

# ملخص محاضرات.. آلات كهربائية

لجنة  
الميكانيك  
Polytechnic

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## 1 Fundamentals of magnetic fields

### (1) The magnetic field

Mag. fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers.

4 basic principles describe how Mag. fields are used in these devices:—

(1) A current-carrying wire produces a mag. f. in the area around it.

(2) A time changing magnetic field induces a voltage in a coil of wire if it passes through that coil (Transformer action).

(3) A current-carrying wire in the presence of a magnetic field has a force induced on it.

(4) a moving wire in the presence of a magnetic field has a force induced on it. (motor action)

### Production of a magnetic field:—

The basic law giving the production of magnetic field by a current is Ampere's law.

$$\oint H \cdot dL = I_{net}$$

where:  $H$  = Mag. field intensity ~~is~~ produced by the current  $I_{net}$  (Amp-turn/meter)

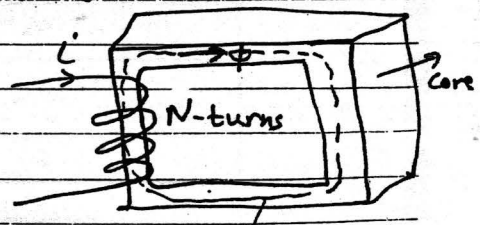
$dL$  = differential element of length along of the path of integration.

Ampere's law will:—

$$Hl_c = Ni$$

magnitude of mag. field intensity

$$H = Ni/l_c$$



$l_c$  - path length

core, ferromagnetic material.



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The relationship between the mag. field intensity  $H$  & the resulting magnetic flux density  $B$  produced within a material is given by:

$$B = \mu H.$$

where  $H$  - mag. field intensity: (Amp-turn/meter).

$\mu$  - mag. permeability of material: (henry/meter)

$B$  - resulting mag. flux density produced. ~~unit~~  
[weber/m<sup>2</sup> = Tesla T]

The permeability of free space is called  $\mu_0$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m.}$$

relative permeability - the permeability of any other material compared to the permeability of free space.

$$\mu_r = \mu / \mu_0.$$

The magnitude of the flux density is given by:-

$$B = \mu H = \frac{\mu N I}{l_c}$$

The Total flux in a given area is given by:-

$$\phi = \int_A B \, dA.$$

where  $dA$  - The differential unit of Area.

& when the flux density vector is perpendicular to a plane of area  $A$  & when the flux density is constant throughout the area then the equation reduces to:-

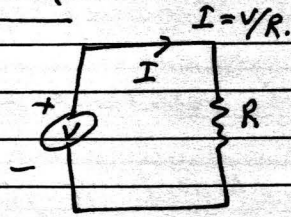
$$\phi = B \cdot A. \quad \text{where } A - \text{The cross sectional area of core.}$$
$$\phi = BA = \mu H \cdot A = \frac{\mu \cdot N \cdot I \cdot A}{l_c}$$



## Ohm's Law & Magnetic circuits :-

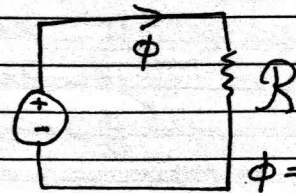
The relationship between these quantities (V, I, R) is given by Ohm's Law

$$V = IR.$$



electric circuit.

Magnetomotive force (mmf)  $f = Ni$



of the mag. circuit is equal to the

effective current flow applied to the core or.

$f = \mathcal{F} = Ni$  where  $\mathcal{F}$  - magnetomotive force  
Amp turns

the  $\Phi$  exits from the positive end of mmf.

by right-hand rule :- the fingers of the right hand curl in the direction of the current flow in a coil of wire then the thumb will point in the direction of the positive mmf.

\* in the electric circuit The  $V_{applied}$  causes a current (I) to flow, in a magnetic circuit the applied mmf causes flux ( $\Phi$ ) to be produced.

\* The relationship between mmf &  $\Phi$  is :-

$$\mathcal{F} = \Phi R, \quad \mathcal{F} - \text{mmf.}$$

$\Phi$  - Flux of circuit.

R - reluctance of circuit.  
Amp - turns / Weber.



What is the reluctance of the core in the first figure?

$$\phi = BA = \frac{\mu N i A}{l_c}$$

$$= Ni \left( \frac{\mu A}{l_c} \right) = f \left( \frac{\mu A}{l_c} \right)$$

$$R = l_c / \mu A$$

→ compare with

$$\rightarrow \mathcal{F} = \phi R$$

Equivalent Reluctance of a number of reluctances in series :-

$$R_{eq} = R_1 + R_2 + \dots$$

in parallel :-

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

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FARADAY'S LAW - Induced voltage from a time-changing magnetic field :-

Faraday's law is the basis of transformer operation. It states that if a flux passes through a turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the rate of change in the flux with respect to time.

$$E_{ind} = - d\phi/dt = -N d\phi/dt$$

where :-  $\phi$  - flux passing through coil

$N$  - Number of turns of wire in coil.

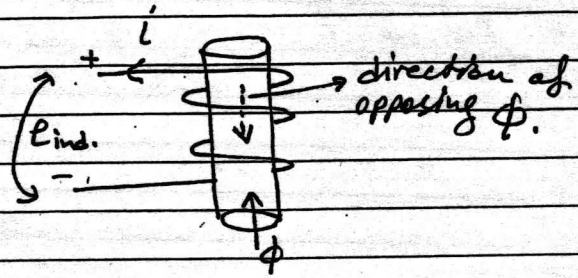
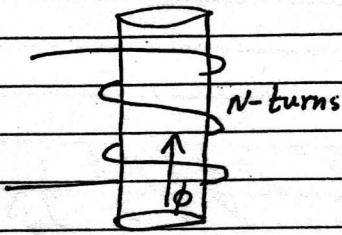
$E_{ind}$  - induced voltage in the coil.

- sign (-) → is an expression of Lenz's law, Lenz's law states that the direction of the voltage buildup in the coil is such that if the coil ends were short circuited,



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it would produce current that would cause a flux opposing the original flux change.



$$\mathcal{E}_{ind} = \frac{d(\Phi_i)}{dt} \Rightarrow \mathcal{E}_{ind} = \sum_{i=1}^N \mathcal{E}_i$$

$$= \sum_{i=1}^n \frac{d(\Phi_i)}{dt} = \frac{d}{dt} \left( \sum_{i=1}^n \Phi_i \right)$$

$$\Rightarrow \mathcal{E}_{ind} = \frac{d\lambda}{dt} \quad \text{where } \lambda - \text{the flux linkage}$$

$$\lambda = \sum_{i=1}^N \Phi_i$$

## Production of induced force on a wire.

A 2<sup>nd</sup> major effect of a magnetic field on its surroundings is that it induces a force on a current-carrying wire<sup>s</sup> within a field.

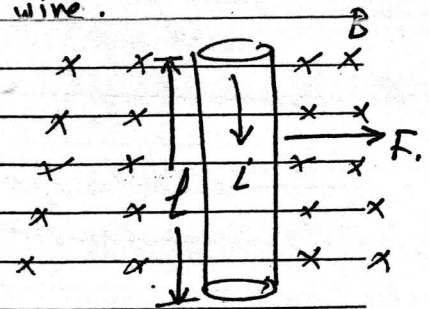
$$F = i (l \times B)$$

where :-  $i$  - Magnitude of current in wire.

$l$  - length of wire

$B$  - mag. flux density.

$$F = i \cdot l \cdot B \cdot \sin \theta$$



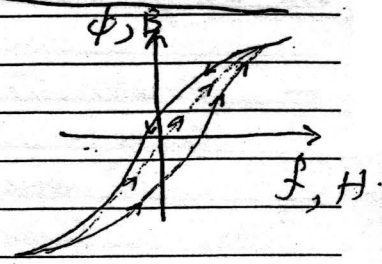
$\theta$  - is the angle between the wire & the flux density vector.



Induced Voltage on a conductor moving in a magnetic field :-

$$E_{ind} = (v \cdot B) \cdot l$$

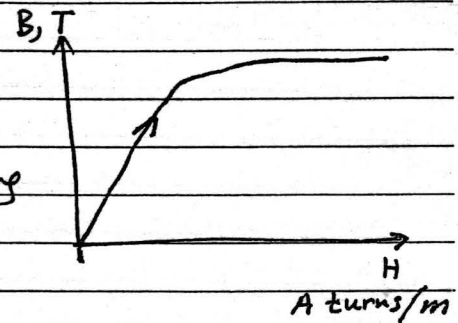
- v - Velocity of the wire.
- B - magnetic flux density vector.
- l - length of conductor.



Hysteresis loop.

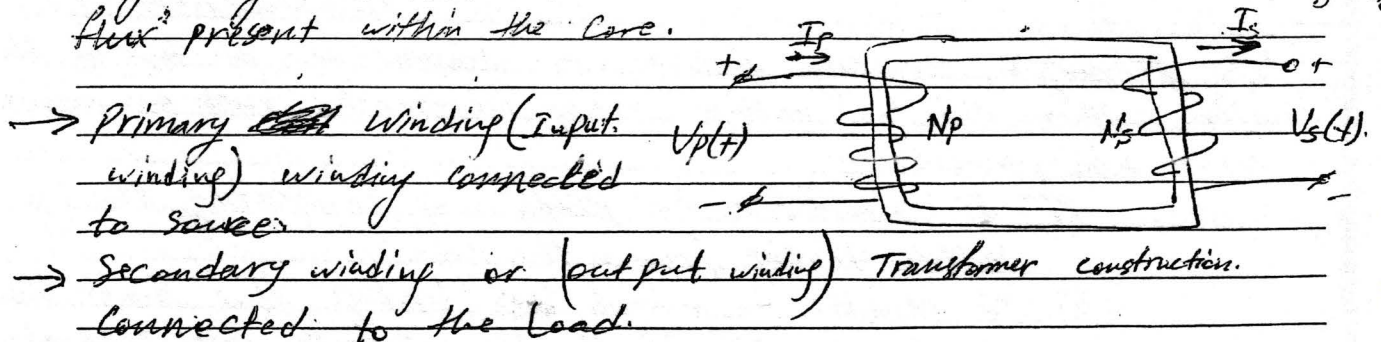
B-H Relation :-

Magnetization curve :- expressed in terms of flux density & magnetizing intensity.



## II Single phase Transformer :-

A transformer is a device that changes ac electric power at one voltage level to ac electric power at another voltage level. through the action of a magnetic field. It consists of 2 or more coils of wire wrapped around a common ferromagnetic core, These coils are not directly connected, They only connection between the coils is the common magnetic flux present within the core.



→ Primary ~~coil~~ winding (input winding) connected to source.

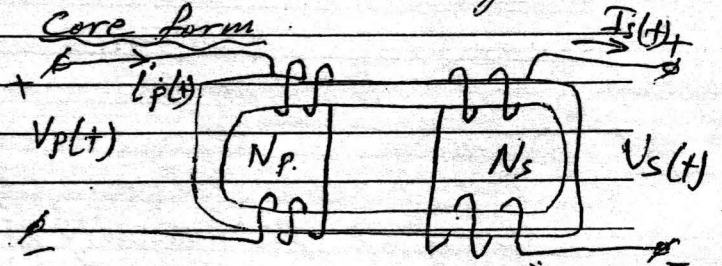
→ Secondary winding or (output winding) connected to the load.

## \* Types & construction of Transformers

The purpose of a Tr-rs is to convert AC power at one voltage level to AC power of the same frequency at another

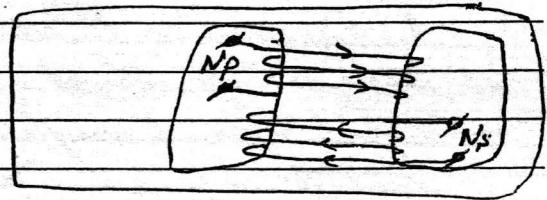
power Tr-rs are constructed on one of two types of cores :-

- ① one type consists of a simple rectangular laminated piece of steel with Tr-rs windings wrapped around two sides of the rectangular. This type is called Core form.



Core form Tr-r construction.

- ② consists of a three-legged laminated core with the windings wrapped around the center leg & is called as Shell form.



Shell form Tr-r construction.

The primary & secondary windings are wrapped one on top of the other with the low voltage winding innermost. Such an arrangement serves two purposes :-

- ① It simplifies the problem of insulating the high-voltage winding from the core.
- ② It results in much less leakage flux than than would be the case if the two windings were separated by a distance on the core.





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\* Power Transformers have different names depending on their use in power systems.

1) Unit Transformer:- Connected to the output of a generator & used to step its voltage up to transmission levels (110KV)

2) Substation transformers :- at the other end of the transmission line, which step the voltage down.

3) Distribution Transformers :- takes the distribution voltage & steps it down to the final voltage

\* There is two special-purpose transformers are used with electric machinery & power systems :-

1) Potential Transformer :- to sample a high voltage & produce a low secondary voltage

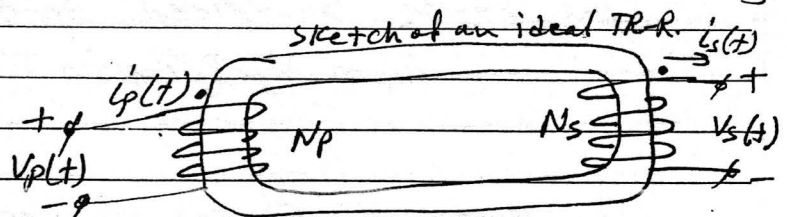
2) Power Transformer :- produces a secondary voltage directly proportional to its primary voltage.

The difference between them is that the potential Tr-r is designed to handle only a very small current & for the second type is designed to provide a secondary current much smaller than & called current Tr-r.

## \* THE IDEAL TR-R :-

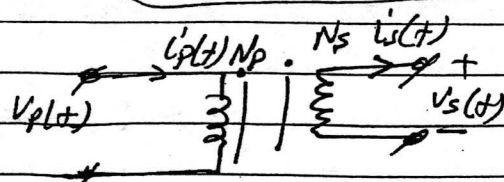
An ideal Tr-r is lossless device with an input winding & an output winding.

The Relationship between  $V_p(t)$  &  $V_s(t)$  is



$$\frac{V_p(t)}{V_s(t)} = \frac{N_p}{N_s} = a$$

where  $a$  :- turns Ratio



$$a = \frac{N_p}{N_s}$$

$N$  - number of turns



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The Relationship between  $i_s(t)$  &  $i_p(t)$ .

$$N_p i_p(t) = N_s i_s(t).$$

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

$$\boxed{\frac{V_p}{V_s} = a} \implies \boxed{\frac{i_p}{i_s} = \frac{1}{a}}$$

\* The turns Ratio of  $\downarrow$  TR-R affects the magnitudes of voltages & currents, but not their angles.

Dot convention :-

(1) if the Primary (V) is (+) at the dotted end of winding then the secondary V will be (+) at the dotted end also.

(2) if the primary (I) of the Transformer flows into the dotted end of the primary winding, The secondary (I) will be (+)

Power in an Ideal TR-R :-

The power supplied to the primary circuit :-

$$P_{in} = V_p \cdot I_p \cdot \cos \theta_p$$

where  $\theta_p$  - The angle between  $I_p$  &  $V_p$ .

The output power of secondary circuit to its leads :-

$$P_{out} = V_s \cdot I_s \cdot \cos \theta_s.$$

\* The output power of an ideal transformer is equal to its input Power

\* The same relationship applies to reactive power Q.

$$Q_{in} = V_p \cdot I_p \cdot \sin \theta_p = V_s \cdot I_s \cdot \sin \theta_s = Q_{out}.$$

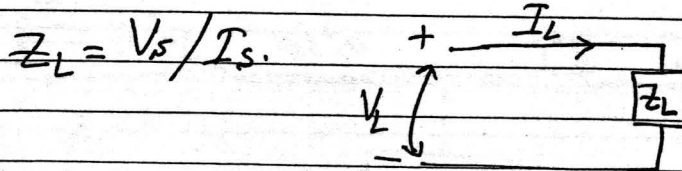


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## Impedance Transformation through a TR-R:-

( $Z_L = V_L / I_L$ ) The Ratio phasor voltage to phasor current

The impedance of the load is given by:-



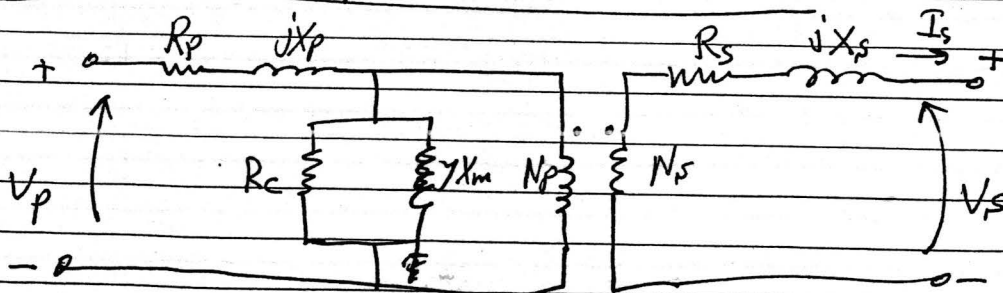
## The Equivalent circuit of a TR-R:-

The major losses to be considered are:-

- ① Copper losses  $I^2R$  - are Resistive heating losses in the primary & secondary windings.
- ② Eddy current losses:- are resistive heating losses in the core of the TR-R, They are proportional to the square of applied voltage to the TR-R.
- ③ Hysteresis losses:- are associated with the rearrangement of the magnetic domains in the core during each half cycle.

④ Leakage flux:- The fluxes  $\Phi_P$  &  $\Phi_S$

## The Exact Equivalent circuit:-



Ideal Transformer.

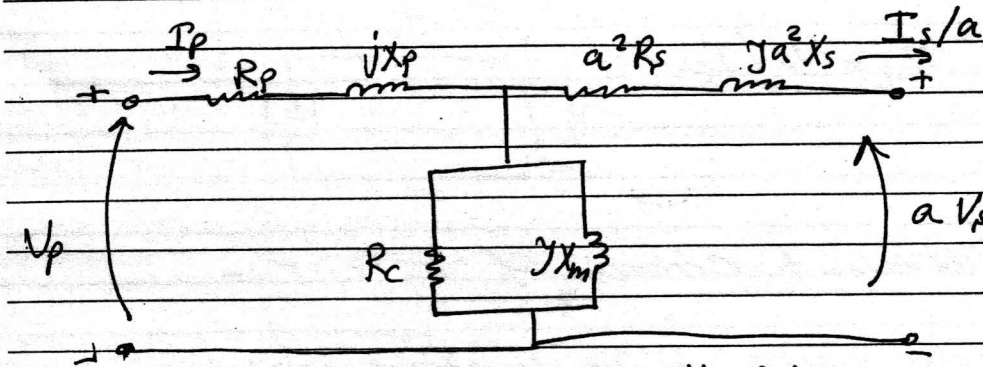
The leakage flux in primary & secondary windings produce a voltage  $\Phi_P$  &  $\Phi_S$



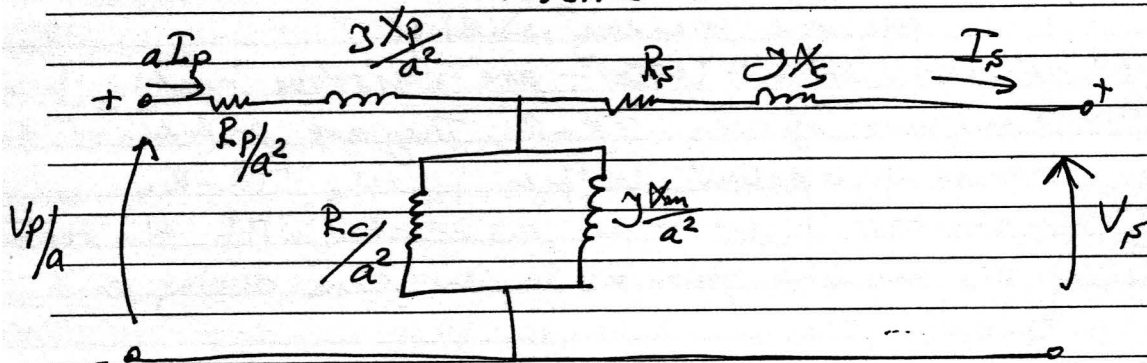
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$$e_{Lp}(t) = N_p \frac{d\phi_p}{dt}$$

$$e_{Ls}(t) = N_s \frac{d\phi_s}{dt}$$



Tr-r model referred to its primary voltage level.



Tr-r model referred to its secondary voltage level.



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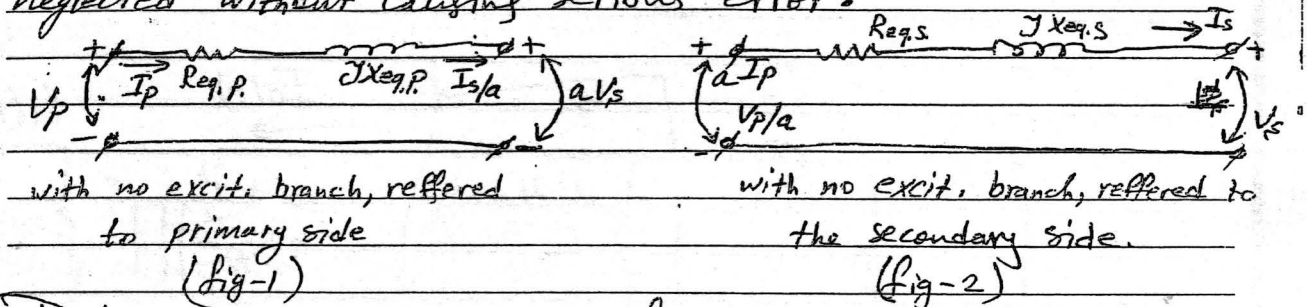
Approximate equivalent circuit of a Transformer:- 

The TR-R models shown before are more complex than ~~more~~ necessary in order to get good results in practical engineering applications.

The excitation branch making the circuit simulation more complex than necessary.

The branch current is very small compared to the load current of TR-Rs & The drop voltage in  $R_p$  &  $X_p$  negligible.

In some applications the excitation branch may be neglected without causing serious error.



Determining the values of components in the TR-R model

It is possible to determine the values of R & X in the Tr-r model experimentally.

Approximation of these values can be obtained <sup>with</sup> only two tests :-

- (a) The open-circuit Test.
- (b) The short-circuit Test.

