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{2.556^2 + (-0.07861)^2 + 0.8161^2} = 2.68 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

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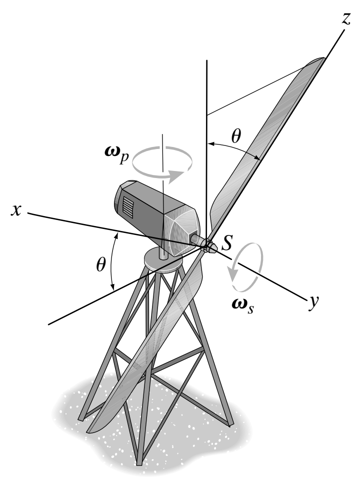
•**21–53.** The blades of a wind turbine spin about the shaft S with a constant angular speed of ω_s, while the frame precesses about the vertical axis with a constant angular speed of ω_p. Determine the x, y, and z components of moment that the shaft exerts on the blades as a function of θ. Consider each blade as a slender rod of mass m and length l.

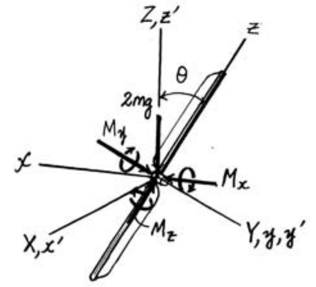


The rotating xyz frame shown in Fig. a will be attached to the blade so that it rotates with an angular velocity of $\Omega = \omega$, where $\omega = \omega_s + \omega_p$. Referring to Fig. b $\omega_p = \omega_p \sin\theta\mathbf{i} + \omega_p \cos\theta\mathbf{k}$. Thus, $\omega = \omega_p \sin\theta\mathbf{i} + \omega_s\mathbf{j} + \omega_p\cos\theta\mathbf{k}$. Then

$$\omega_x = \omega_p \sin\theta \qquad \omega_y = \omega_s \quad \omega_z = \omega_p\cos\theta$$

The angular acceleration of the blade $\dot{\omega}$ with respect to the XYZ frame can be obtained by setting another $x'y'z'$ frame having an angular velocity of $\Omega' = \omega_p = \omega_p\sin\theta\mathbf{i} + \omega_p\cos\theta\mathbf{k}$. Thus,

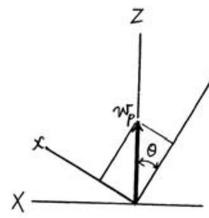
$$\begin{aligned}\dot{\omega} &= \left(\dot{\omega}_{x'y'z'}\right) + \Omega' \times \omega \\ &= (\dot{\omega}_1)_{x'y'z'} + (\dot{\omega}_2)_{x'y'z'} + \Omega' \times \omega_S + \Omega' \times \omega_P \\ &= \mathbf{0} + \mathbf{0} + \left(\omega_p \sin\theta\mathbf{i} + \omega_p\cos\theta\mathbf{k}\right) \times (\omega_s\mathbf{j}) + \mathbf{0} \\ &= -\omega_s\omega_p\cos\theta\mathbf{i} + \omega_s\omega_p\sin\theta\mathbf{k}\end{aligned}$$



(a)

Since $\Omega = \omega$, $\dot{\omega}_{x'y'z'} = \dot{\omega}$. Thus,

$$\dot{\omega}_x = -\omega_s\omega_p\cos\theta \qquad \dot{\omega}_y = 0 \qquad \dot{\omega}_z = \omega_s\omega_p\sin\theta$$



(b)

Also, the x, y, and z axes will remain as principle axes of inertia for the blade. Thus,

$$I_x = I_y = \frac{1}{12}(2m)(2l)^2 = \frac{2}{3}ml^2 \qquad I_z = 0$$

Applying the moment equations of motion and referring to the free-body diagram shown in Fig. a,

$$\Sigma M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z; \qquad M_x = \frac{2}{3}ml^2(-\omega_s\omega_p\cos\theta) - \left(\frac{2}{3}ml^2 - 0\right)(\omega_s)(\omega_p\cos\theta)$$

$$= -\frac{4}{3}ml^2\,\omega_s\omega_p\cos\theta \qquad \textbf{Ans.}$$

$$\Sigma M_y = I_y\dot{\omega}_y - \left(I_z - I_x\right)\omega_z\omega_x; \qquad M_y = 0 - \left(0 - \frac{2}{3}ml^2\right)(\omega_p\cos\theta)(\omega_p\sin\theta)$$

$$= \frac{1}{3}ml^2\,{\omega_p}^2\sin 2\theta \qquad \textbf{Ans.}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y; \qquad M_z = 0 - 0 = 0 \qquad \textbf{Ans.}$$

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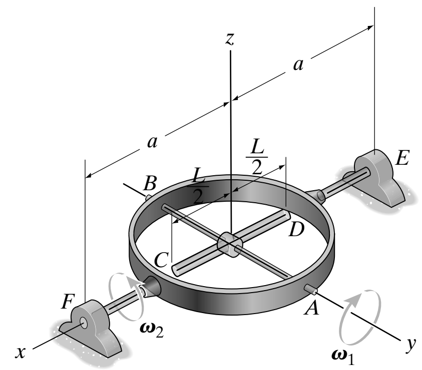
21–54. Rod CD of mass m and length L is rotating with a constant angular rate of ω_1 about axle AB, while shaft EF rotates with a constant angular rate of ω_2. Determine the X, Y, and Z components of reaction at thrust bearing E and journal bearing F at the instant shown. Neglect the mass of the other members.



The rotating xyz frame shown in Fig. a will be attached to the rod so that it rotates with an angular velocity of $\Omega = \omega$, where $\omega = \omega_1 + \omega_2 = \omega_2\mathbf{i} - \omega_1\mathbf{j}$. Thus,

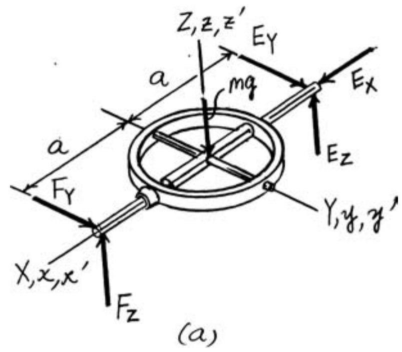
$$\omega_x = \omega_2 \qquad \omega_y = -\omega_1 \qquad \omega_z = 0$$

The angular acceleration of the rod $\dot{\omega}$ with respect to the XYZ frame can be obtained by using another rotating $x'y'z'$ frame having an angular velocity of $\Omega' = \omega_2 = \omega_2\mathbf{i}$. Fig. a. Thus,

$$\begin{aligned}\dot{\omega} &= \left(\dot{\omega}_{x'y'z'}\right) + \Omega' \times \omega \\ &= (\dot{\omega}_1)_{x'y'z'} + (\dot{\omega}_2)_{x'y'z'} + \Omega' \times \omega_1 + \Omega' \times \omega_2 \\ &= 0 + 0 + \left(\omega_2\mathbf{i}\right) \times (-\omega_1\mathbf{j}) + \mathbf{0} \\ &= -\omega_1\omega_2\mathbf{k}\end{aligned}$$

Since $\Omega = \omega, \dot{\omega}_{x'y'z'} = \dot{\omega}$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = 0 \qquad \dot{\omega}_z = -\omega_1\omega_2$$



Also, the x, y, and z axes will remain as principal axes of inertia for the rod. Thus,

$$I_x = 0 \qquad I_y = I_z = \frac{1}{12}mL^2$$

Applying the equations of motion and referring to the free-body diagram shown in Fig. a,

$$\Sigma M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z; \; 0 = 0$$

$$\Sigma M_y = I_y\dot{\omega}_y - \left(I_z - I_x\right)\omega_z\omega_x; \; E_Z(a) - F_Z(a) = 0 - 0$$

$$E_Z - F_Z = 0 \qquad \textbf{(1)}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y; \; F_Y(a) - E_Y(a) = \frac{1}{12}mL^2(-\omega_1\omega_2) - \left(0 - \frac{1}{12}mL^2\right)(\omega_2)(-\omega_1)$$

$$F_Y - E_Y = -\frac{mL^2\omega_1\omega_2}{6a} \qquad \textbf{(2)}$$

Since the mass center G does not move, $\mathbf{a}_G = \mathbf{0}$. Thus,

$$\Sigma F_X = m(a_G)_X; \qquad E_X = 0 \qquad \textbf{Ans.}$$

$$\Sigma F_Y = m(a_G)_Y; \qquad F_Y + E_Y = 0 \qquad \textbf{(3)}$$

$$\Sigma F_Z = m(a_G)_Z; \qquad F_Z + E_Z - mg = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$$F_Y = -\frac{mL^2\,\omega_1\omega_2}{12a} \qquad E_Y = \frac{mL^2\omega_1\omega_2}{12a} \qquad \textbf{Ans.}$$

$$E_Z = F_Z = \frac{mg}{2} \qquad \textbf{Ans.}$$

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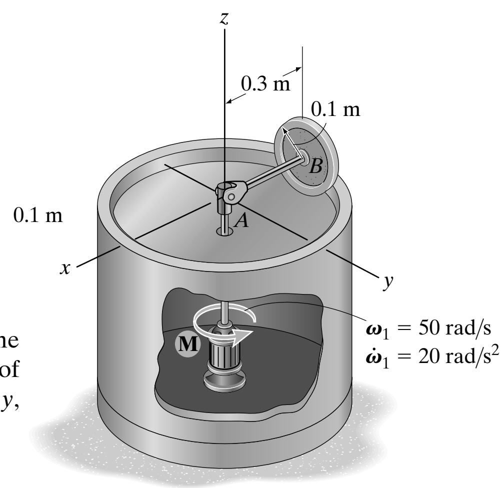
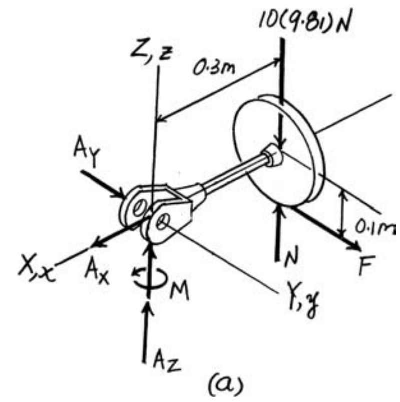
21–55. If shaft AB is driven by the motor with an angular velocity of $\omega_1 = 50$ rad/s and angular acceleration of $\dot{\omega}_1 = 20$ rad/s^2 at the instant shown, and the 10-kg wheel rolls without slipping, determine the frictional force and the normal reaction on the wheel, and the moment **M** that must be supplied by the motor at this instant. Assume that the wheel is a uniform circular disk.



The rotating xyz frame is established to coincide with the fixed XYZ frame at the instant considered, Fig. a. This frame will be set to have an angular velocity of $\Omega = \omega_1 = [50\mathbf{k}]$ rad/s. Since the wheel is symmetric about its spinning axis, the x, y, z axes will remain as the principal axes of inertia. Thus,

$$I_z = I_y = \frac{1}{4}(10)\left(0.1^2\right) + 10\left(0.3^2\right) = 0.925 \text{ kg}\cdot\text{m}^2$$

$$I_x = \frac{1}{2}(10)\left(0.1^2\right) = 0.05 \text{ kg}\cdot\text{m}^2$$



Since the wheel rolls without slipping, the instantaneous axis of zero velocity is shown in Fig. b. Thus

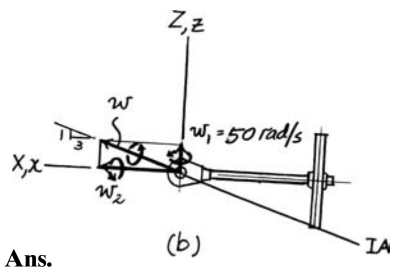
$$\frac{\omega_2}{\omega_1} = \frac{3}{1} \qquad \omega_2 = 3\omega_1 = 3(50) = 150 \text{ rad/s}$$

The angular velocity of the wheel is $\omega = \omega_1 + \omega_2 = [150\mathbf{i} + 50\mathbf{k}]$ rad/s. Then,

$$\omega_x = 150 \text{ rad/s} \qquad \omega_y = 0 \qquad \omega_z = 50 \text{ rad/s}$$

Since the directions of ω_1 and ω_2 do not change with respect to the xyz frame, $\dot{\omega}_{xyz} = (\dot{\omega}_1)_{xyz} + (\dot{\omega}_2)_{xyz}$ where $(\dot{\omega}_2)_{xyz} = 3(\dot{\omega}_1)_{xyz} = 3(20) = 60$ rad/s^2. Thus, $\dot{\omega}_{xyz} = [60\mathbf{i} + 20\mathbf{k}]$ rad/s^2, so that

$$\dot{\omega}_x = 60 \text{ rad/s}^2 \qquad \dot{\omega}_y = 0 \qquad \dot{\omega}_z = 20 \text{ rad/s}^2$$



Applying the equations of motion and referring to the free-body diagram shown in Fig. a,

$\Sigma M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z;$ $\qquad F(0.1) = 0.05(60) - 0 + 0 \qquad F = 30\text{N}$ **Ans.**

$\Sigma M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x;$ $\qquad N(0.3) - 10(9.81)(0.3) = 0 - 0 + 0.05(50)(150)$

$$N = 1348.1 \text{ N} = 1.35 \text{ kN}$$ **Ans.**

$\Sigma M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y;$ $\qquad M - 30(0.3) = 0.925(20) - 0 + 0$

$$M = 27.5 \text{ N}\cdot\text{m}$$ **Ans.**

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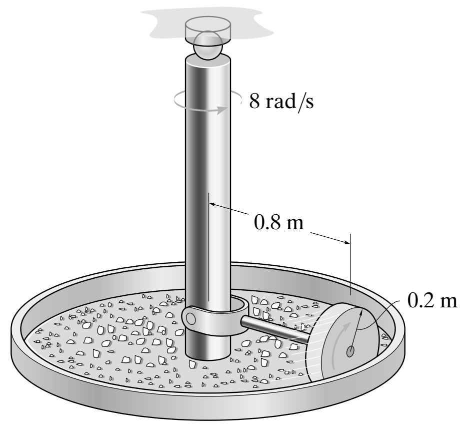
***21–56.** A stone crusher consists of a large thin disk which is pin connected to a horizontal axle. If the axle rotates at a constant rate of 8 rad/s, determine the normal force which the disk exerts on the stones. Assume that the disk rolls without slipping and has a mass of 25 kg. Neglect the mass of the axle.



$$I_x = I_z = \frac{1}{4}(25)(0.2)^2 + 25(0.8)^2 = 16.25 \text{ kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}(25)(0.2)^2 = 0.5 \text{ kg}\cdot\text{m}^2$$

$\omega = -\omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$, where $\omega_z = 8$ rad/s

$$v = 0.8\omega_z = (0.8)(8) = 6.4 \text{ m/s}$$

$$\omega_y = -\frac{6.4}{0.2} = -32 \text{ rad/s}$$

Thus,

$$\omega = -32\mathbf{j} + 8\mathbf{k}$$

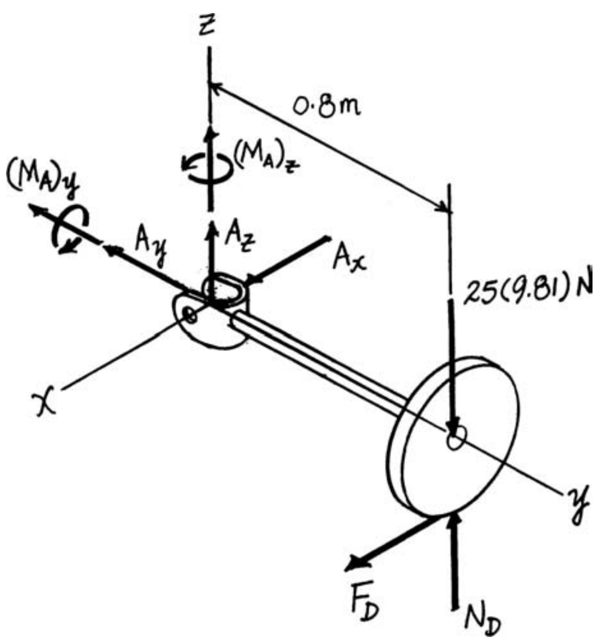
$$\dot{\omega} = \dot{\omega}_{xyz} + \Omega \times \omega = \mathbf{0} + (8\mathbf{k}) \times (-32\mathbf{j} + 8\mathbf{k}) = 256\mathbf{i}$$

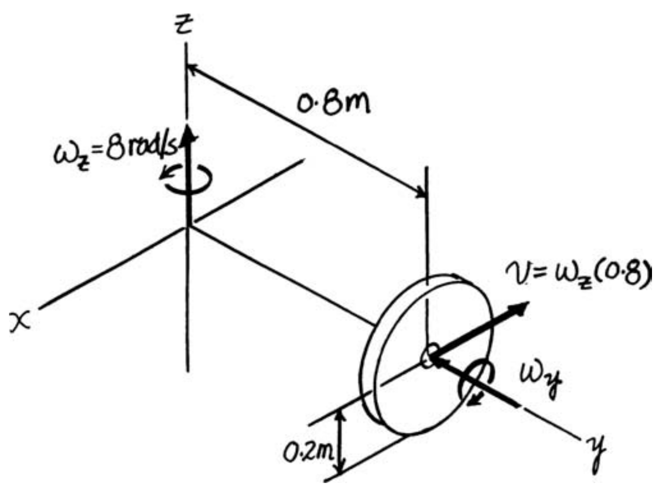
$$\omega_x = 256 \text{ rad/s}^2$$

$$\Sigma M_x = I_x\,\omega_x - (I_y - I_z)\,\omega_y\,\omega_z$$

$$N_D\,(0.8) - 25(9.81)(0.8) = (16.25)(256) - (0.5 - 16.25)(-32)(8)$$

$N_D = 405$ N **Ans.**





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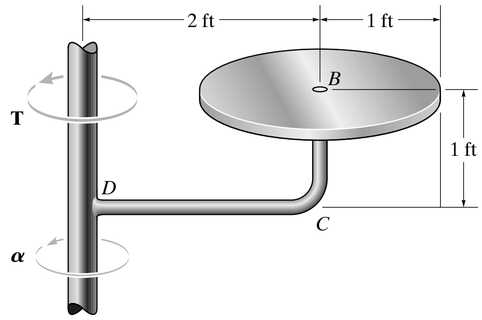
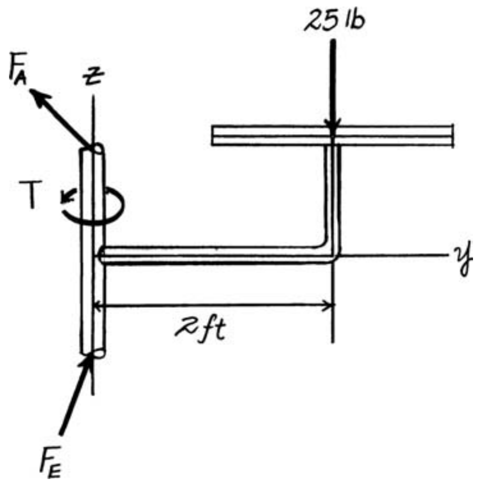
•21–57. The 25-lb disk is *fixed* to rod *BCD*, which has negligible mass. Determine the torque **T** which must be applied to the vertical shaft so that the shaft has an angular acceleration of $\alpha = 6\ \text{rad/s}^2$. The shaft is free to turn in its bearings.



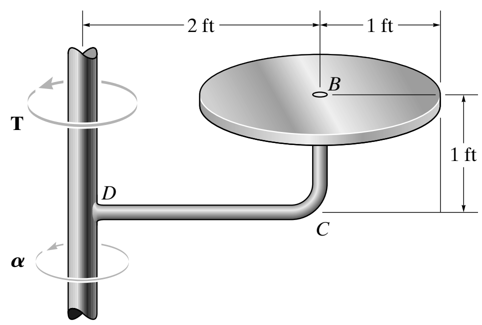
$$I_z = \frac{1}{2}\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(2)^2 = 3.4938\ \text{slug}\cdot\text{ft}^2$$

Applying the third of Eq. 21–25 with $I_x = I_y, \omega_x = \omega_y = 0, \dot{\omega}_z = 6\ \text{rad/s}^2$

$$\Sigma M_z = I_z\,\dot{\omega}_z - (I_x - I_y)\omega_x\,\omega_y\,; \qquad T = 3.4938(6) = 21.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



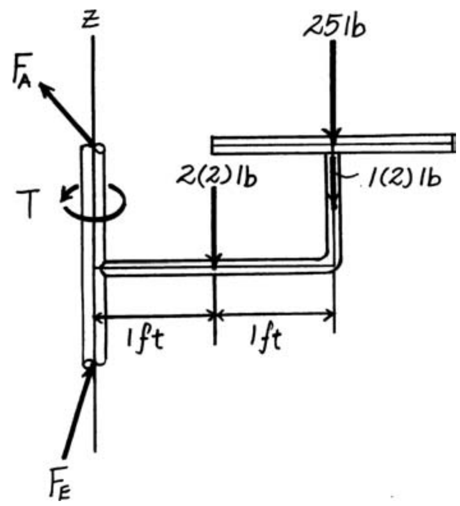
21–58. Solve Prob. 21–57, assuming rod *BCD* has a weight per unit length of 2 lb/ft.



$$I_z = \frac{1}{2}\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(2)^2 + \frac{1}{3}\left(\frac{2(2)}{32.2}\right)(2)^2 + \left(\frac{1(2)}{32.2}\right)(2)^2 = 3.9079\ \text{slug}\cdot\text{ft}^2$$

Applying the third of Eq. 21–25 with $I_x = I_y, \omega_x = \omega_y = 0, \dot{\omega}_z = 6\ \text{rad/s}^2$

$$\Sigma M_z = I_z\,\dot{\omega}_z - (I_x - I_y)\,\omega_x\,\omega_y\,; \qquad T = 3.9079(6) = 23.4\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



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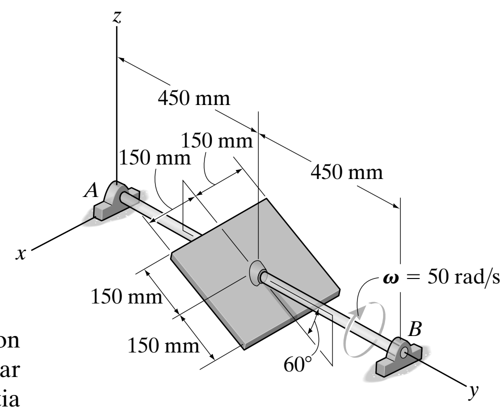
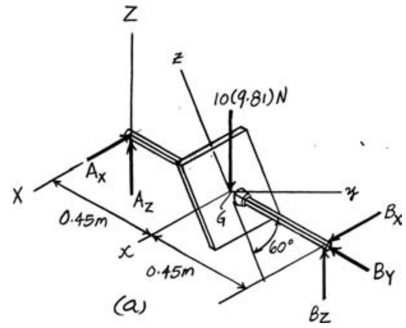
21–59. If shaft AB rotates with a constant angular velocity of $\omega = 50$ rad/s, determine the X, Y, Z components of reaction at journal bearing A and thrust bearing B at the instant shown. The thin plate has a mass of 10 kg. Neglect the mass of shaft AB.



The rotating xyz frame is set with its origin at the plate's mass center as shown on the free-body diagram, Fig. a. This frame will be fixed to the plate so that its angular velocity is $\Omega = \omega$ and the x, y, and z axes will always be the principal axes of inertia of the plate. Referring to Fig. b,

$$\omega = [-50 \sin 60° \, \mathbf{j} + 50 \cos 60° \, \mathbf{k}] \text{ rad/s} = [-43.30\mathbf{j} + 25\mathbf{k}] \text{ rad/s}$$



Thus,

$$\omega_x = 0 \qquad \omega_y = -43.30 \text{ rad/s} \qquad \omega_z = 25 \text{ rad/s}$$

Since ω is always directed towards the $-Y$ axis and has a constant magnitude, $\dot{\omega} = 0$. Also, since $\Omega = \omega, \dot{\omega}_{xyz} = \dot{\omega} = 0$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$

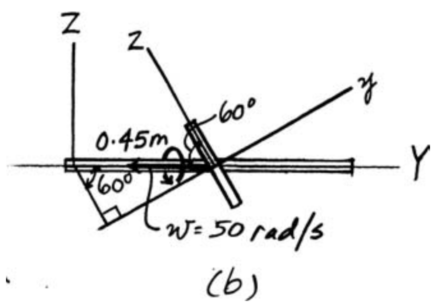
The mass moments of inertia of the plate about the x, y, and z axes are

$$I_x = I_z = \frac{1}{12}(10)\left(0.3^2\right) = 0.075 \text{ kg} \cdot \text{m}^2 \qquad I_y = \frac{1}{12}(10)\left(0.3^2 + 0.3^2\right) = 0.15 \text{ kg} \cdot \text{m}^2$$

Applying the equations of motion,

$$\Sigma M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z; \quad B_Z(0.45) - A_Z(0.45) = 0 - (0.15 - 0.075)(-43.30)(25)$$

$$B_Z - A_Z = 180.42 \qquad \textbf{(1)}$$



$$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \quad -A_X(0.45 \cos 60°) - B_X(0.45 \cos 60°) = 0 - 0$$

$$A_X = -B_X \qquad \textbf{(2)}$$

$$\Sigma M_z = I_z\dot{\omega}_z - \left(I_x - I_y\right)\omega_x\omega_y; \; -A_X(0.45 \sin 60°) - B_X(0.45 \sin 60°) = 0 - 0$$

$$A_X = -B_X$$

$$\Sigma F_X = m(a_G)_X; \qquad B_X - A_X = 0 \qquad \textbf{(3)}$$

$$\Sigma F_Y = m(a_G)_Y; \qquad B_Y = 0 \qquad \textbf{Ans.}$$

$$\Sigma F_Z = m(a_G)_Z; \qquad A_Z + B_Z - 10(9.81) = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$$A_Z = -41.16\text{N} = -41.6 \text{ N} \qquad B_Z = 139.26 \text{ N} = 139 \text{ N} \qquad \textbf{Ans.}$$

$$A_X = B_X = 0 \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{A}_Z$ acts in the opposite sense to that shown on the free-body diagram.

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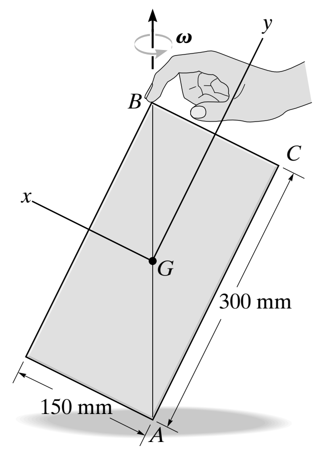
***21–60.** A thin uniform plate having a mass of 0.4 kg spins with a constant angular velocity $\boldsymbol{\omega}$ about its diagonal AB. If the person holding the corner of the plate at B releases his finger, the plate will fall downward on its side AC. Determine the necessary couple moment **M** which if applied to the plate would prevent this from happening.



