Electrical Machines and Power Electronics

ENEE4301

Course Organization

Instructor : Dr. Ali Abdo
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Textbook : **Electric Machinery Fundamentals, Fifth Edition,**
By Stephen J. Chapman

Course Organization

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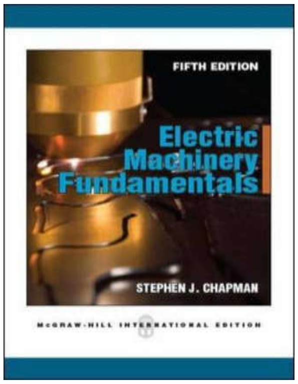

1) Introduction to Machinery Principles
2) Transformers
3) AC Machinery Fundamentals
4) Synchronous Generators
5) Induction Motors
6) DC Machinery Fundamentals
7) DC Motors and Generators
8) Introduction to Power Electronics

Course Organization

Grading:

Midterm Exam	35 %
Final Exam	40 %
Class Work	25 %
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	100 %

Introduction to Machinery Principles

- Electric Machines → mechanical energy to electric energy or vice versa
- Mechanical energy → Electric energy : **GENERATOR**
- Electric energy → mechanical energy : **MOTOR**

Almost all practical motors and generators convert energy from one form to another through the action of a **magnetic field**.

Only machines using magnetic fields to perform such conversions will be considered in this course.

Introduction to Machinery Principles

- When we talk about machines, another related device is the transformer. A transformer is a device that converts ac electric energy at one voltage level to ac electric energy at another voltage level.

- Transformers are usually studied together with generators and motors because they operate on the same principle, the difference is just in **the action of a magnetic field** to accomplish the change in voltage level.

Introduction to Machinery Principles

Why are electric motors and generators so common?

- ✓ Electric power is a clean and efficient energy source that is easy to transmit over long distances and easy to control.
- ✓ An electric motor does not require constant ventilation and fuel the way that an internal-combustion engine does, so the motor is very well suited for use in environments where the pollutants associated with combustion are not desirable.

Introduction to Machinery Principles

Rotational Motion, Newton's Law and Power Relationship

Almost all electric machines rotate about an axis, called the shaft of the machines. It is important to have a basic understanding of rotational motion.

Angular Position θ - is the angle at which it is oriented, measured from some arbitrary reference point. Its measurement units are in radians (rad) or in degrees. It is similar to the linear concept of distance along a line.

Conventional notation: +ve value for counterclockwise
-ve value for clockwise rotation

Angular Velocity ω - Angular velocity (or speed) is the rate of change in angular position with respect to time.

Introduction to Machinery Principles

Similar to the concept of standard velocity where:

$$v = \frac{dr}{dt} \quad \text{(m/sec)}$$

where:

r – distance traverse by the body
t – time taken to travel the distance r

For a rotating body, angular velocity is formulated as:

$$\omega = \frac{d\theta}{dt} \quad \text{(rad/s)}$$

where:

θ - Angular position/ angular distance traversed by the rotating body
t – time taken for the rotating body to traverse the specified distance,

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Introduction to Machinery Principles

The following symbols are used in out text book to describe angular velocity:

w**m** *: angular velocity expressed in radians per second*
f**m** *: angular velocity expressed in revolutions per second*
n**m** *: angular velocity expressed in revolutions per minute*

These measures of shaft speed are related to each other by the following equations:

$$n_m = 60 f_m$$

$$f_m = \frac{\omega_m}{2\pi}$$

Angular acceleration, α - is defined as the rate of change in angular velocity with respect to time. Its formulation is as shown:

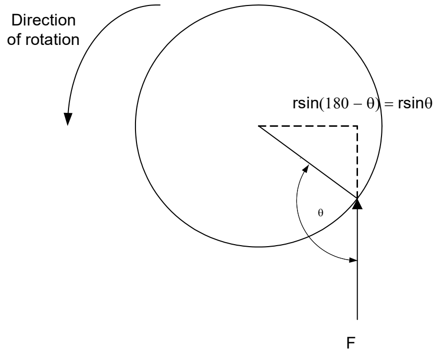
$$\alpha = \frac{d\omega}{dt} \quad rad/s^2$$

Introduction to Machinery Principles

Torque (Twisting Force) τ

The torque on an object is define as the product of the force applied on the object and the smallest distance between the line of the action of force and the axis of rotation.

$$\therefore \tau = \text{Force} \times \text{perpendicular distance}$$
$$= F \times r \sin\theta$$



Introduction to Machinery Principles

Newton's Law of Rotation

$$F = ma$$ (N or kg.m/sec^2)

where:

F – net force applied
m – mass of object
a – resultant acceleration of object

Applying these concept for rotating bodies,

$$\tau = J\alpha$$ (Nm)

where:

τ - Torque
J – moment of inertia
α - angular acceleration

Introduction to Machinery Principles

Work (W)

Is defined as the application of Force through a distance.

- For linear motion

$$W = \int F dr$$ (Joules **OR** foot-pounds)

Assuming that the direction of F is collinear (in the same direction) with the direction of motion.
If ***F*** (force) is constant then:

$$W = Fr$$

- For rotational motion

$$W = \int \tau d\theta$$

if τ is constant, then $$W = \tau\theta$$ (Joules)

Introduction to Machinery Principles

Power (P)

Is defined as rate of doing work. Hence,

$$P = \frac{dW}{dt}$$

(Watt) **OR** (Joules/s) **OR**
hours power (1 hp = 746 watt)
OR foot-pounds/s

Assuming that force is constant and collinear with the direction of motion, power is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(Fr) = F\left(\frac{dr}{dt}\right) = Fv$$

Applying this for rotating bodies,

$$\boxed{\begin{aligned} &P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\left(\frac{d\theta}{dt}\right) = \tau\omega \\ &P = \tau\omega \end{aligned}}$$

This equation can describe the mechanical power on the shaft of a motor or generator.

Introduction to Machinery Principles

Magnetic Field

magnetic fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers.

Basic principles:

1. **A current-carrying wire produces a magnetic field in the area around it.**
2. **A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil.** ***(This is the basis of transformer action.)***
3. **A current-carrying wire in the presence of a magnetic field has a force induced on it.** ***(This is the basis of motor action.)***
4. **A moving wire in the presence of a magnetic field has a voltage induced in it.** ***(This is the basis of generator action.)***

Introduction to Machinery Principles

Production of a Magnetic Field

Ampere's Law – the basic law governing the production of a magnetic field by a current:

$$\oint H\,dl = I_{net}$$

Where,

H is the magnetic field intensity produced by the current I_{net}

dl is a differential element of length along the path of integration.

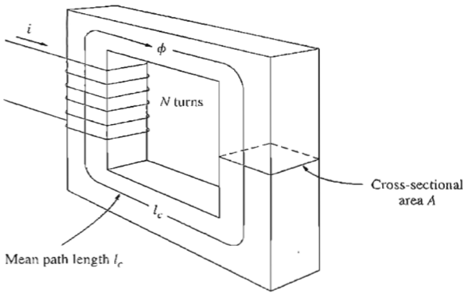
H is measured in Ampere-turns per meter.

To better understanding the previous equation consider the following example:

Introduction to Machinery Principles

Production of a Magnetic Field

Consider a current carrying conductor is wrapped around a ferromagnetic core



Applying Ampere's law, the total amount of magnetic field induced will be proportional to the amount of current flowing through the conductor wound with N turns around the ferromagnetic material as shown. Since the core is made of ferromagnetic material, it is assumed that a majority of the magnetic field will be confined to the core.

Introduction to Machinery Principles

Production of a Magnetic Field

- The path of integration in Ampere's law is the mean path length of the core, l_c
- The current passing within the path of integration I_{net} is then Ni,
- Since the coil of wires cuts the path of integration N times while carrying the current i. Hence Ampere's Law becomes,

$$Hl_c = Ni$$

$$\therefore H = \frac{Ni}{l_c}$$ **(Ampere turns per meter)**

Introduction to Machinery Principles

Production of a Magnetic Field

Magnetic field intensity H is known as the effort required to induce a magnetic field.

- The strength of the magnetic field flux produced in the core also depends on the material of the core. Thus,

$$B = \mu H$$

B = magnetic flux density (webers per square meter, Tesla (T))

μ= magnetic permeability of material (Henrys per meter)

H = magnetic field intensity (ampere-turns per meter)

Introduction to Machinery Principles

Production of a Magnetic Field

The constant μ may be further expanded to include *relative permeability* which can be defined as below:

$$\mu_r = \frac{\mu}{\mu_o}$$

where:

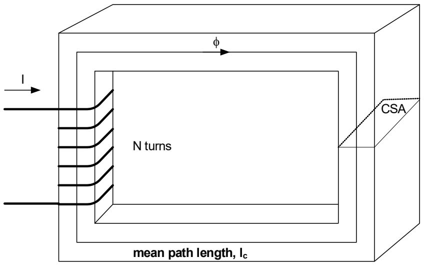
μo – permeability of free space $4\pi \times 10^{-7}$ H/m (Henry/meter)

Note:

- permeability of air = permeability of free space.
- steels used in modern machines have μ_r of 2000 to 6000.

Introduction to Machinery Principles

In a core such as in the figure,



$$B = \mu H = \mu N i / l_c$$

Now, to measure the total flux (Φ) flowing in the ferromagnetic core, consideration has to be made in terms of its cross sectional area (CSA). Therefore,

$$\phi = \int_A B dA$$

Where: A – cross sectional area throughout the core

Introduction to Machinery Principles

Assuming that the flux density in the ferromagnetic core is constant throughout hence constant A, the equation simplifies to be:

$$\phi = BA$$

Taking into account past derivation of **B**,

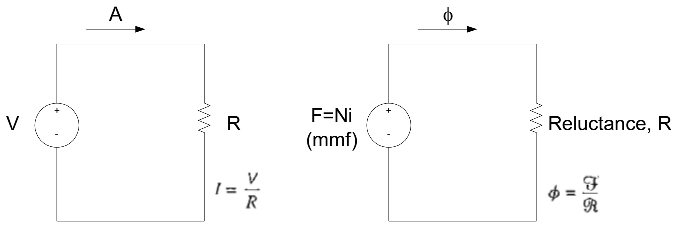
$$\phi = \frac{\mu N i A}{l_c} \qquad \text{weber}$$

Introduction to Machinery Principles

Magnetic Circuit

The *current in a coil of wire wrapped around a core produces* a magnetic flux in the core. This is in some sense analogous to a voltage in an electric circuit producing a current flow.

The analogy is as follows:



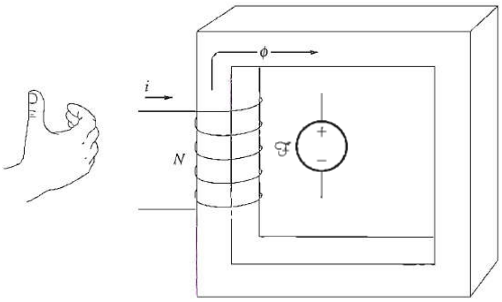
Electric Circuit *Magnetic Circuit*

$\mathcal{F}$ is denoted as **magnetomotive force** (mmf) which is similar to Electromotive force in an electrical circuit (emf).

Introduction to Machinery Principles

Magnetic Circuit

The polarity of the mmf will determine the direction of flux. To easily determine the direction of flux, the 'right hand curl' rule is utilised:



1) The direction of the curled fingers determines the current flow.
2) The resulting thumb direction will show the magnetic flux flow.

Introduction to Machinery Principles

Magnetic Circuit

The element of $\mathcal{R}$ in the magnetic circuit analogy is similar in concept to the electrical resistance.

It is basically the measure of material resistance to the flow of magnetic flux.

Reluctance in this analogy obeys the rule of electrical resistance (Series and Parallel Rules).

Reluctance is measured in **Ampere-turns per weber.**

Series Reluctance,

$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \ldots.$$

Parallel Reluctance,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Introduction to Machinery Principles

Magnetic Circuit

The inverse of electrical resistance is conductance (G) which is a measure of conductivity of a material.

Hence the inverse of reluctance is known as **permeance,** $\mathcal{P}$ where it represents the degree at which the material permits the flow of magnetic flux.

$$P = \frac{1}{R}$$

$$\therefore \text{since } \phi = \frac{F}{R}$$

$$\therefore \phi = FP$$

Also

$$\phi = \frac{\mu NiA}{l_c}$$

$$= Ni\frac{\mu A}{l_c}$$

$$= F\frac{\mu A}{l_c}$$

$$\therefore P = \frac{\mu A}{l_c}, \; R = \frac{l_c}{\mu A}$$

Introduction to Machinery Principles

Inaccuracy in the magnetic circuit approach

By using the magnetic circuit approach, it simplifies calculations related to the magnetic field in a ferromagnetic material, however, this approach has inaccuracy.

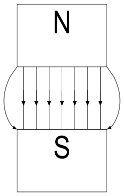
Possible reason of inaccuracy is due to:

1) Assumes that all flux are confined within the core, but in reality a small fraction of the flux escapes from the core into the surrounding low-permeability air, and this flux is called **leakage flux**.

2) Assumes a certain mean path length and **cross sectional area (CSA)** of the core is not accurate especially at the corners.

3) In ferromagnetic materials, **the permeability varies with the amount of flux** already in the material. The material permeability is not constant hence there is an existence of **non-linearity of permeability.**

4) For ferromagnetic core which has air gaps, there are **fringing effects** that should be taken into account as shown:

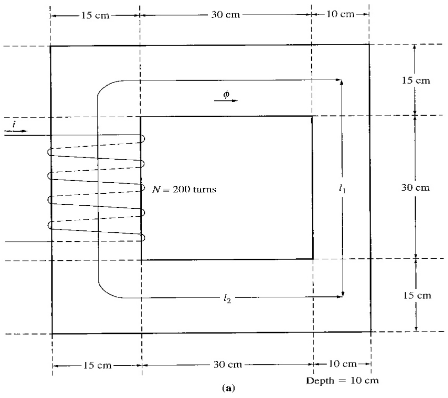


Introduction to Machinery Principles

Example 1-1

A ferromagnetic core is shown. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10cm, and the other dimensions are shown in the figure. There is a 200 turn coil wrapped around the left side of the core.

Assuming relative permeability μ_r of 2500, **how much flux will be produced** by a 1 A input current?



Introduction to Machinery Principles

Example 1-1

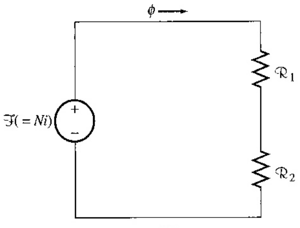
Solution:

3 sides of the core have the same csa, while the 4th side has a different area. Thus the core can be divided into 2 regions:

(1) the single thinner side

(2) the other 3 sides taken together

The magnetic circuit corresponding to this core:



(b)

$$R = \frac{l_c}{\mu A}$$

Solve Examples 1-1, 1-2, 1-3 in the book

Introduction to Machinery Principles

Example 1-2

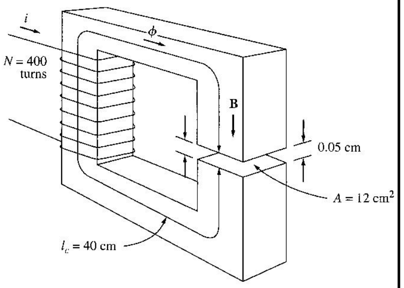
Figure shows a ferromagnetic core whose mean path length is 40cm. There is a small gap of 0.05cm in the structure of the otherwise whole core. The csa of the core is 12cm^2, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective csa of the gap by 5%. Given this information, find

the **total reluctance** of the flux path (iron plus air gap)

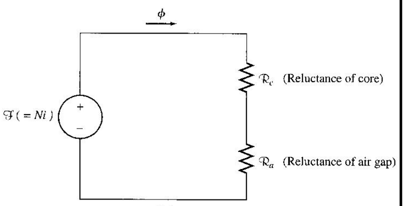
the **current** required to produce a flux density of 0.5T in the air gap.

Solution:

The magnetic circuit corresponding to this core is shown below:



(a)



(b)

Introduction to Machinery Principles

Magnetic Behaviour of Ferromagnetic Materials

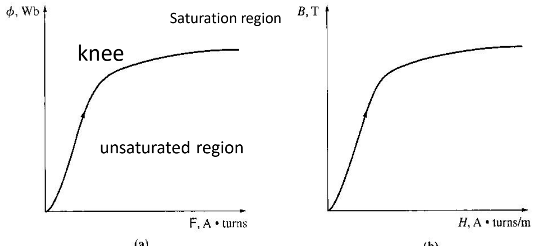
- For magnetic materials, a much larger value of B is produced in these materials than in free space.
- Therefore, the permeability of magnetic materials is much higher than μo.
- However, the permeability is not linear anymore but does depend on the current over a wide range.

The concept of magnetic permeability corresponds to the ability of the material to permit the flow of magnetic flux through it.

In electrical machines and electromechanical devices a somewhat linear relationship between B and I is desired, which is normally approached by limiting the current.

Introduction to Machinery Principles

Look at the magnetization curve and B-H curve.



Note: The curve corresponds to an increase of DC current flow through a coil wrapped around the ferromagnetic core

When the flux produced in the core is plotted versus the mmf producing it. This plot is called a **saturation curve** or a **magnetization curve.**

A small increase in the mmf produces a huge increase in the resulting flux. After a certain point, further increases in the mmf produce relatively smaller increases in the flux. Finally, there will be no change at all as you increase mmf further.

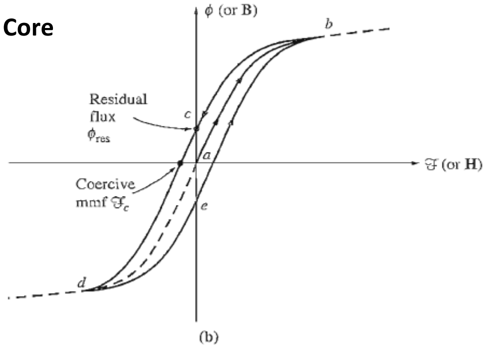
Introduction to Machinery Principles

- Advantage of using a ferromagnetic material for cores in electric machines and transformers is that one gets more flux for a given mmf than with air (free space).
- If the resulting flux has to be proportional to the mmf, then the core must be operated in the unsaturated region.
- Generators and motors depend on magnetic flux to produce voltage and torque, so they need as much flux as possible. So, they operate near the knee of the magnetization curve (flux not linearly related to the mmf). This non-linearity as a result gives peculiar behaviours to machines.
- As magnetizing intensity H increased, the relative permeability first increases and then starts to drop off.

Introduction to Machinery Principles

Energy Losses in Ferromagnetic Core

Hysteresis losses

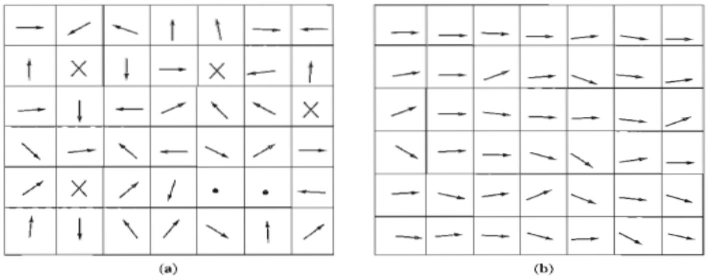


(b)

Notice that the amount of flux present in the core depends not only on the amount of current applied to the windings of the core, but also on the previous history of the flux in the core.

This dependence on the preceding flux history and the resulting failure to retrace flux paths is **called *hysteresis***.

Introduction to Machinery Principles



(a) Magnetic domains oriented randomly. (b) Magnetic domains lined up in the presence of an external magnetic field.

The *hysteresis loss in an iron* core is the energy required to accomplish the reorientation of domains during each cycle of the alternating current applied to the core. It is proportional to hysteresis loop and cause heat in the core.

Introduction to Machinery Principles

FARADAY'S LAW – Induced Voltage from a Time-Changing Magnetic Field

- **It is the basis of transformer operation.**

- **Faraday's law** states that if a flux passes through a turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the *rate of change in the flux with respect to time.*

$$e_{ind} = -\frac{d\phi}{dt}$$

- If there is N number of turns in the coil with the same amount of flux flowing through it, hence:

$$e_{ind} = -N\frac{d\phi}{dt}$$

where:

N – number of turns of wire in coil.

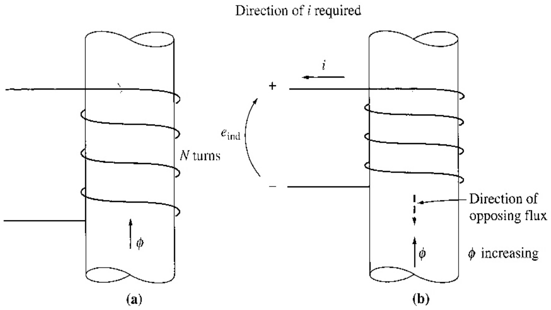
Introduction to Machinery Principles

Note the negative sign at the previous equations which is in accordance to **Lenz' Law** which states:

'The direction of the build-up voltage in the coil is as such that if the coils were short circuited, it would produce current that would cause a flux opposing the original flux change.'

Introduction to Machinery Principles

- If the flux shown is increasing in strength, then the voltage built up in the coil will **tend to establish a flux that will oppose the increase.**
- A current flowing as shown in the figure would produce a flux opposing the increase.
- So, the voltage on the coil must be built up with the polarity required to drive the current through the external circuit. So, $-e_{ind}$



Introduction to Machinery Principles

- In calculations we can omit the minus because it indicates only the physical consideration.

- If there was a leakage flux out of the core, then each turn might have a slightly different flux.

Now consider the induced voltage in the *i*th turn of the coil,

$$e_i = \frac{d\phi_i}{dt}$$

Since there is N number of turns, the equation above may be rewritten into,

$$e_{ind} = \sum_{i=1}^{N} e_i = \sum_{i=1}^{N} \frac{d\phi_i}{dt} = \frac{d}{dt}\left(\sum_{i=1}^{N} \phi_i\right)$$

Introduction to Machinery Principles

Flux linkage - λ

The equation above may be rewritten into,

$$e_{ind} = \frac{d\lambda}{dt}$$

where λ (flux linkage) is defined as:

$$\lambda = \sum_{i=1}^{N} \phi_i \qquad \text{(Weber-turns)}$$

❑ Faraday's law is the fundamental property of magnetic fields involved in transformer operation.

❑ Lenz's Law in transformers is used to predict the polarity of the voltages induced in transformer windings.

Introduction to Machinery Principles

Eddy current: the time changing flux also induces a voltage in the core, which causes eddy currents to flow within the core.
these currents pass through the core (resistive core) causing energy loss called eddy current losses.

Introduction to Machinery Principles

Production of Induced Force on a Wire (motor action principle)

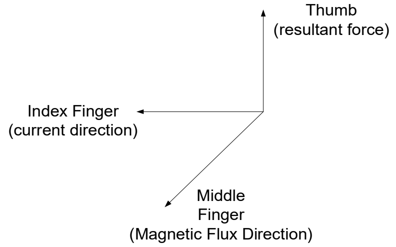
A current carrying wire present in a uniform magnetic field of flux density B, would produce a force on the conductor/wire.
Dependent upon the direction of the surrounding magnetic field, the force induced is given by:

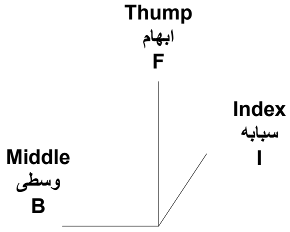
$$F = i\left(l \times B\right)$$

where:
i – represents the current flow in the conductor
l – length of wire, with direction of *l* defined to be in the direction of current flow
B – magnetic field density vector

Introduction to Machinery Principles

The direction of the force is given by **the right-hand rule.**





Right Hand Rule

The induced force formula shown earlier is true if the current carrying conductor is perpendicular to the direction of the magnetic field. If the current carrying conductor is positioned at an angle to the magnetic field, the formula is modified to be as follows:

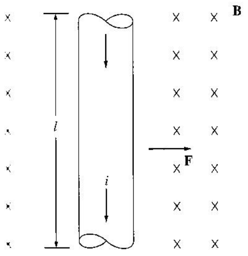
$$F = ilB\sin\theta$$

Where: θ - angle between the conductor and the direction of the magnetic field.

Introduction to Machinery Principles

Example 1-6

The figure shows a wire carrying a current in the presence of a magnetic field. The magnetic flux density is 0.25T, directed into the page. If the wire is 1m long and carries 0.5A of current in the direction from the top of the page to the bottom, what are the magnitude and direction of the force induced on the wire?



Introduction to Machinery Principles

Solution

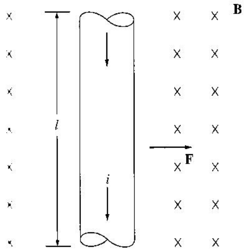
Direction of F is given by the right hand rule ➔ to the right

$$F = ilB\sin\theta$$

= (0.5A)(1.0)m(0.25T) sin 90°

= 0.125 N

Therefore F=0.125 N directed to the right



Introduction to Machinery Principles

Production of Induced voltage on a conductor (Generator action principle)

- If a conductor moves or ‘cuts’ through a magnetic field, voltage will be induced between the terminals of the conductor.

- The magnitude of the induced voltage is dependent upon the velocity of the wire assuming that the magnetic field is constant. This can be summarised in terms of formulation as shown:

$$e_{ind} = (v \times B)\, l$$

where:

v – velocity of the wire

B – magnetic field density

l – length of the wire in the magnetic field

- Note: The value of l (length) is dependent upon the angle at which the wire cuts through the magnetic field. Hence a more complete formula will be as follows:

$$e_{ind} = (v \times B) l \cos\theta$$

where:

θ - smallest angle between the conductor and the direction of (***v x B***)

Introduction to Machinery Principles

The induction of voltages in a wire moving in a magnetic field is fundamental to the operation of all types of generators.

Example 1.8

The figure shows a conductor moving with a velocity of 5m/s to the right in the presence of a magnetic field. The flux density is 0.5T into the page, and the wire is 1m length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

Introduction to Machinery Principles

Solution:

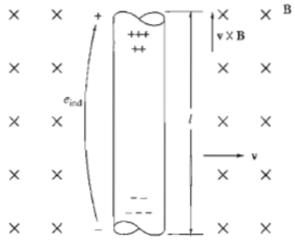
direction of vxB is up and voltage polarity is as shown since positive voltage polarity is in the direction of vxB

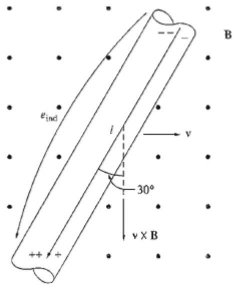
$e_{ind} = (v \times B)\, l = (vB\sin 90^o)\, l \cos 0^o$

$= 5(m/s)(0.5T)\,(1.0m) = 2.5V$

Induced voltage is 2.5V, positive at top of wire

Positive voltage polarity is in the direction of vxB

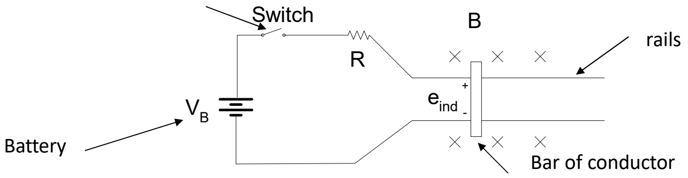




Introduction to Machinery Principles

The Linear DC Machine

Linear DC machine is the simplest form of DC machine which is easy to understand, and it operates according to the same principles and exhibits the same behaviour as motors and generators. Consider the following:



Equations needed to understand linear DC machines are as follows:

❑ **Production of Force on a current carrying conductor in presence of magnetic field**

$$F = i\left(l \times B\right)$$

Introduction to Machinery Principles

❑ **Voltage induced on a current carrying conductor moving in a magnetic field**

$$e_{ind} = (v \times B)\, l$$

❑ **Kirchoff's voltage law**

$$V_B - iR - e_{ind} = 0$$

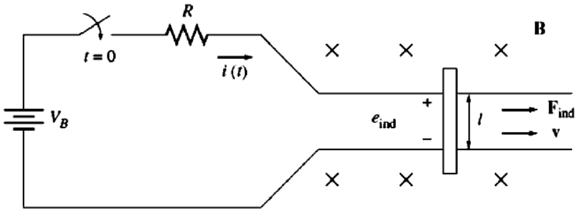
$$\therefore V_B = e_{ind} + iR = 0$$

❑ **Newton's Law for motion**

$$F_{net} = ma$$

Introduction to Machinery Principles

Starting the Linear DC Machine



- To start the machine, the switch is closed.
- Current will flow in the circuit and the equation can be derived from Kirchoff’s law:

$$Since, V_B = iR + e_{ind}$$

$$\therefore \quad i = \frac{V_B - e_{ind}}{R}$$

- At this moment, e_{ind} is 0 due to no movement of the wire (the bar is at rest).

Introduction to Machinery Principles

- As the current flows down through the bar, a force will be induced on the bar.

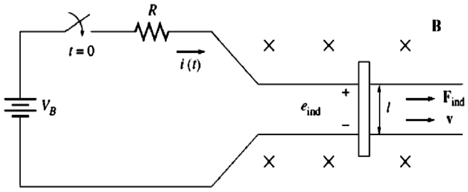
$$\begin{aligned} F &= i\,(l \times B) \\ &= ilB \sin 90 \\ &= ilB \end{aligned}$$

Direction of movement: Right

- When the bar starts to move, its velocity will increase, and a voltage appears across the bar.

$$\begin{aligned} e_{ind} &= (v \times B)l \\ &= vBl \sin 90^\circ \\ &= vlB \end{aligned}$$

Direction of induced potential: positive upwards



Introduction to Machinery Principles

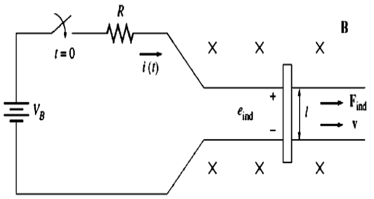
- The voltage now reduces the current flowing in the bar, since by Kirchhoff's voltage law

$$i \downarrow = \frac{V_B - e_{ind} \uparrow}{R}$$

- Eventually; the bar will reach a constant steady-state speed where the net force on the bar is zero. This occurs when e_{ind} has risen all the way up to equal V_B. This is given by:

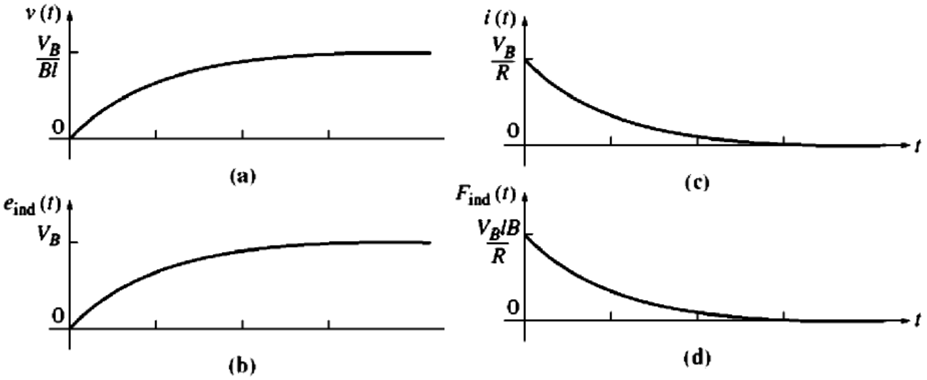
$$V_B = e_{ind} = v_{steady\ state} Bl$$

$$\therefore v_{steady\ state} = \frac{V_B}{Bl}$$



Introduction to Machinery Principles

Summarization of the starting of linear DC machine is sketched in the figures below:



Introduction to Machinery Principles

To summarize, at starting, the linear dc machine behaves as follows:

1.Closing the switch produces a current flow $i = V_B/R$.

2.The current flow produces a force on the bar given by $F = ilB$.

3.The bar accelerates to the right, producing an induced voltage e_{ind} as it speeds up.

4.This induced voltage reduces the current flow

$$i = \frac{V_B - e_{ind} \uparrow}{R}$$

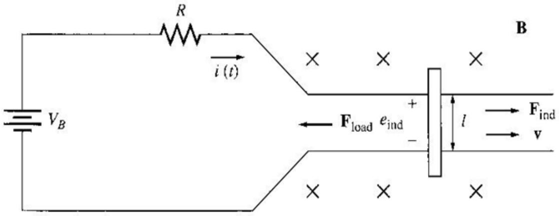
5.The induced force is thus decreased ($F = i\downarrow lB$) until eventually $F = 0$. At that point

6. $e_{ind} = V_B$, $i = 0$, and the bar moves at a constant no- load speed $v_{ss} = V_B/Bl$.

Introduction to Machinery Principles

The Linear DC Machine as a Motor

- Assume the linear machine is initially running at the no-load steady state condition.
- *What happen when an external load is applied?* See figure below:



- A force F_{load} is applied to the bar opposing the direction of motion. Since the bar was initially at steady state, application of the force F_{load} will result in a net force on the bar in the direction opposite the direction of motion.

$$F_{net} = F_{load} - F_{ind}$$

Introduction to Machinery Principles

The Linear DC Machine as a Motor

- Thus, the bar will slow down (the resulting acceleration $a = F_{net}/m$ is negative). As soon as that happen, the induced voltage on the bar drops

  ($e_{ind} = v\downarrow Bl$).
- When the induced voltage drops, the current flow in the bar will rise

$$i\uparrow = \frac{V_B - e_{ind}\downarrow}{R}$$

- Thus, the induced force will rise too.

  ($F_{ind}\uparrow = i\uparrow lB$)
- Final result → the induced force will rise until it is equal and opposite to the load force, and the bar again travels in steady state condition, but at a lower speed.

Introduction to Machinery Principles

The Linear DC Machine as a Motor

- Now, there is an induced force in the direction of motion and power is being converted from **electrical to mechanical form** to keep the bar moving.
- The power converted is

$$P_{conv} = e_{ind}\, I = F_{ind}\, v$$

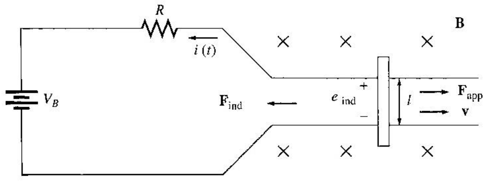
  → An amount of electric power equal to $e_{ind}\, I$ is consumed and is replaced by the mechanical power $F_{ind}\, v$ → **MOTOR**
- The power converted in a real rotating motor is:

$$P_{conv} = \tau_{ind}\, \omega$$

Introduction to Machinery Principles

The Linear DC Machine as a Generator

- Assume the linear machine is operating under no-load steady-state condition. A force in the direction of motion is applied.



- The applied force will cause the bar to accelerate in the direction of motion, and the velocity *v* will increase.
- So; $e_{ind} = v \uparrow Bl$ will increase and will be larger than V_B.
- When $e_{ind} > V_B$ the current reverses direction.
- Since the current now flows up through the bar, it induces a force in the bar ($F_{ind} = ilB$ to the left). This induced force opposes the applied force on the bar.

Introduction to Machinery Principles

The Linear DC Machine as a Generator

- End result → the induced force will be equal and opposite to the applied force, and the bar will move at a higher speed than before. The linear machine is now converting **mechanical power** ($F_{ind}v$) to **electrical power** ($e_{ind}\, i$) → **GENERATOR**
- The amount of power converted : $P_{conv} = \tau_{ind}\, \omega$

NOTE:

☐ The same machine acts as both motor and generator. The only difference is whether the externally applied force is in the direction of motion (generator) or opposite to the direction of motion (motor).

☐ Electrically, $e_{ind} > V_B$ → **Generator**
$e_{ind} < V_B$ → **Motor**

☐ The machine was a generator when it moved rapidly and a motor when it moved more slowly. But, whether it was a motor or a generator, it always moved in the same direction.

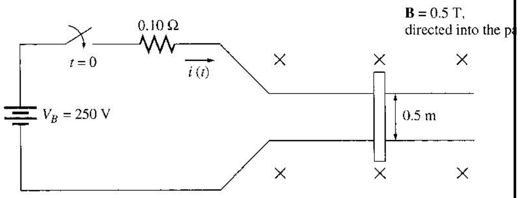
Introduction to Machinery Principles

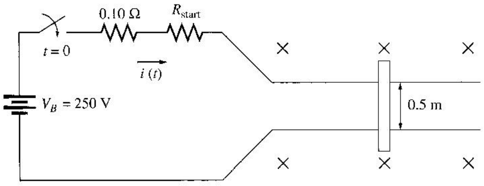
Starting problems with the Linear Machine

- This machine is supplied by a 250V dc source and internal resistance R is 0.1 ohm.
- At starting, the speed of the bar is zero, $e_{ind} = 0$. The current flow at start is:

$$i_{start} = \frac{V_B}{R} = \frac{250}{0.1} = 2500A$$

- This current is very high (10x in excess of the rated current).
- How to prevent? → insert an extra resistance into the circuit during starting to limit current flow until e_{ind} builds up enough to limit it, as shown here:





Introduction to Machinery Principles

Solve Example 1-10 in text book

Homework:

1.6, 1.17, 1.21, and 1.22.

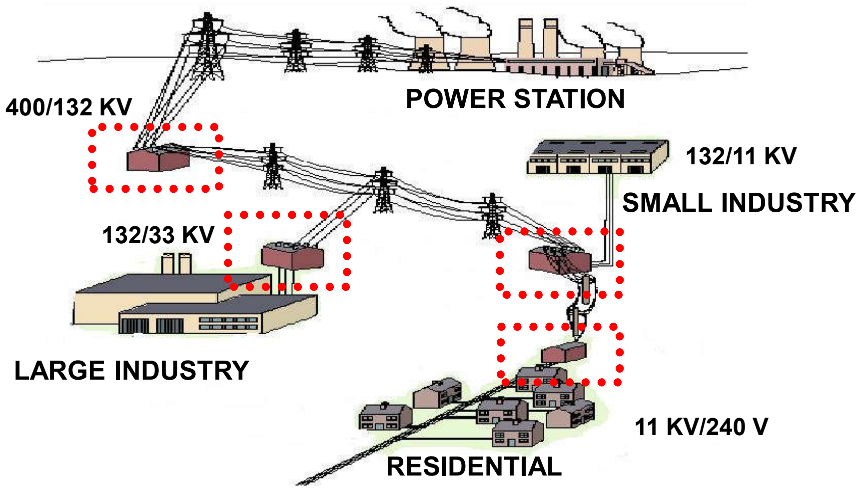
Chapter Two

Transformers

Introduction

- ☐ A *transformer is a device that changes ac electric power at one frequency and* voltage level to ac electric power at the same frequency and another voltage level through the action of a magnetic field.
- ☐ It operates based on Faraday's Law : ***induced voltage on a conductor/coil from a time changing magnetic field.***
- ☐ It consist of two or more coils wrapped around a common Ferromagnetic core.

Introduction



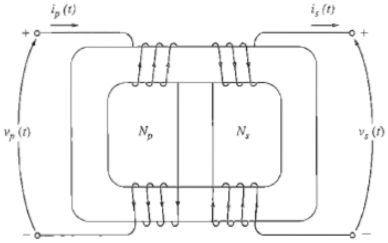
Types of Transformers

- Step up/Unit transformers – Usually located at the output of a generator. Its function is to step up the voltage level so that transmission of power is possible.
- Step down/Substation transformers – Located at main distribution or secondary level transmission substations. Its function is to lower the voltage levels for distribution 1st level purposes.
- Distribution Transformers – located at small distribution substation. It lowers the voltage levels for 2nd level distribution purposes.
- Special Purpose Transformers - E.g. Potential Transformer (PT) , Current Transformer (CT).
- Isolation and Impedance Matching Transformers.

Types and Construction of Transformers

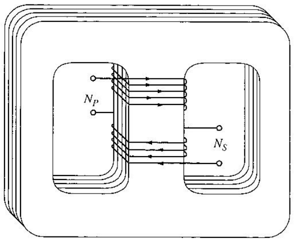
Types of cores for power transformer (both types are constructed from thin laminations electrically isolated from each other –minimize eddy currents)

1) *Core Form* : a simple rectangular laminated piece of steel with the transformer windings wrapped around two sides of the rectangle.



Types and Construction of Transformers

2) *Shell Form* : a three legged laminated core with the windings wrapped around the centre leg.



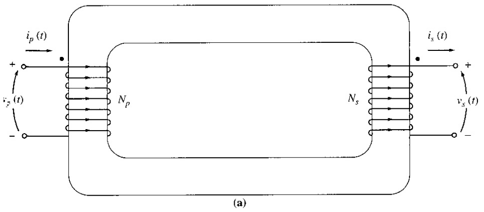
The primary and secondary windings are wrapped one on top of the other with the low-voltage winding innermost, due to 2 purposes:

1) It simplifies the problem of insulating the high-voltage winding from the core.
2) It results in much less leakage flux

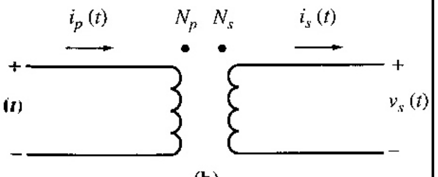
The Ideal Transformer

Definition – a lossless device with an input winding and an output winding.

Figures below show an ideal transformer and schematic symbols of a transformer.



(a)



(b)

The Ideal Transformer

- The transformer has N_p turns of wire on its primary side and N_s turns of wire on its secondary sides. The relationship between the primary and secondary voltage is as follows:

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

where *a* is the turns ratio of the transformer.

