Chapter 4
Inverse Kinematics

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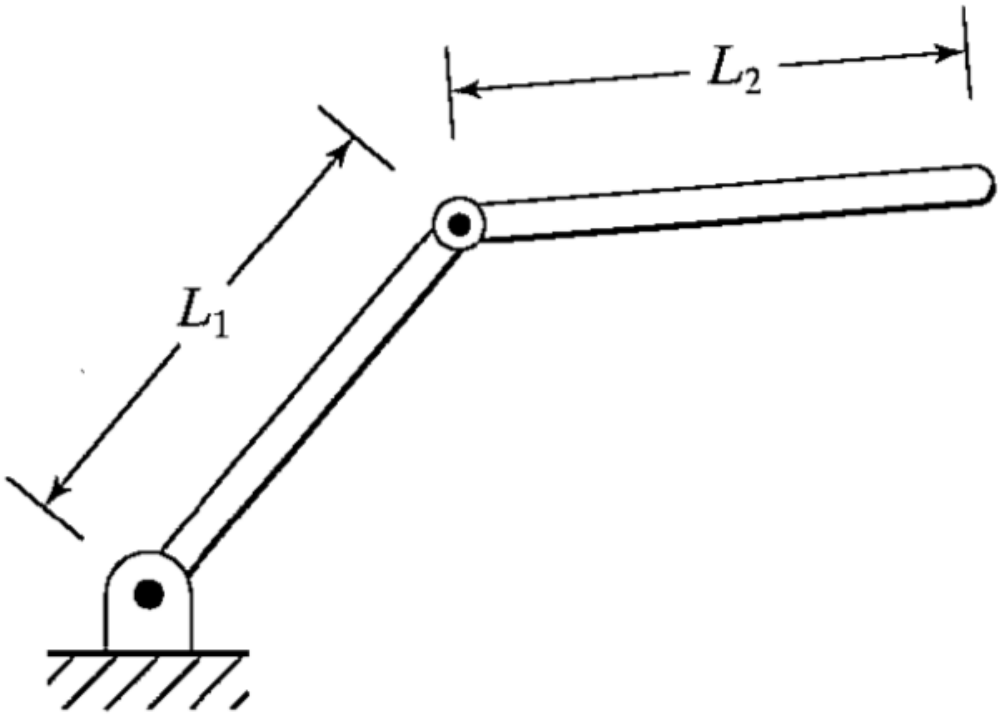
Inverse Kinematics:

- Main Goal: Compute the manipulator's joint variables required to place the end-effector in a specified position and orientation.
- This problem includes equations that are non-linear, transcendental and can be quite difficult to solve.
- Workspace: the volume of space that the end-effector of the manipulator can reach.
- If the desired position and orientation of the wrist frame is in the workspace, then at least one solution exists.

More on the Workspace and Existence of Solutions:

- **Dexterous Workspace**: The volume of space that the end-effector can reach with all orientations.
- Other points form the **Reachable Workspace**.



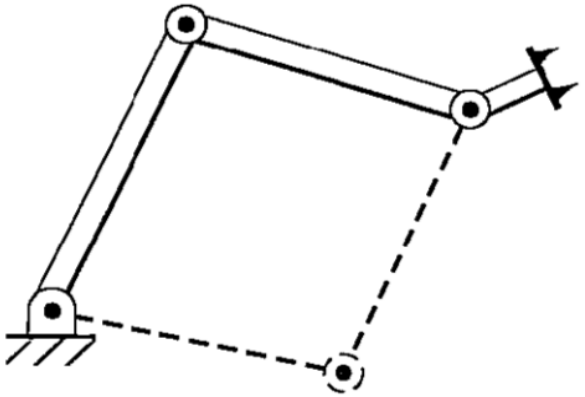
- If $L_1 = L_2$ then the reachable workspace consists of a disk of radius 2 L_1 and the dexterous workspace includes only the origin.

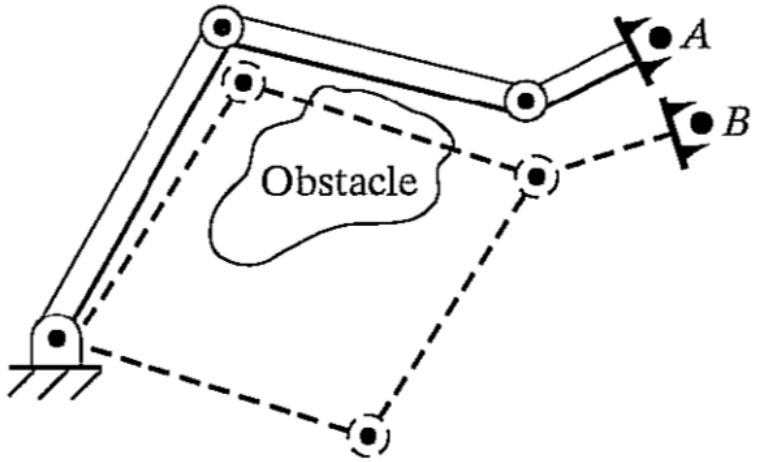
- If $L_1 \neq L_2$ then the reachable workspace consists of a disk that has an outer radius of $L_1 + L_2$ and an inner radius of $L_1 - L_2$, and there is no dexterous workspace.