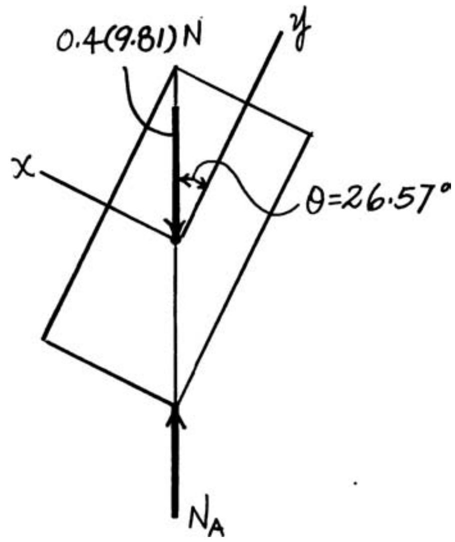
Using the principal axis shown,

$$I_x = \frac{1}{12}(0.4)(0.3)^2 = 3\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{12}(0.4)(0.15)^2 = 0.75\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{12}(0.4)\left[(0.3)^2 + (0.15)^2\right] = 3.75\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$\theta = \tan^{-1}\left(\frac{75}{150}\right) = 26.57°$$



$$\omega_x = \omega \sin 26.57°, \qquad \dot{\omega}_x = 0$$

$$\omega_y = \omega \cos 26.57°, \qquad \omega_y = 0$$

$$\omega_z = 0, \qquad \dot{\omega}_z = 0$$

$$\Sigma M_x = I_x \omega_x - (I_y - I_z)\omega_y \omega_z$$

$$M_x = 0$$

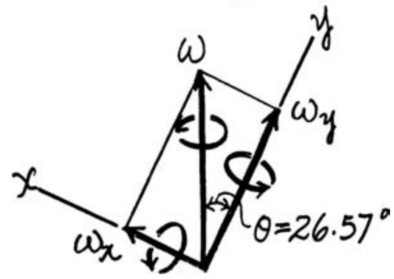
$$\Sigma M_y = I_y \omega_y - (I_z - I_x)\omega_z \omega_z$$

$$M_y = 0$$

$$\Sigma M_z = I_z \omega_z - (I_x - I_y)\omega_x \omega_y$$

$$M_z = 0 - \left[3\left(10^{-3}\right) - 0.75\left(10^{-3}\right)\right]\omega^2 \sin 26.57° \cos 26.57°$$

$$M_z = -0.9\left(10^{-3}\right)\omega^2\ \text{N}\cdot\text{m} = -0.9\omega^2\ \text{mN}\cdot\text{m}$$ **Ans.**



The couple acts outward, perpendicular to the face of the plate.

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21–61. Show that the angular velocity of a body, in terms of Euler angles ϕ, θ, and ψ, can be expressed as $\omega = (\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi)\mathbf{i} + (\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)\mathbf{j} + (\dot{\phi}\cos\theta + \dot{\psi})\mathbf{k}$, where **i**, **j**, and **k** are directed along the x, y, z axes as shown in Fig. 21–15*d*.

From Fig. 21–15*b*. due to rotation ϕ, the x, y, z components of $\dot{\phi}$ are simply $\dot{\phi}$ along z axis.

From Fig 21–15*c*, due to rotation θ, the x, y, z components of $\dot{\phi}$ and $\dot{\theta}$ are $\dot{\phi}\sin\theta$ in the y direction, $\dot{\phi}\cos\theta$ in the z direction, and $\dot{\theta}$ in the x direction.

Lastly, rotation ψ. Fig. 21–15*d*, produces the final components which yields

$$\omega = \left(\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi\right)\mathbf{i} + \left(\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi\right)\mathbf{j} + \left(\dot{\phi}\cos\theta + \dot{\psi}\right)\mathbf{k} \quad \textbf{Q.E.D.}$$

21–62. A thin rod is initially coincident with the Z axis when it is given three rotations defined by the Euler angles $\phi = 30°$, $\theta = 45°$, and $\psi = 60°$. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the X, Y, and Z axes. Are these directions the same for any order of the rotations? Why?

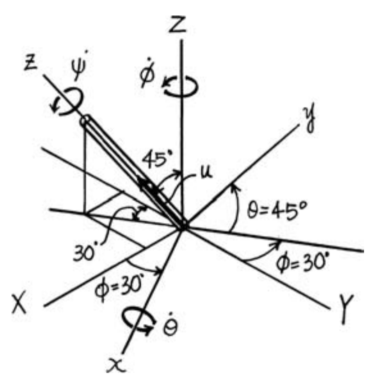
$$\mathbf{u} = (1\sin 45°)\sin 30°\,\mathbf{i} - (1\sin 45°)\cos 30°\mathbf{j} + 1\cos 45°\,\mathbf{k}$$

$$\mathbf{u} = 0.3536\mathbf{i} - 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

$\alpha = \cos^{-1} 0.3536 = 69.3°$ **Ans.**

$\beta = \cos^{-1}(-0.6124) = 128°$ **Ans.**

$\gamma = \cos^{-1}(0.7071) = 45°$ **Ans.**



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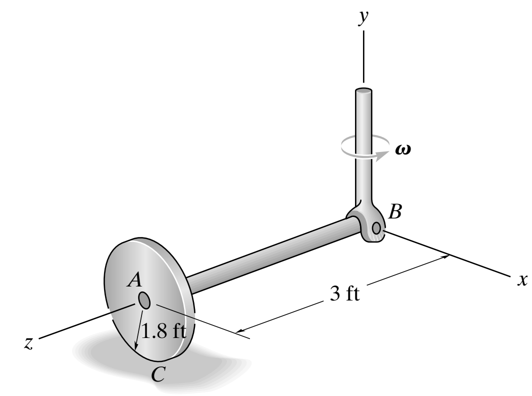
21–63. The 30-lb wheel rolls without slipping. If it has a radius of gyration $k_{AB} = 1.2$ ft about its axle AB, and the vertical drive shaft is turning at 8 rad/s, determine the normal reaction the wheel exerts on the ground at C. Neglect the mass of the axle.

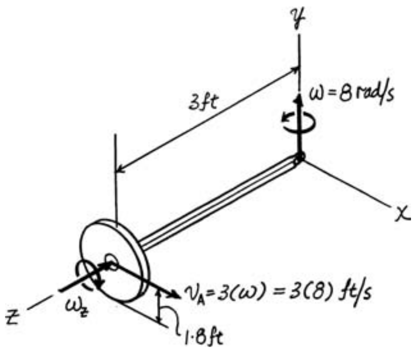


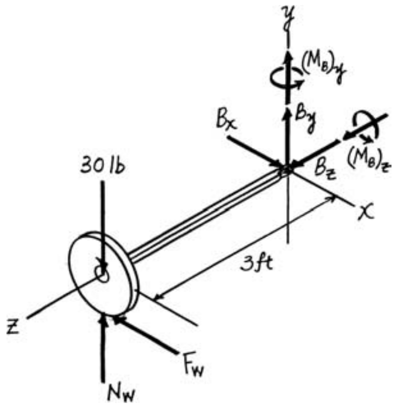
$$\Omega_y = \omega = 8 \text{ rad/s}$$

$$\omega_z = -\frac{3(8)}{1.8} = -13.33 \text{ rad/s}$$

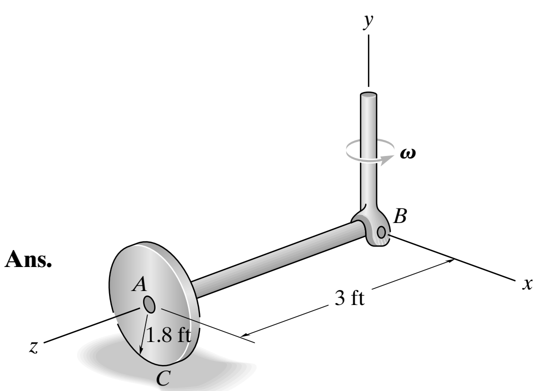
$$\Sigma M_z = I_z \Omega_y \omega_z; \qquad 30(3) - N_w(3) = \left[\left(\frac{30}{32.2}\right)(1.2)^2\right](8)(-13.33)$$

$$N_w = 77.7 \text{ lb}$$ **Ans.**



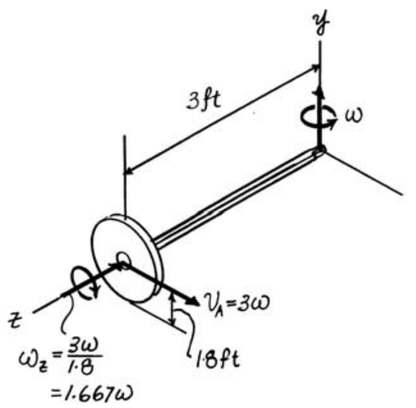


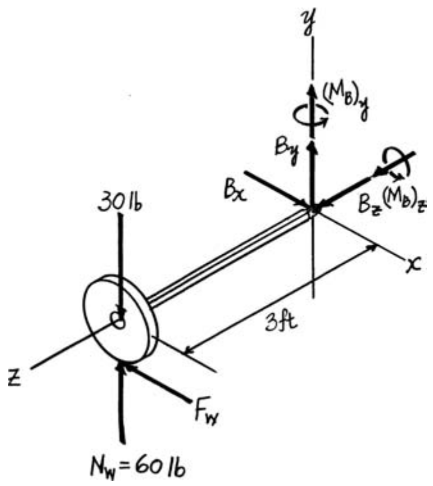
***21–64.** The 30-lb wheel rolls without slipping. If it has a radius of gyration $k_{AB} = 1.2$ ft about its axle AB, determine its angular velocity $\boldsymbol{\omega}$ so that the normal reaction at C becomes 60 lb. Neglect the mass of the axle.



$$\Sigma M_x = I_x \Omega_y \omega_z; \qquad 30(3) - 60(3) = \left[\frac{30}{32.2}(1.2)^2\right]\omega(-1.667\omega)$$

$$\omega = 6.34 \text{ rad/s}$$ **Ans.**





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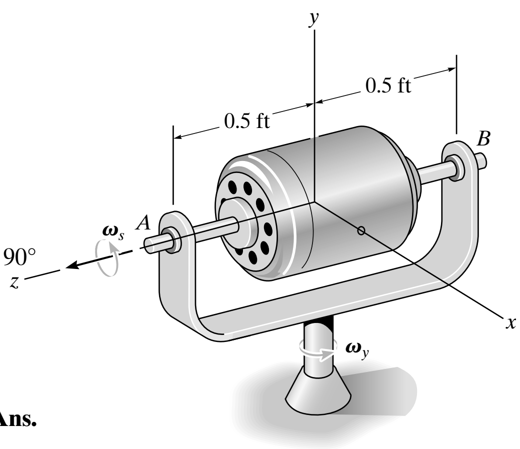
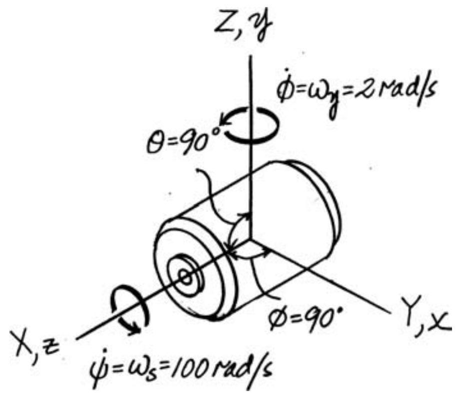
•21–65. The motor weighs 50 lb and has a radius of gyration of 0.2 ft about the z axis. The shaft of the motor is supported by bearings at A and B, and spins at a constant rate of $\boldsymbol{\omega}_s = \{100\mathbf{k}\}$ rad/s, while the frame has an angular velocity of $\boldsymbol{\omega}_y = \{2\mathbf{j}\}$ rad/s. Determine the moment which the bearing forces at A and B exert on the shaft due to this motion.



Applying Eq. 21–30: For the coordinate system shown $\theta = 90°$ $\phi = 90°$ $\dot{\theta} = 0$ $\dot{\phi} = 2$ rad/s $\dot{\psi} = 100$ rad/s.

$\Sigma M_x = -I\dot{\phi}^2 \sin\theta \cos\theta + I_z\,\dot{\phi}\sin\theta(\dot{\phi}\cos\theta + \dot{\psi})$ reduces to

$\Sigma M_x = I_z\dot{\phi}\dot{\psi};\qquad M_x = \left[\left(\dfrac{50}{32.2}\right)(0.2)^2\right](2)(100) = 12.4\ \text{lb}\cdot\text{ft}$ **Ans.**

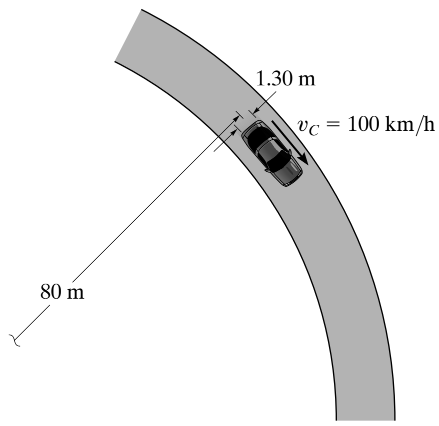


Since $\omega_x = 0$

$\Sigma M_y = 0;\qquad M_y = 0$ **Ans.**

$\Sigma M_z = 0;\qquad M_z = 0$ **Ans.**

21–66. The car travels at a constant speed of $v_C = 100$ km/h around the horizontal curve having a radius of 80 m. If each wheel has a mass of 16 kg, a radius of gyration $k_G = 300$ mm about its spinning axis, and a radius of 400 mm, determine the difference between the normal forces of the rear wheels, caused by the gyroscopic effect. The distance between the wheels is 1.30 m.



$I = 2[16(0.3)^2] = 2.88\ \text{kg}\cdot\text{m}^2$

$\omega_s = \dfrac{100(1000)}{3600(0.4)} = 69.44$ rad/s

$\omega_p = \dfrac{100(1000)}{80(3600)} = 0.347$ rad/s

$M = I\,\omega_s\,\omega_p$

$\Delta F(1.30) = 2.88(69.44)(0.347)$

$\Delta F = 53.4$ N **Ans.**

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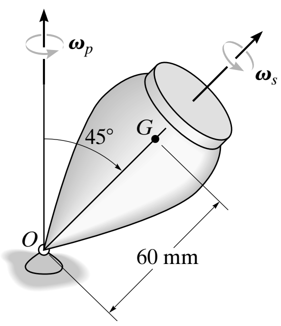
21–67. The top has a mass of 90 g, a center of mass at G, and a radius of gyration $k = 18$ mm about its axis of symmetry. About any transverse axis acting through point O the radius of gyration is $k_t = 35$ mm. If the top is connected to a ball-and-socket joint at O and the precession is $\omega_p = 0.5$ rad/s, determine the spin $\boldsymbol{\omega}_s$.

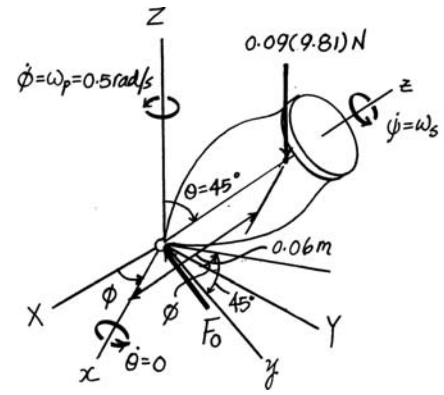


$\omega_p = 0.5$ rad/s

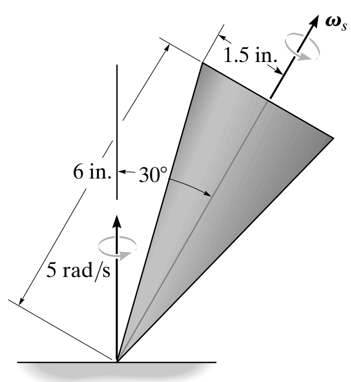
$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

$$0.090(9.81)(0.06)\sin 45^\circ = -0.090(0.035)^2\,(0.5)^2\,(0.7071)^2$$
$$+\ 0.090(0.018)^2(0.5)(0.7071)\left[0.5(0.7071) + \dot{\psi}\right]$$

$\omega_s = \psi = 3.63\left(10^3\right)$ rad/s **Ans.**



***21–68.** The top has a weight of 3 lb and can be considered as a solid cone. If it is observed to precess about the vertical axis at a constant rate of 5 rad/s, determine its spin.



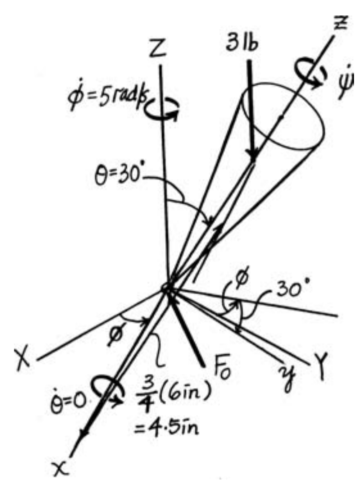
$$I = \frac{3}{80}\left(\frac{3}{32.2}\right)\left[4\left(\frac{1.5}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right] + \frac{3}{32.2}\left(\frac{4.5}{12}\right)^2 = 0.01419\ \text{slug}\cdot\text{ft}^2$$

$$I_z = \frac{3}{10}\left(\frac{3}{32.2}\right)\left(\frac{1.5}{12}\right)^2 = 0.43672\left(10^{-3}\right)\ \text{slug}\cdot\text{ft}^2$$

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\,\dot{\phi}\sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

$$(3)\left(\frac{4.5}{12}\right)(\sin 30^\circ) - (0.01419)(5)^2\sin 30^\circ\cos 30^\circ + 0.43672\left(10^{-3}\right)(5)\sin 30^\circ\left(5\cos 30^\circ + \dot{\psi}\right)$$

$\psi = 652$ rad/s **Ans.**



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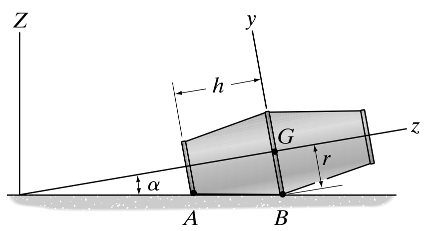
•21–69. The empty aluminum beer keg has a mass of m, center of mass at G, and radii of gyration about the x and y axes of $k_x = k_y = \frac{5}{4}r$, and about the z axis of $k_z = \frac{1}{4}r$, respectively. If the keg rolls without slipping with a constant angular velocity, determine its largest value without having the rim A leave the floor.



Since the beer keg rolls without slipping, the instantaneous axis of zero velocity is indicated in Fig. a. Thus, $\omega_p = \omega_s \sin\alpha$.

Since $\dot{\psi} = \omega_s$, $\dot{\phi} = -\omega_p = -\omega_s \sin\alpha$, and $\theta = 90° - \alpha$ are constant, the beer keg undergoes steady precession.

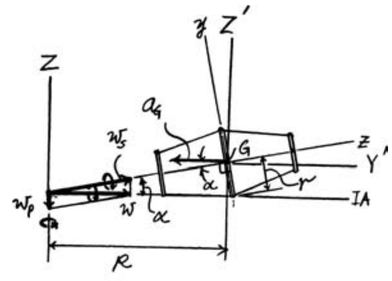
$I = I_x = I_y = m\left(\frac{5}{4}r\right)^2 = \frac{25}{16}mr^2$ and $I_z = m\left(\frac{1}{4}r\right)^2 = \frac{1}{16}mr^2$. Referring to the free-body diagram of the beer keg in Fig. b,

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta(\dot{\phi}\cos\theta + \dot{\psi});$$

$$F_B\cos\alpha(r) - N_B\sin\alpha(r) = -\frac{25}{16}mr^2(-\omega_s\sin\alpha)^2\sin(90° - \alpha)\cos(90° - \alpha)$$

$$+\frac{1}{16}mr^2(-\omega_s\sin\alpha)\sin(90°-\alpha)[(-\omega_s\sin\alpha)\cos(90°-\alpha) + \omega_s]$$

$$F_B\cos\alpha - N_B\sin\alpha = \frac{1}{16}mr\,\omega_s^2\,\sin\alpha\cos\alpha\left(26\sin^2\alpha - 1\right) \qquad \textbf{(1)}$$



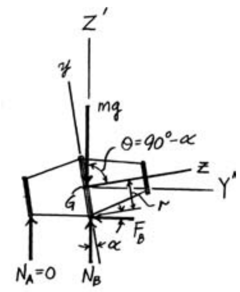
(a)

Since the mass center G of the beer keg rotates about the Z axis, Fig. a, its acceleration can be found from $a_G = \omega_p^2 R = (-\omega_s\sin\alpha)^2\left(\frac{r\cos^2\alpha}{\sin\alpha}\right)$ $= \omega_s^2 r\sin\alpha\cos^2\alpha$ and it is directed towards the negative Y' axis. Fig. a. Since the mass center does not move along the Z', $(a_G)_{Z'} = 0$. Thus,

$$\Sigma F_{Y'} = m(a_G)_{Y'}; \qquad -F_B = -m\left(\omega_s^2 r\,\sin\alpha\cos^2\alpha\right)$$

$$F_B = m\omega_s^2 r\sin\alpha\cos^2\alpha$$

$$\Sigma F_{Z'} = m(a_G)_{Z'}; \qquad N_B - mg = 0 \qquad N_B = mg$$



(b)

Substituting these results into Eq. (1),

$$\omega_S = \sqrt{\frac{16g}{r\cos\alpha\left(16\cos^2\alpha - 26\sin^2\alpha + 1\right)}} \qquad \textbf{Ans.}$$

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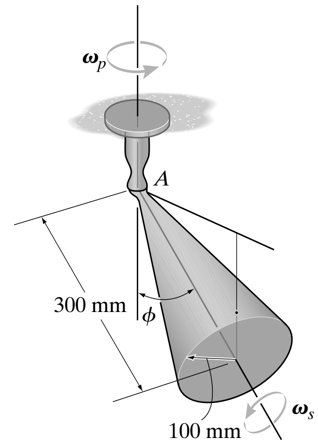
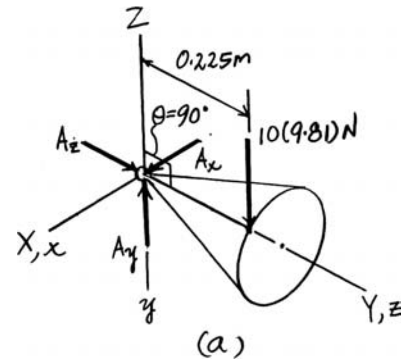
21–70. The 10-kg cone spins at a constant rate of $\omega_s = 150$ rad/s. Determine the constant rate ω_p at which it precesses if $\phi = 90°$.



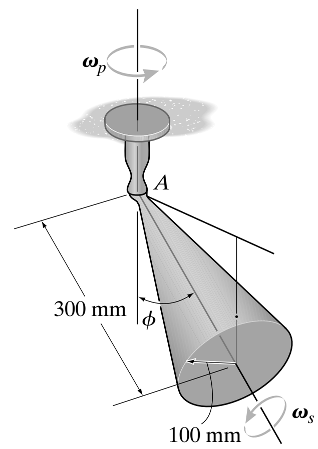
Here, $\theta = 180° - 90° = 90°$, $\dot{\psi} = \omega_s = -150$ rad/s, and $\dot{\phi} = \omega_p$ are constants. Thus, this is a special case of steady precession. $I_z = \frac{3}{10}(10)\left(0.1^2\right) = 0.03\ \text{kg}\cdot\text{m}^2$, $\Omega_y = -\dot{\phi} = -\omega_p$, and $\omega_z = \dot{\psi} = -150$ rad/s. Thus,

$$\Sigma M_x = I_z \Omega_y \omega_z; \qquad -10(9.81)(0.225) = 0.03\left(-\omega_p\right)(-150)$$

$$\omega_p = -4.905 \text{ rad/s} \qquad \textbf{Ans.}$$



21–71. The 10-kg cone is spinning at a constant rate of $\omega_s = 150$ rad/s. Determine the constant rate ω_p at which it precesses if $\phi = 30°$.



Since $\dot{\psi} = \omega_s = -150$ rad/s, $\dot{\phi} = \omega_p$, and $\theta = 180° - 30° = 150°$ are constants, the cone undergoes steady precession. $I_z = \frac{3}{10}(10)\left(0.2^2\right) = 0.03\ \text{kg}\cdot\text{m}^2$ and $I = I_x = I_y = \frac{3}{80}(10)\left[4\left(0.1^2\right) + 0.3^2\right] + 10\left(0.225^2\right) = 0.555\ \text{kg}\cdot\text{m}^2$.

Thus,

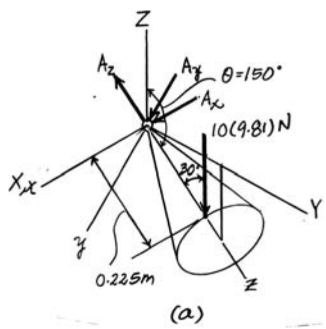
$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

$$-10(9.81)\sin 30°(0.225) = -0.555\omega_p^2 \sin 150° \cos 150° + 0.03\omega_p \sin 150°\left[\omega_p \cos 150° + (-150)\right]$$

$$0.2273\omega_p^2 - 2.25\omega_p + 11.036 = 0$$

Solving,

$$\omega_p = 13.5 \text{ rad/s or } 3.60 \text{ rad/s} \qquad \textbf{Ans.}$$



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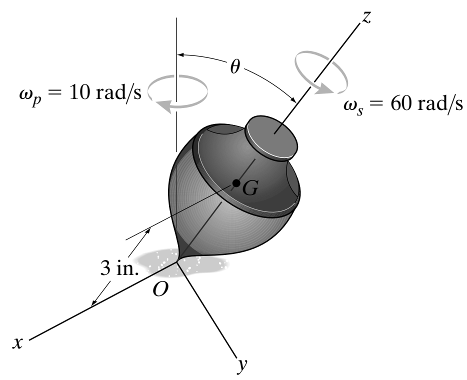
***21–72.** The 1-lb top has a center of gravity at point G. If it spins about its axis of symmetry and precesses about the vertical axis at constant rates of $\omega_s = 60$ rad/s and $\omega_p = 10$ rad/s, respectively, determine the steady state angle θ. The radius of gyration of the top about the z axis is $k_z = 1$ in., and about the x and y axes it is $k_x = k_y = 4$ in.



Since $\dot{\psi} = \omega_s = 60$ rad/s and $\dot{\phi} = \omega_p = -10$ rad/s and θ are constant, the top undergoes steady precession.

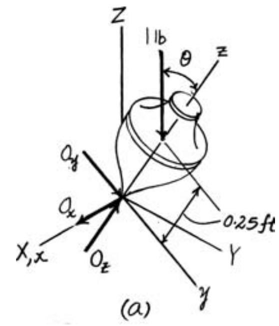
$$I_z = \left(\frac{1}{32.2}\right)\left(\frac{1}{12}\right)^2 = 215.67\left(10^{-6}\right)\text{ slug}\cdot\text{ft}^2 \quad \text{and} \quad I = I_x = I_y = \left(\frac{1}{32.2}\right)\left(\frac{4}{12}\right)^2$$
$$= 3.4507\left(10^{-3}\right)\text{ slug}\cdot\text{ft}^2.$$

Thus,

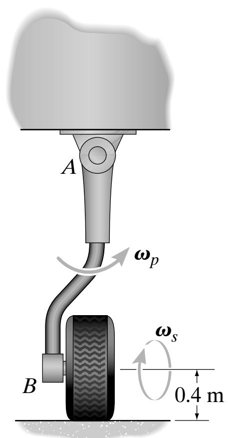
$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos + \dot{\psi}\right)$$

$$-1\sin\theta(0.25) = -3.4507\left(10^{-3}\right)(-10)^2\sin\theta\cos\theta + 215.67\left(10^{-6}\right)(-10)\sin\theta[(-10)\cos\theta + 60]$$

$$\theta = 68.1^\circ \qquad \textbf{Ans.}$$



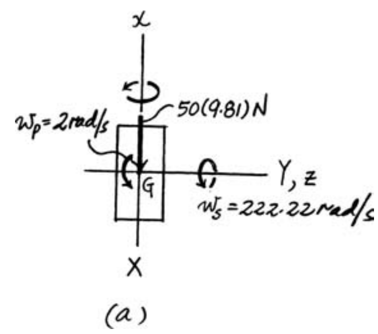
•21–73. At the moment of take off, the landing gear of an airplane is retracted with a constant angular velocity of $\omega_p = 2$ rad/s, while the wheel continues to spin. If the plane takes off with a speed of $v = 320$ km/h, determine the torque at A due to the gyroscopic effect. The wheel has a mass of 50 kg, and the radius of gyration about its spinning axis is $k = 300$ mm.