- The relationship between primary and secondary current is:

$$N_p \, i_p(t) = N_s \, i_s(t)$$

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

- Note that since both type of relations gives a constant ratio, hence the transformer changes ONLY the magnitude value of current and voltage. **Phase angles are not affected.**

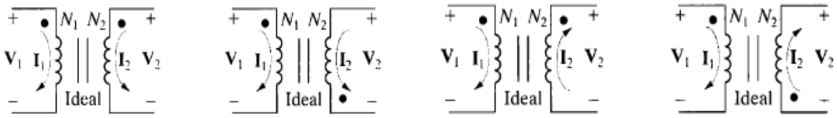
Dot Convention

- The dot convention in schematic diagram for transformers has the following relationship:
  - If the primary **voltage** is +ve at the dotted end of the winding with respect to the undotted end, then the secondary voltage will be positive at the dotted end also. Voltage polarities are the same with respect to the dots on each side of the core.
  - If the primary **current** of the transformer flows **into** the dotted end of the primary winding, the secondary current will flow **out** of the dotted end of the secondary winding.

Dot Convention

☐ If the coil voltages V1and V2 are both positive or negative at the dotmarked terminal, use a plus sign. Otherwise, use a negative sign.

☐ If the coil currents I1 and I2 are both directed into or out of the dotmarked terminal, use a minus sign. Otherwise, use a plus sign.



$\frac{\mathbf{V}_1}{N_1}=\frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1=-N_2\mathbf{I}_2$
(a)

$\frac{\mathbf{V}_1}{N_1}=-\frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1=N_2\mathbf{I}_2$
(b)

$\frac{\mathbf{V}_1}{N_1}=\frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1=N_2\mathbf{I}_2$
(c)

$\frac{\mathbf{V}_1}{N_1}=-\frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1=-N_2\mathbf{I}_2$
(d)

Power in an Ideal Transformer

- The power supplied to the transformer by the primary circuit:

$$P_{in} = V_p I_p \cos \theta_p$$

Where θ_p the angle between the primary voltage and the primary current.

- The power supplied by the transformer secondary circuit to its loads is given by:

$$P_{out} = V_s I_s \cos \theta_s$$

Where θs the angle between the secondary voltage and the secondary current.

- The primary and secondary windings of an ideal transformer have the SAME power factor – because voltage and current angles are unaffected $\theta_p = \theta_s = \theta$

Power in an Ideal Transformer

Input and output power in ideal transformer:

$$P_{out} = V_s I_s \cos \theta$$

We know that, $V_s = V_p / a$ and $I_s = a I_p$

$$P_{out} = \frac{V_p}{a}(aI_p)\cos\theta$$

$$P_{out} = V_p I_p \cos \theta = P_{in}$$

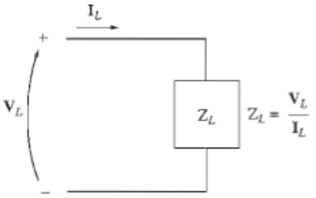
Output Power = Input Power

The same idea can be applied for **reactive power Q** and **apparent power S**.

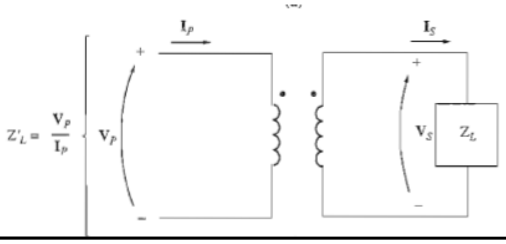
Impedance Transformation

- The impedance is defined as the ratio of the phasor voltage across it to the phasor current flowing through it:

$$Z_L = \frac{V_L}{I_L}$$



- Definition of impedance and impedance scaling through a transformer:



Impedance Transformation

- Hence, the impedance of the load is:

$$Z_L = \frac{V_S}{I_S}$$

- The apparent impedance of the primary circuit of the transformer is:

$$Z_L' = \frac{V_P}{I_P}$$

- Since primary voltage can be expressed as $V_P = aV_S$, and primary current as $I_P = I_S/a$, thus the apparent impedance of the primary is

$$Z_L' = \frac{V_P}{I_P} = \frac{aV_S}{I_S / a} = a^2 \frac{V_S}{I_S}$$

Impedance Transformation

Example 2-1:

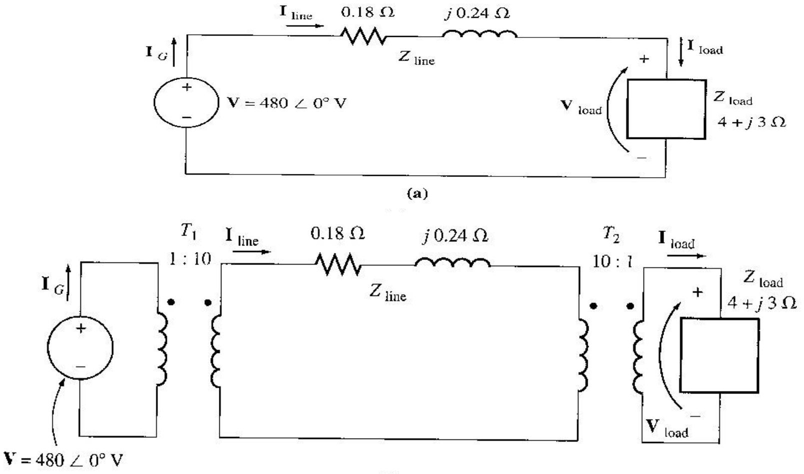
A generator rated at 480 V, 60 Hz is connected a transmission line with an impedance of 0.18+j0.24 Ω. At the end of the transmission line there is a load of 4+j3Ω.

a) If the power system is exactly as described above in Figure (a), what will the voltage at the load be? What will the transmission line losses be?

b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (Figure (b)). What will the load voltage be now? What will the transmission line losses be now?

Impedance Transformation

Example 2-1:

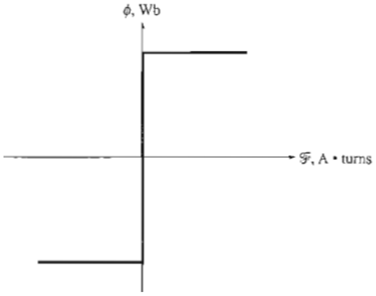


(b)

Real and Ideal Transformer

What assumptions are required to convert a real transformer into the ideal transformer? They are as follows:

1. The core must have no hysteresis or eddy currents.
2. The magnetization curve must have the shape in the following figure.
   Notice that for an unsaturated core the net magnetomotive force $\mathcal{F}_{net} = 0$, implying that $N_p i_p = N_s i_s$.
3. The leakage flux in the core must be zero, implying that all the flux in the core couples both windings.
4. The resistance of the transformer windings must be zero.



The Equivalent Circuit of a Transformer

Taking into account real transformer, there are several losses that has to be taken into account in order to accurately model the transformer, namely:

1. **Copper (I^2R) Losses** – Resistive heating losses in the primary and secondary windings of the transformer.
2. **Eddy current Losses** – resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.
3. **Hysteresis Losses** – these are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are complex, non-linear function of the voltage applied to the transformer.
4. **Leakage flux** – The fluxes and which escape the core and pass through only one of the transformer windings are leakage fluxes. They then produced self-inductance in the primary and secondary coils.

Exact Equivalent Circuit

The Exact equivalent circuit will take into account all the major imperfections in real transformer.

Copper loss

They are modelled by placing a resistor **R_P** in the primary circuit and a resistor **R_S** in the secondary circuit.

Leakage flux

the leakage flux in the primary and secondary windings produces a voltage given by:

$$e_{LP}(t) = N_P \frac{d\phi_{LP}}{dt} \qquad e_{LS}(t) = N_S \frac{d\phi_{LS}}{dt}$$

Since flux is directly proportional to current flow, therefore we can assume that leakage flux is also proportional to current flow in the primary and secondary windings. The following may represent this proportionality:

$$\phi_{LP} = (\mathrm{P}N_P)i_P \qquad \phi_{LS} = (PN_S)i_S$$

Where $\mathscr{P} = 1/\Re$ permeance of flux path

Exact Equivalent Circuit

The constants in these equations can be lumped together. Then,

$$e_{LP}(t) = N_P \frac{d}{dt}(PN_P)i_P = N_P{}^2 P \frac{di_P}{dt}$$

$$e_{LP}(t) = L_P \frac{di_P}{dt}$$

$$e_{LS}(t) = N_S \frac{d}{dt}(PN_S)i_S = N_S{}^2 P \frac{di_S}{dt}$$

$$e_{LS}(t) = L_S \frac{di_S}{dt}$$

Where

$L_P = N_P^2$ P is the self-inductance of the primary coil, and

$L_S = N_S^2$ P is the self-inductance of the secondary coil.

Therefore the **leakage element may be modelled as an inductance** connected together in series with the primary and secondary circuit respectively.

Exact Equivalent Circuit

Core excitation effects: magnetization current and hysteresis & eddy current losses

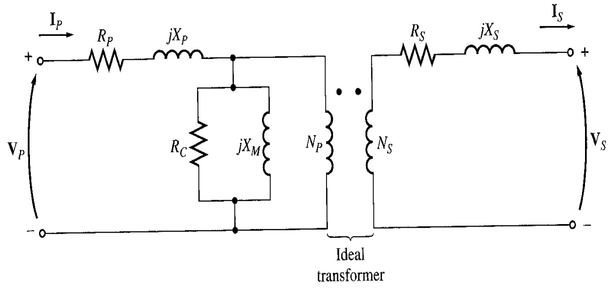
❑ The magnetization current **i_m** is a current proportional (in the unsaturated region) to the voltage applied to the core and lagging the applied voltage by 90° - modeled as reactance **X_m** across the primary voltage source.

❑ The core loss current **i_{h+e}** is a current proportional to the voltage applied to the core that is in phase with the applied voltage – modeled as a resistance **R_C** across the primary voltage source.

Exact Equivalent Circuit

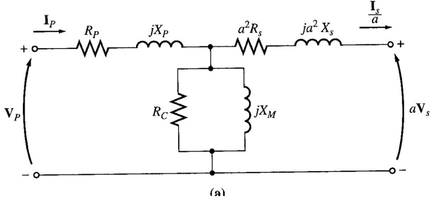
The resulting equivalent circuit:

1) copper loss: **Rp** and **Rs**
2) Leakage Flux: **Xp** and **Xs**
3) Core loss: **Rc**
4) Magnetizing (mutual) Flux : **Xm**

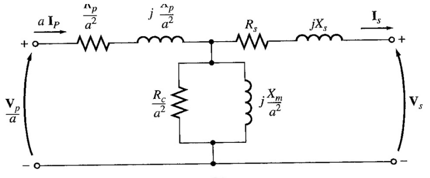


Exact Equivalent Circuit

Based upon the equivalent circuit, in order for mathematical calculation, this transformer equivalent has to be simplified by referring the impedances in the secondary back to the primary or vice versa.



a- Equivalent transformer circuit referred to the primary



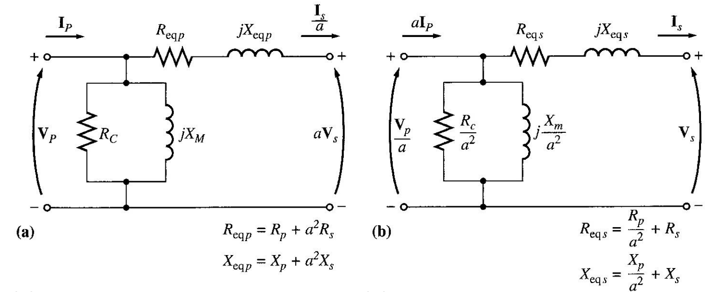
b- Equivalent transformer circuit referred to the secondary

Approximate Equivalent Circuit

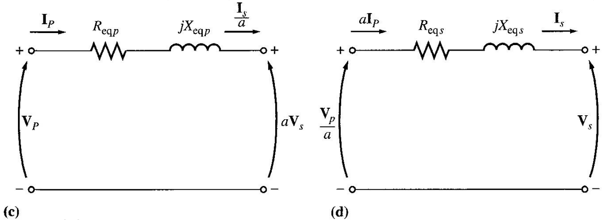
- The derived equivalent circuit is detailed but it is considered to be too complex for practical engineering applications. ***The main problem in calculations will be the excitation and the eddy current and hysteresis loss representation adds an extra branch in the calculations.***

- In practical situations, the excitation current will be relatively small as compared to the load current, which makes the resultant voltage drop across R_p and X_p to be very small, hence R_p and X_p may be lumped together with the secondary referred impedances to form and equivalent impedance. ***In some cases, the excitation current is neglected entirely due to its small magnitude.***

Approximate Equivalent Circuit



(a) Referred to the primary side; (b) Referred to the secondary side



(c) With no excitation branch, referred to the primary side
(d) With no excitation branch, referred to the primary side

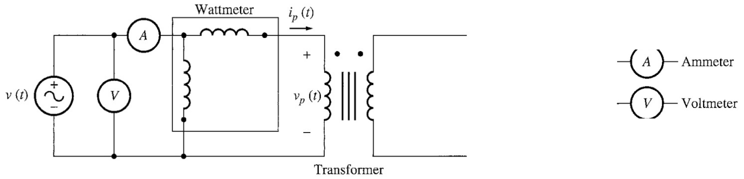
Determining the Values of Components in the Transformer Model

- ➢ The values of the inductances and resistances in the transformer model can be determined experimentally.
- ➢ An adequate approximation of these values can be obtained by:

1. **open-circuit test,**
2. **short-circuit test.**

Open-circuit Test

The transformer's **secondary winding is open-circuited**, and its **primary winding is connected to a full-rated line voltage.**



All the input current will be flowing through the excitation branch of the transformer. The series element R_P and X_P are too small in comparison to R_C and X_M to cause a significant voltage drop. Essentially all input voltage is dropped across the excitation branch.

Determining the values of Components in the Transformer Model

- Full line voltage is applied to the primary – input voltage, input current, input power measured.
- Then, power factor of the input current and magnitude and angle of the excitation impedance can be calculated.
- To obtain the values of R_C and X_M , the easiest way is to find the admittance of the branch:

- Conductance of the core loss resistor, $G_C = \dfrac{1}{R_C}$

- Susceptance of the magnetizing inductor, $B_M = \dfrac{1}{X_M}$

- These two elements are in parallel, thus their admittances add.
- Total excitation admittance, $Y_E = G_C - jB_M = \dfrac{1}{R_C} - j\dfrac{1}{X_M}$
- The magnitude of the excitation admittance (referred to primary),

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

Determining the values of Components in the Transformer Model

- The angle of the admittance can be found from the circuit power factor.

$$PF = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} \qquad \theta = \cos^{-1}\frac{P_{OC}}{V_{OC} I_{OC}}$$

- The power factor is always lagging for a real transformer. Hence,

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\theta$$

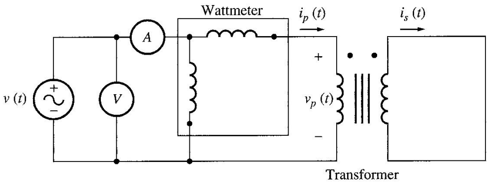
- **This equation can be written in the complex number form and hence the values of R_C and X_M can be determined from the open circuit test data.**

Determining the values of Components in the Transformer Model

Short -circuit Test

➢The secondary terminals are short circuited, and the primary terminals are connected to a fairly low-voltage source.

➢The input voltage is adjusted until the current in the short circuited windings is equal to its rated value. The input voltage, current and power are measured.



Determining the values of Components in the Transformer Model

➢ The excitation branch is ignored, because negligible current flows through it due to low input voltage during this test. Thus, the magnitude of the series impedances referred to the primary is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

➢ Power factor, (lagging) $PF = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}}$

Therefore, $Z_{SE} = \frac{V_{SC}\angle 0^\circ}{I_{SC}\angle -\theta} = \frac{V_{SC}}{I_{SC}}\angle\theta^\circ$

➢ The series impedance

$$Z_{SE} = R_{eq} + jX_{eq}$$
$$= (R_p + a^2 R_S) + j(X_P + a^2 X_S)$$

Determining the values of Components in the Transformer Model

Example 2-2

The equivalent circuit impedances of a 20-kVA, *8000/240 V, 60-Hz* transformer are to be determined. The open-circuit test was performed on the secondary side of the transformer (to reduce the maximum voltage to be measured) and the short circuit test were performed on the primary side of the transformer (to reduce the maximum current to be measured). The following data were taken:

Open circuit test	**Short circuit test**
(on the secondary)	(on the primary)
V_{OC} = 240 V	V_{SC} = 489 V
I_{OC} = 7.133 A	I_{SC} = 2.5 A
P_{OC} = 400 W	P_{SC} = 240 W

Find the impedance of the **approximate equivalent circuit referred to the primary side**, and sketch the circuit

---

Determining the values of Components in the Transformer Model

Solution:

- We should note that the transformer is **step-down** with the following ratio *8000/240V*: Therefore:

➢ The open-circuit test was performed on the secondary side of the transformer (to reduce the maximum voltage to be measured)

➢ the short circuit test were performed on the primary side of the transformer (to reduce the maximum current to be measured).

- The turns ratio of this transformer is *a = 8000/240 = 33.3333.*

-*The power factor during the open-circuit test is*

$$\text{PF} = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}}$$

$$\text{PF} = \cos\theta = \frac{400\ \text{W}}{(240\ \text{V})(7.133\ \text{A})}$$

$$\text{PF} = 0.234\ \text{lagging}$$

- The excitation admittance is given by

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1}\text{PF}$$

$$Y_E = \frac{7.133\ \text{A}}{240\ \text{V}} \angle -\cos^{-1} 0.234$$

$$Y_E = 0.0297 \angle -76.5^\circ\ \text{S}$$

$$Y_E = 0.00693 - j\,0.02888 = \frac{1}{R_C} - j\frac{1}{X_M}$$

Determining the values of Components in the Transformer Model

Solution:

- Therefore , the values of the excitation branch *referred to the Low-voltage (secondary) side are:*

$$R_C = \frac{1}{0.00693} = 144\ \Omega$$

$$X_M = \frac{1}{0.02888} = 34.63\ \Omega$$

- *The power factor during the short-circuit test is*

$$\mathrm{PF} = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}}$$

$$\mathrm{PF} = \cos\theta = \frac{240\ \mathrm{W}}{(489\ \mathrm{V})(2.5\ \mathrm{A})}$$

$$= 0.196\ \text{lagging}$$

- The series impedance is given by

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \cos^{-1} \mathrm{PF}$$

$$Z_{SE} = \frac{489\ \mathrm{V}}{2.5\ \mathrm{A}} \angle 78.7^\circ$$

$$Z_{SE} = 195.6 \angle 78.7^\circ = 38.4 + j192\ \Omega$$

- Therefore, the equivalent resistance and reactance *referred to the high-voltage (primary) side are*

$$R_{eq} = 38.4\ \Omega \qquad X_{eq} = 192\ \Omega$$

Determining the values of Components in the Transformer Model

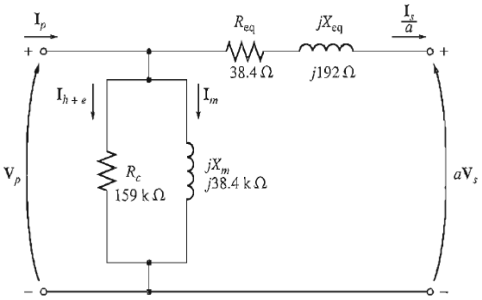
Solution:

- The resulting simplified equivalent circuit referred to the high-voltage (primary) side can be found by converting the excitation branch values to the high-voltage side.

$$R_{C,P} = a^2 R_{C,S} = (33.333)^2\ (144\ \Omega) = 159\ \mathrm{k}\Omega$$

$$X_{M,P} = a^2 X_{M,S} = (33.333)^2\ (34.63\ \Omega) = 38.4\ \mathrm{k}\Omega$$

- The equivalent circuit



The Per-Unit System

Power-system quantities such as voltage, current, power, and impedance are often expressed in per-unit or percent of specified base values.

Advantage of the per-unit system:

1) Simplify the calculations and make the values more meaningful.
2) The transformer equivalent circuit can be simplified.
   The ideal transformer winding can be eliminated. The per-unit system allows us to avoid the possibility of making serious calculation errors when referring quantities from one side of a transformer to the other.

Per-unit quantities are calculated as follows:

$$\boxed{\text{Quantity per unit} = \frac{\text{actual value}}{\text{base value of quantity}}}$$

The Per-Unit System

in order for electrical laws to be valid in the per-unit system, the following relations must be used for other base values:

$$P_{\text{base}1\phi} = Q_{\text{base}1\phi} = S_{\text{base}1\phi}$$

$$I_{\text{base}} = \frac{S_{\text{base}1\phi}}{V_{\text{baseLN}}}$$

$$Z_{\text{base}} = R_{\text{base}} = X_{\text{base}} = \frac{V_{\text{baseLN}}}{I_{\text{base}}} = \frac{V^2_{\text{baseLN}}}{S_{\text{base}1\phi}}$$

$$Y_{\text{base}} = G_{\text{base}} = B_{\text{base}} = \frac{1}{Z_{\text{base}}}$$

The Per-Unit System

Balanced three-phase circuits can be solved in per-unit on a per-phase basis after converting Δ-load impedances to equivalent Y impedances.

$$S_{\text{base}1\phi} = \frac{S_{\text{base}3\phi}}{3}$$

$$V_{\text{baseLN}} = \frac{V_{\text{baseLL}}}{\sqrt{3}}$$

$$S_{\text{base}3\phi} = P_{\text{base}3\phi} = Q_{\text{base}3\phi}$$

$$I_{\text{base}} = \frac{S_{\text{base}1\phi}}{V_{\text{baseLN}}} = \frac{S_{\text{base}3\phi}}{\sqrt{3}V_{\text{baseLL}}}$$

$$Z_{\text{base}} = \frac{V_{\text{baseLN}}}{I_{\text{base}}} = \frac{V^2_{\text{baseLN}}}{S_{\text{base}1\phi}} = \frac{V^2_{\text{baseLL}}}{S_{\text{base}3\phi}}$$

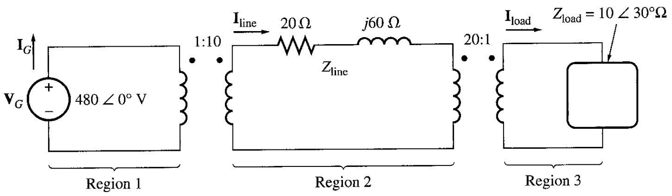
$$R_{\text{base}} = X_{\text{base}} = Z_{\text{base}} = \frac{1}{Y_{\text{base}}}$$

The Per-Unit System

Example 2-3

A simple power system is shown below. This system contains a 480V generator connected to an ideal 1:10 step-up transformer, a transmission line, an ideal 20:1 step-down transformer, and a load. The impedance of the transmission line is 20 + j60Ω, and the impedance of the load is . The base values for this system are chosen to be 480V and 10kVA at the generator.

a) Find the base voltage, current, impedance, and apparent power at every point in the power system.
b) Convert this system to its per-unit equivalent circuit.
c) Find the power supplied to the load in this system.
d) Find the power lost in the transmission line.



The Per-Unit System

- If more than one machine and one transformer are included in a single power system, the system base voltage and power may be chosen arbitrarily, but the *entire system must have the same base.*
- *One common procedure is to choose* the system base quantities to be equal to the base of the largest component in the system.
- Per-unit values given to another base can be converted to the new base by converting them to their actual values (volts, amperes, ohms, etc.) as an in-between step. Alternatively, they can be converted directly by the equations

$$(P,\ Q,\ S)_{\text{pu on base 2}} = (P,\ Q,\ S)_{\text{pu on base 1}} \frac{S_{\text{base 1}}}{S_{\text{base 2}}}$$

$$V_{\text{pu on base 2}} = V_{\text{pu on base 1}} \frac{V_{\text{base 1}}}{V_{\text{base 2}}}$$

$$(R,\ X,\ Z)_{\text{pu on base 2}} = (R,\ X,\ Z)_{\text{pu on base 1}} \frac{(V_{\text{base 1}})^2 (S_{\text{base 2}})}{(V_{\text{base 2}})^2 (S_{\text{base 1}})}$$

Transformer Voltage Regulation and Efficiency

- The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it.

- **Full load Voltage Regulation is a quantity that compares the output voltage at no load with the output voltage at full load**, defined by this equation:

$$VR = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\%$$

- At no load, $V_S = V_P/a$ thus,

$$VR = \frac{(V_P/a) - V_{S,fl}}{V_{S,fl}} \times 100\%$$

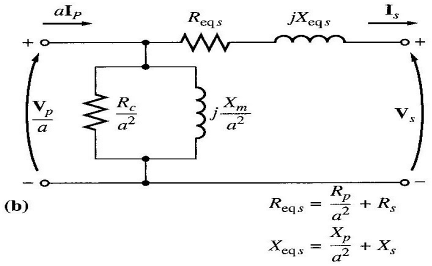
- In per-unit system,

$$VR = \frac{V_{P,pu} - V_{S,fl,pu}}{V_{S,fl,pu}} \times 100\%$$

- Ideal transformer, ***VR = 0%.***

The Transformer Phasor Diagram

- To determine the voltage regulation of a transformer, we must understand the voltage drops within it. Consider the simplified equivalent circuit referred to the secondary side:



$$R_{\text{eq}\,s} = \frac{R_P}{a^2} + R_s$$

$$X_{\text{eq}\,s} = \frac{X_P}{a^2} + X_s$$

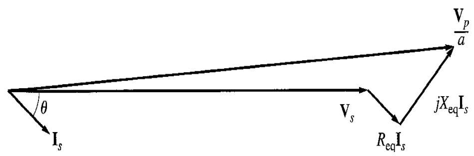
- Ignoring the excitation of the branch, more consideration is given to the series impedances (R_{eq} +jX_{eq}).
- Voltage Regulation depends on magnitude of the series impedance and the phase angle of the current flowing through the transformer.
- Phasor diagrams will determine the effects of these factors on the voltage regulation. A phasor diagram consist of current and voltage vectors.

Transformer Voltage Regulation and Efficiency

- **A phasor diagram consist of current and voltage vectors**.
- Assume that the reference phasor is the secondary voltage V_S. Therefore the reference phasor will have *ϴ* degrees in terms of angle.
- Based upon the equivalent circuit, apply Kirchoff Voltage Law,

$$\frac{V_P}{a} = V_S + R_{eq} I_S + jX_{eq} I_S$$

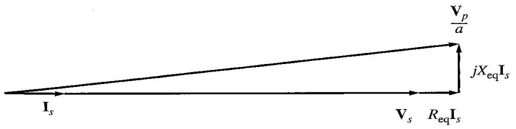
- From this equation, the phasor diagram can be visualised.
- Figure below shows a phasor diagram of a transformer operating at a ***lagging*** power factor. For lagging loads, V_P / a > V_S so the voltage regulation with lagging loads is > 0.



At lagging power factor

Transformer Voltage Regulation and Efficiency

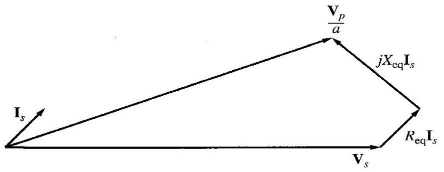
- When the power factor is **unity**, V_S is lower than V_P so $V_R > 0$. But, V_R is smaller than before (during lagging PF).



(a)

Unity PF

- With a **leading** power factor, V_S is higher than the referred V_P so $V_R < 0$.



Leading PF (b)

Transformer Voltage Regulation and Efficiency

- In summary:

Lagging PF	Unity PF	Leading PF
$V_P / a > V_S$	$V_P / a > V_S$	$V_S > V_P / a$
VR > 0	VR > 0 (smaller than VR lag)	VR < 0

- Due to the fact that transformer is usually operated at lagging pf, hence there is an approximate method to simplify calculations.

Transformer Efficiency

- Transformer efficiency is defined as (applies to motors, generators and transformers):

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \qquad \eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

- Types of losses incurred in a transformer:

1) ***Copper I^2R losses***
2) ***Hysteresis losses***
3) ***Eddy current losses***

- Therefore, for a transformer, efficiency may be calculated using the following:

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} \times 100\%$$

Transformer Voltage Regulation and Efficiency

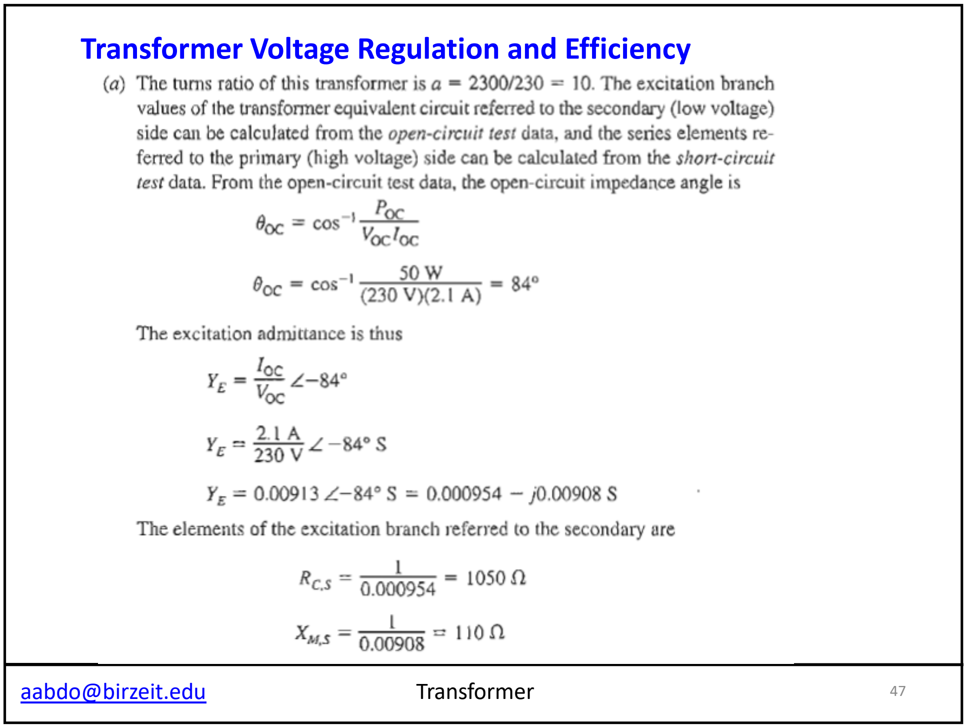
Example 2-5

A 15kVA, 2300/230 V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following data have been taken from the transformer:

Open-circuit test (low voltage side)	Short-circuit test (high voltage side)
$V_{OC} = 230$ V	$V_{SC} = 47$ V
$I_{OC} = 2.1$ A	$I_{SC} = 6.0$ A
$V_{OC} = 50$ W	$P_{SC} = 160$ W

1. Find the equivalent circuit referred to the high voltage side
2. Find the equivalent circuit referred to the low voltage side
3. Calculate the full-load voltage regulation at 0.8 lagging PF, 1.0 PF, and at 0.8 leading PF.
4. Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.
5. Find the efficiency at full load with PF 0.8 lagging.

Important example you need to solve it



Transformer Voltage Regulation and Efficiency

(*a*) The turns ratio of this transformer is $a = 2300/230 = 10$. The excitation branch values of the transformer equivalent circuit referred to the secondary (low voltage) side can be calculated from the *open-circuit test* data, and the series elements referred to the primary (high voltage) side can be calculated from the *short-circuit test* data. From the open-circuit test data, the open-circuit impedance angle is

$$\theta_{OC} = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}}$$

$$\theta_{OC} = \cos^{-1}\frac{50\ \text{W}}{(230\ \text{V})(2.1\ \text{A})} = 84°$$

The excitation admittance is thus

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -84°$$

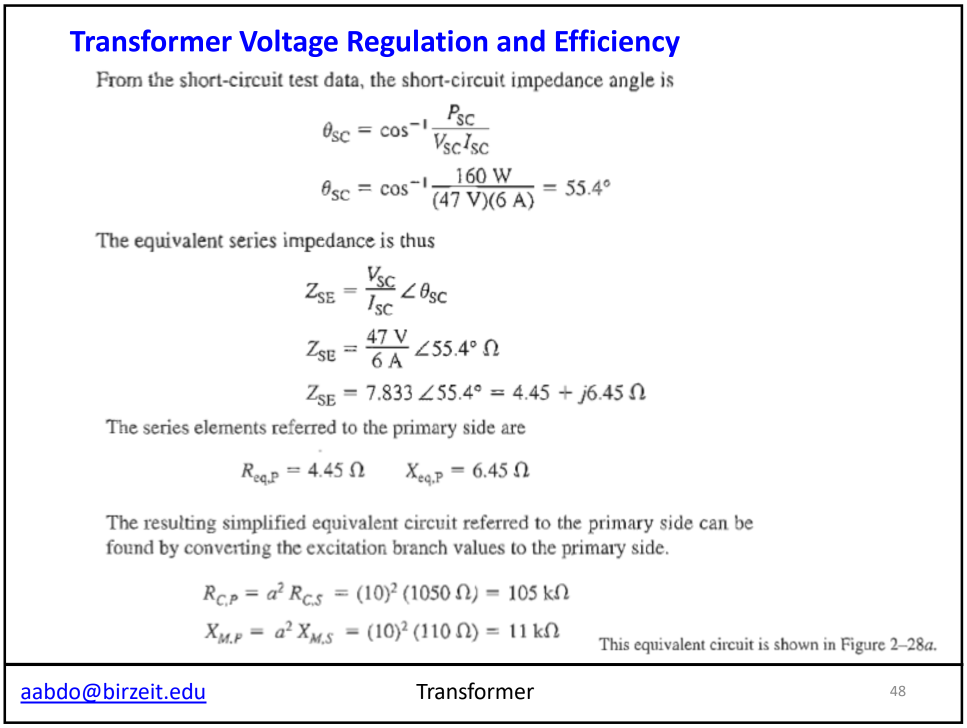
$$Y_E = \frac{2.1\ \text{A}}{230\ \text{V}} \angle -84°\ \text{S}$$

$$Y_E = 0.00913 \angle -84°\ \text{S} = 0.000954 - j0.00908\ \text{S}$$

The elements of the excitation branch referred to the secondary are

$$R_{C,S} = \frac{1}{0.000954} = 1050\ \Omega$$

$$X_{M,S} = \frac{1}{0.00908} = 110\ \Omega$$



Transformer Voltage Regulation and Efficiency

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}}$$

$$\theta_{SC} = \cos^{-1}\frac{160\ \text{W}}{(47\ \text{V})(6\ \text{A})} = 55.4°$$

The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \theta_{SC}$$

$$Z_{SE} = \frac{47\ \text{V}}{6\ \text{A}} \angle 55.4°\ \Omega$$

$$Z_{SE} = 7.833 \angle 55.4° = 4.45 + j6.45\ \Omega$$

The series elements referred to the primary side are

$$R_{eq,P} = 4.45\ \Omega \qquad X_{eq,P} = 6.45\ \Omega$$

The resulting simplified equivalent circuit referred to the primary side can be found by converting the excitation branch values to the primary side.

$$R_{C,P} = a^2 R_{C,S} = (10)^2 (1050\ \Omega) = 105\ \text{k}\Omega$$

$$X_{M,P} = a^2 X_{M,S} = (10)^2 (110\ \Omega) = 11\ \text{k}\Omega$$

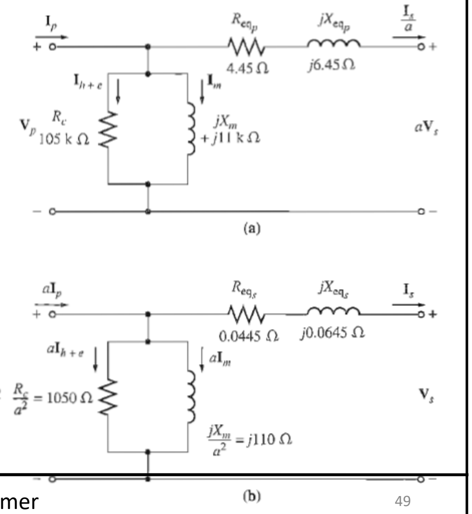
This equivalent circuit is shown in Figure 2–28*a*.

Transformer Voltage Regulation and Efficiency

(b) To find the equivalent circuit referred to the low-voltage side, it is simply neces sary to divide the impedance by a^2. Since $a = N_P/N_S = 10$, the resulting values ar

$$R_C = 1050\ \Omega \qquad R_{eq} = 0.0445\ \Omega$$
$$X_M = 110\ \Omega \qquad X_{eq} = 0.0645\ \Omega$$

The resulting equivalent circuit is shown in Figure 2–28b.



Transformer Voltage Regulation and Efficiency

(c) The full-load current on the secondary side of this transformer is

$$I_{S,\text{rated}} = \frac{S_{\text{rated}}}{V_{S,\text{rated}}} = \frac{15{,}000\ \text{VA}}{230\ \text{V}} = 65.2\ \text{A}$$

To calculate V_P/a, use Equation (2–64):

$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{\text{eq}}\mathbf{I}_S + jX_{\text{eq}}\mathbf{I}_S \qquad (2\text{–}64)$$

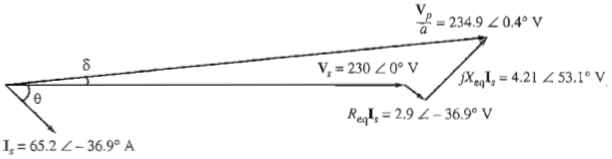
At PF = 0.8 lagging, current $\mathbf{I}_S = 65.2 \angle -36.9°$ A. Therefore,

$$\begin{aligned}\frac{\mathbf{V}_P}{a} &= 230\angle 0°\ \text{V} + (0.0445\ \Omega)(65.2\angle -36.9°\ \text{A}) + j(0.0645\ \Omega)(65.2\angle -36.9°\ \text{A}) \\ &= 230\angle 0°\ \text{V} + 2.90\angle -36.9°\ \text{V} + 4.21\angle 53.1°\ \text{V} \\ &= 230 + 2.32 - j1.74 + 2.52 + j3.36 \\ &= 234.84 + j1.62 = 234.85\angle 0.40°\ \text{V}\end{aligned}$$

The resulting voltage regulation is

$$\text{VR} = \frac{V_P/a - V_{S,\text{fl}}}{V_{S,\text{fl}}} \times 100\% \qquad (2\text{–}62)$$

$$= \frac{234.85\ \text{V} - 230\ \text{V}}{230\ \text{V}} \times 100\% = 2.1\%$$



Three-Phase Transformer Connections

- The primaries and secondaries of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ).
- The important point to note when analyzing any 3-phase transformer is to look at a single transformer in the bank.
- **Any single phase transformer in the bank behaves exactly like the single-phase transformers already studied**.

Figure 3–35 | A three-phase transformer bank composed of independent transformers.

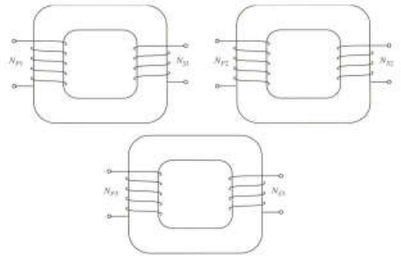
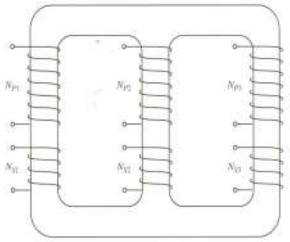


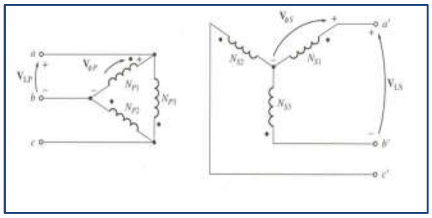
Figure 3–36 | A three-phase transformer wound on a single three-legged core.



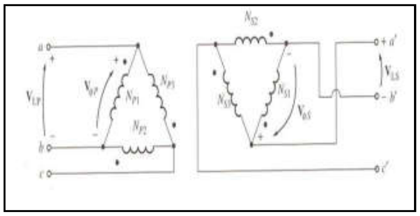
Three-Phase Transformer Connections

Spare power ABB transformer at an HVDC site.

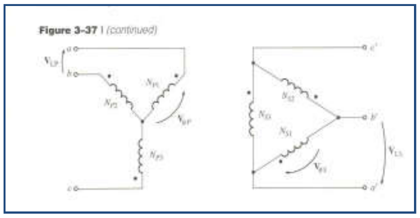
Three-Phase Transformer Connections



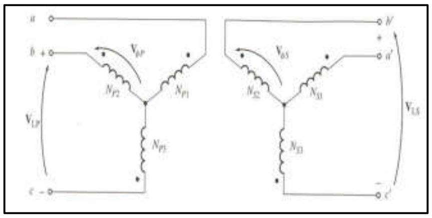
Delta – wye (Δ – Y)



Delta – delta (Δ – Δ)



Wye – delta (Y- Δ)



Wye – wye (Y – Y)

Three-Phase Transformer Connections

- The impedance, voltage regulation, efficiency, and similar calculations for three phase transformers are done on a **per-phase basis**, using same techniques as single-phase transformers.
- A simple concept that all students must remember is that, for a Delta configuration,

$$V_{\phi P} = V_L \qquad I_{\phi P} = \frac{I_L}{\sqrt{3}} \qquad S_{\phi P} = \frac{S}{3}$$

- For Wye configuration,(for balanced case I_N=0 and neutral can be left open and only 3 wires are required)

$$V_{\phi P} = \frac{V_L}{\sqrt{3}} \qquad I_{\phi P} = I_L \qquad S_{\phi P} = \frac{S}{3}$$

Calculating 3-Phase Transformer Turns Ratio

- The basic concept of calculating the turns ratio for a single phase transformer is utilised where,

$$a = \frac{V_{\phi P}}{V_{\phi S}}$$

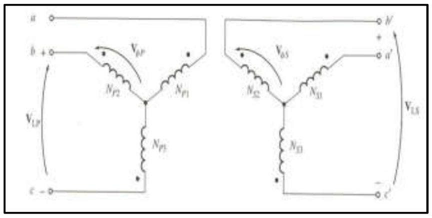
- Therefore to cater for 3 phase transformer, suitable conversion into per phase is needed to relate the turns ratio of the transformer with the **line voltages**.