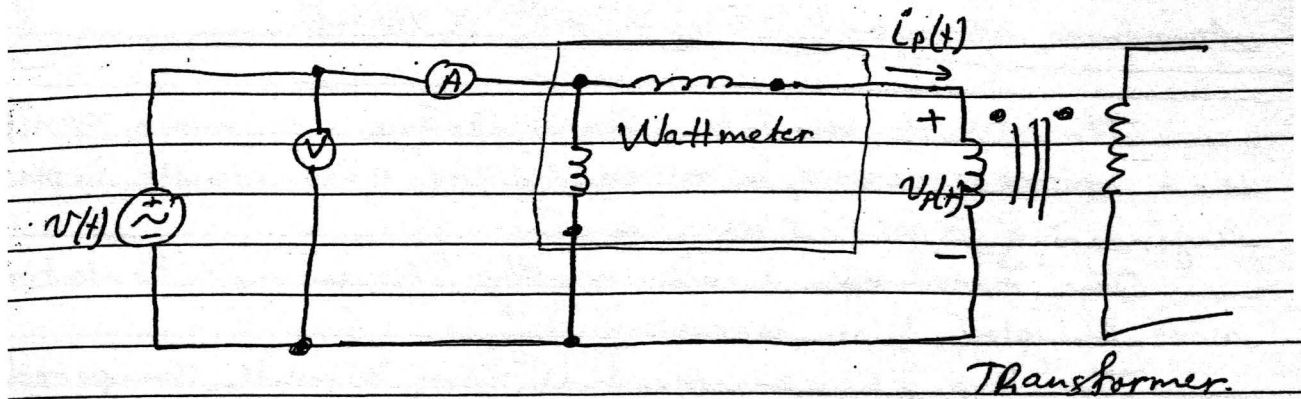
Open-circuit test :-

a TR-Rs secondary winding is open-circuited, and its primary winding is connected to a full-rated line voltage.

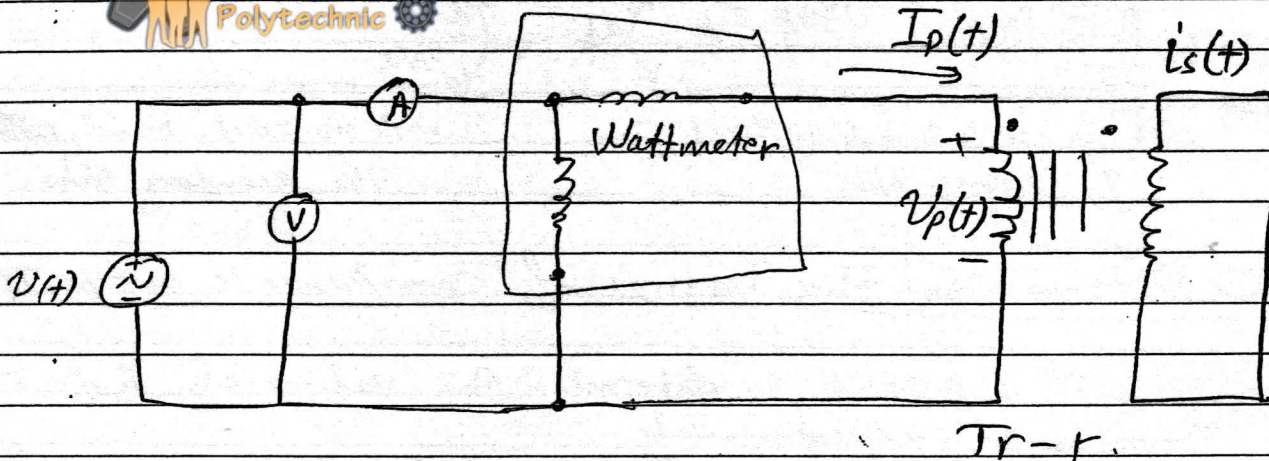
all the input current must be flowing through the excitation branch of the TR-R. The series elements  $R_p$  &  $X_p$  are too small in comparison to  $R_c$  &  $X_m$  to cause a significant voltage drop, so all the input voltage is dropped across the excitation branch.

from figure (3), full line voltage is applied to the Primary of the TR-R, and the Input voltage, Input current

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Open circuit Test



TR-R short circuit Test.



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and Input power to the TR-R are measured & it's poss. to determine the power factor of the input current & therefore both the magnitude & the angle of the excitation Impedance

The best way to calculate the values of  $R_c$  &  $X_m$  is to look first at the admittance of the excitation branch

The conductance of the core-loss resistor is given by:-

$$G_c = 1/R_c$$

& The susceptance of the magnetizing inductor is given by

$$B_m = 1/X_m$$

The Total admittance is

$$Y_E = G_c - jB_m.$$

$$= \frac{1}{R_c} - j \frac{1}{X_m}$$

The magnitude of the excitation admittance (referred to the primary circuit) can be found from the open-circuit test voltage & I.

$$|Y_E| = I_{oc} / V_{oc}.$$

The open-circuit power factor

$$PF = \cos \theta = P_{oc} / V_{oc} \cdot I_{oc}.$$

& The power factor angle  $\theta$  is given by:-

$$\theta = \cos^{-1} P_{oc} / V_{oc} \cdot I_{oc}.$$

The power factor is always lagging for a real TR-R. So the angle of the I always lags the angle of the V by  $\theta$  degrees

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{I_{oc}}{V_{oc}} \angle \cos^{-1} PF.$$



From equations :-

$$Y_E = \frac{1}{R_c} - j \frac{1}{X_m}$$

&

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1} PF$$

it is possible to determine the values of  $R_c$  &  $X_m$  from the open-circuit test data.

Short circuit test :-

the secondary terminals of the TR-R are shorted circuited, and the primary terminals are connected to a fairly low-voltage source

The voltage is adjusted until the current in the short circuited windings is equal to its rated value

The Input  $V$ ,  $I$  & power are again measured

Input ( $V$ ) is so low during the short-circuited test, negligible current flows through the excitation branch, if the excitation current is ignored, then all the voltage drop in the TR-R can be attributed to the series elements in the circuit

The magnitude of the series Impedances referred to the primary side of the TR-R is

$$|Z_{SE}| = V_{sc} / I_{sc}$$

The power factor

$$PF = \cos \theta = P_{sc} / V_{sc} \cdot I_{sc}$$

& is lagging.

The Impedance angle  $\theta$ .

$$\theta = \cos^{-1} (P_{sc} / V_{sc} \cdot I_{sc})$$

The series Impedance  $Z_{SE}$ .

$$\begin{aligned} Z_{SE} &= R_{eq} + jX_{eq} \\ &= (R_p + a^2 R_s) + j(X_p + a^2 X_s) \end{aligned}$$





## The per-unit system of measurements.

In per-unit system, the voltages, currents, powers, Impedance & other quantities are not measured in their usual SI units (volts, amperes, Watts, ohms, --).

Any quantity can be expressed on a per-unit basis by the equation:-

$$\text{Quantity per unit} = \frac{\text{Actual value}}{\text{base value of quantity}}$$

where actual value in volts, amperes, ohms --

$$P_{\text{base}}, Q_{\text{base}} = V_{\text{base}} \cdot I_{\text{base}}$$

$$Z_{\text{base}} = V_{\text{base}} / I_{\text{base}}$$

$$Y_{\text{base}} = I_{\text{base}} / V_{\text{base}}$$

## Transformer voltage Regulation :-

Because the real Tr-r has series Impedances, the output voltage of a Tr-r varies with the load, even if the input voltage remains constant.

Voltage Regulation (VR) :- Full-load voltage regulation is a quantity that compares the output voltage of the Tr-r at no load with the output voltage at full load.

It is defined by the equation :-

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s,fl}} \times 100\%$$

Since at no-load,  $V_s = V_p / a$ , The voltage Regulation can be expressed as

$$VR = \frac{V_p / a - V_{s,fl}}{V_{s,fl}} \times 100\%$$

If the Tr-r equivalent circuit is in the Per-unit system, then voltage regulation can be expressed as :-

$$VR = \frac{V_{p,pu} - V_{s,fl,pu}}{V_{s,fl,pu}} \times 100\%$$

It is good to have a small voltage Regulation as possible. For an Ideal TR-R VR = 0 percent.

## The TR-R Phasor diagram:-

How can the VR of a Tr-r be determined?

To determine the voltage regulation of a TR-R, it's necessary to understand the voltage drops within it. In the shown figure the effects of the excitation branch on Tr-r VR can be ignored. So only the series impedances need be considered.

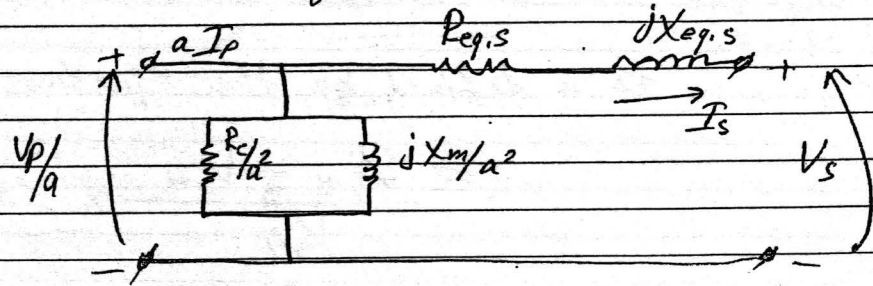


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The VR of a Tr-r depends both on :-

- ① The magnitude of these series impedances
- ② The phase angle of the current flowing through the Tr-r.

The easiest way to determine the effect of the impedances and the current phase angles on the Tr-r VR is to examine a phasor diagram.

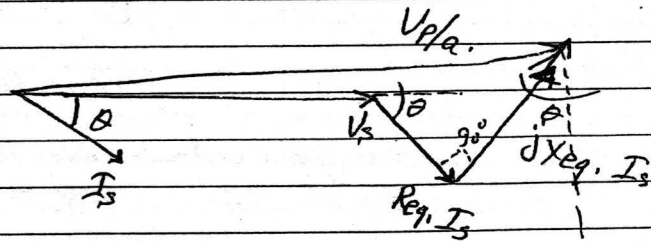


in all the following phasor diagrams, The phasor voltage  $V_s$  is assumed to be at an angle of  $0^\circ$ , and all other voltages & currents are compared to that reference. By applying Kirchhoff's voltage law

$$\frac{V_p}{a} = V_s + R_{eq} \cdot I_s + jX_{eq} \cdot I_s$$

A Tr-r phasor diagram is just a visual representation of this equation.

Phasor diagram of a TR-R operating at a lagging PF. It is easy to see that



$V_p/a > V_s$  for lagging loads

VR for lagging loads must be  $\gg$  zero

A phasor diagram at unity power factor, The voltage at the secondary is lower than the voltage at the primary so,  
 $VR > 0$

VR = smaller number than it was with a lagging current.



$$P_{out} = V_s \cdot I_s \cdot \cos \theta_s$$

$$\eta = \frac{V_s \cdot I_s \cdot \cos \theta}{P_{cu} + P_{core} + V_s \cdot I_s \cdot \cos \theta} \cdot 100\%$$

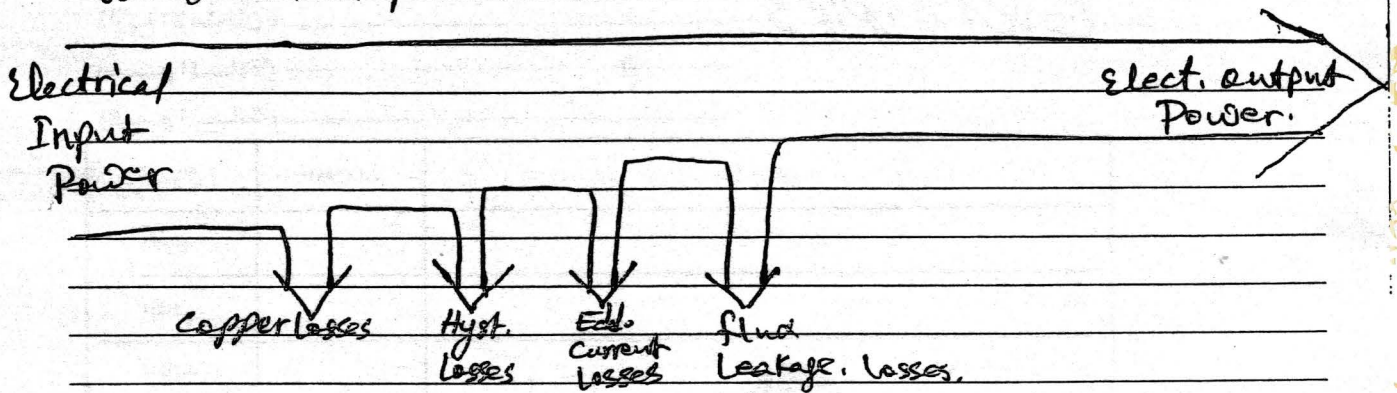


## The power-flow Diagram:-

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Power-flow diagram is The Technique for accounting for power losses in a machine.

A power-flow (P-F) diagram for TR-R is shown below, in this figure, electrical power (primary) is input into the TR-R, and then the copper losses, Eddy current losses, Hysteresis losses & Leakage flux are subtracted. After they have been subtracted, the remaining Power is ideally converted from electrical power one level to electrical power <sup>another</sup> level.



## Example ①.

Find the relative permeability of the typical ferromagnetic material whose magnetization curve is shown below at  $H = 50 \text{ A.turn/m}$ .

Permeability of material  
 $\mu = B/H$

(data)  
 $\left\{ \begin{array}{l} B = 1,23 \text{ T at } H = 300 \\ B = 0,65 \text{ T at } H = 100 \\ B = 0,25 \text{ T at } H = 50 \\ B = 1,58 \text{ T at } H = 2000 \end{array} \right.$

Relative permeability  $\mu_r = \frac{\mu}{\mu_0}$

$$\mu = 0,25 \text{ T} / 50 \text{ A.turn/m} = 0,0050 \text{ H/m}$$

$$\mu_r = \mu / \mu_0 = \frac{0,0050 \text{ H/m}}{4\pi \cdot 10^{-7} \text{ H/m}} = 3980$$



Ex. 2 :- Figure (1) shows a simplified rotor and stator for a dc motor. The mean path length of the stator is (50 cm), and its cross-sectional area is (12 cm<sup>2</sup>). The mean path length of the rotor is (5 cm), and its cross-sectional area also may be assumed to be (12 cm<sup>2</sup>). Each air gap between the rotor & the stator is (0,05 cm) wide, and the cross sectional Area of each air gap is (14 cm<sup>2</sup>). The iron of the core has a relative permeability of (2000), and there are 200 turns of wire on the core. if the current in the wire is adjusted to be (1 A), what will the resulting flux density in the air gaps be?

Solution :-

The reluctance of the stator is :

$$R_s = l_s / \mu_r \mu_0 A_s$$

$$= \frac{0,5 \text{ m}}{(2000)(4\pi \times 10^{-7})(0,0012 \text{ m}^2)} = 166,0 \text{ A. turns/Wb.}$$

The Reluctance of the Rotor is :-

$$R_r = l_r / \mu_r \mu_0 A_r$$

$$= 0,05 \text{ m} / (2000)(4\pi \times 10^{-7})(0,0012 \text{ m}^2) = 16,6 \frac{\text{A. turns}}{\text{Wb.}}$$

The Reluctance of Air Gaps is

$$R_a = l_a / \mu_r \mu_0 A_a$$

$$= 0,0005 \text{ m} / (1)(4\pi \cdot 10^{-7})(0,0014 \text{ m}^2) = 284,00 \frac{\text{A. turns}}{\text{Wb.}}$$

The total Reluctance of the flux Path is:

$$R_{eq} = R_s + R_r + R_{a1} + R_{a2} = 166,0 + 284,00 + 16,6 + 284$$

$$= 751 \text{ A. turns/Wb.}$$

The magnetomotive force applied to the core :-

$$f = NI = (200 \text{ turns}) \times (1,0 \text{ A}) = 200 \text{ A. turns.}$$



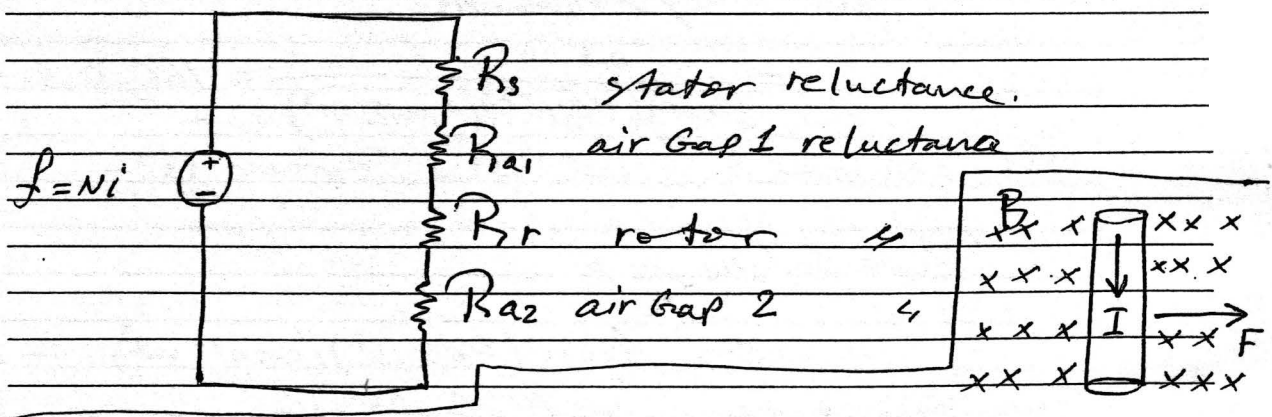
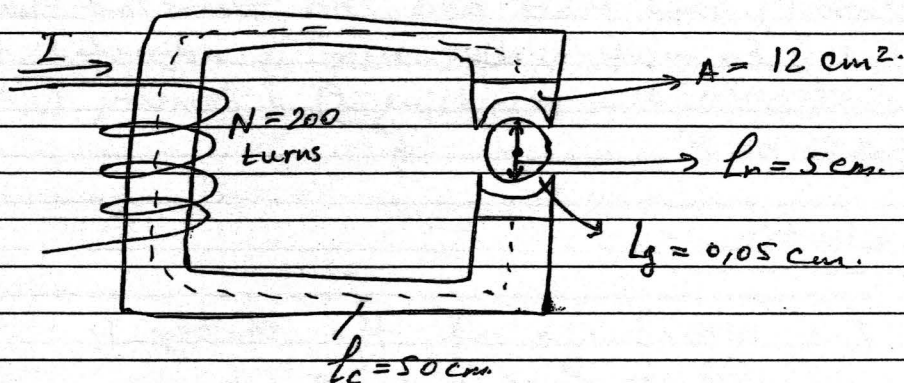
# لجنة الميكانيك - الإتجاه الإسلامي

The Total Flux in the core:-

$$\phi = \mathcal{F} / R = \frac{200 \text{ A.turns}}{751 \text{ A.turns/Wb.}} = 0,00266 \text{ Wb.}$$

The magnetic flux density in the motor's air Gap is

$$B = \phi / A = \frac{0,000266 \text{ Wb}}{0,0014 \text{ m}^2} = 0,19 \text{ T.}$$



Example :- Figure shows a wire carrying a current in the presence of a magnetic field. The magnetic flux density is  $0,25 \text{ T}$ , directed into the page. if the wire is  $1,0 \text{ m}$  long & carries  $0,5 \text{ A}$  of current in the direction from the Top of the page to the bottom of the page, what are the magnitude & direction of the force induced on the wire?

$$F = i.l.B \sin \theta = (0,5 \text{ A}) \cdot (1,0 \text{ m}) \cdot (0,25 \text{ T}) \sin 90^\circ = 0,125 \text{ N}$$

directed to the right. [The basis of motor action]





## The linear DC machine :-

6

A linear DC machine is ~~the~~ an easiest to understand version of a dc machine, it operates according to the same principles as real generators & motors.

A linear dc machine consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. along the bed of this railroad track is a constant, uniform-density magnetic field directed into the page, a bar of conducting metal is lying across the tracks.

behavior of this device can be determined from an application of 4 basic equations to the machine :-

① The equation for the force on a wire in the presence of mag. field.  

$$F = (B \times l) \cdot i$$

② The eq. for the voltage induced on a wire moving in the mag. field.  

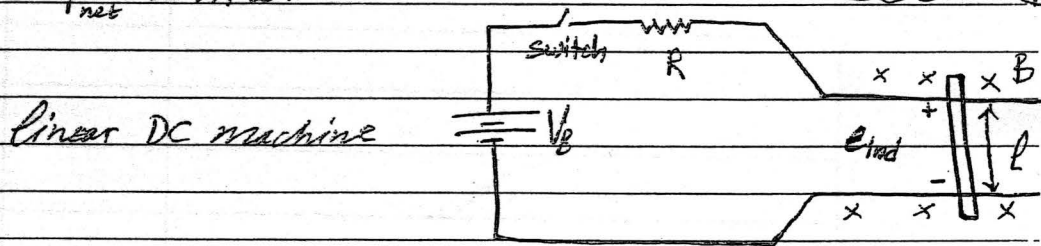
$$E_{ind} = (v \times B) \cdot l$$
  $v$  - speed of the wire

③ Kirchhoff's voltage law

$$V_B - iR - E_{ind} = 0 \Rightarrow V_B = iR + E_{ind}$$

④ Newton's law for the bar across the tracks:

$$F_{net} = ma$$



## Starting The linear DC machine

To start the machine close the switch. The current flows in the bar & by Kirchhoff's voltage law:

$$i = \frac{V_B - E_{ind}}{R}$$

Since the bar is initially at rest,  $E_{ind} = 0$ , so  $i = V_B / R$ . The current flows down through the bar across the tracks, but from equation (1) a current flowing through a wire in the presence

# لجنة الميكانيك - الإتجاه الإسلامي

of a magnetic field induces a force on the wire,

$$F_{ind} = i \cdot l \cdot B \quad \text{to the right.}$$

There for the bar will accelerate to the right (by Newton's law) when the velocity of the bar begins to increase, a voltage appears across the bar

$$\mathcal{E}_{ind} = v B l \quad (\text{positive upward})$$

The voltage now reduces the (I) flowing in the bar:

$$i \downarrow = \frac{V_B - \mathcal{E}_{ind} \uparrow}{R}$$

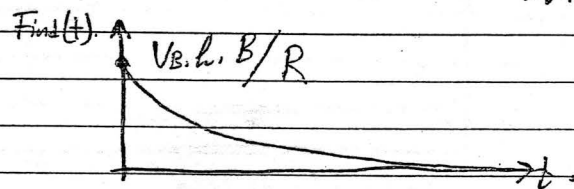
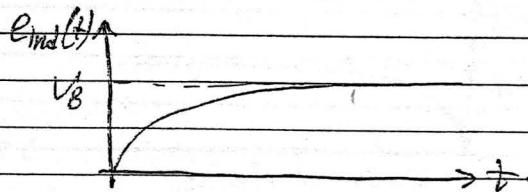
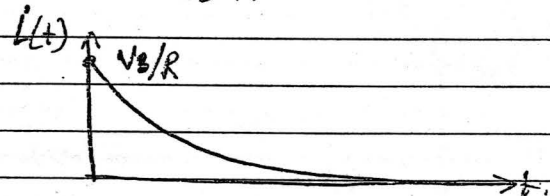
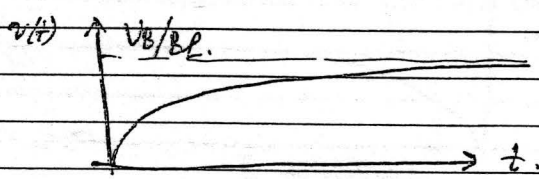
As  $\mathcal{E}_{ind}$  increase the I decreases.