When the plane travels with a speed of $v = \left(320\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 88.89$ m/s, its wheel spins with a constant angular velocity of $\omega_s = \frac{v}{r} = \frac{88.89}{0.4} = 222.22$ rad/s. Here, $\theta = 90^\circ$, $\Omega_y = \omega_p = 2$ rad/s and $\omega_z = \omega_s = 222.22$ rad/s are constants. This is a special case of steady precession.

$I_z = 50\left(0.3^2\right) = 4.5\text{ kg}\cdot\text{m}^2$. Thus,

$$\Sigma M_x = I_z\Omega_y\omega_z; \qquad M_x = 4.5(2)(222.22) = 2000\text{ N}\cdot\text{m} = 2\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



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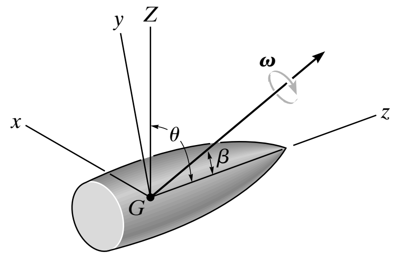
21–74. The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z, respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity $\boldsymbol{\omega}$ and the z axis, show that β and θ are related by the equation $\tan\theta = (I/I_z)\tan\beta$.



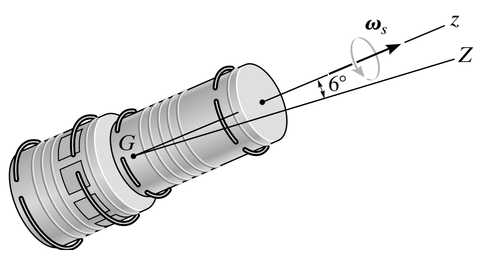
From Eq. 21–34 $\omega_y = \dfrac{H_G \sin\theta}{I}$ and $\omega_z = \dfrac{H_G\cos\theta}{I_z}$ Hence $\dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I}\tan\theta$

However, $\omega_y = \omega\sin\beta$ and $\omega_z = \omega\cos\beta$

$$\frac{\omega_y}{\omega_z} = \tan\beta = \frac{I_z}{I}\tan\theta$$

$$\tan\theta = \frac{I}{I_z}\tan\beta \qquad \textbf{Q.E.D.}$$

21–75. The space capsule has a mass of 3.2 Mg, and about axes passing through the mass center G the axial and transverse radii of gyration are $k_z = 0.90$ m and $k_t = 1.85$ m, respectively. If it spins at $\omega_s = 0.8$ rev/s, determine its angular momentum. Precession occurs about the Z axis.



Gyroscopic Motion: Here, the spinning angular velocity $\psi = \omega_s = 0.8(2\pi) = 1.6\pi$ rad/s. The moment of inertia of the satelite about the z axis is $I_z = 3200\left(0.9^2\right) = 2592\ \text{kg}\cdot\text{m}^2$ and the moment of inertia of the satelite about its transverse axis is $I = 3200\left(1.85^2\right) = 10\,952\ \text{kg}\cdot\text{m}^2$. Applying the third of Eq. 21–36 with $\theta = 6°$, we have

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta$$

$$1.6\pi\left[\frac{10\,952 - 2592}{10\,952(2592)}\right]H_G\cos 6°$$

$$H_G = 17.16\left(10^3\right)\ \text{kg}\cdot\text{m}^2/\text{s} = 17.2\ \text{Mg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

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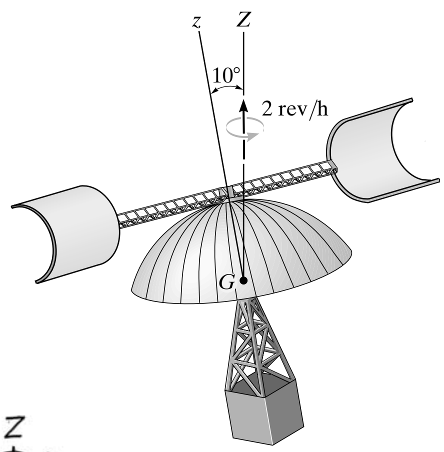
***21–76.** The radius of gyration about an axis passing through the axis of symmetry of the 2.5-Mg satellite is $k_z = 2.3$ m, and about any transverse axis passing through the center of mass G, $k_t = 3.4$ m. If the satellite has a steady-state precession of two revolutions per hour about the Z axis, determine the rate of spin about the z axis.



$$I_z = 2500(2.3)^2 = 13\,225 \text{ kg}\cdot\text{m}^2$$

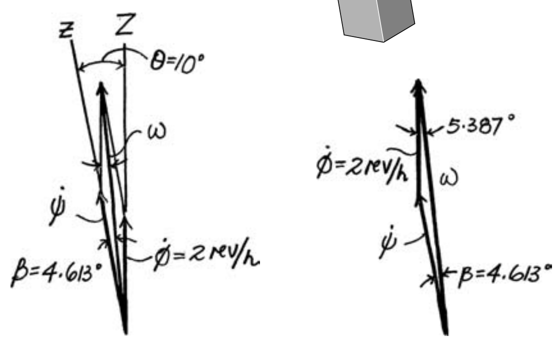
$$I = 2500(3.4)^2 = 28\,900 \text{ kg}\cdot\text{m}^2$$

Use the result of Prob. 21–74.

$$\tan\theta = \left(\frac{I}{I_z}\right)\tan\beta$$

$$\tan 10^\circ = \left(\frac{28\,900}{13\,225}\right)\tan\beta$$

$$\beta = 4.613^\circ$$

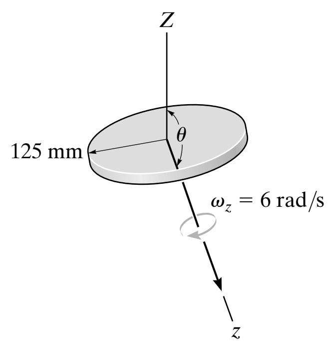


From the law of sines,

$$\frac{\sin 5.387^\circ}{\psi} = \frac{\sin 4.613^\circ}{2}$$

$$\psi = 2.33 \text{ rev/h} \qquad \textbf{Ans.}$$

•21–77. The 4-kg disk is thrown with a spin $\omega_z = 6$ rad/s. If the angle θ is measured as 160°, determine the precession about the Z axis.



$$I = \tfrac{1}{4}(4)(0.125)^2 = 0.015625 \text{ kg}\cdot\text{m}^2 \qquad I_z = \tfrac{1}{2}(4)(0.125)^2 = 0.03125 \text{ kg}\cdot\text{m}^2$$

Applying Eq. 21–36 with $\theta = 160^\circ$ and $\dot{\psi} = 6$ rad/s

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_O \cos\theta$$

$$6 = \frac{0.015625 - 0.03125}{0.015625(0.03125)} H_O \cos 160^\circ$$

$$H_G = 0.1995 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$\phi = \frac{H_G}{I} = \frac{0.1995}{0.015625} = 12.8 \text{ rad/s} \qquad \textbf{Ans.}$$

Note that this is a case of retrograde precession since $I_z > I$.

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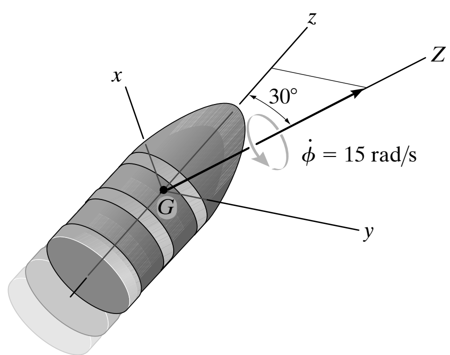
21–78. The projectile precesses about the Z axis at a constant rate of $\dot{\phi} = 15$ rad/s when it leaves the barrel of a gun. Determine its spin $\dot{\psi}$ and the magnitude of its angular momentum $\mathbf{H}_G$. The projectile has a mass of 1.5 kg and radii of gyration about its axis of symmetry (z axis) and about its transverse axes (x and y axes) of $k_z = 65$ mm and $k_x = k_y = 125$ mm, respectively.



Since the only force that acts on the projectile is its own weight, the projectile undergoes torque-free motion. $I_z = 1.5(0.065^2) = 6.3375(10^{-3})\ \text{kg}\cdot\text{m}^2$, $I = I_x = I_y = 1.5(0.125^2) = 0.0234375\ \text{kg}\cdot\text{m}^2$, and $\theta = 30°$. Thus,

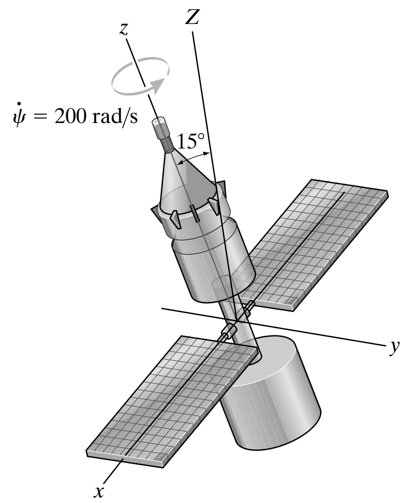
$$\dot{\phi} = \frac{H_G}{I}; \quad H_G = I\dot{\phi} = 0.0234375(15) = 0.352\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

$$\dot{\psi} = \frac{I - I_z}{I_z}\dot{\phi}\cos\theta$$

$$= \frac{0.0234375 - 6.3375(10^{-3})}{6.3375(10^{-3})}(15)\cos 30°$$

$$= 35.1\ \text{rad/s} \qquad \textbf{Ans.}$$

21–79. The satellite has a mass of 100 kg and radii of gyration about its axis of symmetry (z axis) and its transverse axes (x or y axis) of $k_z = 300$ mm and $k_x = k_y = 900$ mm, respectively. If the satellite spins about the z axis at a constant rate of $\dot{\psi} = 200$ rad/s, and precesses about the Z axis, determine the precession $\dot{\phi}$ and the magnitude of its angular momentum $\mathbf{H}_G$.



Since the weight is the only force acting on the satellite, it undergoes torque-free motion.

Here, $I_z = 100(0.3^2) = 9\ \text{kg}\cdot\text{m}^2$, $I = I_x = I_y = 100(0.9^2) = 81\ \text{kg}\cdot\text{m}^2$, and $\theta = 15°$. Then,

$$\dot{\psi} = \frac{I - I_z}{I_z}\dot{\phi}\cos\theta$$

$$200 = \left(\frac{81 - 9}{9}\right)\dot{\phi}\cos 15°$$

$$\dot{\phi} = 25.88\ \text{rad/s}$$

Using this result,

$$\dot{\phi} = \frac{H_G}{I}$$

$$25.88 = \frac{H_G}{81}$$

$$H_G = 2096\ \text{kg}\cdot\text{m}^2/\text{s} = 2.10\ \text{Mg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

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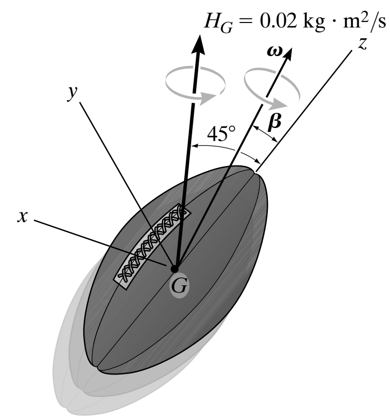
***21–80.** The football has a mass of 450 g and radii of gyration about its axis of symmetry (z axis) and its transverse axes (x or y axis) of $k_z = 30$ mm and $k_x = k_y = 50$ mm, respectively. If the football has an angular momentum of $H_G = 0.02\ \text{kg}\cdot\text{m}^2/\text{s}$, determine its precession $\dot{\phi}$ and spin $\dot{\psi}$. Also, find the angle β that the angular velocity vector makes with the z axis.



Since the weight is the only force acting on the football, it undergoes torque-free motion. $I_z = 0.45\left(0.03^2\right) = 0.405\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$, $I = I_x = I_y = 0.45\left(0.05^2\right) = 1.125\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$, and $\theta = 45°$.

Thus,

$$\dot{\phi} = \frac{H_G}{I} = \frac{0.02}{1.125\left(10^{-3}\right)} = 17.78\ \text{rad/s} = 17.8\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta = \frac{1.125\left(10^{-3}\right) - 0.405\left(10^{-3}\right)}{1.125\left(10^{-3}\right)(0.405)\left(10^{-3}\right)}(0.02)\cos 45°$$

$$= 22.35\ \text{rad/s} = 22.3\ \text{rad/s} \qquad \textbf{Ans.}$$

Also,

$$\omega_y = \frac{H_G \sin\theta}{I} = \frac{0.02 \sin 45°}{1.125\left(10^{-3}\right)} = 12.57\ \text{rad/s}$$

$$\omega_z = \frac{H_G \cos\theta}{I_z} = \frac{0.02 \cos 45°}{0.405\left(10^{-3}\right)} = 34.92\ \text{rad/s}$$

Thus,

$$\beta = \tan^{-1}\left(\frac{\omega_y}{\omega_z}\right) = \tan^{-1}\left(\frac{12.57}{34.92}\right) = 19.8° \qquad \textbf{Ans.}$$

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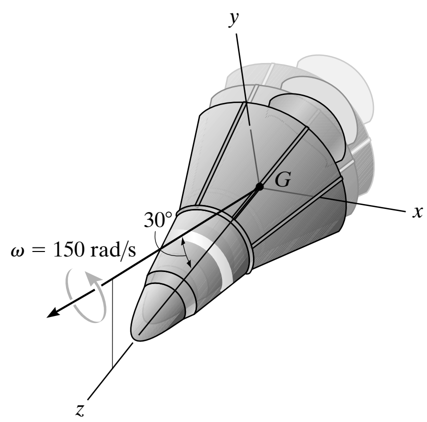
•21–81. The space capsule has a mass of 2 Mg, center of mass at G, and radii of gyration about its axis of symmetry (z axis) and its transverse axes (x or y axis) of $k_z = 2.75$ m and $k_x = k_y = 5.5$ m, respectively. If the capsule has the angular velocity shown, determine its precession $\dot{\phi}$ and spin $\dot{\psi}$. Indicate whether the precession is regular or retrograde. Also, draw the space cone and body cone for the motion.



The only force acting on the space capsule is its own weight. Thus, it undergoes torque-free motion. $I_z = 2000(2.75^2) = 15\,125\ \text{kg}\cdot\text{m}^2$, $I = I_x = I_y = 2000(5.5^2) = 60\,500\ \text{kg}\cdot\text{m}^2$. Thus,

$$\omega_y = \frac{H_G \sin\theta}{I}$$

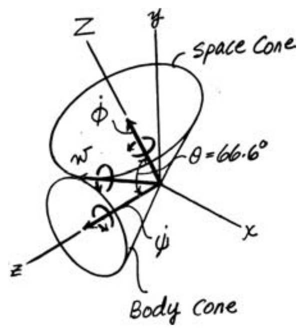
$$150 \sin 30^\circ = \frac{H_G \sin\theta}{60\,500}$$

$$H_G \sin\theta = 4\,537\,500 \qquad \textbf{(1)}$$

$$\omega_z = \frac{H_G \cos\theta}{I_z}$$

$$150 \cos 30^\circ = \frac{H_G \cos\theta}{15\,125}$$

$$H_G \cos\theta = 1\,964\,795.13 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2),

$$H_G = 4.9446(10^6)\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \theta = 66.59^\circ$$

Using these results,

$$\dot{\phi} = \frac{H_G}{I} = \frac{H_G}{60\,500} = \frac{4.9446(10^6)}{60\,500} = 81.7\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta = \left[\frac{60\,500 - 15\,125}{60\,500(15125)}\right] 4.9446(10^6) \cos 30^\circ$$

$$= 212\ \text{rad/s} \qquad \textbf{Ans.}$$

Since $I > I_z$, the motion is *regular precession*. **Ans.**

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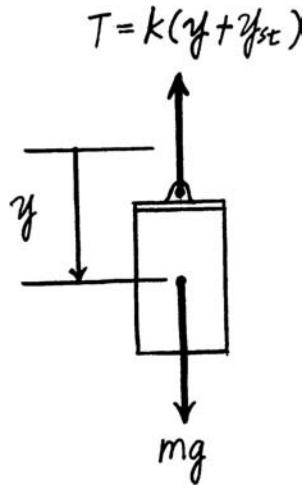
•22–1. A spring is stretched 175 mm by an 8-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 1.50 m/s, determine the differential equation which describes the motion. Assume that positive displacement is downward. Also, determine the position of the block when $t = 0.22$ s.



$$+\downarrow \Sigma F_y = ma_y; \qquad mg - k(y + y_{st}) = m\ddot{y} \qquad \text{where } ky_{st} = mg$$

$$\ddot{y} + \frac{k}{m} y = 0$$

Hence $p = \sqrt{\dfrac{k}{m}}$ Where $k = \dfrac{8(9.81)}{0.175} = 448.46$ N/m

$$= \sqrt{\frac{448.46}{8}} = 7.487$$

$$\therefore \qquad \ddot{y} + (7.487)^2 y = 0 \qquad \ddot{y} + 56.1y = 0$$ **Ans.**

The solution of the above differential equation is of the form:

$$y = A \sin pt + B \cos pt \qquad \textbf{(1)}$$

$$v = \dot{y} = Ap \cos pt - Bp \sin pt \qquad \textbf{(2)}$$

At $t = 0$, $y = 0.1$ m and $v = v_0 = 1.50$ m/s

From Eq. (1) $0.1 = A \sin 0 + B \cos 0 \qquad B = 0.1$ m

From Eq. (2) $v_0 = Ap \cos 0 - 0 \qquad A - \dfrac{v_0}{p} = \dfrac{1.50}{7.487} = 0.2003$ m

Hence $\qquad y = 0.2003 \sin 7.487t + 0.1 \cos 7.487t$

At $t = 0.22$ s, $\quad y = 0.2003 \sin \left[7.487(0.22)\right] + 0.1 \cos \left[7.487(0.22)\right]$

$$= 0.192 \text{ m}$$ **Ans.**

22–2. When a 2-kg block is suspended from a spring, the spring is stretched a distance of 40 mm. Determine the frequency and the period of vibration for a 0.5-kg block attached to the same spring.

$$k = \frac{F}{y} = \frac{2(9.81)}{0.040} = 490.5 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.5}} = 31.321$$

$$f = \frac{p}{2\pi} = \frac{31.321}{2\pi} = 4.985 \text{ Hz}$$ **Ans.**

$$\tau = \frac{1}{f} = \frac{1}{4.985} = 0.201 \text{ s}$$ **Ans.**

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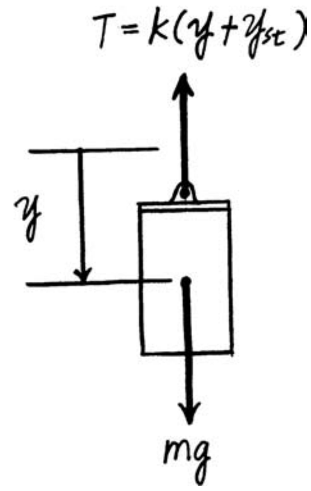
22–3. A block having a weight of 8 lb is suspended from a spring having a stiffness $k = 40$ lb/ft. If the block is pushed $y = 0.2$ ft upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What are the amplitude and the natural frequency of the vibration? Assume that positive displacement is downward.

$+\downarrow \Sigma F_y = ma_y; \qquad mg - k(y + y_{st}) = m\ddot{y} \qquad \text{where } ky_{st} = mg$



$$\ddot{y} + \frac{k}{m}y = 0$$

Hence $\qquad p = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{8/32.2}} = 12.689$

$$f = \frac{p}{2\pi} = \frac{12.689}{2\pi} = 2.02 \text{ Hz} \qquad \textbf{Ans.}$$

The solution of the above differential equation is of the form:

$$v = A \sin pt + B \cos pt \qquad \textbf{(1)}$$

$$v = \dot{y} = Ap \cos pt - Bp \sin pt \qquad \textbf{(2)}$$

At $t = 0$, $y = -0.2$ ft and $v = v_0 = 0$

From Eq. (1) $-0.2 = A \sin 0° + B \cos 0° \qquad B = -0.2$ ft

From Eq. (2) $\quad v_0 = Ap \cos 0° - 0 \qquad A = \frac{v_0}{p} = \frac{0}{12.689} = 0$

Hence $\qquad y = -0.2 \cos 12.7t$ **Ans.**

Amplitude $\qquad C = 0.2$ ft **Ans.**

***22–4.** A spring has a stiffness of 800 N/m. If a 2-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation that describes the block's motion. Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{2}} = 20$$

$$y = A \sin pt + B \cos pt$$

$$y = -0.05 \text{ m when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = 0 \text{ when } t = 0,$$

$$0 = A(20) - 0; \qquad A = 0$$

Thus,

$$y = -0.05 \cos (20t) \qquad \textbf{Ans.}$$

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•22–5. A 2-kg block is suspended from a spring having a stiffness of 800 N/m. If the block is given an upward velocity of 2 m/s when it is displaced downward a distance of 150 mm from its equilibrium position, determine the equation which describes the motion. What is the amplitude of the motion? Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{2}} = 20$$

$$x = A \sin pt + B \cos pt$$

$$x = 0.150 \text{ m when } t = 0,$$

$$0.150 = 0 + B; \qquad B = 0.150$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = -2 \text{ m/s when } t = 0,$$

$$-2 = A(20) - 0; \qquad A = -0.1$$

Thus,

$$x = 0.1 \sin (20t) + 0.150 \cos (20t) \qquad \textbf{Ans.}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(0.1)^2 + (0.150)^2} = 0.180 \text{ m} \qquad \textbf{Ans.}$$

22–6. A spring is stretched 200 mm by a 15-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 0.75 m/s, determine the equation which describes the motion. What is the phase angle? Assume that positive displacement is downward.

$$k = \frac{F}{y} = \frac{15(9.81)}{0.2} = 735.75 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{735.75}{15}} = 7.00$$

$$y = A \sin pt + B \cos pt$$

$$y = 0.1 \text{ m when } t = 0,$$

$$0.1 = 0 + B; \qquad B = 0.1$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = 0.75 \text{ m/s when } t = 0,$$

$$0.75 = A(7.00)$$

$$A = 0.107$$

$$y = 0.107 \sin (7.00t) + 0.100 \cos (7.00t) \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{0.100}{0.107}\right) = 43.0^\circ \qquad \textbf{Ans.}$$

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22–7. A 6-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 75 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{6}} = 5.774$$

$$x = A \sin pt + B \cos pt$$

$$x = 0.075 \text{ m when } t = 0.$$

$$-0.075 = 0 + B; \qquad B = -0.075$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = -0.4 \text{ m/s when } t = 0,$$

$$-0.4 = A(5.774) - 0; \qquad A = -0.0693$$

Thus,

$$x = -0.0693 \sin (5.77t) - 0.075 \cos (5.77t) \qquad \textbf{Ans.}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(-0.0693)^2 + (-0.075)^2} = 0.102 \text{ m} \qquad \textbf{Ans.}$$

***22–8.** A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the frequency of the vibration? Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{3}} = 8.165$$

$$f = \frac{p}{2\pi} = \frac{8.165}{2\pi} = 1.299 = 1.30 \text{ Hz} \qquad \textbf{Ans.}$$

$$x = A \sin pt + B \cos pt$$

$$x = -0.05 \text{ m when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = 0 \text{ when } t = 0,$$

$$0 = A(8.165) - 0; \qquad A = 0$$

Hence,

$$x = -0.05 \cos (8.16t) \qquad \textbf{Ans.}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(0)^2 + (-0.05)} = 0.05 \text{ m} = 50 \text{ mm} \qquad \textbf{Ans.}$$

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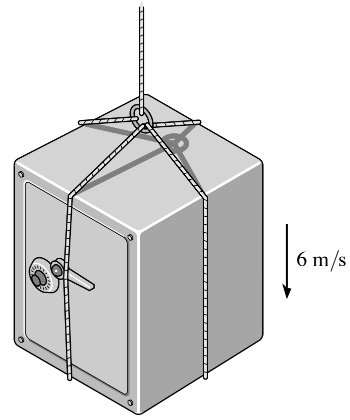
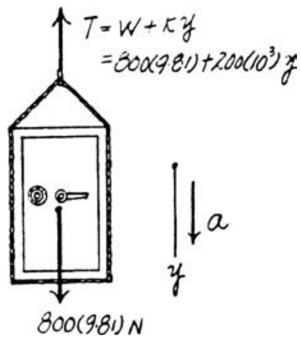
•22–9. A cable is used to suspend the 800-kg safe. If the safe is being lowered at 6 m/s when the motor controlling the cable suddenly jams (stops), determine the maximum tension in the cable and the frequency of vibration of the safe. Neglect the mass of the cable and assume it is elastic such that it stretches 20 mm when subjected to a tension of 4 kN.



Free-body Diagram: Here the stiffness of the cable is $k = \dfrac{4000}{0.02} = 200\left(10^3\right)$ N/m. When the safe is being displaced by an amount y downward vertically from its equilibrium position, the *restoring force* that developed in the cable $T = W + ky = 800(9.81) + 200\left(10^3\right) y$.



Equation of Motion:

$$+\uparrow \Sigma F_x = 0; \qquad 800(9.81) + 200\left(10^3\right) y - 800(9.81) = -800a \qquad \textbf{[1]}$$

Kinematics: Since $a = \dfrac{d^2y}{dt^2} = \ddot{y}$, then substituting this value into Eq. [1], we have

$$200\left(10^3\right) y = -800\ddot{y}$$

$$\ddot{y} + 250x = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = 250$, thus, $p = 15.81$ rad/s. Applying Eq. 22–14, we have

$$f = \frac{p}{2\pi} = \frac{15.81}{2\pi} = 2.52 \text{ Hz} \qquad \textbf{Ans.}$$

The solution of the above differential equation (Eq. [2]) is in the form of

$$y = C \sin (15.81t + \phi) \qquad \textbf{[3]}$$

Taking the time derivative of Eq. [3], we have

$$\dot{y} = 15.81\, C \cos (15.81t + \phi) \qquad \textbf{[4]}$$

Applying the initial condition of $y = 0$ and $\dot{y} = 6$ m/s at $t = 0$ to Eqs. [3] and [4] yields

$$0 = C \sin \phi \qquad \textbf{[5]}$$

$$6 = 15.81\, C \cos \phi \qquad \textbf{[6]}$$

Solving Eqs. [5] and [6] yields

$$\phi = 0^\circ \qquad C = 0.3795 \text{ m}$$

Since $v_{\max} = C = 0.3795$ m, the maximum cable tension is given by

$$T_{\max} = W + ky_{\max} = 800(9.81) + 200\left(10^3\right)(0.3795) = 83.7 \text{ kN} \qquad \textbf{Ans.}$$

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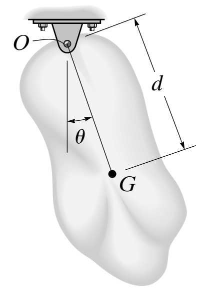
22–10. The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.



$$\circlearrowleft +\Sigma M_O = I_O\,\alpha; \qquad -mgd\sin\theta = \left[mk_G^2 + md^2\right]\ddot{\theta}$$

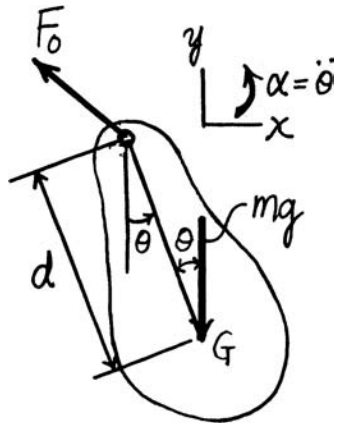
$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\sin\theta = 0$$

However, for small rotation $\sin\theta \approx \theta$. Hence

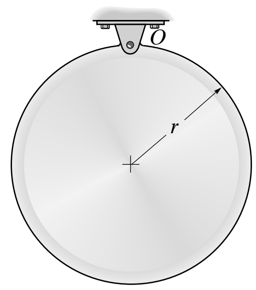
$$\ddot{\theta} + \frac{gd}{k^2 + d^2}\theta = 0$$

From the above differential equation, $p = \sqrt{\dfrac{gd}{k_G^2 + d^2}}$.

