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Solutions Manual

to accompany

Heat Transfer

tenth edition

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Chapter 1

1–1

$$\Delta T = \frac{(3000)(0.025)}{(0.2)(0.6)} = 625°\text{C}$$

1–2

$$\frac{q}{A} = \frac{(0.035)(85)}{0.13} = 22.885 \ \text{J/s} \cdot \text{m}^2$$

$$= 82{,}386 \ \text{J/h} \cdot \text{m}^2$$

1–3

$$q = -kA\frac{dT}{dx} \qquad \frac{dx}{\pi r^2} = -k\frac{dT}{q}$$

$r = ax + b;\ x = 0;\ r = 0.0375$
$x = 0.3,\ r = 0.0625$
$r = 0.0833x + 0.0375$

$$\int \frac{dx}{\pi(0.0833x + 0.0375)^2} = -\frac{(204)(93 - 540)}{q}$$

$$\frac{-1}{\pi(0.0833)}\left(\frac{1}{0.0833x + 0.0375}\right)\Bigg|_{x=0}^{x=0.3} = -\frac{(204)(-447)}{q}$$

$$q = 2238 \text{ W}$$

1–4

$$\frac{q}{A} = \frac{(0.78)(375 - 85)}{0.15} = 1508 \ \text{W/m}^2$$

1–5

$$A = \pi r^2 \qquad q = -k4\pi r^2\frac{dT}{dr} \qquad q = \frac{-4\pi k(T_0 - T_i)}{\frac{1}{r_i} - \frac{1}{r_0}}$$

$$q = \frac{-4\pi(2 \times 10^{-4})(21 + 196)}{\frac{1}{0.26} - \frac{1}{0.285}} = 1.617 \text{ W}$$

$$\text{mass evaporated} = \frac{1.617}{199{,}000} = 0.813 \times 10^{-5} \ \text{kg/s}$$

$$= 0.702 \ \text{kg/day}$$

1–7

$$\frac{q}{L}=\frac{T_i-T_\infty}{\frac{\ln(r_0/r_i)}{2\pi k}+\frac{1}{h\pi d_0}}=\frac{30+20}{\frac{\ln(\frac{30}{25})}{2\pi(7)(10^{-3})}+\frac{1}{9\pi(0.6)}}=11.89\ \text{W/m}$$

1-8

Like many kinds of homespun advice, this is bad advice. *All* types of heat transfer; conduction, convection, and radiation vary directly with area. The surface area of the head is much less than that of the other portion of the body and thus will lose less heat. This may be shown experimentally by comparing exposure in cold weather wearing heavy clothing and no hat, to that wearing a heavy hat and only undergarments!

1-9

$$\frac{q}{A} = \frac{(0.161)(200-100)}{0.05} = 322 \ \text{W/m}^2$$

1-10

$$\begin{aligned}\Delta x &= \frac{kA\Delta T}{q} \\ &= \frac{\left(10\times10^{-3}\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\right)(500)}{400\ \frac{\text{W}}{\text{m}^2}} \\ &= 0.0125\ \text{m} = 1.25\ \text{cm}\end{aligned}$$

1-11

$$2.158 = (2.7)(4\pi)\left(\frac{13}{12}\right)(0.3048)^2\,\Delta T$$

$$\Delta T = 0.632^\circ\text{C}$$

1-12

$$\frac{q}{A} = (5.669\times10^{-8})[(1073)^4-(523)^4] = 70.9\ \frac{\text{kW}}{\text{m}^2}$$

1-13

$$\frac{q}{A} = (5.669\times10^{-8})[(1373)^4-(698)^4] = 188\ \frac{\text{kW}}{\text{m}^2}$$

1-14

$$q = (5.669\times10^{-8})(4\pi)(0.35)^2[(300)^4-(70)^4] = 704.8\ \text{W}$$

1-15

a. $q = (5.669\times10^{-8})[(773)^4-(373)^4] = 1.914\times10^4\ \text{W/m}^2$

b. $q = (5.669\times10^{-8})[(773)^4-(T_p)^4]$

$= (5.669\times10^{-8})[(T_p)^4-(373)^4]$

$T_p = 641\ \text{K}$

$q = 8474\cdot3\ \text{W/m}^2$

Reduced by 44.3%

1-16

$q = hA(T_w - T_{fluid})$

From Table 1–2 $\quad h = 3500 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q = (3500)\pi dL(40) = (3500)\pi(0.025)(3)(40) = 32987 \text{ W}$

$q = mc_p \Delta T_{fluid}$

$32{,}987 \text{ W} = (0.5 \text{ kg/s})(4180 \text{ J/kg} \cdot °\text{C})\Delta T$

$\Delta T = 15.78°\text{C}$

1-17

$h_{fg} = 2257 \text{ kJ/kg}$

$q = \dot{m}h_{fg} = (3.78 \text{ kg/hr})(2257 \text{ kJ/kg}) = 8531 \ \dfrac{\text{kJ}}{\text{hr}} = 2.37 \ \dfrac{\text{kJ}}{\text{s}} = 2.37 \text{ kW}$

From Table 1–2 $\quad h \sim 7500 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q = hA(T_w - T_{fluid})$

$2370 \text{ W} = (7500)(0.3)^2(T_w - 100)$

$T_w \approx 96.5°\text{C}$

1-18

$q = hA\Delta T$

$3 \times 10^4 \ \dfrac{\text{Btu}}{\text{hr} \cdot \text{ft}^2} = h(232 - 212)°\text{F}$

$h = 1500 \ \dfrac{\text{Btu}}{\text{hr} \cdot \text{ft}^2} = 8517 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

1-19

$q = \sigma\varepsilon A[(T_1)^4 - (T_2)^4]$

$2000 \text{ W} = (5.669 \times 10^{-8})(0.85)(0.006)(3)[(T_1)^4 - (298)^4]$

$T_1 = 1233 \text{ K}$

1-20

$\dfrac{q}{A} = \sigma T^4 = (5.669 \times 10^{-8})(1000 + 273)^4 = 1.489 \times 10^5 \ \dfrac{\text{W}}{\text{m}^2}$

1-21

$\dfrac{q}{A} = \sigma T^4$

$54 \times 10^6 = (5.669 \times 10^{-8})T^4$

$T = 5556 \text{ K}$

1-22

$$q = \sigma \varepsilon A_1 (T_1^{\,4} - T_2^{\,4}) = (5.669 \times 10^{-8})(0.6)(4\pi)(0.04)^2(473^4 - 293^4) = 29.19 \text{ W}$$

1-23

$$q = kA\frac{\Delta T}{\Delta x} = hA(T_0 - T_\infty)$$
$$\frac{(1.4)(315-41)}{0.025} = h(41-38)$$
$$h = 5114 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

1-25

$$q = (1.6)\frac{100 - T_{w_2}}{0.4} = 10(T_{w_2} - 10)$$
$$T_{w_2} = 35.7^\circ\text{C}$$
$$q = 10(35.7-10) = 257 \ \frac{\text{W}}{\text{m}^2}$$

1-26

From Table 1–2 $\quad h = 4.5 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$ for $\Delta T = 30^\circ\text{C}$

$$q = hA\Delta T = (4.5)(0.3)^2(30) = 12.15 \text{ W}$$

Conduction

$$q = kA\frac{\Delta T}{\Delta x}$$
$$k \text{ for air } = 0.03 \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$$
$$q = \frac{(0.03)(0.3)^2(30)}{0.025} = 3.24 \text{ W}$$

1-27

$$\Delta T = \frac{(1500)\left(\frac{0.25}{12}\right)}{25} = 1.25^\circ\text{F}$$
$$T = 100 - 1.25 = 98.75^\circ\text{F}$$

1-28

$$700 = (11)(T_w - 30)$$
$$T_w = 93.6^\circ\text{C}$$

1-29

$q = q_{conv} + q_{rad}$
$q_{conv} = hA(T_w - T_\infty)$

From Table 1–2 $\quad h = 180 \dfrac{W}{m^2 \cdot °C}$

$q_{conv} = (180)\pi(0.05)(1)(200 - 30) = 4807 \dfrac{W}{m}$ length

$q_{rad} = \sigma\varepsilon A_1(T_1^{\,4} - T_2^{\,4})$

$= (5.669 \times 10^{-8})(0.7)\pi(0.05)(1)(473^4 - 283^4)$

$= 272 \dfrac{W}{m}$ length

$q_{total} = 4807 + 272 = 5079 \dfrac{W}{m}$

Most heat transfer is by convection.

1-30

$q = q_{conv} + q_{rad}$
$q_{conv} = hA(T_w - T_\infty)$

From Table 1–2 $\quad h = 4.5 \dfrac{W}{m^2 \cdot °C}$

$q_{conv} = (4.5)(0.3)^2(50 - 20)$ (2 sides)

$= 24.3$ W

$q_{rad} = \sigma\varepsilon A_1(T_1^{\,4} - T_2^{\,4}) = (5.669 \times 10^{-8})(0.8)(0.3^2)(323^4 - 293^4)$ (2 sides)

$= 28.7$ W

$q_{total} = 24.3\text{ W} + 28.7\text{ W} = 53\text{ W}$

Convection and radiation are about the same magnitude.

1-31

$q = q_{conv} + q_{rad} = 0$ (insulated)
$q_{conv} = hA(T_w - T_\infty)$

From Table 1–2 $\quad h = 12 \dfrac{W}{m^2 \cdot °C}$

$q_{rad} = \sigma\varepsilon A_1(T_1^{\,4} - T_2^{\,4}),\ \varepsilon = 1.0,\ T_2 = 35°C = 308\text{ K}$

$0 = hA_1(T_1 - T_\infty) + \sigma\varepsilon A_1(T_1^{\,4} - T_2^{\,4})$

$0 = (12)(T_1 - 273) + (5.669 \times 10^{-8})(1.0)(T_1^{\,4} - 308^4)$

Solution by iteration:

$T_1 = T_w = 285\text{ K} = 12°C$

1-32

$$(100)(353 - T_{w_1}) = (5.669 \times 10^{-8})(T_{w_1}{}^4 - T_{w_2}{}^4) = 15(T_{w_2} - 293)$$

$$15(T_{w_2} - 293) - (5.669 \times 10^{-8})[(397 - 0.15T_{w_2})^4 - (T_{w_2})^4] = 0 = f(T_{w_2})$$

T_{w_2}	$f(T_{w_2})$
320	158.41
350	907.22
310	–77.03
313.3	0.058

$$T_{w_1} = 397 - (0.15)(313.3) = 350 \text{ K}$$

1-36

$$h = 4.5 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}} \ \text{(plate)}$$

$$h = 6.5 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}} \ \text{(cylinder)}$$

$$T_\infty = 20^\circ\text{C} = 293 \text{ K}$$

$$hA(T - T_\infty) = \sigma\varepsilon A(T^4 - T_\infty{}^4)$$

<u>Plate</u>

$$(4.5)(T - 293) = (5.668 \times 10^{-8})(T^4 - 293^4)$$

T = no realistic value ($T = 247$ K, heat gained)

<u>Cylinder</u>

$$(6.5)(T - 293) = (5.668 \times 10^{-8})(T^4 - 293^4)$$

$$T = 320 \text{ K} = 47^\circ\text{C}$$

1-37

The woman is probably correct. Her perceived comfort is based on both radiation and convection exchange with the surroundings. Even though a fan does not blow cool air on her from the refrigerator, her body will radiate to the cold interior and thereby contribute to her feeling of "coolness."

1-38

This is an old story. All things being equal, hot water does not freeze faster than cold water. The only explanation for the observed faster cooling is that the refrigerator might be a non-self defrost model which accumulated an ice layer on the freezing coils. Then, when the hot water tray was placed on the ice layer, it melted and reduced the thermal insulation between the cooling coil and the ice tray.

1-39

As in problem 1–36, it must be observed that a person's comfort depends on total heat exchange with the surroundings by both radiation and convection. In the winter the walls of the room will presumably be cooler than the room air and increase the heat loss from the bodies. In the summer the walls are probably hotter than the room air temperature and thereby increase the heat gain or reduce the heat loss from the people in the room.

1-40

$q = q_{\text{conv}} + q_{\text{rad}}$

$q_{\text{conv}} = hA(T_w - T_\infty) = (2)\pi(1)(6)(78-68) = 377 \text{ Btu/hr}$

For $T_2 = 45°\text{F} = 505°\text{R}$

$$q_{\text{rad}} = \sigma\varepsilon A_1(T_1^4 - T_2^4)$$
$$= (0.1714\times10^{-8})(0.9)\pi(1)(6)(538^4 - 505^4)$$
$$= 544 \text{ Btu/hr}$$

$q_{\text{total}} = 377 + 544 = 921 \text{ Btu/hr}$

For $T_2 = 80°\text{F} = 540°\text{R}$

$q_{\text{rad}} = (0.1714\times10^{-8})(0.9)\pi(1)(6)(538^4 - 540^4) = -36.4 \text{ Btu/hr}$

$q_{\text{total}} = 377 - 36.4 = 340.6 \text{ Btu/hr}$

Conclusion: Radiation plays a very important role in "thermal comfort."

1-41

$T_i = 0°\text{C} = 273 \text{ K} \qquad \varepsilon_{\text{ice}} = 0.95$

$A = (12)(40) = 480 \text{ m}^2 \qquad T_s = 25°\text{C} = 298 \text{ K} \qquad T_a = 22°\text{C}$

$q_{\text{rad}} = \sigma\varepsilon A_1(T_s^4 - T_i^4) = (5.668\times10^{-8})(0.95)(480)(298^4 - 273^4) = 60262 \text{ W}$

$q_{\text{conv}} = hA(T_a - T_i) = (10)(480)(22-0) = 105{,}600 \text{ W}$

$q_{\text{total}} = 60{,}262 + 105{,}600 = 165{,}862 \text{ W}$

For ice $i_{fg} = 80 \text{ cal/g} = 3.348\times10^5 \text{ J/kg}$

$$\text{Mass rate melted } = \frac{165{,}862}{3.348\times10^5} = 0.495 \text{ kg/sec}$$

density of ice $\sim 1000 \text{ kg/m}^3$

$$\text{volume rate melted } = \frac{0.495}{1000} = 4.95\times10^{-4} \text{ m}^3/\text{s}$$

$$\text{Volume} = A\times thk = (480)(0.003) = 1.44 \text{ m}^3$$

$$\text{Time to melt} = \frac{1.44 \text{ m}^3}{4.95\times10^{-4} \text{ m}^3/\text{s}} = 2909 \text{ sec} = 0.808 \text{ hr}$$

1-42

The price of fuel and electric energy varies widely with time of year and location throughout the world, so individual answers can differ substantially for this problem.

1-43

$k_{glass\ wool} = 0.038$ thickness = 0.15 m

$A = 144 + (4)(5)(12) = 384\ m^2$

T (inside building surface) = -10 + 30 = 20°C

q lost (without insulation) $= hA\Delta T = (13)(384)(30) = 149{,}760$ W

q lost (with insulation) $= A\Delta T/[\Delta x/k + 1/h]$

$= (384)(30)/[0.15/0.038 + 1/13] = 2862$ W

Energy saving by installing insulation = 146,897 W

This number must be combined with the energy costs obtained in Problem 1-42 to obtain the cost saving per hour (or per day, etc.

1-44

This problem is quite open-ended and the answers will strongly depend on the assumptions cot/bunk materials etc.

Chapter 2

2–1

$$q = \frac{T_i - T_0}{\left.\frac{\Delta x}{kA}\right)_w + \left.\frac{\Delta x}{kA}\right)_{\text{ins}}} \qquad 1830 = \frac{1300 - 30}{\frac{0.02}{1.3} + \frac{\Delta x}{0.35}}$$

$\Delta x = 0.238$ m

2–2

Assume Linear variation: $k = k_0(1 + \beta T)$

$$q = -\frac{k_0 A}{\Delta x}\left[T_3 - T_1 + \frac{\beta}{2}(T_3^2 - T_1^2)\right] = -\frac{k_0 A}{\Delta x/2}\left[T_2 - T_1 + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$$

$T_3 = 95°\text{C},\ T_2 = 62°\text{C},\ T_1 = 35°\text{C},\ \Delta x = 0.025$

$$2\left[62 - 35 + \frac{\beta}{2}(62^2 - 35^2)\right] = \left[95 - 35 + \frac{\beta}{2}(95^2 - 35^2)\right]$$

$\beta = -4.68 \times 10^{-3}$

$$1000 = q = \frac{k_0(0.1)}{0.025}\left[95 - 35 - \frac{4.68 \times 10^{-3}}{2}(95^2 - 35^2)\right]$$

$k_0 = 5.988$

$$k = 5.988(1 - 4.68 \times 10^{-3}\ \text{T})\ \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

2–3

$$\frac{q}{A} = \frac{560}{\frac{0.025}{386} + \frac{3.2\times10^{-3}}{0.16} + \frac{0.05}{0.038}} = 419\ \frac{\text{W}}{\text{m}^2}$$

2–4

$$R = \frac{\Delta x}{kA}$$

$$R_A = \frac{0.025}{(150)(0.1)} = 1.667 \times 10^{-3}$$

$$R_B = \frac{0.075}{(30)(0.05)} = 0.05$$

$$R_C = \frac{0.05}{(50)(0.1)} = 0.01$$

$$R_D = \frac{0.075}{(70)(0.05)} = 0.02143$$

$$R = R_A + R_C + \frac{1}{\frac{1}{R_B} + \frac{1}{R_D}} = 2.667 \times 10^{-2}$$

$$q = \frac{\Delta T}{R} = \frac{370 - 66}{2.667 \times 10^{-2}} = 11{,}400 \text{ W}$$

2–5

$$\frac{44{,}000}{A} = \frac{250 - 35}{\frac{0.05}{386} + \frac{0.025}{0.038}} \qquad A = 134.7 \text{ m}^2$$

2–6

$$\frac{q}{A} = \frac{75}{\frac{0.10}{0.69} + \frac{0.025}{0.05}} = 38.76 \text{ W/m}^2$$

2–7

$$\frac{q}{A} = \frac{\Delta T}{R}$$

$$\frac{300}{A} = \frac{175 - 80}{\frac{0.04}{386} + \frac{0.015}{0.038}}$$

$$A = 1.247 \text{ m}^2$$

2–8

Assume one directional no heat sources

$$q = -kA\frac{dT}{dx} = -k_0A[1 + \beta T^2]\frac{dT}{dx} = -k_0A\frac{dT}{dx} - k_0\beta AT^2\frac{dT}{dx}$$

Integrating: $q\Delta x = -k_0A\int_{T_1}^{T_2} dT - Ak_0\beta\int_{T_1}^{T_2} T^2 dT$

$$q = -\frac{k_0A}{\Delta x}\left[(T_2 - T_1) + \frac{\beta}{3}(T_2^3 - T_1^3)\right]$$

2–9

$$\frac{1}{h_i A_i} = \frac{1}{(1500)\pi(0.03)} = 0.00709 \qquad r_0 = 0.015 + 0.002 = 0.017$$

$$\frac{\ln(r_0/r_i)}{2\pi k} = \frac{\ln(0.017/0.015)}{2\pi(46)} 0.000433$$

$$\frac{1}{h_0 A_0} = \frac{1}{(197)\pi(0.034)} = 0.0475$$

$$\sum R = 0.05502 \ \frac{^\circ\mathrm{C}\cdot \mathrm{m}}{\mathrm{W}}$$

$$\frac{q}{L} = \frac{223 - 57}{0.05502} = 3017 \ \mathrm{W/m}$$

2–10

$$\frac{\partial T}{\partial x} = 300x - 30$$

$$\frac{\partial^2 T}{\partial x^2} = 300 = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad \text{heating up}$$

$$\frac{\partial T}{\partial x} = -30 \quad \text{at} \quad x = 0$$

$$\frac{\partial T}{\partial x} = 60 \quad \text{at} \quad x = 0.3$$

$$\frac{q}{A} = -(0.04)(-30) = +1.2 \ \frac{\mathrm{W}}{\mathrm{m}^2} \ \text{at} \ x = 0$$

$$\frac{q}{A} = -(0.04)(60) = -2.4 \ \frac{\mathrm{W}}{\mathrm{m}^2} \ \text{at} \ x = 0.3$$

2-11

d = 0.000025; k = 16; L = 0.8m; h = 500; $T_\infty = 20°C$; $T_0 = T_L = 200°C$

$m = [(500)(4)/(16)(0.000025)]^{1/2} = 2236$

$\theta'_0 = \theta'_L = 200 - 20 = 180$

$q = -2kA(\partial\theta'/\partial x)_0 = -2kA(-m\theta'_0)$

$= (2)(16)\pi(0.000025)^2(2236)(180)/4 = 0.0063 \text{ W}$

2-12

$$R_{Cu} = \frac{0.02}{374} = 5.35 \times 10^{-5}$$
$$R_{As} = \frac{0.003}{0.166} = 0.0181$$
$$R_{Fi} = \frac{0.06}{0.038} = 1.579$$
$$\frac{q}{A} = \frac{500}{\sum R} = 313 \ \frac{\text{W}}{\text{m}^2}$$

2-13

$$R_c = \frac{\frac{6}{12}}{(1.2)(0.5778)} = 0.721$$
$$R_f = \frac{\frac{2}{12}}{(0.038)(0.5778)} = 7.59$$
$$R_g = \frac{\frac{0.375}{12}}{(0.05)(0.5778)} = 1.082$$
$$R_i = \frac{1}{2.0} = 0.5 \qquad R_0 = \frac{1}{7} = 0.143$$
$$\frac{q}{A} = \frac{72-20}{\sum R} = 5.18 \ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2}$$
$$U = \frac{1}{\sum R} = 0.0996$$

2-14

$$R_{ss} = \frac{\Delta x}{k} = \frac{0.004}{16} = 0.0025$$
$$R_{\text{overall}} = \frac{1}{U} = \frac{1}{120} = 0.00833$$
$$\frac{\Delta T_{ss}}{\Delta T_{\text{overall}}} = \frac{R_{ss}}{R_{\text{overall}}} = \frac{0.0025}{0.00833} = 0.3$$
$$\Delta T_{ss} = (0.3)(60) = 18°\text{C}$$

2-15

Ice at 0°C $\quad \rho = 999.8 \ \text{kg/m}^3$

$V = (0.25)(0.4)(1.0) = 0.1 \ \text{m}^3$

$m = 100 \ \text{kg}$

$q = (100)(330 \times 10^3) = 3.3 \times 10^7 \ \text{J}$

$A_i = (2)(0.25)(0.4) + (2)(0.4)(1.0) + (2)(0.25)(1.0) = 1.5 \ \text{m}^2$

$A_0 = (2)(0.35)(0.5) + (2)(0.5)(1.1) + (2)(0.35)(1.1) = 2.22 \ \text{m}^2$

$A_m = 1.86 \ \text{m}^2$

$$R_s = \frac{\Delta x}{kA} = \frac{0.05}{(0.033)(1.86)} = 0.8146$$

$$R_0 = \frac{1}{hA_0} = 0.045$$

$R = 0.8596$

$$\frac{Q}{\Delta T} = \frac{3.3 \times 10^7}{\Delta T} = \frac{25 - 0}{0.8596}$$

$$\Delta \tau = 1.135 \times 10^6 \ \text{sec}$$
$$= 315 \ \text{hr}$$
$$= 13 \ \text{days}$$

2-16

$q \text{ (no ins.)} = hA(T_w - T_\infty) = (25)(4\pi)(0.5)^2(120 - 15) = 8247 \ \text{W}$

$$k_{\text{foam}} = \frac{18 \ \text{mW}}{\text{m} \cdot {}^\circ\text{C}}$$

$$q = \frac{4\pi k(T_i - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}} = h4\pi r_0^2(T_0 - T_\infty)$$

$$\frac{(0.018)(120 - 40)}{\frac{1}{0.5} - \frac{1}{r_0}} = (25) r_0^2 (40 - 15)$$

$r_0 = 0.5023 \ \text{m}$

$\text{thk} = r_0 - r_i = 0.023 \ \text{m}$

$q \text{ (w/ ins)} = (25)(4\pi)(0.5023)^2(40 - 15) = 1982 \ \text{W}$

2-17

$$q = \frac{4\pi k(T_i - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}} \qquad k = 204 \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$$

$$= \frac{(4)\pi(204)(100 - 50)}{\frac{1}{0.02} - \frac{1}{0.04}} = 5127 \ \text{W}$$

2-18

$$q = \frac{\Delta T}{\sum R}$$

$$R_{alum} = \frac{\frac{1}{0.02} - \frac{1}{0.04}}{4\pi(204)} = 9.752 \times 10^{-3}$$

$$R_{ins} = \frac{\frac{1}{0.04} - \frac{1}{0.05}}{4\pi(0.05)} = 7.958$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{(20)(4\pi)(0.05)^2} = 1.592$$

$$q = \frac{100 - 10}{0.00975 + 7.958 + 1.592} = 9.41 \text{ W}$$

2-19

$$d_i = 2.90 \text{ in.} \quad d_0 = 3.50 \text{ in.} \quad k = 43 \ \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

$$R_{steel} = \frac{\ln(3.5/2.9)}{(2\pi)(43)(1)} = 6.96 \times 10^{-4}$$

$$R_{ins} = \frac{\ln(5.5/3.5)}{(2\pi)(0.06)(1)} = 1.1999$$

$$R_{conv} = \frac{1}{hA_0} = \frac{1}{(10)\pi(5.5)(0.0254)} = 0.2278$$

$$R_{tot} = 1.427$$

$$q = \frac{\Delta T}{R} = \frac{250 - 20}{1.427} = 161.1 \ \frac{\text{W}}{\text{m}}$$

2-20

$$k_A = 0.166 \qquad k_f = 0.0485 \ \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

$$\frac{315 - T_i}{\frac{\ln(31.4/25)}{0.166}} = \frac{T_i - 38}{\frac{\ln(56.4/31.4)}{0.0485}}$$

$$0.7283(315 - T_i) = 0.0828(T_i - 38)$$

$$T_i = 286.7°\text{C}$$

2-21

$$q_r = -k4\pi r^2 \frac{dT}{dr}$$

$$q_r \int_{r_i}^{r_0} \frac{1}{r^2} dr = -k4\pi \int_{T_i}^{T_0} dT$$

$$q_r\left(\frac{1}{r_0} - \frac{1}{r_i}\right) = -4\pi k(T_0 - T_i)$$

$$q = \frac{-4\pi k(T_0 - T_i)}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right)}$$

$$R_{th} = \frac{\left(\frac{1}{r_i} - \frac{1}{r_0}\right)}{4\pi k}$$

2-22

$$r_i = 0.5 \text{ mm} = 5\times 10^{-4} \text{ m}$$

$$r_0 = \frac{k}{h} = 5\times 10^{-4} + 2\times 10^{-4} = 7\times 10^{-4}$$

$$k = (7\times 10^{-4})(120) = 0.084 \ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

$$q(\text{bare wire}) = \pi(0.001)(120)(400 - 40) = 135.7 \ \text{W/m}$$

$$q(\text{insulated}) = (135.7)(0.25) = 33.93 \ \text{W/m}$$

$$q = \frac{400 - 40}{\frac{\ln(r_0/5\times 10^{-4})}{2\pi(0.084)} + \frac{1}{\pi(2)(120)r_0}} = 33.93$$

By iteration: $r_0 = 135$ mm

thickness = 134.5 mm

2-23

$$R_i = \frac{(1)(12)}{(30)\pi(2.067)} = 6.16\times 10^{-2}$$

$$R_p = \frac{\ln(2.375/2.067)}{2\pi(27)} = 8.188\times 10^{-4}$$

$$R_i = \frac{\ln(3.375/2.375)}{2\pi(0.023)} = 2.432$$

$$R_0 = \frac{(1)(12)}{2\pi(3.375)} = 5.659\times 10^{-1}$$

$$\frac{q}{L} = \frac{320 - 70}{\sum R} = 81.7 \ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}}$$

2-24

$$q = -k4\pi r^2 \frac{dT}{dr} = \frac{k4\pi(T_0 - T_i)}{\frac{1}{r_0} - \frac{1}{r_i}} = k4\pi r_0^2 (T_0 - T_\infty)$$

$$q = \frac{T_i - T_\infty}{\frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_0}\right) + \frac{1}{4\pi r_0^2 h}}$$

Take $\frac{dq}{dr_0} = 0$

Result is: $r_0 = 2\frac{k}{h}$

2-25

M_w at 90% full $= (0.9)(970)\pi(0.8)^2(2) = 3511$ kg

at 2°C/hr $q = \frac{(3511)(4191)(2)}{3600} = 8174$ W

$A = 2\pi(0.8)^2 + \pi(0.8)(2) = 9.048\ \text{m}^2$

Fiberglass boards with $k = 40$ mW/m · °C

$$\Delta x = \frac{(40 \times 10^{-3})(9.048)(80 - 20)}{8174} = 2.66 \times 10^{-3}\ \text{m}$$

2-26

For 1 m length

$$R(\text{pipe}) = \frac{\ln(9.1/8)}{2\pi(47)} = 4.363 \times 10^{-4}$$

$$R(\text{ins}(1)) = \frac{\ln(27.1/9.1)}{2\pi(0.5)} = 0.3474$$

$$R(\text{ins}(2)) = \frac{\ln(35.1/27.1)}{2\pi(0.25)} = 0.8246$$

$R(\text{tot}) = 1.172$

$$q = \frac{\Delta T}{R} = \frac{250 - 20}{1.172} = 196.2\ \frac{\text{W}}{\text{m}}$$

2-27

Fiberglass	$k = 0.038$	$\Delta x = 1.2\ \text{cm} \times 2$	
Asbestos	$k = 0.154$	$\Delta x = 8.0$ cm	
brick	$k = 0.69$	$\Delta x = 10.0$ cm	$h = 15\ \frac{\text{W}}{\text{m} \cdot °\text{C}} \times 2$

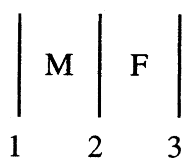
$$U = \frac{1}{\frac{2}{12} + \frac{(2)(0.012)}{0.038} + \frac{0.08}{0.154} + \frac{0.1}{0.69}} = 0.684\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

2-28

$$R = \frac{1}{k}$$

	k	R
Fiberglass	0.046	21.74
Urethane	0.018	55.6
Mineral Wool	0.091	11.0
Calcium Silicate	0.058	17.2

2-29



$T_1 = 1000°\text{C}$

$T_2 = 400°\text{C}$

$T_3 = 55°\text{C}$

$k_m = 90 \ \dfrac{\text{mW}}{\text{m} \cdot °\text{C}}$ $\qquad h = 15 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$k_F = 42 \ \dfrac{\text{mW}}{\text{m} \cdot °\text{C}}$ $\qquad T_\infty = 40°\text{C}$

$$\frac{q}{A} = h(T_3 - T_\infty) = (15)(55-40) = 225 \ \frac{\text{W}}{\text{m}^2}$$

$$\frac{q}{A} = k_m \frac{(1000-400)}{\Delta x_m} \qquad \Delta x_m = 0.24 \text{ m}$$

$$\frac{q}{A} = k_F \frac{(400-55)}{\Delta x_F} \qquad \Delta x_F = 0.0644 \text{ m}$$

2-30

Uniformly distributed heat sources

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0 \quad T=T_1 \quad \text{at} \quad x=-L$$
$$T=T_2 \quad \text{at} \quad x=+L$$

$$T=-\frac{\dot{q}x^2}{2k}+c_1x+c_2$$

$$T_1=-\frac{\dot{q}L^2}{2k}-c_1L+c_2$$

$$T_2=-\frac{\dot{q}L^2}{2k}+c_1L+c_2$$

$$T=\frac{\dot{q}}{2k}(L^2-x^2)+\frac{T_2-T_1}{2L}x+\frac{T_1+T_2}{2}$$

2-31

$r_1=2.5 \qquad r_2=3.5 \qquad r_3=6.5$

$$R_1=\frac{\ln(r_2/r_1)}{0.22(2\pi)}=\frac{\ln(3.5/2.5)}{0.22(2\pi)}=0.2433$$

$$R_2=\frac{\ln(r_3/r_2)}{0.06(2\pi)}=1.642$$

$$R_\infty=\frac{1}{hA}=\frac{1}{(60)\pi(2)(0.065)}=0.0408$$

$$\sum R=1.9262\ \frac{°\text{C}\cdot\text{m}}{\text{W}}$$

$$q=\frac{L\Delta T}{R}=\frac{(20)(400-15)}{1.9262}=3997\ \text{W}$$

2-32

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0 \qquad \dot{q}=\dot{q}_w[1+\beta(T-T_w)]$$

$T=T_w$ at $x=\pm L$

General solution

$$T-T_w=c_1\left[\cos\left(\sqrt{\frac{\dot{q}_w\beta}{k}}x\right)\right]+c_2\left[\sin\left(\sqrt{\frac{\dot{q}_w\beta}{k}}x\right)\right]-\frac{1}{\beta}$$

From boundary conditions

$$c_2=0 \qquad c_1=\frac{1}{\beta\cos\left(\sqrt{\frac{\dot{q}_w\beta}{k}}L\right)}$$

$$T-T_w=\frac{\cos\sqrt{\frac{\dot{q}_w\beta}{k}}x}{\beta\cos\sqrt{\frac{\dot{q}_w\beta}{k}}L}$$

2-33

k = 43; r_1 = 0.015; r_2 = 0.04; T_0 = 250°C; T_∞ = 35°C; h = 43

L = 0.025

L_c = 0.0255

r_{2c} = 0.0405

r_{2c}/r_1 = 2.7

$$L_c^{3/2}(h/kA_m)^{1/2} = 0.825$$

Fig. 2-12 η_f = 0.59

$q = (45)(2)\pi(0.0405^2 - 0.015^2)(250 - 35)(0.59) = 5.08$ W

2-34

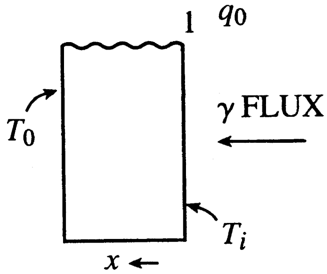
$\dot{q}=0.30\ \frac{\text{MW}}{\text{m}^3}$ Same as half of wall 15 cm thick with convection on each side.

$$T_0-T_w=\frac{\dot{q}L^2}{2k}=\frac{(0.30\times10^6)(0.060)^2}{(2)(21)}=25.7°\text{C}$$

$$\dot{q}LA=hA(T_w-T_\infty) \qquad T_w-T_\infty=\frac{(0.30\times10^6)(0.060)}{570}=31.6°\text{C}$$

$$T_0=T_{max}=93+25.7+31.6=150.3°\text{C}$$

2-35



$\dot{q}_x = \dot{q}_0 e^{-ax}$ $\qquad \dfrac{d^2T}{dx^2} = \dfrac{-\dot{q}_0 e^{-ax}}{k}$

$T = c_1 + c_2 x - \dfrac{\dot{q}_0}{a^2 k} e^{-ax}$

Boundary conditions:

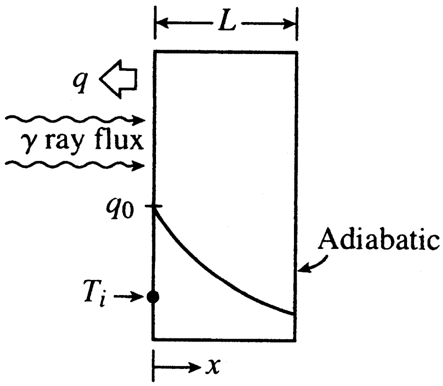
(1) $T = T_i$ at $x = 0$

(2) $T = T_0$ at $x = L$

$c_1 = T_i + \dfrac{\dot{q}_0}{a^2 k}$ $\qquad c_2 = \dfrac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})}{L}$

$$T = T_i + \frac{\dot{q}_0}{a^2 k} + \frac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})x}{L} + \frac{-\dot{q}_0}{a^2 k} e^{-ax}$$

2-36



$\dot{q} = \dot{q}_0 e^{-ax}$ $\qquad\qquad \dfrac{d^2T}{dx^2} = \dfrac{-\dot{q}_0}{k} e^{-ax}$

$$T = c_1 + c_2 x - \frac{\dot{q}_0}{a^2 k} e^{-ax}$$

Boundary conditions:

(1) at $x = L$ $\quad \dfrac{dT}{dx} = 0$ (adiabatic)

(2) at $x = 0$ $\quad T = T_i$

$c_1 = T_i + \dfrac{\dot{q}_0}{a^2 k}$ $\qquad\qquad c_2 = -\dfrac{\dot{q}_0}{ak} e^{-aL}$

$$T = T_i + \frac{\dot{q}_0}{a^2 k} - \frac{q_0 e^{-aL}}{ak} x - \frac{\dot{q}_0}{a^2 k} e^{-ax}$$

2-37

$$T - T_w = c_1 \cos\sqrt{q_w \frac{\beta}{k}}x + c_2 \sin\sqrt{q_w \frac{\beta}{\text{k}}}x - \frac{1}{\beta}$$

$T = T_1$ at $x = \pm L \qquad c_2 = 0$

$$-kA\frac{\partial T}{\partial x}\bigg|_{x=L} = hA(T_1 - T_\infty) \qquad c_1 = \frac{T_1 - T_w + \frac{1}{\beta}}{\cos\sqrt{q_w \frac{\beta}{k}}L}$$

$$T = T_w + \frac{T_1 - T_w + \frac{1}{\beta}}{\cos\sqrt{q_w \frac{\beta}{k}}L}\cos\sqrt{q_w \frac{\beta}{k}}x - \frac{1}{\beta}$$

Solving for $\frac{\partial T}{\partial x}\bigg|_{x=L}$ and substituting in above equation:

$$T_1 = \frac{1}{1-h}\left[T_w - \frac{1}{\beta} - \frac{hT_\infty}{k\sqrt{q_w \frac{\beta}{k}}\tan\sqrt{q_w \frac{\beta}{k}}L}\right]$$

2-38

$$\dot{q}AL = hPL(T_w - T_\infty)$$

$$(35.3\times 10^6)(0.025)^2 = (4000)(4)(0.025)(T_w - 20)$$

$$T_w = 75.16°\text{C}$$

2-39

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_0\cos(ax)}{k} = 0 \qquad \frac{dT}{dx} = \frac{-\dot{q}_0}{ak}\sin(ax) + c_1$$

$T = T_w$ at $x = \pm L \qquad \therefore c_1 = 0 \qquad T = \frac{\dot{q}_0}{a^2k}\cos(ax) + c_1x + c_2$

$$T_w = \frac{\dot{q}_0}{a^2k}\cos(aL) + c_2 \qquad T - T_w = \frac{\dot{q}_0}{a^2k}[\cos(ax) - \cos(aL)]$$

$$\frac{q}{A} = -2k\frac{dT}{dx}\bigg|_{x=L} = -2k\frac{\dot{q}_0}{ak}[-\sin(aL)] = \frac{2k\dot{q}_0}{ak}\sin(aL)$$

2-40

$$k = 0.0124\ \frac{\text{W}}{\text{cm}\cdot{}^\circ\text{C}} = 1.24\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

$$\rho = 1.5\times10^{-3}\ \Omega\cdot\text{cm}$$

$$R = (1.5\times10^{-3})\left(\frac{3}{1}\right) = 4.5\times10^{-3}$$

$$q = I^2R = (50)^2(4.5\times10^{-3}) = 11.25\ \text{W}$$

$$q = \frac{q}{V} = \frac{11.25}{3\times10^{-6}} = 3.75\ \frac{\text{MW}}{\text{m}^3} \qquad T = -\frac{\dot{q}}{2k}x^2 + c_1x + c_2$$

$L = 1.5\ \text{cm} = 0.015\ \text{m}$ $\qquad T = 300$ at $x = -0.015$

$T = 100$ at $x = +0.015$

$300 - 100 = c_1(-0.015 - 0.015) \qquad c_1 = -6667$

$$300 = \frac{(-3.75\times10^6)(0.015)^2}{(2)(1.24)} - (6667)(-0.015) + c_2 \qquad c_2 = 540.2$$

at $x = 0 \qquad T = c_2 = 540.2^\circ\text{C}$

2-41

$k =$ constant $\dot{q} = \dot{q}_0$ at $x = 0$ Assume one directional with no heat storage.