The bar will reach a constant steady-state speed

$$V_B = \mathcal{E}_{ind} = v_{ss} \cdot B \cdot l$$

$$v_{ss} = V_B / B l$$

The bar will continue to ~~move~~ <sup>coast</sup> along at this no-load speed forever unless some external force disturbs it



To Summarize:-

- ① closing the switch produces a current flow  $i = V_B / R$ .
- ② The current flow produces a force on the bar given by  $F = i l B$ .
- ③ The bar accelerates to the right, producing an induced voltage  $\mathcal{E}_{ind}$  as it speeds up.
- ④ This induced voltage reduces the current flow  $i = (V_B - \mathcal{E}_{ind}) / R$ .
- ⑤ The induced force is decreased ( $F = i l B$ ) until  $F = 0$ . At that point,  $\mathcal{E}_{ind} = V_B$  ;  $i = 0$  ; The bar moves at const no-load speed  $v_{ss} = V_B / B l$ .



# لجنة الميكانيك - الإتجاه الإسلامي

The ~~linear~~ linear dc machine as a motor :-

if the linear machine is initially running at the no-load steady-state conditions what will happen to this machine if an external load is applied to it?

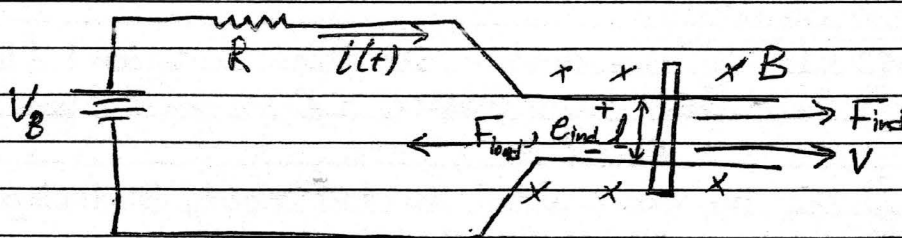
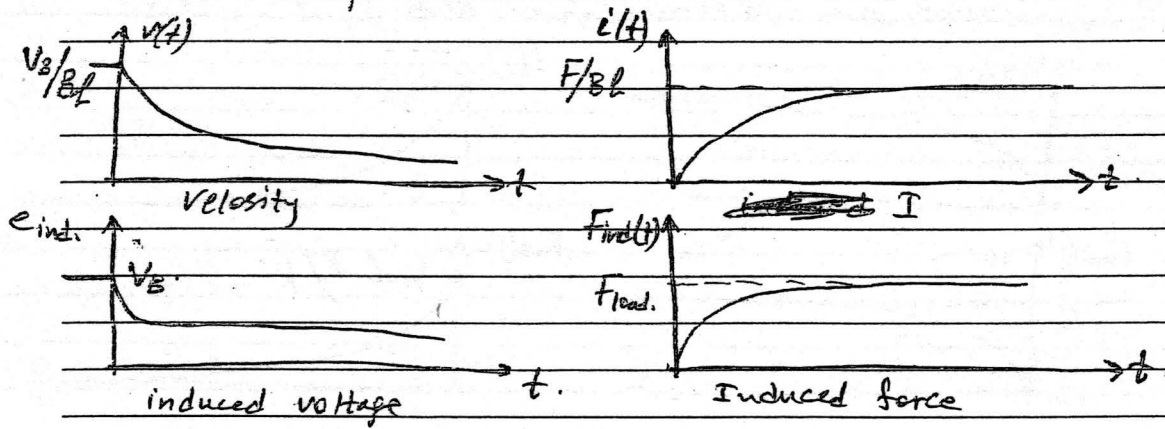
if a force  $F_{load}$  is applied to the bar opposite the direction of motion, since the bar initially at st. state speed, application of the  $F_{load}$  will result in a net force on the bar in the direction opposite the direction of motion

( $F_{net} = F_{load} - F_{ind}$ ) the effect of the force will slow the bar & when the speed of bar slow the induced voltage will be decreased. (drops) ( $e_{ind} = v \downarrow B l$ ) as the  $e_{ind}$  decreases the current flow in the bar rises

$$i \uparrow = \frac{V_B - e_{ind} \downarrow}{R}$$

& The induced ~~voltage~~ force rises too ( $F_{ind} = i \uparrow B l$ )

The result of this events is that the induced force rises until it is equal and opposite to the  $F_{load}$ , and the bar again travels in steady state, but at a lower speed.



There is now an induced force in the direction of motion of the bar, and power is being converted from electrical form to mechanical form to keep the bar moving.

$$P_{conv} = e_{ind} \cdot i = F_{ind} \cdot v$$

Since the power is converted from electrical to mechanical form this bar is operating as a motor

Summarize: -

- ① A force  $F_{load}$  is applied opposite to the direction of motion, which causes a net force  $F_{net}$  opposite to the direction of motion.
- ② The resulting acceleration ( $a = F_{net}/m$ ) is negative so the bar slows down ( $v \downarrow$ )
- ③ The voltage  $e_{ind} = v \downarrow \cdot B \cdot l$  falls, and so ( $i = (V_B - e_{ind})/R$ ) increases.
- ④ the induced force  $F_{ind} = i \uparrow \cdot l \cdot B$  increases until  $|F_{ind}| = |F_{load}|$  at a lower speed  $v$ .
- ⑤ An amount of electric power equal to  $(P_{ind} i)$  is now being converted to mechanical power equal to  $F_{ind} v$ , and the machine is acting as a motor.

A real dc motor when it's loaded:-

As a load is added to its shaft, the motor begins to slow down, which reduces its internal voltage, increasing its current flow. The increased current flow increases its induced torque, and the induced torque will equal the load torque of the motor at a new, slower speed.

$$P_{conv} = T_{ind} \cdot \omega \quad T_{ind} - \text{induced Torque}$$

The linear dc machine as a Generator:-

if the linear machine is operating under no-load steady-state conditions. This time apply a force in the direction of motion & see what happens?.

The applied force will cause the bar to accelerate in the direction of motion, and the velocity  $v$  of the bar will increase. As the velocity increases, ( $e_{ind} = v \uparrow \cdot B \cdot l$ ) will increase and will be larger than the battery voltage ( $V_B$ )

With  $e_{ind} > V_B$  the current reverses direction and is now given by the equation  $i = \frac{e_{ind} - V_B}{R}$



# لجنة الميكانيك - الإتجاه الإسلامي

since this current now flows up through the bar, it induces a force in the bar given by:

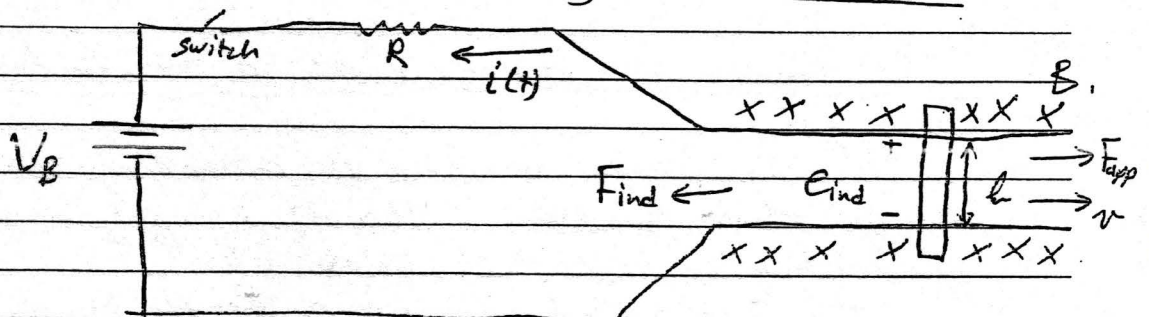
$$F_{ind} = i l B \quad \text{to the left.}$$

The direction of the force is given by the right-hand rule, This induced force opposes the applied force on the bar.

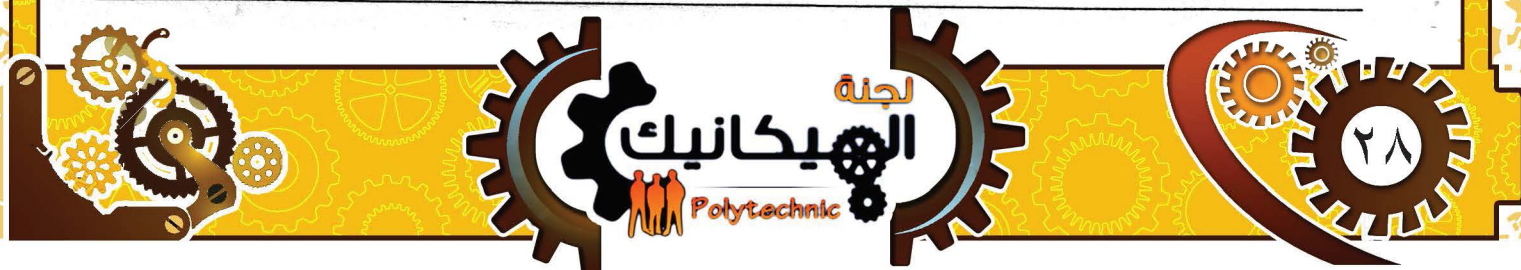
finally the ( $F_{ind} = F_{app}$ ) but opposite  $F_{app}$   $\rightarrow$  The bar will be moving at a higher speed than before. Notice that now the battery is charging, The ~~is~~ linear machine is now serving as a generator, converting mechanical power ( $F_{ind} \cdot v$ ) into electric power ( $\epsilon_{ind} \cdot i$ )

Summarize :-

- ① A force  $F_{app}$  is applied ~~to~~ in the direction of motion,  $F_{net}$  is in the direction of motion.
- ② Acceleration ( $a = F_{net}/m$ ) is positive so the bar speeds up ( $v \uparrow$ )
- ③ The voltage  $\epsilon_{ind} = v \uparrow B l$  increases, and so ( $i = \epsilon_{ind} - V_B$ ) /  $R$  increases.
- ④ The induced force  $F_{ind} = i l B$  increases until  $|F_{ind}| = |F_{load}|$  at a higher speed  $v$ .
- ⑤ An amount of mechanical power equal to ( $F_{ind} v$ ) is now being converted to electric power ( $\epsilon_{ind} i$ )  $\rightarrow$  The machine is acting as a Generator



linear dc machine as a generator.

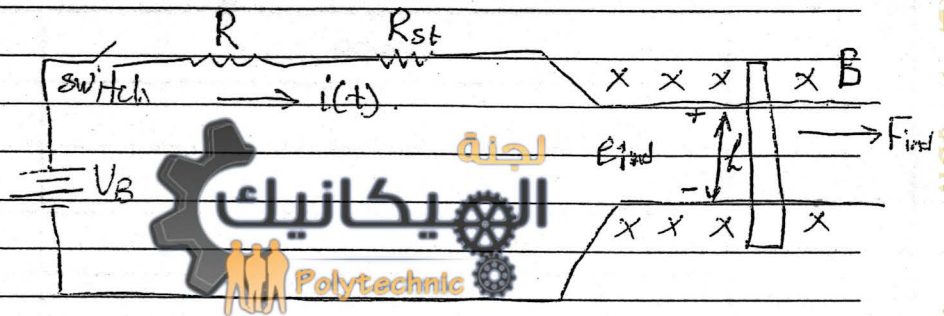


Starting problems with linear machine:-

- ⊕ starting current very high.  
because of the (speed of bar &  $P_{ind} = 0$ )  
The current flow at starting condition is.

$$I_{st} \uparrow = V_B / R \quad (\text{often more than rated in 10 times})$$

- \* The easiest method to decrease the starting current is to insert an extra resistance into the circuit during starting to limit the current flow.



## The Autotransformer :-

Some times we need to change voltage levels by only a small amount. For examples- it may be necessary to increase a voltage from 110 to 120 V or from 13,2 to 13,8 KV. These small rises may be made necessary by voltage drops that occur in power systems a long way from the generators. A special-purpose tr-r, called an autotransformer.

A diagram of step-up Autotransformer is shown in Fig-1. The relationship between the voltage on the first winding & The voltage on the second winding is given by the turns ratio of the Tr-r, the voltage at the output of the whole Tr-r is the sum of the voltage on the first winding & The voltage on the second winding. The first winding here is called Common winding, because its voltage appears on both sides of the Tr-r. The smaller winding is called the series winding, because it is connected in series with the common winding.

A diagram of a step-down autotransformer is shown in Fig-2. Here the voltage at the input is the sum of the voltages on the series winding and the common winding, while the voltage at the output is just the voltage on the common winding.

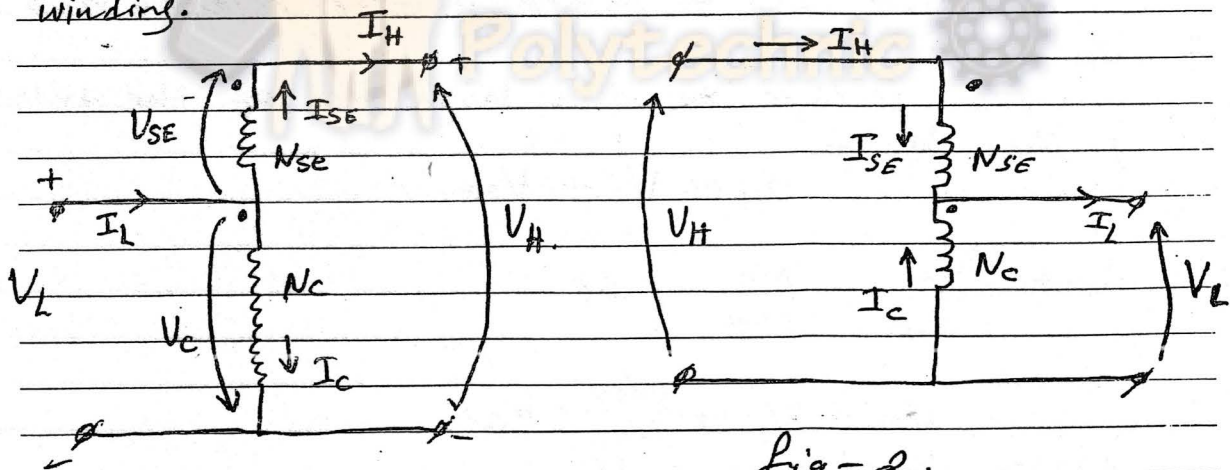


Fig-1

Fig-2.

The voltage on the common coil is called the Common Voltage  $V_c$ , and the current in that coil is called the Common current  $I_c$ , The voltage on the series coil is called

# لجنة الميكانيك - الإتجاه الإسلامي

The series voltage  $V_{SE}$  & The current is called the series current  $I_{SE}$ . The voltage & current on the low-voltage side of the Tr-r are called  $V_L$  &  $I_L$  while these quantities on the high side voltage side of the Tr-r are called  $V_H$  &  $I_H$ . The primary side of the autotransformer can be either the high-voltage side or low-voltage side depending on whether the auto-tr is acting as a step-down or a step-up Tr-r.

From fig-1 The voltages & currents in the coils are related by the equations:

$$\frac{V_C}{V_{SE}} = \frac{N_C}{N_{SE}}$$

$$N_C I_C = N_{SE} \cdot I_{SE}.$$

$$V_L = V_C$$

$$V_H = V_C + V_{SE}.$$

$$I_L = I_C + I_{SE}.$$

$$I_H = I_{SE}.$$

## Voltages & current Relationships in an Allot-r:-

It is quite easy to determine the relationship between  $V_H$  &  $V_L$

The voltage on the high side of the auto-tr-r is given by:-

$$V_H = V_C + V_{SE}.$$

but  $\frac{V_C}{V_{SE}} = \frac{N_C}{N_{SE}}$ , so.

$$V_H = V_C + \frac{N_{SE}}{N_C} \cdot V_C$$

Noting that  $V_C = V_L$

$$V_H = V_L + \frac{N_{SE}}{N_C} \cdot V_L.$$

$$= \frac{N_{SE} + N_C}{N_C} \cdot V_L \Rightarrow \frac{V_L}{V_H} = \frac{N_C}{N_{SE} + N_C}$$





# لجنة الميكانيك - الإتجاه الإسلامي

The current relationship between the two sides of the Tr-r can be found by noting that:

$$I_L = I_c + I_{SE}$$

From equation  $N_c I_c = N_{SE} I_{SE}$ .

$$I_c = \frac{N_{SE}}{N_c} I_{SE}$$

So 
$$I_L = \frac{N_{SE}}{N_c} I_{SE} + I_{SE}$$

Finally noting that  $I_H = I_{SE}$ .

$$I_L = \frac{N_{SE}}{N_c} I_H + I_H$$

$$I_L = \frac{N_{SE} + N_c}{N_c} I_H$$

$$\boxed{\frac{I_L}{I_H} = \frac{N_{SE} + N_c}{N_c}}$$

## The Power Rating Advantage of Auto-tr-r:-

not all the power traveling from the primary to the secondary in the autot-r goes through the windings. As a result, if a conventional tr-r is reconnected as an autot-r, it can handle much more power than it was originally rated for.

To understand this idea, refer again to fig-1 notice that the apparent power to the autot-r is given by.

$$S_{in} = V_L I_L$$

& The output is given by

$$S_{out} = V_H I_H$$

It is easy to show, by using the voltage & current equations that the input apparent power is equal again to output apparent power.

$$S_{in} = S_{out} = S_{IO}$$

where  $S_{IO}$  - the input & output apparent powers of the tr-r.



# لجنة الميكانيك - الإتجاه الإسلامي

The apparent Power in the Tr-r windings is:-


$$S_W = V_c I_c = V_{SE} \cdot I_{SE}$$

The relation ship between the power going into the primary (and out the secondary) of the tr-r & the power in the tr-r's actual windings can be found as follows:-

$$\begin{aligned} S_W &= V_c I_c \\ &= V_L (I_L - I_H) \\ &= V_L I_L - V_L I_H \end{aligned}$$

From equation  $\frac{I_L}{I_H} = \frac{N_{SE} + N_c}{N_c}$  we get.

$$S_W = V_L \cdot I_L - V_L I_H \frac{N_c}{N_{SE} + N_c}$$


$$\begin{aligned} &= V_L I_L \frac{(N_{SE} + N_c) - N_c}{N_{SE} + N_c} \\ &= S_{IO} \frac{N_{SE}}{N_{SE} + N_c} \end{aligned}$$

Therefore, the ratio of the apparent power in the primary & secondary of the aut-r to the apparent power actually traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_c}{N_{SE}}$$

$S_{IO}$  - the apparent power entering the primary & leaving the secondary of the tr-r.

$S_W$  - the apparent power actually traveling through the Tr-r's windings

~~the~~ Note that the smaller the series winding, the greater the advantage.



# لجنة الميكانيك - الإتجاه الإسلامي

- DC Machines are Generators that convert mechanical energy to dc electric energy & motors that convert dc electric energy to mechanical energy.
- = DC machines have a dc output only because a mechanism exists that converts the internal AC voltages to DC voltages at their terminals. Since this mechanism is called a commutator, dc machinery is also known as commutating machinery.

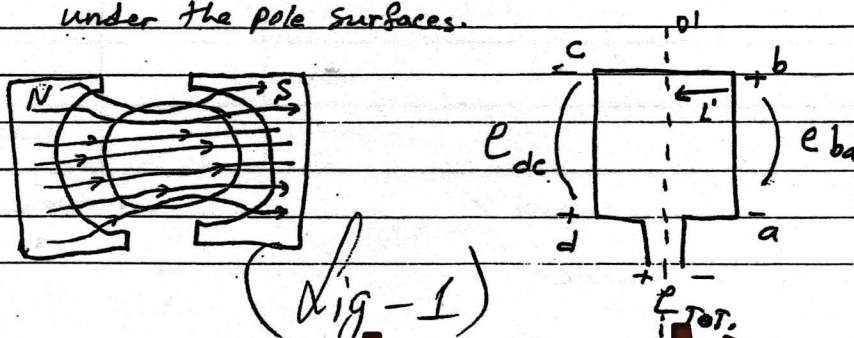
Possible

- The simplest <sup>Possible</sup> rotating dc machine is shown in figure (1) it consists of a single loop of wire rotating about a fixed axis, the rotating part of this machine is ~~shown in fig (1)~~ called the rotor and the stationary part is called the stator. The magnetic field for the machine is supplied by the magnetic north and south poles shown in the stator & in fig. (1).

- The loop of rotor wire lies in a slots carved in a ferromagnetic core, between rotor & stator constant-width air gap.
- The reluctance of air is much higher than the reluctance of iron in the machine.

- To minimize the reluctance of the flux path through the machine the magnetic flux must take the shortest possible path through the air between the pole faces & the rotor surface & the flux will perpendicular to the rotor surface everywhere under the pole faces

- The Reluctance is the same everywhere under the pole faces & The magnetic flux density is constant everywhere under the pole surfaces.



