

# Instructors' Solution Manual

# THE SCIENCE AND ENGINEERING OF MATERIALS

Fourth Edition

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# 1

## Introduction to Materials Science and Engineering

- 1–5** Iron is often coated with a thin layer of zinc if it is to be used outside. What characteristics do you think the zinc provides to this coated, or galvanized, steel? What precautions should be considered in producing this product? How will the recyclability of the product be affected?

**Solution:** The zinc provides corrosion resistance to the iron in two ways. If the iron is completely coated with zinc, the zinc provides a barrier between the iron and the surrounding environment, therefore protecting the underlying iron. If the zinc coating is scratched to expose the iron, the zinc continues to protect the iron because the zinc corrodes preferentially to the iron (see Chapter 22). To be effective, the zinc should bond well to the iron so that it does not permit reactions to occur at the interface with the iron and so that the zinc remains intact during any forming of the galvanized material. When the material is recycled, the zinc will be lost by oxidation and vaporization, often producing a “zinc dust” that may pose an environmental hazard. Special equipment may be required to collect and either recycle or dispose of the zinc dust.

- 1–6** We would like to produce a transparent canopy for an aircraft. If we were to use a ceramic (that is, traditional window glass) canopy, rocks or birds might cause it to shatter. Design a material that would minimize damage or at least keep the canopy from breaking into pieces.

**Solution:** We might sandwich a thin sheet of a transparent polymer between two layers of the glass. This approach, used for windshields of automobiles, will prevent the “safety” glass from completely disintegrating when it

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fails, with the polymer holding the broken pieces of glass together until the canopy can be replaced.

Another approach might be to use a transparent, “glassy” polymer material such as polycarbonate. Some polymers have reasonably good impact properties and may resist failure. The polymers can also be toughened to resist impact by introducing tiny globules of a rubber, or elastomer, into the polymer; these globules improve the energy-absorbing ability of the composite polymer, while being too small to interfere with the optical properties of the material.

- 1–7 Coiled springs ought to be very strong and stiff.  $\text{Si}_3\text{N}_4$  is a strong, stiff material. Would you select this material for a spring? Explain.

**Solution:** Springs are intended to resist high elastic forces, where only the atomic bonds are stretched when the force is applied. The silicon nitride would satisfy this requirement. However, we would like to also have good resistance to impact and at least some ductility (in case the spring is overloaded) to assure that the spring will not fail catastrophically. We also would like to be sure that all springs will perform satisfactorily. Ceramic materials such as silicon nitride have virtually no ductility, poor impact properties, and often are difficult to manufacture without introducing at least some small flaws that cause to fail even for relatively low forces. The silicon nitride is NOT recommended.

- 1–8 Temperature indicators are sometimes produced from a coiled metal strip that uncoils a specific amount when the temperature increases. How does this work; from what kind of material would the indicator be made; and what are the important properties that the material in the indicator must possess?

**Solution:** Bimetallic materials are produced by bonding two materials having different coefficients of thermal expansion to one another, forming a laminar composite. When the temperature changes, one of the materials will expand or contract more than the other material. This difference in expansion or contraction causes the bimetallic material to change shape; if the original shape is that of a coil, then the device will coil or uncoil, depending on the direction of the temperature change. In order for the material to perform well, the two materials must have very different coefficients of thermal expansion and should have high enough modulus of elasticity so that no permanent deformation of the material occurs.

- 1–9 You would like to design an aircraft that can be flown by human power nonstop for a distance of 30 km. What types of material properties would you recommend? What materials might be appropriate?

**Solution:** Such an aircraft must possess enough strength and stiffness to resist its own weight, the weight of the human “power source”, and any aerodynamic forces imposed on it. On the other hand, it must be as light as possible to assure that the human can generate enough work to operate the aircraft. Composite materials, particularly those based on a polymer matrix, might comprise the bulk of the aircraft. The polymers have a light weight (with densities of less than half that of aluminum) and can be strengthened by introducing strong, stiff fibers made of glass, carbon, or other polymers. Composites having the strength and stiffness

of steel, but with only a fraction of the weight, can be produced in this manner.

- 1–10** You would like to place a three-foot diameter microsatellite into orbit. The satellite will contain delicate electronic equipment that will send and receive radio signals from earth. Design the outer shell within which the electronic equipment is contained. What properties will be required and what kind of materials might be considered?

**Solution:** The shell of the microsatellite must satisfy several criteria. The material should have a low density, minimizing the satellite weight so that it can be lifted economically into its orbit; the material must be strong, hard, and impact resistant in order to assure that any “space dust” that might strike the satellite does not penetrate and damage the electronic equipment; the material must be transparent to the radio signals that provide communication between the satellite and earth; and the material must provide some thermal insulation to assure that solar heating does not damage the electronics.

One approach might be to use a composite shell of several materials. The outside surface might be a very thin reflective metal coating that would help reflect solar heat. The main body of the shell might be a light weight fiber-reinforced composite that would provide impact resistance (preventing penetration by dust particles) but would be transparent to radio signals.

- 1–11** What properties should the head of a carpenter’s hammer possess? How would you manufacture a hammer head?

**Solution:** The head for a carpenter’s hammer is produced by forging, a metal-working process; a simple steel shape is heated and formed in several steps while hot into the required shape. The head is then heat treated to produce the required mechanical and physical properties.

The striking face and claws of the hammer should be hard—the metal should not dent or deform when driving or removing nails. Yet these portions must also possess some impact resistance, particularly so that chips do not flake off the striking face and cause injuries.

- 1–12** The hull of the space shuttle consists of ceramic tiles bonded to an aluminum skin. Discuss the design requirements of the shuttle hull that led to the use of this combination of materials. What problems in producing the hull might the designers and manufacturers have faced?

**Solution:** The space shuttle experiences extreme temperatures during re-entry into earth’s atmosphere; consequently a thermal protection system must be used to prevent damage to the structure of the shuttle (not to mention its contents!). The skin must therefore be composed of a material that has an exceptionally low thermal conductivity. The material must be capable of being firmly attached to the skin of the shuttle and to be easily repaired when damage occurs.

The tiles used on the space shuttle are composed of silica fibers bonded together to produce a very low density ceramic. The thermal conductivity is so low that a person can hold on to one side of the tile while the opposite surface is red hot. The tiles are attached to the shuttle

skin using a rubbery polymer that helps assure that the forces do not break the tile loose, which would then expose the underlying skin to high temperatures.

- 1–13** You would like to select a material for the electrical contacts in an electrical switching device which opens and closes frequently and forcefully. What properties should the contact material possess? What type of material might you recommend? Would  $\text{Al}_2\text{O}_3$  be a good choice? Explain.

**Solution:** The material must have a high electrical conductivity to assure that no electrical heating or arcing occurs when the switch is closed. High purity (and therefore very soft) metals such as copper, aluminum, silver or gold provide the high conductivity. However the device must also have good wear resistance, requiring that the material be hard. Most hard, wear resistant materials have poor electrical conductivity.

One solution to this problem is to produce a particulate composite material composed of hard ceramic particles embedded in a continuous matrix of the electrical conductor. For example, silicon carbide particles could be introduced into pure aluminum; the silicon carbide particles provide wear resistance while aluminum provides conductivity. Other examples of these materials are described in Chapter 16.

$\text{Al}_2\text{O}_3$  by itself would not be a good choice—alumina is a ceramic material and is an electrical insulator. However alumina particles dispersed into a copper matrix might provide wear resistance to the composite.

- 1–14** Aluminum has a density of  $2.7 \text{ g/cm}^3$ . Suppose you would like to produce a composite material based on aluminum having a density of  $1.5 \text{ g/cm}^3$ . Design a material that would have this density. Would introducing beads of polyethylene, with a density of  $0.95 \text{ g/cm}^3$ , into the aluminum be a likely possibility? Explain.

**Solution:** In order to produce an aluminum-matrix composite material with a density of  $1.5 \text{ g/cm}^3$ , we would need to select a material having a density considerably less than  $1.5 \text{ g/cm}^3$ . While polyethylene's density would make it a possibility, the polyethylene has a very low melting point compared to aluminum; this would make it very difficult to introduce the polyethylene into a solid aluminum matrix—processes such as casting or powder metallurgy would destroy the polyethylene. Therefore polyethylene would NOT be a likely possibility.

One approach, however, might be to introduce *hollow* glass beads. Although ceramic glasses have densities comparable to that of aluminum, a hollow bead will have a very low density. The glass also has a high melting temperature and could be introduced into liquid aluminum for processing as a casting.

- 1–15** You would like to be able to identify different materials without resorting to chemical analysis or lengthy testing procedures. Describe some possible testing and sorting techniques you might be able to use based on the physical properties of materials.

**Solution:** Some typical methods might include: measuring the density of the material (may help in separating metal groups such as aluminum, copper, steel, magnesium, etc.), determining the electrical conductivity

of the material (may help in separating ceramics and polymers from metallic alloys), measuring the hardness of the material (perhaps even just using a file), and determining whether the material is magnetic or nonmagnetic (may help separate iron from other metallic alloys).

- 1–16** You would like to be able to physically separate different materials in a scrap recycling plant. Describe some possible methods that might be used to separate materials such as polymers, aluminum alloys, and steels from one another.

**Solution:** Steels can be magnetically separated from the other materials; steel (or carbon-containing iron alloys) are ferromagnetic and will be attracted by magnets. Density differences could be used—polymers have a density near that of water; the specific gravity of aluminum alloys is around 2.7; that of steels is between 7.5 and 8. Electrical conductivity measurements could be used—polymers are insulators, aluminum has a particularly high electrical conductivity.

- 1–17** Some pistons for automobile engines might be produced from a composite material containing small, hard silicon carbide particles in an aluminum alloy matrix. Explain what benefits each material in the composite may provide to the overall part. What problems might the different properties of the two materials cause in producing the part?

**Solution:** Aluminum provides good heat transfer due to its high thermal conductivity. It has good ductility and toughness, reasonably good strength, and is easy to cast and process. The silicon carbide, a ceramic, is hard and strong, providing good wear resistance, and also has a high melting temperature. It provides good strength to the aluminum, even at elevated temperatures. However there may be problems producing the material—for example, the silicon carbide may not be uniformly distributed in the aluminum matrix if the pistons are produced by casting. We need to assure good bonding between the particles and the aluminum—the surface chemistry must therefore be understood. Differences in expansion and contraction with temperature changes may cause debonding and even cracking in the composite.



# 2

## Atomic Structure

- 2–6(a)** Aluminum foil used for storing food weighs about 0.3 g per square inch. How many atoms of aluminum are contained in this sample of foil?

**Solution:** In a one square inch sample:

$$\text{number} = \frac{(0.3 \text{ g})(6.02 \times 10^{23} \text{ atoms/mol})}{26.981 \text{ g/mol}} = 6.69 \times 10^{21} \text{ atoms}$$

- 2–6(b)** Using the densities and atomic weights given in Appendix A, calculate and compare the number of atoms per cubic centimeter in (a) lead and (b) lithium.

**Solution:** (a) In lead:

$$\frac{(11.36 \text{ g/cm}^3)(1 \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{207.19 \text{ g/mol}} = 3.3 \times 10^{22} \text{ atoms/cm}^3$$

(b) In lithium:

$$\frac{(0.534 \text{ g/cm}^3)(1 \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{6.94 \text{ g/mol}} = 4.63 \times 10^{22} \text{ atoms/cm}^3$$

- 2–7(a)** Using data in Appendix A, calculate the number of iron atoms in one ton (2000 pounds).

**Solution:** 
$$\frac{(2000 \text{ lb})(454 \text{ g/lb})(6.02 \times 10^{23} \text{ atoms/mol})}{55.847 \text{ g/mol}} = 9.79 \times 10^{27} \text{ atoms/ton}$$

- 2–7(b)** Using data in Appendix A, calculate the volume in cubic centimeters occupied by one mole of boron.

**Solution:** 
$$\frac{(1 \text{ mol})(10.81 \text{ g/mol})}{2.3 \text{ g/cm}^3} = 4.7 \text{ cm}^3$$



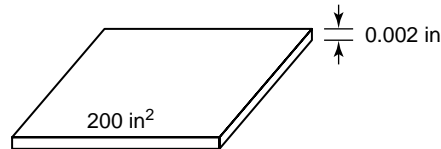
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- 2-8** In order to plate a steel part having a surface area of 200 in.<sup>2</sup> with a 0.002 in. thick layer of nickel, (a) how many atoms of nickel are required and (b) how many moles of nickel are required?

**Solution:** Volume = (200 in.<sup>2</sup>)(0.002 in.)(2.54 cm/in.)<sup>3</sup> = 6.555 cm<sup>3</sup>

$$(a) \frac{(6.555 \text{ cm}^3)(8.902 \text{ g/cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{58.71 \text{ g/mol}} = 5.98 \times 10^{23} \text{ atoms}$$

$$(b) \frac{(6.555 \text{ cm}^3)(8.902 \text{ g/cm}^3)}{58.71 \text{ g/mol}} = 0.994 \text{ mol Ni required}$$



- 2-9** Suppose an element has a valence of 2 and an atomic number of 27. Based only on the quantum numbers, how many electrons must be present in the 3d energy level?

**Solution:** We can let  $x$  be the number of electrons in the 3d energy level. Then:

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^x 4s^2 \quad (\text{must be 2 electrons in } 4s \text{ for valence} = 2)$$

$$\text{Since } 27 - (2+2+6+2+6+2) = 7 = x$$

there must be 7 electrons in the 3d level.

- 2-10** Indium, which has an atomic number of 49, contains no electrons in its 4f energy level. Based only on this information, what must be the valence of indium?

**Solution:** We can let  $x$  be the number of electrons in the outer  $sp$  energy level. Then:

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^0 5(sp)^x$$

$$49 - (2+2+6+2+6+10+2+6+10+0) = 3$$

Therefore the outer 5sp level must be:

$$5s^2 5p^1 \text{ or valence} = 3$$

- 2-11** Without consulting Appendix C, describe the quantum numbers for each of the 18 electrons in the M shell of copper, using a format similar to that in Figure 2-9.

**Solution:** For the M shell:  $n = 3$ ;  $\ell = 0, 1, 2$ ;  $m_\ell = 2\ell + 1$

$$\left. \begin{array}{lll} n = 3 & \ell = 0 & m_\ell = 0 \\ & & m_s = +\frac{1}{2} \\ & & m_s = -\frac{1}{2} \end{array} \right\} 3s^2$$

$$\left. \begin{array}{lll} & \ell = 1 & m_\ell = -1 \\ & & m_s = +\frac{1}{2} \\ & & m_s = -\frac{1}{2} \\ & & m_s = +\frac{1}{2} \\ & & m_s = -\frac{1}{2} \\ & m_\ell = 0 & \\ & & m_s = +\frac{1}{2} \\ & & m_s = -\frac{1}{2} \\ & m_\ell = +1 & \\ & & m_s = +\frac{1}{2} \\ & & m_s = -\frac{1}{2} \end{array} \right\} 3p^6$$

$$\ell = 2 \quad \left. \begin{array}{l} m_l = -2 \\ m_l = -1 \\ m_l = 0 \\ m_l = +1 \\ m_l = +2 \end{array} \right\} \begin{array}{l} m_s = +\frac{1}{2} \\ m_s = -\frac{1}{2} \\ m_s = +\frac{1}{2} \\ m_s = -\frac{1}{2} \\ m_s = +\frac{1}{2} \\ m_s = -\frac{1}{2} \\ m_s = +\frac{1}{2} \\ m_s = -\frac{1}{2} \\ m_s = +\frac{1}{2} \\ m_s = -\frac{1}{2} \end{array} \quad 3d^{10}$$

- 2-12** Electrical charge is transferred in metals by movement of valence electrons. How many potential charge carriers are there in an aluminum wire 1 mm in diameter and 100 m in length?

**Solution:** Aluminum has 3 valence electrons per atom; the volume of the wire is:

$$\begin{aligned} \text{Volume} &= (\pi/4)d^2\ell = (\pi/4)(0.1 \text{ cm})^2(10,000 \text{ cm}) = 78.54 \text{ cm}^3 \\ n &= \frac{(78.54 \text{ cm}^3)(2.699 \text{ g/cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})(3 \text{ electrons/atom})}{26.981 \text{ g/mol}} \\ n &= 1.42 \times 10^{25} \text{ carriers} \end{aligned}$$

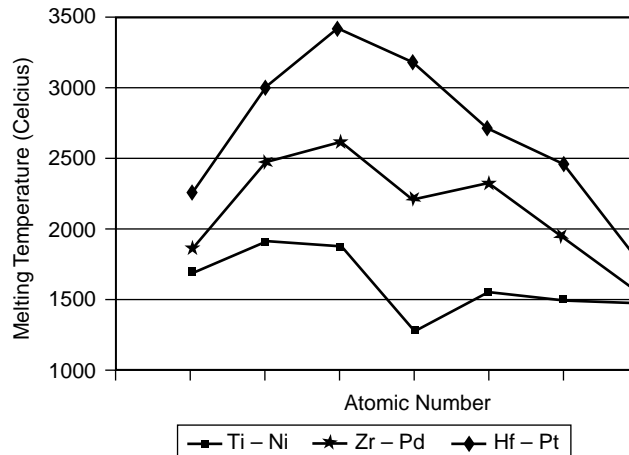
- 2-14** Bonding in the intermetallic compound  $\text{Ni}_3\text{Al}$  is predominantly metallic. Explain why there will be little, if any, ionic bonding component. The electronegativity of nickel is about 1.8.

**Solution:** The electronegativity of Al is 1.5, while that of Ni is 1.8. These values are relatively close, so we wouldn't expect much ionic bonding. Also, both are metals and prefer to give up their electrons rather than share or donate them.

- 2-15** Plot the melting temperatures of elements in the 4A to 8-10 columns of the periodic table versus atomic number (i.e., plot melting temperatures of Ti through Ni, Zr through Pd, and Hf through Pt). Discuss these relationships, based on atomic bonding and binding energy, (a) as the atomic number increases in each row of the periodic table and (b) as the atomic number increases in each column of the periodic table.

**Solution:**

Ti -1668	Zr -1852	Hf -2227
V -1900	Nb -2468	Ta -2996
Cr -1875	Mo -2610	W -3410
Mn -1244	Tc -2200	Re -3180
Fe -1538	Ru -2310	Os -2700
Co -1495	Rh -1963	Ir -2447
Ni -1453	Pd -1552	Pt -1769

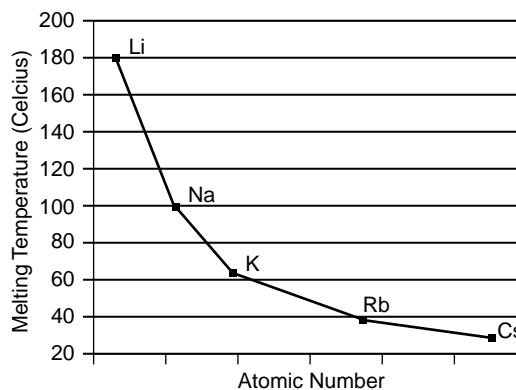


For each row, the melting temperature is highest when the outer “*d*” energy level is partly full. In Cr, there are 5 electrons in the 3*d* shell; in Mo, there are 5 electrons in the 4*d* shell; in W there are 4 electrons in the 5*d* shell. In each column, the melting temperature increases as the atomic number increases—the atom cores contain a larger number of tightly held electrons, making the metals more stable.

- 2–16** Plot the melting temperature of the elements in the 1A column of the periodic table versus atomic number (i.e., plot melting temperatures of Li through Cs). Discuss this relationship, based on atomic bonding and binding energy.

Solution:  $T(^{\circ}\text{C})$

Li –180.7  
 Na– 97.8  
 K – 63.2  
 Rb– 38.9



As the atomic number increases, the melting temperature decreases, opposite that found in Problem 2–15.

- 2–17** Increasing the temperature of a semiconductor breaks covalent bonds. For each broken bond, two electrons become free to move and transfer electrical charge.
- (a) What fraction of valence electrons are free to move and (b) what fraction of the

covalent bonds must be broken in order that  $5 \times 10^{15}$  electrons conduct electrical charge in 50 g of silicon? (c) What fraction of the total silicon atoms must be replaced by arsenic atoms to obtain one million electrons that are free to move in one pound of silicon?

**Solution:** (a)  $\frac{(50 \text{ g})(6.02 \times 10^{23} \text{ atoms/mol})}{28.08 \text{ g/mol}} = 1.072 \times 10^{24} \text{ atoms of Si}$

$$\begin{aligned} \text{Total valence electrons} &= (4 \text{ electrons/atom})(1.072 \times 10^{24} \text{ atoms}) \\ &= 4.288 \times 10^{24} \text{ electrons} \end{aligned}$$

$$\text{Fraction free electrons} = 5 \times 10^{15} / 4.288 \times 10^{24} = 1.17 \times 10^{-9}$$

(b) On average, there is one covalent bond per Si atom (each Si atom is bonded to four others). Thus, there are a total of  $1.072 \times 10^{24}$  bonds. Each bond has 2 electrons, so the number of broken bonds needed to give  $5 \times 10^{15}$  electrons is  $2.5 \times 10^{15}$ . The fraction of broken bonds is:

$$f = \frac{2.5 \times 10^{15}}{1.072 \times 10^{24}} = 2.33 \times 10^{-9}$$

(c)  $\frac{(1 \text{ lb Si})(454 \text{ g/lb})(6.02 \times 10^{23} \text{ atoms/mol})}{28.08 \text{ g/mol}} = 9.733 \times 10^{24} \text{ Si atoms/lb}$

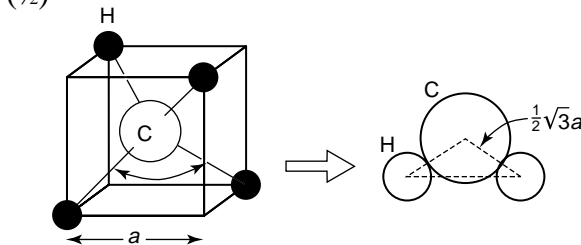
As has a valence of 5; therefore, to get  $10^6$  electrons, we need to replace  $10^6$  Si atoms. In one pound of Si, the fraction of As must be:

$$f = \frac{1 \times 10^6 \text{ replaced atoms}}{9.733 \times 10^{24} \text{ total Si atoms}} = 1.03 \times 10^{-19}$$

**2-18** Methane ( $\text{CH}_4$ ) has a tetrahedral structure similar to that of  $\text{SiO}_2$  (Figure 2-16), with a carbon atom of radius  $0.77 \times 10^{-8} \text{ cm}$  at the center and hydrogen atoms of radius  $0.46 \times 10^{-8} \text{ cm}$  at four of the eight corners. Calculate the size of the tetrahedral cube for methane.

**Solution:**

$$\begin{aligned} \sqrt{3}a &= r_C + r_H \\ \sqrt{3}a &= 0.77 \times 10^{-8} + 0.46 \times 10^{-8} \\ (\frac{1}{2}) \quad a &= 1.42 \times 10^{-8} \text{ cm} \end{aligned}$$



**2-19** The compound aluminum phosphide (AlP) is a compound semiconductor material having mixed ionic and covalent bonding. Estimate the fraction of the bonding that is ionic.

**Solution:**

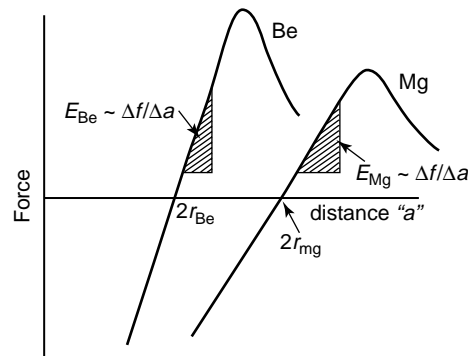
$$\begin{aligned} E_{\text{Al}} &= 1.5 & E_{\text{P}} &= 2.1 \\ f_{\text{covalent}} &= \exp(-0.25 \Delta E^2) \\ f_{\text{covalent}} &= \exp[(-0.25)(2.1 - 1.5)^2] = \exp[-0.09] = 0.914 \\ f_{\text{ionic}} &= 1 - 0.914 = 0.086 \therefore \text{bonding is mostly covalent} \end{aligned}$$

**2-20** Calculate the fraction of bonding of MgO that is ionic.

**Solution:**  $E_{\text{Mg}} = 1.2$        $E_{\text{O}} = 3.5$   
 $f_{\text{covalent}} = \exp[(-0.25)(3.5 - 1.2)^2] = \exp(-1.3225) = 0.266$   
 $f_{\text{ionic}} = 1 - 0.266 = 0.734 \therefore$  bonding is mostly ionic

**2-29** Beryllium and magnesium, both in the 2A column of the periodic table, are light-weight metals. Which would you expect to have the higher modulus of elasticity? Explain, considering binding energy and atom radii and using appropriate sketches of force versus interatomic spacing.

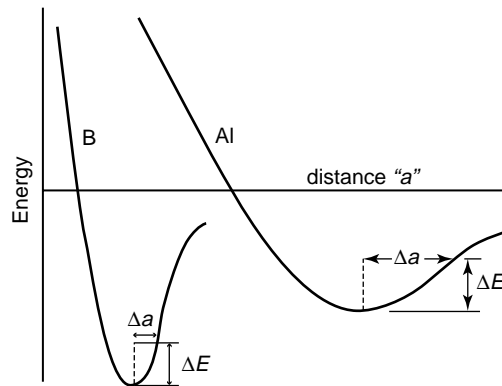
**Solution:**      4 Be     $1s^2 2s^2$        $E = 42 \times 10^6$  psi       $r_{\text{Be}} = 1.143 \text{ \AA}$   
                   12 Mg     $1s^2 2s^2 2p^6 3s^2$        $E = 6 \times 10^6$  psi       $r_{\text{Mg}} = 1.604 \text{ \AA}$



The smaller Be electrons are held closer to the core  $\therefore$  held more tightly, giving a higher binding energy.

**2-30** Boron has a much lower coefficient of thermal expansion than aluminum, even though both are in the 3B column of the periodic table. Explain, based on binding energy, atomic size, and the energy well, why this difference is expected.

**Solution:**      5 B     $1s^2 2s^2 2p^1$        $r_{\text{B}} = 0.46 \text{ \AA}$   
                   13 Al     $s^2 2s^2 2p^6 3s^2 3p^1$        $r_{\text{Al}} = 1.432 \text{ \AA}$



Electrons in Al are not as tightly bonded as those in B due to the smaller size of the boron atom and the lower binding energy associated with its size.

- 2-31** Would you expect MgO or magnesium to have the higher modulus of elasticity? Explain.

**Solution:** MgO has ionic bonds, which are strong compared to the metallic bonds in Mg. A higher force will be required to cause the same separation between the ions in MgO compared to the atoms in Mg. Therefore, MgO should have the higher modulus of elasticity. In Mg,  $E \approx 6 \times 10^6$  psi; in MgO,  $E = 30 \times 10^6$  psi.

- 2-32** Would you expect  $\text{Al}_2\text{O}_3$  or aluminum to have the higher coefficient of thermal expansion? Explain.

**Solution:**  $\text{Al}_2\text{O}_3$  has stronger bonds than Al; therefore,  $\text{Al}_2\text{O}_3$  should have a lower thermal expansion coefficient than Al. In Al,  $\alpha = 25 \times 10^{-6}$  cm/cm°C; in  $\text{Al}_2\text{O}_3$ ,  $\alpha = 6.7 \times 10^{-6}$  cm/cm°C.

- 2-33** Aluminum and silicon are side by side in the periodic table. Which would you expect to have the higher modulus of elasticity ( $E$ )? Explain.

**Solution:** Silicon has covalent bonds; aluminum has metallic bonds. Therefore, Si should have a higher modulus of elasticity.

- 2-34** Explain why the modulus of elasticity of simple thermoplastic polymers, such as polyethylene and polystyrene, is expected to be very low compared with that of metals and ceramics.

**Solution:** The chains in polymers are held to other chains by Van der Waals bonds, which are much weaker than metallic, ionic, and covalent bonds. For this reason, much less force is required to shear these weak bonds and to unkink and straighten the chains.

- 2-35** Steel is coated with a thin layer of ceramic to help protect against corrosion. What do you expect to happen to the coating when the temperature of the steel is increased significantly? Explain.

**Solution:** Ceramics are expected to have a low coefficient of thermal expansion due to strong ionic/covalent bonds; steel has a high thermal expansion coefficient. When the structure heats, steel expands more than the coating, which may crack and expose the underlying steel to corrosion.



# 3

## Atomic and Ionic Arrangements

- 3–25** Calculate the atomic radius in cm for the following: (a) BCC metal with  $a_o = 0.3294$  nm and one atom per lattice point; and (b) FCC metal with  $a_o = 4.0862$  Å and one atom per lattice point.

**Solution:** (a) For BCC metals,

$$r = \frac{(\sqrt{3})a_o}{4} = \frac{(\sqrt{3})(0.3294 \text{ nm})}{4} = 0.1426 \text{ nm} = 1.426 \times 10^{-8} \text{ cm}$$

(b) For FCC metals,

$$r = \frac{(\sqrt{2})a_o}{4} = \frac{(\sqrt{2})(4.0862 \text{ Å})}{4} = 1.4447 \text{ Å} = 1.4447 \times 10^{-8} \text{ cm}$$

- 3–26** Determine the crystal structure for the following: (a) a metal with  $a_o = 4.9489$  Å,  $r = 1.75$  Å and one atom per lattice point; and (b) a metal with  $a_o = 0.42906$  nm,  $r = 0.1858$  nm and one atom per lattice point.

**Solution:** We want to determine if “ $x$ ” in the calculations below equals  $\sqrt{2}$  (for FCC) or  $\sqrt{3}$  (for BCC):

$$(a) (x)(4.9489 \text{ Å}) = (4)(1.75 \text{ Å})$$

$$x = \sqrt{2}, \text{ therefore FCC}$$

$$(b) (x)(0.42906 \text{ nm}) = (4)(0.1858 \text{ nm})$$

$$x = \sqrt{3}, \text{ therefore BCC}$$

- 3–27** The density of potassium, which has the BCC structure and one atom per lattice point, is  $0.855 \text{ g/cm}^3$ . The atomic weight of potassium is  $39.09 \text{ g/mol}$ . Calculate (a) the lattice parameter; and (b) the atomic radius of potassium.



Solution: (a) Using Equation 3-5:

$$0.855 \text{ g/cm}^3 = \frac{(2 \text{ atoms/cell})(39.09 \text{ g/mol})}{(a_o)^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$a_o^3 = 1.5189 \times 10^{-22} \text{ cm}^3 \text{ or } a_o = 5.3355 \times 10^{-8} \text{ cm}$$

(b) From the relationship between atomic radius and lattice parameter:

$$r = \frac{(\sqrt{3})(5.3355 \times 10^{-8} \text{ cm})}{4} = 2.3103 \times 10^{-8} \text{ cm}$$

**3-28** The density of thorium, which has the FCC structure and one atom per lattice point, is  $11.72 \text{ g/cm}^3$ . The atomic weight of thorium is  $232 \text{ g/mol}$ . Calculate (a) the lattice parameter and (b) the atomic radius of thorium.

Solution: (a) From Equation 3-5:

$$11.72 \text{ g/cm}^3 = \frac{(4 \text{ atoms/cell})(232 \text{ g/mol})}{(a_o)^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$a_o^3 = 1.315297 \times 10^{-22} \text{ cm}^3 \text{ or } a_o = 5.0856 \times 10^{-8} \text{ cm}$$

(b) From the relationship between atomic radius and lattice parameter:

$$r = \frac{(\sqrt{2})(5.0856 \times 10^{-8} \text{ cm})}{4} = 1.7980 \times 10^{-8} \text{ cm}$$

**3-29** A metal having a cubic structure has a density of  $2.6 \text{ g/cm}^3$ , an atomic weight of  $87.62 \text{ g/mol}$ , and a lattice parameter of  $6.0849 \text{ \AA}$ . One atom is associated with each lattice point. Determine the crystal structure of the metal.

Solution:  $2.6 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(87.62 \text{ g/mol})}{(6.0849 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$

$x = 4$ , therefore FCC

**3-30** A metal having a cubic structure has a density of  $1.892 \text{ g/cm}^3$ , an atomic weight of  $132.91 \text{ g/mol}$ , and a lattice parameter of  $6.13 \text{ \AA}$ . One atom is associated with each lattice point. Determine the crystal structure of the metal.

Solution:  $1.892 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(132.91 \text{ g/mol})}{(6.13 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$

$x = 2$ , therefore BCC

**3-31** Indium has a tetragonal structure with  $a_o = 0.32517 \text{ nm}$  and  $c_o = 0.49459 \text{ nm}$ . The density is  $7.286 \text{ g/cm}^3$  and the atomic weight is  $114.82 \text{ g/mol}$ . Does indium have the simple tetragonal or body-centered tetragonal structure?

Solution:

$$7.286 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(114.82 \text{ g/mol})}{(3.2517 \times 10^{-8} \text{ cm})^2(4.9459 \times 10^{-8} \text{ cm})(6.02 \times 10^{23} \text{ atoms/mol})}$$

$x = 2$ , therefore BCT (body-centered tetragonal)

- 3-32** Bismuth has a hexagonal structure, with  $a_o = 0.4546$  nm and  $c_o = 1.186$  nm. The density is  $9.808$  g/cm<sup>3</sup> and the atomic weight is  $208.98$  g/mol. Determine (a) the volume of the unit cell and (b) how many atoms are in each unit cell.

**Solution:** (a) The volume of the unit cell is  $V = a_o^2 c_o \cos 30$ .

$$V = (0.4546 \text{ nm})^2 (1.186 \text{ nm}) (\cos 30) = 0.21226 \text{ nm}^3$$

$$= 2.1226 \times 10^{-22} \text{ cm}^3$$

(b) If “ $x$ ” is the number of atoms per unit cell, then:

$$9.808 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(208.98 \text{ g/mol})}{(2.1226 \times 10^{-22} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 6 \text{ atoms/cell}$$

- 3-33** Gallium has an orthorhombic structure, with  $a_o = 0.45258$  nm,  $b_o = 0.45186$  nm, and  $c_o = 0.76570$  nm. The atomic radius is  $0.1218$  nm. The density is  $5.904$  g/cm<sup>3</sup> and the atomic weight is  $69.72$  g/mol. Determine (a) the number of atoms in each unit cell and (b) the packing factor in the unit cell.

**Solution:** The volume of the unit cell is  $V = a_o b_o c_o$  or

$$V = (0.45258 \text{ nm})(0.45186 \text{ nm})(0.76570 \text{ nm}) = 0.1566 \text{ nm}^3$$

$$= 1.566 \times 10^{-22} \text{ cm}^3$$

(a) From the density equation:

$$5.904 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(69.72 \text{ g/mol})}{(1.566 \times 10^{-22} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 8 \text{ atoms/cell}$$

(b) From the packing factor (PF) equation:

$$\text{PF} = \frac{(8 \text{ atoms/cell})(4\pi/3)(0.1218 \text{ nm})^3}{0.1566 \text{ nm}^3} = 0.387$$

- 3-34** Beryllium has a hexagonal crystal structure, with  $a_o = 0.22858$  nm and  $c_o = 0.35842$  nm. The atomic radius is  $0.1143$  nm, the density is  $1.848$  g/cm<sup>3</sup>, and the atomic weight is  $9.01$  g/mol. Determine (a) the number of atoms in each unit cell and (b) the packing factor in the unit cell.

**Solution:**  $V = (0.22858 \text{ nm})^2 (0.35842 \text{ nm}) \cos 30 = 0.01622 \text{ nm}^3 = 16.22 \times 10^{-24} \text{ cm}^3$

(a) From the density equation:

$$1.848 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(9.01 \text{ g/mol})}{(16.22 \times 10^{-24} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 2 \text{ atoms/cell}$$

(b) The packing factor (PF) is:

$$\text{PF} = \frac{(2 \text{ atoms/cell})(4\pi/3)(0.1143 \text{ nm})^3}{0.01622 \text{ nm}^3} = 0.77$$

- 3-39** Above 882°C, titanium has a BCC crystal structure, with  $a = 0.332$  nm. Below this temperature, titanium has a HCP structure, with  $a = 0.2978$  nm and  $c = 0.4735$  nm. Determine the percent volume change when BCC titanium transforms to HCP titanium. Is this a contraction or expansion?

**Solution:** We can find the volume of each unit cell. Two atoms are present in both BCC and HCP titanium unit cells, so the volumes of the unit cells can be directly compared.

$$V_{\text{BCC}} = (0.332 \text{ nm})^3 = 0.03659 \text{ nm}^3$$

$$V_{\text{HCP}} = (0.2978 \text{ nm})^2(0.4735 \text{ nm})\cos 30 = 0.03637 \text{ nm}^3$$

$$\Delta V = \frac{V_{\text{HCP}} - V_{\text{BCC}}}{V_{\text{BCC}}} \times 100 = \frac{0.03637 \text{ nm}^3 - 0.03659 \text{ nm}^3}{0.03659 \text{ nm}^3} \times 100 = -0.6\%$$

Therefore titanium *contracts* 0.6% during cooling.

- 3-40**  $\alpha$ -Mn has a cubic structure with  $a_o = 0.8931$  nm and a density of 7.47 g/cm<sup>3</sup>.  $\beta$ -Mn has a different cubic structure, with  $a_o = 0.6326$  nm and a density of 7.26 g/cm<sup>3</sup>. The atomic weight of manganese is 54.938 g/mol and the atomic radius is 0.112 nm. Determine the percent volume change that would occur if  $\alpha$ -Mn transforms to  $\beta$ -Mn.

**Solution:** First we need to find the number of atoms in each unit cell so we can determine the volume change based on equal numbers of atoms. From the density equation, we find for the  $\alpha$ -Mn:

$$7.47 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(54.938 \text{ g/mol})}{(8.931 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 58 \text{ atoms/cell} \quad V_{\alpha\text{-Mn}} = (8.931 \times 10^{-8} \text{ cm})^3 = 7.12 \times 10^{-22} \text{ cm}^3$$

For  $\beta$ -Mn:

$$7.26 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(54.938 \text{ g/mol})}{(6.326 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 20 \text{ atoms/cell} \quad V_{\beta\text{-Mn}} = (6.326 \times 10^{-8} \text{ cm})^3 = 2.53 \times 10^{-22} \text{ cm}^3$$

The volume of the  $\beta$ -Mn can be adjusted by a factor of 58/20, to account for the different number of atoms per cell. The volume change is then:

$$\Delta V = \frac{(58/20)V_{\beta\text{-Mn}} - V_{\alpha\text{-Mn}}}{V_{\alpha\text{-Mn}}} \times 100 = \frac{(58/20)(2.53) - 7.12}{7.12} \times 100 = +3.05\%$$

The manganese *expands* by 3.05% during the transformation.

- 3-35** A typical paper clip weighs 0.59 g and consists of BCC iron. Calculate (a) the number of unit cells and (b) the number of iron atoms in the paper clip. (See Appendix A for required data)

**Solution:** The lattice parameter for BCC iron is  $2.866 \times 10^{-8}$  cm. Therefore

$$V_{\text{unit cell}} = (2.866 \times 10^{-8} \text{ cm})^3 = 2.354 \times 10^{-23} \text{ cm}^3$$

(a) The density is 7.87 g/cm<sup>3</sup>. The number of unit cells is:

$$\text{number} = \frac{0.59 \text{ g}}{(7.87 \text{ g/cm}^3)(2.354 \times 10^{-23} \text{ cm}^3/\text{cell})} = 3.185 \times 10^{21} \text{ cells}$$

(b) There are 2 atoms/cell in BCC iron. The number of atoms is:

$$\text{number} = (3.185 \times 10^{21} \text{ cells})(2 \text{ atoms/cell}) = 6.37 \times 10^{21} \text{ atoms}$$

- 3-36** Aluminum foil used to package food is approximately 0.001 inch thick. Assume that all of the unit cells of the aluminum are arranged so that  $a_0$  is perpendicular to the foil surface. For a 4 in.  $\times$  4 in. square of the foil, determine (a) the total number of unit cells in the foil and (b) the thickness of the foil in number of unit cells. (See Appendix A)

**Solution:** The lattice parameter for aluminum is  $4.04958 \times 10^{-8}$  cm. Therefore:

$$V_{\text{unit cell}} = (4.04958 \times 10^{-8})^3 = 6.6409 \times 10^{-23} \text{ cm}^3$$

The volume of the foil is:

$$V_{\text{foil}} = (4 \text{ in.})(4 \text{ in.})(0.001 \text{ in.}) = 0.016 \text{ in.}^3 = 0.262 \text{ cm}^3$$

(a) The number of unit cells in the foil is:

$$\text{number} = \frac{0.262 \text{ cm}^3}{6.6409 \times 10^{-23} \text{ cm}^3/\text{cell}} = 3.945 \times 10^{21} \text{ cells}$$

(b) The thickness of the foil, in number of unit cells, is:

$$\text{number} = \frac{(0.001 \text{ in.})(2.54 \text{ cm/in.})}{4.04958 \times 10^{-8} \text{ cm}} = 6.27 \times 10^4 \text{ cells}$$

- 3-51** Determine the Miller indices for the directions in the cubic unit cell shown in Figure 3-48.

**Solution:**  $A: 0, 1, 0 - 0, 1, 1 = 0, 0, -1 = [00\bar{1}]$

$B: \frac{1}{2}, 0, 0 - 0, 1, 0 = \frac{1}{2}, -1, 0 = [\frac{1}{2}\bar{1}0]$

$C: 0, 1, 1 - 1, 0, 0 = -1, 1, 1 = [\bar{1}11]$

$D: 1, 0, \frac{1}{2} - 0, \frac{1}{2}, 1 = 1, -\frac{1}{2}, -\frac{1}{2} = [2\bar{1}\bar{1}]$

- 3-52** Determine the indices for the directions in the cubic unit cell shown in Figure 3-49.

**Solution:**  $A: 0, 0, 1 - 1, 0, 0 = -1, 0, 1 = [\bar{1}01]$

$B: 1, 0, 1 - \frac{1}{2}, 1, 0 = \frac{1}{2}, -1, 1 = [\frac{1}{2}\bar{1}2]$

$C: 1, 0, 0 - 0, \frac{3}{4}, 1 = 1, -\frac{3}{4}, -1 = [4\bar{3}\bar{4}]$

$D: 0, 1, \frac{1}{2} - 0, 0, 0 = 0, 1, \frac{1}{2} = [021]$

- 3-53** Determine the indices for the planes in the cubic unit cell shown in Figure 3-50.

**Solution:**  $A: \begin{array}{ll} x = 1 & 1/x = 1 \\ y = -1 & 1/y = -1 \\ z = 1 & 1/z = 1 \end{array} \quad (1\bar{1}1)$

$B: \begin{array}{ll} x = \infty & 1/x = 0 \\ y = \frac{1}{3} & 1/y = 3 \\ z = \infty & 1/z = 0 \end{array} \quad (030)$

$$\begin{array}{lll} C: & x = 1 & 1/x = 1 \\ & y = \infty & 1/y = 0 \\ & z = -1/2 & 1/z = -2 \end{array} \quad (10\bar{2}) \quad (\text{origin at } 0,0,1)$$

**3-54** Determine the indices for the planes in the cubic unit cell shown in Figure 3-51.

$$\begin{array}{lll} \text{Solution: } A: & x = -1 & 1/x = -1 \times 3 = -3 \\ & y = 1/2 & 1/y = 2 \times 3 = 6 \\ & z = 3/4 & 1/z = 4/3 \times 3 = 4 \end{array} \quad (\bar{3}64) \quad (\text{origin at } 1,0,0)$$

$$\begin{array}{lll} B: & x = 1 & 1/x = 1 \times 3 = 3 \\ & y = -3/4 & 1/y = -4/3 \times 3 = -4 \\ & z = \infty & 1/z = 0 \times 3 = 0 \end{array} \quad (3\bar{4}0) \quad (\text{origin at } 0,1,0)$$

$$\begin{array}{lll} C: & x = 2 & 1/x = 1/2 \times 6 = 3 \\ & y = 3/2 & 1/y = 2/3 \times 6 = 4 \\ & z = 1 & 1/z = 1 \times 6 = 6 \end{array} \quad (346)$$

**3-55** Determine the indices for the directions in the hexagonal lattice shown in Figure 3-52, using both the three-digit and four-digit systems.

$$\begin{array}{l} \text{Solution: } A: \quad 1, -1, 0 - 0, 0, 0 = 1, -1, 0 = [1\bar{1}0] \\ \quad \quad \quad h = 1/3(2 + 1) = 1 \\ \quad \quad \quad k = 1/3(-2 - 1) = -1 = [1\bar{1}00] \\ \quad \quad \quad i = -1/3(1 - 1) = 0 \\ \quad \quad \quad l = 0 \end{array}$$

$$\begin{array}{l} B: \quad 1, 1, 0 - 0, 0, 1 = 1, 1, -1 = [11\bar{1}] \\ \quad \quad \quad h = 1/3(2 - 1) = 1/3 \\ \quad \quad \quad k = 1/3(2 - 1) = 1/3 = [11\bar{2}\bar{3}] \\ \quad \quad \quad i = -1/3(1 + 1) = -2/3 \\ \quad \quad \quad l = -1 \end{array}$$

$$\begin{array}{l} C: \quad 0, 1, 1 - 0, 0, 0 = 0, 1, 1 = [011] \\ \quad \quad \quad h = 1/3(0 - 1) = -1/3 \\ \quad \quad \quad k = 1/3(2 - 0) = 2/3 \\ \quad \quad \quad i = -1/3(0 + 1) = -1/3 = [\bar{1}2\bar{1}3] \\ \quad \quad \quad l = 1 \end{array}$$

**3-56** Determine the indices for the directions in the hexagonal lattice shown in Figure 3-53, using both the three-digit and four-digit systems.

$$\begin{array}{l} \text{Solution: } A: \quad 0, 1, 1 - 1/2, 1, 0 = -1/2, 0, 1 = [\bar{1}02] \\ \quad \quad \quad h = 1/3(-2 - 0) = -2/3 \\ \quad \quad \quad k = 1/3(0 + 1) = 1/3 = [\bar{2}116] \\ \quad \quad \quad i = -1/3(-1 + 0) = 1/3 \\ \quad \quad \quad l = 2 \end{array}$$

$$\begin{array}{l} B: \quad 1, 0, 0 - 1, 1, 1 = 0, -1, -1 = [0\bar{1}\bar{1}] \\ \quad \quad \quad h = 1/3(0 + 1) = 1/3 \\ \quad \quad \quad k = 1/3(-2 + 0) = -2/3 = [1\bar{2}\bar{1}3] \\ \quad \quad \quad i = -1/3(0 - 1) = 1/3 \\ \quad \quad \quad l = -1 \end{array}$$

$$\begin{aligned}
 C: 0,0,0 - 1,0,1 &= -1,0,-1 = [\bar{1}0\bar{1}] \\
 h &= \frac{1}{3}(-2 + 0) = -\frac{2}{3} \\
 k &= \frac{1}{3}(0 + 1) = \frac{1}{3} = [\bar{2}11\bar{3}] \\
 i &= -\frac{1}{3}(-1 + 0) = \frac{1}{3} \\
 l &= -1
 \end{aligned}$$

**3-57** Determine the indices for the planes in the hexagonal lattice shown in Figure 3-54.

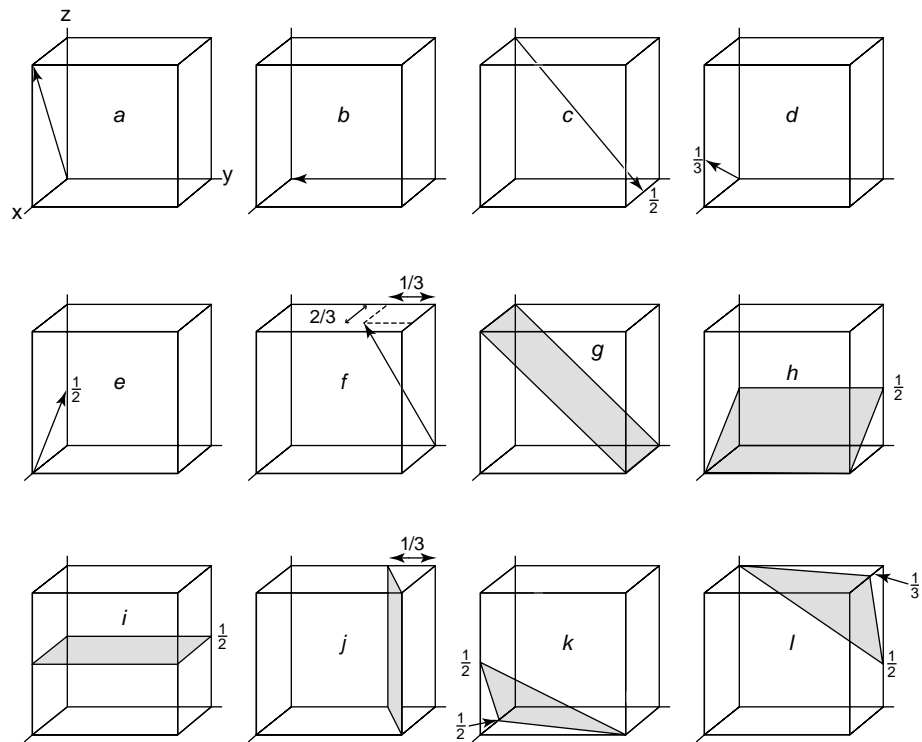
$$\begin{aligned}
 \text{Solution: } A: a_1 &= 1 & 1/a_1 &= 1 \\
 a_2 &= -1 & 1/a_2 &= -1 & (1\bar{1}01) & \text{(origin at } a_2 = 1) \\
 a_3 &= \infty & 1/a_3 &= 0 \\
 c &= 1 & 1/c &= 1 \\
 B: a_1 &= \infty & 1/a_1 &= 0 \\
 a_2 &= \infty & 1/a_2 &= 0 & (0003) \\
 a_3 &= \infty & 1/a_3 &= 0 \\
 c &= \frac{2}{3} & 1/c &= \frac{3}{2} \\
 C: a_1 &= 1 & 1/a_1 &= 1 \\
 a_2 &= -1 & 1/a_2 &= -1 & (1\bar{1}00) \\
 a_3 &= \infty & 1/a_3 &= 0 \\
 c &= \infty & 1/c &= 0
 \end{aligned}$$

**3-58** Determine the indices for the planes in the hexagonal lattice shown in Figure 3-55.

$$\begin{aligned}
 \text{Solution: } A: a_1 &= 1 & 1/a_1 &= 1 \\
 a_2 &= -1 & 1/a_2 &= -1 & (1\bar{1}02) \\
 a_3 &= \infty & 1/a_3 &= 0 \\
 c &= \frac{1}{2} & 1/c &= 2 \\
 B: a_1 &= \infty & 1/a_1 &= 0 \\
 a_2 &= 1 & 1/a_2 &= 1 & (01\bar{1}1) \\
 a_3 &= -1 & 1/a_3 &= -1 \\
 c &= 1 & 1/c &= 1 \\
 C: a_1 &= -1 & 1/a_1 &= -1 \\
 a_2 &= \frac{1}{2} & 1/a_2 &= 2 & (\bar{1}2\bar{1}0) \\
 a_3 &= -1 & 1/a_3 &= -1 \\
 c &= \infty & 1/c &= 0
 \end{aligned}$$

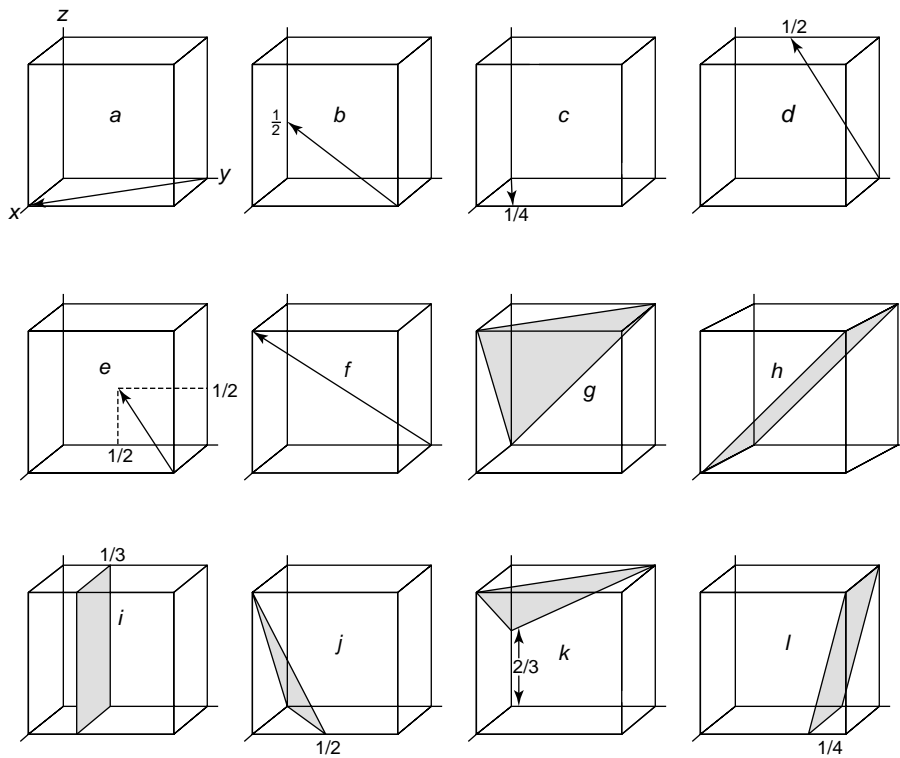
**3-59** Sketch the following planes and directions within a cubic unit cell.

$$\begin{aligned}
 \text{Solution: (a) } [101] \quad & \text{(b) } [0\bar{1}0] \quad & \text{(c) } [12\bar{2}] \quad & \text{(d) } [301] \quad & \text{(e) } [\bar{2}01] \quad & \text{(f) } [2\bar{1}3] \\
 \text{(g) } (0\bar{1}\bar{1}) \quad & \text{(h) } (102) \quad & \text{(i) } (002) \quad & \text{(j) } (1\bar{3}0) \quad & \text{(k) } (\bar{2}12) \quad & \text{(l) } (3\bar{1}\bar{2})
 \end{aligned}$$



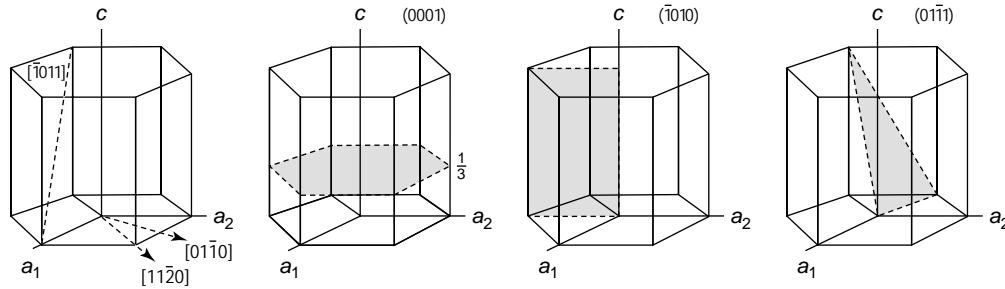
**3-60** Sketch the following planes and directions within a cubic unit cell.

**Solution:** (a)  $[1\bar{1}0]$  (b)  $[2\bar{2}1]$  (c)  $[410]$  (d)  $[0\bar{1}2]$  (e)  $[3\bar{2}1]$  (f)  $[1\bar{1}1]$   
 (g)  $(11\bar{1})$  (h)  $(01\bar{1})$  (i)  $(030)$  (j)  $(\bar{1}21)$  (k)  $(11\bar{3})$  (l)  $(0\bar{4}1)$



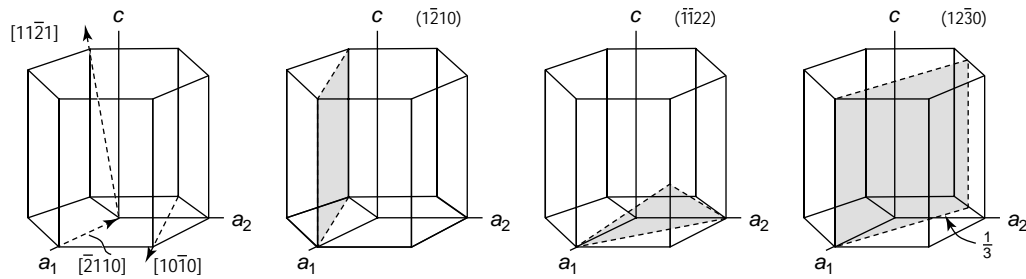
**3-61** Sketch the following planes and directions within a hexagonal unit cell.

**Solution:** (a)  $[01\bar{1}0]$  (b)  $[11\bar{2}0]$  (c)  $[\bar{1}011]$  (d)  $(0001)$  (e)  $(\bar{1}010)$  (f)  $(01\bar{1}1)$



**3-62** Sketch the following planes and directions within a hexagonal unit cell.

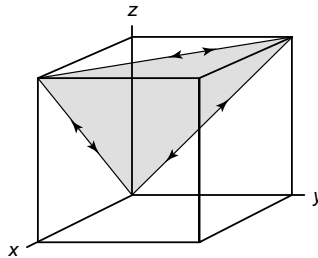
**Solution:** (a)  $[\bar{2}110]$  (b)  $[11\bar{2}1]$  (c)  $[10\bar{1}0]$  (d)  $(1\bar{2}10)$  (e)  $(\bar{1}122)$  (f)  $(12\bar{3}0)$



**3-63** What are the indices of the six directions of the form  $\langle 110 \rangle$  that lie in the  $(11\bar{1})$  plane of a cubic cell?

**Solution:**

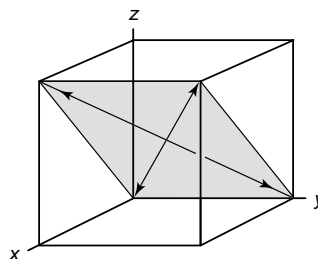
$[\bar{1}10]$	$[101]$	$[011]$
$[1\bar{1}0]$	$[\bar{1}01]$	$[0\bar{1}1]$



**3-64** What are the indices of the four directions of the form  $\langle 111 \rangle$  that lie in the  $(\bar{1}01)$  plane of a cubic cell?

**Solution:**

$[111]$	$[\bar{1}\bar{1}\bar{1}]$
$[1\bar{1}1]$	$[\bar{1}1\bar{1}]$





- 3-65** Determine the number of directions of the form  $\langle 110 \rangle$  in a tetragonal unit cell and compare to the number of directions of the form  $\langle 110 \rangle$  in an orthorhombic unit cell.

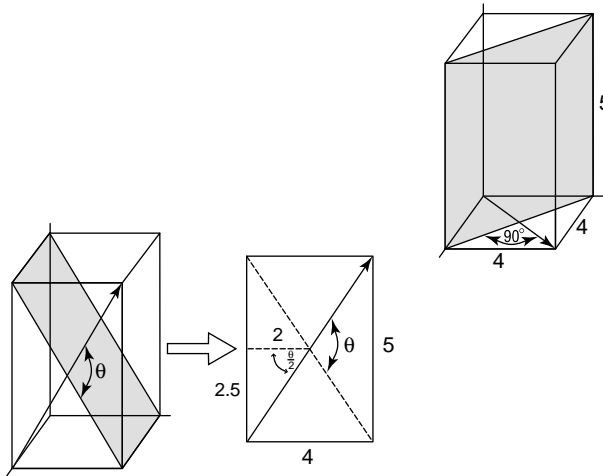
**Solution:** Tetragonal:  $[110], [\bar{1}\bar{1}0], [\bar{1}10], [1\bar{1}0] = 4$

Orthorhombic:  $[110], [\bar{1}\bar{1}0] = 2$

Note that in cubic systems, there are 12 directions of the form  $\langle 110 \rangle$ .

- 3-66** Determine the angle between the  $[110]$  direction and the  $(110)$  plane in a tetragonal unit cell; then determine the angle between the  $[011]$  direction and the  $(011)$  plane in a tetragonal cell. The lattice parameters are  $a_o = 4 \text{ \AA}$  and  $c_o = 5 \text{ \AA}$ . What is responsible for the difference?

**Solution:**  $[110] \perp (110)$



$$\tan(\theta/2) = 2.5 / 2 = 1.25$$

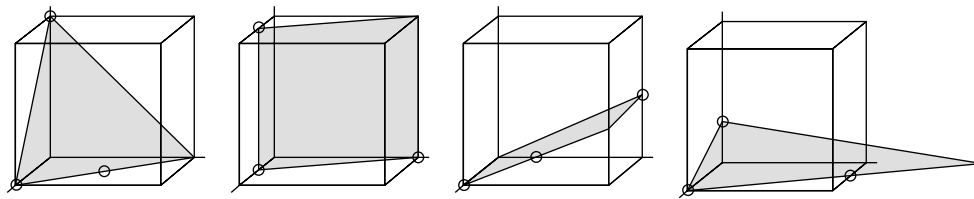
$$\theta/2 = 51.34^\circ$$

$$\theta = 102.68^\circ$$

The lattice parameters in the  $x$  and  $y$  directions are the same; this allows the angle between  $[110]$  and  $(110)$  to be  $90^\circ$ . But the lattice parameters in the  $y$  and  $z$  directions are different!

- 3-67** Determine the Miller indices of the plane that passes through three points having the following coordinates.

**Solution:** (a)  $0,0,1$ ;  $1,0,0$ ; and  $\frac{1}{2}, \frac{1}{2}, 0$   
 (b)  $\frac{1}{2}, 0, 1$ ;  $\frac{1}{2}, 0, 0$ ; and  $0, 1, 0$   
 (c)  $1, 0, 0$ ;  $0, 1, \frac{1}{2}$ ; and  $1, \frac{1}{2}, \frac{1}{4}$   
 (d)  $1, 0, 0$ ;  $0, 0, \frac{1}{4}$ ; and  $\frac{1}{2}, 1, 0$



(a)  $(111)$

(b)  $(210)$

(c)  $(0\bar{1}2)$

(d)  $(218)$

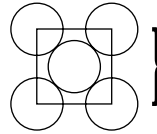
- 3-68** Determine the repeat distance, linear density, and packing fraction for FCC nickel, which has a lattice parameter of 0.35167 nm, in the [100], [110], and [111] directions. Which of these directions is close-packed?

**Solution:**  $r = (\sqrt{2})(0.35167) / 4 = 0.1243 \text{ nm}$

For [100]: repeat distance =  $a_o = 0.35167 \text{ nm}$

linear density =  $1/a_o = 2.84 \text{ points/nm}$

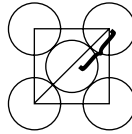
linear packing fraction =  $(2)(0.1243)(2.84) = 0.707$



For [110]: repeat distance =  $\sqrt{2} a_o / 2 = 0.2487 \text{ nm}$

linear density =  $2 / \sqrt{2} a_o = 4.02 \text{ points/nm}$

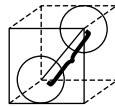
linear packing fraction =  $(2)(0.1243)(4.02) = 1.0$



For [111]: repeat distance =  $\sqrt{3} a_o = 0.6091 \text{ nm}$

linear density =  $1 / \sqrt{3} a_o = 1.642 \text{ points/nm}$

linear packing fraction =  $(2)(0.1243)(1.642) = 0.408$



Only the [110] is close packed; it has a linear packing fraction of 1.

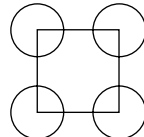
- 3-69** Determine the repeat distance, linear density, and packing fraction for BCC lithium, which has a lattice parameter of 0.35089 nm, in the [100], [110], and [111] directions. Which of these directions is close-packed?

**Solution:**  $r = \sqrt{3} (0.35089) / 4 = 0.1519 \text{ nm}$

For [100]: repeat distance =  $a_o = 0.35089 \text{ nm}$

linear density =  $1/a_o = 2.85 \text{ points/nm}$

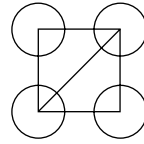
linear packing fraction =  $(2)(0.1519)(2.85) = 0.866$



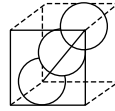
For [110]: repeat distance =  $\sqrt{2} a_o = 0.496 \text{ nm}$

linear density =  $1 / \sqrt{2} a_o = 2.015 \text{ points/nm}$

linear packing fraction =  $(2)(0.1519)(2.015) = 0.612$



For [111]: repeat distance =  $\sqrt{3} a_o / 2 = 0.3039 \text{ nm}$   
 linear density =  $2 / \sqrt{3} a_o = 3.291 \text{ points/nm}$   
 linear packing fraction =  $(2)(0.1519)(3.291) = 1$



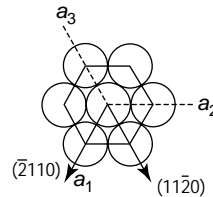
The [111] direction is close packed; the linear packing factor is 1.

- 3-70** Determine the repeat distance, linear density, and packing fraction for HCP magnesium in the  $[\bar{2}110]$  direction and the  $[11\bar{2}0]$  direction. The lattice parameters for HCP magnesium are given in Appendix A.

Solution:  $a_o = 3.2087 \text{ \AA}$   $r = 1.604 \text{ \AA}$

For  $[\bar{2}110]$ :

repeat distance =  $a_o = 3.2087 \text{ \AA}$   
 linear density =  $1/a_o = 0.3116 \text{ points/nm}$   
 linear packing fraction =  $(2)(1.604)(0.3116) = 1$   
 (Same for  $[11\bar{2}0]$ )



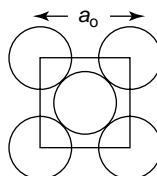
- 3-71** Determine the planar density and packing fraction for FCC nickel in the (100), (110), and (111) planes. Which, if any, of these planes is close-packed?

Solution:  $a_o = 3.5167 \text{ \AA}$

For (100):

$$\text{planar density} = \frac{2}{(3.5167 \times 10^{-8} \text{ cm})^2} = 0.1617 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi r^2}{(4r/\sqrt{2})^2} = 0.7854$$

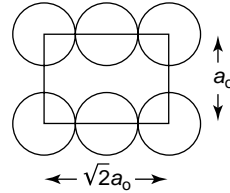


For (110):

$$\text{planar density} = \frac{2 \text{ points}}{(3.5167 \times 10^{-8} \text{ cm}) \left( \sqrt{2} \right) (3.5167 \times 10^{-8} \text{ cm})}$$

$$= 0.1144 \times 10^{-16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi r^2}{\sqrt{2} \left( 4r/\sqrt{2} \right)^2} = 0.555$$



For (111):

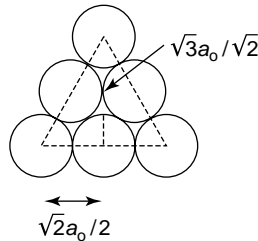
From the sketch, we can determine that the area of the (111) plane is  $\left( \sqrt{2}a_0/2 \right) \left( \sqrt{3}a_0/\sqrt{2} \right) = 0.866a_0^2$ . There are  $(3)(\frac{1}{2}) + (3)(\frac{1}{6}) = 2$  atoms in this area.

$$\text{planar density} = \frac{2 \text{ points}}{0.866(3.5167 \times 10^{-8} \text{ cm})^2}$$

$$= 0.1867 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi \left( \sqrt{2}a_0/4 \right)^2}{0.866a_0^2} = 0.907$$

The (111) is close packed.



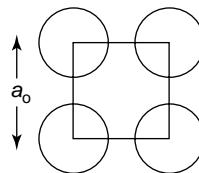
- 3-72** Determine the planar density and packing fraction for BCC lithium in the (100), (110), and (111) planes. Which, if any, of these planes is close-packed?

**Solution:**  $a_0 = 3.5089 \text{ \AA}$

For (100):

$$\text{planar density} = \frac{1}{(3.5089 \times 10^{-8} \text{ cm})^2} = 0.0812 \times 10^{16} \text{ points/cm}^2$$

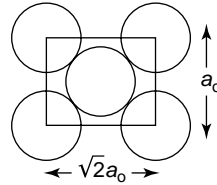
$$\text{packing fraction} = \frac{\pi \left[ \sqrt{3}a_0/4 \right]^2}{a_0^2} = 0.589$$



For (110):

$$\text{planar density} = \frac{2}{\sqrt{2}(3.5089 \times 10^{-8} \text{ cm})^2} = 0.1149 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi[\sqrt{3}a_o/4]^2}{\sqrt{2}a_o^2} = 0.833$$



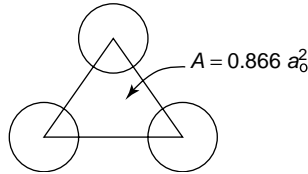
For (111):

There are only  $(3)(\frac{1}{6}) = \frac{1}{2}$  points in the plane, which has an area of  $0.866a_o^2$ .

$$\text{planar density} = \frac{\frac{1}{2}}{0.866(3.5089 \times 10^{-8} \text{ cm})^2} = 0.0469 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{\frac{1}{2} \pi [\sqrt{3}a_o/4]^2}{0.866a_o^2} = 0.34$$

There is no close-packed plane in BCC structures.



- 3-73** Suppose that FCC rhodium is produced as a 1 mm thick sheet, with the (111) plane parallel to the surface of the sheet. How many (111) interplanar spacings  $d_{111}$  thick is the sheet? See Appendix A for necessary data.

**Solution:** 
$$d_{111} = \frac{a_o}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{3.796 \text{ Å}}{\sqrt{3}} = 2.1916 \text{ Å}$$

$$\text{thickness} = \frac{(1 \text{ mm}/10 \text{ mm/cm})}{2.1916 \times 10^{-8} \text{ cm}} = 4.563 \times 10^6 d_{111} \text{ spacings}$$

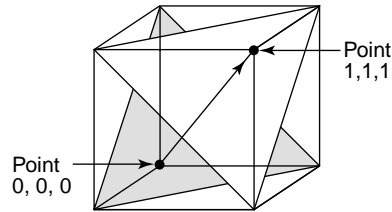
- 3-74** In a FCC unit cell, how many  $d_{111}$  are present between the 0,0,0 point and the 1,1,1 point?

**Solution:** The distance between the 0,0,0 and 1,1,1 points is  $\sqrt{3}a_o$ . The interplanar spacing is

$$d_{111} = a_o/\sqrt{1^2 + 1^2 + 1^2} = a_o/\sqrt{3}$$

Therefore the number of interplanar spacings is

$$\text{number of } d_{111} \text{ spacings} = \sqrt{3}a_o/(a_o/\sqrt{3}) = 3$$



- 3-79** Determine the minimum radius of an atom that will just fit into (a) the tetrahedral interstitial site in FCC nickel and (b) the octahedral interstitial site in BCC lithium.

**Solution:** (a) For the tetrahedral site in FCC nickel ( $a_o = 3.5167 \text{ \AA}$ ):

$$r_{\text{Ni}} = \frac{\sqrt{2}(3.5167 \text{ \AA})}{4} = 1.243 \text{ \AA}$$

$r/r_{\text{Ni}} = 0.225$  for a tetrahedral site. Therefore:

$$r = (1.243 \text{ \AA})(0.225) = 0.2797 \text{ \AA}$$

(b) For the octahedral site in BCC lithium ( $a_o = 3.5089 \text{ \AA}$ ):

$$r_{\text{Li}} = \frac{\sqrt{3}(3.5089)}{4} = 1.519 \text{ \AA}$$

$r/r_{\text{Li}} = 0.414$  for an octahedral site. Therefore:

$$r = (1.519 \text{ \AA})(0.414) = 0.629 \text{ \AA}$$

- 3-86** What is the radius of an atom that will just fit into the octahedral site in FCC copper without disturbing the lattice?

**Solution:**  $r_{\text{Cu}} = 1.278 \text{ \AA}$

$r/r_{\text{Cu}} = 0.414$  for an octahedral site. Therefore:

$$r = (1.278 \text{ \AA})(0.414) = 0.529 \text{ \AA}$$

- 3-87** Using the ionic radii given in Appendix B, determine the coordination number expected for the following compounds.

**Solution:** (a)  $\text{Y}_2\text{O}_3$  (b)  $\text{UO}_2$  (c)  $\text{BaO}$  (d)  $\text{Si}_3\text{N}_4$   
(e)  $\text{GeO}_2$  (f)  $\text{MnO}$  (g)  $\text{MgS}$  (h)  $\text{KBr}$

$$(a) \ r_{\text{Y}^{+3}}/r_{\text{O}^{-2}} = \frac{0.89}{1.32} = 0.67 \quad \text{CN} = 6 \quad (e) \ r_{\text{Ge}^{+4}}/r_{\text{O}^{-2}} = \frac{0.53}{1.32} = 0.40 \quad \text{CN} = 4$$

$$(b) \ r_{\text{U}^{+4}}/r_{\text{O}^{-2}} = \frac{0.97}{1.32} = 0.73 \quad \text{CN} = 6 \quad (f) \ r_{\text{Mn}^{+2}}/r_{\text{O}^{-2}} = \frac{0.80}{1.32} = 0.61 \quad \text{CN} = 6$$

$$(c) \ r_{\text{O}^{-2}}/r_{\text{Ba}^{+2}} = \frac{1.32}{1.34} = 0.99 \quad \text{CN} = 8 \quad (g) \ r_{\text{Mg}^{+2}}/r_{\text{S}^{-2}} = \frac{0.66}{1.32} = 0.50 \quad \text{CN} = 6$$

$$(d) \ r_{\text{N}^{-3}}/r_{\text{Si}^{+4}} = \frac{0.15}{0.42} = 0.36 \quad \text{CN} = 4 \quad (h) \ r_{\text{K}^{+1}}/r_{\text{Br}^{-1}} = \frac{1.33}{1.96} = 0.68 \quad \text{CN} = 6$$

- 3-88** Would you expect NiO to have the cesium chloride, sodium chloride, or zinc blende structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

**Solution:**  $r_{\text{Ni}}^{+2} = 0.69 \text{ \AA}$   $r_{\text{O}}^{-2} = 1.32 \text{ \AA}$   $\frac{r_{\text{Ni}}^{+2}}{r_{\text{O}}^{-2}} = 0.52$  CN = 6

A coordination number of 8 is expected for the CsCl structure, and a coordination number of 4 is expected for ZnS. But a coordination number of 6 is consistent with the *NaCl structure*.

(a)  $a_o = 2(0.69) + 2(1.32) = 4.02 \text{ \AA}$

(b)  $\rho = \frac{(4 \text{ of each ion/cell})(58.71 + 16 \text{ g/mol})}{(4.02 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 7.64 \text{ g/cm}^3$

(c)  $\text{PF} = \frac{(4\pi/3)(4 \text{ ions/cell})[(0.69)^3 + (1.32)^3]}{(4.02)^3} = 0.678$

- 3-89** Would you expect  $\text{UO}_2$  to have the sodium chloride, zinc blende, or fluorite structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

**Solution:**  $r_{\text{U}}^{+4} = 0.97 \text{ \AA}$   $r_{\text{O}}^{-2} = 1.32 \text{ \AA}$   $\frac{r_{\text{U}}^{+4}}{r_{\text{O}}^{-2}} = 0.97/1.32 = 0.735$

valence of U = +4, valence of O = -2

The radius ratio predicts a coordination number of 8; however there must be twice as many oxygen ions as uranium ions in order to balance the charge. The *fluorite* structure will satisfy these requirements, with:

U = FCC position (4) O = tetrahedral position (8)

(a)  $\sqrt{3} a_o = 4r_u + 4r_o = 4(0.97 + 1.32) = 9.16$  or  $a_o = 5.2885 \text{ \AA}$

(b)  $\rho = \frac{4(238.03 \text{ g/mol}) + 8(16 \text{ g/mol})}{(5.2885 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 12.13 \text{ g/cm}^3$

(c)  $\text{PF} = \frac{(4\pi/3)[4(0.97)^3 + 8(1.32)^3]}{(5.2885)^3} = 0.624$

- 3-90** Would you expect BeO to have the sodium chloride, zinc blende, or fluorite structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

**Solution:**  $r_{\text{Be}}^{+2} = 0.35 \text{ \AA}$   $r_{\text{O}}^{-2} = 1.32 \text{ \AA}$

$r_{\text{Be}}/r_{\text{O}} = 0.265$  CN = 4  $\therefore$  Zinc Blende

(a)  $\sqrt{3} a_o = 4r_{\text{Be}}^{+2} + 4r_{\text{O}}^{-2} = 4(0.35 + 1.32) = 6.68$  or  $a_o = 3.8567 \text{ \AA}$

(b)  $\rho = \frac{4(9.01 + 16 \text{ g/mol})}{(3.8567 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 2.897 \text{ g/cm}^3$

(c)  $\text{PF} = \frac{(4\pi/3)(4)[(0.35)^3 + 8(1.32)^3]}{(3.8567)^3} = 0.684$

- 3-91** Would you expect CsBr to have the sodium chloride, zinc blende, fluorite, or cesium chloride structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

**Solution:**  $r_{\text{Cs}}^{+1} = 1.67 \text{ \AA}$   $r_{\text{Br}}^{-1} = 1.96 \text{ \AA}$

$$\frac{r_{\text{Cs}}^{+1}}{r_{\text{Br}}^{-1}} = 0.852 \quad \text{CN} = 8 \quad \therefore \text{CsCl}$$

(a)  $\sqrt{3} a_o = 2r_{\text{Cs}}^{+1} + 2r_{\text{Br}}^{-1} = 2(1.96 + 1.67) = 7.26$  or  $a_o = 4.1916 \text{ \AA}$

(b)  $\rho = \frac{79.909 + 132.905 \text{ g/mol}}{(4.1916 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 4.8 \text{ g/cm}^3$

(c)  $\text{PF} = \frac{(4\pi/3)[(1.96)^3 + (1.67)^3]}{(4.1916)^3} = 0.693$

- 3-92** Sketch the ion arrangement on the (110) plane of ZnS (with the zinc blende structure) and compare this arrangement to that on the (110) plane of  $\text{CaF}_2$  (with the fluorite structure). Compare the planar packing fraction on the (110) planes for these two materials.

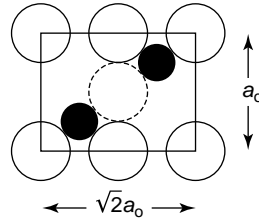
**Solution:** ZnS:

$$\sqrt{3} a_o = 4r_{\text{Zn}}^{+2} + 4r_{\text{S}}^{-2}$$

$$\sqrt{3} a_o = 4(0.074 \text{ nm}) + 4(0.184 \text{ nm})$$

$$a_o = 0.596 \text{ nm}$$

$$\text{PPF} = \frac{(2)(\pi r_{\text{Zn}}^2) + (2)(\pi r_{\text{S}}^2)}{(\sqrt{2} a_o) a_o} = \frac{2\pi(0.074)^2 + 2\pi(0.184)^2}{\sqrt{2}(0.596 \text{ nm})^2} = 0.492$$



$\text{CaF}_2$ :

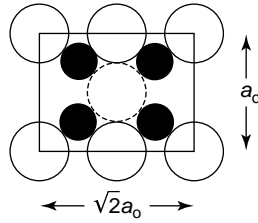
$$\sqrt{3} a_o = 4r_{\text{Ca}}^{+2} + 4r_{\text{F}}^{-1}$$

$$\sqrt{3} a_o = 4(0.099 \text{ nm}) + 4(0.133 \text{ nm})$$

$$a_o = 0.536 \text{ nm}$$

$$\text{PPF} = \frac{(2)(\pi r_{\text{Ca}}^2) + (4)(\pi r_{\text{F}}^2)}{(\sqrt{2} a_o) a_o} = \frac{2\pi(0.099)^2 + 4\pi(0.133)^2}{\sqrt{2}(0.536 \text{ nm})^2} = 0.699$$





- 3-93** MgO, which has the sodium chloride structure, has a lattice parameter of 0.396 nm. Determine the planar density and the planar packing fraction for the (111) and (222) planes of MgO. What ions are present on each plane?

**Solution:** As described in the answer to Problem 3-71, the area of the (111) plane is  $0.866a_o^2$ .

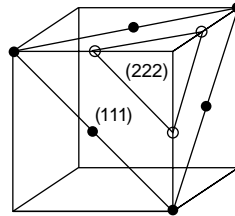
$$a_o = 2r_{\text{Mg}^{+2}} + 2r_{\text{O}^{-2}} = 2(0.66 + 1.32) = 3.96 \text{ \AA}$$

$$(111): \text{P.D.} = \frac{2 \text{ Mg}}{(0.866)(3.96 \times 10^{-8} \text{ cm})^2} = 0.1473 \times 10^{16} \text{ points/cm}^2$$

$$\text{PPF} = \frac{2\pi(0.66)^2}{(0.866)(3.96)^2} = 0.202$$

$$(222): \text{P.D.} = 0.1473 \times 10^{16} \text{ points/cm}^2$$

$$\text{PPF} = \frac{2\pi(1.32)^2}{(0.866)(3.96)^2} = 0.806$$



- 3-100** Polypropylene forms an orthorhombic unit cell with lattice parameters of  $a_o = 1.450 \text{ nm}$ ,  $b_o = 0.569 \text{ nm}$ , and  $c_o = 0.740 \text{ nm}$ . The chemical formula for the propylene molecule, from which the polymer is produced, is  $\text{C}_3\text{H}_6$ . The density of the polymer is about  $0.90 \text{ g/cm}^3$ . Determine the number of propylene molecules, the number of carbon atoms, and the number of hydrogen atoms in each unit cell.

