# Solution – Assignment 1:

## Problem 1:

$$R = \mathrm{rot}(\hat{x}, 45°)\,\mathrm{rot}(\hat{y}, 30°)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & .707 & -.707 \\ 0 & .707 & .707 \end{bmatrix} \begin{bmatrix} .866 & 0 & .5 \\ 0 & 1 & 0 \\ -.5 & 0 & .866 \end{bmatrix}$$

$$= \begin{bmatrix} .866 & 0 & .5 \\ .353 & .707 & -.612 \\ -.353 & .707 & .612 \end{bmatrix}$$

## Problem 2:

$$R = \mathrm{rot}(\hat{z}, 30°)\,\mathrm{rot}(\hat{x}, 45)$$

$$= \begin{bmatrix} .866 & -.353 & .353 \\ .50 & .612 & -.612 \\ 0 & .707 & .707 \end{bmatrix}$$

## Problem 3:

$${}^{A}_{B}T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{B}_{C}T = \begin{bmatrix} -0.866 & -0.5 & 0 & 3 \\ 0 & 0 & +1 & 0 \\ -0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}_{C}T = \begin{bmatrix} 0.866 & 0.5 & 0 & -3 \\ 0.5 & -0.866 & 0 & 4 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 4:

(a)

$$
{}_0B_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}_1B_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 50 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_2B_3 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 100\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}_3B_4 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 100 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}_4B_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}_5B_6 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 100 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(b)

$$
{}_0B_6 = ({}_0B_1)({}_1B_2)\dots({}_5B_6) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 150\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 250\frac{\sqrt{2}}{2} \\ 0 & 0 & -1 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.70711 & -0.70711 & 0 & 106.07 \\ -0.70711 & -0.70711 & 0 & 176.78 \\ 0 & 0 & -1.0 & 200.0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}
$$

( ${}_0B_6$ directions are easy to see, the distance is not obvious. )

(c) (i)

Denote the new frame 6 after a rotation of $\theta$ about $x_6$ as frame $6'$. The two frames are related by,

$$
{}_6B_{6'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ so } {}_0B_{6'} = ({}_0B_6)({}_6B_{6'}) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\cos\theta & \frac{\sqrt{2}}{2}\sin\theta & 150\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\cos\theta & \frac{\sqrt{2}}{2}\sin\theta & 250\frac{\sqrt{2}}{2} \\ 0 & -\sin\theta & -\cos\theta & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.70711 & -0.70711\cos\theta & 0.70711\sin\theta & 106.07 \\ -0.70711 & -0.70711\cos\theta & 0.70711\sin\theta & 176.78 \\ 0 & -1.0\sin\theta & -1.0\cos\theta & 200.0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}
$$

(c) (ii)

In frame 6′ the position of $c$ is ${}_{6'}P(c) = \begin{bmatrix} {}_{6'}p(o_{6'}c) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \\ 50 \\ 1 \end{bmatrix}$

$$ {}_0P(c) = ({}_{06'}B)\ {}_{6'}P(c) = ({}_0B_{6'})\ {}_{6'}P(c) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\cos\theta & \frac{\sqrt{2}}{2}\sin\theta & 150\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\cos\theta & \frac{\sqrt{2}}{2}\sin\theta & 250\frac{\sqrt{2}}{2} \\ 0 & -\sin\theta & -\cos\theta & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -20 \\ 50 \\ 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 10\sqrt{2}\cos\theta + 25\sqrt{2}\sin\theta + 75\sqrt{2} \\ 10\sqrt{2}\cos\theta + 25\sqrt{2}\sin\theta + 125\sqrt{2} \\ 20\sin\theta - 50\cos\theta + 200 \\ 1 \end{bmatrix} = \begin{bmatrix} {}_0p(o_0c) \\ 1 \end{bmatrix} $$

$$ \Rightarrow {}_0p(o_0c) = \begin{bmatrix} 10\sqrt{2}\cos\theta + 25\sqrt{2}\sin\theta + 75\sqrt{2} \\ 10\sqrt{2}\cos\theta + 25\sqrt{2}\sin\theta + 125\sqrt{2} \\ 20\sin\theta - 50\cos\theta + 200 \end{bmatrix} = \begin{bmatrix} 14.142\cos\theta + 35.355\sin\theta + 106.07 \\ 14.142\cos\theta + 35.355\sin\theta + 176.78 \\ 20.0\sin\theta - 50.0\cos\theta + 200.0 \end{bmatrix} $$

## Problem 5:

$$ {}_0B_1 = \begin{bmatrix} c_1 & 0 & s_1 & a \\ 0 & 1 & 0 & 0 \\ -s_1 & 0 & c_1 & -b \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}_1B_2 = \begin{bmatrix} c_2 & -s_2 & 0 & d \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_2B_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & e \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}_3B_4 = \begin{bmatrix} 1 & 0 & 0 & f \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Then by multiplying, ${}_0B_4 = ({}_0B_1)({}_1B_2)({}_2B_3)({}_3B_4)$

$$ {}_0B_4 = \begin{bmatrix} c_1c_{2+3} & s_1 - c_1s_{2+3} & s_1 & a + dc_1 - ec_1s_2 + fc_1c_{2+3} \\ s_{2+3} & c_{2+3} & 0 & ec_2 + fs_{2+3} \\ -s_1c_{2+3} & c_1 + s_1s_{2+3} & c_1 & es_1s_2 - ds_1 - b - fs_1c_{2+3} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$