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\dfrac{gd}{k_G^2 + d^2}}} = 2\pi\sqrt{\frac{k_G^2 + d^2}{gd}}$$ **Ans.**



22–11. The circular disk has a mass m and is pinned at O. Determine the natural period of vibration if it is displaced a small amount and released.

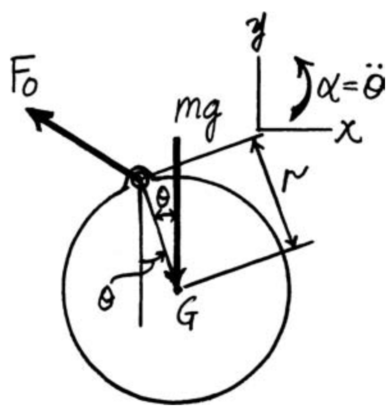


$$\circlearrowleft +\Sigma M_O = I_O\,\alpha; \qquad -mgr\theta = \left(\frac{3}{2}mr^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \left(\frac{2g}{3r}\right)\theta = 0$$

$$p = \sqrt{\frac{2g}{3r}}$$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{3r}{2g}}$$ **Ans.**



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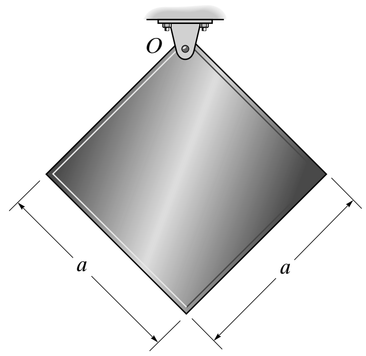
***22–12.** The square plate has a mass m and is suspended at its corner from a pin O. Determine the natural period of vibration if it is displaced a small amount and released.



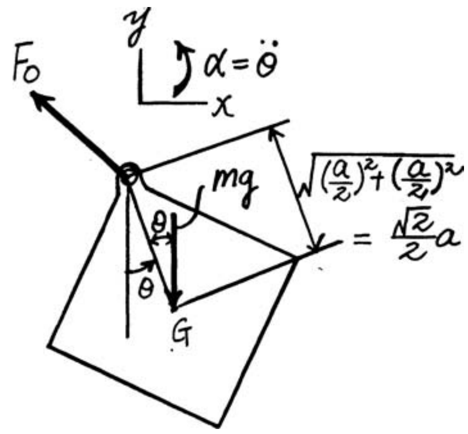
$$I_O = \frac{1}{12}m(a^2 + a^2) + m\left(\frac{\sqrt{2}}{2}a\right)^2 = \frac{1}{6}ma^2 + \frac{1}{2}ma^2 = \frac{2}{3}ma^2$$

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad -mg\left(\frac{\sqrt{2}}{2}a\right)\theta = \left(\frac{2}{3}ma^2\right)\ddot{\theta}$$

$$\dot{\theta} + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0$$

$$p = \sqrt{\frac{3\sqrt{2}g}{4a}}$$

$$\tau = \frac{2\pi}{p} = 6.10\sqrt{\frac{a}{g}} \qquad \textbf{Ans.}$$



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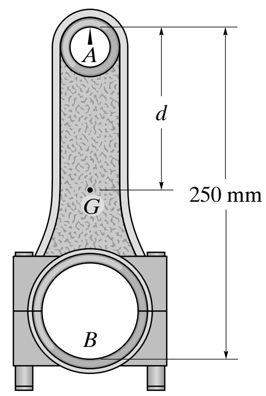
•22–13. The connecting rod is supported by a knife edge at A and the period of vibration is measured as $\tau_A = 3.38$ s. It is then removed and rotated 180° so that it is supported by the knife edge at B. In this case the perod of vibration is measured as $\tau_B = 3.96$ s. Determine the location d of the center of gravity G, and compute the radius of gyration k_G.



Free-body Diagram: When an object of arbitary shape having a mass m is pinned at O and is displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum monent about point O to eliminate O_x and O_y.

$$\zeta+\Sigma M_O = I_O\alpha; \qquad -mg\sin\theta(l) = I_O\,\alpha \qquad \textbf{[1]}$$

Kinematics: Since $\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgl\theta = I_O\,\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = \frac{mgl}{I_O}$, thus, $p = \sqrt{\frac{mgl}{I_O}}$. Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad \textbf{[3]}$$

When the rod is rotating about B, $\tau = \tau_A = 3.38$ s and $l = d$. Substitute these values into Eq. [3], we have

$$3.38 = 2\pi\sqrt{\frac{I_A}{mgd}} \qquad I_A = 0.2894mgd$$

When the rod is rotating about B, $\tau = \tau_B = 3.96$ s and $l = 0.25 - d$. Substitute these values into Eq. [3], we have

$$3.96 = 2\pi\sqrt{\frac{I_B}{mg\,(0.25 - d)}} \qquad I_B = 0.3972mg\,(0.25 - d)$$

However, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - mg^2 = I_B - m(0.25 - d)^2$$

Then,

$$0.2894mgd - md^2 = 0.3972mg\,(0.25 - d) - m\,(0.25 - d)^2$$

$$d = 0.1462\text{ m} = 146\text{ mm} \qquad \textbf{Ans.}$$

Thus, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - md^2 = 0.2894m\,(9.81)(0.1462) - m\left(0.1462^2\right) = 0.3937\,m$$

The radius of gyration is

$$k_G = \sqrt{\frac{I_G}{m}} = \sqrt{\frac{0.3937m}{m}} = 0.627\text{ m} \qquad \textbf{Ans.}$$

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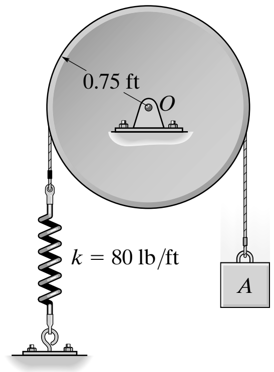
22–14. The disk, having a weight of 15 lb, is pinned at its center O and supports the block A that has a weight of 3 lb. If the belt which passes over the disk does not slip at its contacting surface, determine the natural period of vibration of the system.



For equilibrium:

$T_{st} = 3$ lb

$\zeta \Sigma M_O = I_O \alpha + ma(0.75)$

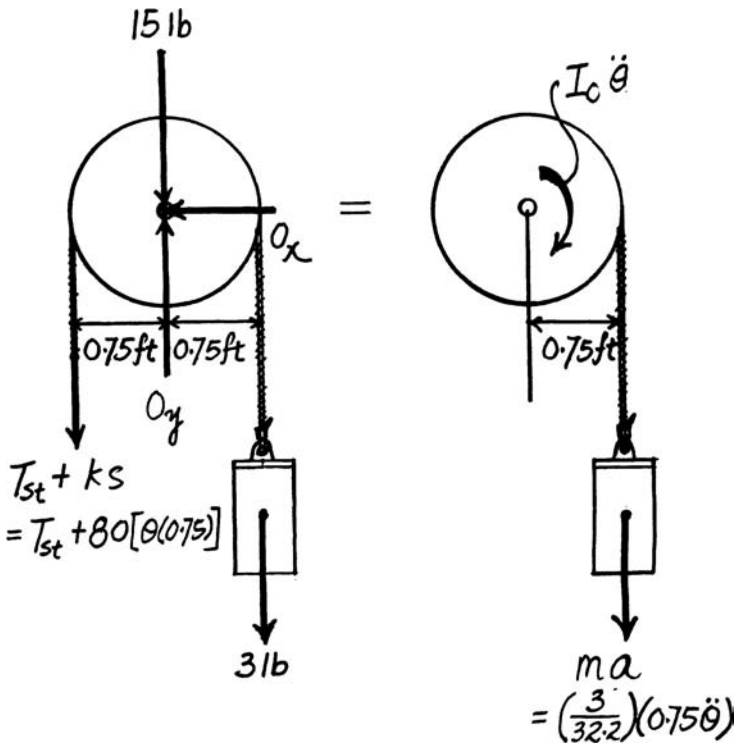
$a = 0.75\alpha$

$$-T_{st}(0.75) - (80)(\theta)(0.75)(0.75) + (3)(0.75) = \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\ddot{\theta} + \left(\frac{3}{32.2}\right)(0.75)\ddot{\theta}(0.75)$$

$$-2.25 - 45\theta + 2.25 = 0.131\ddot{\theta} + 0.05241\ddot{\theta}$$

$$\ddot{\theta} + 245.3\theta = 0$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{245.3}} = 0.401 \text{ s}$$ **Ans.**



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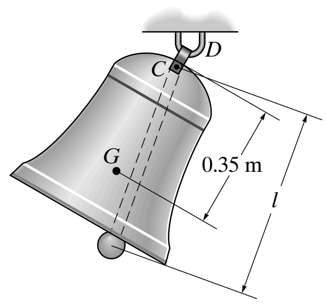
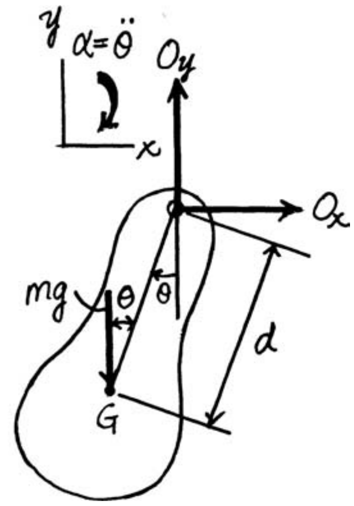
22–15. The bell has a mass of 375 kg, a center of mass at G, and a radius of gyration about point D of $k_D = 0.4$ m. The tongue consists of a slender rod attached to the inside of the bell at C. If an 8-kg mass is attached to the end of the rod, determine the length l of the rod so that the bell will "ring silent," i.e., so that the natural period of vibration of the tongue is the same as that of the bell. For the calculation, neglect the small distance between C and D and neglect the mass of the rod.



For an arbitrarily shaped body which rotates about a fixed point.



$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad mgd \sin\theta = -I_O\,\ddot{\theta}$

$$\ddot{\theta} + \frac{mgd}{I_O}\sin\theta = 0$$

However, for small rotation $\sin\theta \approx \theta$. Hence

$$\ddot{\theta} + \frac{mgd}{I_O}\theta = 0$$

From the above differential equation, $p = \sqrt{\dfrac{mgd}{I_O}}$.

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\dfrac{mgd}{I_O}}} = 2\pi\sqrt{\frac{I_O}{mgd}}$$

In order to have an equal period

$$\tau = 2\pi\sqrt{\frac{(I_O)_T}{m_T g d_T}} = 2\pi\sqrt{\frac{(I_O)_B}{m_B\, g d_B}}$$

$(I_O)_T =$ moment of inertia of tongue about O.

$(I_O)_B =$ moment of inertia of bell about O.

$$\frac{(I_O)_T}{m_T g d_T} = \frac{(I_O)_B}{m_B\, g d_B}$$

$$\frac{8(l^2)}{8gl} = \frac{375(0.4)^2}{375g(0.35)}$$

$$l = 0.457 \text{ m}$$ **Ans.**

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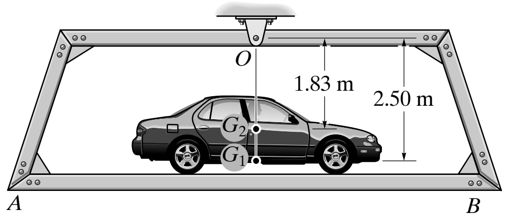
***22–16.** The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



Free-body Diagram: When an object of arbitrary shape having a mass m is pinned at O and being displaced by and angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum monent about point O to eliminate O_x and O_y.

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad -mg\sin\theta(l) = I_O\,\alpha \qquad \textbf{[1]}$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta \approx \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgI\theta = I_O\,\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = \dfrac{mgl}{I_O}$, thus, $p = \sqrt{\dfrac{mgl}{I_O}}$. Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad \textbf{[3]}$$

When the platform is empty, $\tau = \tau_1 = 2.38$ s. $m = 400$ kg and $l = 250$ m. Substitute these values into Eq. [3], we have

$$2.38 = 2\pi\sqrt{\frac{(I_O)_p}{400(9.81)(2.50)}} \qquad (I_O)_p = 1407.55\ \text{kg}\cdot\text{m}^2$$

When the car is on the platform, $\tau = \tau_2 = 3.16$ s, $m = 400\ \text{kg} + 1200\ \text{kg} = 1600\ kg$, $l = \dfrac{2.50(400) + 1.83(1200)}{1600} = 1.9975$ m and $I_O = (I_O)_C + (I_O)_p = (I_O)_C + 1407.55$. Substitute these values into Eq. [3], we have

$$3.16 = 2\pi\sqrt{\frac{(I_O)_C + 1407.55}{1600(9.81)(1.9975)}} \qquad (I_O)_C = 6522.76\ \text{kg}\cdot\text{m}^2$$

Thus, the mass moment inertia of the car about its mass center is

$$(I_G)_C = (I_O)_C - m_C d^2$$
$$= 6522.76 - 1200\left(1.83^2\right) = 2.50\left(10^3\right)\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

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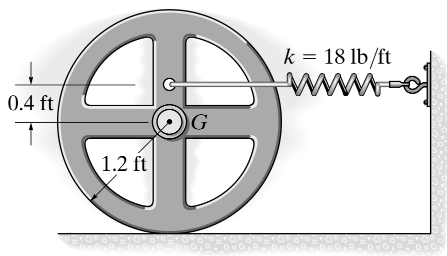
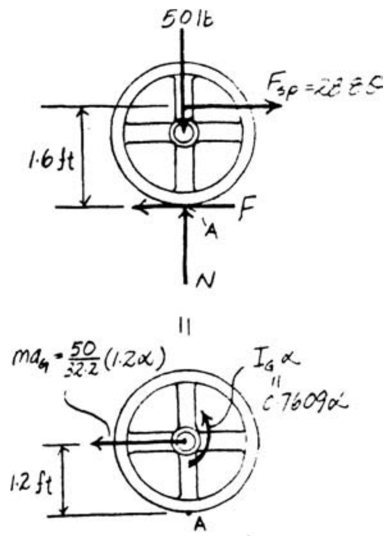
•22–17. The 50-lb wheel has a radius of gyration about its mass center G of $k_G = 0.7$ ft. Determine the frequency of vibration if it is displaced slightly from the equilibrium position and released. Assume no slipping.



Kinematics: Since the wheel rolls without slipping, then $a_G = \alpha r = 1.2\alpha$. Also when the wheel undergoes a small angular displacement θ about point A, the spring is stretched by $x = 1.6 \sin\theta\,\theta$. Since θ us small, then $\sin\theta - \theta$. Thus, $x = 1.6\,\theta$.

Free-body Diagram: The spring force $F_{sp} = kx = 18(1.6\theta) = 28.8\theta$ will create the *restoring moment* about point A.



Equation of Motion: The mass moment inertia of the wheel about its mass center is $I_G = mk_G^2 = \dfrac{50}{32.2}\left(0.7^2\right) = 0.7609\ \text{slug}\cdot\text{ft}^2$.

$$\circlearrowleft +\Sigma M_A = (M_k)_A; \qquad -28.8\theta(1.6) = \frac{50}{32.2}(1.2\alpha)(1.2) + 0.7609\alpha$$

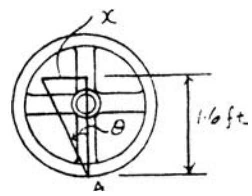
$$\alpha + 15.376\theta = 0 \qquad \textbf{[1]}$$

Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$, then substitute this values into Eq. [1], we have

$$\ddot{\theta} + 15.376\theta = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = 15.376$, thus, $p = 3.921$ rad/s. Applying Eq. 22–14, we have

$$f = \frac{p}{2\pi} = \frac{3.921}{2\pi} = 0.624\ \text{Hz} \qquad \textbf{Ans.}$$



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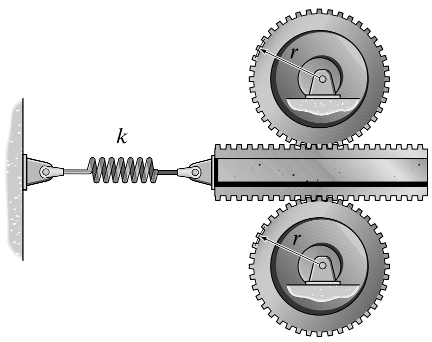
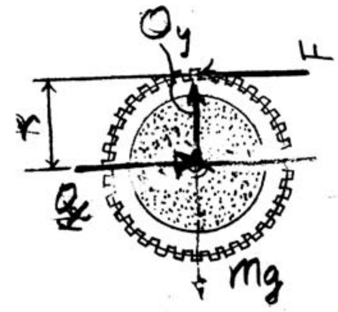
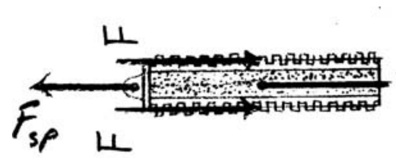
22–18. The two identical gears each have a mass of m and a radius of gyration about their center of mass of k_0. They are in mesh with the gear rack, which has a mass of M and is attached to a spring having a stiffness k. If the gear rack is displaced slightly horizontally, determine the natural period of oscillation.



Equation of Motion: When the gear rack is displaced horizontally downward by a small distance x, the spring is stretched by $s_1 = x$ Thus, $F_{sp} = kx$. Since the gears rotate about fixed axes, $\bar{x} = \dot{\theta} r$ or $\dot{\theta} = \dfrac{\bar{x}}{r}$. The mass moment of inertia of a gear about its mass center is $I_O = mk_O{}^2$. Referring to the free-body diagrams of the rack and gear in Figs. *a* and *b*,



(a)



(b)

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2F - (kx) = M\bar{x}$$

$$2F - kx = M\bar{x} \qquad \textbf{(1)}$$

and

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad -F(r) = mk_O{}^2\left(\frac{\bar{x}}{r}\right)$$

$$F = -\frac{mk_O{}^2}{r^2}\bar{x} \qquad \textbf{(2)}$$

Eliminating **F** from Eqs. (1) and (2),

$$M\bar{x} + \frac{2mk_O{}^2}{r^2}\bar{x} + kx = 0$$

$$\bar{x} + \left(\frac{kr^2}{Mr^2 + 2mk_O{}^2}\right)x = 0$$

Comparing this equation to that of the standard from, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{kr^2}{Mr^2 + 2mk_O{}^2}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{Mr^2 + 2mk_O{}^2}{kr^2}} \qquad \textbf{Ans.}$$

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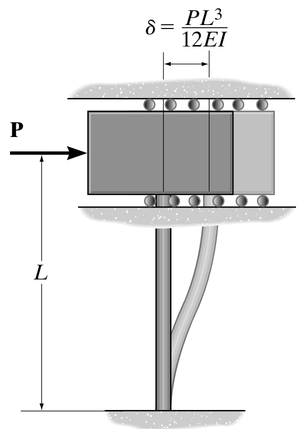
22–19. In the "lump mass theory", a single-story building can be modeled in such a way that the whole mass of the building is lumped at the top of the building, which is supported by a cantilever column of negligible mass as shown. When a horizontal force **P** is applied to the model, the column deflects an amount of $\delta = PL^3/12EI$, where L is the effective length of the column, E is Young's modulus of elasticity for the material, and I is the moment of inertia of the cross section of the column. If the lump mass is m, determine the frequency of vibration in terms of these parameters.



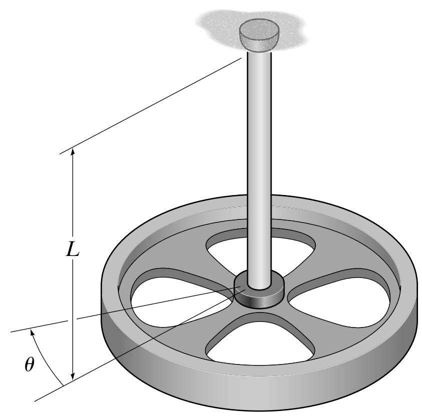
Since δ very small, the vibration can be assumed to occur along the horizontal. Here, the equivalent spring stiffness of the cantilever column is $k_{eq} = \dfrac{P}{\delta} = \dfrac{P}{PL^3/12EI} = \dfrac{12EI}{I^3}$. Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\frac{12EI}{I^3}}{m}} = \sqrt{\frac{12EI}{mL^3}}$$

Then the natural frequency of the system is

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{12EI}{mL^3}} \qquad \textbf{Ans.}$$

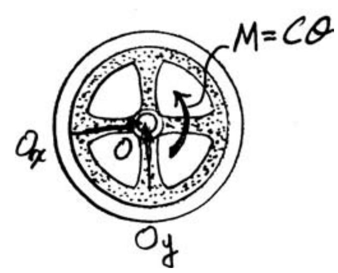
*__22–20.__ A flywheel of mass m, which has a radius of gyration about its center of mass of k_O, is suspended from a circular shaft that has a torsional resistance of $M = C\theta$. If the flywheel is given a small angular displacement of θ and released, determine the natural period of oscillation.



Equation of Motion: The mass moment of inertia of the wheel about point O is $I_O = mk_O{}^2$. Referring to Fig. a,

$$\circlearrowleft + \qquad \Sigma M_O = I_O\,\alpha; \qquad -C\theta = mk_O{}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{C}{mk_O{}^2}\theta = 0$$



Comparing this equation to the standard equation, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{C}{mk_O{}^2}} = \frac{1}{k_O}\sqrt{\frac{C}{m}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi k_O\sqrt{\frac{m}{C}} \qquad \textbf{Ans.}$$

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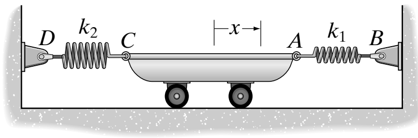
•22–21. The cart has a mass of m and is attached to two springs, each having a stiffness of $k_1 = k_2 = k$, unstretched length of l_0, and a stretched length of l when the cart is in the equilibrium position. If the cart is displaced a distance of $x = x_0$ such that both springs remain in tension $(x_0 < l - l_0)$, determine the natural frequency of oscillation.



Equation of Motion: When the cart is displaced x to the right, the stretch of springs AB and CD are $s_{AB} = (l - l_0) - x_0$ and $s_{AC} = (l - l_0) + x$. Thus, $F_{AB} = ks_{AB} = k[(l - l_0) - x]$ and $F_{AC} = ks_{AC} = k[(l - l_0) + x]$. Referring to the free-body diagram of the cart shown in Fig. *a*,

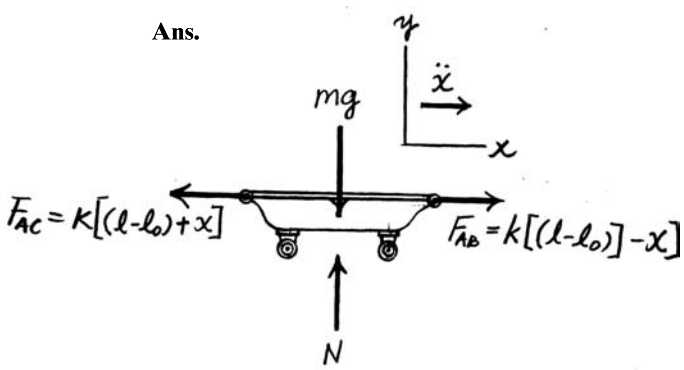
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad k[(l - l_0) - x] - k[(l - l_0) + x] = m\ddot{x}$$

$$-2kx = m\ddot{x}$$

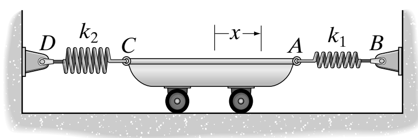
$$\ddot{x} + \frac{2k}{m}x = 0$$

Simple Harmonic Motion: Comparing this equation with that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{2k}{m}} \qquad \textbf{Ans.}$$



22–22. The cart has a mass of m and is attached to two springs, each having a stiffness of k_1 and k_2, respectively. If both springs are unstretched when the cart is in the equilibrium position shown, determine the natural frequency of oscillation.



Equation of Motion: When the cart is displaced x to the right, spring CD stretches $s_{CD} = x$ and spring AB compresses $s_{AB} = x$. Thus, $F_{CD} = k_2 s_{CD} = k_2 x$ and $F_{AB} = k_1 s_{AB} = k_1 x$. Referring to the free-body diagram of the cart shown in Fig. *a*,

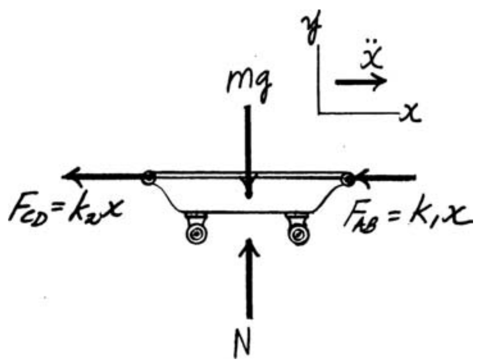
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -k_1 x - k_2 x = m\ddot{x}$$

$$-(k_1 + k_2)x = m\ddot{x}$$

$$\ddot{x} + \left(\frac{k_1 + k_2}{m}\right)x = 0$$

Simple Harmonic Motion: Comparing this equation with that of the standard equation, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}} \qquad \textbf{Ans.}$$



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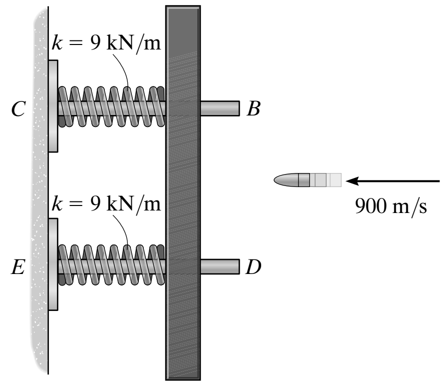
22–23. The 3-kg target slides freely along the smooth horizontal guides BC and DE, which are 'nested' in springs that each have a stiffness of $k = 9$ kN/m. If a 60-g bullet is fired with a velocity of 900 m/s and embeds into the target, determine the amplitude and frequency of oscillation of the target.



Conservation of Linear Momentum: The velocity of the target after impact can be determined from

$$m_b(v_b)_1 = (m_b + m_A)v$$

$$0.06(900) = (0.06 + 3)v$$

$$v = 17.65 \text{ m/s}$$

Since the springs are arranged in parallel, the equivalent stiffness of a single spring is $k_{eq} = 2k = 2(9000 \text{ N/m}) = 18000 \text{ N/m}$. Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{18000}{3.06}} = 76.70 \text{ rad/s} = 76.7 \text{ rad/s} \qquad \textbf{Ans.}$$

The equation that describes the oscillation of the system is

$$y = C \sin (76.70t + \phi) \text{ m} \qquad \textbf{(1)}$$

Since $y = 0$ when $t = 0$,

$$0 = C \sin \phi$$

Since $C \neq 0$, $\sin \phi = 0$. Then $\phi = 0°$. Thus, Eq. (1) becomes

$$y = C \sin (76.70t) \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2),

$$\dot{y} = v = 76.70C \cos (76.70t) \text{ m/s} \qquad \textbf{(3)}$$

Here, $v = 17.65$ m/s when $t = 0$. Thus, Eq. (3) gives

$$17.65 = 76.70C \cos 0$$

$$C = 0.2301 \text{ m} = 230 \text{ mm} \qquad \textbf{Ans.}$$

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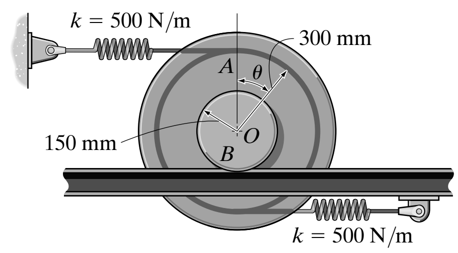
***22–24.** If the spool undergoes a small angular displacement of θ and is then released, determine the frequency of oscillation. The spool has a mass of 50 kg and a radius of gyration about its center of mass O of $k_O = 250$ mm. The spool rolls without slipping.