**Solution:**  $\text{MW}_{\text{PP}} = 3 \text{ C} + 6 \text{ H} = 3(12) + 6 = 42 \text{ g/mol}$

$$0.90 \text{ g/cm}^3 = \frac{(x \text{ C}_3\text{H}_6)(42 \text{ g/mol})}{(14.5 \text{ cm})(5.69 \text{ cm})(7.40 \text{ cm})(10^{-24})(6.02 \times 10^{23} \text{ molecules/mol})}$$

$$x = 8 \text{ C}_3\text{H}_6 \text{ molecules or } 24 \text{ C atoms and } 48 \text{ H atoms}$$

- 3-101** The density of cristobalite is about  $1.538 \text{ g/cm}^3$ , and it has a lattice parameter of  $0.8037 \text{ nm}$ . Calculate the number of  $\text{SiO}_2$  ions, the number of silicon ions, and the number of oxygen ions in each unit cell.

$$\text{Solution: } 1.538 \text{ g/cm}^3 = \frac{(x \text{ SiO}_2)[28.08 + 2(16) \text{ g/mol}]}{(8.037 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ ions/mol})}$$

$$x = 8 \text{ SiO}_2 \text{ or } 8 \text{ Si}^{+4} \text{ ions and } 16 \text{ O}^{-2} \text{ ions}$$

- 3-105** A diffracted x-ray beam is observed from the (220) planes of iron at a  $2\theta$  angle of  $99.1^\circ$  when x-rays of 0.15418 nm wavelength are used. Calculate the lattice parameter of the iron.

**Solution:**  $\sin \theta = \lambda / 2d_{220}$

$$\sin(99.1/2) = \frac{0.15418\sqrt{2^2 + 2^2 + 0^2}}{2a_o}$$

$$a_o = \frac{0.15418\sqrt{8}}{2\sin(49.55)} = 0.2865 \text{ nm}$$

- 3-106** A diffracted x-ray beam is observed from the (311) planes of aluminum at a  $2\theta$  angle of  $78.3^\circ$  when x-rays of 0.15418 nm wavelength are used. Calculate the lattice parameter of the aluminum.

**Solution:**  $\sin \theta = \lambda / d_{311}$

$$a_o = \frac{0.15418\sqrt{3^2 + 1^2 + 1^2}}{2\sin(78.3/2)} = 0.40497 \text{ nm}$$

- 3-107** Figure 3-56 shows the results of an x-ray diffraction experiment in the form of the intensity of the diffracted peak versus the  $2\theta$  diffraction angle. If x-rays with a wavelength of 0.15418 nm are used, determine (a) the crystal structure of the metal, (b) the indices of the planes that produce each of the peaks, and (c) the lattice parameter of the metal.

**Solution:** The  $2\theta$  values can be estimated from Figure 3-56:

	$2\theta$	$\sin^2\theta$	$\sin^2\theta/0.0077$	Planar indices	$d = \lambda/2\sin\theta$	$a_o = d\sqrt{h^2 + k^2 + l^2}$
1	17.5	0.023	3	(111)	0.5068	0.8778
2	20.5	0.032	4	(200)	0.4332	0.8664
3	28.5	0.061	8	(220)	0.3132	0.8859
4	33.5	0.083	11	(311)	0.2675	0.8872
5	35.5	0.093	12	(222)	0.2529	0.8761
6	41	0.123	16	(400)	0.2201	0.8804
7	45	0.146	19	(331)	0.2014	0.8779
8	46.5	0.156	20	(420)	0.1953	0.8734

The  $\sin^2\theta$  values must be divided by 0.077 (one third the first  $\sin^2\theta$  value) in order to produce a possible sequence of numbers)

(a) The 3,4,8,11, . . . sequence means that the material is FCC

(c) The average  $a_o = 0.8781 \text{ nm}$

**3-108** Figure 3-57 shows the results of an x-ray diffraction experiment in the form of the intensity of the diffracted peak versus the  $2\theta$  diffraction angle. If x-rays with a wavelength of 0.0717 nm are used, determine (a) the crystal structure of the metal, (b) the indices of the planes that produce each of the peaks, and (c) the lattice parameter of the metal.

**Solution:** The  $2\theta$  values can be estimated from the figure:

	$2\theta$	$\sin^2\theta$	$\sin^2\theta/0.047$	Planar indices	$d = \lambda/2\sin\theta$	$a_o = d\sqrt{h^2 + k^2 + l^2}$
1	25.5	0.047	1	(111)	0.1610	0.2277
2	36	0.095	2	(200)	0.1150	0.2300
3	44.5	0.143	3	(211)	0.0938	0.2299
4	51.5	0.189	4	(220)	0.0818	0.2313
5	58	0.235	5	(310)	0.0733	0.2318
6	64.5	0.285	6	(222)	0.0666	0.2307
7	70	0.329	7	(321)	0.06195	0.2318
8	75.5	0.375	8	(400)	0.0580	0.2322

(a) The sequence 1,2,3,4,5,6,7,8 (which includes the “7”) means that the material is BCC.

(c) The average  $a_o = 0.2307$  nm

# 4

## Imperfections in the Atomic and Ionic Arrangements

- 4-1** Calculate the number of vacancies per  $\text{cm}^3$  expected in copper at  $1080^\circ\text{C}$  (just below the melting temperature). The activation energy for vacancy formation is  $20,000 \text{ cal/mol}$ .

**Solution:**

$$n = \frac{(4 \text{ atoms/u.c.})}{(3.6151 \times 10^{-8} \text{ cm})^3} = 8.47 \times 10^{22} \text{ atoms/cm}^3$$

$$n_v = 8.47 \times 10^{22} \exp[-20,000/(1.987)(1353)]$$

$$= 8.47 \times 10^{22} \exp(-7.4393) = 4.97 \times 10^{19} \text{ vacancies/cm}^3$$

- 4-2** The fraction of lattice points occupied by vacancies in solid aluminum at  $660^\circ\text{C}$  is  $10^{-3}$ . What is the activation energy required to create vacancies in aluminum?

**Solution:**

$$n_v/n = 10^{-3} = \exp[-Q/(1.987)(933)]$$

$$\ln(10^{-3}) = -6.9078 = -Q/(1.987)(933)$$

$$Q = 12,800 \text{ cal/mol}$$

- 4-3** The density of a sample of FCC palladium is  $11.98 \text{ g/cm}^3$  and its lattice parameter is  $3.8902 \text{ \AA}$ . Calculate (a) the fraction of the lattice points that contain vacancies and (b) the total number of vacancies in a cubic centimeter of Pd.

**Solution:**

$$(a) \ 11.98 \text{ g/cm}^3 = \frac{(x)(106.4 \text{ g/mol})}{(3.8902 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 3.9905$$

$$\text{fraction} = \frac{4.0 - 3.9905}{4} = 0.002375$$

$$(b) \text{ number} = \frac{0.0095 \text{ vacancies/u.c.}}{(3.8902 \times 10^{-8} \text{ cm})^3} = 1.61 \times 10^{20} \text{ vacancies/cm}^3$$

- 4-4** The density of a sample of HCP beryllium is 1.844 g/cm<sup>3</sup> and the lattice parameters are  $a_o = 0.22858 \text{ nm}$  and  $c_o = 0.35842 \text{ nm}$ . Calculate (a) the fraction of the lattice points that contain vacancies and (b) the total number of vacancies in a cubic centimeter.

**Solution:**  $V_{\text{u.c.}} = (0.22858 \text{ nm})^2(0.35842 \text{ nm})\cos 30 = 0.01622 \text{ nm}^3$   
 $= 1.622 \times 10^{-23} \text{ cm}^3$

(a) From the density equation:

$$1.844 \text{ g/cm}^3 = \frac{(x)(9.01 \text{ g/mol})}{(1.622 \times 10^{-23} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})} \quad x = 1.9984$$

$$\text{fraction} = \frac{2 - 1.9984}{2} = 0.0008$$

$$(b) \text{ number} = \frac{0.0016 \text{ vacancies/uc}}{1.622 \times 10^{-23} \text{ cm}^3} = 0.986 \times 10^{20} \text{ vacancies/cm}^3$$

- 4-5** BCC lithium has a lattice parameter of  $3.5089 \times 10^{-8} \text{ cm}$  and contains one vacancy per 200 unit cells. Calculate (a) the number of vacancies per cubic centimeter and (b) the density of Li.

**Solution:**

(a)  $\frac{1 \text{ vacancy}}{(200)(3.5089 \times 10^{-8} \text{ cm})^3} = 1.157 \times 10^{20} \text{ vacancies/cm}^3$

(b) In 200 unit cells, there are 399 Li atoms. The atoms/cell are 399/200:

$$\rho = \frac{(399/200)(6.94 \text{ g/mol})}{(3.5089 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 0.532 \text{ g/cm}^3$$

- 4-6** FCC lead has a lattice parameter of  $0.4949 \text{ nm}$  and contains one vacancy per 500 Pb atoms. Calculate (a) the density and (b) the number of vacancies per gram of Pb.

**Solution:**

(a) The number of atoms/cell =  $(499/500)(4 \text{ sites/cell})$

$$\rho = \frac{(499/500)(4)(207.19 \text{ g/mol})}{(0.4949 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 11.335 \text{ g/cm}^3$$

(b) The 500 Pb atoms occupy  $500 / 4 = 125$  unit cells:

$$\left( \frac{1 \text{ vacancy}}{125 \text{ cells}} \right) \times [(1/11.335 \text{ g/cm}^3)] = 5.82 \times 10^{18} \text{ vacancies/g}$$

- 4-7** A niobium alloy is produced by introducing tungsten substitutional atoms in the BCC structure; eventually an alloy is produced that has a lattice parameter of  $0.32554 \text{ nm}$  and a density of  $11.95 \text{ g/cm}^3$ . Calculate the fraction of the atoms in the alloy that are tungsten.

**Solution:**

$$11.95 \text{ g/cm}^3 = \frac{(x_w)(183.85 \text{ g/mol}) + (2 - x_w)(92.91 \text{ g/mol})}{(0.32554 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$248.186 = 183.85x_w + 185.82 - 92.91x_w$$

$$90.94x_w = 62.366 \quad \text{or} \quad x_w = 0.69 \text{ W atoms/cell}$$

There are 2 atoms per cell in BCC metals. Thus:

$$f_w = 0.69/2 = 0.345$$

- 4-8** Tin atoms are introduced into a FCC copper crystal, producing an alloy with a lattice parameter of  $3.7589 \times 10^{-8}$  cm and a density of  $8.772 \text{ g/cm}^3$ . Calculate the atomic percentage of tin present in the alloy.

**Solution:**

$$8.772 \text{ g/cm}^3 = \frac{(x_{\text{Sn}})(118.69 \text{ g/mol}) + (4 - x_{\text{Sn}})(63.54 \text{ g/mol})}{(3.7589 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})}$$

$$280.5 = 55.15x_{\text{Sn}} + 254.16 \quad \text{or} \quad x_{\text{Sn}} = 0.478 \text{ Sn atoms/cell}$$

There are 4 atoms per cell in FCC metals; therefore the at% Sn is:

$$(0.478/4) = 11.95\%$$

- 4-9** We replace 7.5 atomic percent of the chromium atoms in its BCC crystal with tantalum. X-ray diffraction shows that the lattice parameter is  $0.29158 \text{ nm}$ . Calculate the density of the alloy.

**Solution:**

$$\rho = \frac{(2)(0.925)(51.996 \text{ g/mol}) + 2(0.075)(180.95 \text{ g/mol})}{(2.9158 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 8.265 \text{ g/cm}^3$$

- 4-10** Suppose we introduce one carbon atom for every 100 iron atoms in an interstitial position in BCC iron, giving a lattice parameter of  $0.2867 \text{ nm}$ . For the Fe-C alloy, find (a) the density and (b) the packing factor.

**Solution:** There is one carbon atom per 100 iron atoms, or 1 C/50 unit cells, or 1/50 C per unit cell:

(a)

$$\rho = \frac{(2)(55.847 \text{ g/mol}) + (1/50)(12 \text{ g/mol})}{(2.867 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 7.89 \text{ g/cm}^3$$

(b)

$$\text{Packing Factor} = \frac{2(4\pi/3)(1.241)^3 + (1/50)(4\pi/3)(0.77)^3}{(2.867)^3} = 0.681$$

- 4-11** The density of BCC iron is  $7.882 \text{ g/cm}^3$  and the lattice parameter is  $0.2866 \text{ nm}$  when hydrogen atoms are introduced at interstitial positions. Calculate (a) the atomic fraction of hydrogen atoms and (b) the number of unit cells required on average to contain one hydrogen atom.

**Solution:**

(a)

$$7.882 \text{ g/cm}^3 = \frac{2(55.847 \text{ g/mol}) + x(1.00797 \text{ g/mol})}{(2.866 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 0.0081 \text{ H atoms/cell}$$

The total atoms per cell include 2 Fe atoms and 0.0081 H atoms.  
Thus:

$$f_{\text{H}} = \frac{0.0081}{2.0081} = 0.004$$

- (b) Since there is 0.0081 H/cell, then the number of cells containing H atoms is:

$$\text{cells} = 1/0.0081 = 123.5 \quad \text{or} \quad 1 \text{ H in } 123.5 \text{ cells}$$

- 4-12** Suppose one Schottky defect is present in every tenth unit cell of MgO. MgO has the sodium chloride crystal structure and a lattice parameter of 0.396 nm. Calculate (a) the number of anion vacancies per cm<sup>3</sup> and (b) the density of the ceramic.

**Solution:** In 10 unit cells, we expect 40 Mg + 40 O ions, but due to the defect:

$$40 \text{ Mg} - 1 = 39$$

$$40 \text{ O} - 1 = 39$$

$$(a) \text{ 1 vacancy}/(10 \text{ cells})(3.96 \times 10^{-8} \text{ cm})^3 = 1.61 \times 10^{21} \text{ vacancies/cm}^3$$

$$(b) \rho = \frac{(39/40)(4)(24.312 \text{ g/mol}) + (39/40)(4)(16 \text{ g/mol})}{(3.96 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 4.205 \text{ g/cm}^3$$

- 4-13** ZnS has the zinc blende structure. If the density is 3.02 g/cm<sup>3</sup> and the lattice parameter is 0.59583 nm, determine the number of Schottky defects (a) per unit cell and (b) per cubic centimeter.

**Solution:** Let  $x$  be the number of each type of ion in the unit cell. There normally are 4 of each type.

$$(a) \quad 3.02 \text{ g/cm}^3 = \frac{x(65.38 \text{ g/mol}) + x(32.064 \text{ g/mol})}{(5.9583 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ ions/mol})} \quad x = 3.9465$$

$$4 - 3.9465 = 0.0535 \text{ defects/u.c.}$$

$$(b) \quad \# \text{ of unit cells/cm}^3 = 1/(5.9683 \times 10^{-8} \text{ cm})^3 = 4.704 \times 10^{21}$$

$$\text{Schottky defects per cm}^3 = (4.704 \times 10^{21})(0.0535) = 2.517 \times 10^{20}$$

- 4-14** Suppose we introduce the following point defects. What other changes in each structure might be necessary to maintain a charge balance? Explain.

- (a) Mg<sup>2+</sup> ions substitute for yttrium atoms in Y<sub>2</sub>O<sub>3</sub>
- (b) Fe<sup>3+</sup> ions substitute for magnesium ions in MgO
- (c) Li<sup>1+</sup> ions substitute for magnesium ions in MgO
- (d) Fe<sup>2+</sup> ions replace sodium ions in NaCl

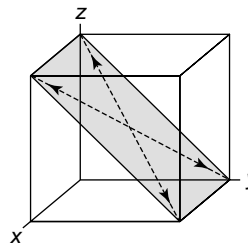
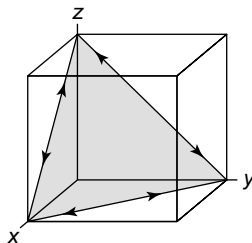
**Solution:**

- (a) Remove 2 Y<sup>3+</sup> and add 3 Mg<sup>2+</sup> – create cation interstitial.
- (b) Remove 3 Mg<sup>2+</sup> and add 2 Fe<sup>3+</sup> – create cation vacancy.
- (c) Remove 1 Mg<sup>2+</sup> and add 2 Li<sup>+</sup> – create cation interstitial.
- (d) Remove 2 Na<sup>+</sup> and add 1 Fe<sup>2+</sup> – create cation vacancy.

- 4-22** What are the Miller indices of the slip directions
- (a) on the (111) plane in an FCC unit cell
  - (b) on the (011) plane in a BCC unit cell?

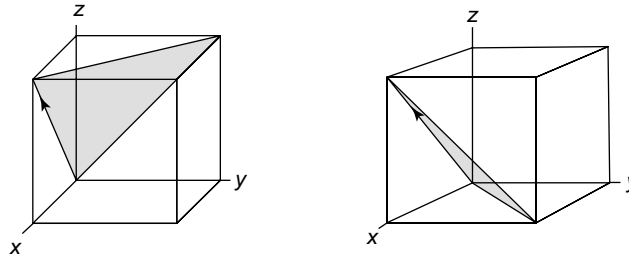
**Solution:**

[0 $\bar{1}$ 1], [01 $\bar{1}$ ]	[1 $\bar{1}$ 1], [ $\bar{1}$ 1 $\bar{1}$ ]
[ $\bar{1}$ 10], [1 $\bar{1}$ 0]	[ $\bar{1}$ $\bar{1}$ 1], [11 $\bar{1}$ ]
[ $\bar{1}$ 01], [10 $\bar{1}$ ]	



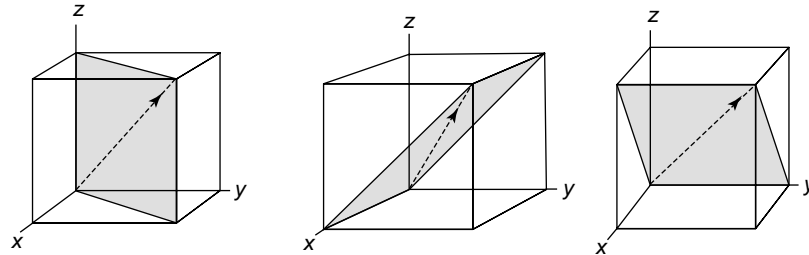
- 4-23** What are the Miller indices of the slip planes in FCC unit cells that include the  $[101]$  slip direction?

**Solution:**  $(11\bar{1})$ ,  $(\bar{1}11)$   $(\bar{1}11)$ ,  $(1\bar{1}\bar{1})$



- 4-24** What are the Miller indices of the  $\{110\}$  slip planes in BCC unit cells that include the  $[111]$  slip direction?

**Solution:**  $(1\bar{1}0)$ ,  $(\bar{1}10)$   $(0\bar{1}1)$ ,  $(01\bar{1})$   $(10\bar{1})$ ,  $(\bar{1}01)$



- 4-25** Calculate the length of the Burgers vector in the following materials:

(a) BCC niobium (b) FCC silver (c) diamond cubic silicon

**Solution:** (a) The repeat distance, or Burgers vector, is half the body diagonal, or:

$$b = \text{repeat distance} = (\frac{1}{2}) (\sqrt{3}) (3.294 \text{ \AA}) = 2.853 \text{ \AA}$$

(b) The repeat distance, or Burgers vector, is half of the face diagonal, or:

$$b = (\frac{1}{2}) (\sqrt{2} a_0) = (\frac{1}{2}) (\sqrt{2}) (4.0862 \text{ \AA}) = 2.889 \text{ \AA}$$

(c) The slip direction is  $[110]$ , where the repeat distance is half of the face diagonal:

$$b = (\frac{1}{2}) (\sqrt{2}) (5.4307 \text{ \AA}) = 3.840 \text{ \AA}$$

- 4-26** Determine the interplanar spacing and the length of the Burgers vector for slip on the expected slip systems in FCC aluminum. Repeat, assuming that the slip system is a  $(110)$  plane and a  $[\bar{1}\bar{1}1]$  direction. What is the ratio between the shear stresses required for slip for the two systems? Assume that  $k = 2$  in Equation 4-2.

**Solution:** (a) For  $(111)/[110]$ ,

$$b = (\frac{1}{2}) (\sqrt{2}) (4.04958 \text{ \AA}) = 2.863 \text{ \AA} \quad d_{111} = \frac{4.04958 \text{ \AA}}{\sqrt{1+1+1}} = 2.338 \text{ \AA}$$

(b) If  $(110)/[111]$ , then:

$$b = \sqrt{3} (4.04958 \text{ \AA}) = 7.014 \text{ \AA} \quad d_{110} = \frac{4.04958 \text{ \AA}}{\sqrt{1^2 + 1^2 + 0^2}} = 2.863 \text{ \AA}$$



(c) If we assume that  $k = 2$  in Equation 4-2, then

$$(d/b)_a = \frac{2.338}{2.863} = 0.8166 \quad (d/b)_b = \frac{2.863}{7.014} = 0.408$$

$$\therefore \frac{\tau_a}{\tau_b} = \frac{\exp(-2(0.8166))}{\exp(-2(0.408))} = 0.44$$

- 4-27** Determine the interplanar spacing and the length of the Burgers vector for slip on the (110)/[1 $\bar{1}$ 1] slip system in BCC tantalum. Repeat, assuming that the slip system is a (111)/[1 $\bar{1}$ 0] system. What is the ratio between the shear stresses required for slip for the two systems? Assume that  $k = 2$  in Equation 4-2.

**Solution:** (a) For (110)/[1 $\bar{1}$ 1]:

$$b = (\frac{1}{2})(\sqrt{3})(3.3026 \text{ \AA}) = 2.860 \text{ \AA} \quad d_{110} = \frac{3.3026 \text{ \AA}}{\sqrt{1^2 + 1^2 + 0^2}} = 2.335 \text{ \AA}$$

(b) If (111)/[1 $\bar{1}$ 0], then:

$$b = \sqrt{2}(3.3026 \text{ \AA}) = 4.671 \text{ \AA} \quad d_{111} = \frac{3.3026 \text{ \AA}}{\sqrt{1^2 + 1^2 + 1^2}} = 1.907 \text{ \AA}$$

(c) If we assume that  $k = 2$  in Equation 4-2, then:

$$(d/b)_a = \frac{2.335}{2.86} = 0.8166 \quad (d/b)_b = \frac{1.907}{4.671} = 0.408$$

$$\frac{\tau_a}{\tau_b} = \frac{\exp(-2(0.8166))}{\exp(-2(0.408))} = 0.44$$

- 4-37** How many grams of aluminum, with a dislocation density of  $10^{10} \text{ cm/cm}^3$ , are required to give a total dislocation length that would stretch from New York City to Los Angeles (3000 miles)?

**Solution:**  $(3000 \text{ mi})(5280 \text{ ft/mi})(12 \text{ in./ft})(2.54 \text{ cm/in.}) = 4.828 \times 10^8 \text{ cm}$

$$\frac{(4.828 \times 10^8 \text{ cm})(2.699 \text{ g/cm}^3)}{(10^{10} \text{ cm/cm}^3)} = 0.13 \text{ g}$$

- 4-38** The distance from Earth to the Moon is 240,000 miles. If this were the total length of dislocation in a cubic centimeter of material, what would be the dislocation density?

**Solution:**  $(240,000 \text{ mi})(5280 \text{ ft/mi})(12 \text{ in./ft})(2.54 \text{ cm/in.}) = 3.86 \times 10^{10} \text{ cm/cm}^3$

- 4-41** Suppose you would like to introduce an interstitial or large substitutional atom into the crystal near a dislocation. Would the atom fit more easily above or below the dislocation line shown in Figure 4-8(b)? Explain.

**Solution:** The atom would fit more easily into the area just below the dislocation due to the atoms being pulled apart; this allows more space into which the atom can fit.

- 4-42** Compare the  $c/a$  ratios for the following HCP metals, determine the likely slip processes in each, and estimate the approximate critical resolved shear stress. Explain. (See data in Appendix A)

(a) zinc                      (b) magnesium                      (c) titanium  
(d) zirconium              (e) rhenium                      (f) beryllium

**Solution:** We expect metals with  $c/a > 1.633$  to have a low  $\tau_{\text{crss}}$ :

- (a) Zn:  $\frac{4.9470}{2.6648} = 1.856$  – low  $\tau_{crss}$       (b) Mg:  $\frac{5.209}{3.2087} = 1.62$  – medium  $\tau_{crss}$
- (c) Ti:  $\frac{4.6831}{2.9503} = 1.587$  – high  $\tau_{crss}$       (d) Zr:  $\frac{5.1477}{3.2312} = 1.593$  – high  $\tau_{crss}$
- (e) Rh:  $\frac{4.458}{2.760} = 1.615$  – medium  $\tau_{crss}$       (f) Be:  $\frac{3.5842}{2.2858} = 1.568$  – high  $\tau_{crss}$

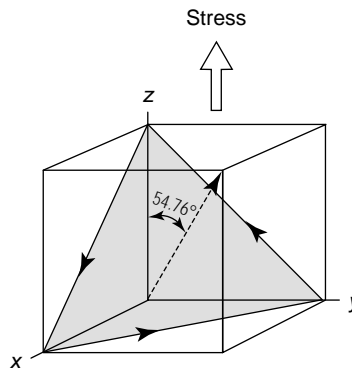
**4-43** A single crystal of an FCC metal is oriented so that the  $[001]$  direction is parallel to an applied stress of 5000 psi. Calculate the resolved shear stress acting on the  $(111)$  slip plane in the  $[\bar{1}10]$ ,  $[0\bar{1}1]$ , and  $[10\bar{1}]$  slip directions. Which slip system(s) will become active first?

**Solution:**       $\phi = 54.76^\circ$        $\tau = 5000 \cos 54.76 \cos \lambda$

$\lambda_{110} = 90^\circ$        $\tau = 0$

$\lambda_{011} = 45^\circ$        $\tau = 2,040$  psi active

$\lambda_{101} = 45^\circ$        $\tau = 2,040$  psi active



**4-44** A single crystal of a BCC metal is oriented so that the  $[001]$  direction is parallel to the applied stress. If the critical resolved shear stress required for slip is 12,000 psi, calculate the magnitude of the applied stress required to cause slip to begin in the  $[1\bar{1}1]$  direction on the  $(110)$ ,  $(011)$ , and  $(10\bar{1})$  slip planes.

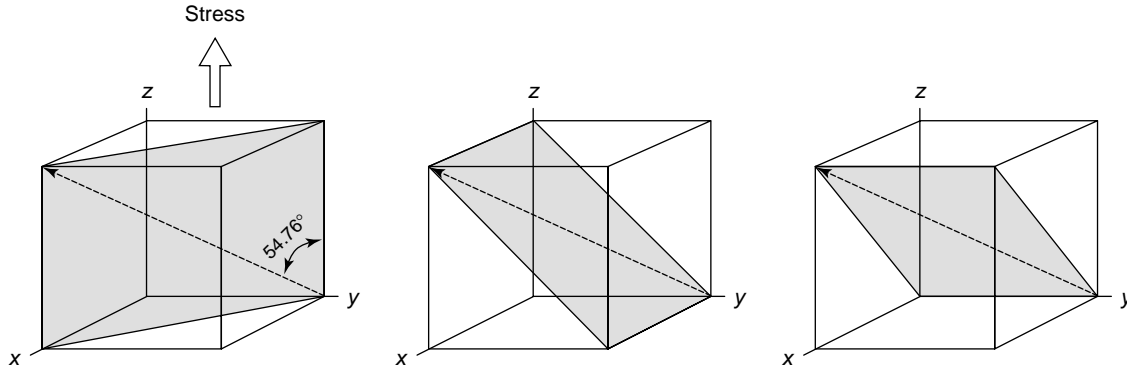
**Solution:**       $CRSS = 12,000 \text{ psi} = \sigma \cos \phi \cos \lambda$

$\lambda = 54.76^\circ$        $\frac{12,000 \text{ psi}}{\cos \phi \cos \lambda} = \sigma$

$\phi_{110} = 90^\circ$        $\sigma = \infty$

$\phi_{011} = 45^\circ$        $\sigma = 29,412$  psi

$\phi_{101} = 45^\circ$        $\sigma = 29,412$  psi



- 4-45** Our discussion of Schmid's law dealt with single crystals of a metal. Discuss slip and Schmid's law in a polycrystalline material. What might happen as the grain size gets smaller and smaller?

**Solution:** With smaller grains, the movement of the dislocations is impeded by frequent intersections with the grain boundaries. The strength of metals is not nearly as low as might be predicted from the critical resolved shear stress as a consequence of these interactions.

- 4-49** The strength of titanium is found to be 65,000 psi when the grain size is  $17 \times 10^{-6}$  m and 82,000 psi when the grain size is  $0.8 \times 10^{-6}$  m. Determine (a) the constants in the Hall-Petch equation and (b) the strength of the titanium when the grain size is reduced to  $0.2 \times 10^{-6}$  m.

**Solution:**  $65,000 = \sigma_o + K \frac{1}{\sqrt{17 \times 10^{-6}}} = \sigma_o + 242.5 K$

$$82,000 = \sigma_o + K \frac{1}{\sqrt{0.8 \times 10^{-6}}} = \sigma_o + 1118.0 K$$

(a) By solving the two simultaneous equations:

$$K = 19.4 \text{ psi} / \sqrt{m} \quad \sigma_o = 60,290 \text{ psi}$$

(b)  $\sigma = 60,290 + 19.4 / \sqrt{0.2 \times 10^{-6}} = 103,670 \text{ psi}$

- 4-50** A copper-zinc alloy has the following properties:

grain diameter (mm)	strength (MPa)	$d^{-1/2}$
0.015	170 MPa	8.165
0.025	158 MPa	6.325
0.035	151 MPa	5.345
0.050	145 MPa	4.472

Determine (a) the constants in the Hall-Petch equation and (b) the grain size required to obtain a strength of 200 MPa.

**Solution:** The values of  $d^{-1/2}$  are included in the table; the graph shows the relationship. We can determine  $K$  and  $\sigma_o$  either from the graph or by using two of the data points.

$$(a) \quad 170 = \sigma_o + K(8.165)$$

$$145 = \sigma_o + K(4.472)$$

$$25 = 3.693K$$

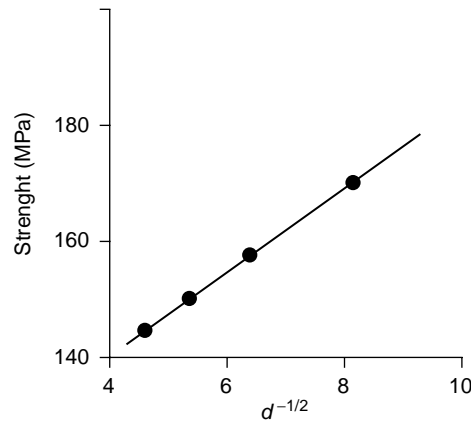
$$K = 6.77 \text{ MPa} / \sqrt{\text{mm}} \quad \sigma_o = 114.7 \text{ MPa}$$

(b) To obtain a strength of 200 MPa:

$$200 = 114.7 + 6.77 / \sqrt{d}$$

$$85.3 = 6.77 / \sqrt{d}$$

$$d = 0.0063 \text{ mm}$$



**4-51** For an ASTM grain size number of 8, calculate the number of grains per square inch  
(a) at a magnification of 100 and (b) with no magnification.

**Solution:** (a)  $N = 2^{n-1}$   $N = 2^{8-1} = 2^7 = 128 \text{ grains/in.}^2$

(b) No magnification means that the magnification is "1":

$$(2^7)(100/1)^2 = 1.28 \times 10^6 \text{ grains/in.}^2$$

**4-52** Determine the ASTM grain size number if 20 grains/square inch are observed at a magnification of 400.

**Solution:**  $(20)(400/100)^2 = 2^{n-1}$   $\log(320) = (n-1)\log(2)$

$$2.505 = (n-1)(0.301) \text{ or } n = 9.3$$

**4-53** Determine the ASTM grain size number if 25 grains/square inch are observed at a magnification of 50.

**Solution:**  $25(50/100)^2 = 2^{n-1}$   $\log(6.25) = (n-1)\log(2)$

$$0.796 = (n-1)(0.301) \text{ or } n = 3.6$$

- 4-54** Determine the ASTM grain size number for the materials in  
(a) Figure 4-18 (b) Figure 4-23

**Solution:** (a) There are about 26 grains in the photomicrograph, which has the dimensions 2.375 in.  $\times$  2 in. The magnification is 100, thus:

$$\frac{26}{(2.375)(2)} = 2^{n-1} \quad \log(5.47) = 0.738 = (n-1)\log(2) \quad n = 3.5$$

(b) There are about 59 grains in the photomicrograph, which has the dimensions 2.25 in.  $\times$  2 in. The magnification is 500, thus:

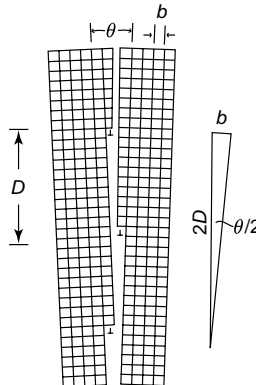
$$\frac{59(500/100)^2}{(2.25)(2)} = 2^{n-1} \quad \log(328) = 2.516 = (n-1)\log(2) \quad n = 9.4$$

There are about 28 grains in the photomicrograph, which has the dimensions 2 in.  $\times$  2.25 in. The magnification is 200, thus:

$$\frac{28(200/100)^2}{(2.25)(2)} = 2^{n-1} \quad \log(24.889) = 1.396 = (n-1)\log(2) \quad n = 5.6$$

- 4-58** The angle  $\theta$  of a tilt boundary is given by  $\sin(\theta/2) = b/2D$  (see Figure 4-19). Verify the correctness of this equation.

**Solution:** From the figure, we note that the grains are offset one Burgers vector,  $b$ , only for two spacings  $D$ . Then it is apparent that  $\sin(\theta/2)$  must be  $b$  divided by two  $D$ .



- 4-59** Calculate the angle  $\theta$  of a small angle grain boundary in FCC aluminum when the dislocations are 5000 Å apart. (See Figure 4-19 and equation in Problem 4-58.)

**Solution:**  $b = (\frac{1}{2}) (\sqrt{2}) (4.04958) = 2.8635 \text{ Å}$  and  $D = 5000 \text{ Å}$

$$\sin(\theta/2) = \frac{2.8635}{(2)(5000)} = 0.000286$$

$$\theta/2 = 0.0164$$

$$\theta = 0.0328^\circ$$

- 4-60** For BCC iron, calculate the average distance between dislocations in a small angle grain boundary tilted 0.50°. (See Figure 4-19.)

**Solution:**  $\sin(0.5/2) = \frac{\frac{1}{2} (\sqrt{3}) (2.866)}{2D}$

$$0.004364 = 1.241/D$$

$$D = 284 \text{ Å}$$

# 5

## Atom and Ion Movements in Materials

- 5-12** Atoms are found to move from one lattice position to another at the rate of  $5 \times 10^5$  jumps/s at  $400^\circ\text{C}$  when the activation energy for their movement is 30,000 cal/mol. Calculate the jump rate at  $750^\circ\text{C}$ .

Solution:

$$\text{Rate} = \frac{5 \times 10^5}{x} = \frac{c_o \exp[-30,000/(1.987)(673)]}{c_o \exp[-30,000/(1.987)(1023)]} = \exp(-22.434 + 14.759)$$

$$\frac{5 \times 10^5}{x} = \exp(-7.675) = 4.64 \times 10^{-4}$$

$$x = \frac{5 \times 10^5}{4.64 \times 10^{-4}} = 1.08 \times 10^9 \text{ jumps/s}$$

- 5-13** The number of vacancies in a material is related to temperature by an Arrhenius equation. If the fraction of lattice points containing vacancies is  $8 \times 10^{-5}$  at  $600^\circ\text{C}$ , determine the fraction at  $1000^\circ\text{C}$ .

Solution:  $8 \times 10^{-5} = \exp[-Q/(1.987)(873)] \quad Q = 16,364 \text{ cal/mol}$

$$f = n_v/n = \exp[-16,364/(1.987)(1273)] = 0.00155$$

- 5-24** The diffusion coefficient for  $\text{Cr}^{+3}$  in  $\text{Cr}_2\text{O}_3$  is  $6 \times 10^{-15} \text{ cm}^2/\text{s}$  at  $727^\circ\text{C}$  and is  $1 \times 10^{-9} \text{ cm}^2/\text{s}$  at  $1400^\circ\text{C}$ . Calculate (a) the activation energy and (b) the constant  $D_o$ .

Solution: (a)  $\frac{6 \times 10^{-15}}{1 \times 10^{-9}} = \frac{D_o \exp[-Q/(1.987)(1000)]}{D_o \exp[-Q/(1.987)(1673)]}$

$$6 \times 10^{-6} = \exp[-Q(0.000503 - 0.00030)] = \exp[-0.000203 Q]$$

$$-12.024 = -0.000203 Q \quad \text{or} \quad Q = 59,230 \text{ cal/mol}$$

(b)  $1 \times 10^{-9} = D_o \exp[-59,230/(1.987)(1673)] = D_o \exp(-17.818)$

$$1 \times 10^{-9} = 1.828 \times 10^{-8} D_o \quad \text{or} \quad D_o = 0.055 \text{ cm}^2/\text{s}$$

- 5-25** The diffusion coefficient for  $O^{2-}$  in  $Cr_2O_3$  is  $4 \times 10^{-15} \text{ cm}^2/\text{s}$  at  $1150^\circ\text{C}$  and  $6 \times 10^{-11} \text{ cm}^2/\text{s}$  at  $1715^\circ\text{C}$ . Calculate (a) the activation energy and (b) the constant  $D_o$ .

**Solution:**

$$\frac{4 \times 10^{-15}}{6 \times 10^{-11}} = \frac{D_o \exp[-Q/(1.987)(1423)]}{D_o \exp[-Q/(1.987)(1988)]}$$

$$6.67 \times 10^{-5} = \exp[-0.0001005 Q]$$

$$-9.615 = -0.0001005 Q \quad \text{or} \quad Q = 95,700 \text{ cal/mol}$$

$$4 \times 10^{-15} = D_o \exp[-95,700/(1.987)(1423)] = D_o(2.02 \times 10^{-15})$$

$$D_o = 1.98 \text{ cm}^2/\text{s}$$

- 5-42** A 0.2-mm thick wafer of silicon is treated so that a uniform concentration gradient of antimony is produced. One surface contains 1 Sb atom per  $10^8$  Si atoms and the other surface contains 500 Sb atoms per  $10^8$  Si atoms. The lattice parameter for Si is  $5.407 \text{ \AA}$  (Appendix A). Calculate the concentration gradient in (a) atomic percent Sb per cm and (b) Sb atoms/ $\text{cm}^3\text{-cm}$ .

**Solution:**

$$\Delta c/\Delta x = \frac{(1/10^8 - 500/10^8)}{0.02 \text{ cm}} \times 100\% = -0.02495 \text{ at\%/cm}$$

$$a_o = 5.4307 \text{ \AA} \quad V_{\text{unit cell}} = 160.16 \times 10^{-24} \text{ cm}^3$$

$$c_1 = \frac{(8 \text{ Si atoms/u.c.})(1 \text{ Sb}/10^8 \text{ Si})}{160.16 \times 10^{-24} \text{ cm}^3/\text{u.c.}} = 0.04995 \times 10^{16} \text{ Sb atoms/cm}^3$$

$$c_2 = \frac{(8 \text{ Si atoms/u.c.})(500 \text{ Sb}/10^8 \text{ Si})}{160.16 \times 10^{-24} \text{ cm}^3/\text{u.c.}} = 24.975 \times 10^{16} \text{ Sb atoms/cm}^3$$

$$\Delta c/\Delta x = \frac{(0.04995 - 24.975) \times 10^{16}}{0.02 \text{ cm}} = -1.246 \times 10^{19} \text{ Sb atoms/cm}^3 \text{ cm}$$

- 5-43** When a Cu-Zn alloy solidifies, one portion of the structure contains 25 atomic percent zinc and another portion 0.025 mm away contains 20 atomic percent zinc. If the lattice parameter for the FCC alloy is  $3.63 \times 10^{-8} \text{ cm}$ , determine the concentration gradient in (a) atomic percent Zn per cm, (b) weight percent Zn per cm, and (c) Zn atoms/ $\text{cm}^3\text{-cm}$ .

**Solution:**

(a)  $\Delta c/\Delta x = \frac{20\% - 25\%}{(0.025 \text{ mm})(0.1 \text{ cm/mm})} = -2000 \text{ at\% Zn/cm}$

(b) We now need to determine the wt% of zinc in each portion:

$$\text{wt\% Zn} = \frac{(20)(65.38 \text{ g/mol})}{(20)(65.38) + (80)(63.54)} \times 100 = 20.46$$

$$\text{wt\% Zn} = \frac{(25)(65.38 \text{ g/mol})}{(25)(65.38) + (75)(63.54)} \times 100 = 25.54$$

$$\Delta c/\Delta x = \frac{20.46\% - 25.54\%}{0.0025 \text{ cm}} = -2032 \text{ wt\% Zn/cm}$$

(c) Now find the number of atoms per  $\text{cm}^3$ :

$$c_1 = \frac{(4 \text{ atoms/cell})(0.2 \text{ Zn fraction})}{(3.63 \times 10^{-8} \text{ cm})^3} = 0.0167 \times 10^{24} \text{ Zn atoms/cm}^3$$

$$c_2 = \frac{(4 \text{ atoms/cell})(0.25 \text{ Zn fraction})}{(3.63 \times 10^{-8} \text{ cm})^3} = 0.0209 \times 10^{24} \text{ Zn atoms/cm}^3$$

$$\Delta c/\Delta x = \frac{0.0167 \times 10^{24} - 0.0209 \times 10^{24}}{0.0025 \text{ cm}} = -1.68 \text{ Zn atoms/cm}^3\text{-cm}$$

- 5-44** A 0.001 in. BCC iron foil is used to separate a high hydrogen gas from a low hydrogen gas at 650°C.  $5 \times 10^8$  H atoms/cm<sup>3</sup> are in equilibrium with the hot side of the foil, while  $2 \times 10^3$  H atoms/cm<sup>3</sup> are in equilibrium with the cold side. Determine (a) the concentration gradient of hydrogen and (b) the flux of hydrogen through the foil.

**Solution:**

(a)  $\Delta c/\Delta x = \frac{2 \times 10^3 - 5 \times 10^8}{(0.001 \text{ in.})(2.54 \text{ cm/in.})} = -1969 \times 10^8 \text{ H atoms/cm}^3\text{-cm}$

(b)  $J = -D(\Delta c/\Delta x) = -0.0012 \exp[-3600/(1.987)(923)](-1969 \times 10^8)$   
 $J = 0.33 \times 10^8 \text{ H atoms/cm}^2\text{-s}$

- 5-45** A 1-mm sheet of FCC iron is used to contain nitrogen in a heat exchanger at 1200°C. The concentration of N at one surface is 0.04 atomic percent and the concentration at the second surface is 0.005 atomic percent. Determine the flux of nitrogen through the foil in atoms/cm<sup>2</sup>-s.

**Solution:**

(a)  $\Delta c/\Delta x = \frac{(0.00005 - 0.0004)(4 \text{ atoms per cell})/(3.589 \times 10^{-8} \text{ cm})^3}{(1 \text{ mm})(0.1 \text{ cm/mm})}$   
 $= -3.03 \times 10^{20} \text{ N atoms/cm}^3\text{-cm}$

(b)  $J = -D(\Delta c/\Delta x) = -0.0034 \exp[-34,600/(1.987)(1473)](-3.03 \times 10^{20})$   
 $= 7.57 \times 10^{12} \text{ N atoms/cm}^2\text{-s}$

- 5-46** A 4 cm-diameter, 0.5 mm-thick spherical container made of BCC iron holds nitrogen at 700°C. The concentration at the inner surface is 0.05 atomic percent and at the outer surface is 0.002 atomic percent. Calculate the number of grams of nitrogen that are lost from the container per hour.

**Solution:**

$\Delta c/\Delta x = \frac{[0.00002 - 0.0005](2 \text{ atoms/cell})/(2.866 \times 10^{-8} \text{ cm})^3}{(0.5 \text{ mm})(0.1 \text{ cm/mm})}$   
 $= -8.16 \times 10^{20} \text{ N/cm}^3\text{-cm}$

$J = -0.0047 \exp[-18,300/(1.987)(973)](-8.16 \times 10^{20}) = 2.97 \times 10^{14} \text{ N/cm}^2\text{-s}$

$A_{\text{sphere}} = 4\pi r^2 = 4\pi(2 \text{ cm})^2 = 50.27 \text{ cm}^2 \quad t = 3600 \text{ s/h}$

$\text{N atoms/h} = (2.97 \times 10^{14})(50.27)(3600) = 5.37 \times 10^{19} \text{ N atoms/h}$

$\text{N loss} = \frac{(5.37 \times 10^{19} \text{ atoms})(14.007 \text{ g/mol})}{(6.02 \times 10^{23} \text{ atoms/mol})} = 1.245 \times 10^{-3} \text{ g/h}$

- 5-47** A BCC iron structure is to be manufactured that will allow no more than 50 g of hydrogen to be lost per year through each square centimeter of the iron at 400°C. If the concentration of hydrogen at one surface is 0.05 H atom per unit cell and is 0.001 H atom per unit cell at the second surface, determine the minimum thickness of the iron.

**Solution:**

$c_1 = 0.05 \text{ H}/(2.866 \times 10^{-8} \text{ cm})^3 = 212.4 \times 10^{19} \text{ H atoms/cm}^3$

$c_2 = 0.001 \text{ H}/(2.866 \times 10^{-8} \text{ cm})^3 = 4.25 \times 10^{19} \text{ H atoms/cm}^3$

$\Delta c/\Delta x = \frac{4.25 \times 10^{19} - 212.4 \times 10^{19}}{\Delta x} = \frac{-2.08 \times 10^{21}}{\Delta x}$



$$J = \frac{(50 \text{ g/cm}^2 \text{ y})(6.02 \times 10^{23} \text{ atoms/mol})}{(1.00797 \text{ g/mol})(31.536 \times 10^6 \text{ s/y})} = 9.47 \times 10^{17} \text{ H atoms/cm}^2\text{-s}$$

$$J = 9.47 \times 10^{17} \text{ H atoms/cm}^2\text{-s}$$

$$= (-2.08 \times 10^{21}/\Delta x)(0.0012)\exp[-3600/((1.987)(673))]$$

$$\Delta x = 0.179 \text{ cm}$$

- 5-48** Determine the maximum allowable temperature that will produce a flux of less than 2000 H atoms/cm<sup>2</sup>-s through a BCC iron foil when the concentration gradient is  $-5 \times 10^{16}$  atoms/cm<sup>3</sup>-cm. (Note the negative sign for the flux.)

**Solution:**

$$2000 \text{ H atoms/cm}^2\text{-s} = -0.0012 \exp[-3600/1.987T] [-5 \times 10^{16} \text{ atoms/cm}^3\text{-cm}]$$

$$\ln(3.33 \times 10^{-11}) = -3600/1.987T$$

$$T = -3600/((-24.12)(1.987)) = 75 \text{ K} = -198^\circ\text{C}$$

- 5-53** Explain why a rubber balloon filled with helium gas deflates over time.

**Solution:** Helium atoms diffuse through the chains of the polymer material due to the small size of the helium atoms and the ease at which they diffuse between the loosely-packed chains.

- 5-59** The electrical conductivity of Mn<sub>3</sub>O<sub>4</sub> is  $8 \times 10^{-18} \text{ ohm}^{-1}\text{-cm}^{-1}$  at 140°C and is  $1 \times 10^{-7} \text{ ohm}^{-1}\text{-cm}^{-1}$  at 400°C. Determine the activation energy that controls the temperature dependence of conductivity. Explain the process by which the temperature controls conductivity.

**Solution:**

$$\frac{8 \times 10^{-18}}{1 \times 10^{-7}} = \frac{C_o \exp[-Q/(1.987)(413)]}{C_o \exp[-Q/(1.987)(673)]}$$

$$8 \times 10^{-11} = \exp(-0.000471Q) \quad \text{or} \quad -23.25 = -0.000471Q$$

$$Q = 49,360 \text{ cal/mol}$$

Electrical charge is carried by the diffusion of the atoms; as the temperature increases, more rapid diffusion occurs and consequently the electrical conductivity is higher.

- 5-60** Compare the rate at which oxygen ions diffuse in Al<sub>2</sub>O<sub>3</sub> with the rate at which aluminum ions diffuse in Al<sub>2</sub>O<sub>3</sub> at 1500°C. Explain the difference.

**Solution:**

$$D_{\text{O}}^{-2} = 1900 \exp[-152,000/(1.987)(1773)] = 3.47 \times 10^{-16} \text{ cm}^2/\text{s}$$

$$D_{\text{Al}}^{+3} = 28 \exp[-114,000/(1.987)(1773)] = 2.48 \times 10^{-13} \text{ cm}^2/\text{s}$$

The ionic radius of the oxygen ion is 1.32 Å, compared with the aluminum ionic radius of 0.51 Å; consequently it is much easier for the smaller aluminum ion to diffuse in the ceramic.

- 5-61** Compare the diffusion coefficients of carbon in BCC and FCC iron at the allotropic transformation temperature of 912°C and explain the difference.

**Solution:**

$$D_{\text{BCC}} = 0.011 \exp[-20,900/(1.987)(1185)] = 1.51 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$D_{\text{FCC}} = 0.23 \exp[-32,900/(1.987)(1185)] = 1.92 \times 10^{-7} \text{ cm}^2/\text{s}$$

Packing factor of the BCC lattice (0.68) is less than that of the FCC lattice; consequently atoms are expected to be able to diffuse more rapidly in the BCC iron.

- 5-62** Compare the diffusion coefficients for hydrogen and nitrogen in FCC iron at 1000°C and explain the difference in their values.

**Solution:**  $D_{\text{H in BCC}} = 0.0063 \exp[-10,300/(1.987)(1273)] = 1.074 \times 10^{-4} \text{ cm}^2/\text{s}$   
 $D_{\text{N in FCC}} = 0.0034 \exp[-34,600/(1.987)(1273)] = 3.898 \times 10^{-9} \text{ cm}^2/\text{s}$

Nitrogen atoms have a larger atoms radius (0.71 Å) compared with that of hydrogen atoms (0.46 Å); the smaller hydrogen ions are expected to diffuse more rapidly.

- 5-66** A carburizing process is carried out on a 0.10% C steel by introducing 1.0% C at the surface at 980°C, where the iron is FCC. Calculate the carbon content at 0.01 cm, 0.05 cm, and 0.10 cm beneath the surface after 1 h.

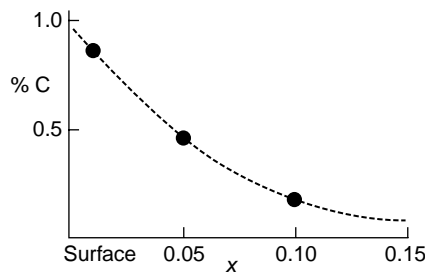
**Solution:**  $D = 0.23 \exp[-32,900/(1.987)(1253)] = 42 \times 10^{-8} \text{ cm}^2/\text{s}$

$$\frac{1 - c_x}{1 - 0.1} = \text{erf}[x / (2\sqrt{(42 \times 10^{-8})(3600)})] = \text{erf}[x / 0.0778]$$

$$x = 0.01: \text{erf}[0.01/0.0778] = \text{erf}(0.1285) = \frac{(1 - c_x)}{0.9} = 0.144 \quad c_x = 0.87\% \text{ C}$$

$$x = 0.05: \text{erf}[0.05/0.0778] = \text{erf}(0.643) = \frac{(1 - c_x)}{0.9} = 0.636 \quad c_x = 0.43\% \text{ C}$$

$$x = 0.10: \text{erf}[0.10/0.0778] = \text{erf}(1.285) = \frac{(1 - c_x)}{0.9} = 0.914 \quad c_x = 0.18\% \text{ C}$$



- 5-67** Iron containing 0.05% C is heated to 912°C in an atmosphere that produces 1.20% C at the surface and is held for 24 h. Calculate the carbon content at 0.05 cm beneath the surface if (a) the iron is BCC and (b) the iron is FCC. Explain the difference.

**Solution:**  $t = (24 \text{ h})(3600 \text{ s/h}) = 86,400 \text{ s}$

$$D_{\text{BCC}} = 0.011 \exp[-20,900/(1.987)(1185)] = 1.54 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$D_{\text{FCC}} = 0.23 \exp[-32,900/(1.987)(1185)] = 1.97 \times 10^{-7} \text{ cm}^2/\text{s}$$

**BCC:**  $\frac{1.2 - c_x}{1.2 - 0.05} = \text{erf}[0.05 / (2\sqrt{(1.54 \times 10^{-6})(86,400)})] = \text{erf}[0.0685] = 0.077$

$$c_x = 1.11\% \text{ C}$$

$$\text{FCC: } \frac{1.2 - c_x}{1.2 - 0.05} = \text{erf}[0.05 / (2\sqrt{(1.97 \times 10^{-7})(86,400)})] = \text{erf}[0.192] = 0.2139$$

$$c_x = 0.95\% \text{ C}$$

Faster diffusion occurs in the looser packed BCC structure, leading to the higher carbon content at point "x".

- 5-68** What temperature is required to obtain 0.50% C at a distance of 0.5 mm beneath the surface of a 0.20% C steel in 2 h. when 1.10% C is present at the surface? Assume that the iron is FCC.

$$\begin{aligned} \text{Solution: } \frac{1.1 - 0.5}{1.1 - 0.2} &= 0.667 = \text{erf}[0.05 / 2\sqrt{Dt}] \\ 0.05 / 2\sqrt{Dt} &= 0.685 \quad \text{or} \quad \sqrt{Dt} = 0.0365 \quad \text{or} \quad Dt = 0.00133 \\ t &= (2 \text{ h})(3600 \text{ s/h}) = 7200 \text{ s} \\ D &= 0.00133 / 7200 = 1.85 \times 10^{-7} = 0.23 \exp[-32,900 / 1.987T] \\ \exp(-16,558/T) &= 8.043 \times 10^{-7} \\ T &= 1180\text{K} = 907^\circ\text{C} \end{aligned}$$

- 5-69** A 0.15% C steel is to be carburized at 1100° C, giving 0.35% C at a distance of 1 mm beneath the surface. If the surface composition is maintained at 0.90% C, what time is required?

$$\begin{aligned} \text{Solution: } \frac{0.9 - 0.35}{0.9 - 0.15} &= 0.733 = \text{erf}[0.1 / 2\sqrt{Dt}] \\ 0.1 / 2\sqrt{Dt} &= 0.786 \quad \text{or} \quad \sqrt{Dt} = 0.0636 \quad \text{or} \quad Dt = 0.00405 \\ D &= 0.23 \exp[-32,900 / (1.987)(1373)] = 1.332 \times 10^{-6} \text{ cm}^2/\text{s} \\ t &= 0.00405 / 1.332 \times 10^{-6} = 3040 \text{ s} = 51 \text{ min} \end{aligned}$$

- 5-70** A 0.02% C steel is to be carburized at 1200°C in 4 h, with a point 0.6 mm beneath the surface reaching 0.45% C. Calculate the carbon content required at the surface of the steel.

$$\begin{aligned} \text{Solution: } \frac{c_s - 0.45}{c_s - 0.02} &= \text{erf}[0.06 / 2\sqrt{Dt}] \\ D &= 0.23 \exp[-32,900 / (1.987)(1473)] = 3.019 \times 10^{-6} \text{ cm}^2/\text{s} \\ t &= (4 \text{ h})(3600) = 14,400 \text{ s} \\ \sqrt{Dt} &= \sqrt{(3.019 \times 10^{-6})(14,400)} = 0.2085 \\ \text{erf}[0.06 / (2)(0.2085)] &= \text{erf}(0.144) = 0.161 \\ \frac{c_s - 0.45}{c_s - 0.02} &= 0.161 \quad \text{or} \quad c_s = 0.53\% \text{ C} \end{aligned}$$

- 5-71** A 1.2% C tool steel held at 1150°C is exposed to oxygen for 48 h. The carbon content at the steel surface is zero. To what depth will the steel be decarburized to less than 0.20% C?

$$\text{Solution: } \frac{0 - 0.2}{0 - 1.2} = 0.1667 \quad \therefore x / 2\sqrt{Dt} = 0.149$$

$$D = 0.23 \exp[-32,900/(1.987)(1423)] = 2.034 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$t = (48 \text{ h})(3600 \text{ s/h}) = 17.28 \times 10^4 \text{ s}$$

$$\sqrt{Dt} = 0.5929$$

$$\text{Then from above, } x = (0.149)(2)(0.5929) = 0.177 \text{ cm}$$

- 5-72** A 0.80% C steel must operate at 950°C in an oxidizing environment, where the carbon content at the steel surface is zero. Only the outermost 0.02 cm of the steel part can fall below 0.75% C. What is the maximum time that the steel part can operate?

$$\text{Solution: } \frac{0 - 0.75}{0 - 0.8} = 0.9375 = \text{erf}[x/2\sqrt{Dt}] \quad \therefore x/2\sqrt{Dt} = 1.384$$

$$0.02/2\sqrt{Dt} = 1.384 \quad \text{or} \quad \sqrt{Dt} = 0.007226 \quad \text{or} \quad Dt = 5.22 \times 10^{-5}$$

$$D = 0.23 \exp[-32,900/(1.987)(1223)] = 3.03 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$t = 5.22 \times 10^{-5} / 3.03 \times 10^{-7} = 172 \text{ s} = 2.9 \text{ min}$$

- 5-73** A BCC steel containing 0.001% N is nitrided at 550°C for 5 h. If the nitrogen content at the steel surface is 0.08%, determine the nitrogen content at 0.25 mm from the surface.

$$\text{Solution: } \frac{0.08 - c_s}{0.08 - 0.001} = \text{erf}[0.025/2\sqrt{Dt}] \quad t = (5 \text{ h})(3600 \text{ s/h}) = 1.8 \times 10^4 \text{ s}$$

$$D = 0.0047 \exp[-18,300/(1.987)(823)]$$

$$= 6.488 \times 10^{-8} \text{ cm}^2/\text{s}$$

$$\sqrt{Dt} = 0.0342$$

$$\text{erf}[0.025/(2)(0.0342)] = \text{erf}(0.3655) = 0.394$$

$$\frac{0.08 - c_s}{0.079} = 0.394 \quad \text{or} \quad c_s = 0.049\% \text{ N}$$

- 5-74** What time is required to nitride a 0.002% N steel to obtain 0.12% N at a distance of 0.002 in. beneath the surface at 625°C? The nitrogen content at the surface is 0.15%.

$$\text{Solution: } \frac{0.15 - 0.12}{0.15 - 0.002} = 0.2027 = \text{erf}[x/2\sqrt{Dt}] \quad \therefore x/2\sqrt{Dt} = 0.2256$$

$$D = 0.0047 \exp[-18,300/(1.987)(898)] = 1.65 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$x = 0.002 \text{ in.} = 0.00508 \text{ cm}$$

$$\frac{0.00508}{2\sqrt{(1.65 \times 10^{-7})t}} = 0.2256$$

$$Dt = 1.267 \times 10^{-4} \quad \text{or} \quad t = 1.267 \times 10^{-4} / 1.65 \times 10^{-7} = 768 \text{ s} = 12.8 \text{ min}$$

- 5-75** We currently can successfully perform a carburizing heat treatment at 1200°C in 1 h. In an effort to reduce the cost of the brick lining in our furnace, we propose to reduce the carburizing temperature to 950°C. What time will be required to give us a similar carburizing treatment?

$$\text{Solution: } D_{1200} = 0.23 \exp[-32,900/(1.987)(1473)] = 3.019 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$D_{950} = 0.23 \exp[-32,900/(1.987)(1223)] = 3.034 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$t_{1200} = 1 \text{ h}$$

$$t_{950} = D_{1200} t_{1200} / D_{950} = \frac{(3.019 \times 10^{-6})(1)}{3.034 \times 10^{-7}} = 9.95 \text{ h}$$

- 5-86** During freezing of a Cu-Zn alloy, we find that the composition is nonuniform. By heating the alloy to 600°C for 3 hours, diffusion of zinc helps to make the composition more uniform. What temperature would be required if we wished to perform this *homogenization* treatment in 30 minutes?

**Solution:**  $D_{600} = 0.78 \exp[-43,900/(1.987)(873)] = 7.9636 \times 10^{-12}$   $t_{600} = 3 \text{ h}$   
 $t_x = 0.5 \text{ h}$

$$D_x = D_{600} t_{600} / t_x = (7.9636 \times 10^{-12})(3)/0.5$$

$$D_x = 4.778 \times 10^{-11} = 0.78 \exp[-43,900/1.987T]$$

$$\ln(6.1258 \times 10^{-11}) = -23.516 = -43,900/1.987 T$$

$$T = 940 \text{ K} = 667^\circ\text{C}$$

- 5-87** A ceramic part made of MgO is sintered successfully at 1700°C in 90 minutes. To minimize thermal stresses during the process, we plan to reduce the temperature to 1500°C. Which will limit the rate at which sintering can be done: diffusion of magnesium ions or diffusion of oxygen ions? What time will be required at the lower temperature?

**Solution:** Diffusion of oxygen is the slower of the two, due to the larger ionic radius of the oxygen.

$$D_{1700} = 0.000043 \exp[-82,100/(1.987)(1973)] = 3.455 \times 10^{-14} \text{ cm}^2/\text{s}$$

$$D_{1500} = 0.000043 \exp[-82,100/(1.987)(1773)] = 3.255 \times 10^{-15} \text{ cm}^2/\text{s}$$

$$t_{1500} = D_{1700} t_{1700} / D_{1500} = \frac{(3.455 \times 10^{-14})(90)}{3.255 \times 10^{-15}} = 955 \text{ min} = 15.9 \text{ h}$$

- 5-88** A Cu-Zn alloy has an initial grain diameter of 0.01 mm. The alloy is then heated to various temperatures, permitting grain growth to occur. The times required for the grains to grow to a diameter of 0.30 mm are

**Solution:**

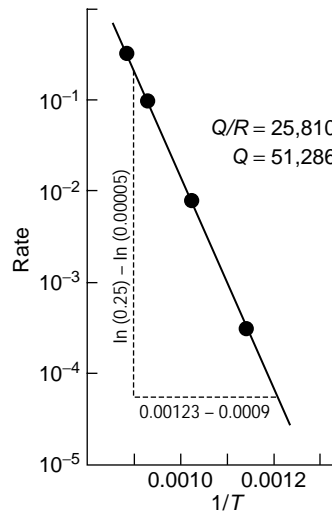
Temperature (°C)	Time (min)
500	80,000
600	3,000
700	120
800	10
850	3

Determine the activation energy for grain growth. Does this correlate with the diffusion of zinc in copper? (Hint: Note that rate is the reciprocal of time.)

**Solution:**

Temperature (°C)	Temperature (K)	1/T (K <sup>-1</sup> )	Time (min)	Rate (min <sup>-1</sup> )
500	773	0.00129	80,000	1.25 × 10 <sup>-5</sup>
600	873	0.00115	3,000	3.33 × 10 <sup>-4</sup>
700	973	0.001028	120	8.33 × 10 <sup>-3</sup>
800	1073	0.000932	10	0.100
850	1123	0.000890	3	0.333

From the graph, we find that  $Q = 51,286$  cal/mol, which does correlate with the activation energy for diffusion of zinc in copper.



- 5-91** A sheet of gold is diffusion-bonded to a sheet of silver in 1 h at 700°C. At 500°C, 440 h are required to obtain the same degree of bonding, and at 300°C, bonding requires 1530 years. What is the activation energy for the diffusion bonding process? Does it appear that diffusion of gold or diffusion of silver controls the bonding rate? (Hint - note that rate is the reciprocal of time.)

**Solution:**

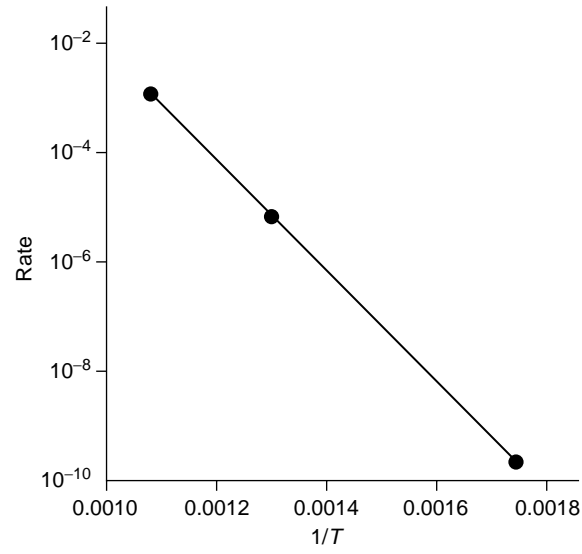
Temperature (°C)	Temperature (K)	1/T (K <sup>-1</sup> )	Time (s)	Rate (sec <sup>-1</sup> )
700	973	0.001007	3600	$0.278 \times 10^{-3}$
500	773	0.001294	$1.584 \times 10^6$	$0.631 \times 10^{-6}$
300	573	0.001745	$4.825 \times 10^{10}$	$0.207 \times 10^{-10}$

$$\frac{0.278 \times 10^{-3}}{0.207 \times 10^{-10}} = \frac{\exp[-Q/(1.987)(973)]}{\exp[-Q/(1.987)(573)]} = \frac{\exp[-0.0005172Q]}{\exp[-0.0008783Q]}$$

$$\ln(1.343 \times 10^7) = 16.413 = 0.0003611 Q$$

$$Q = 45,450 \text{ cal/mol.}$$

The activation energy for the diffusion of gold in silver is 45,500 cal/mole; thus the diffusion of gold appears to control the bonding rate.



# 6

## Mechanical Properties and Behavior

- 6–24** A 850-lb force is applied to a 0.15-in. diameter nickel wire having a yield strength of 45,000 psi and a tensile strength of 55,000 psi. Determine (a) whether the wire will plastically deform and (b) whether the wire will experience necking.

**Solution:** (a) First determine the stress acting on the wire:

$$\sigma = F/A = 850 \text{ lb} / (\pi/4)(0.15 \text{ in.})^2 = 48,100 \text{ psi}$$

Because  $\sigma$  is greater than the yield strength of 45,000 psi, the wire will plastically deform.

- (b) Because  $\sigma$  is less than the tensile strength of 55,000 psi, no necking will occur.

- 6–25** A force of 100,000 N is applied to a 10 mm × 20 mm iron bar having a yield strength of 400 MPa and a tensile strength of 480 MPa. Determine (a) whether the bar will plastically deform and (b) whether the bar will experience necking.

**Solution:** (a) First determine the stress acting on the wire:

$$\sigma = F/A = 100,000 \text{ N} / (10 \text{ mm})(20 \text{ mm}) = 500 \text{ N/mm}^2 = 500 \text{ MPa}$$

Because  $\sigma$  is greater than the yield strength of 400 MPa, the wire will plastically deform.

- (b) Because  $\sigma$  is greater than the tensile strength of 480 MPa, the wire will also neck.

- 6–25(c)** Calculate the maximum force that a 0.2-in. diameter rod of  $\text{Al}_2\text{O}_3$ , having a yield strength of 35,000 psi, can withstand with no plastic deformation. Express your answer in pounds and newtons.

**Solution:**  $F = \sigma A = (35,000 \text{ psi})(\pi/4)(0.2 \text{ in.})^2 = 1100 \text{ lb}$

$$F = (1100 \text{ lb})(4.448 \text{ N/lb}) = 4891 \text{ N}$$



- 6-26** A force of 20,000 N will cause a 1 cm × 1 cm bar of magnesium to stretch from 10 cm to 10.045 cm. Calculate the modulus of elasticity, both in GPa and psi.

**Solution:** The strain  $\epsilon$  is  $\epsilon = (10.045 \text{ cm} - 10 \text{ cm}) / 10 \text{ cm} = 0.0045 \text{ cm/cm}$

The stress  $\sigma$  is  $\sigma = 20,000 \text{ N} / (10 \text{ mm})(10 \text{ mm}) = 200 \text{ N/mm}^2 = 200 \text{ MPa}$

$$E = \sigma / \epsilon = 200 \text{ MPa} / 0.0045 \text{ cm/cm} = 44,444 \text{ MPa} = 44.4 \text{ GPa}$$

$$E = (44,444 \text{ MPa})(145 \text{ psi/MPa}) = 6.44 \times 10^6 \text{ psi}$$

- 6-27** A polymer bar's dimensions are 1 in. × 2 in. × 15 in. The polymer has a modulus of elasticity of 600,000 psi. What force is required to stretch the bar elastically to 15.25 in.?

**Solution:** The strain  $\epsilon$  is  $\epsilon = (15.25 \text{ in.} - 15 \text{ in.}) / (15 \text{ in.}) = 0.01667 \text{ in./in.}$

The stress  $\sigma$  is  $\sigma = E\epsilon = (600,000 \text{ psi})(0.01667 \text{ in./in.}) = 10,000 \text{ psi}$

The force is then  $F = \sigma A = (10,000 \text{ psi})(1 \text{ in.})(2 \text{ in.}) = 20,000 \text{ lb}$

- 6-28** An aluminum plate 0.5 cm thick is to withstand a force of 50,000 N with no permanent deformation. If the aluminum has a yield strength of 125 MPa, what is the minimum width of the plate?

**Solution:** The area is  $A = F / \sigma = 50,000 \text{ N} / 125 \text{ N/mm}^2 = 400 \text{ mm}^2$

The minimum width is  $w = A / t = (400 \text{ mm}^2) / (0.5 \text{ cm}) = 8 \text{ cm}$

- 6-29** A 3-in.-diameter rod of copper is to be reduced to a 2-in.-diameter rod by being pushed through an opening. To account for the elastic strain, what should be the diameter of the opening? The modulus of elasticity for the copper is  $17 \times 10^6 \text{ psi}$  and the yield strength is 40,000 psi.

**Solution:** The strain is  $\epsilon = \sigma / E = 40,000 \text{ psi} / 17 \times 10^6 \text{ psi} = 0.00235 \text{ in./in.}$

The strain is also  $\epsilon = (2 \text{ in.} - d_o) / d_o = 0.00235 \text{ in./in.}$

$$2 - d_o = 0.00235 d_o$$

$$d_o = 2 / 1.00235 = 1.995 \text{ in.}$$

The opening in the die must be smaller than the final diameter.

- 6-30** A steel cable 1.25 in. in diameter and 50 ft long is to lift a 20 ton load. What is the length of the cable during lifting? The modulus of elasticity of the steel is  $30 \times 10^6 \text{ psi}$ .

**Solution:** The stress is  $\sigma = F / A = \frac{(20 \text{ ton})(2000 \text{ lb/ton})}{(\pi/4)(1.25 \text{ in.})^2} = 32,595 \text{ psi}$

The strain is  $\epsilon = \sigma / E = 32,595 \text{ psi} / 30 \times 10^6 \text{ psi} = 0.0010865 \text{ in./in.}$

$$\epsilon = (\ell_f - 50 \text{ ft}) / 50 \text{ ft} = 0.0010865 \text{ ft/ft}$$

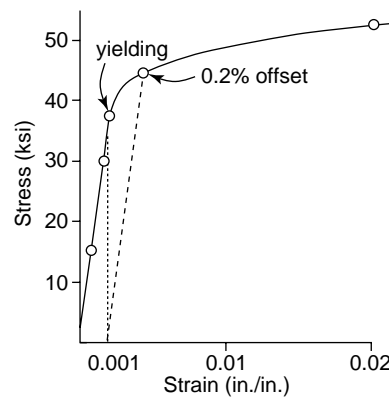
$$\ell_f = 50.0543 \text{ ft}$$

- 6-33** The following data were collected from a standard 0.505-in.-diameter test specimen of a copper alloy (initial length  $(\ell_o) = 2.0 \text{ in.}$ ):

**Solution:**  $\sigma = F / (\pi/4)(0.505)^2 = F/0.2$

$$\epsilon = (\ell - 2) / 2$$

Load (lb)	Gage Length (in.)	Stress (psi)	Strain (in./in.)
0	2.00000	0	0.0
3,000	2.00167	15,000	0.000835
6,000	2.00333	30,000	0.001665
7,500	2.00417	37,500	0.002085
9,000	2.0090	45,000	0.0045
10,500	2.040	52,500	0.02
12,000	2.26	60,000	0.13
12,400	2.50 (max load)	62,000	0.25
11,400	3.02 (fracture)	57,000	0.51



After fracture, the gage length is 3.014 in. and the diameter is 0.374 in. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the %Elongation, (e) the %Reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

(a) 0.2% offset yield strength = 45,000 psi

(b) tensile strength = 62,000 psi

(c)  $E = (30,000 - 0) / (0.001665 - 0) = 18 \times 10^6$  psi

(d) %Elongation =  $\frac{(3.014 - 2)}{2} \times 100 = 50.7\%$

(e) %Reduction in area =  $\frac{(\pi/4)(0.505)^2 - (\pi/4)(0.374)^2}{(\pi/4)(0.505)^2} \times 100 = 45.2\%$

(f) engineering stress at fracture = 57,000 psi

(g) true stress at fracture =  $11,400 \text{ lb} / (\pi/4)(0.374)^2 = 103,770$  psi

(h) From the graph, yielding begins at about 37,500 psi. Thus:

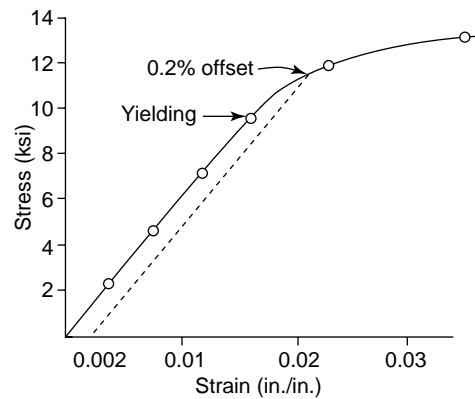
$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(37,500)(0.002085) = 39.1 \text{ psi}$$

**6-34** The following data were collected from a 0.4-in. diameter test specimen of polyvinyl chloride ( $\ell_o = 2.0$  in.):

**Solution:**  $\sigma = F / (\pi/4)(0.4)^2 = F/0.1257$

$$\varepsilon = (\ell - \ell_o) / \ell_o$$

Load (lb)	Gage Length (in.)	Stress (psi)	Strain (in./in.)
0	2.00000	0	0.0
300	2.00746	2,387	0.00373
600	2.01496	4,773	0.00748
900	2.02374	7,160	0.01187
1200	2.032	9,547	0.016
1500	2.046	11,933	0.023
1660	2.070 (max load)	13,206	0.035
1600	2.094	12,729	0.047
1420	2.12 (fracture)	11,297	0.06



After fracture, the gage length is 2.09 in. and the diameter is 0.393 in. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the %Elongation, (e) the %Reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

(a) 0.2% offset yield strength = 11,600 psi

(b) tensile strength = 12,729 psi

(c)  $E = (7160 - 0) / (0.01187 - 0) = 603,000$  psi

(d)  $\% \text{Elongation} = \frac{(2.09 - 2)}{2} \times 100 = 4.5\%$

(e)  $\% \text{Reduction in area} = \frac{(\pi/4)(0.4)^2 - (\pi/4)(0.393)^2}{(\pi/4)(0.4)^2} \times 100 = 3.5\%$

(f) engineering stress at fracture = 11,297 psi

(g) true stress at fracture =  $1420 \text{ lb} / (\pi/4)(0.393)^2 = 11,706$  psi

(h) From the figure, yielding begins near 9550 psi. Thus:

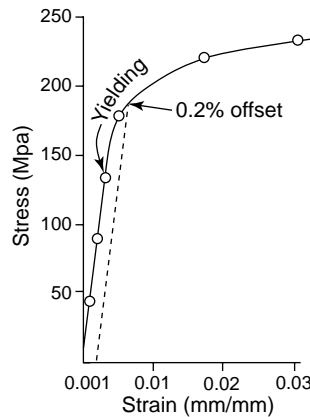
$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(9550)(0.016) = 76.4 \text{ psi}$$

**6-35** The following data were collected from a 12-mm-diameter test specimen of magnesium ( $\ell_o = 30.00$  mm):

**Solution:**  $\sigma = F / (\pi/4)(12 \text{ mm})^2 = F/113.1$

$$\varepsilon = (\ell - 30)/30$$

Load (N)	Gage Length (mm)	Stress (MPa)	Strain (mm/mm)
0	30.0000	0	0.0
5,000	30.0296	44.2	0.000987
10,000	30.0592	88.4	0.001973
15,000	30.0888	132.6	0.00296
20,000	30.15	176.8	0.005
25,000	30.51	221.0	0.017
26,500	30.90	234.3	0.030
27,000	31.50 (max load)	238.7	0.050
26,500	32.10	234.3	0.070
25,000	32.79 (fracture)	221.0	0.093



After fracture, the gage length is 32.61 mm and the diameter is 11.74 mm. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the %Elongation, (e) the %Reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

(a) 0.2% offset yield strength = 186 MPa

(b) tensile strength = 238.7 MPa

(c)  $E = (132.6 - 0) / (0.00296 - 0) = 44,800 \text{ MPa} = 44.8 \text{ GPa}$

(d)  $\% \text{Elongation} = \frac{(32.61 - 30)}{30} \times 100 = 8.7\%$

(e)  $\% \text{Reduction in area} = \frac{(\pi/4)(12)^2 - (\pi/4)(11.74)^2}{(\pi/4)(12)^2} \times 100 = 4.3\%$

(f) engineering stress at fracture = 221 MPa

(g) true stress at fracture =  $25,000 \text{ N} / (\pi/4)(11.74)^2 = 231 \text{ MPa}$

(h) From the figure, yielding begins near 138 MPa psi. Thus:

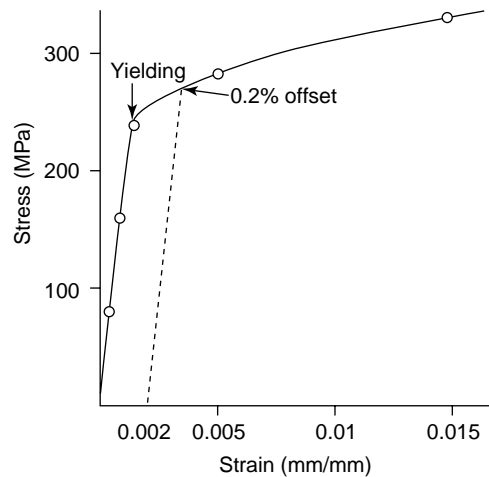
$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(138)(0.00296) = 0.2 \text{ MPa}$$

**6-36** The following data were collected from a 20 mm diameter test specimen of a ductile cast iron ( $\ell_o = 40.00 \text{ mm}$ ):

**Solution:**  $\sigma = F/(\pi/4)(20 \text{ mm})^2 = F/314.2$

$$\varepsilon = (\ell - 40)/40$$

Load (N)	Gage Length (mm)	Stress (MPa)	Strain (mm/mm)
0	40.0000	0	0.0
25,000	40.0185	79.6	0.00046
50,000	40.0370	159.2	0.000925
75,000	40.0555	238.7	0.001388
90,000	40.20	286.5	0.005
105,000	40.60	334.2	0.015
120,000	41.56	382.0	0.039
131,000	44.00 (max load)	417.0	0.010
125,000	47.52 (fracture)	397.9	0.188



After fracture, the gage length is 47.42 mm and the diameter is 18.35 mm. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the %Elongation, (e) the %Reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

(a) 0.2% offset yield strength = 274 MPa

(b) tensile strength = 417 MPa

(c)  $E = (238.7 - 0) / (0.001388 - 0) = 172,000 \text{ MPa} = 172 \text{ GPa}$

(d)  $\% \text{Elongation} = \frac{(47.42 - 40)}{40} \times 100 = 18.55\%$

(e)  $\% \text{Reduction in area} = \frac{(\pi/4)(20)^2 - (\pi/4)(18.35)^2}{(\pi/4)(20)^2} \times 100 = 15.8\%$

(f) engineering stress at fracture = 397.9 MPa

(g) true stress at fracture =  $125,000 \text{ N} / (\pi/4)(18.35)^2 = 473 \text{ MPa}$

(h) From the figure, yielding begins near 240 MPa. Thus:

$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(240)(0.001388) = 0.17 \text{ MPa}$$

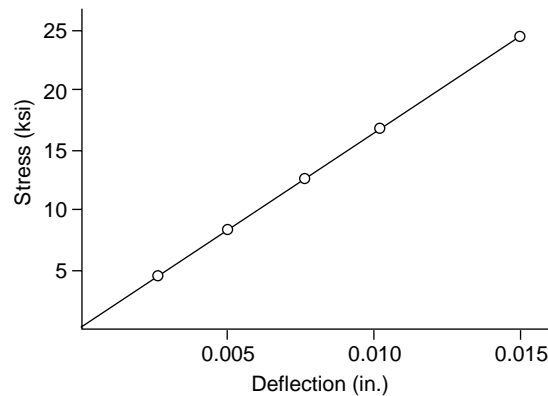
**6-39** A bar of  $\text{Al}_2\text{O}_3$  that is 0.25 in. thick, 0.5 in. wide, and 9 in. long is tested in a three-point bending apparatus, with the supports located 6 in. apart. The deflection of the center of the bar is measured as a function of the applied load. The data are shown below. Determine the flexural strength and the flexural modulus.

**Solution:**  $\text{stress} = 3LF/2wh^2$  (6-13)

$$= (3)(6 \text{ in.})F / (2)(0.5 \text{ in.})(0.25 \text{ in.})^2$$

$$= 288F$$

Force (lb)	Deflection (in.)	Stress (psi)
14.5	0.0025	4,176
28.9	0.0050	8,323
43.4	0.0075	12,499
57.9	0.0100	16,675
86.0	0.0149 (fracture)	24,768



The flexural strength is the stress at fracture, or 24,768 psi.