(Fig - 1)

The voltage Induced in a rotating loop:-

① The Total Voltage

$$E_{tot} = (v \cdot B) \cdot l$$

1.1) - Segment ab

$$e_{ba} = (v \cdot B) \cdot l$$

1.2) - Segment bc

$$e_{bc} = 0 \quad \text{because the quantity } (v \cdot B) \text{ is perpendicular to } l.$$

1.3) segment cd

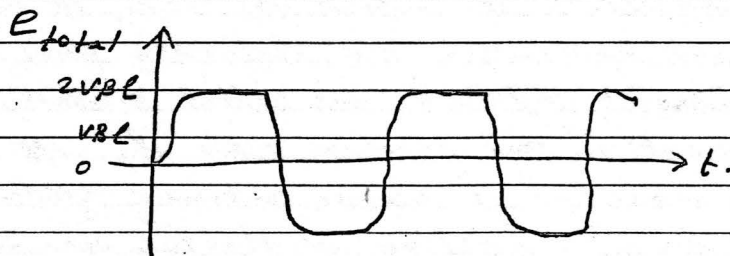
$$e_{dc} = (v \cdot B) \cdot l$$

1.4) segment da

$$e_{ad} = 0 \quad \text{because of the } (v \cdot B) \perp l.$$

$$E_{ind} = e_{ba} + e_{cd} + e_{dc} + e_{ad}$$

$$E_{ind} = 2 \cdot v \cdot B \cdot l$$



The voltage generated in the machine is equal to the product of the flux inside the machine & The speed of rotating of the machine, multiplied by a constant representing the mechanical construction of the machine in General The voltage in any real machine will depend on the same 3 factors

- ① The flux in the machine.
- ② The speed of rotation.
- ③ A constant representing the construction of the machine.

The induced Torque in the rotating loop:-

$$\tau_{ind} = 2 r i l B.$$

$$A_p = \pi r l \quad \& \quad \phi = A_p \cdot B$$

$$= \frac{2}{\pi} \phi i$$

The Torque in any machine will depend on the:-

- ① The flux in the machine.
- ② The current. " " "
- ③ A constant representing the construction of the machine.

Commutation is

is the process of converting the AC voltages & currents in the rotor of a DC machine to DC voltages & currents at its terminals, it is the most critical part of the design & operation of any DC machine

The rotating segments to which the loops are attached are called commutator segments & the stationary pieces that ride on top of the moving segments are called brushes, The commutator segments in real machine made of copper bars. The brushes are made of a mixture containing graphite so that they cause very little friction as they rub over the rotating commutator segments.

Commutation & Armature construction in real DC machine

There are several ways in which the loops on the rotor can be connected to its commutator segments, These different connections affect :-

- ① The number of parallel current paths within the rotor.
- ② The output voltage of the rotor.
- ③ The Number & position of the brushes.



## THE Rotor (armature) coils :-

Each coil consists of a number of turns (loops) of wire each turn insulated from the other turns & from the Rotor slots each side of turn is called a conductor.

The Number of conductors in machine armature is given by

$$Z = 2CN_c$$

Z - number of conductors on rotor.

C - " of coils on Rotor.

N<sub>c</sub> - " " turns per coil.

Normally, a coil spans 180° electrical degrees, The relationship between the electrical angle & mechanical angle.

$$\theta_e = \frac{P}{2} \theta_m$$

$\theta_e$  - electrical angle; degrees

$\theta_m$  - mechanical angle; degrees

P - Number of magnetic poles.

Full-pitch coil :- if a coil spans 180° electrical degrees, The voltages in the conductors on either side of the coil will be exactly the same in magnitude & opposite in direction at all times.

Fractional-pitch coil :- sometimes a coil is built that spans less than 180 electrical degrees.

Pitch factor :- The amount of a chording in a winding is described by a pitch factor.

$$p = \frac{\text{electrical angle of coil}}{180^\circ} \times 100\%$$

Some times a small amount of chording will be used in dc rotor windings to improve commutation.



## Connections to the Commutator segments:-

armature windings are classified according to the sequence of their connections to the commutator segments:-

- ① Lap winding
- ② wave winding
- ③ The frog-leg winding.

## Problems with Commutation in real Machine:-

- ① Armature reaction.
- ②  $L \frac{di}{dt}$ .

## THE construction of DC Machines:-

The structure of dc machine consists of 2 parts:- Stator or stationary part and the rotor or rotating part. The stationary part of the machine consists of the frame for physical support & The pole pieces as flux path in machine.

The ends of the pole pieces that are near the rotor to distribute mag. flux over the rotor surface. & These ends are called the pole shoes. & The distance between the pole & the rotor is called the air Gap.

There are 2 principal windings on a dc machine:- The armature windings & field windings. The armature windings are defined as the windings in which a winding voltage is induced. & the field windings as a windings that produce the main magnetic flux in the machine.

## Solutions to the Problems with Commutation:-

- ① Brush shifting.
- ② Commutating poles or interpoles.
- ③ Compensating windings



## \* Armature Reaction:-

If connect a load to the terminals of the machine, and a current will flow in its armature windings. This current flow will produce a magnetic field of its own, which will distort the original magnetic field from the machine's poles. This distortion of the flux in a machine as a load is increased is called armature reaction. it causes 2 serious problems in DC machines:-

① The first caused by armature reaction is neutral-plane shift. &  $E_{ind} = 0$  because of the velocity of the rotor wires is parallel to the magnetic flux lines.

② The second major problem caused by armature reaction is called flux weakening.

$V = L \frac{di}{dt}$  The rate of change in  $I$  with respect to time in the shorted loop

This high voltage will causes sparking at the brushes & resulting in the same arcing problems that the neutral-plane shift causes.

## \* The internal Generated voltage & induced Torque Equations of DC machine:

The induced voltage in any machine depends on:-

- ① The flux in the machine.
- ② The speed of rotor.
- ③ A constant depending on the construction of the machine.

$E_{ind} = e = v B L$  in single conductor.

The Total output voltage:-

$$E_A = \frac{Z v B L}{a}$$

$Z$  - number of conductors  
 $a$  - number of path current





# لجنة الميكانيك - الإتجاه الإسلامي

2

$$E_A = \frac{Z r \omega B l}{a}$$

$v = r\omega$ ,  $r$  - is radius of rotor.

$$\phi = B A_p$$

$A_p$  - the Area of poles as a cylinder. (pole).

$$A = 2\pi r l.$$

$$A_p = \frac{A}{p} = \frac{2\pi r l}{p}$$

for  $p$ - poles.

$$E_A = \frac{Z r \omega B \cdot l}{a}$$

$$= \left( \frac{Z p}{2\pi a} \right) \left( \frac{2\pi r l B}{p} \right) \omega.$$

$$E_A = \frac{Z p}{2\pi a} \cdot \phi \omega.$$

$$E_A = K \phi \omega.$$

$$K = \frac{Z p}{2\pi a}.$$

The conversion from revolutions per minute to radians per second is: -

$$\omega = \frac{2\pi n}{60}$$

The induced Torque in DC machine depends on:-

- ① The Flux  $\phi$ .
- ②  $\propto$  armature current  $I_A$ .
- ③ A constant depending on the construction of the machine.

The Torque in any conductor under poles

$$T_{\text{cond}} = r I_{\text{cond}} l \cdot B.$$



The Total current in (a) current paths.

$$I_{cond} = I_A / a.$$

then

$$\tau_{cond} = \frac{r I_A l B}{a}.$$

for (Z) conductors the  $\tau_{ind} =$

$$\tau_{ind} = Z r l B I_A / a$$

The flux per pole

$$\phi = B A_p = \frac{B (2\pi r l)}{p} = \frac{2\pi r l B}{p}$$

$$\tau_{ind} = \frac{Z P}{2\pi a} \phi I_A = [K \phi I_A] \text{ Finally.}$$

$$K = \frac{Z P}{2\pi a}$$

Power Flow & Losses in DC machines: -

The efficiency of dc machine: -

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out} - P_{loss}}{P_{in}} \times 100\%$$

The losses in dc machines: -

- ① Electrical or copper losses  $(I_A^2 R_a + I_f^2 R_f)$
- ② Brush losses.  $P_{BD} = V_{BD} I_A$ .
- ③ core losses Hysteresis & eddy currents
- ④ mechanical losses. friction & windage.
- ⑤ stray losses (load). = 1% of full load.

# لجنة الميكانيك - الإتجاه الإسلامي

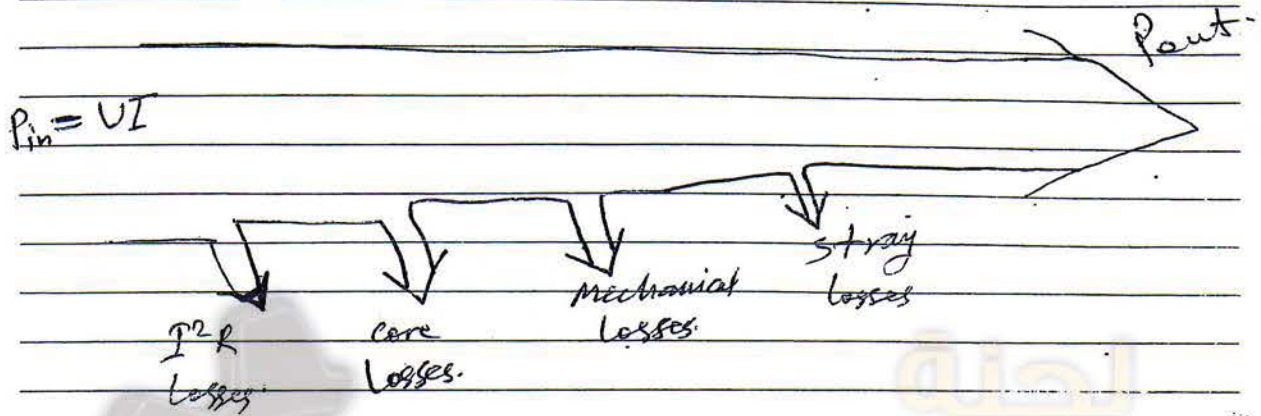
4

Brush losses

$$P_{BD} = V_{BD} \cdot I_a$$

$V_{BD}$  - brush voltage drop  
 $P_{BD}$  - brush drop losses

The power-flow Diagram:-



The Mechanical power that is converted is given by:-

$$P_{conv} = T_{ind} \cdot \omega_m$$

electric power produced is given by:-

$$P_{conv} = E_A \cdot I_A$$

# لجنة الميكانيك - الإتجاه الإسلامي

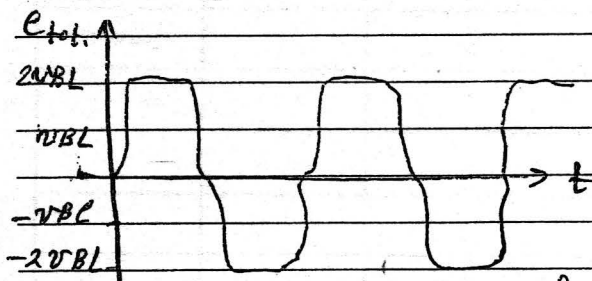
① How can the machine be made to produce a dc voltage instead of the AC voltage?

② Armature construction.

③ The construction of DC Machines.

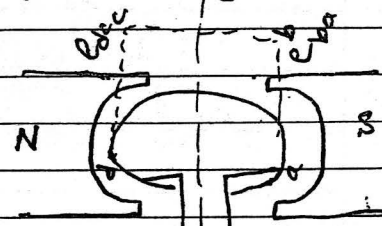
④ Energy losses.

① From fig-1, the voltage out of the loop is alternately a constant positive value and a constant negative value. How can the machine be made to produce a dc voltage instead of the AC voltage? One way to do this is shown in fig-2. Here two semi-circular conducting segments are added to the end of the loop, & two fixed contacts are set up at an angle such that at the instant when the voltage in the loop is zero, the contacts short-circuit the two segments. Every time the voltage of the loop switches direction, the contacts also switch connections, and the output of the contacts is always built up in the same way (fig-3) this connection-switching process is known as commutation. The rotating semi-circular segments are called commutator segments & the fixed contacts are called brushes.

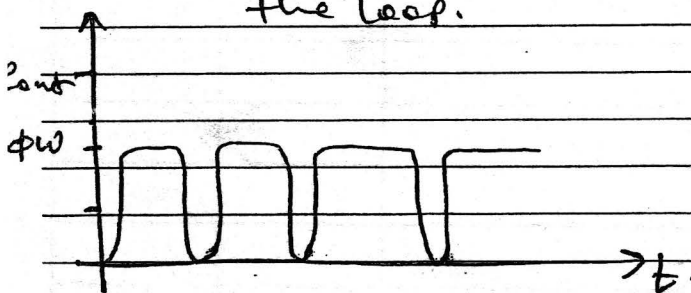


The output voltage of the loop.

(Fig-1)



Producing a dc output from the machine.



(Fig-3)

the resulting output voltage

(Fig-2)

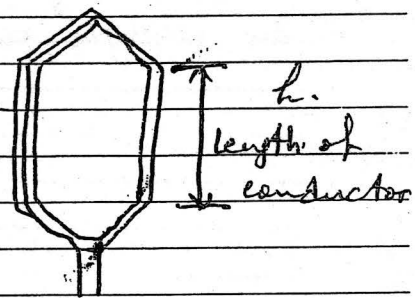
## ② Armature construction:-

\* The rotor coils :-

- each coil consists of a number of turns (loops) of wire.
- each side of a turn is called a conductor.
- coils are inserted into the armature slots.

i

$N_c$  - turns  
insulated from  
each other



The number of conductors

$$Z = 2C N_c$$

$Z$  - number of conductors on rotor.

$C$  - " " " coils " "

$N_c$  - " " " turns per coil.

Normally, a coil spans  $180^\circ$  electrical degrees, This means that when one side is under the center of a given magnetic pole, the other side is under the center of ~~the~~ a pole of opposite polarity

The relationship between the electrical angle & mechanical angle

$$\theta_e = \frac{P}{2} \theta_{mech}$$

where  $P$  - number of magnetic poles

if a coil spans  $180^\circ$  electrical degrees The voltage in the conductor on either side of the coil will be exactly the same in magnitude & opposite in direction at all times. such a coil is called full-pitch coil

some times a coil is built that spans less than  $180^\circ$  such a coil is called a fractional-pitch coil, & a rotor winding wound with fractional-pitch coils is called a chorded winding The amount of chording in a winding is described by a pitch factor  $p$ .

$$p = \frac{\text{electrical angle of coil}}{180^\circ} \cdot 100\%$$

some times a small amount of  $\phi$  chording will be used in dc rotor windings to improve commutation.

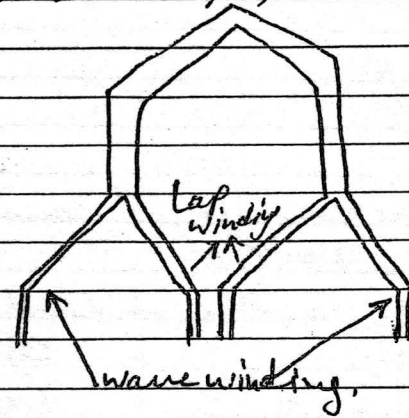


## \* Connections to the Commutator Segments:-

There are two basic sequences of armature winding winding connections:-

- Lap windings
- wave " (series).

& in addition, there is a ~~third~~ a third of winding, called a frog-leg winding which combines lap & wave windings on a single rotor.



## ③ The construction of DC machine:-

The physical structure of the machine consists of 2 parts :-

Stator & Rotor.

Stator consists of the frame for physical support & pole pieces - provide magnetic flux path in the machine & pole shoes - ends of poles, over the rotor surfaces to distribute the flux over the rotor surface.

The distance between the rotor surface & pole face is called air gap.

Rotor consists of a shaft from steel bar, armature windings

Commutator & Brushes - commutator is made of copper bars insulated by a mica-type material. The brushes of the machine are made of a carbon, graphite, or Graphite or a mixture of Graphite & Carbon. they have a high conductivity to reduce electrical losses & also friction to reduce wearing.

winding insulation material.



# لجنة الميكانيك - الإتجاه الإسلامي

## Losses in DC machine:-

- electrical or copper losses.

$$\text{Armature losses : } P_A = I_A^2 R_A$$

$$\text{Field losses : } P_F = I_F^2 R_F$$

- Brush losses

$$P_{BD} = V_{BD} \cdot I_A$$

∴ The brush voltage drop = 2V. (about).

- Core losses:- are the hysteresis & eddy currents occurring in the metal of motor.

$$\& \text{ These losses } \propto B^2 \& n^{1.5}$$

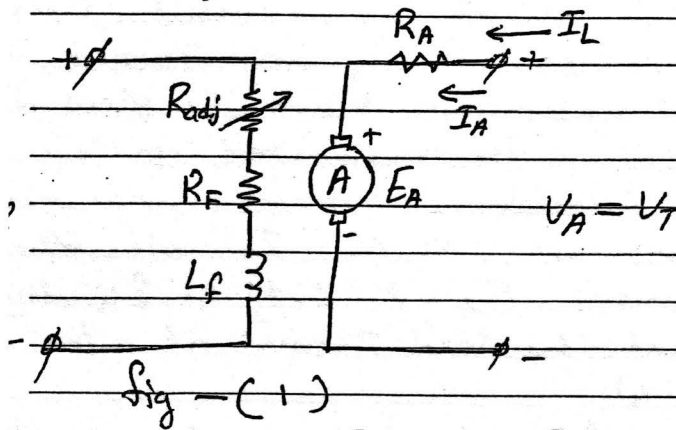
- Mechanical losses : associated with mechanical effects  
two types of mech. losses :- Friction & Windage.  
friction from bearings is caused but windage losses from the friction between the rotating parts in the machine & the air inside the motor casing.

- Stray losses : are the losses cannot be placed in one of the previous categories, for most machines stray losses are taken to be 1 percent of full-load.



# لجنة الميكانيك - الإتجاه الإسلامي

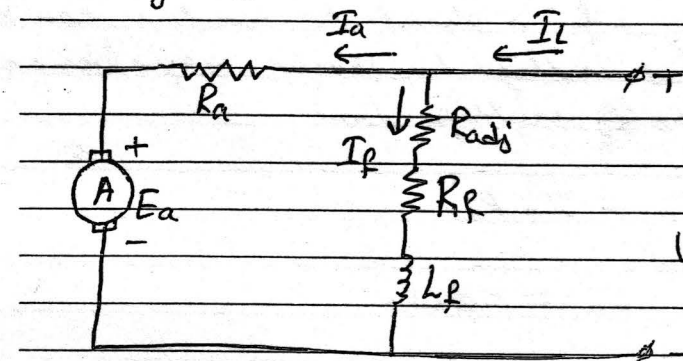
Separately excited & shunt motors:-



$$I_f = V_f / R_f$$

$$V_T = E_a + I_a R_a$$

$$I_L = I_a$$



$$I_f = V_T / R_f$$

$$V_T = E_a + I_a R_a$$

$$I_L = I_a + I_f$$

Fig - (2)

Equivalent circuit of a separately excited dc motor & a shunt dc motor.

sep. exc. motor is a motor whose field circuit is supplied from a separate const-voltage power supply while a shunt dc motor is a motor whose field circuit gets its power directly across the armature terminals of the motor. when the supply voltage to a motor is assumed constant, there is no practical difference in behavior between these two machines.

The Kirchhoff's voltage law (KVL) equation for the arm. circuit of these motors is:-

$$V_T = E_a + I_a R_a$$

Torque - speed ch-c :-

for a motor the output quantities are shaft Torque & Speed.





How does a shunt dc motor respond to a load ?

If the load on the shaft is increased, then the load Torque  $T_L$  will exceed the induced Torque  $T_{ind}$  in the machine & the motor will start to slow down. when the motor slows down, its Generated voltage drops ( $E_a = K\phi\omega$ ), so the armature current in the motor ( $I_a = (V_T - E_a)/R_a$ ) increases. As the  $I_a$  rise the induced torque in the motor increases ( $T_{ind} = K\phi I_a$ ) & finally the induced Torque will equal the load Torque at a lower mechanical speed of rotation  $\omega$ .

The output ch-c of a shunt dc motor can be derived from the induced voltage & Torque equations of the motor plus Kirchhoff's voltage law.

$$V_T = E_a + I_a R_a.$$

The induced voltage  $E_a = K\phi\omega$ .

$$V_T = K\phi\omega + I_a R_a.$$

Since  $T_{ind} = K\phi I_a \Rightarrow$  current can be expressed as.

$$I_a = T_{ind} / K\phi.$$

$$K = PN / 2\pi a.$$

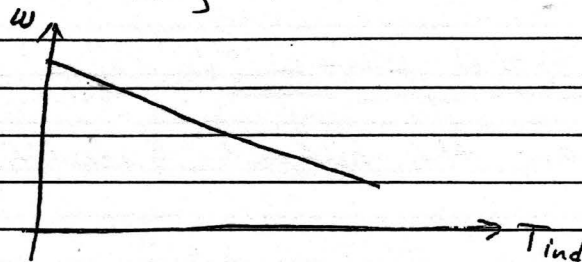
Combining equations :-

$$V_T = K\phi\omega + \frac{T_{ind}}{K\phi} \cdot R_a.$$

Finally

$$\omega = \frac{V_T}{K\phi} - \frac{R_a}{(K\phi)^2} T_{ind}.$$

This equation is just a straight line with a negative slope. The resulting T-s. ch-c of a shunt dc motor :-



\* if the terminal voltage is not constant then the voltage variations will affect the shape of the Torque-speed ch-c.

\* another effect internal to the motor that can also affect the shape of the T-S. ch-c curve is armature reaction, if a motor has armature reaction then as its load increases, the flux weakening effects reduce its flux & the speed will increase at any given load over the speed it would run at without armature reaction

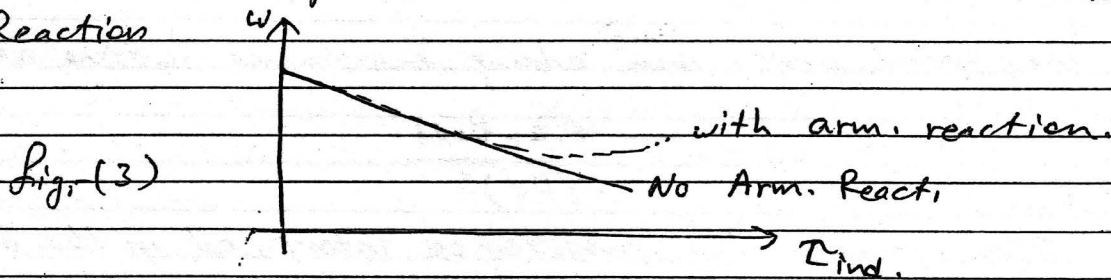


Fig. (3)

if a motor has compensating winding, there will be no flux-weakening problems in the machine & the flux in the machine will be constant.

in The Fig. (4) can be seen a set of speed-Torque ch-cs for a shunt dc motor obtained by inserting different values of resistance in the arm. circuit. from the speed equation it is also seen that when the  $T_{ind} = 0$  all the ch-cs path through the same point on the ordinate axis. The motor speed at this point has a definite value independent of the resistance included in the circuit. This speed is termed the ideal no load speed,  $\omega_0$  & is found from the equation

$$\omega_0 = V / K\phi.$$

at  $\omega_0$  when the armature current is zero the emf. of the armature is equal & opposite to the voltage applied to the armature.

