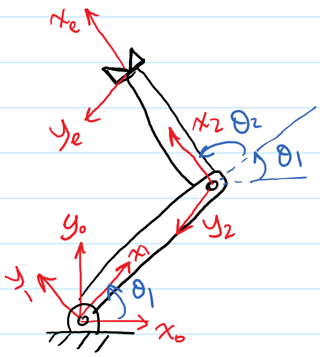
Jacobian Matrix

Sunday, November 12, 2017 5:36 PM

This is an alternative method for calculating the Jacobian Matrix:

$$ {}^0J = \left[\begin{array}{c:c:c:c} \underbrace{{}^0Z_1 \times ({}^0P_{ee} - {}^0P_1)}_{3\times1} & {}^0Z_2 \times ({}^0P_{ee} - {}^0P_2) & \cdots & {}^0Z_6 \times ({}^0P_{ee} - {}^0P_6) \\ \underbrace{{}^0Z_1}_{3\times1} \quad {\scriptstyle 6\times1} & {}^0Z_2 & & {}^0Z_6 \end{array}\right]_{6\times6} $$

<u>Example:</u>



To find the Jacobian

$$ {}^0Z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad {}^0Z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ {}^0P_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^0P_2 = \begin{bmatrix} l_1 C_1 \\ l_1 S_1 \\ 0 \end{bmatrix} $$

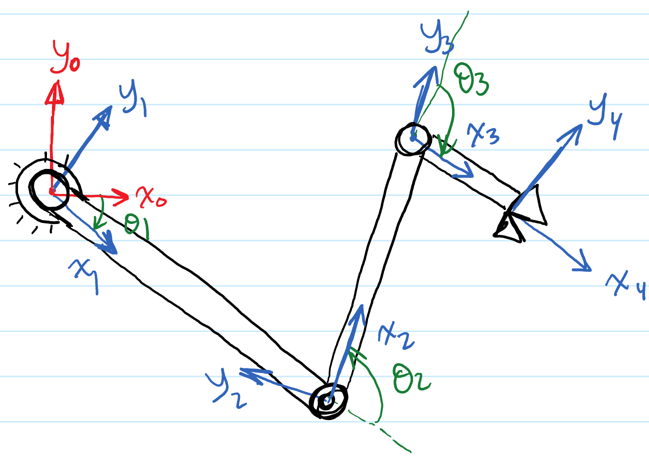
$$ {}^0P_{ee} = {}^0_2T \; {}^2P_{ee} $$

$$ = \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & l_1 C_1 \\ S_{1+2} & C_{1+2} & 0 & l_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_2 C_{1+2} + l_1 C_1 \\ l_2 S_{1+2} + l_1 S_1 \\ 0 \\ 1 \end{bmatrix} $$

$$ {}^0J = \left[\begin{array}{c:c} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 C_{1+2} + l_1 C_1 \\ l_2 S_{1+2} + l_1 S_1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 C_{1+2} + l_1 C_1 - l_1 C_1 \\ l_2 S_{1+2} + l_1 S_1 - l_1 S_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{array}\right] $$

$$
{}^{0}J = \begin{bmatrix} -l_2 S_{1+2} + l_1 S_1 & -l_2 S_{1+2} \\ l_2 C_{1+2} + l_1 C_1 & l_2 C_{1+2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}
$$

Example



DH - Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	10	0	θ_2
3	0	6	0	θ_3
4	0	2	0	0

$$
{}^{0}_{1}T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^{2}_{3}T = \begin{bmatrix} C_3 & -S_3 & 0 & 6 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} C_2 & -S_2 & 0 & 10 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^{3}_{4}T = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$ {}^{0}_{2}T = {}^{0}_{1}T \, {}^{1}_{2}T = \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & 10 \\ S_{1+2} & C_{1+2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{3}T = {}^{0}_{2}T \, {}^{2}_{3}T = \begin{bmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & 6C_{1+2} + 10 \\ S_{1+2+3} & C_{1+2+3} & 0 & 6S_{1+2} + 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{4}T = {}^{0}_{3}T \, {}^{3}_{4}T = \begin{bmatrix} C_{1+2+3} & -S_{1+2+3} & 0 & 2C_{1+2+3} + 6C_{1+2} + 10 \\ S_{1+2+3} & C_{1+2+3} & 0 & 2S_{1+2+3} + 6S_{1+2} + 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

To find the Jacobian

$$ {}^{0}Z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad {}^{0}Z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad {}^{0}Z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ {}^{0}P_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^{0}_{2}P = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \qquad {}^{0}P_3 = \begin{bmatrix} 6C_{1+2} + 10 \\ 6S_{1+2} + 10 \\ 0 \end{bmatrix} $$

$$ {}^{0}P_e = \begin{bmatrix} 2C_{1+2+3} + 6C_{1+2} + 10 \\ 2S_{1+2+3} + 6S_{1+2} + 10 \\ 0 \end{bmatrix} $$

$$ {}^{0}J = \left[\begin{array}{c:c:c} -2S_{1+2+3} - 6S_{1+2} - 10 & -2S_{1+2+3} - 6S_{1+2} & -2S_{1+2+3} \\ 2C_{1+2+3} + 6C_{1+2} + 10 & 2C_{1+2+3} + 6S_{1+2} & 2C_{1+2+3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{array}\right] $$

Try finding the Jacobian using the Recursive method for this robot

See attached Matlab file for finding it using Robotic Toolbox.