The flexural modulus can be calculated from the linear curve; picking the first point as an example:

$$\text{FM} = \frac{FL^3}{4wh^3\delta} = \frac{(14.5 \text{ lb})(6 \text{ in.})^3}{(4)(0.5 \text{ in.})(0.25 \text{ in.})^3(0.0025 \text{ in.})} = 40 \times 10^6 \text{ psi}$$

(6-14)

**6-40(a)** A 0.4-in. diameter, 12-in. long titanium bar has a yield strength of 50,000 psi, a modulus of elasticity of  $16 \times 10^6$  psi, and Poisson's ratio of 0.30. Determine the length and diameter of the bar when a 500-lb load is applied.

**Solution:** The stress is  $\sigma = F/A = 500 \text{ lb} / (\pi/4)(0.4 \text{ in.})^2 = 3,979 \text{ psi}$

The applied stress is much less than the yield strength; therefore Hooke's law can be used.

The strain is  $\epsilon = \sigma/E = 3,979 \text{ psi} / (16 \times 10^6 \text{ psi}) = 0.00024868 \text{ in./in.}$

$$\frac{\ell_f - \ell_o}{\ell_o} = \frac{\ell_f - 12 \text{ in.}}{12 \text{ in.}} = 0.00024868 \text{ in./in.}$$

$$\ell_f = 12.00298 \text{ in.}$$

From Poisson's ratio,  $\mu = -\epsilon_{\text{lat}} / \epsilon_{\text{long}} = 0.3$

$$\epsilon_{\text{lat}} = -(0.3)(0.00024868) = -0.0000746 \text{ in./in.}$$

$$\frac{d_f - d_o}{d_o} = \frac{d_f - 0.4 \text{ in.}}{0.4} = -0.0000746 \text{ in./in.}$$

$$d_f = 0.39997 \text{ in.}$$

**6-40(b)** When a tensile load is applied to a 1.5-cm diameter copper bar, the diameter is reduced to 1.498-cm diameter. Determine the applied load, using the data in Table 6-3.

**Solution:** From Table 6-3,  $\mu = -\epsilon_{\text{lat}} / \epsilon_{\text{long}} = 0.36$

$$\epsilon_{\text{lat}} = \frac{1.498 - 1.5}{1.5} = -0.001333$$

$$\epsilon_{\text{long}} = -\epsilon_{\text{lat}} / \mu = -(-0.001333) / 0.36 = 0.0037 \text{ in./in.}$$

$$\sigma = E\epsilon = (124.8 \text{ GPa})(1000 \text{ MPa/GPa})(0.0037 \text{ in./in.}) = 462 \text{ MPa}$$

$$F = \sigma A = (462 \text{ MPa})(\pi/4)(15 \text{ mm})^2 = 81,640 \text{ N}$$

**6-41** A three-point bend test is performed on a block of  $\text{ZrO}_2$  that is 8 in. long, 0.50 in. wide, and 0.25 in. thick and is resting on two supports 4 in. apart. When a force of 400 lb is applied, the specimen deflects 0.037 in. and breaks. Calculate (a) the flexural strength and (b) the flexural modulus, assuming that no plastic deformation occurs.

**Solution:** (a) flexural strength  $= 3FL/2wh^2 = \frac{(3)(400 \text{ lb})(4 \text{ in.})}{(2)(0.5 \text{ in.})(0.25 \text{ in.})^2} = 76,800 \text{ psi}$

$$\begin{aligned} \text{(b) flexural modulus} &= FL^3/4wh^3\delta \\ &= \frac{(400 \text{ lb})(4 \text{ in.})^3}{(4)(0.5 \text{ in.})(0.25 \text{ in.})^3(0.037 \text{ in.})} \\ &= 22.14 \times 10^6 \text{ psi} \end{aligned}$$

**6-42** A three-point bend test is performed on a block of silicon carbide that is 10 cm long, 1.5 cm wide, and 0.6 cm thick and is resting on two supports 7.5 cm apart. The sample breaks when a deflection of 0.09 mm is recorded. Calculate (a) the force that caused the fracture and (b) the flexural strength. The flexural modulus for silicon carbide is 480 GPa. Assume that no plastic deformation occurs.

**Solution:** (a) The force  $F$  required to produce a deflection of 0.09 mm is

$$F = (\text{flexural modulus})(4wh^3\delta)/L^3$$

$$F = (480,000 \text{ MPa})(4)(15 \text{ mm})(6 \text{ mm})^3(0.09 \text{ mm}) / (75 \text{ mm})^3$$

$$F = 1327 \text{ N}$$

$$\begin{aligned} \text{(b) flexural strength} &= 3FL/2wh^2 = (3)(1327 \text{ N})(75 \text{ mm})/(2)(15 \text{ mm})(6 \text{ mm})^2 \\ &= 276 \text{ MPa} \end{aligned}$$

- 6-43(a)** A thermosetting polymer containing glass beads is required to deflect 0.5 mm when a force of 500 N is applied. The polymer part is 2 cm wide, 0.5 cm thick, and 10 cm long. If the flexural modulus is 6.9 GPa, determine the minimum distance between the supports. Will the polymer fracture if its flexural strength is 85 MPa? Assume that no plastic deformation occurs.

**Solution:** The minimum distance  $L$  between the supports can be calculated from the flexural modulus.

$$L^3 = 4wh^3\delta(\text{flexural modulus})/F$$

$$L^3 = (4)(20 \text{ mm})(5 \text{ mm})^3(0.5 \text{ mm})(6.9 \text{ GPa})(1000 \text{ MPa/GPa}) / 500 \text{ N}$$

$$L^3 = 69,000 \text{ mm}^3 \quad \text{or} \quad L = 41 \text{ mm}$$

The stress acting on the bar when a deflection of 0.5 mm is obtained is

$$\sigma = 3FL/2wh^2 = (3)(500 \text{ N})(41 \text{ mm}) / (2)(20 \text{ mm})(5 \text{ mm})^2 = 61.5 \text{ MPa}$$

The applied stress is less than the flexural strength of 85 MPa; the polymer is not expected to fracture.

- 6-43(b)** The flexural modulus of alumina is  $45 \times 10^6$  psi and its flexural strength is 46,000 psi. A bar of alumina 0.3 in. thick, 1.0 in. wide, and 10 in. long is placed on supports 7 in. apart. Determine the amount of deflection at the moment the bar breaks, assuming that no plastic deformation occurs.

**Solution:** The force required to break the bar is

$$F = 2wh^2(\text{flexural strength})/3L$$

$$F = (2)(1 \text{ in.})(0.3 \text{ in.})^2(46,000 \text{ psi}) / (3)(7 \text{ in.}) = 394 \text{ lb}$$

The deflection just prior to fracture is

$$\delta = FL^3/4wh^3(\text{flexural modulus})$$

$$\delta = (394 \text{ lb})(7 \text{ in.})^3/(4)(1 \text{ in.})(0.3 \text{ in.})^3(45 \times 10^6 \text{ psi}) = 0.0278 \text{ in.}$$

- 6-52** A Brinell hardness measurement, using a 10-mm-diameter indenter and a 500 kg load, produces an indentation of 4.5 mm on an aluminum plate. Determine the Brinell hardness number HB of the metal.

**Solution:** 
$$HB = \frac{500 \text{ kg}}{(\pi / 2)(10 \text{ mm})[10 - \sqrt{10^2 - 4.5^2}]} = 29.8$$

- 6-53** When a 3000 kg load is applied to a 10-mm-diameter ball in a Brinell test of a steel, an indentation of 3.1 mm is produced. Estimate the tensile strength of the steel.

**Solution:** 
$$HB = \frac{3000 \text{ kg}}{(\pi / 2)(10 \text{ mm})[10 - \sqrt{10^2 - 3.1^2}]} = 388$$

$$\text{Tensile strength} = 500 \text{ HB} = (500)(388) = 194,000 \text{ psi}$$

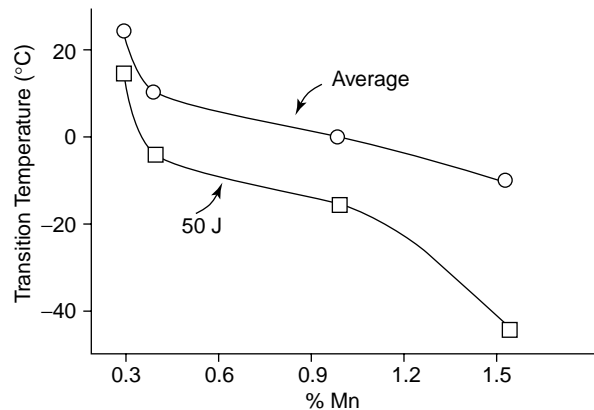
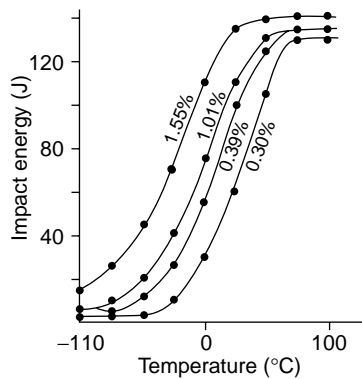
- 6-55** The data below were obtained from a series of Charpy impact tests performed on four steels, each having a different manganese content. Plot the data and determine (a) the transition temperature (defined by the mean of the absorbed energies in the



ductile and brittle regions) and (b) the transition temperature (defined as the temperature that provides 50 J absorbed energy). Plot the transition temperature versus manganese content and discuss the effect of manganese on the toughness of steel. What would be the minimum manganese allowed in the steel if a part is to be used at 0°C?

Solution:

Test temperature °C	Impact energy (J)			
	0.30% Mn	0.39% Mn	1.01% Mn	1.55% Mn
-100	2	5	5	15
-75	2	5	7	25
-50	2	12	20	45
-25	10	25	40	70
0	30	55	75	110
25	60	100	110	135
50	105	125	130	140
75	130	135	135	140
100	130	135	135	140



(a) Transition temperatures defined by the mean of the absorbed energies are:

$$0.30\% \text{ Mn: mean energy} = 2 + (130 + 2)/2 = 68 \text{ J}; \quad T = 27^\circ\text{C}$$

$$0.39\% \text{ Mn: mean energy} = 5 + (135 + 5)/2 = 75 \text{ J}; \quad T = 10^\circ\text{C}$$

$$1.01\% \text{ Mn: mean energy} = 5 + (135 + 5)/2 = 75 \text{ J}; \quad T = 0^\circ\text{C}$$

$$1.55\% \text{ Mn: mean energy} = 15 + (140 + 15)/2 = 92.5 \text{ J}; \quad T = -12^\circ\text{C}$$

(b) Transition temperatures defined by 50 J are:

$$0.30\% \text{ Mn: } T = 15^\circ\text{C}$$

$$0.39\% \text{ Mn: } T = -5^\circ\text{C}$$

$$1.01\% \text{ Mn: } T = -15^\circ\text{C}$$

$$1.55\% \text{ Mn: } T = -45^\circ\text{C}$$

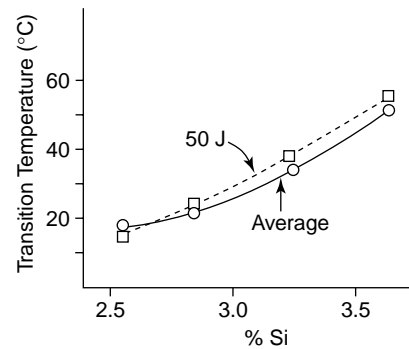
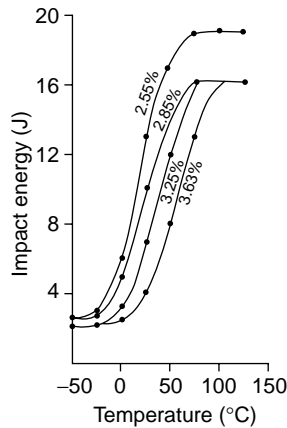
Increasing the manganese increases the toughness and reduces the transition temperature; manganese is therefore a desirable alloying element for improving the impact properties of the steel.

If the part is to be used at 25°C, we would want at least 1.0% Mn in the steel based on the mean absorbed energy criterion or 0.36% Mn based on the 50 J criterion.

- 6-57** The following data were obtained from a series of Charpy impact tests performed on four ductile cast irons, each having a different silicon content. Plot the data and determine (a) the transition temperature (defined by the mean of the absorbed energies in the ductile and brittle regions) and (b) the transition temperature (defined as the temperature that provides 10 J absorbed energy). Plot the transition temperature versus silicon content and discuss the effect of silicon on the toughness of the cast iron. What would be the maximum silicon allowed in the cast iron if a part is to be used at 25°C?

**Solution:**

Test temperature °C	Impact energy (J)			
	2.55% Si	2.85% Si	3.25% Si	3.63% Si
-50	2.5	2.5	2	2
-25	3	2.5	2	2
0	6	5	3	2.5
25	13	10	7	4
50	17	14	12	8
75	19	16	16	13
100	19	16	16	16
125	19	16	16	16



- (a) Transition temperatures defined by the mean of the absorbed energies are:

$$2.55\% \text{ Si: mean energy} = 2.5 + (19 + 2.5)/2 = 13.2 \text{ J; } T = 26^\circ\text{C}$$

$$2.85\% \text{ Si: mean energy} = 2.5 + (16 + 2.5)/2 = 11.8 \text{ J; } T = 35^\circ\text{C}$$

$$3.25\% \text{ Si: mean energy} = 2 + (16 + 2)/2 = 11 \text{ J; } T = 45^\circ\text{C}$$

$$3.63\% \text{ Si: mean energy} = 2 + (16 + 2)/2 = 11 \text{ J; } T = 65^\circ\text{C}$$

(b) Transition temperatures defined by 10 J are:

2.55% Si:  $T = 15^{\circ}\text{C}$

2.85% Si:  $T = 25^{\circ}\text{C}$

3.25% Si:  $T = 38^{\circ}\text{C}$

3.63% Si:  $T = 56^{\circ}\text{C}$

Increasing the silicon decreases the toughness and increases the transition temperature; silicon therefore reduces the impact properties of the cast iron.

If the part is to be used at  $25^{\circ}\text{C}$ , we would want a maximum of about 2.9% Si in the cast iron.

**6-58** FCC metals are often recommended for use at low temperatures, particularly when any sudden loading of the part is expected. Explain.

**Solution:** FCC metals do not normally display a transition temperature; instead the impact energies decrease slowly with decreasing temperature and, in at least some cases (such as some aluminum alloys), the energies even increase at low temperatures. The FCC metals can obtain large ductilities, giving large areas beneath the true stress-strain curve.

**6-59** A steel part can be made by powder metallurgy (compacting iron powder particles and sintering to produce a solid) or by machining from a solid steel block. Which part is expected to have the higher toughness? Explain.

**Solution:** Parts produced by powder metallurgy often contain considerable amounts of porosity due to incomplete sintering; the porosity provides sites at which cracks might easily nucleate. Parts machined from solid steel are less likely to contain flaws that would nucleate cracks, therefore improving toughness.

**6-62** A number of aluminum-silicon alloys have a structure that includes sharp-edged plates of brittle silicon in the softer, more ductile aluminum matrix. Would you expect these alloys to be notch-sensitive in an impact test? Would you expect these alloys to have good toughness? Explain your answers.

**Solution:** The sharp-edged plates of the brittle silicon may act as stress-raisers, or notches, thus giving poor toughness to the alloy. The presence of additional notches, such as machining marks, will not have a significant effect, since there are already very large numbers of "notches" due to the microstructure. Consequently this type of alloy is expected to have poor toughness but is not expected to be notch sensitive.

**6-67** Alumina  $\text{Al}_2\text{O}_3$  is a brittle ceramic with low toughness. Suppose that fibers of silicon carbide SiC, another brittle ceramic with low toughness, could be embedded within the alumina. Would doing this affect the toughness of the ceramic matrix composite? Explain. (These materials are discussed in later chapters.)

**Solution:** The SiC fibers may improve the toughness of the alumina matrix. The fibers may do so by several mechanisms. By introducing an interface (between the fibers and the matrix), a crack may be blocked; to continue growing, the crack may have to pass around the fiber, thus increasing the total energy of the crack and thus the energy that can be absorbed by the material. Or extra energy may be required to force the crack through the

interface in an effort to continue propagating. In addition, the fibers may begin to pull out of the matrix, particularly if bonding is poor; the fiber pull-out requires energy, thus improving toughness. Finally, the fibers may bridge across the crack, helping to hold the material together and requiring more energy to propagate the crack.

- 6-68** A ceramic matrix composite contains internal flaws as large as 0.001 cm in length. The plane strain fracture toughness of the composite is  $45 \text{ MPa}\sqrt{\text{m}}$  and the tensile strength is 550 MPa. Will the flaw cause the composite to fail before the tensile strength is reached? Assume that  $f = 1$ .

**Solution:** Since the crack is internal,  $2a = 0.001 \text{ cm} = 0.00001 \text{ m}$ . Therefore

$$a = 0.000005 \text{ m}$$

$$K_{Ic} = f\sigma\sqrt{\pi a} \quad \text{or} \quad \sigma = K_{Ic} / f\sqrt{\pi a}$$

$$\sigma = (45 \text{ MPa}\sqrt{\text{m}}) / (1)\sqrt{\pi(0.000005 \text{ m})} = 11,354 \text{ MPa}$$

The applied stress required for the crack to cause failure is much larger than the tensile strength of 550 MPa. Any failure of the ceramic should be expected due to the massive overload, not because of the presence of the flaws.

- 6-69** An aluminum alloy that has a plane strain fracture toughness of  $25,000 \text{ psi}\sqrt{\text{in.}}$  fails when a stress of 42,000 psi is applied. Observation of the fracture surface indicates that fracture began at the surface of the part. Estimate the size of the flaw that initiated fracture. Assume that  $f = 1.1$ .

**Solution:**  $K_{Ic} = f\sigma\sqrt{\pi a} \quad \text{or} \quad a = (1/\pi)[K_{Ic} / f\sigma]^2$

$$a = (1/\pi)[25,000 \text{ psi}\sqrt{\text{in.}} / (1.1)(42,000 \text{ psi})]^2 = 0.093 \text{ in.}$$

- 6-70** A polymer that contains internal flaws 1 mm in length fails at a stress of 25 MPa. Determine the plane strain fracture toughness of the polymer. Assume that  $f = 1$ .

**Solution:** Since the flaws are internal,  $2a = 1 \text{ mm} = 0.001 \text{ m}$ ; thus  $a = 0.0005 \text{ m}$

$$K_{Ic} = f\sigma\sqrt{\pi a} = (1)(25 \text{ MPa})\sqrt{\pi(0.0005 \text{ m})} = 0.99 \text{ MPa}\sqrt{\text{m}}$$

- 6-71** A ceramic part for a jet engine has a yield strength of 75,000 psi and a plane strain fracture toughness of  $5,000 \text{ psi}\sqrt{\text{in.}}$ . To be sure that the part does not fail, we plan to assure that the maximum applied stress is only one third the yield strength. We use a nondestructive test that will detect any internal flaws greater than 0.05 in. long. Assuming that  $f = 1.4$ , does our nondestructive test have the required sensitivity? Explain.

**Solution:** The applied stress is  $\sigma = (1/3)(75,000 \text{ psi}) = 25,000 \text{ psi}$

$$a = (1/\pi)[K_{Ic}/f\sigma]^2 = (1/\pi)[5,000 \text{ psi}\sqrt{\text{in.}} / (1.4)(25,000 \text{ psi})]^2$$

$$a = 0.0065 \text{ in.}$$

The length of internal flaws is  $2a = 0.013 \text{ in.}$

Our nondestructive test can detect flaws as small as 0.05 in. long, which is not smaller than the critical flaw size required for failure. Thus our NDT test is not satisfactory.

- 6-86** A cylindrical tool steel specimen that is 6 in. long and 0.25 in. in diameter rotates as a cantilever beam and is to be designed so that failure never occurs. Assuming that the maximum tensile and compressive stresses are equal, determine the maximum load that can be applied to the end of the beam. (See Figure 6-50.)

**Solution:** The stress must be less than the endurance limit, 60,000 psi.

$$\sigma = 10.18LF/d^3 \quad \text{or} \quad F = (\text{endurance limit})d^3/10.18L$$

$$F = (60,000 \text{ psi})(0.25 \text{ in.})^3 / (10.18)(6 \text{ in.}) = 15.35 \text{ lb}$$

- 6-87** A 2 cm-diameter, 20-cm-long bar of an acetal polymer (Figure 6-61) is loaded on one end and is expected to survive one million cycles of loading, with equal maximum tensile and compressive stresses, during its lifetime. What is the maximum permissible load that can be applied?

**Solution:** From the figure, we find that the fatigue strength must be 22 MPa in order for the polymer to survive one million cycles. Thus, the maximum load is

$$F = (\text{fatigue strength})d^3/10.18L$$

$$F = (22 \text{ MPa})(20 \text{ mm})^3 / (10.18)(200 \text{ mm}) = 86.4 \text{ N}$$

- 6-88** A cyclical load of 1500 lb is to be exerted at the end of a 10-in. long aluminum beam (Figure 6-50). The bar must survive for at least  $10^6$  cycles. What is the minimum diameter of the bar?

**Solution:** From the figure, we find that the fatigue strength must be 35,000 psi in order for the aluminum to survive  $10^6$  cycles. Thus, the minimum diameter of the bar is

$$d = \sqrt[3]{10.18LF / \text{fatigue strength}}$$

$$d = \sqrt[3]{10.18(10 \text{ in.})(1500 \text{ lb}) / 35,000 \text{ psi}} = 1.634 \text{ in.}$$

- 6-89** A cylindrical acetal polymer bar 20 cm long and 1.5 cm in diameter is subjected to a vibrational load at a frequency of 500 vibrations per minute with a load of 50 N. How many hours will the part survive before breaking? (See Figure 6-61)

**Solution:** The stress acting on the polymer is

$$\sigma = 10.18LF/d^3 = (10.18)(200 \text{ mm})(50 \text{ N}) / (15 \text{ mm})^3 = 30.16 \text{ MPa}$$

From the figure, the fatigue life at 30.16 MPa is about  $2 \times 10^5$  cycles.

Based on 500 cycles per minute, the life of the part is

$$\text{life} = 2 \times 10^5 \text{ cycles} / (500 \text{ cycles/min})(60 \text{ min/h}) = 6.7 \text{ h}$$

- 6-90** Suppose that we would like a part produced from the acetal polymer shown in Figure 6-61 to survive for one million cycles under conditions that provide for equal compressive and tensile stresses. What is the fatigue strength, or maximum stress amplitude, required? What are the maximum stress, the minimum stress, and the mean stress on the part during its use? What effect would the frequency of the stress application have on your answers? Explain.

**Solution:** From the figure, the fatigue strength at one million cycles is 22 MPa.

The maximum stress is +22 MPa, the minimum stress is -22 MPa, and the mean stress is 0 MPa.

A high frequency will cause heating of the polymer. As the temperature of the polymer increases, the fatigue strength will decrease. If the applied stress is not reduced, then the polymer will fail in a shorter time.

- 6-91** The high-strength steel in Figure 6-52 is subjected to a stress alternating at 200 revolutions per minute between 600 MPa and 200 MPa (both tension). Calculate the growth rate of a surface crack when it reaches a length of 0.2 mm in both m/cycle and m/s. Assume that  $f = 1.0$ .

**Solution:** For the steel,  $C = 1.62 \times 10^{-12}$  and  $n = 3.2$ . The change in the stress intensity factor  $\Delta K$  is

$$\Delta K - f\Delta\sigma\sqrt{\pi a} = (1.2)(600 \text{ MPa} - 200 \text{ MPa})\sqrt{\pi(0.0002 \text{ m})} = 12.03 \text{ MPa}\sqrt{\text{m}}$$

The crack growth rate is

$$da/dN = 1.62 \times 10^{-12}(\Delta K)^{3.2}$$

$$da/dN = 1.62 \times 10^{-12}(12.03)^{3.2} = 4.638 \times 10^{-9} \text{ m/cycle}$$

$$da/dt = (4.638 \times 10^{-9} \text{ m/cycle})(200 \text{ cycles/min}) / 60 \text{ s/min}$$

$$da/dt = 1.55 \times 10^{-8} \text{ m/s}$$

- 6-92** The high-strength steel in Figure 6-52, which has a critical fracture toughness of  $80 \text{ MPa}\sqrt{\text{m}}$ , is subjected to an alternating stress varying from  $-900 \text{ MPa}$  (compression) to  $+900 \text{ MPa}$  (tension). It is to survive for  $10^5$  cycles before failure occurs. Calculate (a) the size of a surface crack required for failure to occur and (b) the largest initial surface crack size that will permit this to happen. Assume that  $f = 1$ .

**Solution:** (a) Only the tensile portion of the applied stress is considered in  $\Delta\sigma$ . Based on the applied stress of  $900 \text{ MPa}$  and the fracture toughness of  $80 \text{ MPa}\sqrt{\text{m}}$ , the size of a surface crack required for failure to occur is

$$K = f\sigma\sqrt{\pi a_c} \quad \text{or} \quad a_c = (1/\pi)[K/f\sigma]^2$$

$$a_c = (1/\pi)[80 \text{ MPa}\sqrt{\text{m}} / (1)(900 \text{ MPa})]^2 = 0.0025 \text{ m} = 2.5 \text{ mm}$$

(b) The largest initial surface crack tolerable to prevent failure within  $10^5$  cycles is

$$N = 10^5 \text{ cycles} = \frac{2[(0.0025 \text{ m})^{(2-3.2)/2} - a_i^{(2-3.2)/2}]}{(2-3.2)(1.62 \times 10^{-12})(1)^{3.2}(900)^{3.2}(\pi)^{3.2/2}}$$

$$10^5 = \frac{2[36.41 - (a_i)^{-0.60}]}{(-1.2)(1.62 \times 10^{-12})(1)(2.84 \times 10^9)(6.244)}$$

$$(a_i)^{-0.60} = 1760$$

$$a_i = 3.9 \times 10^{-6} \text{ m} = 0.0039 \text{ mm}$$

- 6-93** The acrylic polymer from which Figure 6-62 was obtained has a critical fracture toughness of  $2 \text{ MPa}\sqrt{\text{m}}$ . It is subjected to a stress alternating between  $-10$  and  $+10 \text{ MPa}$ . Calculate the growth rate of a surface crack when it reaches a length of  $5 \times 10^{-6} \text{ m}$  if  $f = 1.0$ .

**Solution:**  $\Delta\sigma = 10 \text{ MPa} - 0 = 10 \text{ MPa}$ , since the crack doesn't propagate for compressive loads.

$$\Delta K - f\Delta\sigma\sqrt{\pi a} = (1.3)(10 \text{ MPa})\sqrt{\pi(5 \times 10^{-6} \text{ m})} = 0.0515 \text{ MPa}\sqrt{\text{m}}$$

From the graph,  $da/dN = 3 \times 10^{-7} \text{ m/cycle}$

- 6-94** Calculate the constants “C” and “n” in the Equation 6-36 for the crack growth rate of an acrylic polymer. (See Figure 6-62.)

**Solution:** Let's pick two points on the graph:

$$da / dN = 2 \times 10^{-6} \text{ m / cycle when } \Delta K = 0.1 \text{ MPa}\sqrt{\text{m}}$$

$$da / dN = 1 \times 10^{-7} \text{ m / cycle when } \Delta K = 0.037 \text{ MPa}\sqrt{\text{m}}$$

$$\frac{2 \times 10^{-6}}{1 \times 10^{-7}} = \frac{C(0.1)^n}{C(0.037)^n}$$

$$20 = (0.1 / 0.037)^n = (2.703)^n$$

$$\ln(20) = n \ln(2.703) \quad 2.9957 = 0.994n \quad n = 3.01$$

$$2 \times 10^{-6} = C(0.1)^{3.01} = 0.000977C \quad C = 2.047 \times 10^{-3}$$

- 6-95** The acrylic polymer from which Figure 6-62 was obtained is subjected to an alternating stress between 15 MPa and 0 MPa. The largest surface cracks initially detected by nondestructive testing are 0.001 mm in length. If the critical fracture toughness of the polymer is  $2 \text{ MPa}\sqrt{\text{m}}$ , calculate the number of cycles required before failure occurs. Let  $f = 1.0$ . (*Hint:* Use the results of Problem 6-94.)

**Solution:** From Problem 6-94,  $C = 2.047 \times 10^{-3}$  and  $n = 3.01$

The critical flaw size  $a_c$  is

$$a_c = (1/\pi)[K_{Ic} / f\sigma]^2 = (1/\pi)[(2 \text{ MPa}\sqrt{\text{m}}) / (1.2)(15 \text{ MPa})]^2$$

$$a_c = 0.00393 \text{ m} = 3.93 \text{ mm}$$

Then

$$N = \frac{2[(0.00393 \text{ m})^{(2-3.01)/2} - (0.000001 \text{ m})^{(2-3.01)/2}]}{(2-3.01)(2.047 \times 10^{-3})(1.2)^{3.01}(15 \text{ mPa})^{3.01}(\pi)^{3.01/2}}$$

$$N = \frac{2(16.3995 - 1071.52)}{(-1.01)(2.047 \times 10^{-3})(1.7312)(3467.65)(5.6)} = 30.36 \text{ cycles}$$

- 6-97** Verify that integration of  $da/dN = C(\Delta K)^n$  will give Equation 6-38.

**Solution:**  $dN = (1/cf^n \Delta\sigma^n \pi^{n/2})(da/a^{n/2})$  or  $N = (1/cf^n \Delta\sigma^n \pi^{n/2}) \int (da/a^{n/2})$

since  $\int a^p da = [1/(1+p)](a^{p+1})$

$$\text{then if } p = -n/2, \int da/a^{n/2} = \frac{1}{1-n/2} [a^{-n/2+1}]_{a_i}^{a_c} = (2/2-n)[a_c^{(2-n)/2} - a_i^{(2-n)/2}]$$

$$\text{thus } N = \frac{2[a_c^{(2-n)/2} - a_i^{(2-n)/2}]}{(2-n)cf^n \Delta\sigma^n \pi^{n/2}}$$

- 6-102** The activation energy for self-diffusion in copper is 49,300 cal/mol. A copper specimen creeps at 0.002 in./in.-h when a stress of 15,000 psi is applied at 600°C. If the creep rate of copper is dependent on self-diffusion, determine the creep rate if the temperature is 800°C.

**Solution:** The creep rate is governed by an Arrhenius relationship of the form rate  $= A \exp(-Q/RT)$ . From the information given,

$$\frac{x}{0.002 \text{ in./in.} \cdot h} = \frac{A \exp[-49,300/(1.987)(800+273)]}{A \exp[-49,300/(1.987)(600+273)]} = \frac{9.07 \times 10^{-11}}{4.54 \times 10^{-13}}$$

$$x = (0.002)(9.07 \times 10^{-11} / 4.54 \times 10^{-13}) = 0.4 \text{ in./in.} \cdot h$$

**6-103** When a stress of 20,000 psi is applied to a material heated to 900°C, rupture occurs in 25,000 h. If the activation energy for rupture is 35,000 cal/mol, determine the rupture time if the temperature is reduced to 800°C.

**Solution:** The rupture time is related to temperature by an Arrhenius relationship of the form  $t_r = A \exp(+Q/RT)$ ; the argument of the exponential is positive because the rupture time is inversely related to the rate. From the information given

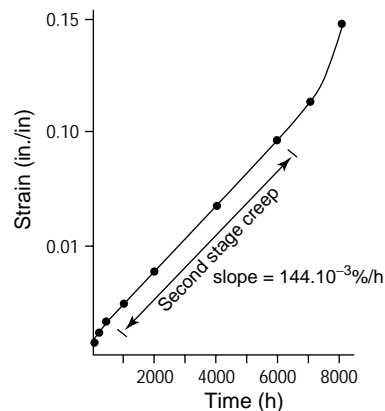
$$\frac{t_r}{25,000 \text{ h}} = \frac{A \exp[35,000/(1.987)(800+273)]}{A \exp[35,000/(1.987)(900+273)]} = \frac{1.35 \times 10^7}{3.32 \times 10^6}$$

$$t_r = (25,000)(1.35 \times 10^7 / 3.32 \times 10^6) = 101,660 \text{ h}$$

**6-104** The following data were obtained from a creep test for a specimen having an initial gage length of 2.0 in. and an initial diameter of 0.6 in. The initial stress applied to the material is 10,000 psi. The diameter of the specimen after fracture is 0.52 in.

**Solution:**

Length Between Gage Marks (in.)	Time (h)	Strain (in./in.)
2.004	0	0.002
2.01	100	0.005
2.02	200	0.010
2.03	400	0.015
2.045	1000	0.0225
2.075	2000	0.0375
2.135	4000	0.0675
2.193	6000	0.0965
2.23	7000	0.115
2.30	8000 (fracture)	0.15





Determine (a) the load applied to the specimen during the test, (b) the approximate length of time during which linear creep occurs, (c) the creep rate in in./in.h and in %/h, and (d) the true stress acting on the specimen at the time of rupture.

(a) The load is  $F = \sigma A = (10,000 \text{ psi})(\pi/4)(0.6 \text{ in.})^2 = 2827 \text{ lb}$

(b) The plot of strain versus time is linear between approximately 500 and 6000 hours, or a total of 5500 hours.

(c) From the graph, the strain rate is the slope of the linear portion of the curve.

$$\Delta\epsilon/\Delta t = \frac{0.095 - 0.03}{6000 - 1500} = 1.44 \times 10^{-5} \text{ in./in.h} = 1.44 \times 10^{-3} \%/\text{h}$$

(d) At the time of rupture, the force is still 2827 lb, but the diameter is reduced to 0.52 in. The true stress is therefore

$$\sigma_t = F/A = 2827 \text{ lb} / (\pi/4)(0.52 \text{ in.})^2 = 13,312 \text{ psi}$$

**6-105** A stainless steel is held at 705°C under different loads. The following data are obtained:

Solution:

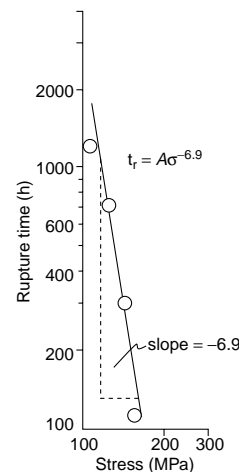
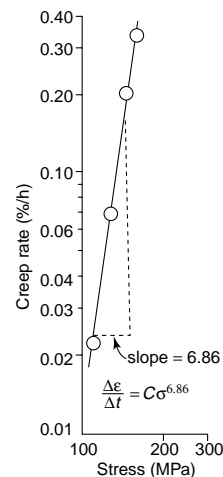
Applied Stress (MPa)	Rupture Time (h)	Creep Rate (%/h)
106.9	1200	0.022
128.2	710	0.068
147.5	300	0.201
160.0	110	0.332

Determine the exponents “ $n$ ” and “ $m$ ” in Equations 6-40 and 6-41 that describe the dependence of creep rate and rupture time on applied stress.

Plots describing the effect of applied stress on creep rate and on rupture time are shown below. In the first plot, the creep rate is given by  $\Delta\epsilon/\Delta t = C\sigma^n$  and the graph is a log-log plot. In the second plot, rupture time is given by  $t_r = A\sigma^m$ , another log-log plot.

The exponents “ $n$ ” and “ $m$ ” are the slopes of the two graphs. In this case,

$$n = 6.86 \quad m = -6.9$$



- 6-106** Using the data in Figure 6-59(a) for an iron-chromium-nickel alloy, determine the activation energy  $Q_r$  and the constant “ $m$ ” for rupture in the temperature range 980 to 1090°C.

**Solution:** The appropriate equation is  $t_r = K\sigma^m \exp(Q_r/RT)$ .

From Figure 6-59(a), we can determine the rupture time versus temperature for a fixed stress, say  $\sigma = 1000$  psi:

$$t_r = 2,400 \text{ h at } 1090^\circ\text{C} = 1363 \text{ K}$$

$$t_r = 14,000 \text{ h at } 1040^\circ\text{C} = 1313 \text{ K}$$

$$t_r = 100,000 \text{ h at } 980^\circ\text{C} = 1253 \text{ K}$$

From this data, the equation becomes  $t_r = K' \exp(Q_r/RT)$  and we can find  $Q_r$  by simultaneous equations or graphically.

$$Q_r = 117,000 \text{ cal/mol}$$

We can also determine the rupture time versus applied stress for a constant temperature, say 1090°C:

$$t_r = 10^5 \text{ h for } \sigma = 450 \text{ psi}$$

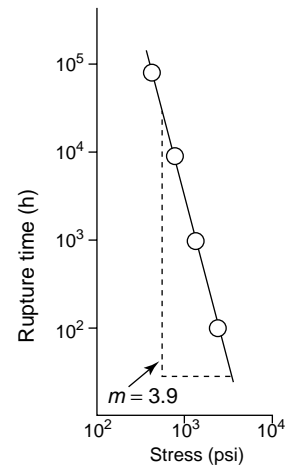
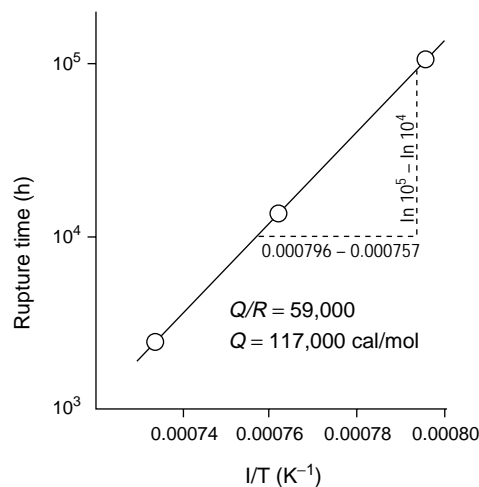
$$t_r = 10^4 \text{ h for } \sigma = 800 \text{ psi}$$

$$t_r = 10^3 \text{ h for } \sigma = 1200 \text{ psi}$$

$$t_r = 10^2 \text{ h for } \sigma = 2100 \text{ psi}$$

With this approach, the equation becomes  $t_r = K''\sigma^m$ , where “ $m$ ” is obtained graphically or by simultaneous equations:

$$m = 3.9$$



- 6-107** A 1-in. diameter bar of an iron-chromium-nickel alloy is subjected to a load of 2500 lb. How many days will the bar survive without rupturing at 980°C? [See Figure 6-59(a).]

**Solution:** The stress is  $\sigma = F/A = 2500 \text{ lb} / (\pi/4)(1 \text{ in.})^2 = 3183 \text{ psi}$

From the graph, the rupture time is 700 h / 24 h/day = 29 days

- 6-108** A 5 mm × 20 mm bar of an iron-chromium-nickel alloy is to operate at 1040°C for 10 years without rupturing. What is the maximum load that can be applied? [See Figure 6-59(a).]

**Solution:** The operating time is (10 years)(365 days/year)(24 h/day) = 87,600 h  
 From the graph, the stress must be less than 500 psi. The load is then  

$$F = \sigma A = (500 \text{ psi})(5 \text{ mm}/25.4 \text{ mm/in.})(20 \text{ mm}/25.4 \text{ mm/in.}) = 77.5 \text{ lb}$$

- 6-109** An iron-chromium-nickel alloy is to withstand a load of 1500 lb at 760°C for 6 years. Calculate the minimum diameter of the bar. [See Figure 6-59(a).]

**Solution:** The operating time is (6 years)(365 days/year)(24 h/day) = 52,560 h  
 From the graph, the stress must be less than 7000 psi. The minimum diameter of the bar is then  

$$d = \sqrt{(4 / \pi)(F / \sigma)} = \sqrt{(4 / \pi)(1500 \text{ lb} / 7000 \text{ psi})} = 0.52 \text{ in.}$$

- 6-110** A 1.2-in.-diameter bar of an iron-chromium-nickel alloy is to operate for 5 years under a load of 4000 lb. What is the maximum operating temperature? [See Figure 6-59(a).]

**Solution:** The operating time is (5 years)(365 days/year)(24 h/day) = 43,800 h  
 The stress is  $\sigma = F/A = 4000 \text{ lb} / (\pi/4)(1.2 \text{ in.})^2 = 3537 \text{ psi}$   
 From the figure, the temperature must be below 850°C in order for the bar to survive five years at 3537 psi.

- 6-111** A 1 in. × 2 in. ductile cast iron bar must operate for 9 years at 650°C. What is the maximum load that can be applied? [See Figure 6-59(b).]

**Solution:** The operating time is (9 year)(365 days/year)(24 h/day) = 78,840 h.  
 The temperature is  $650 + 273 = 923 \text{ K}$   

$$LM = (923/1000)[36 + 0.78 \ln(78,840)] = 41.35$$
  
 From the graph, the stress must be no more than about 1000 psi. The load is then  

$$F = \sigma A = (1000 \text{ psi})(2 \text{ in.}^2) = 2000 \text{ lb}$$

- 6-112** A ductile cast iron bar is to operate at a stress of 6000 psi for 1 year. What is the maximum allowable temperature? [See Figure 6-59(b).]

**Solution:** The operating time is (1 year)(365 days/year)(24 h/day) = 8760 h  
 From the graph, the Larson-Miller parameter must be 34.4 at a stress of 6000 psi. Thus  

$$34.4 = (T / 1000)[36 + 0.78 \ln(8760)] = 0.043T$$
  

$$T = 800\text{K} = 527^\circ\text{C}$$

# 7

## Strain Hardening and Annealing

- 7-5 A 0.505-in.-diameter metal bar with a 2-in. gage length  $l_0$  is subjected to a tensile test. The following measurements are made in the plastic region:

Force (lb)	Change in Gage length (in.)	Diameter (in.)
27,500	0.2103	0.4800
27,000	0.4428	0.4566
25,700	0.6997	0.4343

Determine the strain hardening exponent for the metal. Is the metal most likely to be FCC, BCC, or HCP? Explain.

Solution:

Force (lb)	Gage length (in.)	Diameter (in.)	True stress (psi)	True strain (in./in.)
27,500	2.2103	0.4800	151,970	0.100
27,000	2.4428	0.4566	164,893	0.200
25,700	2.6997	0.4343	173,486	0.300

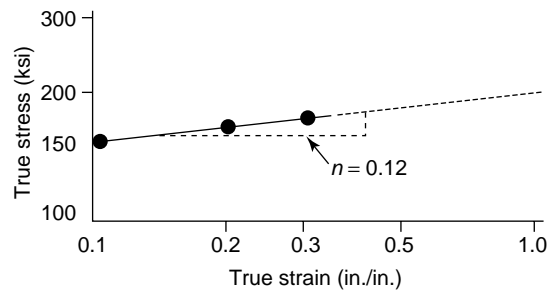
$$\sigma_t = K \epsilon_t^n \quad \text{or} \quad \ln \sigma = \ln K + n \ln \epsilon$$

$$\ln(151,970) = \ln K + n \ln(0.1) \quad 11.9314 = \ln K - n (2.3026)$$

$$\ln(173,486) = \ln K + n \ln(0.3) \quad 12.0639 = \ln K - n (1.2040)$$

$$-0.1325 = -1.0986 n$$

$$n = 0.12 \quad \text{which is in the range of BCC metals}$$



- 7-7 A 1.5-cm-diameter metal bar with a 3-cm gage length is subjected to a tensile test. The following measurements are made.

Force (N)	Change in Gage length (cm)	Diameter (cm)
16,240	0.6642	1.2028
19,066	1.4754	1.0884
19,273	2.4663	0.9848

Determine the strain hardening coefficient for the metal. Is the metal most likely to be FCC, BCC, or HCP? Explain.

Solution:

Force (N)	Gage length (cm)	Diameter (mm)	True stress (MPa)	True strain (cm/cm)
16,240	3.6642	12.028	143	0.200
19,066	4.4754	10.884	205	0.400
19,273	5.4663	9.848	249	0.600

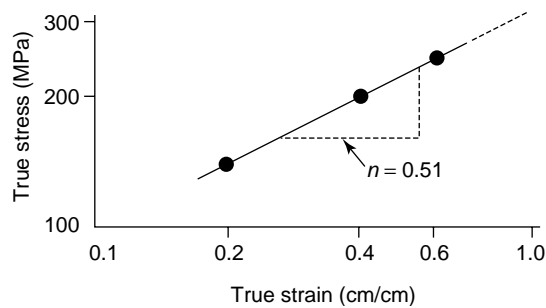
$$\sigma_t = K \epsilon_t^n \quad \ln 143 = \ln K + n \ln 0.2$$

$$\ln 249 = \ln K + n \ln 0.6$$

$$(4.962 - 5.517) = n(-1.609 + 0.511)$$

$$n = 0.51$$

A strain hardening coefficient of 0.51 is typical of FCC metals.



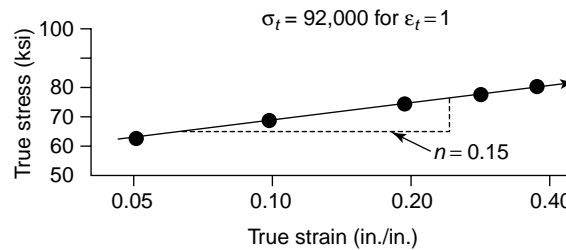
- 7-9** A true stress-true strain curve is shown in Figure 7-22. Determine the strain hardening exponent for the metal.

**Solution:**  $\sigma_t = K\varepsilon_t^n$

$\varepsilon_t$	$\sigma_t$
0.05 in./in.	60,000 psi
0.10 in./in.	66,000 psi
0.20 in./in.	74,000 psi
0.30 in./in.	76,000 psi
0.40 in./in.	81,000 psi

From graph:  $K = 92,000$  psi

$n = 0.15$



- 7-10** A Cu-30% Zn alloy bar has a strain hardening coefficient of 0.50. The bar, which has an initial diameter of 1 cm and an initial gage length of 3 cm, fails at an engineering stress of 120 MPa. After fracture, the gage length is 3.5 cm and the diameter is 0.926 cm. No necking occurred. Calculate the true stress when the true strain is 0.05 cm/cm.

**Solution:**  $\varepsilon_t = \ln(\ell_f/\ell_o) = \ln(3.5/3.0) = 0.154$

$$\sigma_E = 120 \text{ MPa} = \frac{F}{(\pi/4)(10\text{mm})^2}$$

$$F = 9425 \text{ N}$$

$$\sigma_t = \frac{9425 \text{ N}}{(\pi/4)(9.26 \text{ mm})^2} = 139.95 \text{ MPa}$$

$$\sigma_t = K(0.154)^{0.5} = 139.95 \text{ MPa} \quad \text{or} \quad K = 356.6$$

The true stress at  $\varepsilon_t = 0.05$  cm/cm is:

$$\sigma_t = 356.6 (0.05)^{0.5} \quad \text{or} \quad \sigma_t = 79.7 \text{ MPa}$$

- 7-14** The Frank-Read source shown in Figure 7-5(e) has created four dislocation loops from the original dislocation line. Estimate the total dislocation line present in the photograph and determine the percent increase in the length of dislocations produced by the deformation.

**Solution:** If the length of the original dislocation line is 1 mm on the photograph, then we can estimate the circumference of the dislocation loops. The loops are not perfect circles, so we might measure the smallest and largest diameters, then use the average:

first loop:  $D_{\text{small}} = 10 \text{ mm}$ ,  $D_{\text{large}} = 14 \text{ mm}$ ;  $D_{\text{avg}} = 12 \text{ mm}$   
circumference =  $12.0\pi$

second loop:  $D_{\text{small}} = 18 \text{ mm}$ ;  $D_{\text{large}} = 20 \text{ mm}$ ;  $D_{\text{avg}} = 19 \text{ mm}$   
circumference =  $19.0\pi$

third loop:  $D_{\text{small}} = 28 \text{ mm}$ ;  $D_{\text{large}} = 30 \text{ mm}$ ;  $D_{\text{avg}} = 29 \text{ mm}$   
circumference =  $29.0\pi$

fourth loop:  $D_{\text{small}} = 42 \text{ mm}$ ;  $D_{\text{large}} = 45 \text{ mm}$ ;  $D_{\text{avg}} = 43.5 \text{ mm}$   
circumference =  $43.5\pi$

Therefore in the photograph itself:

$$\text{total length} = 1 + (12.0 + 19.0 + 29.0 + 43.5)\pi = 326 \text{ mm}$$

The magnification in the photograph is 30,000. Therefore:

$$\text{total length} = 326 / 30,000 = 0.0109 \text{ mm}$$

The original dislocation line is  $1 \text{ mm} / 30,000 = 3.33 \times 10^{-5} \text{ mm}$

$$\% \text{ increase} = (0.0109 - 0.0000333) / 3.33 \times 10^{-5} \text{ mm} \times 100 = 32,630\%$$

**7-19** A 0.25-in.-thick copper plate is to be cold worked 63%. Find the final thickness.

Solution: (See Figure 7-7.)  $63 = \frac{0.25 - t_f}{0.25} \times 100\%$  or  $t_f = 0.0925 \text{ in.}$

**7-20** A 0.25-in.-diameter copper bar is to be cold worked 63%. Find the final diameter.

Solution:  $63 = \frac{(0.25)^2 - d_f^2}{(0.25)^2} \times 100\%$  or  $d_f^2 = 0.023$  or  $d_f = 0.152 \text{ in.}$

**7-21** A 2-in.-diameter copper rod is reduced to 1.5 in. diameter, then reduced again to a final diameter of 1 in. In a second case, the 2-in.-diameter rod is reduced in one step from 2 in. to a 1 in. diameter. Calculate the %CW for both cases.

Solution:  $\%CW = \frac{(2)^2 - (1)^2}{(2)^2} \times 100 = 75\%$  in both cases

**7-22** A 3105 aluminum plate is reduced from 1.75 in. to 1.15 in. Determine the final properties of the plate. (See Figure 7-23.)

Solution:  $\%CW = \frac{1.75 - 1.15}{1.75} \times 100\% = 34.3\%$

$$TS = 26 \text{ ksi} \quad YS = 22 \text{ ksi} \quad \% \text{elongation} = 5\%$$

**7-23** A Cu-30% Zn brass bar is reduced from 1-in. diameter to a 0.45-in. diameter. Determine the final properties of the bar. (See Figure 7-24.)

Solution:  $\%CW = \frac{(1)^2 - (0.45)^2}{(1)^2} \times 100 = 79.75\%$

$$TS = 105 \text{ ksi} \quad YS = 68 \text{ ksi} \quad \% \text{elongation} = 1\%$$

- 7-24** A 3105 aluminum bar is reduced from a 1-in. diameter, to a 0.8-in. diameter, to a 0.6-in. diameter, to a final 0.4-in. diameter. Determine the %CW and the properties after each step of the process. Calculate the total percent cold work. (See Figure 7-23.)

**Solution:** If we calculated the percent deformation in each step separately, we would find that 36% deformation is required to go from 1 in. to 0.8 in. The deformation from 0.8 in. to 0.6 in. (using 0.8 in. as the initial diameter) is 43.75%, and the deformation from 0.6 in. to 0.4 in. (using 0.6 in. as the initial diameter) is 55.6%. If we added these three deformations, the total would be 135.35%. This would not be correct. Instead, we must always use the original 1 in. diameter as our starting point. The following table summarizes the actual deformation and properties after each step.

	TS ksi	YS ksi	% elongation
$\frac{(1)^2 - (0.8)^2}{(1)^2} \times 100 = 36\%$	26	23	6
$\frac{(1)^2 - (0.6)^2}{(1)^2} \times 100 = 64\%$	30	27	3
$\frac{(1)^2 - (0.4)^2}{(1)^2} \times 100 = 84\%$	32	29	2

The total percent cold work is actually 84%, not the 135.35%.

- 7-25** We want a copper bar to have a tensile strength of at least 70,000 psi and a final diameter of 0.375 in. What is the minimum diameter of the original bar? (See Figure 7-7.)

**Solution:** %CW  $\geq$  50% to achieve the minimum tensile strength

$$50 = \frac{d_o^2 - (0.375)^2}{d_o^2} \times 100$$

$$0.5 d_o^2 = 0.140625 \quad \text{or} \quad d_o = 0.53 \text{ in.}$$

- 7-26** We want a Cu-30% Zn brass plate originally 1.2-in. thick to have a yield strength greater than 50,000 psi and a %Elongation of at least 10%. What range of final thicknesses must be obtained? (See Figure 7-24.)

**Solution:** YS > 50,000 psi requires CW > 20%

%E > 10% requires CW < 35%

$$\frac{1.2 - t_f}{1.2} = 0.20 \qquad \frac{1.2 - t_f}{1.2} = 0.35$$

$$t_f = 0.96 \text{ in.} \qquad t_f = 0.78 \text{ in.}$$

$$t_f = 0.78 \text{ to } 0.96 \text{ in.}$$



- 7-27** We want a copper sheet to have at least 50,000 psi yield strength and at least 10% Elongation, with a final thickness of 0.12 in. What range of original thicknesses must be used? (See Figure 7-7.)

**Solution:**  $YS > 50 \text{ ksi}$  requires  $CW \geq 25\%$

$\%E > 10\%$  requires  $CW \leq 30\%$

$$\frac{t_o - 0.12}{t_o} = 0.25 \qquad \frac{t_o - 0.12}{t_o} = 0.30$$

$$t_o = 0.16 \text{ in.}$$

$$t_o = 0.17 \text{ in.}$$

$$t_o = 0.16 \text{ to } 0.17 \text{ in.}$$

- 7-28** A 3105 aluminum plate previously cold worked 20% is 2-in. thick. It is then cold worked further to 1.3 in. Calculate the total percent cold work and determine the final properties of the plate? (See Figure 7-23.)

**Solution:** The original thickness (before the 20% cold work) must have been:

$$\frac{t_o - 2}{t_o} = 0.20 \qquad t_o = 2.5 \text{ in.}$$

The total cold work is then based on the prior 2.5 in. thickness:

$$CW = \frac{2.5 - 1.3}{2.5} \times 100\% = 48\% \qquad \begin{array}{l} TS = 28 \text{ ksi} \\ YS = 25 \text{ ksi} \\ \%E = 4\% \end{array}$$

- 7-29** An aluminum-lithium strap 0.25-in. thick and 2-in. wide is to be cut from a rolled sheet, as described in Figure 7-10. The strap must be able to support a 35,000 lb load without plastic deformation. Determine the range of orientations from which the strap can be cut from the rolled sheet.

**Solution:**  $\sigma = \frac{35,000}{(0.25)(2)} \geq 70,000 \text{ psi}$

The properties can be obtained at angles of 0 to 20° from the rolling direction of the sheet.

- 7-43** We want to draw a 0.3-in.-diameter copper wire having a yield strength of 20,000 psi into 0.25-in.-diameter wire. (a) Find the draw force, assuming no friction. (b) Will the drawn wire break during the drawing process? Show. (See Figure 7-7.)

**Solution:** (a) Before drawing (0% CW), the yield strength is 20 ksi = 20,000 psi.

$$CW = \frac{(0.3)^2 - (0.25)^2}{(0.3)^2} = 30.6\% \qquad \text{which gives } YS = 53,000 \text{ psi in the drawn wire}$$

(b) The force needed to draw the original wire is :

$$20,000 \text{ psi} = F/(\pi/4)(0.3)^2 \quad \text{or} \quad F = 1414 \text{ lb}$$

(c) The stress acting on the drawn wire is:

$$\sigma = 1414/(\pi/4)(0.25)^2 = 28,806 \text{ psi} < 53,000 \text{ psi}$$

Since the actual stress (28,806 psi) acting on the drawn wire is less than the yield strength (53,000 psi) of the drawn wire, the wire will not break during manufacturing.

- 7-44** A 3105 aluminum wire is to be drawn to give a 1-mm diameter wire having a yield strength of 20,000 psi. (a) Find the original diameter of the wire, (b) calculate the draw force required, and (c) determine whether the as-drawn wire will break during the process. (See Figure 7-23.)

**Solution:** (a) We need to cold work 25% to obtain the required yield strength:

$$\frac{d_o^2 - 1^2}{d_o^2} = 0.25 \quad d_o = \sqrt{1 / 0.75} = 1.1547 \text{ mm} = 0.04546 \text{ in.}$$

(b) The initial yield strength of the wire (with 0% cold work) is 8000 psi, so the force required to deform the initial wire is:

$$F = 8000[(\pi/4)(0.04546)^2] = 12.98 \text{ lb}$$

(c) The stress acting on the drawn wire (which has a smaller diameter but is subjected to the same drawing force) is:

$$\sigma = \frac{12.98 \text{ lb}}{(\pi/4)(1 \text{ mm}/25.4 \text{ mm/in})^2} = 10,662 \text{ psi} < 20,000 \text{ psi}$$

Since the actual stress is less than the 20,000 psi yield strength of the drawn wire, the process will be successful and the wire will not break.

- 7-53** A titanium alloy contains a very fine dispersion of tiny  $\text{Er}_2\text{O}_3$  particles. What will be the effect of these particles on the grain growth temperature and the size of the grains at any particular annealing temperature? Explain.

**Solution:** These particles, by helping pin the grain boundaries, will increase the grain growth temperature and decrease the grain size.

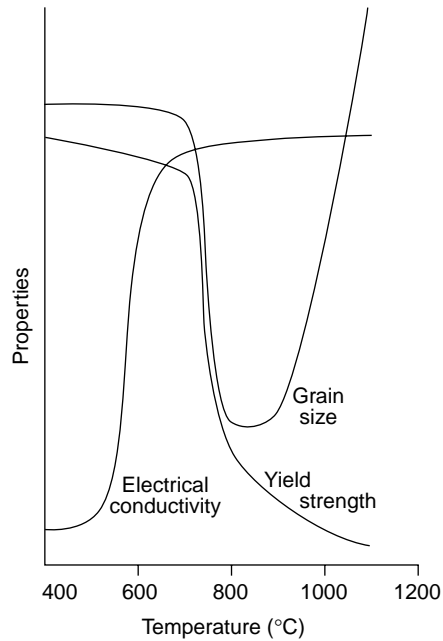
- 7-55** The following data were obtained when a cold-worked metal was annealed.  
 (a) Estimate the recovery, recrystallization, and grain growth temperatures.  
 (b) Recommend a suitable temperature for a stress relief heat treatment.  
 (c) Recommend a suitable temperature for a hot-working process. (d) Estimate the melting temperature of the alloy.

Annealing Temperature (°C)	Electrical Conductivity ( $\text{ohm}^{-1}\text{cm}^{-1}$ )	Yield Strength (MPa)	Grain Size (mm)
400	$3.04 \times 10^5$	86	0.10
500	$3.05 \times 10^5$	85	0.10
600	$3.36 \times 10^5$	84	0.10
700	$3.45 \times 10^5$	83	0.098
800	$3.46 \times 10^5$	52	0.030
900	$3.46 \times 10^5$	47	0.031
1000	$3.47 \times 10^5$	44	0.070
1100	$3.47 \times 10^5$	42	0.120

- Solution:** (a) recovery temperature  $\approx 550^\circ\text{C}$   
 recrystallization temperature  $\approx 750^\circ\text{C}$   
 grain growth temperature  $\approx 950^\circ\text{C}$   
 (b) Stress relief temperature =  $700^\circ\text{C}$

(c) Hot working temperature = 900°C

(d)  $0.4 T_{mp} \cong 750^{\circ}\text{C} = 1023 \text{ K}$   
 $T_{mp} \cong 1023 / 0.4 = 2558 \text{ K} = 2285^{\circ}\text{C}$



- 7-56** The following data were obtained when a cold worked metal was annealed.
- Estimate the recovery, recrystallization, and grain growth temperatures.
  - Recommend a suitable temperature for obtaining a high strength, high electrical conductivity wire.
  - Recommend a suitable temperature for a hot-working process.
  - Estimate the melting temperature of the alloy.

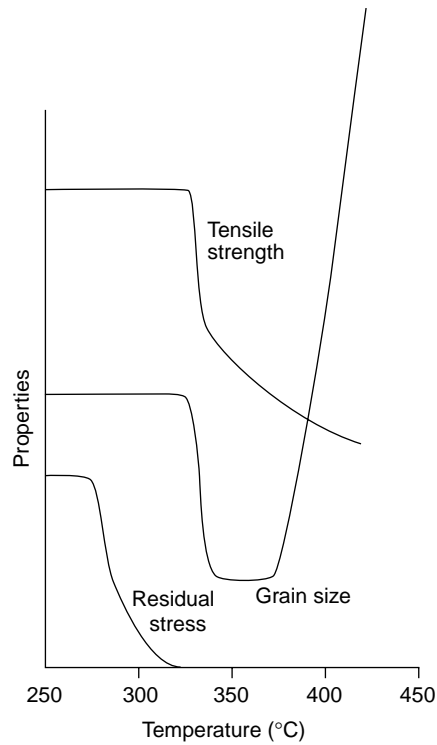
Annealing Temperature (°C)	Residual Stresses (psi)	Tensile Strength (psi)	Grain Size (in.)
250	21,000	52,000	0.0030
275	21,000	52,000	0.0030
300	5,000	52,000	0.0030
325	0	52,000	0.0030
350	0	34,000	0.0010
375	0	30,000	0.0010
400	0	27,000	0.0035
425	0	25,000	0.0072

**Solution:**

- recovery temperature  $\approx 280^{\circ}\text{C}$   
 recrystallization temperature  $\cong 330^{\circ}\text{C}$   
 grain growth temperature  $\cong 380^{\circ}\text{C}$
- For a high strength, high conductivity wire, we want to heat into the recovery range. A suitable temperature might be  $320^{\circ}\text{C}$ .
- Hot working temperature =  $375^{\circ}\text{C}$

(d)  $0.4 T_{\text{mp}} \cong 330^{\circ}\text{C} = 603 \text{ K}$

$T_{\text{mp}} \cong 603 / 0.4 = 1508 \text{ K} = 1235^{\circ}\text{C}$



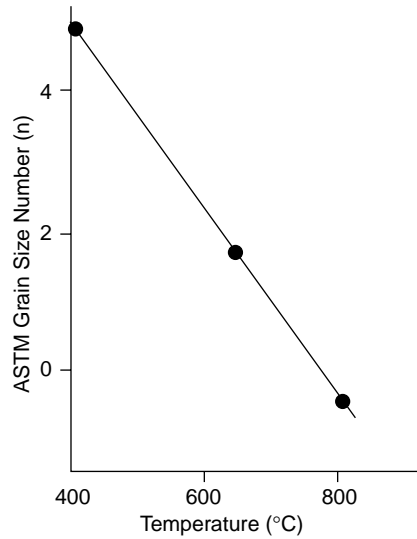
**7-58** Determine the ASTM grain size number for each of the micrographs in Figure 7-16 and plot the grain size number versus the annealing temperature.

**Solution:** The approximate number of grains per square inch in each photomicrograph at 75x is:

$$\begin{aligned} 400^{\circ}\text{C}: N &= (26 \text{ grains/in.}^2)(75/100)^2 \\ &= 14.6 \text{ grains/in.}^2 = 2^{n-1} \\ \log(14.6) &= 2.683 = (n-1)(0.301) \\ n &= 4.9 \end{aligned}$$

$$\begin{aligned} 650^{\circ}\text{C}: N &= (3 \text{ grains/in.}^2)(75/100)^2 \\ &= 1.7 \text{ grains/in.}^2 = 2^{n-1} \\ \log(1.7) &= 0.23 = (n-1)(0.301) \\ n &= 1.8 \end{aligned}$$

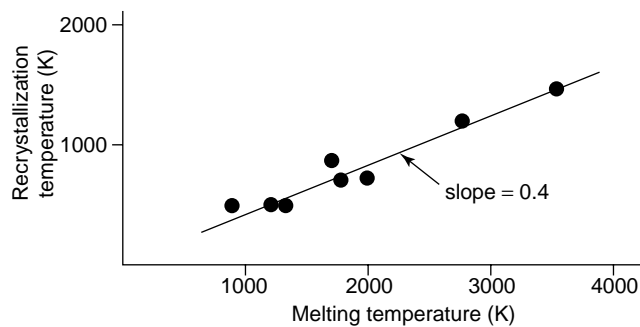
$$\begin{aligned} 800^{\circ}\text{C}: N &= (0.7 \text{ grains/in.}^2)(75/100)^2 \\ &= 0.4 \text{ grains/in.}^2 = 2^{n-1} \\ \log(0.4) &= -0.40 = (n-1)(0.301) \\ n &= -0.3 \end{aligned}$$



- 7-66** Using the data in Table 7-4, plot the recrystallization temperature versus the melting temperature of each metal, using absolute temperatures (Kelvin). Measure the slope and compare with the expected relationship between these two temperatures. Is our approximation a good one?

**Solution:** Converting the recrystallization and melting temperatures to Kelvin, we can obtain the graph shown. The relationship of  $T_r = 0.4T_m$  (K) is very closely followed.

	$T_m$	$T_r$
Al	933 K	423 K
Mg	923 K	473 K
Ag	1235 K	473 K
Cu	1358 K	473 K
Fe	1811 K	723 K
Ni	1726 K	873 K
Mo	2883 K	1173 K
W	3683 K	1473 K



- 7-67** We wish to produce a 0.3-in.-thick plate of 3105 aluminum having a tensile strength of at least 25,000 psi and a %elongation of at least 5%. The original thickness of the plate is 3 in. The maximum cold work in each step is 80%. Describe the cold working and annealing steps required to make this product. Compare this process with that you would recommend if you could do the initial deformation by hot working. (See Figure 7-23.)

**Solution:** For TS  $\geq 25000$  CW  $\geq 30\%$ ; For %elongation  $\geq 5\%$  CW  $\leq 30\%$   
 $\therefore$  required CW = 30%

$$\frac{t_i - 0.3}{t_i} = 0.30 \quad \text{or} \quad t_i = 0.429 \text{ in.}$$

Cold work/anneal treatment

CW 75% from 3.0 to 0.75 in.

anneal

CW 42.8% from 0.75 to 0.429 in.

anneal

CW 30% from 0.429 to 0.3 in.

Hot work treatment

HW 85.7% from 3.0 to 0.429 in.

CW 30% from 0.429 to 0.3 in.

- 7-68** We wish to produce a 0.2-in. diameter wire of copper having a minimum yield strength of 60,000 psi and a minimum %Elongation of 5%. The original diameter of the rod is 2 in. and the maximum cold work in each step is 80%. Describe the cold working and annealing steps required to make this product. Compare this process with that you would recommend if you could do the initial deformation by hot working. (See Figure 7-7.)

**Solution:** For YS  $> 60$  ksi, CW  $\geq 40\%$ ; For %elongation  $> 5$  CW  $\geq 45\%$   
 $\therefore$  pick CW = 42%, the middle of the allowable range

$$\frac{d_i^2 - (0.2)^2}{d_i^2} = 0.42 \quad \text{or} \quad d_i = \sqrt{0.04 / 0.58} = 0.263 \text{ in.}$$

Cold work/anneal treatment

CW 75% from 2 to 1 in-diameter

anneal

CW 75% from 1 to 0.5 in.

anneal

CW 72.3% from 0.5 to 0.263 in.

anneal

CW 42% from 0.263 to 0.2 in.

Hot work treatment

HW 98.3% from 2 to 0.263 in.

CW 42% from 0.263 to 0.2 in.



# 8

## Principles of Solidification

- 8-10** Suppose that liquid nickel is undercooled until homogeneous nucleation occurs. Calculate (a) the critical radius of the nucleus required, and (b) the number of nickel atoms in the nucleus. Assume that the lattice parameter of the solid FCC nickel is 0.356 nm.

**Solution:** From Table 8-1,  $\Delta T_{\max} = 480^{\circ}\text{C}$

$$r^{*} = \frac{(2)(255 \times 10^{-7} \text{ J/cm}^2)(1453 + 273)}{(2756 \text{ J/cm}^3)(480)} = 6.65 \times 10^{-8} \text{ cm}$$

$$a_o = 3.56 \text{ \AA}$$

$$V = 45.118 \times 10^{-24} \text{ cm}^3$$

$$V_{\text{nucleus}} = (4\pi/3)(6.65 \times 10^{-8} \text{ cm})^3 = 1232 \times 10^{-24} \text{ cm}^3$$

$$\text{number of unit cells} = 1232/45.118 = 27.3$$

$$\text{atoms per nucleus} = (4 \text{ atoms/cell})(27.3 \text{ cells}) = 109 \text{ atoms}$$

- 8-11** Suppose that liquid iron is undercooled until homogeneous nucleation occurs. Calculate (a) the critical radius of the nucleus required, and (b) the number of iron atoms in the nucleus. Assume that the lattice parameter of the solid BCC iron is 2.92 Å.

**Solution:** 
$$r^{*} = \frac{(2)(204 \times 10^{-7} \text{ J/cm}^2)(1538 + 273)}{(1737 \text{ J/cm}^3)(420)} = 10.128 \times 10^{-8} \text{ cm}$$

$$V = (4\pi/3)(10.128)^3 = 4352 \text{ \AA}^3 = 4352 \times 10^{-24} \text{ cm}^3$$

$$V_{\text{uc}} = (2.92 \text{ \AA})^3 = 24.897 \text{ \AA}^3 = 24.897 \times 10^{-24} \text{ cm}^3$$

$$\text{number of unit cells} = 4352/24.897 = 175$$

$$\text{atoms per nucleus} = (175 \text{ cells})(2 \text{ atoms/cell}) = 350 \text{ atoms}$$

- 8-12** Suppose that solid nickel was able to nucleate homogeneously with an undercooling of only  $22^{\circ}\text{C}$ . How many atoms would have to group together spontaneously for this to occur? Assume that the lattice parameter of the solid FCC nickel is 0.356 nm.



Solution:  $r^* = \frac{(2)(255 \times 10^{-7} \text{ J/cm}^2)(1453 + 273)}{(2756 \text{ J/cm}^3)(22)} = 145.18 \times 10^{-8} \text{ cm}$

$V_{uc} = 45.118 \times 10^{-24} \text{ cm}^3$  (see Problem 8-10)

$V_{nuc} = (4\pi/3)(145.18 \times 10^{-8} \text{ cm})^3 = 1.282 \times 10^{-17} \text{ cm}^3$

number of unit cells =  $1.282 \times 10^{-17} / 45.118 \times 10^{-24} = 2.84 \times 10^5$

atoms per nucleus =  $(4 \text{ atoms/cells})(2.84 \times 10^5 \text{ cell}) = 1.136 \times 10^6$

- 8-13** Suppose that solid iron was able to nucleate homogeneously with an undercooling of only 15°C. How many atoms would have to group together spontaneously for this to occur? Assume that the lattice parameter of the solid BCC iron is 2.92 Å.

Solution:  $r^* = \frac{(2)(204 \times 10^{-7} \text{ J/cm}^2)(1538 + 273)}{(1737 \text{ J/cm}^3)(15)} = 283.6 \times 10^{-8} \text{ cm}$

$V_{uc} = 24.897 \times 10^{-24} \text{ cm}^3$  (see Problem 8-10)

$V_{nuc} = (4\pi/3)(283.6 \times 10^{-8} \text{ cm})^3 = 95,544,850 \times 10^{-24} \text{ cm}^3$

number of unit cells =  $95,544,850 / 24.897 = 3.838 \times 10^6$

atoms per nucleus =  $(2 \text{ atoms/cells})(3.838 \times 10^6 \text{ cell}) = 7.676 \times 10^6$

- 8-14** Calculate the fraction of solidification that occurs dendritically when iron nucleates (a) at 10°C undercooling, (b) at 100°C undercooling, and (c) homogeneously. The specific heat of iron is 5.78 J/cm<sup>3</sup>·°C.

Solution:  $f = \frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \text{ J/cm}^3 \cdot ^\circ\text{C})(10^\circ\text{C})}{1737 \text{ J/cm}^3} = 0.0333$

$\frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \text{ J/cm}^3 \cdot ^\circ\text{C})(100^\circ\text{C})}{1737 \text{ J/cm}^3} = 0.333$

$\frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \text{ J/cm}^3 \cdot ^\circ\text{C})(420^\circ\text{C})}{1737 \text{ J/cm}^3}$ , therefore, all dendritically

- 8-28** Calculate the fraction of solidification that occurs dendritically when silver nucleates (a) at 10°C undercooling, (b) at 100°C undercooling, and (c) homogeneously. The specific heat of silver is 3.25 J/cm<sup>3</sup>·°C.

Solution:  $f = \frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \text{ J/cm}^3 \cdot ^\circ\text{C})(10^\circ\text{C})}{965 \text{ J/cm}^3} = 0.0337$

$\frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \text{ J/cm}^3 \cdot ^\circ\text{C})(100^\circ\text{C})}{965 \text{ J/cm}^3} = 0.337$

$\frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \text{ J/cm}^3 \cdot ^\circ\text{C})(250^\circ\text{C})}{965 \text{ J/cm}^3} = 0.842$

- 8-29** Analysis of a nickel casting suggests that 28% of the solidification process occurred in a dendritic manner. Calculate the temperature at which nucleation occurred. The specific heat of nickel is 4.1 J/cm<sup>3</sup>·°C.

Solution:  $f = \frac{c\Delta T}{\Delta H_f} = \frac{(4.1 \text{ J/cm}^3 \cdot ^\circ\text{C})(\Delta T)}{2756 \text{ J/cm}^3} = 0.28$

$\Delta T = 188^\circ\text{C}$  or  $T_n = 1453 - 188 = 1265^\circ\text{C}$

- 8-31** A 2-in. cube solidifies in 4.6 min. Calculate (a) the mold constant in Chvorinov's rule and (b) the solidification time for a 0.5 in.  $\times$  0.5 in.  $\times$  6 in. bar cast under the same conditions. Assume that  $n = 2$ .

**Solution:** (a) We can find the volume and surface area of the cube:

$$V = (2)^3 = 8 \text{ in.}^3 \quad A = 6(2)^2 = 24 \text{ in.}^2 \quad t = 4.6 = B(8/24)^2$$

$$B = 4.6/(0.333)^2 = 41.48 \text{ min/in.}^2$$

(b) For the bar, assuming that  $B = 41.48 \text{ min/in.}^2$ :

$$V = (0.5)(0.5)(6) = 1.5 \text{ in.}^2$$

$$A = 2(0.5)(0.5) + 4(0.5)(6) = 12.5 \text{ in.}^2$$

$$t = (41.48)(1.5/12.5)^2 = 0.60 \text{ min}$$

- 8-32** A 5-cm diameter sphere solidifies in 1050 s. Calculate the solidification time for a 0.3 cm  $\times$  10 cm  $\times$  20 cm plate cast under the same conditions. Assume that  $n = 2$ .

**Solution:**  $t = 1050 \text{ s} = B \left[ \frac{(4\pi/3)(2.5)^3}{4\pi(2.5)^2} \right]^2 = B[2.5/3]^2$  or  $B = 1512 \text{ s/cm}^2$

$$t = \frac{(1512)(0.3 \times 10 \times 20)^2}{[2(0.3)(10) + 2(0.3)(20) + 2(10)(20)]^2} = 1512[60/418]^2 = 31.15 \text{ s}$$

- 8-33** Find the constants  $B$  and  $n$  in Chvorinov's rule by plotting the following data on a log-log plot:

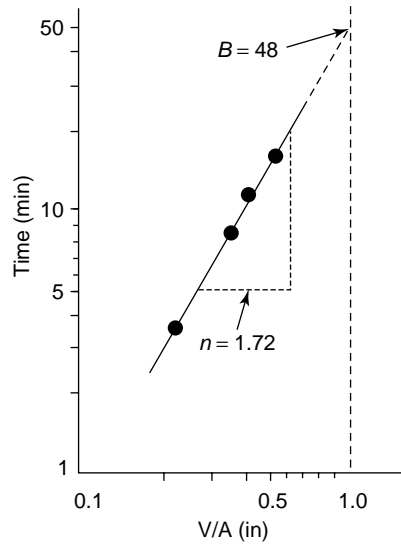
Casting dimensions (in.)	Solidification time (min)
0.5 $\times$ 8 $\times$ 12	3.48
2 $\times$ 3 $\times$ 10	15.78
2.5 cube	10.17
1 $\times$ 4 $\times$ 9	8.13

**Solution:**

V(in. <sup>3</sup> )	A(in. <sup>2</sup> )	V/A (in.)
48	212	0.226
60	112	0.536
15.6	37.5	0.416
36	98	0.367

From the graph, we find that

$$B = 48 \text{ min/in.}^2 \text{ and } n = 1.72$$



- 8-34** Find the constants  $B$  and  $n$  in Chvorinov's rule by plotting the following data on a log-log plot.

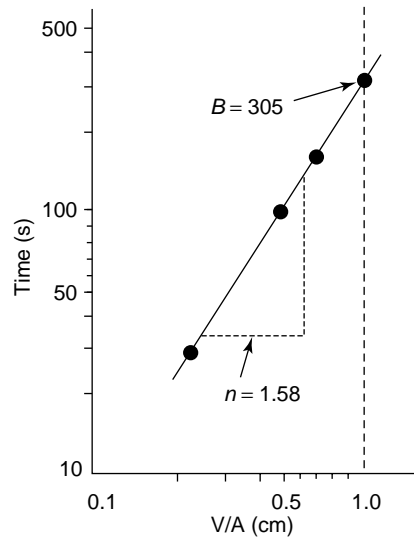
Casting dimensions (cm)	Solidification time (s)
$1 \times 1 \times 6$	28.58
$2 \times 4 \times 4$	98.30
$4 \times 4 \times 4$	155.89
$8 \times 6 \times 5$	306.15

Solution:

V(cm <sup>3</sup> )	A(cm <sup>2</sup> )	V/A (cm)
6	26	0.23
32	64	0.5
64	96	0.67
240	236	1.02

From the graph, we find that

$B = 305 \text{ s/cm}^2$  and  $n = 1.58$ .



- 8-35** A 3-in.-diameter casting was produced. The times required for the solid-liquid interface to reach different distances beneath the casting surface were measured and are shown in the following table.

Distance from surface (in.)	Time (s)	$\sqrt{t}$
0.1	32.6	5.71
0.3	73.5	8.57
0.5	130.6	11.43
0.75	225.0	15.00
1.0	334.9	18.22

Determine (a) the time at which solidification begins at the surface and (b) the time at which the entire casting is expected to be solid. (c) Suppose the center of the casting actually solidified in 720 s. Explain why this time might differ from the time calculated in part (b).

**Solution:** We could plot  $d$  versus  $\sqrt{t}$ , as shown, finding  $\sqrt{t_{\text{surface}}}$  from where the plot intersects the  $x$ -axis and  $\sqrt{t_{\text{center}}}$  where the plot intersects  $d = 1.5$  in. Or we could take two of the data points and solve for  $c$  and  $k$ .

$$d = k\sqrt{t} - c$$

$$0.1 = k\sqrt{32.6} - c$$

$$0.5 = k\sqrt{130.6} - c$$

$$-0.4 = k[\sqrt{32.6} - \sqrt{130.6}] = -5.718k$$

$$k = 0.070$$

$$c = 0.070\sqrt{32.6} - 0.1 = 0.30$$

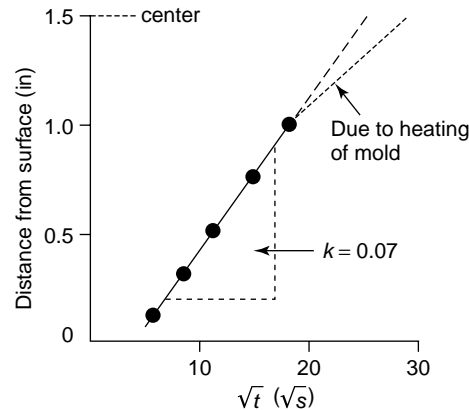
$$(a) \quad d = 0 = 0.070\sqrt{t} - 0.30$$

$$t_{\text{surface}} = (0.3/0.07)^2 = 18.4 \text{ s}$$

$$(b) 1.5 = 0.070\sqrt{t} - 0.3$$

$$t_{\text{center}} = (1.8/0.07)^2 = 661 \text{ s}$$

- (c) The mold gets hot during the solidification process, and consequently heat is extracted from the casting more slowly. This in turn changes the constants in the equation and increases the time required for complete solidification.



- 8-36** Figure 8-9(b) shows a photograph of an aluminum alloy. Estimate (a) the secondary dendrite arm spacing and (b) the local solidification time for that area of the casting.

**Solution:** (a) The distance between adjacent dendrite arms can be measured. Although most people doing these measurements will arrive at slightly different numbers, the author's calculations obtained from four different primary arms are:

$$16 \text{ mm} / 6 \text{ arms} = 2.67 \text{ mm}$$

$$9 \text{ mm} / 5 \text{ arms} = 1.80 \text{ mm}$$

$$13 \text{ mm} / 7 \text{ arms} = 1.85 \text{ mm}$$

$$18 \text{ mm} / 9 \text{ rms} = 2.00 \text{ mm}$$

$$\text{average} = 2.08 \text{ mm} = 0.208 \text{ cm}$$

Dividing by the magnification of  $\times 50$ :

$$\text{SDAS} = 0.208 \text{ cm} / 50 = 4.16 \times 10^{-3} \text{ cm}$$

(b) From Figure 8-10, we find that local solidification time (LST) = 90 s

- 8-37** Figure 8-31 shows a photograph of FeO dendrites that have precipitated from a glass (an undercooled liquid). Estimate the secondary dendrite arm spacing.