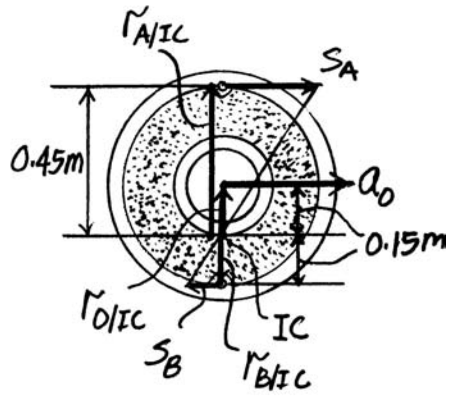


Equation of Motion: Referring to the kinematic diagram of the spool, Fig. *a*, the stretch of the spring at A and$B$ when the spool rotates through a small angle θ is $s_A = \theta r_{A/IC} = \theta(0.45)$ and $s_B = \theta r_{B/IC} = \theta(0.15)$. Thus, $\left(F_{\text{sp}}\right)_A = ks_A = 500[\theta(0.45)] = 225\theta$ and $\left(F_{\text{sp}}\right)_B = ks_B = 500[\theta(0.15)] = 75\theta$. Also, $a_O = \ddot{\theta} r_{O/IC} = \ddot{\theta}(0.15)$. The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = 50\left(0.25^2\right) = 3.125 \text{ kg} \cdot \text{m}^2$. Referring the free-body and kinetic diagrams of the spool, Fig. *b*,

$$+ \qquad \Sigma M_{IC} = \Sigma(M_k)_{IC}; \quad -225\theta(0.045) - 75\theta(0.15) = 50\left[\ddot{\theta}(0.15)\right](0.15) + 3.125\ddot{\theta}$$

$$-112.5\theta = 4.25\ddot{\theta}$$

$$\ddot{\theta} + 26.47\theta = 0$$



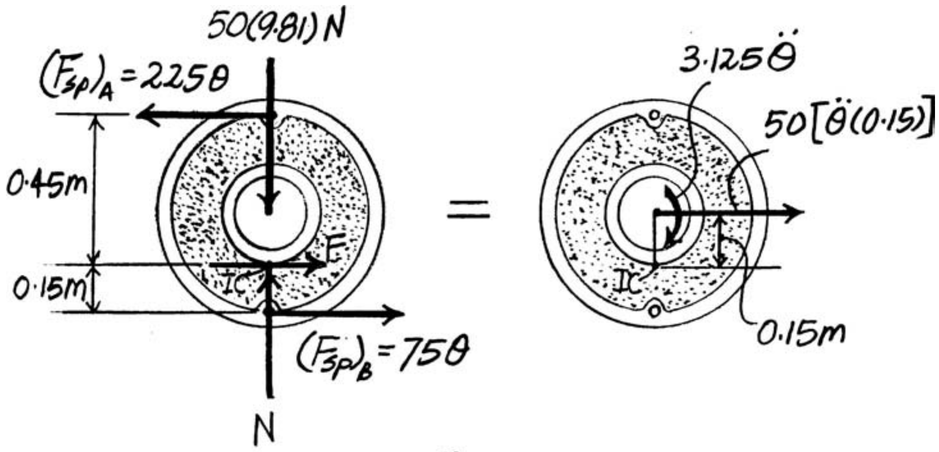
(a)

Comparing this equation to that of the standard equation, the natural circular frequency of the spool is

$$\omega_n = \sqrt{26.47} \text{ rad/s} = 5.145 \text{ rad/s}$$

Thus, the natural frequency of the oscillation is

$$f_n = \frac{\omega_n}{2\pi} = \frac{5.145}{2\pi} = 0.819 \text{ Hz} \qquad \textbf{Ans.}$$



(b)

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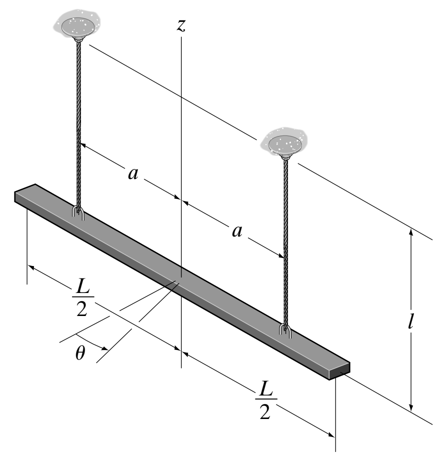
•22–25. The slender bar of mass m is supported by two equal-length cords. If it is given a small angular displacement of θ about the vertical axis and released, determine the natural period of oscillation.



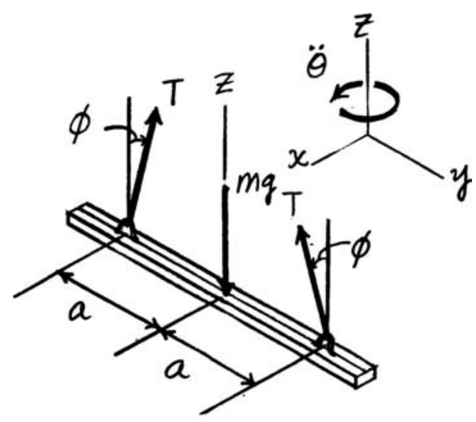
Equation of Motion: The mass moment of inertia of the bar about the z axis is $I_z = \frac{1}{12}mL^2$. Referring to the free-body diagram of the bar shown in Fig. a,

$$+\uparrow \Sigma F_z = ma_z; \qquad 2T\cos\phi - mg = 0 \qquad T = \frac{mg}{2\cos\phi}$$

Then,

$$\uparrow + \Sigma M_z = I_z\alpha; \qquad -2\left(\frac{mg}{2\cos\phi}\right)\sin\phi(a) = \left(\frac{1}{12}mL^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{12ga}{L^2}\tan\phi = 0 \qquad \textbf{(1)}$$



(a)

Since θ is very small, from the geometry of Fig. b,

$$l\phi = a\theta$$

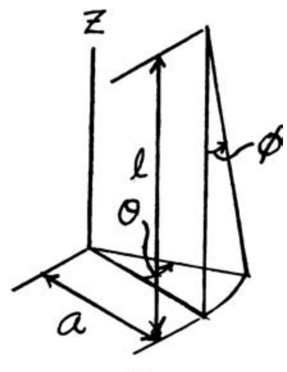
$$\phi = \frac{a}{l}\theta$$

Substituting this result into Eq. (1)

$$\ddot{\theta} + \frac{12ga}{L^2}\tan\left(\frac{a}{l}\theta\right) = 0$$

Since θ is very small, $\tan\left(\frac{g}{l}\theta\right) \cong \frac{g}{l}\theta$. Thus,

$$\ddot{\theta} + \frac{12ga^2}{IL^2}\theta = 0$$



(b)

Comparing this equation to that of the standard form, the natural circular frequency of the bar is

$$\omega_n = \sqrt{\frac{12ga^2}{IL^2}} = \frac{a}{L}\sqrt{\frac{12g}{l}}$$

Thus, the natural period of oscillation is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi L}{a}\sqrt{\frac{l}{12g}} \qquad \textbf{Ans.}$$

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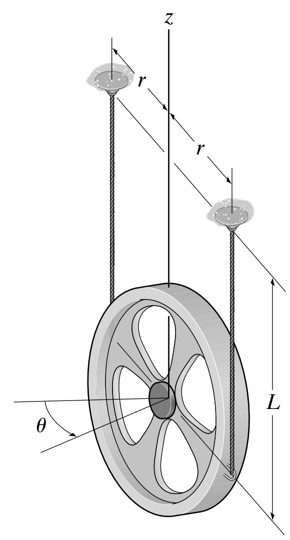
22–26. A wheel of mass m is suspended from two equal-length cords as shown. When it is given a small angular displacement of θ about the z axis and released, it is observed that the period of oscillation is τ. Determine the radius of gyration of the wheel about the z axis.



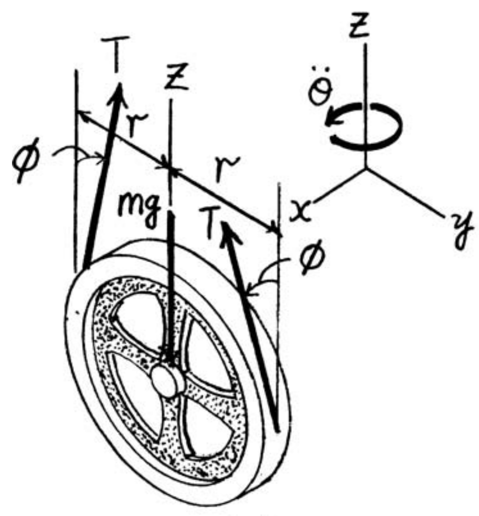
Equation of Motion: The mass moment of inertia of the wheel about is z axis is $I_z = mk_z^2$. Referring to the free-body diagram of the wheel shown in Fig. a,

$$+\uparrow \Sigma F_z = ma_z; \qquad 2T\cos\phi - mg = 0 \qquad\qquad T = \frac{mg}{2\cos\phi}$$

Then,

$$\uparrow + \Sigma M_z = I_z\,\alpha; \qquad -2\left(\frac{mg}{2\cos\phi}\right)\sin\phi(\mathrm{r}) = \left(\mathrm{m}k_z^{\,2}\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{gr}{k_z^{\,2}}\tan\phi = 0 \qquad \textbf{(1)}$$



(a)

Since θ is very small, from the geometry of Fig. b,

$$L\phi = r\theta$$

$$\phi = \frac{r}{L}\theta$$

Substituting this result into Eq. (1)

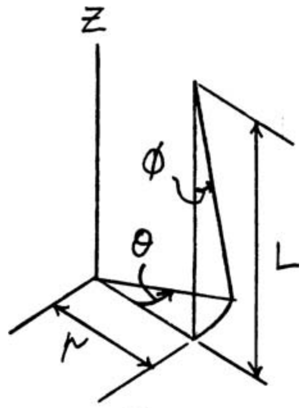
$$\ddot{\theta} + \frac{gr}{k_z^{\,2}}\tan\left(\frac{r}{L}\theta\right) = 0$$

Since θ is very small, $\tan\left(\frac{r}{L}\theta\right) \cong \frac{r}{L}\theta$. Thus,

$$\ddot{\theta} + \frac{gr^2}{k_z^{\,2}L}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{gr^2}{k_z^{\,2}L}} = \frac{r}{k_z}\sqrt{\frac{g}{L}}$$



(b)

Thus, the natural period of oscillation is

$$\tau = \frac{2\pi}{\omega_n}$$

$$\tau = 2\pi\left(\frac{k_z}{r}\sqrt{\frac{L}{g}}\right)$$

$$k_z = \frac{\tau r}{2\pi}\sqrt{\frac{g}{L}} \qquad \textbf{Ans.}$$

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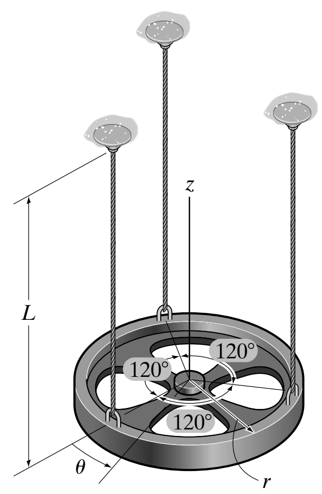
22–27. A wheel of mass m is suspended from three equal-length cords. When it is given a small angular displacement of θ about the z axis and released, it is observed that the period of oscillation is τ. Determine the radius of gyration of the wheel about the z axis.



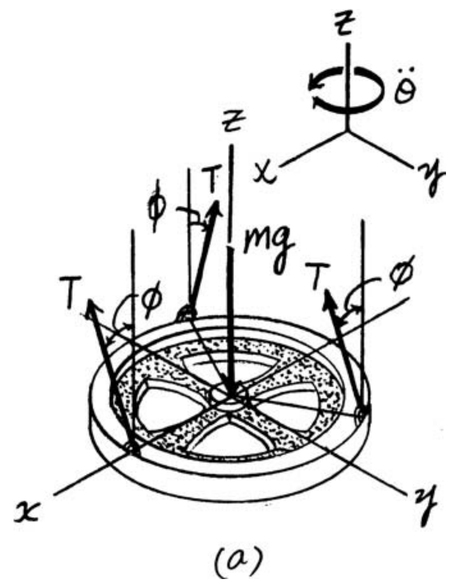
Equation of Motion: Due to symmetry, the force in each cord is the same. The mass moment of inertia of the wheel about is z axis is $I_z = mk_z{}^2$. Referring to the free-body diagram of the wheel shown in Fig. a,

$$+\uparrow \Sigma F_z = ma_z; \qquad 3T\cos\phi - mg = 0 \qquad\qquad T = \frac{mg}{3\cos\phi}$$

Then,

$$\uparrow + \Sigma M_z = I_z\,\alpha; \qquad -3\left(\frac{mg}{3\cos\phi}\right)\sin\phi(\mathrm{r}) = mk_z{}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{gr}{k_z{}^2}\tan\phi = 0 \qquad \textbf{(1)}$$



Since θ is very small, from the geometry of Fig. b,

$$r\theta = L\phi$$

$$\phi = \frac{r}{L}\theta$$

Substituting this result into Eq. (1)

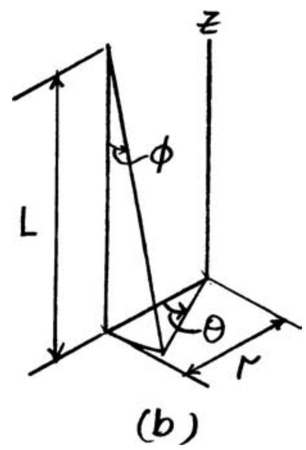
$$\ddot{\theta} + \frac{gr}{k_z{}^2}\tan\left(\frac{r}{L}\theta\right) = 0 \qquad \textbf{(2)}$$

Since θ is very small, $\tan\left(\frac{r}{L}\theta\right) \cong \frac{r}{L}\theta$. Thus,

$$\ddot{\theta} + \frac{gr^2}{kz^2L}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{gr^2}{k_z{}^2L}} = \frac{r}{k_z}\sqrt{\frac{g}{L}}$$



Thus, the natural period of oscillation is

$$\tau = \frac{2\pi}{\omega_n}$$

$$\tau = 2\pi\left(\frac{k_z}{r}\sqrt{\frac{L}{g}}\right)$$

$$k_z = \frac{\tau r}{2\pi}\sqrt{\frac{g}{L}} \qquad \textbf{Ans.}$$

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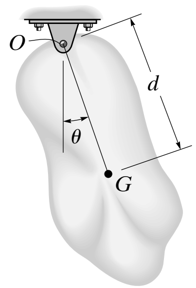
***22–28.** Solve Prob. 22–10 using energy methods.



$$T + V = \frac{1}{2}\left[mk_G^2 + md^2\right]\theta^2 + mg(d)(1 - \cos\theta)$$

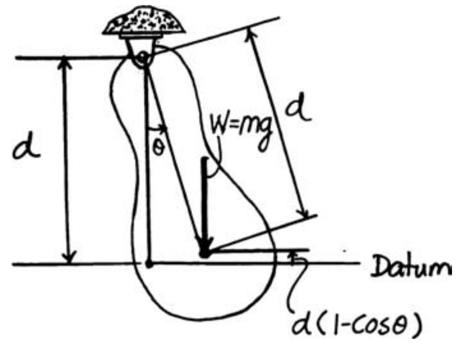
$$\left(k_G^2 + d^2\right)\dot{\theta}\,\ddot{\theta} + gd(\sin\theta)\dot{\theta} = 0$$

$$\sin\theta = \theta$$

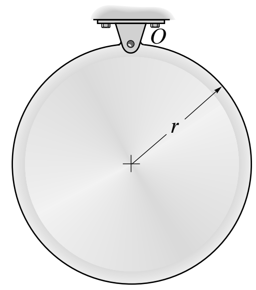
$$\ddot{\theta} + \frac{gd}{\left(k_G^2 + d^2\right)}\theta = 0$$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{\left(k_G^2 + d^2\right)}{gd}}$$

Ans.



•22–29. Solve Prob. 22–11 using energy methods.



$$T + V = \frac{1}{2}\left[\frac{3}{2}mr^2\right]\dot{\theta}^2 + mg(r)(1 - \cos\theta)$$

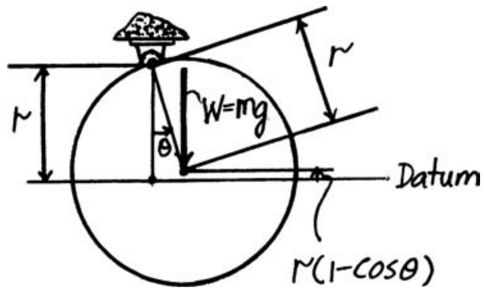
$$\frac{3}{2}mr^2\,\dot{\theta}\dot{\theta} + mg(r)(\sin\theta)\dot{\theta} = 0$$

$$\sin\theta = \theta$$

$$\theta + \left(\frac{2}{3}\right)\left(\frac{g}{r}\right)\theta = 0$$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{3r}{2g}}$$

Ans.



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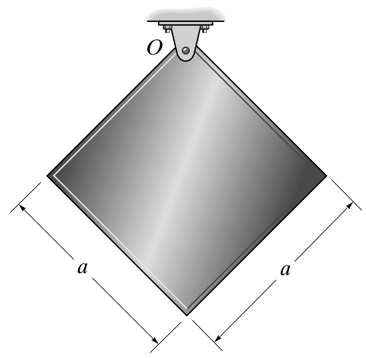
22–30. Solve Prob. 22–12 using energy methods.



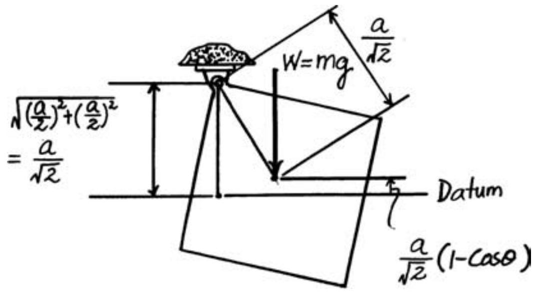
$$T + V = \frac{1}{2}\left[\frac{1}{12}m\left(a^2 + a^2\right) + m\left(\frac{a}{\sqrt{2}}\right)^2\right]\dot{\theta}^2 + mg\left(\frac{a}{\sqrt{2}}\right)(1 - \cos\theta)$$

$$\frac{2}{3}\,ma^2\,\dot{\theta}\,\ddot{\theta} + mg\left(\frac{a}{\sqrt{2}}\right)(\sin\theta)\dot{\theta} = 0$$

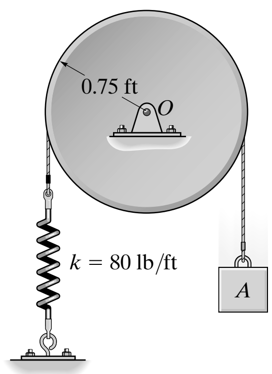
$$\sin\theta = \theta$$

$$\theta + \frac{3g}{2\sqrt{2}a}\,\theta = 0$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{1.0299}\left(\sqrt{\frac{a}{g}}\right) = 6.10\sqrt{\frac{a}{g}}$$ **Ans.**



22–31. Solve Prob. 22–14 using energy methods.



$$s = 0.75\,\theta, \qquad \dot{s} = 0.75\dot{\theta}$$

$$T + V = \frac{1}{2}\left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\dot{\theta}^2 + \frac{1}{2}\left(\frac{3}{32.2}\right)\left(0.75\,\dot{\theta}\right)^2$$

$$+ \frac{1}{2}(80)\,(s_{eq} + 0.75\,\theta)^2 - 3(0.75\,\theta)$$

$$0 = 0.1834\dot{\theta}\,\ddot{\theta} + 80(s_{eq} + 0.75\,\theta)\left(0.75\,\dot{\theta}\right) - 2.25\dot{\theta}$$

$$F_{eq} = 80s_{eq} = 3$$

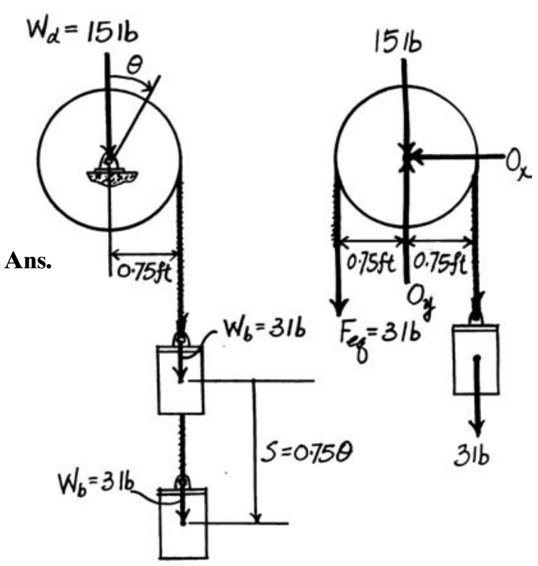
$$s_{eq} = 0.0375\text{ ft}$$

Thus,

$$0.1834\ddot{\theta} + 45\,\theta = 0$$

$$\ddot{\theta} + 245.36 = 0$$

$$\tau = \frac{2\,\pi}{p} = \frac{2\pi}{\sqrt{245.3}} = 0.401\text{ s}$$ **Ans.**



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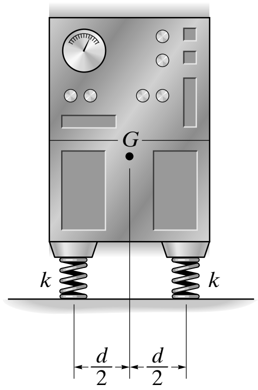
***22–32.** The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.



$$T + V = \text{const.}$$

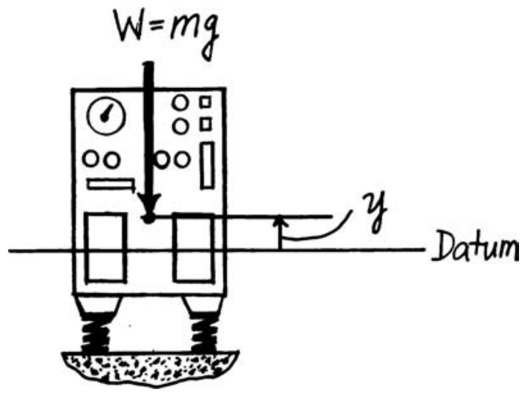
$$T = \frac{1}{2}m(\dot{y})^2$$

$$V = m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$T + V = \frac{1}{2}m(\dot{y})^2 + m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$m\,\dot{y}\,\ddot{y} + m\,g\,\dot{y} - 4k(\Delta s y)\dot{y} = 0$$

$$m\,\ddot{y} + m\,g + 4ky - 4k\Delta s = 0$$



Since $\Delta s = \dfrac{mg}{4k}$

Then

$$m\ddot{y} + 4ky = 0$$

$$y + \frac{4k}{m}y = 0$$

$$p = \sqrt{\frac{4k}{m}}$$

$$\tau = \frac{2\pi}{p} = \pi\sqrt{\frac{m}{k}}$$ **Ans.**

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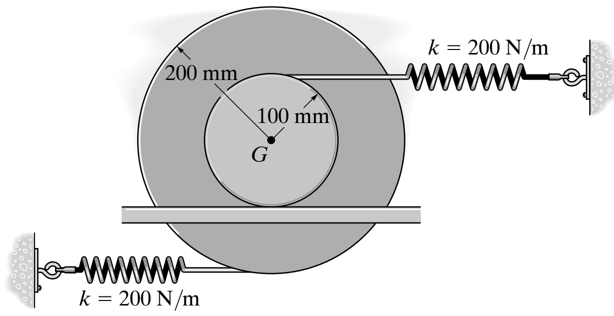
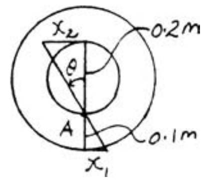
•22–33. Determine the differential equation of motion of the 15-kg spool. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is $k_G = 125$ mm. The springs are originally unstretched.



Energy Equation: Since the spool rolls without slipping, the stretching of both springs can be approximated as $x_1 = 0.1\theta$ and $x_2 = 0.2\theta$ when the spool is being displaced by a small angular displacement θ. Thus, the elastic potential energy is $V_p = \frac{1}{2}k\,x_1^2 + \frac{1}{2}k\,x_2^2 = \frac{1}{2}(200)(0.1\theta)^2 + \frac{1}{2}(200)\,(0.2\theta)^2 = 5\theta^2$. Thus,



$$V = V_e = 5\,\theta^2$$

The mass moment inertia of the spool about point A is $I_A = 15\left(0.125^2\right) + 15\left(0.1^2\right) = 0.384375\ \text{kg}\cdot\text{m}^2$. The kinetic energy is

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(0.384375)\,\dot{\theta}^2 = 0.19218750\dot{\theta}^2$$

The total energy of the system is

$$U = T + V = 0.19218750\dot{\theta}^2 + 5\theta^2 \qquad \textbf{[1]}$$

Time Derivative: Taking the time derivative of Eq. [1], we have

$$0.384375\dot{\theta}\,\ddot{\theta} + 10\,\theta\,\dot{\theta} = 0$$

$$\dot{\theta}\,(0.384375\,\ddot{\theta} + 10\,\theta) = 0$$

Since $\dot{\theta} \neq 0$, then $0.384375\,\ddot{\theta} + 10\,\theta = 0$

$$\ddot{\theta} + 26.0\,\theta = 0 \qquad \textbf{Ans.}$$

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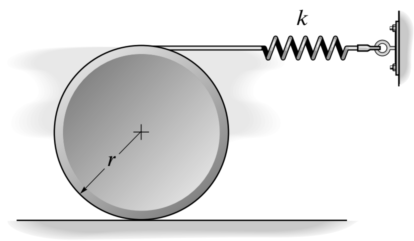
22–34. Determine the natural period of vibration of the disk having a mass m and radius r. Assume the disk does not slip on the surface of contact as it oscillates.



$$T + V = \text{const.}$$

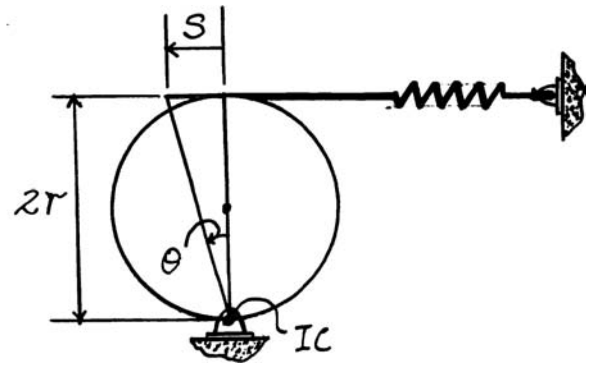
$$s = (2r)\,\theta$$

$$T + V = \frac{1}{2}\left[\frac{1}{2}mr^2 + mr^2\right]\dot{\theta}^2 + \frac{1}{2}k(2r\,\theta)^2$$

$$0 = \frac{3}{2}mr^{\,2}\,\theta\dot{\theta} + 4\,kr^{\,2}\,\theta\dot{\theta}$$

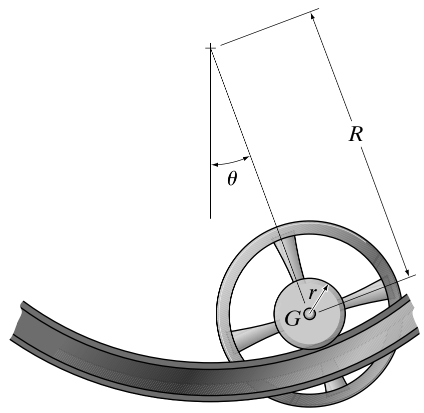
$$\theta + \frac{8k}{3m}\theta = 0$$

$$\tau = \frac{2\,\pi}{p} = \frac{2\pi}{\sqrt{\frac{8k}{3m}}} = 3.85\sqrt{\frac{m}{k}}$$



Ans.