The second member of the speed equation represents the drop in the speed with respect to the ideal no-load speed

$$\Delta\omega = T \frac{R}{(K\phi)^2}$$



# لجنة الميكانيك - الإتجاه الإسلامي

The speed equation can be expressed as:-

$$\omega = \omega_0 - \Delta\omega$$

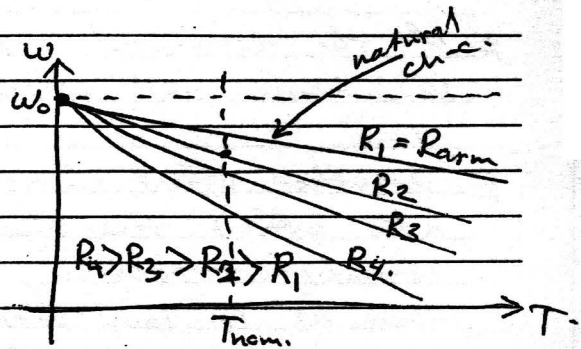


Fig-(4)

The speed drop when adding Resistance in the arm. circuit.

$$\Delta\omega = T \frac{R_a + R_{add}}{(K\phi)^2}$$

The greater the resistance interposed in the armature circuit the less hardness of the ch-c.

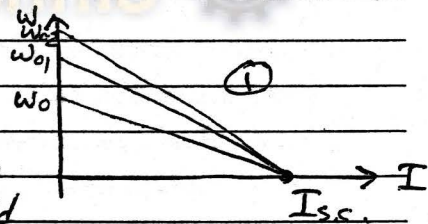
Speed control of shunt d.c. motors:-

The ways in which the speed can be controlled are:-

- ① by varying the excitation  $I$ .
- ② " field weakening.
- ③ " varying the applied voltage.
- ④ " Inserting  $R$  in armature circuit.

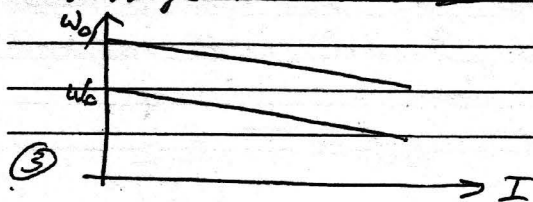
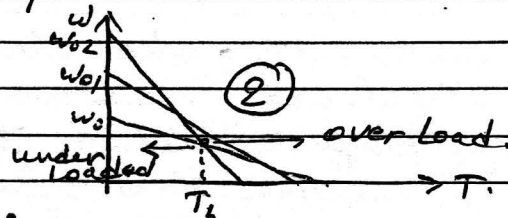
① when  $\omega = 0 \Rightarrow 0 = \frac{V - IR}{K\phi} \Rightarrow I = V/R \Rightarrow I = I_{sc}$

② These  $\omega$ - $T$  ch-c do not meet at the same point on the abscissa axis because the torque corresponding to short circuit falls with decrease in field flux as it is determined from equation.



$$T_{sc} = K\phi I_{sc}$$

③ by changing the applied voltage to the motor



# لجنة الميكانيك - الإتجاه الإسلامي

## DC - Series motors :-

D.C. series motors whose field windings consist of a relatively few turns connected in series with the armature circuit, equivalent circuit shown in Fig-1 the armature, field & line current are all the same.

The Kirchhoff's voltage law equation for this motor is :-

$$V_T = E_A + I_A (R_A + R_S)$$

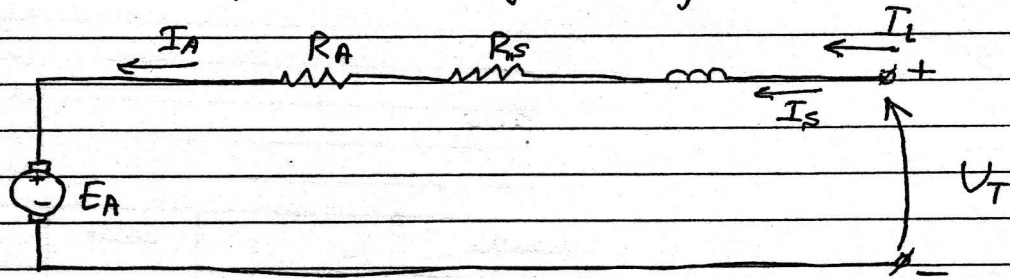


Fig-1

$$I_A = I_S = I_L$$

$$V_T = E_A + I_A (R_A + R_S)$$

The T-S ch-c of a series dc motor is very different from that of the shunt motor, due to the fact that the flux is directly proportional to the armature current, at least until saturation is reached.

The load on the motor increases, its flux increases too, an increase in  $\phi$  in the motor causes a decrease in its speed.

The dc series motor has a sharply drooping T-S ch-c.

The induced induced Torque in this machine :-

$$\tau_{ind} = K \phi I$$

The  $\phi$  proportional directly to its  $I_A$  (until the metal saturation).

$$\phi = c I_A$$

where :-  $c$  - the constant of proportionality.

The induced Torque =

$$\tau_{ind} = K \phi I_A = K.c.I_A^2$$



# لجنة الميكانيك - الإتجاه الإسلامي

Series motor gives more torque per ampere than any other dc motor. It's used in applications requiring very high torques. Examples of applications are the starter motors in cars, elevators, tractor motors in locomotives.

The T-S ch-c. of ~~the~~ series dc motor:

analysis will be based on the assumption of a linear magnetization curve.

$$\phi = c I_A$$

$$V_T = E_A + I_A (R_A + R_S)$$

$$\tau_{ind} = K_c I_A^2 \Rightarrow I_A = \sqrt{\tau_{ind} / K_c}$$

$$V_T = K \phi \omega + \sqrt{\frac{\tau_{ind}}{K_c}} (R_A + R_S)$$

$$I_A = \phi / c$$

$$\tau_{ind} = \frac{K}{c} \phi^2$$

$$\phi = \sqrt{\frac{c}{K}} \sqrt{\tau_{ind}}$$

$$V_T = K \sqrt{\frac{c}{K}} \sqrt{\tau_{ind}} \omega + \sqrt{\frac{\tau_{ind}}{K_c}} (R_A + R_S)$$

$$\sqrt{Kc} \sqrt{\tau_{ind}} \omega = V_T - \frac{R_A + R_S}{\sqrt{Kc}} \sqrt{\tau_{ind}}$$

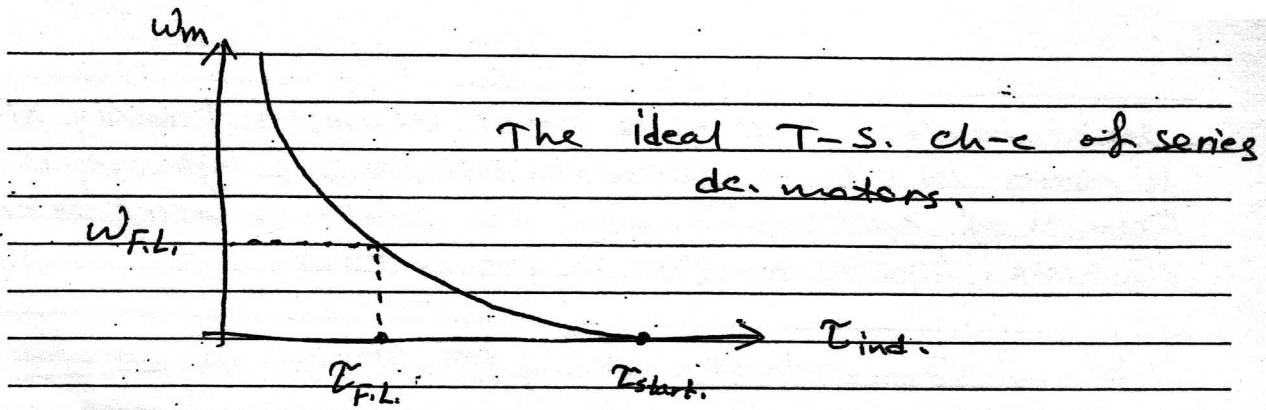
$$\omega = \frac{V_T}{\sqrt{Kc} \sqrt{\tau_{ind}}} - \frac{R_A + R_S}{Kc}$$

The resulting T-S. ch-c is:-

$$\omega = \frac{V_T}{\sqrt{Kc}} \frac{1}{\sqrt{\tau_{ind}}} - \frac{R_A + R_S}{Kc}$$



# لجنة الميكانيك - الإتجاه الإسلامي



- \* when the torque goes to zero the speed of this motor goes to infinity.
- \* The Torque can never go to zero because of the mechanical & stray & core losses that must be overcome.
- \* if no load is connected to the motor, it can turn fast enough to ~~be~~ seriously damage it self.

## Speed control of series dc. motors:-

① by changing the terminal voltage of the motor

if the  $V_T$  is increased, the first term in equation of speed is increased, resulting in a higher speed for any given torque.

② by the insertion of a series resistor into the motor circuit.

but this techniques is very wasteful of power & is used only ~~for~~ during the start-up of some motors.

## DC motor starters:-

dc motor must have some special control & protect equipment, the purposes of this equipment are:-

- ① To protect motor against damage due to short circuits
- ② " " " " " " from long term overload
- ③ " " " " " " Starting current



# لجنة الميكانيك - الإتجاه الإسلامي

for example 50-hp, 250V motor, has an armature resistance  $R_A = 0.06 \Omega$  and full-load  $I$  less than 200 A.

$$I_{starting} = 250V / 0.06 \Omega = 4167 A.$$

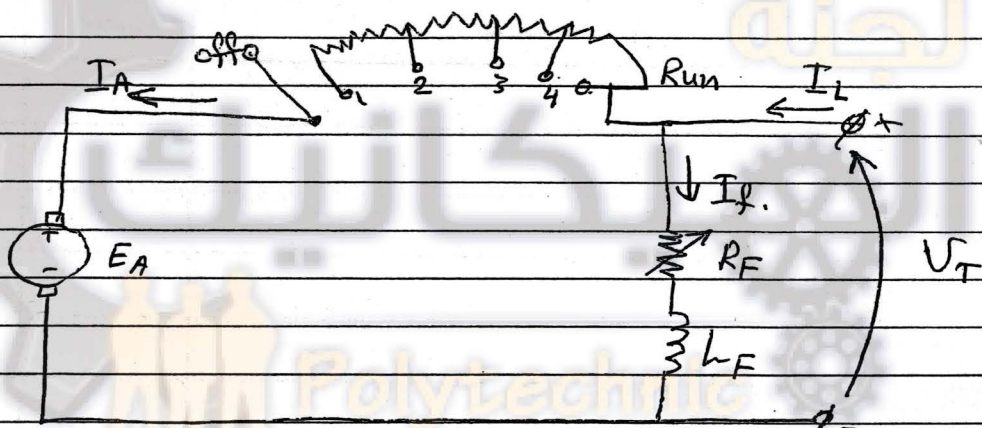
$I_{st}$  is over 20 times the rated current.

a solution to the problem is to insert a starting resistor in series with the armature to limit the current flow until  $E_a$  can build up to do the limiting. [Inserting  $R$  just for starting period due to losses & drop the S-T. ch-c].

starting resistor can be cut out of the circuit:-

- by automatic starter circuits.

- " a person - (human error).



A manual dc motor starter.

## D.C. Generators :-

Are dc machines used as Generators, the difference between a generator & motor except for the direction of power flow.

## Types of dc. Generators:-

According to the manner in which their field flux is produced :-

- ① Separately excited Generator
- ② Shunt Generator
- ③ Series "
- ④ Compounded " (additive).
- ⑤ Differentially Compounded Generator (subtractive)

The Types of Generators differ in their terminal voltage - current characteristics.

DC Generators compared by their :-

- ① Voltages
- ② power Ratings
- ③ efficiencies
- ④ Voltage Regulations

$$\text{Voltage Regulation } VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \cdot 100\%$$

(+) positive VR - means a drooping ch-c.

(-) negative VR - " a rising " .

Generators are driven by a mechanical power which called prime mover :- may be -

- ① steam turbine
- ② diesel engine.
- ③ electric motor.

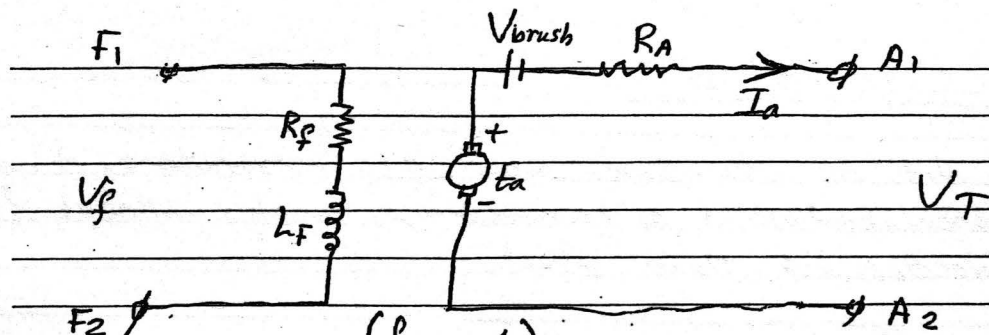


The differ from motors is the direction of current ( $I_a$ ) & brush loss are reversed.





# لجنة الميكانيك - الإتجاه الإسلامي



(fig-1)  
Equivalent circuit of DC Generator,

## Control of terminal voltage:-

$$V_T = E_a - I_a R_a$$

By controlling ( $E_a$ ), can ( $V_T$ ) be controlled

$$E_a = K\phi\omega$$

Two ways to control the voltage of Generator

① change the speed of rotation.

$$\omega \uparrow \Rightarrow E_a \uparrow = K\phi\omega \uparrow, \text{ so } V_T = E_a \uparrow - I_a R_a \Rightarrow V_T - \text{increasing.}$$

② change the field flux.

$$I_F = V_F / R_F \downarrow \Rightarrow I_F \uparrow = \phi \uparrow$$

$$E_a \uparrow = K\phi \uparrow \omega \Rightarrow V_T \uparrow = E_a \uparrow - I_a R_a \Rightarrow \text{Increasing.}$$

The speed range of prime mover is limited so the ( $V_T$ ) commonly controlled by changing ( $I_f$ ).

## Analysis of a separately excited D.C. Generator :-

$$V_T \xrightarrow[\text{function}]{\text{nonlinear}} E$$

magnetomotive force  $I_{net} = N_F I_F - f_{AR}$   
 $f_{AR}$  - of armature reaction.

The equivalent  $I_F^* = I_F - \frac{f_{AR}}{N_F}$  (effective field current)  
 difference between speeds.

$$\frac{E_a}{E_{a0}} = \frac{n}{n_0}$$



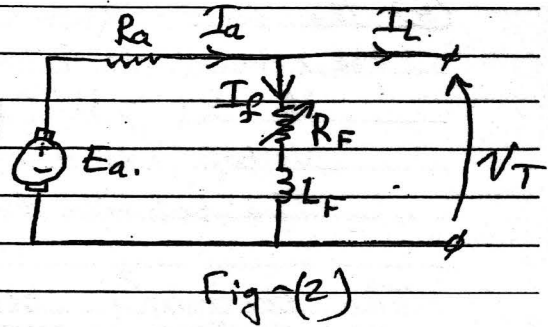
The Shunt dc Generator:-

$$I_A = I_f + I_L$$

$$V_T = E_a - I_A R_a$$

$$I_f = V_T / R_f$$

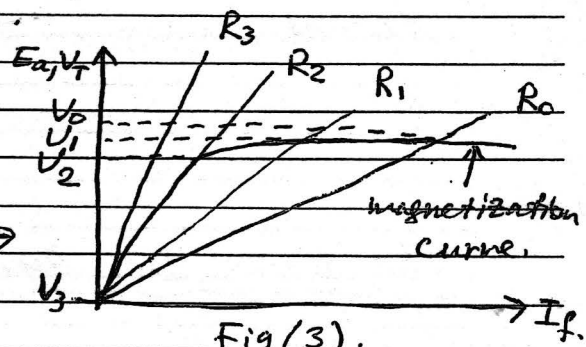
$$E_a = K\phi\omega$$



Voltage control:-

- ① changing the shaft speed  $\omega_m$ .
- ② changing  $R_f \Rightarrow I_f$ .

Field resistance with Generator  
 $V_T$  →



what if a shunt Generator is started & no voltage builds up? what could be wrong? There are several possible causes:

- ① There may be no residual magnetic flux in the Generator to start the process going. if the residual flux = 0 Then  $E_a = 0$ . & The (V) never buildup, if this problem occurs disconnect the field from arm. circuit & connect it directly to an external source dc such as a battery the current flow from external source will leave a residual flux in the poles & then will be normal starting.
- ② The direction of rotation of the Generator may have been reversed. or the connections of the field may have been reversed.

note voltage can buildup because of the residual  $\phi$  produces  $E_a$ .  $E_a$  produces  $I_f$  which produces a flux  $\phi$  opposing the  $\phi_{res}$ , Instead of adding to it. & The flux actually decreases below  $\phi_{res}$ .

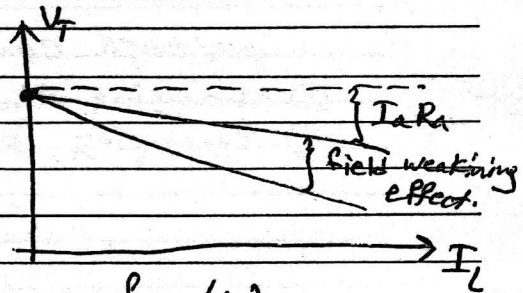


# لجنة الميكانيك - الإتجاه الإسلامي

- ③ The field resistance may be adjusted to a value greater than the critical resistance. Fig(3)  
 if  $R_f > R_c$  (the critical Resistance) then the Generator voltage will never build up. ~~and~~ such as ( $R_c$ )  
 The solution to this problem is to reduce ( $R_f$ ).

## The terminal ch-c of a shunt dc Generator:-

The voltage regulation is worse than the VR of the separately excited.

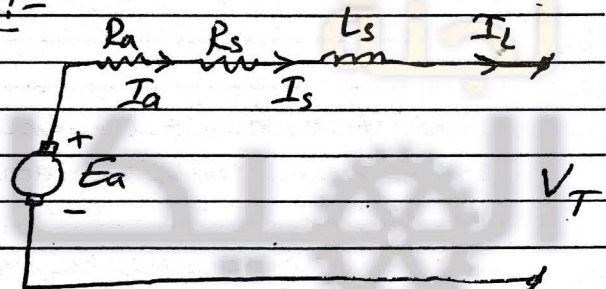


Fig(4)

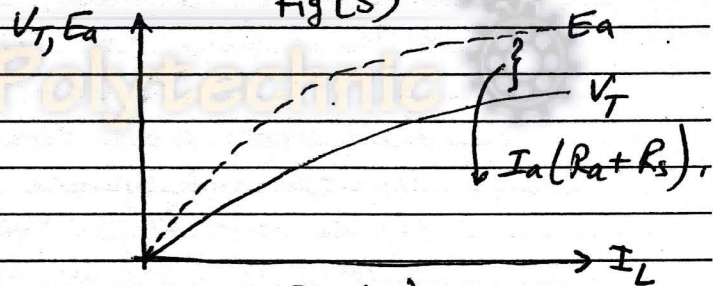
## The series DC Generators:-

$$I_a = I_s = I_L$$

$$V_T = E_a - I_a(R_a + R_s)$$



Fig(5)



Fig(6)

## Example:-

A sep. excited dc Generator is rated at 172 kW, 430V, 400 A & 1800 RPM, is shown in Fig.(7) & its Mag. curve shown in Fig-(8). This machine has the following data:-

$$R_a = 0.05 \Omega ; R_f = 20 \Omega ; R_{adj} = 0 \text{ to } 300 \Omega ; V_T = 430 \text{ V}$$

$$N_f = 1000 \text{ turns per pole.}$$

- ① if the  $R_{adj}$  in the Generators field circuit is adjusted to 63Ω & Generators prime mover is driving it at 1600 rpm. what

$$R_{F_{tot}} = R_F + R_{adj} = 83 \Omega$$

Field current  $I_F = V_F / R_F = 430 / 83 \Omega = 5,2 \text{ A}$ .

from mag. curve this much current would produce a voltage  $E_{A0} = 430 \text{ V}$  at a speed 1800 rpm, since this Generator is actually turning at  $n_m = 1600 \text{ rpm}$ ,  $E_a$  will be

$$\frac{E_a}{E_{A0}} = \frac{n}{n_0} \Rightarrow E_a = \frac{1600}{1800} \cdot 430 = 382 \text{ V}$$

$E_A$	100	200	300	380	410	440	430	440	450	460	480
$I_F$	1	2	3	4	4,75	5	5,2	6	6,15	7	10

at no-load conditions  $V_T = E_a \Rightarrow V_T = 382 \text{ V}$

② if a 360 A load is connected to its terminals, what would its voltage be? (assume that the Generator has compensating winding)

$$V_T = E_a - I_a R_a = 382 - 360 \cdot 0,05 = 364 \text{ V}$$

③ Point 2 but the Generator does not have compensating windings, assume that its armature reaction at this load is 450 A.turns

the effective field current

$$I_F^* = I_F - \frac{I_a R_{AR}}{N_F} = 5,2 \text{ A} - \frac{450 \text{ A.turns}}{1000 \text{ turns}} = 4,75 \text{ A}$$

from mag. curve  $E_{A0} = 410$

so  $E_a$  at 1600 rpm =

$$\frac{E_a}{E_{A0}} = \frac{n}{n_0} \Rightarrow E_a = \frac{1600}{1800} \cdot 410 = 364 \text{ V}$$

$$V_T = E_a - I_a R_a = 364 - (360 \times 0,05) = 346 \text{ V}$$

$V_T$  lower than before due to armature reaction.

④ what adjustment could be made to the Generator to restore its  $V_T$  to the value found in part ②.

6

The  $V_T$  has fallen, so to restore it to original value the voltage of generator must be increased, This requires an increase in  $E_a$  which implies that  $R_{adj}$  must be decreased to increase field current of the generator.