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \qquad T - T_1 = (T_2 - T_1)(c_1 + c_2x^2 + c_3x^3)$$

$$T = T_1 + (T_2 - T_1)(c_1 + c_2x^2 + c_3x^3)$$

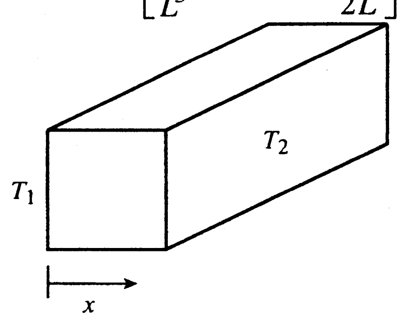
$$\frac{d^2T}{dx^2} = (T_2 - T_1)(2c_2 + 3c_3x)$$

$T = T_1$ at $x = 0 \qquad T = T_2$ at $x = L \qquad \dot{q} = \dot{q}_0$ at $x = 0$

$$q = -k(T_2 - T_1)(2c_2 + 3c_3x)$$

so: $c_1 = 0 \qquad c_2 = -\dfrac{\dot{q}_0}{2k(T_2 - T_1)} \qquad c_3 = \dfrac{1}{L^3} + \dfrac{\dot{q}_0}{2kL(T_2 - T_1)}$

$$\dot{q}_x = \dot{q}_0 - \left[\frac{3k}{L^3}(T_2 - T_1) + \frac{3\dot{q}_0}{2L}\right]x$$



2-42

$k = 2.5$ $\quad h_1 = 75$ (left) $\quad h_2 = 50$ (right)

$T_{1\infty} = 50°C \quad T_{2\infty} = 30°C$

$$T = -\frac{\dot{q}x^2}{2k} + c_1 x + c_2$$

$T = T_1$ at $x = -0.04$; $T = T_2$ at $x = +0.04$

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}x}{k} + c_1$$

$$T = T_{max} = 300 \text{ at } x = c_1 \frac{k}{\dot{q}} \qquad (1)$$

$$h_1(T_{1\infty} - T_1) = -k\frac{\partial T}{\partial x}\bigg|_{x=-0.04} \qquad (2)$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=+0.04} = h_2(T_2 - T_{2\infty}) \qquad (3)$$

$$300 = -\frac{\dot{q}}{2k}\left[c_1\frac{k}{\dot{q}}\right]^2 + c_1\left[c_1\frac{k}{\dot{q}}\right] + c_2 \qquad (1)$$

$$75\left[50 + \frac{\dot{q}}{2k}(0.04)^2 + c_1(0.04) - c_2\right] = -k\left(\frac{+\dot{q}(0.04)}{2k}\right) \qquad (2)$$

$$-k\left[\frac{-\dot{q}(0.04)}{2k}\right] = 50\left[\frac{-\dot{q}(0.04)^2}{2k} + c_1(0.04) + c_2 - 30\right] \qquad (3)$$

3 Equations, 3 Unknowns, c_1, c_2, $\dot{q}$

Solve for $\dot{q} = 2.46 \times 10^5 \text{ W/m}^3$

2-43

d = 0.05; L = 0.02; ΔT = 200°C; k = 204

shim 0.025 mm = 0.001 inch

Table 2-3 ; $1/h_c = 3.52 \times 10^{-4}$

Bars: $\Delta x/kA = (4)(0.02)/\pi(0.05)^2 = 10.19$

Joint: $1/h_cA = 0.179$

$\Sigma R = (2)(10.19) + 0.179 = 20.559$

$\Delta T_{joint} = (200)(0.179/20.559) = 1.74°C$

2-44

Use solution from Prob. 2–28

$T = T_0$ at $x = 0$

$$T_0 = \frac{\dot{q}}{2k}L^2 + \frac{T_1 + T_2}{2} = \frac{(5 \times 10^5)(0.015)^2}{(2)(16)} + \frac{220 + 45}{2} = 136°\text{C}$$

2-45

Use solution from Prob. 2–28

$$T_0 = \frac{\dot{q}}{2k}L^2 + \frac{T_1 + T_2}{2} = \frac{(500 \times 10^6)(0.005)^2}{(2)(20)} + \frac{100 + 200}{2} = 462.5°\text{C}$$

2-46

Behaves like half a plate having a thickness of 8 mm. Max Temp is at $x = 0$

$$T_0 = \frac{\dot{q}L^2}{2k} + T_w$$

$L = 0.004$ m $\quad T_w = 100°\text{C}$

$$T_0 = \frac{(200\times 10^6)(0.004)}{(2)(25)} + 100 = 164°\text{C}$$

2-47

$$\dot{q}\pi r^2 L = \frac{E^2}{R} = \frac{(10)^2\pi(0.16)^2}{(7\times 10^{-6})(30)}$$

$$\dot{q} = \frac{(10)^2\pi(0.16)^2}{(7\times 10^{-6})(30)(0.3)\pi(1.6\times 10^{-3})^2} = 1587\ \frac{\text{MW}}{\text{m}^3}$$

$$T_0 = \frac{\dot{q}r^2}{4k} + T_w = \frac{(1.587\times 10^9)(1.6\times 10^{-3})^2}{(4)(22.5)} + 93 = 138.1°\text{C}$$

2-48

$$(200)^2(0.099) = (5700)\pi(3\times 10^{-3})(1)(T_w - 93)$$

$T_w = 166.7°\text{C} \qquad T_0 = 16.6 + 166.7 = 183.3°\text{C}$

2-49

$$q = EI = \dot{q}\pi(r_0^2 - r_i^2)L = h2\pi rL(T_i - T_f)$$

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}}{k} = 0 \qquad T = \frac{-\dot{q}r}{4k} + c_1 \ln r + c_2$$

$$T = T_0 \text{ at } r = r_0 \qquad \frac{dT}{dr} = 0 \text{ at } r = r_0 \qquad T_0 = \frac{-\dot{q}r}{4k} + c_1 \ln r_0 + c_2$$

$$\frac{dT}{dr} = 0 = \frac{-\dot{q}r}{2k} + \frac{c_1}{r_0} \qquad c_1 = \frac{\dot{q}r_0^2}{2k} \qquad T_i = \frac{-\dot{q}r}{4k} + \frac{\dot{q}r_0^2}{2k}\ln r_1 + c_2$$

$$T_0 = \frac{-\dot{q}r_0^2}{4k} + \frac{\dot{q}r_0^2}{2k}\ln r_0 + c_2 \qquad T_i = T_0 - \frac{\dot{q}}{4k}(r_1^2 - r_0^2) + \frac{\dot{q}r_0^2}{2k}\ln\left(\frac{r_1}{r_0}\right) \quad \text{(a)}$$

$$\dot{q} = \frac{EI}{\pi(r_0^2 - r_i^2)L} \quad \text{(b)}$$

Insert (a) and (b) in $EI = 2\pi rLh(T_i - T_f)$ and solve for h.

2-50

$\dot{q}$ uniform $T = T_w$ at $r = R$ steady state, T varies only with r.

$$\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\theta^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

this reduces to:

$$\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2} + \frac{\dot{q}}{k} = 0 \qquad \frac{\partial^2(rT)}{\partial r^2} = \frac{-\dot{q}r}{k} \qquad \text{Integrating } T = \frac{-\dot{q}r^2}{6k} + c_1 + \frac{c_2}{r}$$

Boundary conditions: $\dot{q}\frac{4}{3}\pi R^3 = -k4\pi R^2 \left.\frac{dT}{dr}\right|_{r=R}$

(1) $\frac{dT}{dr} = -\frac{\dot{q}r}{3k}$ (2) $T = T_w$ at $r = R$

(3) $\left.\frac{dT}{dr}\right|_{r=R} = 0$ then $c_1 = T_w + \frac{\dot{q}R^2}{6k}$ $\qquad c_2 = 0$

$$T - T_w = \frac{\dot{q}}{6k}(R^2 - r^2)$$

2-51

From Prob. 2-~~47~~ 50

$$T - T_w = \frac{\dot{q}}{6k}(R^2 - r^2)$$

$$T_0 - T_w = \frac{(1\times10^6)(0.02)^2}{(6)(16)} = 4.17°\text{C}$$

$$q = \dot{q}V = \dot{q}\frac{4}{3}\pi R^3 = h4\pi r^2(T_w - T_\infty)$$

$$T_w - T_\infty = \frac{(1\times10^6)(0.02)}{(3)(15)} = 444.4°\text{C}$$

$$T_0 - T_\infty = 444.4 + 4.17 = 448.6°\text{C}$$

$$T_0 = 448.6 + 20 = 468.6°\text{C}$$

2-52

$$\frac{R}{L} = \frac{\rho}{\pi r_0^2} = \frac{2.9\times10^{-6}}{\pi(1.5)^2} = 4.1\times10^{-7}\ \frac{\Omega}{\text{cm}} = 4.1\times10^{-5}\ \frac{\Omega}{\text{m}}$$

$$\dot{q} = \frac{I^2R}{V} = \frac{(230)^2(4.1\times10^{-5})}{\pi(0.015)^2} = 3.07\times10^3\ \frac{\text{W}}{\text{m}^3}$$

$$T_0 = \frac{(3.07\times10^3)(0.015)^2}{(4)(190)} + 180 = 180.0009°\text{C}$$

2-53

$$\dot{q} = a + br \qquad \frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}r}{k} = -\frac{ar + br^2}{k}$$

$$r\frac{dT}{dr} = -\frac{1}{k}\left(\frac{ar^2}{2} + \frac{br^3}{3}\right) + c_1 \qquad T = -\frac{1}{k}\left(\frac{ar^2}{4} + \frac{br^3}{9}\right) + c_1 \ln r + c_2$$

$$\frac{dT}{dr} = -\frac{1}{k}\left(\frac{ar}{2} + \frac{br^2}{3}\right) + \frac{c_1}{r}$$

$T = T_i$ at $r = r_1$ $\qquad$ $T = T_0$ at $r = r_0$

Solving for constants gives:

$$c_1 = \frac{T_i - T_0 - \frac{1}{k}\left[\frac{a(r_0{}^2 - r_i{}^2)}{4} + \frac{b(r_0{}^3 - r_i{}^3)}{9}\right]}{\ln\left(\frac{r_i}{r_0}\right)}$$

$$c_2 = T_0 + \frac{1}{k}\left(\frac{ar_0{}^2}{4} + \frac{br_0{}^3}{9}\right) + \frac{T_i - T_0 - \frac{1}{k}\left[\frac{a(r_0{}^2 - r_i{}^2)}{4} + \frac{b(r_0{}^3 - r_i{}^3)}{9}\right]}{\ln r_0}$$

Also $\dot{q} = \dot{q}_i$ at $r = r_i = a + br_i$

2-54

$d = 2$ mm $\qquad T_\infty = 100°\text{C} \qquad h = 5000\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$T_0 = 150°\text{C} \qquad \rho_e = 1.67\ \mu\Omega \cdot \text{cm} \qquad k = 386\ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

$$T_0 - T_w = \frac{\dot{q}R^2}{4k}$$

$$q = \dot{q}(\pi R^2)L = h(2\pi RL)(T_w - T_\infty)$$

$$T_w - T_\infty = \frac{\dot{q}R^2}{2h}$$

$$T_0 - T_\infty = \frac{\dot{q}R^2}{4k} + \frac{\dot{q}R}{2h}$$

$$150 - 100 = \dot{q}\left[\frac{0.001^2}{(4)(386)} + \frac{0.001}{(2)(5000)}\right] \qquad \dot{q} = 4.97 \times 10^8\ \frac{\text{W}}{\text{m}^3}$$

$$q = I^2 \rho_e \frac{L}{A} = \dot{q}AL$$

$$I^2 = \frac{\dot{q}A^2}{\rho_e} = \frac{(4.97 \times 10^8)(\pi)^2(0.001)^4}{1.67 \times 10^{-8}}$$

$I = 542$ amp

2-55

$r_i = 0.0125$ m $\qquad r_0 = 0.0129$ m

Assume inner surface is insulated

$$\frac{dT}{dr} = -\frac{\dot{q}r}{2k} + \frac{c_1}{r} = 0 \text{ at } r = r_i$$

$$c_1 = \frac{\dot{q}r_i^2}{2k} \qquad \text{(a)}$$

$$T = \frac{-\dot{q}r^2}{4k} + c_1 \ln r + c_2 \qquad T_i = \frac{-\dot{q}r_i^2}{4k} c_1 \ln r_i + c_2$$

$$T_0 = \frac{-\dot{q}r_0^2}{4k} c_1 \ln r_0 + c_2$$

$$T_i - T_0 = \frac{-\dot{q}}{4k}(r_0^2 - r_i^2) + c_1 \ln\left(\frac{r_i}{r_0}\right) \qquad \text{(b)}$$

Heat Transfer is:

$$q = \dot{q}V = \dot{q}\pi(r_0^2 - r_i^2) = h\pi(2r_0)(T_0 - T_\infty) \qquad \text{(c)}$$

Inserting (a) in (b) gives

$$T_i - T_0 = \frac{q}{4k}(r_0^2 - r_i^2) + \frac{\dot{q}r_i^2}{2k} \ln\left(\frac{r_i}{r_0}\right) \qquad \text{(d)}$$

We take: $T_i = 250°\text{C} \quad h = 100 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}} \qquad T_\infty = 40°\text{C} \qquad k = 24 \ \frac{\text{W}}{\text{m} \cdot °\text{C}}$

Inserting the numerical values in Equations (c) and (d) and solving gives:

$$\dot{q} = 53.26 \ \frac{\text{MW}}{\text{m}^3}$$

$$T_0 = 249.76°\text{C}$$

2-56

$$k = 43 \ \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

$$U_i = \frac{1}{\frac{1}{500} + \frac{\ln(1.45/1.25)\pi(0.025)}{(2\pi)(43)} + \frac{0.025}{0.029}\left(\frac{1}{12}\right)} = \frac{1}{2 \times 10^{-3} + 4.31 \times 10^{-5} + 71.84 \times 10^{-3}}$$

$$= 13.54 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

2-57

$$r_0 = \frac{k}{h} = \frac{0.18}{12} = 0.015 \text{ m} = 1.5 \text{ cm}$$

(a) $r_0 = 1.25 + 0.05 = 1.3$ cm $\qquad$ Increased

(b) $r_0 = 1.25 + 1.0 = 2.25$ cm $\qquad$ Decreased

2-58

$$U = \frac{1}{R} = \frac{1}{3.114 \times 10^{-2}} = 32.11 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

2-59

$$A = 130.4 \text{ m}^2$$

$$U = \frac{q}{A\Delta T} = \frac{44{,}000}{(130.4)(260 - 38)} = 1.52 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

2-60

For $L = 1$ m $\quad \frac{1}{h_i A_i} = \frac{1}{(65)\pi(0.025)} = 0.1959$

$$\frac{\ln(r_0/r_i)}{2\pi k} = \frac{\ln(2.58/2.5)}{2\pi(18)} = 2.79 \times 10^{-4}$$

$$\frac{1}{h_0 A_0} = \frac{1}{(6.5)\pi(0.0258)} = 1.898$$

$$UA = \frac{1}{\sum R} = 2.094$$

$$\frac{q}{L} = (2.094)(120 - 15) = 219.9 \ \frac{\text{W}}{\text{m}}$$

2-61

$A = 1 \text{ m}^2 \qquad R_{\text{glass}} = \frac{\Delta x}{k} = \frac{0.005}{0.78} = 6.41 \times 10^{-3}$

$$R_{\text{air}} = \frac{\Delta x}{k} = \frac{0.004}{0.026} = 0.1538$$

$$R_{\text{conv}_1} = \frac{1}{h} = \frac{1}{12} = 0.0833$$

$$R_{\text{conv}_2} = \frac{1}{50} = 0.02$$

$$U = \frac{1}{(2)(6.41 \times 10^{-3}) + 0.1538 + 0.0833 + 0.02} = \frac{1}{0.2699} = 3.705 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$R = 0.2699$

single glass plate: $R = 6.41 \times 10^{-3} + 0.0833 + 0.02 = 0.1097$

$$U = \frac{1}{R} = 9.11 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

2-62

$$R_{Cu} = \frac{0.001}{386} = 2.59 \times 10^{-6} \qquad \Delta T_{Cu} = (52)(2.59 \times 10^{-6}) = 1.35 \times 10^{-4}\,°\text{C}$$

$$R_{St} = \frac{0.004}{43} = 9.3 \times 10^{-5} \qquad \Delta T_{St} = (52)(9.3 \times 10^{-5}) = 4.84 \times 10^{-3}\,°\text{C}$$

$$R_{As} = \frac{0.01}{0.166} = 0.0602 \qquad \Delta T_{As} = (52)(0.0602) = 3.13°\text{C}$$

$$R_F = \frac{0.1}{0.038} = 2.632 \qquad \Delta T_F = (52)(2.632) = 136.9°\text{C}$$

$$\sum R = 2.692 \qquad U = \frac{1}{R} = 0.371 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = U\Delta T = (0.371)(150 - 10) = 52 \ \frac{\text{W}}{\text{m}^2}$$

Inside of copper = 150°C

2-63

$$r_1 = 0.015 \qquad L_c = 0.02 + 0.00035 = 0.02035$$

$$r_{2c} = 0.03535 \qquad r_{2c}/r_1 = 2.357$$

$$L_c^{3/2}\left[\frac{h}{kA_m}\right]^{1/2} = (0.02035)^{3/2}\left[\frac{524}{(386)(0.0007)(0.02035)}\right]^{1/2} = 0.896$$

$$\eta_f = 0.55$$

$$q = \eta_f hA\theta_0 = (0.55)\pi(0.03535^2 - 0.015^2)(2)(524)(200 - 100) = 186 \text{ W}$$

2-64

The general solution of eq. 2–19 (b) is:

$\theta = T - T_\infty = c_1 e^{-mx} + c_2 e^{mx}$ (1)

boundary conditions:

(1) at $x = 0$ $\qquad \theta = \theta_1 = T_1 - T_\infty$

(2) at $x = L$ $\qquad \theta = \theta_2 = T_2 - T_\infty$

from:

(1) $\theta_1 = c_1 + c_2 \qquad c_1 = \theta_1 - c_2$

(2) $\theta_2 = c_1 e^{-mL} + c_2 e^{mL}$

$$\theta_2 = (\theta_1 - c_2)e^{-mL} + c_2 e^{mL} = \theta_1 e^{-mL} - c_2(e^{-mL} - e^{mL})$$

$$c_2 = \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}} \quad (2)$$

$$c_1 = \theta_1 - c_2 = \theta_1 - \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}}$$

$$c_1 = \frac{\theta_2 - \theta_1 e^{mL}}{e^{-mL} - e^{mL}} \qquad (3)$$

Then eq. (1) becomes

$$\theta = \frac{\theta_2 - \theta_1 e^{mL}}{e^{-mL} - e^{mL}} e^{-mx} + \frac{\theta_2 - \theta_1 e^{-mL}}{e^{mL} - e^{-mL}} e^{mx}$$

$$\theta = \frac{e^{-mx}(\theta_2 - \theta_1 e^{mL}) + e^{mx}(\theta_1 e^{-mL} - \theta_2)}{e^{-mL} - e^{mL}}$$

Part heat lost by rod:

$$q = -kA\left.\frac{d\theta}{dx}\right|_{x=0} + kA\left.\frac{d\theta}{dx}\right|_{x=L}$$

$$\frac{d\theta}{dx} = m\left[\frac{-e^{-mx}(\theta_2 - \theta_1 e^{mL}) + e^{mx}(\theta_1 e^{-mL} - \theta_2)}{e^{-mL} - e^{mL}}\right]$$

$$q = \frac{kAm[-e^{-mL}(\theta_2 - \theta_1 e^{mL}) + e^{mL}(\theta_1 e^{-mL} - \theta_2)]}{e^{-mL} - e^{mL}}$$

$$+ \frac{kAm[-(\theta_2 - \theta_1 e^{mL}) + (\theta_1 e^{-mL} - \theta_2)]}{e^{-mL} - e^{mL}}$$

$$q = \frac{kAm[(\theta_2 - \theta_1 e^{mL})(1 - e^{-mL}) + (\theta_2 - \theta_1 e^{-mL})(1 - e^{mL})]}{e^{-mL} - e^{mL}}$$

2-64

Part A:

$$\dot{q}A = -kA\frac{d^2T}{dx^2} + hP(T - T_\infty)$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) + \frac{\dot{q}}{k} = 0$$

let $\theta = T - T_\infty$

$$\left(D^2 - \frac{hP}{kA}\right)\theta = \frac{-\dot{q}}{k}$$

$$\theta = c_1 e^{\sqrt{hP/kA}\,x} + c_2 e^{-\sqrt{hP/kA}\,x} + \frac{\dot{q}A}{hP}$$

let $\sqrt{\frac{hP}{kA}} = m$

$\theta = \theta_0$ at $x = 0$ $\qquad \therefore\ c_1 = \theta_0 - \frac{\dot{q}A}{hP} - c_2$

$$-kA\frac{dT}{dx}\bigg|_{x=L} = hA(T - T_\infty)\big|_{x=L} = hA\theta_L$$

$$\theta = \frac{\left[e^{-mL}\left(\theta_0 - \frac{\dot{q}A}{hP}\right) - \frac{h\theta_L}{km}\right]e^{mx}}{e^{mL} + e^{-mL}} + \frac{\left[e^{mL}\left(\theta_0 - \frac{\dot{q}A}{hP}\right) + \frac{h\theta_L}{km}\right]e^{-mx}}{e^{mL} + e^{-mL}} + \frac{\dot{q}A}{hP}$$

Part B:

$$q = \int_0^L [hP(T - T_\infty)]dx + hA\theta_L$$

$$q = \frac{hP}{m}\left\{-\frac{h\theta_L}{kL} + \frac{\left(\theta_0 - \frac{\dot{q}A}{hP}\right)(e^{mL} - e^{-mL}) + \frac{2h\theta_L}{km}}{e^{mL} + e^{-mL}}\right\} + \dot{q}AL + hA\theta_L$$

Part C:

$$-kA\frac{dT}{dx}\bigg|_{x=0} = 0 = q_0$$

$$0 = [c_1 m e^{mx} - c_2 m e^{-mx}]_{x=0}$$

$\therefore\ c_1 = c_2$

$$2c_1 = \theta_0 - \frac{\dot{q}A}{hP}$$

$$\dot{q} = \frac{hP}{A}\left[\theta_0 + \frac{2h\theta_L}{km(e^{mL} - e^{-mL})}\right]$$

2-66

$$\frac{d^2\theta}{dx^2}-\frac{hP}{kA}\theta=0 \text{ let } m=\sqrt{\frac{hP}{kA}}$$

$T_\infty = 38$ $\quad d = 12.5$ mm $\quad L = 30$ cm $\quad h = 17$

$\theta = c_1e^{mx}+c_2e^{-mx}$ at $x=0$ $\quad \theta = 200-38=162 \quad k=386$

$P=\pi d \qquad A=\dfrac{\pi d^2}{4} \qquad x=0.3 \qquad \theta = 93-38=55$

$$m=\left[\frac{(17)\pi(0.0125)(4)}{(386)\pi(0.0125)^2}\right]^{1/2}=3.754 \qquad 162=c_1+c_2$$

$55=3.084c_1+0.324c_2 \qquad c_1=0.91 \qquad c_2=161.09$

$\theta = 0.91e^{mx}+161.09e^{-mx}$

$$q\int_0^L hP\theta\,dx = hP\frac{1}{m}[0.91e^{mx}-161.09e^{-mx}]_0^L=\sqrt{hPkA}[0.91e^{mx}-161.09e^{-mx}]_0^{0.3}$$

$$=[(17)\pi(0.0125)(386)\pi(0.0125)^2]^{1/2}\times[0.91e^{mx}-161.09e^{-mx}]_0^{0.3}$$

$$=122.7 \text{ W}$$

2-68

$k=204\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad L_c=L+\dfrac{d}{4}=12+\dfrac{2}{4}=12.5 \qquad T_0=250^\circ\text{C} \qquad T_\infty=15^\circ\text{C}$

$h=12\ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}} \qquad A=\dfrac{\pi d^2}{4} \qquad P=\pi d$

$$m=\sqrt{\frac{hP}{kA}}=\left[\frac{(12)\pi(0.02)(4)}{(204)\pi(0.02)^2}\right]^{1/2}=3.43$$

$mL_c=(3.43)(0.125)=0.429 \qquad q=\sqrt{hPkA}\theta_0\tanh(mL_c)$

$$q=\left[(12)\pi(0.02)(204)\pi\frac{(0.02)^2}{4}\right]^{1/2}(250-15)\tanh(0.429)=20.89 \text{ W}$$

2-69

$$q=\int_0^\infty hP(T-T_\infty)dx=\int_0^\infty hP\theta\,dx=hP\int_0^\infty e^{-mx}dx=\frac{hP\theta_0}{-m}[e^{-mx}]_0^\infty=\sqrt{hPkA\theta_0}$$

$$=\frac{hP\theta_0}{-\sqrt{\frac{hP}{kA}}}\left[\cancel{e^{-\sqrt{\frac{hP}{kA}}L}}_{0}-1\right]$$

$$q=\frac{hP\theta_0}{\sqrt{\frac{hP}{kA}}}=\sqrt{hPkA\theta_0}$$

2-70

$\theta = c_1 e^{-mx} + c_2 e^{mx}$ In case II the end of the fin is insulated.

$\left.\dfrac{dT}{dx}\right|_{x=L} = 0$ the boundary conditions are $\theta = \theta_0$ at $x = 0$

$\dfrac{d\theta}{dx} = 0$ at $x = L$ then $\theta = \theta_0 \dfrac{\cosh[m(L-x)]}{\cosh(mL)}$

$dq_{\text{conv}} = hP\,dx(T - T_\infty)$ or $hPdx\theta$

$$q_{\text{conv}} = \int_0^L hP\,dx = \int_0^L \frac{hP\theta \cosh[m(L-x)]dx}{\cosh(mL)}$$

$$q_{\text{conv}} = \frac{hP\theta_0}{\cosh(mL)} \frac{1}{m} [\sinh(m(L-x))]_0^L = \sqrt{hPkA}\theta_0 \tanh(mL)$$

2-71

$$q = \sqrt{hPkA}\theta_0 = \left[\frac{(20)\pi(0.0005)(372)\pi(0.0005)^2}{4}\right]^{1/2} (120-20) = 0.152 \text{ W}$$

2-72

$$q = \sqrt{hPkA}\theta_0 = \left[\frac{(3.5)\pi(0.025)(372)\pi(0.025)^2}{4}\right]^{1/2} (90-40) = 11.2 \text{ W}$$

2-73

$T_0 = 150°\text{C}$ $T_\infty = 15°\text{C}$ $r_1 = 1.35$ cm $L = 6.0$ mm $t = 1.5$ mm

$h = 20 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $k = 210 \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

$L_c = L + \dfrac{t}{2} = 6.0 + 0.75 = 6.75$ mm

$r_{2e} = r_1 + L_c = 1.35 + 0.675 = 2.025$ cm $\dfrac{r_{2c}}{r_1} = 1.50$

$A_m = t(r_{2c} - r_1) = (0.0015)(0.00675) = 1.012 \times 10^{-5}\ \text{m}^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.00675)^{3/2}\left[\frac{20}{(210)(1.012\times10^{-5})}\right]^{1/2} = 0.0538$$

From Fig. 2–11 $\eta_f = 97\%$

$q_{\max} = 2h\pi(r_{2c}^2 - r_1^2)(T_0 - T_\infty) = 3.86$ W

$q = (0.97)(3.86) = 3.75$ W

2-74

$L_c = 23 + 1 = 24$ mm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.024)^{3/2}\left[\frac{25}{(14)(0.002)(0.024)}\right]^{1/2} = 0.717$$

$\eta_f = 0.77$

$q = \eta_f hA\theta_0 = (0.77)(25)(0.024)(2)(220 - 23) = 182\ \text{W/m}$

2-75

$L_c = 3 + 0.1 = 3.1$ cm $\qquad r_{2c} = 1.5 + 3.1 = 4.6$ cm

$r_1 = 1.5 \qquad \dfrac{r_{2c}}{r_1} = 3.067$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.031)^{3/2}\left[\frac{68}{(55)(0.002)(0.031)}\right]^{1/2} = 0.77$$

$\eta_f = 0.58$

$q = \eta_f hA\theta_0 = (0.58)(68)(2\pi)[0.046^2 - 0.015^2](100 - 20) = 37.49$ W

2-76

η = total efficiency $\quad A_f$ = surface area of all fins $\quad \eta_f$ = fin efficiency
A = total heat transfer area including fins and exposed tube or other surface.
T_0 = base temp $\qquad T_\infty$ = environment temp

$q_{\text{act}} = h(A - A_f)(T_0 - T_\infty) + \eta_f A_f h(T_0 - T_\infty)$

$q_{\text{ideal}} = hA(T_0 - T_\infty)$

$$\eta_t = \frac{q_{\text{act}}}{q_{\text{ideal}}} = \frac{A - A_c + A_f\eta_f}{A} = 1 - \frac{A_f}{A}(1 - \eta_f)$$

2-77

$T_0 = 460 \qquad t = 6.4$ mm $\qquad L = 2.5$ cm $\qquad T_\infty = 93°\text{C} \qquad h = 28$

$k = 16.3 \qquad A_m = L\left(\dfrac{t}{2}\right) = 8 \times 10^{-5}\ \text{m}^2 \qquad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.579$

$\eta_f = 0.85$

$q = (0.85)(28)(1)(2)(0.025)(460 - 93) = 437$ W

2-78

$T_0 = 200°\text{C}$ $\quad T_\infty = 93°\text{C}$ $\quad L = 12.5 \text{ mm}$ $\quad t = 0.8 \text{ mm}$ $\quad r_1 = 1.25 \text{ cm}$

$k = 204$ $\quad L_c = 12.9 \text{ mm}$ $\quad h = 110$ $\quad r_{2c} = 2.54$ $\quad \dfrac{r_{2c}}{r_1} = 2.03$

$A_m = 1.03 \times 10^{-5} \text{ m}^2$ $\quad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.335$ $\quad \eta_f = 0.87$

$$\text{No. of Fins} = \frac{1.0}{0.0095} = 105.3$$

Tube surface area $= (105.3)\pi(0.025)(9.5 - 0.8)(10^{-3}) = 0.0719 \text{ m}^2$

Tube heat transfer $= (110)(0.0719)(200 - 93) = 846.6 \text{ W}$

$$\frac{q}{\text{fin}} = (0.87)(2)\pi(110)(0.0254^2 - 0.0125^2)(200 - 93) = 31.46 \text{ W}$$

Total fin heat transfer $= (31.46)(105.3) = 3312 \text{ W}$

Total heat transfer $= 846.6 + 3312 = 4159 \text{ W}$

2-79

$r_1 = 1.0 \text{ cm}$ $\quad L = 5 \text{ mm}$ $\quad t = 2.5 \text{ mm}$ $\quad h = 25$ $\quad T_0 = 260°\text{C}$

$T_\infty = 93°\text{C}$ $\quad k = 43$ $\quad L_c = 5 + 1.25 = 6.25 \text{ mm}$ $\quad r_{2c} = 1.625 \text{ cm}$

$\dfrac{r_{2c}}{r_1} = 1.625$ $\quad A_m = (0.0025)(0.00625) = 1.56 \times 10^{-5} \text{ m}^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.00625^{3/2}\left[\frac{25}{(43)(1.56 \times 10^{-5})}\right]^{1/2} = 0.095 \qquad \eta_f = 97\%$$

$$q = (0.97)(25)(2)\pi(0.01625^2 - 0.01^2)(260 - 93) = 4.17 \text{ W}$$

2-80

$k = 43$ $\quad t = 2 \text{ cm}$ $\quad h = 20$ $\quad L_c = 15 \text{ cm}$

$L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.723$ $\quad \eta_f = 0.75$

$$q = (0.75)(20)(2)(0.15)(200 - 15) = 833 \text{ W/m depth}$$

2-81

$t = 1.6$ mm $\quad r_1 = 1.25$ cm $\quad L = 12.5$ mm $\quad T_0 = 200°C$

$T_\infty = 20°C \quad h = 60\ \dfrac{W}{m^2 \cdot °C} \quad k = 204\ \dfrac{W}{m \cdot °C} \quad L_c = 13.3$ mm

$r_{2c} = 2.58$ cm $\quad \dfrac{r_{2c}}{r_1} = 2.064 \quad A_m = (0.0016)(0.0133) = 2.128 \times 10^{-5}\ m^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.18 \qquad \eta_f = 95\%$$

$$q = (0.95)(60)(2)\pi(0.0258^2 - 0.0125^2)(200 - 20) = 32.84\ W$$

2-83

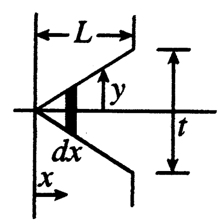
$$A = 2y = \frac{tx}{L} \qquad y = \frac{\frac{t}{2}}{L}x = \frac{tx}{2L}$$

$$-kA\frac{dT}{dx} = hPdx(T - T_\infty) - \left[kA\frac{dT}{dx} + \frac{d}{dx}\left(kA\frac{dT}{dx}\right)dx\right]$$

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0 \qquad \theta = T - T_\infty$$

$$\frac{ktx}{L}\frac{d^2\theta}{dx^2} + \frac{kt}{L}\frac{d\theta}{dx} - hP\theta = 0$$

$$x\frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - \frac{hPL}{kt}\theta = 0$$



2-84

$r_1 = 0.05 \quad r_2 = 0.2 \quad L = 0.15$

$L_c = 0.1 + 0.001 = 0.101 \quad r_{2c} = 0.201$

$$\frac{r_{2c}}{r_1} = 4$$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.101)^{3/2}\left[\frac{60}{(120)(0.101)(0.002)}\right]^{1/2} = 2.388$$

$\eta_f = 0.16$

$$q = \eta_f hA\theta_0 = (0.16)(60)\pi(0.201^2 - 0.05^2)(2)(120 - 23) = 222\ W$$

2-85

$k = 16 \quad h = 40 \quad T_0 = 250°C \quad T_\infty = 90°C$

$P = (4)(0.0125) = 0.05$ m $\quad A = (0.0125)^2 = 1.565 \times 10^{-4}\ m^2$

$$q = \sqrt{hPkA}\,\theta_0 = [(40)(0.05)(16)(1.565 \times 10^{-4})]^{1/2}(250 - 90) = 11.31\ W$$

2-86

$t = 2.1$ mm $\quad L = 17$ mm $\quad h = 75 \quad k = 164 \quad T_0 = 100°C$

$T_\infty = 30°C \quad L_c = 17 + 1.05 = 18.05$ mm

$A_m = (0.0021)(0.01805) = 3.79 \times 10^{-5}\ \text{m}^2$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.01805)^{3/2}\left[\frac{75}{(164)(3.79\times10^{-5})}\right] = 0.266 \qquad \eta_f = 94\%$$

$q = (0.94)(75)(2)(0.01805)(100 - 30) = 178.2$ W

2-87

$L_c = 0.0574$ ft $\qquad r_{2c} = 2.688$ in.