Three-Phase Transformer Connections

Wye – Wye (Y – Y)

Voltage Ratio

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a \qquad \text{Y}-\text{Y}}$$



Problems:

- If loads are unbalanced, then voltages on the phases can become unbalanced
- Third harmonic voltages can be large since third harmonics are in phase and add up and can be larger than the fundamental voltage itself

These problems can be solved by:

- Solidly grounding the neutrals of the transformers , especially primary winding neutral to allow path for 3rd harmonic currents
- Adding a third winding in delta to the transformer which causes a circulating current in this winding to suppress the 3rd harmonic current.

Three-Phase Transformer Connections

Wye – Delta (Y- Δ)

$$\frac{V_{\text{LP}}}{V_{\text{LS}}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}}$$

$$\boxed{\frac{V_{\text{LP}}}{V_{\text{LS}}} = \sqrt{3}a \qquad \text{Y}-\Delta}$$

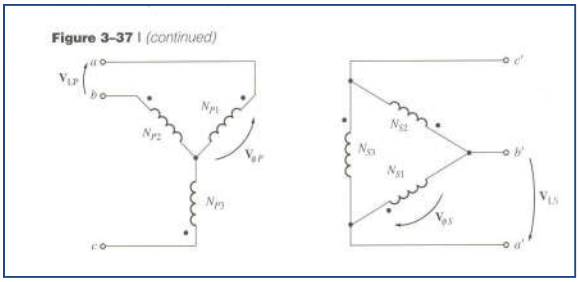


Figure 3–37 | *(continued)*

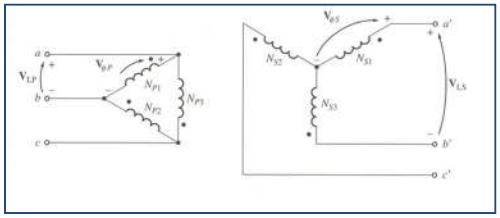
- Does not have 3rd harmonic problem
- It has one problem: the secondary voltage is shifted 30 degree relative to primary which might cause problems when paralleling transformers which requires special attention to this phase shift which might be leading or lagging depending on the phase sequence

Three-Phase Transformer Connections

Delta – Wye (Δ – Y)

$$\frac{V_{\text{LP}}}{V_{\text{LS}}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}}$$

$$\boxed{\frac{V_{\text{LP}}}{V_{\text{LS}}} = \frac{a}{\sqrt{3}} \qquad \Delta-\text{Y}}$$

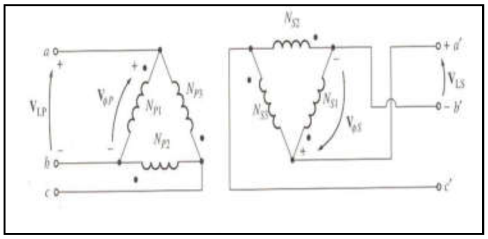


Has the same advantages and phase shift as the Y-D transformer.

Three-Phase Transformer Connections

Delta – Delta (Δ – Δ)

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad \Delta - \Delta}$$



Has no phase shift and no problem with 3rd harmonic.

Solve Example 2-9

Instrument Transformers

- Special transformers used to take measurements: Potential Transformer (PT) and Current Transformer (CT)
- PT is a specially wound transformer with high-voltage primary and low-voltage secondary. It has a very low power rating and it is used to provide a an accurate sample of the power system voltage to the monitoring instrument without affecting the true voltage values
- CT sample the current an a line and reduce it to safe and measurable level.

Homework

- Homework problems are: **2.1, 2.6, 2.12 and 2.24**;
- These problems are from the 5th edition text book.

Chapter Three

AC Machine Fundamentals

Introduction

- **Ac machines** are generators that convert mechanical energy to ac electrical energy and motors that convert ac electrical energy to mechanical energy.

There are two major classes of ac machines:

1. Synchronous machines:

Synchronous machines are motors and generators whose magnetic field current is supplied by a separate dc power source,

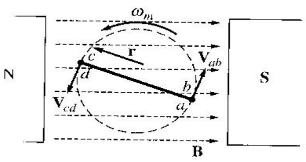
2 . Induction machines:

Induction machines are motors and generators whose field current is supplied by magnetic induction (transformer action) into their field windings.

- The field circuits of most synchronous and induction machines are located on their rotors.

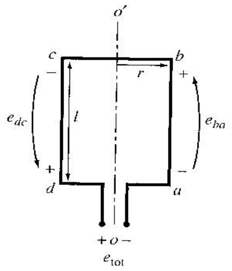
Simple Rotating Loop in a Uniform Magnetic Field

- A loop of wire in a uniform magnetic field is the simplest possible machine that produces a sinusoidal ac voltage.
- The figure below shows a simple rotating loop in a uniform magnetic field.
(a) is the front view and (b) is the view of the coil.
- The rotating part is called **the rotor**, and the stationary part is called **the stator**.



B is a uniform magnetic field, aligned as shown.

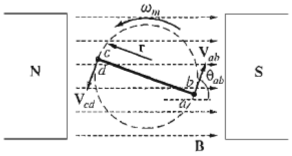
(a)



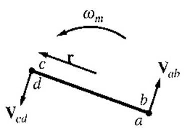
(b)

Voltage Induced in a Simple Rotating Loop

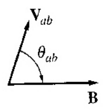
- **Rotor rotation** will **induce a voltage** in the wire loop .
- To determine **total voltage induced** in loop e_{tot} ➔
- examine each segment **separately** using:



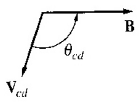
$$e_{ind} = (\vec{v} \times \vec{B}) \bullet \vec{l}$$



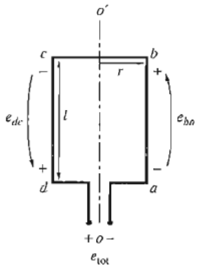
(a)



(b)



(c)

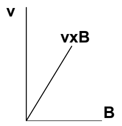


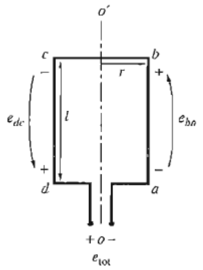
Voltage Induced in a Simple Rotating Loop

1. Segment ***ab***

- The velocity of the wire is tangential to the path of rotation, while the magnetic field **B** points to the right.
- The quantity ***v x B*** points into the page, which is the same direction as segment ***ab***. Thus, the induced voltage on this segment is:

$e_{ba} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot l = vBl \sin \theta_{ab}$ into the page



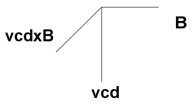


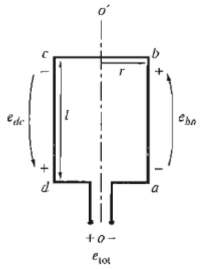
Voltage Induced in a Simple Rotating Loop

2. Segment *cd*

- The velocity of the wire is tangential to the path of rotation, while **B** points to the right.
- The quantity ***v x B*** points into the page, which is the same direction as segment *cd*. Thus,

$e_{cd} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot l = vBl \sin \theta_{cd}$ out of the page



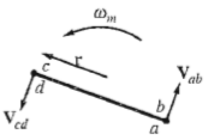


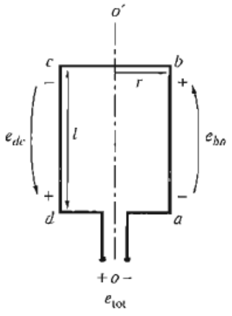
Voltage Induced in a Simple Rotating Loop

3. Segment *bc*

In the first half of this segment, the quantity ***v x B*** points into the page, and in the second half of this segment, the quantity ***v x B*** points out of the page. Since the length *l* is in the plane of the page, ***v x B*** is perpendicular to *l* for both portions of the segment. Thus,

$e_{bc} = 0$





4. Segment *da*

Same as segment *bc*, ***v x B*** is perpendicular to *l*. Thus,

$e_{da} = 0$

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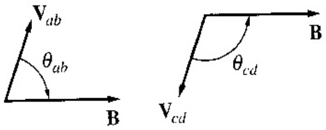
Voltage Induced in a Simple Rotating Loop

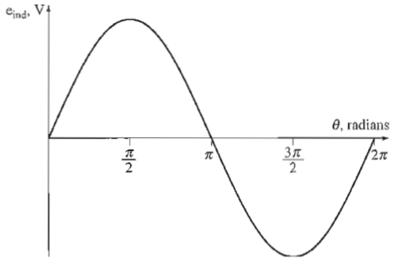
- Total induced voltage on the loop

$$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$
$$= vBl \sin\theta_{ab} + vBl \sin\theta_{cd}$$

- since $\theta_{ab} = 180º - \theta_{cd}$ and $\sin\theta = \sin(180º - \theta)$

$$\boxed{e_{ind} = 2\,vBl \sin\theta}$$





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Voltage Induced in a Simple Rotating Loop

Alternative way to express e_{ind}:

- If the loop is rotating at a constant angular velocity ω, then the angle θ of the loop will increase linearly with time.

$$\theta = \omega t$$

- also, the tangential velocity v of the edges of the loop is:

$$v = r\,\omega$$

- where **r** is the radius from axis of rotation out to the edge of the loop and **ω** is the angular velocity of the loop.
- Therefore,

$$e_{ind} = 2\,vBl\,\sin\theta$$
$$e_{ind} = 2r\,\omega Bl\,\sin\omega t$$
$$= AB\omega\,\sin\omega t$$

since area, $A = 2rl$,

- Finally, since maximum flux through the loop occurs when the loop is perpendicular to the magnetic flux density lines, so

$$\phi_{\max} = AB$$

Voltage Induced in a Simple Rotating Loop

- Thus,

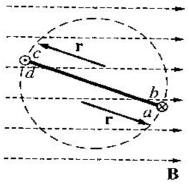
$$e_{ind} = \phi_{\max}\,\omega \sin \omega t$$

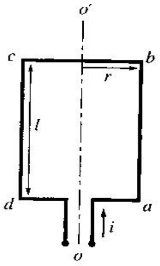
- From here we may conclude that the induced voltage is dependent upon:

1. Flux level (**the B component**)
2. Speed of Rotation (**the v component**)
3. Machine Constants (**the *l* component and machine materials**)

The Torque Induced in a Current-Carrying Loop

- ➢ Assume that the rotor loop is at some arbitrary angle θ with respect to the magnetic field, and that current is flowing in the loop.





- ➢ To determine the magnitude and direction of the torque, examine the phasors below:
- ➢ The force on each segment of the loop is given by:

F = i (***l* x *B***)

Torque on that segment,

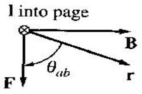
$$\tau = rF\sin\theta$$

The Torque Induced in a Current-Carrying Loop

1. Segment *ab*

- The direction of the current is into the page, while the magnetic field B points to the right. (***l* x *B***) points down. Thus,

F = i (***l* x *B***)= ***ilB*** down



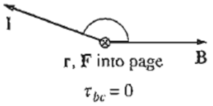
- Resulting torque, clockwise

$$\tau_{ab} = (F)(r\sin\theta_{ab})$$

2. Segment *bc*

- The direction of the current is in the plane of the page, while the magnetic field B points to the right. (***l* x *B***) points into the page. Thus,

F = i (***l* x *B***) = ***ilB*** into the page



- Resulting torque is zero, since vector r and *l* are parallel and the angle θ_{bc} is 0.

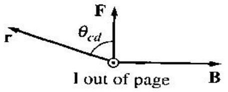
$$\tau_{bc} = (F)(r\sin\theta_{bc}) = 0$$

The Torque Induced in a Current-Carrying Loop

3. Segment *cd*

The direction of the current is out of the page, while the magnetic field B points to the right. (***l* x *B***) points up. Thus,

F = i (***l* x *B***)= ***ilB*** up



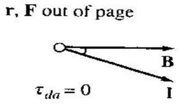
Resulting torque,

$$\tau_{cd} = (F)(r\sin\theta_{cd})$$
$$= rilB\sin\theta_{cd} \qquad \text{clockwise}$$

4. Segment *da*

The direction of the current is in the plane of the page, while the magnetic field B points to the right. (***l* x *B***) points out of the page. Thus,

F = i (***l* x *B***)= ***ilB*** out of the page



Resulting torque is zero, since vector r and *l* are parallel and the angle θ_{da} is 0.

$$\tau_{da} = (F)(r\sin\theta_{da})$$
$$= 0$$

The Torque Induced in a Current-Carrying Loop

- The total induced torque on the loop:

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$
$$= rilB\sin\theta_{ab} + rilB\sin\theta_{cd}$$
$$= 2rilB\sin\theta$$
$$where\ \theta_{ab} = \theta_{cd} = \theta$$

- Note : The **torque is maximum** when the plane of the loop is parallel to the magnetic field, and the **torque is zero** when the plane of the loop is perpendicular to the magnetic field.

The Torque Induced in a Current-Carrying Loop

- **Alternative way to express e_{ind}:**
- The current flowing in the wire loop will generate a magnetic flux density :

$$\left|\vec{B}_{loop}\right| = \frac{\mu i}{G}$$

where G = factor depending on loop geometry.

- Hence, since loop area *A=2rl*, by substituting :

$$\begin{aligned}\tau_{ind} &= 2rliB\sin\theta \\ &= A(G/\mu)B_{loop}B_s\sin\theta\end{aligned}$$

Or $$\therefore \tau_{ind} = kB_{loop}B_s\sin\theta$$

where *k = AG/μ*.

$$\tau_{ind} = k\left(\vec{B}_{loop} \times \vec{B}_s\right)$$

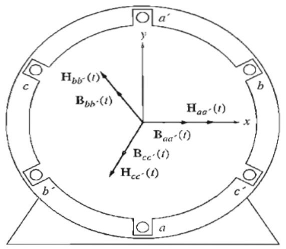
Factors Affecting Torque in Real Machines

1. Strength of the **rotor magnetic field**
2. Strength of **external (stator) magnetic field**
3. **Angle** between the two fields
4. **Machine constants**

The Rotating Magnetic Field

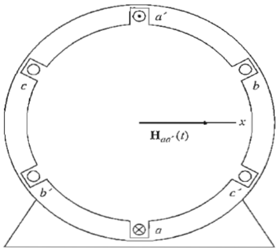
- **Torque** is produced to **align rotor** (loop) magnetic **field with stator** magnetic **field**.
- If **stator magnetic field** is **rotated**, torque will cause rotor to **'chase'** the rotating stator magnetic field.
- How to create a rotating stator magnetic field?
  Use a set of **three-phase windings displaced by 120° electrical** around the machine circumference.
- Fundamental principle – ***a 3-phase set of currents, each of equal magnitude and differing in phase by 120º, flows in a 3-phase winding, then it will produce a rotating magnetic field of constant magnitude***

The Rotating Magnetic Field



(a)

(a) A simple three phase stator. Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils. The magnetizing intensities produced by each coil are also shown.



(b)

(b) The magnetizing intensity vector H*aa*'(t) produced by a current flowing in coil *aa*'.

- Let's apply a set of currents to the stator above and see what happens at specific instants of time. Assume currents in the 3 coils are:

$$i_{aa'}(t) = I_M \sin \omega t \ A$$

$$i_{bb'}(t) = I_M \sin(\omega t - 120°) A$$

$$i_{cc'}(t) = I_M \sin(\omega t - 240°) A$$

The Rotating Magnetic Field

- The current in coil aa’ flows into the “a” end of the coil and out the ”a’” end of the coil. It produces the magnetic field intensity:

$$H_{aa'}(t) = H_M \sin \omega t \angle 0^\circ \ A \bullet turns \ / m$$
$$H_{bb'}(t) = H_M \sin(\omega t - 120^\circ) \angle 120^\circ \ A \bullet turns \ / m$$
$$H_{cc'}(t) = H_M \sin(\omega t - 240^\circ) \angle 240^\circ \ A \bullet turns \ / m$$

- The flux densities equations are:

$$B_{aa'}(t) = B_M \sin \omega t \angle 0^\circ \ T$$
$$B_{bb'}(t) = B_M \sin(\omega t - 120^\circ) \angle 120^\circ \ T$$
$$B_{cc'}(t) = B_M \sin(\omega t - 240^\circ) \angle 240^\circ \ T$$

Where $B_M = \mu H_M$

The Rotating Magnetic Field

At

$$\omega t = 0^\circ$$

$$B_{aa'} = 0$$
$$B_{bb'} = B_M \sin(-120^\circ) \angle 120^\circ \ T$$
$$B_{cc'} = B_M \sin(-240^\circ) \angle 240^\circ \ T$$

The total magnetic field from all three coils added together will be

$$\mathrm{B_{net}} = \mathrm{B_{aa'}} + \mathrm{B_{bb'}} + \mathrm{B_{cc'}}$$
$$= 0 + \left(-\frac{\sqrt{3}}{2} B_M\right) \angle 120^\circ + \left(\frac{\sqrt{3}}{2} B_M\right) \angle 240^\circ$$
$$= 1.5 B_M \angle -90^\circ$$

$$\omega t = 90^\circ$$

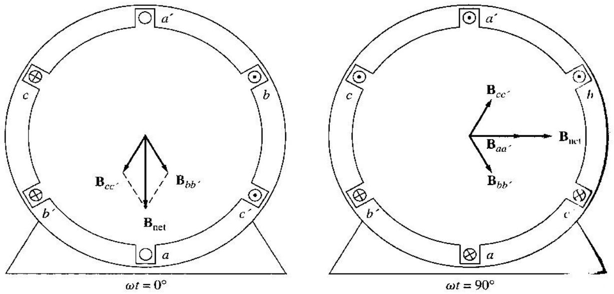
$$B_{aa'} = B_M \angle 0^\circ$$
$$B_{bb'} = -0.5 B_M \angle 120^\circ \ T$$
$$B_{cc'} = -0.5 B_M \angle 240^\circ \ T$$

The total magnetic field from all three coils added together will be

$$\mathrm{B_{net}} = \mathrm{B_{aa'}} + \mathrm{B_{bb'}} + \mathrm{B_{cc'}}$$
$$= B_M + (-0.5 B_M) \angle 120^\circ + (-0.5 B_M) \angle 240^\circ$$
$$= 1.5 B_M \angle 0^\circ$$

The Rotating Magnetic Field

- The resulting magnetic flux

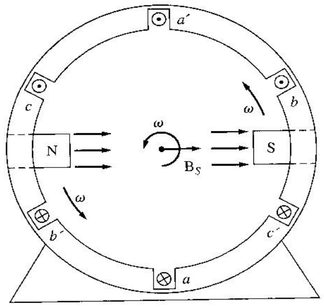


$\omega t = 0°$ (a)

$\omega t = 90°$ (b)

- ***At any time t, the magnetic field will have the same magnitude 1.5 B_M and it will continue to rotate at angular velocity ω.***

The Rotating Magnetic Field



- The following figure shows that the rotating magnetic field in this stator can be represented as a north pole (the flux leaves the stator) and a south pole (flux enters the stator).

- These magnetic poles complete one mechanical rotation around the stator surface for each electrical cycle of the applied current.

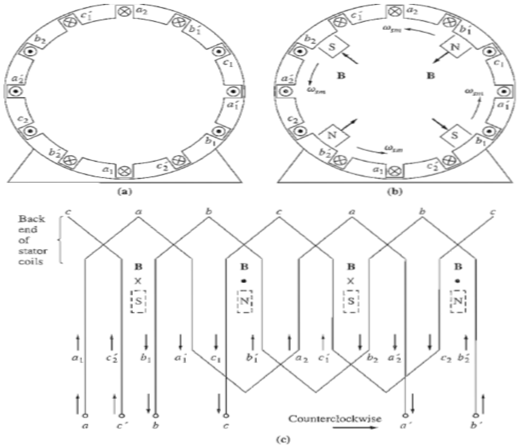
- The mechanical speed of rotation of the magnetic field in revolutions per second is equal to electric frequency in hertz:
  - f_e (hertz) = f_m (revolutions per second) two poles
  - ω_e (radians per second) = ω_m (radians per second) two poles
  - The windings on the 2-pole stator above occur in the order a – c′ – b – a′ – c – b′

The Rotating Magnetic Field

- If we were to double the amount of windings, hence the sequence of windings will be as follows:

  a1 – c2’ – b1 – a1’ – c1 – b1’ – a2 – c1’ – b2 – a2’ – c2 – b2’

- For a three-phase set of currents, this stator will have 2 north poles and 2 south poles produced in the stator winding, (refer figure (b)):



(a) A simple four-pole stator winding. (b) The resulting stator magnetic poles. Notice that there are moving poles of alternating polarity every 90° around the stator surface. (c) a winding diagram of the stator as seen from its inner surface, showing how the stator currents produce north and south magnetic poles.

The Rotating Magnetic Field

- In this winding, a pole moves only halfway around the stator surface in one electrical cycle.
- Since one electrical cycle is 360 electrical degrees, and mechanical motion is 180 mechanical degrees, the relationship between the electrical angle θ_e and the mechanical θ_m in this stator is

$$\theta s_e = 2\theta s_m$$

- Thus, for a four pole winding, the electrical frequency of the current is twice the mechanical frequency of rotation:

$$f_e = 2f_m$$
$$\omega_e = 2\omega_m$$

The Rotating Magnetic Field

Therefore the general format will be as follows:

$$\theta_e = \frac{P}{2}\theta_m$$

$$f_e = \frac{P}{2} f_m$$

$$\omega_e = \frac{P}{2}\omega_m$$

Also, $since \quad f_m = \frac{n_m}{60} \quad where\ n\ is\ the\ number\ of\ rotation$

$$\therefore f_e = \frac{n_m}{120} P$$

Reversing the direction of Magnetic Field Rotation

If the current in any two of the 3 coils is swapped, the direction of the magnetic field's rotation will be reversed.

Magnetomotive Force and Flux Distribution on AC Machines

Assumptions in previous sections:

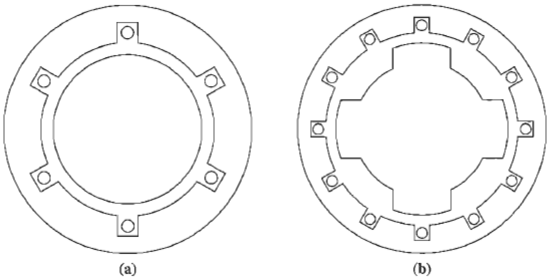
- Flux produced inside an ac machine is in free space
- Direction of flux density produced by a coil of wire is perpendicular to the plane of the coil
- Direction of flux given by the right hand rule.

However, the flux in a real machine does not follow these assumptions, since there is a ferromagnetic rotor in the centre of the machine with a small air gap between the rotor and the stator.

Magnetomotive Force and Flux Distribution on AC Machines

The rotor:

1. can be cylindrical (a) (nonsalient-pole), or
2. it can have pole faces projecting out from its surface (b) (salient pole).



Magnetomotive Force and Flux Distribution on AC Machines

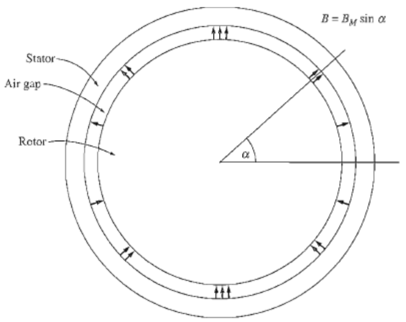
- The reluctance of the air gap in this machine is much higher than the reluctances of either the rotor or the stator, so the flux density vector **B** takes the shortest possible path across the air gap and jumps perpendicularly between the rotor and the stator.

- To produce a sinusoidal voltage in a machine like this, the magnitude of the flux density vector **B** must vary in a sinusoidal manner along the surface of the air gap. The flux density will vary sinusoidally only if the magnetizing intensity **H** (and mmf) varies in a sinusoidal manner along the surface of the air gap.

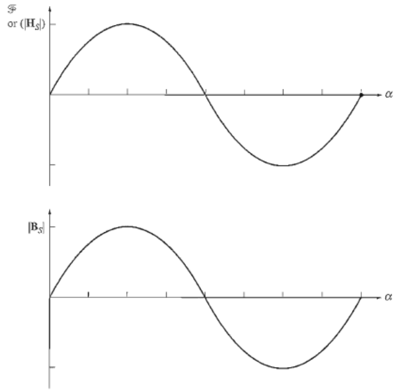
Magnetomotive Force and Flux Distribution on AC Machines

To achieve a sinusoidal variation of mmf along the surface of the air gap is

- to distribute the turns of the winding that produces the mmf in closely spaced slots around the surface of the machine and
- to vary the number of conductors in each slot in a sinusoidal manner.



A cylindrical rotor with sinusoidally varying air-gap flux density

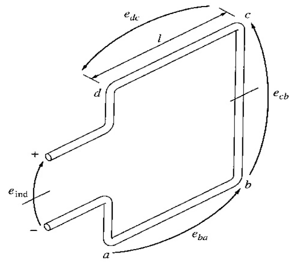


The mmf or H or B as a function of angle in the air gap

Induced Voltage in AC Machines

The induced voltage in a Coil on a Two-Pole Stator

Previously, discussions were made related to induced 3 phase currents producing a rotating magnetic field. Now, lets look into the fact that a rotating magnetic field may produce voltages in the stator. The Figures below show a rotating rotor with a sinusoidally distributed magnetic field in the centre of a stationary coil.



(a)

A rotating rotor magnetic field inside a stationary stator coil

Induced Voltage in AC Machines

- The previous equation was derived for the case of a moving wire in a stationary magnetic field.

In this case, the wire is stationary and the magnetic field is moving, so the equation for induced voltage does not directly apply.

- To use it, we must be in a frame of reference where the magnetic field appears to be stationary. If we ***"sit on the magnetic field"*** so that the held appears to be stationary, the sides of the coil will appear to go by at an apparent velocity v_{rel}, and the equation can be applied.

- The total voltage induced in the coil will be the sum of the voltages induced in each of its four sides.

Induced Voltage in AC Machines

Therefore, the total voltage on the coil will be

$$e_{induced} = e_{ba} + e_{dc} = 2VB_M l \cos \omega_m t$$

Since, $v = r\omega_m$

Therefore, $e_{induced} = 2rlB_M \omega_m \cos \omega_m t$

Since, $\phi = 2rlB_m$

And the angular mechanical velocity should be equal to the angular electrical velocity,

$$e_{induced} = \phi\omega \cos \omega t$$

or (taking into account number of turns of windings),

$$e_{induced} = N_c \phi\omega \cos \omega t$$

The Induced Voltage in a 3-Phase Set of Coils

- If the stator now has 3 sets of different windings as such that the stator voltage induced due to the rotating magnetic field produced by the rotor will have a phase difference of 120°, the induced voltages at each phase will be as follows:

$$e_{aa'} = N\phi\omega \sin \omega t \quad V$$
$$e_{bb'} = N\phi\omega \sin(\omega t - 120^o) \quad V$$
$$e_{cc'} = N\phi\omega \sin(\omega t - 240^o) \quad V$$

- Therefore, a 3 phase set of currents flowing into the stator windings and hence generating a rotating magnetic field (earlier case), and at the same time, a rotating magnetic field produced by the rotor will be able to generate 3 phase voltages in a stator.

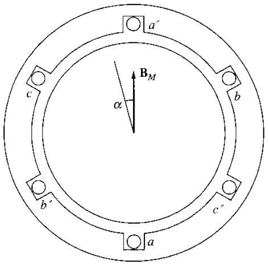
The Induced Voltage in a 3-Phase Set of Coils

- Referring to the induced voltage derived earlier, the maximum induced voltage is when sin has a value of 1, hence,

$$E_{\max} = N\phi\omega \quad , \quad \text{since } \omega = 2\pi f,$$
$$\therefore E_{\max} = 2\pi N\phi f$$

- Therefore, the rms voltage at the 3 phase stator:

$$E_A = \frac{2\pi}{\sqrt{2}} N_C \phi f$$
$$\boxed{E_A = \sqrt{2}\pi N_C \phi f}$$



The production of three-phase voltages from three coils spaced 120º apart

- Note: These are induced voltages at each phase, as for the line-line voltage values; it will depend upon how the stator windings are connected, whether as Y or D.

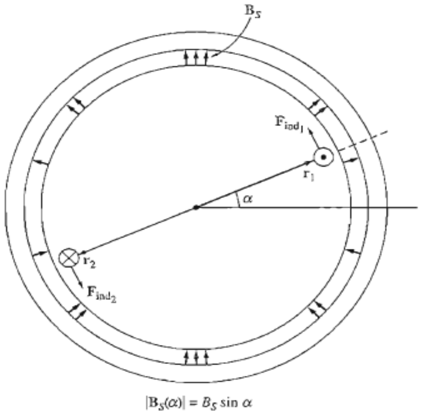
Solve Example 3-2

Induced Torque in an AC Machines

- In ac machines under normal operating conditions, there are 2 magnetic fields present :

1. a magnetic field from the rotor circuit and,
2. another magnetic field from the stator circuit.

The interaction of these two magnetic fields produces the torque in the machine, just as 2 permanent magnets near each other will experience a torque, which causes them to line up.



A simplified ac machine with a sinusoidal stator flux distribution and a single coil of wire mounted in the rotor.

Induced Torque in an AC Machines

- Therefore the torque equation may be represented in the following form:

$$\tau_{ind} = KH_r B_s \sin\alpha = KH_r \times B_s$$

Note that K is a constant value.

Since $B_R = \mu H_R$,

- The constant k is a value which will be dependent upon the permeability of the machine's material. Since the total magnetic field density will be the summation of the BS and BR, hence:

$$\tau_{ind} = kB_r \times \left(B_{net} - B_r\right) = kB_r \times B_{net}$$

- If there is an angle δ between B_{net} and B_R,

$$\tau_{ind} = kB_r B_{net} \sin\delta$$

- These 3 equations will be used to help develop a qualitative understanding of the torque in ac machines.

Homework

From our text book the 5th edition try to solve the following problems:

3.1, 3.4 and 3.8

Chapter Four

Synchronous Generator

Introduction

Synchronous Generators or Alternators are synchronous machines used to **convert mechanical power to electrical power**.

- A DC current is applied to the rotor winding, which then produces a rotor magnetic field.
- The rotor is then turned by a prime mover (eg. Steam, water etc.) producing a rotating magnetic field.
- This rotating magnetic field **induces** a 3-phase set of voltages within the stator windings of the generator.

Introduction

Generally, there are **two windings on a machine**:

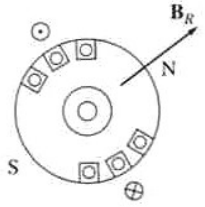
1. **Field windings** – windings that **produce the main magnetic field (rotor windings)**
2. **Armature windings** – windings **where main voltage is induced (Stator windings)**

Synchronous Machine Construction

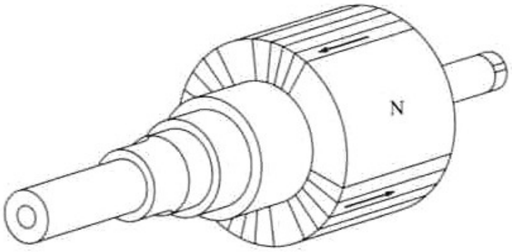
The **rotor** of a synchronous machine:

- essentially a **large electromagnet but it can also a natural magnet**
- its magnetic poles are either:

1. **nonsalient (cylindrical) poles**



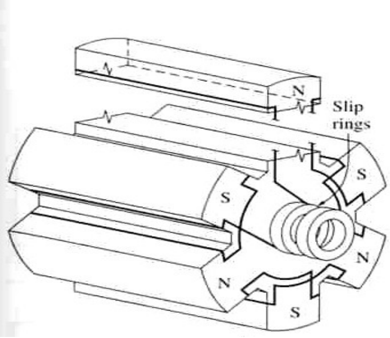
End View

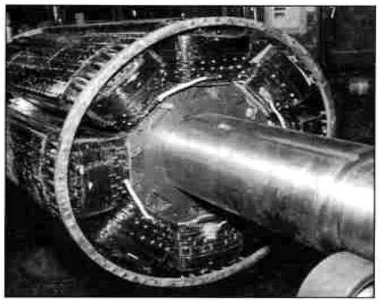


Side View

Synchronous Machine Construction

2. **salient poles,** i.e. poles that are “protruding” or “sticking out” from the rotor surface. Normally used for rotors with 4 or more poles.





- ➢constructed of **thin laminations** to reduce eddy current losses
- ➢contains **field windings supplied with dc current**

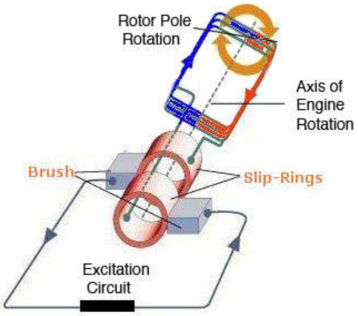
Synchronous Machine Construction

There are **two common approaches** to supplying this dc current:

1) from an **external dc source** by means of *slip rings* and *brushes*

A **slip ring** is an electromechanical device that allows the transmission of power and electrical signals from a stationary to a rotating structure.

Brush – a **block of graphite-like carbon compound that conducts electricity freely** but has very low friction such that it doesn’t wear down the slip ring.



Synchronous Machine Construction

In a synchronous machine,
- one end of the dc rotor winding is tied to each of the two slip rings, and
- a stationary brush rides on each slip ring.

- If positive end of a dc voltage source is connected to one brush and the negative end is connected to the other, then the **same dc voltage will be applied to the field winding at all times** regardless of the angular position or speed of rotor.

Problems with slip rings and brushes:

1. Increased maintenance required – check brushes for wear regularly.
2. Brush voltage drop can cause significant power losses on machines with larger field currents.

Therefore, slip rings and brushes are used on all small synchronous machines

Synchronous Machine Construction

2) from a **special dc power source** mounted directly **on the shaft** of the synchronous machine

- **This Techniques is used on larger machines**

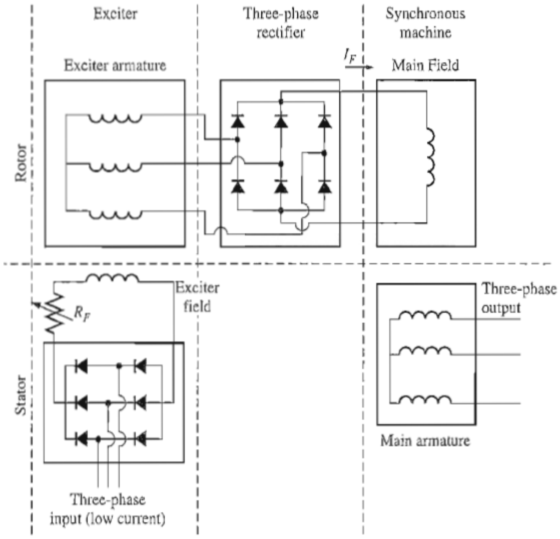
Brushless exciters **–** a **small ac generator** with its field circuit mounted on the stator and its armature circuit mounted on the rotor shaft.

- The **three-phase output of the exciter generator** is **rectified** to direct current **by a three-phase rectifier** circuit **and fed to the main dc field circuit** of the generator.
- By controlling the small dc field current of the exciter generator (located on the stator), we can adjust the field current on the main machine without slip rings and brushes.
- **Brushless exciter requires much less maintenance** since there is no mechanical contacts.

Synchronous Machine Construction

A brushless exciter circuit.

- A small three-phase current is rectified and used to supply the field circuit of the exciter, which is located on the stator.
- The output of the armature circuit of the exciter (on the rotor) is then rectified and used to supply the field current of the main machine.



Synchronous Machine Construction

Pilot exciter – without external power source

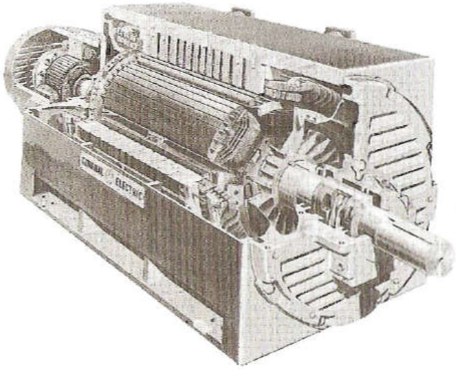
To make the **excitation generator** completely **independent of any external power sources**, a small **pilot exciter** is often included.

Pilot exciter – a small ac generator with **permanent magnets mounted on the rotor shaft** and a three-phase winding on the stator.

- Permanent magnets produce power for the field circuit of the exciter. Exciter, in turn, produces the field current of the main machine.

- Many synchronous generators having brushless exciters **also have slip rings and brushes as an auxiliary source of dc field current in emergencies**.

Synchronous Machine Construction



A cutaway diagram of a large synchronous machine. Note the salient-pole construction and the on-shaft exciter.

The Speed of Rotation of a Synchronous Generator

- **Synchronous generators are by definition *synchronous*, meaning that the electrical frequency produced is locked in or synchronized with the mechanical rate of rotation of the generator.**
- A synchronous generator's rotor consists of an electromagnet to which direct current is supplied.
- The rotor's magnetic field points in the direction the rotor is turned. Hence, the rate of rotation of the magnetic field in the machine is related to the stator electrical frequency by:

$$f_e = \frac{n_m P}{120}$$

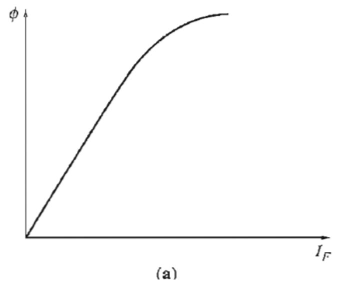
The Internal Generated Voltage of a Synchronous Generator

- ➢ Voltage induced is dependent upon flux and speed of rotation, hence from what we have learnt so far, the induced voltage can be found as follows:

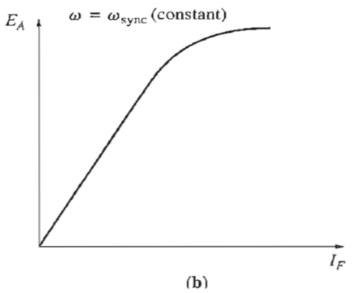
$$E_A = \sqrt{2}\pi N_C \phi f$$

- ➢ For simplicity, it may be simplified to as follows:

$$E_A = K\phi\omega$$



(a) Plot of flux versus field current for a synchronous generator



(b) The magnetization curve for the synchronous generator.

The Equivalent Circuit of a Synchronous Generator

- ➢ The voltage E_A is the internal generated voltage produced in one phase of a synchronous generator.
- ➢ If the machine is not connected to a load (no armature current flowing), the terminal voltage will be equivalent to the voltage induced at the stator coils.
- ➢ This is due to the fact that there are no current flow in the stator coils hence no losses. When there is a load connected to the generator, there will be differences between E_A and $V\phi$.

These differences are due to:

a) Distortion of the air gap magnetic field by the current flowing in the stator called **armature reaction**.
b) Self inductance of the armature coil
c) Resistance of the armature coils
d) The effect of salient pole rotor shapes.

- ➢ We will explore factors a, b, and c and derive a machine model from them. The effect of salient pole rotor shape will be ignored, and all machines in this chapter are assumed to have non-salient or cylindrical rotors.

The Equivalent Circuit of a Synchronous Generator

a) Armature Reaction

- When the rotor is spun ➔ a voltage E_A is induced in the stator windings.
- If a load is attached to the terminals of the generator, a current flows. But a 3-phase stator current flow will produce a magnetic field of its own.
- This stator magnetic field will distort the original rotor magnetic field, changing the resulting phase voltage.
-
- **This effect is called armature reaction because the armature (stator) current affects the magnetic field, which produced it in the first place.**
- Refer to the diagrams below, showing a two-pole rotor spinning inside a 3-phase stator.

The Equivalent Circuit of a Synchronous Generator

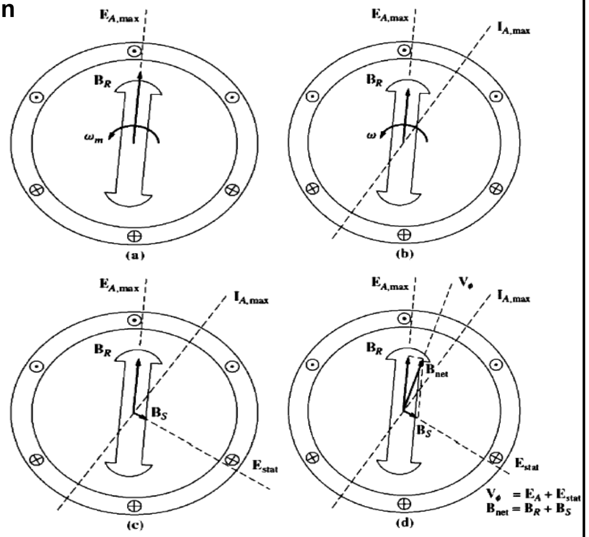
Armature Reaction Explanation

(a) A rotating magnetic field produces the internal generated voltage E_A.

(b) The resulting voltage produces a lagging current flow when connected to a lagging load.

(c) The stator current produces its own magnetic field B_S which produces its own E_{stat} in the stator windings.

(d) The field B_S adds to B_R distorting it into B_{net}. The voltage E_{stat} adds to E_A, producing Vf at the output of the phase.