- Usually joints cannot rotate a full 360^o which makes the workspace even smaller.

Multiple Solutions:

- If the manipulator has good link lengths and large joint ranges, then the same position and orientation can be reached with 2 sets of joint angles.
- The system has to be able to choose one.
- Usually: choose **shortest obstacle free solution.**
- In general there can be up to eight solutions for a single goal.
- Because of joint angle limits some of the eight solutions could be inaccessible.

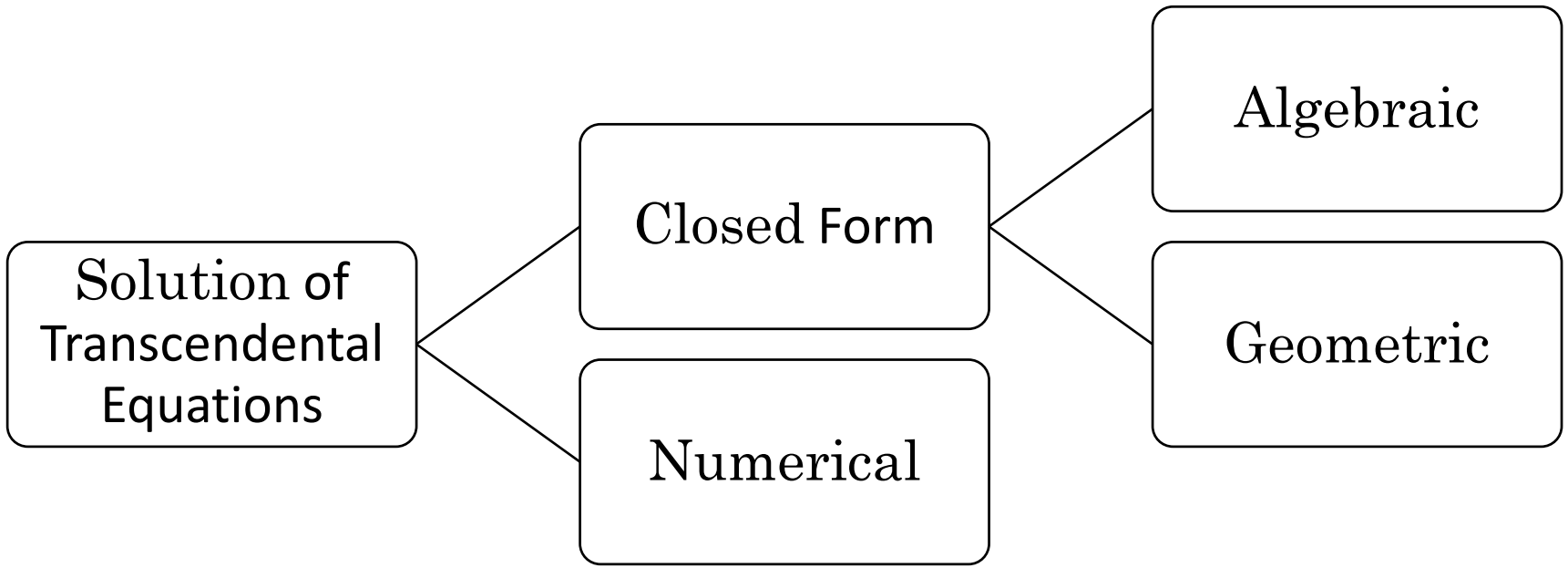




Notes on Space, Subspace, and Workspace

- If a manipulator has less than 6 DOF, then it can only access a portion of the space (subspace)
- Depending on the robot geometry and joint angle limits, it can only access a portion of the subspace (workspace)
- Example: The subspace of the 2 link planar robot is the xy-plane, the workspace depends on the link lengths and joint angle limits.

Methods of Solution:



Spherical Wrist:

- 3 joint axes intersecting at the same point.

- Position of these origins with respect to the base is the same.
- Orientations are different.

- Pieper (PhD Thesis) – 1968
  Stanford University

Proved that if a manipulator has 3 intersecting axes → there is an analytical solution to the inverse kinematic problem (no need for numerical solutions)

- Almost every manipulator with six DOF built today has 3 intersecting axes (Puma 560, KUKA 1000 …)