$A_m = (0.125)(0.688) = 0.081\ \text{in}^2 = 5.97 \times 10^{-4}\ \text{ft}^2$

$$\frac{r_{2c}}{r_1} = 1.34 \qquad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.329 \qquad \eta_f = 87\%$$

$$q_{\max} = 2h\pi(r_{2c}^2 - r_1^2)(450 - 100) = 591\ \frac{\text{Btu}}{\text{hr}}$$

$$q = (0.87)(591) = 514\ \frac{\text{Btu}}{\text{hr}}$$

2-88

Calculate heat lost (not temp. at tip)

$d = 1.5$ mm $\quad k = 19 \quad L = 12$ mm $\quad T_0 = 45°C \quad T_\infty = 20°C$

$h = 500$

Use insulated tip solution

$$L_c = L + \frac{d}{4} = 12 + 0.375 = 12.375\ \text{mm}$$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left[\frac{(500)\pi(0.0015)}{(19)\pi(0.0015)^2(4)}\right]^{1/2} = 264.9$$

$mL_c = (0.012375)(264.9) = 3.278$

$q = \sqrt{hPkA}\,\theta_0 \tanh(mL)$

$$= \left[(500)\pi(0.0015)(19)\pi(0.0015)\left(\frac{2}{4}\right)\right]^{1/2}(45 - 20)\tanh(3.278) = 0.177\ \text{W}$$

For $h = 200 \qquad mL_c = 2.073 \qquad \tanh(mL_c) = 0.969$

$$q = \left(\frac{200}{500}\right)^{1/2}(0.177)\left(\frac{0.969}{0.997}\right) = 0.109\ \text{W}$$

For $h = 1500 \qquad mL_c = 5.677 \qquad \tanh(mL_c) = 1.0$

$$q = \left(\frac{1500}{500}\right)^{1/2}(0.177)\left(\frac{1.0}{0.997}\right) = 0.307\ \text{W}$$

2-89

$k = 204$; $T_\infty = 20°C$; $T_0 = 70°C$

$L = 25$ mm

$h = 13.2$; $d = 2$ mm; $N = 225$ pins

$m = [(13.2)(4)/(204)(0.002)]^{1/2} = 11.38$

$L_c = 0.025 + 0.002/4 = 0.0255$

$q/\text{pin} = (hPkA)^{1/2}\,\theta_0 \tanh(mL_c)$

$= [(13.2)\pi(0.002)(204)\pi(0.001)^2]^{1/2}\,(70 - 20)\tanh[(11.38)(0.0255)]$

$= 0.1029$ W/ pin fin

total = (225)(0.1029) = 23.1`5 W

2-90

$k = 204$; $N = 8$; $T_0 = 100°C$; $T_\infty = 30°C$; $h = 15$; $L = 0.02$; $t = 0.002$

$P = (2)(0.15 + 0.002) = 0.304$

$A = (0.002)(0.15) = 0.0003$

$m = [(15)(0.304)/(294)(0.0003)]^{1/2} = 8.632$

$L_c = 0.02 + 0.001 = 0.021$

$q/\text{fin} = (hPkA)^{1/2}\, \theta_0 \tanh(mL_c)$

$= [(15)(0.304)(204)(0.0003)]^{1/2}(100 - 30) \tanh[(8.632)(0.021)]$

$= 6.62$ W/fin

Total = (8)(6.62) = 53 W

2-91

Surface area from Prob. 2-90 = (8)(0.304)(0.02 = 0.04864

Area per circular fin = $(2)\pi(0.0325^2 - 0.0125^2) = 0.00565$

Number of circular fins = 0.04865/0.0565 = 8.6 Round off to 9 fins

$r_1 = 0.0125$; $r_2 = 0.0325$; $L_c = 0.02 + 0.001 = 0.021$

$r_{2c} = 0.0335$; $r_{2c}/r_1 = 2.68$

$L_c^{3/2}(h/kA_m)^{1/2} = 0.1273$

$\eta_f = 0.98$

For 9 fins;

$q = (9)(0.98)((15)\pi(0.0335^2 - 0.0125^2)(100 - 30)(2)$

$= 56.2$ W

2-92

$\theta = c_1 e^{-mx} + c_2 e^{+mx}$

$\theta = 100 - 20 = 80$ at $x = 0$

$\theta = 35 - 20 = 15$ at $x = 0.06$

$m = \sqrt{\frac{hP}{kA}}$

$80 = c_1 + c_2$ (1)

$15 = c_1 e^{-m(0.06)} + c_2 e^{+m(0.06)}$ (2)

$-k[c_1 e^{-m(0.06)}(-m) + c_2 e^{+m(0.06)}(+m)] = h(15)$ (3)

$m = \left[\frac{h\pi(0.02)(4)}{k(0.02)^2}\right]^{1/2}$ (4)

4 Equations, 4 unknowns, c_1, c_2, m, h.

Solve, and then evaluate q from Eq. (2–37) or (2–36) using L_c

2-93

$L = 2.5$ cm $\quad t = 1.5$ mm $\quad k = 50\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ $\quad T_\infty = 20^\circ\text{C}$ $\quad h = 500$

$T_0 = 200^\circ\text{C}$

$L_c = 0.025 + 0.00075 = 0.02575$

$q_{\max} = (2)(500)(0.02595)(200 - 20) = 4635$ W/m

$A_m = (0.0015)(0.02575) = 3.863 \times 10^{-5}$

$L_c^{3/2}\left(\frac{h}{kA_m}\right) = (0.02578)^{3/2}\left[\frac{500}{(50)(3.863 \times 10^{-5})}\right]^{1/2} = 2.1$

$\eta_f = 0.36$

$q = (0.36)(4635) = 1669$ W

2-94

$L = 3.5$ cm $\quad t = 1.4$ mm $\quad L_c = 3.57$ cm $\quad k = 55$

$q_{\max} = hA\theta_0 = (500)(2)(0.0357)(150 - 20) = 4641$ W/m

$mL_c = \left(\frac{2h}{kA_m}\right)^{1/2} L_c^{3/2} = 4.068$

$\eta_f = \frac{\tanh(mL_c)}{mL_c} = 0.246 \qquad q_{act} = (0.246)(4641) = 1140$ W/m

2-95

$k = 43$ $h = 100$ $r_1 = 2.5$ cm $r_2 = 7.5$ cm $L = 5$ cm
$T_\infty = 20°C$ $t = 2$ mm $r_{2c} = 7.51$ cm $L_c = 5.1$ cm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 1.74 \qquad \frac{r_{2c}}{r_1} \cong 3 \qquad \eta_f = 0.27$$

$$q = \eta_f 2h\pi(r_{2c}^2 - r_1^2)\theta_0 = (0.27)(100)(2)\pi(0.0751^2 - 0.025^2)(150-20)$$
$$= 110.6 \text{ W}$$

2-96

$r_1 = 1.5$ cm $L = 2$ cm $r_2 = 3.5$ cm $t = 1$ mm $h = 80$
$k = 200$ $L_c = 2.05$ cm $r_{2c} = 3.55$ cm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.41 \qquad \frac{r_{2c}}{r_1} = 2.37 \qquad \eta_f = 0.81$$

$$q = (80)\pi(0.0355^2 - 0.015^2)(2)(200-20)(0.81) = 75.9 \text{ W}$$

2-97

$h = 50$ $k = 20$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left[\frac{(50)\pi(0.01)(4)}{(20)\pi(0.01)^2}\right]^{1/2} = 31.62$$

$mL = (0.2)(31.62) = 6.324$

$e^{mL} = 557.8$ $e^{-mL} = 0.00179$

$e^{mx} = 2362$ $e^{-mx} = 0.0423$

$\theta_1 = 50 - 20 = 30$ $\theta_2 = 100 - 20 = 80$

Using solution from Prob 2–61

$$\theta_{(x=10\text{ cm})} = \frac{(0.0423)[80-(30)(557.8)] + (23.62)[(30)(0.00179)-80]}{0.00179 - 557.8}$$
$$= \frac{-704.46 - 1888.33}{-557.8}$$
$$= 4.65°C$$

$T = 20 + 4.65 = 24.65°C$

2-98

$k = 386$ $\quad r_1 = 0.625$ cm $\quad L = 0.6$ cm $\quad h = 55$

$t = 0.3$ mm $\quad L_c = 0.6 + 0.015 = 0.615$ cm

$r_{2c} = 0.625 + 0.615 = 1.24$

$$\frac{r_{2c}}{r_1} = \frac{1.24}{0.625} \cong 20$$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.00615)^{3/2}\left[\frac{55}{(386)(0.00615)(0.0003)}\right]^{1/2} = 0.134$$

$\eta_f = 0.95$

$$q = \eta_f hA\theta_0 = (0.95)(55)\pi(2)(0.0124^2 - 0.00625^2)(100 - 20) = 3.012 \text{ W}$$

2-99

$t = 2$ cm $\quad L = 17$ cm $\quad k = 43$ $\quad h = 23$ $\quad L_c = 18$ cm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.93 \qquad \eta_f = 0.64$$

$$q = (0.64)(2)(23)(0.18)(230 - 25) = 1086 \text{ W/m}$$

2-100

$L = 5$ cm $\quad L_c = 5$ cm $\quad t = 4$ mm $\quad k = 23$ $\quad h = 20$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 1.042 \qquad \eta_f = 0.68 \qquad q = \eta_f hA\theta_0$$

$$A = (2)(0.002^2 + 0.05^2)^{1/2} = 0.10008 \ \frac{\text{m}^2}{\text{m depth}}$$

$$q = (0.68)(20)(0.10008)(200 - 40) = 217.8 \text{ W/m}$$

2-101

$t = 1.0$ mm $\quad r_1 = 1.27$ cm $\quad L = 1.27$ cm $\quad L_c = 1.32$ cm $\quad r_2 = 2.54$ cm

$r_{2c} = 2.59$ cm $\quad h = 56$ $\quad k = 204$ $\quad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.219$

$$\frac{r_{2c}}{r_1} = 2.04 \qquad \eta_f = 0.93$$

$$q = (0.93)(2)\pi(0.0259^2 - 0.0127^2)(56)(125 - 30) = 15.84 \text{ W}$$

2-102

$t = 2$ mm $\quad r_1 = 2.0$ cm $\quad r_2 = 10.0$ cm $\quad L = 8$ cm $\quad L_c = 8.1$ cm

$r_{2c} = 10.2$ cm $\quad h = 20 \quad k = 17 \quad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 1.96$

$\dfrac{r_{2c}}{r_1} = 5.1 \quad \eta_f = 0.19$

$q = (0.19)(20)\pi(0.102^2 - 0.02^2)(2)(135 - 15) = 28.7$ W

2-103

$L = 2.5$ cm $\quad t = 1.1$ mm $\quad k = 55 \quad h = 500 \quad L_c = 2.555$ cm

$L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 2.32 \quad \eta_f = 0.33$

$q = (0.33)(2)(0.02555)(500)(125 - 20) = 885$ W/m

2-104

$t = 1.0$ mm $\quad r_1 = 1.25$ cm $\quad r_2 = 2.5$ cm $\quad r_{2c} = 2.55$ cm

$h = 25 \quad k = 204 \quad L = 1.25$ cm $\quad L_c = 1.3$ cm

$L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.249 \quad \eta_f = 0.91$

$q = (0.91)(2)(75)\pi(0.0255^2 - 0.0125^2)(100 - 30) = 4.94$ W

2-105

$d = 1$ cm $\quad L = 5$ cm $\quad h = 20 \quad k = 0.78 \quad T_0 = 180$

$T_\infty = 20$

$L_c = L + \dfrac{d}{4} = 5 + 0.25 = 5.25$ cm

$$\left(\frac{hP}{kA}\right)^{1/2} = m = \left[\frac{(20)\pi(0.01)(4)}{(0.78)\pi(0.01)^2}\right]^{1/2} = 101.3$$

$mL_c = (101.3)(0.0525) = 5.317$

$\tanh(5.317) = 1.0$

$$q = (hPkA)^{1/2}\theta_0 \tanh(mL_c) = \left[\frac{(20)\pi(0.01)(0.78)\pi(0.01)^2}{4}\right]^{1/2}(180 - 20)(1.0)$$

$$= 0.993 \text{ W}$$

2-106

$A = 1\times 1\ \text{cm}^2$ $\quad L = 8\ \text{cm}$ $\quad L_c = 8.5\ \text{cm}$ $\quad m = \left[\dfrac{hP}{kA}\right]^{1/2} = 31.62$

$mL_c = 2.688$ $\quad \eta_f = \dfrac{\tanh(mL_c)}{mL_c} = 0.369$

$q = (0.369)(45)(0.085)(4)(0.01)(300-50) = 14.11\ \text{W}$

2-107

$t = 1.0\ \text{mm}$ $\quad r_1 = 1.25\ \text{cm}$ $\quad L = 12\ \text{mm}$ $\quad T_0 = 275°\text{C}$

$T_\infty = 25°\text{C}$ $\quad h = 120$ $\quad k = 386$

$L_c = 0.012 + 0.0005 = 0.0125$ $\quad r_{2c} = 0.025$ $\quad \dfrac{r_{2c}}{r_1} = 2.0$

$A_m = (0.0125)(0.001) = 1.25\times 10^{-5}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0125)^{3/2}\left[\frac{120}{(386)(1.25\times 10^{-5})}\right]^{1/2} = 0.22$$

$\eta_f = 0.93$

$q = (0.93)(120)\pi(0.025^2 - 0.0125^2)(2)(275-25) = 82.12\ \text{W}$

2-108

$k = 17$ $\quad h = 47$ $\quad L = 5\ \text{cm}$ $\quad t = 2.5\ \text{cm}$ $\quad L_c = 6.25\ \text{cm}$

$L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.657$ $\quad \eta_f = 0.8$

$q = (0.8)(47)(2)(0.0625)(100-20) = 376\ \text{W/m}$

2-109

$r_1 = 1.5\ \text{cm}$ $\quad L = 2\ \text{cm}$ $\quad r_2 = 3.5\ \text{cm}$ $\quad t = 1\ \text{mm}$ $\quad r_{2c} = 3.55\ \text{cm}$

$h = 80$ $\quad k = 204$ $\quad L_c = 2.05\ \text{cm}$ $\quad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.453$

$\dfrac{r_{2c}}{r_1} = 2.37$ $\quad \eta_f = 0.79$

$q = (0.79)(80)(2)\pi(0.0355^2 - 0.015^2)(200-20) = 74\ \text{W}$

2-110

$r_1 = 1.5$ cm $\quad r_2 = 4.5$ cm $\quad t = 1.0$ mm $\quad h = 50 \quad r_{2c} = 4.55$ cm

$L_c = 3.05$ cm $\quad k = 204 \quad \eta_f = 0.6 \quad \dfrac{r_{2c}}{r_1} = 3$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.78$$

$$k = \frac{1}{(0.001)(0.0305)}(0.0305)^3 \frac{50}{(0.78)^2} = 76.5 \ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

2-111

$t = 1.0$ mm $\quad L = 2.0$ cm $\quad r_1 = 1.0$ cm $\quad h = 150 \quad k = 204$

$L_c = 2.05$ cm $\quad r_{2c} = 3.05$ cm $\quad \dfrac{r_{2c}}{r_1} = 3.05 \quad T_0 = 150$

$T_\infty = 20$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0205)^{3/2}\left[\frac{150}{(204)(0.001)(0.0205)}\right]^{1/2} = 0.556$$

$\eta_f = 0.75$

$$q = (0.75)(150)(2)\pi(0.0305^2 - 0.01^2)(150 - 20) = 76.3 \text{ W}$$

2-112

$k_A = k_B = 17 \quad A_c = 0.001A \quad \Delta T = 300^\circ\text{C} \quad A = \dfrac{\pi d^2}{4} = 5.067\times10^{-4} \text{ m}^2$

$\dfrac{L_g}{2} = 1.3\times10^{-6}$ m $\quad k_f = 0.035 \quad L_A = L_B = 7.5$ cm

$$h_c = \frac{1}{L_g}\left[\frac{A_c}{A}\left(\frac{2k_Ak_B}{k_A+k_B}\right) + \frac{A_v}{A}k_f\right] = 19{,}986 \ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$\frac{1}{h_cA} = \frac{1}{(19{,}986)(5.067\times10^{-4})} = 0.0987 \ {}^\circ\text{C/W}$$

$$q = \frac{300}{\frac{(0.075)(2)}{(17)(5.067\times10^{-4})} + 0.0987} = 17.31 \text{ W} \quad \text{w/no contact resistance}$$

$$q = kA\frac{\Delta T}{\Delta x} = \frac{(17)(5.067\times10^{-4})(300)}{0.15} = 17.228 \text{ W}$$

2-114

$$R_{th}=\frac{\Delta x}{kA}=\frac{5\times10^{-3}}{204}=2.45\times10^{-5}\qquad R_c=\frac{1}{h_cA}=0.88\times10^{-4}$$

$$\sum R_{th}=(2)(2.45\times10^{-5})+0.88+10^{-4}=1.37\times10^{-4}$$

$$\Delta T_c=\frac{(80)(0.88\times10^{-4})}{1.37\times10^{-4}}=51.4°\text{C}$$

2-115

$t=1\times10^{-3}$ $\quad r_i=0.0125$ $\quad L=0.0125$ $\quad L_c=0.0130$

$r_{2c}=0.0255$ $\quad \dfrac{r_0}{r_i}=2.04$ $\quad A_m=(0.001)(0.0130)=1.3\times10^{-5}\text{ m}^2$

$L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2}=0.322$ $\quad \eta_f=0.86$ $\quad \dfrac{1}{h_c}=0.88\times10^{-4}$ $\quad T_0=200°\text{C}$

$T_\infty=20°\text{C}$

$$R_{\text{fin}}=\frac{1}{2\eta_f\pi(r_{2c}^2-r_1^2)h}=2.997\ °\text{C/W}$$

$$R_c=\frac{1}{h_cA}=1.1205\ °\text{C/W}$$

$$q=\frac{\theta_0}{R_{\text{fin}}+R_c}=43.72\text{ W}\qquad \text{w/o contact resistance}$$

$$q=\frac{\theta_0}{R_{\text{fin}}}=60.06\text{ W}\quad \%\text{ Reduction}=\frac{60.06-43.72}{60.06}\times100\%=27.2\%$$

2-116

$\dfrac{1}{h_c}=0.9\times10^{-4}$ $\quad A_c=0.5\text{ cm}^2$ $\quad q=300\text{ mW}$

Assume fin at 27°C

$$q=(300\times10^{-3})=\frac{1}{0.9\times10^{-4}}(0.5)(10^{-4})(T_t-27)$$

$T_t=27.54°\text{C}$

2-117

$2L = 20\text{ cm} = 0.2\text{ m} \qquad T_\infty = 50°\text{C}$

$$\frac{q}{A} = \dot{q}(2L) = (2\times10^5)(-0.2) = 40{,}000\ \text{W/m}^2 = 2h(T_w - T_\infty) = (400)(T_w - 50)(2)$$

$T_w = 100°\text{C}$

$$T_0 - T_w = \frac{\dot{q}L^2}{2k} = \frac{(2\times10^5)(0.1)^2}{(2)(20)} = 50°\text{C}$$

$T_0 = 150°\text{C}$

2-118

$\frac{1}{2}$ heat generated, max temperature at insulated surface which is the same as in Problem 2–~~111~~ ١١٧.

2-119

$L_c = 0.03 + 0.001 = 0.031 \qquad k = 204 \qquad h = 220$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.031)^{3/2}\left[\frac{220}{(204)(0.031)(0.002)}\right]^{1/2} = 0.715$$

$\eta_f = 0.69$

$$q = \eta_f hA\theta_0 = (0.69)\pi(0.061^2 - 0.03^2)(2)(220)(120-20) = 269\text{ W}$$

2-120

$L_c = 0.008 \qquad k = 204\ \frac{\text{W}}{\text{m}\cdot°\text{C}} \qquad h = 45\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.008)^{3/2}\left[\frac{(45)(2)}{(204)(0.008)(0.002)}\right]^{1/2} = 0.119$$

$\eta_f = 0.97$

$$q = (2)(0.97)(0.008^2 + 0.001^2)^{1/2}(45)(200-25) = 123.2\ \text{W/m}$$

2-121

$r_1 = 1.25$ cm $\quad r_2 = 2.25$ cm $\quad t = 2.0$ mm $\quad r_{2c} = 2.35$ cm

$\dfrac{r_{2c}}{r_1} = 1.88 \quad L_c = 1.1$ cm $\quad T_0 = 180°\text{C} \quad T_\infty = 20°\text{C}$

$h = 50 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad k = 204 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

L of tube $= 1.0$ m

bare length $= 1.0 - (100)(0.002) = 0.8$ m

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.011)^{3/2}\left[\frac{50}{(204)(0.002)(0.011)}\right]^{1/2} = 0.122$$

$\eta_f = 0.95$

$q(\text{100 fins}) = (100)(2)(0.95)\pi(0.0235^2 - 0.0125^2)(50)(180 - 20) = 1869$ W

$q(\text{bare tube}) = (50)\pi(0.025)(0.8)(180 - 20) = 503$ W

$q(\text{total}) = 1869 + 503 = 2372$ W

2-122

$k = 100 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad q = kA\dfrac{\Delta T}{\Delta x}$

$A = (1.7 - 1.5)\ \text{cm} = \dfrac{0.2}{100} = 0.002 \ \text{m}^2/\text{m}$

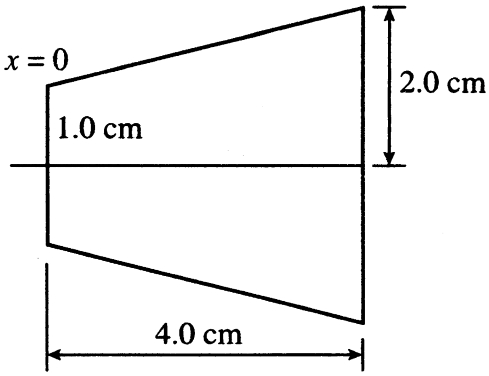
$\Delta x =$ circumference at $r = 1.6$ cm for $\dfrac{\pi}{4}$

$$\Delta x = (2)(1.6)\left(\frac{\pi}{4}\right) = 2.513 \text{ cm} = 0.02513 \text{ m}$$

$$R = \frac{\Delta x}{kA} = \frac{0.02513}{(100)(0.002)} = 0.1256°\text{C}^{-1}$$

$$\frac{q}{L} = \frac{50}{0.1256} = 398 \ \text{W/m}$$

2-123



$k = 386 \ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ $\quad r = ax + b \quad$ $r = 0.01$ at $x = 0$

$r = 0.02$ at $x = 0.04$

$0.01 = b \qquad a = 0.25$

$0.02 = (0.04)a + 0.01$

$r = 0.25x + 0.01 \qquad A = 2\pi r(0.0005)$

$$q = -k2\pi(0.25x + 0.01)(0.0005)\frac{dT}{dx}$$

$$\int_0^{0.04} \frac{dx}{0.25x + 0.01} = \int -2\pi k(0.0005)\frac{dT}{q}$$

$$\frac{1}{0.25}\ln\left[\frac{(0.25)(0.04) + 0.01}{0.01}\right] = 2.773 = 2\pi(386)(0.0005)\frac{\Delta T}{q} = 1.213\frac{\Delta T}{q}$$

$q = \dfrac{\Delta T}{2.287} \qquad R = 2.287$

For $\Delta T = 300°\text{C}$ $\qquad q = \dfrac{300}{2.287} = 131.2\ \text{W}$

2-124

Tube $d_i = 1.25 \text{ cm} = 12.5 \text{ mm}$ $\quad T_{\text{inside}} = 100°\text{C}$ $\quad \text{thk} = 0.8 \text{ mm}$

$d_0 = 14.1 \text{ mm}$ $\quad T_\infty = 20°\text{C}$

Fins: $T = 0.3 \text{ mm}$ $\quad L = 3 \text{ mm}$ $\quad L_c = 3 + 0.15 = 3.15 \text{ mm}$

$r_1 = 6.25 + 0.8 = 7.05 \text{ mm}$ $\quad r_{2c} = 7.05 + 3.15 = 10.20 \text{ mm}$

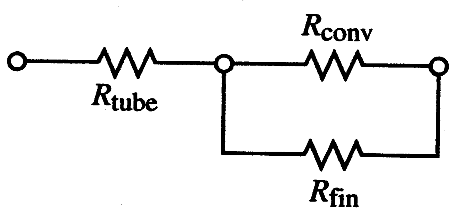
$h = 50 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad k = 386 \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

No. of fins $= \dfrac{300}{6} = 50$ for 30 cm length

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.00315)^{3/2}\left[\frac{50}{(386)(0.00315)(0.0003)}\right]^{1/2} = 0.0654$$

$$\frac{r_{2c}}{r_1} = \frac{10.2}{7.05} = 1.447 \qquad \eta_f = 0.98$$

$$\frac{R_{\text{tube}}}{L} = \frac{\ln(14.1/12.5)}{2\pi(386)} = 4.97 \times 10^{-5} \qquad \frac{R_{\text{conv}}}{L} = \frac{1}{(50)\pi(0.0141)} = 0.4515$$



For 6 mm length

$L_{\text{tube}} = 6 - 0.3 = 5.7 \text{ mm}$

$$R_{\text{tube}} = \frac{4.97 \times 10^{-5}}{0.0057} = 8.72 \times 10^{-3} \qquad R_{\text{conv}} = \frac{0.4515}{0.0057} = 79.2$$

$$q_{\text{fin}} = \frac{\Delta T}{R_{\text{fin}}} = (0.98)(50)(2\pi)(0.0102^2 - 0.00705^2)\Delta T = 0.01672\Delta T$$

$R_{\text{fin}} = 59.78$

$$R_{\text{overall}} = 8.72 \times 10^{-3} + \frac{1}{\frac{1}{79.2} + \frac{1}{59.78}} = 34.07$$

$$q = \frac{100 - 20}{34.07} = 2.348 \text{ W/6 mm length}$$

For 30 cm length

$$q = (2.348)\left(\frac{300}{6}\right) = 117.4 \text{ W} \qquad R = \frac{100 - 20}{117.4} = 0.6814$$

For 1 m length

$$q = (117.4)\left(\frac{1}{0.3}\right) = 392 \text{ W}$$

2-125

For 6 mm length total surface area

$$= A_{\text{tube}} + A_{\text{fin}} = \pi(0.0141)(0.0057) + (2)\pi(0.0102^2 - 0.00705^2)$$

$$= 5.939 \times 10^{-4}\ \text{m}^2$$

$$\frac{q}{A} = \frac{2.348}{5.939 \times 10^{-4}} = 3953\ \text{W/m}^2$$

$$= \frac{\Delta T}{R\text{-value}}$$

$$R\text{-value} = \frac{100 - 20}{3953} = 0.0202$$

2-126

$$k = 204\ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}} \qquad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0654)\left(\frac{386}{204}\right)^{1/2} = 0.09$$

$$\eta_f = 0.96$$

For 6 mm length

$$R_{\text{tube}} = 8.72 \times 10^{-3}$$

$$R_{\text{conv}} = 79.2$$

$$R_{\text{fin}} = (59.78)\left(\frac{0.98}{0.96}\right) = 61.03$$

$$R_{\text{overall}} = 8.72 \times 10^{-3} + \frac{1}{\frac{1}{79.2} + \frac{1}{61.03}} = 34.47$$

$$q = \frac{100 - 20}{34.47} = 2.32\ \text{W/ 6 mm length}$$

For 30 cm length

$$q = (2.32)\left(\frac{300}{6}\right) = 116\ \text{W}$$

$$R = \frac{100 - 20}{116} = 0.6896$$

For 1 m length $q = (116)\left(\frac{1}{0.3}\right) = 387\ \text{W}$

2-127

$$\frac{q}{A} = \frac{2.32}{5.939 \times 10^{-4}} = 3906\ \text{W/m}^2 = \frac{\Delta T}{R\text{-value}}$$

$$R\text{-value} = \frac{110 - 20}{3906} = 0.0205$$

2-128

$h = 100 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad d = 2 \text{ mm} \quad T_\infty = 20°\text{C} \quad T_0 = 100°\text{C} \quad L = 10 \text{ cm}$

$k = 16 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$ $\quad \theta = c_1 e^{-mx} + c_2 e^{mx}$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left[\frac{(100)\pi(0.002)}{(16)\pi(0.001)^2}\right]^{1/2} = 111.8$$

$\theta = 100 - 20 = 80$ at $x = 0$ and $x = 0.1$

$80 = c_1 + c_2$

$80 = 1.395 \times 10^{-5} c_1 + 71{,}682 c_2$

$c_1 = 79.999 \quad c_2 = 1.117 \times 10^{-3}$

$q = \sqrt{hPkA}[-c_1 e^{-mx} + c_2 e^{mx}]$

$\sqrt{hPkA} = [(100)\pi(0.002)(16)\pi(0.001)^2]^{1/2} = 5.62 \times 10^{-3}$

$q_0 = -(5.62 \times 10^{-3})(-79.999 + 1.12 \times 10^{-3}) = 0.45 \text{ W} = -q_L$

$q_{\text{total}} = (2)(0.45) = 0.9 \text{ W}$

2-129

$T_\infty = 20°\text{C} \quad h = 100 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad k = 16 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad d = 2 \text{ mm}$

$T_0 = 100°\text{C}$

$\theta = c_1 e^{-mx} + c_2 e^{mx}$

For very long rod $c_2 = 0$

Acts like two long rods 100°C on each end

$q = 2\sqrt{hPkA}\theta_0 = (2)[(100)\pi(0.002)(16)\pi(0.001)^2]^{1/2}(100 - 20) = 0.9 \text{ W}$

2-130

$A = (0.017 - 0.015)(1) = 0.002 \ \text{m}^2/\text{m}$ depth

$k = 100 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad h = 75 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad T_\infty = 30°\text{C} \quad T_0 = 100°\text{C}$

$T_L = 50°\text{C} \quad L = \dfrac{2\pi}{4}(0.016) = 0.02513 \text{ m}$

$\theta = T - T_\infty = c_1 e^{-mx} + c_2 e^{mx}$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left(\frac{2h}{kt}\right)^{1/2} = \left[\frac{(2)(75)}{(100)(0.002)}\right]^{1/2} = 27.39$$

$mL = 0.6882$

$100 - 30 = c_1 + c_2$

$50 - 30 = c_1 e^{-0.6882} + c_2 e^{0.6882}$

$c_1 = 80.2 \qquad c_2 = -10.2$

$q_x = -kAm(-c_1 e^{-mx} + c_2 e^{mx})$

$q_{x=0} = -(100)(0.002)(27.39)(-80.2 - 10.2) = 495 \text{ W}$

$q_{x=L} = -(100)(0.002)(27.39)[-(80.2)(0.5024) - (10.2)(1.99)] = 332 \text{ W}$

$q_{\text{net lost}} = 495 - 332 = 163 \text{ W/m depth}$

2-131

$t = 3.0 \text{ mm} \qquad k = 204 \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad h = 50 \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}} \qquad \rho = 2707 \text{ kg/m}^3$

Take $L = 2$ cm

Rect. Fin $\qquad L_c = 2 + 0.15 = 2.15$ cm

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0215)^{3/2}\left[\frac{50}{(204)(0.003)(0.0215)}\right]^{1/2} = 0.1943$$

$\eta_f = 0.96$

$A_{\text{surf}} = (2)L_c = 0.043$

For same weight A_m = same

$$t(0.0215) = \frac{t}{2}L(\text{triang})$$

$$L(\text{triang}) = 0.043$$

$$(\text{triang})L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.5496$$

$\eta_f = 0.85$

$q \sim A_{\text{surf}}\eta_f$

$A_{\text{surf}}(\text{triang}) = 2(4.3^2 + 0.15^2)^{1/2} = 8.6 = 0.086 \text{ m}^2$

$A_{\text{surf}}\eta(\text{triangle}) = (0.086)(0.85) = 0.0731$

$A_{\text{surf}}\eta(\text{rect}) = (0.043)(0.96) = 0.0413$

Triangle fin produces more heat transfer for given weight.

2-132

$r_1 = 1.0 \qquad r_2 = 2.0 \text{ cm} \qquad h = 160 \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}} \qquad k = 204 \dfrac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$

<u>1.0 mm Fin</u>

$L_c = 1.05 \text{ cm}$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0105)^{3/2}\left[\frac{160}{(204)(0.001)(0.0105)}\right]^{1/2} = 0.294$$

$\eta_f = 0.88$

$q = (6)(160)\pi(0.0205^2 - 0.01^2)(2)(\Delta T)(0.88) = 1.7\Delta T$

<u>2.0 mm Fin</u>

$L_c = 1.1 \text{ cm} \qquad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.218 \qquad \eta_f = 0.92$

$q = (3)(160)\pi(0.021^2 - 0.01^2)(2)\Delta T(0.92) = 0.95\Delta T$

<u>3.0 mm Fin</u>

$L_c = 1.15 \text{ cm} \qquad L_c^{3/2}\left(\dfrac{h}{kA_m}\right)^{1/2} = 0.186 \qquad \eta_f = 0.95$

$q = (2)(160)\pi(0.0215^2 - 0.01^2)(2)\Delta T(0.95) = 0.69\Delta T$

<u>Conclusion:</u> Several thin fins are better than a few thick fins. More heat transfer for the same weight of fins.