The Equivalent Circuit of a Synchronous Generator

- If there is no load connected to the stator. The rotor magnetic field B_R produces an internal generated voltage E_A whose peak coincides with direction of B_R. With no load, there is no armature current and E_A will be equal to the phase voltage Vϕ.
- When a lagging load is connected, the peak current will occur at an angle behind the peak voltage.
- The current flowing in the stator windings produces a magnetic field of its own. This stator magnetic field B_S and its direction are given by the right-hand rule. The stator field produces a voltage of its own called E_{stat}.
- With 2 voltages and 2 magnetic fields present in the stator windings, the total voltage and the net magnetic field are:

$$V_\phi = E_A + E_{Stat}$$

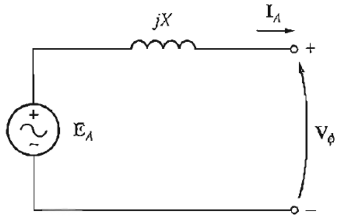
$$B_{net} = B_R + B_S$$

The Equivalent Circuit of a Synchronous Generator

How can the effects of armature reaction on the phase voltage be modelled?

- The voltage E_{stat} lies at an angle of 90° behind the plane of I_A.
- The voltage E_{stat} is directly proportional to the current I_A.
- If X is a constant of proportionality, then the armature reaction voltage can be expressed as:

$$E_{stat} = -\,jXI_A$$



- Therefore:

$$V_\phi = E_A - jXI_A$$

- Thus, the armature reaction voltage can be **modelled as an inductor in series with the internal generated voltage**.

The Equivalent Circuit of a Synchronous Generator

Self-inductance and Resistance of the Armature Coils

- If the stator self-inductance is called L_A (reactance is X_A)
- If the stator resistance is called R_A, then the total difference between E_A and $V\phi$ is:

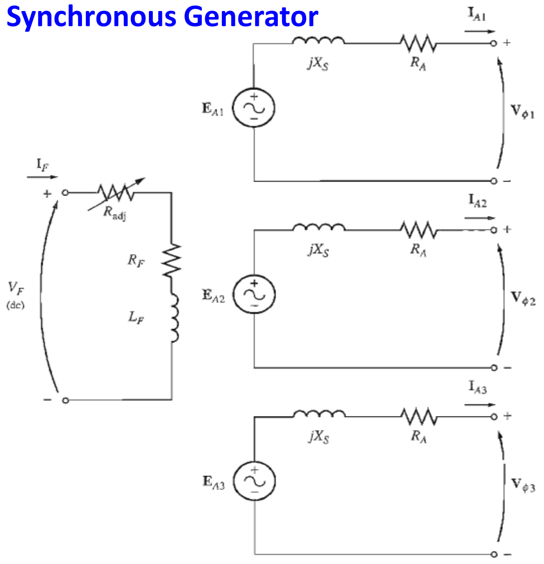
$$\begin{aligned} V_\phi &= E_A - jXI_A - jX_AI_A - R_AI_A \\ &= E_A - jX_sI_A - R_AI_A \end{aligned}$$

- Where

$X_S = X + X_A$

The Equivalent Circuit of a Synchronous Generator

- A dc power source is supplying the rotor field circuit, which is modelled by the coil's inductance and resistance in series. In series with R_F is an adjustable resistor R_{adj} which controls the flow of the field current.
- The rest of the equivalent circuit consists of the models for each phase.
- Each phase has an internal generated voltage with a series inductance X_S (consisting of the sum of the armature reactance and the coil's self-inductance) and a series resistance R_A



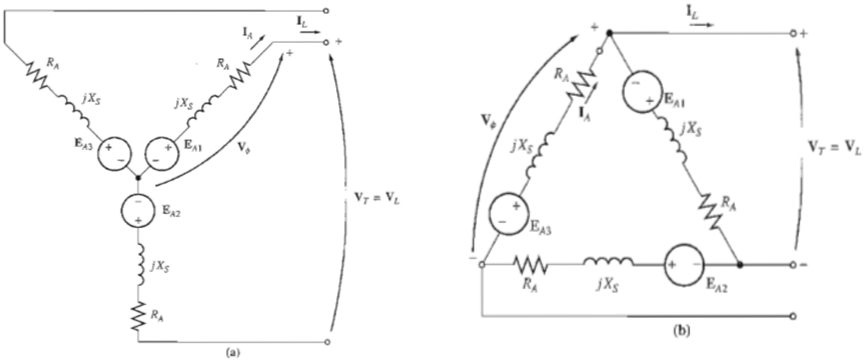
The full equivalent circuit of a three-phase synchronous generator.

The Phasor Diagram of a Synchronous Generator

- If the 3 phases are connected in Y or Δ, the terminal voltage may be found as follows:

$$V_T = \sqrt{3}V_\phi \quad (for\ Y\ connection)$$

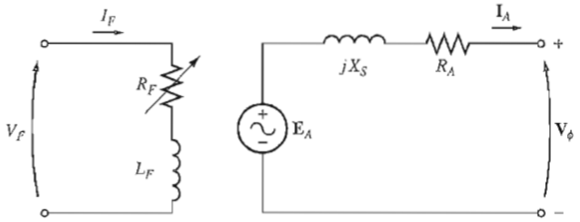
$$V_T = V_\phi \quad (for\ \Delta\ connection)$$



The Phasor Diagram of a Synchronous Generator

- Ideally, the terminal voltage for all 3 phases should be identical since we assume that the load connected is balanced. If it is not balanced, a more in-depth technique is required.

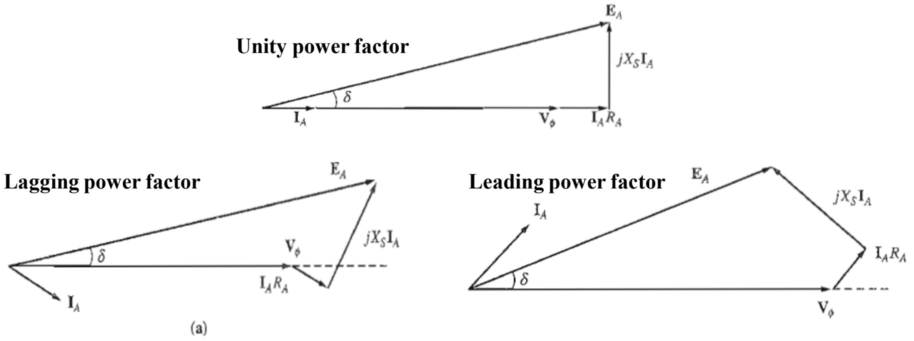
- The per-phase equivalent circuit:



The Phasor Diagram of a Synchronous Generator

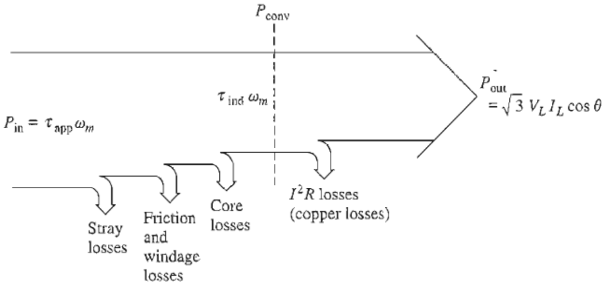
Similar concept as applied for Transformers:

- For a given phase voltage and armature current, a larger internal voltage E_A is needed for lagging loads than for leading loads. ➔ Thus, a larger field current is needed to get the same terminal voltage because $E_A = k\phi\omega$ because ω must be kept constant to keep constant frequency.
- Alternatively, for a given field current and magnitude of load current, the terminal voltage is lower for lagging loads and higher for leading loads.



Power and Torque in Synchronous Generators

- A generator converts mechanical energy into electrical energy, hence the input power will be a mechanical prime mover, e.g. diesel engine, steam turbine, water turbine or anything similar.
- **Regardless of the type of prime mover, the rotor velocity must remain constant to maintain a stable system frequency.**
- The power-flow diagram for a synchronous generator is shown:



Power and Torque in Synchronous Generators

Input:

$$P_{in} = \tau_{app}\omega_m$$

Losses: Stray losses, friction and windage losses, core loss

Converted power: $P_{conv} = \tau_{ind}\omega_m = 3E_A I_A \cos\gamma$

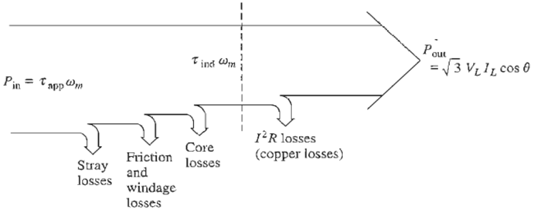
Where γ is the angle between E_A and I_A.

Losses: Copper losses

Output: using line quantities or phase quantities

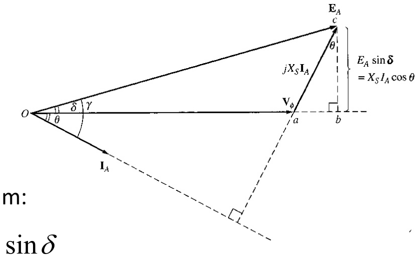
$P_{out} = \sqrt{3}V_T I_L \cos\theta$ or $P_{out} = 3V_\phi I_A \cos\theta$

$Q_{out} = \sqrt{3}V_T I_L \sin\theta$ or $Q_{out} = 3V_\phi I_A \sin\theta$



Simplified Phasor Diagram

- Simplifying the phasor diagram, an assumption may be made whereby the armature resistance R_A is considered to be negligible and assuming that load connected to it is lagging in nature ➔



- Based upon this simplified phasor diagram:

$$I_A\cos\theta = \frac{E_A\sin\delta}{X_s}$$

- Which gives another form of output power expression (since R_A assumed to be zero):

$$P = \frac{3V_\phi E_A \sin\delta}{X_s}$$

Simplified Phasor Diagram

- From the above equation, it can be seen that power is dependent upon:
- The angle between Vϕ and E_A which is δ.
- δ is known as the torque angle of the machine.
- maximum torque may be found when sin δ is 1 which gives the maximum power (a.k.a. static stability limit) to be:

$$P_{\max} = \frac{3V_\phi E_A}{X_s}$$

- The basic torque equation:

$$\tau_{ind} = kB_R \times B_s = kB_R \times B_{net} = kB_R B_{net} \sin\delta$$

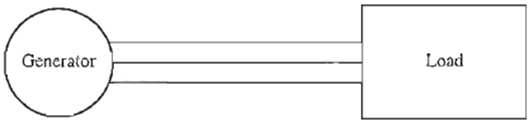
- An alternative expression can be derived from the power expression since $P_{out}=P_{conv}$ when R_A assumed to be zero. Because $P_{conv}=\tau_{ind}\omega_m$, the induced torque is:

$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_s}$$

The Synchronous Generator Operating Alone

- The behaviour of a synchronous generator under load varies greatly depending on the power factor of the load and on whether the generator is operating alone or in parallel with other synchronous generator.

- **In the next discussion,**
- R_A=0
- Rotor flux is assumed to be constant (unless it is stated that the field current I_F is changed).
- The speed of the generator will be assumed constant, and all terminal characteristics are drawn assuming constant speed.



The Effect of Load Changes on a Synchronous Generator Operating Alone

- Assume a generator is connected to a load.

- **Load increases:**

  An increase of load is an increase in real and reactive power drawn from the generator. Such a load increase increases the load current drawn from the generator.

- **Assumptions:**

- Field resistor (R_F) has not been changed, field current is kept constant, ➔ flux is constant.
- Generator rotor speed is maintained constant ➔ Therefore E_A is constant.

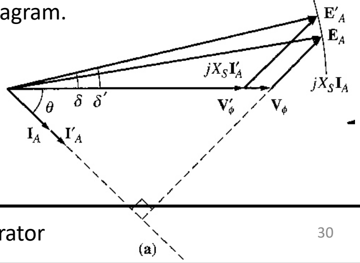
The Effect of Load Changes on a Synchronous Generator Operating Alone

If E_A is constant, what actually varies with a changing load??

- **Initially lagging load:**
- Load is increased with the lagging power factor maintained.
- Magnitude of I_A will increase but will maintain the same angle with reference to Vϕ. (due to power factor is maintained lagging)
- $X_S I_A$ will also increase and will maintain the same angle. Since

$$E_A = V_\phi + jX_s I_A$$

- j $X_S I_A$ must stretch between Vϕ at an angle of 0° and E_A, which is constrained to be of the same magnitude as before the load increase.
- Note that E_A has to remain constant (refer to the assumption stated earlier)
- Hence the only element which would change to compensate would be Vϕ. This change (reduction) may be seen in the phasor diagram.

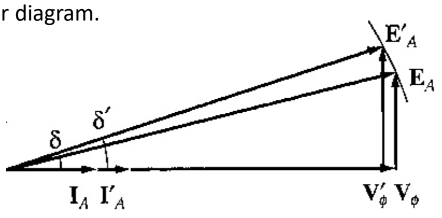


The Effect of Load Changes on a Synchronous Generator Operating Alone

- **Initially unity power factor load:**
- Load is increased with the unity power factor maintained.
- Magnitude of I_A will increase but will maintain the same angle with reference to Vϕ. (due to power factor is maintained unity)
- $X_S I_A$ will also increase and will maintain the same angle. Since

$$E_A = V_\phi + jX_s I_A$$

- Note that E_A has to remain constant (refer to the assumption stated earlier)
- Hence the only element which would change to compensate would be Vϕ (↓).
- This change may be seen in the phasor diagram.



(b)

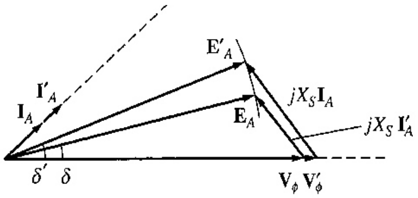
The Effect of Load Changes on a Synchronous Generator Operating Alone

Initially leading power factor Load

- Load is increased with the leading power factor maintained.
- Magnitude of I_A will increase but will maintain the same angle with reference to Vϕ. (due to power factor is maintained leading)
- $X_S I_A$ will also increase and will maintain the same angle. Since

$$E_A = V_\phi + jX_s I_A$$

- Note that E_A has to remain constant (as assumed earlier)
- Hence the only element which would change to compensate would be Vϕ.



(c)

Voltage Regulation of Generators

- **Voltage regulation:** $$\boxed{\mathrm{VR} = \frac{V_{\mathrm{nl}} - V_{\mathrm{fl}}}{V_{\mathrm{fl}}} \times 100\%}$$
- For lagging loads, VR would be very large & positive.
- For leading loads, VR would be very large & negative.
- For unity loads, VR would be small & positive.
- However, in practice it is best to keep the **output voltage of a generator constant**, hence **E_A has to be controlled** which can be done by **controlling the field current I_F**.
- **Varying I_F will vary the flux in the core which then will vary E_A accordingly**

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Voltage Regulation of Generators

How must a generator's field current be adjusted to keep V_T constant as the load changes?

Answer: Increase or decrease R_F

$$\downarrow R_F \Longrightarrow \uparrow I_F \Longrightarrow \uparrow \phi \Longrightarrow \uparrow E_A \Longrightarrow \uparrow V_\phi$$

$$or\ vise\ versa$$

$$\uparrow R_F \Longrightarrow \downarrow I_F \Longrightarrow \downarrow \phi \Longrightarrow \downarrow E_A \Longrightarrow \downarrow V_\phi$$

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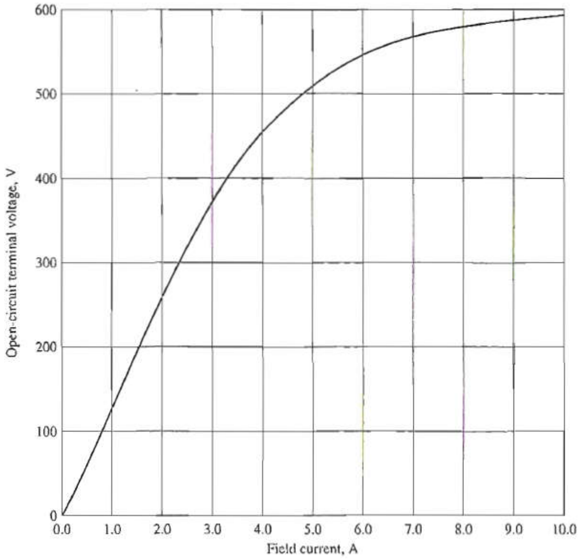
Example 4-2

A 480-V, 60-Hz, Δ-connected, four-pole synchronous generator has the *OCC shown in Figure 4-23a. This generator has a synchronous reactance of 0.1 Ω and* an mature resistance of 0.0 15 Ω, At full load, the machine supplies 1200 A at 0.8 PF lagging. Under full-load conditions, the friction and windage losses are 40 kW, and the core losses are 30 kW. Ignore any field circuit losses.

(a) What is the speed of rotation of this generator?
(b) How much field current must be supplied to the generator to make the terminal voltage 480 V at no load?
(c) If the generator is now connected to a load and the load draws 1200 A at 0.8 PF lagging, how much field current will be required to keep the terminal voltage equal to 480 V?
(d) How much power is tile generator now supplying? How much power is supplied to the generator by the prime mover? What is this machine's overall efficiency?
(e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage?
(f) Finally, suppose that the generator is connected to a load drawing 1200 A at 0.8 PF leading. How much field current would be required to keep V_T at 480 V?

Example 4-2



Example 4-2

Solution

This synchronous generator is Δ-connected, so its phase voltage is equal to its line voltage $V_\phi = V_T$, while its phase current is related to its line current by the equation $I_L = \sqrt{3} I_\phi$.

(a) The relationship between the electrical frequency produced by a synchronous generator and the mechanical rate of shaft rotation is given by Equation (3–34):

$$f_{se} = \frac{n_m P}{120} \tag{3–34}$$

Therefore,

$$n_m = \frac{120 f_{se}}{P}$$

$$= \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

(b) In this machine, $V_T = V_\phi$. Since the generator is at no load, $\mathbf{I}_A = 0$ and $\mathbf{E}_A = \mathbf{V}_\phi$. Therefore, $V_T = V_\phi = E_A = 480$ V, and from the open-circuit characteristic, $I_F = 4.5$ A.

Example 4-2

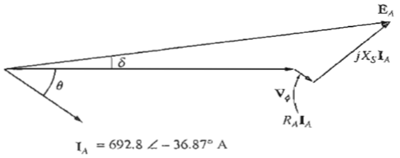
(c) If the generator is supplying 1200 A, then the armature current in the machine is

$$I_A = \frac{1200\text{ A}}{\sqrt{3}} = 692.8\text{ A}$$

The phasor diagram for this generator is shown in Figure 4–23b. If the terminal voltage is adjusted to be 480 V, the size of the internal generated voltage $\mathbf{E}_A$ is given by

$$\begin{aligned}
\mathbf{E}_A &= \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A \\
&= 480\angle 0^\circ\text{ V} + (0.015\ \Omega)(692.8\angle -36.87^\circ\text{ A}) + (j0.1\ \Omega)(692.8\angle -36.87^\circ\text{ A}) \\
&= 480\angle 0^\circ\text{ V} + 10.39\angle -36.87^\circ\text{ V} + 69.28\angle 53.13^\circ\text{ V} \\
&= 529.9 + j49.2\text{ V} = 532\angle 5.3^\circ\text{ V}
\end{aligned}$$

To keep the terminal voltage at 480 V, $\mathbf{E}_A$ must be adjusted to 532 V. From Figure 4–23, the required fie



Example 4-2

(*d*) The power that the generator is now supplying can be found from Equation (4–16):

$$P_{\text{out}} = \sqrt{3}V_L I_L \cos\theta \qquad (4\text{–}16)$$
$$= \sqrt{3}(480\text{ V})(1200\text{ A})\cos 36.87^\circ$$
$$= 798\text{ kW}$$

To determine the power input to the generator, use the power-flow diagram (Figure 4–15). From the power-flow diagram, the mechanical input power is given by

$$P_{\text{in}} = P_{\text{out}} + P_{\text{elec loss}} + P_{\text{core loss}} + P_{\text{mech loss}} + P_{\text{stray loss}}$$

The stray losses were not specified here, so they will be ignored. In this generator, the electrical losses are

$$P_{\text{elec loss}} = 3I_A^2 R_A$$
$$= 3(692.8\text{ A})^2(0.015\ \Omega) = 21.6\text{ kW}$$

The core losses are 30 kW, and the friction and windage losses are 40 kW, so the total input power to the generator is

$$P_{\text{in}} = 798\text{ kW} + 21.6\text{ kW} + 30\text{ kW} + 40\text{ kW} = 889.6\text{ kW}$$

Therefore, the machine's overall efficiency is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{798\text{ kW}}{889.6\text{ kW}} \times 100\% = 89.75\%$$

(*e*) If the generator's load were suddenly disconnected from the line, the current $\mathbf{I}_A$ would drop to zero, making $\mathbf{E}_A = \mathbf{V}_\phi$. Since the field current has not changed, $|\mathbf{E}_A|$ has not changed and $\mathbf{V}_\phi$ and V_T must rise to equal $\mathbf{E}_A$. Therefore, if the load were suddenly dropped, the terminal voltage of the generator would rise to 532 V.

Example 4-2

(*f*) If the generator were loaded down with 1200 A at 0.8 PF leading while the terminal voltage was 480 V, then the internal generated voltage would have to be

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$= 480\angle 0^\circ\text{ V} + (0.015\ \Omega)(692.8\angle 36.87^\circ\text{ A}) + (j0.1\ \Omega)(692.8\angle 36.87^\circ\text{ A})$$
$$= 480\angle 0^\circ\text{ V} + 10.39\angle 36.87^\circ\text{ V} + 69.28\angle 126.87^\circ\text{ V}$$
$$= 446.7 + j61.7\text{ V} = 451\angle 7.1^\circ\text{ V}$$

Therefore, the internal generated voltage E_A must be adjusted to provide 451 V if V_T is to remain 480 V. Using the open-circuit characteristic, the field current would have to be adjusted to 4.1 A.

Example 4-3

A 480V, 50Hz, Y-connected, 6-pole synchronous generator has a per-phase synchronous reactance of 1Ω. Its full-load armature current is 60A at 0.8PF lagging. This generator has friction and windage losses of 1.5kW and core losses of 1 kW at 50Hz at full load. Since the armature resistance is being ignored, assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480V at no load.

(a) What is the speed of rotation of this generator?

(b) What is the terminal voltage of this generator if the following are true?

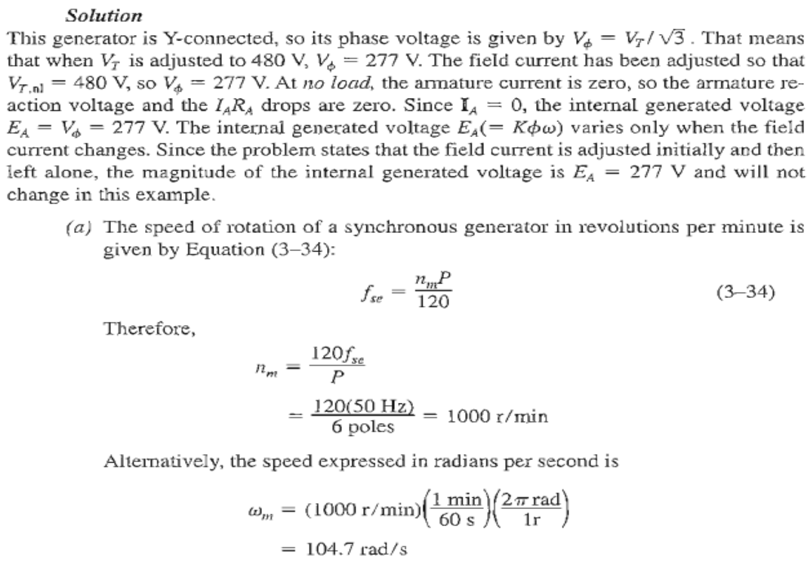
1. It is loaded with the rated current at 0.8 PF lagging.
2. It is loaded with the rated current at 1.0 PF.
3. It is loaded with the rated current at 0.8 PF leading.

(c) What is the efficiency of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?

(d) How much shaft torque must applied by the prime mover at full load? How large is the induced counter-torque?

(e) What is the voltage regulation of this generator at 0.8 PF lagging? At 1.0 PF? At 0.8 PF leading?

Example 4-3



Solution

This generator is Y-connected, so its phase voltage is given by $V_\phi = V_T/\sqrt{3}$. That means that when V_T is adjusted to 480 V, $V_\phi = 277$ V. The field current has been adjusted so that $V_{T,\text{nl}} = 480$ V, so $V_\phi = 277$ V. At *no load*, the armature current is zero, so the armature reaction voltage and the I_AR_A drops are zero. Since $\mathbf{I}_A = 0$, the internal generated voltage $E_A = V_\phi = 277$ V. The internal generated voltage $E_A(= K\phi\omega)$ varies only when the field current changes. Since the problem states that the field current is adjusted initially and then left alone, the magnitude of the internal generated voltage is $E_A = 277$ V and will not change in this example.

(a) The speed of rotation of a synchronous generator in revolutions per minute is given by Equation (3–34):

$$f_{se} = \frac{n_m P}{120} \tag{3–34}$$

Therefore,

$$n_m = \frac{120 f_{se}}{P}$$

$$= \frac{120(50\text{ Hz})}{6\text{ poles}} = 1000\text{ r/min}$$

Alternatively, the speed expressed in radians per second is

$$\omega_m = (1000\text{ r/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{r}}\right)$$

$$= 104.7\text{ rad/s}$$

Example 4-3

(b) 1. If the generator is loaded down with rated current at 0.8 PF lagging, the resulting phasor diagram looks like the one shown in Figure 4–24a. In this phasor diagram, we know that $\mathbf{V}_\phi$ is at an angle of 0°, that the magnitude of $\mathbf{E}_A$ is 277 V, and that the quantity $jX_S\mathbf{I}_A$ is

$$jX_S\mathbf{I}_A = j(1.0\ \Omega)(60\angle -36.87°\ \mathrm{A}) = 60\angle 53.13°\ \mathrm{V}$$

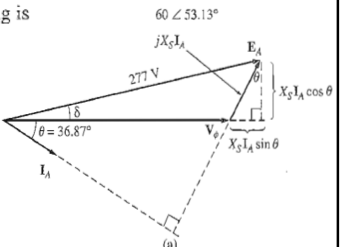
The two quantities not known on the voltage diagram are the magnitude of $\mathbf{V}_\phi$ and the angle δ of $\mathbf{E}_A$. To find these values, the easiest approach is to construct a right triangle on the phasor diagram, as shown in the figure. From Figure 4–24a, the right triangle gives

$$E_A^2 = (V_\phi + X_S I_A \sin\theta)^2 + (X_S I_A \cos\theta)^2$$

Therefore, the phase voltage at the rated load and 0.8 PF lagging is

$$(277\ \mathrm{V})^2 = [V_\phi + (1.0\ \Omega)(60\ \mathrm{A})\sin 36.87°]^2 + [(1.0\ \Omega)(60\ \mathrm{A})\cos 36.87°]^2$$
$$76{,}729 = (V_\phi + 36)^2 + 2304$$
$$74{,}425 = (V_\phi + 36)^2$$
$$272.8 = V_\phi + 36$$
$$V_\phi = 236.8\ \mathrm{V}$$

Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 410$ V.



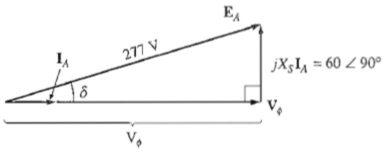
(a)

Example 4-3

2. If the generator is loaded with the rated current at unity power factor, then the phasor diagram will look like Figure 4–24b. To find $\mathbf{V}_\phi$ here the right triangle is

$$E_A^2 = V_\phi^2 + (X_S I_A)^2$$
$$(277\ \mathrm{V})^2 = V_\phi^2 + [(1.0\ \Omega)(60\ \mathrm{A})]^2$$
$$76{,}729 = V_\phi^2 + 3600$$
$$V_\phi^2 = 73{,}129$$
$$V_\phi = 270.4\ \mathrm{V}$$

Therefore, $V_T = \sqrt{3}V_\phi = 468.4$ V.



(b)

Example 4-3

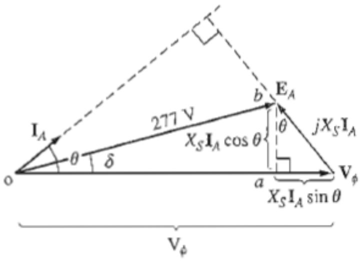
3. When the generator is loaded with the rated current at 0.8 PF leading, the resulting phasor diagram is the one shown in Figure 4–24c. To find $\mathbf{V}_\phi$ in this situation, we construct the triangle *OAB* shown in the figure. The resulting equation is

$$E_A^2 = (V_\phi - X_S I_A \sin\theta)^2 + (X_S I_A \cos\theta)$$

Therefore, the phase voltage at the rated load and 0.8 PF leading is

$$(277\text{ V})^2 = [V_\phi - (1.0\ \Omega)(60\text{ A})\sin 36.87°]^2 + [(1.0\ \Omega)(60\text{ A})\cos 36.87°]^2$$
$$76{,}729 = (V_\phi - 36)^2 + 2304$$
$$74{,}425 = (V_\phi - 36)^2$$
$$272.8 = V_\phi - 36$$
$$V_\phi = 308.8\text{ V}$$

Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 535$ V.



(c)

Example 4-3

(*c*) The output power of this generator at 60 A and 0.8 PF lagging is

$$P_{\text{out}} = 3V_\phi I_A \cos\theta$$
$$= 3(236.8\text{ V})(60\text{ A})(0.8) = 34.1\text{ kW}$$

The mechanical input power is given by

$$P_{\text{in}} = P_{\text{out}} + P_{\text{elec loss}} + P_{\text{core loss}} + P_{\text{mech loss}}$$
$$= 34.1\text{ kW} + 0 + 1.0\text{ kW} + 1.5\text{ kW} = 36.6\text{ kW}$$

The efficiency of the generator is thus

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{34.1\text{ kW}}{36.6\text{ kW}} \times 100\% = 93.2\%$$

(*d*) The input torque to this generator is given by the equation

$$P_{\text{in}} = \tau_{\text{app}}\omega_m$$

so
$$\tau_{\text{app}} = \frac{P_{\text{in}}}{\omega_m} = \frac{36.6\text{ kW}}{125.7\text{ rad/s}} = 291.2\text{ N}\cdot\text{m}$$

The induced countertorque is given by

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

so
$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_V} = \frac{34.1\text{ kW}}{125.7\text{ rad/s}} = 271.3\text{ N}\cdot\text{m}$$

Example 4-3

(e) The voltage regulation of a generator is defined as

$$\mathrm{VR} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\% \qquad (3\text{–}67)$$

By this definition, the voltage regulation for the lagging, unity, and leading power-factor cases are

1. Lagging case: $\mathrm{VR} = \frac{480\ \mathrm{V} - 410\ \mathrm{V}}{410\ \mathrm{V}} \times 100\% = 17.1\%$
2. Unity case: $\mathrm{VR} = \frac{480\ \mathrm{V} - 468\ \mathrm{V}}{468\ \mathrm{V}} \times 100\% = 2.6\%$
3. Leading case: $\mathrm{VR} = \frac{480\ \mathrm{V} - 535\ \mathrm{V}}{535\ \mathrm{V}} \times 100\% = -10.3\%$

In Example 4–3, lagging loads resulted in a drop in terminal voltage, unity-power-factor loads caused little effect on V_T, and leading loads resulted in an increase in terminal voltage.

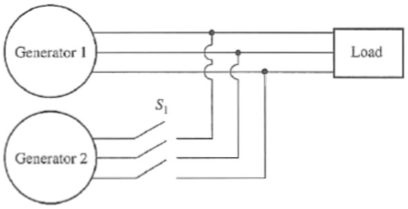
Parallel Operation of AC Generators

Reasons for operating in parallel:

- Handling larger loads.
- Maintenance can be done without power disruption
- Increasing power system reliability,
- Increased efficiency (operating near full load results in better efficiency)

Conditions required for Paralleling

- The figure below shows a synchronous generator G1 supplying power to a load, with another generator G2 about to be paralleled with G1 by closing switch S1. What conditions must be met before the switch can be closed and the 2 generators connected?



Parallel Operation of AC Generators

Effects of wrong operations of Parallel Generators

- If the switch is closed arbitrarily at some moment, the generators are liable to be severely damaged, and the load may lose power.
- If the voltages are not exactly the same in each conductor being tied together, there will be a very large current flow when the switch is closed. To avoid this problem, each of the three phases must have exactly the same voltage magnitude and phase angle as the conductor to which it is connected.

Thus, paralleling 2 or more generators must be done carefully as to avoid generator or other system component damage.

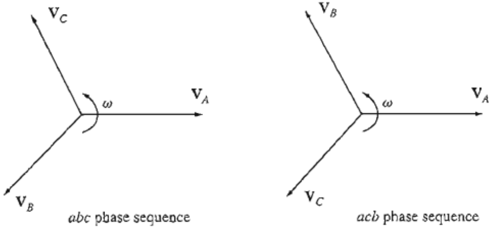
Parallel Operation of AC Generators

Conditions required for Paralleling

Conditions are as follows (4 conditions):

1) RMS line voltages must be equal.
2) The phase angles must be the same for any two similar phases.
- since for two voltages to be equal their rms magnitude and phase must be equal.

3) The generators to be paralleled must have the same phase sequence.
- If the phase sequence is different (as shown here), then even though one pair of voltages (the *a* phase) is in phase, the other 2 pairs of voltages are 120° out of phase. If the generators were connected in this manner, there would be no problem with phase *a*, but huge currents would flow in phases *b* and *c*, damaging both machines.



Parallel Operation of AC Generators

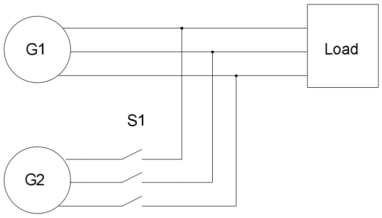
Conditions are as follows (4 conditions):

4) The **oncoming generator** (the new generator) must have a slightly higher operating frequency as compared to the system frequency.

- This is done so that the phase angles of the incoming machine will change slowly with respect to the phase angles of the running system.

The General Procedure for Paralleling Generators

- Suppose that generator G2 is to be connected to the running system as shown below :

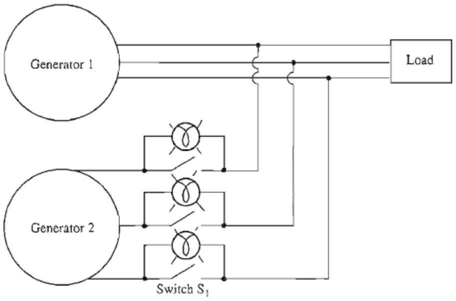


1) Using Voltmeters, the field current of the oncoming generator should be adjusted until its terminal voltage is equal to the line voltage of the running system.
2) Check and verify phase sequence to be identical to the system phase sequence.

There are 2 methods to do this:

The General Procedure for Paralleling Generators

- **First:** Alternately connect a small induction motor to the terminals of each of the 2 generators. If the motor rotates in the same direction each time, then the phase sequence is the same for both generators. If the motor rotates in opposite directions, then the phase sequences differ, and 2 of the conductors on the incoming generator must be reversed.
- **Second:** Using the 3 light bulb method, where the bulbs are stretched across the open terminals of the switch connecting the generator to the system (as shown in the figure below).



Parallel Operation of AC Generators

- As the phase changes between the 2 systems, the light bulbs first get bright (large phase difference) and then get dim (small phase difference).
- If all 3 bulbs get bright and dark together, then the systems have the same phase sequence.
- If the bulbs brighten in succession, then the systems have the opposite phase sequence, and one of the sequences must be reversed.
- When the three bulbs all go out , the voltage difference across them is zero , this means that they are in phase and switch can be closed.
- **Another Method to check when the two systems are in phase:**
  Using a Synchroscope – a meter that measures the difference in phase angles of a single phase (it does not check phase sequences only phase angles).

Parallel Operation of AC Generators

3) Check and verify **generator frequency** to be slightly higher than the system frequency. This is done by watching a frequency meter until the frequencies are close and then by observing changes in phase between the systems.

4) Once the frequencies are nearly equal, the voltages in the 2 systems will change phase with respect to each other very slowly. The phase changes are observed, and when the phase angles are equal, the switch connecting the 2 systems is shut.

- In Large generators belonging to power systems, this whole process of paralleling is automated and a computer does this job.
- For smaller generators an operator goes through the paralleling process described.

Parallel Operation of AC Generators

Suggested questions to be solved:

4.2(a,b,c,d,e,f), 4.7, 4.27

Chapter Six

Induction Motors

Introduction

- **Induction machine** – rotor voltage (that produces the rotor current and the rotor magnetic field) is **induced** in rotor windings.
- No need for physical wires or dc field current (like in synchronous machine).
- It can be motor or generator. But **rarely used as generator** due to many disadvantages (eg: it always consumes reactive power, low PF, not stand alone).

- **The main difference between an induction motor and a synchronous motor:**

1. In synchronous motor a current is supplied onto the rotor. This then creates a magnetic field which, through magnetic interaction, links to the rotating magnetic field in the stator which in turn causes the rotor to turn. It is called synchronous because at steady state the speed of the rotor is the same as the speed of the rotating magnetic field in the stator.

Induction Motor Construction

The main difference between an induction motor and a synchronous motor:

2. The induction motor does not have any direct supply onto the rotor; instead, a secondary current is induced in the rotor. To achieve this, stator windings are arranged around the rotor so that when energized with a poly-phase supply they create a rotating magnetic field pattern which sweeps past the rotor. This changing magnetic field pattern induces current in the rotor conductors. These currents interact with the rotating magnetic field created by the stator and in effect causes a rotational motion on the rotor.

3. However, for these currents to be induced, the speed of the physical rotor and the speed of the rotating magnetic field in the stator must be different, or else the magnetic field will not be moving relative to the rotor conductors and no currents will be induced.

4. This difference between the speed of the rotor and speed of the rotating magnetic field in the stator is called *slip*. Due to this an induction motor is sometimes referred to as an asynchronous machine.

Induction Motor Construction

- Induction motors have both a **stator and rotor**.

There are basically two types of rotor construction:

1. **Squirrel Cage** – A cage induction motor rotor consists of a series of conducting bars laid into slots carved in the face of the rotor and shorted at either end by large *shorting rings.*

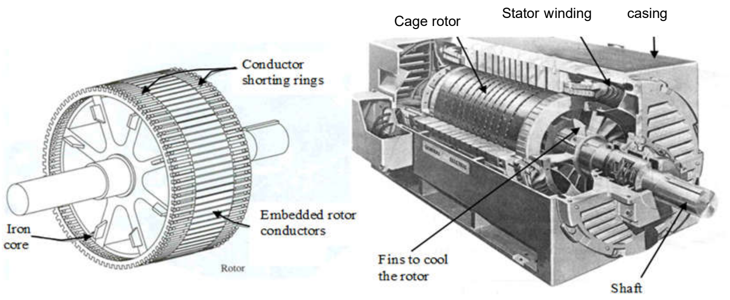
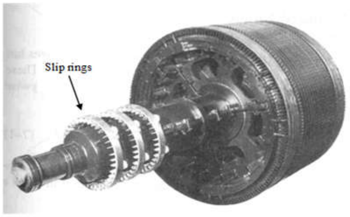


Diagram of a cage rotor induction motor.

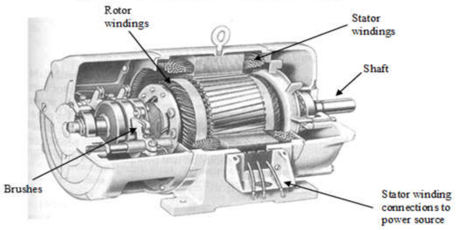
Induction Motor Construction

2. **Wound rotor** - It has 3 phase windings, usually Y connected, and the winding ends are connected via slip rings on the rotor shaft.

- ➢ The rotor windings are shorted through brushes riding on the slip rings.
- ➢ Wound rotor motors have their rotor currents accessible at the stator brushes, where they can be examined and extra resistance can be inserted into the rotor circuit to modify the motor torque-speed characteristic
- ➢ Wound rotor are known to be more expensive due to its maintenance cost to keep the slip rings, carbon brushes and also rotor windings.
- ➢ They are used to start high inertia loads



Typical wound rotor for induction motors.



Cutaway diagram of a wound rotor induction motor.