**Solution:** We can find 13 SDAS along a 3.5 cm distance on the photomicrograph. The magnification of the photomicrograph is  $\times 450$ , while we want the actual length (at magnification  $\times 1$ ). Thus:

$$\text{SDAS} = (13 \text{ SDAS} / 3.5 \text{ cm})(1/450) = 8.25 \times 10^{-3} \text{ cm}$$

- 8-38** Find the constants  $c$  and  $m$  relating the secondary dendrite arm spacing to the local solidification time by plotting the following data on a log-log plot:

Solidification Time (s)	SDAS (cm)
156	0.0176
282	0.0216
606	0.0282
1356	0.0374

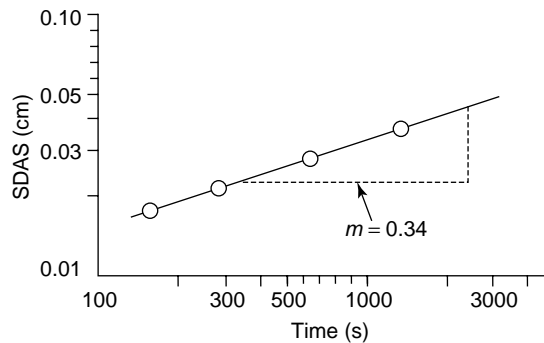
**Solution:** From the slope of the graph:

$$m = 34/100 = 0.34$$

We can then pick a point off the graph (say SDAS = 0.0225 cm when LST = 300 s) and calculate “c”:

$$0.0225 = c(300)^{0.34} = 6.954c$$

$$c = 0.0032$$



**8-39** Figure 8-32 shows dendrites in a titanium powder particle that has been rapidly solidified. Assuming that the size of the titanium dendrites is related to solidification time by the same relationship as in aluminum, estimate the solidification time of the powder particle.

**Solution:** The secondary dendrite arm spacing can be estimated from the photomicrograph at several locations. The author’s calculations, derived from measurements at three locations, are

$$11 \text{ mm} / 8 \text{ arms} = 1.375 \text{ mm}$$

$$13 \text{ mm} / 8 \text{ arms} = 1.625 \text{ mm}$$

$$13 \text{ mm} / 8 \text{ arms} = 1.625 \text{ mm}$$

$$\text{average} = 1.540 \text{ mm}$$

Dividing by the magnification of 2200:

$$\text{SDAS} = (1.540 \text{ mm})(0.1 \text{ cm/mm}) / 2200 = 7 \times 10^{-5} \text{ cm}$$

The relationship between SDAS and solidification time for aluminum is:

$$\text{SDAS} = 8 \times 10^{-4} t^{0.42} = 7 \times 10^{-5}$$

$$t = (0.0875)^{1/0.42} = 0.003 \text{ s}$$

- 8-40** The secondary dendrite arm spacing in an electron beam weld of copper is  $9.5 \times 10^{-4}$  cm. Estimate the solidification time of the weld.

**Solution:** From Figure 8-10, we can determine the equation relating SDAS and solidification time for copper:

$$n = 19/50 = 0.38 \quad c = 4 \times 10^{-3} \text{ cm}$$

Then for the copper weld:

$$9.5 \times 10^{-4} = 4 \times 10^{-3}(\text{LST})^{0.38}$$

(Note: LST is local solidification time)

$$0.2375 = (\text{LST})^{0.38} \quad \text{or} \quad -1.438 = 0.38 \ln \text{LST}$$

$$\ln \text{LST} = -3.783 \quad \text{or} \quad \text{LST} = 0.023 \text{ s}$$

- 8-45** A cooling curve is shown in Figure 8-33. Determine (a) the pouring temperature, (b) the solidification temperature, (c) the superheat, (d) the cooling rate just before solidification begins, (e) the total solidification time, (f) the local solidification time, and (g) the probable identity of the metal. (h) If the cooling curve was obtained at the center of the casting sketched in the figure, determine the mold constant, assuming that  $n = 2$ .

**Solution:**

(a) $T_{\text{pour}} = 475^\circ\text{C}$	(e) $t_s = 470 \text{ s}$
(b) $T_{\text{sol}} = 320^\circ\text{C}$	(f) $\text{LST} = 470 - 130 = 340 \text{ s}$
(c) $\Delta T_s = 475 - 320 = 155^\circ\text{C}$	(g) Cadmium (Cd)
(d) $\Delta T/\Delta t = \frac{475 - 320}{130 - 0} = 1.2^\circ\text{C/s}$	(h) $t_s = 470 = B[38.4/121.6]^2$ $B = 4713 \text{ s/cm}^2$

- 8-46** A cooling curve is shown in Figure 8-34. Determine (a) the pouring temperature, (b) the solidification temperature, (c) the superheat, (d) the cooling rate just before solidification begins, (e) the total solidification time, (f) the local solidification time, (g) the undercooling, and (h) the probable identity of the metal. (i) If the cooling curve was obtained at the center of the casting sketched in the figure, determine the mold constant, assuming  $n = 2$ .

**Solution:**

(a) $T_{\text{pour}} = 900^\circ\text{C}$	(e) $t_s = 9.7 \text{ min}$
(b) $T_{\text{sol}} = 420^\circ\text{C}$	(f) $\text{LST} = 9.7 - 1.6 = 8.1 \text{ min}$
(c) $\Delta T_s = 900 - 420 = 480^\circ\text{C}$	(g) $420 - 360 = 60^\circ\text{C}$
(d) $\Delta T/\Delta t = \frac{900 - 400}{1.6 - 0} = 312^\circ\text{C/min}$	(h) Zn
(i) $t_s = 9.7 = B[8/24]^2$	or $B = 87.5 \text{ min/in.}^2$

- 8-47** Figure 8-35 shows the cooling curves obtained from several locations within a cylindrical aluminum casting. Determine the local solidification times and the SDAS at each location, then plot the tensile strength versus distance from the casting surface. Would you recommend that the casting be designed so that a large or small amount of material must be machined from the surface during finishing? Explain.

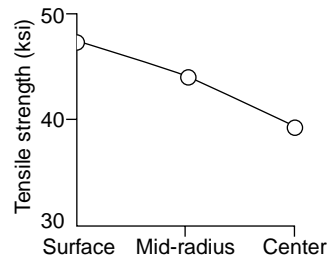
**Solution:** The local solidification times can be found from the cooling curves and can be used to find the expected SDAS values from Figure 8-10. The SDAS values can then be used to find the tensile strength, using Figure 8-11.

Surface:  $LST = 10 \text{ s} \Rightarrow SDAS = 1.5 \times 10^{-3} \text{ cm} \Rightarrow TS = 47 \text{ ksi}$

Midradius:  $LST = 100 \text{ s} \Rightarrow SDAS = 5 \times 10^{-3} \text{ cm} \Rightarrow TS = 44 \text{ ksi}$

Center:  $LST = 500 \text{ s} \Rightarrow SDAS = 10 \times 10^{-3} \text{ cm} \Rightarrow TS = 39.5 \text{ ksi}$

You prefer to machine as little material off the surface of the casting as possible; the surface material has the finest structure and highest strength; any excessive machining simply removes the “best” material.



- 8-48** Calculate the volume, diameter, and height of the cylindrical riser required to prevent shrinkage in a 4 in.  $\times$  10 in.  $\times$  20 in. casting if the  $H/D$  of the riser is 1.5.

**Solution:**

$$(V/A)_c = \frac{(4)(10)(20)}{2(4)(10) + 2(4)(20) + 2(10)(20)} = 800/640 = 1.25$$

$$(V/A)_r = \frac{(\pi/4)D^2H}{2(\pi/4)D^2 + \pi DH} = \frac{(\pi/4)(3/2)D^3}{(\pi/2)D^2 + (3\pi/2)D^2} = \frac{3D/8}{2} = 3D/16 \geq 1.25$$

$$D \geq 6.67 \text{ in.} \quad H \geq 10 \text{ in.} \quad V \geq 349 \text{ in.}^3$$

- 8-55** Calculate the volume, diameter, and height of the cylindrical riser required to prevent shrinkage in a 1 in.  $\times$  6 in.  $\times$  6 in. casting if the  $H/D$  of the riser is 1.0.

**Solution:**

$$V = (1)(6)(6) = 36 \text{ in.}^3 \quad A = 2(1)(6) + 2(1)(6) + 2(6)(6) = 96 \text{ in.}^2$$

$$(V/A)_c = 36/96 = 0.375$$

$$(V/A)_r = \frac{(\pi/4)D^2H}{2(\pi/4)D^2 + \pi DH} = \frac{(\pi/4)D^3}{(3\pi/2)D^2} = D/6 \geq 0.375$$

$$D \geq 2.25 \text{ in.} \quad H \geq 2.25 \text{ in.} \quad V \geq 8.95 \text{ in.}^3$$

- 8-56** Figure 8-36 shows a cylindrical riser attached to a casting. Compare the solidification times for each casting section and the riser and determine whether the riser will be effective.

**Solution:**

$$(V/A)_{\text{thin}} = \frac{(8)(6)(3)}{(3)(6) + 2(3)(8) + 2(6)(8)} = 0.889$$

$$(V/A)_{\text{thick}} = \frac{(6)(6)(6)}{(6)(3) + 5(6)(6) - (\pi/4)(3)^2} = 1.13$$

$$(V/A)_{\text{riser}} = \frac{(\pi/4)(3)^2(7)}{\pi(3)(7) + (\pi/4)(3)^2} = 0.68$$

Note that the riser area in contact with the casting is not included in either the riser or casting surface area; no heat is lost across this



interface. In a like manner, the area of contact between the thick and thin portions of the casting are not included in the calculation of the casting area.

The riser will not be effective; the thick section of the casting has the largest  $V/A$  ratio and therefore requires the longest solidification time. Consequently the riser will be completely solid before the thick section is solidified; no liquid metal will be available to compensate for the solidification shrinkage.

- 8-57** Figure 8-37 shows a cylindrical riser attached to a casting. Compare the solidification times for each casting section and the riser and determine whether the riser will be effective.

**Solution:**

$$(V/A)_{\text{thick}} = \frac{(4)(4)(4)}{5(4)(4) + 1(2)(4)} = 0.73$$

$$(V/A)_{\text{thin}} = \frac{(2)(2)(4)}{3(2)(4) + 2(2)(2)} = 0.50$$

$$(V/A)_R = \frac{(\pi/4)(4^2)(8)}{\pi(4)(8) + 2(\pi/4)4^2} = 0.8$$

The area between the thick and thin sections of the casting are not included in calculating casting area; no heat is lost across this interface.

The riser will not be effective; the thin section has the smallest  $V/A$  ratio and therefore freezes first. Even though the riser has the longest solidification time, the thin section isolates the thick section from the riser, preventing liquid metal from feeding from the riser to the thick section. Shrinkage will occur in the thick section.

- 8-58** A 4-in.-diameter sphere of liquid copper is allowed to solidify, producing a spherical shrinkage cavity in the center of the casting. Compare the volume and diameter of the shrinkage cavity in the copper casting to that obtained when a 4-in. sphere of liquid iron is allowed to solidify.

**Solution:** Cu: 5.1% Fe: 3.4%  $r_{\text{sphere}} = 4/2 = 2$  in.

$$\text{Cu: } V_{\text{shrinkage}} = (4\pi/3)(2)^3 (0.051) = 1.709 \text{ in.}^3$$

$$(4\pi/3)r^3 = 1.709 \text{ in.}^3 \quad \text{or} \quad r = 0.742 \text{ in.} \quad d_{\text{pore}} = 1.48 \text{ in.}$$

$$\text{Fe: } V_{\text{shrinkage}} = (4\pi/3)(2)^3 (0.034) = 1.139 \text{ in.}^3$$

$$(4\pi/3)r^3 = 1.139 \text{ in.}^3 \quad \text{or} \quad r = 0.648 \text{ in.}$$

$$d_{\text{cavity}} = 1.30 \text{ in.}$$

- 8-59** A 4-in. cube of a liquid metal is allowed to solidify. A spherical shrinkage cavity with a diameter of 1.49 in. is observed in the solid casting. Determine the percent volume change that occurs during solidification.

**Solution:**

$$V_{\text{liquid}} = (4 \text{ in.})^3 = 64 \text{ in.}^3$$

$$V_{\text{shrinkage}} = (4\pi/3)(1.49/2)^3 = 1.732 \text{ in.}^3$$

$$V_{\text{solid}} = 64 - 1.732 = 62.268 \text{ in.}^3$$

$$\% \text{ Volume change} = \frac{64 - 62.268}{64} \times 100 = 2.7\%$$

- 8-60** A 2 cm × 4 cm × 6 cm magnesium casting is produced. After cooling to room temperature, the casting is found to weigh 80 g. Determine (a) the volume of the shrinkage cavity at the center of the casting and (b) the percent shrinkage that must have occurred during solidification.

**Solution:** The density of the magnesium is 1.738 g/cm<sup>3</sup>

$$(a) V_{\text{initial}} = (2)(4)(6) = 48 \text{ cm}^3$$

$$V_{\text{final}} = 80 \text{ g} / 1.738 \text{ g/cm}^3 = 46.03 \text{ cm}^3$$

$$(b) \% \text{shrinkage} = \frac{48 - 46.03}{48} \times 100\% = 4.1\%$$

- 8-61** A 2 in. × 8 in. × 10 in. iron casting is produced and, after cooling to room temperature, is found to weigh 43.9 lb. Determine (a) the percent shrinkage that must have occurred during solidification and (b) the number of shrinkage pores in the casting if all of the shrinkage occurs as pores with a diameter of 0.05 in.

**Solution:** The density of the iron is 7.87 g/cm<sup>3</sup>

$$(a) V_{\text{actual}} = \frac{(43.9 \text{ lb})(454 \text{ g})}{7.87 \text{ g/cm}^3} = 2532.5 \text{ cm}^3$$

$$V_{\text{intended}} = (2)(8)(10) = 160 \text{ in.}^3 \times (2.54 \text{ cm/in.})^3 = 2621.9 \text{ cm}^3$$

$$\text{shrinkage} = \frac{2621.9 - 2532.5}{2621.9} \times 100\% = 3.4\%$$

$$(b) V_{\text{pores}} = 2621.9 - 2532.5 = 89.4 \text{ cm}^3$$

$$r_{\text{pores}} = (0.05 \text{ in.}/2)(2.54 \text{ cm/in.}) = 0.0635 \text{ cm}$$

$$\# \text{ pores} = \frac{89.4 \text{ cm}^3}{(\pi/3)(0.0635 \text{ cm})^3} = 83,354 \text{ pores}$$

- 8-65** Liquid magnesium is poured into a 2 cm × 2 cm × 24 cm mold and, as a result of directional solidification, all of the solidification shrinkage occurs along the length of the casting. Determine the length of the casting immediately after solidification is completed.

**Solution:**  $V_{\text{initial}} = (2)(2)(24) = 96 \text{ cm}^3$

$$\% \text{ contraction} = 4 \quad \text{or} \quad 0.04 \times 96 = 3.84 \text{ cm}^3$$

$$V_{\text{final}} = 96 - 3.84 = 92.16 \text{ cm}^3 = (2)(2)(L)$$

$$\text{Length (L)} = 23.04 \text{ cm}$$

- 8-66** A liquid cast iron has a density of 7.65 g/cm<sup>3</sup>. Immediately after solidification, the density of the solid cast iron is found to be 7.71 g/cm<sup>3</sup>. Determine the percent volume change that occurs during solidification. Does the cast iron expand or contract during solidification?

$$\text{Solution:} \quad \frac{1/7.65 - 1/7.71}{1/7.65} \times 100\% = \frac{0.1307 \text{ cm}^3 - 0.1297 \text{ cm}^3}{0.1307 \text{ cm}^3} \times 100\% = 0.77\%$$

The casting contracts.

- 8-67** From Figure 8-18, find the solubility of hydrogen in liquid aluminum just before solidification begins when the partial pressure of hydrogen is 1 atm. Determine the

solubility of hydrogen (in  $\text{cm}^3/100 \text{ g Al}$ ) at the same temperature if the partial pressure were reduced to 0.01 atm.

**Solution:**  $0.46 \text{ cm}^3 \text{ H}_2/100 \text{ g Aluminum}$

$$0.46/x = \frac{\sqrt{1}}{\sqrt{0.01}}$$

$$x = 0.46\sqrt{0.01} = 0.046 \text{ cm}^3/100 \text{ g AL}$$

**8-68** The solubility of hydrogen in liquid aluminum at  $715^\circ\text{C}$  is found to be  $1 \text{ cm}^3/100 \text{ g Al}$ . If all of this hydrogen precipitated as gas bubbles during solidification and remained trapped in the casting, calculate the volume percent gas in the solid aluminum.

**Solution:**  $(1 \text{ cm}^3 \text{ H}_2/100 \text{ g Al})(2.699 \text{ g/cm}^3) = 0.02699 \text{ cm}^3 \text{ H}_2/\text{cm}^3 \text{ Al} = 2.699\%$

# 9

## Solid Solutions and Phase Equilibrium

- 9–15** The unary phase diagram for  $\text{SiO}_2$  is shown in Figure 9–3(c). Locate the triple point where solid, liquid, and vapor coexist and give the temperature and the type of solid present. What do the other “triple” points indicate?

**Solution:** (a) The solid-liquid-vapor triple point occurs at  $1713^\circ\text{C}$ ; the solid phase present at this point is  $\beta$ -cristobalite.

(b) The other triple points describe the equilibrium between two solids and a vapor phase.

- 9–34** Based on Hume-Rothery’s conditions, which of the following systems would be expected to display unlimited solid solubility? Explain.

- (a) Au–Ag      (b) Al–Cu      (c) Al–Au      (d) U–W  
(e) Mo–Ta      (f) Nb–W      (g) Mg–Zn      (h) Mg–Cd

**Solution:**

(a)	$r_{\text{Au}} = 1.442$	$v = +1$	FCC
	$r_{\text{Ag}} = 1.445$	$v = +1$	FCC
	$\Delta r = 0.2\%$		Yes
(b)	$r_{\text{Al}} = 1.432$	$v = +3$	FCC
	$r_{\text{Cu}} = 1.278$	$v = +1$	FCC
	$\Delta r = 10.7\%$		No
(c)	$r_{\text{Al}} = 1.432$	$v = +3$	FCC
	$r_{\text{Au}} = 1.442$	$v = +1$	FCC
	$\Delta r = 0.7\%$		No
(d)	$r_{\text{U}} = 1.38$	$v = +4$	Ortho
	$r_{\text{W}} = 1.371$	$v = +4$	FCC
	$\Delta r = 0.7\%$		No

- (e)  $r_{\text{Mo}} = 1.363$   $v = +4$  BCC  
 $r_{\text{Ta}} = 1.43$   $v = +5$  BCC  
 $\Delta r = 4.7\%$  No
- (f)  $r_{\text{Nb}} = 1.426$   $v = +4$  BCC  
 $r_{\text{W}} = 1.371$   $v = +4$  BCC  
 $\Delta r = 3.9\%$  Yes
- (g)  $r_{\text{Mg}} = 1.604$   $v = +2$  HCP  
 $r_{\text{Zn}} = 1.332$   $v = +2$  HCP  
 $\Delta r = 17\%$  No
- (h)  $r_{\text{Mg}} = 1.604$   $v = +2$  HCP  
 $r_{\text{Cd}} = 1.490$   $v = +2$  HCP  
 $\Delta r = 7.1\%$  Yes

The Au–Ag, Mo–Ta, and Mg–Cd systems have the required radius ratio, the same crystal structures, and the same valences. Each of these might be expected to display complete solid solubility. [The Au–Ag and Mo–Ta do have isomorphous phase diagrams. In addition, the Mg–Cd alloys all solidify like isomorphous alloys; however a number of solid state phase transformations complicate the diagram.]

- 9–35** Suppose 1 at% of the following elements is added to copper (forming a separate alloy with each element) without exceeding the solubility limit. Which one would be expected to give the higher strength alloy? Is any of the alloying elements expected to have unlimited solid solubility in copper?

(a) Au    (b) Mn    (c) Sr    (d) Si    (e) Co

Solution: For copper:  $r_{\text{Cu}} = 1.278 \text{ \AA}$

- |                     |  |                              |
|---------------------|--|------------------------------|
| (a) Au: $r = 1.442$ | $\Delta r = \frac{r_{\text{Au}} - r_{\text{Cu}}}{r_{\text{Cu}}} = +12.8\%$ | May be Unlimited Solubility. |
| (b) Mn: $r = 1.12$  | $\Delta r = -12.4\%$   | Different structure.         |
| (c) Sr: $r = 2.151$ | $\Delta r = +68.3\%$   | Highest Strength             |
| (d) Si: $r = 1.176$ | $\Delta r = -8.0\%$  | Different structure.         |
| (e) Co: $r = 1.253$ | $\Delta r = -2.0\%$  | Different structure.         |

The Cu–Sr alloy would be expected to be strongest (largest size difference). The Cu–Au alloy satisfies Hume-Rothery's conditions and might be expected to display complete solid solubility—in fact it freezes like an isomorphous series of alloys, but a number of solid state transformations occur at lower temperatures.

- 9–36** Suppose 1 at% of the following elements is added to aluminum (forming a separate alloy with each element) without exceeding the solubility limit. Which one would be expected to give the least reduction in electrical conductivity? Is any of the alloy elements expected to have unlimited solid solubility in aluminum?

(a) Li    (b) Ba    (c) Be    (d) Cd    (e) Ga

**Solution:** For aluminum:  $r = 1.432 \text{ \AA}$  (FCC structure with valence of 3)

- |                     |                      |              |             |
|---------------------|----------------------|--------------|-------------|
| (a) Li: $r = 1.519$ | $\Delta r = 6.1\%$   | BCC          | valence = 1 |
| (b) Ba: $r = 2.176$ | $\Delta r = -52.0\%$ | BCC          | valence = 2 |
| (c) Be: $r = 1.143$ | $\Delta r = -20.2\%$ | HCP          | valence = 2 |
| (d) Cd: $r = 1.49$  | $\Delta r = 4.1\%$   | HCP          | valence = 2 |
| (e) Ga: $r = 1.218$ | $\Delta r = 14.9\%$  | Orthorhombic | valence = 3 |

The cadmium would be expected to give the smallest reduction in electrical conductivity, since the Cd atoms are most similar in size to the aluminum atoms.

None are expected to have unlimited solid solubility, due either to difference in valence, atomic radius or crystal structure.

**9–37** Which of the following oxides is expected to have the largest solid solubility in  $\text{Al}_2\text{O}_3$ ?

- (a)  $\text{Y}_2\text{O}_3$       (b)  $\text{Cr}_2\text{O}_3$       (c)  $\text{Fe}_2\text{O}_3$

**Solution:** The ionic radius of  $\text{Al}^{3+} = 0.51 \text{ \AA}$

- |                                 |   |
|---------------------------------|---|
| (a) $r_{\text{Y}^{3+}} = 0.89$  | $\Delta r = \frac{0.63 - 0.51}{0.51} \times 100 = 74.5\%$ |
| (b) $r_{\text{Cr}^{3+}} = 0.63$ | $\Delta r = 23.5\%$                                       |
| (c) $r_{\text{Fe}^{3+}} = 0.64$ | $\Delta r = 25.5\%$                                       |

We would expect  $\text{Cr}_2\text{O}_3$  to have a high solubility in  $\text{Al}_2\text{O}_3$ ; in fact, they are completely soluble in one another.

**9–41** Determine the liquidus temperature, solidus temperature, and freezing range for the following NiO–MgO ceramic compositions. [See Figure 9–10(b).]

- (a) NiO–30 mol% MgO      (b) NiO–45 mol% MgO  
(c) NiO–60 mol% MgO      (d) NiO–85 mol% MgO

- Solution:**
- |                                |                            |                          |
|--------------------------------|----------------------------|--------------------------|
| (a) $T_L = 2330^\circ\text{C}$ | $T_S = 2150^\circ\text{C}$ | FR = $180^\circ\text{C}$ |
| (b) $T_L = 2460^\circ\text{C}$ | $T_S = 2250^\circ\text{C}$ | FR = $210^\circ\text{C}$ |
| (c) $T_L = 2570^\circ\text{C}$ | $T_S = 2380^\circ\text{C}$ | FR = $190^\circ\text{C}$ |
| (d) $T_L = 2720^\circ\text{C}$ | $T_S = 2610^\circ\text{C}$ | FR = $110^\circ\text{C}$ |

**9–42** Determine the liquidus temperature, solidus temperature, and freezing range for the following MgO–FeO ceramic compositions. (See Figure 9–21.)

- (a) MgO–25 wt% FeO      (b) MgO–45 wt% FeO  
(c) MgO–65 wt% FeO      (d) MgO–80 wt% FeO

Solution: (a)  $T_L = 2600^\circ\text{C}$      $T_S = 2230^\circ\text{C}$      $\text{FR} = 370^\circ\text{C}$   
 (b)  $T_L = 2340^\circ\text{C}$      $T_S = 1900^\circ\text{C}$      $\text{FR} = 440^\circ\text{C}$   
 (c)  $T_L = 2000^\circ\text{C}$      $T_S = 1610^\circ\text{C}$      $\text{FR} = 390^\circ\text{C}$   
 (d)  $T_L = 1750^\circ\text{C}$      $T_S = 1480^\circ\text{C}$      $\text{FR} = 270^\circ\text{C}$

**9–43** Determine the phases present, the compositions of each phase, and the amount of each phase in mol% for the following NiO–MgO ceramics at  $2400^\circ\text{C}$ . [See Figure 9–10(b).]

- (a) NiO–30 mol% MgO    (b) NiO–45 mol% MgO  
 (c) NiO–60 mol% MgO    (d) NiO–85 mol% MgO

Solution: (a)  $L$ : NiO–30 mol% MgO    100%  $L$   
 (b)  $L$ : 38% MgO     $\%L = \frac{62 - 45}{62 - 38} \times 100\% = 70.8\%$   
        $S$ : 62% MgO     $\%L = \frac{45 - 38}{62 - 38} \times 100\% = 29.2\%$   
 (c)  $L$ : 38% MgO     $\%L = \frac{62 - 60}{62 - 38} \times 100\% = 8.3\%$   
        $S$ : 62% MgO     $\%L = \frac{60 - 38}{62 - 38} \times 100\% = 91.7\%$   
 (d)  $S$ : 85% MgO    100%  $S$

**9–44(a)** Determine the phases present, the compositions of each phase, and the amount of each phase in wt% for the following MgO–FeO ceramics at  $2000^\circ\text{C}$ . (See Figure 9–21.)

- (a) MgO–25 wt% FeO    (b) MgO–45 wt% FeO  
 (c) MgO–60 wt% FeO    (d) MgO–80 wt% FeO

Solution: (a)  $S$ : 25% FeO    100%  $S$   
 (b)  $S$ : 39% FeO     $\%S = \frac{65 - 45}{65 - 39} \times 100\% = 76.9\%$   
        $L$ : 65% FeO     $\%L = \frac{45 - 39}{65 - 39} \times 100\% = 23.1\%$   
 (c)  $S$ : 39% FeO     $\%S = \frac{65 - 60}{65 - 39} \times 100\% = 19.2\%$   
        $L$ : 65% MgO     $\%L = \frac{60 - 39}{65 - 39} \times 100\% = 80.8\%$   
 (d)  $S$ : 80% MgO    100%  $L$

- 9–44(b)** Consider an alloy of 65 wt% Cu and 35 wt% Al. Calculate the composition of the alloy in at%.

**Solution:**

$$\text{At\% Cu} = \frac{65/63.54}{(65/63.54) + (35/26.981)} \times 100\% = 44.1\%$$

$$\text{At\% Al} = \frac{35/26.981}{(65/63.54) + (35/26.981)} \times 100\% = 55.9\%$$

- 9–45** Consider a ceramic composed of 30 mol% MgO and 70 mol% FeO. Calculate the composition of the ceramic in wt%.

**Solution:**

$$\text{MW}_{\text{MgO}} = 24.312 + 16 = 40.312 \text{ g/mol}$$

$$\text{MW}_{\text{FeO}} = 55.847 + 16 = 71.847 \text{ g/mol}$$

$$\text{wt\% MgO} = \frac{(30)(40.312)}{(30)(40.312) + (70)(71.847)} \times 100\% = 19.4\%$$

$$\text{wt\% FeO} = \frac{(70)(71.847)}{(30)(40.312) + (70)(71.847)} \times 100\% = 80.6\%$$

- 9–46** A NiO–20 mol% MgO ceramic is heated to 2200°C. Determine (a) the composition of the solid and liquid phases in both mol% and wt% and (b) the amount of each phase in both mol% and wt%. (c) assuming that the density of the solid is 6.32 g/cm<sup>3</sup> and that of the liquid is 7.14 g/cm<sup>3</sup>, determine the amount of each phase in vol% (see Figure 9–10(b)).

**Solution:**

$$\text{MW}_{\text{MgO}} = 24.312 + 16 = 40.312 \text{ g/mol}$$

$$\text{MW}_{\text{NiO}} = 58.71 + 16 = 74.71 \text{ g/mol}$$

(a) *L*: 15 mol% MgO

$$\text{wt\% MgO} = \frac{(15)(40.312)}{(15)(40.312) + (85)(74.71)} \times 100\% = 8.69\%$$

*S*: 38 mol% MgO

$$\text{wt\% MgO} = \frac{(38)(40.312)}{(38)(40.312) + (62)(74.71)} \times 100\% = 24.85\%$$

(b)  $\text{mol\% } L = \frac{38 - 20}{38 - 15} \times 100\% = 78.26\% \quad \text{mol\% } S = 21.74\%$

The original composition, in wt% MgO, is:

$$\frac{(20)(40.312)}{(20)(40.312) + (80)(74.71)} \times 100\% = 11.9\%$$

$$\text{wt\% } L = \frac{24.85 - 11.9}{24.85 - 8.69} \times 100\% = 80.1\% \quad \text{wt\% } S = 19.9\%$$

(c)  $\text{Vol\% } L = \frac{80.1/7.14}{(80.1/7.14) + (19.9/6.32)} \times 100\% = 78.1\%$

$$\text{Vol\% } S = 21.9\%$$



- 9-47** A Nb–60 wt% W alloy is heated to 2800°C. Determine (a) the composition of the solid and liquid phases in both wt% and at% and (b) the amount of each phase in both wt% and at%. (c) Assuming that the density of the solid is 16.05 g/cm<sup>3</sup> and that of the liquid is 13.91 g/cm<sup>3</sup>, determine the amount of each phase in vol%. (See Figure 9-22.)

**Solution:** (a)  $L$ : 49 wt% W

$$\text{at\% W} = \frac{49/183.85}{(49/183.85) + (51/92.91)} \times 100\% = 32.7\%$$

$\alpha$ : 70 wt% W

$$\text{at\% W} = \frac{(70/183.85)}{(70/183.85) + (30/92.91)} \times 100\% = 54.1\%$$

$$(b) \text{ wt\% } L = \frac{70 - 60}{70 - 49} \times 100\% = 47.6\% \quad \text{wt\% } \alpha = 52.4\%$$

The original composition, in wt% MgO, is:

$$\frac{60/183.85}{(60/183.85) + (40/92.91)} \times 100\% = 43.1\%$$

$$\text{at\% } L = \frac{54.1 - 43.1}{54.1 - 32.7} \times 100\% = 51.4\% \quad \text{wt\% } \alpha = 48.6\%$$

$$(c) \text{ Vol\% } L = \frac{47.6/13.91}{(47.6/13.91) + (52.4/16.05)} \times 100\% = 51.2\%$$

$$\text{Vol\% } \alpha = 48.8\%$$

- 9-48** How many grams of nickel must be added to 500 grams of copper to produce an alloy that has a liquidus temperature of 1350°C? What is the ratio of the number of nickel atoms to copper atoms in this alloy?

**Solution:** We need 60 wt% Ni to obtain the correct liquidus temperature.

$$\% \text{Ni} = 60 = \frac{x}{x + 500 \text{ g}} \times 100\% \quad \text{or} \quad x = 750 \text{ g Ni}$$

$$\frac{\text{Ni atoms}}{\text{Cu atoms}} = \frac{(750 \text{ g})(N_A)/58.71 \text{ g/mol}}{(500 \text{ g})(N_A)/63.54 \text{ g/mol}} = 1.62$$

- 9-49** How many grams of nickel must be added to 500 grams of copper to produce an alloy that contains 50 wt%  $\alpha$  at 1300°C?

**Solution:** At 1300°C, the composition of the two phases in equilibrium are

$L$ : 46 wt% Ni and  $\alpha$ : 58 wt% Ni

The alloy required to give 50%  $\alpha$  is then

$$\frac{x - 46}{58 - 46} \times 100 = 50\% \alpha \quad \text{or} \quad x = 52 \text{ wt\% Ni}$$

The number of grams of Ni must be:

$$\frac{x}{x + 500} \times 100\% = 52 \quad \text{or} \quad x = 541.7 \text{ g Ni}$$

- 9–50** How many grams of MgO must be added to 1 kg of NiO to produce a ceramic that has a solidus temperature of 2200°C?

**Solution:**  $MW_{\text{MgO}} = 40.312 \text{ g/mol}$       $MW_{\text{NiO}} = 74.71 \text{ g/mol}$

38 mol% MgO is needed to obtain the correct solidus temperature.

$$\text{wt\% MgO} = \frac{(38)(40.312)}{(38)(40.312) + (62)(74.71)} \times 100\% = 24.9\%$$

The number of grams required is:

$$\frac{x}{x + 1000} \times 100\% = 24.9\% \quad \text{or} \quad x = 332 \text{ g of MgO}$$

- 9–51** How many grams of MgO must be added to 1 kg of NiO to produce a ceramic that contains 25 mol% solid at 2400°C?

**Solution:**  $L$ : 38 mol% MgO      $MW_{\text{MgO}} = 40.312 \text{ g/mol}$

$S$ : 62 mol% MgO      $MW_{\text{NiO}} = 74.71 \text{ g/mol}$

$$\frac{x - 38}{62 - 38} \times 100\% = 25\% S \quad \text{or} \quad x = 44 \text{ mol\% MgO}$$

$$\text{wt\% MgO} = \frac{(44)(40.312)}{(44)(40.312) + (56)(74.71)} \times 100\% = 29.77\%$$

The number of grams of MgO is then:

$$\frac{x}{x + 1000} \times 100\% = 29.77\% \quad \text{or} \quad x = 424 \text{ g MgO}$$

- 9–52** We would like to produce a solid MgO–FeO ceramic that contains equal mol percentages of MgO and FeO at 1200°C. Determine the wt% FeO in the ceramic. (See Figure 9–21.)

**Solution:** Only solid is present at 1200°C.      $MW_{\text{MgO}} = 40.312 \text{ g/mol}$

$MW_{\text{FeO}} = 71.847 \text{ g/mol}$

$$50 \text{ mol\% FeO: } \frac{(50)(71.847)}{(50)(40.312) + (50)(71.847)} = 64.1 \text{ wt\% FeO}$$

- 9–53** We would like to produce a MgO–FeO ceramic that is 30 wt% solid at 2000°C. Determine the original composition of the ceramic in wt%. (See Figure 9–21.)

**Solution:**  $L$ : 65 wt% FeO      $S$ : 38 wt% FeO

$$30 \text{ wt\%} = \frac{65 - x}{65 - 38} \times 100\% \quad \text{or} \quad x = 56.9 \text{ wt\% FeO}$$

- 9-54** A Nb–W alloy held at 2800°C is partly liquid and partly solid. (a) If possible, determine the composition of each phase in the alloy; and (b) if possible, determine the amount of each phase in the alloy. (See Figure 9–22.)

**Solution:** (a)  $L$ : 49 wt% W       $\alpha$ : 70 wt% W

(b) Not possible unless we know the original composition of the alloy.

- 9-55** A Nb–W alloy contains 55%  $\alpha$  at 2600°C. Determine (a) the composition of each phase; and (b) the original composition of the alloy. (See Figure 9–22.)

**Solution:** (a)  $L$ : 22 wt% W       $\alpha$ : 42 wt% W

$$(b) 0.55 = \frac{x - 22}{42 - 22} \quad \text{or} \quad x = 33 \text{ wt\% W}$$

- 9-56** Suppose a 1200 lb bath of a Nb–40 wt% W alloy is held at 2800°C. How many pounds of tungsten can be added to the bath before any solid forms? How many pounds of tungsten must be added to cause the entire bath to be solid? (See Figure 9–22.)

**Solution:** Solid starts to form at 2800°C when 49 wt% W is in the alloy. In 1200 lb of the original Nb–40% W alloy, there are  $(0.4)(1200) = 480$  lb W and 720 lb Nb. The total amount of tungsten that must be in the final alloy is:

$$0.49 = \frac{x}{x + 720} \quad \text{or} \quad x = 692 \text{ lb W total}$$

or  $692 - 480 = 212$  additional pounds of W must be added

To be completely solid at 2800°C, the alloy must contain 70 wt% W. The total amount of tungsten required in the final alloy is:

$$0.70 = \frac{x}{x + 720} \quad \text{or} \quad x = 1680 \text{ lb W total}$$

or  $1680 - 480 = 1200$  additional pounds of W must be added

- 9-57** A fiber-reinforced composite material is produced, in which tungsten fibers are embedded in a Nb matrix. The composite is composed of 70 vol% tungsten. (a) Calculate the wt% of tungsten fibers in the composite. (b) Suppose the composite is heated to 2600°C and held for several years. What happens to the fibers? Explain. (See Figure 9–22.)

$$\text{Solution: (a) wt\%} = \frac{(70 \text{ cm}^3)(19.254 \text{ g/cm}^3)}{(70)(19.254) + (30)(8.57)} = 83.98 \text{ wt\% W}$$

(b) The fibers will dissolve. Since the W and Nb are completely soluble in one another, and the temperature is high enough for rapid diffusion, a single solid solution will eventually be produced.

- 9-58** Suppose a crucible made of pure nickel is used to contain 500 g of liquid copper at 1150°C. Describe what happens to the system as it is held at this temperature for several hours. Explain.

**Solution:** Cu dissolves Ni until the Cu contains enough Ni that it solidifies completely. When 10% Ni is dissolved, freezing begins:

$$0.10 = \frac{x}{x + 500} \quad \text{or} \quad x = 55.5 \text{ g Ni}$$

When 18% Ni dissolved, the bath is completely solid:

$$0.18 = \frac{x}{x + 500} \quad \text{or} \quad x = 109.8 \text{ g Ni}$$

- 9–61** Equal moles of MgO and FeO are combined and melted. Determine (a) the liquidus temperature, the solidus temperature, and the freezing range of the ceramic and (b) determine the phase(s) present, their composition(s), and their amount(s) at 1800°C. (See Figure 9–21.)

**Solution:**  $MW_{\text{MgO}} = 40.312 \text{ g/mol}$        $MW_{\text{FeO}} = 71.847 \text{ g/mol}$

$$\text{wt\% FeO} = \frac{(1 \text{ mol FeO})(71.847 \text{ g/mol})}{(1 \text{ mol FeO})(71.847) + (1 \text{ mol MgO})(40.312)} = 64.1\%$$

$$(a) \quad T_{\text{Liq}} = 2000^\circ\text{C} \quad T_s = 1620^\circ\text{C} \quad \text{FR} = 380^\circ\text{C}$$

$$(b) \quad L: 75\% \text{ FeO} \quad S: 50\% \text{ FeO}$$

$$\%L = \frac{64.1 - 50}{75 - 50} \times 100\% = 56.4\% \quad \%S = 43.6\%$$

- 9–62** Suppose 75 cm<sup>3</sup> of Nb and 45 cm<sup>3</sup> of W are combined and melted. Determine (a) the liquidus temperature, the solidus temperature, and the freezing range of the alloy and (b) determine the phase(s) present, their composition(s), and their amount(s) at 2800°C. (See Figure 9–22.)

**Solution:**  $\text{wt\% W} = \frac{(45 \text{ cm}^3)(19.254 \text{ g/cm}^3)}{(45)(19.254) + (75)(8.57)} \times 100 = 57.4 \text{ wt\% W}$

$$(a) \quad T_{\text{Liq}} = 2900^\circ\text{C} \quad T_{\text{Sol}} = 2690^\circ\text{C} \quad \text{FR} = 210^\circ\text{C}$$

$$(b) \quad L: 49\% \text{ W} \quad \%L = \frac{70 - 57.4}{70 - 49} = 60\%$$

$$\alpha: 70\% \text{ W} \quad \% \alpha = 40\%$$

- 9–63** A NiO–60 mol% MgO ceramic is allowed to solidify. Determine (a) the composition of the first solid to form and (b) the composition of the last liquid to solidify under equilibrium conditions.

**Solution:** (a) 1st  $\alpha$ : 80% MgO      (b) Last L: 35% MgO

- 9–64** A Nb–35% W alloy is allowed to solidify. Determine (a) the composition of the first solid to form and (b) the composition of the last liquid to solidify under equilibrium conditions. (See Figure 9–22.)

**Solution:** (a) 1st  $\alpha$ : 55% W      (b) Last L: 18% W

- 9–65** For equilibrium conditions and a MgO–65 wt% FeO ceramic, determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1800°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1600°C. (See Figure 9–21.)

**Solution:** (a) Liquidus = 2000°C

(b) Solidus = 1605°C

(c) Freezing range = 2000 – 1605 = 395°C

(d) First solid: 40% FeO

(e) Last liquid: 88% FeO

(f)  $L$ : 75% FeO     $\%L = \frac{65 - 51}{75 - 51} \times 100\% = 58\%$

$\alpha$ : 51% FeO     $\%\alpha = 42\%$

(g)  $\alpha$ : 65% FeO    100%  $\alpha$

- 9–66** Figure 9–23 shows the cooling curve for a NiO–MgO ceramic. Determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the pouring temperature, (e) the superheat, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the ceramic.

**Solution:** (a) Liquidus = 2690°C

(b) Solidus = 2570°C

(c) Freezing range = 2690 – 2570 = 120°C

(d) Pouring temperature = 2775°C

(e) Superheat = 2775 – 2690 = 85°C

(f) Local solidification time = 27 – 5 = 22 min

(g) Total solidification time = 27 min

(h) 80% MgO

- 9–67** For equilibrium conditions and a Nb–80 wt% W alloy, determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 3000°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 2800°C. (see Figure 9–22.)

**Solution:** (a) Liquidus = 3100°C

(b) Solidus = 2920°C

(c) Freezing range = 3100 – 2920 = 180°C

(d) First solid: 90% W

(e) Last liquid: 64% W

$$(f) L: 70\% W \quad \%L = \frac{85 - 80}{85 - 70} \times 100\% = 33.3\%$$

$$\alpha: 85\% W \quad \% \alpha = 66.7\%$$

(g)  $\alpha: 80\% W \quad 100\% \alpha$

**9–68** Figure 9–24 shows the cooling curve for a Nb–W alloy. Determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the pouring temperature, (e) the superheat, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

**Solution:** (a) Liquidus = 2900°C

(b) Solidus = 2710°C

(c) Freezing range = 2900 – 2710 = 190°C

(d) Pouring temperature = 2990°C

(e) Superheat = 2990 – 2900 = 90°C

(f) Local solidification time = 340 – 40 = 300 s

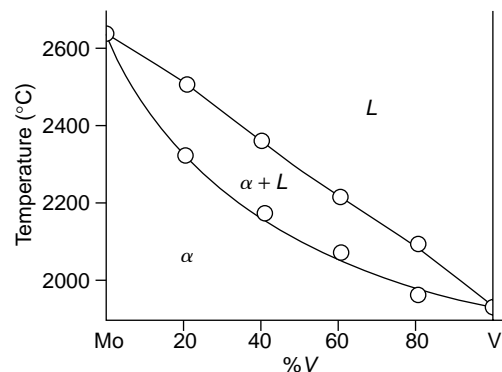
(g) Total solidification time = 340 min

(h) 60% W

**9–69** Cooling curves are shown in Figure 9–25 for several Mo–V alloys. Based on these curves, construct the Mo–V phase diagram.

**Solution:**

	$T_{\text{Liquidus}}$	$T_{\text{Solidus}}$
0% V	2630°C	
20% V	2500°C	2320°C
40% V	2360°C	2160°C
60% V	2220°C	2070°C
80% V	2100°C	1970°C
100% V	1930°C	



- 9-71** For the nonequilibrium conditions shown for the MgO–65 wt% FeO ceramic, determine (a) the liquidus temperature, (b) the nonequilibrium solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1800°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1600°C. (See Figure 9–21.)

**Solution:** (a) Liquidus = 2000°C

(b) Solidus = 1450°C

(c) Freezing range = 2000 – 1450 = 550°C

(d) First solid: 40% FeO

(e) Last liquid: 92% FeO

(f)  $L$ : 75% FeO     $\%L = \frac{65 - 46}{75 - 46} \times 100\% = 65.5\%$

$S$ : 46% FeO     $\%S = 34.5\%$

(g)  $L$ : 88% FeO     $\%L = \frac{65 - 55}{88 - 55} \times 100\% = 30.3\%$

$S$ : 55% FeO     $\%S = 69.7\%$

- 9-72** For the nonequilibrium conditions shown for the Nb–80 wt% W alloy, determine (a) the liquidus temperature, (b) the nonequilibrium solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 3000°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 2800°C. (See Figure 9–22.)

**Solution:** (a) Liquidus = 3100°C

(b) Solidus = 2720°C

(c) Freezing range = 3100 – 2720 = 380°C

(d) First solid: 90% W

(e) Last liquid: 40% W

(f)  $L$ : 70% W     $\%L = \frac{88 - 80}{88 - 70} \times 100\% = 44.4\%$

$\alpha$ : 88% W     $\%\alpha = 55.6\%$

(g)  $L$ : 50% W     $\%L = \frac{83 - 80}{83 - 50} \times 100\% = 9.1\%$

$\alpha$ : 83% W     $\%\alpha = 90.9\%$

# 10

## Dispersion Strengthening and Eutectic Phase Diagrams

- 10–22** A hypothetical phase diagram is shown in Figure 10–32. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. (b) Identify the solid solutions present in the system. Is either material *A* or *B* allotropic? Explain. (c) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

**Solution:** (a)  $\theta$  = non-stoichiometric intermetallic compound.

(b)  $\alpha$ ,  $\eta$ ,  $\gamma$ , and  $\beta$ ; material *B* is allotropic, existing in three different forms at different temperatures

(c) 1100°C:  $\gamma + L \rightarrow \beta$ ; peritectic;  $L$ : 82% *B*  
 $\gamma$ : 97% *B*  $\beta$ : 90% *B*  
 900°C:  $L_1 \rightarrow L_2 + \alpha$ ; monotectic;  $L_1$ : 28% *B*  
 $L_2$ : 50% *B*  $\alpha$ : 5% *B*  
 680°C:  $L \rightarrow \alpha + \beta$ ; eutectic;  $L$ : 60% *B*  
 $\alpha$ : 5% *B*  $\beta$ : 90% *B*  
 600°C:  $\alpha + \beta \rightarrow \theta$ ; peritectoid;  $\alpha$ : 5% *B*  
 $\beta$ : 80% *B*  $\theta$ : 37% *B*  
 300°C:  $\beta \rightarrow \theta + \eta$ ; eutectoid;  $\beta$ : 90% *B*  
 $\theta$ : 40% *B*  $\eta$ : 95% *B*

- 10–23** The Cu–Zn phase diagram is shown in Figure 10–33. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. (b) Identify the solid solutions present in the system. (c) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, and the name of the reaction.



Solution: (a)  $\beta, \beta', \gamma, \delta, \varepsilon$ : all nonstoichiometric.

(b)  $\alpha, \theta$

(c) 900°C:  $\alpha + L \rightarrow \beta$ ; peritectic

830°C:  $\beta + L \rightarrow \gamma$ ; peritectic

700°C:  $\gamma + L \rightarrow \delta$ ; peritectic

600°C:  $\delta + L \rightarrow \varepsilon$ ; peritectic

550°C:  $\delta \rightarrow \gamma + \varepsilon$ ; eutectoid

420°C:  $\varepsilon + L \rightarrow \theta$ ; peritectic

250°C:  $\beta' \rightarrow \alpha + \gamma$ ; eutectoid

**10–24** A portion of the Al–Cu phase diagram is shown in Figure 10–34. (a) Determine the formula for the  $\theta$  compound. (b) Identify the three-phase reaction by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

Solution: (a)  $\theta$  at 54% Cu;  $\frac{54 \text{ g}/63.54 \text{ g/mol}}{54/63.54 + 46/26.981} = 33 \text{ at\% Cu}$ ;  $\text{CuAl}_2$

(b) 548°C;  $L \rightarrow \alpha + \theta$ ; eutectic;  $L$ : 33.2% Cu,  
 $\alpha$ : 5.65% Cu,  $\theta$ : 52.5% Cu.

**10–25** The Al–Li phase diagram is shown in Figure 10–35. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. Determine the formula for each compound. (b) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

Solution: (a)  $\beta$  is non-stoichiometric @ 21 wt% Li:

$$\text{at\% Li} = \frac{21 \text{ g}/6.94 \text{ g/mol}}{21/6.94 + 79/26.981} \times 100\% = 50 \text{ at\% Li} \therefore \text{AlLi}$$

$\gamma$ , is stoichiometric @ 34 wt% Li:

$$\text{at\% Li} = \frac{34 \text{ g}/6.94 \text{ g/mol}}{34/6.94 + 66/26.981} \times 100\% = 66.7\% \text{ Li} \therefore \text{AlLi}_2$$

(b) 600°C:  $L \rightarrow \alpha + \beta$  eutectic  $L$ : 9.9% Li

$\alpha$ : 4% Li  $\beta$ : 20.4% Li

510°C:  $\beta + L \rightarrow \gamma$  peritectic  $\beta$ : 25% Li

$L$ : 47% Li  $\gamma$ : 34% Li

170°C:  $L \rightarrow \gamma + \alpha(\text{Li})$  eutectic  $L$ : 98% Li

$\gamma$ : 34% Li  $\alpha(\text{Li})$ : 99% Li

**10–26** An intermetallic compound is found for 10 wt% Si in the Cu–Si phase diagram. Determine the formula for the compound.

Solution:  $\text{at\% Si} = \frac{10 \text{ g}/28.08 \text{ g/mol}}{10/28.08 + 90/63.54} = 0.20$  or  $\text{SiCu}_4$

- 10–27** Using the phase rule, predict and explain how many solid phases will form in an eutectic reaction in a ternary (three-component) phase diagram, assuming that the pressure is fixed.

**Solution:**  $F = C - P + 1$

At the eutectic,  $F = 0$ ,  $C = 3$   $0 = 3 - P + 1$  or  $P = 4$

Therefore,  $L \rightarrow \alpha + \beta + \gamma$  and 3 solid phases form.

- 10–30** Consider a Pb–15% Sn alloy. During solidification, determine (a) the composition of the first solid to form, (b) the liquidus temperature, solidus temperature, solvus temperature, and freezing range of the alloy, (c) the amounts and compositions of each phase at 260°C, (d) the amounts and compositions of each phase at 183°C, and (e) the amounts and compositions of each phase at 25°C.

**Solution:** (a) 8% Sn

(b) liquidus = 290°C, solidus = 240°C,  
solvus = 170°C, freezing range = 50°C

(c)  $L$ : 30% Sn  $\alpha$ : 12% Sn;

$$\%L = \frac{15 - 12}{30 - 12} \times 100\% = 17\% \quad \% \alpha = 83\%$$

(d)  $\alpha$ : 15% Sn 100%  $\alpha$

(e)  $\alpha$ : 2% Pb  $\beta$ : 100% Sn

$$\% \alpha = \frac{100 - 15}{100 - 2} \times 100 = 87\% \quad \% \beta = 13\%$$

- 10–31** Consider an Al–12% Mg alloy (Figure 10–36). During solidification, determine (a) the composition of the first solid to form, (b) the liquidus temperature, solidus temperature, solvus temperature, and freezing range of the alloy, (c) the amounts and compositions of each phase at 525°C, (d) the amounts and compositions of each phase at 450°C, and (e) the amounts and compositions of each phase at 25°C.

**Solution:** (a) 2.5% Mg

(b) liquidus = 600°C, solidus = 470°C,  
solvus = 400°C, freezing range = 130°C

(c)  $L$ : 26% Mg  $\alpha$ : 7% Mg;

$$\% \alpha = \frac{26 - 12}{26 - 7} \times 100\% = 74\% \quad \%L = 26\%$$

(d)  $\alpha$ : 12% Mg 100%  $\alpha$

(e)  $\alpha$ : 1% Mg  $\beta$ : 34% Mg

$$\% \alpha = \frac{34 - 12}{34 - 1} \times 100\% = 67\% \quad \% \beta = 33\%$$

- 10–32** Consider a Pb–35% Sn alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 184°C, (d) the amounts and compositions of each phase at 182°C, (e) the amounts and compositions of each microconstituent at 182°C, and (f) the amounts and compositions of each phase at 25°C.

**Solution:** (a) hypoeutectic (b) 14% Sn

(c)  $\alpha$ : 19% Sn  $L$ : 61.9% Sn

$$\% \alpha = \frac{61.9 - 35}{61.9 - 19} \times 100\% = 63\% \quad \% L = 37\%$$

(d)  $\alpha$ : 19% Sn  $\beta$ : 97.5% Sn

$$\% \alpha = \frac{97.5 - 35}{97.5 - 19} \times 100\% = 80\% \quad \% \beta = 20\%$$

(e) primary  $\alpha$ : 19% Sn  $\%$ primary  $\alpha = 63\%$   
eutectic: 61.9% Sn  $\%$ eutectic = 37%

(f)  $\alpha$ : 2% Sn  $\beta$ : 100% Sn

$$\% \alpha = \frac{100 - 35}{100 - 2} \times 100\% = 66\% \quad \% \beta = 34\%$$

- 10–33** Consider a Pb–70% Sn alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 184°C, (d) the amounts and compositions of each phase at 182°C, (e) the amounts and compositions of each microconstituent at 182°C, and (f) the amounts and compositions of each phase at 25°C.

**Solution:** (a) hypereutectic (b) 98% Sn

(c)  $\beta$ : 97.5% Sn  $L$ : 61.9% Sn

$$\% \beta = \frac{70 - 61.9}{97.5 - 61.9} \times 100\% = 22.8\% \quad \% L = 77.2\%$$

(d)  $\alpha$ : 19% Sn  $\beta$ : 97.5% Sn

$$\% \alpha = \frac{97.5 - 70}{97.5 - 19} \times 100\% = 35\% \quad \% \beta = 65\%$$

(e) primary  $\beta$ : 97.5% Sn  $\%$ primary  $\beta = 22.8\%$   
eutectic: 61.9% Sn  $\%$ eutectic = 77.2%

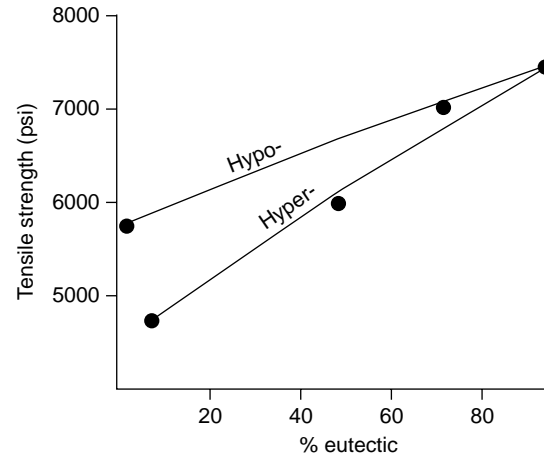
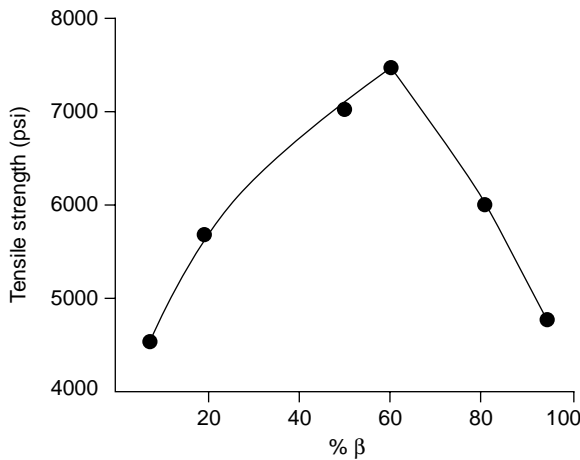
(f)  $\alpha$ : 2% Sn  $\beta$ : 100% Sn

$$\% \alpha = \frac{100 - 70}{100 - 2} \times 100\% = 30\% \quad \% \beta = 70\%$$

- 10–34** Calculate the total %  $\beta$  and the % eutectic microconstituent at room temperature for the following lead-tin alloys: 10% Sn, 20% Sn, 50% Sn, 60% Sn, 80% Sn, and 95% Sn. Using Figure 10–22, plot the strength of the alloys versus the %  $\beta$  and the % eutectic and explain your graphs.

Solution:

	$\% \beta$	$\% \text{eutectic}$
10% Sn	$\frac{10 - 2}{99 - 2} = 8.2\%$	0%
20% Sn	$\frac{20 - 2}{99 - 2} = 18.6\%$	$\frac{20 - 19}{61.9 - 19} = 2.3\%$
50% Sn	$\frac{50 - 2}{99 - 2} = 49.5\%$	$\frac{50 - 19}{61.9 - 19} = 72.3\%$
60% Sn	$\frac{60 - 2}{99 - 2} = 59.8\%$	$\frac{60 - 19}{61.9 - 19} = 95.6\%$
80% Sn	$\frac{80 - 2}{99 - 2} = 80.4\%$	$\frac{97.5 - 80}{97.5 - 61.9} = 49.2\%$
95% Sn	$\frac{95 - 2}{99 - 2} = 95.9\%$	$\frac{97.5 - 95}{97.5 - 61.9} = 7.0\%$



**10-35** Consider an Al-4% Si alloy. (See Figure 10-23.) Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 578°C, (d) the amounts and compositions of each phase at 576°C, the amounts and compositions of each microconstituent at 576°C, and (e) the amounts and compositions of each phase at 25°C.

Solution: (a) hypoeutectic

(b) 1% Si

(c)  $\alpha$ : 1.65% Si  $L$ : 12.6% Si

$$\% \alpha = \frac{12.6 - 4}{12.6 - 1.65} = 78.5\% \quad \% L = 21.5\%$$

(d)  $\alpha$ : 1.65% Si  $\beta$ : 99.83% Si

$$\% \alpha = \frac{99.83 - 4}{99.83 - 1.65} = 97.6\% \quad \% \beta = 2.4\%$$

$$\begin{array}{ll} \text{(e) primary } \alpha: 1.65\% \text{ Si} & \% \text{primary } \alpha = 78.5\% \\ \text{eutectic: } 12.6\% \text{ Si} & \% \text{eutectic} = 21.5\% \end{array}$$

$$\alpha: 0\% \text{ Si} \quad \beta: 100\% \text{ Si} \quad \% \alpha = \frac{100 - 4}{100 - 0} = 96\% \quad \% \beta = 4\%$$

**10–36** Consider a Al–25% Si alloy. (See Figure 10–23.) Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 578°C, (d) the amounts and compositions of each phase at 576°C, (e) the amounts and compositions of each microconstituent at 576°C, and (f) the amounts and compositions of each phase at 25°C.

**Solution:** (a) hypereutectic

(b) 100% Si

(c)  $\beta$ : 99.83% Si  $L$ : 12.6% Si

$$\% L = \frac{99.83 - 25}{99.83 - 12.6} = 85.8\% \quad \% \beta = 14.2\%$$

(d)  $\alpha$ : 1.65% Si  $\beta$ : 99.83% Si

$$\% \alpha = \frac{99.83 - 25}{99.83 - 1.65} = 76.2\% \quad \% \beta = 23.8\%$$

(e) primary  $\beta$ : 99.83% Si  $\%$ primary  $\beta = 14.2\%$   
eutectic: 12.6% Si  $\%$ eutectic = 85.8%

$$\text{(f) } \alpha: 0\% \text{ Si} \quad \beta: 100\% \text{ Si} \quad \% \alpha = \frac{100 - 25}{100 - 0} = 75\% \quad \% \beta = 25\%$$

**10–37** A Pb–Sn alloy contains 45%  $\alpha$  and 55%  $\beta$  at 100°C. Determine the composition of the alloy. Is the alloy hypoeutectic or hypereutectic?

$$\text{Solution:} \quad \% \alpha = 45 = \frac{98.0 - x}{98.0 - 5} \times 100 \quad \text{or} \quad x = 56.15\% \text{ Sn} \quad \text{Hypoeutectic}$$

**10–38** An Al–Si alloy contains 85%  $\alpha$  and 15%  $\beta$  at 500°C. Determine the composition of the alloy. Is the alloy hypoeutectic or hypereutectic?

$$\text{Solution:} \quad \% \alpha = 85 = \frac{100 - x}{100 - 1} \times 100 \quad \text{or} \quad x = 15.85\% \text{ Si} \quad \text{Hypereutectic}$$

**10–39** A Pb–Sn alloy contains 23% primary  $\alpha$  and 77% eutectic microconstituent. Determine the composition of the alloy.

$$\text{Solution:} \quad \% \text{primary } \alpha = 23 = \frac{61.9 - x}{61.9 - 19} \times 100 \quad \text{or} \quad x = 52\% \text{ Sn}$$

**10–40** An Al–Si alloy contains 15% primary  $\beta$  and 85% eutectic microconstituent. Determine the composition of the alloy.

**Solution:**  $\% \text{ eutectic} = 85 = \frac{100 - x}{100 - 12.6} \times 100 \quad \text{or} \quad x = 25.71\% \text{ Si}$

**10–41** Determine the maximum solubility for the following cases.

- (a) lithium in aluminum (Figure 10–35),
- (b) aluminum in magnesium (Figure 10–37),
- (c) copper in zinc (Figure 10–33), and
- (d) carbon in  $\gamma$ -iron (Figure 10–38)

**Solution:** (a) 4% Li dissolves in aluminum  
 (b) 12.7% Al dissolves in magnesium  
 (c) 3% Cu dissolves in zinc  
 (d) 2.11% C dissolves in  $\gamma$ -iron

**10–42** Determine the maximum solubility for the following cases.

- (a) magnesium in aluminum (Figure 10–36),
- (b) zinc in copper (Figure 10–33),
- (c) beryllium in copper (Figure 10–33), and
- (d)  $\text{Al}_2\text{O}_3$  in MgO (Figure 10–39)

**Solution:** (a) 14.9% Mg dissolves in aluminum  
 (b) 40% Zn dissolves in copper  
 (c) 2.5% Be dissolves in copper  
 (d) 18%  $\text{Al}_2\text{O}_3$  dissolves in MgO

**10–43** Observation of a microstructure shows that there is 28% eutectic and 72% primary  $\beta$  in an Al–Li alloy (Figure 10–35). (a) Determine the composition of the alloy and whether it is hypoeutectic or hypereutectic. (b) How much  $\alpha$  and  $\beta$  are in the eutectic microconstituent?

**Solution:** (a)  $28 = \frac{20.4 - x}{20.4 - 9.9} \times 100 \quad \text{or} \quad x = 17.46\% \text{ Li} \quad \text{Hypereutectic}$

(b)  $\% \alpha_{\text{Eut}} = \frac{20.4 - 9.9}{20.4 - 4} \times 100\% = 64\% \quad \text{and} \quad \% \beta_{\text{Eut}} = 36\%$

**10–44** Write the eutectic reaction that occurs, including the compositions of the three phases in equilibrium, and calculate the amount of  $\alpha$  and  $\beta$  in the eutectic microconstituent in the Mg–Al system, (Figure 10–36).

**Solution:**  $L_{32.3} \rightarrow \alpha_{12.7} + \gamma_{40.2}$

$\therefore \% \alpha_{\text{Eut}} = \frac{40.2 - 32.3}{40.2 - 12.7} \times 100\% = 28.7\% \quad \text{and} \quad \% \gamma_{\text{Eut}} = 71.3\%$

- 10–45** Calculate the total amount of  $\alpha$  and  $\beta$  and the amount of each microconstituent in a Pb–50% Sn alloy at 182°C. What fraction of the total  $\alpha$  in the alloy is contained in the eutectic microconstituent?

**Solution:**

$$\alpha_{\text{total}} = \frac{97.5 - 50}{97.5 - 19} \times 100\% = 60.5\% \quad \beta_{\text{Total}} = 39.5\%$$

$$\alpha_{\text{Primary}} = \frac{61.9 - 50}{61.9 - 19} \times 100\% = 27.7\% \quad \text{Eutectic} = 72.3\%$$

$$\alpha_{\text{in eutectic}} = 60.5 - 27.7 = 32.8\%$$

$$f = 32.8/60.5 = 0.54$$

- 10–46** Figure 10–40 shows a cooling curve for a Pb–Sn alloy. Determine (a) the pouring temperature, (b) the superheat, (c) the liquidus temperature, (d) the eutectic temperature, (e) the freezing range, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

**Solution:** (a) pouring temperature = 360°C

(b) superheat = 360 – 250 = 110°C

(c) liquidus temperature = 250°C

(d) eutectic temperature = 183°C

(e) freezing range = 250 – 183 = 67°C

(f) local solidification time = 600 – 110 = 490 s

(g) total solidification time = 600 s

(h) approximately 32% Sn

- 10–47** Figure 10–41 shows a cooling curve for an Al–Si alloy. Determine (a) the pouring temperature, (b) the superheat, (c) the liquidus temperature, (d) the eutectic temperature, (e) the freezing range, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

**Solution:** (a) pouring temperature = 1150°C

(b) superheat = 1150 – 1000 = 150°C

(c) liquidus temperature = 1000°C

(d) eutectic temperature = 577°C

(e) freezing range = 1000 – 577 = 423°C

(f) local solidification time = 11.5 – 1 = 10.5 min

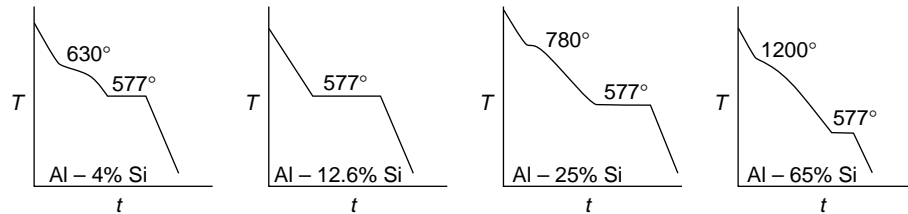
(g) total solidification time = 11.5 min

(h) approximately 45% Si

**10-48** Draw the cooling curves, including appropriate temperatures, expected for the following Al-Si alloys.

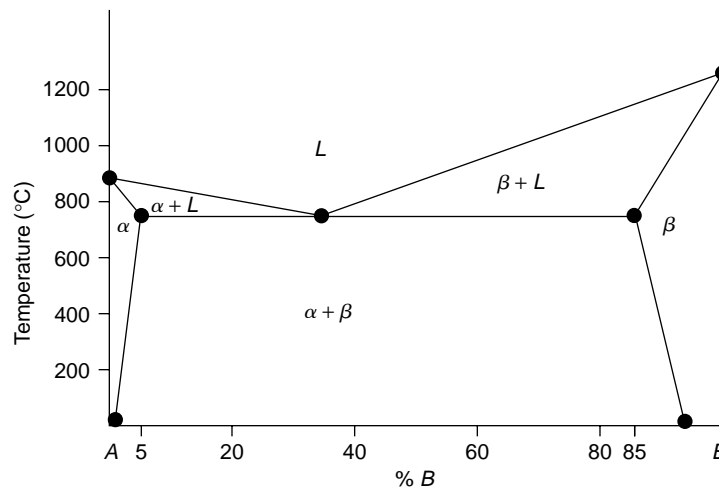
- (a) Al-4% Si      (b) Al-12.6% Si      (c) Al-25% Si      (d) Al-65% Si

Solution:



**10-49** Based on the following observations, construct a phase diagram. Element *A* melts at 850°C and element *B* melts at 1200°C. Element *B* has a maximum solubility of 5% in element *A*, and element *A* has a maximum solubility of 15% in element *B*. The number of degrees of freedom from the phase rule is zero when the temperature is 725°C and there is 35% *B* present. At room temperature 1% *B* is soluble in *A* and 7% *A* is soluble in *B*.

Solution:

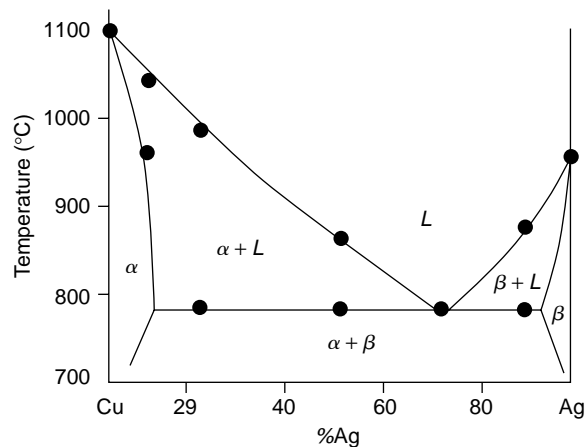




- 10–50** Cooling curves are obtained for a series of Cu–Ag alloys, (Figure 10–42). Use this data to produce the Cu–Ag phase diagram. The maximum solubility of Ag in Cu is 7.9% and the maximum solubility of Cu in Ag is 8.8%. The solubilities at room temperature are near zero.

Solution:

		$T_{\text{liq}}$	$T_{\text{sol}}$
0% Ag	→	1085°C	
8% Ag	→	1030°C	950°C
20% Ag	→	975°C	780°C
50% Ag	→	860°C	780°C
71.9% Ag	→	780°C	780°C
90% Ag	→	870°C	780°C
100% Ag	→	961°C	



- 10–51** The SiO<sub>2</sub>–Al<sub>2</sub>O<sub>3</sub> phase diagram is included in Figure 10–27(b). A refractory is required to contain molten metal at 1900°C. (a) Will pure Al<sub>2</sub>O<sub>3</sub> be a potential candidate? Explain. (b) Will Al<sub>2</sub>O<sub>3</sub> contaminated with 1% SiO<sub>2</sub> be a candidate? Explain.

Solution: (a) Yes.  $T_m = 2040^\circ\text{C} > 1900^\circ\text{C}$  No liquid will form.

(b) No. Some liquid will form.

$$\%L = \frac{100 - 99}{100 - 80} \times 100\% = 5\% L$$

This liquid will weaken the refractory.

**10–66** Consider the ternary phase diagram shown in Figures 10–30 and 10–31. Determine the liquidus temperature, the first solid to form, and the phases present at room temperature for the following compositions.

- (a) 30% *B*–20% *C*, balance *A*      (b) 10% *B*–25% *C*, balance *A*  
 (c) 60% *B*–10% *C*, balance *A*

**Solution:** (a)  $T_{\text{Liq}} = 220^{\circ}\text{C}$ ;  $\beta$ ;  $\alpha + \gamma + \beta$

(b)  $T_{\text{Liq}} = 330^{\circ}\text{C}$ ;  $\alpha$ ;  $\alpha + \gamma$

(c)  $T_{\text{Liq}} = 390^{\circ}\text{C}$ ;  $\beta$ ;  $\alpha + \beta$

**10–67** Consider the ternary phase diagram shown in Figures 10–30 and 10–31. Determine the liquidus temperature, the first solid to form, and the phases present at room temperature for the following compositions.

- (a) 5% *B*–80% *C*, balance *A*      (b) 50% *B*–5% *C*, balance *A*  
 (c) 30% *B*–35% *C*, balance *A*

**Solution:** (a)  $T_{\text{Liq}} = 390^{\circ}\text{C}$ ;  $\gamma$ ;  $\alpha + \gamma$

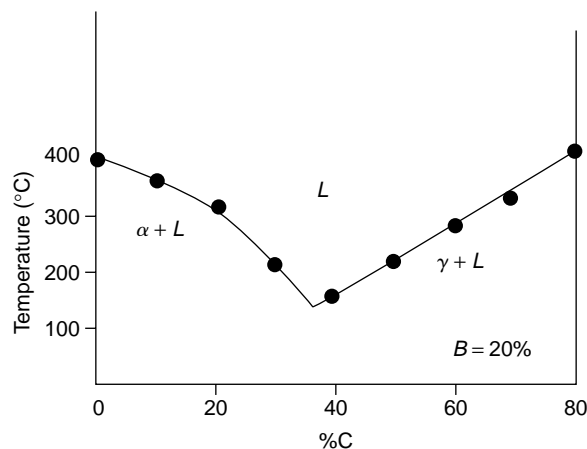
(b)  $T_{\text{Liq}} = 330^{\circ}\text{C}$ ;  $\beta$ ;  $\alpha + \beta$

(c)  $T_{\text{Liq}} = 290^{\circ}\text{C}$ ;  $\beta$ ;  $\alpha + \beta + \gamma$

**10–68** Consider the liquidus plot in Figure 10–30. (a) For a constant 20% *B*, draw a graph showing how the liquidus temperature changes from 20% *B*–0% *C*, balance *A* to 20% *B*–80% *C*, balance *A*, (b) What is the composition of the ternary eutectic in this system? (c) Estimate the temperature at which the ternary eutectic reaction occurs.

**Solution:**

%A	%B	%C	$T_{\text{liquidus}}$
80	20	0	390°C
70	20	10	355°C
60	20	20	300°C
50	20	30	210°C
40	20	40	150°C
30	20	50	210°C
20	20	60	270°C
10	20	70	320°C
0	20	80	400°C

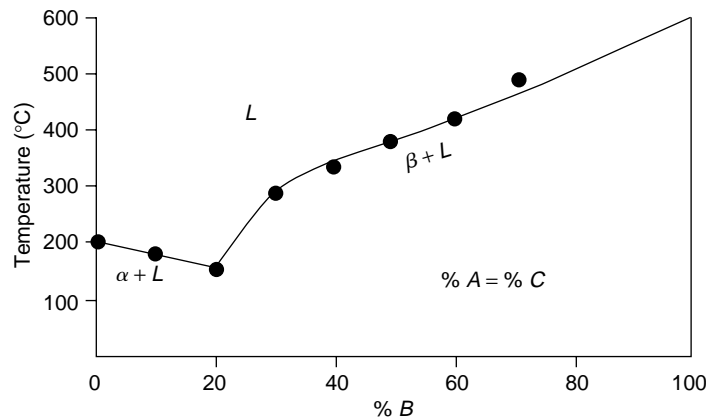


- (b) The composition of the ternary eutectic is about 40% 20%  $B$ –40%  $C$ , balance  $A$
- (c) The ternary eutectic temperature is about 150°C

**10–69** From the liquidus plot in Figure 10–30, prepare a graph of liquidus temperature versus percent  $B$  for a constant ratio of materials  $A$  and  $C$  (that is, from pure  $B$  to 50%  $A$ –50%  $C$  on the liquidus plot). Material  $B$  melts at 600°C.

Solution:

$A$	$B$	$C$	
50–	0–50		200°C
45–	10–45		180°C
40–	20–40		150°C
35–	30–35		280°C
30–	40–30		330°C
25–	50–25		375°C
20–	60–20		415°C
15–	70–15		485°C
0–	100–0		580°C



# 11

## Dispersion Strengthening by Phase Transformations and Heat Treatment

**11–2** Determine the constants  $c$  and  $n$  in Equation 11–2 that describe the rate of crystallization of polypropylene at  $140^\circ\text{C}$ . (See Figure 11–31)

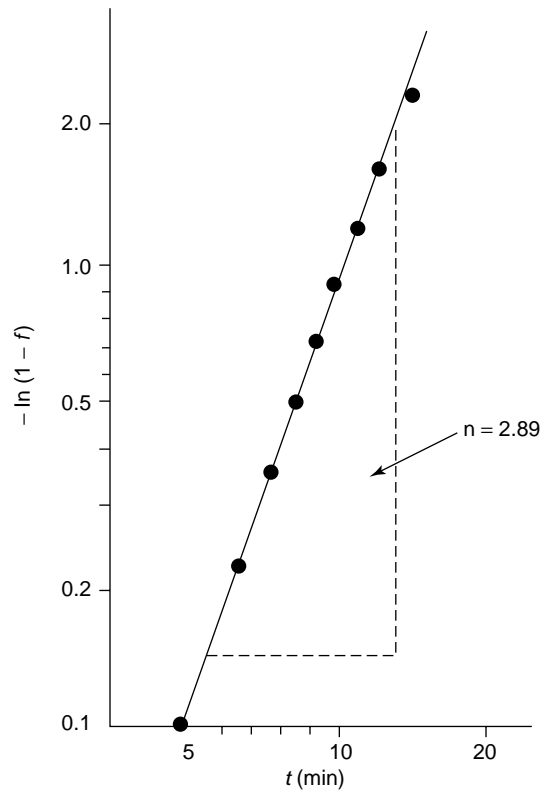
**Solution:**  $f = 1 - \exp(-ct^n)$   $T = 140^\circ\text{C} = 413 \text{ K}$

We can rearrange the equation and eliminate the exponential by taking natural logarithms of both sides of the rearranged equation. We can then note that  $\ln(1 - f)$  versus  $t$  is a power equation; if these terms are plotted on a log-log plot, we should obtain a linear relationship, as the graph of the data below indicates. Note that in setting up the equation for plotting, we switch the minus sign from the right hand to the left hand side, since we don't have negative numbers on the log-log paper.

$1 - f = \exp(-ct^n)$	$f$	$t(\text{min})$	$-\ln(1 - f)$
$\ln(1 - f) = -ct^n$	0.1	28	0.1
$\ln[-\ln(1 - f)] = \ln(ct^n)$	0.2	37	0.22
$\ln[-\ln(1 - f)] = \ln(c) + n \ln(t)$	0.3	44	0.36
	0.4	50	0.51
	0.5	55	0.69
	0.6	60	0.92
	0.7	67	1.20
	0.8	73	1.61
	0.9	86	2.302

A log-log plot of “ $-\ln(1 - f)$ ” versus “ $t$ ” is shown. From the graph, we find that the slope  $n = 2.89$  and the constant  $c$  can be found from one of the points from the curve:

$$\begin{aligned} \text{if } f = 0.5, t = 55. \text{ Then} \\ 1 - 0.5 &= \exp[-c(55)^{2.89}] \\ c &= 6.47 \times 10^{-6} \end{aligned}$$



**11-3** Determine the constants  $c$  and  $n$  in Equation 11-2 that describe the rate of recrystallization of copper at  $135^{\circ}\text{C}$ . (See Figure 11-2)

**Solution:**  $f = 1 - \exp(-ct^n)$   $T = 135^{\circ}\text{C} = 408 \text{ K}$

We can rearrange the equation and eliminate the exponential by taking natural logarithms of both sides of the rearranged equation. We can then note that  $\ln(1 - f)$  versus  $t$  is a power equation and should give a linear relationship in a log-log plot. Note that in setting up the equation for plotting, we switch the minus sign from the right hand to the left hand side, since we don't have negative numbers on the log-log paper.

$$1 - f = \exp(-ct^n)$$

$$\ln(1 - f) = -ct^n$$

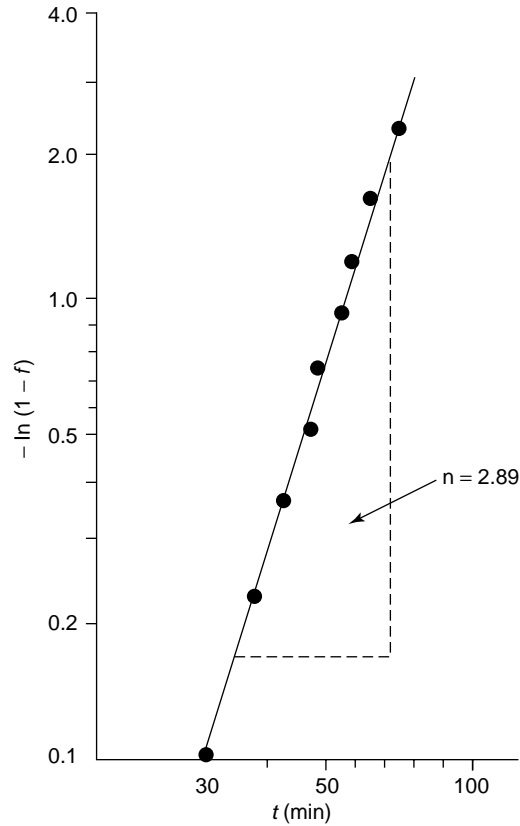
$$\ln[-\ln(1 - f)] = \ln(ct^n)$$

$$\ln[-\ln(1 - f)] = \ln(c) + \ln(t)$$

$f$	$t \text{ (min)}$	$-\ln(1 - f)$
0.1	5.0	0.10
0.2	6.6	0.22
0.3	7.7	0.36
0.4	8.5	0.51
0.5	9.0	0.69
0.6	10.0	0.92
0.7	10.5	1.20
0.8	11.5	1.61
0.9	13.7	2.30

A log-log plot of “ $-\ln(1 - f)$ ” versus “ $t$ ” is shown. From the graph, we find that the slope  $n = 3.1$  and the constant  $c$  can be found from one of the points from the curve:

$$\begin{aligned} \text{If } f &= 0.6, \text{ then } t = 10. \text{ Then} \\ 1 - 0.6 &= \exp[-c(10)^{3.1}] \\ c &= 7.28 \times 10^{-4}. \end{aligned}$$



**11-4** Determine the activation energy for crystallization of polypropylene, using the curves in Figure 11-36.

**Solution:** We can determine how the rate (equal to  $1/\tau$ ) changes with temperature:

$$\text{rate} = 1/\tau = c \exp(-Q/RT)$$

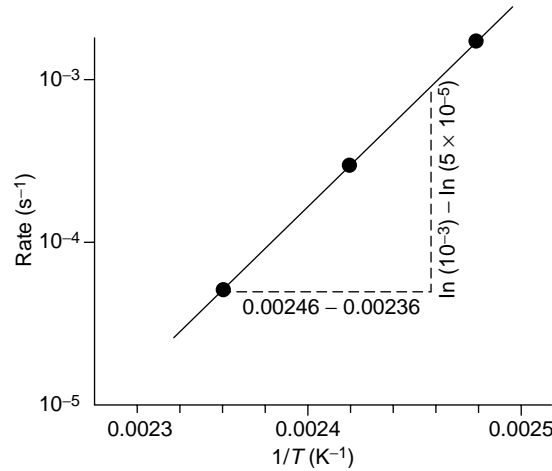
$1/\tau \text{ (s}^{-1}\text{)}$	$1/T \text{ (K}^{-1}\text{)}$
$1/(9 \text{ min})(60 \text{ s/min}) = 1.85 \times 10^{-3}$	$1/(130 + 273) = 2.48 \times 10^{-3}$
$1/(55 \text{ min})(60 \text{ s/min}) = 3.03 \times 10^{-4}$	$1/(140 + 273) = 2.42 \times 10^{-3}$
$1/(316 \text{ min})(60 \text{ s/min}) = 5.27 \times 10^{-5}$	$1/(150 + 273) = 2.36 \times 10^{-3}$

From the semilog graph of rate versus reciprocal temperature, we find that the slope is:

$$Q/R = \frac{\ln(10^{-3}) - \ln(5 \times 10^{-5})}{0.00246 - 0.00236}$$

$$Q/R = 29,957$$

$$Q = 59,525 \text{ cal/mol}$$



- 11–16** (a) Recommend an artificial age-hardening heat treatment for a Cu–1.2% Be alloy (see Figure 11–34). Include appropriate temperatures. (b) Compare the amount of the  $\gamma_2$  precipitate that forms by artificial aging at 400°C with the amount of the precipitate that forms by natural aging.