2-133

$L = 5 \text{ cm} \qquad d = 2,\ 5,\ 10 \text{ mm} \qquad T_\infty = 20^\circ\text{C} \qquad h = 40 \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

$k = 204 \dfrac{\text{W}}{\text{m} \cdot {}^\circ\text{C}} \qquad T_0 = 200^\circ\text{C} \qquad L_c = L + \dfrac{d}{4}$

<u>2 mm pin</u>

$L_c = 5 + 0.05 = 5.05 \text{ cm} = 0.0505 \text{ m}$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left(\frac{h\pi d}{k\pi \frac{d^2}{4}}\right)^{1/2} = \left(\frac{4h}{kd}\right)^{1/2} = \left[\frac{(4)(40)}{(204)(0.002)}\right]^{1/2} = 19.8$$

$mL_c = (19.8)(0.0505) = 1.0$

$$\eta = \frac{\tanh(mL_c)}{mL_c} = 0.762$$

$q = (0.762)(40)\pi(0.002)(0.0505)(200 - 20) = 1.74 \text{ W}$

<u>5 mm pin</u>

$L_c = 5 + 0.125 \text{ cm} = 0.05125 \text{ m} \qquad m = 12.52 \qquad mL_c = 0.6419$

$$\eta = \frac{\tanh(mL_c)}{mL_c} = 0.882$$

$q = (0.882)(40)\pi(0.005)(0.05125)(200 - 20) = 5.11\text{ W}$

<u>10 mm pin</u>

$L_c = 5 + 0.25\text{ cm} = 0.0525\text{ m} \qquad m = 8.856 \qquad mL_c = 0.46495$

$$\eta = \frac{\tanh(mL_c)}{mL_c} = 0.934$$

$q = (0.934)(40)\pi(0.01)(0.0525)(200 - 20) = 11.09\text{ W}$

d (mm)	q (W)	$\frac{q}{d\text{ (mm)}}$	$\frac{q}{d^2}$ (per weight)
2	1.74	0.87	0.435
5	5.11	1.022	0.2044
10	11.09	1.109	0.1109

<u>Conclusion:</u> Smaller pins produce more heat transfer per unit weight.

2-134

See <u>conclusion</u> at end of Problem 2–~~127~~ 133.

2-137

Once an insulation material is selected for this problem a commercial vendor must be consulted to determine the cost, as no cost figures are given in the problem statement. Cost figures will also have to be determined for the material with reflective coating. When the reflective material is installed it may have an emissivity of 0.1, but after a period of time the surface may oxidize or become coated with foreign matter such that its emissivity will increase. In that case the economic benefit of the coated material will be reduced.

2-138

For this problem, the net heat generated in the tube will be equal to the heat which will be delivered to the fluid by convection. The temperature gradient at the other surface of the tube will be zero. The problem does not state whether the fluid is on the outside or inside of the tube so both cases must be examined. In either case the maximum tube temperature will occur at the insulated surface. To effect the design one might first assume a surface temperature for the tube surface in contact with the fluid. This will then determine the surface area. Suitable combinations of tube length and diameter may then be examined to equal the total surface area. The heat generation equation for a hollow cylinder may then be solved for the other tube surface temperature if a tube wall thickness is assumed (i.e., establishing the other diameter). The resultant value of temperature must be reasonable, i.e., low enough. Obviously, there are many combinations which will be satisfactory.

2-145

$d^2T/dx^2 - (\varepsilon\sigma P/kA)(T^4 - T_s^4) = 0$

Boundary conditions:

Base: $T = 0$ at $x = 0$

Tip: $-kA(dT/dx)_{x=L} = \sigma\varepsilon A(T^4_{x=L} - T_s^4)$

2-146

Insulated tip: $dT/dx = 0$ at $x = L$

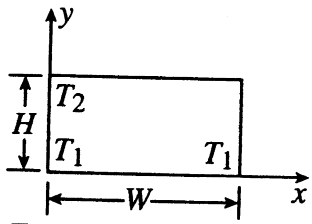
Very long fin: $T \to T_s$ as $x \to \infty$

2-147

Set $T_s = 0$ in differential equations

Chapter 3

3–1



$T = T_1$ at $y = 0$ $T = T_1$ at $x = 0$ $T = T_1$ at $x = W$

$T = T_2$ at $y = H = XY$ For $\lambda^2 = 0$ $X = c_1 + c_2 x$

$Y = c_3 + c_4 y$ at $y = 0$

$\therefore T - T_1 = (c_1 + c_2 x)[c_3 + 0(c_4)] = \theta = 0$ hence $c_3 = 0$

at $x = 0$ $c_1 = 0$ also $c_2 = 0$ $T_2 - T_1 = [0 + (0)x](0 + c_4 H) = 0$ which isn't true

For $\lambda^2 < 0$: $\theta = T - T_1$

$T - T_1 = (c_5 e^{-\lambda x} + c_6 e^{\lambda x})[c_7 \cos(\lambda y) + c_8 \sin(\lambda y)]$

$\theta = 0$ at $y = 0$ $0 = (c_5 e^{-\lambda x} + c_6 e^{\lambda x})(c_7)$ $\therefore$ $c_7 = 0$

$\theta = 0$ at $x = 0$ $0 = (c_5 + c_6)[c_8 \sin(\lambda y)]$ $\therefore$ either $c_5 + c_6 = 0$

or $c_8 = 0$ If $c_8 = 0$ then have trivial soln. then,

$c_5 + c_6 = 0$ and $c_5 = -c_6$ $\theta = 0$ at $x = W$

$0 = (c_5 e^{-\lambda W} + c_6 e^{\lambda W})[c_8 \sin(\lambda y)]$ $\therefore$ $x = 0$ but it was stated that $\lambda^2 < 0$

3–2

$$\frac{T - T_1}{T_2 - T_1} = \frac{2}{\pi} \sum \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{W} \frac{\sinh\left(\frac{n\pi y}{W}\right)}{\sinh\left(\frac{n\pi H}{W}\right)} \quad \text{at } y = H \quad x = \frac{W}{2}$$

$\frac{T - T_1}{T_2 - T_1} = 1$ (Ref Fig 3–2) Also, non-zero terms for $n = 1, 3, 5, \ldots$ First four terms: $n = 1, 3, 5, 7$

$$\frac{T - T_1}{T_2 - T_1} = \frac{2}{\pi}\left[2 \sin \frac{\pi}{2} + \frac{2}{3} \sin \frac{3\pi}{2} + \frac{2}{5} \sin \frac{5\pi}{2} + \frac{2}{7} \sin \frac{7\pi}{2}\right]$$

$\frac{T - T_1}{T_2 - T_1} = 0.92$ Error is $(1.00 - 0.92) \times 100\% = 8\%$

3–3

$$S = \frac{2\pi(1)}{\cosh^{-1}(1.2/0.125)} = 2.128$$

$q = kS\Delta T = (2.128)(1.8)(67 - 15) = 199.2$ W/m length

3-4

$q = kS\Delta T \qquad \Delta T = 210 - 15 = 195°\text{C}$

$r = 0.08 \qquad D = 0.225 \qquad k = 1.37$

$$S = \frac{2\pi L}{\ln(40/r)} = \frac{2\pi(1)}{\ln(0.9/0.03)} = 1.847$$

$q = (1.37)(1.847)(195) = 494 \text{ W/m}$

3-5

$T_{steam} = 100°\text{C}, \quad r = 0.0125; \quad D = 0.025 \quad T_\infty = 27°\text{C}, \; h = 5.1, \quad k = 0.1$

$\sum R = 1/kS + 1/hA$

$A \to \infty$ for inf plate

$S = 2\pi/\ln(0.1/0.0125) = 3.022$

$q = 0.1)(3.022)(100 - 27) = 22.06$ W/m length

3-6

$$k_m = 11 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot °\text{F}} = 19.04 \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

$$k_{\text{glass}} = 0.038 \frac{\text{W}}{\text{m} \cdot °\text{C}}$$

$$q = \frac{300 - 40}{\frac{\ln(2.5/1.5)}{2\pi(19.04)} + \frac{\ln(5/2.5)}{2\pi(0.038)}} = 89.38 \text{ W/m}$$

3-8

$L = 0.5 \text{ ft} = 0.1524 \text{ m}$

$$S_{\text{walls}} = \frac{2}{0.1524}[(0.3)(0.6) + (0.3)(0.9) + (0.6)(0.9)] = 12.992$$

$S_{\text{edges}} = (4)(0.54)(0.3 + 0.6 + 0.9) = 3.888$

$S_{\text{corners}} = (8)(0.1524) = 1.219$

$S_{\text{total}} = 18.099$

$q = kS\Delta T = (0.5)(1.73)(18.099)(500 - 100) = 6262 \text{ W}$

3-9

$D = 35 - 10 = 25$ cm $\quad L = 5$ cm $\quad A = (0.25)^2 = 0.0625\ \text{m}^2$

$k = 1.04\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \quad S_{\text{wall}} = \dfrac{A}{L} = 1.25 \quad S_{\text{edge}} = 0.59D = 0.135$

$S_{\text{corner}} = 0.15L = 7.5 \times 10^{-3}$

$S = 6(1.25) + 12(0.135) + 8(7.5 \times 10^{-3}) = 9.18$

$q = kS\Delta T = (1.04)(9.18)(500 - 80) = 4009.8$ W

3-10

$r_1 = 4$ cm $\quad r_2 = 1.50$ cm $\quad D = 10$ cm $\quad k = 1.4\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$

$$S = \frac{2\pi}{\cosh^{-1}\left[\frac{10^2 - 4^2 - 1.5^2}{(2)(4)(1.5)}\right]} = 2.411$$

$q = ks\Delta T = (1.4)(2.411)(200 - 35) = 556.8$ W/m

Chapter 3

3-11

$q = kS\Delta T \qquad k = 1.2\ \dfrac{\mathrm{W}}{\mathrm{m \cdot {}^\circ C}} \qquad r = 5\ \mathrm{cm} \qquad \Delta T = 30 - 0 = 30^\circ\mathrm{C}$

$D = 24\ \mathrm{cm}$

$S = \dfrac{4\pi r}{1 - r/20} = \dfrac{4\pi(0.05)}{1 - 5/48} = 0.7013$

$q = (1.2)(0.7013)(30) = 25.24\ \mathrm{W}$

3-12

$S = 4\pi r = 1.257\ \mathrm{m} \qquad k = 0.038\ \dfrac{\mathrm{W}}{\mathrm{m \cdot {}^\circ C}}$

$q = (1.257)(0.038)(170 - 20) = 7.165\ \mathrm{W}$

3–12

$r = 1\ \mathrm{m} \qquad k = 1.5\ \dfrac{\mathrm{W}}{\mathrm{m \cdot {}^\circ C}} \qquad S = 4\pi r = 12.56\ \mathrm{m}$

$q = (1.5)(12.56)(20 - 0) = 377\ \mathrm{W}$

3-15

$k = 0.04$; $T_{steam} = 100^\circ\mathrm{C}$; $L = 2$ m; $T_\infty = 27^\circ\mathrm{C}$; $h = 5.1$

$r = 0.0125$, $W = 0.05$

$S = 2\pi(2.0)/\ln[(0.54)(0.05/0.0125)] = 16.318$

$1/hA = 1/(5.1)(2.0)(0.05)(4) = 0.4902$

$\Sigma R = 1/(0.04)(16.318) + 0.4902 = 2.022$

$q = (100 - 27)/2.022 = 36.1\ \mathrm{W}$

3-16

$D > 3r \qquad \frac{q}{L} = \frac{2\pi(1.2)(110-25)}{\ln\left[\frac{(2)(1.3)}{0.04}\right]} = 153.5 \text{ W/m} = I^2R = I^2(1.1\times10^{-4})$

$I = 1181$ amp

3-17

$S = 4\pi r$
$q = (1.3)(4\pi)(0.02)(70-12) = 18.95$ W

3-18

$r_1 = 0.025$ m $\quad r_2 = 0.01$ m $\quad D = 0.055$ m $\quad S = \dfrac{2\pi L}{\cosh^{-1}\left(\frac{r_1^2+r_2^2-D^2}{2r_1r_2}\right)}$

$k = 2.5 \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad \Delta T = 100-20 \qquad S = \dfrac{2\pi}{\cosh^{-1}\left(\frac{2.5^2+10^2-5.5^2}{2(2.5)(10)}\right)} = 6.41$

$q = kS\Delta T = (2.5)(6.41)(100-20) = 1282$ W/m

3-19

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{40^2-9^2-4.5^2}{2(9)(4.5)}\right)} = 1.74$$

$$\frac{q}{r} = (1.74)(1.1)(200-100) = 191.4 \text{ W/m}$$

3-20

$$S = \frac{4\pi(0.75)}{1-\frac{0.75}{(2)(3.75)}} = 10.47$$

$$q = (10.47)(1.2)(300-30) = 3393 \text{ W}$$

3-21

$r = 1.25$ cm $\quad L = 1$ m $\quad T_r = 55°C \quad T_s = 10°C \quad k = 1.7\ \frac{\text{W}}{\text{m}\cdot°\text{C}}$

$$S = \frac{2\pi L}{\ln\left(\frac{2L}{r}\right)} = \frac{2\pi(1.0)}{\ln\left[\frac{(2)(1.0)}{0.0125}\right]} = 1.238$$

$$q = (1.238)(1.7)(55-10) = 94.71 \text{ W}$$

3-22

$k = 0.8\ \frac{\text{W}}{\text{m}\cdot°\text{C}} \quad r_1 = 5$ cm $\quad r_2 = 1.4$ cm $\quad D = 12$ cm

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{12^2-1.4^2-5^2}{(2)(1.4)(5)}\right)} = 2.813$$

$$q = (2.813)(0.8)(300-15) = 641.4 \text{ W/m}$$

3-23

$D = 5$ m $\quad r = 1.0$ m $\quad k = 1.5\ \frac{\text{W}}{\text{m}\cdot°\text{C}}$

$$S = \frac{4\pi r}{1-\frac{r}{20}} = \frac{4\pi(1.0)}{1-\frac{1}{10}} = 13.96$$

$$q = (13.96)(1.5)(25-5) = 418.9 \text{ W}$$

3-24

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{100^2-10^2-20^2}{(2)(10)(20)}\right)} = 1.6278$$

$r_1 = 10 \quad r_2 = 20 \quad D = 100$

$$q = kS\Delta T = (3.5)(1.6278)(120-20) = 570 \text{ W/m}$$

3-25

$r = 5$ cm $\quad L = 100$ m $\quad D = 23$ cm

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{D}{r}\right)} = \frac{2\pi(100)}{\cosh^{-1}\left(\frac{23}{5}\right)} = 284.7$$

$q = kS\Delta T = (1.2)(284.7)(150 - 15) = 46{,}120$ W

3-26

$T_{steam} = 150°C$, $\quad T_\infty = 20°C$ $\quad r = 0.025$; $\quad D = 0.075$

$k = 0.65$

$S = 2\pi/\ln(0.3/0.0125) = 1.977$

$q = (0.65)(1.977)(150 - 20) = 167.06$ W/m

3-27

$T_{steam} = 100°C$; $\quad T_{surface} = 25°C$; $\quad k = 1.2$; $\quad D = 0.05$; $\quad r = 0.005$;

$l = 0.03$

$S = 2\pi/\ln[(0.03/\pi\times0.005)\sinh(2\pi\times0.05/0.03)] = 0.603$

$q = (1.2)(0.603)(100 - 25) = 54.24$ W/m

3-28

$r_1 = 5 \text{ cm} \qquad r_2 = 2.5 \text{ cm} \qquad D = 20 \text{ cm} \qquad k = 0.15 \ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$

$$S = \frac{2\pi}{\cosh^{-1}\left[\frac{20^2-5^2-2.5^2}{(2)(2.5)(5)}\right]} = 1.857$$

$q = (0.15)(1.857)(110-3) = 29.81 \ \text{W/m}$

3-29

$r = 1.5 \text{ cm} \qquad D = 5 \text{ cm} \qquad k = 15.5 \ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{D}{r}\right)} = \frac{2\pi}{\cosh^{-1}\left(\frac{5}{1.5}\right)} = 3.353$$

$q = (15.5)(3.353)(135-46) = 4626 \ \text{W/m}$

3-30

$\Delta T = 30 - 0 = 30^\circ\text{C} \qquad k = 0.2 \qquad D = 8.5 \text{ m} \qquad r = 1.25 \text{ m}$

$$S = \frac{4\pi r}{1-\frac{r}{2D}} = \frac{4\pi(1.25)}{1-\frac{1.25}{17}} = 16.95$$

$q = kS\Delta T = (0.2)(16.95)(30) = 101.7 \ \text{W}$

3-31

$k = 0.74 \ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad D = 0 \qquad W = 100 \text{ cm} \qquad L = 50 \text{ cm}$

$$S = \frac{\pi W}{\ln\left(\frac{4W}{L}\right)} = \frac{\pi(1.0)}{\ln\left(\frac{400}{50}\right)} = 1.511$$

$q = kS\Delta T = (0.74)(1.511)(120-15) = 117.4 \ \text{W}$

3-32

$T_{disk} = 40^\circ\text{C}; \ k = 0.8; \ T_{surface} = 15^\circ\text{C}$

$r = 0.009; \quad D = 0.02$

$S = 4\pi(0.009)/[\pi/2 - \tan^{-1}(0.009/0.04)] = 0.0795$

$q = (0.8)((0.0795)(40 - 15) = 1.591 \ \text{W}$

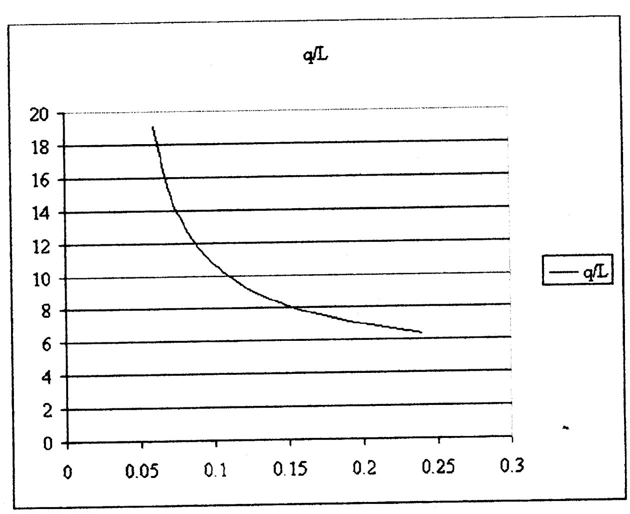
3-33

$r_1 = r_2 = 0.025 \quad \Delta T = 115^\circ\text{C} \quad k = 0.04$

0.05<D<;large

$S = 2\pi/\cosh^{-1}[(0.5)(D/0.025)^2 - 0.5]$

$q/L = 0.04S(115) = 4.6S$



3-34

$L = 5$ cm

$$S_{\text{walls}} = \frac{(2)(0.6)(0.7) + 2(0.6)(0.8) + (0.7)(0.8)}{0.05} = 58.4$$

$S_{\text{edges}} = (4)(0.54)(0.6 + 0.7 + 0.8) = 4.536$

$S_{\text{corners}} = (8)(0.15)(0.05) = 0.06$

$S_{\text{total}} = 62.996$

3-35

$r_1 = 7.5$ cm $T_1 = 150°C$ $T_2 = 5°C$ $k = 0.7\ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$ $r_2 = 2.5$ cm

$D = 15$ cm

$$S = \frac{2\pi}{\cosh^{-1}\left[\frac{15^2 - 7.5^2 - 2.5^2}{(2)(7.5)(2.5)}\right]} = 2.928$$

$q = (0.7)(2.928)(150 - 5) = 297.2\ \text{W/m}$

3-36

The temperature gradients are written:

$$\left.\frac{\partial T}{\partial x}\right|_{m+\frac{1}{2},\ n,\ p} \approx \frac{T_{m+1,\ n,\ p} - T_{m,\ n,\ p}}{\Delta x} \qquad \left.\frac{\partial T}{\partial x}\right|_{m-\frac{1}{2},\ n,\ p} \approx \frac{T_{m,\ n,\ p} - T_{m-1,\ n,\ p}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial y}\right|_{m,\ n+\frac{1}{2},\ p} \approx \frac{T_{m,\ n+1,\ p} - T_{m,\ n,\ p}}{\Delta y} \qquad \left.\frac{\partial T}{\partial y}\right|_{m,\ n-\frac{1}{2},\ p} \approx \frac{T_{m,\ n,\ p} - T_{m,\ n-1,\ p}}{\Delta y}$$

$$\left.\frac{\partial T}{\partial z}\right|_{m,\ n,\ p+\frac{1}{2}} \approx \frac{T_{m,\ n,\ p+1} - T_{m,\ n,\ p}}{\Delta z} \qquad \left.\frac{\partial T}{\partial z}\right|_{m,\ n,\ p-\frac{1}{2}} \approx \frac{T_{m,\ n,\ p} - T_{m,\ n,\ p-1}}{\Delta z}$$

For 3 dimensions 2g places eqn. is

$\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2} = 0$ also if $\Delta x = \Delta y = \Delta z$, the finite difference approximations become:

$$T_{m+1,\ n,\ p} + T_{m-1,\ n,\ p} + T_{m,\ n+1,\ p} + T_{m,\ n-1,\ p} + T_{m,\ n,\ p+1} + T_{m,\ n,\ p-1} - 6T_{m,\ n,\ p} = 0$$

3-37

the Temp. gradient is written:

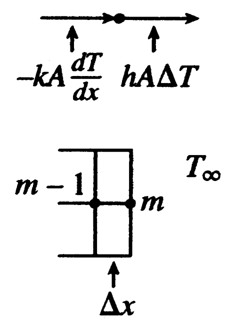
$$\left.\frac{dT}{dx}\right|_{m+\frac{1}{2},\ n} \approx \frac{T_{m+1,\ n} - T_{m,\ n}}{\Delta x} \qquad \left.\frac{dT}{dx}\right|_{m-\frac{1}{2},\ n} \approx \frac{T_{m,\ n} - T_{m-1,\ n}}{\Delta x}$$

for 1 dimension: $\dfrac{d^2 T}{dx^2} = 0$ and the finite difference approximation is:

$$T_{m+1,\ n} + T_{m-1,\ n} - 2T_{m,\ n} = 0$$

3-38

$$kA\frac{T_{m-1}-T_m}{\Delta x}-hA(T_m-T_\infty)=0$$



then:

$$T_m\left[\frac{h}{k}(\Delta x)+1\right]-T_\infty\left[\frac{h}{k}(\Delta x)\right]-T_{m-1}=0$$

3-39

$$\frac{d^2T}{dx^2}-\frac{hP}{kA}(T-T_\infty)=0 \qquad \left.\frac{d^2T}{dx^2}\right|_{m,\,n}\approx\frac{T_{m+1,\,n}-T_{m,\,n}-T_{m,\,n}+T_{m-1,\,n}}{(\Delta x)^2}$$

Substituting T_m for T above:

$$T_m\left[\frac{hP(\Delta x)^2}{kA}+2\right]-\left[\frac{hP(\Delta x)^2}{kA}\right]T_\infty-(T_{m-1}+T_{m+1})=\bar{q}_m$$

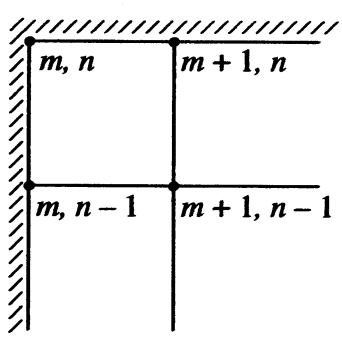
3-40

$$T_{m,\,n+1}+T_{m,\,n-1}+2T_{m-1,\,n}-4T_{m,\,n}=\bar{q}_{m,\,n}$$

$$\frac{hA(T_{m-1,\,n}-T_{m,\,n})}{\Delta x}+\frac{hA(T_{m,\,n+1}-T_{m,\,n})}{2\Delta y}+\frac{hA(T_{m,\,n-1}-T_{m,\,n})}{2\Delta y}=\bar{q}_{m,\,n}$$

for $\Delta x=\Delta y$: $T_{m,\,n+1}+T_{m,\,n-1}+2T_{m-1,\,n}-4T_{m,\,n}=\bar{q}_{m,\,n}$

3-42



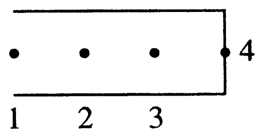
$$T_{m,\,n-1}+T_{m+1,\,n}-2T_{m,\,n}=\bar{q}_{m,\,n}$$

3–37

$$k(T_{m-1,\,n}-T_{m,\,n})+\frac{k}{2}(T_{m,\,n+1}-T_{m,\,n})+\frac{k}{2}(T_{m,\,n-1}-T_{m,\,n})+q''\Delta x=\bar{q}_{m,\,n}$$

3-46

$k = 204 \qquad A = \dfrac{\pi(0.025)^2}{4} = 4.91\times10^{-4} \qquad T_1 = 300°\text{C}$



$T_\infty = 38°\text{C} \qquad L = 15 \text{ cm} \qquad \Delta x = 0.05 \text{ m} \qquad h = 17$

$$\frac{1}{R_{21}} = \frac{kA}{\Delta x} = \frac{(204)(4.91\times10^{-4})}{0.05} = 2.003 = \frac{1}{R_{23}}$$

$$\frac{1}{R_{1-\infty}} = hA_c = (17)\pi(0.025)(0.05) = 0.0668$$

$$\frac{1}{R_{4-\infty}} = \left(\frac{1}{2}\right)(0.0668) + \frac{(17)\pi(0.025)^2}{4} = 0.0417$$

$$\sum \frac{1}{R_{2j}} = (2)(2.003) + 0.0668 = 4.0728 = \sum \frac{1}{R_{3j}}$$

$$\sum \frac{1}{R_{4j}} = 2.003 + 0.0417 = 2.045$$

$$T_2 = \frac{(2.003)(300 + T_3) + (0.0668)(38)}{4.0728}$$

$$T_3 = \frac{(2.003)(T_2 + T_3) + (0.0668)(38)}{4.0728}$$

$$T_4 = \frac{(2.003)T_3 + (0.0417)(38)}{2.045}$$

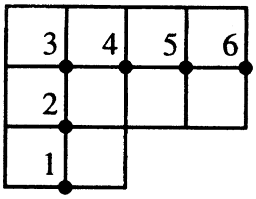
	A	B
1	T2=	=(2.003*(300+B2)+0.0668*38)/4.0728
2	T3=	=(2.003*(B1+B3)+0.0668*38)/4.0728
3	T4=	=(2.003*B2+0.0417*38)/2.045

<u>Solution:</u>

	A	B
1	T2=	279.5847538
2	T3=	267.2265034
3	T4=	262.5130984

$$q = -\frac{kA}{\Delta x}(T_2 - 300) = -(2.003)(279.5848 - 300) = 40.89 \text{ W}$$

3-48



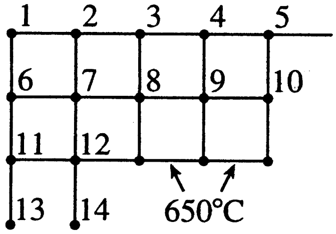
$\Delta x = \Delta y = 0.5$ m All internal nodes $k = 1.4$

	A	B
1	T1=	=(2*B2+800)/4
2	T2=	=(B1+B3+800)/4
3	T3=	=(B2+B4+300)/4
4	T4=	=(B3+B5+800)/4
5	T5=	=(B4+B6+800)/4
6	T6=	=(2*B5+800)/4

	A	B
1	T1=	379.2663
2	T2=	358.5327
3	T3=	254.8644
4	T4=	360.9250
5	T5=	388.8357
6	T6=	394.4179

q (inside) = 6781.2 W/m depth
q (outside) = 6187.2 W/m
q (avg) = 6484.2 W/m

3-49



$\Delta x = \Delta y = 0.5$ m $k = 1.4$ $h = 17$ $T_\infty = 38°C$

Cond $\dfrac{1}{R} = \dfrac{kA}{\Delta x} = 0.7$ conv. $\dfrac{1}{R} = hA = 8.5$

Node	$\sum \frac{1}{R_{ij}}$
1	9.9
2, 3, 4, 5, 6, 11, 13	11.3

Using the Gauss Seidel format:
The Equations:

	A	B
1	T1=	=(0.7*(B2+B6)+(8.5*38))/9.899999
2	T2=	=(0.7*(B1+B3)+(1.4*B7)+(8.5*38))/11.3
3	T3=	=(0.7*(B2+B4)+(1.4*B8)+(8.5*38))/11.3
4	T4=	=(0.7*(B3+B5)+(1.4*B9)+(8.5*38))/11.3
5	T5=	=(1.4*(B4+B10)+(8.5*38))/11.3
6	T6=	=(0.7*(B1+B11)+(1.4*B7)+(8.5*38))/11.3
7	T7=	=(B2+B6+B8+B12)/4
8	T8=	=(B3+B7+B9+650)/4
9	T9=	=(B4+B8+B10+650)/4
10	T10=	=(2*B9+B5+650)/4
11	T11=	=(0.7*(B6+B13)+(1.4*B12)+(8.5*38))/11.3
12	T12=	=(B7+B11+B14+650)/4
13	T13=	=(1.4*(B11+B14)+(8.5*38))/11.3
14	T14=	=(2*B12+B13+650)/4

The Solution:

	A	B
1	T1=	40.96808
2	T2=	59.00557
3	T3=	76.51266
4	T4=	82.08207
5	T5=	83.26541
6	T6=	58.97151
7	T7=	186.80459
8	T8=	316.30837
9	T9=	351.91623
10	T10=	359.27447
11	T11=	75.96281
12	T12=	312.93290
13	T13=	79.99096
14	T14=	338.96419

3-51

$$\frac{1}{R_{14}} = \frac{1}{R_{12}} = \frac{hA}{\Delta x} = 2.6$$

$$\frac{1}{R_{1-\infty}} = hA = 2.1$$

$$\frac{1}{R_{2-\infty}} = 4.2$$

$$\frac{1}{R_{2-5}} = 5.2$$

Node	$\sum \frac{1}{R_{ij}}$
1	7.3
2	14.6
4	10.4

The Equations:

	A	B
1	T1=	=(2.6*B2+2.6*B4)/7.3
2	T2=	=(2.6*B1+2.6*10+5.2*B5)/14.6
3		
4	T4=	=(2.6*B1+2.6*38+5.2*B5)/10.4
5	T5=	=(10+38+B2+B4)/4

The Solution:

	A	B
1	T1=	12.11864
2	T2=	11.24257
3		
4	T4=	22.78283
5	T5=	20.50635

3-53

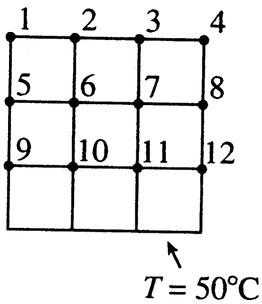
The Equations:

	A	B
1	T1=	=(1100+B3+B4)/4
2	T2=	=(600+B3+B4)/4
3	T3=	=(900+B1+B2)/4
4	T4=	=(800+B1+B2)/4

The Solution:

	A	B
1	T1=	487.5
2	T2=	362.5
3	T3=	437.5
4	T4=	412.5

3-55



$h = 40$ $\quad T_\infty = 300°C$ $\quad k_A = 20$ $\quad k_B = 1.2$ $\quad k_C = 0.5$

$\frac{1}{R_{12}} = 2.5$ $\quad \frac{1}{R_{26}} = 80$ $\quad \frac{1}{R_{1-\infty}} = 0.5$ $\quad \frac{1}{R_{2-\infty}} = 0.8$ $\quad \frac{1}{R_{15}} = 40$

$\frac{1}{R_{9-\infty}} = 0.7$ $\quad \frac{1}{R_{5-\infty}} = 0.4$ $\quad \frac{1}{R_{56}} = 2.95$ $\quad \frac{1}{R_{9-10}} = 0.70$ $\quad \frac{1}{R_{6-10}} = 1.6$

$R_{5-9} = 0.8$ $\quad R_{10-50} = 0.5$ $\quad R_{9-50} = 0.25$

The Equations:

	A	B
1	T1=	=(0.5*300+2.5*B2+40*B5)/43
2	T2=	=(0.8*300+2.5*B1+2.5*B3+80*B6)/85.8
3	T3=	=(0.8*300+2.5*B2+2.5*B4+80*B7)/85.8
4	T4=	=(0.5*300+2.5*B3+40*B8)/43
5	T5=	=(40*B1+2.95*B6+0.8*B9+0.4*300)/44.15
6	T6=	=(80*B2+2.95*B5+2.95*B7+1.6*B10)/87.5
7	T7=	=(80*B3+2.95*B6+2.95*B8+1.6*B11)/87.5
8	T8=	=(40*B4+2.95*B7+0.8*B12+0.4*300)/44.15
9	T9=	=(0.25*50+0.8*B5+0.7*B10+0.7*300)/2.45
10	T10=	=(0.5*50+1.6*B6+0.7*B9+0.7*B11)/3.5
11	T11=	=(0.5*50+1.6*B7+0.7*B10+0.7*B12)/3.5
12	T12=	=(0.25*50+0.8*B8+0.7*B11+0.7*300)/2.45

The Solution:

	A	B
1	T1=	254.9386
2	T2=	250.6814
3	T3=	250.6819
4	T4=	254.9397
5	T5=	254.6422
6	T6=	250.0561
7	T7=	250.0565
8	T8=	254.6434
9	T9=	234.0597
10	T10=	210.3326
11	T11=	210.3329
12	T12=	234.0603

3-56

$$\frac{1}{R_{12}}=0.25 \quad \frac{1}{R_{1-\infty}}=0.8 \quad \frac{1}{R_{1-5}}=2.0 \quad \sum\frac{1}{R_1}=3.3=\sum\frac{1}{R_2}=\sum\frac{1}{R_3}$$

$$\frac{1}{R_{4-\infty}}=0.6 \quad \frac{1}{R_{8-\infty}}=0.4 \quad \frac{1}{R_{5-6}}=0.5 \quad \frac{1}{R_{4-8}}=1.0$$

$$\sum\frac{1}{R_4}=1.85 \qquad \sum\frac{1}{R_5}=5=\sum\frac{1}{R_6}=\sum\frac{1}{R_7} \qquad \sum\frac{1}{R_8}=2.9$$

The Equations:

	A	B
1	T1=	=(75+0.25*B2+2*B5+16)/3.3
2	T2=	=(0.25*B1+0.25*B3+2*B6+16)/3.3
3	T3=	=(0.25*B2+0.25*B4+2*B7+16)/3.3
4	T4=	=(0.25*B3+B8+12)/1.85
5	T5=	=(150+0.5*B6+4*B1)/5
6	T6=	=(0.5*B5+0.5*B7+4*B2)/5
7	T7=	=(0.5*B6+0.5*B8+4*B3)/5
8	T8=	=(0.5*B7+2*B4+8)/2.9

The Solution:

	A	B
1	T1=	101.594256
2	T2=	45.79543347
3	T3=	28.45020861
4	T4=	23.35101566
5	T5=	116.4060956
6	T6=	51.30691016
7	T7=	30.29954059
8	T8=	24.08682814

3-57

The Equations:

