Dynamic Model

Monday, December 18, 2017 4:27 PM

The general form for the dynamic equations of the robot is given by

$$\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta)$$

where:

(n is the # of joints)

τ is the torque vector $(n \times 1)$
M is the mass / inertia matrix $(n \times n)$
V is the centrifugal / Coriolis force vector $(n \times 1)$
G is the gravity vector

$\theta, \dot{\theta}, \ddot{\theta} \rightarrow$ joint positions / velocities / accelerations.

Procedure

1) Find the # of joints (n) then determine the dimensions of all vectors & matrices

$\theta \rightarrow n \times 1$
$\dot{\theta} \rightarrow n \times 1$
$\ddot{\theta} \rightarrow n \times 1$

$\dot{\theta}\dot{\theta} \rightarrow n \times \frac{n(n-1)}{2}$
$\dot{\theta}^2 \rightarrow n \times 1$

$V(\theta, \dot{\theta}) \rightarrow n \times 1$
$M(\theta) \rightarrow n \times n$
$G(\theta) \rightarrow n \times 1$

$C(\theta) \rightarrow n \times n$
$B(\theta) \rightarrow n \times \frac{n(n-1)}{2}$

2) Calculate the inertia tensor of each link about its center of gravity.

3) Calculate the vectors ${}^0P_{c_i}$ for $i = 1 \ldots n$

${}^0P_{c_i}$: vector pointing from $\{0\}$ to the center of mass of link i.

4) Compute J_{v_i} , J_{ω_i} for all joints

$$J_{v_i} = \begin{bmatrix} \frac{\partial P_{c_i}}{\partial \theta_1} & \frac{\partial P_{c_i}}{\partial \theta_2} & \cdots & \frac{\partial P_{c_i}}{\partial \theta_i} & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$J_{\omega_i} = \begin{bmatrix} {}^0z_1 & {}^0z_2 & \cdots & {}^0z_i & 0 & 0 & \cdots & 0 \end{bmatrix}$$

5) Calculate the mass matrix using

$$M(\theta) = \sum_{i=1}^{n} \left(m_i J_{v_i}^T J_{v_i} + J_{\omega_i}^T I_{c_i} J_{\omega_i} \right)$$

6) Calculate the gravity vector using

$$G(\theta) = \begin{bmatrix} J_{v_1}^T & J_{v_2}^T & \cdots & J_{v_n}^T \end{bmatrix} \begin{bmatrix} m_1 \vec{g} \\ m_2 \vec{g} \\ \vdots \\ m_n \vec{g} \end{bmatrix}$$

7) Calculate the Coriolis force vector

$$V = \begin{bmatrix} b_{1,11} & b_{1,22} & \cdots & b_{1,nn} \\ b_{2,11} & b_{2,22} & \cdots & b_{2,nn} \\ \vdots & & & \\ b_{n,11} & \cdots & \cdots & b_{n,nn} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \\ \vdots \\ \dot{\theta}_n^2 \end{bmatrix} + \begin{bmatrix} 2b_{1,12} & 2b_{1,13} & \cdots & 2b_{1,(n-1)n} \\ 2b_{2,12} & 2b_{2,13} & & 2b_{2,(n-1)n} \\ \vdots & \vdots & & \vdots \\ 2b_{n,12} & 2b_{n,13} & & 2b_{n,(n-1)n} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_3 \\ \vdots \\ \dot{\theta}_{n-1} \dot{\theta}_n \end{bmatrix}$$

all combinations

$$b_{i,jk} = \frac{1}{2} \left(\frac{\partial m_{ij}}{\partial \theta_k} + \frac{\partial m_{ik}}{\partial \theta_j} - \frac{\partial m_{jk}}{\partial \theta_i} \right)$$

8) Write τ as $\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$ then substitute all values in the equation

<u>Example</u> : For the 2R planar Robot discussed before

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$n = 2$, Inertia tensors $= 0$

$$ {}^{0}P_{c_1} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} , \quad {}^{0}P_{c_2} = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}P_{c_2} = \begin{bmatrix} l_2 c_{1+2} + l_1 c_1 \\ l_2 s_{1+2} + l_1 s_1 \\ 0 \end{bmatrix} $$

$$ J_{v_1} = \begin{bmatrix} \dfrac{\partial P_{c_1}}{\partial \theta_1} & 0 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 & 0 \\ l_1 c_1 & 0 \\ 0 & 0 \end{bmatrix} $$

$$ J_{v_2} = \begin{bmatrix} \dfrac{\partial P_{c_2}}{\partial \theta_1} & \dfrac{\partial P_{c_2}}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -l_2 s_{1+2} - l_1 s_1 & -l_2 s_{1+2} \\ l_2 c_{1+2} + l_1 c_1 & l_2 c_{1+2} \\ 0 & 0 \end{bmatrix} $$

$$ J_{\omega_1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} , \quad J_{\omega_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} $$

$$ M = m_1 J_{v_1}^T J_{v_1} + m_2 J_{v_2}^T J_{v_2} $$ (See matlab)

$$ G = \begin{bmatrix} J_{v_1}^T & J_{v_2}^T \end{bmatrix} \begin{bmatrix} 0 \\ m_1 g \\ 0 \\ 0 \\ m_2 g \\ 0 \end{bmatrix} $$ (See matlab)

$$ C(\theta) = \begin{bmatrix} b_{1,11} & b_{1,22} \\ b_{2,11} & b_{2,22} \end{bmatrix} $$

$$ b_{1,11} = \frac{1}{2}\left(\frac{\partial m_{11}}{\partial \theta_1} + \frac{\partial m_{11}}{\partial \theta_1} - \frac{\partial m_{11}}{\partial \theta_1} \right) $$

$$ b_{1,22} = \frac{1}{2}\left(\frac{\partial m_{12}}{\partial \theta_1} + \frac{\partial m_{12}}{\partial \theta_2} - \frac{\partial m_{22}}{\partial \theta_1} \right) $$

See matlab.

$$\ldots 2\left(\ \partial\theta_1 \qquad \partial\theta_2 \qquad \partial\theta_1\ \right) \qquad \text{mat}\ldots$$

$$b_{2,11} = \frac{1}{2}\left(\frac{\partial m_{21}}{\partial\theta_1} + \frac{\partial m_{21}}{\partial\theta_1} - \frac{\partial m_{11}}{\partial\theta_2}\right)$$

$$b_{2,22} = \frac{1}{2}\left(\frac{\partial m_{22}}{\partial\theta_2} + \frac{\partial m_{22}}{\partial\theta_2} - \frac{\partial m_{22}}{\partial\theta_2}\right)$$

$$B(\theta) = \begin{bmatrix} 2b_{1,12} \\ 2b_{2,12} \end{bmatrix} \dot{\theta}_1\dot{\theta}_2$$

$$b_{1,12} = \frac{1}{2}\left(\frac{\partial m_{11}}{\partial\theta_2} + \frac{\partial m_{12}}{\partial\theta_1} - \frac{\partial m_{12}}{\partial\theta_1}\right)$$

$$b_{2,12} = \frac{1}{2}\left(\frac{\partial m_{21}}{\partial\theta_2} + \frac{\partial m_{22}}{\partial\theta_1} - \frac{\partial m_{12}}{\partial\theta_2}\right)$$

See matlab

$$V = C\begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} + B\,\dot{\theta}_1\dot{\theta}_2$$

See matlab.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = M(\theta)\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + V(\theta,\dot{\theta}) + G(\theta).$$