Basic Induction Motor Concept

The Development of Induced Torque in an IM:

- ➢ The rotating stator magnetic field $\mathbf{B_S}$ ➔ induces voltage $\mathbf{e_{ind}}$ in the rotor bars (windings) ➔ this $\mathbf{e_{ind}}$ produces rotor current flow $\mathbf{I_R}$ (lagging) ➔ $\mathbf{I_R}$ produces $\mathbf{B_R}$ lagging by 90° ➔ $\mathbf{B_R}$ interacts with $\mathbf{B_{net}}$ to produce torque.
- ➢ When current flows in the stator, it will produce a magnetic field in stator as such that **Bs** (stator magnetic field) will rotate at a speed:

$$n_{sync} = \frac{120 f_e}{P}$$

where f_e = system frequency in Hz
P = number of poles in machine

Basic Induction Motor Concept

- This rotating magnetic field **Bs** passes over the rotor bars and induces a voltage in them. The voltage induced in the rotor is given by:

  $$\mathbf{e_{ind} = (v \times B)}\ \boldsymbol{l}$$

- *Hence there will be rotor current flow which would be lagging due to the fact that the rotor has an inductive element. And this rotor current will produce a magnetic field at the rotor,* $\boldsymbol{B_R}$*.*

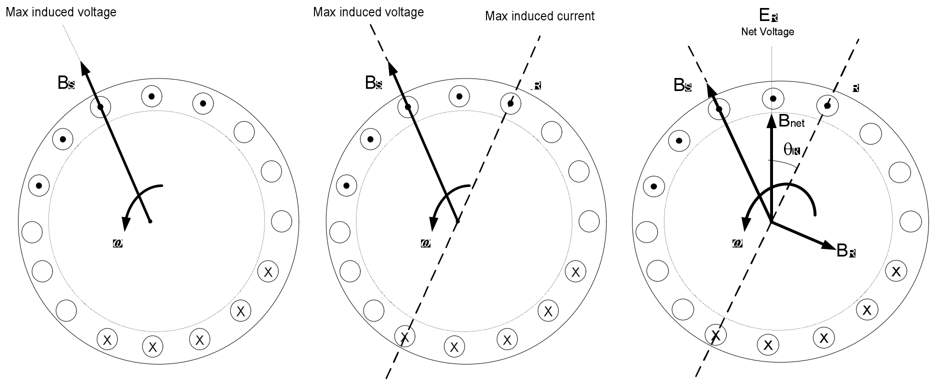
- *Hence the interaction between both magnetic field would give torque:*

  $$\tau_{ind} = kB_R \times B_S$$

- *The torque induced would generate acceleration to the rotor, hence the rotor will spin.*

- *However, there is a finite upper limit to the motor's speed.*

Basic Induction Motor Concept



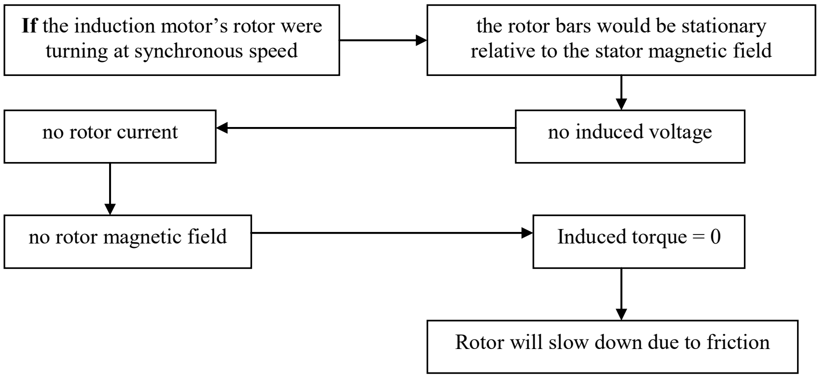
Rotating Magnetic field Bs produces a voltage in the rotor bars

rotor voltage produces a lagging I_R due to inductance of the rotor

$_R$ produces B_R (lagging 90 deg), B_R interacts with Bnet to produce a CCW torque

Basic Induction Motor Concept

Is there upper limit to the motor's speed ?



If the induction motor's rotor were turning at synchronous speed

the rotor bars would be stationary relative to the stator magnetic field

no induced voltage

no rotor current

no rotor magnetic field

Induced torque = 0

Rotor will slow down due to friction

Conclusion : An induction motor can thus speed up to near synchronous speed but it can never reach synchronous speed.

Basic Induction Motor Concept

Is there upper limit to the motor's speed ?

If **rotor speed = synchronous speed**

⇩

Rotor **bars appear stationary** relative to $\vec{B}_S$

⇩

Hence, $e_{ind} = \underline{\ 0\ }$, since $v = 0$

⇩

No rotor current present

⇩

Therefore, $\vec{B}_R = 0$_____

⇩

Since $T_{ind} = k\vec{B}_R \times \vec{B}_S$,

$\therefore T_{ind} = \underline{\ 0\ }$

⇩

Rotor slows down due to friction

Conclusion: Induction motor can speed up to *near synchronous speed* but never actually reach it.

The Concept of Rotor Slip

- **The induced voltage at the rotor bar is dependent upon** the ***relative speed between the stator magnetic field and the rotor***. This can be easily termed as ***slip speed***:

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip} = slip speed of the machine
n_{sync} = speed of the magnetic field.
n_m = mechanical shaft speed

- From this we can define **slip** (relative speed expressed on a percentage basis):

$$Slip,\, s = \frac{n_{slip}}{n_{sync}} \times 100\% = \frac{n_{sync} - n_m}{n_{sync}} \times 100\%$$

The Concept of Rotor Slip

- Slip may also be described in terms of angular velocity, ω.

$$s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} x100\,\%$$

- Using the ratio of slip, we may also determine the rotor speed:

$$n_m = (1 - s)\,n_{sync} \qquad or \qquad \omega_m = (1 - s)\,\omega_{sync}$$

- **Notice:**

1. rotor rotates at synchronous speed, *s* = 0
2. rotor is stationary, *s* = 1
3. All normal motor speeds fall between *s* = 0 and *s* = 1

The Electrical Frequency on the Rotor

An induction motor is like a rotating transformer, i.e.

→ **Stator (primary) induces voltage in the rotor (secondary)**

However, in induction motor:
secondary frequency is not the same as primary frequency

When **rotor is locked**, $\boldsymbol{n_m}$ **= 0 r/min,**
s = 1 → $f_r = sf_e = f_e$

If **rotor** rotates **synchronous to field**, $\boldsymbol{n_m = n_{sync}}$,
s = 0 → $f_r = 0$

The Electrical Frequency on the Rotor

Hence, at other rotor speeds, i.e. $0 < n_m < n_{sync}$,

$$f_r = sf_e$$

By substituting for *s*,

$$f_r = \frac{n_{sync} - n_m}{n_{sync}} f_e$$

Alternatively, since $n_{sync} = 120f_e/P$,

$$f_r = \frac{P}{120}(n_{sync} - n_m)$$

This shows that the relative difference between synchronous speed and rotor speed will determine the rotor frequency.

Example 7-1

A 208V, 10-hp, 4-pole, 60Hz, Y-connected induction motor has a full load slip of 5%.

(a) What is the synchronous speed of the motor?

(b) What is the rotor speed of the motor at rated load?

(c) What is the rotor frequency of the motor at rated load?

(d) What is the shaft torque of this motor at rated load?

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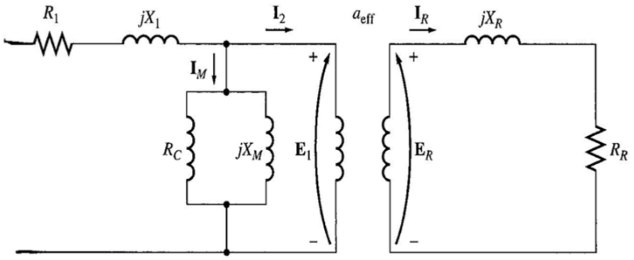
The Equivalent Circuit of Induction Motor

- **The operation of induction motor relies on induction of rotor voltages and currents due to stator circuit, i.e. transformer action.**
- Hence, **induction motor equivalent circuit similar to that of a transformer**.
- As in any transformer, there is certain resistance and self-inductance in the primary (stator) windings, which must be represented in the equivalent circuit of the machine ➔ R1 - stator resistance and X1 – stator leakage reactance



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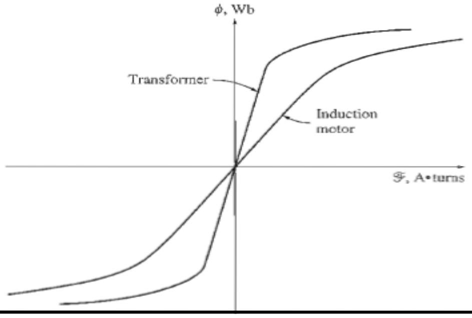
The Equivalent Circuit of Induction Motor

- Also, like any transformer with an iron core, the flux in the machine is related to the integral of the applied voltage E1. The curve of mmf vs flux (magnetization curve) for this machine is compared to a similar curve for a transformer, as shown below:
- The slope of the induction motor’s mmf-flux curve is much shallower than the transformer ➔ This is because there must be an air gap in an induction motor, which greatly increases the reluctance of the flux path and thus reduces the coupling between primary and secondary windings.



The Equivalent Circuit of Induction Motor

- The magnetizing reactance Xm in the equivalent circuit will have a much smaller value than it would in a transformer.
- The primary internal stator voltage is E1 is coupled to the secondary E_R by an ideal transformer with an effective turns ratio a_{eff}.
- The turns ratio for a wound rotor is basically the ratio of the conductors per phase on the stator to the conductors per phase on the rotor.
- It is rather difficult to see a_{eff} clearly in the cage rotor because there are no distinct windings on the cage rotor.
- E_R in the rotor produces current flow in the shorted rotor (or secondary) circuit of the machine.
- The primary impedances and the magnetization current of the induction motor are very similar to the corresponding components in a transformer equivalent circuit.

The Rotor Circuit Model

- When the voltage is applied to the stator windings, a voltage is induced in the rotor windings.
- In general, **the greater the relative motion between the rotor and the stator magnetic fields, the greater the resulting rotor voltage and rotor frequency.**
- The largest relative motion occurs when the rotor is stationary, called the *locked-rotor* or *blocked-rotor* condition, so the largest voltage and rotor frequency are induced in the rotor at that condition.
- The smallest voltage and frequency occur when the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion.
- The magnitude and frequency of the voltage induced in the rotor at any speed between these extremes is directly proportional to the slip of the rotor.

The Rotor Circuit Model

- Therefore, if the magnitude of the induced rotor voltage at locked-rotor conditions is called $\mathbf{E_{R0}}$, the magnitude of the induced voltage at any slip will be given by:

$$\mathbf{E_R = sE_{R0}}$$

- And the frequency of the induced voltage at any slip is:

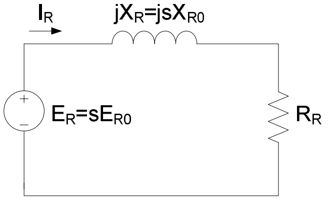
$$f_r = sf_e$$

- The rotor resistance $\mathbf{R_R}$ is a constant, independent of slip, while the rotor reactance is affected in a more complicated way by slip.

The Rotor Circuit Model

- **The rotor contains both resistance and reactance**. However, **only the reactance** will be affected by the frequency of the rotor voltage and current. Hence,

$$\begin{aligned} X_R &= \omega_r L_R = 2\pi f_R L_R \\ &= 2\pi (s f_e) L_R \\ &= s(2\pi f_e L_R) \\ \therefore\ X_R &= sX_{R0} \end{aligned}$$



where X_{R0} = blocked-rotor rotor reactance.

- Hence, the resulting rotor equivalent circuit is

The rotor current flow is:

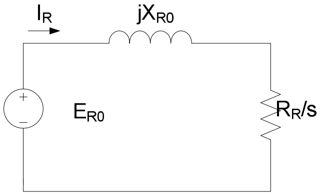
$$I_R = \frac{E_R}{R_R + jX_R} = \frac{E_R}{R_R + jsX_{R0}} = \frac{E_{R0}}{R_R/s + jX_{R0}}$$

The Rotor Circuit Model

- Therefore, the overall rotor impedance talking into account rotor slip would be:

$$Z_{R,eq} = R_R/s + jX_{R0}$$

- And the rotor equivalent circuit using this convention is:



- The rotor circuit model with all the frequency (slip) effects concentrated in resistor R_R.
- In this equivalent circuit, the rotor voltage is a constant E_{R0} V and the rotor impedance $Z_{R,eq}$ contains all the effects of varying rotor slip.

The Rotor Circuit Model

Notice:

- at **very low slips**, ***$R_R/s \gg X_{R0}$***

  rotor current varies linearly with slip

- at **high slips**, ***$X_{R0} \gg R_R/s$***

  rotor current approaches steady state value

The Final Equivalent Circuit (refer rotor to stator)

- To produce the final per-phase equivalent circuit for an induction motor, it is necessary to refer the rotor part of the model over to the stator side.
- In an ordinary transformer, the voltages, currents and impedances on the secondary side can be referred to the primary by means of the turns ratio of the transformer.
- Exactly the same sort of transformation can be done for the induction motor's rotor circuit. If the effective turns ratio of an induction motor is a_{eff} , then the transformed rotor voltage becomes

$$E_1 = E_R^{'} = a_{eff} E_{R0}$$

- The rotor current:

$$I_2 = \frac{I_R}{a_{eff}}$$

- And the rotor impedance:

$$Z_2 = a_{eff}^2 \left(\frac{R_R}{s} + jX_{R0} \right)$$

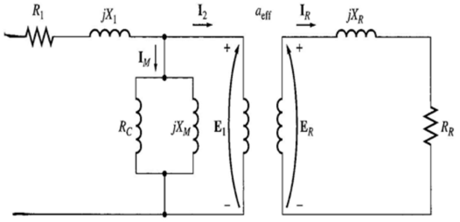
The Final Equivalent Circuit (refer rotor to stator)

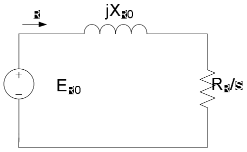
- If we make the following definitions:

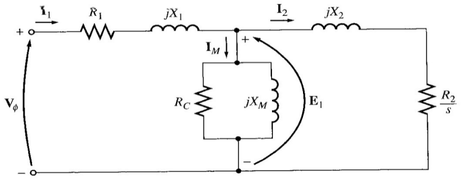
  $R_2 = a^2_{eff}\, R_R$ (rotor impedance referred to stator)

  $X_2 = a^2_{eff}\, X_{R0}$ (reactance referred to stator)

- The final per-phase equivalent circuit is as shown below:







Power and Torque in an Induction Motor

Losses and Power-Flow diagram

- An induction motor can be basically described as a rotating transformer.
- Its input is a 3 phase system of voltages and currents.
- For an ordinary transformer, the output is electric power from the secondary windings.
- The secondary windings in an induction motor (the rotor) are **shorted out**, so no electrical output exists from normal induction motors. Instead, the output is mechanical.
- The relationship between the input electric power and the output mechanical power of this motor is shown below:

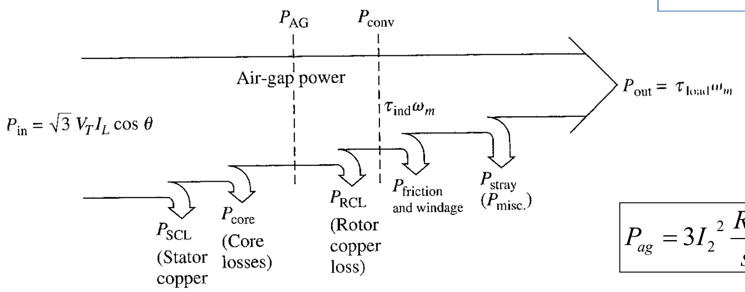
Power and Torque in an Induction Motor

Power Flow Diagram

Note:

$$\boxed{P_{conv} = (1-s)P_{ag}}$$

$$\boxed{P_{RCL} = sP_{ag}}$$



$$\boxed{P_{ag} = 3I_2^2 \frac{R_2}{s}}$$

$$\boxed{P_{SCL} = 3I_s^2 R_s}$$

$$\boxed{P_{RCL} = 3I_2^2 R_2}$$

$$\boxed{P_{conv} = 3I_2^2 R_2 \left(\frac{1-s}{s}\right)}$$

Power and Torque in an Induction Motor

- The **input power (electrical)** of an induction motor:

$$P_{in} = \sqrt{3}\, V_T I_L \cos\theta$$

- **Losses encountered on stator side:**

- **Stator copper loss P_{SCL}**, i.e. I^2R loss in stator windings.
- **Hysteresis and eddy current losses P_{core}**
- **Air gap power P_{AG} ⇒ power transferred** to the rotor **across** the **air gap**

- **Losses encountered on rotor side:**

- **Rotor copper loss P_{RCL}**, i.e. I^2R loss in rotor windings.

- What is left?

Power converted from electrical to mechanical form, P_{conv}.

- **Final losses:**

- **Friction and windage losses, $P_{F\&W}$**
- **Stray losses, P_{misc}**

Power and Torque in an Induction Motor

- The **output power (mechanical)** of the **induction motor**:

$$P_{out} = \tau_{load}\, w_m$$

- **Special note on P_{core}:**
- The core losses do **not always** appear after P_{SCL}.
- P_{core} comes partially from the stator circuit and partially from the rotor circuit. Usually the rotor core losses are very small compared to the stator core losses.
- P_{core} are represented in the induction motor equivalent circuit by the **resistor R_C** (or the conductance G_C).
- If R_C is **not given** but **P_{core} = X watts** is **given**, then often **add it together with $P_{F\&W}$** at the end of the power flow diagram.
- Note: P_{core} , $P_{F\&W}$ and P_{misc} are sometimes lumped together and called **rotational losses P_{rot}**.

Example 6-2

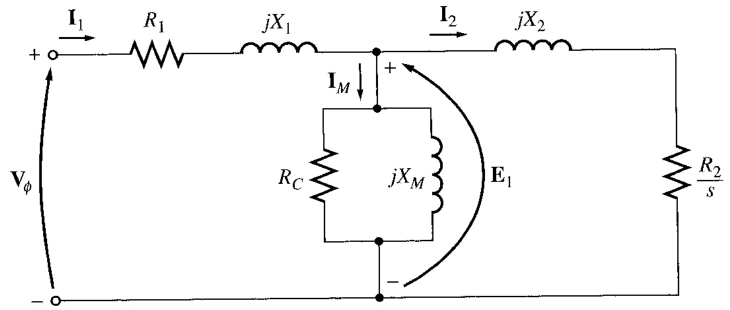
A 480V, 60Hz, 50hp, 3 phase induction motor is drawing 60A at 0.85 PF lagging. The stator copper losses are 2kW, and the rotor copper losses are 700W. The friction and windage losses are 600W, the core losses are 1800W, and the stray losses are negligible.

Find:

1. The air gap power P_{AG}
2. The power converted P_{conv}
3. The output power P_{out}
4. The efficiency of the motor

Power and Torque in an Induction Motor

By examining the per-phase equivalent circuit, the power and torque equations governing the operation of the motor can be derived.



Power and Torque in an Induction Motor

- The input current to a phase of the motor is: $I_1 = \dfrac{V_\phi}{Z_{eq}}$
-
- Where $$Z_{eq} = R_1 + jX_1 + \cfrac{1}{G_C - jB_M + \cfrac{1}{\cfrac{R_2}{s} + jX_2}}$$

- Thus, the stator copper losses, the core losses, and the rotor copper losses can be found.
- The stator copper are: $\mathbf{P_{SCL} = 3\, I_1^2\, R_1}$
- The core losses : $\mathbf{P_{core} = 3\, E_1^2\, G_C}$
- So, the air gap power: $\mathbf{P_{AG} = P_{in} - P_{SCL} - P_{core}}$

Power and Torque in an Induction Motor

- Also, the only element in the equivalent circuit where the air-gap power can be consumed is in the resistor R_2/s. Thus, the air-gap power:

$$P_{AG} = 3 I_2^{\,2} \frac{R_2}{s}$$

- The actual resistive losses in the rotor circuit are given by:

$$\mathbf{P_{RCL} = 3\, I_R^{\,2}\, R_R}$$

- Since power is unchanged when referred across an ideal transformer, the rotor copper losses can also be expressed as:

$$\mathbf{P_{RCL} = 3\, I_2^{\,2}\, R_2}$$

Power and Torque in an Induction Motor

- After stator copper losses, core losses and rotor copper losses are subtracted from the input power to the motor, the remaining power is converted from electrical to mechanical form. The power converted, which is called developed mechanical power is given as:

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= 3I_2^{\,2} \frac{R_2}{s} - 3I_2^{\,2} R_2$$

$$= 3I_2^{\,2} R_2 \left(\frac{1}{s} - 1 \right)$$

$$P_{conv} = 3I_2^{\,2} R_2 \left(\frac{1-s}{s} \right)$$

- And the rotor copper losses are noticed to be equal to the air gap power times the slip → $P_{RCL} = s\, P_{AG}$

Power and Torque in an Induction Motor

- Hence, the lower the slip of the motor ➔ the lower the rotor losses.
- Also, if the rotor is not turning, the slip is **s = 1** and the air gap power is entirely consumed in the rotor.
- This is logical, since if the rotor is not turning, the output power $\mathbf{P_{out}\ (= \tau_{load}\ \omega_m)}$ must be zero.
- Since $P_{conv} = P_{AG} - P_{RCL}$, this also gives another relationship between the air-gap power and the power converted from electrical and mechanical form:

$$\begin{aligned} \mathbf{P_{conv}} &= \mathbf{P_{AG} - P_{RCL}} \\ &= \mathbf{P_{AG} - sP_{AG}} \\ \mathbf{P_{conv}} &= \mathbf{(1\text{-}s)\ P_{AG}} \end{aligned}$$

- Finally, if the friction and windage losses and the stray losses are known, the output power:

$$\mathbf{P_{out} = P_{conv} - P_{F\&W} - P_{misc}}$$

Power and Torque in an Induction Motor

- The inducd torque in a machine was defined as the torque generated by the internal electric to mechanical power conversion. This torque differs from the torque actually available at the terminals of the motor by an amount equal to the friction and windage torques in the machine. Hence, the developed torque is:

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

- Other ways to express torque:

$$\tau_{ind} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}}$$

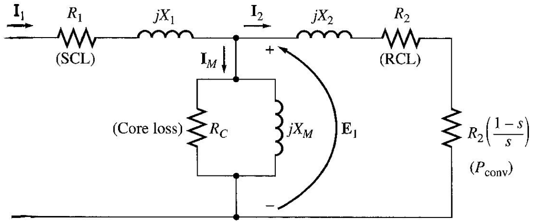
$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

Separating the Rotor Copper Losses and the Power Converted in an Induction Motor's Equivalent Circuit

- A portion of power transferred via the air gap will be consumed by the rotor copper loss and also converted into mechanical power.
- Hence it may be useful to separate the rotor copper loss element since rotor resistance are both used for calculating rotor copper loss and also the output power.
- Since Air Gap power would require R_2/s and rotor copper loss require R_2 element. The difference between the air gap power and the rotor copper loss would give the converted power, hence;

$$R_{conv} = \frac{R_2}{s} - R_2 = R_2\left(\frac{1-s}{s}\right)$$

- Therefore the equivalent circuit would be modified to be as follows:



Example 6-3

A 460V, 25hp, 60Hz, 4 pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

R1 = 0.641 Ω	R2 = 0.332 Ω	
X1 = 1.106 Ω	X2 = 0.464 Ω	Xm = 26.3 Ω

The total rotational losses are 1100W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2% at the rated voltage and rated frequency, find the motor's

(a) speed
(b) stator current
(c) power factor
(d) Pconv and Pout
(e) τind and τload
(d) efficiency

See the solution form the text book

Induction Motor Torque-Speed Characteristics

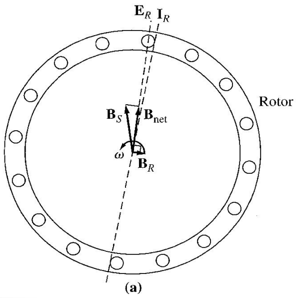
- The torque-speed relationship will be examined first from the physical viewpoint of the **motor's magnetic field** behaviour and then, a general equation for **torque as a function of slip** will be derived from the induction motor equivalent circuit.

From physical viewpoint:

- Assume IM is rotating at no load conditions → its rotating speed is near to synchronous speed.
- The net magnetic field B_{net} is produced by the magnetization current I_M.
- The magnitude of I_M and B_{net} is directly proportional to voltage E1.
- If E1 is constant, then B_{net} is constant.
- In an actual machine, E1 varies as the load changes due to the stator impedances R1 and X1 which cause varying voltage drops with varying loads.
- However, the volt drop at R1 and X1 is so small, that E1 is assumed to remain constant throughout.

Induction Motor Torque-Speed Characteristics

- **At no-load**, slip is very small, and so the relative motion between rotor and magnetic field is very small, and f_r is also very small.
- Since the relative motion is small, $\mathbf{E_R}$ induced in the bars of the rotor is very small, and $\mathbf{I_R}$ is also very small.
- Since f_r is small, the reactance of the rotor is nearly zero, and the max rotor current $\mathbf{I_R}$ is almost in phase with the rotor voltage $\mathbf{E_R}$.
- The rotor current produces a small magnetic field $\mathbf{B_R}$ at an angle slightly greater than **90 degrees** behind $\mathbf{B_{net}}$.



Induction Motor Torque-Speed Characteristics

- The stator current must be quite large even at no-load since it must supply most of B_{net} .
- The induced torque which is keeping the rotor running,

$$\tau_{ind} = kB_R \times B_{net}$$

- and its magnitude is

$$\tau_{ind} = kB_R B_{net} \sin \delta$$

- In terms of magnitude, the induced torque will be small due to small rotor magnetic field.

At heavy load:

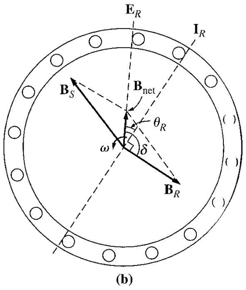
- As the motor's load increases ➔ its slip increases, and the rotor speed falls.
- Since the rotor speed is slower, there is now more relative motion between rotor and stator magnetic fields.
- Greater relative motion means a stronger rotor voltage E_R which in turn produces a larger rotor current I_R .

Induction Motor Torque-Speed Characteristics

- With large rotor current, the rotor magnetic field B_R also increases. However, the angle between rotor current and B_R changes as well.
- Since the rotor slip is larger, the rotor frequency rises ($f_r = sf_e$) and the rotor reactance increases (ωL_R).
- Therefore, the rotor current now lags further behind the rotor voltage, and the rotor magnetic field shifts with the current.
- The rotor current now has increased compared to no-load and the angle δ has increased.
- The increase in B_R tends to increase the torque, while the increase in angle δ tends to decrease the torque (τ_{ind} is proportional to sin δ, and δ>90º).
- Since the first effect is larger than the second one, the overall induced torque increases to supply the motor's increased load.

Induction Motor Torque-Speed Characteristics

- As the load on the shaft is increased, the sin δ term decreases more than the B_R term increases (the value is going towards the 0 cross over point for a sine wave).
- At that point, a further increase in load decreases τ_{ind} and the motor stops. This effect is known as **pullout torque**.



Modelling the Torque-Speed Characteristics of an Induction Motor

- Note:

$$\tau_{ind} = kB_R B_{net} \sin\delta$$

- Each Term can be considered separately to derive the overall torque behaviour:

a) $\vec{B}_R \propto \bar{I}_R$ (provided the rotor **core is unsaturated**). Hence, $\vec{B}_R$ **increases with** $\bar{I}_R$ which in turn **increases with slip** (decrease in speed).

b) $\vec{B}_{net} \propto \bar{E}_1$ and will remain **approximately constant**.

c) The **angle** δ **increases with slip**. Hence, **'sin δ' term decreases**. From the *on load condition*,

Modelling the Torque-Speed Characteristics of an Induction Motor

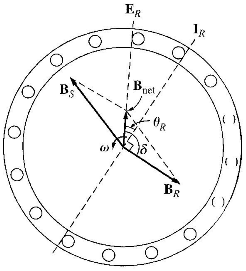
Note:

$$\delta = \theta_R + 90^o$$

$$\sin\delta = \sin(\theta_R + 90^o) = \cos\theta_R = \text{power factor of rotor}$$

where,

$$\theta_R = \tan^{-1}\frac{sX_{R0}}{R_R}$$



(b)

Modelling the Torque-Speed Characteristics of an Induction Motor

- The **torque-speed characteristic** can be constructed from the graphical manipulation of the three properties (a)-(c) which is shown on the next page.
- The characteristic curve can be divided into three regions:

1. ***Low-slip region*** ($s \uparrow$ linearly, $n_m \downarrow$ linearly) **:**
    - X_R negligible $\Rightarrow$ $\text{PF}_\text{R} \approx 1$
    - $\bar{I}_R$ increases linearly with s

    Contains the entire **normal operating range** of an induction motor.

2. ***Moderate-slip region***:
    - X_R same order of magnitude as R_R $\Rightarrow$ PF_R droops
    - $\bar{I}_R$ doesn't increase as rapidly as in low-slip region

    Peak torque (**pullout torque**) occurs **in this region**.

Modelling the Torque-Speed Characteristics of an Induction Motor

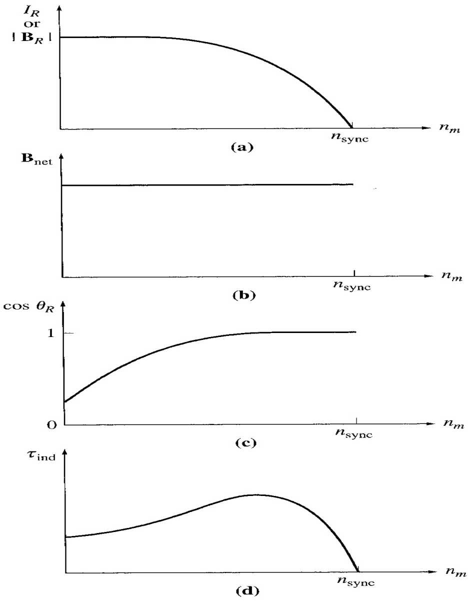
3. High-slip region:

- Increase in $\bar{I}_R$ completely overshadowed by decrease in PF_R.
- T_{ind} decreases with increase in load

Note:

- Typical pullout torque ≈ 200% to 250% of T_{rated}.
- The starting torque ≈150% of the T_{rated}.
  Hence induction motor may be started at full load.

Modelling the Torque-Speed Characteristics of an Induction Motor



Deriving Induction Torque Equation

- Only the final result is shown without going through the derivation

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} \qquad or \qquad \tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

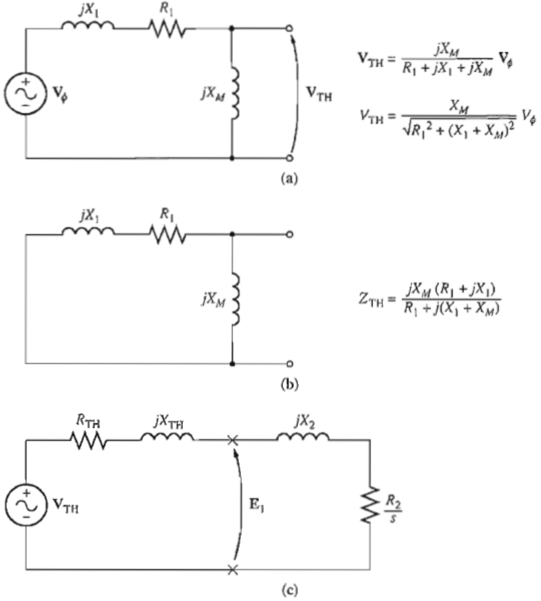
- Second equation is more useful since it is referred to w_{sync}
- Using the equivalent circuit of the motor and using thevenin's equivalent circuit techniques, an expression for Torque can be found by first finding an expression for the P_{AG}

Deriving Induction Torque Equation

$$P_{AG\ per\ phase} = I_2^2 \frac{R_2}{s}$$

hence, total air gap power:

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$



$$V_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M} V_\phi$$

$$V_{TH} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}} V_\phi$$

$$Z_{TH} = \frac{jX_M (R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

Deriving Induction Torque Equation

1. Derive the thevenin voltage (potential divider rule):

$$V_{TH} = V_\phi \frac{jX_m}{R_1 + jX_1 + jX_m}$$

- Hence the magnitude of thevenin voltage:

$$V_{TH} = V_\phi \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}}$$

- Since $X_m >> X_1$, $X_m >> R_1$, therefore the magnitude may be approximated to:

$$V_{TH} \approx V_\phi \frac{X_m}{X_1 + X_m}$$

Deriving Induction Torque Equation

2. Find the thevenin impedance

- Take out the source and replace it with a short circuit, and derive the equivalent impedances.

$$Z_{TH} = \frac{jX_m(R_1 + jX_1)}{R_1 + jX_1 + jX_m}$$

- Since Xm >> X1, Xm >> R1,

$$R_{TH} \approx R_1 \left(\frac{X_m}{X_1 + X_m}\right)^2$$

$$X_{TH} \approx X_1$$

- Representing the stator circuit by the thevenin equivalent, and adding back the rotor circuit, we can derive I2,

$$I_2 = \frac{V_{TH}}{R_{TH} + R_2/s + j(X_{TH} + X_2)}$$

- Hence the magnitude will be,

$$I_2 = \frac{V_{TH}}{\sqrt{\left(R_{TH} + R_2/s\right)^2 + (X_{TH} + X_2)^2}}$$

Deriving Induction Torque Equation

- Hence air gap power,

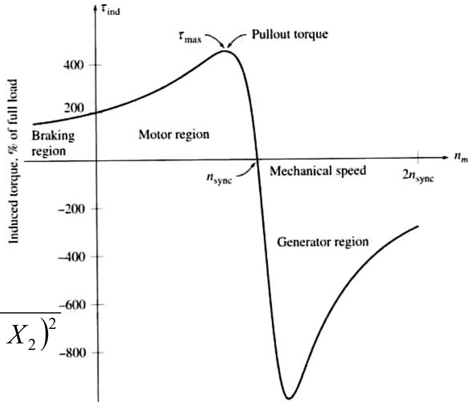
$$P_{AG} = 3\left(\frac{V_{TH}}{\sqrt{\left(R_{TH} + {R_2}/{s}\right)^2 + \left(X_{TH} + X_2\right)^2}}\right)^2 \frac{R_2}{s}$$

- Therefore, induced torque,

$$\tau_{ind} = \frac{3\left(\frac{V_{TH}}{\sqrt{\left(R_{TH} + {R_2}/{s}\right)^2 + \left(X_{TH} + X_2\right)^2}}\right)^2 \frac{R_2}{s}}{\omega_{sync}}$$

- If a graph of Torque and speed were plotted based upon changes in slip, we would get a similar graph as we had derived earlier.

Deriving Induction Torque Equation



$$P_{AG} = 3I_2^2\frac{R_2}{s} = \frac{3V_{TH}^2\, R_2/s}{\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

$$= \frac{3V_{TH}^2\, R_2/s}{\omega_{sync}[\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2]}$$

Deriving Induction Torque Equation

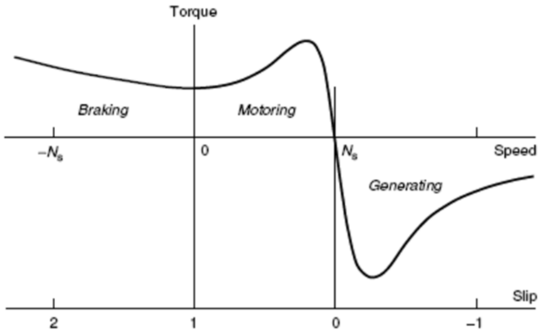


Figure 6.15 *Torque–speed curve over motoring region (slip between 0 and 1), braking region (slip greater than 1) and generating region (negative slip)*

Comments on the induction motor torque-speed curve

- At **synchronous speed, τ_{ind} = 0**.
- The **curve is linear** between **no load and full load**.
- The maximum torque is known as **pullout torque** or **breakdown torque**. It is approximately 2 to 3 times the rated full-load torque of the motor.
- The **starting torque** is slightly **larger** than its full-load torque. So, IM will start carrying any load it can supply at full power.
- Torque for a given slip varies as **square** of the **applied voltage**. This is useful as one form of IM speed control.

Comments on the induction motor torque-speed curve

- If rotor is driven **faster than synchronous speed**, ➔ t_{ind} direction reverses and machine becomes a generator

- If motor is **turning backward relative** to the **direction** of magnetic fields (achieved by reversing the magnetic field rotation direction), ➔ t_{ind} will **stop the machine** very **rapidly (braking)** and try to rotate in the other direction.

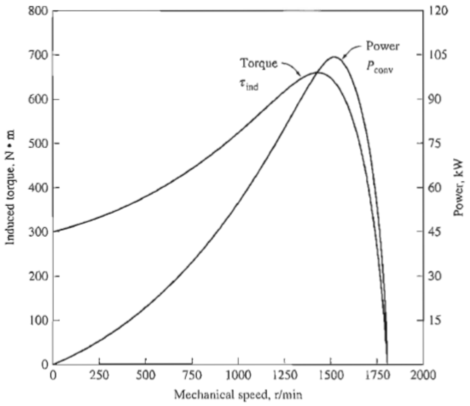
  This can be achieved by switching two stator phases, which causes the machine to change direction of rotation which is called ***plugging (plug reversal)***.

Converted Power

- The power converted to mechanical form in an induction motor is:

$$P_{conv} = \tau_{ind}\omega_m$$

- Hence, a characteristic to show the variation of converted power with speed (i.e. load) can be obtained.



Note that:

Peak power supplied by the induction motor occurs at **different speed** to **maximum torque**.

No power is converted **when rotor speed = 0**.

Maximum (Pullout) Torque in an Induction Motor

- Since induced torque is equal to P_{AG} / ω_{sync}, the maximum pullout torque may be found by finding the maximum air gap power. And maximum air gap power is during which the power consumed by the R_2/s resistor is the highest.

- Based upon the maximum power transfer theorem, maximum power transfer will be achieved when the magnitude of source impedance matches the load impedance. Since the source impedance is as follows:

$$Z_{source} = R_{TH} + jX_{TH} + jX_2$$

Maximum (Pullout) Torque in an Induction Motor

Hence maximum power transfer occurs during:

$$\frac{R_2}{s} = \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}$$

Hence max power transfer is possible when slip is as follows:

$$s_{\max} = \frac{R_2}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}}$$

Put in the value of $S_{\max}$ into the torque equation,

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

Maximum (Pullout) Torque in an Induction Motor

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

- ➢ From this equation we can notice that:

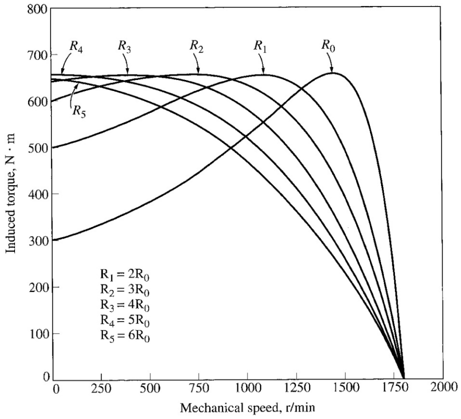
A. **Torque is related to the square of the applied voltage**
B. **Torque is also inversely proportional to the machine impedances**
C. **Slip during maximum torque is dependent upon rotor resistance**
D. **Torque is also independent to rotor resistance as shown in the maximum torque equation.**

- ➢ By adding more resistance to the machine impedances, we can vary:

i. Starting torque
ii. Max pull out speed

Maximum (Pullout) Torque in an Induction Motor

- ➢ Torque is related to the square of supplied voltage.
- ➢ Torque is inversely proportional to stator impedances and rotor reactance.
- ➢ s_{max} **is directly proportional to *R2*.**
- ➢ t_{max} **is independent of *R2*.**
- ➢ As **increase *R2*** (i.e. increase s_{max}):
- ➢ **pullout speed** of motor **decreases**
- ➢ **maximum torque** remains **constant**
- ➢ **starting torque increases**



Example 6-4

A 2 pole, 50 Hz induction motor supplies 15kW to a load at a speed of 2950 r/min.