	A	B
1	T1=	=(1.5+2*(B2/0.1)+B3/0.2)/25.15
2	T2=	=(2.25+B1/0.1+B4/0.4)/12.725
3	T3=	=(B1/0.2+2*(B4/0.05)+B5/0.2)/50
4	T4=	=(B2/0.4+B3/0.05+B6/0.4+3)/25.3
5	T5=	=(B3/0.2+2*(B6/0.05)+B7/0.2)/50
6	T6=	=(B4/0.4+B5/0.05+B8/0.4+3)/25.3
7	T7=	=(B5/0.2+2*(B8/0.05)+B9/0.2)/50
8	T8=	=(B6/0.4+B7/0.05+B10/0.4+3)/25.3
9	T9=	=(B7/0.2+2*(B10/0.05)+B13/0.2)/50
10	T10=	=(B8/0.4+B9/0.05+B14/0.1+B11/0.3+3.75)/36.21
11	T11=	=(B10/0.3+B15/0.08+B12/0.2+3.75)/21.21
12	T12=	=(B11/0.2+B16/0.2+1.5)/10.15
13	T13=	=(B9/0.2+2*(B14/0.05)+1000)/50
14	T14=	=(B10/0.1+B13/0.05+B15/0.15+2000)/46.67
15	T15=	=(B11/0.08+B14/0.15+B16/0.1+2500)/41.67
16	T16=	=(B12/0.2+B15/0.1+1000)/20

The Solution:

	A	B
1	T1=	109.7101
2	T2=	108.9655
3	T3=	115.6835
4	T4=	114.8946
5	T5=	127.9737
6	T6=	127.1008
7	T7=	147.2517
8	T8=	146.3765
9	T9=	173.5343
10	T10=	175.0162
11	T11=	184.3020
12	T12=	185.5951
13	T13=	187.9629
14	T14=	188.2619
15	T15=	191.5146
16	T16=	192.1561

3-58

$$\frac{1}{R_{12}} = \frac{(2.3)(0.125)}{0.25} = 1.15 = \frac{1}{R_{14}} \qquad \frac{1}{R_{1-\infty}} = (25)(0.125) = 3.125$$

$$\sum \frac{1}{R_{1-j}} = 5.425$$

The Equations:

	A	B
1	T1=	=(5*3.125+1.15*B2+1.15*B4)/5.425
2	T2=	=(1.15*B1+1.15*B3+6.25*5+2.3*B5)/10.85
3	T3=	=(1.15*B2+115+6.25*5+2.3*B6)/10.85
4	T4=	=(1.15*B1+2.3*B5+1.15*B7)/4.6
5	T5=	=(B2+B4+B6+B8)/4
6	T6=	=(B3+B5+B9+100)/4
7	T7=	=(1.15*B4+115+2.3*B8)/4.6
8	T8=	=(B5+B7+B9+100)/4
9	T9=	=(B6+B8+200)/4

The Solution:

	A	B
1	T1=	17.86873
2	T2=	19.51926
3	T3=	29.93323
4	T4=	51.18759
5	T5=	54.59206
6	T6=	67.86016
7	T7=	77.69751
8	T8=	79.80123
9	T9=	86.91535

3-59

$$\frac{1}{R_{1-\infty}} = (12)(0.25) = 3 \qquad \frac{1}{R_{12}} = \frac{(1.5)(0.125)}{0.25} = 0.75$$

The Equations:

	A	B
1	T1=	=(0.75*B2+37.5+45+1.5*B3)/6
2	T2=	=(0.75*B1+37.5+45+1.5*B4)/6
3	T3=	=(B1+B4+B5+50)/4
4	T4=	=(B2+B3+B6+50)/4
5	T5=	=(B3+B6+100)/4
6	T6=	=(B4+B5+100)/4

The Solution:

	A	B
1	T1=	27.6
2	T2=	27.6
3	T3=	41.6
4	T4=	41.6
5	T5=	47.2
6	T6=	47.2

3-60

$$\frac{1}{R_{12}} = \frac{(10)(0.005)}{0.01} = 5 \qquad \frac{1}{R_{1-\infty}} = (125)(0.01) = 1.25 \qquad \frac{1}{R_{1-5}} = 10$$

$$\Sigma = 21.25 \qquad \frac{1}{R_{59}} = 40 \qquad \frac{1}{R_{5-6}} = \frac{(40)(0.005)}{0.01} + \frac{10}{2} = 25$$

The Equations:

	A	B
1	T1=	=(10*B2+12.5+10*B5)/21.25
2	T2=	=(5*B1+5*B3+12.5+10*B6)/21.25
3	T3=	=(5*B2+5*B4+12.5+10*B7)/21.25
4	T4=	=(5*B3+12.5+5*B8)/11.25
5	T5=	=(10*B1+12000+50*B6)/100
6	T6=	=(25*B5+12000+10*B2+25*B7)/100
7	T7=	=(25*B6+10*B3+25*B11+10*B8)/70
8	T8=	=(10*B7+5*B4+5*B12+12.5)/21.25
9		
10		
11	T11=	=(12000+25*B7+10*B12+25*B14)/100
12	T12=	=(10*B11+12.5+5*B8+5*B15)/21.25
13		
14	T14=	=(50*B11+10*B15+12000)/100
15	T15=	=(10*B14+10*B12+12.5)/21.25

The Solution:

	A	B
1	T1=	253.5476
2	T2=	249.8672
3	T3=	236.6211
4	T4=	211.4409
5	T5=	287.6715
6	T6=	284.6335
7	T7=	270.9157
8	T8=	236.6211
9		
10		
11	T11=	284.6335
12	T12=	249.8672
13		
14	T14=	287.6715
15	T15=	253.5477

3-61

$$\frac{1}{R_{12}} = 0.5498 \qquad \frac{1}{R_{1-\infty}} = 0.05027 \qquad q_1 = (50\times 10^6)\pi(0.01)^2(0.02) = 3.142$$

$$\sum \frac{1}{R_1} = (2)(0.5498) + 0.05027 = 1.1499$$

$$\frac{1}{R_{5-\infty}} = (40)[\pi(0.01)^2 + \pi(0.01)(0.02)] = 0.0377$$

The Equations:

	A	B
1	T1=	=(200*0.5498+0.5498*B2+1.257+3.142)/1.1499
2	T2=	=(0.5498*(B1+B3)+1.257+3.142)/1.1499
3	T3=	=(0.5498*(B2+B4)+1.257+3.142)/1.1499
4	T4=	=(0.5498*(B3+B5)+1.257+3.142)/1.1499
5	T5=	=(0.5498*B4+0.0377*25+1.571)/0.5875

The Solution:

	A	B
1	T1=	172.9506
2	T2=	153.723
3	T3=	140.5582
4	T4=	132.2515
5	T5=	128.0432

3-62

$$\frac{1}{R_{12}} = 0.2 \qquad \frac{1}{R_{16}} = 0.05 \qquad \frac{1}{R_{1-\infty_2}} = 0.25 \qquad \frac{1}{R_{1-\infty_6}} = 0.5$$

$$\sum \frac{1}{R_{1-j}} = 1.0$$

The Equations:

	A	B
1	T1=	=(10+5+0.2*B2+0.05*B6)/1
2	T2=	=(0.2*B1+0.2*B3+0.1*B7+10)/1
3	T3=	=(0.2*B2+0.1*B4+0.15*B8+15)/1.2
4	T4=	=(0.1*B3+0.1*B5+0.2*B9+20)/1.4
5	T5=	=(0.2*B4+0.2*B10+20)/1.4
6	T6=	=(0.05*B1+0.05*B11+0.4*B7+20)/1.5
7	T7=	=(0.4*B6+0.4*B8+0.1*B2+0.1*B12)/1
8	T8=	=(0.4*B7+0.15*B3+75+0.2*B9)/0.9
9	T9=	=(0.2*B8+0.2*B4+100+0.2*B10)/0.8
10	T10=	=(0.4*B9+0.2*B5+100)/0.8
11	T11=	=(0.4*B12+0.05*B6+0.05*B13+20)/1.5
12	T12=	=(0.4*B11+0.1*B7+0.1*B14+200)/1
13	T13=	=(0.2*B11+0.4*B14+20)/1.5
14	T14=	=(0.4*B13+0.2*B12+200)/1

The Solution:

	A	B
1	T1=	25.497
2	T2=	38.959
3	T3=	50.304
4	T4=	59.435
5	T5=	61.412
6	T6=	54.101
7	T7=	137.990
8	T8=	210.866
9	T9=	260.187
10	T10=	270.446
11	T11=	93.603
12	T12=	281.077
13	T13=	105.378
14	T14=	298.367

3-63

$$\frac{1}{R_{12}} = \frac{(4.0)(0.00125)}{0.01} = 0.5 \qquad \frac{1}{R_{1-\infty}} = (75)(0.01) = 0.75$$

$$\frac{1}{R_{13}} = \frac{(4.0)(0.01)}{0.0025} = 16 \qquad \Sigma = 17.75$$

The Equations:

	A	B
1	T1=	=(0.5*B2+50+16*B3+0)/17.75
2	T2=	=(0.5*B1+50+16*B4)/17.75
3	T3=	=(1*B4+100+16*B5+16*B1)/34
4	T4=	=(1*B3+100+16*B6+16*B2)/34
5	T5=	=(0.5*B6+50+16*B3)/17
6	T6=	=(0.5*B5+50+16*B4)/17

The Solution:

	A	B
1	T1=	71.2456
2	T2=	71.2458
3	T3=	73.6871
4	T4=	73.6873
5	T5=	74.4844
6	T6=	74.4846

3-64

$$\frac{1}{R_{12}} = \frac{(20)(0.005)}{0.01} = 10 = \frac{1}{R_{14}}$$

$$q_1 = (90)(10^6)(0.005)(0.005) = 2250$$

$$\frac{1}{R_{1-\infty}} = (100)(0.005) = 0.5 \qquad \sum \frac{1}{R_{1-j}} = 20.5$$

The Equations:

	A	B
1	T1=	=(10*B2+10*B4+10+2250)/20.5
2	T2=	=(10*B1+10*B3+20*B5+20+4500)/41
3	T3=	=(10*B2+1000+20*B6+20+4500)/41
4	T4=	=(10*B1+10*B7+20*B5+4500)/40
5	T5=	=(20*(B4+B2+B6+B8)+9000)/80
6	T6=	=(20*(B3+B5+100+B9)+9000)/80
7	T7=	=(10*B4+10*B10+20*B8+4500)/40
8	T8=	=(20*(B5+B7+B9+B11)+9000)/80
9	T9=	=(20*(B6+B8+100+B12)+9000)/80
10	T10=	=(10*B7+10*B11+2250)/20
11	T11=	=(10*B10+10*B12+20*B8+4500)/40
12	T12=	=(10*B11+1000+20*B9+4500)/40

The Solution:

	A	B
1	T1=	1923.864
2	T2=	1723.906
3	T3=	1120.150
4	T4=	1994.015
5	T5=	1786.001
6	T6=	1158.355
7	T7=	2030.197
8	T8=	1817.727
9	T9=	1177.269
10	T10=	2041.320
11	T11=	1827.442
12	T12=	1182.995

3-65

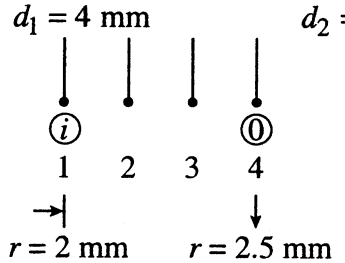
$r = 15\text{ cm} \qquad D = 80\text{ cm} \qquad D > 5r$

$$S = \frac{4\pi r}{\frac{\pi}{2} - \tan^{-1}\left(\frac{r}{D}\right)} = \frac{(4\pi)(0.15)}{\frac{\pi}{2} - \tan^{-1}\left(\frac{15}{80}\right)} = 1.3605$$

$q = kS\Delta T = (3.4)(1.3605)(120 - 34) = 398\text{ W}$

3-67

$d_1 = 4\text{ mm} \qquad d_2 = 5\text{ mm} \qquad k = 20 \qquad \dot{q} = 500 \times 10^6\ \text{W/m}^3$



node	r	$\frac{1}{R_+}$	$\frac{1}{R_-}$	$\sum\frac{1}{R}$	ΔV	q_i
1	2	1508	0	1508	1.05×10^{-6}	525
2	2.1667	1634	1508	3142	2.26×10^{-6}	1130
3	2.3333	1760	1634	3394	2.46×10^{-6}	1220
4	2.5	0	1760	1760	1.31×10^{-6}	654

$$\frac{1}{R} = \frac{kA}{\Delta x} = \frac{(20)(2\pi r)(1)}{\Delta r} = 7.54 \times 10^5 r \qquad \Delta r = 0.16667 \text{ mm} \qquad \Delta V = 2\pi r \Delta r$$

$$T_4 = 100°\text{C} \qquad T_i = \frac{q_i + \sum \frac{T_j}{R_{ij}}}{\sum \frac{1}{R_{ij}}}$$

The Equations:

	A	B
1	T1=	=(525+1508*B2)/1508
2	T2=	=(1130+1508*B1+1634*B3)/3142
3	T3=	=(1220+1634*B2+176000)/3394

The Solution:

	A	B
1	T1=	102.994
2	T2=	102.646
3	T3=	101.633

3-68

Inside surface, node 1

$$\frac{1}{R\_} = hA = 2\pi r h = 2\pi(0.002)(40) = 0.503$$

$$\sum \frac{1}{R} = 0.503 + 1508 = 1508.503$$

Must specify convection temperature
set at 100°C
Equation for node 1 becomes

$$T_1 = \frac{525 + 0.503(100) + 1508 T_2}{1508.503}$$

Other nodal equations are the same as in Prob. 3–67

The Equations:

	A	B
1	T1=	=(525+(0.503*100)+1508*B2)/1508.503
2	T2=	=(1130+1508*B1+1634*B3)/3142
3	T3=	=(1220+1634*B2+176000)/3394

The Solution:

	A	B
1	T1=	102.992
2	T2=	102.645
3	T3=	101.633

3-69

The Equations:

	A	B
1	T1=	=(20*B2+5*B3+1.5)/25
2	T2=	=(10*B1+2.5*B4+2.25)/12.5
3	T3=	=(40*B4+5*B1+5*B5)/50
4	T4=	=(20*B3+2.5*B2+2.5*B6+3)/25
5	T5=	=(40*B6+5*B3+5*B7)/50
6	T6=	=(20*B5+2.5*B4+2.5*B8+3)/25
7	T7=	=(40*B8+5*B5+5*B9)/50
8	T8=	=(20*B7+2.5*B6+2.5*B10+3)/25
9	T9=	=(40*B10+5*B7+5*B13)/50
10	T10=	=(20*B9+(10*B11/3)+10*B14+3.75)*(3/100)
11	T11=	=((10*B10/3)+5*B12+12.5*B15+3.75)/20.8333
12	T12=	=(5*B11+2.5*B16+1.5)/7.5
13	T13=	=(5*B9+1000+40*B14)/50
14	T14=	=(20*B13+10*B10+2000+(10*B15/1.5))/46.6667
15	T15=	=((10*B14/1.5)+12.5*B11+2500+10*B16)/41.6667
16	T16=	=(10*B15+2.5*B12+500)/15

The Solution:

	A	B
1	T1=	207.2562
2	T2=	207.3315
3	T3=	206.6584
4	T4=	206.7337
5	T5=	205.4644
6	T6=	205.5389
7	T7=	203.6795
8	T8=	203.7402
9	T9=	201.4115
10	T10=	201.2273
11	T11=	200.8919
12	T12=	200.9536
13	T13=	200.6177
14	T14=	200.5958
15	T15=	200.4772
16	T16=	200.4771

3-70

$$\frac{1}{R_{9-10}} = \frac{(40)(0.005)}{0.01} = 20 = \frac{1}{R_{10-13}} \qquad \frac{1}{R_{9-5}} = \frac{(40)(0.01)}{0.01} = 40$$

The Equations:

	A	B
1	T1=	=(10*B2+12.5+10*B5)/21.25
2	T2=	=(5*B1+5*B3+12.5+10*B6)/21.25
3	T3=	=(5*B2+5*B4+12.5+10*B7)/21.25
4	T4=	=(5*B3+12.5+5*B8)/11.25
5	T5=	=(10*B1+12000+50*B6)/100
6	T6=	=(25*B5+12000+10*B2+25*B7)/100
7	T7=	=(25*B6+10*B3+25*B11+10*B8)/70
8	T8=	=(10*B7+5*B4+5*B12+12.5)/21.25
9	T9=	=(40*B5+40*B10+3)/80
10	T10=	=(40*B6+20*B9+20*B13+40*B11+3)/120
11	T11=	=(12000+25*B7+10*B12+25*B14)/100
12	T12=	=(10*B11+12.5+5*B8+5*B15)/21.25
13	T13=	=(40*B10+40*B14+3)/80
14	T14=	=(50*B11+10*B15+12000)/100
15	T15=	=(10*B14+10*B12+12.5)/21.25

The Solution:

	A	B
1	T1=	253.5476
2	T2=	249.8672
3	T3=	236.6210
4	T4=	211.4409
5	T5=	287.6715
6	T6=	284.6335
7	T7=	270.9156
8	T8=	236.6210
9	T9=	286.5163
10	T10=	285.2861
11	T11=	284.6335
12	T12=	249.8672
13	T13=	286.5163
14	T14=	287:6715
15	T15=	253.5476

3-71

Add nodes 13, 14, 15, 16 on right side

$q_{13} = (500)(0.005)^2 + 2250 = 2252.5 = q_{16}$

$q_{14} = 5 + 4500 = 4505 = q_{15}$

Equations for nodes 1, 2, 4, 5, 7, 8, 10, 11 remain the same as in Prob. 3–~~58~~ 64.

The Equations:

	A	B
1	T1=	=(10*B2+10*B4+10+2250)/20.5
2	T2=	=(10*B1+10*B3+20*B5+20+4500)/41
3	T3=	=(10*B2+10*B13+20*B6+20+4500)/41
4	T4=	=(10*B1+10*B7+20*B5+4500)/40
5	T5=	=(20*(B4+B2+B6+B8)+9000)/80
6	T6=	=(20*(B3+B5+B9+B14)+9000)/80
7	T7=	=(10*B4+10*B10+20*B8+4500)/40
8	T8=	=(20*(B5+B7+B9+B11)+9000)/80
9	T9=	=(20*(B6+B8+B12+B15)+9000)/80
10	T10=	=(10*B7+10*B11+2250)/20
11	T11=	=(10*B10+10*B12+20*B8+4500)/40
12	T12=	=(10*B11+10*B16+20*B9+4500)/40
13	T13=	=(10*B3+10*B14+2262.5)/20.5
14	T14=	=(10*B13+20*B6+10*B15+4505)/40
15	T15=	=(10*B14+20*B9+10*B16+4505)/40
16	T16=	=(10*B12+10*B15+2252.5)/20

The Solution:

	A	B
1	T1=	27024.76
2	T2=	27024.80
3	T3=	27024.92
4	T4=	28149.96
5	T5=	28150.01
6	T6=	28150.13
7	T7=	28825.09
8	T8=	28825.13
9	T9=	28825.25
10	T10=	29050.13
11	T11=	29050.17
12	T12=	29050.29
13	T13=	27025.12
14	T14=	28150.33
15	T15=	28825.46
16	T16=	29050.50

3-72

Take resistance in terms of mean areas between nodes.

$k = 210 \ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ $\quad h = 200 \ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$ $\quad r_0 = 0.5 \text{ cm}$ $\quad r_4 = 0.25 \text{ cm}$

$T_\infty = 10^\circ\text{C}$ $\quad r_2 = 0.375 \text{ cm}$ $\quad r_1 = 0.4375 \text{ cm}$ $\quad r_3 = 0.3125 \text{ cm}$

$A_0 = \pi(0.005)^2 = 7.85 \times 10^{-5}$

$A_1 = \pi(0.004375)^2 = 6.013 \times 10^{-5} \text{ m}^2$

$A_2 = \pi(0.00375)^2 = 4.418 \times 10^{-5}$

$A_3 = \pi(0.003125)^2 = 3.068 \times 10^{-5} \text{ m}^2$

$A_4 = \pi(0.0025)^2 = 1.963 \times 10^{-5}$

Assume convection areas are $\pi d \Delta x$ at node.

$$\frac{1}{R_{10}} = \frac{kA}{\Delta x} = \frac{(210)(6.013 + 7.85)(10^{-5})}{(2)(0.015)} = 0.9704$$

$$\frac{1}{R_{12}} = \frac{(210)(6.013 + 4.418)(10^{-5})}{(2)(0.015)} = 0.7302$$

Also,

$$\frac{1}{R_{23}} = \frac{(210)(4.418+3.068)(10^{-5})}{(2)(0.015)} = 0.524$$

$$\frac{1}{R_{34}} = \frac{(210)(3.068+1.963)(10^{-5})}{(2)(0.015)} = 0.3522$$

$$\frac{1}{R_{4+}} = 0$$

$$\frac{1}{R_{1-\infty}} = \pi(2)(0.4375)(0.015)(200)(0.01) = 0.0825$$

$$\frac{1}{R_{2-\infty}} = \pi(2)(0.375)(0.015)(200)(0.01) = 0.0707$$

$$\frac{1}{R_{3-\infty}} = \pi(2)(0.3125)(0.015)(200)(0.01) = 0.0589$$

$$\frac{1}{R_{4-\infty}} = \pi(2)(0.25)(0.0075)(200)(0.01) = 0.0236$$

$$0.9704(200-T_1)+0.7302(T_2-T_1)+0.0825(10-T_1)=0$$
$$0.7302(T_1-T_2)+0.524(T_3-T_2)+0.0707(10-T_2)=0$$
$$0.524(T_2-T_3)+0.3522(T_4-T_3)+0.0589(10-T_3)=0$$
$$0.3522(T_3-T_4)+0.0236(10-T_4)=0$$

$$\begin{array}{lllll}
-1.7831T_1 & +0.7302T_2 & & & =-194.905 \\
0.7302T_1 & -1.3249T_2 & +0.524T_3 & & =-0.707 \\
 & 0.524T_2 & -0.9351T_3 & +0.3522T_4 & =-0.589 \\
 & & 0.3522T_3 & -0.3758T_4 & =-0.236
\end{array}$$

Solution:

$T_1 = 167.45°C$ $T_2 = 141.99°C$ $T_3 = 124.32°C$ $T_4 = 117.14°C$

3-73

$$Bi = \frac{n\Delta x}{k}$$

Node

(1) $BiT_\infty + T_3 + T_2 - (Bi+2)T_1 = 0$

(2) $2BiT_\infty + T_1 + T_4 - 2(Bi+1)T_2 = 0$

(3) $T_1 + 2T_4 + 100 - 4T_3 = 0$

(4) $2BiT_\infty + (2)(100) + 2T_3 + T_2 - 2(Bi+2)T_4 = 0$

(5) $BiT_\infty + \frac{1}{2}(200 + T_4 + T_6) - (Bi+2)T_5 = 0$

(6) $BiT_\infty + \frac{1}{2}(200 + T_5 + T_7) - (Bi+2)T_6 = 0$

(7) $T_6 + 100 - (Bi+2)T_7 = 0$

3-74

Initial Equations:

$$140 + T_2 + T_3 - 4T_1 = 0$$
$$T_1 + T_4 + T_5 + 100 - 4T_3 = 0$$
$$40 + T_1 + T_4 - 4T_2 = 0$$
$$T_2 + T_3 + T_6 - 4T_4 = 0$$
$$200 + T_3 + T_6 - 4T_5 = 0$$
$$100 + T_4 + T_5 - 4T_6 = 0$$

Matrix Form:

T_1	T_2	T_3	T_4	T_5	T_6	C
–4	1	1	0	0	0	–140
1	–4	0	1	0	0	–40
1	0	–4	1	1	0	–100
0	1	1	–4	0	1	0
0	0	1	0	–4	1	–200
0	0	0	1	1	–4	–100

Solution:

$T_1 = 61.615$ $T_2 = 35.528$ $T_3 = 70.932$ $T_4 = 40.969$ $T_5 = 81.615$

$T_5 = 55.528°C$

3-75

Node:

(1) $100 + 2T_2 + T_4 - 4T_1 = 0$

(2) $T_1 + 100 + T_3 + T_5 - 4T_2 = 0$

(3) $T_2 + 150 + T_6 - 4T_3 = 0$

(4) $T_1 + T_7 + 2T_5 - 4T_4 = 0$

(5) $T_4 + T_2 + T_6 + T_8 - 4T_5 = 0$

(6) $T_5 + T_3 + 50 + T_9 - 4T_6 = 0$

(7) $\frac{k}{2}[T_4 + T_8 - 2T_7] + q''\frac{\Delta x}{2} = 0$

(8) $\frac{k}{2}(T_7 + 2T_5 + T_9 - 4T_8) + q''\frac{\Delta x}{2} = 0$

(9) $\frac{k}{2}[T_8 + 2T_6 + 50 - 4T_9] + q''\frac{\Delta x}{2} = 0$

3-76

Node:

(1) $$k\frac{\Delta x/2}{\Delta y}(T_5-T_1)+h\frac{\Delta x}{2}(T_\infty-T_1)+\frac{k\,\Delta y/2}{\Delta x}(T_2-T_1)=0$$

$$T_5-T_1+\frac{h\Delta y}{k}(T_\infty-T_1)+T_2-T_1=0$$

$$T_1=\frac{T_5+T_2+\frac{h\Delta y}{k}}{2+\frac{h\Delta y}{k}}$$

Let $Bi=\dfrac{h\Delta y}{k}$

$k=10\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ $\qquad h=30\ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$ $\qquad T_\infty=15^\circ\text{C}$ $\qquad \Delta x=\Delta y=0.01\ \text{m}$

$$Bi=\frac{h\Delta y}{k}=0.03$$

(2) $$T_2=\frac{T_6+\frac{T_1+T_3}{2}+BiT_\infty}{2+Bi}$$

(3) $$T_3=\frac{T_7+\frac{T_2+T_4}{2}+BiT_\infty}{2+Bi}$$

(4) $$T_4=\frac{T_8+\frac{T_3+50}{2}+BiT_\infty}{2+Bi}$$

(5) $$T_5=\frac{T_1+T_9+2T_6}{4}$$

(6) $$T_6=\frac{T_5+T_2+T_7+T_{10}}{4}$$

(7) $$T_7=\frac{T_6+T_3+T_8+T_{11}}{4}$$

(8) $$T_8=\frac{T_7+T_4+100+T_{12}}{4}$$

(9) $$T_9=\frac{T_5+T_{10}}{2}$$

(10) $$T_{10}=\frac{T_9+T_{11}+2T_6}{4}$$

(11) $$T_{11}=\frac{T_{10}+T_{12}+2T_7}{4}$$

(12) $$T_{12}=\frac{T_{11}+150+2T_8}{4}$$

3-77

$x^2 + y^2 = 100$ circle, $r = 10$

at $x = 6,\ y = 8$ cm $\quad 6 + b\Delta y = 8 \quad b = \dfrac{2}{3}$

at $y = 6,\ x = 8$ and $\quad a = \dfrac{2}{3}$

at $y = 9,\ x = \sqrt{19} = 4.359$ cm $\quad 9 - 8 = 1 = c\Delta y \quad c = \dfrac{1}{3}$

Using Table 3–2(f) for node 4

$$0 = \frac{2}{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}T_3 + \frac{2}{\frac{4}{3}}T_6 + \frac{2}{\frac{5}{3}}T_5 + \frac{2}{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)} - 2\left(\frac{3}{2} + \frac{3}{2}\right)T_4$$

Using Table 3–2(g) for node 3

$$\frac{h\Delta x}{k} = \frac{(30)(0.03)}{10} = 0.09$$

$$\frac{\frac{2}{3}}{\left[\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right]^{1/2}}T_2 + \frac{\frac{2}{3}}{\left(\frac{1}{3}\right)^2 + 1}T_7 + \frac{\frac{4}{3}}{\frac{2}{3}}T_4$$

$$+ (0.09)\left\{\left[\left(\frac{1}{3}\right)^2 + 1\right]^{1/2} + \left[\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right]^{1/2}\right\}(20)$$

$$-\text{[Insert values of } a,\ b,\ c, \text{ etc.]}T_3 = 0$$

3-78

$b = \dfrac{1}{2} \qquad a = \dfrac{1}{2} \qquad c = \dfrac{3}{2}$

Nomenclature fits Table 3–2(f, g) exactly. Insert above values for final equations.

3-79

Same as Prob. 3–7~~0~~6 except that insulated surface is equivalent to $h = 0$, inserted in the equations.

3-80

The same as Prob. 3–7~~0~~6 except that T_∞ would be inserted for T_1 and T_3.

3-81

Assume $\Delta x = \Delta y$
Nodes 3 and 6 correspond to Node m, n of Table 3–2(f) but for different media.
The correspondence with the equations in Table 3–2(f) is as follows:

<u>Node 3, Material A</u>

$a = 0.5 \qquad b = 0.5 \qquad T_{m,\,n} = T_3 \qquad T_{m,\,n-1} = T_2 \qquad T_{m+1,\,n} = T_1$

$T_2 = T_4 \qquad T_1 = T_5$

<u>Node 6, Material A</u>

$a = 0.5 \qquad b = 0.5 \qquad T_{m,\,n} = T_6 \qquad T_{m,\,n-1} = T_8 \qquad T_{m+1,\,n} = T_7$

$T_2 = T_4 \qquad T_1 = T_9$

For nodes 4 and 5, the connecting resistances between the two materials must be taken into account $a = \frac{1}{2},\ b = \frac{1}{2},\ c = \frac{1}{2}$ for Material A.

<u>Node 4, Material A</u>

$$R_{43_A} = \frac{2b}{k_A(a+1)} \qquad R_{49_A} = \frac{2(c^2+1)^{1/2}}{bk_A} \qquad R_{45_A} = \frac{2(a^2+b^2)^{1/2}}{bk_A}$$

<u>Node 4, Material B</u>

$$a = \frac{1}{2} \qquad b = \frac{1}{2} \qquad c = \frac{1}{2}$$

$$R_{46_B} = \frac{2a}{(1+b)k_B} \qquad R_{45_B} = \frac{2(c^2+1)^{1/2}}{bk_B} \qquad R_{49_B} = \frac{2(a^2+b^2)^{1/2}}{bk_B}$$

$$R_{49}\big|_{\text{Tot}} = \frac{1}{\frac{1}{R_{49_A}} + \frac{1}{R_{49_B}}} = R_{49} \qquad R_{45}\big|_{\text{Tot}} = \frac{1}{\frac{1}{R_{45_A}} + \frac{1}{R_{45_B}}} = R_{45}$$

<u>Nodal Equation for Node 4:</u>

$$\frac{T_6 - T_4}{R_{46_B}} + \frac{T_3 - T_4}{R_{43_A}} + \frac{T_9 - T_4}{R_{49}} + \frac{T_5 - T_4}{R_{45}} = 0$$

Similar equation for Node 5 in terms of nodes 3, 4, 10, and 11.

3-82

$W > L \qquad W = 50 \text{ cm} \qquad L = 25 \text{ cm}$

$$S = \frac{2\pi W}{\ln(4W/L)} = \frac{2\pi(0.5)}{\ln(200/25)} = 1.511$$

$q = kS\Delta T = (1.511)(2.8)(78-15) = 266.5 \text{ W}$

3-83

$L = 0.2$ m $\quad S = 8.24L = 1.648$
$q = kS\Delta T = (2.3)(1.648)(80-10) = 265.3$ W
Sphere
$S = 4\pi r = 4\pi(0.1) = 1.2567$
$q = kS\Delta T = (2.3)(1.2567)(80-10) = 202.32$ W
$V\,(\text{cube}) = (0.2)^3 = 8\times10^{-3}\ \text{m}^3$

$$V\,(\text{sphere}) = \frac{4}{3}\pi(0.1)^3 = 4.189\times10^{-3}\ \text{m}^3$$

$$\frac{q}{V}\,(\text{cube}) = \frac{265.3}{8\times10^{-3}} = 33{,}162\ \text{W/m}^3$$

$$\frac{q}{V}\,(\text{sphere}) = \frac{202.32}{4.189\times10^{-3}} = 48{,}300\ \text{W/m}^3$$

3-84

$r = 5$ cm $\quad D = 15$ cm

$$\frac{S}{L} = \frac{2\pi}{\ln(4D/r)} = \frac{2\pi}{\ln(60/5)} = 2.529$$

$$\frac{q}{L} = kS\Delta T = (10)(2.529)(100-20) = 2023\ \text{W}$$

3-86

$W = 90$ cm $\quad L = 10$ cm $\quad D = 2.0$ m $\quad \Delta T = 50-10 = 40°\text{C} \quad k = 1.5$
$D > 2W \quad W >> L$

$$S = \frac{2\pi W}{\ln\left(\frac{2\pi D}{L}\right)} = \frac{2\pi(0.9)}{\ln\left(\frac{2\pi(2)}{0.1}\right)} = 1.17$$

$q = kS\Delta T = (1.5)(1.17)(40) = 70.19$ W

3-87

$\Delta T = 75-15 = 60°\text{C} \quad r = 5$ cm $\quad S = 4r = 0.2$ m
$q = kS\Delta T = (3)(0.2)(60) = 36$ W

3-88

$\Delta T = 60°\text{C} \qquad W = L = 0.1$

Use $S = \dfrac{\pi W}{\ln\left(\frac{4W}{L}\right)} = \dfrac{\pi(0.1)}{\ln(4)} = 0.227$

$q = kS\Delta T = (3)(0.227)(60) = 40.86$

$$\frac{q}{A_{\text{square}}} = \frac{40.86}{(0.1)^2} = 4086 \ \text{W/m}^2$$

$$\frac{q}{A_{\text{disc}}} = \frac{(36)(4)}{\pi(0.1)^2} = 4584 \ \text{W/m}^2$$

3-89

$S = \dfrac{2\pi L}{\ln\left(\frac{0.54W}{r}\right)} \qquad r = 5 \text{ cm} \qquad L = 20 \text{ m} \qquad W = 20 \text{ cm}$

$$S = \frac{2\pi(20)}{\ln\left(\frac{(0.54)(20)}{5}\right)} = 163.2$$

$q = kS\Delta T = (50 \times 10^{-3})(163.2)(200 - 35) = 1346 \text{ W}$

3-90

$q = kS\Delta T \qquad \Delta T = 100 - 24 \qquad k = 0.76 \ \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \qquad r = 5 \text{ cm}$

$D = 35 \text{ cm} \qquad L = 1 \text{ m}$

$$S = \frac{2\pi}{\ln\left(\frac{4D}{r}\right)} = \frac{2\pi(1)}{\ln\left[\frac{(4)(0.35)}{0.05}\right]} = 1.886$$

$q = (0.76)(1.886)(100 - 24) = 108.9 \text{ W}$

3-91

$A = 2\pi rt$ (conduction) $\qquad \Delta V = 2\pi r\Delta rt \qquad A_{\text{conv}} = 2\pi r\Delta r(2)$

$T_0 =$ base temperature $\qquad T_\infty =$

<u>Base node (adjacent)</u>

$$k(T_0 - T_m)\left(r_m - \frac{\Delta r}{2}\right)\frac{2\pi t}{\Delta r} + \dot{q}(2\pi)r_m\Delta rt + h(2)(2\pi)r_m\Delta r(T_\infty - T_m)$$

$$= k(T_m - T_{m+1})\left(r_m + \frac{\Delta r}{2}\right)\left(\frac{2\pi t}{\Delta r}\right)$$

<u>Middle node</u>

$$k(T_{m-1} - T_m)\left(r_m - \frac{\Delta r}{2}\right)\frac{2\pi t}{\Delta r} + \dot{q}(2\pi)(r_m\Delta rt) + h(2)(2\pi r_m\Delta r)(T_\infty - T_m)$$

$$= k(T_m - T_{m+1})\left(r_m + \frac{\Delta r}{2}\right)\left(\frac{2\pi t}{\Delta r}\right)$$

Tip node

$$k(T_{m-1}-T_m)\left(r_m-\frac{\Delta r}{2}\right)\frac{2\pi t}{\Delta r}+\dot{q}(2\pi)r_m\left(\frac{\Delta r}{2}\right)t+h(2)(2\pi r_m)\frac{\Delta r}{2}(T_\infty-T_m)$$
$$=h(2\pi r_m)t(T_m-T_\infty)$$

3-92

$A_{\text{cond}}=2\pi rt$ $\qquad r=0.25x+0.01$ $\qquad x=0$ at top of cone

$$kt(2\pi r_{x-\frac{\Delta x}{2}})\frac{T_{m-1}-T_m}{\Delta x}+kt(2\pi r_{x+\frac{\Delta x}{2}})\frac{T_{m+1}-T_m}{\Delta x}=0$$

$$\left[0.25\left(x-\frac{\Delta x}{2}\right)+0.01\right](T_{m-1}-T_m)+\left[0.25\left(x+\frac{\Delta x}{2}\right)+0.01\right](T_{m+1}-T_m)=0$$

3-93

$T_0=50\text{ K}$ $\qquad T_6=0\text{ K}$ $\qquad r_1=\frac{1.5}{2}=0.75\text{ cm}$ $\qquad r_2=\frac{1.7}{2}=0.85\text{ cm}$

$r_m=\frac{r_1+r_2}{2}$ $\qquad \Delta\theta=\frac{\pi}{12}$ $\qquad rd\theta=r_m\Delta\theta$ $\qquad A=(r_2-r_1)(1)$ per unit depth

$k=100\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$

Node 1

$$kA\frac{T_0-T_1}{rd\theta}+kA\frac{T_2-T_1}{rd\theta}=0$$

$T_1=\frac{T_0+T_2}{2}$ $\qquad T_2=\frac{T_1+T_3}{2}$ $\qquad T_3=\frac{T_2+T_4}{2}$ $\qquad T_5=\frac{T_4+T_6}{2}$

Solution

$T_0=50$
$T_1=41.667$
$T_2=33.333$
$T_3=25$
$T_4=16.667$
$T_5=8.333$
$T_6=0$

Because outside surface is insulated a linear temperature distribution results.