⑤ How much  $I_f$  would be needed to restore the terminal voltage to its no-load value? (machine has compensating winding what is the required value for the resistor are adjustable to accomplish this.

For  $V_T$  to go back up to 382V

The req.  $E_a$

$$E_a = V_T + I_a R_a$$

$$= 382 + 300 * 0.05 = 400 \text{ V}$$

to get a voltage 400 V at 1600 RPM

The voltage at 1800 RPM

$$\frac{E_a}{E_{a0}} = \frac{n}{n_0} \Rightarrow E_{a0} = \frac{1800}{1600} * 400 = 450$$

From mag. curve  $V_T \Rightarrow I_f = 0.15$

$$R_f + R_{ext, adj} = \frac{V_f}{I_f}$$

$$20 \Omega + R_{ext, adj} = \frac{470}{0.15}$$

$$R_{adj} = 49.9 \Omega \approx 50 \Omega$$

For the same  $R_f$  and  $I_f$ , the generator with Arm. reaction has a lower output voltage than a generator without Ar.



Example: (1)

Fig (1) shows a 100-hp, 250-V, 1200 r/min shunt dc motor with an  $R_a = 0,03 \Omega$  &  $R_f = 41,67 \Omega$ . The Arm. reaction ignored mechanical & core losses may be assumed to be negligible for the purposes of this problem, The motor is assumed to be driving a load with a line current of 126 A & an initial speed of 1103 r/min,  $I_a$  drawn by the motor remains constant.

(1) if a machine's mag. curve is shown in fig (2), what is the motor speed if the field resistance is raised to  $50 \Omega$ ?

(2) Calculate & plot the speed of this motor as a function of the field resistance  $R_f$  assuming a constant load current.

The motor has an initial <sup>line</sup> current of 126 A, so the initial arm. current is

$$I_{a1} = I_L - I_{f1} = 126 \text{ A} - \frac{250 \text{ V}}{41,67 \Omega} = 120 \text{ A.}$$

The internal Generated Voltage :-

$$E_{a1} = V_T - I_{a1} R_a = 250 - (120 \text{ A} \times 0,03 \Omega) = 246,4 \Omega$$

After the  $R_f$  is increased to  $50 \Omega$  The  $I_f$  will become

$$I_{f2} = V_T / R_f = 250 / 50 = 5 \text{ A.}$$

The Ratio of internal generated voltage at one speed to the  $E$  at another speed :-

$$\frac{E_{a2}}{E_{a1}} = \frac{k \phi_2 n_2}{k \phi_1 n_1}$$

because of the  $I_a$  is assumed const.

$$E_{a1} = E_{a2} \quad \text{then,}$$

$$1 = \frac{\phi_2 n_2}{\phi_1 n_1} \Rightarrow n_2 = \frac{\phi_1}{\phi_2} n_1$$



# لجنة الميكانيك - الإتجاه الإسلامي

From Mag. Curve :-

at  $I_f = 5A \rightarrow E_{ao} = 250V$

$I_f = 6A \rightarrow E_{ao} = 268V$

Therefore the Flux Ratio

$$\frac{\phi_1}{\phi_2} = \frac{268}{250} = 1,076.$$

& The new speed of the motor is

$$n_2 = \frac{\phi_1}{\phi_2} n_1 = (1,076)(1103 \text{ r/min}) = 1187 \text{ r/min}$$

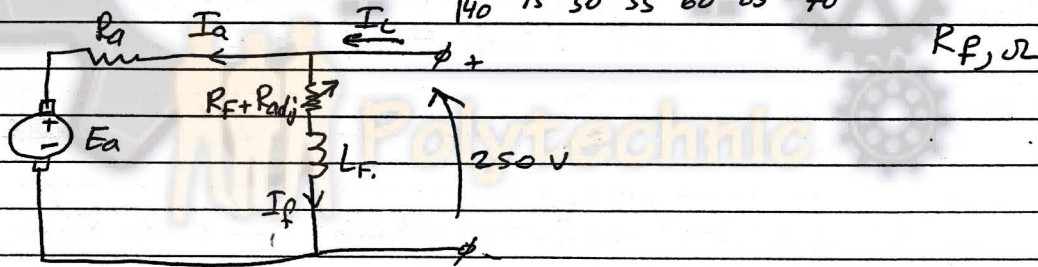
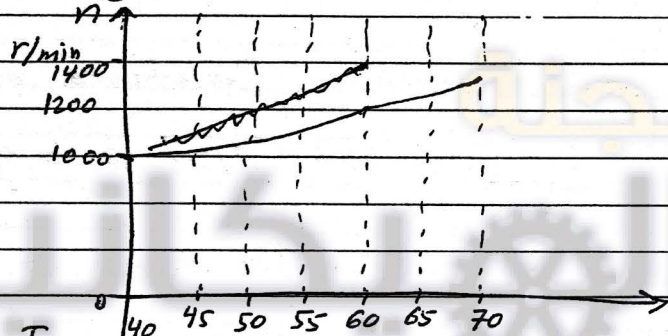


Fig - (1)

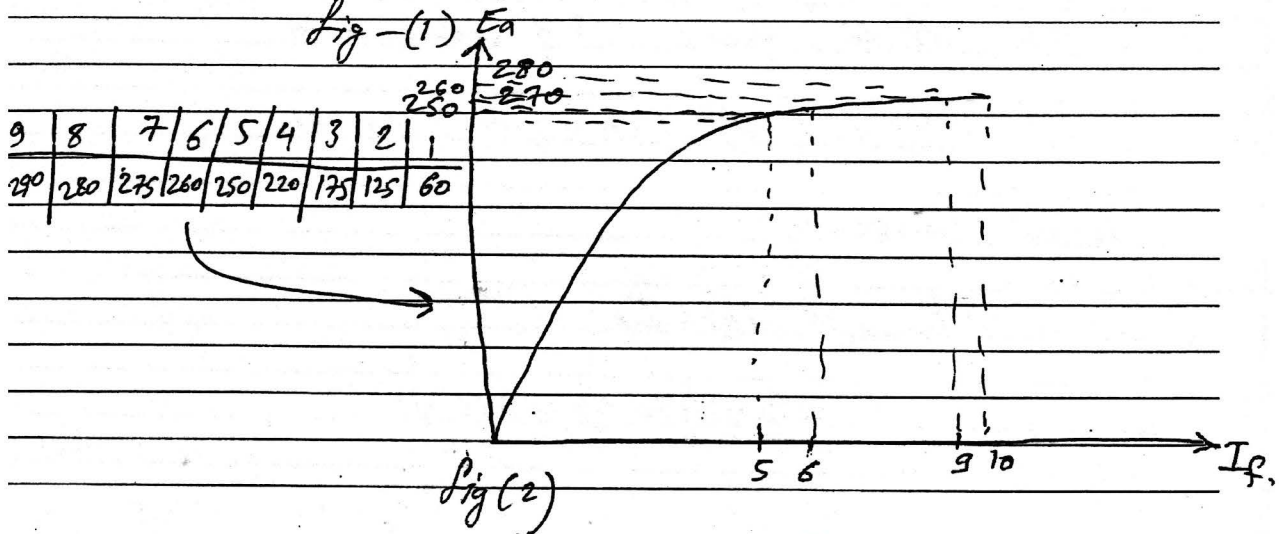


Fig (2)

## Example (2):

A 50 hp, 250 v, 1200 r/min shunt dc motor has a rated armature current of 170 A & a rated field current of a 5 A, when its rotor is blocked, an armature voltage of 10,2 v (exclusive of brushes) produces 170 A of current flow, and a field voltage of 250 v produces a field current flow of 5 A. The brush voltage drop is assumed to be 2 v. At no load with the terminal voltage equal to 240 v, The armature current is equal to 13,2 A, The field current is 4,8 A & The motor's speed is 1150 r/min.

- ① How much power is output from this motor at rated conditions?
- ② what is the motor efficiency?

The armature Resistance of this machine is

$$R_a = 10,2 \text{ v} / 170 \text{ A} = 0,06 \Omega.$$

& The field Resistance is

$$R_f = 250 \text{ v} / 5 \text{ A} = 50 \Omega.$$

At full load the armature  $I^2 R$  losses are:-

$$P_a = (170)^2 (0,06 \Omega) = 1734 \text{ W}.$$

The field circuit  $I^2 R$  losses are

$$P_f = (5 \text{ A})^2 (50 \Omega) = 1250 \text{ W}.$$

Brush losses:-

$$P_{\text{brush}} = V_{BD} \cdot I_a$$

$$= (2 \text{ v}) (170 \text{ A}) = 340 \text{ W}$$





# لجنة الميكانيك - الإتجاه الإسلامي

$$P_{tot} = P_{core} + P_{mech} = 240 \text{ V} (13,2) \text{ A} = 3168 \text{ W}.$$

The  $P_{in}$  at rated load  $\Rightarrow$

$$P_{in} = V_T I_L = (250 \text{ V}) (175 \text{ A}) = 43,750 \text{ W}.$$

The output power  $\Rightarrow$

$$P_{out} = P_{in} - P_{brush} - P_{cu} - P_{core} - P_{mech} - P_{stray}.$$

$$= 43,750 \text{ W} - 340 \text{ W} - 1734 \text{ W} - 1250 \text{ W} - 1368 \text{ W} - (0,01)(43,750 \text{ W})$$

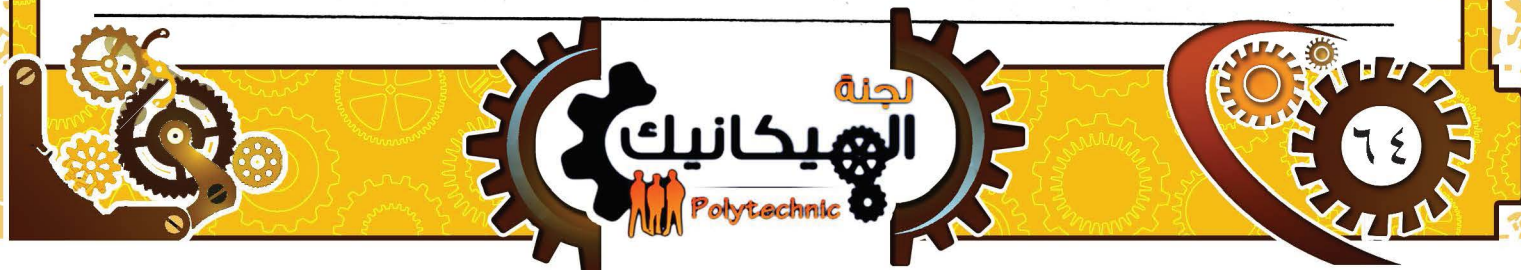
$$= 36,820 \text{ W}.$$

Stray losses = 1% of the input power.

The efficiency at full load of this motor

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\%$$

$$= \frac{36,820 \text{ W}}{43,750 \text{ W}} \cdot 100\% = 84,2\%$$



## Classification :-

There are 2 major classes of AC machines:-

- 1- synchronous machines.
- 2- induction "

→ are motors & Generators whose magnetic field current is supplied by a separate dc power source.

→ are motors & Generators whose field current is supplied by magnetic induction (transformer action) in to their field windings.

## The rotating magnetic field:-

① if two magnetic fields are present in a machine then a torque will be created which will tend to line up the two magnetic fields.

if one mag. field is produced by the stator of an ac machine and the other one is produced by the rotor of the machine, then a torque will be induced in the rotor which will cause the rotor to turn.

② How can the stator magnetic field be made to rotate?

if a 3-phase set of currents, each of equal magnitude and differing in phase by  $120^\circ$ , flows in a 3-phase winding, then it will produce a rotating magnetic field of constant magnitude.

③ To understand the concept of the rotating magnetic field, we will apply a set of currents to the stator & see what happens at specific time. assume that the currents in the 3 coils are given by the equations:-



# لجنة الميكانيك - الإتجاه الإسلامي

$$i_{aa'}(t) = I_m \sin \omega t, A$$

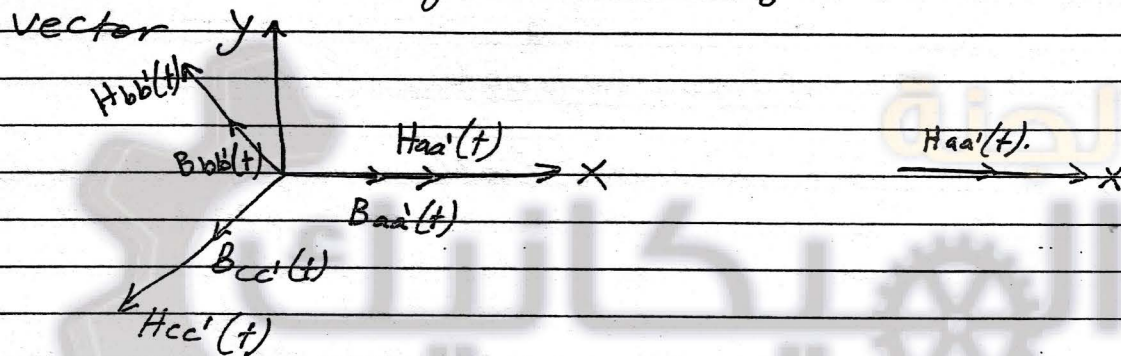
$$i_{bb'}(t) = I_m \sin (\omega t - 120^\circ), A$$

$$i_{cc'}(t) = I_m \sin (\omega t + 240^\circ), A.$$

The  $I$  in coil  $aa'$  flows in to the (a) end of coil & out the  $a'$  end of the coil it produces the mag. field intensity.

$$H_{aa'}(t) = H_m \sin \omega t \angle 0^\circ \text{ A.turn/m.}$$

where  $0^\circ$  - the angle of the magnetic field intensity vector



\* The direction of the mag. field intensity is given by the right-hand rule.

Figure of right hand curl ~~is~~ in the direction of the current flow in the coil, then the resulting magnetic field is in the direction that the thumb points.

\* The magnitude of the mag. field intensity vector  $H_{aa'}(t)$  varies sinusoidally in time but the direction of  $H_{aa'}(t)$  is constant.

\* The mag. field intensity vectors  $H_{bb}(t)$  &  $H_{cc}(t)$  are:-

$$H_{bb}(t) = H_m \sin (\omega t - 120^\circ) \angle 120^\circ \text{ A.turn/m}$$

$$H_{cc}(t) = H_m \sin (\omega t - 240^\circ) \angle 240^\circ \text{ A.turn/m}$$



# لجنة الميكانيك - الإتجاه الإسلامي

The flux densities resulting from these magnetic field intensities are given by equations :-

$$B = \mu \cdot H$$

$$B_{aa'}(t) = B_m \sin \omega t \quad < 0^\circ, T$$

$$B_{bb'}(t) = B_m \sin(\omega t - 120^\circ) \quad < 120^\circ, T$$

$$B_{cc'}(t) = B_m \sin(\omega t - 240^\circ) \quad < 240^\circ, T$$

where  $B_m = \mu H_m$

$$B_{aa'} = 0 \Rightarrow \omega t = 0 \quad \text{at time } \omega t = 0$$

$$B_{bb'} = B_m \sin(-120^\circ) \quad < 120^\circ$$

$$B_{cc'} = B_m \sin(-240^\circ) \quad < 240^\circ$$

The induced voltage (EMF) in the phase coils of the stator:

if the rotor of machine is rotated, a voltage will be induced in the wire loop.

To determine the total voltage  $E_{tot}$  on the loop we will examine each segment of the loop separately & sum all the resulting voltages:-

$$E_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

1-) segment  $ab =$

$$E_{ba} = v \cdot B \cdot l = v B l \sin \theta_{ab} \quad (\text{into the page})$$

2-) segment  $bc \Rightarrow$  because of the  $\mathbf{B} \cdot \mathbf{v}$  is perpendicular to  $\mathbf{l}$

$$E_{cb} = 0$$

3-) segment  $cd$  :-

$$E_{dc} = v B l = v B l \sin \theta_{cd} \quad (\text{out of the page})$$



# لجنة الميكانيك - الإتجاه الإسلامي

4) segment da :-

$$E_{ad} = 0$$

because of  $v_B$  perpendicular to  $l$ .

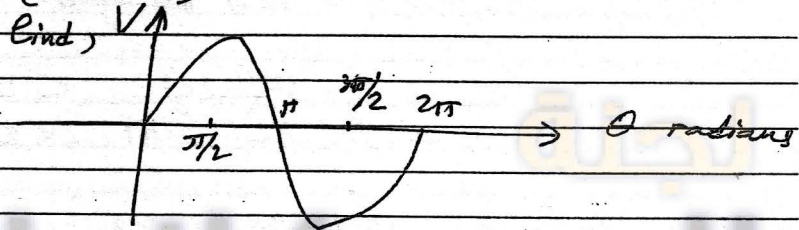
$$E_{ind} = E_{ba} + E_{cb} + E_{dc} + E_{ad}$$

$$= v_B l \sin \theta_{ab} + v_B l \sin \theta_{cd}$$

$$E_{ind} = 2 v_B l \sin \theta$$

$$\theta_{ab} = 180^\circ - \theta_{cd}$$

$$\sin \theta = \sin (180 - \theta)$$



$v = r\omega$  where  $r$  - the radius from axis  
 $\omega$  - angular velocity of the loop

$$E_{ind} = 2r\omega Bl \sin \omega t$$

$$\text{Area } A = 2r l$$

then

$$E_{ind} = A B \omega \sin \omega t$$

$$\phi = A B$$

$$\text{then } E_{ind} = \phi_{max} \omega \sin \omega t$$

The voltage generated in the loop is a sinusoidal whose magnitude is equal to the product of the flux inside the machine & the speed of rotation of the machine.

The induced voltage in machine will depend on:-

- ① The flux in the machine
- ② " speed of rotation.
- ③ A constant representing the construction of the machine (number of loops, - - -)



(1)

The induced Torque in a current-carrying loop:-

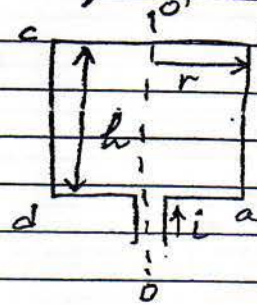
If the current flows in the loop, then a Torque will be induced on the wire loop.

The force on each segment of the loop will be given by equation

$$F = (l \cdot B) \cdot I$$

The Torque on that segment =

$$\tau = F \cdot r \cdot \sin \theta$$



$$\tau = 2 \cdot r \cdot F \cdot \sin \theta$$

induced Torque can be determined by equation:-

$$\tau = K B_{loop} \cdot B_s$$

$$\tau = K B_{rotor} \cdot B_{stator} \cdot \sin \delta \quad \delta = \text{angle between } B_{rot} \text{ \& } B_{stat}$$

The Torque will depend on four factors:-

- ① The strength of the rotor magnetic field.
- ② " " " " external magnetic " .
- ③ The sine of the angle between them.
- ④ a constant representing the construction of the machine.

42) The Relationship between electrical frequency & The speed of magnetic field Rotation:-

$$\omega_e = \frac{P}{2} \omega_m \quad \omega_m = \text{mechanical speed of the magnetic fields in revolutions per minute.}$$

$$f_e = \frac{P}{2} f_m$$

$$\omega_e = \frac{P}{2} \omega_m$$

noting that the  $f_m = \text{rpm}/60$ . (revolution per minute).

$$f_e = \frac{\text{rpm} \cdot P}{120}$$



(2)

AC machines Power flows & losses :-

The efficiency of an AC machine =

$$\eta = \frac{P_{out}}{P_{inP}} \cdot 100\%$$

$$\eta = \frac{P_{inP} - P_{loss}}{P_{inP}} \times 100\%$$

The losses in AC machine :-

4- categories:-

- ① electrical or copper losses ( $I^2R$ )
- ② core losses.
- ③ mechanical losses.
- ④ stray load losses.

⑤ The stator copper losses.

$$P_{scL} = 3 I_A^2 R_A \quad \text{in 3-phase ac machine.}$$

⑥ The rotor copper losses.

$$P_{rcL} = I_f^2 R_f$$

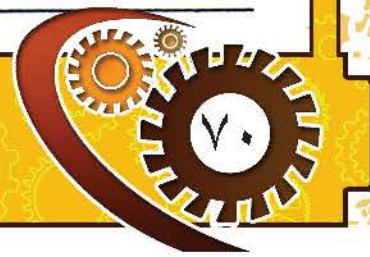
Copper losses are the resistive heating losses.

⑦ Core losses:-

are the hysteresis losses & eddy current losses occurring in the metal of the motor & they are depending upon ( $B^2$ ) & the speed of rotation of the magnetic fields ( $n^{1.5}$ ).

⑧ mechanical losses:- friction & windage

⑨ stray losses: are losses that cannot be placed in one of the previous categories.



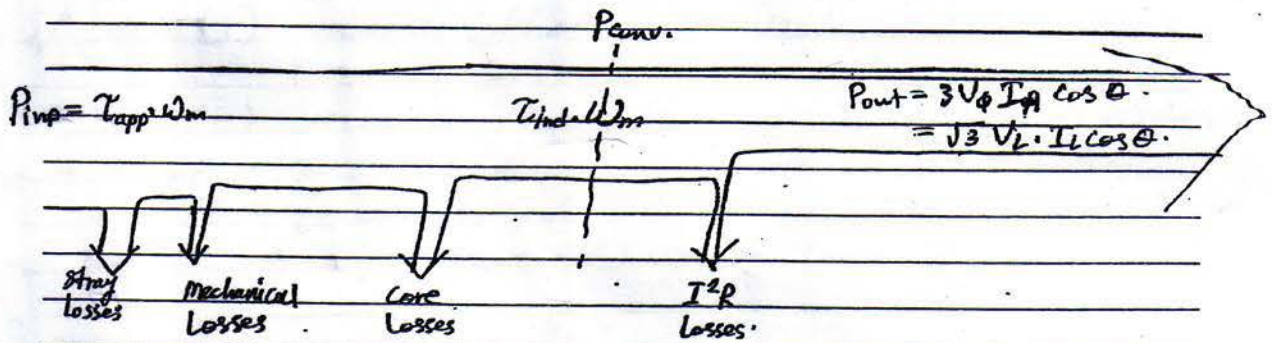
The power-flow diagram :-

- for accounting power losses in a machine.