1. **What is the motor's slip?**
2. **What is the induced torque in the motor in Nm under these conditions?**
3. **What will the operating speed of the motor be if its torque is doubled?**
4. **How much power will be supplied by the motor when the torque is doubled?**

Solve it by Your Self

Example 6-4

Solution

(a) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{2\text{ poles}} = 3000\text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}}(\times 100\%) \qquad (6\text{–}4)$$

$$= \frac{3000\text{ r/min} - 2950\text{ r/min}}{3000\text{ r/min}}(\times 100\%)$$

$$= 0.0167 \text{ or } 1.67\%$$

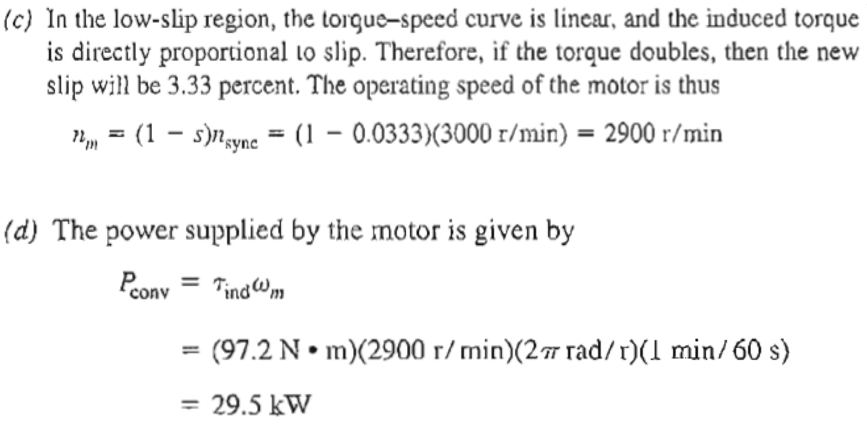
(b) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m}$$

$$= \frac{15\text{ kW}}{(2950\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})}$$

$$= 48.6\text{ N}\cdot\text{m}$$

Example 6-4



(c) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.0333)(3000 \text{ r/min}) = 2900 \text{ r/min}$$

(d) The power supplied by the motor is given by

$$\begin{aligned} P_{\text{conv}} &= \tau_{\text{ind}}\omega_m \\ &= (97.2 \text{ N} \cdot \text{m})(2900 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min}/60 \text{ s}) \\ &= 29.5 \text{ kW} \end{aligned}$$

Example 6-5

A 460V, 25hp, 60Hz, 4-pole, Y-connected wound rotor induction motor has the following impedances in ohms per-phase referred to the stator circuit:

$R1 = 0.641\ \Omega$ $R2 = 0.332\ \Omega$

$X1 = 1.106\ \Omega$ $X2 = 0.464\ \Omega$ $Xm = 26.3\ \Omega$

1. **What is the max torque of this motor? At what speed and slip does it occur?**
2. **What is the starting torque?**
3. **When the rotor resistance is doubled, what is the speed at which the max torque now occurs? What is the new starting torque?**

Solve it by Your Self

Example 6-5

Solution

The Thevenin voltage of this motor is

$$V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}} \qquad (6\text{–}41a)$$

$$= \frac{(266\ \text{V})(26.3\ \Omega)}{\sqrt{(0.641\ \Omega)^2 + (1.106\ \Omega + 26.3\ \Omega)^2}} = 255.2\ \text{V}$$

The Thevenin resistance is

$$R_{TH} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2 \qquad (6\text{–}44)$$

$$\approx (0.641\ \Omega)\left(\frac{26.3\ \Omega}{1.106\ \Omega + 26.3\ \Omega}\right)^2 = 0.590\ \Omega$$

The Thevenin reactance is

$$X_{TH} \approx X_1 = 1.106\ \Omega$$

(a) The slip at which maximum torque occurs is given by Equation (6–53):

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \qquad (6\text{–}53)$$

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = 0.198$$

Example 6-5

This corresponds to a mechanical speed of

$$n_m = (1 - s)n_{sync} = (1 - 0.198)(1800\ \text{r/min}) = 1444\ \text{r/min}$$

The torque at this speed is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]} \qquad (6\text{–}54)$$

$$= \frac{3(255.2\ \text{V})^2}{2(188.5\ \text{rad/s})[0.590\ \Omega + \sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}]}$$

$$= 229\ \text{N} \cdot \text{m}$$

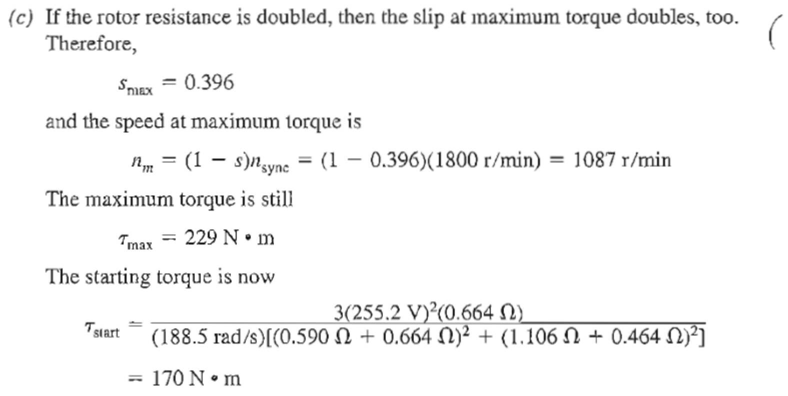
(b) The starting torque of this motor is found by setting $s = 1$ in Equation (6–50):

$$\tau_{start} = \frac{3V_{TH}^2 R_2}{\omega_{sync}[(R_{TH} + R_2)^2 + (X_{TH} + X_2)^2]}$$

$$= \frac{3(255.2\ \text{V})^2(0.332\ \Omega)}{(188.5\ \text{rad/s})[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 104\ \text{N} \cdot \text{m}$$

Example 6-5



(c) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{\max} = 0.396$$

and the speed at maximum torque is

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.396)(1800 \text{ r/min}) = 1087 \text{ r/min}$$

The maximum torque is still

$$\tau_{\max} = 229 \text{ N} \cdot \text{m}$$

The starting torque is now

$$\tau_{\text{start}} = \frac{3(255.2 \text{ V})^2(0.664\ \Omega)}{(188.5 \text{ rad/s})[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 170 \text{ N} \cdot \text{m}$$

Speed Control of Induction Motors

1. Before advances in Power electronics IM were not good machines for applications requiring considerable speed control.

2. The normal operating range of a typical induction motor was confined to less than 5% slip, and the speed variation is more or less proportional to the load on the motor shaft.

3. If slip is made higher, motor efficiency becomes very poor since rotor copper losses will be high as well ($P_{RCL} = sP_{AG}$).

Speed Control of Induction Motors

Following methods are employed for speed control of induction motors:

1. **Pole changing.**
2. **Stator voltage control.**
3. **Supply frequency control.**
4. **Rotor resistance control.**

Speed control by changing the line frequency

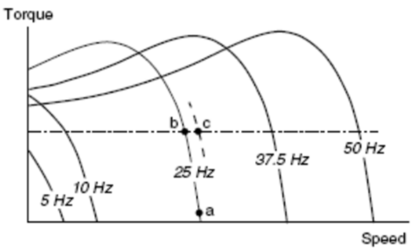
- This is also known as **variable frequency control.**
- **Changing** the **electrical frequency** will **change** the **synchronous speed** of the machine since: **Base speed = synchronous speed of motor at rated conditions**
- Hence, it is possible to adjust speed of motor either above or below base speed.
- **BUT** it also **requires terminal voltage limitation** in order **to maintain** the **same** amount of **flux level** in the machine core.
- **If not** the machine will experience:

1) **Core saturation** (non linearity effects)
2) **Excessive magnetization current will flow**

Variable-frequency speed control in IM

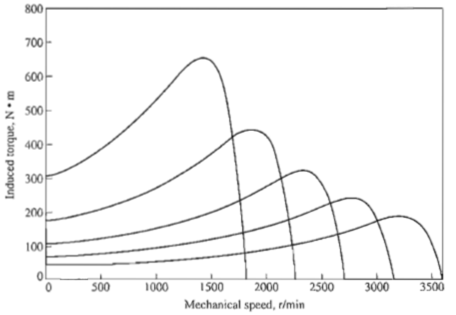
- torque-speed curves for speed below base speed.
- V/F=constant to keep flux constant

below base speed



for V/F (F decreases and V decreases also)

above base speed

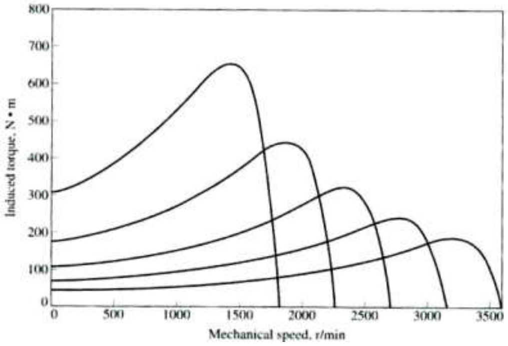


The family of torque-speed characteristic curves for speeds above base speed, assuming that the line voltage is held constant.

Variable-frequency speed control in IM

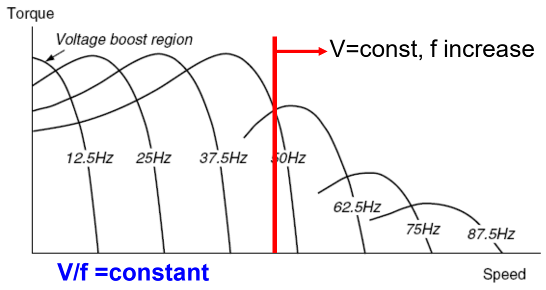
Above Base Speed

- Variable-frequency speed control in an IM
- Above Base Speed ➔ Frequency is increased , while V=Vrated ➔ Flux is decreased ➔ maximum Torque is also decreased



Variable-frequency speed control in IM

- There may also be instances where both characteristics are needed in the motor operation, hence it is **possible to combine both effects**.
- With the arrival of solid state devices/power electronics, variable frequency control has become the method of choice for induction motor speed control.
- **Advantage: can be used with *any* induction motor**



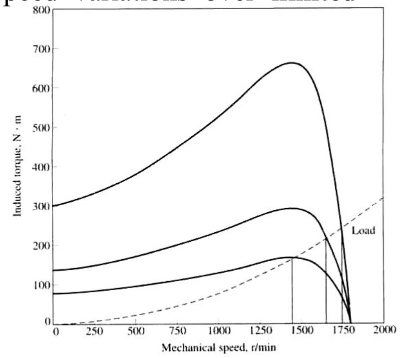
Speed control by changing the line voltage

- Variable-line-voltage speed control in an induction motor

But it **also causes variation** of **operating torque** since $T_{start} \propto V_T^2$

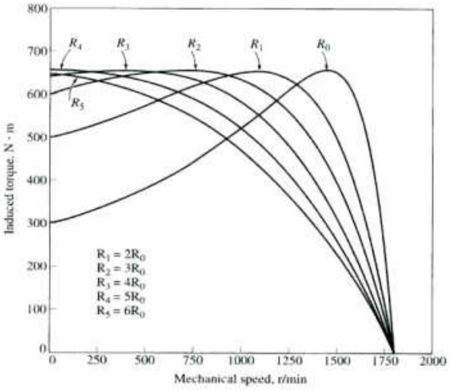
However, it only allows for motor speed variations over **limited range**.

Hence, this method is **only suitable** for **small motors** driving fans



Speed control by changing the rotor resistance

- This is **only possible** for **wound rotor induction motors,** i.e we can add extra rotor resistance to vary the torque-speed curve.
- **BUT** this **causes reduction in motor efficiency**. Hence, it is used **only for short periods**.



Homework

- Try to solve the following problems from our text book 5th edition

6.3, 6.15, 6.16, 6,17

Chapter Seven

DC Machinery Fundamentals

Introduction

- Most electric machines operate on the basis of interaction between current carrying conductors and electromagnetic fields.
- In particular: generator action is based on Faraday's law of electromagnetic induction, which implies that a current carrying conductor moving in a region having flux lines at right angles to the conductor will have emf induced in it $e_{ind} = (v \times B)\, l$

Motor Action

- Motor action is based on

$$F = i\left(l \times B\right)$$

- The linear dc machine behavior studied earlier resembles the behavior of real dc generators and motors
- However, real machines rotate and do not move in straight lines as linear dc machines

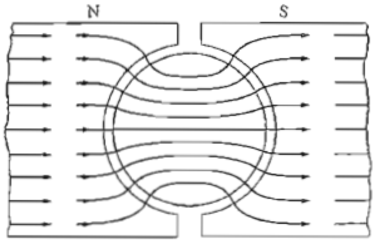
---

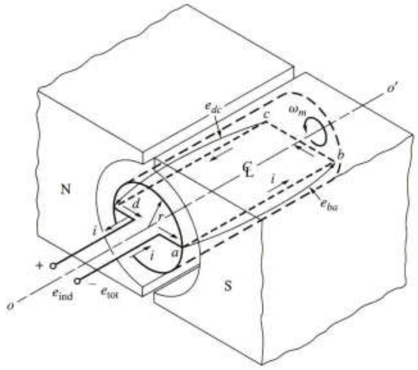
Rotating Loop Between Curved Pole Faces

The simplest rotating dc machine is shown below.

It consists of:

1. A rotating single loop of wire in a slot carved in a cylindrical ferromagnetic core – **rotor**
2. A pair of stationary magnetic poles (north and south) – **stator**



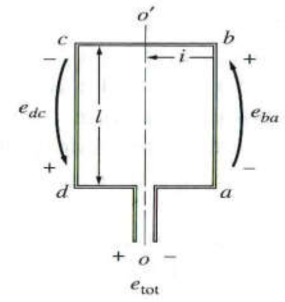


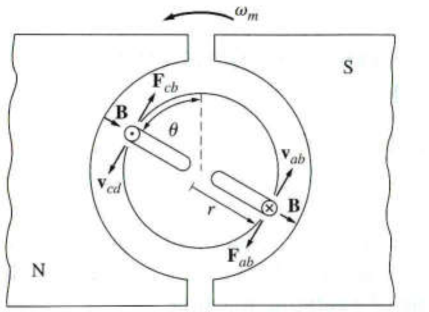
Rotating Loop Between Curved Pole Faces

- The **magnetic field** is supplied by the stator.
- The air gap between the rotor and stator is of **constant width (i.e.** is the **same** everywhere under the pole faces).
- ➔ magnetic **flux density is equal** everywhere **under** the **pole surface**.
- From Chapter 1, we know that the reluctance of the air is much higher than that of the core material, ➔ magnetic flux takes **shortest possible path** through the air gap (i.e. **perpendicular** to the rotor surface).

Voltage Induced in a Rotating Loop

- If the rotor is rotated, a **voltage will be induced** in the wire loop. (Note: No voltage source applied to the rotor terminals.)
- To determine the total voltage e_{tot} on the loop, each segment of the loop (as shown in the figure below) has to be examined separately and the resulting voltages summed up





Voltage Induced in a Rotating Loop

- To determine the total voltage e_{tot} on the loop, examine each segment of the loop separately and sum all the resulting voltages. The voltage on each segment is given by $e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$
- Note that B is 0 beyond the edges of the pole face
- Thus, the total induced voltage on the loop is:

$$e_{ind} = e_{tot} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

- Calculation for each section is shown next

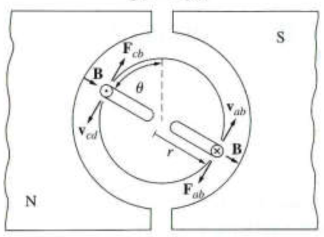
Voltage Induced in a Rotating Loop

Segment *ab*:

Velocity v_{ab} tangential to rotation path

$$\vec{B} = \begin{cases} \perp \text{ to surface} & \text{, under the pole} \\ 0 & \text{, beyond the pole edges} \end{cases}$$

$$e_{ba} = \begin{cases} vBl & \text{positive into page, under the pole face} \\ 0 & \text{, beyond the pole edges} \end{cases}$$

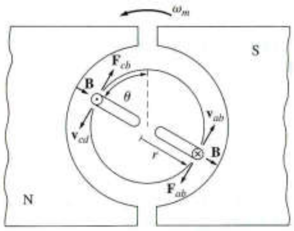


Segment *bc*:

direction of $(\vec{v} \times \vec{B})$ either into or out of page

- $(\vec{v} \times \vec{B}) \perp \vec{l}$
- $e_{cb} = 0$

Voltage Induced in a Rotating Loop



<u>Segment *cd*:</u>

Velocity v_{cd} tangential to rotation path

$$\vec{B} = \begin{cases} \perp \text{ to surface} & \text{, under the pole} \\ 0 & \text{, beyond the pole edges} \end{cases}$$

$$e_{dc} = \begin{cases} vBl & \text{positive out of page, under the pole face} \\ 0 & \text{, beyond the pole edges} \end{cases}$$

<u>Segment *da*:</u>

- $\left(\vec{v} \times \vec{B}\right) \perp \vec{l}$
- $e_{ad} = 0$

Voltage Induced in a Rotating Loop

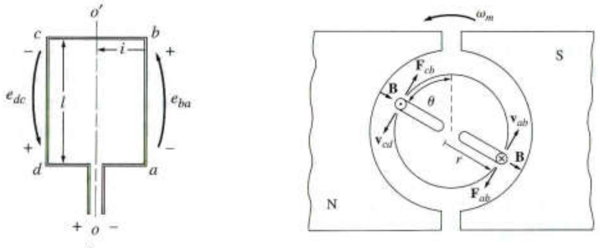
- Thus, the total induced voltage on the loop is:
- $e_{ba} = e_{dc} = vBl$ (since *v* x *B* is parallel to *l*)
- $e_{cb} = e_{ad} = 0$ (since *v* x *B* is perpendicular to *l*)
- Final result:

$$e_{ind} = e_{tot} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

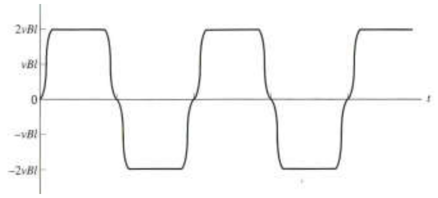
$$\therefore e_{ind} = \begin{cases} 2vBl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

Voltage Induced in a Rotating Loop

- When the loop rotates through 180°, segment *ab* is under the north pole face instead of the south pole face. At that time, the direction of the voltage on the segment reverses, but its magnitude remains constant.



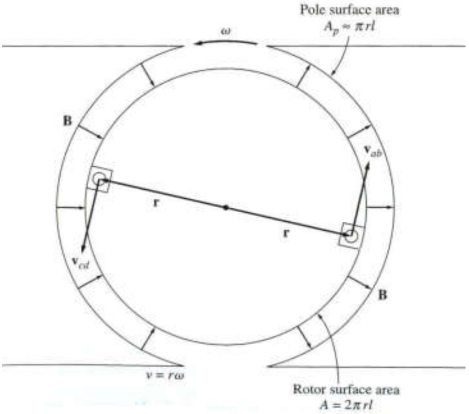
- The resulting voltage e_{tot}



Voltage Induced in a Rotating Loop

- An **alternative expression** for e_{ind} that relates the behaviour of the single loop to the behaviour of larger, real dc machines can be obtained by examining the figure below:
- The tangential velocity *v* of the loop edges can be expressed as $v = r\omega$. Therefore,

$$e_{ind} = \begin{cases} 2r\omega Bl & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$



Voltage Induced in a Rotating Loop

- For a 2-pole machine, if we assume that the gap between the poles is negligible (the gaps at the top and bottom of the diagram) then the surface area of the pole can be written as (area of cylinder /2) :

$$A_P = \frac{2\pi r l}{2} = \pi r l$$

- Therefore,

$$e_{ind} = \begin{cases} \dfrac{2}{\pi} A_P B \omega & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

Voltage Induced in a Rotating Loop

- Since the air gap flux density B is constant everywhere under the pole faces, the total flux under each pole is:

$$\phi = A_P B$$

- Thus, the **final form of the voltage equation** is:

$$e_{ind} = \begin{cases} \dfrac{2}{\pi} \phi \omega & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

- **In general, the voltage in any real machine will depend on the same three factors:**

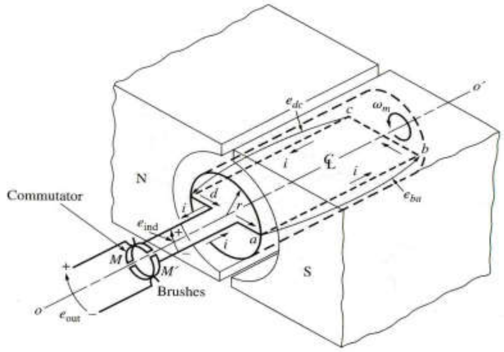
1) The flux in the machine

2) the speed of rotation

3) a constant representing the machine construction

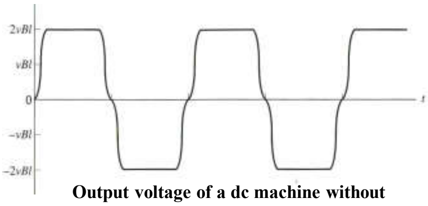
Getting DC voltage out of the rotating loop

- ➢ The voltage out of the loop is alternating between a constant positive value and a constant negative value, i.e. ac voltage.
- ➢ *How can this machine be modified to produce a dc voltage?*
- ➢ Answer: By using a mechanism called **commutator and brushes.**



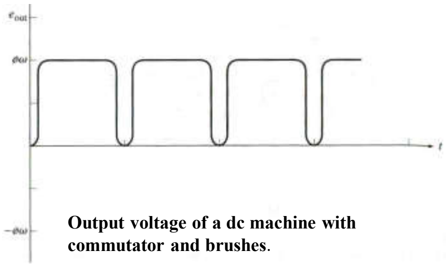
Commutation

- ➢ Two semicircular conducting segments (**commutator segments**) are added to the end of the loop.
- ➢ Two fixed contacts (**brushes**) are placed at an angle such that at the instant the voltage in the loop is zero, the contacts **short-circuit** the two segments.
- ➢ *Every time voltage of the loop changes direction, the contacts also switch connections and the output of the contacts is always built up in the same way (as shown next).*
- ➢ This connection-switching process is known as commutation process



Output voltage of a dc machine without commutator and brushes.

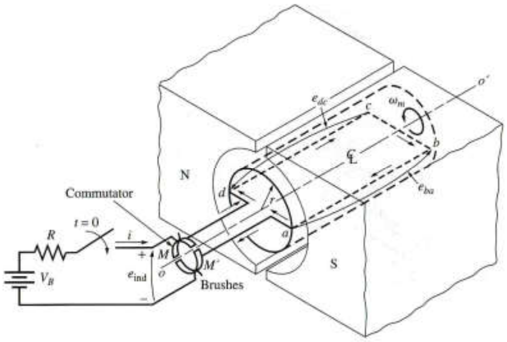
➔



Output voltage of a dc machine with commutator and brushes.

Torque Induced in a Rotating Loop

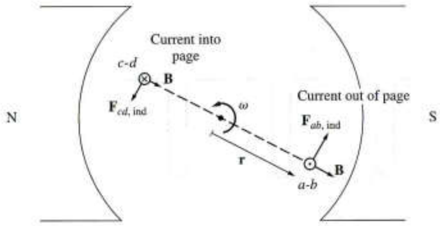
- If the simple machine is connected to a battery, as shown below, **how much torque will be produced** in the stationary loop when the switch is closed and current is allowed to flow?

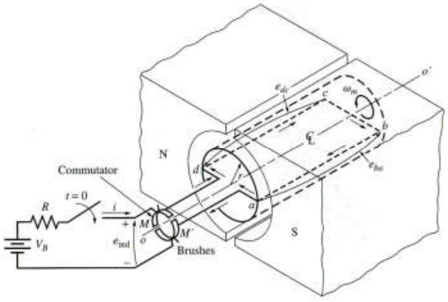


Torque Induced in a Rotating Loop

- As before, the approach is to examine each segment of the loop and then sum the effects of all segments.
- The **force on each segment** is given by

$$F = i(l \times B)$$





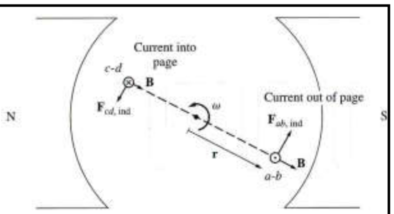
Torque Induced in a Rotating Loop

- And the **torque on the segment** is given by :

$$\tau \ = \vec{r} \times \vec{F} = rF \sin\theta$$

- where θ is the angle between *r* and *F*.
- When the **loop is beyond the pole edges, τ = 0** (since B = 0).

Torque Induced in a Rotating Loop



Segment *ab*:

- Current directed out of the page

$$\vec{B} = \begin{cases} \perp \text{ to surface} & \text{, under the pole} \\ 0 & \text{, beyond the pole edges} \end{cases}$$

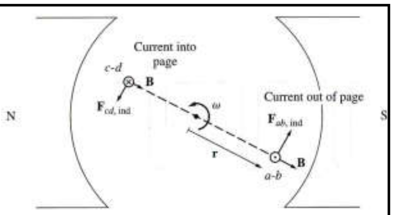
$$\vec{F}_{ab} = \begin{cases} ilB \quad \text{tangent to motion direction} & \text{, under the pole face} \\ 0 & \text{, beyond the pole edges} \end{cases}$$

$$\tau_{ab} = \begin{cases} rilB \text{ counterclockwise} & \text{, under the pole face} \\ 0 & \text{, beyond the pole edges} \end{cases}$$

Segment *bc*:

- Current flowing from upper left to lower right
- $\vec{F}_{bc} = i\left(\vec{l} \times \vec{B}\right) = 0$ since $\vec{l}$ is parallel to $\vec{B}$
- $\boxed{\tau_{bc} = 0}$

Torque Induced in a Rotating Loop



<u>Segment *cd*:</u>

- Current directed into the page
- $\vec{B} = \begin{cases} \perp \text{ to surface} & \text{, under the pole} \\ 0 & \text{, beyond the pole edges} \end{cases}$
- $\vec{F}_{cd} = \begin{cases} ilB \text{ tangent to motion direction} & \text{, under the pole face} \\ 0 & \text{, beyond the pole edges} \end{cases}$

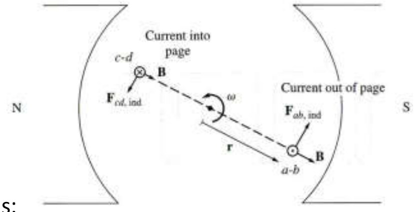
$$\boxed{\tau_{cd} = \begin{cases} rilB \quad \text{counterclockwise} & \text{, under the pole face} \\ 0 & \text{, beyond the pole edges} \end{cases}}$$

<u>Segment *da*:</u>

- Current flowing from upper left to lower right
- $\vec{F}_{da} = i\left(\vec{l} \times \vec{B}\right) = 0$ since $\vec{l}$ is parallel to $\vec{B}$
- $\boxed{\tau_{da} = 0}$

Torque Induced in a Rotating Loop

- ➢ Note that
- ➢ F_{ab}= F_{cd}= *ilB* and t_{ab}=t_{cd}=rFsinq=*rilB* *CCW*
- ➢ *(since q=90 deg)* And
- ➢ F_{bc}=F_{da}=0 (since *l is parallel to B*)
- ➢ t_{bc}=t_{da}=0
- ➢ Hence, the **total induced torque** in the loop is:



$$\tau_{ind} = \tau_{tot} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$

$$\therefore \tau_{ind} = \begin{cases} 2rilB & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

Torque Induced in a Rotating Loop

- By employing the facts that $A_P \approx \pi r l$ and $\phi = A_P B$, the **torque expression** can be reduced to:

$$\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\phi i & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

- **In general, the torque in any real machine will depend on the same three factors:**

1) the flux in the machine

2) the current in the machine

3) a constant representing the machine construction

Example 7-1

☐ A simple rotating loop between curved pole faces connected to a battery and a resistor through a switch. The physical dimensions and characteristics of the machine are:

r=0.5m; *l*=1m; R=0.3 ohm; B=0.25T; V_B=120V

(a) What happens when switch is closed?

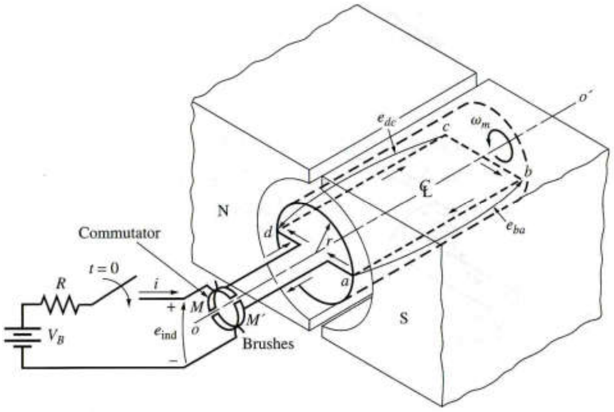
(b) What is the machine's maximum starting current? What is the steady-state angular velocity at no load?

(c) Suppose load torque=10 N.m, what is the new steady-state speed? How much power is supplied to the shaft? How much power is being supplied by the battery? is the machine a motor or a generator?

(d) Suppose the machine is unloaded again, and a torque =7.5 N.m is applied to the shaft in the direction of motion. What is the new steady-state speed? Is the machine acting as a motor or generator?

(a) Suppose the machine is running unloaded, what would be the final steady-state speed of the rotor be if the flux density were reduced to 0.2 T?

Example 7-1



Example 7-1

Solution

(a) When the switch in Figure 7–6 is closed, a current will flow in the loop. Since the loop is initially stationary, $e_{ind} = 0$. Therefore, the current will be given by

$$i = \frac{V_B - e_{\text{ind}}}{R} = \frac{V_B}{R}$$

This current flows through the rotor loop, producing a torque

$$\tau_{\text{ind}} = \frac{2}{\pi}\phi i \qquad \text{CCW}$$

This induced torque produces an angular acceleration in a counterclockwise direction, so the rotor of the machine begins to turn. But as the rotor begins to turn, an induced voltage is produced in the motor, given by

$$e_{\text{ind}} = \frac{2}{\pi}\phi\omega_m$$

so the current i falls. As the current falls, $\tau_{\text{ind}} = (2/\pi)\phi i\downarrow$ decreases, and the machine winds up in steady state with $\tau_{\text{ind}} = 0$, and the battery voltage $V_B = e_{\text{ind}}$.

This is the same sort of starting behavior seen earlier in the linear dc machine.

Example 7-1

(b) At *starting conditions*, the machine's current is

$$i = \frac{V_B}{R} = \frac{120\ \text{V}}{0.3\ \Omega} = 400\ \text{A}$$

At *no-load steady-state conditions*, the induced torque τ_{ind} must be zero. But $\tau_{\text{ind}} = 0$ implies that current i must equal zero, since $\tau_{\text{ind}} = (2/\pi)\phi i$, and the flux is nonzero. The fact that $i = 0$ A means that the battery voltage $V_B = e_{\text{ind}}$. Therefore, the speed of the rotor is

$$V_B = e_{\text{ind}} = \frac{2}{\pi}\phi\omega_m$$

$$\omega = \frac{V_B}{(2/\pi)\phi} = \frac{V_B}{2rlB}$$

$T = \frac{Wb}{m}$

$$= \frac{120\ \text{V}}{2(0.5\ \text{m})(1.0\ \text{m})(0.25\ \text{T})} = 480\ \text{rad/s}$$

(c) If a load torque of 10 N • m is applied to the shaft of the machine, it will begin to slow down. But as ω decreases, $e_{\text{ind}} = (2/\pi)\phi\,\omega\downarrow$ decreases and the rotor current increases $[i = (V_B - e_{\text{ind}}\downarrow)/R]$. As the rotor current increases, $|\tau_{\text{ind}}|$ increases too, until $|\tau_{\text{ind}}| = |\tau_{\text{load}}|$ at a lower speed ω.

At steady state, $|\tau_{\text{load}}| = |\tau_{\text{ind}}| = (2/\pi)\phi i$. Therefore,

$$i = \frac{\tau_{\text{ind}}}{(2/\pi)\phi} = \frac{\tau_{\text{ind}}}{2rlB}$$

$$= \frac{10\ \text{N}\cdot\text{m}}{(2)(0.5\ \text{m})(1.0\ \text{m})(0.25\ \text{T})} = 40\ \text{A}$$

Example 7-1

By Kirchhoff's voltage law, $e_{\text{ind}} = V_B - iR$, so

$$e_{\text{ind}} = 120\ \text{V} - (40\ \text{A})(0.3\ \Omega) = 108\ \text{V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{\text{ind}}}{(2/\pi)\phi} = \frac{e_{\text{ind}}}{2rlB}$$

$$= \frac{108\ \text{V}}{(2)(0.5\ \text{m})(1.0\ \text{m})(0.25\ \text{T})} = 432\ \text{rad/s}$$

The power supplied to the shaft is

$$P = \tau\omega_m$$

$$= (10\ \text{N}\cdot\text{m})(432\ \text{rad/s}) = 4320\ \text{W}$$

The power out of the battery is

$$P = V_B i = (120\ \text{V})(40\ \text{A}) = 4800\ \text{W}$$

This machine is operating as a *motor*, converting electric power to mechanical power.

Example 7-1

(d) If a torque is applied in the direction of motion, the rotor accelerates. As the speed increases, the internal voltage e_{ind} increases and exceeds V_B, so the current flows out of the top of the bar and into the battery. This machine is now a *generator*. This current causes an induced torque opposite to the direction of motion. The induced torque opposes the external applied torque, and eventually $|\tau_{load}| = |\tau_{ind}|$ at a higher speed ω_m.

The current in the rotor will be

$$i = \frac{\tau_{ind}}{(2/\pi)\phi} = \frac{\tau_{ind}}{2rlB}$$

$$= \frac{7.5\ \text{N}\cdot\text{m}}{(2)(0.5\ \text{m})(1.0\ \text{m})(0.25\ \text{T})} = 30\ \text{A}$$

The induced voltage e_{ind} is

$$\begin{aligned} e_{ind} &= V_B + iR \\ &= 120\ \text{V} + (30\ \text{A})(0.3\ \Omega) \\ &= 129\ \text{V} \end{aligned}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{129\ \text{V}}{(2)(0.5\ \text{m})(1.0\ \text{m})(0.25\ \text{T})} = 516\ \text{rad/s}$$

Example 7-1

(e) Since the machine is initially unloaded at the original conditions, the speed $\omega_m = 480$ rad/s. If the flux decreases, there is a transient. However, after the transient is over, the machine must again have zero torque, since there is still no load on its shaft. If $\tau_{ind} = 0$, then the current in the rotor must be zero, and $V_B = e_{ind}$. The shaft speed is thus

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{120\ \text{V}}{(2)(0.5\ \text{m})(1.0\ \text{m})(0.20\ \text{T})} = 600\ \text{rad/s}$$

Notice that when the flux in the machine is decreased, its speed increases. This is the same behavior seen in the linear machine and the same way that real dc motors behave.

Chapter Eight

DC Motors

Introduction

- **Why were dc motors so common, when dc power systems themselves were fairly rare?**

1. DC power systems are still common in cars, trucks, and aircraft. When a vehicle has a dc power system, it makes sense to consider using dc motors.

2. Another application for dc motors was a situation in which wide variations in speed are needed. Before the widespread use of power electronic rectifier-inverters, dc motors were unexcelled in speed control applications. Even if no dc power source were available, solid-state rectifier and chopper circuits were used to create the necessary dc power, and dc motors were used to provide the desired speed control

3. Today, induction motors with solid-state drive packages are the preferred choice over dc motors for most speed control applications. However, there are still some applications where dc motors are preferred.

Introduction

- DC motors are often compared by their speed regulations. The ***speed regulation* (SR)** of a motor is defined by

$$\mathrm{SR} = \frac{\omega_{m,nl} - \omega_{m,fl}}{\omega_{m,fl}} \times 100\%$$

$$\mathrm{SR} = \frac{n_{m,nl} - n_{m,fl}}{n_{m,fl}} \times 100\%$$

- It is a rough measure of the shape of a motor's torque- speed characteristic:

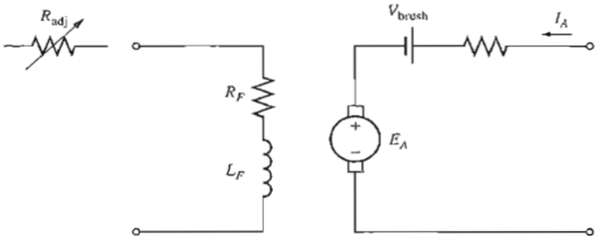
i. A positive speed regulation means that a motor's speed drops with increasing load, and
ii. A negative speed regulation means a motor's speed increases with increasing load.
iii. The magnitude- of the speed regulation tells approximately how steep the slope of the torque-speed curve is.

Introduction

- There are **five major types** of dc motors in general use:
  1) The separately excited dc motor
  2) The shunt dc motor
  3) The permanent-magnet dc motor
  4) The series dc motor
  5) The compounded dc motor

The Equivalent Circuit of a DC Motor

- The equivalent circuit of a dc motor is as follows:

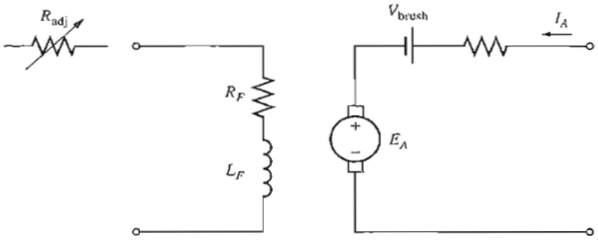


- The **armature (rotor) circuit** is represented by:

1. an ideal **voltage source E_A**
2. a **resistor R_A** (including rotor coils, interpoles and compensating windings, if present)

} a **Thevenin equivalent** of the entire rotor structure

The Equivalent Circuit of a DC Motor

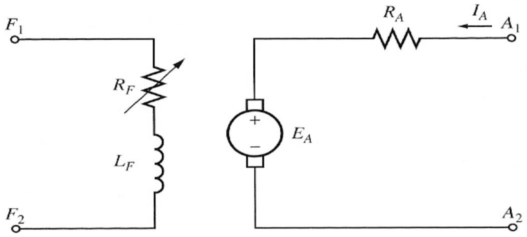


- The **brush voltage drop** is represented by a **small battery V_{brush} opposing** the **direction of current flow** in the machine.
- The **field coils** (producing the magnetic flux in the motor) are represented by:

  1) an **inductor L_F ,** 2) a **resistor R_F**
- Note: resistor ***R_{adj}*** **is an external variable resistor** used to **control** the **amount of current in the field circuit**.

The Equivalent Circuit of a DC Motor

- Some variation and simplifications can be made:
- V_{brush} may be **left out (V_{brush} << E_A) or included in R_A**
- The R_F is sometimes **lumped together with R_{adj}** and the **total is a variable resistor called R_F**
- Hence, the **simplified equivalent circuit** of a dc motor:



The Equivalent Circuit of a DC Motor

- The internal generated voltage is given by:

$$E_A = K\phi\omega$$

- and the torque induced is

$$\tau_{ind} = K\phi I_A$$

- The **tools necessary to analyse** the behaviour and performance of a **dc motor** are:

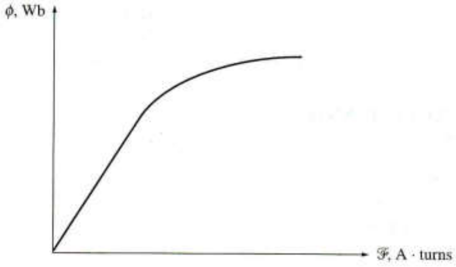
1) **Equations for E_A and τ_{ind}**
2) **Kirchoff's voltage law (KVL) equation of the armature circuit**
3) **The machine's magnetisation curve**

The Magnetisation Curve of a DC Machine

- E_A is directly proportional to the flux in the machine and the speed of rotation of the machine.
- *How is the internal generated voltage related to the field current in the machine?*
- The field current I_F **produces** a field magnetomotive force (**mmf**) given by

$$\mathscr{F} = N_F I_F$$

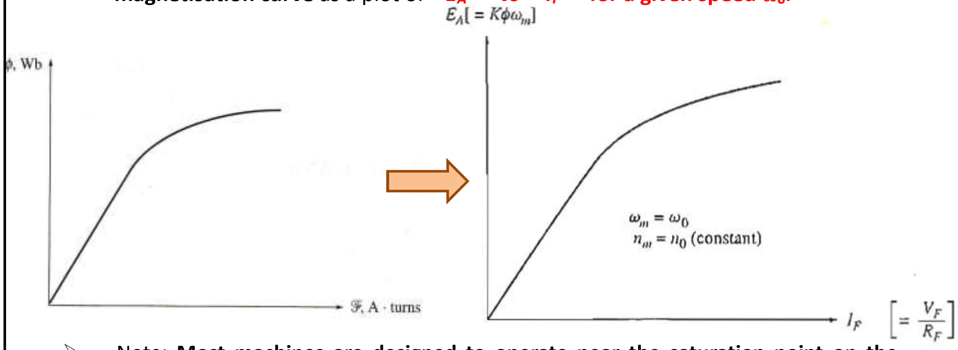
- This **mmf produces a flux** in the machine **in accordance** with its **magnetisation curve** shown below.



The magnetization curve of a ferromagnetic material (ϕ vs. F)

The Magnetisation Curve of a DC Machine

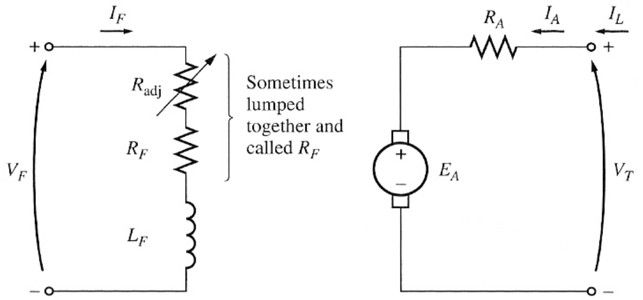
- Since, $I_F \propto \text{mmf}$ and $E_A \propto \text{flux } \phi$, it is customary to present the **magnetisation curve** as a plot of **E_A vs I_F for a given speed ω_0.**



- Note: **Most machines are designed to operate near the saturation point on the magnetisation curve** (at the **knee** of the curve).
- This implies that a fairly large increase in field current is often necessary to get a small increase in E_A when operation is near full load.

Separately Excited and Shunt DC Motors

- A **separately excited dc motor** is a motor whose field circuit is supplied from a separate constant-voltage power supply.



The equivalent circuit of a separately excited dc motor.

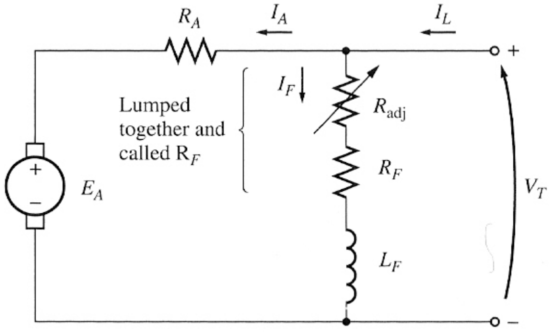
$$I_F = \frac{V_F}{R_F + R_{adj}}$$

$$V_T = E_A + I_A R_A$$

$$I_L = I_A$$

Separately Excited and Shunt DC Motors

- While **a shunt dc motor** is a motor whose field circuit gets its power directly across the armature terminals of the motor



The equivalent circuit of a shunt dc motor.

$$I_F = \frac{V_T}{R_F}$$

$$V_T = E_A + I_A R_A$$

$$I_L = I_A + I_F$$

Separately Excited and Shunt DC Motors

- When the **supply voltage** to a motor is **assumed constant**, there is **no practical difference in behaviour between these two motors**.
- Hence, unless otherwise specified, whenever the behaviour of a shunt motor is described, the separately excited motor is included too.
- The **KVL equation for the armature circuit** of these motors is:

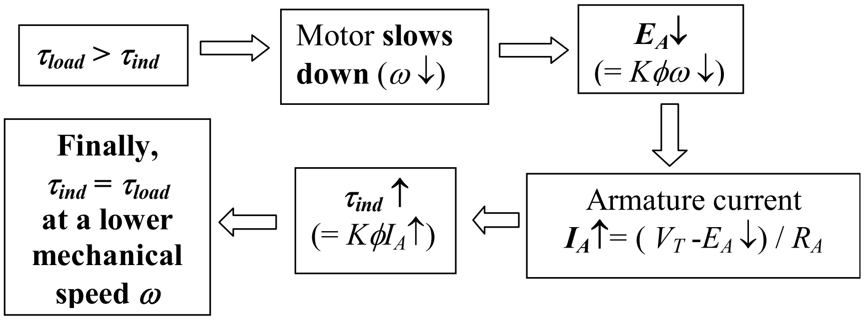
$$V_T = E_A + I_A R_A$$

The Terminal Characteristic of a Shunt DC Motor

- The **terminal characteristic** of a machine is a **plot of the machine's output quantities versus each other**.
- The **terminal characteristic** of **a motor** is A plot of its **output torque vs. Speed**
- ***How does a shunt dc motor respond to a load?***

The Terminal Characteristic of a Shunt DC Motor

If the **load on the shaft** of a shunt motor is **increased**,



$\tau_{load} > \tau_{ind}$

Motor **slows down** ($\omega\downarrow$)

$E_A\downarrow$ ($= K\phi\omega\downarrow$)

Armature current $I_A\uparrow = (V_T - E_A\downarrow) / R_A$

$\tau_{ind}\uparrow$ ($= K\phi I_A\uparrow$)

Finally, $\tau_{ind} = \tau_{load}$ at a lower mechanical speed ω

The output characteristic of a shunt dc motor can be derived from the induced voltage and torque equations of the motor plus the KVL.