22–35. If the wheel is given a small angular displacement of θ and released from rest, it is observed that it oscillates with a natural period of τ. Determine the wheel's radius of gyration about its center of mass G. The wheel has a mass of m and rolls on the rails without slipping.



Potential and Kinetic Energy: With reference to the datum established in Fig. *a*, the gravitational potential energy of the wheel is

$$V = V_g = -Wy_G = -mgR\cos\theta$$

As shown in Fig. *b*, $v_G = \dot{\theta}R$. Also, $v_G = \omega r_{G/IC} = \omega r$. Then, $\omega r = \dot{\theta}R$ or $\omega = \left(\frac{R}{r}\right)\dot{\theta}$. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2$. Thus, the kinetic energy of the wheel is

$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\,\omega^2$$

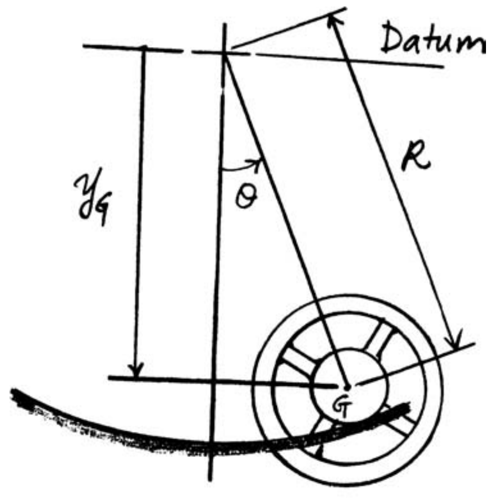
$$= \frac{1}{2}m\left(\dot{\theta}R\right)^2 + \frac{1}{2}\left(mk_G^2\right)\left[\left(\frac{R}{r}\right)\dot{\theta}\right]^2$$

$$= \frac{1}{2}mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\dot{\theta}^2$$

The energy function of the wheel is

$$T + V = \text{constant}$$

$$\frac{1}{2}mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\dot{\theta}^2 - mgR\cos\theta = \text{constant}$$



(a)

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Taking the time derivative of this equation,

$$mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\dot{\theta}\,\ddot{\theta} + mgR\sin\theta\dot{\theta} = 0$$

$$\dot{\theta}\left[mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\dot{\theta} + mgR\sin\theta\right] = 0$$

Since $\dot{\theta}$ is not always equal to zero, then

$$mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\ddot{\theta} + mgR\sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{R}\left(\frac{r^2}{r^2 + k_G{}^2}\right)\sin\theta = 0$$

Since θ is small, $\sin\theta \cong \theta$. This equation becomes

$$\ddot{\theta} + \frac{g}{R}\left(\frac{r^2}{r^2 + k_G{}^2}\right)\theta = 0$$

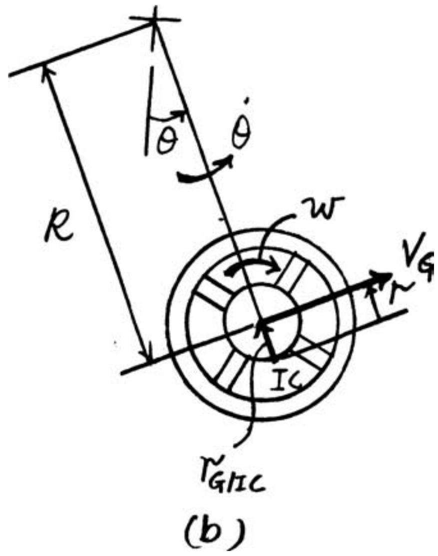
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{g}{R}\left(\frac{r^2}{r^2 + k_G{}^2}\right)}$$

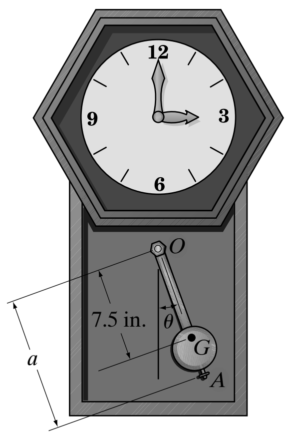
The natural period of the oscillation is therefore

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{R}{g}\left(\frac{r^2 + k_G{}^2}{r^2}\right)}$$

$$k_G = \frac{r}{2\pi}\sqrt{\frac{\tau^2 g - 4\pi^2 R}{R}}$$ **Ans.**



***22–36.** Without an adjustable screw, A, the 1.5-lb pendulum has a center of gravity at G. If it is required that it oscillates with a period of 1 s, determine the distance a from pin O to the screw. The pendulum's radius of gyration about O is $k_O = 8.5$ in. and the screw has a weight of 0.05 lb.



Potential and Kinetic Energy: With reference to the datum established in Fig. a, the gravitational potential energy of the system is

$$V = V_g = -W_P\,y_G - W_A\,y_A$$

$$= -1.5(0.625\cos\theta) - 0.05(a\cos\theta)$$

$$= -(0.9375 + 0.05\,a)\cos\theta$$

The mass moment of inertia of the pendulum about point O is $I_O = mk_O{}^2 = \frac{1.5}{32.2}\left(\frac{8.5}{12}\right)^2 = 0.02337\ \text{slug}\cdot\text{ft}^2$. Since the pendulum rotates about point O, $v_A = \dot{\theta}r_{OA} = \dot{\theta}a$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_O\dot{\theta}^2 + \frac{1}{2}m_A v_A{}^2$$

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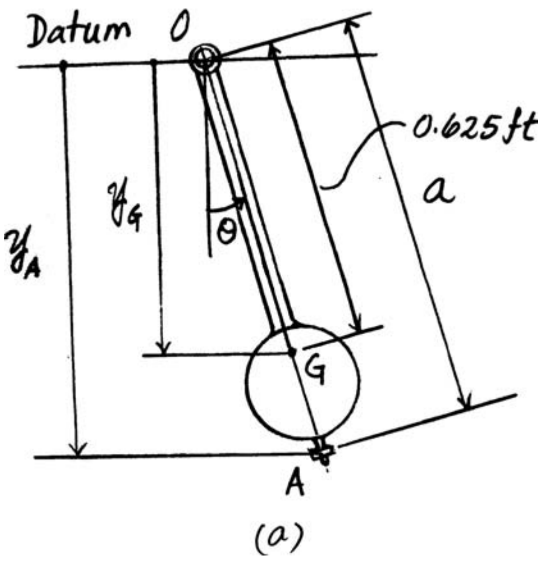
***22–36. Continued**



$$= \frac{1}{2}(0.02337)\dot{\theta}^2 + \frac{1}{2}\left(\frac{0.05}{32.2}\right)(\dot{\theta}a)^2$$

$$= \left(0.01169 + 0.0007764\, a^2\right)\dot{\theta}^2$$

Thus, the energy function of the system is

$$T + V = \text{constant}$$

$$\left(0.01169 + 0.0007764a^2\right)\dot{\theta}^2 - (0.9375 + 0.05a)\cos\theta = \text{constant}$$

Taking the time derivative of this equation,

$$\left(0.02337 + 0.001553a^2\right)\dot{\theta}\,\ddot{\theta} + (0.9375 + 0.05a)\sin\theta\,\dot{\theta} = 0$$

$$\dot{\theta}\left[\left(0.02337 + 0.001553\, a^2\right)\ddot{\theta} + (0.9375 + 0.05a)\sin\theta\right] = 0$$

Since θ is not always equal to zero, then

$$\left(0.02337 + 0.001553a^2\right)\ddot{\theta} + (0.9375 + 0.05a)\sin\theta = 0$$

$$\ddot{\theta} + \left(\frac{0.9375 + 0.05a}{0.02337 + 0.001553a^2}\right)\sin\theta = 0$$

Since θ is small, $\sin\theta \cong 0$. This equation becomes

$$\ddot{\theta} + \left(\frac{0.9375 + 0.05a}{0.02337 + 0.001553a^2}\right)\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{0.9375 + 0.05a}{0.02337 + 0.001553a^2}}$$

The natural period of the oscillation is therefore

$$\tau = \frac{2\pi}{\omega_n}$$

$$1 = 2\pi\sqrt{\frac{0.02337 + 0.001553a^2}{0.9375 + 0.05a}}$$

$$0.06130a^2 - 0.05a - 0.01478 = 0$$

Solving for the positive root,

$$a = 1.05 \text{ ft}$$ **Ans.**

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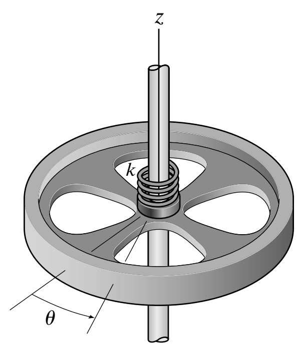
•22–37. A torsional spring of stiffness k is attached to a wheel that has a mass of M. If the wheel is given a small angular displacement of θ about the z axis determine the natural period of oscillation. The wheel has a radius of gyration about the z axis of k_z.



Potential and Kinetic Energy: The elastic potential energy of the system is

$$V = V_e = \frac{1}{2}k\theta^2$$

The mass moment of inertia of the wheel about the z axis is $I_z = Mk_z^{\,2}$. Thus, the kinetic energy of the wheel is

$$T_1 = \frac{1}{2}I_z\dot{\theta}^2 = \frac{1}{2}Mk_z^{\,2}\dot{\theta}^2$$

The energy function of the wheel is

$$T_1 + V = \text{constant}$$

$$\frac{1}{2}Mk_z^{\,2}\dot{\theta}^2 + \frac{1}{2}k\theta^2 = \text{constant}$$

Taking the time derivative of this equation,

$$Mk_z^{\,2}\dot{\theta}\,\ddot{\theta} + k\theta\dot{\theta} = 0$$

$$\dot{\theta}\left(Mk_z^{\,2}\dot{\theta} + k\theta\right) = 0$$

Since $\dot{\theta}$ is not always equal to zero, then

$$Mk_z^{\,2}\ddot{\theta} + k\theta = 0$$

$$\dot{\theta} + \frac{k}{Mk_z^{\,2}}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$(\omega_n) = \sqrt{\frac{k}{Mk_z^{\,2}}}$$

The natural period of the oscillation is therefore

$$\tau = \frac{2\pi}{(\omega_n)_1}$$

$$\tau = 2\pi\sqrt{\frac{Mk_z^{\,2}}{k}}$$ **Ans.**

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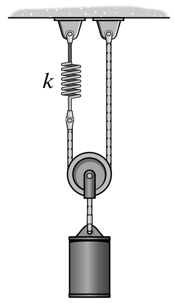
22–38. Determine the frequency of oscillation of the cylinder of mass m when it is pulled down slightly and released. Neglect the mass of the small pulley.



Potential and Kinetic Energy: Referring to the free-body diagram of the system at its equilibrium position, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2(F_{sp})_{st} - mg = 0 \qquad (F_{sp})_{st} = \frac{mg}{2}$$

Thus, the initial stretch of the spring is $s_0 = \dfrac{(F_{sp})_{st}}{k} = \dfrac{mg}{2k}$. Referring to Fig. *b*,

$$(+\downarrow) \qquad s_C + (s_C - s_P) = l$$

$$2s_C - s_P = l$$

$$2\Delta s_C - \Delta s_P = 0$$

$$\Delta s_P = 2\Delta s_C$$

When the cylinder is displaced vertically downward a distance $\Delta s_C = y$, the spring is stretched further by $s_1 = \Delta s_P = 2y$.

Thus, the elastic potential energy of the spring is

$$V_e = \frac{1}{2}k(s_0 + s_1)^2 = \frac{1}{2}k\left(\frac{mg}{2k} + 2y\right)^2$$

With reference to the datum established in Fig. *b*, the gravitational potential energy of the cylinder is

$$V_g = -Wy = -mgy$$

The kinetic energy of the system is $T = \dfrac{1}{2}m\dot{y}^2$. Thus, the energy function of the system is

$$T + V = \text{constant}$$

$$\frac{1}{2}m\dot{y}^2 + \frac{1}{2}k\left(\frac{mg}{2k} + 2y\right)^2 - mgy = 0$$

Taking the time derivative of this equation,

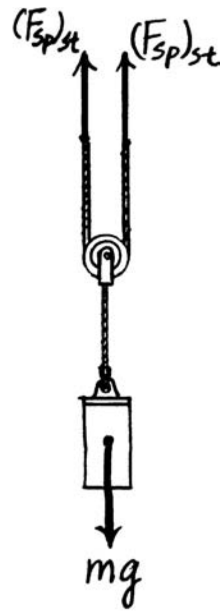
$$m\dot{y}\,\ddot{y} + k\left(\frac{mg}{2k} + 2y\right)(2\dot{y}) - mg\dot{y} = 0$$

$$\dot{y}(m\ddot{y} + 4ky) = 0$$

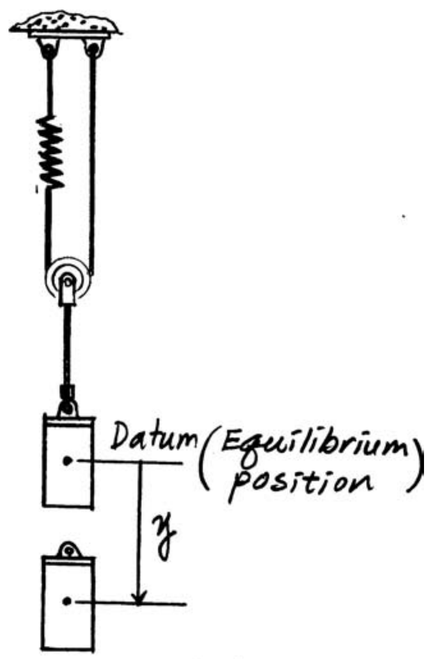
Since $\dot{y}$ is not equal to zero,

$$m\ddot{y} + 4\,ky = 0$$

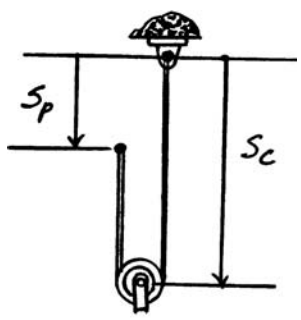
$$\ddot{y} + \frac{4k}{m}y = 0$$



(a)



(c)



(b)

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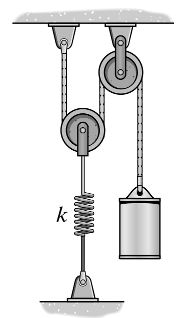
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$$

Thus, the frequency of the oscillation is

$$f = \frac{\omega_n}{2\pi} = \frac{1}{\pi}\sqrt{\frac{k}{m}}$$ **Ans.**

22–39. Determine the frequency of oscillation of the cylinder of mass m when it is pulled down slightly and released. Neglect the mass of the small pulleys.



Potential and Kinetic Energy: Referring to the free-body diagram of the system at its equilibrium position, Fig. *a*,

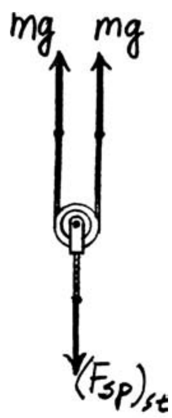
$$+\uparrow \Sigma F_y = 0; \qquad 2mg - (F_{sp})_{st} = 0 \qquad (F_{sp})_{st} = 2mg$$

Thus, the initial stretch of the spring is $s_0 = \frac{(F_{sp})_{st}}{k} = \frac{2mg}{k}$. Referring to Fig. *b*,

$$(+\downarrow) \qquad 2s_P + s_C = l$$

$$2\Delta s_P - \Delta s_C = 0$$

$$\Delta s_P = -\frac{\Delta s_C}{2} = \frac{\Delta s_C}{2} \quad \uparrow$$



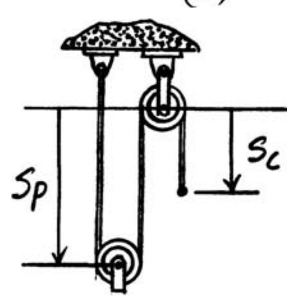
(a)

When the cylinder is displaced vertically downward a distance $\Delta s_C = y$, the spring stretches further by $s_1 = \Delta s_P = \frac{y}{2}$. Thus, the elastic potential energy of the spring is

$$V_e = \frac{1}{2}k(s_0 + s_1)^2 = \frac{1}{2}k\left(\frac{2mg}{k} + \frac{y}{2}\right)^2$$

With reference to the datum established in Fig. *c*, the gravitational potential energy of the cylinder is

$$V_g = -Wy = -mgy$$



(b)

The kinetic energy of the cylinder is $T = \frac{1}{2}m\dot{y}^2$. Thus, the energy function of the system is

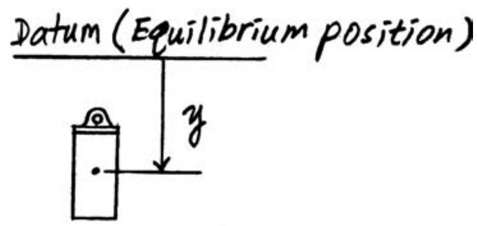
$$T + V = \text{constant}$$

$$\frac{1}{2}m\dot{y}^2 + k\left(\frac{2mg}{2k} + \frac{y}{2}\right)^2 - mgy = \text{constant}$$

Taking the time derivative of this equation,

$$m\dot{y}\,\ddot{y} + k\left(\frac{2mg}{k} + \frac{y}{2}\right)\left(\frac{\dot{y}}{2}\right) - mg\,\dot{y} = 0$$

$$\dot{y}\left(m\ddot{y} + \frac{k}{4}y\right) = 0$$



(c)

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Since $\dot{y}$ is not always equal to zero,

$$m\ddot{y} + \frac{k}{4}y = 0$$

$$\ddot{y} + \frac{1}{4}\left(\frac{k}{m}\right)y = 0$$

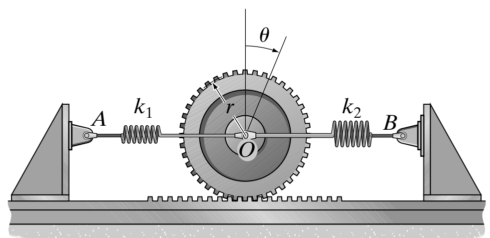
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{1}{4}\left(\frac{k}{m}\right)} = \frac{1}{2}\sqrt{\frac{k}{m}}$$

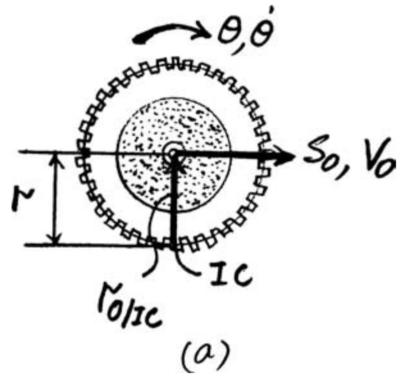
Thus, the natural frequency of oscillation is

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{4\pi}\sqrt{\frac{k}{m}} \qquad \textbf{Ans.}$$

***22–40.** The gear of mass m has a radius of gyration about its center of mass O of k_O. The springs have stiffnesses of k_1 and k_2, respectively, and both springs are unstretched when the gear is in an equilibrium position. If the gear is given a small angular displacement of θ and released, determine its natural period of oscillation.



Potential and Kinetic Energy: Since the gear rolls on the gear rack, springs AO and BO stretch and compress $s_O = r_{O/IC}\theta = r\theta$. When the gear rotates a small angle θ, Fig. a, the elastic potential energy of the system is



$$V = V_e = \frac{1}{2}k_1 s_O{}^2 + \frac{1}{2}k_2 s_O{}^2$$

$$= \frac{1}{2}k_1(r\theta)^2 + \frac{1}{2}k_2(r\theta)^2$$

$$= \frac{1}{2}r^2(k_1 + k_2)\theta^2$$

Also, from Fig. a, $v_O = \dot{\theta}r_{O/IC} = \dot{\theta}r$. The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2$.

Thus, the kinetic energy of the system is

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\,\omega^2$$

$$= \frac{1}{2}m(\dot{\theta}r)^2 + \frac{1}{2}\left(mk_O{}^2\right)\dot{\theta}^2$$

$$= \frac{1}{2}m(r^2 + k_O{}^2)\dot{\theta}^2$$

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The energy function of the system is therefore

$$T + V = \text{constant}$$

$$\frac{1}{2}m\left(r^2 + k_O{}^2\right)\dot{\theta}^2 + \frac{1}{2}r^2(k_1 + k_2)\theta^2 = \text{constant}$$

Taking the time derivative of this equation,

$$m\left(r^2 + k_O{}^2\right)\dot{\theta}\,\ddot{\theta} + r^2(k_1 + k_2)\theta\dot{\theta} = 0$$

$$\dot{\theta}\left[m\left(r^2 + k_O{}^2\right)\ddot{\theta} + r^2(k_1 + k_2)\theta\right] = 0$$

Since $\dot{\theta}$ is not always equal to zero, then

$$m\left(r^2 + k_O{}^2\right)\ddot{\theta} + r^2(k_1 + k_2)\theta = 0$$

$$\ddot{\theta} + \frac{r^2(k_1 + k_2)}{m\left(r^2 + k_O{}^2\right)}\theta = 0$$

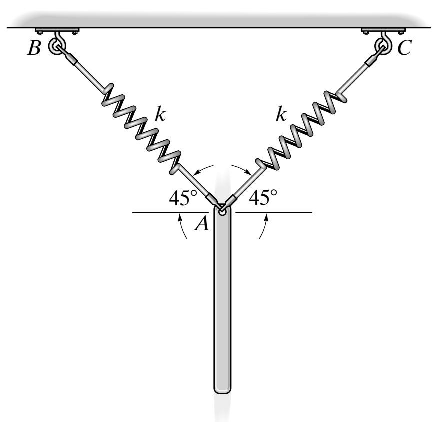
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{r^2(k_1 + k_2)}{m\left(r^2 + k_O{}^2\right)}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m\left(r^2 + k_O{}^2\right)}{r^2(k_1 + k_2)}}$$ **Ans.**

22–41. The bar has a mass of 8 kg and is suspended from two springs such that when it is in equilibrium, the springs make an angle of 45° with the horizontal as shown. Determine the natural period of vibration if the bar is pulled down a short distance and released. Each spring has a stiffness of $k = 40$ N/m.



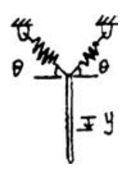
$$E = 2\left(\frac{1}{2}\right)k(s_{eq} + y\sin\theta)^2 - mgy + \frac{1}{2}m\dot{y}^2$$

$$\dot{E} = 2k(s_{eq} + y\sin\theta)\dot{y}\sin\theta - mg\dot{y} + m\dot{y}\,\ddot{y} = 0$$

$$2k\left(\frac{mg}{2k\sin\theta} + y\sin\theta\right)\sin\theta - mg + m\ddot{y} = 0$$

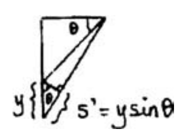
$$2ky\sin^2\theta + m\ddot{y} = 0$$

$$\ddot{y} + \frac{2k\sin^2\theta}{m}y = 0$$



$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sin\theta}\sqrt{\frac{m}{2k}} = \frac{2\pi}{\sin 45^\circ}\sqrt{\frac{8}{2(40)}}$$

$$\tau = 2.81\text{ s}$$ **Ans.**



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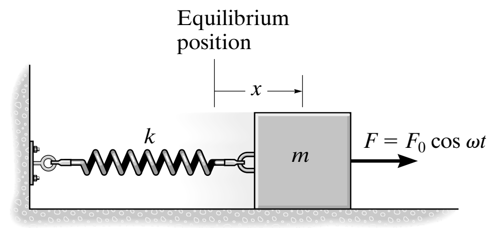
22–42. If the block-and-spring model is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{x} + (k/m)x = (F_0/m) \cos \omega t$, where x is measured from the equilibrium position of the block. What is the general solution of this equation?



$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \cos \omega t - kx = m\ddot{x}$

$$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m} \cos \omega t \qquad \textbf{(Q.E.D.)}$$

$$\ddot{x} + p^2 x = \frac{F_0}{m} \cos \omega t \qquad \text{Where } p = \sqrt{\frac{k}{m}} \qquad \textbf{(1)}$$

The general solution of the above differential equation is of the form of $x = x_c + x_p$.

The complementary solution:

$x_c = A \sin pt + B \cos pt$

The particular solution:

$s_p = .C \cos \omega t$ **(2)**

$\ddot{x}_P = -C\omega^2 \cos \omega\, t$ **(3)**

Substitute Eqs. (2) and (3) into (1) yields:

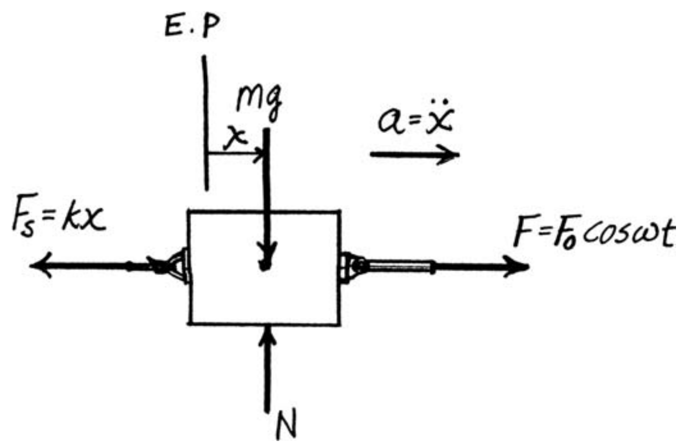
$$-C\omega^2 \cos \omega t + p^2 (C \cos \omega t) = \frac{F_0}{m} \cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{p^2 - \omega^2} = \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2}$$

The general solution is therefore

$$s = A \sin pt + B \cos pt + \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2} \cos \omega t \qquad \textbf{Ans.}$$

The constants A and B can be found from the initial conditions.



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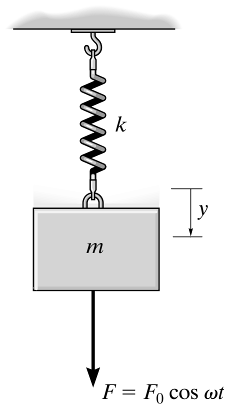
22–43. If the block is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{y} + (k/m)y = (F_0/m) \cos \omega t$, where y is measured from the equilibrium position of the block. What is the general solution of this equation?