3-94

$S=8.246$ $\qquad L=0.3$
$=2.472$
$q=kS\Delta T=(1.8)(2.472)(30-10)=88.99\text{ W}$

3-95

$D = 0.2 \qquad r = 0.025$

$$S = \frac{4\pi(0.075)}{\frac{\pi}{2} + \tan^{-1}\left(\frac{0.075}{0.4}\right)} = 0.5367$$

$q = kS\Delta T = (2.7)(0.5367)(87 - 13) = 107.2\ \text{W}$

3-96

$$s = \frac{4\pi L}{\ln\left(\frac{2L}{r}\right)} = \frac{2\pi(0.4)}{\ln\left(\frac{80}{1}\right)} = 0.5734$$

$q = kS\Delta T = (3.4)(0.5734)(50 - 20) = 58.5\ \text{W}$

3-97

$T_0 = 323\ \text{K} \qquad T_6 = 373\ \text{K} \qquad T_\infty = 303\ \text{K} \qquad r_1 = \frac{1.5}{2} \qquad r_2 = \frac{1.7}{2}$

$r_m = \frac{r_1 + r_2}{2} \qquad rd\theta = r_m \Delta\theta \qquad \Delta\theta = \frac{\pi}{12} \qquad A = (r_2 - r_1)(1)$ per m depth

$k = 100 \qquad h = 75$

Node 1

$$kA\frac{T_0 - T_1}{rd\theta} + kA\frac{T_2 - T_1}{rd\theta} + 2hrd\theta(1)(T_\infty - T_1) = 0$$

Let $C = \dfrac{2hr_m(\Delta\theta)^2}{kA}$

$$T_1 = \frac{T_0 + T_2 + CT_\infty}{2 + C}$$

$$T_2 = \frac{T_1 + T_3 + CT_\infty}{2 + C}$$

$$T_3 = \frac{T_2 + T_4 + CT_\infty}{2 + C}$$

$$T_4 = \frac{T_3 + T_5 + CT_\infty}{2 + C}$$

$$T_5 = \frac{T_4 + T_6 + CT_\infty}{2 + C}$$

Solution

$T_1 = 330.7$
$T_2 = 338.6$
$T_3 = 346.7$
$T_4 = 355.1$
$T_5 = 363.9$

Analytical solution is obtained from temperature equation of Example 2–10 with $\dot{q} = 0$ and

$$m = \left(\frac{hP}{kA}\right)^{1/2} \quad P = 2[(r_2 - r_1) + 1] \quad \theta_1 = T_0 - T_\infty \quad \theta_2 = T_6 - T_\infty \quad x_1 = r_m \Delta\theta$$

$x_2 = 2r_m \Delta\theta$; etc.

Numerical solution agrees with analytical solution.

3-99

$$q = \frac{\Delta T}{R_{\text{overall}}}$$

$$R_{\text{overall}} = R_{\text{inside convection}} + R_{\text{wall}} + R_{\text{outside}}$$

$$R_{\text{inside convection}} = \frac{1}{hA_{\text{inside}}}$$

$$R_{\text{wall}} = \frac{\Delta x}{kA_{\text{wall mean}}}$$

$$A_{\text{wall mean}} = \frac{A_{\text{inside}} + A_{\text{outside}}}{2}$$

$$R_{\text{outside}} = \frac{1}{k_{\text{soil}} S}$$

S = shape factor from outside wall to soil which must be estimated by considering several different geometries.

3-100

For this problem the overall thermal resistance will be the sum of the resistance of the pipe wall plus the resistance resulting from the shape factor for a buried pipe. From a practical standpoint one might want to use several small pipes operating in series to minimize pressure losses due to fluid friction, but that is not a consideration in this problem. Properties of plastic pipe may be taken as that of PVC from the appendix. The overall temperature difference may be taken as the mean water temperature minus the soil temperature at depth. To start the selection process take the outside wall temperature as that of the water and choose some convenient depth for the pipe. Then select a standard diameter (1 inch, 2 inch) and determine the shape factor per unit length. Then determine the required surface area and pipe length. Then refine calculation by including pipe wall thermal resistance. Repeat the calculation for several pipe sizes for both steel and plastic. If possible, obtain some cost information for the pipe as a function of diameter and examine the influence on the total cost. Something like copper tubing is probably not a viable alternative because of the high cost. PVC pipe will probably represent the lowest installed cost option.

3-101

For this problem only consider conventional electric burners which have spiral heating coils. Halogen and radiant burners have different characteristics. To simplify the study, one might consider just the aluminum layer for an initial calculation. The major spreading effect is caused by this layer. After the heat spreads, the stainless steel acts mainly as a series thermal resistance and the objective here is to evaluate the effectiveness of the aluminum in bringing about uniform heating. Although the pan is circular, an initial study could consider just the spreading effect of approximately 6 mm heater elements spaced 18 mm on centers in intimate contact with an infinite aluminum plate exposed to the given convection condition on the top surface. The objective is to determine the uniformity of the surface temperature in contact with the water. The analysis can then be refined to include a contact conductance between the heater element and the pan, but this should mainly be evidenced as a lowering of the effective burner temperature. Keep in mind that the given value of $h = 1500 \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$ could vary by ±40% or more, so elaborate numerical models are not necessary.

3-103

The analysis of this problem requires that the heat gained by the building from all sources be equal to the cooling supplied by the refrigeration system. The heat gained from the ground passes through two thermal resistances: that of the concrete slab and that resulting from the shape factor of the slab in contact with the ground. In addition, of course, there is an internal convection resistance. The heat gained from the outside environment air passes through thermal resistances of the outside convection, concrete wall, inside convection, and whatever resistance results from installation of the outside insulating material. Note that the heat gained through the floor slab will not depend on the outside insulation, as long as the interior conditions are maintained constant.

Chapter 4

4–1

$T_\infty = T_m + A_m \sin \omega\tau$

Energy balance: $q = hA(T - T_\infty) = \rho c V\left(\dfrac{dT}{d\tau}\right)$

Let $K = \dfrac{hA}{\rho c V}$ along with initial condition $T = T_0$ at $\tau = 0$;
Solution is:

$$T - T_m = (T_0 - T_m)e^{-K\tau} + \left(\frac{KA_m}{\omega^2 + K^2}\right)[\omega(e^{-K\tau} - \cos\omega\tau) + K\sin\omega\tau]$$

4–2

$\alpha = 1.8\times10^{-6}\ \mathrm{m^2/sec}$ $\quad 2L = 2.5$ cm $\quad T_i = 150°\mathrm{C}$ $\quad T_1 = 30°\mathrm{C}$

$\tau = 1$ min $= 60$ sec $\quad \dfrac{\pi x}{2L} = \dfrac{\pi}{2};\ \left(\dfrac{\pi}{2L}\right)^2 \alpha\tau = 1.705$

1st four nonzero terms $n = 1,\ 3,\ 5,\ 7$

$$\frac{T - T_1}{T_i - T_1} = \frac{4}{\pi}[0.1818 - 7.22\times10^{-8} + 6.15\times10^{28}] = 0.231$$

$T = 30 + (0.231)(150 - 30) = 57.8°\mathrm{C}$ $\quad \dfrac{\alpha\tau}{L^2} = 0.69$ $\quad \dfrac{\theta_0}{\theta_i} = 0.25$

4–3

at $\tau = 0$ $\quad \dfrac{x}{2L} = \dfrac{1}{2}$ $\quad \dfrac{\pi x}{2L} = \dfrac{\pi}{2}$

$$\frac{T - T_1}{T_i - T_1} = \frac{4}{\pi}\left(\sin\frac{\pi}{2} + \frac{1}{3}\sin\frac{3\pi}{2} + \frac{1}{5}\sin\frac{5\pi}{2} + \frac{1}{7}\sin\frac{7\pi}{2}\right) = 0.9216$$

correct value is 1.0 $\quad$ Error = 7.84%

4–4

$$q = \sigma A(T^4 - T_\infty{}^4) + hA(T - T_\infty) = -c\rho V\frac{dT}{d\tau}$$

4–5

$T_0 = 250°C \qquad T = 90°C \qquad T_\infty = 35°C$

$$R_{th} = \frac{1}{2}\left(\frac{\Delta x}{kA}\right) = \left(\frac{1}{2}\right)\left[\frac{0.05}{(0.2)^2(370)}\right] = 1.69 \times 10^{-3}$$

$$C_{th} = \rho c V = (8900)(380)(0.05)(0.2)^2 = 6764$$

$$\frac{1}{R_{th}C_{th}} = 0.0875$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp(-0.0875\tau) = \frac{90-35}{250-35} = 0.2558$$

$\tau = 15.58$ sec

4–6

$m = \rho V \qquad \rho = 2707 \ \text{kg/m}^3 \qquad c = 896 \ \frac{\text{J}}{\text{kg}\cdot°\text{C}} \qquad h = 58$

$\frac{4}{3}\pi r^3(2700) = 6 \qquad r = 0.0807$ m

$A = 4\pi r^2 = 0.0822 \ \text{m}^2$

$$\frac{hA}{\rho cV} = \frac{(58)(0.0822)}{(6)(896)} = 8.87 \times 10^{-4}$$

$$\frac{90-20}{300-20} = \exp(-8.87 \times 10^{-4}\tau) = 0.25$$

$\tau = 1563$ sec

4–9

$$\frac{q}{A} = \sigma\varepsilon(T^4 - T_s^4) = -\rho c(2L)\frac{dT}{d\tau}$$

$$\frac{dT}{T_s^4 - T^4} = \frac{\sigma\varepsilon}{2\rho cL}d\tau$$

$$\int_{T_i}^{T} \frac{dT}{T_s^4 - T^4} = \left[\frac{1}{4T_s^3}\log\left|\frac{T_s + T}{T_s - T}\right| + \frac{1}{2T_s^3}\tan^{-1}\left(\frac{T}{T_s}\right)\right]_{T_i}^{T} \qquad \text{(a)}$$

$$\int_0^{\tau} \frac{\sigma\varepsilon d\tau}{2\rho cL} = \frac{\sigma\varepsilon\tau}{2\rho cL} \qquad \text{(b)}$$

Setting (a) = (b) produces an equation for T as a function of τ. For specific problems the answer is more easily obtained with numerical methods.

4–10

$T_0 = 25°C$ $T_\infty = 150°C$ $h = 120 \dfrac{W}{m^2 \cdot °C}$ $T = 120°C$

$\rho = 7817$ $c = 460$ $d = 6.4$ mm

$$\frac{A}{V} = \frac{4}{d}$$

$$\frac{hA}{\rho c V} = \frac{(120)(4)}{(0.0064)(7817)(460)} = 0.02086$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hA}{\rho c V}\tau\right)$$

$$\frac{120 - 25}{150 - 25} = 0.76 = e^{-0.02086\tau}$$

$\tau = 13.16$ sec

4–11

$T_\infty = 20°C$ $T_0 = 200°C$ $h = 28$ $d = 5$ cm $T = 90°C$

$\rho = 8954$ $c = 383$

$$\frac{A}{V} = \frac{3}{r}$$

$$\frac{hA}{\rho c V} = \frac{(28)(3)}{(0.025)(8954)(383)} = 9.8 \times 10^{-4}$$

$$\frac{90 - 20}{200 - 20} = 0.3889 = e^{-9.8\times10^{-4}\tau}$$

$\tau = 964$ sec

4–13

Lumped Capacity $\rho = 8954$ $c = 383$

$$\frac{hA}{\rho c V} = \frac{(15)(4\pi)(0.015)^2}{(8954)(383)\left[\frac{4}{3}\pi(0.015)^3\right]} = 8.75 \times 10^{-4}$$

$$\frac{25 - 10}{50 - 10} = e^{-8.75\times10^{-4}\tau}$$ $\tau = 1121$ sec

4–14

$\rho = 2707$ $c = 896$ $\sigma A T^4 = -\rho c V \dfrac{dT}{d\tau}$ T in °K

$$\frac{dT}{T^4} = \frac{-\sigma A}{\rho c V} d\tau \qquad \frac{1}{T^3} - \frac{1}{T_0^3} = \frac{\sigma A \tau}{\rho c V}$$

$T = -240 + 273 = 33$ K $T_0 = 40 + 273 = 313$ K $\tau = 9.9 \times 10^6$ sec

4–15

$\rho = 999.8 \qquad c = 4225 \qquad L = 2d \qquad A = 2.5\pi d^2 \qquad V = \frac{1}{2}\pi d^3$

$d = 6.06 \text{ cm} \qquad A = 288.5 \text{ cm}^2$

$$\frac{hA}{\rho c V} = \frac{(15)(288.5)(10^{-4})}{(999.8)(4225)(350)(10^{-6})} = 2.927 \times 10^{-4}$$

$$\frac{15-20}{1-20} = e^{-2.927\times10^{-4}\tau} \qquad \tau = 456 \text{ sec}$$

4–16

$$\frac{h(V/A)}{k} = \frac{(10)(0.006)}{(3)(204)} = 9.8 \times 10^{-5}$$

lumped capacity $\qquad \rho = 2707 \qquad c = 896$

$$\frac{200-20}{400-20} = e^{-\left[\frac{(10)(3)\tau}{(0.006)(896)(2707)}\right]} = 0.4737 \qquad \tau = 362 \text{ sec}$$

4–17

$$\frac{h(V/A)}{k} = \frac{hV}{3k} = \frac{(20)(0.02)}{(3)(380)} = 3.5 \times 10^{-4}$$

lumped capacity $\qquad c = 383 \qquad \rho = 8954$

$$\frac{80-30}{220-30} = e^{-\left[\frac{(20)(3)\tau}{(0.02)(383)(8954)}\right]} = 0.263 \qquad \tau = 1494 \text{ sec}$$

4–18

$A = \frac{90-35}{2} = 27.5°\text{C} \qquad x = 5 \text{ cm} = 0.05 \text{ m} \qquad k = 1.37 \ \frac{\text{W}}{\text{m}\cdot°\text{C}}$

$a = 7 \times 10^{-7} \ \text{m}^2/\text{sec} \qquad n = \frac{1 \text{ cyc}}{15 \text{ min}} = 1.1111 \times 10^{-3} \ \text{cyc/sec}$

$$x = \sqrt{\frac{\pi n}{\alpha}} = 0.05\left[\frac{\pi(1.1111\times10^{-3})}{7\times10^{-7}}\right]^{1/2} = 3.531$$

$$2n\pi\tau = 2\pi(1.1111\times10^{-3})(2)(3600) = 50.26$$

$$2\pi n\tau - x\sqrt{\frac{\pi n}{\alpha}} = 46.734 \text{ radians} = 2677.66°$$

$\cos(2677.66) = -0.925 \qquad \sin(2677.66) = 0.3801$

$$\frac{q}{A} = kAe^{-x\sqrt{\frac{\pi n}{\alpha}}}\left(\sqrt{\frac{\pi n}{\alpha}}\right)[\cos(2677.66) + \sin(2677.66)]$$

$$\frac{q}{A} = (1.37)(27.5)(e^{-3.531})(-0.925 + 0.3801) = -0.601 \ \text{W/m}^2$$

4–19

Maximum points when sine function is max, i.e.:

$$2\pi n\tau - x\sqrt{\frac{\pi n}{\alpha}} = \frac{\pi}{2} \quad \text{at } x=0 \qquad \tau = \frac{\pi}{4\pi n} = \frac{1}{4n}$$

$$\text{at } x = x_1 \qquad \tau = \frac{\pi}{4\pi n} + \frac{x\sqrt{\frac{\pi n}{\alpha}}}{2\pi n} = \frac{1}{4n} + \frac{x}{2}\sqrt{\frac{1}{\pi\alpha n}}$$

$$\Delta\tau = \frac{x}{2}\sqrt{\frac{1}{\pi\alpha n}}$$

4–20

$T_i = 54°C \qquad T_\infty = 10°C \qquad h = 10\ \frac{W}{m^2\cdot °C} \qquad x = 7\text{ cm} \qquad \tau = 30\text{ min}$

$\alpha = 7\times10^{-7}\ m^2/sec \qquad k = 1.37\ \frac{W}{m\cdot °C}$

$$\frac{x}{2\sqrt{\alpha\tau}} = \frac{0.07}{2[(7\times10^{-7})(30)(60)]^{1/2}} = 0.986 \qquad \frac{T-T_i}{T_\infty - T_i} = 0.021$$

$$\frac{h\sqrt{\alpha\tau}}{k} = \frac{(10)[(7.7\times10^{-7})(30)(60)]^{1/2}}{1.37} = 0.258 \qquad T = 53.08°C$$

4–21

$T_i = 300°C \qquad T_0 = 35°C \qquad x = 7.5\text{ cm} \qquad \tau = 4\text{ min} = 240\text{ sec}$

$\alpha = 11.23\times10^{-5}\ m^2/s$

$$X = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.075}{2[(11.23\times10^{-5})(240)]^{1/2}} = 0.2284$$

$$\text{erf } X = 0.2533 = \frac{T-T_0}{T_1-T_0} \qquad T = 102.1°C$$

4–22

$\alpha = 7\times10^{-7}\ m^2/s \qquad T_i = 55°C \qquad T_0 = 15°C \qquad T = 25°C$

$x = 5\text{ cm}$

$$\frac{T-T_0}{T_i-T_0} = \frac{25-15}{55-15} = 0.25 = \text{erf } X \qquad X = 0.2253 = \frac{x}{2\sqrt{\alpha\tau}}$$

$\tau = 17{,}589\text{ sec} = 4.89\text{ hr}$

4–23

$\frac{q_0}{A} = 0.5 \times 10^6 \text{ W/m}^2 \qquad \tau = 300 \text{ s} \qquad T_i = 20°\text{C} \qquad k = 386$

$\alpha = 11.23 \times 10^{-5} \text{ m}^2/\text{s}$

$\underline{x = 0}$

$$T = 20 + \frac{(2)(0.5 \times 10^6)\left[\frac{(11.23\times10^{-5})(300)}{\pi}\right]^{1/2}}{386} e^0 = 288.3°\text{C}$$

$\underline{x = 15 \text{ cm}}$

$$X = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.15}{2[(11.23 \times 10^{-5})(300)]^{1/2}} = 0.4086$$

$\frac{x^2}{4\alpha\tau} = 0.167 \qquad \text{erf } X = 0.4173$

$$T - T_i = \frac{2(0.5 \times 10^6)\left[\frac{(11.23\times10^{-5})(300)}{\pi}\right]^{1/2}}{386} e^{-0.167} - \frac{(0.5 \times 10^6)(0.15)}{386}(1 - 0.4173)$$

$= 113.8$

$T = 133.8°\text{C}$

4-24 All cases remain at 20°C because $x/2(\alpha\tau)^{1/2}$ is so large.

4-26

$T_i = 90°\text{C} \qquad T_0 = 30°\text{C} \qquad x = 7.5 \text{ cm} \qquad \tau = 10 \text{ sec} \qquad k = 386 \frac{\text{W}}{\text{m} \cdot °\text{C}}$

$\alpha = 11.23 \times 10^{-5} \text{ m}^2/\text{s}$

$$\frac{q}{A} = \frac{-k(T_i - T_0)}{\sqrt{\pi\alpha\tau}} e^{\frac{-x^2}{4\alpha\tau}}$$

$$\frac{q}{A} = \frac{(-386)(90 - 30)}{[\pi(11.23 \times 10^{-5})(10)]^{1/2}} \exp\left[\frac{-(0.075)^2}{4(11.23 \times 10^{-5})(10)}\right] = -111.3 \text{ kW/m}^2$$

4-27

$T_i = 30°\text{C} \qquad \dfrac{q_0}{A} = 15{,}000\ \text{W/m}^2 \qquad x = 2.5\ \text{cm} \qquad \tau = 120\ \text{sec}$

$k = 204 \qquad \alpha = 8.42 \times 10^{-5}$

$$T - T_i = \frac{15{,}000}{204}\left\{2\left[\frac{(8.42\times10^{-5})(120)}{\pi}\right]^{1/2} \exp\left[\frac{-(0.025)^2}{4(8.42\times10^{-5})(120)}\right] - (0.025)\left[1 - \text{erf}\left(\frac{0.025}{2\sqrt{8.42\times10^{-5}(120)}}\right)\right]\right\}$$

$= 6.59$

$T = 36.59°\text{C}$

4-30

$$\frac{h(V/A)}{k} = \frac{h(r/3)}{k} = \frac{(78)\left(\frac{0.028}{3}\right)}{204} = 0.00357$$

Therefore: Lumped capacity $\rho = 2707,\ c = 896$

$$\frac{hA}{\rho c V} = \frac{3h}{r\rho c} = \frac{(3)(78)}{(0.028)(2707)(896)} = 0.00345$$

$$\frac{73-23}{355-23} = e^{-0.00345\tau}$$

$\tau = 549\ \text{sec}$

4-31

$$\frac{T - T_0}{T_i - T_0} = \frac{-1-(-1)}{-20-(-1)} = 0.5263 = \text{erf}\,\frac{x}{2\sqrt{\alpha\tau}}$$

$$\frac{x}{2\sqrt{\alpha\tau}} = 0.5267$$

$$\tau = \frac{1}{0.048}\left[\frac{(0.015)(3.2808)}{2(0.5267)}\right]^2 = 0.04547\ \text{hr} = 163.7\ \text{sec}$$

4-32

$$\frac{q}{A} = 900\ \mathrm{W/m^2} \qquad T_i = 20°\mathrm{C} \qquad x = 10\ \mathrm{cm} \qquad \tau = 9\ \mathrm{hr} = 32{,}400\ \mathrm{sec}$$

$$k = 1.37\ \frac{\mathrm{W}}{\mathrm{m \cdot °C}} \qquad \alpha = 7.5 \times 10^{-7}\ \mathrm{m^2/s}$$

$$T = 20 + \frac{(2)(900)\left[(7.5 \times 10^{-7})\frac{32{,}400}{\pi}\right]^{1/2}}{1.37}\exp\left[\frac{-(0.1)^2}{4(7.57 \times 10^{-7})(32{,}400)}\right]$$

$$-\frac{(900)(0.1)}{1.37}\left[1 - \mathrm{erf}\left(\frac{0.1}{(2)(7.5 \times 10^{-7})(32{,}400)^{1/2}}\right)\right]$$

$$= 81.5°\mathrm{C}$$

4-33

$$T_i = 300°\mathrm{C} \qquad T_0 = 100°\mathrm{C} \qquad x = 0.03\ \mathrm{m} \qquad T = 200°\mathrm{C}$$

$$\frac{200-100}{300-100} = 0.5 = \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha\tau}}\right) \qquad \alpha = 0.444 \times 10^{-5}$$

$$\frac{x}{2\sqrt{\alpha\tau}} = 0.48 \qquad \tau = \frac{\left[\frac{0.03}{(2)(0.48)}\right]^2}{0.444 \times 10^{-5}} = 2200\ \mathrm{sec}$$

4-34

$$T_i = 40°\mathrm{C} \qquad h = 25\ \frac{\mathrm{W}}{\mathrm{m^2 \cdot °C}} \qquad T_\infty = 2°\mathrm{C} \qquad x = 0.08\ \mathrm{m} \qquad T(x) = 20°\mathrm{C}$$

$$\alpha = 5.2 \times 10^{-7} \qquad k = 0.69\ \frac{\mathrm{W}}{\mathrm{m \cdot °C}}$$

$$\frac{T - T_i}{T_\infty - T_i} = \frac{20-40}{2-40} = 0.5263 \qquad \frac{x}{2\sqrt{\alpha\tau}} \approx 0.4$$

$$\tau = \frac{\left[\frac{0.08}{(2)(0.4)}\right]^2}{5.2 \times 10^{-7}} = 19{,}231\ \mathrm{sec}$$

4-35

$T_i = 30°C \qquad \frac{q}{A} = 3\times10^4 \qquad \tau = 10 \text{ min} = 600 \text{ sec} \qquad x = 3 \text{ cm}$

$k = 2.32 \ \frac{W}{m\cdot°C} \qquad \alpha = 9.2\times10^{-7} \ m^2/s$

$$T = 30 + \frac{(2)(3\times10^4)\left[(9.2\times10^{-7})\frac{600}{\pi}\right]^{1/2}}{2.32}\exp\left[\frac{-(0.03)^2}{(4)(9.2\times10^{-7})(600)}\right]$$

$$-\frac{(3\times10^4)(0.03)}{2.32}\left\{1-\text{erf}\left[\frac{0.03}{(2)(9.2\times10^{-7})(600)}\right]\right\}$$

$= 30 + 228 = 258°C$

4-36

From symmetry same as inf. plate 6 cm thick

$\tau = 360 \text{ sec} \qquad L = 30 \text{ cm} \qquad \alpha = 11.23\times10^{-5} \qquad k = 370 \qquad \frac{\alpha\tau}{L^2} = 44.92$

$\frac{\theta_{x=L}}{\theta_i} = \frac{150-100}{250-100} = 0.33$

<u>Iterative Solution:</u>

$\frac{k}{hL}$	$\frac{\theta_0}{\theta_i}$	$\frac{\theta}{\theta_0}$	$\frac{\theta}{\theta_i}$	$\frac{\theta}{\theta_i} - 0.33$
100	0.65	1.0	0.65	0.32
50	0.42	0.98	0.41	0.08
45	0.38	0.98	0.37	0.04
40	0.34	0.98	0.33	0

$h = \frac{370}{(40)(0.03)} = 308.3 \ \frac{W}{m^2\cdot°C}$

4-37

$L = 5 \text{ cm} \qquad h = 1400 \qquad k = 230 \qquad T_i = 400 \qquad \alpha = 8.42\times10^{-5}$

$T_\infty = 90 \qquad T_0 = 180 \qquad \frac{k}{hL} = 3.29 \qquad \frac{\theta_0}{\theta_i} = 0.29 \qquad \frac{\alpha\tau}{L^2} = 5.0$

$\tau = \frac{(0.05)^2(5)}{8.42\times10^{-5}} = 148 \text{ sec}$

4-38

$T_i = 350°C \quad T_\infty = 80°C \quad T_0 = 150°C \quad \tau = 6\text{ min} = 360\text{ sec}$

$k = 374\ \dfrac{W}{m \cdot °C} \quad \alpha = 11.23 \times 10^{-5} \quad L = 0.1\text{ m} \quad \dfrac{\alpha\tau}{L^2} = 4.04$

$\dfrac{\theta_0}{\theta_i} = \dfrac{150-80}{250-80} = \dfrac{70}{170} = 0.412 \quad \dfrac{x}{L} = 1.0 \quad \dfrac{k}{hL} \approx 4.0$

$h = \dfrac{374}{(4.0)(0.1)} = 935\ \dfrac{W}{m^2 \cdot °C}$

4-39

$L = 5\text{ cm} \quad T_i = 400°C \quad T_\infty = 90°C \quad h = 1400\ \dfrac{W}{m^2 \cdot °C}$

$k = 204\ \dfrac{W}{m \cdot °C} \quad \alpha = 8.4 \times 10^{-5}\ m^2/s \quad \dfrac{\theta_0}{\theta_i} = \dfrac{180-90}{400-90} = 0.29$

$\dfrac{k}{hL} = \dfrac{204}{(1400)(0.05)} = 2.91 \quad \dfrac{\alpha\tau}{L^2} = 4.2 \quad \tau = \dfrac{(4.2)(0.05)^2}{8.4 \times 10^{-5}} = 125\text{ sec}$

4-40

$L = 0.015\text{ m} \quad T_i = 500°C \quad T_\infty = 40°C \quad h = 150\ \dfrac{W}{m^2 \cdot °C}$

$k = 16.3\ \dfrac{W}{m \cdot °C} \quad \alpha = 0.44 \times 10^{-5}\ m^2/s \quad \dfrac{k}{hL} = \dfrac{6.3}{(150)(0.015)} = 7.24$

at $\dfrac{x}{L} = 1.0 \quad \dfrac{\theta}{\theta_0} = 0.93$

For $\dfrac{\theta_0}{\theta_i} = \dfrac{120-40}{500-40} = 0.174 \quad \dfrac{\alpha\tau}{L^2} = 13.9$

$\tau = \dfrac{(13.9)(0.015)^2}{0.44 \times 10^{-5}} = 711\text{ sec}$

For $\dfrac{\theta}{\theta_i} = 0.174 \quad \dfrac{\theta_0}{\theta_i} = \dfrac{0.174}{0.93} = 0.187 \quad \dfrac{\alpha\tau}{L^2} = 13$

$\tau = \dfrac{(13)(0.015)^2}{0.44 \times 10^{-5}} = 665\text{ sec}$

4-41

$r_0 = 5 \text{ cm} \qquad L = 5 \text{ cm} \qquad T_i = 250°\text{C} \qquad T_\infty = 30°\text{C} \qquad h = 280 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$k = 43 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \alpha = 1.172 \times 10^{-5} \ \text{m}^2/\text{sec} \qquad \tau = 2 \text{ min} = 120 \text{ sec}$

$\dfrac{\alpha\tau}{L^2} = \dfrac{\alpha\tau}{r_0^{\,2}} = 0.563 \qquad \dfrac{k}{hL} = \dfrac{k}{hr_0} = 3.071$

cylinder: $\dfrac{\theta_0}{\theta_i} = 0.86 \qquad$ plate: $\dfrac{\theta_0}{\theta_i} = 0.93 \qquad \dfrac{\theta_2}{\theta_0} = 0.86$

center: $\dfrac{\theta}{\theta_i} = (0.86)(0.93) = 0.8 \qquad T = 175°\text{C}$

End center: $\dfrac{\theta}{\theta_i} = (0.86)(0.93)(0.86) = 0.688 \qquad T = 151°\text{C}$

4-42

$$T - T_i = \left[\frac{Q_0}{A\rho c(\pi\alpha\tau)^{1/2}}\right]\exp\left(\frac{-x^2}{4\alpha\tau}\right)$$

$$\frac{q}{A} = -k\frac{\partial T}{\partial x}$$

$$= -k\left[\frac{Q_0}{A\rho c(\pi\alpha\tau)^{1/2}}\right]\exp\left(\frac{-x^2}{4\alpha\tau}\right)\left(\frac{-2x}{4\alpha\tau}\right)$$

$$= \frac{1}{2}\frac{x}{\tau}\left[\frac{Q_0}{A(\pi\alpha\tau)^{1/2}}\right]\exp\left(\frac{-x^2}{4\alpha\tau}\right)$$

4-43

Assume behaves like center of 20 cm thick wall with $T_i = 15°\text{C}$,

$c = 900$, $\rho = 2200$, $k = 2.32$, $\alpha = 1.17\times10^{-6}$

$h = 65$, $\theta_0/\theta_i = (5-(-10))/(15-(-10)) = 0.6$

$k/hL = 0.356$

Fig. 4-7(b) $\alpha\tau/L^2 = 0.48$

$\tau = (0.48)()0.1)^2/1.17\times10^{-6} = 4100$ sec

4-44

$Q/A = 1.0\ MJ/m^2$; $T_i = 20°C$; $x = 0.023$; $\tau = 1.8$ s

$\alpha = 8.4\times10^{-5}$; $\rho = 2700$; $c = 896$

Eq. (4-13b)

$T = 20 + \{10^{6}/(2700)(896)[\pi(8.4\times10^{-5})(1.8)]^{1/2}\}\exp[-0.023^2/(4)(8.4\times10^{-5})(1.8)]$

$=27.91°C$

4-45

$\rho = 7817 \qquad c = 460 \qquad \alpha = 0.44\times10^{-5} \qquad x = 0.01 \qquad \tau = 3$

$\dfrac{Q_0}{A} = 10^7\ \text{J/m}^2 \qquad T_i = 0°C$

$$T - 0 = \frac{10^7}{(7817)(460)[\pi(0.44\times10^{-5})(3)]^{1/2}}\exp\left[\frac{-(0.01)^2}{(4)(0.44\times10^{-5})(3)}\right]$$

$T = 431.9e^{-1.894} = 64.99°C$

4-46

$$64.99 = \frac{431.9}{1\times10^7}\left(\frac{Q_0}{A}\right)\exp\left[-\left(\frac{1.2}{1}\right)^2(1.894)\right]$$

$\dfrac{Q_0}{A} = 2.3\times10^7\ \text{J/m}^2$

4-47

From Prob. 4–40

$$\frac{q}{A}=\frac{1}{2}\frac{x}{\tau}\left[\frac{Q_0}{A(\pi\alpha\tau)^{1/2}}\right]\exp\left(\frac{-x^2}{4\alpha\tau}\right)$$

$$=\frac{0.01}{(2)(3)}\frac{(10^7)}{[\pi(0.44\times10^{-5})(3)]^{1/2}}e^{-1.894}$$

$$=(2.588\times10^6)(0.1505)=3.89\times10^5\ \text{W/m}^2$$

4-48

$\rho=2700$ $\quad c=896$ $\quad \alpha=8.42\times10^{-5}$ $\quad x=0.002$ $\quad \tau=0.2$

$T_i=30°\text{C}$ $\quad T=600°\text{C}$

$$600-30=\frac{Q_0/A}{(2700)(896)[\pi(8.42\times10^{-5})(0.2)]^{1/2}}\times\exp\left[\frac{-(0.002)^2}{(4)(8.42\times10^{-5})(0.2)}\right]$$

$$570=\frac{\left(\frac{Q_0}{A}\right)e^{-0.0594}}{17{,}596}$$

$$\frac{Q_0}{A}=10.64\ \text{MJ/m}^2$$

4-49

$\rho=4000$ $\quad c=760$ $\quad \alpha=120\times10^{-7}$ $\quad x=0.0002$ $\quad \tau=0.2$

$T_i=40°\text{C}$ $\quad T=900°\text{C}$

$$900-40=\frac{Q_0/A}{(2700)(760)[\pi(120\times10^{-7})(0.2)]^{1/2}}\exp\left[\frac{-(0.0002)^2}{(4)(120\times10^{-7})(0.2)}\right]$$

$$860=\frac{(Q_0/A)e^{-0.0042}}{8347}$$

$$\frac{Q_0}{A}=7.2\ \text{MJ/m}^2$$

4-50

$\rho=2700$ $\quad c=840$ $\quad \alpha=3.4\times10^{-7}$ $\quad x=0.0002$ $\quad \tau=0.2$

$$900-40=\frac{Q_0/A}{(2700)(840)[\pi(3.4\times10^{-7})(0.2)]^{1/2}}\exp\left[\frac{-(0.0002)^2}{(4)(3.4\times10^{-7})(0.2)}\right]$$

$$860=\frac{(Q_0/A)e^{-0.1471}}{1048}$$

$$\frac{Q_0}{A}=1.04\ \text{MJ/m}^2$$

4-51

$r_0 = 5.5 \text{ cm} \quad T_i = 300°\text{C} \quad T_\infty = 50°\text{C} \quad h = 1200 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad T_0 = 80°\text{C}$