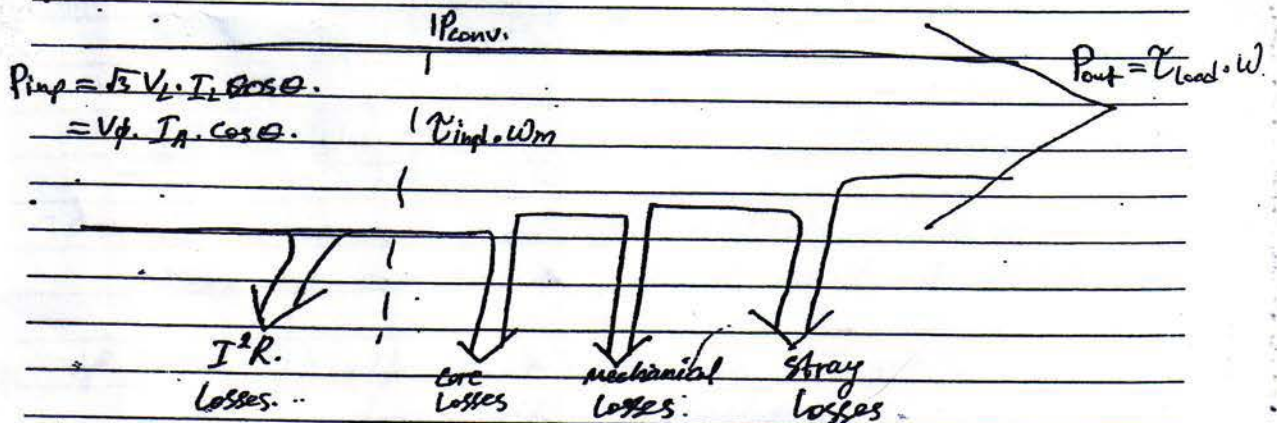
① Power Flow diagram for an AC Generator (Fig-1), mechanical power is input into the machine, and then the losses are subtracted. after they have been subtracted, the remaining power is converted from mechanical to electrical form at the point labeled  $P_{conv}$ . The mechanical power that is converted is given by :-

$$P_{conv} = T_{ind} \cdot \omega_m$$

From  $P_{conv}$  is subtracted electrical losses  $I^2R$ .



(Fig-1) Flow-diagram for Generator





(4)

## Voltage Regulation & Speed Regulation:-

VR- is a measure of the ability of a Generator to keep a constant voltage at its terminals as load varies.

& The Generators are compared to each other by voltage Regulation.

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \cdot 100\%$$

$V_{NL}$  - no-load voltage (terminal)

$V_{FL}$  - full-load voltage. (terminal)

\* if VR (+) means that the voltage-current ch-c is dropping.

VR (-) means that the V-I ch-c is rising.

\* a small VR is better - it means that the voltage at the terminals of the Generator is more constant with variations in load.

## Speed Regulation: SR :-

$$SR = \frac{N_{NL} - N_{FL}}{N_{FL}} \cdot 100\%$$

$$SR = \frac{\omega_{NL} - \omega_{FL}}{\omega_{FL}} \cdot 100\%$$

SR- is a measure of the ability of a motor to keep a constant speed as load varies.

\* if SR (+)  $\Rightarrow$  It means that the Torque-speed ch-c drops with increasing load.

SR (-)  $\Rightarrow$  it means that the motor's speed increases with increasing load.

# لجنة الميكانيك - الإتجاه الإسلامي

## ① Losses in the machine :-

① the stator copper losses

$$P_{scL} = I^2 R \Rightarrow P_{scL} = 3 I_1^2 R_1$$

② stator core losses (some amount)

$$P_{core} = 3 E_1^2 G_c$$

③ Rotor copper losses

$$P_{rcL} = I^2 R \Rightarrow P_{rcL} = 3 I_2^2 R_2$$

④ Friction & windage losses  $P_{f \& w}$ .

⑤ Stray losses.

## ② Induced Torque equation :-

The induced Torque in an Induction motor :-

$$T_{ind} = P_{conv} / \omega_m$$

$$T_{ind} = P_{air.gap} / \omega_{syn}$$

where :  $P_{air.gap} = 3 I_2^2 R_2 / s$ .

Thevenin voltage by the voltage divider rule :-

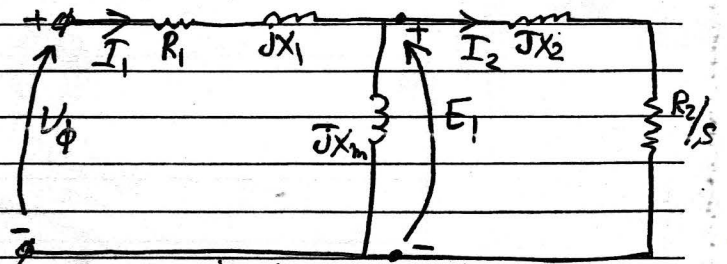
$$V_{Th} = V_\phi \frac{Z_m}{Z_m + Z_1}$$

$$= V_\phi \frac{jX_m}{jX_m + jX_1 + R_1}$$

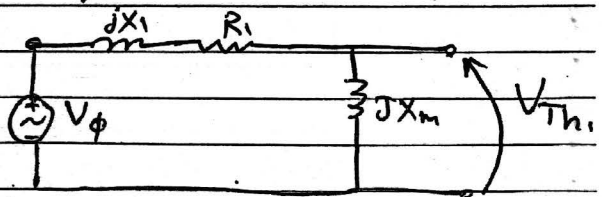
$$X_m \gg X_1 \text{ \& } X_m \gg R_1$$

$$V_{Th} \approx V_\phi \frac{X_m}{X_m + X_1}$$

$$Z_m \quad Z_1$$



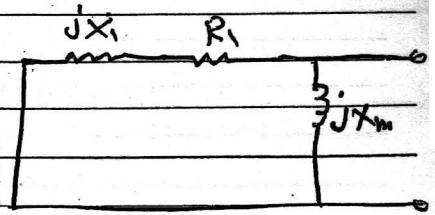
Equivalent circuit of an Ind. motor



# لجنة الميكانيك - الإتجاه الإسلامي

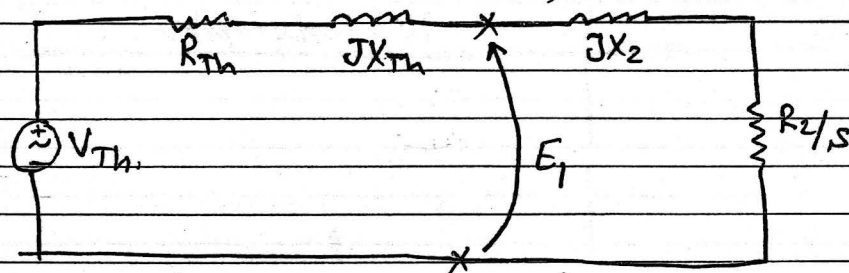
The magnitude of of the  $V_{Th}$  is:-

$$V_{Th} = V_{\phi} \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}}$$



$$Z_{Th} = \frac{Z_1 Z_m}{Z_1 + Z_m} = \frac{jX_m (R_1 + jX_1)}{R_1 + j(X_m + X_1)}$$

Thevenin equivalent circuit of Impedance.



Simplified equivalent circuit of an Induction motor.

from simplified eq. circuit the current  $I_2 \Rightarrow$

$$I_2 = \frac{V_{Th}}{Z_{Th} + Z_2} = \frac{V_{Th}}{(R_{Th} + R_2/s) + j(X_{Th} + X_2)}$$

The magnitude of this current is

$$I_2 = \frac{V_{Th}}{\sqrt{(R_{Th} + R_2/s)^2 + (X_{Th} + X_2)^2}}$$

The air Gap power is

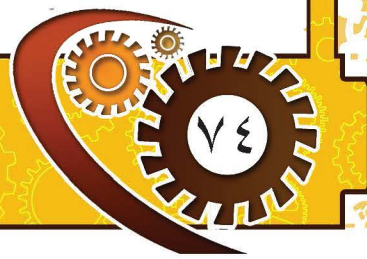
$$P_{air\ gap} = 3 I_2^2 \cdot R_2/s$$

$$= \frac{3 V_{Th}^2 R_2/s}{(R_{Th} + R_2/s)^2 + (X_{Th} + X_2)^2}$$

& The Rotor induced torque is given by

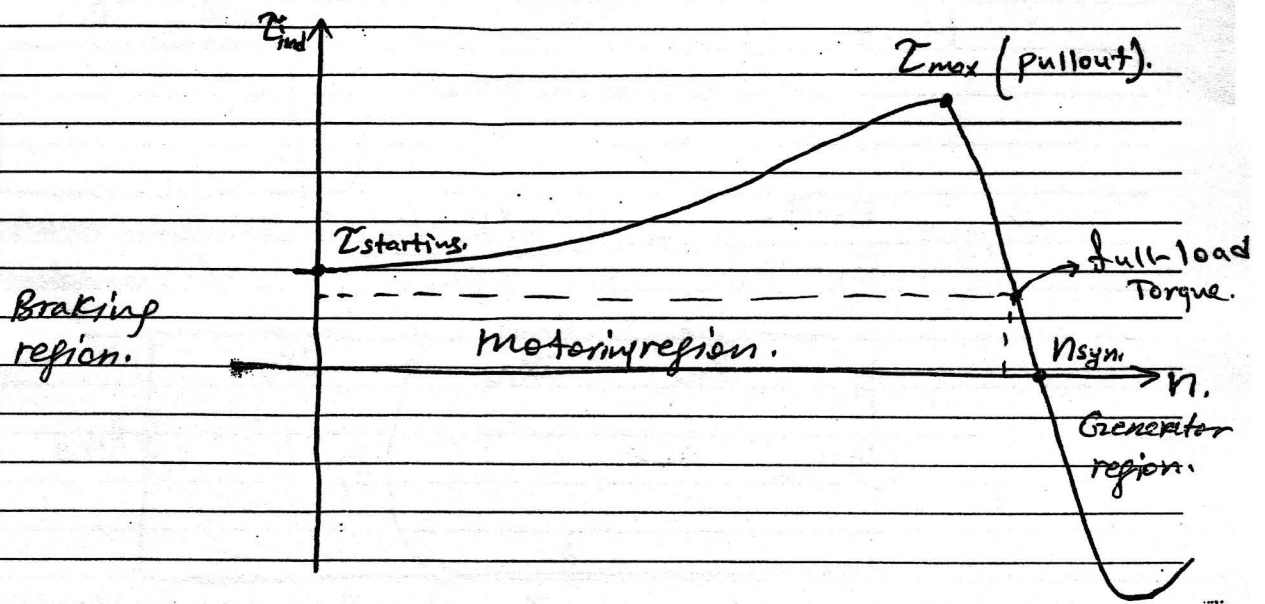
$$T_{ind} = \frac{P_{AG}}{\omega_{syn}}$$

3 112 n 1



# لجنة الميكانيك - الإتجاه الإسلامي

Induction Motor Torque - speed characteristic curve.



Maximum Torque occurs when the air-gap power is Max. & The air-gap power consumed in the resistor  $R_2/s$ , The max. induced torque will occur when the power consumed by that resistor is Maximum.

$$Z_{source} = Z_{Th} + jX_2 = R_{Th} + jX_{Th} + jX_2$$

The Max. power transfer to the load Resistor  $R_2/s$  will occur when the magnitude of source impedance equal to the magnitude of that impedance ( $R_2/s$ ),

$$\frac{R_2}{s} = \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}$$

Max slip is the slip at Max. Torque.

$$s_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$

The max Torque can be found by inserting  $s_{max}$  into the torque equation.

$$T_{max} = \frac{3 V_{Th}^2}{\pi \omega_{mec} R_2} \cdot \frac{1}{1 + \sqrt{1 + \frac{R_{Th}^2 + (X_{Th} + X_2)^2}{R_2^2}}}$$



- ① The  $\tau_{ind} = 0$  at  $\omega_{sync}$ .
- ② The T-S curve is nearly linear between no-load & full load. &  $R_2$  larger than  $X_{rotor}$ .
- ③ There's a maximum torque that cannot be exceeded. This called Torque pullout or breakdown torque.
- ④ The  $\tau_{starting} > \tau_{Full-load}$ .
- ⑤  $\tau \propto V_{\phi}^2$
- ⑥ if the rotor of the Induction motor is driven faster than  $\omega_{sync}$ , then the direction of the  $\tau_{ind}$  in the machine reverse & The machine becomes a generator.

## Starting Methods of An Induction Motors:-

- ① in many cases directly by connecting to the power line.
- ② For Wound-rotor ind. motors starting can be achieved at low currents by inserting resistance in the rotor circuit during starting. (increasing the starting torque but also reduces the starting current).
- ③ For cage induction motors the starting current can vary widely depending on the Motors rated power & on the effective rotor resistance at starting conditions.

all cage motors have a starting code letter.

Example:-

$$S_{start} = (15) hp \cdot (5.6) = 84 \text{ KVA.}$$

$$5.6 = \text{code} = F$$

S - maximum ~~ampere~~ kilovoltampere of the motor.

Then the starting current  $I_1 = S_{start} / \sqrt{3} V_T$ .

- ④ by inserting a resistance or inductors into the power line during starting.
- ⑤ by reducing the terminal voltage.

## Speed control of Induction motors:-

- ① by pole changing.
- ② by changing the line frequency
- ③ " " " " voltage.
- ④ " " " " the rotor resistance.



## ① Types & Construction of Induction Motors:-

There are 2 Types of Ind. motors:-

① Cage rotor.

② Wound rotor.

→ A cage ind. motor rotor consists of a series of conductive bars laid into slots carved in the face of the rotor & shorted at either end by large shorting rings.

→ A wound rotor has a complete set of 3-ph. windings that are mirror images of the windings on the stator, and the 3-ph. of the rotor windings are usually Y-connected & the ends of the 3 rotor wires are tied to slip rings on the rotor's shaft. The rotor windings are shorted through brushes riding on the sliprings.

Wound-rotor induction motors are more expensive than cage induction motors, and they require much more maintenance because of the wear associated with their brushes & slip rings, As a result, wound-rotor induction motors are rarely used.

## ② Principle of operation:-

the same as that on synchronous motors.

## \* Induced Torque :-

The speed of the produced magnetic field  $B_s$  by ~~wrapping~~ wrapping of the stator windings

$$n_{syn.} = 120 f_e / p.$$

$f_e$  - system frequency, Hertz

$p$  - Number of poles in the machine.



# لجنة الميكانيك - الإتجاه الإسلامي

This rotating mag. field  $B_s$  passes over the rotor bars & induces a voltage in them

$$E_{ind} = v \cdot B \cdot l$$

$v$  - speed (velocity).

$B$  - mag. field density vector.

$l$  - length of conductor in the magnetic field.

The induced Torque in the machine :-

$$*T_{ind} = k B_r \cdot B_s$$

$B_r$  - rotor magnetic field. (from rotor current,

$B_s$  - stator " " " " " "

in normal operation both the rotor & stator magnetic fields  $B_r$  &  $B_s$  rotate together at synchronous speed  $N_{sync}$ , while the rotor itself turns at a slower speed.

\* The concept of rotor slip:-

The induced voltage in a rotor depends on the speed of the rotor relative to the magnetic fields.

Two terms are used to define the relative motion of the rotor & the magnetic fields. One is slip speed, defined as the difference between synchronous speed & rotor speed.

$$N_{slip} = N_{sync} - N_m$$

$N_{slip}$  - slip speed of the machine.

$N_{sync}$  - speed of the mag. field.

$N_m$  - mechanical shaft speed of motor.

The other term used to describe the relative motion is slip

$$S = \frac{N_{sync} - N_m}{N_{sync}} \cdot 100\%$$

& for angular velocity ( $\omega$ )

$$S = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} \cdot 100\%$$



# لجنة الميكانيك - الإتجاه الإسلامي

$$S = (0 - 1) \quad \text{when } \omega_m = \omega_{sync} \Rightarrow S = 0.$$

$$" \quad \omega_m = 0 \text{ (stationary)} \Rightarrow S = 1$$

All normal rotating speeds fall somewhere between those two limits.

it is possible to express the mechanical speed of the rotor shaft in terms of synchronous speed & slip.

$$n_m = (1 - S) n_{sync}.$$

$$\omega_m = (1 - S) \omega_{sync}.$$

These equations are useful in the derivation of induction motor torque & power relationships.

The Electrical Frequency on the rotor:-

- if the rotor is locked. (cannot move)  $n_m = 0$  the rotor frequency  $f_r = f_e$  & the slip  $S = 1$

- if the rotor rotate at synchronous speed  $\omega_m = \omega_{sync}$  the rotor frequency  $f_r = 0$  & the slip  $S = 0$ .

- for any speed in between the  $f_r$  is directly proportional to the difference between the speed of the magnetic field  $n_{sync}$  & The speed of rotor  $n_m$ .

$$S = \frac{n_{sync} - n_m}{n_{sync}}$$

the rotor frequency can be expressed as:-

$$f_r = S f_e.$$

$$f_r = \frac{n_{sync} - n_m}{n_{sync}} \cdot f_e.$$

$$n_{sync} = 120 f / p;$$

$$f_r = \frac{p}{120 f_e} \cdot f_e (n_{sync} - n_m)$$

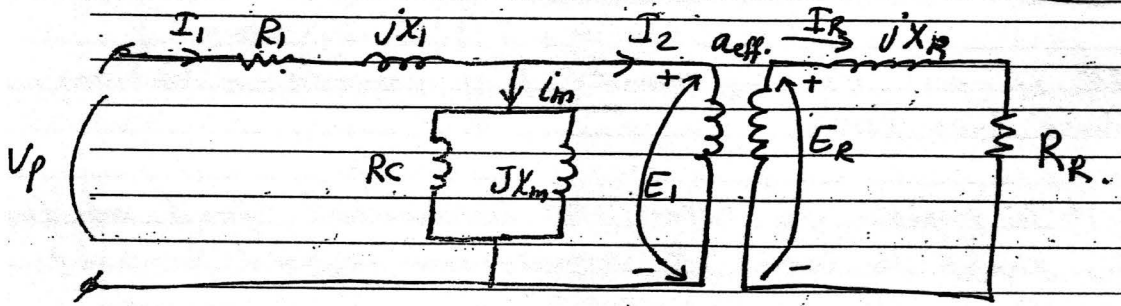
$$f_r = \frac{p}{120} (n_{sync} - n_m).$$





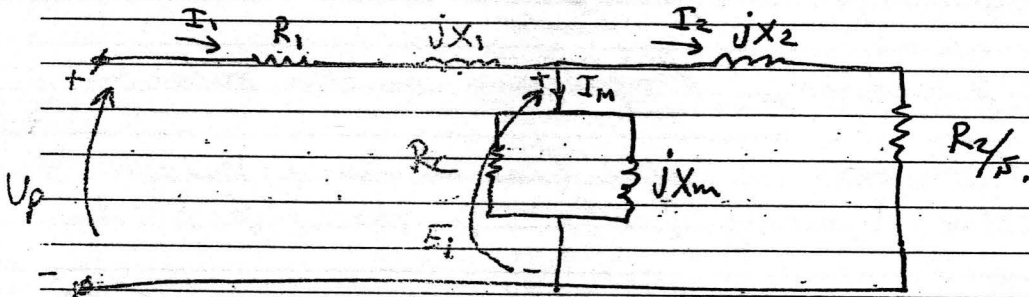
# لجنة الميكانيك - الإتجاه الإسلامي

\* The Equivalent circuit of An Ind. motor:-

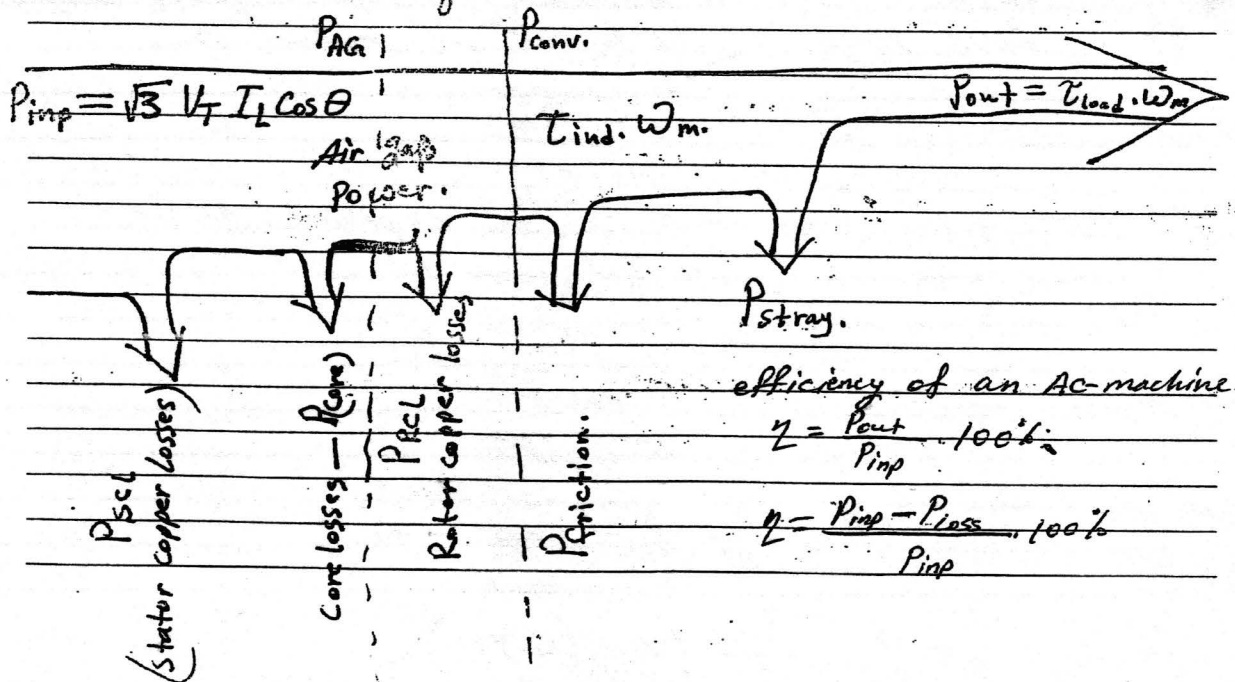


Ind. motor with refer & stator connected by an ideal transformer of turns ratio  $a_{eff}$ .

The final Equivalent circuit of an Ind. motor:-



\* Power flow diagram of an ind. motor:-



## Types & constructions of synchronous machines.

Synch. Generators are synch. machines used to convert mechanical power to (ac) electrical power.

In a synchronous Generators, a DC current is applied to the rotor winding, which produces a rotor magnetic field. The rotor of the Generator is turned by a prime mover, producing a mag. field within the machine & This rotating field induces a 3-phase set of voltages within the stator windings of the Generator.