$+\downarrow \Sigma F_y = ma_y; \qquad F_0 \cos \omega t + W - k\delta_{st} - ky = m\ddot{y}$

Since $W = k\delta_{st}$,

$$\ddot{y} + \left(\frac{k}{m}\right)y = \frac{F_0}{m} \cos \omega t \qquad \textbf{(1) (Q.E.D.)}$$

$y_c = A \sin py + B \cos py$ (complementary solution)

$y_p = C \cos \omega t$ (particular solution)

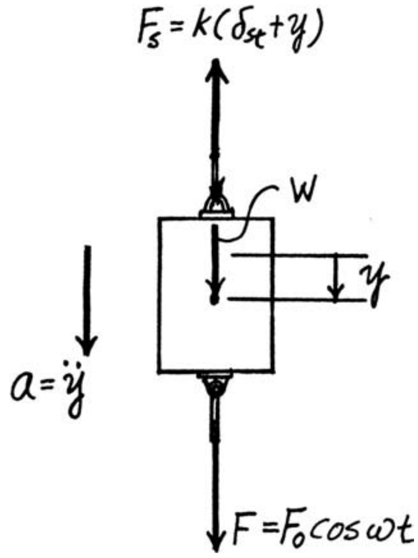
Substitute y_p into Eq. (1).

$$C\left(-\omega^2 + \frac{k}{m}\right)\cos \omega t = \frac{F_0}{m} \cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{\left(\frac{k}{m} - \omega^2\right)}$$

$$y = y_c + y_p$$

$$y = A \sin pt + B \cos pt + \left(\frac{F_0}{(k - m\omega^2)}\right)\cos \omega t \qquad \textbf{Ans.}$$



*__22–44.__ A block having a mass of 0.8 kg is suspended from a spring having a stiffness of 120 N/m. If a dashpot provides a damping force of 2.5 N when the speed of the block is 0.2 m/s, determine the period of free vibration.

$$F = cv \qquad c = \frac{F}{v} = \frac{2.5}{0.2} = 12.5 \text{ N} \cdot \text{s/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{120}{0.8}} = 12.247 \text{ rad/s}$$

$$C_c = 2mp = 2(0.8)(12.247) = 19.60 \text{ N} \cdot \text{s/m}$$

$$p_d = p\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 12.247\sqrt{1 - \left(\frac{12.5}{19.6}\right)^2} = 9.432 \text{ rad/s}$$

$$\tau_d = \frac{2\pi}{p_d} = \frac{2\pi}{9.432} = 0.666 \text{ s} \qquad \textbf{Ans.}$$

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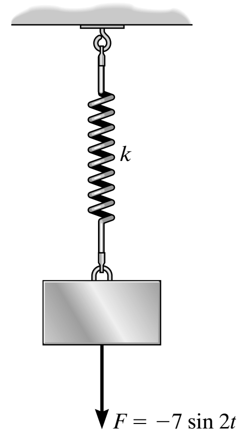
•22–45. The spring shown stretches 6 in. when it is loaded with a 50-lb weight. Determine the equation which describes the position of the weight as a function of time if the weight is pulled 4 in. below its equilibrium position and released from rest at $t = 0$. The weight is subjected to the periodic force of $F = (-7 \sin 2t)$ lb, where t is in seconds.



$$+\uparrow \Sigma F_y = ma_y; \qquad k(y_{st} + y) - mg - F_O \sin \omega t = -m\ddot{y}$$

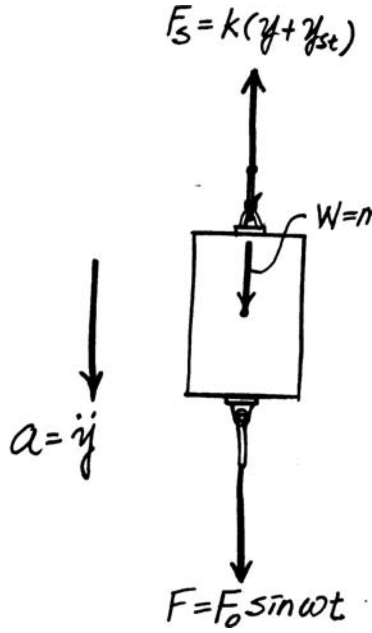
$$m\ddot{y} + ky + ky_{st} - mg = F_O \sin \omega t$$

However, from equilibrium $ky_{st} - mg = 0$, therefore

$$m\ddot{y} + ky = F_O \sin \omega t$$

$$\ddot{y} + \frac{k}{m} y = \frac{F_O}{m} \sin \omega t \qquad \text{where } p = \sqrt{\frac{k}{m}}$$

$$\ddot{y} + p^2 y = \frac{F_O}{m} \sin \omega t$$



From the text, the general solution of the above differential equation is

$$y = A \sin pt + B \cos pt + \frac{F_O/k}{1 - \left(\frac{\omega}{p}\right)^2} \sin \omega t$$

$$v = \dot{y} = Ap \cos pt - Bp \sin pt + \frac{(F_O/k)\omega}{1 - \left(\frac{\omega}{p}\right)^2} \cos \omega t$$

The initial condition when $t = 0$, $y = y_0$ and $v = v_0$.

$$y_0 = 0 + B + 0 \qquad B = y_0$$

$$v_0 = Ap - 0 + \frac{(F_O/k)\omega}{1 - \left(\frac{\omega}{p}\right)^2} \qquad A = \frac{v_0}{p} - \frac{(F_O/k)\omega}{p - \frac{\omega^2}{p}}$$

The solution is therefore

$$y = \left(\frac{v_0}{p} - \frac{(F_O/k)\omega}{p - \frac{\omega^2}{p}}\right) \sin pt + y_0 \cos pt + \frac{F_O/k}{1 - \left(\frac{\omega}{p}\right)^2} \sin \omega t$$

For this problem:

$$k = \frac{50}{6/12} = 100 \text{ lb/ft} \qquad p = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{50/32.2}} = 8.025 \text{ rad/s}$$

$$\frac{F_O/k}{1 - \left(\frac{\omega}{p}\right)^2} = \frac{-7/100}{1 - \left(\frac{2}{8.025}\right)^2} = -0.0746 \qquad y_0 = 0.333$$

$$\frac{v_0}{p} - \frac{(F_O/k)\omega}{p - \frac{\omega^2}{p}} = 0 - \frac{(-7/100)2}{8.025 - \frac{2^2}{8.025}} = 0.0186$$

$$y = (0.0186 \sin 8.02t + 0.333 \cos 8.02t - 0.0746 \sin 2t) \text{ ft} \qquad \textbf{Ans.}$$

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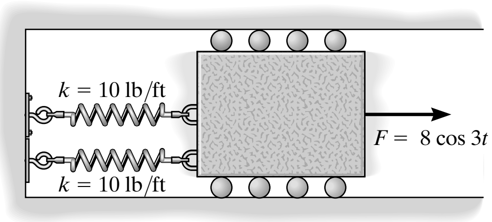
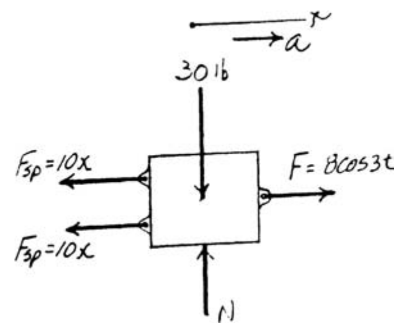
22–46. The 30-lb block is attached to two springs having a stiffness of 10 lb/ft. A periodic force $F = (8 \cos 3t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



Free-body Diagram: When the block is being displaced by amount x to the right, the *restoring force* that develops in both springs is $F_{sp} = kx = 10x$.



Equation of Motion:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -2(10x) + 8\cos 3t = \frac{30}{32.2}a$$

$$a + 21.47x = 8.587 \cos 3t \qquad \textbf{[1]}$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substituting this value into Eq. [1], we have

$$\ddot{x} + 21.47x = 8.587 \cos 3t \qquad \textbf{[2]}$$

Since the friction will eventually dampen out the free vibration, we are only interested in the *particular solution* of the above differential equation which is in the form of

$$x_p = C \cos 3t$$

Taking second time derivative and substituting into Eq. [2], we have

$$-9C \cos 3t + 21.47C \cos 3t = 8.587 \cos 3t$$

$$C = 0.6888 \text{ ft}$$

Thus,

$$x_p = 0.6888 \cos 3t \qquad \textbf{[3]}$$

Taking the time derivative of Eq. [3], we have

$$v_p = \dot{x}_p = -2.0663 \sin 3t$$

Thus,

$$(v_p)_{max} = 2.07 \text{ ft/s} \qquad \textbf{Ans.}$$

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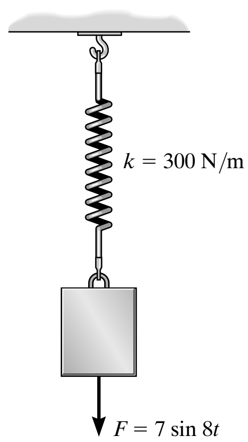
22–47. A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical periodic force $F = (7 \sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Consider positive displacement to be downward.



The general solution is defined by:

$$v = A \sin pt + B \cos pt + \left(\frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right) \sin \omega t$$

Since

$$F = 7 \sin 8t, \qquad F_0 = 7 \text{ N} \qquad \omega = 8 \text{ rad/s}, \qquad k = 300 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}} = 7.746 \text{ rad/s}$$

Thus,

$$y = A \sin 7.746t + B \cos 7.746t + \left(\frac{\frac{7}{300}}{1 - \left(\frac{8}{7.746}\right)^2} \right) \sin 8t$$

$$y = 0.1 \text{ m when } t = 0,$$

$$0.1 = 0 + B - 0; \qquad B = 0.1 \text{ m}$$

$$v = A(7.746) \cos 7.746 - B(7.746) \sin 7.746t - (0.35)(8) \cos 8t$$

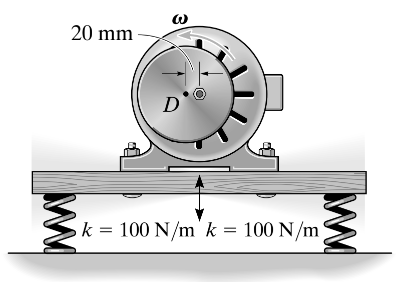
$$v = y = 0 \text{ when } t = 0,$$

$$v = A(7.746) - 2.8 = 0; \qquad A = 0.361$$

Expressing the results in mm, we have

$$y = (361 \sin 7.75t + 100 \cos 7.75t - 350 \sin 8t) \text{ mm}$$ **Ans.**

***22–48.** The electric motor has a mass of 50 kg and is supported by *four springs,* each spring having a stiffness of 100 N/m. If the motor turns a disk D which is mounted eccentrically, 20 mm from the disk's center, determine the angular velocity ω at which resonance occurs. Assume that the motor only vibrates in the vertical direction.



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4(100)}{50}} = 2.83 \text{ rad/s}$$

$$\omega_n = \omega = 2.83 \text{ rad/s}$$ **Ans.**

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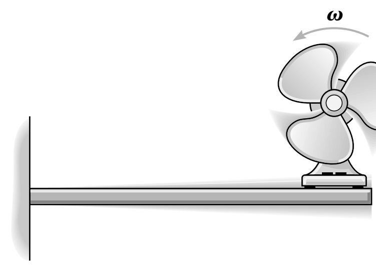
•22–49. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the angular velocity of the fan blade at which resonance will occur. *Hint:* See the first part of Example 22.8.



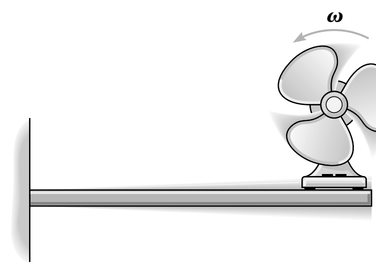
$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905\text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01\text{ rad/s}$$

Resonance occurs when

$$\omega = p = 14.0\text{ rad/s} \qquad \textbf{Ans.}$$

22–50. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the amplitude of steady-state vibration of the fan if the angular velocity of the fan blade is 10 rad/s. *Hint:* See the first part of Example 22.8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905\text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01\text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\,\omega^2 = 3.5(0.1)(10)^2 = 35\text{ N}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left|\frac{\frac{F_0}{k}}{1 - (\frac{\omega}{p})^2}\right|$$

$$(x_p)_{\max} = \left|\frac{\frac{35}{4905}}{1 - (\frac{10}{14.01})^2}\right| = 0.0146\text{ m}$$

$$(x_p)_{\max} = 14.6\text{ mm} \qquad \textbf{Ans.}$$

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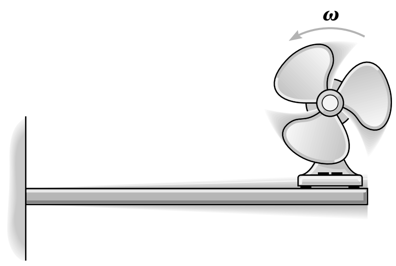
22–51. What will be the amplitude of steady-state vibration of the fan in Prob. 22–50 if the angular velocity of the fan blade is 18 rad/s? *Hint:* See the first part of Example 22.8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(18)^2 = 113.4 \text{ N}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

$$(x_p)_{\max} = \left| \frac{\frac{113.4}{4905}}{1 - \left(\frac{18}{14.01}\right)^2} \right| = 0.0355 \text{ m}$$

$$(x_p)_{\max} = 35.5 \text{ mm}$$ **Ans.**

***22–52.** A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which can be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \omega = 2$$

$$\phi' = \tan^{-1}\left(\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)}{1 - \left(\frac{\omega}{p}\right)^2}\right) = \tan^{-1}\left(\frac{2(0.8)\left(\frac{2}{18.57}\right)}{1 - \left(\frac{2}{18.57}\right)^2}\right)$$

$$\phi' = 9.89^\circ$$ **Ans.**

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•22–53. Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22–52.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \omega = 2$$

$$\text{MF} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)\right]^2}} = \frac{1}{\sqrt{\left[1 - \left(\frac{2}{18.57}\right)^2\right]^2 + \left[2(0.8)\left(\frac{2}{18.57}\right)\right]^2}}$$

$$\text{MF} = 0.997 \qquad \textbf{Ans.}$$

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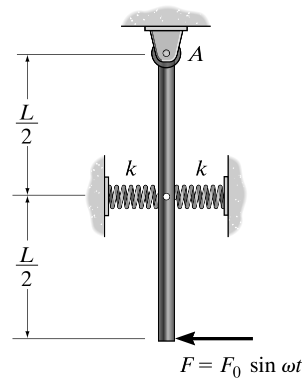
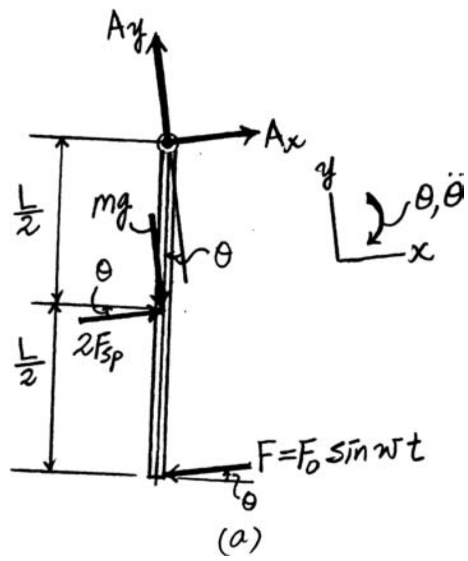
22–54. The uniform rod has a mass of m. If it is acted upon by a periodic force of $F = F_0 \sin \omega t$, determine the amplitude of the steady-state vibration.



Equation of Motion: When the rod rotates through a small angle θ, the springs compress and stretch $s = r_{AG}\theta = \frac{L}{2}\theta$. Thus, the force in each spring is $F_{sp} = ks = \frac{kL}{2}\theta$. The mass moment of inertia of the rod about point A is $I_A = \frac{1}{3}mL^2$. Referring to the free-body diagram of the rod shown in Fig. *a*,

$$+\Sigma M_A = I_A\alpha; \qquad F_O \sin \omega t \cos \theta(L) - mg \sin \theta\left(\frac{L}{2}\right) - 2\left(\frac{kL}{2}\theta\right)\cos\theta\left(\frac{L}{2}\right)$$

$$= \frac{1}{3}mL^2\dot{\theta}$$



Since θ is small, $\sin \theta \cong 0$ and $\cos \theta \cong 1$. Thus, this equation becomes

$$\frac{1}{3}mL\ddot{\theta} + \frac{1}{2}(mg + kL)\theta = F_O \sin \omega t$$

$$\ddot{\theta} + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)\theta = \frac{3F_O}{mL}\sin \omega t \qquad \textbf{(1)}$$

The particular solution of this differential equation is assumed to be in the form of

$$\theta_p = C \sin \omega t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2) twice,

$$\ddot{\theta}_p = -C\omega^2 \sin \omega t \qquad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$-C\omega^2 \sin \omega t + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)(C \sin \omega t) = \frac{3F_O}{mL}\sin \omega t$$

$$C\left[\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2\right]\sin \omega t = \frac{3F_O}{mL}\sin \omega t$$

$$C = \frac{3F_O/mL}{\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2}$$

$$C = \frac{3F_O}{\frac{3}{2}(mg + Lk) - mL\omega^2} \qquad \textbf{Ans.}$$

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22–55. The motion of an underdamped system can be described by the graph in Fig. 20–16. Show that the relation between two successive peaks of vibration is given by $\ln(x_n/x_{n+1}) = 2\pi(c/c_c)/\sqrt{1-(c/c_c)^2}$, where c/c_c is the *damping factor* and $\ln(x_n/x_{n+1})$ is called the *logarithmic decrement*.

If the first peak occurs when $t_n = t$, then the successive peaks occur when $t_{n+1} = l + \tau_d = t + \dfrac{2\pi}{\omega_d} = t + \dfrac{2\pi}{\omega_n\sqrt{1-(c/c_c)^2}}$.

Thus, the two successive peaks are

$$y_n = De^{-((c/c_c)\omega_n)}$$

and

$$y_{n+1} = De^{-\left[(c/c_c)\omega_n\left(t+\frac{2\pi}{\omega_n\sqrt{1-(c/c_c)^2}}\right)\right]}$$

$$= De^{-((c/c_c)\omega_n)t}\, e^{-\frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}}$$

Thus,

$$\frac{y_n}{y_n + 1} = e^{\left(\frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}\right)}$$

$$\ln\left(\frac{y_n}{y_n + 1}\right) = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}} \qquad \textbf{(Q.E.D.)}$$

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***22–56.** Two successive amplitudes of a spring-block underdamped vibrating system are observed to be 100 mm and 75 mm. Determine the damping coefficient of the system. The block has a mass of 10 kg and the spring has a stiffness of $k = 1000$ N/m. Use the result of Prob. 22–55.

Using the result of Prob. 22-55,

$$\ln\left(\frac{y_n}{y_n + 1}\right) = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$\ln\left(\frac{100}{75}\right) = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$\frac{c}{c_c} = 0.04574$$

However,

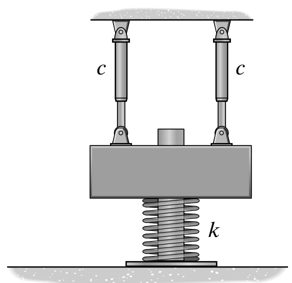
$$c_c = 2m\omega_n = 2(10)\sqrt{\frac{1000}{10}} = 200 \text{ N}\cdot\text{s/m}$$

Thus,

$$\frac{c}{200} = 0.04574$$

$$c = 9.15 \text{ N}\cdot\text{s/m} \qquad \textbf{Ans.}$$

•22–57. Two identical dashpots are arranged parallel to each other, as shown. Show that if the damping coefficient $c < \sqrt{mk}$, then the block of mass m will vibrate as an underdamped system.



When the two dash pots are arranged in parallel, the piston of the dashpots have the same velocity. Thus, the force produced is

$$F = c\dot{y} + c\dot{y} = 2c\dot{y}$$

The equivalent damping coefficient c_{eq} of a single dashpot is

$$c_{eq} = \frac{F}{\dot{y}} = \frac{2c\dot{y}}{\dot{y}} = 2c$$

For the vibration to occur (underdamped system), $c_{eq} < c_c$. However, $c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}}$. Thus,

$$c_{eq} < c_c$$

$$2c < 2m\sqrt{\frac{k}{m}}$$

$$c < \sqrt{mk} \qquad \textbf{Ans.}$$

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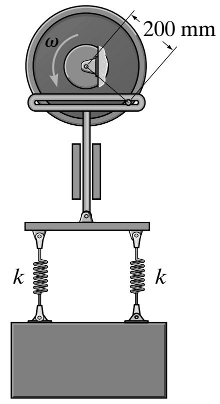
22–58. The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\boldsymbol{\omega}$. If the amplitude of the steady-state vibration is observed to be 400 mm, and the springs each have a stiffness of $k = 2500$ N/m, determine the two possible values of $\boldsymbol{\omega}$ at which the wheel must rotate. The block has a mass of 50 kg.



In this case, $k_{eq} = 2k = 2(2500) = 5000$ N/m Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{5000}{50}} = 10 \text{ rad/s}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{\max} = \pm 0.4$ m, so that

$$(Y_P)_{\max} = \frac{\delta_O}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\frac{\omega}{10}\right)^2}$$

$$\frac{\omega^2}{100} = 1 \pm 0.5$$

Thus,

$$\frac{\omega^2}{100} = 1.5 \qquad \omega = 12.2 \text{ rad/s} \qquad \textbf{Ans.}$$

or

$$\frac{\omega^2}{100} = 0.5 \qquad \omega = 7.07 \text{ rad/s} \qquad \textbf{Ans.}$$

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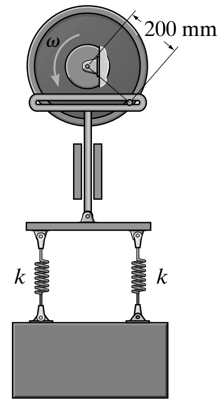
22–59. The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\omega = 5$ rad/s. If the amplitude of the steady-state vibration is observed to be 400 mm, determine the two possible values of the stiffness k of the springs. The block has a mass of 50 kg.



In this case, $k_{eq} = 2k$ Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2k}{50}} = \sqrt{0.04k}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{\max} = \pm 0.4$ m, so that

$$(Y_P)_{\max} = \frac{\delta_O}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\frac{5}{\sqrt{0.04k}}\right)^2}$$

$$\frac{625}{k} = 1 \pm 0.5$$

Thus,

$$\frac{625}{k} = 1.5 \qquad k = 417 \text{ N/m} \qquad \textbf{Ans.}$$

or

$$\frac{625}{k} = 0.5 \qquad k = 1250 \text{ N/m} \qquad \textbf{Ans.}$$

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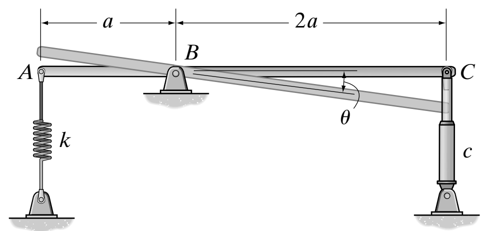
***22–60.** Find the differential equation for small oscillations in terms of θ for the uniform rod of mass m. Also show that if $c < \sqrt{mk}/2$, then the system remains underdamped. The rod is in a horizontal position when it is in equilibrium.



Equation of Motion: When the rod is in equilibrium, $\theta = 0°$, $F_c = c\dot{y}_c = 0$ and $\ddot{\theta} = 0$. writing the moment equation of motion about point B by referring to the free-body diagram of the rod, Fig. a,

$$+\Sigma M_B = 0; \qquad -F_A\,(a) - mg\left(\frac{a}{2}\right) = 0 \qquad F_A = \frac{mg}{2}$$

Thus, the initial stretch of the spring is $s_O = \dfrac{F_A}{k} = \dfrac{mg}{2k}$. When the rod rotates about point B through a small angle θ, the spring stretches further by $s_1 = a\theta$. Thus, the force in the spring is $F_A = k(s_0 + s_1) = k\left(\dfrac{mg}{2k} + a\theta\right)$. Also, the velocity of end C of the rod is $v_c = \dot{y}_c = 2a\dot{\theta}$. Thus, $F_c = c\dot{y}_c = c(2a\dot{\theta})$. The mass moment of inertia of the rod about B is $I_B = \dfrac{1}{12}m(3a)^2 + m\left(\dfrac{a}{2}\right)^2 = ma^2$. Again, referring to Fig. a and writing the moment equation of motion about B,

$$\Sigma M_B = I_B\,\alpha; \qquad k\left(\frac{mg}{2k} + a\theta\right)\cos\theta(a) + \left(2a\dot{\theta}\right)\cos\theta(2a) - mg\cos\theta\left(\frac{a}{2}\right)$$

$$= -ma^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{4c}{m}\cos\theta\dot{\theta} + \frac{k}{m}(\cos\theta)\theta = 0$$

Since θ is small, $\cos\theta \cong 1$. Thus, this equation becomes

$$\ddot{\theta} + \frac{4c}{m}\dot{\theta} + \frac{k}{m}\theta = 0$$ **Ans.**

Comparing this equation to that of the standard form,

$$\omega_n = \sqrt{\frac{k}{m}} \qquad c_{eq} = 4c$$

Thus,

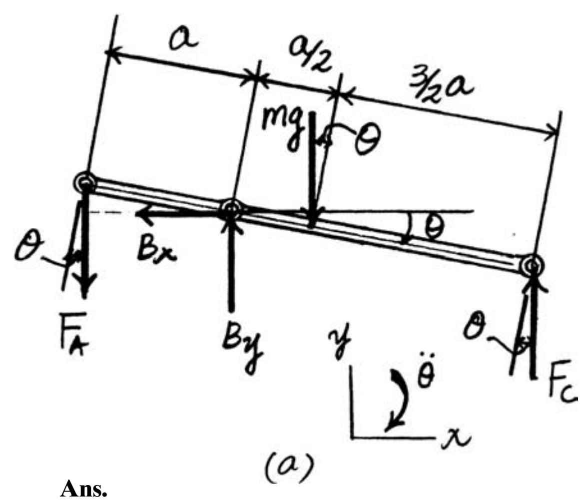
$$c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk}$$

For the system to be underdamped,

$$c_{eq} < c_c$$

$$4c < 2\sqrt{mk}$$

$$c < \frac{1}{2}\sqrt{mk}$$ **Ans.**



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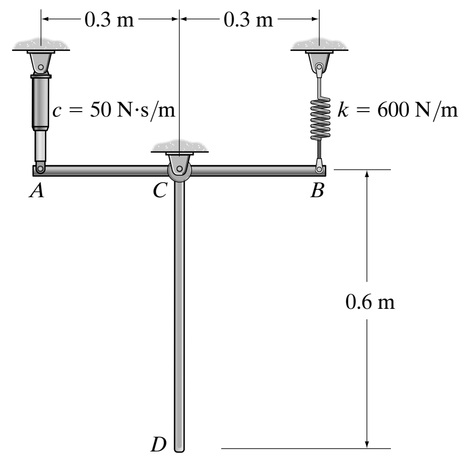
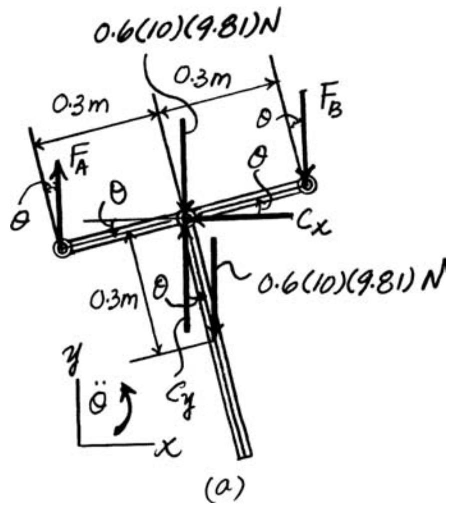
•22–61. If the dashpot has a damping coefficient of $c = 50\ \text{N}\cdot\text{s/m}$, and the spring has a stiffness of $k = 600\ \text{N/m}$, show that the system is underdamped, and then find the pendulum's period of oscillation. The uniform rods have a mass per unit length of 10 kg/m.



Equation of Motion: When the pendulum rotates point C through a small angle θ, the spring compresses $s = 0.3\theta$. Thus, the force in the spring is $F_B = ks = 600(0.3\theta) = 180\theta$. Also, the velocity of end A is $v_A = \dot{y}_A = 0.3\dot{\theta}$. Thus, $F_A = c\dot{y}_A = 50(0.3\dot{\theta}) = 15\dot{\theta}$. The mass moment of inertia of the pendulum about point C is $I_C = \frac{1}{12}\left[0.6(10)\left(0.6^2\right)\right] + \frac{1}{3}\left[0.6(10)\left(0.6^2\right)\right] = 0.9\ \text{kg}\cdot\text{m}^2$. Referring to the free-body diagram of the pendulum, Fig. a,



$$\Sigma M_C = I_C\,\alpha; \qquad -180\theta\cos\theta(0.3) - 15\dot{\theta}\cos\theta(0.3) - 0.6(10)(9.81)\sin\theta(0.3) = 0.9\ddot{\theta}$$

$$\ddot{\theta} + 5\cos\theta\dot{\theta} + 60\cos\theta\theta + 16.62\sin\theta = 0$$

Since θ is small, $\sin\theta \cong \theta$ and $\cos\theta \cong 1$. Thus, this equation becomes

$$\ddot{\theta} + 5\dot{\theta} + 79.62 = 0$$

Comparing this equation to that of the standard form,

$$c_{eq} = 5 \qquad\qquad \omega_n = 8.923\ \text{rad/s}$$

Here, $m = 2\left[(10)(0.6)\right] = 12\ \text{kg}$. Thus, $c_{eq} = 5m = 5(12) = 60\ \text{N}\cdot\text{m/s}$. Also, $c_c = 2m\omega_n = 2(12)8.923 = 214.15\ \text{N}\cdot\text{m/s}$. Since $c_{eq} < c_c$, the system is *underdamped* (Q.E.D.). Thus,

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c_{eq}}{c_c}\right)^2}$$

$$= 8.923\left(\sqrt{1 - \left(\frac{60}{214.15}\right)^2}\right)$$

$$= 8.566\ \text{rad/s}$$

Thus, the period of under-damped oscillation of the pendulum is

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{8.566} = 0.734\ \text{s} \qquad \textbf{Ans.}$$

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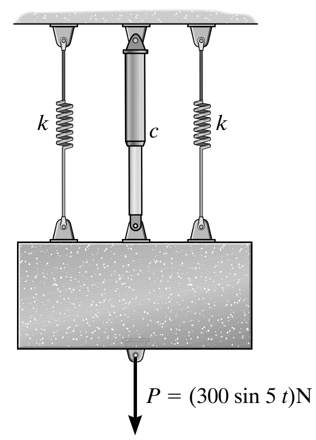
22–62. If the 30-kg block is subjected to a periodic force of $P = (300 \sin 5t)$ N, $k = 1500$ N/m, and $c = 300$ N·s/m, determine the equation that describes the steady-state vibration as a function of time.