$\rho = 2707 \quad c = 896 \ \dfrac{\text{J}}{\text{kg} \cdot °\text{C}} \quad k = 204 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad \alpha = 8.4 \times 10^{-5} \ \text{m}^2/\text{s}$

$\dfrac{k}{hr_0} = \dfrac{204}{(1200)(0.055)} = 3.09 \quad \dfrac{\theta_0}{\theta_i} = \dfrac{80-50}{300-50} = 0.12 \quad \dfrac{\alpha\tau}{r_0^2} = 3.7$

$\tau = \dfrac{(3.7)(0.055)^2}{8.4 \times 10^{-5}} = 133 \text{ sec} \quad \dfrac{h^2\alpha\tau}{k^2} = \dfrac{(1200)^2(8.4 \times 10^{-5})(133)}{(204)^2} = 0.386$

$\dfrac{hr_0}{k} = 0.324 \quad \dfrac{Q}{Q_0} = 0.85$

$Q_0 = \rho c V \theta_i = (2707)(896)(300-50)\pi(0.055)^2 = 5.76 \text{ MJ}$

$Q = (0.85)(5.76) = 4.9 \text{ MJ}$

4-52

$\alpha = 9.5 \times 10^{-7} \ \text{m}^2/\text{s} \quad r_0 = 1.25 \text{ cm} \quad k = 1.52 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

$T_i = 25°\text{C} \quad T_\infty = 200°\text{C} \quad h = 110 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad \tau = 3 \text{ min} = 180 \text{ sec}$

$\dfrac{k}{hr_0} = \dfrac{1.52}{(110)(0.0125)} = 1.105 \quad \dfrac{r}{r_0} = \dfrac{0.64}{1.25} = 0.51$

$\dfrac{\alpha\tau}{r_0^2} = \dfrac{(9.5 \times 10^{-7})(180)}{(0.0125)^2} = 1.094 \quad \dfrac{\theta_0}{\theta_i} = 0.12 \quad \dfrac{\theta_r}{\theta_0} = 0.89$

center $T = (25-200)(0.12) + 200 = 179°\text{C} \quad r = 6.4 \text{ mm}$

$T = (25-200)(0.12)(0.89) + 200 = 181.3°\text{C}$

4-53

$T_i = 300°\text{C} \quad T_0 = 120°\text{C} \quad d = 1.5 \text{ mm} \quad r_0 = 0.75 \text{ mm} \quad T_\infty = 100°\text{C}$

$h = 5000 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad k = 35 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad \alpha = 2.34 \times 10^{-5} \ \text{m}^2/\text{s}$

$\dfrac{k}{hr_0} = \dfrac{35}{(5000)(0.00075)} = 9.33 \quad \dfrac{\theta_0}{\theta_i} = \dfrac{120-100}{300-100} = 0.1$

$\dfrac{\alpha\tau}{r_0^2} = 7.3 \quad \tau = \dfrac{(7.3)(0.00075)^2}{2.34 \times 10^{-5}} = 0.175 \text{ sec}$

4-54

$r_0 = 5 \text{ cm} \qquad T_\infty = 10°\text{C} \qquad T_i = 250°\text{C} \qquad h = 280 \qquad \alpha = 1.172 \times 10^{-5}$

$T_0 = 150°\text{C} \qquad k = 43 \qquad \dfrac{\theta_0}{\theta_i} = \dfrac{150-10}{250-10} = 0.583$

$\dfrac{k}{hr_0} = \dfrac{43}{(280)(0.05)} = 3.07 \qquad \dfrac{\alpha\tau}{r_0^2} = 0.75$

$$\tau = \frac{(0.75)(0.05)^2}{1.172 \times 10^{-5}} = 160 \text{ sec} = 2.67 \text{ min}$$

4-55

$T_i = 200°\text{C} \qquad T_\infty = 20°\text{C} \qquad h = 14 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \qquad r_0 = 0.0075 \text{ m}$

$T_0 = 50°\text{C} \qquad k = 0.78 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \alpha = 3.4 \times 10^{-7}$

$\dfrac{k}{hr_0} = \dfrac{0.78}{(14)(0.0075)} = 7.43 \qquad \dfrac{\theta_0}{\theta_i} = \dfrac{50-20}{200-20} = 0.167$

$\dfrac{\alpha\tau}{r_0^2} = 4.5 \qquad \tau = \dfrac{(4.5)(0.0075)^2}{3.4 \times 10^{-7}} = 744 \text{ sec}$

4-56

$r_0 = 0.0075 \text{ m} \qquad T_i = 200°\text{C} \qquad h = 5000 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \qquad T_\infty = 100°\text{C}$

$T_0 = 120°\text{C} \qquad k = 35 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \alpha = 2.34 \times 10^{-5}$

$\dfrac{k}{hr_0} = \dfrac{35}{(5000)(0.00075)} = 9.33 \qquad \dfrac{\theta_0}{\theta_i} = \dfrac{120-100}{200-100} = 0.2$

$\dfrac{\alpha\tau}{r_0^2} = 5.2 \qquad \tau = \dfrac{(5.2)(0.00075)^2}{2.34 \times 10^{-5}} = 0.125 \text{ sec}$

4-57

$T_i = 250°\text{C} \qquad T_\infty = 30°\text{C} \qquad h = 570 \qquad \tau = 120 \text{ sec} \qquad L_1 = L_2 = 1.25 \text{ cm}$

$L_3 = 3.75 \text{ cm} \qquad k = 43 \ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \alpha = 1.172 \times 10^{-5} \ \text{m}^2/\text{sec}$

$\left.\dfrac{k}{hL}\right|_{1,\,2} = 6.035 \qquad \left.\dfrac{k}{hL}\right|_3 = 2.01 \qquad \left.\dfrac{\alpha\tau}{L^2}\right|_{1,\,2} = 9.00 \qquad \left.\dfrac{\alpha\tau}{L^2}\right|_3 = 1.00$

$\left.\dfrac{\theta_0}{\theta_i}\right|_{1,\,2} = 0.25 \qquad \left.\dfrac{\theta_0}{\theta_i}\right|_3 = 0.7 \qquad \text{Center } \dfrac{\theta_0}{\theta_i} = (0.25)^2(0.07) = 0.0438$

$T = 39.6°\text{C}$

4-58

$T_\infty = 30°C$ $\quad \tau = 120$ sec $\quad T_i = 220°C$ $\quad h = 570\ \dfrac{W}{m^2 \cdot °C}$

$\alpha = 1.17 \times 10^{-5}\ m^2/s$ $\quad k = 43\ \dfrac{W}{m \cdot °C}$ $\quad L_1 = L_2 = 0.025$

$L_3 = 0.075$

$$\frac{\alpha\tau}{L_1^2} = \frac{(1.17\times10^{-5})(120)}{(0.025)^2} = 2.246$$

$$\frac{\alpha\tau}{L_3^2} = 0.25$$

$$\frac{k}{hL_1} = \frac{43}{(570)(0.025)} = 3.02$$

$$\frac{k}{hL_3} = 1.01$$

$$\left(\frac{\theta_0}{\theta_i}\right)_{L_1} = 0.54 \qquad \left(\frac{\theta_0}{\theta_i}\right)_{L_3} = 0.92 \qquad \left(\frac{\theta_0}{\theta_i}\right)_{\text{bar}} = (0.54)^2(0.92) = 0.268$$

$T_0 = 30 + (0.268)(220 - 30) = 81°C$

4-59

$L = 5$ cm $\quad T_i = 300°C$ $\quad T_\infty = 100°C$ $\quad h = 900\ \dfrac{W}{m^2 \cdot °C}$ $\quad \tau = 60$ sec

$k = 204\ \dfrac{W}{m \cdot °C}$ $\quad \alpha = 8.42 \times 10^{-5}\ m^2/s$ $\quad \dfrac{k}{hL} = 4.53$ $\quad \dfrac{\alpha\tau}{L^2} = 2.02$

$\dfrac{\theta_0}{\theta_i} = 0.7$ $\quad \dfrac{\theta_L}{\theta_0} = 0.9$ $\quad$ Center of face: $\dfrac{\theta}{\theta_i} = (0.7)(0.9) = 0.63$

$T = (0.63)(300 - 100) + 100 = 226°C$

4-60

$r_0 = 7.5$ cm $\quad L = 15$ cm $\quad T_i = 25°\text{C} \quad T_\infty = 0°\text{C} \quad T_0 = 6°\text{C}$

$h = 17 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad \alpha = 7\times10^{-7}\ \text{m}^2/\text{s} \quad k = 1.37 \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \quad \dfrac{k}{hL} = 0.54$

$$\frac{k}{hr_0} = 1.075 \quad \frac{\theta_0}{\theta_i} = \frac{6-0}{25-0} = 0.24 = \left(\frac{\theta_0}{\theta_i}\right)_{\text{cyl}}\left(\frac{\theta_0}{\theta_i}\right)_{\text{plate}}$$

Iterative Solution:

τ	$\dfrac{\alpha\tau}{r_0^2}$	c	$\dfrac{\alpha\tau}{L^2}$	ρ	CP
3600	0.448	0.65	0.112	1.0	0.65
7200	0.896	0.28	0.224	0.9	0.252

$\tau = 7200$ sec $= 2$ hr

4-61

$$\frac{1}{R_{m+1}} = \frac{1}{R_{m-1}} = \frac{kA}{\Delta x} = \frac{(0.8)\pi(0.02)^2}{(4)(0.01)} = 0.02513$$

$$\frac{1}{R_\infty} = hA = (50)\pi(0.02)(0.01) = 0.031416$$

$$\sum\frac{1}{R} = 0.08168$$

$$C_m = (2700)(840)\pi(0.01)^2(0.01) = 7.125$$

$$\Delta\tau_{\max} = \frac{7.125}{0.08168} = 87.2 \text{ sec}$$

$$T_m^{p+1} = \frac{\Delta\tau}{7.125}[0.025(T_{m-1}{}^p - T_m{}^p) + (0.025)(T_{m+1}{}^p - T_m{}^p) + 0.03142(35 - T_m{}^p)] + T_m{}^p$$

4-62

$$k = 290 \qquad \frac{h(V/A)}{k} = \frac{(120)(0.04)^3}{(6)(0.04)^2(240)} = 3.3\times10^{-3} < 0.1$$

Lumped capacity: $\rho = 2707 \quad c = 896$

$$\frac{hA}{\rho cV} = \frac{(120)(6)(0.04)^2}{(2707)(896)(0.04)^3} = 7.42\times10^{-3}$$

$$\frac{250-100}{450-100} = e^{-7.42\times10^{-3}\tau} \qquad \tau = 114 \text{ sec}$$

4-63

$L = 5.5\text{ cm} \quad T_i = 400°\text{C} \quad T_\infty = 85°\text{C} \quad h = 1100\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad \tau = 60\text{ sec}$

$k = 204\ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad \alpha = 8.4 \times 10^{-5}\ \text{m}^2/\text{s} \quad \dfrac{k}{hL} = \dfrac{204}{(1100)(0.055)} = 3.37$

$\dfrac{\alpha\tau}{L^2} = \dfrac{(8.4 \times 10^{-5})(60)}{(0.055)^2} = 1.67 \qquad \dfrac{\theta_0}{\theta_i} = 0.7$

at $\dfrac{x}{L} = 1.0 \quad \dfrac{\theta}{\theta_0} = 0.86 \quad \dfrac{\theta}{\theta_i}$ at center of face $= (0.7)^3(0.86) = 0.295$

$T = (0.295)(400 - 85) + 85 = 178°\text{C}$

4-64

$L = 0.025 \quad T_i = 100°\text{C} \quad T_\infty = 25°\text{C} \quad h = 20\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad T_0 = 50°\text{C}$

$k = 204\ \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \quad \alpha = 8.4 \times 10^{-5}\ \text{m}^2/\text{s}$

$\dfrac{h(V/A)}{k} = \dfrac{(20)(0.05)^3}{(6)(0.05)^2(204)} = 8.16 \times 10^{-4}$

Lumped Capacity: $\dfrac{50 - 25}{100 - 25} = \exp\left[\dfrac{-(20)(6)(0.05)^2\tau}{(2707)(896)(0.05)^3}\right]$

$\tau = 1111\text{ sec}$

4-65

$T_i = 200°\text{C} \quad T_\infty = 30°\text{C} \quad \tau = 600\text{ sec} \quad h = 200\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad d = 10\text{ cm}$

$L = 15\text{ cm} \quad T_0 = 100°\text{C} \quad k = 16.3 \quad \alpha = 0.44 \times 10^{-5} \quad \rho = 7817$

$c = 460$

$\dfrac{k}{hr_0} = \dfrac{16.3}{(200)(0.05)} = 1.63 \qquad \dfrac{k}{hL} = \dfrac{16.3}{(200)(0.075)} = 1.09$

$\dfrac{\alpha\tau}{r_0^2} = 1.056 \qquad \dfrac{\alpha\tau}{L^2} = 0.47$

$\left.\dfrac{\theta_0}{\theta_i}\right|_c = 0.45 \qquad \left.\dfrac{\theta_0}{\theta_i}\right|_p = 0.9 \qquad \dfrac{\theta_0}{\theta_i} = (0.45)(0.9) = 0.405$

$T = (0.405)(200 - 30) + 30 = 98.8°\text{C}$

Plate

$\dfrac{hL}{k} = 0.92 \qquad \dfrac{h^2\alpha\tau}{k^2} = 0.397$

Cyl

$\frac{hr_0}{k} = 0.61$

$\left.\frac{Q}{Q_0}\right|_{\text{cyl}} = 0.55 \qquad \left.\frac{Q}{Q_0}\right|_{\text{plate}} = 0.2$

$\left.\frac{Q}{Q_0}\right|_{\text{total}} = 0.55 + 0.2(1 - 0.55) = 0.64$

$Q_0 = \rho c V \theta_i = (7817)(460)\pi(0.05)^2(0.15)(200 - 30) = 0.72 \text{ MJ}$

$Q = (0.64)(0.72) = 0.46 \text{ MJ}$

4-66

$L = 0.15 \qquad r_0 = 0.075 \qquad T_i = 300°\text{C} \qquad T_\infty = 20°\text{C} \qquad h = 35 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$T = 120°\text{C} \qquad k = 2.3 \ \frac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \rho = 300 \ \text{kg/m}^3 \qquad c = 840 \ \frac{\text{J}}{\text{kg} \cdot °\text{C}}$

$\frac{k}{hL} = \frac{2.3}{(35)(0.15)} = 0.44 \qquad \frac{k}{hr_0} = 0.88$

$\frac{\theta}{\theta_i} = \frac{120 - 20}{300 - 20} = 0.36 \qquad \alpha = \frac{k}{\rho c} = 9.12 \times 10^{-6}$

Geometric Center

At $\tau = 300$ sec $\qquad \frac{\alpha\tau}{L^2} = \frac{(9.12 \times 10^{-6})(300)}{(0.15)^2} = 0.12 \qquad \frac{\alpha\tau}{r_0^2} = 0.36$

$\left.\begin{array}{r} (\theta_0/\theta_i)_{\text{cyl}} = 0.7 \\ (\theta_0/\theta_i)_{\text{plate}} = 0.96 \end{array}\right\} \frac{\theta}{\theta_i} = 0.67$ therefore τ is too small

At $\tau \approx 500$ sec $\qquad \frac{\alpha\tau}{r_0^2} = 0.6 \qquad \frac{\theta}{\theta_i} \approx 0.4$

Therefore $\tau \approx 500$ sec

Center of Face

$\frac{x}{L} = 1.0 \qquad \frac{\theta}{\theta_0} = 0.43$

$\left(\frac{\theta_0}{\theta_i}\right)_{\text{cyl}} \times \left(\frac{\theta_0}{\theta_i}\right)_{\text{plate}} \times 0.43 = 0.36 \qquad \left(\frac{\theta_0}{\theta_i}\right)_{\text{cyl}} \times \left(\frac{\theta_0}{\theta_i}\right)_{\text{plate}} = 0.84$

$\tau \approx 250$ sec

4-73

$$\frac{1}{R_{12}} = \frac{kA}{\Delta x} = \frac{(20)(1)}{0.0025} = 8000 = \frac{1}{R_{23}}$$

$$\frac{1}{R_{1-\infty}} = hA = 70(1) = 70$$

$$\frac{1}{R_{34}} = \frac{(1.2)(1)}{0.0075} = 160 = \frac{1}{R_{45}}$$

$$\frac{1}{R_{56}} = \frac{(0.5)(1)}{0.01} = 50 = \frac{1}{R_{67}}$$

$$C_1 = (7800)(1)\left(\frac{0.005}{4}\right)(460) = 4485$$

$$C_2 = (2)(4485) = 8970$$

$$C_3 = 4485 + (1600)(1)\left(\frac{0.015}{4}\right)(850) = 9585$$

$$C_4 = (2)(5100) = 10,200$$

$$C_5 = 5100 + (2500)(1)(0.005)(800) = 15,100$$

$$C_6 = (2)(10,000) = 20,000$$

$$C_7 = 10,000$$

$$\sum \frac{1}{R_1} = 8070 \qquad \sum \frac{1}{R_2} = 16,600 \qquad \sum \frac{1}{R_3} = 8000 + 160 = 8160$$

$$\sum \frac{1}{R_4} = (2)(160) = 320 \qquad \sum \frac{1}{R_5} = 160 + 50 = 210$$

$$\sum \frac{1}{R_6} = (2)(50) = 100 \qquad \sum \frac{1}{R_7} = 50$$

Nodes	$\dfrac{C_i}{\sum \frac{1}{R_i}}$
1	$4485/8070 = 0.5558$ sec
2	$8970/16,000 = 0.5606$
3	$9585/8160 = 1.175$
4	$10,200/320 = 31.88$
5	$15,100/210 = 71.9$
6	$20,000/100 = 200$
7	$10,000/50 = 200$

Use $\Delta\tau = 0.5558$ sec
Compute for 2, 20, 120 time increments.

The Equations

	A	B	C	D
1	T1=			=(70*(10-C1)+8000*(C2-C1))*$C$9/4485+C1
2	T2=			=(8000*(C1-C2)+8000*(C3-C2))*$C$9/8970+C2
3	T3=			=(8000*(C2-C3)+160*(C4-C3))*$C$9/9585+C3
4	T4=			=(160*(C3-C4)+160*(C5-C4))*$C$9/10200+C4
5	T5=			=(160*(C4-C5)+50*(C6-C5))*$C$9/15100+C5
6	T6=			=(50*(C5-C6)+50*(C7-C6))*$C$9/2000+C6
7	T7=			=(50*(C6-C7))*$C$9/2000+C7
8				
9	Dt=		0.5558	

The Solution

	G	H	I	J	K	L	M	N
1		T1=	T2=	T3=	T4=	T5=	T6=	T7=
2	Time							
3	iner	100	100	100	100	100	100	100
4	1	99.21928	100	100	100	100	100	100
5	2	99.21933	99.613	100	100	100	100	100
6	3	98.83566	99.60969	99.82047	100	100	100	100
7	4	98.83241	99.33049	99.72436	99.99843	100	100	100
8	5	98.55561	99.27883	99.54419	99.99606	99.99999	100	100
9	6	98.50442	99.05187	99.42529	99.99215	99.99997	100	100
10	7	98.27941	98.9656	99.25732	99.98728	99.99992	100	100
11	8	98.1959	98.77006	99.12877	99.98103	99.99985	100	100
12	9	98.00005	98.66227	98.97027	99.97376	99.99974	100	100
13	10	97.8932	98.48469	98.8367	99.96524	99.99958	99.99999	100
14	11	97.71913	98.366	98.6848	99.9557	99.99938	99.99999	100
15	12	97.59949	98.20538	98.5487	99.945	99.99913	99.99998	100
16	13	97.43828	98.07521	98.40146	99.9333	99.99881	99.99997	100
17	14	97.31123	97.92121	98.26433	99.92051	99.99843	99.99995	100
18	15	97.15856	97.78893	98.12053	99.90675	99.99797	99.99993	100
19	16	97.02743	97.64083	97.98327	99.89197	99.99744	99.9999	100
20	17	96.88061	97.50652	97.84212	99.87625	99.99682	99.99987	100
21	18	96.74747	97.36262	97.70531	99.85957	99.99612	99.99983	99.99999
22	19	96.60482	97.22756	97.56633	99.84198	99.99532	99.99978	99.99999
23	20	96.47093	97.08679	97.43029	99.82348	99.99442	99.99972	99.99999
24	21	96.33138	96.95178	97.29315	99.8041	99.99343	99.99965	99.99999
25	22	96.19754	96.81347	97.15809	99.78386	99.99232	99.99957	99.99998
26	23	96.06043	96.67898	97.02258	99.76278	99.99111	99.99948	99.99998
27	24	95.92711	96.54269	96.88861	99.74089	99.98978	99.99937	99.99997
28	25	95.792	96.40902	96.7546	99.71819	99.98833	99.99924	99.99996
29	26	95.65949	96.27447	96.62179	99.69471	99.98676	99.9991	99.99995
30	27	95.52611	96.14179	96.48918	99.67046	99.98506	99.99894	99.99994
31	28	95.39458	96.0088	96.35755	99.64547	99.98324	99.99876	99.99992
32	29	95.26274	95.87721	96.22627	99.61975	99.98127	99.99856	99.99991
33	30	95.13229	95.74565	96.09583	99.59331	99.97918	99.99834	99.99989
34	31	95.00187	95.61519	95.96583	99.56618	99.97694	99.9981	99.99987
35	32	94.87255	95.48498	95.83657	99.53838	99.97456	99.99783	99.99984
36	33	94.74347	95.35568	95.70782	99.50991	99.97203	99.99753	99.99982
37	34	94.61529	95.22676	95.57974	99.48079	99.96936	99.99721	99.99978
38	35	94.48749	95.09863	95.45219	99.45103	99.96653	99.99686	99.99975
39	36	94.36046	94.97095	95.32528	99.42067	99.96355	99.99648	99.99971
40	37	94.23389	94.84397	95.1989	99.38969	99.96042	99.99606	99.99966
41	38	94.10802	94.71749	95.07313	99.35813	99.95712	99.99562	99.99961
42	39	93.98264	94.59167	94.94791	99.326	99.95366	99.99514	99.99956
43	40	93.8579	94.46636	94.82327	99.2933	99.95004	99.99462	99.9995
44	41	93.73368	94.34167	94.69918	99.26005	99.94626	99.99407	99.99943
45	42	93.61007	94.21751	94.57565	99.22627	99.94231	99.99348	99.99935
46	43	93.48699	94.09393	94.45266	99.19197	99.93818	99.99285	99.99927
47	44	93.36449	93.97089	94.33022	99.15715	99.93389	99.99218	99.99918
48	45	93.24251	93.84842	94.20831	99.12184	99.92942	99.99147	99.99909
49	46	93.1211	93.72647	94.08695	99.08605	99.92478	99.99071	99.99898
50	47	93.00021	93.60508	93.96611	99.04977	99.91996	99.98991	99.99886
51	48	92.87987	93.48421	93.84579	99.01304	99.91497	99.98906	99.99874
52	49	92.76005	93.36388	93.726	98.97585	99.90979	99.98817	99.99861
53	50	92.64076	93.24406	93.60672	98.93822	99.90443	99.98723	99.99846
54	51	92.52199	93.12478	93.48795	98.90017	99.8989	99.98623	99.9983
55	52	92.40374	93.006	93.36969	98.86169	99.89317	99.98519	99.99814
56	53	92.28599	92.88774	93.25193	98.8228	99.88727	99.98409	99.99796
57	54	92.16876	92.76998	93.13467	98.78351	99.88118	99.98293	99.99776
58	55	92.05202	92.65273	93.01791	98.74383	99.8749	99.98173	99.99756
59	56	91.93579	92.53598	92.90163	98.70377	99.86844	99.98046	99.99734
60	57	91.82004	92.41972	92.78584	98.66334	99.86178	99.97914	99.9971
61	58	91.70479	92.30394	92.67053	98.62254	99.85494	99.97776	99.99685
62	59	91.59002	92.18866	92.55569	98.5814	99.84791	99.97632	99.99659
63	60	91.47574	92.07386	92.44134	98.5399	99.84069	99.97481	99.99631
64	61	91.36193	91.95953	92.32745	98.49808	99.83327	99.97325	99.99601
65	62	91.2486	91.84568	92.21402	98.45592	99.82567	99.97162	99.99569
66	63	91.13573	91.73229	92.10106	98.41344	99.81787	99.96993	99.99536
67	64	91.02333	91.61938	91.98856	98.37065	99.80988	99.96817	99.995
68	65	90.91139	91.50692	91.87651	98.32756	99.80169	99.96634	99.99463
69	66	90.79992	91.39492	91.76491	98.28417	99.79332	99.96445	99.99424
70	67	90.68889	91.28338	91.65376	98.24049	99.78474	99.96248	99.99382
71	68	90.57831	91.17229	91.54306	98.19652	99.77598	99.96045	99.99339
72	69	90.46819	91.06165	91.43279	98.15229	99.76701	99.95834	99.99293
73	70	90.3585	90.95145	91.32296	98.10778	99.75786	99.95617	99.99245
74	71	90.24926	90.84169	91.21357	98.06301	99.7485	99.95391	99.99195
75	72	90.14045	90.73236	91.1046	98.01799	99.73895	99.95159	99.99142
76	73	90.03207	90.62347	90.99606	97.97272	99.72921	99.94919	99.99086
77	74	89.92413	90.51501	90.88795	97.92721	99.71927	99.94671	99.99029
78	75	89.81661	90.40698	90.78026	97.88146	99.70914	99.94415	99.98968
79	76	89.70951	90.29937	90.67298	97.83549	99.6988	99.94152	99.98905
80	77	89.60283	90.19218	90.56612	97.78929	99.68828	99.93881	99.98839
81	78	89.49657	90.0854	90.45966	97.74287	99.67755	99.93602	99.9877
82	79	89.39073	89.97904	90.35362	97.69624	99.66664	99.93314	99.98698
83	80	89.28529	89.87309	90.24798	97.6494	99.65552	99.93019	99.98623
84	81	89.18026	89.76755	90.14274	97.60236	99.64421	99.92715	99.98545
85	82	89.07563	89.66241	90.0379	97.55513	99.63271	99.92403	99.98464
86	83	88.9714	89.55768	89.93346	97.5077	99.62101	99.92082	99.9838
87	84	88.86758	89.45334	89.82941	97.46009	99.60912	99.91753	99.98293
88	85	88.76414	89.3494	89.72575	97.4123	99.59703	99.91416	99.98202
89	86	88.6611	89.24585	89.62248	97.36433	99.58474	99.91069	99.98107
90	87	88.55845	89.14269	89.51959	97.31619	99.57227	99.90714	99.9801
91	88	88.45619	89.03991	89.41708	97.26789	99.5596	99.9035	99.97908
92	89	88.3543	88.93752	89.31495	97.21942	99.54673	99.89977	99.97803
93	90	88.2528	88.83552	89.2132	97.1708	99.53368	99.89596	99.97695
94	91	88.15168	88.73389	89.11183	97.12202	99.52043	99.89205	99.97582
95	92	88.05093	88.63263	89.01082	97.07309	99.50699	99.88805	99.97466
96	93	87.95056	88.53175	88.91018	97.02402	99.49336	99.88396	99.97345
97	94	87.85055	88.43124	88.80991	96.97481	99.47953	99.87977	99.97221
98	95	87.75092	88.3311	88.71	96.92546	99.46552	99.8755	99.97092
99	96	87.65164	88.23133	88.61046	96.87598	99.45131	99.87113	99.9696
100	97	87.55273	88.13191	88.51127	96.82637	99.43692	99.86666	99.96823
101	98	87.45418	88.03286	88.41243	96.77664	99.42234	99.8621	99.96682
102	99	87.35599	87.93416	88.31395	96.72678	99.40756	99.85744	99.96536
103	100	87.25815	87.83582	88.21583	96.67681	99.3926	99.85269	99.96386
104	101	87.16066	87.73784	88.11805	96.62672	99.37746	99.84784	99.96232
105	102	87.06352	87.6402	88.02061	96.57652	99.36212	99.8429	99.96073
106	103	86.96674	87.54291	87.92352	96.52621	99.3466	99.83786	99.95909
107	104	86.87029	87.44597	87.82678	96.4758	99.3309	99.83271	99.95741
108	105	86.77419	87.34937	87.73037	96.42528	99.315	99.82747	99.95567
109	106	86.67843	87.25311	87.6343	96.37467	99.29893	99.82214	99.95389
110	107	86.58301	87.1572	87.53856	96.32397	99.28267	99.8167	99.95206
111	108	86.48792	87.06161	87.44316	96.27317	99.26623	99.81116	99.95018
112	109	86.39317	86.96637	87.34809	96.22228	99.2496	99.80552	99.94825
113	110	86.29875	86.87145	87.25334	96.1713	99.2328	99.79978	99.94627
114	111	86.20465	86.77687	87.15893	96.12024	99.21581	99.79393	99.94423
115	112	86.11089	86.68261	87.06483	96.0691	99.19865	99.78799	99.94214
116	113	86.01745	86.58868	86.97106	96.01788	99.1813	99.78194	99.94
117	114	85.92433	86.49507	86.87761	95.96659	99.16377	99.77579	99.9378
118	115	85.83154	86.40178	86.78448	95.91522	99.14607	99.76954	99.93555
119	116	85.73906	86.30881	86.69166	95.86378	99.12819	99.76318	99.93325
120	117	85.6469	86.21616	86.59916	95.81228	99.11014	99.75672	99.93088
121	118	85.55505	86.12383	86.50697	95.76071	99.0919	99.75016	99.92846
122	119	85.46352	86.03181	86.41509	95.70907	99.0735	99.74349	99.92599
123	120	85.3723	85.9401	86.32352	95.65737	99.05492	99.73672	99.92345

4-74

$\rho = 2000\ \text{kg/m}^3 \qquad c = 960\ \dfrac{\text{J}}{\text{kg}\cdot°\text{C}} \qquad k = 1.04\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}}$

$$\frac{1}{R_{12}} = \frac{kA}{\Delta x} = \frac{(1.04)(0.005)}{0.02} = 0.26$$

$$\frac{1}{R_{13}} = \frac{(1.04)(0.02)}{0.01} = 2.08$$

$$C_i = \rho_i c_i V_i$$

Node	V_i	C_i	$\sum \frac{1}{R_{ij}}$	$\Delta\tau_{max}$
1	0.0001	192	3.6	53.3
2	0.00005	96	2.05	46.8
3	0.0002	384	5.2	73.8
4	0.0001	192	3.1	61.9

$$\frac{1}{R_{1-\infty}} = (50)(0.02) = 1.0$$

$$\frac{1}{R_{2-\infty}} = (50)(0.015) = 0.75$$

$$\frac{1}{R_{4-\infty}} = (50)(0.01) = 0.5$$

$$\frac{1}{R_{34}} = \frac{(1.04)(0.01)}{0.02} = 0.52$$

$$\frac{1}{R_{24}} = \frac{(1.04)(0.01)}{0.01} = 1.04$$

4-75

$k = 204$ $c = 896$ $\rho = 2707$ $T_\infty = 20°C$ $d = 2.5$ cm

$\Delta x = 4$ cm $h = 50$

$$\frac{1}{R_{m-1}} = \frac{kA}{\Delta x} = \frac{(204)\pi(0.0125)^2}{0.04} = 2.503$$

$$\frac{1}{R_\infty} = hA = 50\left[\pi\left(\frac{0.04}{2}\right)(0.025) + \pi(0.0125)^2\right] = 0.1031$$

$$\sum\frac{1}{R} = 2.6065$$

$$C = \rho c\Delta V = (2707)(896)\pi(0.0125)^2(0.02) = 23.812$$

$$\Delta\tau_{\max} = \frac{23.812}{2.6065} = 9.1356 \text{ sec}$$

$$T_m{}^{p+1} = \frac{\Delta\tau}{23.812}[2.503T_{m-1}{}^p + 0.1031(20)] + \left(1 - \frac{\Delta\tau}{9.1356}\right)T_m{}^p$$

4-76

$$\frac{1}{R_{31}} = \frac{kA}{\Delta x} = \frac{(2.32)(0.01)}{0.01} = 2.32$$

$$\frac{1}{R_{35}} = \frac{(0.48)(0.01)}{0.01} = 0.48$$

$$\frac{1}{R_{3-\infty}} = hA = (50)(0.01) = 0.5$$

$$\sum\frac{1}{R_3} = 2.32 + 0.48 + 0.5 = 3.3$$

$$C_3 = (3000)(0.005)(0.01)(840) + (1440)(0.005)(0.01)(1000) = 198$$

$$\Delta\tau_{\max} = \frac{198}{3.3} = 60 \text{ sec}$$

$$T_3{}^{p+1} = [2.32(T_1{}^p - T_3{}^p) + 0.48(T_5{}^p - T_3{}^p) + 0.05(40 - T_3{}^p)]\frac{\Delta\tau}{198} + T_3{}^p$$

4-77

$$\frac{1}{R_{m+}} = \frac{kA}{\Delta x} = \frac{1}{2}k = \frac{1}{R_{m-}}$$

$$\frac{1}{R_{n-}} = k \qquad \frac{1}{R_{n+}} = hA = h\Delta x$$

$$T_{m,\ n}{}^{p+1} = \frac{\Delta\tau}{C_{m,\ n}}\left[\frac{k}{2}(T_{m-1,\ n}{}^{p} + T_{m+1,\ n}{}^{p} + 2T_{m,\ n-1}{}^{p}) + h\Delta x T_\infty\right]$$
$$+\left[1-\frac{\Delta\tau}{\Delta\tau_{\max}}\right]T_{m,\ n}{}^{p}$$

4-78

$$\left.\frac{1}{R_{12}}\right|_A = \frac{kA}{\Delta x} = \frac{(20)(0.02)}{0.02} = 20 \qquad T_\infty = 30°\mathrm{C} \qquad \frac{1}{R_{13}} = \frac{(20)(0.01)}{0.04} = 5$$

$$\left.\frac{1}{R_{12}}\right|_B = \frac{(1.2)(0.02)}{0.02} = 1.2 \qquad \frac{1}{R_{14}} = \frac{(1.2)(0.01)}{0.04} = 0.3$$

$$\frac{1}{R_{1-\infty}} = hA = (40)(0.02) = 0.8$$

$$\sum\frac{1}{R_{1-j}} = 27.3$$

$$C_1 = \sum\rho c\Delta V = (7800)(460)(0.01)(0.02) + (1600)(850)(0.01)(0.02) = 989.6$$

$$\Delta\tau_{\max} = \frac{989.6}{27.3} = 36.25\ \mathrm{sec}$$

$$T_1{}^{p+1} = \frac{\Delta\tau}{C_1}[5T_3{}^p + 21.2T_2{}^p + 0.3T_4{}^p + (0.8)(30)] + \left[1-\frac{\Delta\tau}{36.25}\right]T_1{}^p$$

4-79

$$\frac{1}{R_{13}} = \frac{(1.2)(0.0225)}{0.02} = 1.35$$

$$\left.\frac{1}{R_{1-7}}\right|_B = \frac{(1.2)(0.01)}{0.03} = 0.4$$

$$\left.\frac{1}{R_{1-7}}\right|_C = \frac{(0.5)(0.005)}{0.03} = 0.0833$$

$$\left.\frac{1}{R_{12}}\right|_B = \frac{(1.2)(0.01)}{0.015} = 0.8$$

$$\left.\frac{1}{R_{1-2}}\right|_A = \frac{(20)(0.05)}{0.015} = 6.6667$$

$$\left.\frac{1}{R_{1-4}}\right|_A = \frac{(20)(0.0075)}{0.01} = 15.0$$

$$\left.\frac{1}{R_{1-4}}\right|_C = \frac{(0.5)(0.015)}{0.01} = 0.75$$

$$\sum \frac{1}{R_1} = 25.05$$

$$\begin{aligned} C = C_A + C_B + C_C &= (7800)(460)(0.005)(0.0075) + (1600)(850)(0.0225)(0.01) \\ &\quad + (2500)(800)(0.005)(0.015) \\ &= 590.55 \end{aligned}$$

$$\Delta\tau_{\max} = \frac{590.55}{25.05} = 23.57 \text{ sec}$$

$$\begin{aligned} T_1^{p+1} = \frac{\Delta\tau}{590.55}&[(T_3^p - T_1^p)(1.35) + (T_7^p - T_1^p)(0.4833) + (7.4667)(T_2^p - T_1^p) \\ &+ (15.75)(T_4^p - T_1^p)] + T_1^p \end{aligned}$$