- ① field windings:- produces the main magnetic field in the machine & They are on the rotor.
- ② armature windings:- applies to the windings where the main voltage is induced

The magnetic poles on the rotor can be either salient or nonsalient construction.

- ① salient means sticking out, that sticks out of the surface of the rotor, are used for 4 & more poles.
- ② nonsalient pole is a magnetic pole constructed flush with the surface of the rotor, & are used for 2-4 pole rotors. to reduce eddy currents.

A dc current must be supplied to the field on the rotor. There are 2 common approaches to supplying this dc power:-

- ① supply the DC power from an external dc source to the rotors by means of sliprings & brushes.
- ② supply the DC power from a special dc power source mounted directly on the shaft of the synch. Generator.

Sliprings & brushes create a few problems :-

- ① They increase the amount of maintenance.
- ② The brushes must be checked for wear regularly.
- ③ brush voltage drop can be the cause of power losses. That's why the sliprings & Brushes are used on all smaller synchronous machines for cost-effective.



# لجنة الميكانيك - الإتجاه الإسلامي

On larger Generators & motors brushless exciters are used to supply the DC field current to the machine.

Brushless exciter :- is a small AC Generator with its field circuit mounted on the stator & its armature circuit mounted on the rotor shaft.

The 3-phase output of the exciter generator is rectified to dc current by a 3-phase rectifier circuit also mounted on the shaft of the Generator. using exciter it's possible to adjust the field current on the main machine without slip rings and brushes.

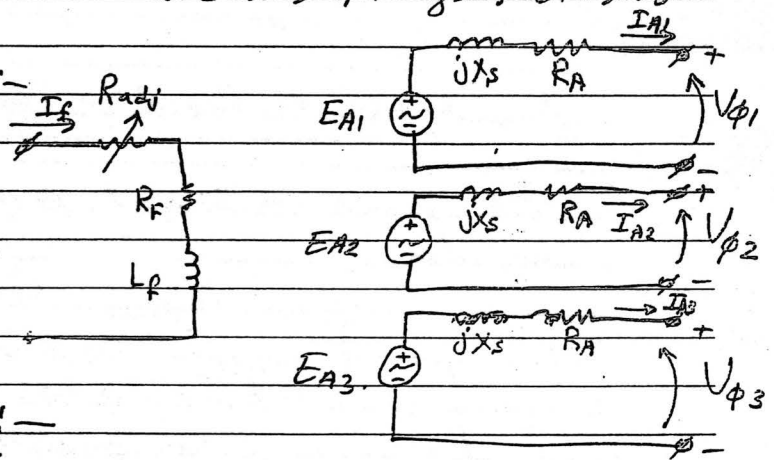
To make the excitation of Generator completely independent of any external power sources, a small pilot exciter is included in the system.

pilot exciter :- is a small AC Generator mounted on the rotor shaft & a 3-phase winding on the stator.

For emergency cases many synch. Generators that include brushless exciters also have a sliprings & brushes.

## Equivalent circuit :-

$E_A$  - internal Generated voltage in one phase of Generator.  $V_F$  (dc)



## Why The $E_A \neq V_\phi$ :-

- ① The distortion of the air-gap magnetic field by the current flowing in the stator, called armature reaction.
- ② The self-inductance of the armature coils.
- ③ The resistance " " " " " "
- ④ The effect of salient-pole rotor shapes.

$$V_\phi = E_A - jX_s I_A - R_A I_A$$

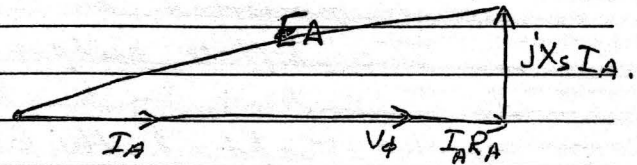
# لجنة الميكانيك - الإتجاه الإسلامي

The 3-phases can be either  $\gamma$  or  $\Delta$  connected  
if there are  $\gamma$ -connected then the terminal voltage

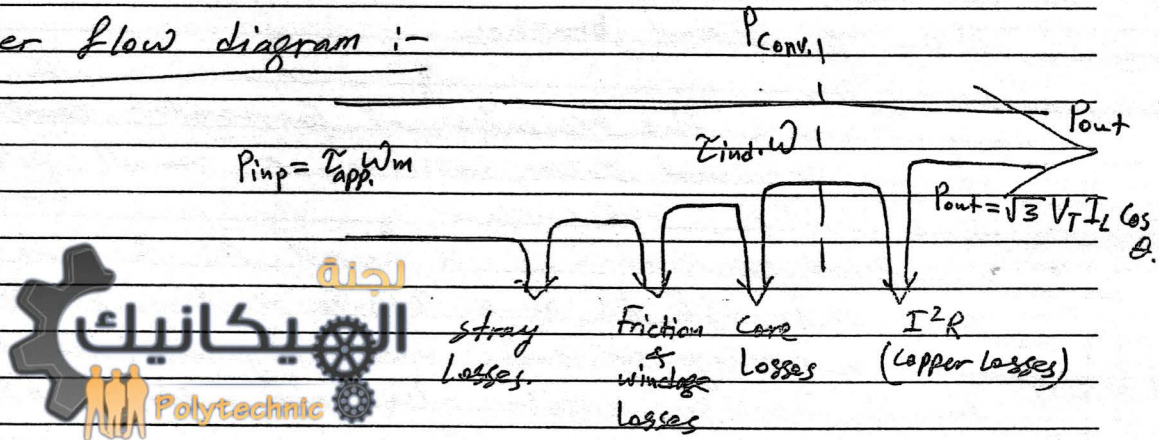
$$V_T = \sqrt{3} V_\phi$$

& if they are  $\Delta$ -connected then:-

$$V_T = V_\phi$$



Power flow diagram :-



Active & Reactive powers:-

The input mechanical power is the shaft power in the Generator.

$$P_{in} = T_{applied} \cdot W_m$$

$$P_{converted} = T_{ind} \cdot W_m$$

$$= 3 E_A I_A \cos \delta$$

$\delta$  - the angle between  $E_A$  &  $I_A$

→ the power converted from mechanical to electrical.

electrical output power of the synch. Generator

$$P_{out} = \sqrt{3} V_T I_L \cos \theta$$

& In phase quantities

$$P_{out} = 3 V_\phi \cdot I_A \cos \theta$$

The reactive power output in line quantities :-

$$Q_{out} = \sqrt{3} V_T I_L \sin \theta$$

& in phase quantities :-

$$Q_{out} = 3 V_{\phi} I_A \sin \theta$$

Finally the  $P_{out}$  can be expressed as :-  $P = \frac{3 V_{\phi} E_A \sin \delta}{X_s}$   
Electromagnetic torque :-

induced torque in this Generator :-

$$T_{ind} = K B_r \times B_s$$

or as :-

$$T_{ind} = K B_r \cdot B_{net}$$

& The magnitude of equation above :-

$$T_{ind} = K B_r \cdot B_{net} \sin \delta$$

where  $\delta$  - The torque angle between the rotor & net magnetic fields.

$B_r$  - Produces the voltage  $E_A$ .

$B_{net}$  - " " " "  $V_{\phi}$ .

then The angle  $\delta$  - between  $E_A$  &  $V_{\phi}$  is the same angle between  $B_r$  &  $B_{net}$ .

Finally The induced Torque can be expressed as :-

$$T_{ind} = \frac{3 V_{\phi} E_A \sin \delta}{\omega_m X_s}$$

## ① Starting Synchronous Motors:-

Three basic approaches can be used to safely start a synchronous motor:-

A) Motor starting by Reducing electrical frequency (Reduce The speed of the stator magnetic field) of the applied electrical power.

B) ~~Motor~~ Motor starting with an external prime mover to accelerate the synch. motor up to synch. speed then disconnecting the prime mover will make the synch. machine a motor.

C) Motor starting by using damper windings or Amortisseur winding :- Are special bars laid into curved in the face of a synch. motors rotor and then shorted out on each end by a large shorting Ring.

~~if~~ if a machine has amortisseur windings, it can be started by the following procedure:-

1- Disconnect the field windings from their DC power source & short them out.

2- Apply a 3<sup>Ph</sup> voltage to the stator of the motor & let the rotor accelerate up to near-synchronous speed

3- connect the DC field circuit to its power source, After this the motor will lock into step at synch. speed.

Amortisseur increase stability in machine.

## ② Active & Reactive powers

$$P = 3 V_{\phi} I_a \cos \theta \quad \text{KW,}$$

$$= \sqrt{3} V_T I_L \cos \theta$$

$$Q = 3 V_{\phi} I_a \sin \theta \quad \text{KVAR}$$

$$= \sqrt{3} V_T I_L \sin \theta.$$



## 3) Electromagnetic Torque :-

The induced Torque

$$T_{ind} = K B_r \cdot B_{net} \cdot$$

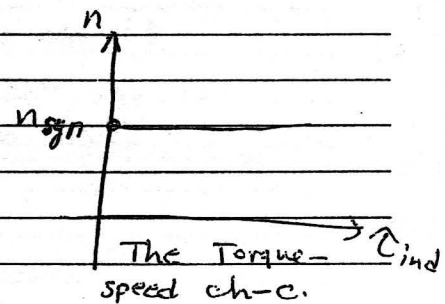
$$T_{ind} = K B_r B_{net} \cdot \sin^2 \delta$$

The induced Torque in this machine is clockwise.

$$T_{ind} = \frac{3 V \phi \cdot E_A \cdot \sin \delta}{\omega_m X_s}$$

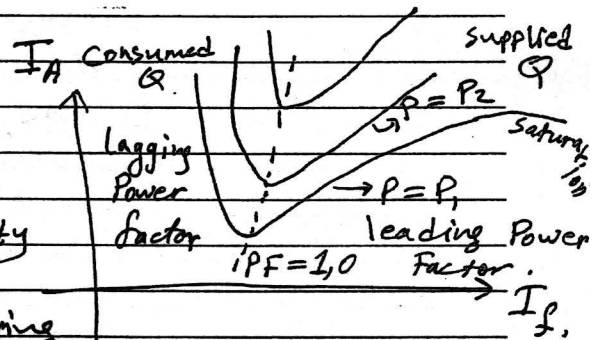
$$T_{max} = K B_r \cdot B_{net} \cdot$$

$$T_{max} = \frac{3 V \phi E_A}{\omega_m X_s}$$



The equation indicate that the larger the field current (and  $E_A$ ), the greater the maximum torque of the motor. & There for a stability advantage in operating the motor with a large field current or a large ( $E_A$ ).

## 4) V-Curves :-



1) For each curve the minimum armature current occurs at unity power factor.

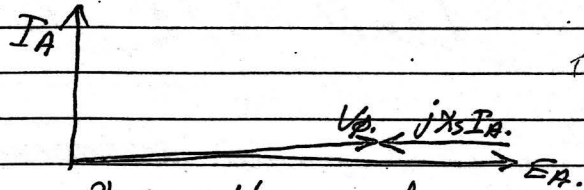
2) For  $I_f <$  than the value giving minimum  $I_A$  The  $I_A$  is lagging. Consuming  $Q$ .

3) For  $I_f >$  than the value giving the Minimum  $I_A$ . The  $I_A$  is leading, supplying  $Q$  to power system

5) Power flow Diagram  $\rightarrow$  like synch. Generators

## 5) The synchronous condenser or capacitor :-

A synchronous motor purchased to drive a load can be operated over excited to supply Q for power supply. Some times the synch. motors run without a load simply for power factor correction.



Phasor diagram of synch. capacitor, operating over excited at no load

The Kirchoff's ~~law~~ voltage law equation for synch. motor is

$$V_{\phi} = E_A + jX_s I_a$$

Some synch. motors used to be sold specifically for power-factor correction, these machines had shafts that did not even come through the frame of the motor. No-load could be connected to them. Such special synch. motors were often called synch. condensers or synchronous capacitors (condenser is an old name of capacitor)

Static capacitors are more economical to buy and use than synch. capacitors.

## Power factor correction :-

The lower the ~~loss~~ cosφ of a system, the greater the losses in the power lines feeding it.

most loads on power systems are induction motors (lagging) & Having 1 or more leading loads on the system can be useful for the following reasons :-

- ① A leading load can supply some Q lagging loads. instead of it coming from the generator.
- ② Transmission lines carry less current they can be smaller & it's Reduce the cost of lines.
- ③ The motor Max Torque will increase.



# لجنة الميكانيك - الإتجاه الإسلامي

## Angle characteristic & V-curve.

The stator & Rotor heat limits with any external limits can be expressed in graphical form by a Generator capability diagram. A capability diagram is a plot of complex power

$$S = P + jQ$$

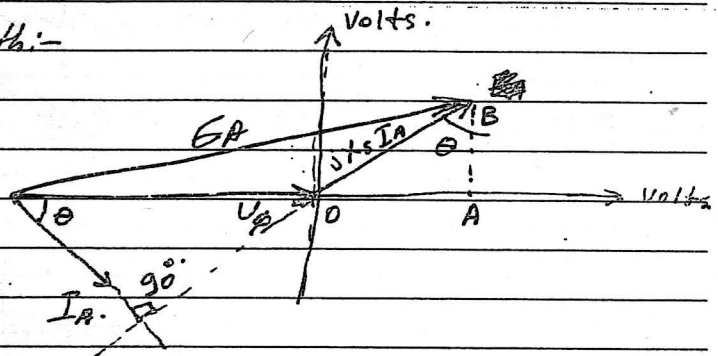
it's derived from the phasor diagram of the Generator, assuming that  $V_{\phi} = \text{const.}$

Phasor diagram of a synch. Generator operating at a lagging power factor and its rated voltage.

segment (AB) has a length:-  
 $X_s \cdot I_A \cos \theta.$

horizontal segment (OA) has a length:-

$$X_s \cdot I_A \sin \theta.$$



The real output power of the Generator:-

$$P = 3 V_{\phi} I_A \cos \theta.$$

The reactive power output :-

$$Q = 3 V_{\phi} I_A \sin \theta.$$

& The apparent power output

$$S = 3 V_{\phi} \cdot I_A.$$

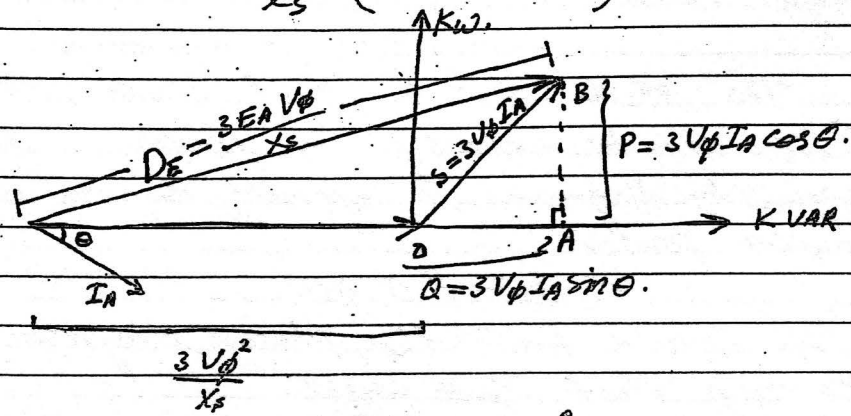
So the vertical & horizontal axes of this figure can be recalibrated in terms of real & reactive power.

$$P = 3 V_{\phi} I_A \cos \theta = \frac{3 V_{\phi}}{X_s} (X_s I_A \cos \theta)$$



# لجنة الميكانيك - الإتجاه الإسلامي

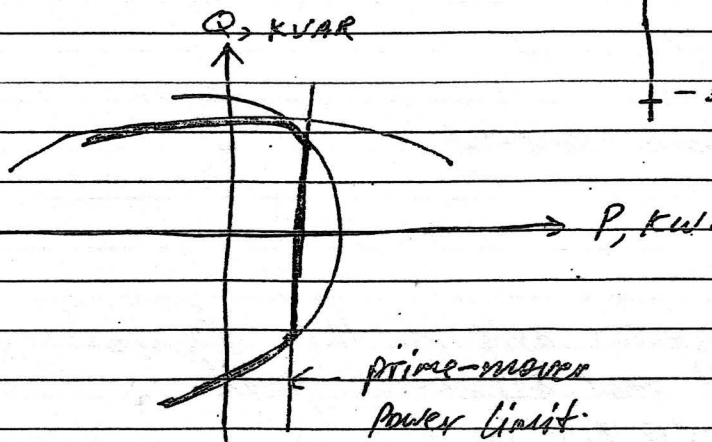
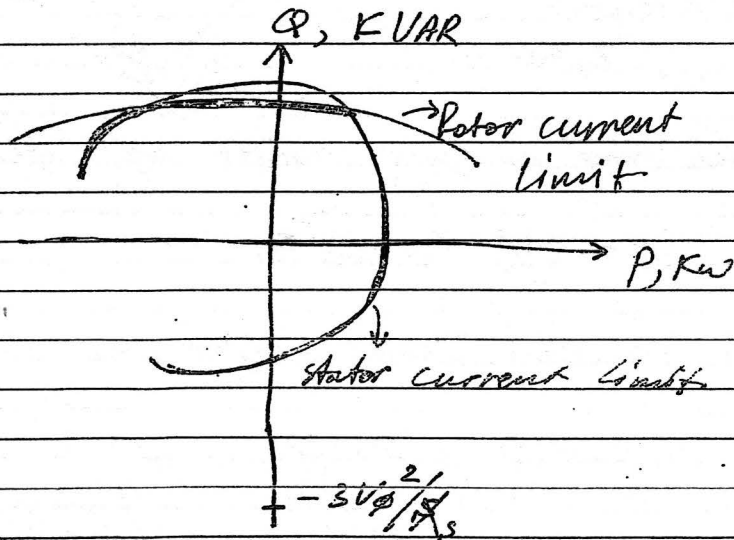
$$Q = 3V_{\phi} I_A \sin \theta = \frac{3V_{\phi}}{X_s} (X_s I_A \sin \theta)$$



Corresponding Power Units.

$3V_{\phi}/X_s$  — to change the scale of the axes from volts to voltamperes. (power units).

$$Q = \frac{3V_{\phi}}{X_s} (-V_{\phi}) = -\frac{3V_{\phi}^2}{X_s}$$



Origin of rotor current cycle.  $Q = -3V_{\phi}^2 / X_s$



# لجنة الميكانيك - الإتجاه الإسلامي

Example:-

A 480-V, 50 MVA, Y connected, six-pole synch. generator is rated at 50 kVA at 0.8 PF lagging, it has  $Z = 1.0 \Omega$  per phase. Assume that this generator is connected to a steam turbine capable of supplying up to 45 kW the friction & windage losses are 1.5 kW & the core losses are 1.0 kW.

① Sketch the capability curve for this generator including the prime mover power limits.

The max. current in this generator.

$$S_{\text{rated}} = 3 V_{\phi, \text{rated}} \cdot I_{A, \text{max}}$$

$$I_{\text{max}} = S_{\text{rated}} / 3 V_{\phi} = 50 \text{ kVA} / 3 \cdot (277) \text{ V} = 60 \text{ A.}$$

$$V_{\phi} = V_T / \sqrt{3} = 480 / \sqrt{3} = 277 \text{ V}$$

The max permissible apparent power is 50 kVA & The center of the EA circles is at.

$$Q = -\frac{3 V_{\phi}^2}{X_s} = \frac{3 \cdot (277)^2}{1.0} = 230 \text{ kVAR.}$$

The max size of  $E_A$  is given by:-

$$E_A = V_{\phi} + j X_s \cdot I_A.$$

$$= 277 \angle 0^\circ \text{ V} + (j \cdot 1.0 \Omega) \cdot (60 \angle -36.87^\circ \text{ A})$$

$$= 313 + j48 \text{ V} = 317 \angle 8.7^\circ \text{ V.}$$

The magnitude of the distance proportional to  $E_A$ .

$$D_E = \frac{3 E_A \cdot V_{\phi}}{X_s} = \frac{3 (317) (277)}{1.0 \Omega} = 263 \text{ kVAR.}$$

The max power output with a prime-mover power of 45 kW. is

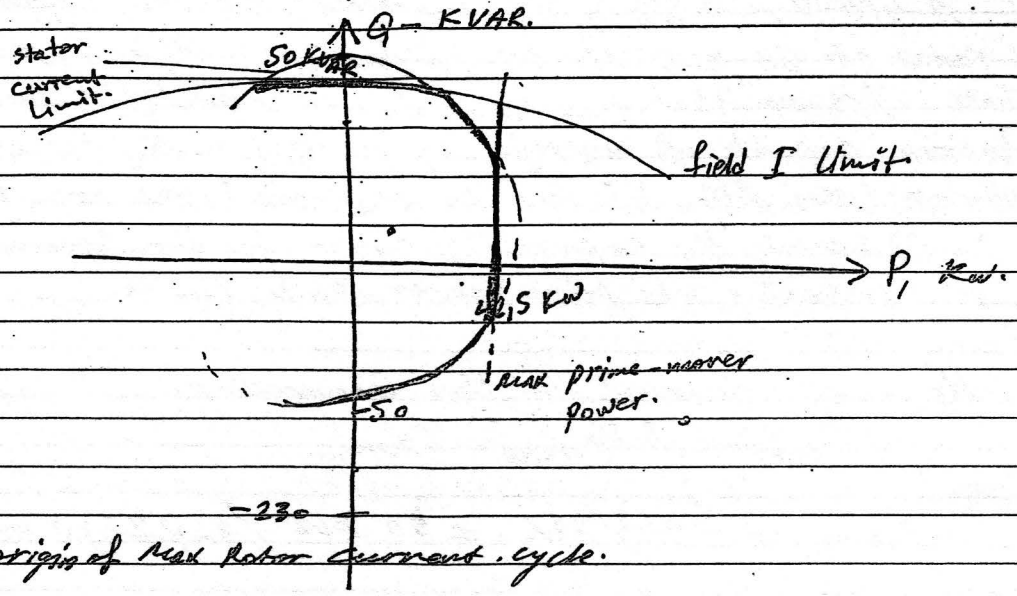
$$P_{\text{max, out}} = P_{\text{max, in}} - P_{\text{mech, loss}} - P_{\text{core loss}}$$



# لجنة الميكانيك - الإتجاه الإسلامي

(20) 10

$$= 45 - 1,5 - 1,0 = 42,5 \text{ KW.}$$



MAX output reactive power.

$$Q = D_E - Q = 263 \text{ KVAR} - 230 \text{ KVAR.}$$

$$Q = 33 \text{ KVAR.}$$

\* operating will be safely if the point will be within the curves.

⊗ operating not safe if the point ~~is~~ outside the curves.