**Solution:** (a) For the Cu–1.2% Be alloy, the peritectic temperature is 870°C; above this temperature, liquid may form. The solvus temperature is about 530°C. Therefore:

- 1) Solution treat between 530°C and 870°C (780°C is typical for beryllium copper alloys)
- 2) Quench
- 3) Age below 530°C (330°C is typical for these alloys)

(b) We can perform lever law calculations at 400°C and at room temperature. The solubility of Be in Cu at 400°C is about 0.6% Be and that at room temperature is about 0.2% Be:

$$\gamma_2 \text{ (at 400°C)} = \frac{1.2 - 0.6}{11.7 - 0.6} \times 100 = 5.4\%$$

$$\gamma_2 \text{ (room } T) = \frac{1.2 - 0.2}{12 - 0.2} \times 100 = 8.5\%$$

- 11–17** Suppose that age hardening is possible in the Al–Mg system (see Figure 11–11). (a) Recommend an artificial age-hardening heat treatment for each of the following alloys, and (b) compare the amount of the  $\beta$  precipitate that forms from your treatment of each alloy. (i) Al–4% Mg (ii) Al–6% Mg (iii) Al–12% Mg (c) Testing of the alloys after the heat treatment reveals that little strengthening occurs as a result of the heat treatment. Which of the requirements for age hardening is likely not satisfied?

**Solution:** (a) The heat treatments for each alloy might be:

	Al-4% Mg	Al-6% Mg	Al-12% Mg
$T_{\text{Eutectic}} =$	451°C	451°C	451°C
$T_{\text{Solvus}} =$	210°C	280°C	390°C
<b>Solution</b>			
Treat at:	210–451°C	280–451°C	390–451°C
	Quench	Quench	Quench
Age at:	<210°C	<280°C	<390°C

(b) Answers will vary depending on aging temperature selected. If all three are aged at 200°C, as an example, the tie line goes from about 3.8 to 35% Mg:

$$\text{Al-4\% Mg: } \% \beta = (4 - 3.8)/(35 - 3.8) \times 100 = 0.6\%$$

$$\text{Al-6\% Mg: } \% \beta = (6 - 3.8)/(35 - 3.8) \times 100 = 7.1\%$$

$$\text{Al-12\% Mg: } \% \beta = (12 - 3.8)/(35 - 3.8) \times 100 = 26.8\%$$

(c) Most likely, a coherent precipitate is not formed; simple dispersion strengthening, rather than age hardening, occurs.

**11–18** An Al-2.5% Cu alloy is solution-treated, quenched, and overaged at 230°C to produce a stable microstructure. If the spheroidal  $\theta$  precipitates so that form has a diameter of 9000 Å and a density of 4.26 g/cm<sup>3</sup>, determine the number of precipitate particles per cm<sup>3</sup>.

$$\text{Solution: } \text{wt\% } \alpha = \frac{53 - 2.5}{53 - 1} = 97.12\% \quad \text{wt\% } \theta = 2.88\%$$

$$\text{vol fraction } \theta = \frac{2.88 \text{ g}/4.26 \text{ g/cm}^3}{2.88/4.26 + 97.12/2.669} = 0.0182 \text{ cm}^3 \theta / \text{cm}^3 \text{ alloy}$$

$$d_{\theta} = 9000 \times 10^{-10} \text{ m} = 9 \times 10^{-5} \text{ cm} \quad r_{\theta} = 4.5 \times 10^{-5} \text{ cm}$$

$$V_{\theta} = (4\pi/3)(4.5 \times 10^{-5} \text{ cm})^3 = 382 \times 10^{-15} \text{ cm}^3$$

$$\# \text{ of particles} = \frac{0.0182 \text{ cm}^3}{382 \times 10^{-15} \text{ cm}^3} = 4.76 \times 10^{10} \text{ particles}$$

**11–38** Figure 11–32 shows a hypothetical phase diagram. Determine whether each of the following alloys might be good candidates for age hardening and explain your answer. For those alloys that might be good candidates, describe the heat treatment required, including recommended temperatures.

- (a) A-10% B      (b) A-20% B      (c) A-55% B  
(d) A-87% B      (e) A-95% B

**Solution:** (a) A-10% B is a good candidate: Solution Treatment @  $T = 290$  to  $400^{\circ}\text{C}$   
quench  
Age @  $T < 290^{\circ}\text{C}$

(b) A-20% B: Some age hardening effect may occur when alloy is solution treated below  $400^{\circ}\text{C}$  and quenched. However, eutectic is also present and the strengthening effect will not be as dramatic as in (a).



- (c) A–55% B: almost all  $\theta$  is formed. The alloy is expected to be very brittle.
- (d) A–87% B: the alloy cools from a two-phase ( $\beta + \theta$ ) region to a one-phase ( $\beta$ ) region, opposite of what we need for age hardening.
- (e) A–95% B: the alloy is single phase ( $\beta$ ) at all temperatures and thus cannot be age hardened.

**11–51** Figure 11–1 shows the sigmoidal curve for the transformation of austenite. Determine the constants  $c$  and  $n$  in Equation 11-2 for this reaction. By comparing this figure with the TTT diagram, Figure 11–21, estimate the temperature at which this transformation occurred.

Solution:

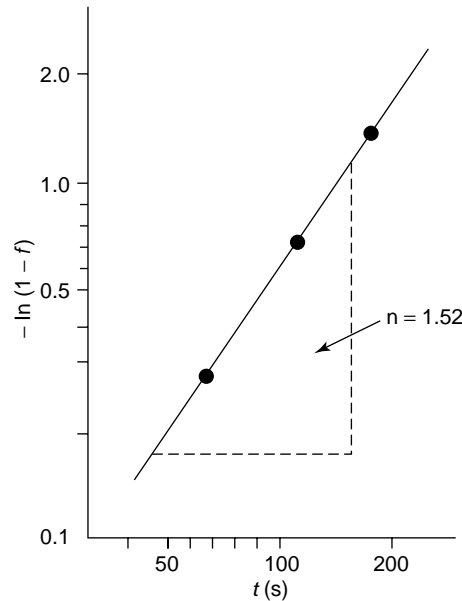
$f$	$1 - f$	$-\ln(1 - f)$	$t(\text{s})$
0.25	0.75	0.288	63 s
0.50	0.50	0.69	110 s
0.75	0.25	1.39	170 s

From the log-log plot of “ $-\ln(1 - f)$ ” versus “ $t$ ”, we find that the slope  $n = 1.52$  and since  $t = 110$  s when  $f = 0.5$ ,

$$0.5 = 1 - \exp[-c(110)^{1.52}]$$

$$c = 5.47 \times 10^{-4}$$

Figure 11–1 shows that the transformation begins at about 20 s and ends at about 720 s. Based on the TTT diagram (Figure 11–21), the transformation temperature must be about 680°C.



- 11–52** For an Fe–0.35%C alloy, determine (a) the temperature at which austenite first begins to transform on cooling, (b) the primary microconstituent that forms, (c) the composition and amount of each phase present at 728°C, (d) the composition and amount of each phase present at 726°C, and (e) the composition and amount of each microconstituent present at 726°C.

**Solution:** (a) 795°C (b) primary  $\alpha$ -ferrite

$$\begin{aligned} \text{(c) } \alpha: 0.0218\% \text{ C} \quad \% \alpha &= \frac{0.77 - 0.35}{0.77 - 0.0218} \times 100 = 56.1\% \\ \gamma: 0.77\% \text{ C} \quad \% \gamma &= 43.9\% \end{aligned}$$

$$\begin{aligned} \text{(d) } \alpha: 0.0218\% \text{ C} \quad \% \alpha &= \frac{6.67 - 0.35}{6.67 - 0.0218} \times 100 = 95.1\% \\ \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{Fe}_3\text{C} &= 4.9\% \end{aligned}$$

$$\begin{aligned} \text{(e) primary } \alpha: 0.0218\% \text{ C} \quad \% \text{ primary } \alpha &= 56.1\% \\ \text{pearlite: } 0.77\% \text{ C} \quad \% \text{ Pearlite} &= 43.9\% \end{aligned}$$

- 11–53** For an Fe–1.15%C alloy, determine (a) the temperature at which austenite first begins to transform on cooling, (b) the primary microconstituent that forms, (c) the composition and amount of each phase present at 728°C, (d) the composition and amount of each phase present at 726°C, and (e) the composition and amount of each microconstituent present at 726°C.

**Solution:** (a) 880°C (b) primary  $\text{Fe}_3\text{C}$

$$\begin{aligned} \text{(c) } \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{Fe}_3\text{C} &= \frac{1.15 - 0.77}{6.67 - 0.77} \times 100 = 6.4\% \\ \gamma: 0.77\% \text{ C} \quad \% \gamma &= 93.6\% \end{aligned}$$

$$\begin{aligned} \text{(d) } \alpha: 0.0218\% \text{ C} \quad \% \alpha &= \frac{6.67 - 1.15}{6.67 - 0.0218} \times 100 = 83\% \\ \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{Fe}_3\text{C} &= 17\% \end{aligned}$$

$$\begin{aligned} \text{(e) primary } \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{ primary } \text{Fe}_3\text{C} &= 6.4\% \\ \text{pearlite: } 0.77\% \text{ C} \quad \% \text{ Pearlite} &= 93.6\% \end{aligned}$$

- 11–54** A steel contains 8% cementite and 92% ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

**Solution:**  $\alpha = 0.92 = \frac{6.67 - x}{6.67 - 0}$   $x = 0.53\% \text{ C}$ ,  $\therefore$  Hypoeutectoid

- 11–55** A steel contains 18% cementite and 82% ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

**Solution:**  $\alpha = 0.82 = \frac{6.67 - x}{6.67 - 0}$   $x = 1.20\% \text{ C}$ ,  $\therefore$  Hypereutectoid

- 11–56** A steel contains 18% pearlite and 82% primary ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

Solution:  $\text{primary } \alpha = 0.82 = \frac{0.77 - x}{0.77 - 0.0218}$ ,  
 $x = 0.156\% \text{ C}$ ,  $\therefore$  Hypoeutectoid

- 11–57** A steel contains 94% pearlite and 6% primary cementite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

Solution:  $\text{Pearlite} = 0.94 = \frac{6.67 - x}{6.67 - 0.77}$ ,  $x = 1.124\% \text{ C}$ ,  $\therefore$  Hypereutectoid

- 11–58** A steel contains 55%  $\alpha$  and 45%  $\gamma$  at 750°C. Estimate the carbon content of the steel.

Solution:  $\alpha = 0.02\% \text{ C}$  and  $\gamma = 0.6\% \text{ C}$  (from the tie line at 750°C)  
 $\% \alpha = 55 = \frac{0.6 - x}{0.6 - 0.02} \times 100$   $x = 0.281\% \text{ C}$

- 11–59** A steel contains 96%  $\gamma$  and 4%  $\text{Fe}_3\text{C}$  at 800°C. Estimate the carbon content of the steel.

Solution:  $\gamma = 0.92\% \text{ C}$  and  $\text{Fe}_3\text{C} = 6.67\% \text{ C}$  (from the tie line at 800°C)  
 $\gamma = 0.96 = \frac{6.67 - x}{6.67 - 0.92}$   $x = 1.15\% \text{ C}$

- 11–60** A steel is heated until 40% austenite, with a carbon content of 0.5%, forms. Estimate the temperature and the overall carbon content of the steel.

Solution: In order for  $\gamma$  to contain 0.5% C, the austenitizing temperature must be about 760°C (from the tie line). At this temperature:  
 $0.4 = \frac{x - 0.02}{0.5 - 0.02}$   $x = 0.212\% \text{ C}$

- 11–61** A steel is heated until 85% austenite, with a carbon content of 1.05%, forms. Estimate the temperature and the overall carbon content of the steel.

Solution: In order for  $\gamma$  to contain 1.05% C, the austenitizing temperature must be about 845°C (from the tie line). At this temperature:  
 $0.85 = \frac{6.67 - x}{6.67 - 1.05}$   $x = 1.893\% \text{ C}$

- 11–62** Determine the eutectoid temperature, the composition of each phase in the eutectoid reaction, and the amount of each phase present in the eutectoid microconstituent for the following systems. For the metallic systems, comment on whether you expect the eutectoid microconstituent to be ductile or brittle.

- (a)  $\text{ZrO}_2\text{--CaO}$  (See Figure 11–33)
- (b)  $\text{Cu--Al}$  at 11.8% Al (See Figure 11–34(c))
- (c)  $\text{Cu--Zn}$  at 47% Zn (See Figure 11–34(a))
- (d)  $\text{Cu--Be}$  (See Figure 11–34(d))

**Solution:** (a) @900°C: Tetragonal<sub>12% CaO</sub> → Monoclinic<sub>3% CaO</sub> + Cubic<sub>14% CaO</sub>

$$\% \text{Monoclinic} = \frac{14 - 12}{14 - 3} \times 100 = 18\% \quad \% \text{Cubic} = 82\%$$

The eutectoid microconstituent (and the entire material, for that matter) will be brittle because the materials are ceramics)

(b) @565°C:  $\beta_{11.8\% \text{ Al}} \rightarrow \alpha_{9.4\% \text{ Al}} + \gamma_{15.6\% \text{ Al}}$

$$\% \alpha = \frac{15.6 - 11.8}{15.6 - 9.4} \times 100 = 61.3\% \quad \% \beta = 38.7\%$$

Most of the eutectoid microconstituent is  $\alpha$  (solid solution strengthened copper) and is expected to be ductile.

(c) @250°C:  $\beta'_{47\% \text{ Zn}} \rightarrow \alpha_{36\% \text{ Zn}} + \gamma_{59\% \text{ Zn}}$

$$\% \alpha = \frac{59 - 47}{59 - 36} \times 100 = 52.2\% \quad \% \gamma = 47.8\%$$

Slightly more than half of the eutectoid is the copper solid solution; there is a good chance that the eutectoid would be ductile.

(d) @605°C:  $\gamma_{16\% \text{ Be}} \rightarrow \alpha_{1.5\% \text{ Be}} + \gamma_{11\% \text{ Be}}$

$$\% \alpha = \frac{11 - 6}{11 - 1.5} \times 100 = 52.6\% \quad \% \beta = 47.4\%$$

Slightly more than half of the eutectoid is the copper solid solution; we might then expect the eutectoid to be ductile.

**11–64** Compare the interlamellar spacing and the yield strength when an eutectoid steel is isothermally transformed to pearlite at (a) 700°C, and (b) 600°C.

**Solution:** We can find the interlamellar spacing from Figure 11–20 and then use this spacing to find the strength from Figure 11–19.

$$(a) \lambda = 7.5 \times 10^{-5} \text{ cm} \quad 1/\lambda = 13,333 \quad \text{YS} = 200 \text{ MPa (29,400 psi)}$$

$$(b) \lambda = 1.5 \times 10^{-5} \text{ cm} \quad 1/\lambda = 66,667 \quad \text{YS} = 460 \text{ MPa (67,600 psi)}$$

**11–72** An isothermally transformed eutectoid steel is found to have a yield strength of 410 MPa. Estimate (a) the transformation temperature and (b) the interlamellar spacing in the pearlite.

**Solution:** We can first find the interlamellar spacing from Figure 11–19; then using this interlamellar spacing, we can find the transformation temperature from Figure 11–20.

$$(a) \text{ transformation temperature} = 615^\circ\text{C}$$

$$(b) 1/\lambda = 60,000 \quad \text{or} \quad \lambda = 1.67 \times 10^{-5} \text{ cm}$$

**11–73** Determine the required transformation temperature and microconstituent if an eutectoid steel is to have the following hardness values:

- (a) HRC 38      (b) HRC 42      (c) HRC 48      (d) HRC 52

Solution: (a) 600°C pearlite (b) 400°C bainite (c) 340°C bainite (d) 300°C bainite

- 11-74** Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C for 1 h, quenched to 350°C and held for 750 s, and finally quenched to room temperature.

Solution: HRC = 47 and the microstructure is all bainite.

- 11-75** Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 650°C and held for 500 s, and finally quenched to room temperature.

Solution: HRC = 25 and the microstructure is all pearlite.

- 11-76** Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 300°C and held for 10 s, and finally quenched to room temperature.

Solution: HRC = 66 and the microstructure is all martensite.

- 11-77** Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 300°C and held for 10 s, quenched to room temperature, and then reheated to 400°C before finally cooling to room temperature again.

Solution: HRC = 42 and the microstructure is all tempered martensite.

- 11-78** A steel containing 0.3% C is heated to various temperatures above the eutectoid temperature, held for 1 h, and then quenched to room temperature. Using Figure 11-35, determine the amount, composition, and hardness of any martensite that forms when the heating temperature is

(a) 728°C (b) 750°C (c) 790°C (d) 850°C

Solution: (a)  $\gamma$ : 0.77% C  $\%M = \frac{0.3 - 0.0218}{0.77 - 0.0218} \times 100\% = 37.2\%$  HRC 65

(b)  $\gamma$ : 0.60% C  $\%M = \frac{0.3 - 0.02}{0.6 - 0.02} \times 100\% = 48.3\%$  HRC 65

(c)  $\gamma$ : 0.35% C  $\%M = \frac{0.3 - 0.02}{0.35 - 0.02} \times 100\% = 84.8\%$  HRC 58

(d)  $\gamma$ : 0.3% C  $\%M = 100\%$  HRC 55

- 11-86** A steel containing 0.95% C is heated to various temperatures above the eutectoid temperature, held for 1 h, and then quenched to room temperature. Using Figure 11-35, determine the amount and composition of any martensite that forms when the heating temperature is

(a) 728°C (b) 750°C (c) 780°C (d) 850°C

**Solution:** (a)  $\gamma = 0.77\% \text{ C}$   $\%M = \frac{6.67 - 0.95}{6.67 - 0.77} \times 100\% = 96.9\% \quad \text{HRC } 65$

(b)  $\gamma = 0.82\% \text{ C}$   $\%M = \frac{6.67 - 0.95}{6.67 - 0.82} \times 100\% = 97.8\% \quad \text{HRC } 65$

(c)  $\gamma = 0.88\% \text{ C}$   $\%M = \frac{6.67 - 0.95}{6.67 - 0.88} \times 100\% = 98.8\% \quad \text{HRC } 65$

(d)  $\gamma = 0.95\% \text{ C}$   $\%M = 100\% \quad \text{HRC } 65$

- 11–87** A steel microstructure contains 75% martensite and 25% ferrite; the composition of the martensite is 0.6% C. Using Figure 11–35, determine (a) the temperature from which the steel was quenched and (b) the carbon content of the steel.

**Solution:** In order for  $\gamma$  (and therefore martensite) to contain 0.6% C, the austenitizing  $T = 750^\circ\text{C}$ . Then:

$$M = \gamma = 0.25 = \frac{0.6 - x}{0.6 - 0.02} \quad x = 0.455\% \text{ C}$$

- 11–88** A steel microstructure contains 92% martensite and 8%  $\text{Fe}_3\text{C}$ ; the composition of the martensite is 1.10% C. Using Figure 11–35, determine (a) the temperature from which the steel was quenched and (b) the carbon content of the steel.

**Solution:** In order for  $\gamma$  (and therefore martensite) to contain 1.10% C, the austenitizing  $T = 865^\circ\text{C}$ . Then:

$$M = \gamma = 0.92 = \frac{6.67 - x}{6.67 - 1.10} \quad x = 1.55\% \text{ C}$$

- 11–89** A steel containing 0.8% C is quenched to produce all martensite. Estimate the volume change that occurs, assuming that the lattice parameter of the austenite is 3.6 Å. Does the steel expand or contract during quenching?

**Solution:**  $V_\gamma = (3.6 \text{ Å})^3 = 46.656 \times 10^{-24} \text{ cm}^3$

$$V_M = a^2c = (2.85 \times 10^{-8} \text{ cm})^2(2.96 \times 10^{-8}) = 24.0426 \times 10^{-24} \text{ cm}^3$$

But to assure that we have the same number of atoms, we need to consider two unit cells of martensite (2 atoms/cell) for each cell of FCC austenite (4 atoms/cell)

$$\% \Delta V = \left[ \frac{(2)(24.0426) - 46.656}{46.656} \right] \times 100\% = 3.06\%, \therefore \text{expansion}$$

- 11–90** Describe the complete heat treatment required to produce a quenched and tempered eutectoid steel having a tensile strength of at least 125,000 psi. Include appropriate temperatures.

**Solution:** Austenitize at approximately  $750^\circ\text{C}$ ,  
 Quench to below  $130^\circ\text{C}$  (the  $M_f$  temperature)  
 Temper at  $620^\circ\text{C}$  or less.

- 11–91** Describe the complete heat treatment required to produce a quenched and tempered eutectoid steel having a HRC hardness of less than 50. Include appropriate temperatures.

**Solution:** Austenitize at approximately 750°C,  
 Quench to below the  $M_f$  (less than 130°C)  
 Temper at a temperature higher than 330°C, but less than 727°C.

- 11–92** In eutectic alloys, the eutectic microconstituent is generally the continuous one, but in the eutectoid structures, the primary microconstituent is normally continuous. By describing the changes that occur with decreasing temperature in each reaction, explain why this difference is expected.

**Solution:** In a eutectoid reaction, the original grain boundaries serve as nucleation sites; consequently the primary microconstituent outlines the original grain boundaries and isolates the eutectoid product as a discontinuous constituent.

In a eutectic reaction, the primary phase nucleates from the liquid and grows. When the liquid composition approaches the eutectic composition, the eutectic constituent forms around the primary constituent, making the eutectic product the continuous constituent.

# 12

## Ferrous Alloys

**12–4** Calculate the amounts of ferrite, cementite, primary microconstituent, and pearlite in the following steels: (a) 1015, (b) 1035, (c) 1095, and (d) 10130.

Solution: (a) 1015:

$$\alpha = \frac{6.67 - 0.15}{6.67 - 0} \times 100 = 97.8\% \quad \text{Fe}_3\text{C} = 2.2\%$$

$$\text{primary } \alpha = \frac{0.77 - 0.15}{0.77 - 0.0218} \times 100 = 82.9\% \quad \text{pearlite} = 17.1\%$$

(b) 1035:

$$\alpha = \frac{6.67 - 0.35}{6.67 - 0} \times 100 = 94.8\% \quad \text{Fe}_3\text{C} = 5.2\%$$

$$\text{primary } \alpha = \frac{0.77 - 0.35}{0.77 - 0.0218} \times 100 = 56.1\% \quad \text{pearlite} = 43.9\%$$

(c) 1095:

$$\alpha = \frac{6.67 - 0.95}{6.67 - 0} \times 100 = 85.8\% \quad \text{Fe}_3\text{C} = 14.2\%$$

$$\text{primary Fe}_3\text{C} = \frac{0.95 - 0.77}{6.67 - 0.77} \times 100 = 3.1\% \quad \text{pearlite} = 96.9\%$$

(d) 10130:

$$\alpha = \frac{6.67 - 1.30}{6.67 - 0} \times 100 = 80.5\% \quad \text{Fe}_3\text{C} = 19.5\%$$

$$\text{primary Fe}_3\text{C} = \frac{1.30 - 0.77}{6.67 - 0.77} \times 100 = 9.0\% \quad \text{pearlite} = 91.0\%$$



**12–5** Estimate the AISI-SAE number for steels having the following microstructures.

- (a) 38% pearlite – 62% primary ferrite
- (b) 93% pearlite – 7% primary cementite
- (c) 97% ferrite – 3% cementite
- (d) 86% ferrite – 14% cementite

**Solution:** (a) 38% pearlite – 62% primary ferrite

$$62\% = \frac{0.77 - x}{0.77 - 0.0218} \times 100 \quad x = 0.306\% \text{ C} \quad 1030 \text{ steel}$$

(b) 93% pearlite – 7% primary cementite

$$93\% = \frac{6.67 - x}{6.67 - 0.77} \times 100 \quad x = 1.183\% \text{ C} \quad 10120 \text{ steel}$$

(c) 97% ferrite – 3% cementite

$$97\% = \frac{6.67 - x}{6.67 - 0} \times 100 \quad x = 0.200\% \text{ C} \quad 1020 \text{ steel}$$

(d) 86% ferrite – 14% cementite

$$86\% = \frac{6.67 - x}{6.67 - 0} \times 100 \quad x = 0.934\% \text{ C} \quad 1095 \text{ steel}$$

**12–6** Complete the following table:

<b>Solution:</b>	1035 steel	10115 steel
$A_1$ temperature	727°C	727°C
$A_3$ or $A_{cm}$ temperature	790°C	880°C
Full annealing temperature	820°C	757°C
Normalizing temperature	845°C	935°C
Process annealing temperature	557–647°C	—
Spheroidizing temperature	—	697°C

**12–10** In a pearlitic 1080 steel, the cementite platelets are  $4 \times 10^{-5}$  cm thick, and the ferrite platelets are  $14 \times 10^{-5}$  cm thick. In a spheroidized 1080 steel, the cementite spheres are  $4 \times 10^{-3}$  cm in diameter. Estimate the total interface area between the ferrite and cementite in a cubic centimeter of each steel. Determine the percent reduction in surface area when the pearlitic steel is spheroidized. The density of ferrite is  $7.87 \text{ g/cm}^3$  and that of cementite is  $7.66 \text{ g/cm}^3$ .

**Solution:** First, we can determine the weight and volume percents of  $\text{Fe}_3\text{C}$  in the steel:

$$\text{wt\% Fe}_3\text{C} = \frac{0.80 - 0.0218}{6.67 - 0.0218} \times 100 = 11.705$$

$$\text{vol\% Fe}_3\text{C} = \frac{11.705/7.66}{(11.705/7.66) + (88.295/7.87)} \times 100 = 11.987$$

Pearlite: Based on the thicknesses of the ferrite and cementite platelets in pearlite, there are two interfaces per  $(4 \times 10^{-5} \text{ cm} + 14 \times 10^{-5} \text{ cm}) = 18 \times 10^{-5} \text{ cm}$ , or:

$$2 \text{ interfaces} / 18 \times 10^{-5} \text{ cm} = 1.1 \times 10^4 \text{ interfaces/cm}$$

If all of the platelets are parallel to one another, then in  $1 \text{ cm}^3$  of pearlite, there is a total of

$$A = (1.1 \times 10^4 / \text{cm})(1 \text{ cm}^3) = 11,000 \text{ cm}^2 \text{ of interface/cm}^3$$

Spheroidite: The volume of an  $\text{Fe}_3\text{C}$  sphere with  $r = 2 \times 10^{-3} \text{ cm}$  is:

$$V = (4\pi/3)(2 \times 10^{-3} \text{ cm})^3 = 3.35 \times 10^{-8} \text{ cm}^3$$

The volume of  $\text{Fe}_3\text{C}$  in  $1 \text{ cm}^3$  of spheroidite is given by the volume fraction of cementite, or 0.11987. The number of spheres in  $1 \text{ cm}^3$  of spheroidite is:

$$\text{number} = 0.11987 \text{ cm}^3 / 3.35 \times 10^{-8} \text{ cm}^3 = 3.58 \times 10^6 \text{ spheres/cm}^3$$

The surface area of the spheres is therefore:

$$\begin{aligned} A &= 4\pi(2 \times 10^{-3} \text{ cm})^2(3.58 \times 10^6 \text{ spheres/cm}^3) \\ &= 180 \text{ cm}^2 \text{ of interface/cm}^3 \end{aligned}$$

The percent reduction in surface area during spheroidizing is then:

$$\% = \frac{(11,000 - 180) \text{ cm}^2}{11,000 \text{ cm}^2} \times 100 = 98.4\%$$

**12–11** Describe the microstructure present in a 1050 steel after each step in the following heat treatments:

- Heat at  $820^\circ\text{C}$ , quench to  $650^\circ\text{C}$  and hold for 90 s, and quench to  $25^\circ\text{C}$
- Heat at  $820^\circ\text{C}$ , quench to  $450^\circ\text{C}$  and hold for 90 s, and quench to  $25^\circ\text{C}$
- Heat at  $820^\circ\text{C}$  and quench to  $25^\circ\text{C}$
- Heat at  $820^\circ\text{C}$ , quench to  $720^\circ\text{C}$  and hold for 100 s, and quench to  $25^\circ\text{C}$
- Heat at  $820^\circ\text{C}$ , quench to  $720^\circ\text{C}$  and hold for 100 s, quench to  $400^\circ\text{C}$  and hold for 500 s, and quench to  $25^\circ\text{C}$
- Heat at  $820^\circ\text{C}$ , quench to  $720^\circ\text{C}$  and hold for 100 s, quench to  $400^\circ\text{C}$  and hold for 10 s, and quench to  $25^\circ\text{C}$
- Heat at  $820^\circ\text{C}$ , quench to  $25^\circ\text{C}$ , heat to  $500^\circ\text{C}$  and hold for  $10^3 \text{ s}$ , and air cool to  $25^\circ\text{C}$

**Solution:** (a) Austenite is present after heating to  $820^\circ\text{C}$ ; both ferrite and pearlite form during holding at  $650^\circ\text{C}$ ; ferrite and pearlite remain after cooling to  $25^\circ\text{C}$ .

(b) Austenite is present after heating to  $820^\circ\text{C}$ ; bainite forms after holding at  $450^\circ\text{C}$ ; and bainite remains after cooling.

(c) Austenite is present after heating to  $820^\circ\text{C}$ ; martensite forms due to the quench.

(d) Austenite is present after heating to  $820^\circ\text{C}$ ; ferrite forms at  $720^\circ\text{C}$ , but some austenite still remains. During quenching, the remaining austenite forms martensite; the final structure is ferrite and martensite.

- (e) Austenite is present after heating to 820°C; ferrite begins to form at 720°C, but austenite still remains. At 400°C, the remaining austenite transforms to bainite; the final structure contains ferrite and bainite.
- (f) Austenite is present after heating to 820°C; ferrite begins to form at 720°C; some of the remaining austenite transforms to bainite at 400°C, but some austenite still remains after 10 s; the remaining austenite transforms to martensite during quenching. The final structure is ferrite, bainite, and martensite.
- (g) Austenite is present after heating to 820°C. The austenite transforms to martensite during quenching. During reheating to 500°C, the martensite tempers. The final structure is tempered martensite. Note that the TTT diagram isn't really needed for this part of the question.

**12–12** Describe the microstructure present in a 10110 steel after each step in the following heat treatments:

- (a) Heat to 900°C, quench to 400°C and hold for  $10^3$  s, and quench to 25°C
- (b) Heat to 900°C, quench to 600°C and hold for 50 s, and quench to 25°C
- (c) Heat to 900°C and quench to 25°C
- (d) Heat to 900°C, quench to 300°C and hold for 200 s, and quench to 25°C
- (e) Heat to 900°C, quench to 675°C and hold for 1 s, and quench to 25°C
- (f) Heat to 900°C, quench to 675°C and hold for 1 s, quench to 400°C and hold for 900 s, and slowly cool to 25°C
- (g) Heat to 900°C, quench to 675°C and hold for 1 s, quench to 300°C and hold for  $10^3$  s, and air cool to 25°C.
- (h) Heat to 900°C, quench to 300°C and hold for 100 s, quench to 25°C, heat to 450°C for 3600 s, and cool to 25°C.

**Solution:** (a) Austenite forms at 900°C. At 400°C, all of the austenite transforms to bainite. The final structure is all bainite.

(b) Austenite forms at 900°C. At 600°C, all of the austenite transforms to cementite and pearlite, which gives the final structure.

(c) Austenite forms at 900°C. All of the austenite transforms to martensite during quenching.

(d) Austenite forms at 900°C. None of the austenite transforms within 200 s at 300°C; consequently all of the austenite forms martensite during quenching. This is a martempering heat treatment.

(e) Austenite forms at 900°C. Cementite begins to form at 675°C; the remainder of the austenite transforms to martensite during quenching to 25°C. The final structure is cementite and martensite.

(f) Austenite forms at 900°C. Cementite begins to form at 675°C. The remaining austenite transforms to bainite at 400°C. The final structure is cementite and bainite.

(g) Austenite forms at 900°C. Cementite begins to form at 675°C. At 300°C, some of the remaining austenite transforms to bainite, but the  $B_f$  line is not crossed. The remaining austenite forms martensite during air cooling. The final structure is cementite, bainite, and martensite.

- (h) Austenite forms at 900°C. No transformation occurs at 300°C, since the time is too short. Consequently all of the austenite transforms to martensite during quenching. Reheating to 450°C for 3600 s (1 hour) tempers the martensite. The final structure is tempered martensite.

**12–13** Recommend appropriate isothermal heat treatments to obtain the following, including appropriate temperatures and times:

- (a) an isothermally annealed 1050 steel with HRC 23
- (b) an isothermally annealed 10110 steel with HRC 40
- (c) an isothermally annealed 1080 steel with HRC 38
- (d) an austempered 1050 steel with HRC 40
- (e) an austempered 10110 steel with HRC 55
- (f) an austempered 1080 steel with HRC 50

**Solution:**

- (a) Austenitize at 820°C  
Quench to 600°C and hold for more than 10 s  
Cool to room temperature
- (b) Austenitize at 900°C  
quench to 640°C and hold for more than 10 s  
Cool to room temperature
- (c) Austenitize at 780°C  
Quench to 600°C for more than 10 s  
Cool to room temperature
- (d) Austenitize at 820°C  
quench to 390°C and hold for 100 s  
Cool to room temperature
- (e) Austenitize at 900°C  
quench to 320°C and hold for 5000 s  
Cool to room temperature
- (f) Austenitize at 780°C  
quench to 330°C and hold for 1000 s  
Cool to room temperature

**12–14** Compare the minimum times required to isothermally anneal the following steels at 600°C. Discuss the effect of the carbon content of the steel on the kinetics of nucleation and growth during the heat treatment.

- (a) 1050      (b) 1080      (c) 10110

**Solution:**

- (a) 1050: The  $P_f$  time is about 5 s, the minimum time
- (b) 1080: The  $P_f$  time is about 10 s, the minimum time
- (c) 10110: The  $P_f$  time is about 3 s, the minimum time

The carbon content has relatively little effect on the minimum annealing time (or the  $P_f$  time). The longest time is obtained for the 1080, or eutectoid, steel.

- 12–16** We wish to produce a 1050 steel that has a Brinell hardness of at least 330 and an elongation of at least 15% (a) Recommend a heat treatment, including appropriate temperatures, that permits this to be achieved. Determine the yield strength and tensile strength that are obtained by this heat treatment. (b) What yield and tensile strength would be obtained in a 1080 steel by the same heat treatment? (c) What yield strength, tensile strength, and %elongation would be obtained in the 1050 steel if it were normalized?

**Solution:** (a) It is possible to obtain the required properties; the Brinell hardness is obtained if the steel is quenched and then tempered at a temperature below 480°C, and the %elongation can be obtained if the tempering temperature is greater than 420°C. Therefore a possible heat treatment would be:

Austenitize at 820°C  
 quench to room temperature  
 Temper between 420°C and 480°C  
 Cool to room temperature

The quench and temper heat treatment will also give a yield strength between 140,000 and 160,000 psi, while the tensile strength will be between 150,000 and 180,000 psi. The higher strengths are obtained for the lower tempering temperatures.

(b) If a 1080 steel is tempered in the same way (Figure 11–26), the yield strength would lie between 130,000 and 135,000 psi, and the tensile strength would be 175,000 to 180,000 psi. The higher strengths are obtained for the lower tempering temperatures.

(c) If the 1050 steel were normalized rather than quench and tempered, the properties would be about (from Figure 12–5):

100,000 psi tensile strength  
 65,000 psi yield strength  
 20% elongation

- 12–17** We wish to produce a 1050 steel that has a tensile strength of at least 175,000 psi and a %Reduction in area of at least 50%. (a) Recommend a heat treatment, including appropriate temperatures, that permits this to be achieved. Determine the Brinell hardness number, %elongation, and yield strength that are obtained by this heat treatment. (b) What yield strength and tensile strength would be obtained in a 1080 steel by the same heat treatment? (c) What yield strength, tensile strength, and %elongation would be obtained in the 1050 steel if it were annealed?

**Solution:** (a) Using a quench and temper heat treatment, we can obtain the minimum tensile strength by tempering below 430°C, and the minimum reduction in area by tempering above 400°C. Our heat treatment is then:

Austenitize at 820°C  
 Quench to room temperature  
 Temper between 400°C and 430°C  
 Cool to room temperature

This heat treatment will also give:

390 to 405 BH  
 14 to 15% elongation  
 160,000 to 165,000 psi yield strength

(b) If the same treatment is used for a 1080 steel, the properties would be:

140,000 psi yield strength  
180,000 psi tensile strength

(c) If the 1050 steel is annealed, the properties are (From Figure 12-5)

52,000 psi yield strength  
85,000 psi tensile strength  
25% elongation

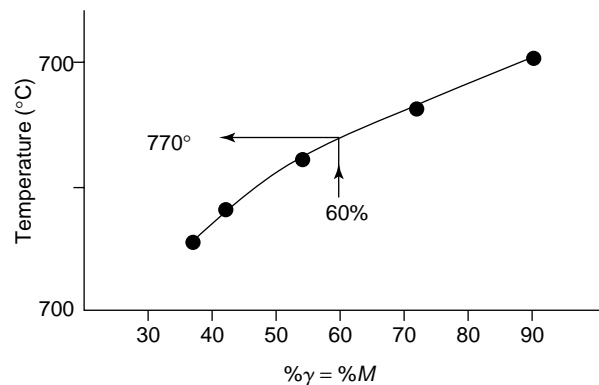
**12-18** A 1030 steel is given an improper quench and temper heat treatment, producing a final structure composed of 60% martensite and 40% ferrite. Estimate the carbon content of the martensite and the austenitizing temperature that was used. What austenitizing temperature would you recommend?

**Solution:** We can work a lever law at several temperatures in the  $\alpha + \gamma$  region of the iron-carbon phase diagram, finding the amount of austenite (and its composition) at each temperature. The composition of the ferrite at each of these temperatures is about 0.02% C. The amount and composition of the martensite that forms will be the same as that of the austenite:

at 800°C:  $\gamma$ : 0.33% C    %  $\gamma$  =  $(0.30 - 0.02)/(0.33 - 0.02) = 90\%$   
at 780°C:  $\gamma$ : 0.41% C    %  $\gamma$  =  $(0.30 - 0.02)/(0.41 - 0.02) = 72\%$   
at 760°C:  $\gamma$ : 0.54% C    %  $\gamma$  =  $(0.30 - 0.02)/(0.54 - 0.02) = 54\%$   
at 740°C:  $\gamma$ : 0.68% C    %  $\gamma$  =  $(0.30 - 0.02)/(0.68 - 0.02) = 42\%$   
at 727°C:  $\gamma$ : 0.77% C    %  $\gamma$  =  $(0.30 - 0.02)/(0.77 - 0.02) = 37\%$

The amount of austenite (equal to that of the martensite) is plotted versus temperature in the graph. Based on this graph, 60% martensite forms when the austenitizing temperature is about 770°C. The carbon content of the martensite that forms is about 0.48%C.

The  $A_3$  temperature of the steel is about 805°C. A proper heat treatment might use an austenitizing temperature of about 805°C + 55°C = 860°C.



**12-19** A 1050 steel should be austenitized at 820°C, quenched in oil to 25°C, and tempered at 400°C for an appropriate time. (a) What yield strength, hardness, and %elongation would you expect to obtain from this heat treatment? (b) Suppose the actual yield strength of the steel is found to be 125,000 psi. What might have gone wrong in the heat treatment to cause this low strength? (c) Suppose the hardness is found to be HB 525. What might have gone wrong in the heat treatment to cause this high hardness?

**Solution:** (a) The properties expected for a proper heat treatment are:

170,000 psi yield strength  
190,000 psi tensile strength  
405 HB  
14% elongation

(b) If the yield strength is 125,000 psi (much lower than expected), then the tempering process might have been done at a tempering temperature greater than 400°C (perhaps as high as 500°C). Another possible problem could be an austenitizing temperature that was lower than 820°C (even lower than about 770°C, the  $A_3$ ), preventing complete austenitizing and thus not all martensite during the quench.

(c) If the hardness is HB 525 (higher than expected), the tempering temperature may have been too low or, in fact, the steel probably was not tempered at all.

**12–20** A part produced from a low alloy, 0.2% C steel (Figure 12–17) has a microstructure containing ferrite, pearlite, bainite, and martensite after quenching. What microstructure would be obtained if we had used a 1080 steel? What microstructure would be obtained if we had used a 4340 steel?

**Solution:** To produce ferrite, pearlite, bainite, and martensite in the same microstructure during continuous cooling, the cooling rate must have been between 10 and 20°C/s. If the same cooling rates are used for the other steels, the microstructures are:

1080 steel: fine pearlite  
4340 steel: martensite

**12–21** Fine pearlite and a small amount of martensite are found in a quenched 1080 steel. What microstructure would be expected if we had used a low alloy, 0.2% C steel? What microstructure would be expected if we had used a 4340 steel?

**Solution:** A cooling rate of about 50°C/s will produce fine pearlite and a small amount of martensite in the 1080 steel. For the same cooling rate, the microstructure in the other steels will be:

low alloy, 0.2% C steel: ferrite, bainite, and martensite  
4340 steel: martensite

**12–26** We have found that a 1070 steel, when austenitized at 750°C, forms a structure containing pearlite and a small amount of grain boundary ferrite that gives acceptable strength and ductility. What changes in the microstructure, if any, would be expected if the 1070 steel contained an alloying element, such as Mo or Cr? Explain.

**Solution:** The alloying element may shift the eutectoid carbon content to below 0.7% C, making the steel hypereutectoid rather than hypoeutectoid. This in turn means that grain boundary  $Fe_3C$  will form rather than grain boundary ferrite. The grain boundary  $Fe_3C$  will embrittle the steel.

**12–27** Using the TTT diagrams, compare the hardenabilities of 4340 and 1050 steels by determining the times required for the isothermal transformation of ferrite and pearlite ( $F_s$ ,  $P_s$ , and  $P_f$ ) to occur at 650°C.

**Solution:** From the diagrams, we can find the appropriate times:

$$\begin{array}{lll} 4340 \text{ steel: } F_s = 200 \text{ s} & P_s = 3,000 \text{ s} & P_f = 15,000 \text{ s} \\ 1050 \text{ steel: } F_s = 3 \text{ s} & P_s = 10 \text{ s} & P_f = 50 \text{ s} \end{array}$$

Because the transformation times are much longer for the 4340 steel, the 4340 steel has the higher hardenability.

**12–28** We would like to obtain a hardness of HRC 38 to 40 in a quenched steel. What range of cooling rates would we have to obtain for the following steels? Are some steels inappropriate?

- (a) 4340      (b) 8640      (c) 9310      (d) 4320      (e) 1050      (f) 1080

**Solution:** (a) 4340: not applicable; the hardnesses are always much higher than the desired range.

(b) 8640: a Jominy distance of about 18/16 to 20/16 is required to give the desired hardness; this corresponds to a cooling rate of about 3 to 4°C/s

(c) 9310: a Jominy distance of 10/16 to 12/16 is required to give the desired hardness; this corresponds to a cooling rate of 8 to 10°C/s

(d) 4320: a Jominy distance of about 6/16 is required to give the desired hardness; this corresponds to a cooling rate of 22°C/s

(e) 1050: a Jominy distance of 4/16 to 4.5/16 is required to give the desired hardness; this corresponds to a cooling rate of 32 to 36°C/s

(f) 1080: a Jominy distance of 5/16 to 6/16 is required to give the desired hardness; this corresponds to a cooling rate of 16 to 28°C/s

**12–29** A steel part must have an as-quenched hardness of HRC 35 in order to avoid excessive wear rates during use. When the part is made from 4320 steel, the hardness is only HRC 32. Determine the hardness if the part were made under identical conditions, but with the following steels. Which, if any, of these steels would be better choices than 4320?

- (a) 4340      (b) 8640      (c) 9310      (d) 1050      (e) 1080

**Solution:** The Jominy distance that gives a hardness of HRC 32 in the 4320 steel is 9/16 in. The cooling rates, and hence Jominy distances, will be the same 9/16 in. for the other steels. From the hardenability curves, the hardnesses of the other steels are

(a) 4340: HRC 60      (b) 8640: HRC 54

(c) 9310: HRC 40      (d) 1050: HRC 28

(e) 1080: HRC 36

All of the steels except the 1050 steel would develop an as-quenched hardness of at least HRC 35 and would be better choices than the 4320 steel. The 1080 steel might be the best choice, since it will likely be the least expensive (no alloying elements present).



- 12–30** A part produced from a 4320 steel has a hardness of HRC 35 at a critical location after quenching. Determine (a) the cooling rate at that location and (b) the microstructure and hardness that would be obtained if the part were made of a 1080 steel.

**Solution:** (a) To obtain the HRC 35 in a 4320 steel, the Jominy distance must be about 7.5/16 in., corresponding to a cooling rate of 16°C/s.

(b) If the part is produced in a 1080 steel, the cooling rate will still be about 16°C/s. From the CCT diagram for the 1080 steel, the part will contain all pearlite, with a hardness of HRC 38.

- 12–31** A 1080 steel is cooled at the fastest possible rate that still permits all pearlite to form. What is this cooling rate? What Jominy distance, and hardness are expected for this cooling rate?

**Solution:** The fastest possible cooling rate that still permits all pearlite is about 40°C/s. This cooling rate corresponds to a Jominy distance of about 3.5/16 in. From the hardenability curve, the hardness will be HRC 46.

- 12–32** Determine the hardness and the microstructure at the center of a 1.5-in.-diameter 1080 steel bar produced by quenching in (a) unagitated oil, (b) unagitated water, and (c) agitated brine.

**Solution:** (a) unagitated oil: the H-factor for the 1.5-in. bar is 0.25. The Jominy distance will be about 11/16 in., or a cooling rate of 9°C/s. From the CCT diagram, the hardness is HRC 36 and the steel is all pearlite.

(b) unagitated water: the H-factor for the bar is 1.0. The Jominy distance will be about 5/16 in., or a cooling rate of 28°C/s. From the CCT diagram, the hardness is HRC 40 and the steel will contain pearlite.

(c) agitated brine: the H-factor is now 5.0. The Jominy distance is about 3.5/16 in., or a cooling rate of 43°C/s. The steel has a hardness of HRC 46 and the microstructure contains both pearlite and martensite.

- 12–33** A 2-in.-diameter bar of 4320 steel is to have a hardness of at least HRC 35. What is the minimum severity of the quench (H coefficient)? What type of quenching medium would you recommend to produce the desired hardness with the least chance of quench cracking?

**Solution:** The hardness of HRC 35 is produced by a Jominy distance of 7.5/16 in. In order to produce this Jominy distance in a 2-in. diameter bar, the H-coefficient must be greater or equal to 0.9. All of the quenching media described in Table 12–2 will provide this Jominy distance *except* unagitated oil. To prevent quench cracking, we would like to use the least severe quenchant; agitated oil and unagitated water, with  $H = 1.0$ , might be the best choices.

- 12–34** A steel bar is to be quenched in agitated water. Determine the maximum diameter of the bar that will produce a minimum hardness of HRC 40 if the bar is:

(a) 1050      (b) 1080      (c) 4320      (d) 8640      (e) 4340

- Solution:** (a) 1050 steel: The H-coefficient for the agitated water is 4.0. For the 1050 steel, the Jominy distance must be at least 3/16 in. to produce the desired hardness. Therefore the maximum diameter that will permit this Jominy distance (or cooling rate) is 1.3 in.
- (b) 1080 steel: Now the Jominy distance must be at least 5/16 in., with the same H-coefficient. The maximum diameter allowed is 1.9 in.
- (c) 4320 steel: The minimum Jominy distance is 5/16 in., and the maximum diameter of the bar is 1.9 in.
- (d) 8640 steel: The minimum Jominy distance is 18/16 in. Consequently bars with a maximum diameter of much greater than 2.5 in. will produce the desired cooling rate and hardness.
- (e) 4340 steel: Bars with a maximum diameter of much greater than 2.5 in. produce the required cooling rate.

- 12–35** The center of a 1-in.-diameter bar of 4320 steel has a hardness of HRC 40. Determine the hardness and microstructure at the center of a 2-in.-bar of 1050 steel quenched in the same medium.

**Solution:** To obtain HRC 40 in the 4320 steel, we need a Jominy distance of 5/16 in. For a 1-in.-diameter bar, the quenching medium must have a minimum H-coefficient of 0.4. Therefore, if a 2-in. diameter bar is quenched in the same medium (i.e.  $H = 0.4$ ), the Jominy distance will be about 11/16 in.; this Jominy distance produces a hardness of HRC 27 in a 1050 steel.

- 12–39** A 1010 steel is to be carburized using a gas atmosphere that produces 1.0% C at the surface of the steel. The case depth is defined as the distance below the surface that contains at least 0.5% C. If carburizing is done at 1000°C, determine the time required to produce a case depth of 0.01 in. (See Chapter 5 for review.)

**Solution:** The diffusion coefficient for carbon in FCC iron at 1000°C is:  

$$D = 0.23 \exp[-32,900/(1.987)(1273)] = 5.16 \times 10^{-7} \text{ cm}^2/\text{s}$$

The case depth “ $x$ ” is to be 0.01 in. = 0.0254 cm.

From Fick’s law:

$$\frac{1.0 - 0.5}{1.0 - 0.01} = 0.505 = \text{erf}(x/2\sqrt{Dt})$$

$$x/2\sqrt{Dt} = 0.583 \quad (\text{from Table 5-3})$$

$$0.0254/(2)\sqrt{(5.16 \times 10^{-7})t} = 0.583$$

$$t = 920 \text{ s} = 0.25 \text{ h}$$

- 12–40** A 1015 steel is to be carburized at 1050°C for 2 h using a gas atmosphere that produces 1.2% C at the surface of the steel. Plot the percent carbon versus the distance from the surface of the steel. If the steel is slowly cooled after carburizing, determine the amount of each phase and microconstituent at 0.002-in. intervals from the surface. (See Chapter 5.)

**Solution:** The diffusion coefficient for carbon in FCC iron at 1050°C is:

$$D = 0.23 \exp[-32,900/(1.987)(1323)] = 8.44 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$t = 2 \text{ h} = 7200 \text{ s}$$

From Fick's law:

$$\frac{1.2 - c_x}{1.2 - 0.15} = \text{erf}(x/2\sqrt{(8.44 \times 10^{-7})(7200)})$$

$$\text{or } \frac{1.2 - c_x}{1.05} = \text{erf}(6.41x)$$

If  $x = 0.002 \text{ in.} = 0.00508 \text{ cm}$ , then

$$\frac{1.2 - c_x}{1.05} = \text{erf}(0.0326) = 0.037$$

$$c_x = 1.161\% \text{ C}$$

These calculations can be repeated for other values of  $x$ , with the results shown below:

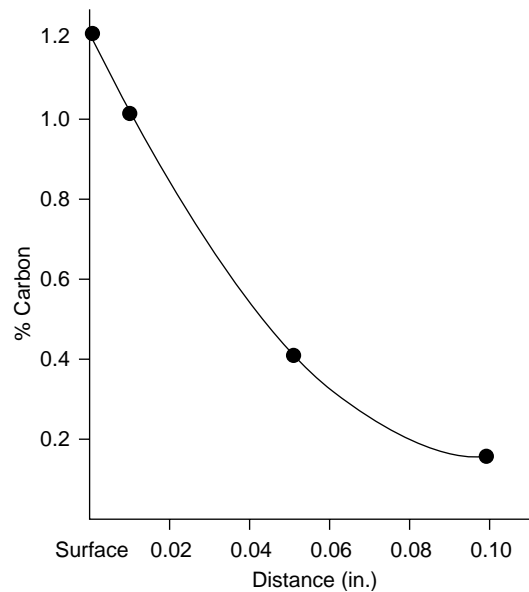
$$x = 0.010 \text{ in.} = 0.0254 \text{ cm} \quad c_x = 1.009\% \text{ °C}$$

$$x = 0.020 \text{ in.} = 0.0508 \text{ cm} \quad c_x = 0.838\% \text{ °C}$$

$$x = 0.050 \text{ in.} = 0.1270 \text{ cm} \quad c_x = 0.413\% \text{ °C}$$

$$x = 0.100 \text{ in.} = 0.2540 \text{ cm} \quad c_x = 0.178\% \text{ °C}$$

The graph shows how the carbon content varies with distance.



- 12-43** A 1050 steel is welded. After cooling, hardnesses in the heat-affected zone are obtained at various locations from the edge of the fusion zone. Determine the hardnesses expected at each point if a 1080 steel were welded under the same conditions. Predict the microstructure at each location in the as-welded 1080 steel.

Distance from edge of Fusion Zone	HRC in 1050 Weld
0.05 mm	50
0.10 mm	40
0.15 mm	32
0.20 mm	28

**Solution:** We can take advantage of the fact that the cooling rate in the two steels will be virtually identical if the welding conditions are the same. Thus at a distance of 0.05 mm from the edge of the fusion zone, the HRC 50 hardness of the 1050 steel is obtained with a Jominy distance of 3/16 in., or a cooling rate of 50°C/s. At the same point in a 1080 steel weldment, the 3/16-in. Jominy distance gives a hardness of HRC 53 (from the hardenability curve) and the 50°C/s cooling rate gives a microstructure of pearlite and martensite (from the CCT curve). The table below shows the results for all four points in the weldment.

distance	Jominy distance	Cooling rate	Hardness	Structure
0.05 mm	3/16 in.	50°C/s	HRC 53	$P + M$
0.10	4/16	36	HRC 46	pearlite
0.15	7/16	17	HRC 38	pearlite
0.20	10/16	10	HRC 36	pearlite

- 12–45** We wish to produce a martensitic stainless steel containing 17% Cr. Recommend a carbon content and austenitizing temperature that would permit us to obtain 100% martensite during the quench. What microstructure would be produced if the martensite were then tempered until the equilibrium phases formed?

**Solution:** We must select a combination of a carbon content and austenitizing temperature that puts us in the all-austenite region of the Fe–Cr–C phase diagram. One such combination is 1200°C and 0.5% C. If a 0.5% C steel is held at 1200°C to produce all austenite, and then is quenched, 100% martensite will form.

If the martensite is tempered until equilibrium is reached, the two phases will be ferrite and  $M_{23}C_6$ . The  $M_{23}C_6$  is typically  $Cr_{23}C_6$ .

- 12–48** Occasionally, when an austenitic stainless steel is welded, the weld deposit may be slightly magnetic. Based on the Fe–Cr–Ni–C phase diagram [Figure 12–30(b)], what phase would you expect is causing the magnetic behavior? Why might this phase have formed? What could you do to restore the nonmagnetic behavior?

**Solution:** The magnetic behavior is caused by the formation of a BCC iron phase, in this case the high temperature  $\delta$ -ferrite. The  $\delta$ -ferrite forms during solidification, particularly when solidification does not follow equilibrium; subsequent cooling is too rapid for the  $\delta$ -ferrite to transform to austenite, and the ferrite is trapped in the microstructure. If the steel is subsequently annealed at an elevated temperature, the  $\delta$ -ferrite can transform to austenite and the steel is no longer magnetic.

- 12–51** A tensile bar of a class 40 gray iron casting is found to have a tensile strength of 50,000 psi. Why is the tensile strength greater than that given by the class number? What do you think is the diameter of the test bar?

**Solution:** The strength of gray iron depends on the cooling rate of the casting; faster cooling rates produce finer microstructures and more pearlite in the microstructure. Although the iron has a nominal strength of 40,000 psi, rapid cooling can produce the fine graphite and pearlite that give the higher 50,000 psi strength.

The nominal 40,000 psi strength is expected for a casting with a diameter of about 1.5 in.; if the bar is only 0.75 in. in diameter, a tensile strength of 50,000 psi might be expected.

**12-52** You would like to produce a gray iron casting that freezes with no primary austenite or graphite. If the carbon content in the iron is 3.5%, what percentage of silicon must you add?

**Solution:** We get neither primary phase when the carbon equivalent (CE) is 4.3%. Thus

$$\begin{aligned} \text{CE} &= 4.3 = \%C + (1/3)\%Si \\ 4.3 &= 3.5 + (1/3)\%Si \quad \text{or} \quad \%Si = 2.4 \end{aligned}$$

**12-53** Compare the expected hardenabilities of a plain carbon steel, a malleable cast iron, and a ductile cast iron. Explain why you expect different hardenabilities.

**Solution:** Plain carbon steels contain very little alloying elements and therefore are expected to have a low hardenability.

Malleable cast irons contain on the order of 1.5% Si; the silicon improves the hardenability of the austenite, making it easier to obtain martensite during quenching.

Ductile cast iron contains more silicon (often 2 to 3%); the higher silicon content gives the ductile iron higher hardenabilities than either plain carbon steels or malleable irons.

# 13

## Nonferrous Alloys

- 13–1** In some cases, we may be more interested in cost per unit volume than in cost per unit weight. Rework Table 13–1 to show the cost in terms of  $\$/\text{cm}^3$ . Does this change/alter the relationship between the different materials?

**Solution:** We can find the density (in  $\text{g}/\text{cm}^3$ ) of each metal from Appendix A. We can convert the cost in  $\$/\text{lb}$  to  $\$/\text{g}$  (using 454  $\text{g}/\text{lb}$ ) and then multiply the cost in  $\$/\text{g}$  by the density, giving  $\$/\text{cm}^3$ . The left hand side of the table shows the results of these conversions, with the metals ranked in order of cost per volume. The right hand side of the table shows the cost per pound.

	cost/volume	rank		cost/lb	rank
Steels	$\$0.0017/\text{cm}^3$	1	Fe	$\$0.10$	1
Al	$\$0.0036/\text{cm}^3$	2	Pb	$\$0.45$	2
Mg	$\$0.0057/\text{cm}^3$	3	Zn	$\$0.40$	3
Zn	$\$0.0063/\text{cm}^3$	4	Al	$\$0.60$	4
Pb	$\$0.0113/\text{cm}^3$	5	Cu	$\$0.71$	5
Cu	$\$0.014/\text{cm}^3$	6	Mg	$\$1.50$	6
Ti	$\$0.04/\text{cm}^3$	7	Ni	$\$4.10$	8
Ni	$\$0.0804/\text{cm}^3$	8	Ti	$\$4.00$	7
W	$\$0.169/\text{cm}^3$	9	W	$\$4.00$	7
Be	$\$1.4262/\text{cm}^3$	10	Be	$\$350.00$	10

The relationship is changed; for example, aluminum is fourth based on weight, but second on the basis of volume. Titanium is more expensive than nickel on a weight basis, but less expensive than nickel on a volume basis.

- 13-2** Assuming that the density remains unchanged, compare the specific strength of the 2090-T6 aluminum alloy to that of a die cast 443-F aluminum alloy. If you considered the actual density, do you think the difference between the specific strengths would increase or become smaller? Explain.

**Solution:** 2090-T6: Tensile strength = 80,000 psi

$$\begin{aligned}\text{Spec. strength} &= \frac{(80,000 \text{ psi})(454 \text{ g/lb})}{(2.7 \text{ g/cm}^3)(2.54 \text{ cm/in.})^3} \\ &= 8.2 \times 10^5 \text{ in.}\end{aligned}$$

443-F: Tensile strength = 33,000 psi

$$\begin{aligned}\text{Spec. strength} &= \frac{(33,000 \text{ psi})(454 \text{ g/lb})}{(2.7 \text{ g/cm}^3)(2.54 \text{ cm/in.})^3} \\ &= 3.39 \times 10^5 \text{ in.}\end{aligned}$$

Both should increase since both Li and Si (the major alloying elements) are less dense than Al.

- 13-3** Explain why aluminum alloys containing more than about 15% Mg are not used.

**Solution:** When more than 15% Mg is added to Al, a eutectic microconstituent is produced during solidification. This eutectic contains

$$\% \beta_{\text{Eut}} = \frac{35 - 14.9}{35.5 - 14.9} = 97.6\%$$

Most of the eutectic is the brittle intermetallic compound  $\beta$ , and it will likely embrittle the eutectic. The brittle eutectic, which is the continuous microconstituent, will then make the entire alloy brittle.

- 13-7** Would you expect a 2024-T9 aluminum alloy to be stronger or weaker than a 2024-T6 alloy? Explain.

**Solution:** The T9 treatment will give the higher strength; in this temper cold working and age hardening are combined, while in T6, only age hardening is done.

- 13-8** Estimate the tensile strength expected for the following aluminum alloys.

(a) 1100-H14 (b) 5182-H12 (c) 3004-H16

**Solution:** (a) The tensile strength for 1100-H14 is the average of the 0 and H18 treatments.

$$1100-0 (0\% \text{ CW}): \text{TS} = 13 \text{ ksi}$$

$$1100-H18 (75\% \text{ CW}): \text{TS} = 24 \text{ ksi}$$

$$\text{TS}_{\text{H14}} = \frac{13 + 24}{2} = 18.5 \text{ ksi}$$

(b) The tensile strength for 5182-H12 is the average of the 0 and H14 treatments, and H14 is the average of the 0 and H18 treatments. We do not have data in Table 13-5 for 5182-H18. However, 5182-H19 has a tensile strength of 61,000 psi and H18 should be 2000 psi less, or 59,000 psi.

$$5182-0 (0\% \text{ CW}): \text{ TS} = 42 \text{ ksi}$$

$$5182-H18 (75\% \text{ CW}): \text{ TS} = 61 - 2 = 59 \text{ ksi}$$

$$\text{TS}_{H14} = \frac{42 + 59}{2} = 50.5 \text{ ksi}$$

$$\text{TS}_{H12} = \frac{42 + 50.5}{2} = 46.25 \text{ ksi}$$

(c) The tensile strength for 3004-H16 is the average of the H14 and H18 treatments, and H14 is the average of the 0 and H18 treatments.

$$3004-H0: \text{ TS} = 26 \text{ ksi}$$

$$3004-H18: \text{ TS} = 41 \text{ ksi}$$

$$\text{TS}_{H14} = \frac{26 + 41}{2} = 33.5 \text{ ksi}$$

$$\text{TS}_{H16} = \frac{41 + 33.5}{2} = 37.25 \text{ ksi}$$

**13-9** Suppose, by rapid solidification from the liquid state, that a supersaturated Al-7% Li alloy can be produced and subsequently aged. Compare the amount of  $\beta$  that will form in this alloy with that formed in a 2090 alloy.

**Solution:** The 2090 alloy contains 2.4% Li; from the Al-Li phase diagram, the composition of the  $\beta$  is about 20.4% Li and that of the  $\alpha$  is approximately 2% Li at a typical aging temperature or at room temperature:

$$\text{Al-7\% Li: } \% \beta = \frac{7 - 2}{20.4 - 2} \times 100\% = 27\%$$

$$2090: \% \beta = \frac{2.4 - 2}{20.4 - 2} \times 100\% = 2.2\%$$

**13-10** Determine the amount of  $\text{Mg}_2\text{Al}_3$  ( $\beta$ ) expected to form in a 5182-O aluminum alloy. (See Figure 13-5.)

**Solution:** The 5182 alloy contains 4.5% Mg. Thus from the Mg-Al phase diagram, which shows the  $\alpha$  contains about 0% Mg and  $\beta$  contains about 35% Mg:

$$\% \beta = \frac{4.5 - 0}{35 - 0} \times 100\% = 12.9\%$$

**13-11** Based on the phase diagrams, which of the following alloys would be most suited for thixocasting? Explain your answer. (See Figures 13-3 and phase diagrams from Chapters 10 and 11.)

(a) Al-12% Si      (b) Al-1% Cu      (c) Al-10% Mg

**Solution:** Alloys best suited for thixocasting are those with a large freezing range. Of the alloys listed, Al-10% Mg has a freezing range of 110°C, which is the largest freezing range of the three and is therefore most desirable. Al-12% Si is a eutectic alloy (approximately 0°C freezing range), and Al-1% Mg has a freezing range of only 10°C.



- 13–12** From the data in Table 13–6, estimate the ratio by which the yield strength of magnesium can be increased by alloying and heat treatment and compare with that of aluminum alloys.

**Solution:** The exact values will differ depending on the alloys we select for comparison. The table below provides an example. Strengthening of Mg is only about 1/10 as effective as in Al.

	Magnesium			Aluminum	
	YS	YS/YS <sub>Mg</sub>		YS	YS/YS <sub>Al</sub>
Pure Mg:	13 ksi	—	Pure Al	2.5 ksi	—
Cold Worked	17 ksi	1.3	CW 1100–0	22 ksi	8.8
Casting & T6 (ZK61A–T6)	28 ksi	2.2	5182–0 Alloy	19 ksi	7.6
Wrought & T5 (AZ80A–T5)	40 ksi	3.1	2090–T6	75 ksi	30.0

- 13–13** Suppose a 24-in.-long round bar is to support a load of 400 lb without any permanent deformation. Calculate the minimum diameter of the bar if it is made of (a) AZ80A–T5 magnesium alloy and (b) 6061–T6 aluminum alloy. Calculate the weight of the bar and the approximate cost (based on pure Al and Mg) in each case.

**Solution:**  $A = F/\text{Yield Strength}$

(a) AZ80A–T5: YS = 40 ksi

$$A = 400/40,000 = 0.01 \text{ in.}^2$$

$$d = \sqrt{4A/\pi} = 0.113 \text{ in.}$$

$$\text{Weight} = (24 \text{ in.})(0.01 \text{ in.}^2)(0.0628 \text{ lb/in.}^3) = 0.0151 \text{ lb}$$

$$\text{cost} = (\$1.4/\text{lb})(0.0151 \text{ lb}) = \$0.021$$

(b) 6061–T6: YS = 40 ksi therefore;

$$A = 0.01 \text{ in.}^2 \quad d = 0.113 \text{ in.} \quad \text{as in part (a), but:}$$

$$\text{Weight} = (24 \text{ in.})(0.01 \text{ in.}^2)(0.097 \text{ lb/in.}^3) = 0.0233 \text{ lb}$$

$$\text{cost} = (\$0.60/\text{lb})(0.0233 \text{ lb}) = \$0.014$$

Al is less costly than Mg, even though Mg is lighter.

- 13–14** A 10 m rod 0.5 cm in diameter must elongate no more than 2 mm under load. What is the maximum force that can be applied if the rod is made of (a) aluminum, (b) magnesium, and (c) beryllium?

**Solution:**  $E = \sigma/\epsilon = F/A\epsilon \therefore F = EA\epsilon$  diameter = 0.5 cm = 0.1969 in.

$$\epsilon = \frac{10,002 \text{ m} - 10,000 \text{ m}}{10,000 \text{ m}} = 0.0002 \text{ m/m} = 0.0002 \text{ in./in.}$$

$$F_{\text{Al}} = (10 \times 10^6 \text{ psi})(\pi/4)(0.1969 \text{ in.})^2(0.0002 \text{ in./in.})$$

$$= 60.9 \text{ lb} = 271 \text{ N}$$

$$F_{\text{Mg}} = (6.5 \times 10^6 \text{ psi})(\pi/4)(0.1969 \text{ in.})^2(0.0002 \text{ in./in.}) \\ = 39.6 \text{ lb} = 176 \text{ N}$$

$$F_{\text{Be}} = (42 \times 10^6 \text{ psi})(\pi/4)(0.1969 \text{ in.})^2(0.0002 \text{ in./in.}) \\ = 256 \text{ lb} = 1138 \text{ N}$$

- 13–16** We say that copper can contain up to 40% Zn or 9% Al and still be single phase. How do we explain this statement in view of the phase diagrams in Figure 13–6?

**Solution:** This is possible due to slow kinetics of transformation at low temperatures.

- 13–17** Compare the percentage increase in the yield strength of commercially pure annealed aluminum, magnesium, and copper by strain hardening. Explain the differences observed.

**Solution:** Al:  $\frac{1100 - \text{H18}}{1100 - 0} = \frac{22,000}{5,000} \times 100\% = 440\%$

Mg:  $\frac{\text{C.W.}}{\text{Annealed}} = \frac{17,000}{13,000} \times 100\% = 130\%$

Cu:  $\frac{70\% \text{ C.W.}}{\text{Annealed}} = \frac{53,000}{4,800} \times 100\% = 1100\%$

Both Al and Cu (with an FCC structure) have high strain hardening coefficients and can be cold worked a large amount (due to their good ductility). Mg has the HCP structure, a low strain hardening coefficient, and a limited ability to be cold worked.

- 13–18** We would like to produce a quenched and tempered aluminum bronze containing 13% Al. Recommend a heat treatment, including appropriate temperatures. Calculate the amount of each phase after each step of the treatment.

**Solution:** Heat to above about 710°C to get all  $\beta$ ; 100%  $\beta$ ,  $\beta$ : 13% Al

Quench; still all  $\beta$  containing 13% Al.

Reheat; temper at 400°C to allow  $\gamma_2$  to form.

$$\% \gamma_2 = \frac{13 - 9.4}{16 - 9.4} = 54.5\% \quad \gamma_2: 16\% \text{ Al}, \quad \alpha: 9.4\% \text{ Al}$$

We want to be sure to temper above 400°C so we obtain  $\gamma_2$  in a matrix of  $\alpha$  rather than a structure containing  $\gamma + \gamma_2$ .

- 13–19** A number of casting alloys have very high lead contents; however the Pb content in wrought alloys is comparatively low. Why isn't more lead added to the wrought alloys? What precautions must be taken when a leaded wrought alloy is hot worked or heat treated?

**Solution:** The lead rich phase may melt during hot working or may form stringers during cold working.

We must be sure that the temperature is low enough to avoid melting of the lead phase.

- 13–20** Would you expect the fracture toughness of quenched and tempered aluminum bronze to be high or low? Would there be a difference in the resistance of the alloy to crack nucleation compared to crack growth? Explain.

**Solution:** The fracture toughness should be relatively good. The acicular, or Widmanstätten, microstructure forces a crack to follow a very tortuous path, which consumes a large amount of energy.

This microstructure is less resistant to crack nucleation. The acicular structure may concentrate stresses that lead to easier formation of a crack.

- 13–21** Based on the photomicrograph in Figure 13–8(a), would you expect the  $\gamma'$  precipitate or the carbides to provide a greater strengthening effect in superalloys at low temperatures? Explain.

**Solution:** The  $\gamma'$  phase is more numerous and also more uniformly and closely spaced; consequently the  $\gamma'$  should be more effective than the smaller number of coarse carbides at blocking slip at low temperatures.

- 13–22** The density of  $\text{Ni}_3\text{Al}$  is  $7.5 \text{ g/cm}^3$ . Suppose a Ni–5 wt% Al alloy is heat treated so that all of the aluminum reacts with nickel to produce  $\text{Ni}_3\text{Al}$ . Determine the volume percent of the  $\text{Ni}_3\text{Al}$  precipitate in the nickel matrix.

**Solution:** Let's assume that the density of the Ni–5 wt% Al alloy is the same as that of pure Ni ( $8.902 \text{ g/cm}^3$ ).

In 100 g of the alloy, the total atoms present are:

$$\begin{aligned} \text{atoms} &= \frac{(95 \text{ g/Ni})N_A}{58.71 \text{ g/mol}} + \frac{(5 \text{ g Al})N_A}{26.981 \text{ g/mol}} \\ &= 1.6181 N_A + 0.1853 N_A = 1.803 N_A \end{aligned}$$

If all of the Al reacts to form  $\text{Ni}_3\text{Al}$ , then the number of atoms in the compound is  $0.1853 N_A$  of Al and  $(3)(0.1853 N_A) = 0.5559 N_A$  of Ni.

The weight of the  $\text{Ni}_3\text{Al}$  is then:

$$\begin{aligned} \text{wt} &= \frac{(0.1853 N_A \text{ of Al})(26.981 \text{ g/mol})}{N_A} \\ &\quad + \frac{(0.5559 N_A \text{ of Ni})(58.71 \text{ g/mol})}{N_A} \\ &= 37.64 \text{ g of Ni}_3\text{Al} \end{aligned}$$

The wt of the Ni matrix is thus 62.36 g. The vol%  $\text{Ni}_3\text{Al}$  is thus:

$$\begin{aligned} \text{vol\% Ni}_3\text{Al} &= \frac{37.64 \text{ g}/6.56 \text{ g/cm}^3}{(37.64 \text{ g}/6.56 \text{ g/cm}^3) + (62.36 \text{ g}/8.902 \text{ g/cm}^3)} \\ &\quad \times 100\% = 45\% \end{aligned}$$

Even a small amount (5 wt% aluminum) produces a very large volume percent of precipitate in the microstructure.

- 13–23** Figure 13–8(b) shows a nickel superalloy containing two sizes of  $\gamma'$  precipitates. Which precipitate likely formed first? Which precipitate formed at the higher temperature? What does our ability to perform this treatment suggest concerning the effect of temperature on the solubility of Al and Ti in nickel? Explain.

**Solution:** The larger precipitate forms first and at the higher temperatures

The solubility of Al and Ti in Ni decreases as temperature decreases; at a high temperature, the Al and Ti form the  $\gamma'$ , but some Al and Ti still remain in solution in the matrix. As the temperature decreases, the solubility decreases as well and more of the  $\gamma'$  can form.

- 13–24** When steel is joined using arc welding, only the liquid fusion zone must be protected by a gas or flux. However, when titanium is welded, both the front and back sides of the welded metal must be protected. Why must these extra precautions be taken when joining titanium?

**Solution:** The titanium may be contaminated or embrittled anytime the temperature is above about 535°C. Therefore the titanium must be protected until the metal cools below this critical temperature. Since both sides of the titanium plate will be heated by the welding process, special provisions must be made to shield all sides of the titanium until the metal cools sufficiently.

- 13–25** Both a Ti–15% V alloy and a Ti–35% V alloy are heated to a temperature at which all  $\beta$  just forms. They are then quenched and reheated to 300°C. Describe the changes in microstructures during the heat treatment for each alloy, including the amount of each phase. What is the matrix and what is the precipitate in each case? Which is an age-hardening process? Which is a quench and temper process? [See Figure 13–14(a)]

**Solution:** Ti–15% V: 100%  $\beta \rightarrow 100\% \alpha' \rightarrow \beta$  precipitates in a matrix.

$$\% \alpha_{300^\circ\text{C}} = \frac{46 - 15}{46 - 5} \times 100\% = 76\% \quad \beta = 24\%$$

This is a quench and temper process.

Ti–35% V: 100%  $\beta \rightarrow 100\% \beta_{ss} \rightarrow \alpha$  precipitates in  $\beta$  matrix.

$$\% \alpha_{300^\circ\text{C}} = \frac{46 - 35}{46 - 5} \times 100\% = 27\% \quad \beta = 73\%$$

This is an age hardening process.

- 13–26** The  $\theta$  phase in the Ti–Mn phase diagram has the formula MnTi. Calculate the amount of  $\alpha$  and  $\theta$  in the eutectoid microconstituent. [See Figure 13–10(d)]

**Solution:**

$$\text{wt\% Mn in } \theta = \frac{(1 \text{ atom of Mn})(54.938 \text{ g/mol})}{(1 \text{ atom of Mn})(54.938) + (1 \text{ atom of Ti})(47.9)} = 53.4\%$$

$$\% \alpha_{\text{eutectoid}} = \frac{53.4 - 20}{53.4 - 1} \times 100\% = 63.7\% \quad \% \theta = 36.3\%$$

- 13–28** Determine the specific strength of the strongest Al, Mg, Cu, Ti, and Ni alloys. Use the densities of the pure metals, in lb/in.<sup>3</sup>, in your calculations. Try to explain their order.

Solution:

	Strength (psi)	Density	Strength-to- weight ratio
Ti	176,000	4.505 g/cm <sup>3</sup> = 0.162 lb/in. <sup>3</sup>	10.9 × 10 <sup>5</sup> in.
Al	73,000	2.7 g/cm <sup>3</sup> = 0.097 lb/in. <sup>3</sup>	7.5 × 10 <sup>5</sup> in.
Mg	40,000	1.74 g/cm <sup>3</sup> = 0.063 lb/in. <sup>3</sup>	6.3 × 10 <sup>5</sup> in.
Cu	175,000	8.93 g/cm <sup>3</sup> = 0.32 lb/in. <sup>3</sup>	5.5 × 10 <sup>5</sup> in.
Monel	110,000	8.93 g/cm <sup>3</sup> = 0.32 lb/in. <sup>3</sup>	3.4 × 10 <sup>5</sup> in.
W	220,000	19.25 g/cm <sup>3</sup> = 0.69 lb/in. <sup>3</sup>	3.2 × 10 <sup>5</sup> in.

Titanium is both strong and relatively low density. Cu, Ni, W are strong but dense. Al and Mg have modest strength but light weight.

- 13–29** Based on the phase diagrams, estimate the solubilities of Ni, Zn, Al, Sn, and Be in copper at room temperature. Are these solubilities expected in view of Hume-Rothery's conditions for solid solubility? Explain.

Solution:

	Solubility	Structure	Valence	Atom size difference
Cu–Ni	100% Ni	FCC	1	$\frac{1.278 - 1.243}{1.278} \times 100 = 2.7\%$
Cu–Zn	30% Zn	HCP	2	$\frac{1.278 - 1.332}{1.278} \times 100 = -4.2\%$
Cu–Al	8% Al	FCC	3	$\frac{1.278 - 1.432}{1.278} \times 100 = -12.1\%$
Cu–Be	0.2% Be	hex	2	$\frac{1.278 - 1.143}{1.278} \times 100 = 10.6\%$
Cu–Sn	0% Sn	DC	4	$\frac{1.278 - 1.405}{1.278} \times 100 = -9.9\%$

Hume-Rothery's conditions do help to explain the differences in solubility. Solubilities tend to decrease as atom size difference increases.

- 13–31** The temperature of a coated tungsten part is increased. What happens when the protective coating on a tungsten part expands more than the tungsten? What happens when the protective coating on a tungsten part expands less than the tungsten?

**Solution:** If the protective coating expands more than tungsten, compressive stresses will build up in the coating and the coating will flake.

If the protective coating expands less than tungsten, tensile stresses will build up in the coating and the coating will crack and become porous.

# 14

## Ceramic Materials

- 14–42** The specific gravity of  $\text{Al}_2\text{O}_3$  is  $3.96 \text{ g/cm}^3$ . A ceramic part is produced by sintering alumina powder. It weighs 80 g when dry, 92 g after it has soaked in water, and 58 g when suspended in water. Calculate the apparent porosity, the true porosity, and the closed pores.

**Solution:** From the problem statement,  $\rho = 3.96$ ,  $W_d = 80 \text{ g}$ ,  $W_w = 92$ , and  $W_s = 58$ . From the equations,

$$\text{apparent porosity} = \frac{W_w - W_d}{W_w - W_s} \times 100 = \frac{92 - 80}{92 - 58} \times 100 = 35.29\%$$

The bulk density is  $B = W_d / (W_w - W_s) = 80 / (92 - 58) = 2.3529 \text{ g/cm}^3$ . Therefore:

$$\text{true porosity} = \frac{\rho - B}{\rho} \times 100 = \frac{3.96 - 2.3529}{3.96} \times 100 = 40.58\%$$

$$\text{closed porosity} = 40.58 - 35.29 = 5.29\%$$

- 14–43** Silicon carbide (SiC) has a specific gravity of  $3.1 \text{ g/cm}^3$ . A sintered SiC part is produced, occupying a volume of  $500 \text{ cm}^3$  and weighing 1200 g. After soaking in water, the part weighs 1250 g. Calculate the bulk density, the true porosity, and the volume fraction of the total porosity that consists of closed pores.