Here, $k_{eq} = 2k = 2(1500) = 3000$ N/m. Thus, the circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{3000}{30}} = 10 \text{ rad/s}$$

The critical damping coefficient is

$$c_c = 2m\omega_n = 2(30)(10) = 600 \text{ N}\cdot\text{s/m}$$

Then, the damping factor is

$$\frac{c}{c_c} = \frac{300}{600} = 0.5$$

Here, $F_O = 300$ N and $\omega = 5$ rad/s.

$$Y = \frac{F_O / k_{eq}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[\left(2\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$= \frac{300/3000}{\sqrt{\left[1 - \left(\frac{5}{10}\right)^2\right]^2 + \left[\frac{2(0.5)(5)}{10}\right]^2}}$$

$$= 0.1109 \text{ m}$$

$$\phi' = \tan^{-1}\left[\frac{2\frac{c}{c_c}\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right] = \tan^{-1}\left[\frac{2(0.5)\left(\frac{5}{10}\right)}{1 - \left(\frac{5}{10}\right)^2}\right] = 33.69° = 0.588 \text{ rad}$$

Thus,

$$y_P = 0.111 \sin (5t - 0.588) \text{ m}$$ **Ans.**

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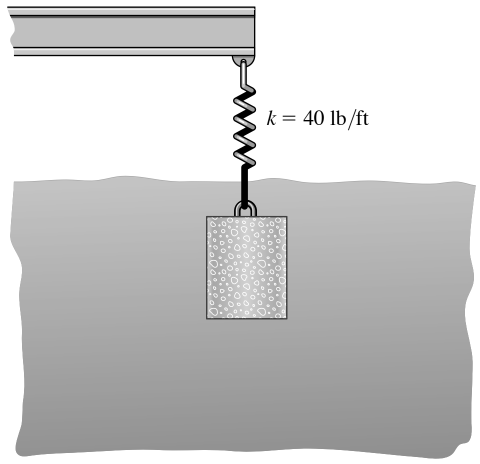
22–63. The block, having a weight of 15 lb, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.8|v|)$ lb, where v is the velocity of the block in ft/s. If the block is pulled down 0.8 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 40$ lb/ft. Consider positive displacement to be downward.



Viscous Damped Free Vibration: Here $c = 0.8 \text{ lb}\cdot\text{s/ft}$, $p = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{40}{15/32.2}}$ $= 9.266$ rad/s and $c_c = 2mp = 2\left(\dfrac{15}{32.2}\right)(9.266) = 8.633 \text{ lb}\cdot\text{s/ft}$. Since $c < c_c$, the system is underdamped and the solution of the differential equation is in the form of

$$y = D\left[e^{-(c/2m)t}\sin(p_d t + \phi)\right] \qquad \textbf{[1]}$$

Taking the time derivative of Eq. [1], we have

$$v = \dot{y} = D\left[-\left(\frac{c}{2m}\right)e^{-(c/2m)t}\sin(p_d t + \phi) + p_d e^{-(c/2m)t}\cos(p_d t + \phi)\right]$$

$$= De^{-(c/2m)t}\left[-\left(\frac{c}{2m}\right)\sin(p_d t + \phi) + p_d\cos(p_d t + \phi)\right] \qquad \textbf{[2]}$$

Applying the initial condition $v = 0$ at $t = 0$ to Eq. [2], we have

$$0 = De^{-0}\left[-\left(\frac{c}{2m}\right)\sin(0 + \phi) + p_d\cos(0 + \phi)\right]$$

$$0 = D\left[-\left(\frac{c}{2m}\right)\sin\phi + p_d\cos\phi\right] \qquad \textbf{[3]}$$

Here, $\dfrac{c}{2m} = \dfrac{0.8}{2(15/32.2)} = 0.8587$ and $p_d = p\sqrt{1 - \left(\dfrac{c}{c_c}\right)^2} = 9.266\sqrt{1 - \left(\dfrac{0.8}{8.633}\right)^2}$ $= 9.227$ rad/s. Substituting these values into Eq. [3] yields

$$0 = D[-0.8587\sin\phi + 9.227\cos\phi] \qquad \textbf{[4]}$$

Applying the initial condition $y = 0.8$ ft at $t = 0$ to Eq. [1], we have

$$0.8 = D\left[e^{-0}\sin(0 + \phi)\right]$$

$$0.8 = D\sin\phi \qquad \textbf{[5]}$$

Solving Eqs. [4] and [5] yields

$$\phi = 84.68° = 1.50 \text{ rad} \qquad D = 0.8035 \text{ ft}$$

Substituting these values into Eq. [1] yields

$$y = 0.803\left[e^{-0.8597}\sin(9.23t + 1.48)\right] \qquad \textbf{Ans.}$$

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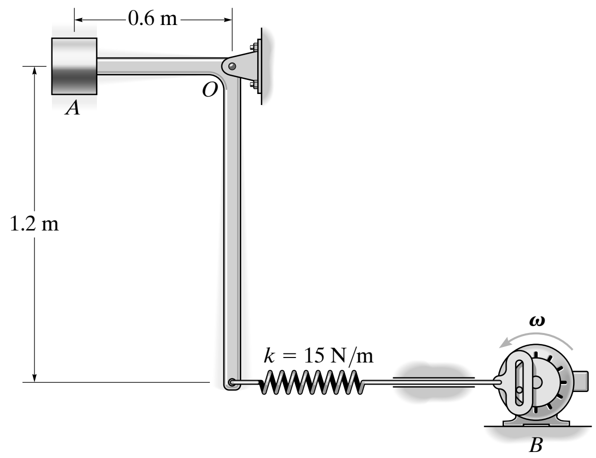
***22–64.** The small block at A has a mass of 4 kg and is mounted on the bent rod having negligible mass. If the rotor at B causes a harmonic movement $\delta_B = (0.1 \cos 15t)$ m, where t is in seconds, determine the steady-state amplitude of vibration of the block.



$$+\Sigma M_O = I_O\,\alpha; \qquad 4(9.81)(0.6) - F_s(1.2) = 4(0.6)^2\ddot{\theta}$$

$$F_s = kx = 15(x + x_{st} - 0.1 \cos 15t)$$

$$x_{st} = \frac{4(9.81)(0.6)}{1.2(15)}$$

Thus,

$$-15(x - 0.1 \cos 15t)(1.2) = 4(0.6)^2\ddot{\theta}$$

$$x = 1.2\theta$$

$$\theta + 15\theta = 1.25 \cos 15t$$

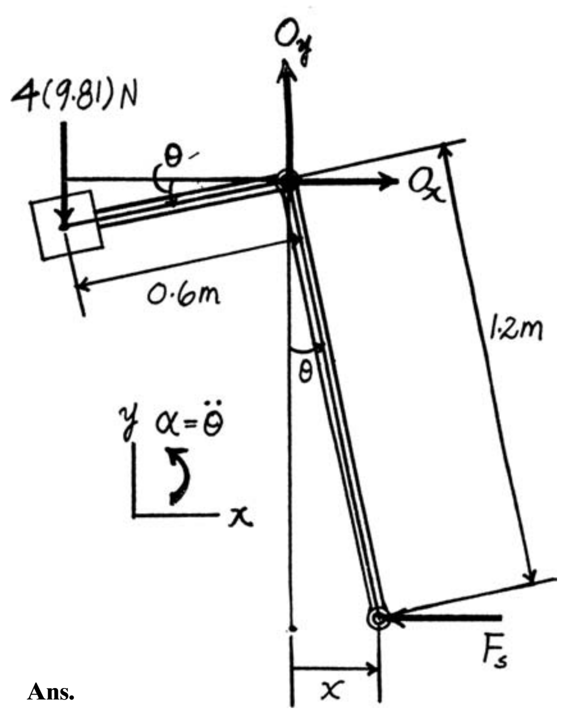
Set $x_p = C \cos 15t$

$$-C(15)^2 \cos 15t + 15(C \cos 15t) = 1.25 \cos 15t$$

$$C = \frac{1.25}{15 - (15)^2} = -0.00595 \text{ m}$$

$$\theta_{max} = C = 0.00595 \text{ rad}$$

$$y_{max} = (0.6 \text{ m})(0.00595 \text{ rad}) = 3.57 \text{ rad} \qquad \textbf{Ans.}$$



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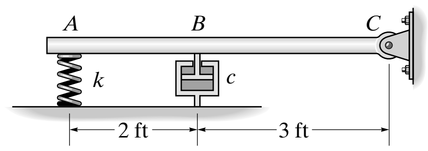
•22–65. The bar has a weight of 6 lb. If the stiffness of the spring is $k = 8\ \text{lb/ft}$ and the dashpot has a damping coefficient $c = 60\ \text{lb}\cdot\text{s/ft}$, determine the differential equation which describes the motion in terms of the angle θ of the bar's rotation. Also, what should be the damping coefficient of the dashpot if the bar is to be critically damped?



$$\zeta + \Sigma M_A = I_A\,\alpha; \qquad 6(2.5) - (60\dot{y}_2)(3) - 8(y_1 + y_{st})(5) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(5)^2\right]\ddot{\theta}$$

$$1.5528\ddot{\theta} + 180\dot{y}_2 + 40y_1 + 40y_{st} - 15 = 0 \qquad \textbf{[1]}$$

From equilibrium $40y_{st} - 15 = 0$. Also, for small θ, $y_1 = 5\theta$ and $y_2 = 3\theta$ hence $\dot{y}_2 = 3\theta$.

From Eq. [1] $\qquad 1.5528\ddot{\theta} + 180(3\dot{\theta}) + 40(5\theta) = 0$

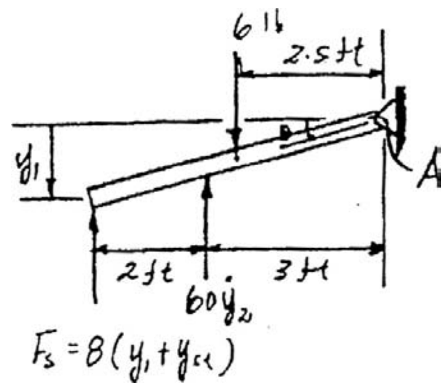
$$1.55\dot{\theta} + 540\dot{\theta} + 200\theta = 0 \qquad \textbf{Ans.}$$

By comparing the above differential equation to Eq. 22-27

$$m = 1.55 \qquad k = 200 \qquad \omega_n = \sqrt{\frac{200}{1.55}} = 11.35\ \text{rad/s} \qquad c = 9c_{d\cdot p}$$

$$\left(\frac{9\left(c_{d\cdot p}\right)_c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$\left(c_{d\cdot p}\right)_c = \frac{2}{9}\sqrt{km} = \frac{2}{9}\sqrt{200(1.55)} = 3.92\ \text{lb}\cdot\text{s/ft} \qquad \textbf{Ans.}$$



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22–66. A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$ N, where v is the velocity of the block in m/s.

$$c = 50 \text{ N s/m} \quad k = 600 \text{ N/m} \quad m = 7 \text{ kg}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{7}} = 9.258 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(7)(9.258) = 129.6 \text{ N} \cdot \text{s/m}$$

Since $c < c_z$, the system is underdamped,

$$\omega_d = \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.258 \sqrt{1 - \left(\frac{50}{129.6}\right)^2} = 8.542 \text{ rad/s}$$

$$\frac{c}{2m} = \frac{50}{2(7)} = 3.751$$

From Eq. 22-32

$$y = D\left[e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t} \omega_d \cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right) e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d \cos(\omega_d t + \phi) - \frac{c}{2m} \sin(\omega_d t + \phi)\right]$$

Applying the initial condition at $t = 0$, $y = 0$ and $v = -0.6$ m/s.

$$0 = D[e^{-0} \sin(0 + \phi)] \qquad \text{since} \quad D \neq 0$$

$$\sin\phi = 0 \qquad \phi = 0°$$

$$-0.6 = De^{-0}[8.542 \cos 0° - 0]$$

$$D = -0.0702 \text{ m}$$

$$y = \{-0.0702[e^{-3.57t} \sin(8.540)]\} \text{ m}$$ **Ans.**

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22–67. A 4-lb weight is attached to a spring having a stiffness $k = 10\text{ lb/ft}$. The weight is drawn downward a distance of 4 in. and released from rest. If the support moves with a vertical displacement $\delta = (0.5 \sin 4t)$ in., where t is in seconds, determine the equation which describes the position of the weight as a function of time.

From Prob. 22-46

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_0}\right)^2} \sin \omega_0 t$$

$$v = \dot{y} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t + \frac{\delta_0\, \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \cos \omega_0 t$$

The initial condition when $t = 0$, $y = y_0$ and $v = v_0$

$$y_0 = 0 + B + 0 \qquad B = y_0$$

$$v_0 = A\omega_n - 0 + \frac{\delta_0\, \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \qquad A = \frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}$$

Thus,

$$y = \left(\frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}\right) \sin \omega_n t + y_0 \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{4/32.2}} = 8.972$$

$$\frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} = \frac{0.5/12}{1 - \left(\frac{4}{8.972}\right)^2} = 0.0520$$

$$\frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}} = 0 - \frac{(0.5/12)4}{8.972 - \frac{4^2}{8.972}} = -0.0232$$

$$y = \{-0.0232 \sin 8.97t + 0.333 \cos 8.97t + 0.0520 \sin 4t\}\text{ ft} \qquad \textbf{Ans.}$$

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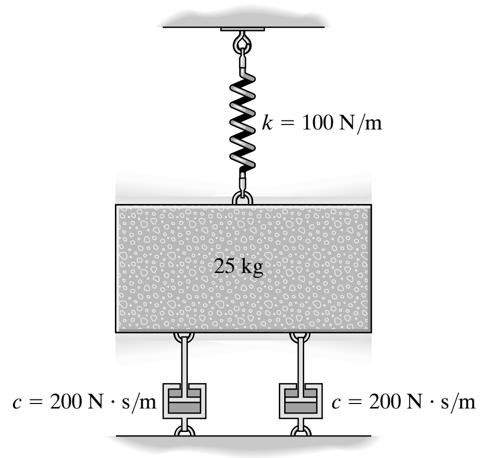
***22–68.** Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs?



$+\downarrow \Sigma F_y = ma_y;$ $\quad mg - k(y + y_{st}) - 2c\dot{y} = m\ddot{y}$

$$m\ddot{y} + ky + 2c\dot{y} + ky_{st} - mg = 0$$

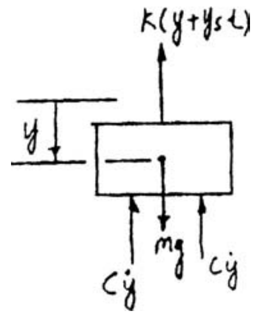
Equilibrium $\quad ky_{st} - mg = 0$

$$m\ddot{y} + 2c\dot{y} + ky = 0 \text{ Here } m = 25\text{ kg} \quad k = 100\text{ N/m}$$

$$c = 200\text{ N}\cdot\text{s/m}$$

$$25\ddot{y} + 400\dot{y} + 100y = 0 \qquad \textbf{(1)}$$

$$\ddot{y} + 16\dot{y} + 4y = 0 \qquad \textbf{Ans.}$$



By comparing Eq. (1) to Eq. 22–27

$$m = 25 \qquad k = 100 \qquad c = 400 \qquad \omega_n = \sqrt{\frac{4}{1}} = 2\text{ rad/s}$$

$$c_c = 2m\omega_n = 2(25)(2) = 100\text{ N}\cdot\text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore, it is **overdamped**. **Ans.**

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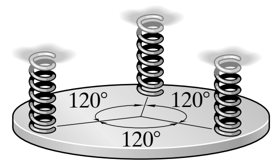
•22–69. The 4-kg circular disk is attached to three springs, each spring having a stiffness $k = 180\ \text{N/m}$. If the disk is immersed in a fluid and given a downward velocity of 0.3 m/s at the equilibrium position, determine the equation which describes the motion. Consider positive displacement to be measured downward, and that fluid resistance acting on the disk furnishes a damping force having a magnitude $F = (60|v|)\ \text{N}$, where v is the velocity of the block in m/s.



$$k = 540\ \text{N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{540}{4}} = 11.62\ \text{rad/s}$$

$$c_c = 2m\omega_n = 2(4)(11.62) = 92.95$$

$$F = 60v, \text{ so that } c = 60$$

Since $c < c_c$, system is underdamped.

$$\omega_d = \omega_n\sqrt{1 - (\frac{c}{c_c})^2}$$

$$= 11.62\sqrt{1 - (\frac{60}{92.95})^2}$$

$$= 8.87\ \text{rad/s}$$

$$y = A[e^{-(\frac{c}{2m})t}\sin(\omega_d t + \phi)] \qquad \textbf{(1)}$$

$$y = 0, v = 0.3 \text{ at } t = 0$$

$$0 = A\sin\phi$$

$$A \neq 0 \text{ (trivial solution) so that } \phi = 0$$

$$v = y = A[-\frac{c}{2m}e^{-(\frac{c}{2m})t}\sin(\omega_d t + \phi) + e^{-(\frac{c}{2m})t}\cos(\omega_d t + \phi)(\omega_d)]$$

Since $\phi = 0$

$$0.3 = A[0 + 1(8.87)]$$

$$A = 0.0338$$

Substituting into Eq. (1)

$$y = 0.0338[e^{-(\frac{60}{2(4)})t}\sin(8.87)t]$$

Expressing the result in mm

$$y = 33.8[e^{-7.5t}\sin(8.87t)]\ \text{mm} \qquad \textbf{Ans.}$$

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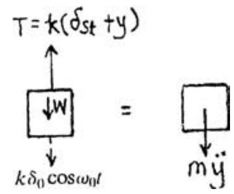
22–70. Using a block-and-spring model, like that shown in Fig. 22–13*a*, but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.



$+\downarrow \Sigma F_y = ma_y; \qquad k\delta_0 \cos \omega_0 t + W - k\delta_{st} - ky = m\ddot{y}$

Since $W = k\delta_{st}$,

$$\ddot{y} + \frac{k}{m} y = \frac{k\delta_0}{m} \cos \omega_0 t \qquad \textbf{(1)}$$

$$y_C = A \sin \omega_n y + B \cos \omega_n y \text{ (General sol.)}$$

$$y_p = C \cos \omega_0 t \text{ (Particular sol.)}$$

Substitute y_p into Eq. (1)

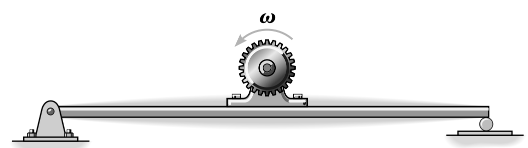
$$C(-\omega_0{}^2 + \frac{k}{m}) \cos \omega_0 t = \frac{k\delta_0}{m} \cos \omega_0 t$$

$$C = \frac{\frac{k\delta_0}{m}}{(\frac{k}{m} - \omega_0{}^2)}$$

Thus, $y = y_C + y_P$

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\frac{k\delta_0}{m}}{(\frac{k}{m} - \omega_0{}^2)} \cos \omega_0 t \qquad \textbf{Ans.}$$

22–71. The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. due to the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weights 150 lb. Neglect the mass of the beam.



$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.66$$

Resonance occurs when $\qquad \omega_0 = \omega_n = 19.7 \text{ rad/s} \qquad$ **Ans.**

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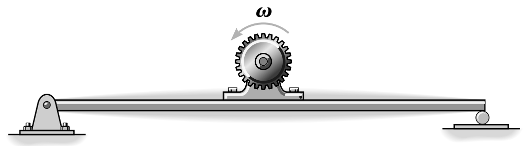
***22–72.** What will be the amplitude of steady-state vibration of the motor in Prob. 22–71 if the angular velocity of the flywheel is 20 rad/s?



The constant value F_0 of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_0 = ma_n = mr\omega_0{}^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)(20)^2 = 2.588 \text{ lb}$$

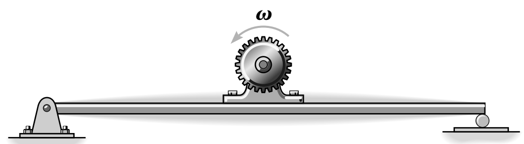
Hence $F = 2.588 \sin 20t$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22–21, the amplitude of the steady state motion is

$$C = \left|\frac{F_0/k}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right| = \left|\frac{2.588/1800}{1 - \left(\frac{20}{19.657}\right)^2}\right| = 0.04085 \text{ ft} = 0.490 \text{ in.} \qquad \textbf{Ans.}$$

•22–73. Determine the angular velocity of the flywheel in Prob. 22–71 which will produce an amplitude of vibration of 0.25 in.



The constant value F_0 of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)\omega^2 = 0.006470\omega^2$$

$$F = 0.006470\omega^2 \sin \omega t$$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad p = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22.21, the amplitude of the steady state motion is

$$C = \left|\frac{F_O/k}{1 - \left(\frac{\omega}{p}\right)^2}\right|$$

$$\frac{0.25}{12} = \left|\frac{0.006470\left(\frac{\omega^2}{1800}\right)}{1 - \left(\frac{\omega}{19.657}\right)^2}\right|$$

$$\omega = 19.0 \text{ rad/s} \qquad \textbf{Ans.}$$

Or,

$$\theta_0 = 20.3 \text{ rad/} \qquad \textbf{Ans.}$$

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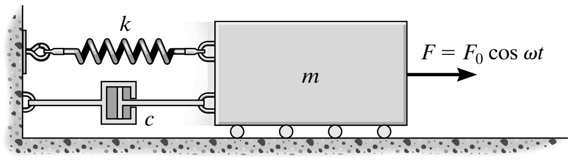
22–74. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.

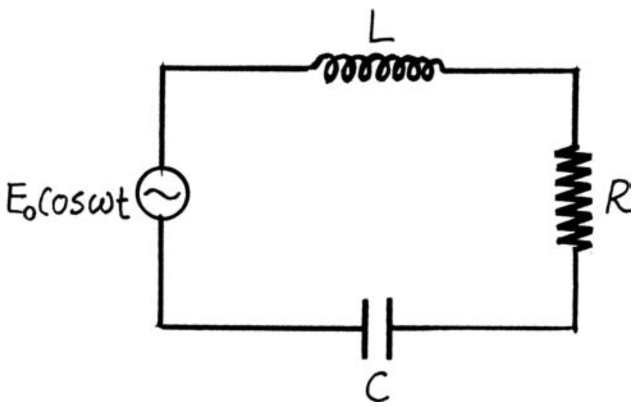


For the block,

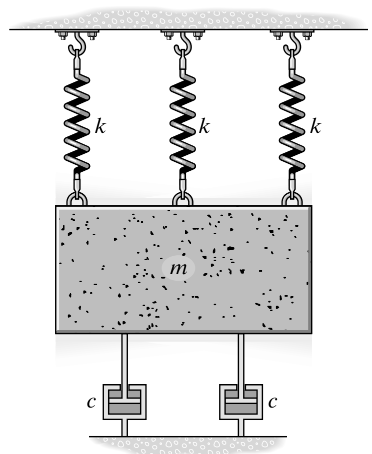
$$mx + cx + kx = F_0 \cos \omega t$$

Using Table 22–1,

$$Lq + Rq + \left(\frac{1}{C}\right)q = E_0 \cos \omega t \qquad \textbf{Ans.}$$



22–75. Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N·s/m, $m = 25$ kg.



Free-body Diagram: When the block is being displaced by an amount y vertically downward, the *restoring force* is developed by the three springs attached the block.

Equation of Motion:

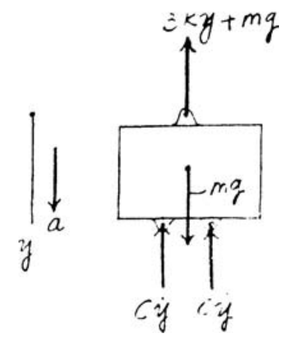
$+\uparrow \Sigma F_x = 0; \qquad 3ky + mg + 2c\dot{y} - mg = -m\ddot{y}$

$$m\ddot{y} + 2c\dot{y} + 3ky = 0 \qquad [1]$$

Here, $m = 25$ kg, $c = 200$ N·s/m and $k = 100$ N/m. Substituting these values into Eq. [1] yields

$$25\ddot{y} + 400\dot{y} + 300y = 0$$

$$\ddot{y} + 16\dot{y} + 12y = 0 \qquad \textbf{Ans.}$$



Comparing the above differential equation with Eq. 22–27, we have $m = 1$kg, $c = 16$ N·s/m and $k = 12$ N/m. Thus, $p = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{12}{1}} = 3.464$ rad/s.

$$c_c = 2mp = 2(1)(3.464) = 6928 \text{ N}\cdot\text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped.** **Ans.**

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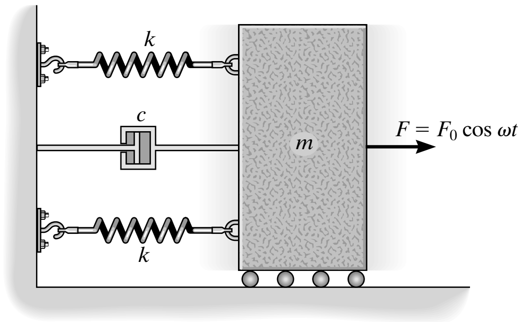
***22–76.** Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?

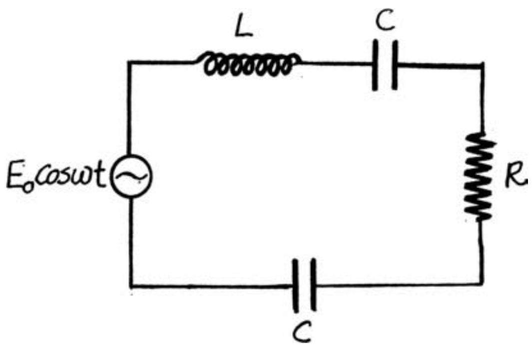


Electrical Circuit Analogs: The differential equation that deseribes the motion of the given mechanical system is

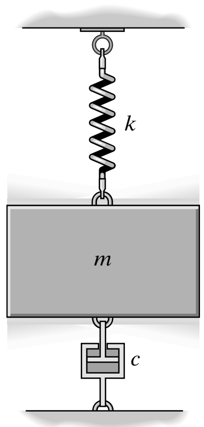
$$m\ddot{x} + c\dot{x} + 2kx = F_0 \cos \omega t$$

From Table 22–1 of the text, the differential equation of the analog electrical circuit is

$$L\ddot{q} + R\dot{q} + \left(\frac{2}{C}\right)q = E_0 \cos \omega t \qquad \textbf{Ans.}$$



•22–77. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



For the block

$$m\ddot{y} + c\dot{y} + ky = 0$$

Using Table 22–1

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0 \qquad \textbf{Ans.}$$