The Terminal Characteristic of a Shunt DC Motor

From KVL, $V_T = E_A + I_A R_A$.

The induced voltage $E_A = K\phi\omega$, so

$$V_T = K\phi\omega + I_A R_A \qquad (1)$$

Since $\tau_{ind} = K\phi I_A$, current I_A can be expressed as

$$I_A = \frac{\tau_{ind}}{K\phi} \qquad (2)$$

Combining equations (1) and (2) yields

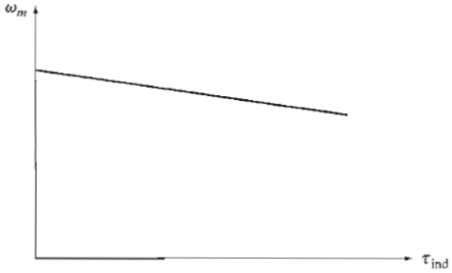
$$V_T = K\phi\omega + \frac{\tau_{ind}}{K\phi} R_A \qquad (3)$$

The Terminal Characteristic of a Shunt DC Motor

- Finally, the motor speed is given by:

$$\boxed{\omega = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}} \qquad (4)$$

- This equation is just **a straight line with a negative slope**.
- The resulting torque-speed characteristic of a shunt dc motor is shown below:

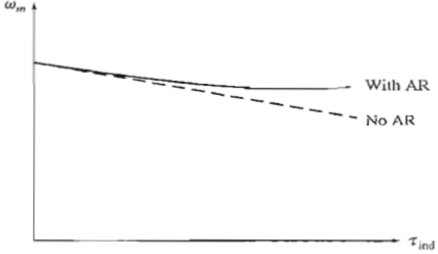


Torque-speed characteristic of a shunt or separately excited dc motor with compensating windings to **eliminate armature reaction**.

The Terminal Characteristic of a Shunt DC Motor

$$\omega = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$

- In order for the speed of the motor to vary linearly with torque, the other terms in this expression must be constant as the load changes.
- The terminal voltage supplied by the dc power source is assumed to be constant - if it is not constant, then the voltage variations will affect the shape of the torque-speed curve.
- If a motor has armature reaction, then as its load increases, the flux-weakening effects reduce its flux. From the motor speed equation above, the effect of reduction in flux is to increase the motor's speed at any given load over the speed it would run at without armature reaction.



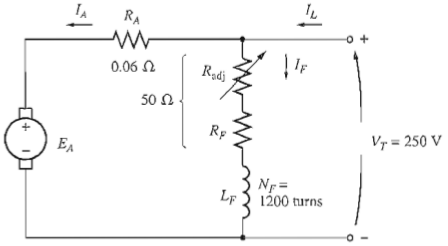
Torque-speed characteristic of the motor with armature reaction present.

The Terminal Characteristic of a Shunt DC Motor

- **If a motor has compensating windings, there will be no flux weakening problems and the flux in the motor will be constant.**
- If a shunt dc motor has compensating windings so that flux is constant regardless of load, and the motor's speed and armature current are known at any one value of load, then it is possible to calculate its speed at any other value of load, as long as the armature current at that load is known or can be determined.

Example 8-1

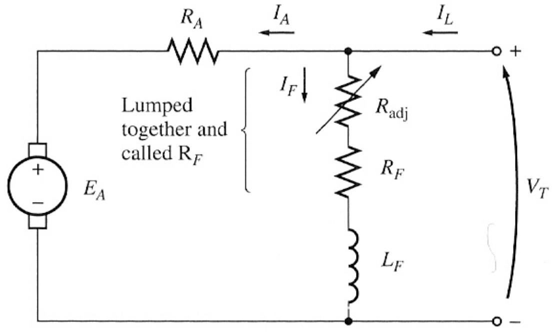
A 50HP, 250V, 1200 r/min DC shunt motor with compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_{adj} + R_F$ of 50 Ω, which produces a no-load speed of 1200r/min. There are 1200 turns per pole on the shunt field winding (Figure below)



(a) Find the speed of this motor when its input current is 100A.
(b) Find the speed of this motor when its input current is 200A.
(c) Find the speed of this motor when its input current is 300A.

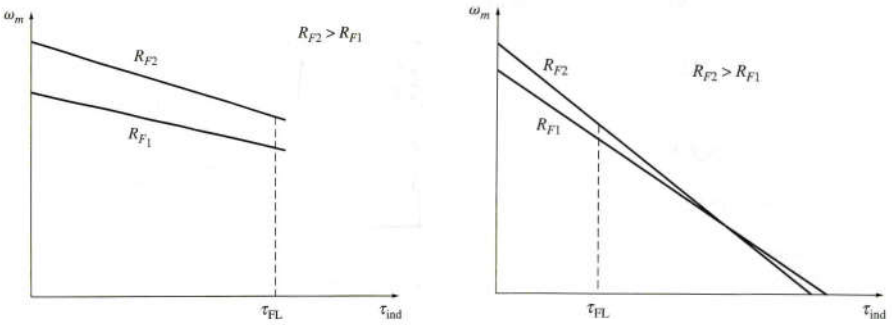
Speed Control of Shunt DC Motors

- **Adjusting the field resistance R_F**
- **Adjusting the terminal voltage** applied to the armature
- **Inserting a resistance** in series with the armature circuit - **less common** method.



Changing the Field Resistance

- The effect of increasing the field resistance on the output characteristic of a shunt motor is shown below.



(a) Over the normal operating range

(b) Over the entire range from no load to stall conditions

Warning about Field Resistance Speed Control

- As the **flux decreases**, the motor's:

1) no-load speed **increases**

2) torque-speed curve **slope** becomes **steeper**

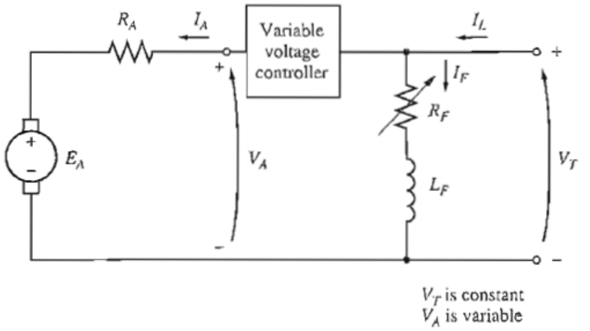
- Figure on previous slide shows the terminal characteristic of the motor **over the whole full range** from no-load to stall conditions (speed = 0).
- It is apparent that **at very slow speed**, an **increase in R_F** will actually **decrease the speed** of the motor.
- This is because at very slow speeds, the increase in I_A (due to decrease in E_A) is **not large enough to compensate** for decrease in ϕ in the t_{ind} equation

Warning about Field Resistance Speed Control

- With ϕ **decrease larger than I_A increase**, t_{ind} **decreases and motor slows down**.
- Some small dc motors used for control purposes actually operate at speeds close to stall conditions. For these motors, an increase in field resistance might have no effect, or it might even decrease the speed of the motor. Since the **results are not predictable**, field resistance control **should not be used** in these types of dc motors.

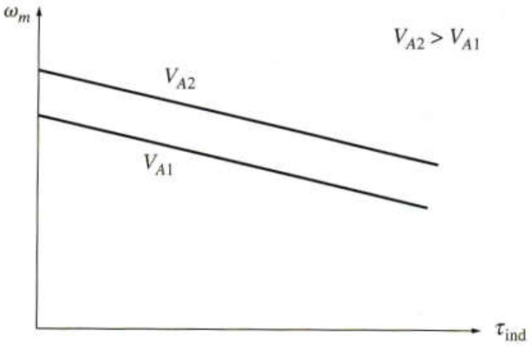
Changing the Armature Voltage

- This method involves changing the voltage applied to the armature circuit ***without changing the voltage applied to the field***.
- In effect the motor must be *separately excited* to use armature voltage control.



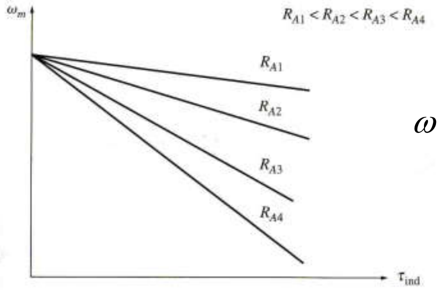
Changing the Armature Voltage

- The effect of increasing V_A on the torque-speed characteristic of a separately excited motor is shown below.
- Notice that the **no-load speed is shifted** by this method of speed control but the **slope of the curve remains constant**.



Inserting a resistor in series with the armature circuit

- If a resistor is inserted in series with the armature circuit (R_A ↑), the effect is to **drastically increase the slope** of the motor's torque-speed characteristic, **making it operate more slowly if loaded.**
- The effect of armature resistance speed control on a shunt motor's torque-speed characteristics is shown below.
- The insertion of a resistor is **very wasteful** since the **losses in the inserted resistor are very large**. Hence, this method for speed control is **rarely used**

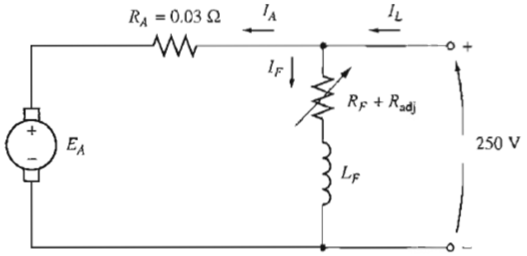


$$\omega = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$

Example 8-3

The following figure shows a 100-hp, 250-V, 1200 rpm shunt de motor with an armature resistance of 0.03 Ω and a field resistance of 41.67 Ω. The motor has compensating windings, so armature reaction can be ignored. Mechanical and core losses may be assumed to be negligible for the purposes of this problem. The motor is assumed to be driving a load with a line current of 126 A and an initial speed of 1103 *rpm . To simplify the problem,* assume that the amount of armature current drawn by the motor remains constant.

1. *If the machine's magnetization curve is shown in Figure 8- 9, what is the motor's* speed if the field resistance is raised to 50Ω?
2. *Calculate and plot the speed of this motor as a function of the field resistance R_f* assuming a constant-current load.



Solve it by your Self

Example 8-3

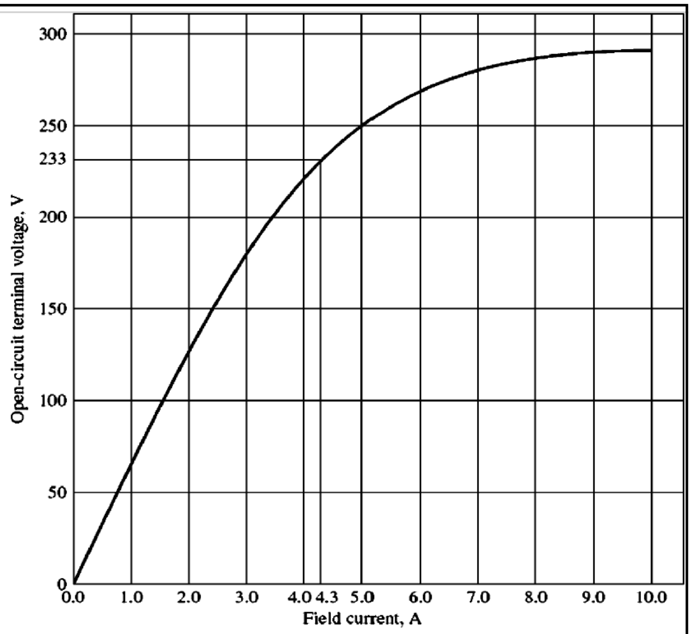
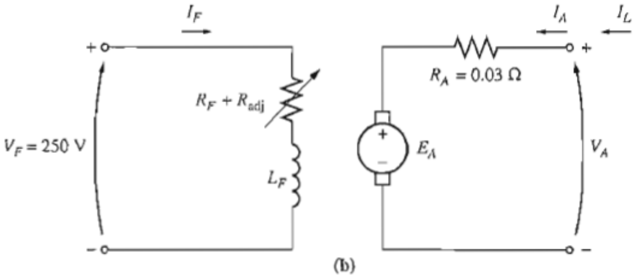


FIGURE 9–9
The magnetization curve of a typical 250-V dc motor, taken at a speed of 1200 r/min.

Example 8-4

The motor in Example 8- 3 is now connected separately excited, as shown in Figure 8-17b. The motor is initially running with *$V_A = 250$ V, $I_A = 120$ A, and n = 1103 r/min, while supplying a constant-torque load. What will the speed of this motor* be if *V_A is reduced to 200 V?*

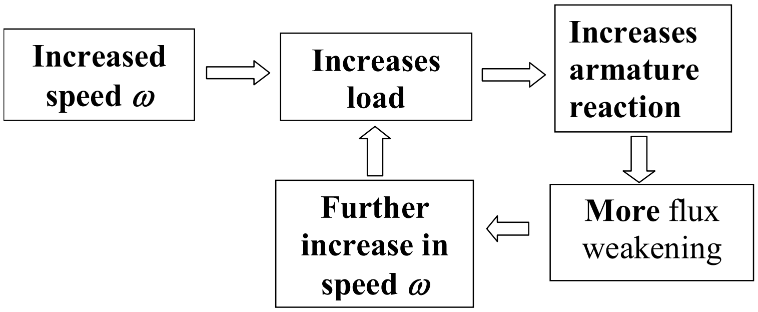


Solve it by your Self

The Effect of an Open Field Circuit

- As R_F is increased, the motor speed increases.
- *What happens if the field circuit were actually opened while the motor is running?*
- The **flux** in the machine would **drop drastically** (i.e. all the way down to ϕ_{res}).
- Hence, $\boldsymbol{E_A}$ (= $K\phi\omega$) **also drops**.
- This causes **really enormous increase** in $\boldsymbol{I_A}$.
- Since $t_{ind} \propto I_A$, the **induced torque** would be quite a bit **higher than load torque** in the motor.
- Therefore, **motor's speed rises and keeps going up**.
- Therefore, in a shunt dc motor operating with **light fields**, **armature reaction** can be **severe** such that increase in loads can **weaken its flux** enough to cause motor's **speed to rise**.
- However, **most loads** have torque-speed curves whose **torque increases with speed**.

Runaway of DC Motors



Increased speed ω

Increases load

Increases armature reaction

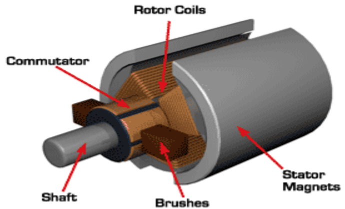
More flux weakening

Further increase in speed ω

This **continues until motor overspeeds**. This condition is known as **runaway**.

The Permanent Magnet DC Motor (PMDC)

- PMDC is a dc motor whose **poles are made of permanent magnets**.



- **Advantage** (compared to shunt dc motor):

1) No external field circuit is required, **no field circuit copper losses**

2) Smaller than shunt dc motors – because of no field circuit

- **Disadvantages:**

1) Cannot produce high flux **density** as an externally supplied shunt dc motor – **lower induced torque** per ampere of armature current compared to a shunt motor of same size.

The Permanent Magnet DC Motor (PMDC)

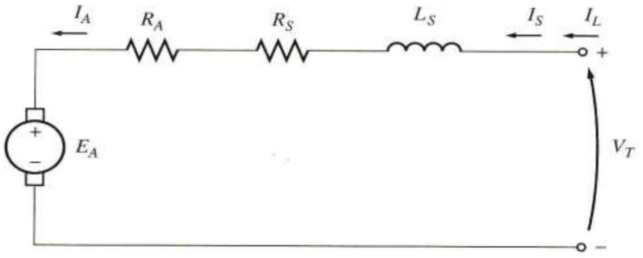
2) Runs the **risk of demagnetisation** due to armature reaction or excessive heating during prolonged periods of overload.

- The PMDC is basically the **same** machine as a shunt dc motor **except** the **flux in the PMDC motor is fixed**.
- Speed control through **varying the field current or flux** is **not possible**.
- Hence, **speed control methods** for PMDC motors are:

1. **Armature voltage control**
2. **Armature resistance control**

The Series DC Motor

- A series dc motor contains **field windings** of relatively few turns connected **in series with** the **armature circuit**.
- The equivalent circuit:



$$I_A = I_S = I_L$$

$$V_T = E_A + I_A(R_A + R_S)$$

The Series DC Motor

In the series dc motor,

$$\boxed{I_A = I_S = I_L}$$

armature current **field current** **line current**

The KVL equation for this motor is:

$$\boxed{V_T = E_A + I_A(R_A + R_S)}$$

Induced Torque in a Series DC Motor

- The basic behaviour of a series dc motor is due to the fact that the **flux** is **directly proportional to *I_A***, at least **until saturation** is reached i.e.

$$\boxed{\phi = cI_S = cI_A}$$

- where *c* = constant of proportionality.
- As **load increases (I_A increases), flux ϕ increases** too.

⇨ This causes **speed ω** to **decrease**.

- Hence, the series dc motor has a **sharply drooping torque-speed characteristic**.

Induced Torque in a Series DC Motor

- The induced torque is given by $\tau_{ind} = K\phi I_A$.
- By substituting for, the **induced torque in the series dc machine** is:

$$\boxed{\tau_{ind} = KcI_A^{2}}$$

⇨ **Torque in a series dc motor is proportional to the square of its armature current.**

- Therefore, this motor is **used in applications requiring very high torques**.
- Example: starter motors in cars, elevator motors and tractor motors in locomotives.

The Terminal Characteristic of a Series DC Motor

- **Assumption: The magnetisation curve is linear (no saturation)**.
- Hence, flux in the machine is given by equation : $\phi = cI_A$.
- *Derivation of torque-speed characteristic curve*:

1. KVL for series dc motor, $V_T = E_A + I_A(R_A + R_S)$
2. From torque equation, $I_A = \sqrt{\tau_{ind}/Kc}$

The Terminal Characteristic of a Series DC Motor

1. Also, $E_A = K\phi\omega$. Hence, by substituting for E_A and I_A into the KVL equation:

$$V_T = K\phi\omega + \sqrt{\frac{\tau_{ind}}{Kc}}(R_A + R_S)$$

2. If the flux can be eliminated from this expression, it will directly relate the torque of a motor to its speed. Notice that $I_A = \phi/c$, thus $\tau_{ind} = (K/c)\phi^2$. Therefore,

$$\phi = \sqrt{\frac{c}{K}}\sqrt{\tau_{ind}}$$

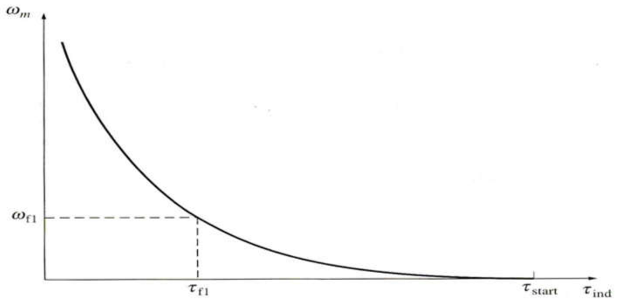
3. By substituting this flux expression into the equation in part 1 and solving for speed, the resulting **torque-speed relationship for the series dc motor** is:

The Terminal Characteristic of a Series DC Motor

$$\boxed{\omega = \frac{V_T}{\sqrt{Kc}}\frac{1}{\sqrt{T_{ind}}} - \frac{(R_A + R_S)}{Kc}}$$

Notice that for an **unsaturated series motor**, $\omega \propto \dfrac{1}{\sqrt{\tau_{ind}}}$

This **ideal torque-speed characteristic** is plotted below:



The Terminal Characteristic of a Series DC Motor

- **Disadvantage of series dc motor:**
When the **torque goes to zero, speed goes to infinity –** if no load is connected to the motor, it can turn fast enough to seriously damage itself.

- **Warning**!
Never completely **unload** a series motor
Never connect motor to a load **by a belt** or other mechanism that could break. **Use steel chains** instead.

Speed Control of Series DC Motors

There is only **one efficient way** to change the speed of a series dc motor:

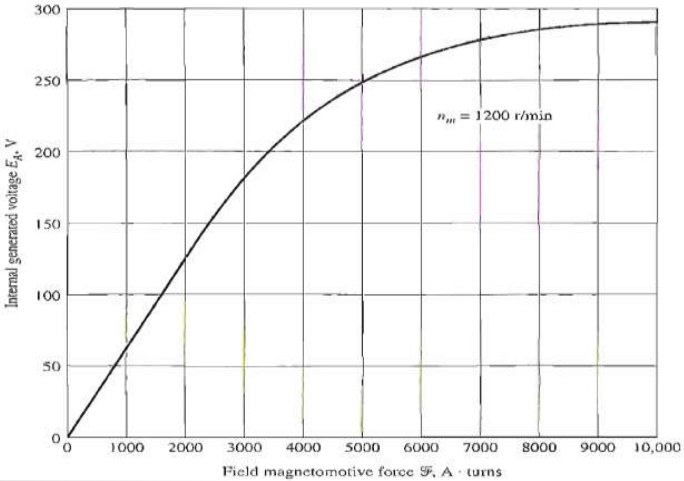
⇨ By **changing the terminal voltage**

If V_T is **increased,** the **speed increases** for any given torque

Example 8-5

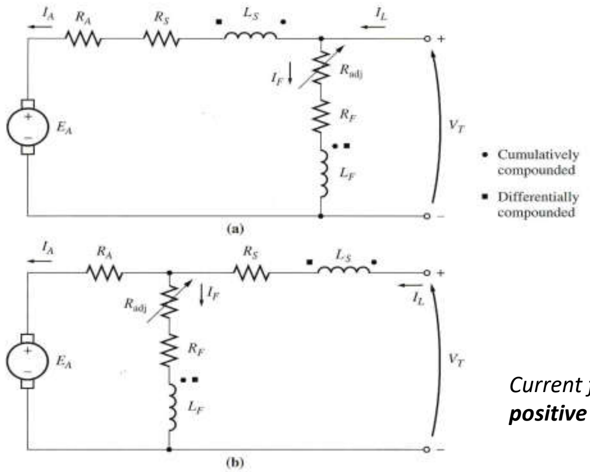
A series dc motor with V_T=250V , total R_A+R_S=0.08 ohm , field circuit consists of 25 turns per pole. Magnetizing curve is given in Fig. 8-22

a) Find the speed and induced torque of this motor for when the armature current is 50A



The Compounded DC Motor

- A compounded dc motor is a motor with both a shunt and a series field. The equivalent circuit of the compounded motor is shown below:



Current flowing ***into*** *a dot produced* ***positive mmf****.*

The Compounded DC Motor

- **Cumulative compounding** – current flows **into dots on both** field coils. Hence, resulting mmfs **add** to give a larger total mmf.

- **Differential compounding** – current flows **into dot on one** field coil and **out of the dot on the other**. Hence, resulting mmfs **subtract**

- There are **two components** of **flux**:

1. **one** is **constant** and
2. **another** which is **proportional to I_A** (and hence to the load)

The Cumulative Compound DC Motor

Compounded DC motor has:

- **higher** starting torque than a **shunt motor** (whose flux is constant)
- but a **lower** starting torque than a **series motor** (whose entire flux is proportional to I_A)

⇨ **Combines best features of both the shunt and series motors.**

The Cumulative Compound DC Motor

Advantages of CC motor**:**

- **extra torque for starting** (like series motor)
- **does not overspeed at no load** (like shunt motor)

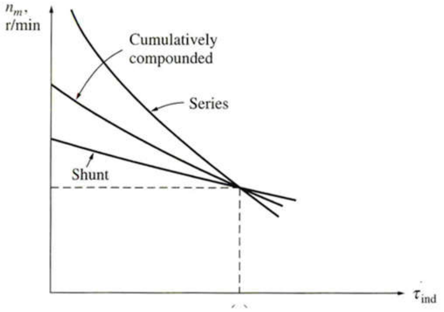
At **light loads**:

- series field has very small effect
- motor behaves approximately **like a shunt dc motor**

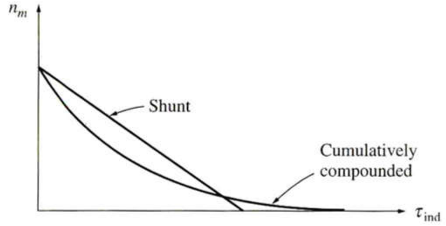
As **load gets very large**:

- series flux becomes quite important
- torque-speed curve begins to look **like a series motor's** characteristic

The Compounded DC Motor



The torque-speed characteristic of a cumulatively compounded dc motor compared to series and shunt motors with the **same full-load rating**.



The torque-speed characteristic of a cumulatively compounded dc motor compared to series and shunt motors with the **same no-load speed**.

The Torque-Speed Characteristics of a Differentially Compounded DC Motor

- In a differentially compounded dc motor, the shunt mmf and series mmf **subtract** from each other.

- Therefore, as the **load increases**:
  I_A increases ($\phi_{series}\uparrow \propto I_A\uparrow$)
  flux ϕ decreases ($\phi_{net}\downarrow = \phi_{shunt} - \phi_{series}\uparrow$)

  ⇨ Since ϕ decreases**, speed ω increases**.

- The increase in speed causes **another increase in load**, which further increases I_A, further decreasing the flux, and increasing the speed again

The Torque-Speed Characteristics of a Differentially Compounded DC Motor

- **The result: differentially compounded motor is unstable and tends to runaway**.
- This instability is **much worse** than that of a shunt dc motor with armature reaction. It is so bad that a differentially compounded motor is **unsuitable for any application**.

The Torque-Speed Characteristics of a Differentially Compounded DC Motor

- It is also **impossible to start** a differentially compounded motor.
- **At starting** conditions:
- The armature current and series field current are **very high.**
- Since the series flux subtracts from the shunt flux, the series field can actually **reverse** the magnetic polarity of the machine's poles.
- The motor will typically **remain still** or turn slowly in the **wrong direction** while **burning up** because of the excessive armature current.
- Hence, **when starting** this type of motor, the **series field must be short-circuited**, so that it behaves as an ordinary shunt motor during the starting period

Speed Control in the Cumulatively Compounded DC Motor

- The techniques available for speed control of a cumulatively compounded motor are the **same as those available for a shunt motor**:

  1. Change the field resistance R_F
  2. Change the armature voltage V_A
  3. Change the armature resistance R_A

  $\}$ **The arguments describing the effects of these methods are very similar to the arguments given**

- Theoretically, the differentially compounded dc motor could be controlled in a similar manner.

- However, since the differentially compounded motor is almost never used, its speed control method hardly matters.

Chapter Nine

Introduction To Power Electronics

Introduction

POWER ELECTRONIC COMPONENTS

I. The Diode

2. The Two-Wire Thyristor (or PNPN diode)

3. The Three-Wire Thyristor (or silicon controlled rectifier (SCR))

4. The Gate TurnOff (GTO) Thyristor

5. The DIAC

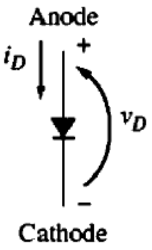
6. The TRIAC

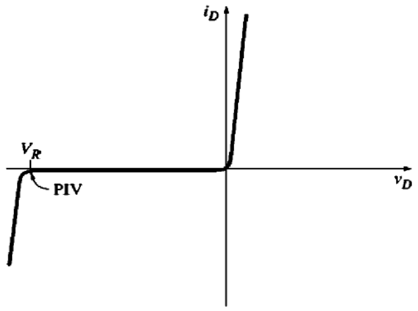
7. The Power Transistor (PTR)

8. The Insulated-gate Bipolar Transistor (IGBT)

The Diode

- A *diode is a semiconductor device designed to conduct current in one direction* only.
- When a voltage is applied to the diode in the forward direction, a large current flow results. When a voltage is applied to the diode in the reverse direction, the current flow is limited to a very small value (on the order of microamperes or less).





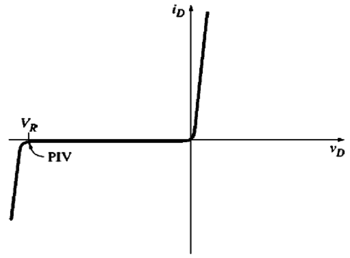
Voltage-current characteristic of a diode.

The Diode

- Diodes are rated by:
  1. The amount of power they can safely dissipate.
  2. The maximum reverse voltage that they can take before breaking down.
- The power dissipated by a diode during forward operation is equal to the forward voltage drop across the diode times the current flowing through it.
- This power must be limited to protect the diode from **overheating**.

The Diode

- The maximum reverse voltage of a diode is known as its **peak inverse voltage** *(PIV).*
- It must be high enough to ensure that the diode does not break down in a circuit and conduct in the reverse direction.
- Diodes are also rated by their switching time.
- switching time: the time it takes to go from the off state to the on state, and vice versa.



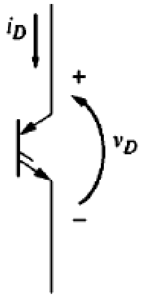
Voltage-current characteristic of a diode.

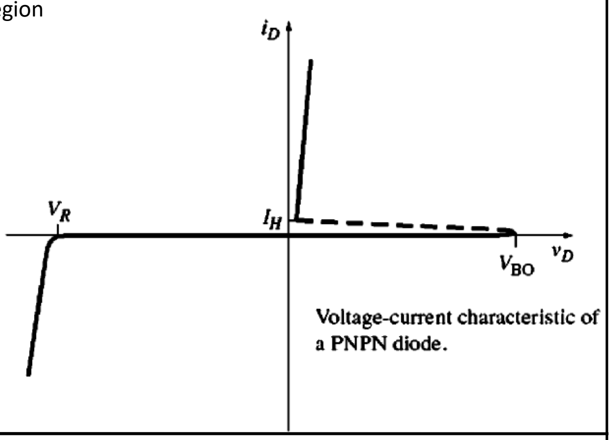
The Two-Wire Thyristor (or PNPN Diode)

- **Thyristor** is the generic name given to a family of semiconductor devices which are made up of four semiconductor layers.
- **The Two-Wire Thyristor** known as the *PNPN diode or trigger diode.*
- The PNPN diode is a rectifier or diode with an unusual voltage-current characteristic in the forward-biased region.

The Two-Wire Thyristor (or PNPN Diode)

- The characteristic curve consists of three regions:
    1. The reverse-blocking region
    2. The forward-blocking region
    3. The conducting region



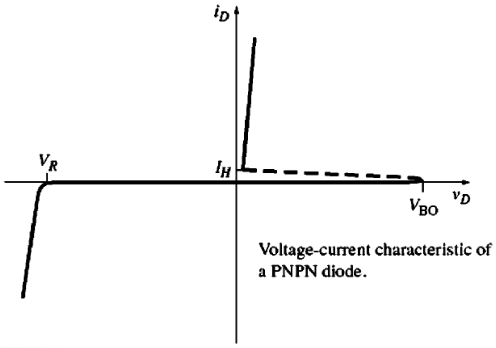


Voltage-current characteristic of a PNPN diode.

The Two-Wire Thyristor (or PNPN Diode)

- In the reverse-blocking region, the PNPN diode behaves as an ordinary diode and blocks all current flow until the reverse breakdown voltage is reached.

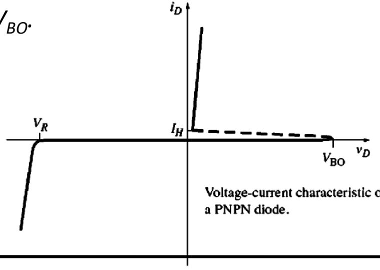
- In the conducting region, the PNPN diode again behaves as an ordinary diode, allowing large amounts of current to flow with very little voltage drop. It is the forward-blocking region that distinguishes a PNPN diode from an ordinary diode.



Voltage-current characteristic of a PNPN diode.

The Two-Wire Thyristor (or PNPN Diode)

- When a PNPN diode is forward-biased, no current flows until the forward voltage drop exceeds a certain value called the ***breakover voltage V_{BO}.***
- When the forward voltage across the PNPN diode exceeds V_{BO} the PNPN diode turns on and remains *on* until the current flowing through it falls below a certain minimum value (typically a few milliamperes).
- If the current is reduced to a value below this minimum value (called the *holding current I_H),* the *PNPN* diode turns *off* and will not conduct until the forward voltage drop again exceeds V_{BO}.

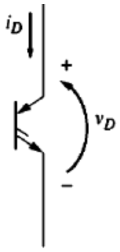


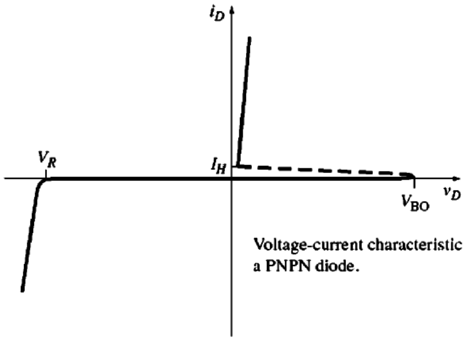
Voltage-current characteristic of a PNPN diode.

The Two-Wire Thyristor (or PNPN Diode)

- **In summary, a PNPN diode**

1. Turns on when the applied voltage v_D exceeds V_{BO}
2. Turns off when the current I_D drops below I_H
3. Blocks all current flow in the reverse direction until the maximum reverse voltage is exceeded.

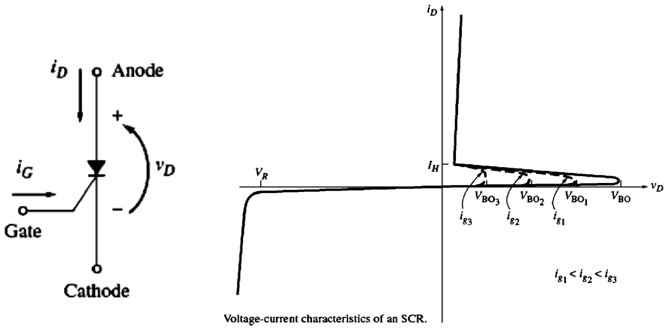




Voltage-current characteristic of a PNPN diode.

The Three-Wire Thyristor (or SCR)

- The most important member of the thyristor family is the three-wire thyristor.
- It is also known as **the *silicon controlled rectifier or SCR.***
- What makes an SCR especially useful in motor-control applications is that the break-over or turn-on voltage of the device can be adjusted by a current flowing into its gate lead. The larger the gate current, the lower V_{BO} becomes.

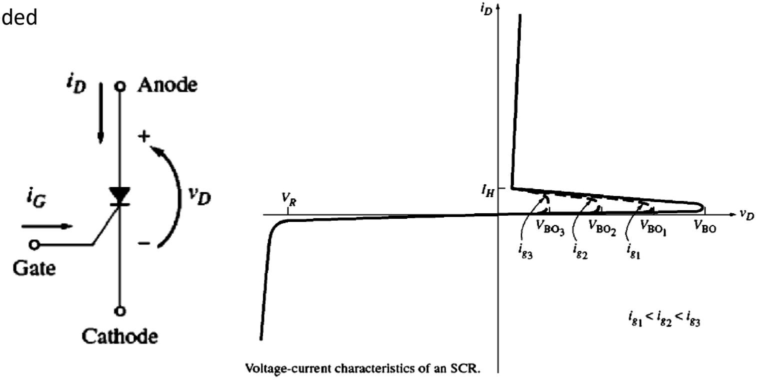


Voltage-current characteristics of an SCR.

The Three-Wire Thyristor (or SCR)

- **In summary, a SCR**

1. Turns on when the voltage $\mathbf{v_D}$ applied to it exceeds $\mathbf{V_{BO}}$
2. Has a breakover voltage $\mathbf{V_{BO}}$ whose level is controlled by the amount of gate current $\mathbf{i_G}$ present in the SCR
3. Turns off when the current $\mathbf{i_D}$ flowing through it drops below $\mathbf{I_H}$
4. Blocks all current flow in the reverse direction until the maximum reverse voltage is exceeded

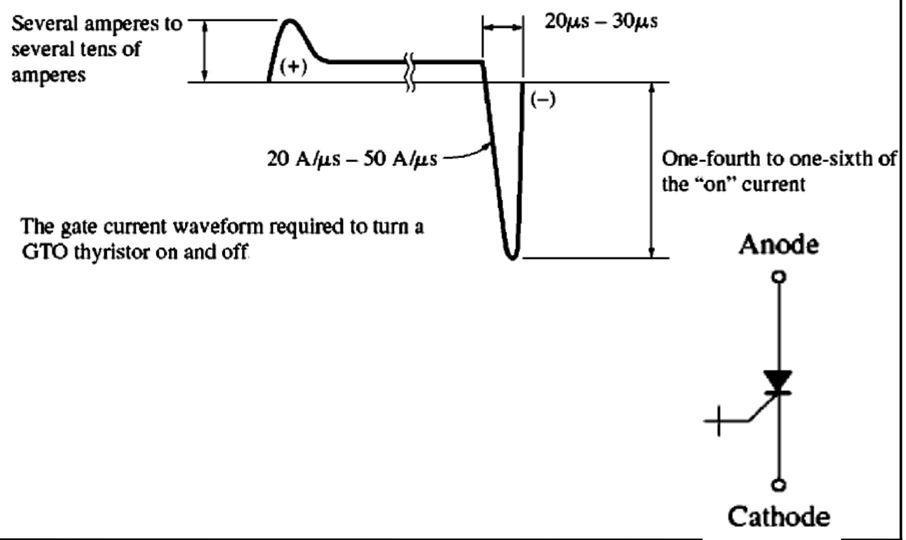


Voltage-current characteristics of an SCR.

The Gate Turnoff (GTO) Thyristor

- A GTO thyristor is SCR that can be **turned off** by a large enough **negative pulse** at its gate lead even if the current ***i_D*** *exceeds* ***I_H***.
- A GTO thyristor typically requires a larger gate current for turn-on than an ordinary SCR.
- To turn off the device, a large negative current pulse of 20- to 30-µs duration is required.
- The magnitude of the negative current pulse must be one-fourth to one-sixth that of the current flowing through the device.

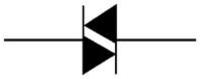
The Gate Turnoff (GTO) Thyristor

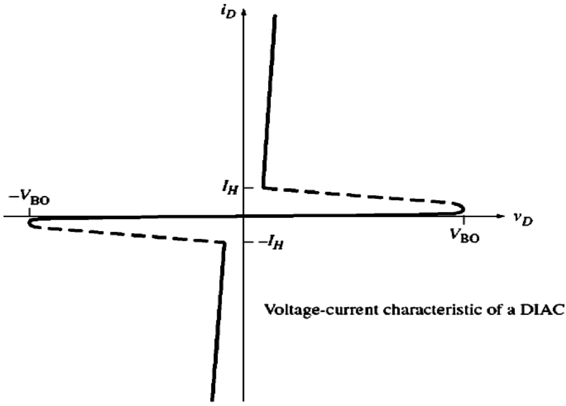


The gate current waveform required to turn a GTO thyristor on and off

The DIAC

- It can conduct in either direction once the break-over voltage is exceeded.
- It turns on when the applied voltage *in either direction exceeds* V_{BO}*. Once it is turned on, a* DIAC remains on until its current falls below I_H.

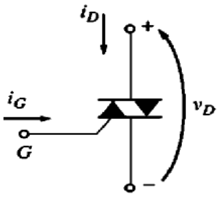


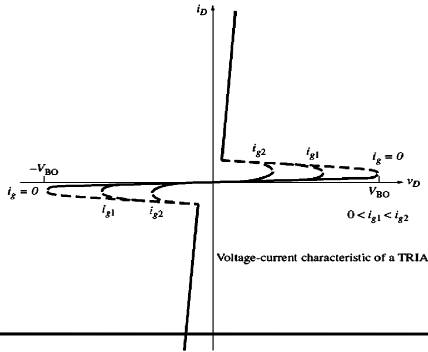


Voltage-current characteristic of a DIAC.

The TRIAC

- A TRIAC is a device that behaves like two SCRs connected back to back **with a common gate lead**. It can conduct in either direction once its break-over voltage is exceeded.
- The break-over voltage in a TRIAC decreases with increasing gate current in just the same manner as it does in an SCR, except that a TRIAC responds to either positive or negative pulses at its gate.
- Once it is turned on, a TRIAC remains on until its current falls below I_H.

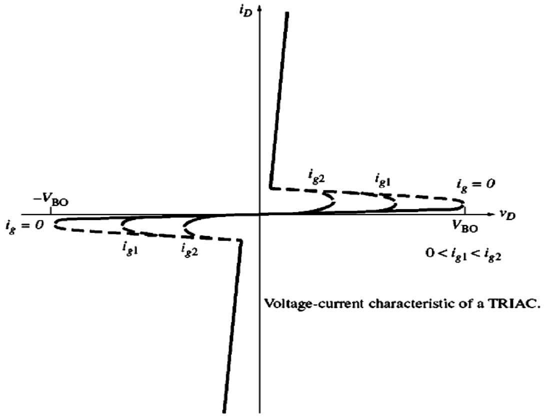




Voltage-current characteristic of a TRIAC.