**Solution:** The appropriate constants required for the equations are:

$$\rho = 3.1 \text{ g/cm}^3 \quad B = 1200 \text{ g} / 500 \text{ cm}^3 = 2.4 \text{ g/cm}^3$$

$$W_w = 1250 \text{ g} \quad W_d = 1200 \text{ g}$$

Therefore:

$$B = 2.4 = W_d / (W_w - W_s) = 1200 / (1250 - W_s) \quad \text{or} \quad W_s = 750 \text{ g}$$

$$\text{apparent porosity} = \frac{W_w - W_d}{W_w - W_s} \times 100 = \frac{1250 - 1200}{1250 - 750} \times 100 = 10\%$$

$$\text{true porosity} = \frac{(\rho - B)}{\rho} = \frac{(3.1 - 2.4)}{3.1} \times 100 = 22.58\%$$

$$\text{closed porosity} = 22.58 - 10 = 12.58\%$$

$$f_{\text{closed}} = 12.58/22.58 = 0.44$$

- 14–54** Calculate the O:Si ratio when 20 wt% Na<sub>2</sub>O is added to SiO<sub>2</sub>. Explain whether this material will provide good glass forming tendencies. Above what temperature must the ceramic be heated to be all-liquid?

**Solution:**  $MW_{\text{soda}} = 2(22.99) + 16 = 61.98 \text{ g/mol}$

$$MW_{\text{silica}} = 28.08 + 2(16) = 60.08 \text{ g/mol}$$

$$\text{mole fraction Na}_2\text{O} = \frac{20 \text{ g}/61.98 \text{ g/mol}}{20/61.98 + 80/60.08} = 0.1951$$

$$\text{O/Si} = \frac{(1 \text{ O/Na}_2\text{O})(0.1951) + (2 \text{ O/SiO}_2)(0.8049)}{(1 \text{ Si/SiO}_2)(0.8049)} = 2.24$$

Since the O/Si ratio is less than 2.5, it should be possible to produce a glass.

From the Na<sub>2</sub>O–SiO<sub>2</sub> phase diagram (Figure 14–11), we find that, for 20 wt% Na<sub>2</sub>O, the liquidus temperature is about 1000°C. We must heat the material above 1000°C to begin the glass-making operation.

- 14–55** How many grams of BaO can be added to 1 kg of SiO<sub>2</sub> before the O:Si ratio exceeds 2.5 and glass-forming tendencies are poor? Compare this to the case when Li<sub>2</sub>O is added to SiO<sub>2</sub>.

**Solution:** We can first calculate the required mole fraction of BaO required to produce an O:Si ratio of 2.5:

$$\text{O/Si} = 2.5 = \frac{(1 \text{ O/BaO})f_{\text{BaO}} + (2 \text{ O/SiO}_2)(1 - f_{\text{BaO}})}{(1 \text{ Si/SiO}_2)(1 - f_{\text{BaO}})}$$

$$f_{\text{BaO}} = 0.33 \quad \text{and} \quad f_{\text{silica}} = 0.67$$

The molecular weight of BaO is 137.3 + 16 = 153.3 g/mol, and that of silica is 60.08 g/mol. The weight percent BaO is therefore:

$$\text{wt\% BaO} = \frac{(0.33 \text{ mol})(153.3 \text{ g/mol})}{(0.33)(153.3) + (0.67)(60.08)} \times 100 = 55.69\%$$

For 1 kg of SiO<sub>2</sub>, the amount of BaO is:

$$0.5569 = \frac{x \text{ g BaO}}{x \text{ g BaO} + 1000 \text{ g SiO}_2} \quad \text{or} \quad x = 1257 \text{ g BaO}$$

The mole fraction of Li<sub>2</sub>O required is:

$$\text{O/Si} = 2.5 = \frac{(1 \text{ O/Li}_2\text{O})f_{\text{Li}_2\text{O}} + (2 \text{ O/SiO}_2)(1 - f_{\text{Li}_2\text{O}})}{(1 \text{ Si/SiO}_2)(1 - f_{\text{Li}_2\text{O}})}$$

$$f_{\text{Li}_2\text{O}} = 0.33 \quad \text{and} \quad f_{\text{silica}} = 0.67$$

The molecular weight of  $\text{Li}_2\text{O}$  is  $2(6.94) + 16 = 29.88 \text{ g/mol}$ , and that of silica is  $60.08 \text{ g/mol}$ . The weight percent  $\text{Li}_2\text{O}$  is therefore:

$$\text{wt\% Li}_2\text{O} = \frac{(0.33 \text{ mol})(29.88 \text{ g/mol})}{(0.33)(29.88) + (0.67)(60.08)} \times 100 = 19.7\%$$

For 1 kg of  $\text{SiO}_2$ , the amount of  $\text{Li}_2\text{O}$  is:

$$0.197 = \frac{x \text{ g Li}_2\text{O}}{x \text{ g Li}_2\text{O} + 1000 \text{ g SiO}_2} \quad \text{or} \quad x = 245 \text{ g Li}_2\text{O}$$

Much larger amounts of  $\text{BaO}$  can be added compared to  $\text{Li}_2\text{O}$  and still retain the ability to form a glass.

- 14–56** Calculate the O:Si ratio when 30 wt%  $\text{Y}_2\text{O}_3$  is added to  $\text{SiO}_2$ . Will this material provide good glass-forming tendencies?

**Solution:**  $\text{MW}_{\text{yttria}} = 2(88.91) + 3(16) = 225.82 \text{ g/mol}$

$$\text{MW}_{\text{silica}} = 60.08 \text{ g/mol}$$

The mole fraction of yttria is (assuming a base of 100 g of ceramic):

$$f_{\text{yttria}} = \frac{30 \text{ g}/225.82 \text{ g/mol}}{30/225.82 + 70/60.08} = 0.102$$

The O/Si ratio is then:

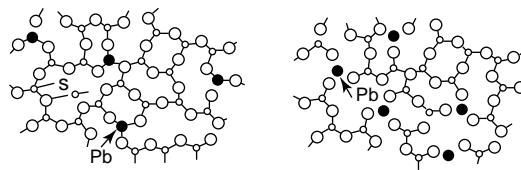
$$\text{O/Si} = \frac{(3 \text{ O/Y}_2\text{O}_3)(0.102) + (2 \text{ O/SiO}_2)(0.898)}{(1 \text{ Si/SiO}_2)(0.898)} = 2.34$$

The material will produce a glass.

- 14–57** Lead can be introduced into a glass either as  $\text{PbO}$  (where the Pb has a valence of +2) or as  $\text{PbO}_2$  (where the Pb has a valence of +4). Such leaded glasses are used to make what is marketed as “crystal glass” for dinnerware. Draw a sketch (similar to Figure 14–10) showing the effect of each of these oxides on the silicate network. Which oxide is a modifier and which is an intermediate?

**Solution:**  $\text{PbO}_2$  provides the same number of metal and oxygen atoms to the network as does silica; the  $\text{PbO}_2$  does not disrupt the silicate network; therefore the  $\text{PbO}_2$  is a *intermediate*.

$\text{PbO}$  does not provide enough oxygen to keep the network intact; consequently  $\text{PbO}$  is a *modifier*.





- 14–58** A glass composed of 65 mol% SiO<sub>2</sub>, 20 mol% CaO, and 15 mol% Na<sub>2</sub>O is prepared. Calculate the O:Si ratio and determine whether the material has good glass-forming tendencies. Estimate the liquidus temperature of the material using Figure 14–16.

**Solution:** Based on the mole fractions, we can determine the O:Si ratio:

$$\text{O/Si} = \frac{(2 \text{ O/SiO}_2)(0.65) + (1 \text{ O/CaO})(0.20) + (1 \text{ O/Na}_2\text{O})(0.15)}{(1 \text{ Si/SiO}_2)(0.65)}$$

$$\text{O/Si} = 2.54$$

The glass-forming tendencies are relatively poor and special attention to the cooling rate may be required.

To determine the liquidus, we must find the weight percentages of each constituent. The molecular weights are:

$$\text{MW}_{\text{silica}} = 60.08 \text{ g/mol}$$

$$\text{MW}_{\text{CaO}} = 40.08 + 16 = 56.08 \text{ g/mol}$$

$$\text{MW}_{\text{soda}} = 2(22.99) + 16 = 61.98 \text{ g/mol}$$

$$\begin{aligned} \text{wt\% SiO}_2 &= \frac{(0.65)(60.08)}{(0.65)(60.08) + (0.20)(56.08) + (0.15)(61.98)} \times 100 \\ &= 65.56\% \end{aligned}$$

$$\begin{aligned} \text{wt\% CaO} &= \frac{(0.20)(56.08)}{(0.65)(60.08) + (0.20)(56.08) + (0.15)(61.98)} \times 100 \\ &= 18.83\% \end{aligned}$$

$$\begin{aligned} \text{wt\% Na}_2\text{O} &= \frac{(0.15)(61.98)}{(0.65)(60.08) + (0.20)(56.08) + (0.15)(61.98)} \times 100 \\ &= 15.61\% \end{aligned}$$

From the ternary phase diagram, this overall composition gives a liquidus temperature of about 1140°C.

# 15

## Polymers

- 15–6(a)** Suppose that 20 g of benzoyl peroxide are introduced to 5 kg of propylene monomer (see Table 15–3). If 30% of the initiator groups are effective, calculate the expected degree of polymerization and the molecular weight of the polypropylene polymer if (a) all of the termination of the chains occurs by combination and (b) all of the termination occurs by disproportionation.

**Solution:**  $MW_{\text{propylene}} = 3 \text{ C} + 6 \text{ H} = 42 \text{ g/mol}$   
 $5000 \text{ g}/42 \text{ g/mol} = 119 \text{ mol of propylene}$   
 $MW_{\text{benzoyl peroxide}} = 14 \text{ C} + 4 \text{ O} + 10 \text{ H} = 242 \text{ g/mol}$   
 $20 \text{ g}/242 \text{ g/mol} = 0.0826 \text{ mol of benzoyl peroxide}$

If only 30% of the initiator is effective, the actual number of moles of benzoyl peroxide involved in the polymerization process is

$$(0.3)(0.0826 \text{ mol}) = 0.0248 \text{ mol of benzoyl peroxide}$$

- (a) For combination, 1 mol of benzoyl peroxide produces 1 chain:

$$\text{degree of polymerization} = 119 \text{ mol}/0.0248 \text{ mol} = 4798$$

- (b) For disproportionation, 1 mol of benzoyl peroxide produces 2 chains:

$$\text{degree of polymerization} = (2)(119 \text{ mol})/0.0248 \text{ mol} = 9597$$

- 15–6(b)** Suppose hydrogen peroxide ( $\text{H}_2\text{O}_2$ ) is used as the initiator for 10 kg of vinyl chloride monomer (see Table 15–3). Show schematically how the hydrogen peroxide will initiate the polymer chains. Calculate the amount of hydrogen peroxide (assuming that it is 10% effective) required to produce a degree of polymerization of 4000 if (a) termination of the chains occurs by combination and (b) termination occurs by disproportionation.

**Solution:**  $MW_{\text{vinyl chloride}} = 2 \text{ C} + 3 \text{ H} + 1 \text{ Cl} = 62.453 \text{ g/mol}$   
 $10,000 \text{ g} / 62.453 \text{ g/mol} = 160.12 \text{ mol of vinyl chloride}$   
 $MW_{\text{hydrogen peroxide}} = 2 \text{ H} + 2 \text{ O} = 34 \text{ g/mol}$

- (a) One molecule of hydrogen peroxide decomposes to produce two OH groups; one could initiate a chain and the other could terminate the chain. In order to obtain a degree of polymerization of 4000 by the combination mechanism, the number of mols of hydrogen peroxide ( $x$ ) required for each chain is:

$$4000 = \frac{160.12 \text{ mol of v.c.}}{x \text{ mol H}_2\text{O}_2}$$

or  $x = 0.04 \text{ mol of H}_2\text{O}_2$

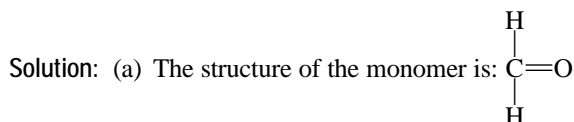
The peroxide is only 10% effective; the amount of peroxide required is therefore:

$$(0.04 \text{ mol})(34 \text{ g/mol}) / 0.1 = 13.6 \text{ g}$$

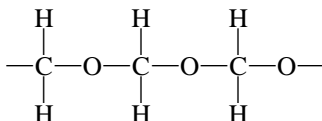
- (b) For disproportionation, 1 mol of peroxide is sufficient for polymerizing two chains of polymer. Only 0.02 mol is required per chain. Therefore the amount required is:

$$(0.02 \text{ mol})(34 \text{ g/mol}) / 0.1 = 6.8 \text{ g}$$

- 15–7** The formula for formaldehyde is HCHO. (a) Draw the structure of the formaldehyde molecule and repeat unit. (b) Does formaldehyde polymerize to produce an acetal polymer (see Table 15–4) by the addition mechanism or the condensation mechanism? Try to draw a sketch of the reaction and the acetal polymer by both mechanisms.



- (b) In the addition mechanism, the double (unsaturated) bond between the carbon and oxygen is replaced by a single bond, permitting repeat units to be joined:



Polymerization by the condensation mechanism cannot occur with only the formaldehyde monomer.

- 15–8** You would like to combine 5 kg of dimethyl terephthalate with ethylene glycol to produce polyester (PET). Calculate (a) the amount of ethylene glycol required, (b) the amount of byproduct evolved, and (c) the amount of polyester produced.

**Solution:**  $MW_{\text{ethylene glycol}} = 2 \text{ C} + 2 \text{ O} + 6 \text{ H} = 62 \text{ g/mol}$   
 $MW_{\text{terephthalate}} = 10 \text{ C} + 4 \text{ O} + 10 \text{ H} = 194 \text{ g/mol}$   
 $5000 \text{ g} / 194 \text{ g/mol} = 25.773 \text{ mol of dimethyl terephthalate}$

- (a) Equal numbers of moles of the two monomers are required for polymerization to occur:

$$5 \text{ kg}/194 \text{ g/mol} = x \text{ kg}/62 \text{ g/mol}$$

$$x = 1.598 \text{ kg} = 1,598 \text{ g of ethylene glycol}$$

- (b) The byproduct of the condensation reaction between the two monomers is methyl alcohol,  $\text{COH}_4$ . One molecule of the alcohol is produced for each monomer that is attached to the growing polymer chain. Thus the amount of byproduct produced is  $2 \times 25.773 \text{ mol}$ , or 51.546 mol:

$$\text{MW}_{\text{alcohol}} = 1 \text{ C} + 1 \text{ O} + 4 \text{ H} = 32 \text{ g/mol}$$

$$\text{amount} = (51.546 \text{ mol})(32 \text{ g/mol}) = 1649 \text{ g of methyl alcohol}$$

- (c) The total weight of the PET produced is the sum of the two monomers minus the weight of the byproduct:

$$\text{amount} = 5 \text{ kg} + 1.598 \text{ kg} - 1.649 \text{ kg} = 4.949 \text{ kg}$$

- 15–9** Would you expect polyethylene to polymerize at a faster or slower rate than polymethyl methacrylate? Explain. Would you expect polyethylene to polymerize at a faster or slower rate than a polyester? Explain.

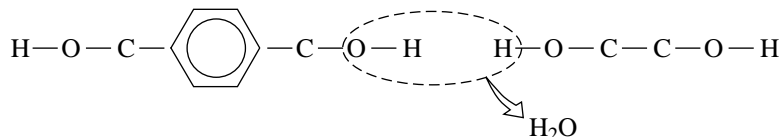
**Solution:** In both cases, we would expect polyethylene to polymerize at a faster rate. The ethylene monomer is smaller than the methyl methacrylate monomer and therefore should diffuse more quickly to the active ends of the growing chains. In polyester, two different monomers must diffuse to the active end of the chain in order for polymerization to continue; this would also be expected to occur at a slower rate than diffusion of only ethylene monomer.

- 15–10** You would like to combine 10 kg of ethylene glycol with terephthalic acid to produce a polyester. The monomer for terephthalic acid is shown below. (a) Determine the byproduct of the condensation reaction and (b) calculate the amount of terephthalic acid required, the amount of byproduct evolved, and the amount of polyester produced.

**Solution:**  $\text{MW}_{\text{glycol}} = 2 \text{ C} + 2 \text{ O} + 6 \text{ H} = 62 \text{ g/mol}$

$$\text{MW}_{\text{acid}} = 8 \text{ C} + 4 \text{ O} + 6 \text{ H} = 166 \text{ g/mol}$$

- (a) The O—H group from the terephthalic acid combines with H from ethylene glycol, producing water as the byproduct. One molecule of water is produced for each monomer that is attached to the polymer chain.



(b) The number of moles of ethylene glycol present is

$$10,000 \text{ g}/62 \text{ g/mol} = 161.29 \text{ mol of ethylene glycol}$$

The amount of terephthalic acid must also be 161.29 mol. Its mass is:

$$x \text{ g of acid}/166 \text{ g/mol} = 161.29 \text{ mol}$$

$$x = 26,770 \text{ g} = 26.77 \text{ kg of terephthalic acid}$$

Two moles of water are produced for each mole of ethylene glycol. The amount of water evolved is then:

$$(2)(161.29 \text{ mol of glycol})(18 \text{ g/mol}) = 5,810 \text{ g} = 5.81 \text{ kg}$$

The total weight of the polymer is:

$$\text{weight} = 10 \text{ kg} + 26.77 \text{ kg} - 5.81 \text{ kg} = 30.96 \text{ kg}$$

- 15–13** The molecular weight of polymethyl methacrylate (see Table 15–3) is 250,000 g/mol. If all of the polymer chains are the same length, calculate (a) the degree of polymerization and (b) the number of chains in 1 g of the polymer.

**Solution:** The molecular weight of methyl methacrylate is:

$$\text{MW} = 5 \text{ C} + 2 \text{ O} + 8 \text{ H} = 100 \text{ g/mol}$$

$$(a) \text{ Degree of polymerization} = 250,000/100 = 2,500$$

(b) In 1 g of the polymer:

$$\frac{(1 \text{ g})(6.02 \times 10^{23} \text{ chains/mol})}{250,000 \text{ g/mol}} = 2.408 \times 10^{18} \text{ chains}$$

- 15–14** The degree of polymerization of polytetrafluoroethylene (see Table 15–3) is 7,500. If all of the polymer chains are the same length, calculate (a) the molecular weight of the chains and (b) the total number of chains in 1000 g of the polymer.

**Solution:** The molecular weight of tetrafluoroethylene is:

$$\text{MW} = 2 \text{ C} + 4 \text{ F} = 100 \text{ g/mol}$$

$$(a) \text{ MW}_{\text{chains}} = (7500)(100) = 750,000 \text{ g/mol}$$

(b) In 1000 g of the polymer:

$$\frac{(1000 \text{ g})(6.02 \times 10^{23} \text{ chains/mol})}{750,000 \text{ g/mol}} = 8.03 \times 10^{20} \text{ chains}$$

- 15–15** A polyethylene rope weighs 0.25 lb per foot. If each chain contains 7000 repeat units, calculate (a) the number of polyethylene chains in a 10-ft length of rope and (b) the total length of chains in the rope, assuming that carbon atoms in each chain are approximately 0.15 nm apart.

**Solution:** The molecular weight of ethylene is 28 g/mol, so the molecular weight of the polyethylene is  $7000 \times 28 = 196,000 \text{ g/mol}$ . The weight of the 10 ft length of rope is  $(0.25 \text{ lb/ft})(10 \text{ ft})(454 \text{ g/lb}) = 1135 \text{ g}$ .

(a) The number of chains is:

$$\frac{(1135 \text{ g})(6.02 \times 10^{23} \text{ chains/mol})}{196,000 \text{ g/mol}} = 34.86 \times 10^{20} \text{ chains}$$

(b) The length of one repeat unit is 0.24495 nm (from Problem 15–3).

Therefore the length of each chain, which contain 7000 repeat units, is

$$\text{one chain} = (7000)(0.24495 \text{ nm}) = 1715 \text{ nm} = 1.715 \times 10^{-4} \text{ cm}$$

$$\text{all chains} = (1.715 \times 10^{-4} \text{ cm/chain})(34.86 \times 10^{20} \text{ chains})$$

$$= 5.978 \times 10^{17} \text{ cm}$$

$$= 3.7 \times 10^{12} \text{ miles}$$

**15–16** A common copolymer is produced by including both ethylene and propylene monomers in the same chain. Calculate the molecular weight of the polymer produced using 1 kg of ethylene and 3 kg of propylene, giving a degree of polymerization of 5000.

**Solution:** We will consider that each repeat unit—whether ethylene or propylene—counts towards the degree of polymerization. We can calculate the number of moles of each monomer in the polymer:

$$\text{MW}_{\text{ethylene}} = 28 \text{ g/mol} \quad 1000 \text{ g}/28 \text{ g/mol} = 35.71 \text{ mol of ethylene}$$

$$\text{MW}_{\text{propylene}} = 42 \text{ g/mol} \quad 3000 \text{ g}/42 \text{ g/mol} = 71.43 \text{ mol of propylene}$$

The mole fraction of each monomer is:

$$f_{\text{ethylene}} = 35.71/(35.71 + 71.43) = 0.333$$

$$f_{\text{propylene}} = 71.43/(35.71 + 71.43) = 0.667$$

The molecular weight of the polymer is then:

$$\text{MW}_{\text{polymer}} = 5000[(0.333)(28) + (0.667)(42)] = 186,690 \text{ g/mol}$$

**15–17** Analysis of a sample of polyacrylonitrile (PAN) (see Table 15–3) shows that there are six lengths of chains, with the following number of chains of each length. Determine (a) the weight average molecular weight and degree of polymerization and (b) the number average molecular weight and degree of polymerization.

**Solution:**

Number of chains	Mean Molecular weight of chains (g/mol)	$x_i$	$x_i M_i$	weight	$f_i$	$f_i M_i$
10,000	3,000	0.137	411	$30 \times 10^6$	0.044	132
18,000	6,000	0.247	1482	$108 \times 10^6$	0.159	954
17,000	9,000	0.233	2097	$153 \times 10^6$	0.226	2034
15,000	12,000	0.205	2460	$180 \times 10^6$	0.265	3180
9,000	15,000	0.123	1845	$135 \times 10^6$	0.199	2985
4,000	18,000	0.055	990	$72 \times 10^6$	0.106	1908
73,000	sum =		9,285	$678 \times 10^6$		11,193

The molecular weight of the acrylonitrile monomer is:

$$MW_{\text{acrylonitrile}} = 3 \text{ C} + 1 \text{ N} + 3 \text{ H} = 53 \text{ g/mol}$$

(a) The weight average molecular weight and degree of polymerization are:

$$MW_w = 11,193 \text{ g/mol} \quad DP_w = 11,193/53 = 211$$

(b) The number average molecular weight and degree of polymerization are:

$$MW_n = 9,285 \text{ g/mol} \quad DP_n = 9285/53 \text{ g/mol} = 175$$

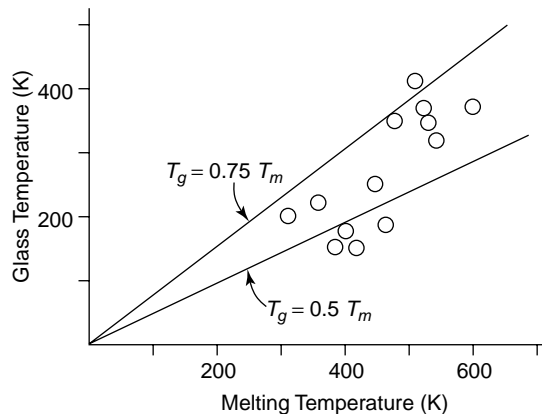
**15–18** Explain why you would prefer that the number average molecular weight of a polymer be as close as possible to the weight average molecular weight.

**Solution:** We do not want a large number of small chains in the polymer; the small chains, due to less entanglement, will reduce the mechanical properties.

**15–20** Using Table 15–5, plot the relationship between the glass temperatures and the melting temperatures of the addition thermoplastics. What is the approximate relationship between these two critical temperatures? Do the condensation thermoplastics and the elastomers also follow the same relationship?

Converting temperature to Kelvin:

	$T_m$	$T_g$
LD polyethylene	388	153
HD polyethylene	410	153
PVC	448–485	360
Polypropylene	441–449	257
Polystyrene	513	358–398
PAN	593	380
Acetal	454	188
6,6 Nylon	538	323
Polycarbonate	503	416
Polyester	528	348
Polybutadiene	393	183
Polychloroprene	353	223
Polyisoprene	303	200



Most of the polymers fall between the lines constructed with the relationships  $T_g = 0.5T_m$  and  $T_g = 0.75T_m$ . The condensation thermoplastics and elastomers also follow this relationship.

- 15–21** List the addition polymers in Table 15–5 that might be good candidates for making the bracket that holds the rear view mirror onto the outside of an automobile, assuming that temperatures frequently fall below zero degrees Celsius. Explain your choices.

**Solution:** Because of the mounting of the rear view mirror, it is often subject to being bumped; we would like the mounting material to have reasonable ductility and impact resistance so that the mirror does not break off the automobile. Therefore we might want to select a material that has a glass transition temperature below 0°C. Both polyethylene and polypropylene have low glass transition temperatures and might be acceptable choices. In addition, acetal (polyoxymethylene) has a low glass transition temperature and (from Table 15–5) is twice as strong as polyethylene and polypropylene. Finally all of the elastomers listed in Table 15–8 might be appropriate choices.

- 15–22** Based on Table 15–8, which of the elastomers might be suited for use as a gasket in a pump for liquid CO<sub>2</sub> at –78°C? Explain.

**Solution:** We wish to select a material that will not be brittle at very low temperatures, that is, the elastomer should have a glass transition temperature below –78°C. Of the materials listed in Table 15–8, only polybutadiene and silicone have glass transition temperatures below –78°C.

- 15–23** How do the glass temperatures of polyethylene, polypropylene, and polymethyl methacrylate compare? Explain their differences, based on the structure of the monomer.

**Solution:** From Table 15–5:

polyethylene . . . . .	$T_g = -120^\circ\text{C}$
polypropylene . . . . .	$T_g = -16^\circ\text{C}$
polymethyl methacrylate . .	$T_g = +90 \text{ to } 105^\circ\text{C}$

The side groups in polyethylene are small hydrogen atoms; in polypropylene, more complex side groups are present; in PMMA, the side groups are even more extensive (see Table 15–5). As the complexity of the monomers increases, the glass transition temperature increases.

- 15–24** Which of the addition polymers in Table 15–5 are used in their leathery condition at room temperature? How is this condition expected to affect their mechanical properties compared with those of glassy polymers?

**Solution:** Both polyethylene and polypropylene have glass transition temperatures below room temperature and therefore are presumably in the leathery condition. As a consequence, they are expected to have relatively low strengths compared to most other thermoplastic polymers.

- 15–25** The density of polypropylene is approximately 0.89 g/cm<sup>3</sup>. Determine the number of propylene repeat units in each unit cell of crystalline polypropylene.

**Solution:** From Table 15–6, we find the lattice parameters for orthorhombic polypropylene. The volume of the unit cell is:

$$V_{\text{cell}} = (14.5 \times 10^{-8} \text{ cm})(5.69 \times 10^{-8} \text{ cm})(7.40 \times 10^{-8} \text{ cm}) \\ = 6.10537 \times 10^{-22} \text{ cm}^3$$

The molecular weight of propylene is 3 C + 6 H = 42 g/mol. The number of repeat units “x” is:



$$0.89 \text{ g/cm}^3 = \frac{(x)(42 \text{ g/mol})}{(6.10537 \times 10^{-22} \text{ cm}^3)(6.02 \times 10^{23} \text{ units/mol})}$$

$$x = 7.8$$

Therefore there are 8 propylene repeat units in one unit cell of crystalline polypropylene.

- 15–26** The density of polyvinyl chloride is approximately  $1.4 \text{ g/cm}^3$ . Determine the number of vinyl chloride repeat units, hydrogen atoms, chlorine atoms, and carbon atoms in each unit cell of crystalline PVC.

**Solution:** From Table 15–3, we can find the lattice parameters for orthorhombic polyvinyl chloride. The volume of the unit cell is

$$V_{\text{cell}} = (10.4 \times 10^{-8} \text{ cm})(5.3 \times 10^{-8} \text{ cm})(5.1 \times 10^{-8} \text{ cm})$$

$$= 2.811 \times 10^{-22} \text{ cm}^3$$

The molecular weight of vinyl chloride is  $2 \text{ C} + 3 \text{ H} + 1 \text{ Cl} = 62.453 \text{ g/mol}$ . The number of repeat units “ $x$ ” is therefore:

$$1.4 \text{ g/cm}^3 = \frac{(x)(62.453 \text{ g/mol})}{(2.811 \times 10^{-22} \text{ cm}^3)(6.02 \times 10^{23} \text{ units/mol})}$$

$$x = 3.8$$

Therefore in each unit cell, there are:

4 vinyl chloride repeat units  
8 carbon atoms  
12 hydrogen atoms  
4 chlorine atoms

- 15–27** A polyethylene sample is reported to have a density of  $0.97 \text{ g/cm}^3$ . Calculate the percent crystallinity in the sample. Would you expect that the structure of this sample has a large or small amount of branching? Explain.

**Solution:** From Example 15–7, we find that the density of completely crystalline polyethylene is  $0.9932 \text{ g/cm}^3$ . The density of completely amorphous polyethylene was also given in the example as  $0.87 \text{ g/cm}^3$ . Therefore:

$$\% \text{crystallinity} = \frac{(0.9932)(0.97 - 0.87)}{(0.97)(0.9932 - 0.87)} \times 100 = 83.1\%$$

Because the %crystallinity is very high, it is likely that the sample has a very small amount of branching; increasing the branching decreases crystallinity.

- 15–28** Amorphous polyvinyl chloride is expected to have a density of  $1.38 \text{ g/cm}^3$ . Calculate the %crystallization in PVC that has a density of  $1.45 \text{ g/cm}^3$ . (Hint: find the density of completely crystalline PVC from its lattice parameters, assuming four repeat units per unit cell.)

**Solution:** The molecular weight of vinyl chloride is  $2 \text{ C} + 3 \text{ H} + 1 \text{ Cl} = 62.453 \text{ g/mol}$ . The lattice parameters are given in Table 15–3. The density of completely crystalline PVC is therefore:

$$\rho_{\text{PVC}} = \frac{(4 \text{ units/cell})(62.453 \text{ g/mol})}{(10.4 \times 10^{-8} \text{ cm})(5.3 \times 10^{-8} \text{ cm})(5.1 \times 10^{-8} \text{ cm})(6.02 \times 10^{23} \text{ units/mol})}$$

$$= 1.476 \text{ g/cm}^3$$

The percent crystallization is therefore:

$$\% \text{ crystallization} = \frac{(1.476)(1.45 - 1.38)}{(1.45)(1.476 - 1.38)} \times 100 = 74.2\%$$

**15–30** Describe the relative tendencies of the following polymers to crystallize. Explain your answer.

- (a) branched polyethylene versus linear polyethylene
- (b) polyethylene versus polyethylene–polypropylene copolymer
- (c) isotactic polypropylene versus atactic polypropylene
- (d) polymethyl methacrylate versus acetal (polyoxymethylene).

**Solution:** (a) *Linear polyethylene* is more likely to crystallize than *branched polyethylene*. The branching prevents close packing of the polymer chains into the crystalline structure.

(b) *Polyethylene* is more likely to crystallize than the *polyethylene–propylene copolymer*. The propylene monomers have larger side groups than polyethylene and, of course, different repeat units are present in the polymer chains. These factors make close packing of the chains more difficult, reducing the ease with which crystallization occurs.

(c) *Isotactic polypropylene* is more likely to crystallize than *atactic polypropylene*. In isotactic polypropylene, the side groups are aligned, making the polymer chain less random, and permitting the chains to pack more closely in a crystalline manner.

(d) *Acetal, or polyoxymethylene*, is more symmetrical and has smaller side groups than *polymethyl methacrylate*; consequently acetal polymers are more likely to crystallize.

**15–32** A stress of 2500 psi is applied to a polymer serving as a fastener in a complex assembly. At a constant strain, the stress drops to 2400 psi after 100 h. If the stress on the part must remain above 2100 psi in order for the part to function properly, determine the life of the assembly.

**Solution:** First we can determine the relaxation constant  $\lambda$ :

$$\sigma = \sigma_0 \exp(-t/\lambda)$$

$$2400 = 2500 \exp(-100/\lambda) \quad \text{or} \quad \ln(2400/2500) = -100/\lambda$$

$$-0.0408 = -100/\lambda \quad \text{or} \quad \lambda = 2451 \text{ h}$$

Then we can determine the time required before the stress relaxes to 2100 psi:

$$2100 = 2500 \exp(-t/2451) \quad \text{or} \quad \ln(2100/2500) = -t/2451$$

$$-0.1744 = -t/2451 \quad \text{or} \quad t = 427 \text{ h}$$

- 15–33** A stress of 1000 psi is applied to a polymer that operates at a constant strain; after six months, the stress drops to 850 psi. For a particular application, a part made of the same polymer must maintain a stress of 900 psi after 12 months. What should be the original stress applied to the polymer for this application?

**Solution:** First we can determine the relaxation constant  $\lambda$ , using a time of 6 months = 4380 h;

$$\sigma = \sigma_o \exp(-t/\lambda)$$

$$850 = 1000 \exp(-4380/\lambda) \quad \text{or} \quad \ln(850/1000) = -4380/\lambda$$

$$-0.1625 = -4380/\lambda \quad \text{or} \quad \lambda = 26,954 \text{ h}$$

Then we can determine the initial required stress that will assure a stress of only 900 psi after 12 months = 8760 h:

$$900 = \sigma_o \exp(-8760/26,954) = \sigma_o(0.722)$$

$$\sigma_o = 1246 \text{ psi}$$

- 15–34** Data for the rupture time of polyethylene are shown in Figure 15-19. At an applied stress of 700 psi, the figure indicates that the polymer ruptures in 0.2 h at 90°C but survives for 10,000 h at 65°C. Assuming that the rupture time is related to the viscosity, calculate the activation energy for the viscosity of polyethylene and estimate the rupture time at 23°C.

**Solution:** We expect the rupture time to follow the expression:

$$t_r = a \exp(Q_\eta/RT)$$

For  $T = 90^\circ\text{C} = 363 \text{ K}$ ,  $t_r = 0.2 \text{ h}$ , while for  $T = 65^\circ\text{C} = 338 \text{ K}$ ,  $t_r = 10,000 \text{ h}$ . By solving simultaneous equations, we can find the constant “a” and the activation energy  $Q$ :

$$\begin{aligned} 0.2 &= a \exp[Q_\eta/(1.987)(363)] = a \exp(0.0013864 Q_\eta) \\ 10,000 &= a \exp[Q_\eta/(1.987)(338)] = a \exp(0.0014890 Q_\eta) \end{aligned}$$

$$0.00002 = \exp[(0.0013864 - 0.0014890)Q_\eta] = \exp(-0.0001026 Q_\eta)$$

$$\ln(0.00002) = -10.8198 = -0.0001026 Q_\eta$$

$$Q_\eta = 105,456 \text{ cal/mol}$$

$$0.2 = a \exp[105,456/(1.987)(363)] = a \exp(146.21)$$

$$a = 0.2/3.149 \times 10^{63} = 6.35 \times 10^{-65}$$

The rupture time at  $23^\circ\text{C} = 296 \text{ K}$  is therefore:

$$t_r = 6.35 \times 10^{-65} \exp[105,456/(1.987)(296)]$$

$$t_r = 4.70 \times 10^{13} \text{ h}$$

The polyethylene will essentially not rupture at  $23^\circ\text{C}$ .

- 15–35** For each of the following pairs, recommend the one that will most likely have the better impact properties at  $25^\circ\text{C}$ . Explain each of your choices.

- (a) polyethylene versus polystyrene.
- (b) low-density polyethylene versus high-density polyethylene.
- (c) polymethyl methacrylate versus polytetrafluoroethylene.

**Solution:** (a) *Polyethylene* is expected to have better impact properties than *polystyrene*. The polyethylene chains are symmetrical, with small hydrogen side groups, and consequently will deform rapidly when an impact load is applied.

(b) *Low density polyethylene*, which contains substantial branching, is expected to have better impact properties than *high density polyethylene*. The more loosely packed chains in LD polyethylene can more easily move when an impact load is applied.

(c) *Polytetrafluoroethylene* is expected to have better impact properties than *polymethyl methacrylate* (PMMA). PTFE has symmetrical chains with relatively small (F) side groups compared to the chains in PMMA. Consequently chain sliding will be more quickly accomplished in PTFE.

**15–38** The polymer ABS can be produced with varying amounts of styrene, butadiene, and acrylonitrile monomers, which are present in the form of two copolymers: BS rubber and SAN. (a) How would you adjust the composition of ABS if you wanted to obtain good impact properties? (b) How would you adjust the composition if you wanted to obtain good ductility at room temperature? (c) How would you adjust the composition if you wanted to obtain good strength at room temperature?

**Solution:** (a) Improved impact properties are obtained by increasing the amount of butadiene monomer; the elastomer provides large amounts of elastic strain, which helps to absorb an impact blow.

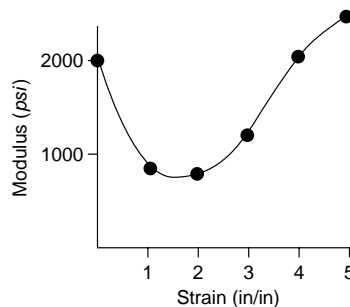
(b) The styrene helps to provide good ductility; the butadiene provides good “elastic” strain but not “plastic” strain. Acrylonitrile has poor ductility when polymerized.

(c) Higher acrylonitrile will help produce higher strengths.

**15–39** Figure 15–24 shows the stress-strain curve for an elastomer. From the curve, calculate and plot the modulus of elasticity versus strain and explain the results.

**Solution:** We obtain the modulus of elasticity by finding the slope of the tangent drawn to the stress-strain curve at different values of strain. Examples of such calculations are shown below:

$$\begin{aligned} \varepsilon = 0 & \quad \Delta\sigma/\Delta\varepsilon = (4000 - 0)/(2 - 0) = 2000 \text{ psi} \\ & \quad = 1 \quad = (5500 - 500)/(6 - 0) = 833 \text{ psi} \\ & \quad = 2 \quad = (5000 - 200)/(6 - 0) = 800 \text{ psi} \\ & \quad = 3 \quad = (6500 - 0)/(6 - 0.6) = 1204 \text{ psi} \\ & \quad = 4 \quad = (8700 - 0)/(6 - 1.7) = 2023 \text{ psi} \\ & \quad = 5 \quad = (9300 - 0)/(6 - 2.2) = 2450 \text{ psi} \end{aligned}$$



The modulus of elasticity is plotted versus strain in the sketch. Initially the modulus decreases as some of the chains become untwisted. However eventually the modulus increases again as the chains become straight and higher stresses are required to stretch the bonds within the chains.

- 15–40** The maximum number of cross-linking sites in polyisoprene is the number of unsaturated bonds in the polymer chain. If three sulfur atoms are in each cross-linking sulfur strand, calculate the amount of sulfur required to provide cross-links at every available site in 5 kg of polymer and the wt% S that would be present in the elastomer. Is this typical?

**Solution:**  $MW_{\text{isoprene}} = 5 \text{ C} + 8 \text{ H} = 68 \text{ g/mol}$        $MW_{\text{sulfur}} = 32 \text{ g/mol}$

After the original chains polymerize, there remains one unsaturated bond per repeat unit within the chain. Each time one double bond is broken, two active sites are created and sulfur atoms then join two repeat units. Therefore, on the average, there is one set of cross-linking sulfur groups per each repeat unit. In other words, the number of moles of isoprene is equal to the number of sulfur atom groups if every cross-linking site is utilized. If just one sulfur atom provided cross-linking at every site, then the amount of sulfur required would be:

$$S/32 \text{ g/mol} = 5000 \text{ g isoprene}/68 \text{ g/mol}$$

$$S = 2,353 \text{ g} = 2.353 \text{ kg}$$

But if there are three sulfur atoms in each cross-linking strand, the total amount of sulfur for complete cross-linking is

$$\text{weight of sulfur} = (3)(2.353 \text{ kg}) = 7.059 \text{ kg}$$

The weight percent sulfur in the polymerized and cross-linked elastomer is:

$$\text{wt\% S} = \frac{7.059}{7.059 + 5} \times 100 = 58.5\%$$

This is far higher than typical elastomers, for which the %S is less than about 5%.

- 15–41** Suppose we vulcanize polychloroprene, obtaining the desired properties by adding 1.5% sulfur by weight to the polymer. If each cross-linking strand contains an average of four sulfur atoms, calculate the fraction of the unsaturated bonds that must be broken.

**Solution:**  $MW_{\text{chloroprene}} = 4 \text{ C} + 5 \text{ H} + 1 \text{ Cl} = 88.453 \text{ g/mol}$

$$MW_{\text{sulfur}} = 32 \text{ g/mol}$$

As in Problem 15–40, one mole of sulfur would be required per each mole of chloroprene if all cross-linking sites were satisfied by one sulfur atom. In 1000 g of chloroprene, the number of moles of chloroprene (and also of sulfur, assuming one sulfur at each site) is:

$$1000 \text{ g}/88.453 \text{ g/mol} = 11.305 \text{ mol of chloroprene}$$

But we have an average of 4 sulfur atoms per strand; therefore for cross-linking at every site, we need  $(4)(11.305) = 45.22 \text{ mol}$  of sulfur. The total weight of sulfur that must be added to 1000 g of the monomer to produce cross-linking at every site with four sulfur atom strands is:

$$\text{maximum sulfur} = (45.22 \text{ mol})(32 \text{ g/mol}) = 1,447 \text{ g}$$

But only 1.5% sulfur is present. If the amount of chloroprene is 1000 g, then the amount of sulfur is:

$$\frac{\text{g of S}}{\text{g of S} + 1000 \text{ g}} \times 100 = 1.5\% \quad \text{or} \quad \text{S} = 15.228 \text{ g}$$

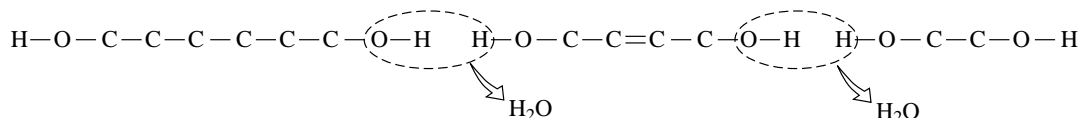
The fraction of the unsaturated bonds that are actually broken is therefore:

$$\text{fraction} = 15.228 \text{ g}/1447 \text{ g} = 0.0105$$

Only a small fraction, about 1%, of the available cross-linking sites are actually used.

- 15-42** The monomers for adipic acid, ethylene glycol, and maleic acid are shown below (see text). These monomers can be joined into chains by condensation reactions, then cross-linked by breaking unsaturated bonds and inserting a styrene molecule as the cross-linking agent. Show how a linear chain composed of these three monomers can be produced.

**Solution:** The original chains are produced by condensation reactions involving the H at the ends of the maleic acid monomer and OH groups at the ends of the other two monomers, producing water as a byproduct:

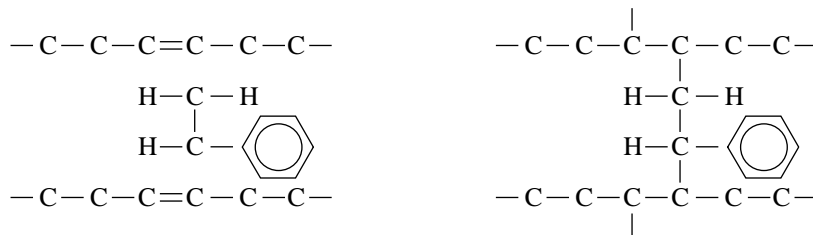


- 15-43** Explain the term thermosetting polymer. Why can't a thermosetting polymer be produced using just adipic acid and ethylene glycol?

**Solution:** Polymers that are heavily cross-linked to produce a strong three dimensional network structure are called thermosetting polymers. Unsaturated bonds are introduced into the linear polymer chain through the maleic acid. If the maleic acid were not present, cross-linking could not occur.

- 15-44** Show how styrene provides cross-linking between the linear chains?

**Solution:** During cross-linking, the unsaturated bonds in the chains, provided by the maleic acid, are broken. This frees up active sites at the two carbon atoms in the maleic acid monomer. When styrene is introduced, the unsaturated bond in styrene is also broken, providing two active sites on it. The active sites on both the chain and the styrene can be satisfied by inserting the styrene as a cross-linking agent:



- 15–45** If 50 g of adipic acid, 100 g of maleic acid, and 50 g of ethylene glycol are combined, calculate the amount of styrene required to completely cross-link the polymer.

**Solution:** For complete cross-linking, we need to introduce one styrene monomer for each maleic acid monomer (assuming only one styrene provides the cross-link). The molecular weights are:

$$MW_{\text{maleic acid}} = 4 \text{ C} + 4 \text{ O} + 4 \text{ H} = 116 \text{ g/mol}$$

$$MW_{\text{styrene}} = 8 \text{ C} + 8 \text{ H} = 104 \text{ g/mol}$$

The amount of styrene is then:

$$\text{g of styrene}/104 \text{ g/mol} = 100 \text{ g maleic acid}/116 \text{ g/mol}$$

$$\text{g of styrene} = 89.655 \text{ g}$$

- 15–46** How much formaldehyde is required to completely cross-link 10 kg of phenol to produce a thermosetting phenolic polymer? How much byproduct is evolved?

**Solution:** To make the chain, we must add 1 mole of formaldehyde per mole of phenol. Then, to completely cross-link the chains (remembering that phenol is really trifunctional), we need an additional mole of formaldehyde for each mole of phenol. The number of moles of phenol added is:

$$MW_{\text{phenol}} = 6 \text{ C} + 6 \text{ H} + 1 \text{ O} = 94 \text{ g/mol}$$

$$\text{moles of phenol} = 10,000 \text{ g}/94 \text{ g/mol} = 106.383 \text{ mol}$$

But we need twice as many moles of formaldehyde, or 212.766 mol. The amount of formaldehyde is therefore:

$$MW_{\text{formaldehyde}} = 1 \text{ C} + 1 \text{ O} + 2 \text{ H} = 30 \text{ g/mol}$$

$$\text{weight of formaldehyde} = (212.766 \text{ mol})(30 \text{ g/mol}) = 6383 \text{ g}$$

The byproduct formed during polymerization is water. For complete polymerization (both chain formation and cross-linking), two moles of water are produced for each mole of phenol. The amount of water is then:

$$\text{weight of water} = (212.766 \text{ mol})(18 \text{ g/mol}) = 3830 \text{ g}$$

- 15–47** Explain why the degree of polymerization is not usually used to characterize thermosetting polymers.

**Solution:** Individual chains are no longer present after the polymer is completely cross-linked and polymerized; instead the entire polymer should be considered continuous.

**15–48** Defend or contradict the choice to use the following materials as hot-melt adhesives for an application in which the assembled part is subjected to impact-type loading:

- (a) polyethylene (b) polystyrene (c) styrene-butadiene thermoplastic elastomer  
(d) polyacrylonitrile and (e) polybutadiene.

**Solution:** (a) Polyethylene is expected to have relatively good impact resistance due to the ease with which chains can move; the polyethylene is well above its glass transition temperature.

(b) Polystyrene is expected to have relatively poor impact resistance due to the resistance to chain sliding by the large benzene ring side groups.

(c) Styrene-butadiene thermoplastic elastomers are expected to have good impact resistance; although the styrene portion may be rather brittle, the high energy absorbing capability of the butadiene component provides good impact properties.

(d) Polyacrylonitrile will have relatively poor impact properties due to the presence of the side groups.

(e) Polybutadiene, an elastomer, will provide good impact properties.

**15–50** Many paints are polymeric materials. Explain why plasticizers are added to paints. What must happen to the plasticizers after the paint is applied?

**Solution:** The plasticizers lower the viscosity and make the paint flow more easily, providing better coverage.

**15–51** You want to extrude a complex component from an elastomer. Should you vulcanize the rubber before or after the extrusion operation? Explain.

**Solution:** The elastomer must be extruded before vulcanization, while it still behaves much like a thermoplastic polymer. After extrusion, vulcanization can occur. Now the polymer develops its high elastic strain, although it can no longer be plastically deformed.

**15–52** Suppose a thermoplastic polymer can be produced in sheet form either by rolling (deformation) or by continuous casting (with a rapid cooling rate). In which case would you expect to obtain the higher strength? Explain.

**Solution:** During rolling, the chains become aligned in the direction of rolling, perhaps even assuming a high degree of crystallinity. The rolled sheet will have a high tensile strength, particularly in the direction of rolling.

During solidification, particularly at a high rate of cooling, crystallization will be suppressed and a relatively low strength, amorphous polymer structure is expected.



- 15–66** The data below were obtained for polyethylene. Determine (a) the weight average molecular weight and degree of polymerization and (b) the number average molecular weight and degree of polymerization.

Solution:

Molecular weight range	$x_i$	$f_i$	$M_i$	$f_i M_i$	$x_i M_i$
0–3,000	0.03	0.01	1,500	15	45
3,000–6,000	0.10	0.08	4,500	360	450
6,000–9,000	0.22	0.19	7,500	1425	1650
9,000–12,000	0.36	0.27	10,500	2835	3780
12,000–15,000	0.19	0.23	13,500	3105	2565
15,000–18,000	0.07	0.11	16,500	1815	1155
18,000–21,000	0.02	0.06	19,500	1170	390
21,000–24,000	0.01	0.05	22,500	1125	225
sum =				11,850	10,260

- (a) The molecular weight of the ethylene monomer is 28 g/mol. Therefore the weight average molecular weight and degree of polymerization are:

$$M_x = 11,850 \text{ g/mol} \quad DP_x = 11,850/28 \text{ g/mol} = 423$$

- (b) The number average molecular weight and degree of polymerization are:

$$M_n = 10,260 \text{ g/mol} \quad DP_n = 10,260/28 \text{ g/mol} = 366$$

# 16

## Composites: Teamwork and Synergy in Materials

- 16–7** Nickel containing 2 wt% thorium is produced in powder form, consolidated into a part, and sintered in the presence of oxygen, causing all of the thorium to produce ThO<sub>2</sub> spheres 80 nm in diameter. Calculate the number of spheres per cm<sup>3</sup>. The density of ThO<sub>2</sub> is 9.86 g/cm<sup>3</sup>.

**Solution:** In 100 g of material, there are  $98 \text{ g} / 8.902 \text{ g/cm}^3 = 11.0088 \text{ cm}^3$  of nickel. From the reaction  $\text{Th} + \text{O}_2 = \text{ThO}_2$ ,

$$2 \text{ g Th} / 232 \text{ g/mol} = x \text{ g ThO}_2 / 264 \text{ g/mol}$$

$$x = 2.2759 \text{ g ThO}_2$$

The total volume of the oxide is:

$$V_{\text{oxide}} = 2.2759 \text{ g} / 9.86 \text{ g/cm}^3 = 0.2308 \text{ cm}^3$$

The volume fraction of the oxide is

$$f_{\text{oxide}} = \frac{0.2308}{0.2308 + 11.0088} = 0.0205$$

The volume of each oxide sphere is:

$$V_{\text{sphere}} = (4\pi/3)r^3 = (4\pi/3)(40 \times 10^{-7} \text{ cm})^3 = 2.68 \times 10^{-16} \text{ cm}^3$$

The total number of oxide particles in 1 cm<sup>3</sup> is:

$$\begin{aligned} \text{particles} &= 0.0205 \text{ cm}^3 \text{ ThO}_2 / 2.68 \times 10^{-16} \text{ cm}^3/\text{particle} \\ &= 7.65 \times 10^{13} \text{ particles/cm}^3 \end{aligned}$$

- 16–8** Spherical aluminum powder (SAP) 0.002 mm in diameter is treated to create a thin oxide layer and is then used to produce a SAP dispersion-strengthened material containing 10 vol% Al<sub>2</sub>O<sub>3</sub>. Calculate the average thickness of the oxide film prior to compaction and sintering of the powders into the part.

**Solution:** The volume of an aluminum powder particle is:

$$V_{\text{Al}} = (4\pi/3)(0.002 \text{ mm}/2)^3 = 4.19 \times 10^{-9} \text{ mm}^3$$

The volume fraction  $\text{Al}_2\text{O}_3$  is

$$0.10 = \frac{V_{\text{oxide}}}{V_{\text{oxide}} + V_{\text{Al}}} = \frac{V_{\text{oxide}}}{V_{\text{oxide}} + 4.19 \times 10^{-9}}$$

$$V_{\text{oxide}} = 4.654 \times 10^{-10} \text{ mm}^3$$

We can then calculate the radius of the particle after oxidation has occurred:

$$V_{\text{oxide}} = (4\pi/3)r^3 - 4.19 \times 10^{-9} = 4.65 \times 10^{-10}$$

$$r = 1.0358 \times 10^{-3} \text{ mm} = 0.0010358 \text{ mm}$$

The thickness of the oxide layer must therefore be:

$$\begin{aligned} \text{thickness} &= 0.0010358 - 0.001 = 0.0000358 \text{ mm} \\ &= 3.58 \times 10^{-5} \text{ mm} \end{aligned}$$

- 16-9** Ytria ( $\text{Y}_2\text{O}_3$ ) particles 750 Å in diameter are introduced into tungsten by internal oxidation. Measurements using an electron microscope show that there are  $5 \times 10^{14}$  oxide particles per  $\text{cm}^3$ . Calculate the wt% Y originally in the alloy. The density of  $\text{Y}_2\text{O}_3$  is  $5.01 \text{ g/cm}^3$ .

**Solution:** The volume of each particle is:

$$V_{\text{oxide}} = (4\pi/3)(750 \times 10^{-8}/2 \text{ cm})^3 = 2.209 \times 10^{-16} \text{ cm}^3$$

The total volume of oxide particles per  $\text{cm}^3$  is given by:

$$V_{\text{yttria}} = (2.209 \times 10^{-16} \text{ cm}^3)(5 \times 10^{14} \text{ particles}) = 0.11 \text{ cm}^3$$

The volume fraction of yttria is therefore

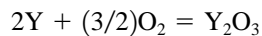
$$f_{\text{oxide}} = 0.11$$

The weight percentages of oxide and tungsten are

$$\text{wt\% } \text{Y}_2\text{O}_3 = \frac{(0.11)(5.01 \text{ g/cm}^3)}{(0.11)(5.01) + (0.89)(19.254)} \times 100 = 3.116\%$$

$$\text{wt\% W} = 96.884\%$$

In 1 g of material, there are 0.03116 g of oxide. From the equation



$$x \text{ g of Y} / (88.91 \text{ g/mol}) = 0.03116 \text{ g of } \text{Y}_2\text{O}_3 / 225.82 \text{ g/mol}$$

$$x = 0.0245 \text{ g of Y}$$

The weight percent Y in the original alloy was therefore:

$$\text{wt\% Y} = \frac{0.0245 \text{ g Y}}{0.0245 \text{ g Y} + 0.96884 \text{ g W}} \times 100 = 2.47\%$$

- 16-10** With no special treatment, aluminum is typically found to have an  $\text{Al}_2\text{O}_3$  layer that is 3 nm thick. If spherical aluminum powder prepared with a total diameter of 0.01 mm is used to produce the SAP dispersion-strengthened material, calculate the volume percent  $\text{Al}_2\text{O}_3$  in the material and the number of oxide particles per  $\text{cm}^3$ . Assume

that the oxide breaks into disk-shaped flakes 3 nm thick and  $3 \times 10^{-4}$  mm in diameter. Compare the number of oxide particles per  $\text{cm}^3$  to the number of solid solution atoms per  $\text{cm}^3$  when 3 at% of an alloying element is added to aluminum.

**Solution:** The total volume of the powder particle is:

$$V_{\text{total}} = (4\pi/3)(0.01/2)^3 = 5.235988 \times 10^{-7} \text{ mm}^3$$

The volume of just the oxide layer is:

$$\begin{aligned} V_{\text{oxide}} &= 5.235988 \times 10^{-7} - (4\pi/3)(0.005 - 3 \times 10^{-6})^3 \\ &= 0.009419 \times 10^{-7} \text{ mm}^3 \end{aligned}$$

The volume fraction of the oxide is:

$$f_{\text{oxide}} = 0.009419 \times 10^{-7} / 5.235988 \times 10^{-7} = 0.001799$$

The volume of one disk-shaped oxide flake is:

$$\begin{aligned} V_{\text{flake}} &= (\pi/4)(3 \times 10^{-4} \text{ mm})^2(3 \times 10^{-6} \text{ mm}) = 2.12 \times 10^{-13} \text{ mm}^3 \\ &= 2.12 \times 10^{-16} \text{ cm}^3 \end{aligned}$$

In one  $\text{cm}^3$  of SAP, there is  $0.001799 \text{ cm}^3$  of oxide. The number of oxide particles per  $\text{cm}^3$  is therefore

$$\begin{aligned} \text{number} &= 0.001799 \text{ cm}^3 / 2.12 \times 10^{-16} \text{ cm}^3/\text{particle} \\ &= 8.49 \times 10^{12} \text{ flakes/cm}^3 \end{aligned}$$

The number of solid solution atoms per  $\text{cm}^3$  in an Al–3 at% alloying element alloy is calculated by first determining the volume of the unit cell:

$$V_{\text{cell}} = (4.04958 \times 10^{-8} \text{ cm})^3 = 66.41 \times 10^{-24} \text{ cm}^3$$

In 25 unit cells of FCC aluminum, there are 100 atom sites. In the alloy, 3 of these sites are filled with substitutional atoms, the other 97 sites by aluminum atoms. The number of solid solution atoms per  $\text{cm}^3$  is therefore:

$$\begin{aligned} \text{number} &= 3 \text{ atoms in 25 cells} / (25 \text{ cells})(66.41 \times 10^{-24} \text{ cm}^3/\text{cell}) \\ &= 18.1 \times 10^{20} \text{ substitutional atoms/cm}^3 \end{aligned}$$

The number of substitutional point defects is eight orders of magnitude larger than the number of oxide flakes.

- 16–13** Calculate the density of a cemented carbide, or cermet, based on a titanium matrix if the composite contains 50 wt% WC, 22 wt% TaC, and 14 wt% TiC. (See Example 16–2 for densities of the carbides.)

**Solution:** We must find the volume fractions from the weight percentages. Using a basis of 100 g of the cemented carbide:

$$f_{\text{WC}} = \frac{50 \text{ g WC}/15.77 \text{ g/cm}^3}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.298$$

$$f_{\text{TaC}} = \frac{22 \text{ g TaC}/14.5 \text{ g/cm}^3}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.143$$

$$f_{\text{TiC}} = \frac{14 \text{ g TiC}/4.94 \text{ g/cm}^3}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.267$$

$$f_{\text{Ti}} = \frac{14 \text{ g Ti}/4.507 \text{ g/cm}^3}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.292$$

The density is then found from the rule of mixtures:

$$\rho_c = (0.298)(15.77) + (0.143)(14.5) + (0.267)(4.94) + (0.292)(4.507) = 9.408 \text{ g/cm}^3$$

- 16–14** A typical grinding wheel is 9 in. in diameter, 1 in. thick, and weighs 6 lb. The wheel contains SiC (density of  $3.2 \text{ g/cm}^3$ ) bonded by a silica glass (density of  $2.5 \text{ g/cm}^3$ ); 5 vol% of the wheel is porous. The SiC is in the form of  $0.04 \text{ cm}$  cubes. Calculate (a) the volume fraction of SiC particles in the wheel and (b) the number of SiC particles lost from the wheel after it is worn to a diameter of 8 in.

**Solution:** (a) To find the volume fraction of SiC:

$$V_{\text{wheel}} = (\pi/4)D^2h = (\pi/4)(9 \text{ in.})^2(1 \text{ in.}) = 63.617 \text{ in.}^3 = 1042.5 \text{ cm}^3$$

$$W_{\text{wheel}} = 6 \text{ lb} = 2724 \text{ g}$$

$$\rho_{\text{wheel}} = 2724 \text{ g}/1042.5 \text{ cm}^3 = 2.6129 \text{ g/cm}^3$$

From the rule of mixtures:

$$2.6129 = f_{\text{pore}}\rho_{\text{pore}} + f_{\text{SiC}}\rho_{\text{SiC}} + f_{\text{glass}}\rho_{\text{glass}}$$

$$2.6129 = (0.05)(0) + f_{\text{SiC}}(3.2) + (1 - 0.05 - f_{\text{SiC}})(2.5)$$

$$f_{\text{SiC}} = 0.34$$

- (b) First we can determine the volume of the wheel that is lost; then we can find the number of particles of SiC per  $\text{cm}^3$ .

$$V_{\text{lost}} = (\pi/4)(9)^2(1) - (\pi/4)(8)^2(1) = 13.352 \text{ in.}^3 = 218.8 \text{ cm}^3$$

$$V_{\text{particles}} = (0.04 \text{ cm})^3 = 6.4 \times 10^{-5} \text{ cm}^3$$

In  $1 \text{ cm}^3$  of the wheel, there are  $(0.34)(1 \text{ cm}^3) = 0.34 \text{ cm}^3$  of SiC. The number of SiC particles per  $\text{cm}^3$  of the wheel is:

$$0.34 \text{ cm}^3 / 6.4 \times 10^{-5} \text{ cm}^3/\text{particle} = 5312.5 \text{ particles/cm}^3$$

The number of particles lost during use of the wheel is:

$$\text{particles lost} = (5312.5/\text{cm}^3)(218.8 \text{ cm}^3) = 1.16 \times 10^6 \text{ particles}$$

- 16–15** An electrical contact material is produced by infiltrating copper into a porous tungsten carbide (WC) compact. The density of the final composite is  $12.3 \text{ g/cm}^3$ . Assuming that all of the pores are filled with copper, calculate (a) the volume fraction of copper in the composite, (b) the volume fraction of pores in the WC compact prior to infiltration, and (c) the original density of the WC compact before infiltration.

**Solution:** (a)  $\rho_c = 12.3 \text{ g/cm}^3 = f_{\text{Cu}}\rho_{\text{Cu}} + f_{\text{WC}}\rho_{\text{WC}} = f_{\text{Cu}}(8.93) + (1 - f_{\text{Cu}})(15.77)$   
 $f_{\text{Cu}} = 0.507$

- (b) The copper fills the pores. Therefore the volume fraction of the pores prior to infiltration is equal to that of the copper, or  $f_{\text{pores}} = 0.507$ .

- (c) Before infiltration, the composite contains tungsten carbide and pores (which have zero density):

$$\rho_{\text{compact}} = f_{\text{WC}}\rho_{\text{WC}} + f_{\text{pore}}\rho_{\text{pore}} = (0.493)(15.77) + (0.507)(0) = 7.775 \text{ g/cm}^3$$

- 16–16** An electrical contact material is produced by first making a porous tungsten compact that weighs 125 g. Liquid silver is introduced into the compact; careful measurement indicates that 105 g of silver is infiltrated. The final density of the composite is 13.8 g/cm<sup>3</sup>. Calculate the volume fraction of the original compact that is interconnected porosity and the volume fraction that is closed porosity (no silver infiltration).

**Solution:** First we can find the volume of the tungsten and silver:

$$V_W = 125 \text{ g} / 19.254 \text{ g/cm}^3 = 6.492 \text{ cm}^3$$

$$V_{Ag} = 105 \text{ g} / 10.49 \text{ g/cm}^3 = 10.010 \text{ cm}^3$$

The volume fractions of each constituent are:

$$f_W = \frac{6.492}{6.492 + 10.010 + V_{\text{pore}}}$$

$$f_{Ag} = \frac{10.010}{6.492 + 10.010 + V_{\text{pore}}}$$

$$f_{\text{pore}} = \frac{V_{\text{pore}}}{6.492 + 10.010 + V_{\text{pore}}}$$

From the rule of mixtures:

$$13.8 = [6.492 / (6.492 + 10.010 + V_{\text{pore}})](19.254) + [10.010 / (6.492 + 10.010 + V_{\text{pore}})](10.49) + 0$$

$$V_{\text{pore}} = 0.165 \text{ cm}^3$$

The total volume is  $6.492 + 10.010 + 0.165 = 16.667 \text{ cm}^3$ . The fraction of the contact material that is interconnected porosity prior to silver infiltration is equal to the volume fraction of silver; the volume fraction of closed porosity is obtained from  $V_{\text{pore}}$ .

$$f_{\text{interconnected}} = 10.010 / 16.667 = 0.6005$$

$$f_{\text{closed}} = 0.165 / 16.667 = 0.0099$$

- 16–17** How much clay must be added to 10 kg of polyethylene to produce a low-cost composite having a modulus of elasticity greater than 120,000 psi and a tensile strength greater than 2000 psi? The density of the clay is 2.4 g/cm<sup>3</sup> and that of polyethylene is 0.92 g/cm<sup>3</sup>.

**Solution:** From Figure 16–6, we find that  $f_{\text{clay}}$  must be greater than 0.3 if the modulus is to exceed 120,000 psi; however the  $f_{\text{clay}}$  must be less than 0.46 to assure that the tensile strength exceeds 2000 psi. Therefore any clay fraction between 0.30 and 0.46 should be satisfactory.

$$f_{\text{clay}} = \frac{W_{\text{clay}} / 2.4 \text{ g/cm}^3}{(W_{\text{clay}} / 2.4) + (10,000 \text{ g} / 0.92)}$$

$$\text{If } f_{\text{clay}} = 0.3, \text{ then } W_{\text{clay}} = 11,200 \text{ g} = 11.2 \text{ kg}$$

$$\text{If } f_{\text{clay}} = 0.46, \text{ then } W_{\text{clay}} = 22,250 \text{ g} = 22.25 \text{ kg}$$

The overall cost of the composite will be reduced as the amount of clay added to the composite increases.

- 16–18** We would like to produce a lightweight epoxy part to provide thermal insulation. We have available hollow glass beads for which the outside diameter is 1/16 in. and the wall thickness is 0.001 in. Determine the weight and number of beads that must be added to the epoxy to produce a one-pound composite with a density of 0.65 g/cm<sup>3</sup>. The density of the glass is 2.5 g/cm<sup>3</sup> and that of the epoxy is 1.25 g/cm<sup>3</sup>.

**Solution:** First we can find the total volume of a glass bead, the volume of the glass portion of the bead, the weight of the glass in the bead, and finally the overall density (weight of glass divided by the total volume) of the bead. The air in the hollow bead is assumed to be weightless.

$$V_{\text{bead}} = (4\pi/3)(1/32)^3 = 1.27832 \times 10^{-4} \text{ in.}^3 = 2.0948 \times 10^{-3} \text{ cm}^3$$

$$V_{\text{glass}} = 1.27832 \times 10^{-4} - (4\pi/3)[0.03125 - 0.001]^3 \\ = 0.1188 \times 10^{-4} \text{ in.}^3 = 1.947 \times 10^{-4} \text{ cm}^3$$

$$W_{\text{glass}} = (1.947 \times 10^{-4} \text{ cm}^3)(2.5 \text{ g/cm}^3) = 4.8675 \times 10^{-4} \text{ g/bead}$$

$$\rho_{\text{bead}} = 4.8675 \times 10^{-4} \text{ g} / 2.0948 \times 10^{-3} \text{ cm}^3 = 0.232 \text{ g/cm}^3$$

Now we can use the rule of mixtures to determine the volume fraction of beads that must be introduced into the epoxy.

$$\rho_c = 0.65 = f_{\text{bead}} \rho_{\text{bead}} + (1 - f_{\text{bead}}) \rho_{\text{epoxy}} \\ = f_{\text{bead}}(0.232) + (1 - f_{\text{bead}})(1.25) \\ f_{\text{bead}} = 0.59$$

We want to produce 1 lb = 454 g = 454 g/0.65 g/cm<sup>3</sup> = 698.46 cm<sup>3</sup> of composite material. The volume of beads required is

$$(698.46 \text{ cm}^3)(0.59) = 412 \text{ cm}^3 \text{ of beads}$$

$$\text{wt of beads} = (412 \text{ cm}^3)(0.232 \text{ g/cm}^3) = 95.58 \text{ g of beads}$$

The number of beads needed is

$$\text{number} = 95.58 \text{ g} / 4.8675 \times 10^{-4} \text{ g/bead} = 1.96 \times 10^5 \text{ beads}$$

- 16–24** Five kg of continuous boron fibers are introduced in a unidirectional orientation into 8 kg of an aluminum matrix. Calculate (a) the density of the composite, (b) the modulus of elasticity parallel to the fibers, and (c) the modulus of elasticity perpendicular to the fibers.

**Solution:**  $f_B = \frac{5 \text{ kg}/2.3 \text{ g/cm}^3}{5 \text{ kg}/2.3 + 8 \text{ kg}/2.699} = 0.423 \quad f_{Al} = 0.577$

$$(a) \rho_c = f_B \rho_B + f_{Al} \rho_{Al} = (0.423)(2.3) + (0.577)(2.699) = 2.530 \text{ g/cm}^3$$

$$(b) E_c = f_B E_B + f_{Al} E_{Al} = (0.423)(55 \times 10^6) + (0.577)(10 \times 10^6) \\ = 29 \times 10^6 \text{ psi}$$

$$(c) 1/E_c = f_B/E_B + f_{Al}/E_{Al} = 0.423/55 \times 10^6 + 0.577/10 \times 10^6 \\ = 0.0654 \times 10^{-6}$$

$$E_c = 15.3 \times 10^6 \text{ psi}$$

- 16–25** We want to produce 10 lbs of a continuous unidirectional fiber-reinforced composite of HS carbon in a polyimide matrix that has a modulus of elasticity of at least  $25 \times 10^6$  psi parallel to the fibers. How many pounds of fibers are required? See Chapter 15 for properties of polyimide.

**Solution:** The modulus for HS carbon is  $40 \times 10^6$  psi and for polyimide is 300,000 psi. From the rule of mixtures, we can determine the required volume fraction of fibers:

$$25 \times 10^6 \text{ psi} = f_{\text{carbon}}(40 \times 10^6 \text{ psi}) + (1 - f_{\text{carbon}})(0.3 \times 10^6 \text{ psi})$$

$$f_{\text{carbon}} = 0.622$$

Then we can find the weight of fibers required to produce 10 lbs of composite:

$$0.622 = \frac{W_{\text{carbon}}/1.75 \text{ g/cm}^3}{W_{\text{carbon}}/1.75 + (10 - W_{\text{carbon}})/1.39} \quad \text{or} \quad W_{\text{carbon}} = 6.75 \text{ lbs}$$

- 16–26** We produce a continuous unidirectionally reinforced composite containing 60 vol% HM carbon fibers in an epoxy matrix. The epoxy has a tensile strength of 15,000 psi. What fraction of the applied force is carried by the fibers?

**Solution:** From Example 16–8, we find that the fraction carried by the fibers is given by:

$$f = \frac{\sigma_f A_f}{\sigma_f A_f + \sigma_m A_m}$$

We can replace the areas by the volume fractions (assuming that the part has a continuous cross-section). Thus  $A_f = 0.6$  and  $A_m = 0.4$ . The tensile strength of the fibers is 270,000 psi and that of the epoxy matrix is 15,000 psi. Thus:

$$f = \frac{(270,000)(0.6)}{(270,000)(0.6) + (15,000)(0.4)} = 0.964$$

Over 96% of the force is carried by the fibers.

- 16–27** A polyester matrix with a tensile strength of 13,000 psi is reinforced with  $\text{Al}_2\text{O}_3$  fibers. What vol% fibers must be added to insure that the fibers carry 75% of the applied load?

**Solution:** The tensile strength of the fibers is 300,000 psi and that of the polyester is 13,000 psi. From Example 16–8, and assuming a total area of one  $\text{cm}^2$ :

$$f = \frac{\sigma_f A_f}{\sigma_f A_f + \sigma_m A_m} = 0.75$$

$$\frac{300,000 A_f}{300,000 A_f + 13,000(1 - A_f)} = 0.75 \quad \text{or} \quad A_f = 0.115$$

Assuming that the area and volume fractions are the same, the volume fraction of fibers is  $f_{\text{alumina}} = 0.115$



- 16–28** An epoxy matrix is reinforced with 40 vol% E-glass fibers to produce a 2-cm diameter composite that is to withstand a load of 25,000 N. Calculate the stress acting on each fiber.

**Solution:** We can assume that the strains in the composite, matrix, and fibers are equal. Thus:

$$\epsilon_c = \epsilon_m = \epsilon_f = \sigma_m/E_m = \sigma_f/E_f$$

The modulus for the E-glass is  $10.5 \times 10^6$  psi and that for the epoxy is  $0.4 \times 10^6$  psi. Therefore the ratio of the stresses is:

$$\sigma_f/\sigma_m = E_f/E_m = 10.5 \times 10^6/0.4 \times 10^6 = 26.25$$

The fraction of the force carried by the fibers (as described in Example 16–8) is (assuming that area and volume fractions are equal):

$$\begin{aligned} f &= \frac{\sigma_f A_f}{\sigma_f A_f + \sigma_m A_m} = \frac{A_f}{A_f + (\sigma_m/\sigma_f)A_m} \\ &= \frac{0.4}{0.4 + (1/26.25)(0.6)} = 0.9459 \end{aligned}$$

Since the total force is 25,000 N, the force carried by the fibers is:

$$F_f = (0.9459)(25,000 \text{ N}) = 23,650 \text{ N}$$

The cross-sectional area of the fibers is:

$$A_f = (f_f)(\pi/4)d^2 = (0.4)(\pi/4)(20 \text{ mm})^2 = 125.66 \text{ mm}^2$$

Thus the stress is:

$$\sigma_f = 23,650 \text{ N}/125.66 \text{ mm}^2 = 188 \text{ MPa}$$

- 16–29** A titanium alloy with a modulus of elasticity of  $16 \times 10^6$  psi is used to make a 1000-lb part for a manned space vehicle. Determine the weight of a part having the same modulus of elasticity parallel to the fibers, if the part is made of (a) aluminum reinforced with boron fibers and (b) polyester (with a modulus of 650,000 psi) reinforced with high-modulus carbon fibers. (c) Compare the specific modulus for all three materials.

**Solution:** The titanium alloy has a density of about  $4.507 \text{ g/cm}^3 = 0.163 \text{ lb/in.}^3$ . The volume of the 1000 lb part is therefore

$$V_{\text{part}} = 1000/0.163 = 6135 \text{ in.}^3$$

$$\begin{aligned} \text{(a) } E_c &= 16 \times 10^6 = f_B E_B + (1 - f_B) E_{Al} \\ &= f_B (55 \times 10^6) + (1 - f_B) (10 \times 10^6) \end{aligned}$$

$$f_B = 0.133$$

$$\begin{aligned} \rho_c &= (0.133)(2.36 \text{ g/cm}^3) + (0.867)(2.699 \text{ g/cm}^3) = 2.654 \text{ g/cm}^3 \\ &= 0.0958 \text{ lb/in.}^3 \end{aligned}$$

To produce a  $6135 \text{ in.}^3$  part of the composite, the part must weigh:

$$\text{Weight} = (6135 \text{ in.}^3)(0.0958 \text{ lb/in.}^3) = 587.7 \text{ lb}$$

$$\begin{aligned} (b) E_C &= 16 \times 10^6 = f_C E_C + (1 - f_C) E_{PET} \\ &= f_V (77 \times 10^6) + (1 - f_C) (0.65 \times 10^6) \\ f_C &= 0.201 \end{aligned}$$

$$\begin{aligned} \rho_C &= (0.201)(1.9 \text{ g/cm}^3) + (0.799)(1.28 \text{ g/cm}^3) = 1.405 \text{ g/cm}^3 \\ &= 0.0507 \text{ lb/in.}^3 \end{aligned}$$

To produce a 6135 in.<sup>3</sup> part of the composite, the part must weigh:

$$\text{Weight} = (6135 \text{ in.}^3)(0.0507 \text{ lb/in.}^3) = 311 \text{ lb}$$

(c) The specific moduli of the three materials are:

$$\text{Ti: } E/\rho = 16 \times 10^6 \text{ psi}/0.163 \text{ lb/in.}^3 = 9.82 \times 10^7 \text{ in.}$$

$$\text{B-Al: } E/\rho = 16 \times 10^6 \text{ psi}/0.0958 \text{ lb/in.}^3 = 16.7 \times 10^7 \text{ in.}$$

$$\text{C-PET: } E/\rho = 16 \times 10^6 \text{ psi}/0.0507 \text{ lb/in.}^3 = 31.6 \times 10^7 \text{ in.}$$

**16–30** Short but aligned Al<sub>2</sub>O<sub>3</sub> fibers with a diameter of 20 μm are introduced into a 6,6-nylon matrix. The strength of the bond between the fibers and the matrix is estimated to be 1000 psi. Calculate the critical fiber length and compare with the case when 1-μm alumina whiskers are used instead of the coarser fibers. What is the minimum aspect ratio in each case?

**Solution:** The critical fiber length is given by  $\ell_c = \sigma_f d / 2\tau_i$ . For the alumina fibers,  $\tau_i = 1000 \text{ psi} = 6.897 \text{ MPa}$ ;  $d = 20 \times 10^{-6} \text{ m}$ ; and  $\sigma_f = 300,000 \text{ psi} = 2069 \text{ MPa}$ . Thus, for alumina fibers:

$$\begin{aligned} \ell_c &= (2069 \text{ MPa})(20 \times 10^{-6} \text{ m}) / (2)(6.897 \text{ MPa}) \\ &= 0.003 \text{ m} = 0.3 \text{ cm} \end{aligned}$$

$$\ell_c / d = 0.3 \text{ cm} / 20 \times 10^{-4} \text{ cm} = 150$$

For alumina whiskers,  $d = 1 \times 10^{-6} \text{ m}$ . The strength of the whiskers can be much higher than that of the fibers; 3,000,000 psi = 20,690 MPa can be achieved. Thus, for alumina whiskers:

$$\begin{aligned} \ell_c &= (20,690 \text{ MPa})(1 \times 10^{-6} \text{ m}) / (2)(6.897 \text{ MPa}) = 0.0015 \text{ m} \\ &= 0.15 \text{ cm} \end{aligned}$$

$$\ell_c / d = 0.15 \text{ cm} / 1 \times 10^{-4} \text{ cm} = 1500$$

**16–31** We prepare several epoxy matrix composites using different lengths of 3-μm-diameter ZrO<sub>2</sub> fibers and find that the strength of the composite increases with increasing fiber length up to 5 mm. For longer fibers, the strength is virtually unchanged. Estimate the strength of the bond between the fibers and the matrix.

**Solution:** We do not expect much change in the strength when  $\ell > 15\ell_c$ . Therefore:

$$15\ell_c = 5 \text{ mm} \quad \text{or} \quad \ell_c = 0.333 \text{ mm}$$

$$\text{In addition, } \ell_c = \sigma_f d / 2\tau_i = 0.333 \text{ mm}$$

For ZrO<sub>2</sub> fibers, the tensile strength = 300,000 psi = 2069 MPa. Therefore:

$$\tau_i = \sigma_f d / 2\ell_c = (2069 \text{ MPa})(0.003 \text{ mm}) / (2)(0.333 \text{ mm}) = 9.32 \text{ MPa}$$

- 16–36** In one polymer-matrix composite, as produced discontinuous glass fibers are introduced directly into the matrix; in a second case, the fibers are first “sized.” Discuss the effect this difference might have on the critical fiber length and the strength of the composite.

**Solution:** By sizing the glass fibers, the surface is conditioned so that improved bonding between the fibers and the matrix is obtained. From Equation 16–9, we expect that improved bonding ( $\tau_i$ ) will reduce the length of fibers required for achieving good strength. Improved bonding will also reduce pull-out of the fibers from the matrix. Therefore the sizing improves the strength and allows small fibers to still be effective.

- 16–37** A Borsic fiber-reinforced aluminum composite is shown in Figure 16–18. Estimate the volume fractions of tungsten, boron, and the matrix for this composite. Calculate the modulus of elasticity parallel to the fibers for this composite. What would the modulus be if the same size boron fiber could be produced without the tungsten precursor?

**Solution:** From the photograph, the diameter of the tungsten core is about 2 mm, the diameter of the boron fiber is 30 mm, and the distance between the centers of adjacent fibers is 33 mm. If we assume that the fibers produce a square arrangement (see sketch), then

$$A_{\text{total}} = (33 \text{ mm})^2 = 1089 \text{ mm}^2$$

$$A_{\text{tungsten}} = (\pi/4)(2 \text{ mm})^2 = 3.14 \text{ mm}^2$$

$$A_{\text{boron}} = (\pi/4)(30 \text{ mm})^2 - 3.14 \text{ mm}^2 = 703.72 \text{ mm}^2$$

$$A_{\text{Al}} = 1089 - 3.14 - 703.72 = 382.14 \text{ mm}^2$$

We can then determine the volume fractions:

$$f_{\text{tungsten}} = 3.14/1089 = 0.0029$$

$$f_{\text{boron}} = 703.72/1089 = 0.6462$$

$$f_{\text{Al}} = 382.14/1089 = 0.3509$$

We can now estimate the modulus of elasticity of the composite using the rule of mixtures:

$$E_{\text{composite}} = (0.0029)(59 \times 10^6) + (0.6462)(55 \times 10^6) + (0.3509)(10 \times 10^6) = 39.22 \times 10^6 \text{ psi}$$

If the tungsten filament was absent, then  $f_{\text{boron}} = 0.6491$  and the modulus is:

$$E_{\text{composite}} = (0.6491)(55 \times 10^6) + (0.3509)(10 \times 10^6) = 39.21 \times 10^6 \text{ psi}$$

The tungsten makes virtually no difference in the stiffness of the overall composite. Its function is to serve as the precursor for the boron.

- 16–38** A silicon nitride matrix reinforced with silicon carbide fibers containing a HS carbon precursor is shown in Figure 16–18. Estimate the volume fractions of the SiC, Si<sub>3</sub>N<sub>4</sub>, and carbon in this composite. Calculate the modulus of elasticity parallel to the fibers for this composite. What would the modulus be if the same size SiC fiber could be produced without the carbon precursor?

**Solution:** From the photograph, the diameter of the carbon core is about 4 mm, the diameter of the SiC fiber is 16 mm, and the fibers produce a “rectangular” area  $29 \text{ mm} \times 31 \text{ mm}$ . Then:

$$A_{\text{total}} = (29 \text{ mm})(31 \text{ mm}) = 899 \text{ mm}^2$$

$$A_{\text{carbon}} = (\pi/4)(4 \text{ mm})^2 = 12.57 \text{ mm}^2$$

$$A_{\text{SiC}} = (\pi/4)(16 \text{ mm})^2 - 12.57 \text{ mm}^2 = 188.5 \text{ mm}^2$$

$$A_{\text{nitride}} = 899 - 12.57 - 188.5 = 697.9 \text{ mm}^2$$

We can then determine the volume fractions:

$$f_{\text{carbon}} = 12.57/899 = 0.014$$

$$f_{\text{SiC}} = 188.5/899 = 0.210$$

$$f_{\text{nitride}} = 697.9/899 = 0.776$$

We can now estimate the modulus of elasticity of the composite using the rule of mixtures:

$$E_{\text{composite}} = (0.014)(40 \times 10^6) + (0.210)(70 \times 10^6) + (0.776)(55 \times 10^6) = 57.94 \times 10^6 \text{ psi}$$

If the carbon filament was absent, then  $f_{\text{SiC}} = 0.224$  and the modulus is:

$$E_{\text{composite}} = (0.224)(70 \times 10^6) + (0.776)(55 \times 10^6) = 58.36 \times 10^6 \text{ psi}$$

The carbon makes virtually no difference in the stiffness of the overall composite. Its function is to serve as the precursor for the silicon carbide.

- 16–39** Explain why bonding between carbon fibers and an epoxy matrix should be excellent, whereas bonding between silicon nitride fibers and a silicon carbide matrix should be poor.

**Solution:** In the carbon/epoxy composite, we are interested in developing high strength, with the stresses carried predominantly by the strong carbon fibers. In order to transfer the applied loads from the weak epoxy to the strong carbon fibers, good bonding is required.

In the  $\text{Si}_3\text{N}_4/\text{SiC}$  composite, we are interested primarily in developing improved fracture toughness. Now we must design the microstructure to absorb and dissipate energy. By assuring that bonding is poor, the silicon nitride fibers can pull out of the silicon carbide matrix. This pull-out requires energy, thus improving the fracture toughness of the ceramic matrix composite.

- 16–41** A polyimide matrix is to be reinforced with 70 vol% carbon fibers to give a minimum modulus of elasticity of  $40 \times 10^6$  psi. Recommend a process for producing the carbon fibers required. Estimate the tensile strength of the fibers that are produced.

**Solution:** The modulus of polyimide is  $0.3 \times 10^6$  psi. The required modulus of the carbon fibers can be found from the rule of mixtures:

$$E_{\text{composite}} = f_{\text{carbon}}E_{\text{carbon}} + f_{\text{PI}}E_{\text{PI}}$$

$$40 \times 10^6 = (0.7)E_{\text{carbon}} + (0.3)(0.3 \times 10^6)$$

$$E_{\text{carbon}} = 57.0 \times 10^6 \text{ psi}$$

From Figure 16–19, we find that, to obtain this modulus in the carbon fibers, they must be pyrolyzed at 2500°C. This in turn means that the tensile strength will be about 250,000 psi.

- 16–44** A microlaminate, Arall, is produced using 5 sheets of 0.4-mm-thick aluminum and 4 sheets of 0.2-mm-thick epoxy reinforced with unidirectionally aligned Kevlar™ fibers. The volume fraction of Kevlar™ fibers in these intermediate sheets is 55%. Calculate the modulus of elasticity of the microlaminate parallel and perpendicular to the unidirectionally aligned Kevlar™ fibers. What are the principle advantages of the Arall material compared with those of unreinforced aluminum?

**Solution:** First we can find the volume fractions of each material. The volumes (expressed in a linear direction) are:

$$\begin{aligned} V_{Al} &= (5 \text{ sheets})(0.4 \text{ mm/sheet}) = 2 \text{ mm} \\ V_{Kevlar} &= (0.55)(4 \text{ sheets})(0.2 \text{ mm/sheet}) = 0.44 \text{ mm} \\ V_{epoxy} &= (0.45)(4 \text{ sheets})(0.2 \text{ mm/sheet}) = 0.36 \text{ mm} \\ &\text{total} = 2.8 \text{ mm} \end{aligned}$$

$$\begin{aligned} f_{Al} &= 2/2.8 = 0.714 \\ f_{Kevlar} &= 0.44/2.8 = 0.157 \\ f_{epoxy} &= 0.36/2.8 = 0.129 \end{aligned}$$

From the rule of mixtures, the modulus parallel to the laminate is:

$$\begin{aligned} E_{\text{composite}} &= (0.714)(10 \times 10^6) + (0.157)(18 \times 10^6) \\ &\quad + (0.129)(0.5 \times 10^6) = 10.031 \times 10^6 \text{ psi} \end{aligned}$$

Perpendicular to the laminate:

$$\begin{aligned} 1/E_{\text{composite}} &= 0.714/10 \times 10^6 + 0.157/18 \times 10^6 + 0.129/0.5 \times 10^6 \\ &= 0.338 \times 10^{-6} \\ E_{\text{composite}} &= 2.96 \times 10^6 \text{ psi} \end{aligned}$$

- 16–45** A laminate composed of 0.1-mm-thick aluminum sandwiched around a 2-cm thick layer of polystyrene foam is produced as an insulation material. Calculate the thermal conductivity of the laminate parallel and perpendicular to the layers. The thermal conductivity of aluminum is 0.57 cal/cm · s · K and that of the foam is 0.000077 cal/cm · s · K.

**Solution:** First we find the volume fractions:

$$\begin{aligned} f_{Al} &= 2(0.01 \text{ cm}) / [(2)(0.01 \text{ cm}) + 2 \text{ cm}] = 0.0099 \\ f_{\text{foam}} &= 0.9901 \end{aligned}$$

The thermal conductivity parallel to the laminate is:

$$\begin{aligned} K_{\text{parallel}} &= (0.0099)(0.57) + (0.9901)(0.000077) \\ &= 0.00572 \text{ cal/cm} \cdot \text{s} \cdot \text{K} \end{aligned}$$

Perpendicular to the laminate:

$$\begin{aligned} 1/K_{\text{perpendicular}} &= 0.0099/0.57 + 0.9901/0.000077 = 12,858 \\ K_{\text{perpendicular}} &= 0.000078 \text{ cal/cm} \cdot \text{s} \cdot \text{K} \end{aligned}$$

- 16–46** A 0.01-cm-thick sheet of a polymer with a modulus of elasticity of  $0.7 \times 10^6$  psi is sandwiched between two 4-mm-thick sheets of glass with a modulus of elasticity of  $12 \times 10^6$  psi. Calculate the modulus of elasticity of the composite parallel and perpendicular to the sheets.

**Solution:** The volume fractions are:

$$f_{\text{polymer}} = 0.01 \text{ cm} / (0.01 \text{ cm} + 0.4 \text{ cm} + 0.4 \text{ cm}) = 0.01234$$

$$f_{\text{glass}} = 0.98765$$

The modulus parallel to the laminate is:

$$E_{\text{parallel}} = (0.01234)(0.7 \times 10^6) + (0.98765)(12 \times 10^6)$$

$$= 11.86 \times 10^6 \text{ psi}$$

Perpendicular to the laminate:

$$1/E_{\text{perpendicular}} = 0.01234/0.7 \times 10^6 + 0.98765/12 \times 10^6$$

$$= 0.09994 \times 10^6$$

$$E_{\text{perpendicular}} = 10.0 \times 10^6 \text{ psi}$$

This material is “safety” glass and is used in automobile windshields to keep the windshield from shattering.

- 16–47** A U.S. quarter is 15/16 in. in diameter and is about 1/16 in. thick. Assuming copper costs about \$1.10 per pound and nickel costs about \$4.10 per pound. Compare the material cost in a composite quarter versus a quarter made entirely of nickel.

**Solution:** In a quarter, the thickness (and hence the volume) ratio is 1/6 Ni: 2/3 Cu: 1/6 Ni. The volume fraction of each is:

$$f_{\text{Cu}} = 0.667 \quad f_{\text{Ni}} = 0.333$$

The volume of the quarter, as well as the copper and nickel, are:

$$V_{\text{quarter}} = (\pi/4)(15/16 \text{ in.})^2(1/16 \text{ in.}) = 0.04314 \text{ in.}^3 = 0.707 \text{ cm}^3$$

$$V_{\text{Cu}} = (0.707 \text{ cm}^3)(0.667) = 0.4716 \text{ cm}^3$$

$$V_{\text{Ni}} = (0.707 \text{ cm}^3)(0.333) = 0.2354 \text{ cm}^3$$

The weights of copper and nickel in the coin are:

$$W_{\text{Cu}} = (0.4716 \text{ cm}^3)(8.93 \text{ g/cm}^3) = 4.211 \text{ g} = 0.00928 \text{ lb}$$

$$W_{\text{Ni}} = (0.2354 \text{ cm}^3)(8.902 \text{ g/cm}^3) = 2.0955 \text{ g} = 0.004616 \text{ lb}$$

The cost of each material in the coin is:

$$$/\text{Cu} = (0.00928 \text{ lb})(\$1.10/\text{lb}) = \$0.0102$$

$$$/\text{Ni} = (0.004616 \text{ lb})(\$4.10/\text{lb}) = \$0.0189$$

The total cost of the composite (for materials only) = \$0.0291

If the entire coin were made of nickel, then

$$(0.707 \text{ cm}^3)(8.902 \text{ g/cm}^3)(1/454 \text{ g/lb})(\$4.10/\text{lb}) = \$0.0568$$

By using the composite coin, the cost of the materials is about half that of a pure nickel coin, yet the coin appears to have the nickel (or silvery) color.

- 16–48** Calculate the density of a honeycomb structure composed of the following elements. The two 2-mm-thick cover sheets are produced using an epoxy matrix containing 55 vol% E-glass fibers. The aluminium honeycomb is 2 cm thick; the cells are in the shape of 0.5 cm squares and the walls of the cells are 0.1 mm thick. Estimate the density of the structure. Compare the weight of a 1 m × 2 m panel of the honeycomb compared to a solid aluminum panel with the same dimensions.

**Solution:** Each cell of aluminum can be considered to be a hollow square shape where the dimensions of the cell are 0.5 cm × 0.5 cm × 2 cm, with a wall thickness belonging uniquely to that cell of 0.1 mm/2 = 0.05 mm = 0.005 cm.

$$V_{Al} = (4 \text{ sides})(0.005 \text{ cm})(0.5 \text{ cm})(2 \text{ cm}) = 0.02 \text{ cm}^3$$

The cover sheet dimensions that just cover the single cell described above are 0.5 cm × 0.5 cm × 2 mm = 0.2 cm. The volume is:

$$V_{cover} = (2 \text{ sheets})(0.5 \text{ cm})(0.5 \text{ cm})(0.2 \text{ cm}) = 0.1 \text{ cm}^3$$

The total “height” of the cell, including the cover sheets, is 2 cm + 2(0.2 cm) = 2.4 cm. The total volume of the cell is:

$$V_{total} = (0.5 \text{ cm})(0.5 \text{ cm})(2.4 \text{ cm}) = 0.6 \text{ cm}^3$$

The volume fractions of these constituents are:

$$\begin{aligned} f_{Al \text{ in Cell}} &= 0.02/0.6 = 0.0333 \\ f_{cover} &= 0.1/0.6 = 0.1667 \\ f_{void} &= 0.80 \end{aligned}$$

The densities of the three constituents can be determined. The density of the aluminum in the cells is 2.699 g/cm<sup>3</sup> and the density of the void space within the cells is zero. But the cover sheets are themselves composites:

$$\begin{aligned} \rho_{cover} &= f_{glass} \rho_{glass} + f_{epoxy} \rho_{epoxy} = (0.55)(2.55 \text{ g/cm}^3) \\ &\quad + (0.45)(1.25 \text{ g/cm}^3) = 1.965 \text{ g/cm}^3 \end{aligned}$$

Therefore the overall density of the honeycomb structure is:

$$\begin{aligned} \rho_{honeycomb} &= f_{Al \text{ in Cell}} \rho_{Al \text{ in Cell}} + f_{cover} \rho_{cover} + f_{void} \rho_{void} \\ &= (0.0333)(2.699) + (0.1667)(1.965) + (0.80)(0) \\ &= 0.417 \text{ g/cm}^3 \end{aligned}$$

The weight of a 1 m × 2 m panel of the honeycomb is:

$$\begin{aligned} W_{honeycomb} &= (2.4 \text{ cm})(100 \text{ cm})(200 \text{ cm})(0.417 \text{ g/cm}^3) = 20,016 \text{ g} \\ &= 20.016 \text{ kg} = 44.1 \text{ lb} \end{aligned}$$

If the panel were made of solid aluminum, with the same dimensions, the panel would weigh:

$$\begin{aligned} W_{solid} &= (2.4 \text{ cm})(100 \text{ cm})(200 \text{ cm})(2.669 \text{ g/cm}^3) = 129,552 \text{ g} \\ &= 129.552 \text{ kg} = 285 \text{ lb} \end{aligned}$$

The weight savings using the honeycomb are enormous.

# 17

## Construction Materials

- 17-1** A sample of wood with dimensions 3 in.  $\times$  4 in.  $\times$  12 in. has a dry density of 0.35 g/cm<sup>3</sup>. (a) Calculate the number of gallons of water that must be absorbed by the sample to contain 120% water. (b) Calculate the density after the wood absorbs this amount of water.

**Solution:**  $V = 3 \times 4 \times 12 = 144 \text{ in.}^3 = 2359.7 \text{ cm}^3$

$$\text{dry weight} = 0.35 \times 2359.7 = 825.9 \text{ g}$$

$$@120\% \text{ water} = \frac{\text{weight of water}}{\text{weight of dry wood}} \times 100$$

$$\begin{aligned} \text{(a) water} &= (1.2)(825.9) = 991 \text{ g} = 2.183 \text{ lb} \\ &= (2.183 \text{ lb})(7.48 \text{ gal/ft}^3)/62.4 \text{ lb/ft}^3 = 0.262 \text{ gal} \end{aligned}$$

(b) If the volume remains the same, then

$$\text{density} = \frac{825.9 \text{ g of dry wood} + 991 \text{ g of water}}{2359.7 \text{ cm}^3} = 0.77 \text{ g/cm}^3$$

- 17-2** The density of a sample of oak is 0.90 g/cm<sup>3</sup>. Calculate (a) the density of completely dry oak and (b) the percent water in the original sample.

**Solution:**  $\rho_{12\% \text{ water}} = 0.68 \text{ g/cm}^3$  (Table 17-1)

(a) Therefore, in 100 cm<sup>3</sup> of wood at 12% H<sub>2</sub>O, there are 68 g.

$$12\% \text{ water} = \frac{\text{green weight} - \text{dry weight}}{\text{dry weight}} = \frac{68 - \text{dry weight}}{\text{dry weight}} \times 100$$

$$\text{dry weight} = 68/1.12 = 60.71 \text{ g}$$



- (b) When the density is  $0.90 \text{ g/cm}^3$ , there are 90 g of green wood per  $100 \text{ cm}^3$ . The water is therefore  $90 - 60.71 \text{ g}$ , or  $29.29 \text{ g}$ .

$$\% \text{H}_2\text{O} = \frac{90 \text{ g} - 60.71 \text{ g}}{60.71 \text{ g}} \times 100 = 48.2\%$$

- 17–3** Boards of maple 1 in. thick, 6 in. wide, and 16 ft. long are used as the flooring for a  $60 \text{ ft} \times 60 \text{ ft}$  hall. The boards were cut from logs with a tangential-longitudinal cut. The floor is laid when the boards have a moisture content of 12%. After some particularly humid days, the moisture content in the boards increases to 45%. Determine the dimensional change in the flooring parallel to the boards and perpendicular to the boards. What will happen to the floor? How can this problem be corrected?

**Solution:** Perpendicular:

$$c_{\text{tangential}} = 0.00353 \text{ in./in.} \cdot \% \text{H}_2\text{O for maple}$$

$$\Delta x = x_o[c(M_f - M_i)] = 6[0.00353(45 - 12)] = 0.699 \text{ in. in 6 in.}$$

$$\text{Over a 60 ft span: } \Delta x = \frac{(60 \text{ ft})(12 \text{ in./ft})(0.699 \text{ in.})}{6 \text{ in.}} = 83.9 \text{ in.}$$

The floor will therefore buckle due to the large amount of expansion of the boards perpendicular to the flooring.

Parallel:

For most woods, only about a 0.2% change in dimensions occurs longitudinally. Thus the total change in the length of the boards will be about

$$\Delta y = (0.002)(60 \text{ ft})(12 \text{ in./ft}) = 1.44 \text{ in.}$$

- 17–4** A wall 30 feet long is built using radial-longitudinal cuts of 5-inch wide pine, with the boards arranged in a vertical fashion. The wood contains a moisture content of 55% when the wall is built; however the humidity level in the room is maintained to give 45% moisture in the wood. Determine the dimensional changes in the wood boards and estimate the size of the gaps that will be produced as a consequence of these changes.

**Solution:**  $c_{\text{tangential}} = 0.00141 \text{ in./in.} \cdot \% \text{H}_2\text{O for pine}$

$$\begin{aligned} \Delta x &= (30 \text{ ft})(12 \text{ in./ft})[(0.00141 \text{ in./in.} \cdot \% \text{H}_2\text{O})(45 - 55)] \\ &= -5.076 \text{ in.} \end{aligned}$$

The total number of boards in the width of the wall is:

$$\# \text{ of boards} = (30 \text{ ft})(12 \text{ in./ft})/5 \text{ in./board} = 72 \text{ boards}$$

Therefore there are 71 gaps between the boards. The average width of the gaps is:

$$\text{gap} = 5.076 \text{ in.}/71 \text{ gaps} = 0.0715 \text{ in.}$$

- 17–5** We have been asked to prepare  $100 \text{ yd}^3$  of normal concrete using a volume ratio of cement-sand-coarse aggregate of 1 : 2 : 4. The water-cement ratio (by weight) is to be 0.5. The sand contains 6 wt% water and the coarse aggregate contains 3 wt% water. No entrained air is expected. (a) Determine the number of sacks of cement that must be ordered, the tons of sand and aggregate required, and the amount of water

needed. (b) Calculate the total weight of the concrete per cubic yard. (c) What is the weight ratio of cement-sand-coarse aggregate?

**Solution:** First we can determine the volume of each material on a “sack” basis, keeping in mind the 1:2:4 volume ratio of solids and the 0.5 water-cement weight ratio:

$$\begin{aligned}\text{cement} &= (1 \text{ sack})(94 \text{ lb/sack})/190 \text{ lb/ft}^3 = 0.495 \text{ ft}^3/\text{sack} \\ \text{sand} &= (2)(0.495 \text{ ft}^3/\text{sack}) = 0.990 \text{ ft}^3/\text{sack} \\ \text{aggregate} &= (4)(0.495 \text{ ft}^3/\text{sack}) = 1.980 \text{ ft}^3/\text{sack} \\ \text{water} &= (0.5)(94 \text{ lb})/62.4 \text{ lb/ft}^3 = 0.753 \text{ ft}^3/\text{sack} \\ \text{total volume of materials/sack} &= 4.218 \text{ ft}^3/\text{sack}\end{aligned}$$

In 100 yd<sup>3</sup>, or (100 yd<sup>3</sup>)(27 ft<sup>3</sup>/yd<sup>3</sup>):

$$\begin{aligned}\text{cement} &= 2700 \text{ ft}^3/4.218 \text{ ft}^3/\text{sack} = 640 \text{ sacks} \\ \text{sand} &= (640 \text{ sacks})(0.990 \text{ ft}^3/\text{sack})(160 \text{ lb/ft}^3) \\ &= 101,376 \text{ lb} = 50.7 \text{ tons} \\ \text{aggregate} &= (640 \text{ sacks})(1.980 \text{ ft}^3/\text{sack})(170 \text{ lb/ft}^3) \\ &= 215,424 \text{ lb} = 107.7 \text{ tons} \\ \text{water} &= (640 \text{ sacks})(0.753 \text{ ft}^3/\text{sack})(62.4 \text{ lb/ft}^3) = 30,072 \text{ lb} \text{ or} \\ &= (640 \text{ sacks})(0.753 \text{ ft}^3/\text{sack})(7.48 \text{ gal/ft}^3) = 3,605 \text{ gal}\end{aligned}$$

But we must make adjustments for the water that is already present in the sand and aggregate. There is 6% water in the sand and 3% water in the aggregate. We can either multiply the dry sand by 1.06, or divide the dry sand by 0.94, to obtain the amount of wet sand that we need to order.

$$\begin{aligned}\text{wet sand} &= (101,376 \text{ lb})(1.06) = 107,459 \text{ lb} = 53.7 \text{ tons} \\ \text{water in sand} &= 107,459 - 101,376 = 6083 \text{ lb} \\ \text{wet aggregate} &= (215,424 \text{ lb})(1.03) = 221,887 \text{ lb} = 110.9 \text{ tons} \\ \text{water in aggregate} &= 221,887 - 215,424 = 6463 \text{ lb}\end{aligned}$$

The actual amount of water that should be added to the concrete mix is:

$$\begin{aligned}\text{water} &= 30,072 - 6083 - 6463 = 17,526 \text{ lb} \\ \text{gal water} &= (17,526 \text{ lb})(7.48 \text{ gal/ft}^3)/62.4 \text{ lb/ft}^3 \\ &= 2101 \text{ gal}\end{aligned}$$

Therefore:

(a) The ingredients of the concrete mix are:

640 sacks of cement  
53.7 tons of sand  
110.9 tons of aggregate  
2101 gal of water

(b) The total weight per yd<sup>3</sup> is:

$$\begin{aligned}\text{wt/yd}^3 &= \frac{(640 \text{ sacks})(94 \text{ lb/sack}) + 107,459 + 221,887 + 17,526}{100 \text{ yd}^3} \\ &= 4070 \text{ lb/yd}^3\end{aligned}$$

(c) The cement-sand-aggregate ratio, on a weight basis, is:

$$\begin{aligned}\text{ratio} &= (640 \text{ sacks})(94 \text{ lb/sack}) : 107,847 \text{ lb} : 221,887 \text{ lb} \\ &= 60,160 : 107,847 : 221,887 \\ &= 1 : 1.79 : 3.69\end{aligned}$$

**17-6** We plan to prepare 10 yd<sup>3</sup> of concrete using a 1 : 2.5 : 4.5 weight ratio of cement-sand-coarse aggregate. The water-cement ratio (by weight) is 0.45. The sand contains 3 wt% water, the coarse aggregate contains 2 wt% water, and 5% entrained air is expected. Determine the number of sacks of cement, tons of sand, and coarse aggregate, and gallons of water required.

**Solution:** First, we can determine the volume of each material required, using the 1 : 2.5 : 4.5 ratio to determine the weights per sack of cement and dividing by the density to determine the volume. Per sack of cement:

$$\begin{aligned}\text{cement: } & 94 \text{ lb/sack} / 190 \text{ lb/ft}^3 = 0.495 \text{ ft}^3/\text{sack} \\ \text{sand: } & (2.5)(94 \text{ lb/sack}) / 160 \text{ lb/ft}^3 = 1.469 \text{ ft}^3/\text{sack} \\ \text{aggregate: } & (4.5)(94 \text{ lb/sack}) / 170 \text{ lb/ft}^3 = 2.488 \text{ ft}^3/\text{sack} \\ \text{water: } & (0.45)(94 \text{ lb/sack}) / 62.4 \text{ lb/ft}^3 = 0.678 \text{ ft}^3/\text{sack} \\ & \text{Volume per sack} = 5.130 \text{ ft}^3/\text{sack}\end{aligned}$$

But 5% of the concrete is expected to be entrained air. The volume of air “x” per sack of cement is:

$$x / (5.130 + x) = 0.05 \quad \text{or} \quad x = 0.27 \text{ ft}^3$$

Therefore the total volume of concrete per sack is:

$$\text{Volume of concrete} = 5.130 + 0.27 = 5.400 \text{ ft}^3/\text{sack}$$

In 10 yd<sup>3</sup> = 270 ft<sup>3</sup>:

$$\begin{aligned}\text{cement} &= 270 \text{ ft}^3 / 5.400 \text{ ft}^3/\text{sack} = 50 \text{ sacks} \\ \text{sand} &= (50 \text{ sacks})(1.469 \text{ ft}^3/\text{sack})(160 \text{ lb/ft}^3) = 11,752 \text{ lb} \\ \text{aggregate} &= (50 \text{ sacks})(2.488 \text{ ft}^3/\text{sack})(170 \text{ lb/ft}^3) = 21,148 \text{ lb} \\ \text{water} &= (50 \text{ sacks})(0.678 \text{ ft}^3/\text{sack})(62.4 \text{ lb/ft}^3) = 2,115 \text{ lb}\end{aligned}$$

But we must also adjust for the water present in the wet sand (3%) and wet aggregate (2%). For example, to find the amount of wet sand, we could either multiply the dry sand by 1.03 or divide by 0.97:

$$\begin{aligned}\text{wet sand} &= 11,752 \text{ lb} / 0.97 = 12,115 \text{ lb}; \quad \text{H}_2\text{O} = 363 \text{ lb} \\ \text{wet aggregate} &= 21,148 \text{ lb} / 0.98 = 21,580 \text{ lb}; \quad \text{H}_2\text{O} = 432 \text{ lb}\end{aligned}$$

Therefore, the ingredients for the concrete mix include:

$$\begin{aligned}\text{cement} &= 50 \text{ sacks} \\ \text{sand} &= 12,115 \text{ lb} = 6.06 \text{ tons} \\ \text{aggregate} &= 21,580 \text{ lb} = 10.8 \text{ tons} \\ \text{water} &= 2115 - 363 - 432 = 1320 \text{ lb} \\ &= (1320 \text{ lb})(7.48 \text{ gal/ft}^3) / 62.4 \text{ lb/ft}^3 = 158 \text{ gal}\end{aligned}$$

# 18

## Electronic Materials

- 18–1** A current of 10 A is passed through a 1-mm-diameter wire 1000 m long. Calculate the power loss if the wire is made of (a) aluminum, (b) silicon, and (c) silicon carbide. (See Table 18–1).

**Solution:**      $\text{Power} = I^2 R = I^2 \ell / \sigma A = (10 \text{ A})^2 (100,000 \text{ cm}) / (\pi/4)(0.1 \text{ cm})^2 \sigma$   
 $\text{Power} = 1.273 \times 10^9 / \sigma$

The electrical conductivity of each material is given in Table 18–1:

- (a)  $P_{\text{Al}} = 1.273 \times 10^9 / 3.77 \times 10^5 = 3380 \text{ watt}$   
 (b)  $P_{\text{Si}} = 1.273 \times 10^9 / 5 \times 10^{-6} = 2.546 \times 10^{14} \text{ watt}$   
 (c)  $P_{\text{SiC}} = 1.273 \times 10^9 / 10^{-1} \text{ to } 10^{-2}$   
 $= 1.273 \times 10^{10} \text{ to } 1.273 \times 10^{11} \text{ watt}$

- 18–4** The power lost in a 2-mm-diameter copper wire is to be less than 250 W when a 5-A current is flowing in the circuit. What is the maximum length of the wire?

**Solution:**      $P = I^2 R = I^2 \ell / \sigma A = 250 \text{ W}$   
 $\ell = 250 \sigma A / I^2 = (250)(5.98 \times 10^5)(\pi/4)(0.2)^2 / (5)^2$   
 $= 1.88 \times 10^5 \text{ cm} = 1.88 \text{ km}$

- 18–5** A current density of 100,000 A/cm<sup>2</sup> is applied to a gold wire 50 m in length. The resistance of the wire is found to be 2 ohm. Calculate the diameter of the wire and the voltage applied to the wire.

**Solution:**      $J = I/A = \sigma V / \ell = 100,000 \text{ A/cm}^2$   
 $V = 100,000 \ell / \sigma = (100,000)(5000 \text{ cm}) / 4.26 \times 10^5 = 1174 \text{ V}$

From Ohm's law,  $I = V/R = 1174/2 = 587 \text{ A}$

$$A = I/J = 587/100,000 = 0.00587 \text{ cm}^2$$

$$(\pi/4)d^2 = 0.00587 \quad \text{or} \quad d^2 = 0.00747 \quad \text{or} \quad d = 0.0865 \text{ cm}$$

- 18–6** We would like to produce a 5000-ohm resistor from boron-carbide fibers having a diameter of 0.1 mm. What is the required length of the fibers?

**Solution:** The electrical conductivity is  $1 \text{ to } 2 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$ .

$$R = \ell/\sigma A = 5000 \text{ ohm}$$

If the conductivity is  $1 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$ :

$$\ell = R\sigma A = (5000)(1 \text{ ohm cm})(\pi/4)(0.01 \text{ cm})^2 = 0.393 \text{ cm}$$

If the conductivity is  $2 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$ :

$$\ell = R\sigma A = (5000)(2 \text{ ohm cm})(\pi/4)(0.01 \text{ cm})^2 = 0.785 \text{ cm}$$

The fibers should be 0.393 to 0.785 cm in length.

- 18–7** Suppose we estimate that the mobility of electrons in silver is  $75 \text{ cm}^2/\text{V} \cdot \text{s}$ . Estimate the fraction of the valence electrons that are carrying an electrical charge.

**Solution:** The total number of valence electrons is:

$$n_T = \frac{(4 \text{ atoms/cell})(1 \text{ electron/atom})}{(4.0862 \times 10^{-8} \text{ cm})^3} = 5.86 \times 10^{22}$$

The number of charge carriers is:

$$n = \sigma/q\mu = (6.80 \times 10^5)/(1.6 \times 10^{-19})(75) = 5.67 \times 10^{22}$$

The fraction of the electrons that carry the electrical charge is:

$$n/n_T = 5.67 \times 10^{22}/5.86 \times 10^{22} = 0.968$$

- 18–8** A current density of  $5000 \text{ A/cm}^2$  is applied to a magnesium wire. If half of the valence electrons serve as charge carriers, calculate the average drift velocity of the electrons.

**Solution:** The total number of valence electrons is:

$$n_T = \frac{(2 \text{ atoms/cell})(2 \text{ electrons/atom})}{(3.2087 \times 10^{-8})^2(5.209 \times 10^{-8})\cos 30} = 8.61 \times 10^{22}$$

The actual number of charge carriers is then  $4.305 \times 10^{22}$ .

$$\bar{v} = J/nq = (5000 \text{ A/cm}^2)/(4.305 \times 10^{22})(1.6 \times 10^{-19}) \\ = 0.7259 \text{ cm/s}$$

- 18–9** We apply a voltage of 10 V to an aluminum wire 2 mm in diameter and 20 m long. If 10% of the valence electrons carry the electrical charge, calculate the average drift velocity of the electrons in km/h and miles/h.

**Solution:** The total number of valence electrons is:

$$n_T = \frac{(4 \text{ atoms/cell})(3 \text{ electrons/atom})}{(4.04958 \times 10^{-8} \text{ cm})^3} = 1.81 \times 10^{23}/\text{cm}^3$$

The number of electrons carrying the electrical charge is one-tenth of the total number, or  $1.81 \times 10^{22}$  electrons/cm<sup>3</sup>. The electric field is

$$\xi = V/\ell = 10 \text{ V}/2000 \text{ cm} = 0.005 \text{ V/cm}$$

$$\sigma \xi = nq\bar{v} \quad \text{or} \quad \bar{v} = \sigma \xi / nq$$

$$\bar{v} = (3.77 \times 10^5)(0.005)/(1.81 \times 10^{22})(1.6 \times 10^{-19}) = 0.651 \text{ cm/s}$$

$$\bar{v} = (0.651 \text{ cm/s})(3600 \text{ s/h})/10^5 \text{ cm/km} = 0.0234 \text{ km/h}$$

$$\begin{aligned} \bar{v} &= (0.651 \text{ cm/s})(3600 \text{ s/h})(1 \text{ in.}/2.54 \text{ cm})(1 \text{ ft}/12 \text{ in.})(1 \text{ mile}/5280 \text{ ft}) \\ &= 0.0146 \text{ miles/h} \end{aligned}$$

- 18–10** In a welding process, a current of 400 A flows through the arc when the voltage is 35 V. The length of the arc is about 0.1 in. and the average diameter of the arc is about 0.18 in. Calculate the current density in the arc, the electric field across the arc, and the electrical conductivity of the hot gases in the arc during welding.

**Solution:**  $R = V/I = 35 \text{ V}/400 \text{ A} = 0.0875 \text{ ohm}$

The electrical conductivity of the gases in the arc is:

$$\begin{aligned} \sigma &= \ell/RA = \frac{(0.1 \text{ in.})(2.54 \text{ cm/in.})}{(0.0875 \text{ ohm})(\pi/4)(0.18 \text{ in.} \times 2.54 \text{ cm/in.})^2} \\ &= 17.68 \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

The current density  $J$  is:

$$J = I/A = 400 \text{ A}/(\pi/4)(0.18 \text{ in.} \times 2.54 \text{ cm/in.})^2 = 2436 \text{ A/cm}^2$$

The electric field is:

$$\xi = V/\ell = 35 \text{ V}/(0.18 \text{ in.})(2.54 \text{ cm/in.}) = 76.6 \text{ V/cm}$$

- 18–12** Calculate the electrical conductivity of nickel at  $-50^\circ\text{C}$  and at  $+500^\circ\text{C}$ .

**Solution:**  $\rho_{\text{room}} = 6.84 \times 10^{-6} \text{ ohm} \cdot \text{cm} \quad \alpha = 0.0069 \text{ ohm} \cdot \text{cm}/^\circ\text{C}$

$$\begin{aligned} \rho_{500} &= (6.84 \times 10^{-6})[1 + (0.0069)(500 - 25)] \\ &= 29.26 \times 10^{-6} \text{ ohm} \cdot \text{cm} \end{aligned}$$

$$\sigma_{500} = 1/\rho = 1/29.26 \times 10^{-6} = 0.34 \times 10^5 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$$

$$\begin{aligned} \rho_{-50} &= (6.84 \times 10^{-6})[1 + (0.0069)(-50 - 25)] \\ &= 3.3003 \times 10^{-6} \text{ ohm} \cdot \text{cm} \end{aligned}$$

$$\sigma_{-50} = 1/3.3003 \times 10^{-6} = 3.03 \times 10^5 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$$

- 18–13** The electrical resistivity of pure chromium is found to be  $18 \times 10^{-6} \text{ ohm} \cdot \text{cm}$ . Estimate the temperature at which the resistivity measurement was made.

**Solution:**  $\rho_{\text{room}} = 12.9 \times 10^{-6} \text{ ohm} \cdot \text{cm} \quad \alpha = 0.0030 \text{ ohm} \cdot \text{cm}/^\circ\text{C}$

$$18 \times 10^{-6} = (12.9 \times 10^{-6})[1 + (0.0030)(T - 25)]$$

$$1.395 - 1 = (0.003)(T - 25)$$

$$T = 156.8^\circ\text{C}$$

**18–14** After finding the electrical conductivity of cobalt at 0°C, we decide we would like to double that conductivity. To what temperature must we cool the metal?

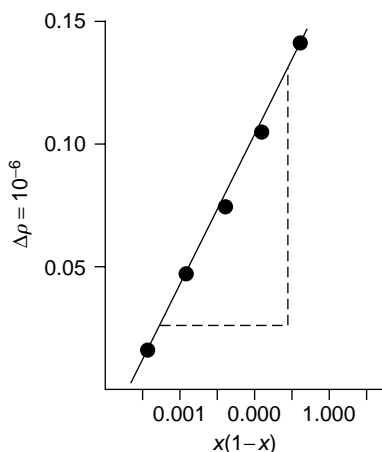
**Solution:**  $\rho_{\text{room}} = 6.24 \times 10^{-6} \text{ ohm} \cdot \text{cm}$      $a = 0.006 \text{ ohm} \cdot \text{cm}/^\circ\text{C}$   
 $\rho_{\text{zero}} = (6.24 \times 10^{-6})[1 + (0.006)(0 - 25)] = 5.304 \times 10^{-6}$   
 We wish to double the conductivity, or halve the resistivity to  $2.652 \times 10^{-6} \text{ ohm} \cdot \text{cm}$ . The required temperature is:  
 $2.652 \times 10^{-6} = (6.24 \times 10^{-6})[1 + (0.006)(T - 25)]$   
 $-0.575 = 0.006(T - 25)$     or     $T = -70.8^\circ\text{C}$

**18–15** From Figure 18–11(b), estimate the defect resistivity coefficient for tin in copper.

**Solution:** The conductivity and resistivity of pure copper are, from Table 18–1:  
 $\sigma = 5.98 \times 10^5 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$      $\rho = 1/\sigma = 0.167 \times 10^{-5} \text{ ohm} \cdot \text{cm}$   
 For 0.2 wt% Sn in copper:  
 $x_{\text{Sn}} = \frac{(0.2/118.69)}{(0.2/118.69) + (99.8/63.54)} = 0.00107$   
 $x_{\text{Sn}}(1 - x_{\text{Sn}}) = (0.00107)(1 - 0.00107) = 0.00107$   
 For 0.2% Sn, Figure 18–11(b) shows that the conductivity is 92% that of pure copper, or  
 $\sigma = (5.98 \times 10^5)(0.92) = 5.50 \times 10^5$   
 $\rho = 1/\sigma = 0.182 \times 10^{-5}$   
 $\Delta\rho = 0.182 \times 10^{-5} - 0.167 \times 10^{-5} = 0.015 \times 10^{-5}$

The following table includes the calculations for other compositions:

wt% Sn	$x_{\text{Sn}}$	$x_{\text{Sn}}(1 - x_{\text{Sn}})$	% $\sigma$	$\sigma$	$\rho$	$\Delta\rho$
0	0	0	100	$5.98 \times 10^5$	$0.167 \times 10^{-5}$	0
0.2	0.00107	0.00107	92	$5.50 \times 10^5$	$0.182 \times 10^{-5}$	$0.015 \times 10^{-5}$
0.4	0.00215	0.00215	78	$4.66 \times 10^5$	$0.215 \times 10^{-5}$	$0.048 \times 10^{-5}$
0.6	0.00322	0.00321	69	$4.13 \times 10^5$	$0.242 \times 10^{-5}$	$0.075 \times 10^{-5}$
0.8	0.00430	0.00428	61	$3.65 \times 10^5$	$0.274 \times 10^{-5}$	$0.107 \times 10^{-5}$
1.0	0.00538	0.00535	54	$3.23 \times 10^5$	$0.310 \times 10^{-5}$	$0.143 \times 10^{-5}$



These data are plotted. The slope of the graph is “b”:

$$b = \frac{0.135 \times 10^{-5} - 0.030 \times 10^{-5}}{0.0050 - 0.0015}$$

$$= 30 \times 10^{-5} \text{ ohm} \cdot \text{cm}$$

- 18–16** The electrical resistivity of a beryllium alloy containing 5 at% of an alloying element is found to be  $50 \times 10^{-6} \text{ ohm} \cdot \text{cm}$  at  $400^\circ\text{C}$ . Determine the contributions to resistivity due to temperature and due to impurities by finding the expected resistivity of pure beryllium at  $400^\circ\text{C}$ , the resistivity due to impurities, and the defect resistivity coefficient. What would be the electrical resistivity if the beryllium contained 10 at% of the alloying element at  $200^\circ\text{C}$ ?

**Solution:** From the data in Table 18–3, the resistivity at  $400^\circ\text{C}$  should be:

$$\rho_\tau = (4 \times 10^{-6})[1 + (0.025)(400 - 25)] = 41.5 \times 10^{-6}$$

Consequently the resistance due to impurities is:

$$\rho = \rho_\tau + \rho_d$$

$$50 \times 10^{-6} = 41.5 \times 10^{-6} + \rho_d$$

$$\rho_d = 8.5 \times 10^{-6} \text{ ohm} \cdot \text{cm}$$

Since there are 5 at% impurities present,  $x = 0.05$ , and the defect resistivity coefficient is:

$$\rho_d = bx(1 - x) \quad \text{or} \quad b = \rho_d/x(1 - x)$$

$$b = 8.5 \times 10^{-6}/(0.05)(1 - 0.05) = 178.9 \times 10^{-6} \text{ ohm} \cdot \text{cm}$$

The resistivity at  $200^\circ\text{C}$  in an alloy containing 10 at% impurities is:

$$\begin{aligned} \rho_{200} &= \rho_\tau + \rho_d \\ &= (4 \times 10^{-6})[1 + (0.025)(200 - 25)] + 178.9 \\ &\quad \times 10^{-6}(0.1)(1 - 0.1) \\ &= 21.5 \times 10^{-6} + 16.1 \times 10^{-6} = 37.6 \times 10^{-6} \text{ ohm} \cdot \text{cm} \end{aligned}$$

- 18–17** Is Equation 18–7 valid for the copper-zinc system? If so, calculate the defect resistivity coefficient for zinc in copper. (See Figure 18–11.)

**Solution:** The conductivity and resistivity of pure copper are:

$$\sigma = 5.98 \times 10^5 \quad \text{or} \quad \rho = 1/\sigma = 0.167 \times 10^{-5} \text{ ohm} \cdot \text{cm}$$

For 10 wt% Zn in copper:

$$x_{\text{Zn}} = \frac{(10/65.38)}{(10/65.38) + (90/63.54)} = 0.0975$$

$$x_{\text{Zn}}(1 - x_{\text{Zn}}) = (0.0975)(1 - 0.0975) = 0.088$$

From Figure 18–11(a), the conductivity of the Cu–10% Zn alloy at zero deformation is about 44% that of pure copper, or

$$\sigma = (5.98 \times 10^5)(0.44) = 2.63 \times 10^5$$

$$\rho = 1/\sigma = 0.38 \times 10^{-5}$$

$$\Delta\rho = 0.38 \times 10^{-5} - 0.167 \times 10^{-5} = 0.213 \times 10^{-5}$$

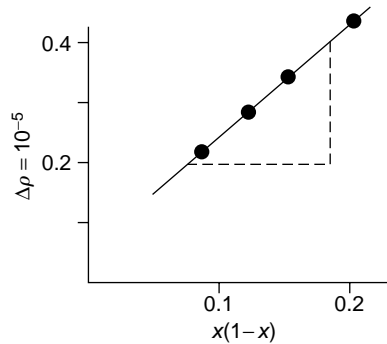


The following table includes the calculations for other compositions:

wt% Zn	$x_{\text{Zn}}$	$x_{\text{Zn}}(1 - x_{\text{Zn}})$	% $\sigma$	$\sigma$	$\rho$	$\Delta\rho$
0	0	0	101	$5.98 \times 10^5$	$0.167 \times 10^{-5}$	0
10	0.0975	0.088	44	$2.63 \times 10^5$	$0.380 \times 10^{-5}$	$0.213 \times 10^{-5}$
15	0.146	0.125	37	$2.21 \times 10^5$	$0.452 \times 10^{-5}$	$0.285 \times 10^{-5}$
20	0.196	0.158	33	$1.97 \times 10^5$	$0.508 \times 10^{-5}$	$0.341 \times 10^{-5}$
30	0.294	0.208	28	$1.67 \times 10^5$	$0.599 \times 10^{-5}$	$0.432 \times 10^{-5}$

These data are plotted. The slope of the graph is “ $b$ ”:

$$b = \frac{0.4 \times 10^{-5} - 0.2 \times 10^{-5}}{0.19 - 0.08} = 1.8 \times 10^{-5} \text{ ohm} \cdot \text{cm}$$



- 18–19**  $\text{GaV}_3$  is to operate as a superconductor in liquid helium (at 4 K). The  $T_c$  is 16.8 K and  $H_o$  is 350,000 oersted. What is the maximum magnetic field that can be applied to the material?

**Solution:**  $T_c = 16.8 \text{ K}$   $H_o = 350,000 \text{ oersted}$

$$H_c = H_o[1 - (T/T_c)^2] = 350,000[1 - (4/16.8)^2] = 330,159 \text{ oersted}$$

- 18–20**  $\text{Nb}_3\text{Sn}$  and  $\text{GaV}_3$  are candidates for a superconductive application when the magnetic field is 150,000 oersted. Which would require the lower temperature in order to be superconductive?

**Solution:**  $150,000 = H_o[1 - (T/T_c)^2]$

$$\text{For } \text{Nb}_3\text{Sn: } 150,000 = 250,000[1 - (T/18.05)^2]$$

$$T = 11.42 \text{ K}$$

$$\text{For } \text{GaV}_3: 150,000 = 350,000[1 - (T/16.8)^2]$$

$$T = 12.7 \text{ K}$$

- 18–21** A filament of  $\text{Nb}_3\text{Sn}$  0.05 mm in diameter operates in a magnetic field of 1000 oersted at 4 K. What is the maximum current that can be applied to the filament in order for the material to behave as a superconductor?

**Solution:** From Figure 18–12, the maximum current density for  $\text{Nb}_3\text{Sn}$  in a field of 1000 oersted is about  $2 \times 10^6 \text{ A/cm}^2$ .

$$I = JA = (2 \times 10^6 \text{ A/cm}^2)(\pi/4)(0.005 \text{ cm})^2 = 39.3 \text{ A}$$

- 18–22** Assume that most of the electrical charge transferred in MgO is caused by the diffusion of  $\text{Mg}^{2+}$  ions. Determine the mobility and electrical conductivity of MgO at 25°C and at 1500°C. (See Table 5–1.)

**Solution:** At 25°C = 298 K:

$$\begin{aligned} D_{\text{Mg}} &= 0.249 \exp[-79,000/(1.987)(25 + 273)] \\ &= 2.84 \times 10^{-57} \text{ cm}^2/\text{s} \\ \mu &= \frac{ZqD}{kT} = \frac{(2)(1.6 \times 10^{-19})(2.84 \times 10^{-57})}{(1.38 \times 10^{-23})(298)} \\ &= 2.21 \times 10^{-59} \text{ cm}^2/\text{V} \cdot \text{s} \end{aligned}$$

We can determine that the lattice parameter for MgO is 3.96 Å (since  $a_0 \times 2r_{\text{Mg}} + 2r_{\text{O}}$ ). There are four Mg ions per unit cell, so the number of Mg ions per  $\text{cm}^3$  is:

$$\begin{aligned} n &= (4)/(3.96 \times 10^{-8})^3 = 6.44 \times 10^{22}/\text{cm}^3 \\ \sigma &= nZq\mu = (6.44 \times 10^{22})(2)(1.6 \times 10^{-19})(2.21 \times 10^{-59}) \\ &= 45.5 \times 10^{-54} \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

At 1500°C = 1773 K:

$$\begin{aligned} D_{\text{Mg}} &= 0.249 \exp[-79,000/(1.987)(1500 + 273)] \\ &= 4.54 \times 10^{-11} \text{ cm}^2/\text{s} \\ \mu &= \frac{(2)(1.6 \times 10^{-19})(4.54 \times 10^{-11})}{(1.38 \times 10^{-23})(1773)} = 5.94 \times 10^{-10} \text{ cm}^2/\text{V} \cdot \text{s} \\ n &= (4)/(3.96 \times 10^{-8})^3 = 6.44 \times 10^{22}/\text{cm}^3 \\ \sigma &= (6.44 \times 10^{22})(2)(1.6 \times 10^{-19})(5.94 \times 10^{-10}) \\ &= 1.22 \times 10^{-5} \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

Conductivity increases about fifty orders of magnitude when the temperature increases to 1500°C.

- 18–23** Assume that most of the electrical charge transferred in  $\text{Al}_2\text{O}_3$  is caused by the diffusion of  $\text{Al}^{3+}$  ions. Determine the mobility and electrical conductivity of  $\text{Al}_2\text{O}_3$  at 500°C and at 1500°C. (See Table 5–1.)

**Solution:** At 500°C = 773 K:

$$\begin{aligned} D_{\text{Al}} &= 28 \exp[-114,000/(1.987)(773)] = 1.63 \times 10^{-31} \text{ cm}^2/\text{s} \\ \mu &= \frac{ZqD}{kT} = \frac{(3)(1.6 \times 10^{-19})(1.63 \times 10^{-31})}{(1.38 \times 10^{-23})(773)} \\ &= 7.3 \times 10^{-30} \text{ cm}^2/\text{V} \cdot \text{s} \end{aligned}$$

Example 14–1 showed that there are 12 Al ions per unit cell. The volume of the unit cell is  $253.82 \times 10^{-24} \text{ cm}^3$ . Thus the number of Al ions per  $\text{cm}^3$  is:

$$\begin{aligned} n &= 12/253.82 \times 10^{-24} = 4.73 \times 10^{22}/\text{cm}^3 \\ \sigma &= nAq\mu = (4.73 \times 10^{22})(3)(1.6 \times 10^{-19})(7.3 \times 10^{-30}) \\ &= 1.66 \times 10^{-25} \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

At  $1500^{\circ}\text{C} = 1773\text{ K}$ :

$$D_{\text{Al}} = 28 \exp[-114,000/(1.987)(1773)] = 2.48 \times 10^{-13} \text{ cm}^2/\text{s}$$

$$\mu = \frac{(3)(1.6 \times 10^{-19})(2.48 \times 10^{-13})}{(1.38 \times 10^{-23})(1773)} = 4.87 \times 10^{-12} \text{ cm}^2/\text{V} \cdot \text{s}$$

$$n = 12/253.82 \times 10^{-24} = 4.73 \times 10^{22}/\text{cm}^3$$

$$\begin{aligned} \sigma &= (4.73 \times 10^{22})(3)(1.6 \times 10^{-19})(4.87 \times 10^{-12}) \\ &= 1.11 \times 10^{-7} \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

Conductivity increases about 18 orders of magnitude when the temperature increases to  $1500^{\circ}\text{C}$ .

- 18–27** Calculate the electrical conductivity of a fiber-reinforced polyethylene part that is reinforced with 20 vol% of continuous, aligned nickel fibers.

**Solution:** From Table 18–1,  $\sigma_{\text{PE}} = 10^{-15}$  and  $\sigma_{\text{Ni}} = 1.46 \times 10^5$

$$\begin{aligned} \sigma_{\text{composite}} &= f_{\text{PE}} \sigma_{\text{PE}} + f_{\text{Ni}} \sigma_{\text{Ni}} = (0.8)(10^{-15}) + (0.2)(1.46 \times 10^5) \\ &= 0.292 \times 10^5 \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

- 18–33** For germanium, silicon, and tin, compare, at  $25^{\circ}\text{C}$ , (a) the number of charge carriers per cubic centimeter, (b) the fraction of the total electrons in the valence band that are excited into the conduction band, and (c) the constant  $n_o$ .

For germanium:

$$n_{\text{Ge}} = \frac{(8 \text{ atoms/cell})(4 \text{ electrons/atom})}{(5.6575 \times 10^{-8} \text{ cm})^3} = 1.767 \times 10^{23}/\text{cm}^3$$

From Table 18–6, we can find the conductivity and mobilities for germanium. The number of excited electrons is then:

$$\begin{aligned} n_{\text{conduction}} &= \sigma/q(\mu_e + \mu_h) = 0.02/(1.6 \times 10^{-19})(3800 + 1820) \\ &= 2.224 \times 10^{13} \end{aligned}$$

$$\text{fraction} = 2.224 \times 10^{13}/1.767 \times 10^{23} = 1.259 \times 10^{-10}$$

$$\begin{aligned} n_o &= n/\exp(-E_g/2kT) \\ &= 2.224 \times 10^{13}/\exp[-0.67/(2)(8.63 \times 10^{-5})(298)] \\ &= 1.017 \times 10^{19} \end{aligned}$$

For silicon:

$$n_{\text{Si}} = \frac{(8 \text{ atoms/cell})(4 \text{ electrons/atom})}{(5.4307 \times 10^{-8} \text{ cm})^3} = 1.998 \times 10^{23}/\text{cm}^3$$

$$\begin{aligned} n_{\text{conduction}} &= \sigma/q(\mu_e + \mu_h) = 5 \times 10^{-6}/(1.6 \times 10^{-19})(1900 + 500) \\ &= 1.302 \times 10^{10} \end{aligned}$$

$$\text{fraction} = 1.302 \times 10^{10}/1.998 \times 10^{23} = 6.517 \times 10^{-14}$$

$$\begin{aligned} n_o &= n/\exp(-E_g/2kT) \\ &= 1.302 \times 10^{10}/\exp[-1.107/(2)(8.63 \times 10^{-5})(298)] \\ &= 2.895 \times 10^{19} \end{aligned}$$

For tin:

$$n_{\text{Sn}} = \frac{(8 \text{ atoms/cell})(4 \text{ electrons/atom})}{(6.4912 \times 10^{-8} \text{ cm})^3} = 1.170 \times 10^{23}/\text{cm}^3$$

$$n_{\text{conduction}} = \sigma/q(\mu_e + \mu_h) = 0.9 \times 10^5/(1.6 \times 10^{-19})(2500 + 2400) \\ = 1.148 \times 10^{20}$$

$$\text{fraction} = 1.148 \times 10^{20}/1.170 \times 10^{23} = 9.812 \times 10^{-4}$$

$$n_o = n/\exp(-E_g/2kT) \\ = 1.148 \times 10^{20}/\exp[-0.08/(2)(8.63 \times 10^{-5})(298)] \\ = 5.44 \times 10^{20}$$

- 18–34** For germanium, silicon, and tin, compare the temperature required to double the electrical conductivity from the room temperature value.

**Solution:** For germanium, we wish to increase the conductivity from 0.02 to 0.04  $\text{ohm}^{-1} \cdot \text{cm}^{-1}$ . From Problem 18–33,  $n_o = 1.017 \times 10^{19}$ :

$$\sigma = nq(\mu_e + \mu_h) = n_o q(\mu_e + \mu_h) \exp(-E_g/2kT) \\ 0.04 = (1.017 \times 10^{19})(1.6 \times 10^{-19})(3800 + 1820) \\ \exp[-0.67/(2)(8.63 \times 10^{-5})T]$$

$$4.374 \times 10^{-6} = \exp(-3882/T) \quad \text{or} \quad T = 325 \text{ K} = 42^\circ\text{C}$$

For silicon, we wish to increase the conductivity from  $5 \times 10^{-6}$  to  $10 \times 10^{-6} \text{ ohm}^{-1} \cdot \text{cm}^{-1}$ . From Problem 18–21,  $n_o = 2.895 \times 10^{19}$ :

$$\sigma = nq(\mu_e + \mu_h) = n_o q(\mu_e + \mu_h) \exp(-E_g/2kT) \\ 10 \times 10^{-6} = (2.895 \times 10^{19})(1.6 \times 10^{-19})(1900 + 500) \\ \exp[-1.107/(2)(8.63 \times 10^{-5})T]$$

$$8.995 \times 10^{-10} = \exp(-6414/T)$$

$$-20.829 = -6414/T \quad \text{or} \quad T = 308 \text{ K} = 35^\circ\text{C}$$

For tin, we wish to increase the conductivity from  $0.9 \times 10^5$  to  $1.8 \times 10^5 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$ . From Problem 18–21,  $n_o = 5.44 \times 10^{20}$ :

$$\sigma = nq(\mu_e + \mu_h) = n_o q(\mu_e + \mu_h) \exp(-E_g/2kT) \\ 1.8 \times 10^5 = (5.44 \times 10^{20})(1.6 \times 10^{-19})(2500 + 2400) \\ \exp[-0.08/(2)(8.63 \times 10^{-5})T]$$

$$0.422 = \exp(-463.499/T)$$

$$-0.863 = -463.499/T \quad \text{or} \quad T = 537 \text{ K} = 264^\circ\text{C}$$

- 18–35** Determine the electrical conductivity of silicon when 0.0001 at% antimony is added as a dopant and compare it to the electrical conductivity when 0.0001 at% indium is added.

**Solution:** 0.0001 at% = 1 impurity atom per  $10^6$  host atoms.

For antimony additions (an  $n$ -type semiconductor):

$$n = \frac{(8 \text{ atoms/cell})(1 \text{ Sb atom}/10^6 \text{ Si atoms})}{(5.4307 \times 10^{-8} \text{ cm})^3} = 5 \times 10^{16}$$

$$\sigma = nq\mu_e = (5 \times 10^{16})(1.6 \times 10^{-19})(1900) = 15.2 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$$

For indium additions (a *p*-type semiconductor):

$$n = \frac{(8 \text{ atoms/cell})(1 \text{ In atom}/10^6 \text{ Si atoms})}{(5.4307 \times 10^{-8} \text{ cm})^3} = 5 \times 10^{16}$$

$$\sigma = nq\mu = (5 \times 10^{16})(1.6 \times 10^{-19})(500) = 4.0 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$$

- 18–36** We would like to produce an extrinsic germanium semiconductor having an electrical conductivity of  $2000 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$ . Determine the amount of phosphorous and the amount of gallium required.

**Solution:** For phosphorous (an *n*-type semiconductor):

$$n = \sigma/q\mu_e = 2000/(1.6 \times 10^{-19})(3800) = 3.29 \times 10^{18}$$

$$3.29 \times 10^{18} = \frac{(8 \text{ atoms/cell})(x \text{ P atoms}/10^6 \text{ Ge atoms})}{(5.6575 \times 10^{-8} \text{ cm})^3}$$

$$x = 74.47 \text{ P atoms}/10^6 \text{ Ge atoms} = 0.007447 \text{ at\% P}$$

For gallium (a *p*-type semiconductor):

$$n = \sigma/q\mu_h = 2000/(1.6 \times 10^{-19})(1820) = 6.868 \times 10^{18}$$

$$6.868 \times 10^{18} = \frac{(8 \text{ atoms/cell})(x \text{ Ga atoms}/10^6 \text{ Ge atoms})}{(5.6575 \times 10^{-8} \text{ cm})^3}$$

$$x = 155.5 \text{ Ga atoms}/10^6 \text{ Ge atoms} = 0.01555 \text{ at\% Ga}$$

- 18–37** Estimate the electrical conductivity of silicon doped with 0.0002 at% arsenic at  $600^\circ\text{C}$ , which is above the exhaustion plateau in the conductivity-temperature curve.

**Solution:** 
$$n_d = \frac{(8 \text{ atoms/cell})(2 \text{ As atoms}/10^6 \text{ Si atoms})}{(5.4307 \times 10^{-8} \text{ cm})^3} = 9.99 \times 10^{16}$$

From Problem 18–21,  $n_o = 2.895 \times 10^{19}$

$$\begin{aligned} \sigma_{600} &= n_d q \mu_e + q(\mu_e + \mu_h) n_o \exp(-E_g/2kT) \\ &= (9.99 \times 10^{16})(1.6 \times 10^{-19})(1900) + (1.6 \times 10^{-19})(1900 + 500) \\ &\quad (2.895 \times 10^{19}) \exp[-1.107/(2)(8.63 \times 10^{-5})(873)] \\ &= 30.37 + 7.167 = 37.537 \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

- 18–38** Determine the amount of arsenic that must be combined with 1 kg of gallium to produce a *p*-type semiconductor with an electrical conductivity of  $500 \text{ ohm}^{-1} \cdot \text{cm}^{-1}$  at  $25^\circ\text{C}$ . The lattice parameter of GaAs is about  $5.65 \text{ \AA}$  and GaAs has the zinc blende structure.

**Solution:** 
$$n = \sigma/q\mu_h = 500/(1.6 \times 10^{-19})(400) = 7.81 \times 10^{18}$$

$$7.81 \times 10^{18} = \frac{(4 \text{ Ga atoms/cell})(x \text{ vacancies/Ga atom})}{(5.65 \times 10^{-8} \text{ cm})^3}$$

$$x = 0.000352 \text{ vacancies/cell}$$

Therefore there are 0.999648 As atoms per one Ga atom.

$$\text{at\% As} = \frac{0.999648}{1 + 0.999648} \times 100 = 49.991\%$$

$$\text{wt\% As} = \frac{(49.991)(74.9216)}{(49.991)(74.9216) + (50.009)(69.72)} \times 100 = 51.789\%$$

$$\frac{x \text{ g Ni}}{x + 1000 \text{ g Ga}} \times 100 = 51.789 \quad \text{or} \quad x = 1074 \text{ g As}$$

- 18–39** A ZnO crystal is produced in which one interstitial Zn atom is introduced for every 500 Zn lattice sites. Estimate (a) the number of charge carriers per cubic centimeter and (b) the electrical conductivity at 25°C. Assume that the lattice parameter for ZnO is 4.757 Å.

**Solution:** A *n*-type semiconductor is produced.

$$\begin{aligned} \text{(a) carriers} &= \frac{(4 \text{ Zn/cell})(1 \text{ interstitial/500 Zn})(2 \text{ electrons/interstitial})}{(4.758 \times 10^{-8} \text{ cm})^3} \\ &= 1.486 \times 10^{20} \text{ interstitials/cm}^3 \\ \text{(b) } \sigma &= nq\mu_e = (1.486 \times 10^{20})(1.6 \times 10^{-19})(180) \\ &= 4.28 \times 10^3 \text{ ohm}^{-1} \cdot \text{cm}^{-1} \end{aligned}$$

- 18–40** Each Fe<sup>3+</sup> ion in FeO serves as an acceptor site for an electron. If there is one vacancy per 750 unit cells of the FeO crystal (with the sodium chloride structure), determine the number of possible charge carriers per cubic centimeter. The lattice parameter of FeO is 0.429 nm.

**Solution:** One vacancy requires that 2 Fe<sup>3+</sup> ions be substituted for 3 Fe<sup>3+</sup> ions. A hole is present for each Fe<sup>3+</sup> ion. In 750 unit cells, there are 4 × 750 = 3000 Fe sites in the NaCl-type crystal structure:

$$\begin{aligned} \text{carriers} &= \frac{(4 \text{ Fe sites/cell})(2 \text{ Fe}^{3+}/3000 \text{ Fe sites})(1 \text{ hole/Fe}^{3+})}{(4.29 \times 10^{-8} \text{ cm})^3} \\ \text{carriers/cm}^3 &= 3.3775 \times 10^{19} \end{aligned}$$

- 18–41** When a voltage of 5 mV is applied to the emitter of a transistor, a current of 2 mA is produced. When the voltage is increased to 8 mV, the current through the collector rises to 6 mA. By what percentage will the collector current increase when the emitter voltage is doubled from 9 mV to 18 mV?

**Solution:** First we can find the constants  $I_o$  and  $B$  in Equation 18–17.

$$\begin{aligned} \frac{2 \text{ mA}}{6 \text{ mA}} &= \frac{I_o \exp(5 \text{ mV}/B)}{I_o \exp(8 \text{ mV}/B)} \\ 0.333 &= \exp(-3/B) \\ -1.0986 &= -3/B \quad \text{or} \quad B = 2.73 \text{ mV} \\ I_o &= 2/\exp(5/2.73) = 0.32 \text{ mA} \\ \text{At 9 mV, } I &= 0.32 \exp(9/2.73) = 8.647 \text{ mA} \\ \text{At 18 mV, } I &= 0.32 \exp(18/2.73) = 233.685 \text{ mA} \\ \text{Therefore the percentage increase in the collector current is:} \\ \Delta &= \frac{233.685 - 8.647}{8.647} \times 100 = 2600\% \end{aligned}$$

**18–48** Calculate the displacement of the electrons or ions for the following conditions:

- (a) electronic polarization in nickel of  $2 \times 10^{-7} \text{ C/m}^2$
- (b) electronic polarization in aluminum of  $2 \times 10^{-8} \text{ C/m}^2$
- (c) ionic polarization in NaCl of  $4.3 \times 10^{-8} \text{ C/m}^2$
- (d) ionic polarization in ZnS of  $5 \times 10^{-8} \text{ C/m}^2$

**Solution:**  $n$  is the number of charge centers per  $\text{m}^3$ :

(a) For FCC nickel,  $a_o = 3.5167 \text{ \AA}$  and the atomic number is 28:

$$n = \frac{(4 \text{ atoms/cell})(28 \text{ electrons/atom})}{(3.5167 \times 10^{-10} \text{ m})^3} = 2.58 \times 10^{30}$$

$$d = P/nq = (2 \times 10^{-7} \text{ C/m}^2) / (2.58 \times 10^{30} \text{ m}^3)(1.6 \times 10^{-19} \text{ C/electron})$$

$$d = 4.84 \times 10^{-19} \text{ m}$$

(b) For FCC aluminum,  $a_o = 4.04988 \text{ \AA}$  and the atomic number is 13:

$$n = \frac{(4 \text{ atoms/cell})(13 \text{ electrons/atom})}{(4.04988 \times 10^{-10} \text{ m})^3} = 0.78 \times 10^{30}$$

$$d = P/nq = (2 \times 10^{-8}) / (0.78 \times 10^{30})(1.6 \times 10^{-19})$$

$$d = 1.603 \times 10^{-19} \text{ m}$$

(c) For NaCl,  $a_o = 5.5 \text{ \AA}$  and there is one charge per ion. There are 4 of each type of ion per cell. The lattice parameter is:

$$a_o = 2r_{\text{Na}} + 2r_{\text{Cl}} = 2(0.97) + 2(1.81) = 5.56 \text{ \AA}$$

$$n = \frac{(4 \text{ Na ions/cell})(1 \text{ charge/ion})}{(5.56 \times 10^{-10} \text{ m})^3} = 0.024 \times 10^{30}$$

$$d = P/nq = (4.3 \times 10^{-8}) / (0.024 \times 10^{30})(1.6 \times 10^{-19})$$

$$d = 1.12 \times 10^{-17} \text{ m}$$

(d) For ZnS,  $a_o = 5.96 \text{ \AA}$  and there are two charges per ion. There are 4 of each type of ion per cell. The lattice parameter is:

$$a_o = (4r_{\text{Zn}} + 4r_{\text{S}}) / \sqrt{3} = [(4)(0.74) + (4)(1.84)] / \sqrt{3} = 5.96 \text{ \AA}$$

$$n = \frac{(4 \text{ ZnS ions/cell})(2 \text{ charge/ion})}{(5.96 \times 10^{-10} \text{ m})^3} = 0.038 \times 10^{30}$$

$$d = P/nq = (5 \times 10^{-8}) / (0.038 \times 10^{30})(1.6 \times 10^{-19})$$

$$d = 8.22 \times 10^{-18} \text{ m.}$$

**18–49** A 2-mm-thick alumina dielectric is used in a 60-Hz circuit. Calculate the voltage required to produce a polarization of  $5 \times 10^{-7} \text{ C/m}^2$ .

**Solution:**  $P = (\kappa - 1)\epsilon_o \xi = (\kappa - 1)\epsilon_o V/\ell$  where  $\ell = 2 \text{ mm} = 0.002 \text{ m}$

$$5 \times 10^{-7} = (9 - 1)(8.85 \times 10^{-12})V / 2 \times 10^{-3}$$

$$V = 14.1 \text{ volts}$$

- 18–50** Suppose we are able to produce a polarization of  $5 \times 10^{-5} \text{ C/cm}^2$  in a cube (5 mm side) of barium titanate. What voltage is produced?

**Solution:**  $P = (\kappa - 1)\epsilon_0\xi = (\kappa - 1)\epsilon_0V/\ell$  where  $\ell = 5 \text{ mm} = 0.005 \text{ m}$   
 $5 \times 10^{-5} = (3000 - 1)(8.85 \times 10^{-12})V/0.005$   
 $V = 9.4 \text{ volts}$

- 18–51** Calculate the thickness of polyethylene required to store the maximum charge in a 24,000-V circuit without breakdown.

**Solution:**  $\xi_{\max} = 20 \times 10^6 \text{ V/m} = 24,000/\ell$   
 $\ell = 0.0012 \text{ m} = 1.2 \text{ mm}$

- 18–57** Calculate the capacitance of a parallel-plate capacitor containing 5 layers of mica, where each mica sheet is  $1 \text{ cm} \times 2 \text{ cm} \times 0.005 \text{ cm}$ .

**Solution:**  $C = \epsilon_0(\kappa)(n - 1)A/d$   
 $= (8.85 \times 10^{-14} \text{ F/cm})(7)(6 - 1)(1 \text{ cm} \times 2 \text{ cm})/0.005 \text{ cm}$   
 $= 1.239 \times 10^{-9} \text{ F} = 0.001239 \mu\text{F}$

- 18–60** Determine the number of  $\text{Al}_2\text{O}_3$  sheets, each  $1.5 \text{ cm} \times 1.5 \text{ cm} \times 0.001 \text{ cm}$ , required to obtain a capacitance of  $0.0142 \mu\text{F}$  in a  $10^6 \text{ Hz}$  parallel plate capacitor.

**Solution:**  $n - 1 = Cd/\epsilon_0\kappa A$   
 $n - 1 = (0.0142 \times 10^{-6} \text{ F})(0.001 \text{ cm})/(8.85 \times 10^{-14} \text{ F/cm})$   
 $\hspace{15em} (6.5)(1.5 \text{ cm})^2$   
 $n - 1 = 11 \text{ Al}_2\text{O}_3 \text{ sheets and } n = 12 \text{ conductor plates}$

- 18–61** We would like to construct a barium titanate device with a 0.1-in. diameter that will produce a voltage of 250 V when a 5 pound force is applied. How thick should the device be?

**Solution:**  $E = 10 \times 10^6 \text{ psi}$   
 $d = (100 \times 10^{-12} \text{ m/V})(100 \text{ cm/m})(1 \text{ in.}/2.54 \text{ cm})$   
 $= 3.937 \times 10^{-9} \text{ in./V}$

If  $t$  = thickness of the device,  $F$  is the applied force, and  $A$  is the area of the device, then:

$$\xi = V/t = g\sigma = \sigma/Ed = F/AEd$$

$$t = VAEd/F$$

$$= (250 \text{ V})(\pi/4)(0.1 \text{ in.})^2(10 \times 10^6 \text{ psi})(3.937 \times 10^{-9} \text{ in./V})/5 \text{ lb}$$

$$= 0.0155 \text{ in.}$$

- 18–62** A force of 20 lb is applied to the face of a  $0.5 \text{ cm} \times 0.5 \text{ cm} \times 0.1 \text{ cm}$  thick quartz crystal. Determine the voltage produced by the force. The modulus of elasticity of quartz is  $10.4 \times 10^6 \text{ psi}$ .



**Solution:**  $d = (2.3 \times 10^{-12} \text{ m/V})(100 \text{ cm/m})(1 \text{ in.}/2.54 \text{ cm})$   
 $= 9.055 \times 10^{-11} \text{ in./V}$   
 $A = (0.5 \text{ cm}/2.54 \text{ cm/in.})^2 = 0.03875 \text{ in.}^2$   
 $t = \text{thickness} = 0.1 \text{ cm} = 0.03937 \text{ in.}$   
 $\xi = V/t = \sigma/Ed = F/AEd$   
 $V = Ft/AEd = \frac{(20 \text{ lb})(0.03937 \text{ in.})}{(0.03875 \text{ in.}^2)(10.4 \times 10^6 \text{ psi})(9.055 \times 10^{-11} \text{ in./V})}$   
 $V = 21,578 \text{ volts}$

- 18–63** Determine the strain produced when a 300 V signal is applied to a barium titanate wafer  $0.2 \text{ cm} \times 0.2 \text{ cm} \times 0.01 \text{ cm}$  thick.

**Solution:**  $d = 100 \times 10^{-12} \text{ m/V}$   
 $\varepsilon = d\xi = (100 \times 10^{-12} \text{ m/V})(300 \text{ V}/0.01 \text{ cm})(100 \text{ cm/m})$   
 $= 0.0003 \text{ cm/cm}$

- 18–64** Figure 18–35 shows the hysteresis loops for two ferroelectric materials: Determine the voltage required to eliminate polarization in a 0.1 cm-thick dielectric made from Material A.

**Solution:** coercive field = 4000 V/m  
 $V = (4000 \text{ V/m})(0.001 \text{ m}) = 4 \text{ volts}$

- 18–65** From Figure 18–35, determine the thickness of a dielectric made from Material B if 10 V is required to eliminate polarization.

**Solution:** coercive field = 3500 V/m  
 $\text{thickness} = 10 \text{ V}/3500 \text{ V/m} = 0.002857 \text{ m} = 0.2857 \text{ cm}$

- 18–66** Using Figure 18–35, what electric field is required to produce a polarization of  $8 \times 10^{-8} \text{ C/m}^2$  in material A; and what is the dielectric constant at this polarization?

**Solution:** field = 5000 V/m  
 $P = (\kappa - 1)\varepsilon_0\xi \quad \text{or} \quad \kappa = 1 + P/\varepsilon_0\xi$   
 $\kappa = 1 + (8 \times 10^{-8} \text{ C/m}^2)/(8.85 \times 10^{-12} \text{ F/m})(5000 \text{ V/m})$   
 $\kappa = 2.81$

- 18–67** An electric field of 2500 V/m is applied to material B in Figure 18–35. Determine the polarization and the dielectric constant at this electric field.

**Solution:** polarization =  $12 \times 10^{-8} \text{ C/m}^2$   
 $\kappa = 1 + P/\varepsilon_0\xi$   
 $\kappa = 1 + (12 \times 10^{-8} \text{ C/m}^2)/(8.85 \times 10^{-12} \text{ F/m})(2500 \text{ V/m})$   
 $\kappa = 6.42$

# 19

## Magnetic Materials

- 19–6** Calculate and compare the maximum magnetization we would expect in iron, nickel, cobalt, and gadolinium. There are seven electrons in the 4f level of gadolinium.

**Solution:** Iron: The number of atoms/m<sup>3</sup> is:

$$\frac{2 \text{ atoms/cell}}{(2.866 \times 10^{-10} \text{ m})^3} = 0.085 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.085 \times 10^{30})(4 \text{ magnetons/atom})(9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2) \\ = 3.15 \times 10^6 \text{ A/m} = 39,600 \text{ oersted}$$

Nickel: The number of atoms/m<sup>3</sup> is:

$$\frac{4 \text{ atoms/cell}}{(3.5167 \times 10^{-10} \text{ m})^3} = 0.09197 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.09197 \times 10^{30})(2 \text{ magnetons/atom})(9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2) \\ = 1.705 \times 10^6 \text{ A/m} = 21,430 \text{ oersted}$$

Cobalt: The number of atoms/m<sup>3</sup> is:

$$\frac{2 \text{ atoms/cell}}{(2.5071 \times 10^{-10} \text{ m})^2 (4.0686 \times 10^{-10}) \cos 30} \\ = 0.0903 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.0903 \times 10^{30})(3 \text{ magnetons/atom})(9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2) \\ = 2.51 \times 10^6 \text{ A/m} = 31,560 \text{ oersted}$$

Gadolinium: The number of atoms/m<sup>3</sup> is:

$$\frac{2 \text{ atoms/cell}}{(3.6336 \times 10^{-10} \text{ m})^2 (5.781 \times 10^{-10} \text{ m}) \cos 30} = 0.0303 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.0303 \times 10^{30})(7 \text{ magnetons/atom})(9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2) = 1.96 \times 10^6 \text{ A/m} = 24,670 \text{ oersted}$$

- 19–11** An alloy of nickel and cobalt is to be produced to give a magnetization of  $2 \times 10^6$  A/m. The crystal structure of the alloy is FCC with a lattice parameter of 0.3544 nm. Determine the atomic percent cobalt required, assuming no interaction between the nickel and cobalt.

**Solution:** Let  $f_{\text{Ni}}$  be the atomic fraction of nickel;  $1 - f_{\text{Ni}}$  is then the atomic fraction of cobalt. The numbers of Bohr magnetons per cubic meter due to nickel and to cobalt atoms are:

$$\text{Ni: } (4 \text{ atoms/cell})(2 \text{ magnetons/atom}) f_{\text{Ni}} / (3.544 \times 10^{-10} \text{ m})^3 = 0.1797 \times 10^{30} f_{\text{Ni}}$$

$$\text{Co: } (4 \text{ atoms/cell})(3 \text{ magnetons/atom})(1 - f_{\text{Ni}}) / (3.544 \times 10^{-10} \text{ m})^3 = 0.2696 \times 10^{30} (1 - f_{\text{Ni}})$$

$$M = [(0.1797 \times 10^{30}) f_{\text{Ni}} + (0.2696 \times 10^{30})(1 - f_{\text{Ni}})](9.27 \times 10^{-24})$$

$$M = -0.833 \times 10^6 f_{\text{Ni}} + 2.499 \times 10^6 = 2 \times 10^6$$

$$f_{\text{Ni}} = 0.60 \quad f_{\text{Co}} = 0.40$$

- 19–12** Estimate the magnetization that might be produced in an alloy containing nickel and 70 at% copper, assuming that no interaction occurs.

**Solution:** We can estimate the lattice parameter of the alloy from those of the pure nickel and copper and their atomic fractions:

$$a_0 = (0.3)(3.294) + (0.7)(3.6151) = 3.52 \text{ \AA}$$

If the copper does not provide magnetic moments that influence magnetization, then

$$M = \frac{(4 \text{ atoms/cell})(0.3 \text{ fraction Ni})(2 \text{ magnetons/Ni atom})(9.27 \times 10^{-24})}{(3.52 \times 10^{-10} \text{ m})^3}$$

$$M = 0.51 \times 10^6 \text{ A/m} = 6410 \text{ oersted}$$

- 19–13** An Fe–80% Ni alloy has a maximum permeability of 300,000 when an inductance of 3500 gauss is obtained. The alloy is placed in a 20-turn coil that is 2 cm in length. What current must flow through the conductor coil to obtain this field?

**Solution:** Since  $B = \mu H$ ,

$$H = B/\mu = 3500 \text{ G}/300,000 \text{ G/Oe} = 0.0117 \text{ Oe} = 0.928 \text{ A/m}$$

Then:

$$I = H\ell/n = (0.928 \text{ A/m})(0.02 \text{ m})/20 \text{ turns} = 0.00093 \text{ A}$$

- 19–14** An Fe–49% Ni alloy has a maximum permeability of 64,000 when a magnetic field of 0.125 oersted is applied. What inductance is obtained and what current is needed to obtain this inductance in a 200-turn, 3-cm long coil?

**Solution:**  $B = \mu H = (64,000 \text{ G/Oe})(0.125 \text{ Oe}) = 8000 \text{ G}$

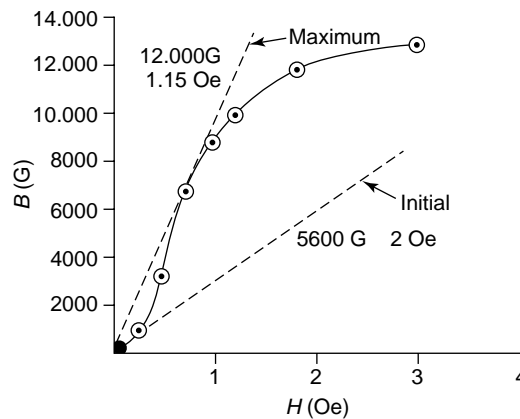
If we convert units,  $H = 0.125 \text{ Oe}/4\pi \times 10^{-3} \text{ Oe/A/m} = 9.947 \text{ A/m}$

$I = H\ell/n = (9.947 \text{ A/m})(0.03 \text{ m})/200 \text{ turns} = 0.00149 \text{ A} = 1.49 \text{ mA}$

- 19–26** The following data describe the effect of the magnetic field on the inductance in a silicon steel. Calculate (a) the initial permeability and (b) the maximum permeability for the material.

**Solution:**

$H$	$B$
0 A/m = 0 Oe	0 T = 0 G
20 A/m = 0.25 Oe	0.08 T = 800 G
40 A/m = 0.50 Oe	0.3 T = 3,000 G
60 A/m = 0.75 Oe	0.65 T = 6,500 G
80 A/m = 1.01 Oe	0.85 T = 8,500 G
100 A/m = 1.26 Oe	0.95 T = 9,500 G
150 A/m = 1.88 Oe	1.10 T = 11,500 G
250 A/m = 3.14 Oe	1.25 T = 12,500 G



The data is plotted; from the graph, the initial and maximum permeability are calculated, as indicated:

(a) initial permeability = 2222 G/Oe

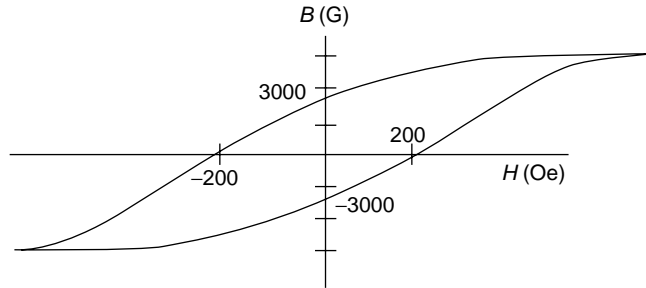
(b) maximum permeability = 8667 G/Oe

- 19–27** A magnetic material has a coercive field of 167 A/m, a saturation magnetization of 0.616 Tesla, and a residual inductance of 0.3 tesla. Sketch the hysteresis loop for the material.