The TRIAC

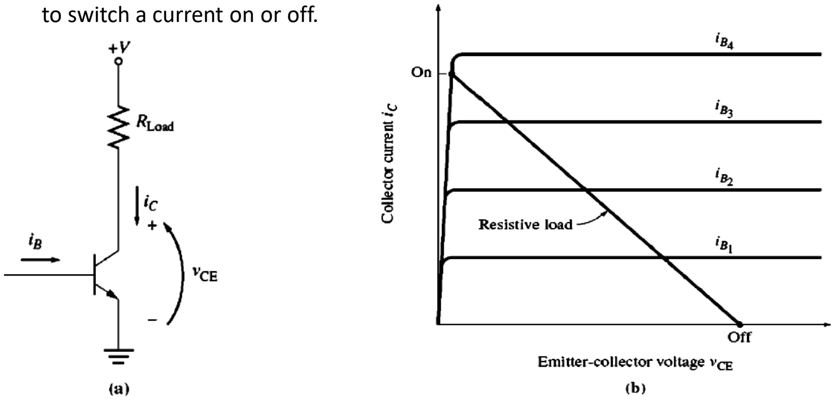
- TRIACs generally **switch more slowly** than SCRs, and are available only at **lower power ratings**. As a result, their use is largely restricted to low- to medium-power applications in 50- or 6O-Hz circuits, such as simple lighting circuits.



Voltage-current characteristic of a TRIAC.

The Power Transistor

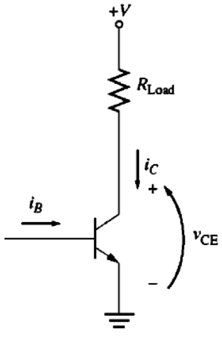
- The transistor is a device whose collector current i_e is directly proportional to its base current i_B over a very wide range of collector-to-emitter voltages (V_{CE}).
- Power transistors (PTRs) are commonly used in machinery-control applications to switch a current on or off.



(a) A transistor with a resistive load. (b) The voltage-current characteristic of this transistor and load.

The Power Transistor

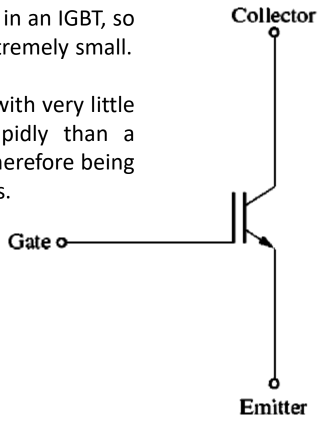
- Power transistors are most often used in inverter circuits. Their major drawback in switching applications is that large power transistors are relatively **slow in changing from the on to the off state and vice versa**, since a relatively large base current has to be applied or removed when they are turned on or off.



(a)

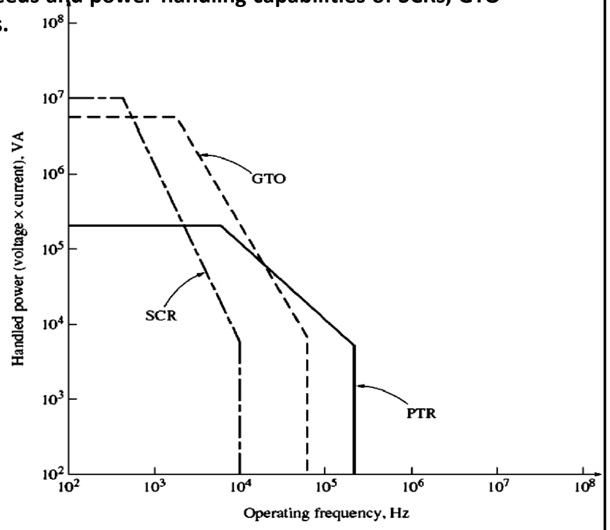
The Insulated-Gate Bipolar Transistor IGBT

- is similar to the power transistor, except that it is controlled by the voltage applied to a gate rather than the current flowing into the base as in the power transistor.
- The impedance of the control gate is very high in an IGBT, so the amount of current flowing in the gate is extremely small.
- Since the IGBT is controlled by a gate voltage with very little current flow, it can switch much more rapidly than a conventional power transistor can. IGBTs are therefore being used in high-power high-frequency applications.



Power Electronics Comparison

A comparison of the relative speeds and power-handling capabilities of SCRs, GTO thyristors, and power transistors.



The Basic Rectifier Circuits

- A rectifier circuit is a circuit that converts ac power to dc power.
- **The four most common rectifier circuits are**
  1. **The half-wave rectifier**
  2. **The full-wave bridge rectifier**
  3. **The three-phase half-wave rectifier**
  4. **The three-phase full-wave rectifier**

The Basic Rectifier Circuits

- ***The ripple factor:***

The ripple factor of the dc output is a measure of the smoothness of the dc voltage out of a rectifier circuit.

- The percentage of ripple in a dc power supply is defined as the ratio of the **rms** value of the ac components in the supply's voltage to the **dc** value of the voltage.

$$\boxed{r = \sqrt{\left(\frac{V_{\mathrm{rms}}}{V_{\mathrm{DC}}}\right)^2 - 1} \times 100\%}$$

- where $\mathbf{V_{rms}}$ is the rms value of the total output voltage from the rectifier and $\mathbf{V_{DC}}$ is the dc or average output voltage from the rectifier.

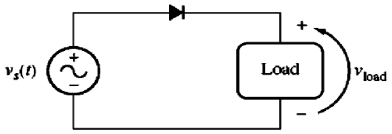
The Basic Rectifier Circuits

- The smaller the ripple factor in a power supply, the smoother the resulting dc waveform.
- The dc component of the output voltage $\mathbf{V_{DC}}$ is

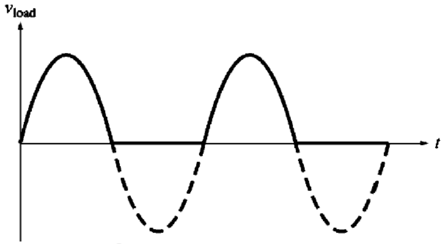
$$\boxed{V_{\mathrm{DC}} = \frac{1}{T}\int v_0(t)\, dt}$$

The Basic Rectifier Circuits
The Half-Wave Rectifier

- **A half-wave rectifier circuit.**



- **The output voltage of the rectifier circuit.**



$$v_{\text{load}}(t) = \begin{cases} V_M \sin \omega t & 0 < \omega t < \pi \\ 0 & \pi \le \omega t \le 2\pi \end{cases}$$

The Basic Rectifier Circuits
The Half-Wave Rectifier

- **The average voltage out of the rectifier is**

$$V_{\text{DC}} = V_{\text{avg}} = \frac{1}{T}\int_0^T v_{\text{load}}(t)\, dt$$

$$= \frac{\omega}{2\pi}\int_0^{\pi/\omega} V_M \sin \omega t\, dt$$

$$= \frac{\omega}{2\pi}\left(-\frac{V_M}{\omega}\cos \omega t\right)\Big|_0^{\pi/\omega}$$

$$= -\frac{V_M}{2\pi}[(-1) - (1)]$$

$$= \frac{V_M}{\pi}$$

$$\boxed{V_{\text{DC}} = \frac{V_M}{\pi}}$$

The Basic Rectifier Circuits

The Half-Wave Rectifier

- The rms value of the total voltage out of the rectifier is

$$
\begin{aligned}
V_{\text{rms}} &= \sqrt{\frac{1}{T}\int_0^T v_{\text{load}}^2(t)\,dt} \\
&= \sqrt{\frac{\omega}{2\pi}\int_0^{\pi/\omega} V_M^2 \sin^2 \omega t\,dt} \\
&= V_M\sqrt{\frac{\omega}{2\pi}\int_0^{\pi/\omega} \frac{1-\cos 2\omega t}{2}\,dt} \\
&= V_M\sqrt{\frac{\omega}{2\pi}\int_0^{\pi/\omega} \frac{1}{2}\,dt - \frac{\omega}{2\pi}\int_0^{\pi/\omega} \frac{1}{2}\cos 2\omega t\,dt} \\
&= V_M\sqrt{\left(\frac{\omega}{4\pi}t - \frac{1}{8\pi}\sin 2\omega t\right)\Bigg|_0^{\pi/\omega}} \\
&= V_M\sqrt{\left(\frac{1}{4} - \frac{1}{8\pi}\sin 2\pi\right) - \left(0 - \frac{1}{8\pi}\sin 0\right)} \\
&= \frac{V_M}{2}
\end{aligned}
$$

$$\boxed{V_{\text{rms}} = \frac{V_M}{2}}$$

The Basic Rectifier Circuits

The Half-Wave Rectifier

- The ripple factor of this rectifier circuit is

$$r = \sqrt{\left(\frac{V_M/2}{V_M/\pi}\right)^2 - 1} \times 100\%$$

$$\boxed{r = 121\%}$$

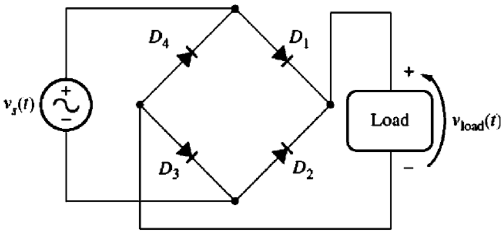
- The PIV on the diode equals

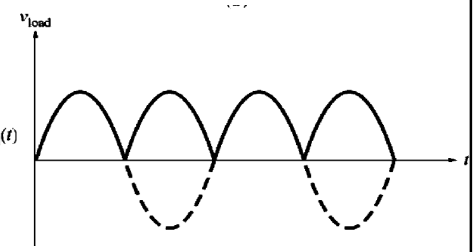
  PIV = V_m

The Basic Rectifier Circuits
The Full-Wave Rectifier

- In the full-wave bridge rectifier circuit; diodes **D1** and **D3** conduct on the **positive half-cycle** of the ac input, and diodes **D2** and **D4** conduct on the **negative half-cycle**.
- The output voltage from this circuit is smoother than the output voltage from the half-wave rectifier.
- The PIV on each diode equals

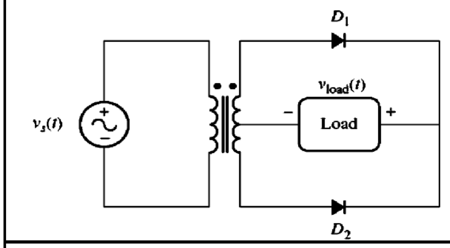
  PIV = V_m
- **A full-wave bridge rectifier circuit.**

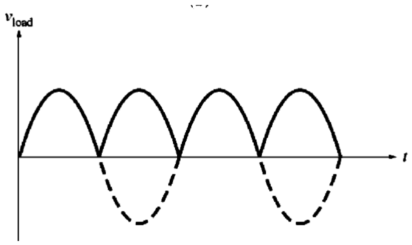




The Basic Rectifier Circuits
The Full-Wave Rectifier

- **Full-wave rectifier circuit using two diodes and a center-tapped transformer**
- The diode **D1** conducts on the **positive half-cycle** of the ac input with the current returning through the center tap of the transformer, and diode **D2** conducts on the **negative half-cycle** of the ac input with the current returning through the center tap of the transformer.
- The PIV on each diode equals: **PIV = V_m**





The Basic Rectifier Circuits
The Full-Wave Rectifier

- **Full-wave rectifier circuit with center-tapped transformer**

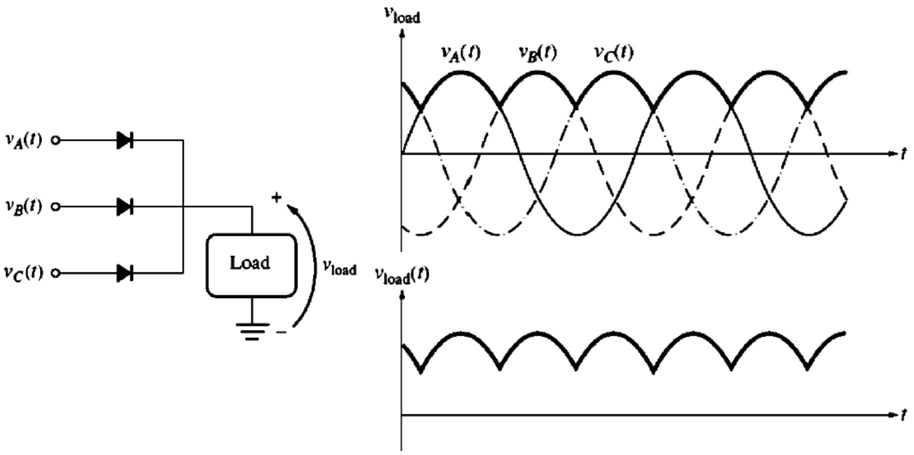
$$\boxed{V_{DC} = 2\frac{V_M}{\pi}}$$

$$\boxed{V_{rms} = \frac{V_M}{\sqrt{2}}}$$

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1} \times 100\%$$

$$\boxed{r = 48.2\%}$$

The Basic Rectifier Circuits
The Three-Phase Half-Wave Rectifier



The Basic Rectifier Circuits

The Three-Phase Half-Wave Rectifier

- The effect of having three diodes with their cathodes connected to a common point is that at any instant the diode with the largest voltage applied to it will conduct, and the other two diodes will be reverse-biased.
- This output voltage is even smoother than that of a full-wave bridge rectifier circuit.
- **Notice that the voltage at the output of the rectifier at any time is just the highest of the three input voltages at that moment.**

The Basic Rectifier Circuits

The Three-Phase Half-Wave Rectifier

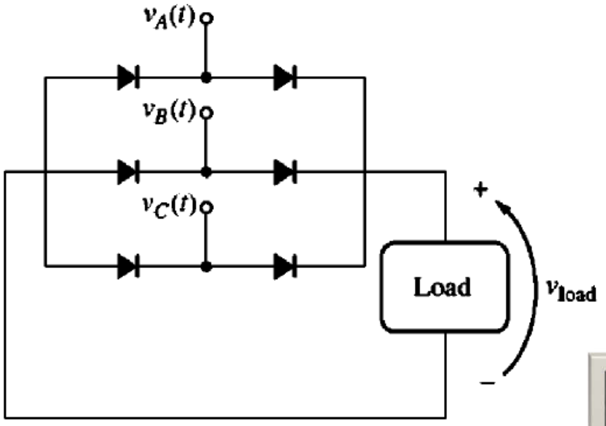
$$\boxed{V_{DC} = 3\sqrt{3}\,\frac{V_m}{2\pi}}$$

$$\boxed{V_{rms} = 0.840\ V_m}$$

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1} \times 100\%$$

$$\boxed{r = 18.3\%}$$

The Basic Rectifier Circuits
The Three-Phase Full-Wave Rectifier



$$\boxed{V_{DC} = 3\sqrt{3}\frac{V_m}{\pi}}$$

The Basic Rectifier Circuits
The Three-Phase Full-Wave Rectifier

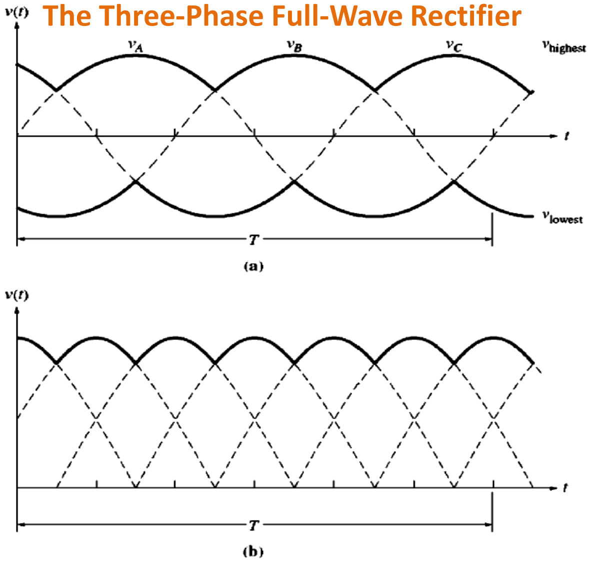
Basically, a circuit of this sort can be divided into two component parts:

1. One part of the circuit looks just like the three-phase half-wave rectifier, and it serves to connect the highest of the three phase voltages at any given instant to the load.

2. The other part of the circuit consists of three diodes oriented with their anodes connected to the load and their cathodes connected to the supply voltages. This arrangement connects the *lowest of the three supply voltages* to the load at any given time.

- The three-phase full-wave rectifier at all times **connects the *highest* of the three voltages to one end of the load** and always **connects the *lowest* of the three voltages to the other end of the load.**

The Basic Rectifier Circuits
The Three-Phase Full-Wave Rectifier



(a) The highest and lowest voltages in the three-phase full-wave rectifier.

(b) The resulting output voltage.

The Basic Rectifier Circuits
The Three-Phase Full-Wave Rectifier

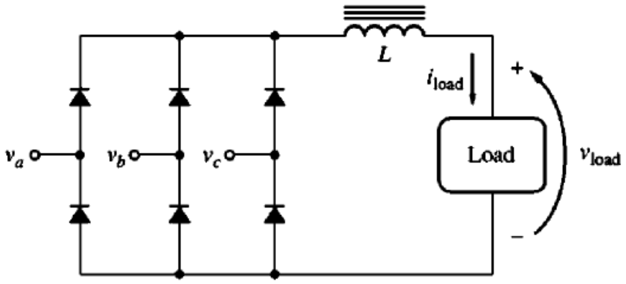
- The output of a three-phase full-wave rectifier is even smoother than the output of a three-phase half-wave rectifier. and the **ripple factor is only 4.2 percent.**

Filtering Rectifier Output

- The output of any of these rectifier circuits may be further smoothed by the use of **low-pass** filters to remove more of the ac frequency components from the output.
- Two types of elements are commonly used to smooth the rectifier's output:
  1. Capacitors connected across the lines to smooth ac voltage changes.
  2. Inductors connected in series with the line to smooth ac current changes.
- ***A common filter in rectifier circuits used with machines is a single series inductor, or choke.***

Filtering Rectifier Output

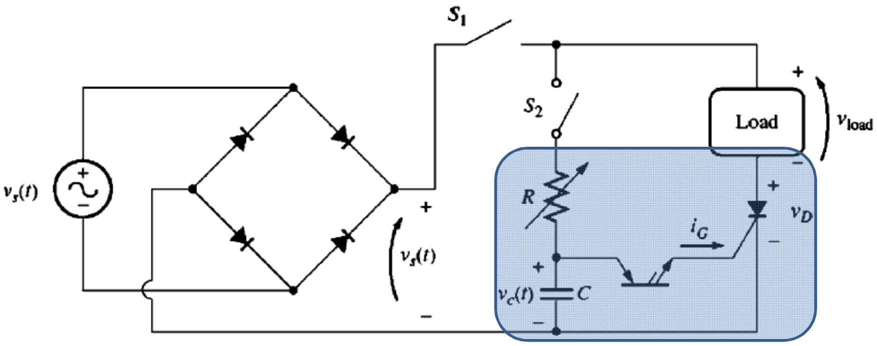
- A three-phase full-wave bridge circuit with an inductive filter for reducing output ripple.



Voltage Variation by AC Phase Control

- The level of voltage applied to a motor is one of the most common variables in motor-control applications.
- The SCR and the TRIAC provide a convenient technique for controlling the average voltage applied to a load by changing the phase angle at which the source voltage is applied to it.

Voltage Variation by AC Phase Control
AC Phase Control for a DC Load Driven from an AC Source



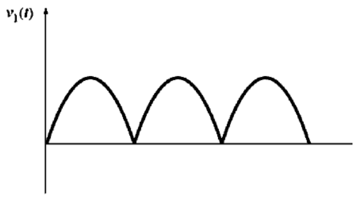
A circuit controlling the voltage to a dc load by phase angle control.

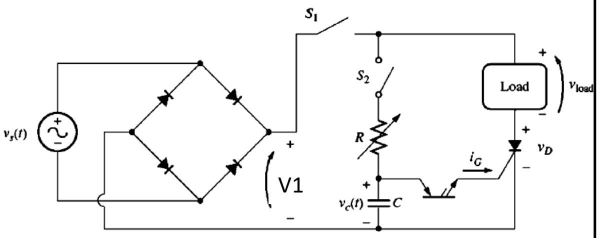
Relaxation Oscillator
OR Pulse Generator

Voltage Variation by AC Phase Control

AC Phase Control for a DC Load Driven from an AC Source

- The figure shows a voltage-phase-control circuit with a resistive dc load supplied by an ac source.
- The SCR in the circuit has a breakover voltage for i_G = 0 A *that is greater* than the highest voltage in the circuit, while the PNPN diode has a very low breakover voltage, perhaps 10 V or so.
- The full-wave bridge circuit ensures that the voltage applied to the SCR and the load will always be dc.
- If the switch S1 in the picture is open, then the voltage V_1 at the terminals of the rectifier will just be a full-wave rectified version of the input voltage



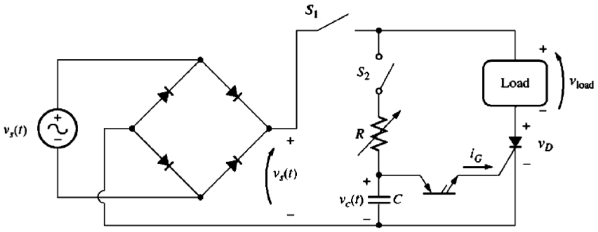


A circuit controlling the voltage to a dc load by phase angle control.

Voltage Variation by AC Phase Control

AC Phase Control for a DC Load Driven from an AC Source

- If switch S1 is shut but switch S2 is left open, then the SCR will always be off.
- This is true because the voltage out of the rectifier will never exceed VBO for the SCR.
- Since the SCR is always an open circuit, the current through it and the load, and hence the voltage on the load, will still be zero.

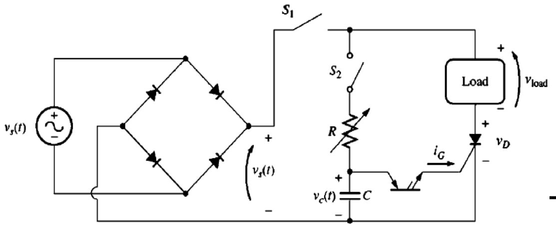


A circuit controlling the voltage to a dc load by phase angle control.

Voltage Variation by AC Phase Control
AC Phase Control for a DC Load Driven from an AC Source

Now suppose that switch S2 is closed.

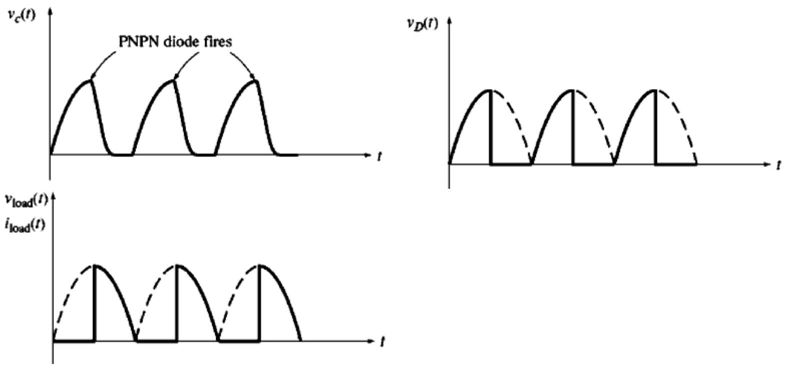
- At the beginning of the first half-cycle after the switch is closed, a voltage builds up across the *RC network, and the* capacitor begins to charge.
- During the time the capacitor is charging, the SCR is off, since the voltage applied to it has not exceeded *VBO As time passes, the capacitor* charges up to the breakover voltage of the PNPN diode, and the PNPN diode conducts.
- The current flow from the capacitor and the PNPN diode flows through the gate of the SCR, lowering *VBO for the SCR and turning it on.*
- *When the* SCR turns on, current flows through it and the load. This current flow continues for the rest of the half-cycle.
- The SCR turns off only when its current falls below the holding current (since *I_H is a few* mille-amperes, this does not occur until the extreme end of the half-cycle).



A circuit controlling the voltage to a dc load by phase angle control.

Voltage Variation by AC Phase Control
AC Phase Control for a DC Load Driven from an AC Source

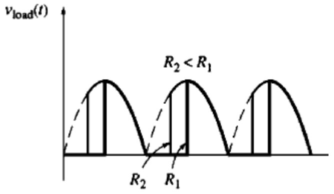
- At the beginning of the next half-cycle,
- The SCR is again off. The *RC circuit* again charges up over a finite period and triggers the PNPN diode.
- The PNPN diode once more sends a current to the gate of the SCR, turning it on. Once on, the SCR remains on for the rest of the cycle again.

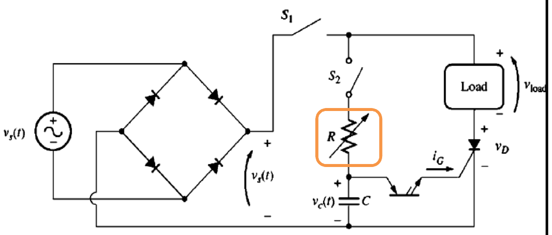


Voltage Variation by AC Phase Control
AC Phase Control for a DC Load Driven from an AC Source

Now for the critical question: How can the power supplied to this load be changed?

- Suppose the value of *R is decreased. Then at the beginning of each half*-cycle, the capacitor will charge more quickly, and the SCR will fire sooner.
- Since the SCR will be on for longer in the half-cycle, *more power will be supplied to the load.*
- ***The resistor R in this circuit controls the power flow to the* load in the circuit.**

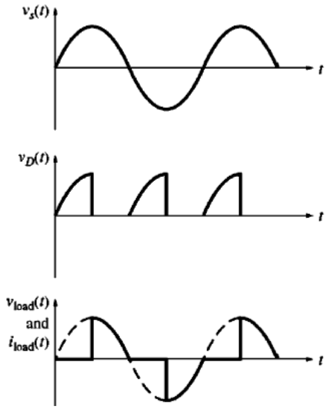


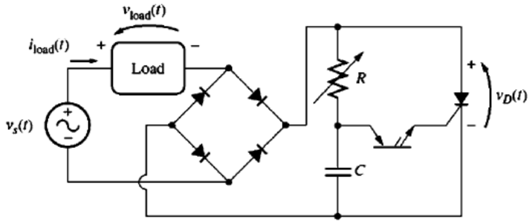


A circuit controlling the voltage to a dc load by phase angle control.

AC Phase Angle Control for an AC Load

- It is possible to control an ac load simply by moving the load from the dc side of the circuit to a point before the rectifiers.

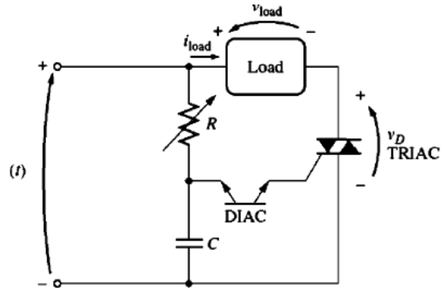




AC Phase Angle Control for an AC Load

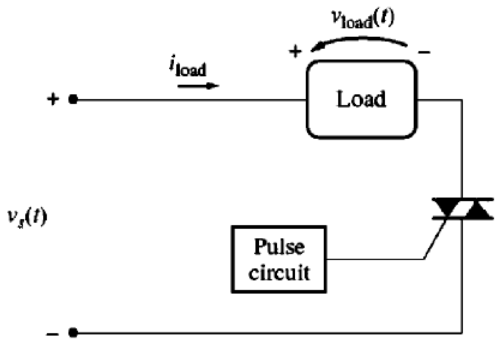
- However, there is a much easier way to make an ac power controller. If the same basic circuit is used with a DIAC in place of the PNPN diode and a TRIAC in place of the SCR, then the diode bridge circuit can be completely taken out of the circuit. Because both the DIAC and the TRIAC are two-way devices, they operate equally well on either half-cycle of the ac source.

- An ac phase angle controller using a DIAC and TRIAC.



AC Phase Angle Control for an AC Load

- An ac phase angle controller using a TRIAC triggered by a digital pulse circuit.



DC-To-DC Power Control- CHOPPERS

- The circuits which vary the voltage of a dc source to different DC voltage levels are called ***DC-to-DC conveners or choppers***

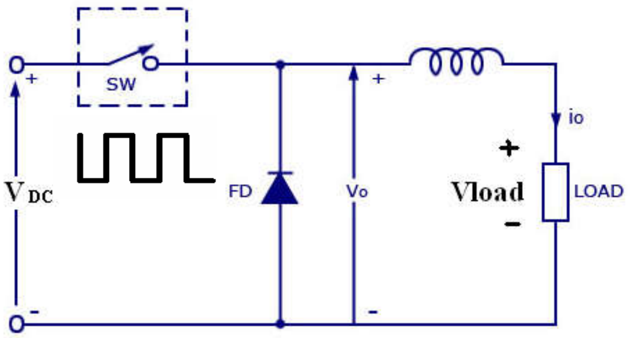
- ***Input:*** constant DC voltage.
- ***Output:*** Variable DC voltage.

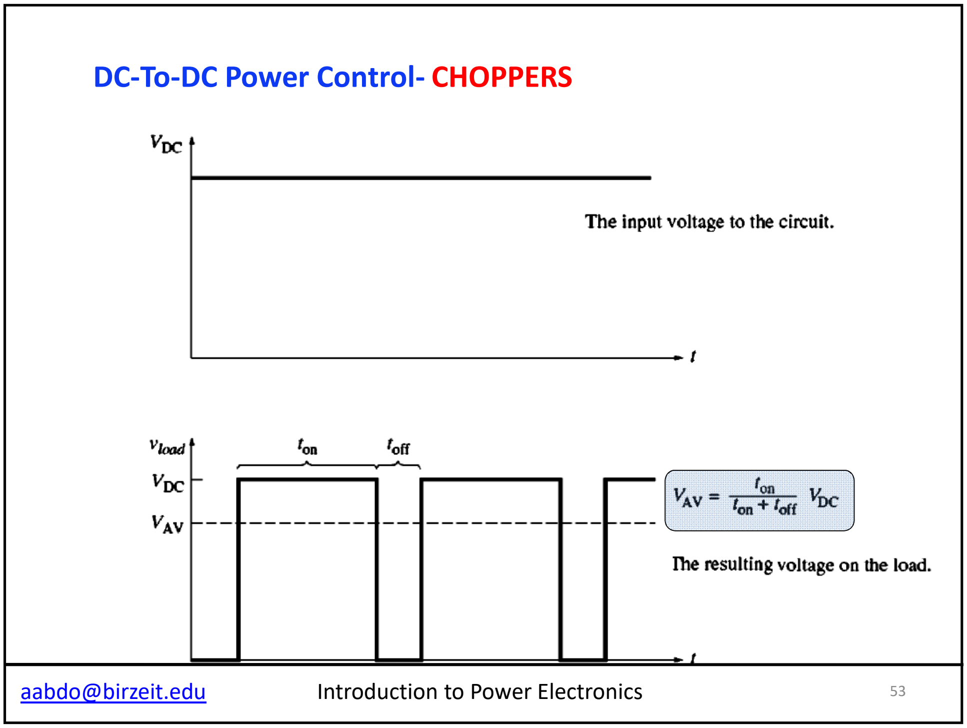
DC-To-DC Power Control- CHOPPERS

Step-down DC chopper circuit

Chopper Circuit

IGBT, SCR….





DC-To-DC Power Control- CHOPPERS

The input voltage to the circuit.

$$V_{AV} = \frac{t_{on}}{t_{on} + t_{off}} V_{DC}$$

The resulting voltage on the load.

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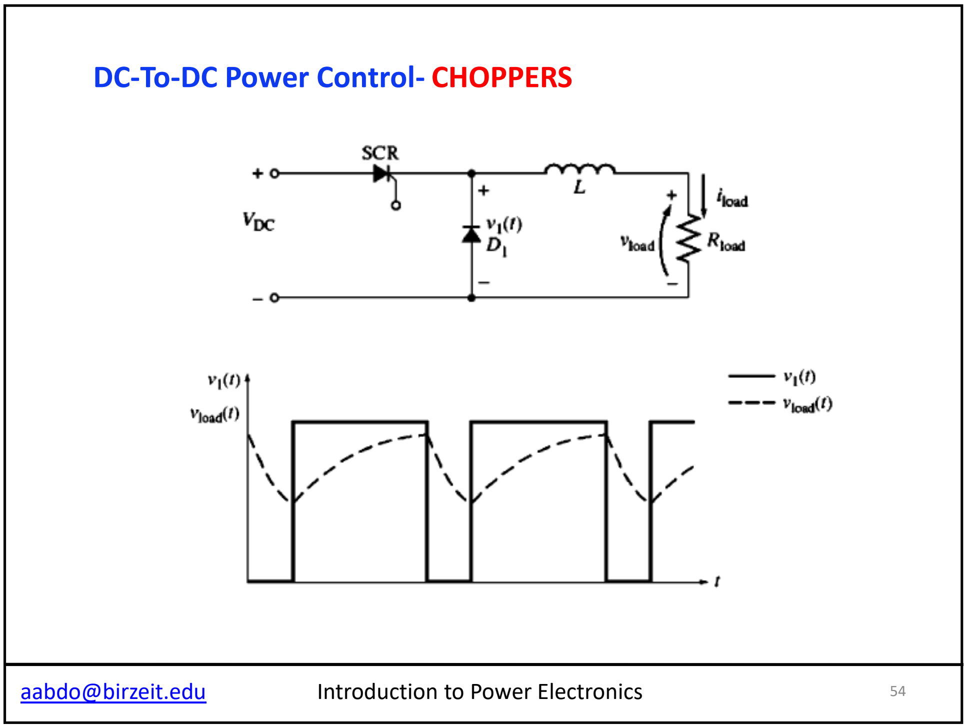

DC-To-DC Power Control- CHOPPERS

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Forced Commutation in Chopper Circuits

When SCRs are used in choppers, a forced-commutation circuit must be included to turn off the SCRs at the desired time.

The process of forcing an SCR to turn off at a desired time is known as *forced commutation.*

Most such forced-commutation circuits depend for their turnoff voltage on a charged capacitor. Two basic versions of capacitor commutation are examined in this brief overview:

1. Series-capacitor commutation circuits
2. Parallel-capacitor commutation circuits

Inverter

- ***Inverters*** are used in the conversion of ac power at one frequency to ac power at another frequency by means of solid-state electronics.
- Traditionally there have been two approaches to static ac frequency conversion:
  - The *cycloconverter.*
  - *The rectifier-inverter.*
- The ***cycloconverter*** is a device for directly converting ac power at one frequency to ac power at another frequency.
- The ***rectifier-inverter*** first converts ac power to dc power and then converts the dc power to ac power again at a different frequency.

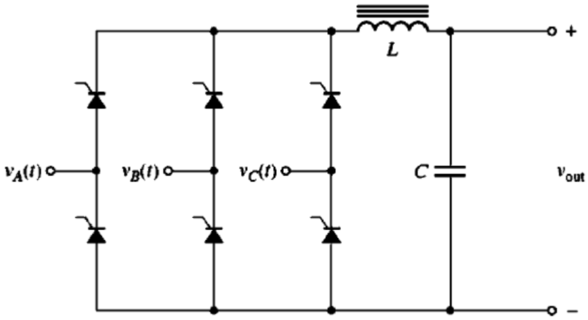
Inverters

- A rectifier-inverter is divided into two parts:

1. **A rectifier to produce dc power.**
2. **An inverter to produce ac power from the dc power.**

Part 1:

- **A three-phase rectifier circuit using SCRs to provide control of the dc output voltage level**



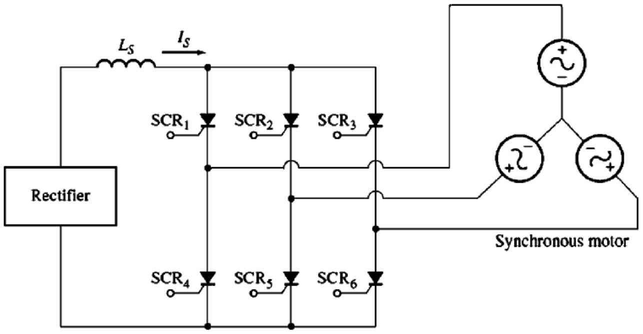
Inverters

- A rectifier-inverter is divided into two parts:

1. **A rectifier to produce dc power.**
2. **An inverter to produce ac power from the dc power.**

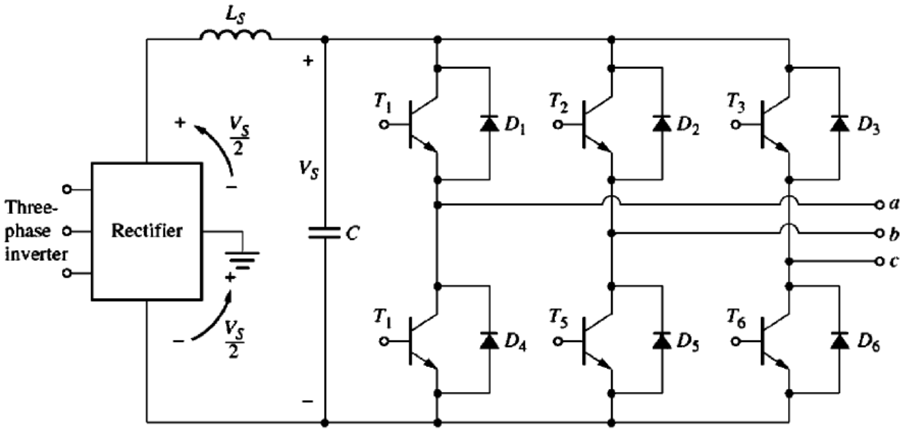
Part 2:

- **An external commutation inverter.**



Inverters

- A three-phase voltage source inverter using power transistors.



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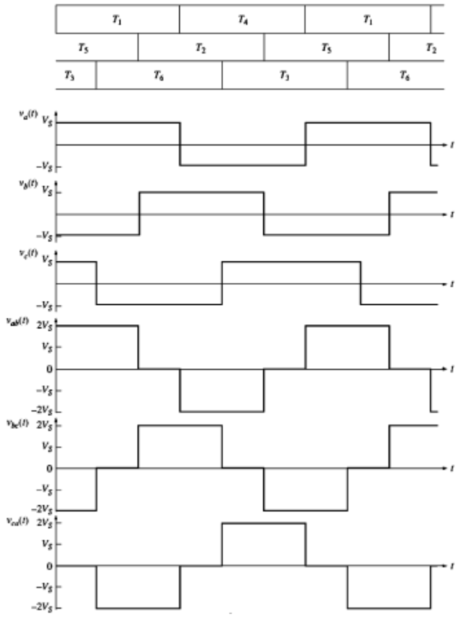


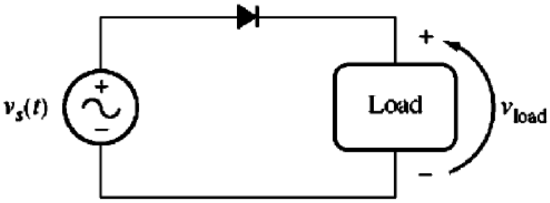
Example 1

A single phase half wave uncontrolled rectifier has purely resistive load R=10 ohm, the peak supply voltage is 170V, and the supply frequency is 60Hz. Draw the circuit then determine

1. The average voltage of the rectifier
2. The rms voltage
3. The average load current

Example 1

Solution



1. $V_{DC} = V_m/\pi$
   $= 170/\pi$
   $= 54.14V$

2. $V_{rms} = V_m/2$
   $= 170/2$
   $= 85V$

3. $I_{DC} = V_{DC} / R$
   $= 54.14/10$
   $= 5.414A$

Example 2

A three phase half wave uncontrolled rectifier has purely resistive load R=10 ohm, the peak supply voltage is 170V, and the supply frequency is 60Hz. Draw the circuit then determine

1. The average voltage of the rectifier
2. The rms voltage
3. The average load current

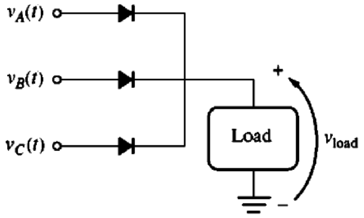
Inverters

Solution:

1. **$V_{DC} = 3(3)^{\wedge 0.5} * V_m/2\pi$**
   $= 40V$

2. **$V_{rms} = 0.840\ V_m$**
   $= 142.8\ V$

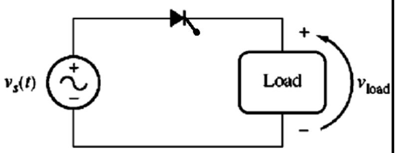
3. **$I_{DC} = V_{DC} / R$**
   $= 140/10$
   $= 14A$



Example 3

A single phase half wave controlled rectifier is operated from $120V_{rms}$.60Hz supply and the resistive load is R = 10 ohm. If the firing angle was $\alpha=30^o$. Calculate the rms and average output current.

Example 3



Solution:

V_{DC} = $(V_m/2\pi)(1+\cos\alpha)$

V_m = $V_{rms}X(2)^{\wedge 0.5}$
= $120\ X(2)^{\wedge 0.5}$
= 170.4 V

V_{DC} = $(170.4/2\pi)(1+\cos 30)$
= 50.631V
I_{DC} = V_{DC} / R
= 50.631/10 =5.631A

$$Vrms = \frac{Vm}{2}\sqrt{\frac{1}{\pi}(\pi-\alpha+\sin\ (\alpha)}$$

Example 4

A step down (Buck) converter (DC Chopper) has the following specification

V_{in} = 84V
V_o =18 V
f=50KHz
R_L = 1.5 ohm

- Draw the circuit and calculate
- The duty cycle
- The Switch on time
- The average output current

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