**Solution:**  $M_{\text{sat}} = B_{\text{sat}} = 0.616 \text{ T} = 6160 \text{ G}$

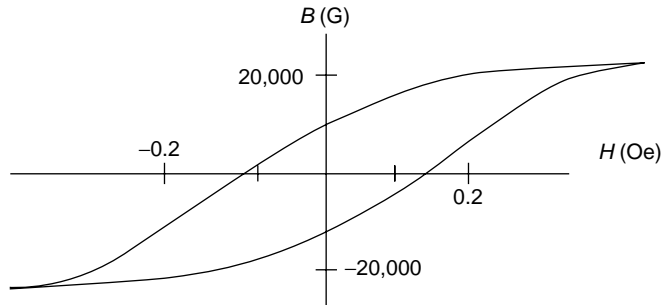
$B_r = 3000 \text{ G}$

$H_c = 167 \text{ A/m} \times 4\pi \times 10^{-3} \text{ Oe/A/m}$   
 $= 2.1 \text{ Oe}$



- 19–28** A magnetic material has a coercive field of 10.74 A/m, a saturation magnetization of 2.158 Tesla, and a remanance induction of 1.183 tesla. Sketch the hysteresis loop for the material.

**Solution:**  $B_{\text{sat}} = M_{\text{sat}} = 2.158 \text{ T} = 21,580 \text{ G}$   
 $B_r = 1.183 \text{ T} = 11,830 \text{ G}$   
 $H_c = 10.74 \text{ A/m} = 0.135 \text{ Oe}$



- 19–29** Using Figure 19–16, determine the following properties of the magnetic material.

- |                              |                                  |
|------------------------------|----------------------------------|
| (a) remanance                | (d) initial permeability         |
| (b) saturation magnetization | (e) maximum permeability         |
| (c) coercive field           | (f) power (maximum $BH$ product) |

**Solution:** (a) remanance = 13,000 G

(b) saturation magnetization = 14,000 G

(c) coercive field = 800 Oe

(d) initial permeability =  $7000 \text{ G}/1200 \text{ Oe} = 5.8 \text{ G/Oe}$

(e) maximum permeability =  $14,000 \text{ G}/900 \text{ Oe} = 15.6 \text{ G/Oe}$

(f) we can try several  $BH$  products in the 4th quadrant:

$$12,000 \text{ G} \times 450 \text{ Oe} = 5.4 \times 10^6 \text{ G} \cdot \text{Oe}$$

$$10,000 \text{ G} \times 680 \text{ Oe} = 6.8 \times 10^6 \text{ G} \cdot \text{Oe}$$

$$8,000 \text{ G} \times 720 \text{ Oe} = 5.76 \times 10^6 \text{ G} \cdot \text{Oe}$$

The maximum  $BH$  product, or power, is about  $6.8 \times 10^6 \text{ G} \cdot \text{Oe}$

**19–30** Using Figure 19–17 (see text), determine the following properties of the magnetic material.

- (a) remanance
- (b) saturation magnetization
- (c) coercive field
- (d) initial permeability
- (e) maximum permeability
- (f) power (maximum  $BH$  product)

**Solution:** (a) remanance = 5500 G

(b) saturation magnetization = 5800 G

(c) coercive field = 44,000 A/m

(d) initial permeability =  $2,000 \text{ G}/(40,000 \text{ A/m})(4\pi \times 10^{-3} \text{ Oe/A/m})$   
= 4.0 G/Oe

(e) maximum permeability =  $5500 \text{ G}/(40,000 \text{ A/m})(4\pi \times 10^{-3} \text{ Oe/A/m})$   
= 10.9 G/Oe

(f) we can try several  $BH$  products in the 4th quadrant:

$$4500 \text{ G} \times 24,000 \text{ A/m} \times 4\pi \times 10^{-3} \text{ Oe/A/m} = 1.36 \times 10^6 \text{ G} \cdot \text{Oe}$$

$$4000 \text{ G} \times 30,000 \text{ A/m} \times 4\pi \times 10^{-3} \text{ Oe/A/m} = 1.51 \times 10^6 \text{ G} \cdot \text{Oe}$$

$$3500 \text{ G} \times 34,000 \text{ A/m} \times 4\pi \times 10^{-3} \text{ Oe/A/m} = 1.50 \times 10^6 \text{ G} \cdot \text{Oe}$$

$$3000 \text{ G} \times 37,000 \text{ A/m} \times 4\pi \times 10^{-3} \text{ Oe/A/m} = 1.39 \times 10^6 \text{ G} \cdot \text{Oe}$$

The maximum  $BH$  product, or power, is about  $1.51 \times 10^6 \text{ G} \cdot \text{Oe}$

**19–36** Estimate the power of the  $\text{Co}_5\text{Ce}$  material shown in Figure 19–14.

Solution:	$H$	$B$	$BH$
	0 Oe	7500 G	0 G · Oe
	2000 Oe	7500 G	$15 \times 10^6 \text{ G} \cdot \text{Oe}$
	2500 Oe	6000 G	$15 \times 10^6 \text{ G} \cdot \text{Oe}$
	3500 Oe	0 G	0 G · Oe

**19–37** What advantage does the Fe–3% Si material have compared to Supermalloy for use in electric motors?

**Solution:** The Fe–3% Si has a larger saturation inductance than Supermalloy, allowing more work to be done. However Fe–3% Si does require larger fields, since the coercive field for Fe–3% Si is large, and the permeability of Fe–3% Si is small compared with that of Supermalloy.

**19–38** The coercive field for pure iron is related to the grain size of the iron by the relationship  $H_c = 1.83 + 4.14/\sqrt{A}$ , where  $A$  is the area of the grain in two dimensions ( $\text{mm}^2$ ) and  $H_c$  is in A/m. If only the grain size influences the 99.95% iron (coercivity 0.9 oersted), estimate the size of the grains in the material. What happens to the coercivity value when the iron is annealed to increase the grain size?

**Solution:**  $H_c = 0.9 \text{ Oe}/4\pi \times 10^{-3} \text{ Oe/A/m} = 71.62 \text{ A/m}$

Thus, from the equation,

$$71.62 = 1.83 + 4.14/\sqrt{A}$$

$$\sqrt{A} = 4.14/69.79 = 0.0593 \quad \text{or} \quad A = 0.0035 \text{ mm}^2$$

When the iron is annealed, the grain size increases,  $A$  increases, and the coercive field  $H_c$  decreases.

- 19–40** Suppose we replace 10% of the  $\text{Fe}^{2+}$  ions in magnetite with  $\text{Cu}^{2+}$  ions. Determine the total magnetic moment per cubic centimeter.

**Solution:** From Example 19–6, the lattice parameter is  $8.37 \text{ \AA}$ .

$$V_{\text{unit cell}} = (8.37 \times 10^{-8} \text{ cm})^3 = 5.86 \times 10^{-22} \text{ cm}^3 \\ = 5.86 \times 10^{-28} \text{ m}^3$$

In the tetrahedral sites, the fraction of copper atoms is 0.1, while the fraction of  $\text{Fe}^{2+}$  ions is 0.9. The magnetic moment is then:

$$\text{moment} = \frac{(8 \text{ subcells})[0.1 \text{ Cu}(1 \text{ magneton}) + 0.9 \text{ Fe}(4 \text{ magneton})](9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2)}{5.86 \times 10^{-28} \text{ m}^3}$$

$$\text{moment} = 4.68 \times 10^5 \text{ A} \cdot \text{m}^2/\text{m}^3 = 4.68 \times 10^5 \text{ A/m} = 0.468 \text{ A} \cdot \text{m}^2/\text{cm}^3$$

- 19–41** Suppose that the total magnetic moment per cubic meter in a spinel structure in which  $\text{Ni}^{2+}$  ions have replaced a portion of the  $\text{Fe}^{2+}$  ions is  $4.6 \times 10^5 \text{ A/m}$ . Calculate the fraction of the  $\text{Fe}^{2+}$  ions that have been replaced and the wt% Ni present in the spinel.

**Solution:** From Example 19–6, the volume of the unit cell is  $5.86 \times 10^{-28} \text{ m}^3$ . If we let  $x$  be the fraction of the tetrahedral sites occupied by nickel, then  $(1 - x)$  is the fraction of the sites occupied by iron. Then:

$$\text{moment} = 4.6 \times 10^5 = \frac{(8)[(x)(2 \text{ magnetons}) + (1 - x)(4 \text{ magnetons})](9.27 \times 10^{-24})}{5.86 \times 10^{-28} \text{ m}^3}$$

$$x = 0.185$$

Thus the number of each type of atom or ion in the unit cell is:

$$\text{oxygen: } (4 \text{ atoms/subcell})(8 \text{ subcells}) = 32$$

$$\text{Fe}^{3+}: (2 \text{ ions/subcell})(8 \text{ subcells}) = 16$$

$$\text{Fe}^{2+}: (0.815)(1 \text{ ion/subcell})(8 \text{ subcells}) = 6.52$$

$$\text{Ni}^{2+}: (0.185)(1 \text{ ion/subcell})(8 \text{ subcells}) = 1.48$$

The total number of ions in the unit cell is 56; the atomic fraction of each ion is:

$$f_{\text{oxygen}} = 32/56 = 0.5714 \quad f_{\text{Fe}^{3+}} = 16/56 = 0.2857$$

$$f_{\text{Fe}^{2+}} = 6.52/56 = 0.1164 \quad f_{\text{Ni}^{2+}} = 1.48/56 = 0.0264$$

The weight percent nickel is (using the molecular weights of oxygen, iron and nickel):

$$\% \text{Ni} = \frac{(0.0264)(58.71)}{(0.5714)(16) + (0.2857)(55.847) + (0.1164)(55.847) + (0.0264)(58.71)} \\ = 4.68 \text{ wt\%}$$

# 20

## Photonic Materials

- 20–10** A beam of photons strikes a material at an angle of  $25^\circ$  to the normal of the surface. Which, if any, of the materials listed in Table 20–1 could cause the beam of photons to continue at an angle of  $18$  to  $20^\circ$  from the normal of the material's surface?

**Solution:** Assuming that the beam originally is passing through air or a vacuum,

$$n = \sin \theta_i / \sin \theta_t = \sin 25^\circ / \sin \beta$$

To exit at an angle of  $18^\circ$ :

$$n = \sin 25^\circ / \sin 18^\circ = 0.4226 / 0.3090 = 1.367$$

To exit at an angle of  $20^\circ$ :

$$n = \sin 25^\circ / \sin 20^\circ = 0.4226 / 0.3420 = 1.236$$

In Table 20–1, only ice, water, and Teflon have an index of refraction between 1.236 and 1.367.

- 20–11** A laser beam passing through air strikes a 5-cm thick polystyrene block at a  $20^\circ$  angle to the normal of the block. By what distance is the beam displaced from its original path when the beam reaches the opposite side of the block?

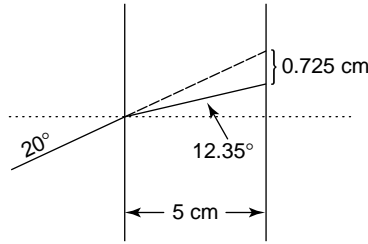
**Solution:** The index of refraction for polystyrene is 1.60. Since the incident angle  $\theta_i$  is  $20^\circ$ , the angle of the beam as it passes through the polystyrene block will be:

$$n = \sin \theta_i / \sin \theta_t = \sin 20^\circ / \sin \theta_t = 1.6$$

$$\sin \theta_t = 0.3420 / 1.6 = 0.2138$$

$$\theta_t = 12.35^\circ$$





From the sketch, we can find the displacement of the beam expected if no refraction occurs:

$$\tan 20^\circ = x/5 \quad \text{or} \quad x = 5 \tan 20^\circ = (5)(0.3640) = 1.820 \text{ cm}$$

We can also find the displacement of the beam when refraction occurs:

$$\tan 12.35^\circ = y/5 \quad \text{or} \quad y = 5 \tan 12.35^\circ = (5)(0.2189) = 1.095 \text{ cm}$$

Because of refraction, the beam is displaced  $1.820 - 1.095 = 0.725 \text{ cm}$  from its path had no refraction occurred.

- 20–12** A beam of photons passes through air and strikes a soda-lime glass that is part of an aquarium containing water. What fraction of the beam is reflected by the front face of the glass? What fraction of the remaining beam is reflected by the back face of the glass?

**Solution:** The fraction of the beam reflected by the front face is:

$$R = \left( \frac{n_{\text{glass}} - n_{\text{air}}}{n_{\text{glass}} + n_{\text{air}}} \right)^2 = \left( \frac{1.50 - 1.00}{1.50 + 1.00} \right)^2 = 0.04$$

The fraction of the remaining beam reflected from the back face of the glass is:

$$R = \left( \frac{n_{\text{water}} - n_{\text{glass}}}{n_{\text{water}} + n_{\text{glass}}} \right)^2 = \left( \frac{1.33 - 1.50}{1.33 + 1.50} \right)^2 = 0.0036$$

- 20–13** We find that 20% of the original intensity of a beam of photons is transmitted from air through a 1-cm thick-material having a dielectric constant of 2.3 and back into air. Determine the fraction of the beam that is (a) reflected at the front surface, (b) absorbed in the material, and (c) reflected at the back surface. (d) Determine the linear absorption coefficient of the photons in the material.

**Solution:** The dielectric material has an index of refraction of:

$$m = \sqrt{\kappa} = \sqrt{2.3} = 1.5166$$

- (a) The fraction of the beam reflected at the front surface is:

$$R = \left( \frac{n_{\text{material}} - n_{\text{air}}}{n_{\text{material}} + n_{\text{air}}} \right)^2 = \left( \frac{1.5166 - 1.00}{1.5166 + 1.00} \right)^2 = 0.04214$$

- (b) The fraction transmitted through the material is 0.2; therefore the linear absorption coefficient of the materials is:

$$\begin{aligned} I_t/I_o &= (1 - R)^2 \exp(-\alpha x) \\ &= (1 - 0.04214)^2 \exp[-\alpha(1 \text{ cm})] = 0.20 \end{aligned}$$

$$\exp(-\alpha) = 0.21798$$

$$-\alpha = \ln(0.21798) = -1.523$$

$$\alpha = 1.523 \text{ cm}^{-1}$$

After reflection, the intensity of the remaining beam is

$$I_{\text{after reflection}} = 1 - 0.04215 = 0.95785I_0$$

Before reflection at the back surface, the intensity of the beam is:

$$I_{\text{after absorption}} = 0.95785 \exp[(-1.523)(1)] = 0.2089I_0$$

The fraction of the beam that is absorbed is therefore

$$I_{\text{absorbed}} = 0.95785 - 0.2089 = 0.74895I_0$$

(c) The fraction of the beam reflected off the back surface is:

$$I_0 = I_{\text{reflected, front}} + I_{\text{absorbed}} + I_{\text{reflected, back}} + I_{\text{transmitted}}$$

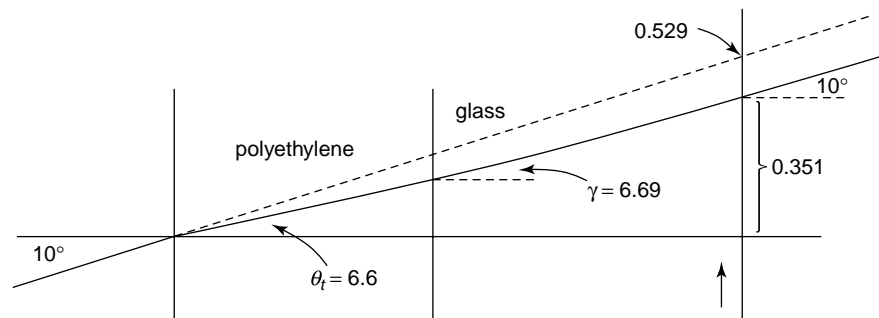
$$I_0 = 0.04214I_0 + 0.74895I_0 + I_{\text{reflected, back}} + 0.20I_0$$

$$I_{\text{reflected, back}} = 0.0089I_0$$

(d) See part b;  $\alpha = 1.523 \text{ cm}^{-1}$

- 20–14** A beam of photons in air strikes a composite material consisting of a 1-cm-thick sheet of polyethylene and a 2-cm-thick sheet of soda-lime glass. The incident beam is  $10^\circ$  from the normal of the composite. Determine the angle of the beam with respect to the normal as the beam (a) passes through the polyethylene, (b) passes through the glass, and (c) passes through air on the opposite side of the composite. (d) By what distance is the beam displaced from its original path when it emerges from the composite?

**Solution:** The figure shows how the beam changes directions, and the amount that the beam is displaced from the normal to the point of entry, as it passes through each interface.



- (a) As the beam passes from air into polyethylene (which has an index of refraction of 1.52),

$$\sin \theta_t = \sin \theta_i / n = \sin 10^\circ / 1.52 = 0.1736 / 1.52 = 0.1142$$

$$\theta_t = 6.6^\circ$$

- (b) When the beam enters the glass (which has an index of refraction of 1.50), the new angle is:

$$\begin{aligned} n_g/n_{PE} &= \sin \theta_i / \sin \gamma \\ 1.50/1.52 &= \sin 6.6^\circ / \sin \gamma \\ \sin \gamma &= 0.11647 \quad \text{or} \quad \gamma = 6.69^\circ \end{aligned}$$

- (c) When the beam emerges from the glass back into air, the final angle is:

$$\begin{aligned} n_{\text{air}}/n_g &= \sin \gamma / \sin x \\ 1.00/1.50 &= \sin 6.69^\circ / \sin x \\ \sin x &= 0.1747 \quad \text{or} \quad x = 10^\circ \end{aligned}$$

- (d) When the beam reaches the polyethylene-glass interface, it has been displaced:

$$\tan 6.6^\circ = x/1 \text{ cm} \quad \text{or} \quad x = 0.116 \text{ cm}$$

When the beam then reaches the glass-air interface, it has been displaced an additional:

$$\tan 6.69^\circ = y/2 \text{ cm} \quad \text{or} \quad y = 0.235 \text{ cm}$$

The total displacement is therefore  $x + y = 0.351 \text{ cm}$ . If the beam had not been refracted, the displacement would have been:

$$\tan 10^\circ = z/3 \text{ cm} \quad \text{or} \quad z = 0.529 \text{ cm}$$

The beam has therefore been displaced  $0.529 - 0.351 = 0.178 \text{ cm}$  from its original path.

- 20–15** A glass fiber ( $n = 1.5$ ) is coated with Teflon™. Calculate the maximum angle that a beam of light can deviate from the axis of the fiber without escaping from the inner portion of the fiber.

**Solution:** To keep the beam from escaping from the fiber, the angle  $\beta$  must be  $90^\circ$ . Therefore the maximum angle that the incoming beam can deviate from the fiber axis is:

$$\begin{aligned} n_{\text{teflon}}/n_{\text{glass}} &= \sin \theta_i / \sin \theta_t \\ 1.35/1.50 &= \sin \alpha / \sin 90^\circ \\ \sin \theta_i &= 0.90 \quad \text{or} \quad \theta_i = 64.16^\circ \end{aligned}$$

The maximum angle is therefore  $90 - 64.16 = 25.84^\circ$ .

- 20–16** A material has a linear-absorption coefficient of  $591 \text{ cm}^{-1}$  for photons of a particular wavelength. Determine the thickness of the material required to absorb 99.9% of the photons.

**Solution:**  $I/I_o = 0.001 = \exp(-\alpha x) = \exp(-591x)$   
 $\ln(0.001) = -6.9078 = -591x$   
 $x = 0.0117 \text{ cm}$

- 20–25** Calcium tungstate ( $\text{CaWO}_4$ ) has a relaxation time of  $4 \times 10^{-6}$  s. Determine the time required for the intensity of this phosphorescent material to decrease to 1% of the original intensity after the stimulus is removed.

**Solution:**

$$\ln(I/I_0) = -t/\tau$$

$$\ln(0.01) = -t/4 \times 10^{-6} \text{ s}$$

$$-4.605 = -t/4 \times 10^{-6}$$

$$t = 18.4 \times 10^{-6} \text{ s}$$

- 20–26** The intensity of a phosphorescent material is reduced to 90% of its original intensity after  $1.95 \times 10^{-7}$  s. Determine the time required for the intensity to decrease to 1% of its original intensity.

**Solution:** We can use the information in the problem to find the relaxation time for the material.

$$\ln(I/I_0) = \ln(0.9) = -(1.95 \times 10^{-7})/\tau$$

$$-0.1054 = -(1.95 \times 10^{-7})/\tau$$

$$\tau = 1.85 \times 10^{-6} \text{ s}$$

Then we can find the time required to reduce the intensity to  $I/I_0 = 0.01$ :

$$\ln(0.01) = -t/1.85 \times 10^{-6}$$

$$-4.605 = -t/1.85 \times 10^{-6}$$

$$t = 8.52 \times 10^{-6} \text{ s}$$

- 20–30** By appropriately doping yttrium aluminum garnet with neodymium, electrons are excited within the  $4f$  energy shell of the Nd atoms. Determine the approximate energy transition if the Nd:YAG serves as a laser, producing a wavelength of 532 nm. What color would the laser beam possess?

**Solution:** The energy transition is:

$$E = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(532 \times 10^{-9} \text{ m})(100 \text{ cm/m})(1.6 \times 10^{-19} \text{ J/eV})} = 2.333 \text{ eV}$$

The wavelength of 532 nm is  $5320 \text{ \AA}$  or  $5.32 \times 10^{-5} \text{ cm}$ . This wavelength corresponds to a color of green.

- 20–31** Determine whether an incident beam of photons with a wavelength of  $7500 \text{ \AA}$  will cause luminescence in the following materials (see Chapter 18).

(a) ZnO    (b) GaP    (c) GaAs    (d) GaSb    (e) PbS

**Solution:** The incident beam must have an energy greater than the energy gap of the material in order for luminescence to occur. The energy of the incident photons is:

$$E = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(7500 \times 10^{-8} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 1.655 \text{ eV}$$

From Chapter 18 and literature values, the energy gaps of the five materials are:

ZnO:	3.2 eV
GaP:	2.24 eV
GaAs:	1.35 eV
GaSb:	0.67 eV
PbS:	0.37 eV

Consequently the photons, having energy 1.655 eV, will be able to excite electrons in GaAs, GaSb, and PbS; however electrons will not be excited in ZnO and GaP.

- 20–32** Determine the wavelength of photons produced when electrons excited into the conduction band of indium-doped silicon (a) drop from the conduction band to the acceptor band and (b) then drop from the acceptor band to the valence band (see Chapter 18).

**Solution:** The acceptor energy in Si–In is 0.16 eV; the energy gap in pure Si is 1.107 eV. The difference between the energy gap and the acceptor energy level is  $1.107 - 0.16 = 0.947$  eV.

- (a) The wavelength of photons produced when an electron drops from the conduction band to the acceptor band, an energy difference of 0.947 eV, is:

$$\lambda = hc/E = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.947 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})} = 13.11 \times 10^{-5} \text{ cm}$$

- (b) The wavelength of photons produced when the electron subsequently drops from the acceptor band to the valence band, an energy difference of 0.16 eV, is:

$$\lambda = hc/E = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.16 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})} = 77.58 \times 10^{-5} \text{ cm}$$

- 20–33** Which, if any, of the semiconducting compounds listed in Chapter 18 are capable of producing an infrared laser beam?

**Solution:** Infrared radiation has a wavelength of between  $10^{-2}$  and  $10^{-4}$  cm. Thus the semiconducting compound must have an energy gap that lies between the energies corresponding to these wavelength limits:

$$E_g = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(10^{-2} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 0.0124 \text{ eV}$$

$$E_g = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(10^{-4} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 1.24 \text{ eV}$$

Of the semiconducting compounds in Table 18–8, the following have energy gaps between 0.0124 and 1.24 eV and can therefore act as infrared lasers:

InSb	InAs	PbS	PbTe	CdSnAs <sub>2</sub>
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- 20–34** What type of electromagnetic radiation (ultraviolet, infrared, visible) is produced from (a) pure germanium and (b) germanium doped with phosphorous? (See Chapter 18.)

**Solution:** (a) For pure germanium, the energy gap is 0.67 eV; the wavelength is:

$$\lambda = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.67 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})} = 1.853 \times 10^{-4} \text{ cm}$$

This corresponds to the infrared region of the spectrum.

(b) For Ge doped with phosphorous, the energy gap is 0.012 eV.

$$\lambda = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.012 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})} = 1.034 \times 10^{-2} \text{ cm}$$

This wavelength is also in the infrared region.

- 20–35** Which, if any, of the dielectric materials listed in Chapter 18 would reduce the speed of light in air from  $3 \times 10^{10}$  cm/s to less than  $0.5 \times 10^{10}$  cm/s?

**Solution:** To reduce the speed of light the required amount, the index of refraction must be greater than:

$$n = c/v = 3 \times 10^{10} \text{ cm/s} / 0.5 \times 10^{10} \text{ cm/s} = 6$$

Consequently the dielectric constant  $\kappa$  of the material must be greater than:

$$\kappa = n^2 = 6^2 = 36$$

From Table 18–9, only  $\text{H}_2\text{O}$ ,  $\text{BaTiO}_3$ , and  $\text{TiO}_2$  have dielectric constants greater than 36.

- 20–36** What filter material would you use to isolate the  $K_\alpha$  peak of the following x-rays: iron, manganese, nickel? Explain your answer.

**Solution:** Iron: use a manganese filter. The absorption edge for Mn is 1.896 Å, which lies between the iron  $K_\alpha$  peak of 1.937 Å and the  $K_\beta$  peak of 1.757 Å.

Manganese: use a chromium filter. The absorption edge for Cr is 2.070 Å, which lies between the manganese  $K_\alpha$  peak of 2.104 Å and the  $K_\beta$  peak of 1.910 Å.

Nickel: use a cobalt filter. The absorption edge for Co is 1.608 Å, which lies between the nickel  $K_\alpha$  peak of 1.660 Å and the  $K_\beta$  peak of 1.500 Å.

- 20–37** What voltage must be applied to a tungsten filament to produce a continuous spectrum of x-rays having a minimum wavelength of 0.09 nm?

**Solution:** 
$$E = \frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.09 \times 10^{-9} \text{ m})(100 \text{ cm/m})} = 2.206 \times 10^{-15} \text{ J}$$

$$E = (2.206 \times 10^{-15} \text{ J}) / (1.6 \times 10^{-19} \text{ J/eV}) = 13,790 \text{ V}$$

- 20–38** A tungsten filament is heated with a 12,400 V power supply. What is (a) the wavelength and (b) frequency of the highest-energy x-rays that are produced?

**Solution:**  $E = (12,400 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = 1.984 \times 10^{-15} \text{ J}$

$$1.984 \times 10^{-15} \text{ J} = hc/\lambda = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{\lambda}$$

(a)  $\lambda = 1.00 \times 10^{-8} \text{ cm} = 1.00 \text{ \AA} = 0.100 \text{ nm}$

(b)  $\nu = c/\lambda = 3 \times 10^{10} \text{ cm/s} / 1.00 \times 10^{-8} \text{ cm} = 3.0 \times 10^{18} \text{ s}^{-1}$

- 20–39** What is the minimum voltage required to produce  $K_\alpha$  x-rays in nickel?

**Solution:** The wavelength of  $K_\alpha$  x-rays in nickel is  $1.66 \text{ \AA} = 1.66 \times 10^{-8} \text{ cm}$

$$E = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(1.66 \times 10^{-8} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 7,477 \text{ V}$$

- 20–40** Based on the characteristic x-rays that are emitted, determine the difference in energy between electrons in tungsten for (a) the  $K$  and  $L$  shells, (b) the  $K$  and  $M$  shells, and (c) the  $L$  and  $M$  shells.

**Solution:** The energy difference between the  $K$  and  $L$  shells produces  $K_\alpha$  x-rays. The wavelength of these x-rays is  $0.211 \text{ \AA}$ :

$$E(K - L) = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.211 \times 10^{-8} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 58,830 \text{ eV}$$

The energy difference between the  $K$  and  $M$  shells produces  $K_\beta$  x-rays. The wavelength of these x-rays is  $0.184 \text{ \AA}$ :

$$E(K - M) = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.184 \times 10^{-8} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 67,459 \text{ eV}$$

The energy difference between the  $L$  and  $M$  shells produces  $L_\alpha$  x-rays. The wavelength of these x-rays is  $1.476 \text{ \AA}$ :

$$E(L - M) = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(1.476 \times 10^{-8} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 8,410 \text{ eV}$$

- 20–41** Figure 20–22 shows the results of an x-ray fluorescence analysis, in which the energy of x-rays emitted from a material are plotted relative to the wavelength of the x-rays. Determine (a) the accelerating voltage used to produce the exciting x-rays and (b) the identity of the elements in the sample.

**Solution:** (a) The highest energy x-rays produced have a wavelength ( $\lambda_{\text{swl}}$ ) of about  $0.5 \text{ \AA}$ . The accelerating voltage is therefore:

$$E = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(0.5 \times 10^{-8} \text{ cm})(1.6 \times 10^{-19} \text{ J/eV})} = 24,825 \text{ V}$$

- (b) The wavelengths of the characteristic x-rays are listed below. By comparison with the wavelengths of characteristic x-rays from different elements, Table 20–2, we can match the observed x-rays with the x-rays of the elements to obtain the composition of the sample.

observed	expected		element
1.4 Å	1.392 Å	—	Cu $K_\beta$
1.55	1.542	—	Cu $K_\alpha$
1.9	1.910	—	Mn $K_\beta$
2.1	2.104	—	Mn $K_\alpha$
6.7	6.768	—	Si $K_\beta$
7.1	7.125	—	Si $K_\alpha$

The alloy must contain copper, manganese, and silicon.

- 20–42** Figure 20–23 shows the energies of x-rays produced from an energy-dispersive analysis of radiation emitted from a specimen in a scanning electron microscope. Determine the identity of the elements in the sample.

**Solution:** The energy of the first observed peak is about 2200 eV; the wavelength corresponding to this energy is:

$$\lambda = hc/E = \frac{(6.62 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^{10} \text{ cm/s})}{(2200 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}$$

$$= 5.642 \times 10^{-8} \text{ cm} = 5.642 \text{ Å}$$

Similarly we can find the wavelength corresponding to the energies of the other characteristic peaks. The table below lists the energies and calculated wavelengths for each peak and compares the wavelength to the characteristic radiation for different elements, from Table 20–2.

energy	calculated $\lambda$	expected $\lambda$		element
2,200 eV	5.642 Å	5.724 Å	—	Mo $L_\alpha$
5,250	2.364	2.291	—	Cr $K_\alpha$
6,000	2.069	2.084	—	Cr $K_\beta$
7,000	1.773	1.790	—	Co $K_\alpha$
7,800	1.591	1.621	—	Co $K_\beta$
17,300	0.717	0.711	—	Mo $K_\alpha$
19,700	0.630	0.632	—	Mo $K_\beta$

The sample must contain molybdenum, chromium, and cobalt.

- 20–43** Figure 20–24 shows the intensity of the radiation obtained from a copper x-ray generating tube as a function of wavelength. The accompanying table shows the linear absorption coefficient for a nickel filter for several wavelengths. If the Ni filter is 0.005 cm thick, calculate and plot the intensity of the transmitted x-ray beam versus wavelength.

**Solution:** The intensity after absorption is

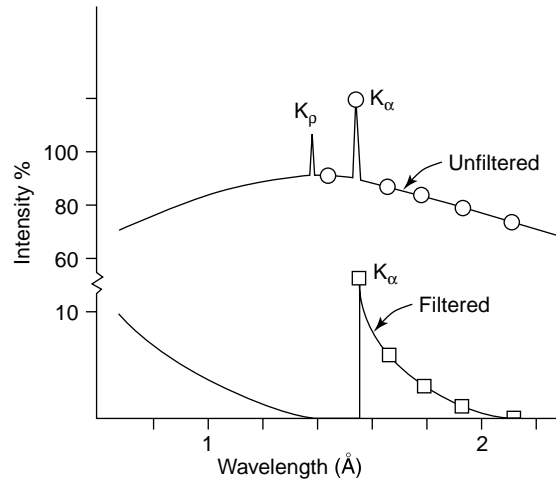
$$I/I_o = \exp(-\alpha x) = \exp(-0.005\alpha)$$

We can then select various wavelengths of x-rays and, from the table, determine the  $\mu$  for each wavelength. From our equation, we can then calculate the  $I/I_o$  expected for each wavelength. Finally we can multiply  $I/I_o$  by the initial intensity, obtained from Figure 20–23. For  $\lambda = 0.711 \text{ Å}$ , these calculations are:

$$\alpha = 422 \text{ cm}^{-1} \quad I_t/I_o = \exp[(-422)(0.005)] = 0.121$$

$$I_o = 72 \quad I_t = (0.121)(72) = 8.7$$





$\lambda$	$\mu$	$I_o$	$I/I_o$	$I_f$
0.711 Å	422 cm <sup>-1</sup>	72	0.121	8.7
1.436	2900	90	$5.04 \times 10^{-7}$	0.000045
1.542	440	120	0.110	13.3
1.659	543	88	0.066	5.8
1.79	670	86	0.035	3.0
1.937	830	80	0.016	1.3
2.103	1030	75	0.006	0.4
2.291	1300	68	0.0015	0.1

The graph compares the original intensity to the final, filtered intensity of the x-ray beam. Note that the characteristic  $K_\beta$  peak from the copper is eliminated, while much of the  $K_\alpha$  peak is transmitted.

# 21

## Thermal Properties of Materials

**21–3** Calculate the heat (in calories and joules) required to raise the temperature of 1 kg of the following materials by 50°C.

- (a) lead      (b) nickel      (c)  $\text{Si}_3\text{N}_4$       (d) 6,6–nylon

**Solution:** The heat is the specific heat times the weight times the temperature change. Calories can be converted to joules by multiplying by 4.184.

$$(a) c_{\text{Pb}} = (0.038 \text{ cal/gK})(1000 \text{ g})(50 \text{ K}) = 1900 \text{ cal} = 7,950 \text{ J}$$

$$(b) c_{\text{Ni}} = (0.106 \text{ cal/gK})(1000 \text{ g})(50 \text{ K}) = 5300 \text{ cal} = 22,175 \text{ J}$$

$$(c) c_{\text{silicon nitride}} = (0.17 \text{ cal/gK})(1000 \text{ g})(50 \text{ K}) = 8,500 \text{ cal} = 35,564 \text{ J}$$

$$(d) c_{6,6 \text{ nylon}} = (0.40 \text{ cal/gK})(1000 \text{ g})(50 \text{ K}) = 20,000 \text{ cal} = 83,680 \text{ J}$$

**21–4** Calculate the temperature of a 100-g sample of the following materials, (originally at 25°C) when 3000 calories are introduced.

- (a) tungsten      (b) titanium      (c)  $\text{Al}_2\text{O}_3$       (d) low-density polyethylene

$$\text{Solution: (a) W: } 3000 \text{ cal} = (0.032 \text{ cal/gK})(100 \text{ g})(T - 25) \quad \text{or } T_{\text{W}} = 962.5^\circ\text{C}$$

$$(b) \text{ Ti: } 3000 \text{ cal} = (0.125 \text{ cal/gK})(100 \text{ g})(T - 25) \quad \text{or } T_{\text{Ti}} = 265^\circ\text{C}$$

$$(c) \text{ Al}_2\text{O}_3: 3000 \text{ cal} = (0.20 \text{ cal/gK})(100 \text{ g})(T - 25) \quad \text{or } T_{\text{alumina}} = 175^\circ\text{C}$$

$$(d) \text{ LDPE: } 3000 \text{ cal} = (0.55 \text{ cal/gK})(100 \text{ g})(T - 25) \quad \text{or } T_{\text{PE}} = 79.5^\circ\text{C}$$

- 21–5** An alumina insulator for an electrical device is also to serve as a heat sink. A 10°C temperature rise in an alumina insulator 1 cm × 1 cm × 0.02 cm is observed during use. Determine the thickness of a high-density polyethylene insulator that would be needed to provide the same performance as a heat sink. The density of alumina is 3.96 g/cm<sup>3</sup>.

**Solution:** The heat absorbed by the alumina is:

$$\begin{aligned}\text{heat} &= (0.20 \text{ cal/gK})(10^\circ\text{C})(3.96 \text{ g/cm}^3)(1 \text{ cm} \times 1 \text{ cm} \times 0.02 \text{ cm}) \\ &= 0.1584 \text{ cal}\end{aligned}$$

The same amount of heat must be absorbed by the polyethylene, which has a density of about 0.96 g/cm<sup>3</sup>:

$$\begin{aligned}\text{heat} &= (0.44 \text{ cal/gK})(10^\circ\text{C})(0.96 \text{ g/cm}^3)(1 \text{ cm} \times 1 \text{ cm} \times t) \\ 0.1584 \text{ cal} &= 4.224t \\ t &= 0.0375 \text{ cm}\end{aligned}$$

- 21–6** A 200-g sample of aluminum is heated to 400°C and is then quenched into 2000 cm<sup>3</sup> of water at 20°C. Calculate the temperature of the water after the aluminum and water reach equilibrium. Assume no temperature loss from the system.

**Solution:** The amount of heat gained by the water equals the amount lost by the aluminum. If the equilibrium temperature is  $T_e$ :

$$\begin{aligned}(0.215 \text{ cal/gK})(400 - T_e)(200 \text{ g}) &= (1.0 \text{ cal/gK})(T_e - 20)(2000 \text{ g}) \\ 17,200 - 43T_e &= 2000T_e - 40,000 \\ T_e &= 28^\circ\text{C}\end{aligned}$$

- 21–7** A 2-m-long soda-lime glass sheet is produced at 1400°C. Determine its length after it cools to 25°C.

$$\begin{aligned}\text{Solution: } \Delta\ell &= \ell_o\alpha\Delta T = (2 \text{ m})(9 \times 10^{-6} \text{ m/m}^\circ\text{C})(1400 - 25) \\ \Delta\ell &= 0.02475 \text{ m} \\ \ell_o &= \ell_f - \Delta\ell = 2 - 0.02475 = 1.97525 \text{ m}\end{aligned}$$

- 21–8** A copper casting is to be produced having the final dimensions of 1 in. × 12 in. × 24 in. Determine the size of the pattern that must be used to make the mold into which the liquid copper is poured during the manufacturing process.

$$\begin{aligned}\text{Solution: } \Delta\ell &= \ell_o\alpha(T_m - T_o) = \ell_o(16.6 \times 10^{-6})(1084.9 - 25) \\ \Delta\ell &= 0.01759\ell_o \\ \ell &= \ell_o + 0.01759\ell_o \\ \ell &= 24 + (0.01759)(24) = 24.422 \text{ in.} \\ \ell &= 12 + (0.01759)(12) = 12.211 \text{ in.} \\ \ell &= 1 + (0.01759)(1) = 1.0176 \text{ in.}\end{aligned}$$

- 21–9** An aluminum casting is made by the permanent mold process. In this process, the liquid aluminum is poured into a gray cast iron mold that is heated to 350°C. We wish to produce an aluminum casting that is 15 in. long at 25°C. Calculate the length of the cavity that must be machined into the gray cast iron mold.

**Solution:** The aluminum casting shrinks between the solidification temperature ( $660.4^{\circ}\text{C}$ ) and room temperature ( $25^{\circ}\text{C}$ ). However, the gray cast iron mold expands when it is heated from  $25^{\circ}\text{C}$  to  $350^{\circ}\text{C}$  during the casting process. The original length of the cavity in the mold is therefore given by the amount of contraction of the aluminum minus the amount of expansion of the mold:

$$\ell = \ell_o + \Delta\ell_{\text{gray iron}} - \Delta\ell_{\text{aluminum}}$$

$$15 = \ell_o + \ell_o[(12 \times 10^{-6})(350 - 25) - (25 \times 10^{-6})(660.4 - 25)]$$

$$15 = \ell_o + 0.0039\ell_o - 0.015885\ell_o = 0.988\ell_o$$

$$\ell_o = 15.182 \text{ in.}$$

**21–10** We coat a 100-cm-long, 2-mm-diameter copper wire with a 0.5-mm-thick epoxy insulation coating. Determine the length of the copper and the coating when their temperature increases from  $25^{\circ}\text{C}$  to  $250^{\circ}\text{C}$ . What is likely to happen to the epoxy coating as a result of this heating?

**Solution:** Both the copper and the epoxy expand when heated. The final length of each material, assuming that they are not bonded to one another, would be:

$$\ell_{\text{Cu}} = (100 \text{ cm})(16.6 \times 10^{-6})(250 - 25) + 100 = 100.3735 \text{ cm}$$

$$\ell_{\text{epoxy}} = (100 \text{ cm})(55 \times 10^{-6})(250 - 25) + 100 = 101.2375 \text{ cm}$$

The epoxy expands nearly 1 cm more than does the underlying copper. If the copper and epoxy are well bonded, the epoxy coating will buckle, debond, and perhaps even flake off.

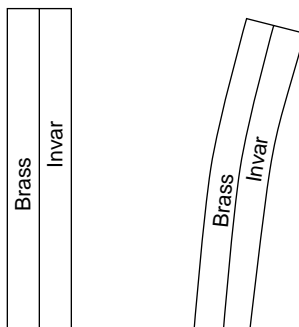
**21–11** We produce a 10-in.-long bimetallic composite material composed of a strip of yellow brass bonded to a strip of Invar. Determine the length to which each material would like to expand when the temperature increases from  $20^{\circ}\text{C}$  to  $150^{\circ}\text{C}$ . Draw a sketch showing what will happen to the shape of the bimetallic strip.

**Solution:** If the two metals are not bonded to one another, the amount each would like to expand is:

$$\pi_{\text{brass}} = 10 + (10)(18.9 \times 10^{-6})(150 - 20) = 10.0246 \text{ in.}$$

$$\ell_{\text{Invar}} = 10 + (10)(1.54 \times 10^{-6})(150 - 20) = 10.0020 \text{ in.}$$

The brass expands more than the Invar; if the two are bonded together, the bimetallic strip will bend, with the Invar on the inside radius of curvature of the strip.



- 21–17** A nickel engine part is coated with SiC to provide corrosion resistance at high temperatures. If no residual stresses are present in the part at 20°C, determine the thermal stresses that develop when the part is heated to 1000°C during use. (See Table 14–3.)

**Solution:** The net difference in the amount of expansion is given by:

$$\Delta\alpha = \alpha_{\text{nickel}} - \alpha_{\text{SiC}} = (13 - 4.3) \times 10^{-6} = 8.7 \times 10^{-6}$$

The thermal stresses  $\sigma_{\tau}$  in the SiC coating are:

$$\begin{aligned}\sigma_{\tau} &= E\Delta\alpha\Delta T \\ &= (60 \times 10^6 \text{ psi})(8.7 \times 10^{-6} \text{ in./in.}^{\circ}\text{C})(1000 - 20) = 511,560 \text{ psi}\end{aligned}$$

The nickel expands more than the SiC; therefore the stresses acting on the SiC are tensile stresses. The tensile strength of SiC is only on the order of 25,000 psi (Table 14–3), so the coating will likely crack.

- 21–18** Alumina fibers 2 cm long are incorporated into an aluminum matrix. Assuming good bonding between the ceramic fibers and the aluminum, estimate the thermal stresses acting on the fiber when the temperature of the composite increases 250°C. Are the stresses on the fiber tensile or compressive? (See Table 14–3.)

**Solution:** The net difference in the expansion coefficients of the two materials is:

$$\Delta\alpha = \alpha_{\text{Al}} - \alpha_{\text{alumina}} = (25 - 6.7) \times 10^{-6} = 18.3 \times 10^{-6}$$

The thermal stresses on the alumina are:

$$\begin{aligned}\sigma_{\tau} &= E\Delta\alpha\Delta T \\ &= (56 \times 10^6 \text{ psi})(18.3 \times 10^{-6} \text{ in./in.}^{\circ}\text{C})(250^{\circ}\text{C}) = 256,200 \text{ psi}\end{aligned}$$

The aluminum expands more than the alumina; thus the alumina fibers are subjected to tensile stresses. The alumina has a tensile strength of only about 30,000 psi (Table 14–3); consequently the fibers are expected to crack.

- 21–19** A 24-in.-long copper bar with a yield strength of 30,000 psi is heated to 120°C and immediately fastened securely to a rigid framework. Will the copper deform plastically during cooling to 25°C? How much will the bar deform if it is released from the framework after cooling?

**Solution:** If room temperature is 25°C, then the thermal stresses that develop in the restrained copper as it cools is:

$$\begin{aligned}\sigma_{\tau} &= E\alpha\Delta T = (18.1 \times 10^6 \text{ psi})(16.6 \times 10^{-6})(120 - 25) \\ \sigma_{\tau} &= 28,544 \text{ psi}\end{aligned}$$

The thermal stresses are less than the yield strength; consequently, no plastic deformation occurs in the copper as it cools. When the copper is released from its restraint, the residual stresses will be relieved by elastic deformation. The strain stored in the material by contraction will be:

$$\epsilon = \sigma/E = 28,544/18.1 \times 10^6 = 0.001577 \text{ in./in.}$$

The change in length of the copper bar is

$$\Delta\ell = (24 \text{ in.})(0.001577 \text{ in./in.}) = 0.0378 \text{ in.}$$

- 21–20** Repeat problem 21–19, but using a silicon carbide rod rather than a copper rod. (See Table 14–3.)

**Solution:** SiC has a modulus of  $60 \times 10^6$  psi (Table 14–3). The thermal stresses are:

$$\sigma_{\tau} = E\alpha\Delta T = (60 \times 10^6 \text{ psi})(4.3 \times 10^{-6})(120 - 25)$$

$$\sigma_{\tau} = 24,510 \text{ psi}$$

The thermal stresses are less than the tensile strength of SiC (about 25,000 psi, Table 14–3). Thus the elastic strain stored in the SiC is:

$$\varepsilon = \sigma/E = 24,510/60 \times 10^6 = 0.0004085 \text{ in./in.}$$

The change in length of the copper bar is

$$\Delta \ell = (24 \text{ in.})(0.0004085 \text{ in./in.}) = 0.0098 \text{ in.}$$

- 21–21** A 3-cm-plate of silicon carbide separates liquid aluminum (held at 700°C) from a water-cooled steel shell maintained at 20°C. Calculate the heat  $Q$  transferred to the steel per cm<sup>2</sup> of silicon carbide each second.

**Solution:** The temperature change through the  $\Delta x = 3$  cm thick SiC is  $\Delta T = 700 - 20 = 680^\circ\text{C}$ . The temperature gradient is thus:

$$\Delta T/\Delta x = 680/3 = 226.7^\circ\text{C/cm}$$

The thermal conductivity is 0.21 cal/cm · s · K; thus:

$$Q/A = (0.21)(226.7) = 47.6 \text{ cal/cm}^2 \cdot \text{s}$$

- 21–22** A sheet of 0.01-in. polyethylene is sandwiched between two 3 ft × 3 ft × 0.125 in. sheets of soda-lime glass to produce a window. Calculate (a) the heat lost through the window each day when room temperature is 25°C and the outside air is 0°C and (b) the heat entering through the window each day when room temperature is 25°C and the outside air is 40°C.

**Solution:** The rule of mixtures will allow us to calculate the thermal conductivity of this laminar composite. The volume fractions of each constituent are determined from the thicknesses:

$$f_{\text{PE}} = 0.01 \text{ in.}/(0.01 + 0.125 + 0.125) = 0.01/0.26 = 0.03846$$

$$f_{\text{g}} = (2)(0.125)/0.26 = 0.96154$$

$$1/K = 0.03846/0.0008 + 0.96154/0.0032 = 348.556$$

$$K = 1/348.556 = 0.00287 \text{ cal/cm} \cdot \text{s} \cdot \text{K}$$

The surface area of the glass is 3 ft × 3 ft, or

$$A = (3 \text{ ft})^2 (12 \text{ in./ft})^2 (2.54 \text{ cm/in.})^2 = 8361 \text{ cm}^2$$

The thickness of the composite is:

$$\Delta x = (0.26 \text{ in.})(2.54 \text{ cm/in.}) = 0.6604 \text{ cm}$$

- (a) The heat loss to the outside is:

$$\begin{aligned} Q &= K_{\text{composite}} A \Delta T / \Delta x \\ &= (0.00287 \text{ cal/cm} \cdot \text{s} \cdot \text{K})(8361 \text{ cm}^2)(25 \text{ K}/0.6604 \text{ cm}) \\ &= 908.39 \text{ cal/s} \end{aligned}$$

or:

$$Q = (908.39 \text{ cal/s})(3600 \text{ s/h})(24 \text{ h/day}) = 78.5 \times 10^6 \text{ cal/day}$$

(b) The heat entering the room from outside is:

$$\begin{aligned} Q &= (0.00287 \text{ cal/cm} \cdot \text{s} \cdot \text{K})(8361 \text{ cm}^2)(40 - 25) / 0.6604 \text{ cm} \\ &= 545.03 \text{ cal/s} = 47.09 \times 10^6 \text{ cal/day} \end{aligned}$$

**21–23** We would like to build a heat-deflection plate that permits heat to be transferred rapidly parallel to the sheet but very slowly perpendicular to the sheet. Consequently we incorporate 1 kg of copper wires, each 0.1 cm in diameter, into 5 kg of a polyimide polymer matrix. Estimate the thermal conductivity parallel and perpendicular to the sheet.

**Solution:** We can first calculate the volume fractions of the two constituents in the composite. The volume of each material is:

$$V_{\text{Cu}} = 1000 \text{ g} / 8.93 \text{ g/cm}^3 = 111.98 \text{ cm}^3$$

$$V_{\text{PI}} = 5000 \text{ g} / 1.14 \text{ g/cm}^3 = 4385.96 \text{ cm}^3$$

$$f_{\text{Cu}} = 111.98 / (111.98 + 4385.96) = 0.025$$

$$f_{\text{PI}} = 0.975$$

Parallel to the wires:

$$\begin{aligned} K &= (0.025)(0.96 \text{ cal/cm} \cdot \text{s} \cdot \text{K}) + (0.975)(0.0005 \text{ cal/cm} \cdot \text{s} \cdot \text{K}) \\ &= 0.0245 \text{ cal/cm} \cdot \text{s} \cdot \text{K} \end{aligned}$$

Perpendicular to the wires:

$$1/K = 0.025/0.96 + 0.975/0.0005 = 0.026 + 1950 = 1950.026$$

$$K = 0.00051 \text{ cal/cm} \cdot \text{s} \cdot \text{K}$$

The thermal conductivity is much higher parallel to the conductive copper wires than perpendicular to the wires.

**21–24** Suppose we just dip a 1-cm-diameter, 10-cm-long rod of aluminum into one liter of water at 20°C. The other end of the rod is in contact with a heat source operating at 400°C. Determine the length of time required to heat the water to 25°C if 75% of the heat is lost by radiation from the bar.

**Solution:** The heat required to raise the temperature of the water by 5°C is:

$$\text{Heat} = (1 \text{ cal/g} \cdot \text{K})(1000 \text{ g})(25 - 20) = 5000 \text{ cal}$$

However, since 75% of the heat is lost by radiation, we must supply a total of

$$\text{Heat} = 4 \times 5000 = 20,000 \text{ cal}$$

The heat flux  $Q$  is cal per area per time; thus

$$\text{Heat}/t = KA\Delta T/\Delta x$$

$$\frac{20,000}{t} = \frac{(0.57 \text{ cal/cm} \cdot \text{s} \cdot \text{K})(\pi/4)(1 \text{ cm})^2(400 - 20)}{10 \text{ cm}}$$

$$t = 1176 \text{ s} = 19.6 \text{ min}$$

- 21–26** Determine the thermal shock parameter for hot-pressed silicon nitride, hot pressed silicon carbide, and alumina and compare it with the thermal-shock resistance as defined by the maximum quenching temperature difference. (See Table 14–3.)

**Solution:** For  $\text{Si}_3\text{N}_4$ :

$$\begin{aligned} \text{TSP} &= \sigma_f K / E \alpha = \frac{(80,000 \text{ psi})(0.035 \text{ cal/cm} \cdot \text{s} \cdot \text{K})}{(45 \times 10^6 \text{ psi})(3.3 \times 10^{-6} \text{ cm/cm} \cdot \text{K})} \\ &= 18.86 \text{ cal} \cdot \text{cm/s} \end{aligned}$$

For SiC:

$$\begin{aligned} \text{TSP} &= \sigma_f K / E \alpha = \frac{(25,000 \text{ psi})(0.21 \text{ cal/cm} \cdot \text{s} \cdot \text{K})}{(60 \times 10^6 \text{ psi})(4.3 \times 10^{-6} \text{ cm/cm} \cdot \text{K})} \\ &= 20.35 \text{ cal} \cdot \text{cm/s} \end{aligned}$$

For alumina:

$$\begin{aligned} \text{TSP} &= \sigma_f K / E \alpha = \frac{(30,000 \text{ psi})(0.038 \text{ cal/cm} \cdot \text{s} \cdot \text{K})}{(56 \times 10^6 \text{ psi})(6.7 \times 10^{-6} \text{ cm/cm} \cdot \text{K})} \\ &= 3.04 \text{ cal} \cdot \text{cm/s} \end{aligned}$$

The maximum quenching difference for silicon nitride is 500°C, for silicon carbide is 350°C, and for alumina is 200°C. The maximum quenching difference correlates reasonably well with the thermal shock parameter.

- 21–27** Gray cast iron has a higher thermal conductivity than ductile or malleable cast iron. Review Chapter 12 and explain why this difference in conductivity might be expected.

**Solution:** The thermal conductivities of the constituents in the cast irons are:

$$K_{\text{graphite}} = 0.8 \text{ cal/cm} \cdot \text{s} \cdot \text{K}$$

$$K_{\text{ferrite}} = 0.18 \text{ cal/cm} \cdot \text{s} \cdot \text{K}$$

$$K_{\text{cementite}} = 0.12 \text{ cal/cm} \cdot \text{s} \cdot \text{K}$$

The gray cast iron contains interconnected graphite flakes, while the graphite nodules in ductile and malleable iron are not interconnected. Graphite, or carbon, has a higher thermal conductivity than does the “steel” matrix of the cast iron. Consequently heat can be transferred more rapidly through the iron-graphite “composite” structure of the gray iron than through the ductile and malleable irons.





# 22

## Corrosion and Wear

- 22–1** A gray cast iron pipe is used in the natural gas distribution system for a city. The pipe fails and leaks, even though no corrosion noticeable to the naked eye has occurred. Offer an explanation for why the pipe failed.

**Solution:** Because no corrosion is noticeable, the corrosion byproduct apparently is still in place on the pipe, hiding the corroded area. The circumstances suggest graphitic corrosion, an example of a selective chemical attack. The graphite flakes in the gray iron are not attacked by the corrosive soil, while the iron matrix is removed or converted to an iron oxide or hydroxide trapped between the graphite flakes. Although the pipe appears to be sound, the attacked area is weak, porous, and spongy. The natural gas can leak through the area of graphitic corrosion and eventually cause gas accumulations leading to an explosion.

- 22–2** A brass plumbing fitting produced from a Cu-30% Zn alloy operates in the hot water system of a large office building. After some period of use, cracking and leaking occur. On visual examination no metal appears to have been corroded. Offer an explanation for why the fitting failed.

**Solution:** The high zinc brasses are susceptible to dezincification, particularly when the temperature is increased, as in the hot water supply of the building. One of the characteristics of dezincification is that copper is redeposited in the regions that are attacked, obscuring the damage. However the redeposited copper is spongy, brittle, and weak, permitting the fitting to fail and leak. Therefore dezincification appears to be a logical explanation.

- 22–3** Suppose 10 g of  $\text{Sn}^{2+}$  are dissolved in 1000 ml of water to produce an electrolyte. Calculate the electrode potential of the tin half-cell.

**Solution:** The concentration of the electrolyte is:

$$C = 10 \text{ g} / 118.69 \text{ g/mol} = 0.0842 \text{ M}$$

The electrode potential from the Nernst equation is:

$$E = -0.14 + (0.0592/2)\log(0.0842) = -0.172 \text{ V}$$

Note that the logarithm is to the base 10 in this equation.

- 22-4** A half-cell produced by dissolving copper in water produces an electrode potential of +0.32 V. Calculate the amount of copper that must have been added to 1000 ml of water to produce this potential.

**Solution:** From the Nernst equation, with  $E_o = 0.34$  and the molecular weight of copper of 63.54 g/mol, we can find the number of grams “x” added to 1000 ml of the solution. For  $\text{Cu}^{2+}$ ,  $n = 2$ .

$$0.32 = 0.34 + (0.0592/2)\log(x / 63.54)$$

$$\log (x / 63.54) = (-0.02)(2) / 0.0592 = -0.67568$$

$$x / 63.54 = 0.211 \quad \text{or} \quad x = 13.4 \text{ g Cu per 1000 ml H}_2\text{O}$$

- 22-5** An electrode potential in a platinum half-cell is 1.10 V. Determine the concentration of  $\text{Pt}^{4+}$  ions in the electrolyte.

**Solution:** From the Nernst equation, with  $E_o = 1.20$  and the molecular weight of platinum of 195.09 g/mol, we can find the amount “x” of Pt added per 1000 ml of solution. For Pt,  $n = 4$ .

$$1.10 = 1.20 + (0.0592/4)\log(x / 195.09)$$

$$\log (x / 195.09) = -6.7568$$

$$x / 195.09 = 0.000000175$$

$$x = 0.00003415 \text{ g Pt per 1000 ml H}_2\text{O}$$

- 22-6** A current density of 0.05 A/cm<sup>2</sup> is applied to a 150-cm<sup>2</sup> cathode. What period of time is required to plate out a 1-mm-thick coating of silver onto the cathode?

**Solution:** The current in the cell is  $I = iA = (0.05 \text{ A/cm}^2)(150 \text{ cm}^2) = 7.5 \text{ A}$

The weight of silver that must be deposited to produce a 1 mm = 0.1 cm thick layer is:

$$w = (150 \text{ cm}^2)(0.1 \text{ cm})(10.49 \text{ g/cm}^3) = 157.35 \text{ g}$$

From the Faraday equation, with  $n = 1$  for silver:

$$157.35 \text{ g} = (7.5 \text{ A})(t)(107.868 \text{ g/mol}) / (1)(96,500 \text{ C})$$

$$t = 18,769 \text{ s} = 5.21 \text{ h}$$

- 22-7** We wish to produce 100 g of platinum per hour on a 1000 cm<sup>2</sup> cathode by electroplating. What plating current density is required? Determine the current required.

**Solution:** In the Faraday equation,  $n = 4$  for platinum, which has an atomic weight of 195.09 g/mol. The weight of platinum that must be deposited per second is  $100 \text{ g} / 3600 \text{ s/h} = 0.02778 \text{ g/s}$ .

$$0.02778 \text{ g/s} = (i)(1000 \text{ cm}^2)(195.09 \text{ g/mol}) / (4)(96,500 \text{ C})$$

$$i = 0.055 \text{ A/cm}^2$$

The current must then be:

$$I = iA = (0.055 \text{ A/cm}^2)(1000 \text{ cm}^2) = 55 \text{ A}$$

- 22-8** A 1-m-square steel plate is coated on both sides with a 0.005-cm-thick layer of zinc. A current density of 0.02 A/cm<sup>2</sup> is applied to the plate in an aqueous solution. Assuming that the zinc corrodes uniformly, determine the length of time required before the steel is exposed.

**Solution:** The surface area includes both sides of the plate:

$$A = (2 \text{ sides})(100 \text{ cm})(100 \text{ cm}) = 20,000 \text{ cm}^2$$

The weight of zinc that must be removed by corrosion is:

$$w = (20,000 \text{ cm}^2)(0.005 \text{ cm})(7.133 \text{ g/cm}^3) = 713.3 \text{ g}$$

From Faraday's equation, where  $n = 2$  for zinc:

$$713.3 \text{ g} = \frac{(0.02 \text{ A/cm}^2)(20,000 \text{ cm}^2)(t)(65.38 \text{ g/cm}^3)}{(2)(96,500 \text{ C})}$$

$$t = 5264 \text{ s} = 1.462 \text{ h}$$

- 22-9** A 2-in.-inside-diameter, 12-ft-long copper distribution pipe in a plumbing system is accidentally connected to the power system of a manufacturing plant, causing a current of 0.05 A to flow through the pipe. If the wall thickness of the pipe is 0.125 in., estimate the time required before the pipe begins to leak, assuming a uniform rate of corrosion.

**Solution:** If the pipe corroded uniformly, essentially all of the pipe would have to be consumed before leaking. The volume of material in the pipe, which has an inside diameter of 2 in. and an outside diameter of 2.25 in., is:

$$V = (\pi/4)[(2.25 \text{ in.})^2 - (2 \text{ in.})^2](12 \text{ ft})(12 \text{ in./ft}) = 120.17 \text{ in.}^3$$

The density of copper is 8.96 g/cm<sup>3</sup> = 0.323 lb/in.<sup>3</sup>. The weight of material to be corroded is:

$$w = V\rho = (120.17 \text{ in.}^3)(0.323 \text{ lb/in.}^3) = 38.81 \text{ lb} = 17,621 \text{ g}$$

From Faraday's law, with  $n = 2$  for copper:

$$17,621 \text{ g} = (0.05 \text{ A})(t)(63.54 \text{ g/mol}) / (2)(96,500 \text{ C})$$

$$t = 1.07 \times 10^9 \text{ s} = 34 \text{ years}$$

- 22-10** A steel surface 10 cm × 100 cm is coated with a 0.002-cm thick layer of chromium. After one year of exposure to an electrolytic cell, the chromium layer is completely removed. Calculate the current density required to accomplish this removal.

**Solution:** The volume and weight of chromium that must be removed are:

$$V = (10 \text{ cm})(100 \text{ cm})(0.002 \text{ cm}) = 2 \text{ cm}^3$$

$$w = (2 \text{ cm}^3)(7.19 \text{ g/cm}^3) = 14.38 \text{ g}$$

There are  $31.536 \times 10^6$  s per year. The surface area of the chromium is (10 cm)(100 cm) = 1000 cm<sup>2</sup>. From Faraday's law, with  $n = 3$  for chromium:

$$14.38 \text{ g} = \frac{(i)(1000 \text{ cm}^2)(31.536 \times 10^6 \text{ s})(51.996 \text{ g/mol})}{(3)(96,500 \text{ C})}$$

$$i = 2.54 \times 10^{-6} \text{ A/cm}^2$$

The current is  $(2.54 \times 10^{-6} \text{ A/cm}^2)(1000 \text{ cm}^2) = 2.54 \times 10^{-3} \text{ A}$   
 $A = 2.54 \text{ mA}$

- 22–11** A corrosion cell is composed of a 300 cm<sup>2</sup> copper sheet and a 20 cm<sup>2</sup> iron sheet, with a current density of 0.6 A/cm<sup>2</sup> applied to the copper. Which material is the anode? What is the rate of loss of metal from the anode per hour?

**Solution:** In the Cu-Fe cell, the iron is the anode.

$$\begin{aligned} I &= i_{\text{Cu}} A_{\text{Cu}} = i_{\text{Fe}} A_{\text{Fe}} \\ (0.6 \text{ A/cm}^2)(300 \text{ cm}^2) &= i_{\text{Fe}} (20 \text{ cm}^2) \\ i_{\text{Fe}} &= 9 \text{ A/cm}^2 \end{aligned}$$

The weight loss of iron per hour (3600 s) is:

$$\begin{aligned} w &= (9 \text{ A/cm}^2)(20 \text{ cm}^2)(3600 \text{ s})(55.847 \text{ g/mol}) / (2)(96,500 \text{ C}) \\ &= 187.5 \text{ g of iron lost per hour} \end{aligned}$$

- 22–12** A corrosion cell is composed of a 20 cm<sup>2</sup> copper sheet and a 400 cm<sup>2</sup> iron sheet, with a current density of 0.7 A/cm<sup>2</sup> applied to the copper. Which material is the anode? What is the rate of loss of metal from the anode per hour?

**Solution:** In the Cu-Fe cell, the iron is the anode.

$$\begin{aligned} I &= i_{\text{Cu}} A_{\text{Cu}} = i_{\text{Fe}} A_{\text{Fe}} \\ (0.7 \text{ A/cm}^2)(20 \text{ cm}^2) &= i_{\text{Fe}} (400 \text{ cm}^2) \\ i_{\text{Fe}} &= 0.035 \text{ A/cm}^2 \end{aligned}$$

The weight loss of iron per hour (3600 s) is:

$$\begin{aligned} w &= (0.035 \text{ A/cm}^2)(400 \text{ cm}^2)(3600 \text{ s})(55.847 \text{ g/mol}) / (2)(96,500 \text{ C}) \\ &= 14.58 \text{ g of iron lost per hour} \end{aligned}$$

Note that the rate of iron lost per hour when the anode area is large is much smaller than the rate of iron loss when the anode area is small (Problem 22–11).

- 22–13** Alclad is a laminar composite composed of two sheets of commercially pure aluminum (alloy 1100) sandwiched around a core of 2024 aluminum alloy. Discuss the corrosion resistance of the composite. Suppose that a portion of one of the 1100 layers was machined off, exposing a small patch of the 2024 alloy. How would this affect the corrosion resistance? Explain. Would there be a difference in behavior if the core material were 3003 aluminum? Explain.

**Solution:** The Alclad composed of 2024 and 1100 alloys should have good corrosion resistance under most circumstances. The 1100 alloy has good corrosion resistance, since it is nearly pure aluminum, when it completely covers the underlying 2024 alloy. Furthermore, if the 1100 layer is disturbed by machining, scratching, or other means, the 1100 alloy serves as the anode and protects the 2024 alloy. The surface area of the 1100 alloy is large, and even corrosion of the 1100 alloy will be slow.

When the 3003 alloy is coated with 1100 alloy, a disturbance of the surface is more critical. The 3003 alloy will serve as the anode; since the surface area of the 3003 anode is likely to be small compared to the surface area of the 1100 alloy, corrosion may occur rapidly.

- 22–14** The leaf springs for an automobile are formed from a high-carbon steel. For best corrosion resistance, should the springs be formed by hot working or cold working? Explain. Would corrosion still occur even if you use the most desirable forming process? Explain.

**Solution:** If we form the springs cold, residual stresses are likely to be introduced into the product, leading to a stress cell and corrosion of the spring. Forming the springs hot will reduce the level of any residual stresses introduced into the spring and minimize the stress cell.

However, the steel will contain ferrite and pearlite (forming a composition cell), not to mention grain boundaries, inclusions, and other potential sites for corrosion cells. Corrosion is still likely to occur even if the springs were produced by hot working.

- 22–15** Several types of metallic coatings are used to protect steel, including zinc, lead, tin, cadmium, aluminum, and nickel. In which of these cases will the coating provide protection even when the coating is locally disrupted? Explain.

**Solution:** Aluminum, zinc, and cadmium are anodic compared to iron; consequently these three elements should provide protection (serving as sacrificial anodes) to the iron even if the coating is disrupted.

Nickel, tin, and lead are cathodic compared to iron; when these coatings are disrupted, small anodic regions of iron are exposed and corrosion may occur rapidly.

- 22–16** An austenitic stainless steel corrodes in all of the heat-affected zone (HAZ) surrounding the fusion zone of a weld. Explain why corrosion occurs and discuss the type of welding process or procedure that might have been used. What might you do to prevent corrosion in this region?

**Solution:** Since the entire heat affected zone has corroded, the entire heat affected region must have been sensitized during the welding process. Sensitization means that chromium carbides have precipitated at the austenite grain boundaries during joining, reducing the chromium content in the austenite near the carbides. The low chromium content austenite serves as the anode and corrosion occurs.

Based on our observation of the corrosion, we can speculate that the austenitic stainless steel is not a low carbon steel (that is, the steel contains more than 0.03%C). The welding process was such that the heat affected zone experienced long exposure times and slow rates of cooling. Nearest the fusion zone, the steel was all austenite at the peak temperatures developed during welding; however the slow rate of cooling provided ample time for carbide precipitation as the region cooled between 870 and 425°C. A bit further from the fusion zone, the steel was exposed to the 870 to 425°C temperature range for a long time, permitting carbides to precipitate and sensitize the steel.

The long times and slow cooling rates may have been a result of the welding process—a low rate of heat input process, or inefficient process, will introduce the heat slowly, which in turn heats up the surrounding base metal which then acts as a poor heat sink. Preheating the steel prior to welding would also result in the same problems.

The stainless steel should be welded as rapidly as possible, using a high rate of heat input process, with no preheat of the steel prior to welding.

The steel should be a low carbon steel to assure that carbides do not precipitate even when the cooling rates are slow. If problems persist, a quench anneal heat treatment might be used to redissolve the carbides.

- 22–17** A steel nut is securely tightened onto a bolt in an industrial environment. After several months, the nut is found to contain numerous cracks, even though no externally applied load acts on the nut. Explain why cracking might have occurred.

**Solution:** When the nut is tightly secured onto the bolt, residual stresses are likely to be introduced into the assembly. The presence of numerous cracks suggests that stress corrosion cracking may have occurred as a result of these stresses.

- 22–18** The shaft for a propellor on a ship is carefully designed so that the applied stresses are well below the endurance limit for the material. Yet after several months, the shaft cracks and fails. Offer an explanation for why failure might have occurred under these conditions.

**Solution:** The propellor is under a cyclical load during operation, but it is also in a marine environment which may provide a relatively aggressive electrolyte. Failure, it is noted, requires some time to occur. Corrosion-fatigue sounds like a strong possibility in this case. Even though the stress is nominally below the endurance limit for the shaft, corrosion encouraged by the stress will lead to loss of material or development of pits in the shaft. This will increase the stress acting on the shaft and further encourage corrosion. The result is the eventual formation of fatigue cracks, encouraged by corrosion, which cause the shaft to fail.

- 22–19** An aircraft wing composed of carbon fiber-reinforced epoxy is connected to a titanium forging on the fuselage. Will the anode for a corrosion cell be the carbon fiber, the titanium, or the epoxy? Which will most likely be the cathode? Explain.

**Solution:** Titanium is expected to serve as the anode and corrode, while carbon is expected to be the cathode. Titanium is more anodic than carbon, or graphite. Both are electrical conductors, they are in physical contact with one another at the junction between the wing and the fuselage, and both can be exposed to the environment.

The epoxy should not participate in the electrochemical cell; epoxy is an electrical insulator.

- 22–20** The inside surface of a cast iron pipe is covered with tar, which provides a protective coating. Acetone in a chemical laboratory is drained through the pipe on a regular basis. Explain why, after several weeks, the pipe begins to corrode.

**Solution:** During use, the acetone serves as a solvent for the tar; the protective tar coating is eventually dissolved and the cast iron pipe is then exposed to any corrosive material that is drained through the pipe.

- 22–21** A cold-worked copper tube is soldered, using a lead-tin alloy, into a steel connector. What types of electrochemical cells might develop due to this connection? Which of the materials would you expect to serve as the anode and suffer the most extensive damage due to corrosion? Explain.

**Solution:** Several cells may develop. Composition cells include those between the solder and the steel, with the steel serving as the anode and the solder as the cathode. The steel then corrodes.

A composition cell may also develop between the copper and the solder. In this case, the solder will act as the anode.

Microcomposition cells may also develop. The steel contains ferrite and cementite; the ferrite will act as the anode. In addition, the lead-tin solder is a two-phase alloy containing nearly pure tin ( $\beta$ ) and a solid solution of tin in lead ( $\alpha$ ). Lead is most likely to serve as the anode with respect to tin.

A concentration cell is also possible, particularly in the crevice between the copper tube and the steel. The material adjacent to the crevice will act as the anode.

Finally, the copper tube is cold worked; the cold working may cause a stress cell to develop. This may be accentuated by the soldering process; during soldering, the copper tube at the soldered joint will heat, perhaps to a high enough temperature to allow stress relieving to occur. This again helps to provide the stress cell between the cold worked and stress relieved portions of the tube.

- 22–22** Pure tin is used to provide a solder connection for copper in many electrical uses. Which metal will most likely act as the anode?

**Solution:** From the galvanic series, we find that tin is anodic to copper; therefore the tin anode is expected to corrode while the copper cathode is protected.

- 22–23** Sheets of annealed nickel, cold-worked nickel, and recrystallized nickel are placed into an electrolyte. Which would be most likely to corrode? Which would be least likely to corrode? Explain.

**Solution:** The cold worked nickel sheet is expected to have the poorest corrosion resistance due to the residual stresses introduced during the cold working process.

The annealed nickel sheet should be most resistant to corrosion; the grain size is expected to be particularly large and no residual stresses are expected; consequently a stress cell is unlikely. In addition, the annealed sheet is expected to have the most uniform composition, that is, the least segregation, so a composition cell is also unlikely.

The recrystallized nickel sheet should have intermediate corrosion resistance; the residual stresses should have been eliminated as a result of the heat treatment but the grain size may be smaller than in the annealed sheet.

- 22–24** A pipeline carrying liquid fertilizer crosses a small creek. A large tree washes down the creek and is wedged against the steel pipe. After some time, a hole is produced in the pipe at the point where the tree touches the pipe, with the diameter of the hole larger on the outside of the pipe than on the inside of the pipe. The pipe then leaks fertilizer into the creek. Offer an explanation for why the pipe corroded.

**Solution:** One possibility for the corrosion is a concentration cell caused by microbial corrosion. The point of contact between the tree and the pipe produces a low oxygen environment and also a location at which various microbes may grow. As the microbes grow on the pipe, a low oxygen environment is further encouraged. A galvanic cell is produced, with the pipe beneath the fallen tree (and thus the microbes) serving as the anode and the remainder of the pipe acting as the cathode. Localized corrosion will then continue until a hole is corroded through the wall of the pipe.



- 22–25** Two sheets of a 1040 steel are joined together with an aluminum rivet (Figure 22–20). Discuss the possible corrosion cells that might be created as a result of this joining process. Recommend a joining process that might minimize some of these cells.

**Solution:** Composition cells: Aluminum may act as the anode in comparison to the steel, thus causing corrosion of the aluminum. In addition, ferrite may serve as the anode to cementite within the steel.

Stress cells: The aluminum rivet is deformed when the joint is produced, causing the most highly cold worked portion of the rivet to act as the anode. In addition, grain boundaries in both the steel and the aluminum may act as anodes for a stress cell.

Concentration cells: Crevice corrosion may occur between the two steel sheets and also between the aluminum rivet and the steel sheets.

A fusion welding process, in which a filler material having a composition similar to that of the 1040 steel, might be the best way to minimize most of these cells.

- 22–26** Figure 22–21 shows a cross-section through an epoxy-encapsulated integrated circuit, including a small microgap between the copper lead frame and the epoxy polymer. Suppose chloride ions from the manufacturing process penetrate the package. What types of corrosion cells might develop? What portions of the integrated circuit are most likely to corrode?

**Solution:** Composition cells can develop between gold and aluminum (with the aluminum serving as the anode and corroding); gold and copper (with the copper serving as the anode and corroding); and aluminum and silicon (with aluminum serving as the anode and corroding).

- 22–27** A current density of  $0.1 \text{ A/cm}^2$  is applied to the iron in an iron-zinc corrosion cell. Calculate the weight loss of zinc per hour (a) if the zinc has a surface area of  $10 \text{ cm}^2$  and the iron has a surface area of  $100 \text{ cm}^2$  and (b) if the zinc has a surface area of  $100 \text{ cm}^2$  and the iron has a surface area of  $10 \text{ cm}^2$ .

**Solution:** (a)  $I = i_{\text{Fe}} A_{\text{Fe}} = (0.1 \text{ A/cm}^2)(100 \text{ cm}^2) = 10 \text{ A}$   
 $w_{\text{Zn}} = (10 \text{ A})(3600 \text{ s})(65.38 \text{ g/mol}) / (2)(96,500 \text{ C})$   
 $= 12.2 \text{ g of Zn lost per hour}$   
 (b)  $I = i_{\text{Fe}} A_{\text{Fe}} = (0.1 \text{ A/cm}^2)(10 \text{ cm}^2) = 1 \text{ A}$   
 $w_{\text{Zn}} = (1 \text{ A})(3600 \text{ s})(65.38 \text{ g/mol}) / (2)(96,500 \text{ C})$   
 $= 1.22 \text{ g of Zn lost per hour}$

The loss of zinc is accelerated when the zinc anode area is small.

- 22–28** Determine the Pilling-Bedworth ratio for the following metals and predict the behavior of the oxide that forms on the surface. Is the oxide protective, does it flake off the metal, or is it permeable? (see Appendix A for the metal density)

**Solution:** The Pilling-Bedworth relation is

$$P-W = MW_{\text{oxide}} \rho_{\text{metal}} / n MW_{\text{metal}} \rho_{\text{oxide}}$$

From metal density data listed in Appendix A, the calculated P-W ratios are shown in the table below.

	Oxide density (g/cm <sup>3</sup> )	Metal density (g/cm <sup>3</sup> )	<i>n</i>	P-W ratio
Mg-MgO	3.6	1.738	1	0.80
Na-Na <sub>2</sub> O	2.27	0.967	2	0.57
Ti-TiO <sub>2</sub>	5.1	4.507	1	1.47
Fe-Fe <sub>2</sub> O <sub>3</sub>	5.3	7.87	2	2.12
Ce-Ce <sub>2</sub> O <sub>3</sub>	6.86	6.6893	2	1.14
Nb-Nb <sub>2</sub> O <sub>3</sub>	4.47	8.57	2	2.74
W-WO <sub>3</sub>	7.3	19.254	1	3.33

The oxides that form in magnesium and sodium are expected to be non-adherent, or tend to flake off, since the oxide volume is substantially larger than the metal volume. A P-W ratio of less than 1 indicates this condition.

The oxides that form on iron, niobium, and tungsten are expected to be adherent but permeable. A P-W ratio of more than 2 indicates that the oxide volume is much less than that of the metal volume.

The oxides that form on titanium and cesium are expected to be protective; a P-W ratio of 1 to 2 indicates this condition.

**22–29** Oxidation of most ceramics is not considered to be a problem. Explain.

**Solution:** Most ceramics are already oxides—thus materials such as MgO and Al<sub>2</sub>O<sub>3</sub> are expected to be inert in an oxidizing atmosphere. Non-oxide ceramics, however, may sometimes be subjected to oxidation problems.

**22–30** A sheet of copper is exposed to oxygen at 1000°C. After 100 h, 0.246 g of copper are lost per cm<sup>2</sup> of surface area; after 250 h, 0.388 g/cm<sup>2</sup> are lost, and after 500 h, 0.550 g/cm<sup>2</sup> are lost. Determine whether oxidation is parabolic, linear, or logarithmic, then determine the time required for a 0.75 cm sheet of copper to be completely oxidized. The sheet of copper is oxidized from both sides.

**Solution:** Let's assume that the rate is parabolic: We can determine the constant "k" in the oxidation equation  $y = \sqrt{kt}$  for each time, first converting the rate in g/cm<sup>2</sup> to thickness *y* in cm:

$$y_1 = V/A = (0.246 \text{ g} / 8.93 \text{ g/cm}^3) / 1 \text{ cm}^2 = 0.0275 \text{ cm}$$

$$y_2 = V/A = (0.388 \text{ g} / 8.93 \text{ g/cm}^3) / 1 \text{ cm}^2 = 0.0434 \text{ cm}$$

$$y_3 = V/A = (0.549 \text{ g} / 8.93 \text{ g/cm}^3) / 1 \text{ cm}^2 = 0.0615 \text{ cm}$$

If oxidation is parabolic, the value for *k* should be the same for each time:

$$0.0275 \text{ cm} = \sqrt{k}(100 \text{ h}) \quad \text{or} \quad k = 7.56 \times 10^6 \text{ cm}^2/\text{h}$$

$$0.0434 \text{ cm} = \sqrt{k}(250 \text{ h}) \quad \text{or} \quad k = 7.53 \times 10^6 \text{ cm}^2/\text{h}$$

$$0.0615 \text{ cm} = \sqrt{k}(500 \text{ h}) \quad \text{or} \quad k = 7.56 \times 10^6 \text{ cm}^2/\text{h}$$

Therefore the rate of oxidation must be parabolic.

To completely oxidize the copper, assuming that the rate of oxidation is the same from both sides of the sheet, we need to determine the time required to oxidize  $0.75 \text{ cm} / 2 \text{ sides} = 0.375 \text{ cm}$ :

$$y = 0.375 \text{ cm} = \sqrt{(7.56 \times 10^6 \text{ cm}^2 / \text{h})(t)} \quad \text{or} \quad t = 18,601 \text{ h}$$

- 22–31** At  $800^\circ\text{C}$ , iron oxidizes at a rate of  $0.014 \text{ g/cm}^2$  per hour; at  $1000^\circ\text{C}$ , iron oxidizes at a rate of  $0.0656 \text{ g/cm}^2$  per hour. Assuming a parabolic oxidation rate, determine the maximum temperature at which iron can be held if the oxidation rate is to be less than  $0.005 \text{ g/cm}^2$  per hour.

**Solution:** The rate is given by an Arrhenius equation,  $\text{rate} = A \exp(-Q/RT)$ . We can find the constants  $A$  and  $Q$  from the data provided.

$$0.014 \text{ g/cm}^2\text{-h} = A \exp(-Q/(1.987 \text{ cal/mol}\cdot\text{K})(800 + 273\text{K}))$$

$$0.0656 \text{ g/cm}^2\text{-h} = A \exp(-Q/(1.987 \text{ cal/mol}\cdot\text{K})(1000 + 273\text{K}))$$

Taking logarithms of both sides:

$$-4.2687 = \ln A - 0.0004690Q$$

$$-2.7242 = \ln A - 0.0003953Q$$

$$1.5445 = 0.0000737 Q \quad \text{or} \quad Q = 20,957 \text{ cal/mol}$$

$$-4.2687 = \ln A - (0.000469)(20,957)$$

$$\ln A = 5.56$$

$$A = 260$$

To keep the oxidation rate below  $0.005 \text{ g/cm}^2\text{-h}$ , the maximum temperature is:

$$\text{rate} = 260 \exp(-20,957/RT) = 0.005$$

$$\ln(0.005/260) = -20,957 / (1.987)(T)$$

$$\ln(0.00001923) = -10.859 = -10,547/T$$

$$T = 971 \text{ K} = 698^\circ\text{C}$$