4-80

	ρ	C	k
A	1440	840	0.48
B	2787	883	164

$$\frac{1}{R_{54}} = \frac{(0.48)(0.005)}{0.02} + \frac{(164)(0.005)}{0.02} = 41.12$$

$$\frac{1}{R_{52}} = \frac{(0.48)(0.02)}{0.01} = 0.96 \qquad \frac{1}{R_{56}} = 0.12 \qquad \frac{1}{R_{58}} = \frac{(164)(0.005)}{0.02} = 41$$

$$\frac{1}{R_{5-\infty}} = (35)(0.005 + 0.01) = 0.525 \qquad \sum \frac{1}{R} = 83.725$$

$$C_5 = (1440)(840)(0.02)(0.005) + (2787)(883)(0.01)(0.005) = 244.006$$

$$\Delta\tau_{\max} = \frac{244.006}{83.725} = 2.914 \text{ sec}$$

$$T_5^{p+1} = [41.12T_4^p + 0.96T_2^p + 0.12T_6^p + 41T_8^p + (55)(0.525)]\frac{\Delta\tau}{C_5} + \left(1 - \frac{\Delta\tau}{2.914}\right)T_5^p$$

4-81

<u>Node 1</u>

$$\frac{1}{R_{12}} = \frac{(1.04)(0.02)}{0.01} = 2.08$$

$$\frac{1}{R_{13}} = \frac{(1.04)(0.01)}{0.02} = 0.52$$

$$\sum \frac{1}{R} = (2)(2.08) + (2)(0.52) = 5.2$$

$$C_1 = (2000)(960)(0.01)(0.02) = 384$$

<u>Node 2</u>

$$C_2 = \frac{384}{2} = 192$$

$$\frac{1}{R_{2-\infty}} = (60)(0.02) = 1.2$$

$$\frac{1}{R_{24}} = \frac{(1.04)(0.005)}{0.02} = 0.26$$

$$\sum \frac{1}{R} = 2.08 + (2)(0.26) + 1.2 = 3.8$$

Node 3

$C_3 = (2000)(960)(0.01)(0.015) = 288$

$$\frac{1}{R_{34}} = \frac{(1.04)(0.015)}{0.01} = 1.56$$

$$\frac{1}{R_{35}} = \frac{(1.04)(0.01)}{0.01} = 1.04$$

$$\sum \frac{1}{R} = (2)(1.56) + 1.04 + 0.52 = 4.68$$

Node 4

$C_4 = (2000)(960)[(0.005)(0.015) + (0.005)(0.01)] = 240$

$$\frac{1}{R_{4+}} = \frac{(1.04)(0.005)}{0.02} = 0.26$$

$$\frac{1}{R_{4-\infty}} = (60)(0.02) = 1.2$$

$$\frac{1}{R_{46}} = \frac{(1.04)(0.015)}{0.01} = 1.56$$

$$\sum \frac{1}{R} = 0.26 + 0.26 + 1.56 + 1.56 + 1.2 = 4.84$$

Node 5

Take all $\frac{1}{R} = \frac{(1.04)(0.01)}{0.01} = 1.04$

$$\sum \frac{1}{R} = 4.16$$

$C_5 = (2000)(960)(0.01)(0.01) = 192$

Node	C	$\sum \frac{1}{R}$	$\Delta\tau_{\text{max, sec.}}$
1	384	5.2	73.85
2	192	3.8	50.53
3	288	4.68	61.54
4	240	4.84	49.59
5	192	4.16	46.15

4-82

$$\frac{1}{R_{35}} = \frac{(15)(0.0025)}{0.02} = \frac{1}{R_{31}} = 1.875$$

$$\frac{1}{R_{34}} = \frac{(15)(0.02)}{0.005} = 60 \qquad \frac{1}{R_{4-\infty}} = (25)(0.02) = 0.5$$

$$\sum \frac{1}{R} = 64.25$$

$$T_3 = \frac{(1.875)(T_1 + T_5) + 60T_4 + 0.5T_\infty}{64.25}$$

4-83

Fraction liquified = $(1/u_{if}\rho\Delta V)\Sigma\ [(T_j - T_i)/R_{ij}$

4-84

$$\frac{1}{R_{76}} = \frac{(16)(0.0125)}{0.01} = 20 \qquad \left.\frac{1}{R_{74}}\right|_A = \frac{(16)(0.005)}{0.01} = 8$$

$$\frac{1}{R_{78}} = \frac{(100)(0.0125)}{0.02} = 62.5 \qquad \left.\frac{1}{R_{74}}\right|_B = \frac{(100)(0.01)}{0.01} = 100$$

$$\left.\frac{1}{R_{7-10}}\right|_A = \frac{(16)(0.005)}{0.015} = 5.333 \qquad \left.\frac{1}{R_{7-10}}\right|_B = \frac{(100)(0.01)}{0.015} = 66.667$$

$$\sum \frac{1}{R_{7-j}} = 262.5$$

$$C_7 = (7800)(800)(0.005)(0.0125) + (2600)(500)(0.01)(0.0125) = 390 + 162.5$$
$$= 552.5$$

$$\Delta\tau_{max} = \frac{552.5}{262.5} = 2.1048 \text{ sec}$$

4-85

$$\frac{1}{R_{21}} = \frac{(10)(0.005)}{0.02} = 2.5 \qquad \frac{1}{R_{23}} = \frac{2}{4} = 0.5 \qquad \frac{1}{R_{25}} = 10 + 2 = 12$$

$$\frac{1}{R_{2-\infty}} = (40)(0.02) = 0.8 \qquad \sum \frac{1}{R} = 15.8$$

$$C_2 = (6500)(300)(0.01)(0.005) + (2000)(700)(0.01)(0.005) = 167.5$$

$$\Delta\tau_{max} = 10.601 \text{ sec}$$

$$T_2^{p+1} = [2.5T_1^p + 0.5T_3^p + 12T_5^p + (0.8)(20)]\frac{\Delta\tau}{167.5} + \left(1 - \frac{\Delta\tau}{10.601}\right)T_2^p$$

4-86

	ρ	C	k
A	1440	840	0.48
B	2787	883	164
C	7817	460	16.3

$$\frac{1}{R_{52}}=\frac{(0.48)(0.01)}{0.01}=0.48$$
$$\frac{1}{R_{54}}=\frac{(0.48)(0.005)}{0.01}+\frac{(164)(0.005)}{0.01}=82.24$$
$$\frac{1}{R_{56}}=\frac{(0.48)(0.005)}{0.01}+\frac{(16.3)(0.005)}{0.01}=8.39$$
$$\frac{1}{R_{58}}=\frac{(164+16.3)(0.005)}{0.01}=90.15$$
$$\sum\frac{1}{R}=181.26$$
$$C_5=(1440)(840)(0.01)(0.005)+[(2787)(883)+(7817)(460)](0.005)(0.005)$$
$$=211.90$$
$$\Delta\tau_{\max}=\frac{211.90}{181.26}=1.169 \text{ sec}$$

4-87

$$\frac{1}{R_{42}}=\frac{(20)(0.005)}{0.01}=10 \qquad \frac{1}{R_{43}}=\frac{(20)(0.01)}{0.01}=20$$
$$\frac{1}{R_{45}}=\frac{(2)(0.005)}{0.02}=0.5 \qquad \frac{1}{R_{47}}=10+\frac{(2)(0.01)}{0.01}=12$$
$$\frac{1}{R_{4-\infty}}=(50)(0.01+0.005)=0.75 \qquad \sum\frac{1}{R}=43.25$$
$$C_4=(7800)(500)(0.01)(0.005)+(1600)(800)(0.005)(0.01)=259$$
$$\Delta\tau_{\max}=\frac{259}{43.25}=5.988 \text{ sec}$$
$$T_4{}^{p+1}=[10T_2{}^p+20T_3{}^p+0.5T_5{}^p+12T_7{}^p+(0.75)(50)]\frac{\Delta\tau}{C_4}+\left(1-\frac{\Delta\tau}{5.988}\right)T_4{}^p$$

4-88

$$\frac{1}{R_{14}} = \frac{(200)(0.005)}{0.01} = 100$$
$$\frac{1}{R_{15}} = \frac{(30)(0.005)}{0.015} = 10$$
$$\frac{1}{R_{12}} = \frac{(200)(0.005)}{0.01} + \frac{(30)(0.0075)}{0.01} = 122.5$$
$$\frac{1}{R_{1-\infty}} = hA = (50)(0.0125) = 0.625$$
$$\sum \frac{1}{R_1} = 233.125$$
$$C_1 = (2700)(900)(0.005)(0.005) + (7800)(800)(0.005)(0.0075) = 294.75$$
$$\Delta\tau_{\max} = \frac{294.75}{233.125} = 1.264 \text{ sec}$$
$$T_1^{p+1} = \frac{\Delta\tau}{294.75}[100(T_4^p - T_1^p) + 10(T_5^p - T_1^p) + 122.5(T_2^p - T_1^p) + 0.625(10 - T_1^p)] + T_1^p$$

4-89

$$\left.\frac{1}{R_{21}}\right|_A = \frac{(2)(0.005)}{0.02} = 0.5 \qquad \left.\frac{1}{R_{23}}\right|_B = \frac{(20)(0.005)}{0.01} = 10$$
$$\left.\frac{1}{R_{25}}\right|_A = \frac{(2)(0.01)}{0.01} = 2 \qquad \left.\frac{1}{R_{25}}\right|_B = \frac{(20)(0.005)}{0.01} = 10$$
$$\frac{1}{R_{2-\infty}} = hA = (120)(0.005) = 0.6 \qquad \sum \frac{1}{R_{2-j}} = 23.1 \text{ W/°C}$$
$$C_2 = (1600)(800)(0.005)(0.01) + (7800)(500)(0.005)(0.005) = 161.5 \text{ J/°C}$$
$$\Delta\tau_{\max} = \frac{161.5}{23.1} = 6.991 \text{ sec}$$
$$T_2^{p+1} = \frac{\Delta\tau}{161.5}[0.5(T_1^p - T_2^p) + 10(T_3^p - T_2^p) + 12(T_5^p - T_2^p) + 0.6(10 - T_2^p)] + T_2^p$$

4-90

$k = 16.3 \qquad \rho = 7817 \qquad C = 460 \qquad$ see Table 4–2(d)

$$Bi = \frac{h\Delta x}{k} = \frac{(60)(0.01)}{16.3} = 0.0368 \qquad F_0 = \frac{\alpha\Delta\tau}{(\Delta x)^2} \qquad F_0(3 + Bi) \le \frac{3}{4}$$

$$\alpha = \frac{16.3}{(7817)(460)} = 4.53 \times 10^{-6} \qquad F_0 < \frac{\frac{3}{4}}{3.0368} = 0.24697$$

$$\Delta\tau \leq \frac{(0.24697)(0.01)^2}{4.53 \times 10^{-6}} = 5.452 \text{ sec}$$

4-91

For $\Delta\tau = 1942$ sec

$T_1 = 11.912$ $\qquad T_2 = 11.097$ $\qquad T_4 = 22.532$ $\qquad T_5 = 20.329$

The Equations

	A	B	C	D
1	T1=			=(((2.6*C2+2.6*C4)*$C$7)/14175+(1-$C$7/1942)*C1)
2	T2=			=(((2.6*C1+26+5.2*C5)*$C$7)/28350+(1-$C$7/1942)*C2)
3				
4	T4=			=(((2.6*C1+2.6*38+5.2*C5)*$C$7)/28350+(1-$C$7/2726)*C4)
5	T5=			=(5.2*((C2+C4+38+10)*$C$7)/56700+(1-$C$7/2726)*C5)
6				
7	Dt=		1942	

The Solution

	G	H	I	J	K	L
1	Time	T1=	T2=		T4=	T5=
2	incr					
3	0	10	10		10	10
4	1	7.124092	7.124092		14.98696	14.98696
5	2	7.87606	8.388266		17.68539	16.7972
6	3	9.287557	9.167008		19.24021	18.02358
7	4	10.11878	9.85524		20.3756	18.7919
8	5	10.76837	10.27696		21.12387	19.33766
9	6	11.18512	10.58706		21.64916	19.703
10	7	11.48269	10.79142		22.0046	19.95685
11	8	11.68209	10.93484		22.25025	20.12956
12	9	11.82068	11.03187		22.41793	20.24853
13	10	11.91497	11.09893		22.53321	20.32989
14	11	11.97992	11.14471		22.61214	20.38577
15	12	12.02434	11.17618		22.66631	20.42405
16	13	12.05485	11.19773		22.70344	20.45031
17	14	12.07575	11.21251		22.72891	20.46831
18	15	12.09009	11.22265		22.74637	20.48066
19	16	12.09992	11.2296		22.75834	20.48913
20	17	12.10666	11.23437		22.76655	20.49493
21	18	12.11128	11.23764		22.77218	20.49891
22	19	12.11445	11.23988		22.77604	20.50164
23	20	12.11663	11.24141		22.77869	20.50351
24	21	12.11812	11.24247		22.7805	20.5048
25	22	12.11914	11.24319		22.78175	20.50568
26	23	12.11984	11.24368		22.7826	20.50628
27	24	12.12032	11.24402		22.78318	20.50669
28	25	12.12065	11.24426		22.78359	20.50698
29	26	12.12087	11.24442		22.78386	20.50717
30	27	12.12103	11.24453		22.78405	20.50731
31	28	12.12113	11.2446		22.78418	20.5074
32	29	12.12121	11.24465		22.78427	20.50746
33	30	12.12126	11.24469		22.78433	20.5075

4-92

The Equations

	A	B	C	D
1	T1=			=(1100+C3+C4)/4
2	T2=			=(600+C3+C4)/4
3	T3=			=(900+C1+C2)/4
4	T4=			=(800+C1+C2)/4

The Solution

	G	H	I	J	K
1	Time	T1=	T2=	T3=	T4=
2	incr				
3	0	1000	1000	1000	1000
4	1	775	650	725	700
5	2	631.25	506.25	581.25	556.25
6	3	559.375	434.375	509.375	484.375
7	4	523.4375	398.4375	473.4375	448.4375
8	5	505.4688	380.4688	455.4688	430.4688
9	6	496.4844	371.4844	446.4844	421.4844
10	7	491.9922	366.9922	441.9922	416.9922
11	8	489.7461	364.7461	439.7461	414.7461
12	9	488.623	363.623	438.623	413.623
13	10	488.0615	363.0615	438.0615	413.0615
14	11	487.7808	362.7808	437.7808	412.7808
15	12	487.6404	362.6404	437.6404	412.6404
16	13	487.5702	362.5702	437.5702	412.5702
17	14	487.5351	362.5351	437.5351	412.5351
18	15	487.5175	362.5175	437.5175	412.5175
19	16	487.5088	362.5088	437.5088	412.5088
20	17	487.5044	362.5044	437.5044	412.5044
21	18	487.5022	362.5022	437.5022	412.5022
22	19	487.5011	362.5011	437.5011	412.5011
23	20	487.5005	362.5005	437.5005	412.5005
24	21	487.5003	362.5003	437.5003	412.5003
25	22	487.5001	362.5001	437.5001	412.5001
26	23	487.5001	362.5001	437.5001	412.5001
27	24	487.500	362.500	437.500	412.500
28	25	487.500	362.500	437.500	412.500
29	26	487.500	362.500	437.500	412.500
30	27	487.500	362.500	437.500	412.500
31	28	487.500	362.500	437.500	412.500
32	29	487.500	362.500	437.500	412.500
33	30	487.500	362.500	437.500	412.500

4-94

	$\Delta\tau_{max}$
$C_1 = 89.7$	3.5666
$C_2 = 44.85$	3.8114
$C_3 = 179.4$	3.5880
$C_4 = 89.7$	3.5455
$C_5 = 179.4$	3.5880
$C_6 = 89.7$	3.5455
$C_7 = 179.4$	3.5880
$C_8 = 89.7$	3.5455
$C_9 = 179.4$	3.5880
$C_{10} = 224.25$	6.6527
$C_{11} = 224.25$	10.5737
$C_{12} = 89.7$	8.8374
$C_{13} = 179.4$	3.5880
$C_{14} = 358.8$	7.6885
$C_{15} = 448.5$	10.7639
$C_{16} = 179.4$	8.9700

The Equations

	A	B	C	D
1	T1=			=C1+($C$18/89.7)*(((C3-C1)/0.2)+2*((C2-C1)/0.1)+((10-C1)/6.67))
2	T2=			=C2+($C$18/44.85)*(((C1-C2)/0.1)+((C4-C2)/0.4)+((10-C2)/4.44))
3	T3=			=C3+($C$18/179.4)*(((C1-C3)/0.2)+2*((C4-C3)/0.05)+((C5-C3)/0.2))
4	T4=			=C4+($C$18/89.7)*(((C2-C4)/0.4)+((C3-C4)/0.05)+((C6-C4)/0.4)+((10-C4)/3.33))
5	T5=			=C5+($C$18/179.4)*(((C3-C5)/0.2)+2*((C6-C5)/0.05)+((C7-C5)/0.2))
6	T6=			=C6+($C$18/89.7)*(((C4-C6)/0.4)+((C5-C6)/0.05)+((C8-C6)/0.4)+((10-C6)/3.33))
7	T7=			=C7+($C$18/179.4)*(((C5-C7)/0.2)+2*((C8-C7)/0.05)+((C9-C7)/0.2))
8	T8=			=C8+($C$18/89.7)*(((C6-C8)/0.4)+((C7-C8)/0.05)+((C10-C8)/0.4)+((10-C8)/3.33))
9	T9=			=C9+($C$18/179.4)*(((C7-C9)/0.2)+2*((C10-C9)/0.05)+((C13-C9)/0.2))
10	T10=			=C10+($C$18/224.25)*(((C8-C10)/0.4)+((C9-C10)/0.05)+((C14-C10)/0.1)+((C11-C10)/0.3)+((10-C10)/2.67))
11	T11=			=C11+($C$18/224.25)*(((C10-C11)/0.3)+((C15-C11)/0.08)+((C12-C11)/0.2)+((10-C11)/2.67))
12	T12=			=C12+($C$18/89.7)*(((C11-C12)/0.2)+((C16-C12)/0.2)+((10-C12)/6.67))
13	T13=			=C13+($C$18/179.4)*(((C9-C13)/0.2)+2*((C14-C13)/0.05)+((200-C13)/0.2))
14	T14=			=C14+($C$18/358.8)*(((C10-C14)/0.1)+((C13-C14)/0.05)+((C15-C14)/0.15)+((200-C14)/0.1))
15	T15=			=C15+($C$18/448.5)*(((C11-C15)/0.08)+((C14-C15)/0.15)+((C16-C15)/0.1)+((200-C15)/0.08))
16	T16=			=C16+($C$18/179.4)*(((C12-C16)/0.2)+((C15-C16)/0.1)+((200-C16)/0.2))
17				
18	Dt=		3.5455	

The Solution

	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1		T1=	T2=	T3=	T4=	T5=	T6=	T7=	T8=	T9=	T10=	T11=	T12=	T13=	T14=	T15=	T16=
2	Time																
3	incr	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
4	1	198.8741	196.6171	200	197.7448	200	197.7448	200	197.7448	200	198.8749	198.8749	198.8741	200	200	200	200
5	2	196.1932	195.3015	198.1059	197.1877	198.2172	197.2991	198.2172	197.4108	199.1106	198.2455	197.9788	197.9775	200	199.8888	199.8888	199.8887
6	3	194.7629	193.0799	197.202	195.5163	197.4804	195.8017	197.668	195.8952	198.4263	197.6171	197.2571	197.2415	199.8242	199.7594	199.7111	199.7109
7	4	192.8196	191.6322	195.6559	194.4343	196.1444	194.9064	195.3229	195.251	197.8498	197.0137	196.651	196.623	199.6522	199.5811	199.4997	199.4955
8	5	191.3581	189.8908	194.4582	192.9804	195.1335	193.6777	195.6088	194.0394	197.216	196.4878	196.1256	196.0903	199.4523	199.3775	199.2716	199.2623
9	6	189.7361	188.4585	193.0504	191.7403	193.9629	192.6152	194.48	193.3017	196.7025	195.9548	195.6615	195.6214	199.2263	199.1613	199.0375	199.0236
10	7	188.316	186.9397	191.7774	190.3809	192.8584	191.4944	193.7171	192.2518	196.1412	195.4767	195.2416	195.2017	199.0019	198.932	198.8044	198.7866
11	8	186.8554	185.5574	190.4382	189.1138	191.7581	190.3832	192.7134	191.4907	195.6591	194.995	194.858	194.8206	198.7626	198.7015	198.5758	198.5558
12	9	185.4894	184.1606	189.1676	187.8088	190.6352	189.313	191.9435	190.5399	195.1497	194.5498	194.5024	194.471	198.5299	198.4673	198.3542	198.3334
13	10	184.1259	182.8311	187.875	186.5606	189.5742	188.2024	191.0215	189.7815	194.6927	194.1051	194.1711	194.1473	198.2917	198.2366	198.1406	198.1205
14	11	182.8114	181.5146	186.6333	185.2976	188.4649	187.1655	190.261	188.899	194.221	193.6865	193.8597	193.846	198.0613	198.0072	197.9355	197.9176
15	12	181.5175	180.2337	185.3807	184.0836	187.4342	186.0765	189.3981	188.154	193.7866	193.2708	193.5662	193.5639	197.8306	197.7831	197.7391	197.7246
16	13	180.2497	178.9785	184.1764	182.8592	186.3521	185.0681	188.6542	187.3232	193.3448	192.8754	193.2882	193.2989	197.6078	197.5628	197.5514	197.5412
17	14	179.0119	177.7418	182.9621	181.6835	185.3496	184.0096	187.838	186.5964	192.9315	192.4843	193.0245	193.0489	197.3873	197.3486	197.3719	197.3669
18	15	177.7869	176.5383	181.7969	180.4968	184.3003	183.0292	187.1139	185.808	192.515	192.1099	192.7733	192.8127	197.1746	197.1391	197.2004	197.2014
19	16	176.5981	175.3426	180.6202	179.3599	183.3261	182.0046	186.3372	185.1017	192.1215	191.7407	192.5341	192.5889	196.9653	196.9359	197.0364	197.0441
20	17	175.4132	174.1852	179.4939	178.2103	182.3116	181.0524	185.6346	184.35	191.7275	191.3857	192.3055	192.3766	196.7633	196.7377	196.8797	196.8944
21	18	174.2686	173.0282	178.3544	177.1115	181.3661	180.0625	184.8928	183.6654	191.3528	191.0363	192.0871	192.1746	196.5653	196.5456	196.7297	196.7519
22	19	173.1221	171.9131	177.2657	175.9986	180.3864	179.1388	184.2123	182.9467	190.9793	190.6995	191.8779	191.9824	196.3741	196.3586	196.5862	196.6162
23	20	172.0186	170.7934	176.1629	174.9365	179.4698	178.1834	183.5024	182.2842	190.6225	190.3683	191.6776	191.7991	196.187	196.1773	196.4486	196.4867
24	21	170.909	169.7179	175.1107	173.8597	178.5246	177.2884	182.8445	181.5959	190.2679	190.0485	191.4853	191.6242	196.0062	196.0008	196.3168	196.3632
25	22	169.8442	168.6343	174.0439	172.8332	177.6369	176.3668	182.1641	180.9557	189.928	189.7344	191.3007	191.457	195.8295	195.8297	196.1904	196.2451
26	23	168.7705	167.5962	173.0269	171.7918	176.7252	175.5004	181.5287	180.2958	189.5909	189.4306	191.1233	191.2971	195.6586	195.6632	196.069	196.1322
27	24	167.7425	166.5478	171.9954	170.7996	175.8662	174.6116	180.8761	179.6779	189.2671	189.1323	190.9527	191.1439	195.4917	195.5017	195.9523	196.0241
28	25	166.7038	165.5453	171.0123	169.7928	174.9869	173.7734	180.263	179.0447	188.9465	188.8435	190.7885	190.9972	195.33	195.3445	195.8402	195.9206
29	26	165.7108	164.5311	170.0152	168.8338	174.1562	172.9163	179.6366	178.4486	188.6378	188.5601	190.6303	190.8564	195.1721	195.1919	195.7323	195.8213
30	27	164.7062	163.5627	169.0651	167.8607	173.3084	172.106	179.0454	177.8408	188.3326	188.2852	190.4778	190.7212	195.0191	195.0434	195.6284	195.726
31	28	163.7469	162.5819	168.1016	166.9339	172.5055	171.2796	178.4439	177.2662	188.0382	188.0157	190.3307	190.5912	194.8698	194.8991	195.5283	195.6345
32	29	162.7755	161.6463	167.1833	165.9936	171.688	170.4965	177.8742	176.6825	187.7474	187.7541	190.1886	190.4663	194.7249	194.7586	195.4318	195.5465
33	30	161.8486	160.6981	166.2524	165.0979	170.9122	169.6997	177.2964	176.1288	187.4666	187.4975	190.0515	190.346	194.5833	194.6221	195.3387	195.4619
34	31	160.9096	159.794	165.365	164.1895	170.1241	168.9432	176.7475	175.5681	187.1893	187.2483	189.9189	190.2301	194.446	194.489	195.2488	195.3805
35	32	160.0139	158.8776	164.4658	163.324	169.3748	168.175	176.1924	175.0348	186.9212	187.004	189.7907	190.1184	194.3118	194.3597	195.162	195.302
36	33	159.1064	158.0039	163.6083	162.4467	168.6149	167.4445	175.6638	174.4959	186.6568	186.7664	189.6667	190.0107	194.1814	194.2335	195.0781	195.2263
37	34	158.241	157.1184	162.7399	161.6103	167.8915	166.7038	175.1303	173.9823	186.4007	186.5334	189.5466	189.9068	194.054	194.1108	194.9969	195.1534
38	35	157.3642	156.2742	161.9115	160.7631	167.1589	165.9986	174.6212	173.4644	186.1482	186.3068	189.4302	189.8064	193.9302	193.9912	194.9184	195.0829
39	36	156.5279	155.4187	161.0729	159.9552	166.4605	165.2846	174.1084	172.9699	185.9035	186.0846	189.3175	189.7093	193.8092	193.8746	194.8424	195.0149
40	37	155.6809	154.603	160.2726	159.1372	165.7542	164.6038	173.6182	172.472	185.6623	185.8683	189.2081	189.6155	193.6915	193.761	194.7687	194.9491
41	38	154.873	153.7766	159.4629	158.3566	165.0802	163.9154	173.1252	171.9958	185.4284	185.6561	189.1021	189.5247	193.5764	193.6502	194.6973	194.8855
42	39	154.0549	152.9886	158.6899	157.567	164.3993	163.2584	172.6532	171.5171	185.1978	185.4495	188.9991	189.4368	193.4644	193.5421	194.6281	194.824
43	40	153.2743	152.1905	157.9084	156.8131	163.7488	162.5949	172.1791	171.0587	184.9741	185.2469	188.8992	189.3516	193.3548	193.4368	194.561	194.7644
44	41	152.4843	151.4292	157.1617	156.0508	163.0925	161.9609	171.7247	170.5983	184.7535	185.0494	188.802	189.2691	193.2481	193.3339	194.4958	194.7067
45	42	151.7303	150.6586	156.4074	155.3228	162.4649	161.3212	171.2687	170.157	184.5394	184.8557	188.7077	189.1891	193.1437	193.2335	194.4325	194.6507
46	43	150.9675	149.9232	155.6864	154.5871	161.8322	160.7095	170.8312	169.7142	184.3283	184.6669	188.6159	189.1115	193.042	193.1355	194.371	194.5965
47	44	150.2392	149.1792	154.9584	153.884	161.2266	160.093	170.3927	169.2892	184.1233	184.4815	188.5267	189.0361	192.9425	193.0399	194.3113	194.5439
48	45	149.5029	148.4689	154.2621	153.1741	160.6168	159.5028	169.9714	168.8633	183.9212	184.3009	188.4398	188.963	192.8454	192.9465	194.2532	194.4928
49	46	148.7794	147.7508	153.5597	152.4952	160.0326	158.9085	169.5495	168.4541	183.7247	184.1235	188.3553	188.8919	192.7504	192.8553	194.1967	194.4432
50	47	148.0887	147.0648	152.8874	151.8102	159.4447	158.3391	169.1439	168.0443	183.5311	183.9505	188.273	188.8229	192.6578	192.7661	194.1417	194.395
51	48	147.4093	146.3717	152.2096	151.1548	158.8812	157.7663	168.7379	167.6504	183.3429	183.7807	188.1929	188.7557	192.5671	192.679	194.0882	194.3482
52	49	146.7234	145.7092	151.5606	150.4939	158.3146	157.2171	168.3474	167.2561	183.1573	183.6151	188.1148	188.6904	192.4786	192.5939	194.0361	194.3027
53	50	146.0674	145.0403	150.9067	149.8611	157.771	156.6649	167.9567	166.8767	182.9768	183.4524	188.0387	188.6269	192.3919	192.5107	193.9854	194.2584
54	51	145.4056	144.4007	150.2802	149.2235	157.2248	156.1352	167.5807	166.4973	182.7989	183.2936	187.9645	188.5651	192.3072	192.4293	193.936	194.2153
55	52	144.7722	143.7553	149.6494	148.6127	156.7005	155.6029	167.2047	166.1319	182.6257	183.1377	187.8921	188.5049	192.2243	192.3498	193.8878	194.1734
56	53	144.1335	143.1377	149.0447	147.9977	156.1741	155.092	166.8425	165.7667	182.4551	182.9854	187.8215	188.4462	192.1434	192.272	193.8408	194.1326
57	54	143.522	142.5149	148.4362	147.4082	155.6684	154.579	166.4806	165.4149	182.2889	182.8359	187.7527	188.3891	192.064	192.1959	193.795	194.0928
58	55	142.9059	141.9187	147.8526	146.8149	155.161	154.0863	166.1318	165.0634	182.1252	182.6899	187.6854	188.3334	191.9865	192.1214	193.7504	194.0541
59	56	142.3155	141.318	147.2657	146.246	154.6733	153.5918	165.7835	164.7245	181.9657	182.5465	187.6198	188.2791	191.9106	192.0486	193.7068	194.0163
60	57	141.7211	140.7424	146.7024	145.6738	154.1842	153.1166	165.4475	164.3861	181.8085	182.4064	187.5558	188.2262	191.8364	191.9773	193.6642	193.9795
61	58	141.1513	140.1629	146.1363	145.1247	153.7139	152.6399	165.1122	164.0597	181.6553	182.2687	187.4932	188.1746	191.7636	191.9076	193.6227	193.9436
62	59	140.5779	139.6074	145.5928	144.5728	153.2424	152.1816	164.7886	163.7339	181.5044	182.1342	187.4321	188.1242	191.6925	191.8393	193.5822	193.9086
63	60	140.028	139.0483	145.0469	144.043	152.7889	151.7223	164.4657	163.4196	181.3572	182.0021	187.3724	188.075	191.6227	191.7725	193.5426	193.8744
64	61	139.4749	138.5122	144.5224	143.5108	152.3345	151.2803	164.154	163.1059	181.2123	181.8729	187.314	188.027	191.5545	191.707	193.5039	193.8411
65	62	138.9441	137.973	143.9959	142.9995	151.8971	150.8375	163.8431	162.8031	181.0709	181.7461	187.257	187.9802	191.4877	191.643	193.4661	193.8085
66	63	138.4107	137.4556	143.4898	142.4863	151.4592	150.4113	163.5428	162.501	180.9316	181.622	187.2012	187.9344	191.4223	191.5802	193.4292	193.7767
67	64	137.8985	136.9356	142.9821	141.993	151.0374	149.9846	163.2435	162.2093	180.7957	181.5001	187.1467	187.8897	191.3581	191.5188	193.3931	193.7457
68	65	137.384	136.4363	142.4939	141.4981	150.6153	149.5736	162.9542	161.9184	180.6619	181.3809	187.0933	187.846	191.2954	191.4586	193.3578	193.7154
69	66	136.8898	135.9347	142.0043	141.0222	150.2085	149.1622	162.6659	161.6373	180.5313	181.2638	187.0412	187.8033	191.2338	191.3996	193.3233	193.6857
70	67	136.3936	135.453	141.5333	140.545	149.8017	148.7659	162.3872	161.3572	180.4026	181.1493	186.9901	187.7616	191.1736	191.3419	193.2896	193.6568
71	68	135.9168	134.9693	141.0612	140.0859	149.4095	148.3695	162.1095	161.0864	180.277	181.0367	186.9402	187.7208	191.1145	191.2853	193.2566	193.6285
72	69	135.4383	134.5045	140.6068	139.6258	149.0173	147.9873	161.841	160.8166	180.1532	180.9266	186.8913	187.6809	191.0566	191.2298	193.2243	193.6008
73	70	134.9782	134.0381	140.1516	139.1828	148.6391	147.6052	161.5735	160.5557	180.0325	180.8184	186.8435	187.6419	190.9999	191.1755	193.1927	193.5738
74	71	134.5168	133.5896	139.7133	138.7392	148.2612	147.2367	161.3148	160.2958	179.9135	180.7125	186.7967	187.6037	190.9443	191.1222	193.1618	193.5473
75	72	134.0729	133.1398	139.2744	138.3119	147.8965	146.8684	161.0572	160.0444	179.7973	180.6084	186.7509	187.5664	190.8898	191.07	193.1315	193.5214
76	73	133.628	132.7072	138.8516	137.8842	147.5322	146.513	160.8079	159.7941	179.6828	180.5066	186.706	187.5299	190.8364	191.0189	193.1019	193.4961
77	74	133.1998	132.2735	138.4285	137.4721	147.1806	146.1581	160.5598	159.5519	179.5711	180.4065	186.6621	187.4941	190.784	190.9687	193.0729	193.4713
78	75	132.7708	131.8561	138.0206	137.0597	146.8295	145.8155	160.3195	159.3108	179.4609	180.3086	186.619	187.4591	190.7327	190.9196	193.0445	193.4471
79	76	132.3577	131.438	137.6127	136.6621	146.4904	145.4733	160.0805	159.0773	179.3534	180.2123	186.5769	187.4249	190.6823	190.8714	193.0167	193.4234
80	77	131.9441	131.0353	137.2192	136.2646	146.152	145.1429	159.849	158.845	179.2474	180.1181	186.5356	187.3914	190.633	190.8241	192.9895	193.4001
81	78	131.5456	130.6322	136.826	135.8811	145.8251	144.8132	159.6187	158.6201	179.1439	180.0255	186.4952	187.3586	190.5846	190.7778	192.9629	193.3774
82	79	131.1468	130.2438	136.4465	135.4978	145.4989	144.4946	159.3956	158.3963	179.0419	179.9348	186.4556	187.3264	190.5372	190.7324	192.9367	193.3551
83	80	130.7624	129.8551	136.0674	135.128	145.1837	144.1768	159.1738	158.1795	178.9423	179.8456	186.4167	187.2949	190.4906	190.6878	192.9112	193.3333
84	81	130.378	129.4805	135.7014	134.7585	144.8694	143.8697	158.9588	157.9639	178.8441	179.7584	186.3787	187.2641	190.445	190.6441	192.8861	193.312
85	82	130.0072	129.1058	135.3359	134.4018	144.5654	143.5634	158.7451	157.755	178.7482	179.6726	186.3414	187.2339	190.4003	190.6013	192.8615	193.2911
86	83	129.6366	128.7444	134.983	134.0456	144.2624	143.2672	158.5378	157.5473	178.6537	179.5886	186.3049	187.2043	190.3564	190.5593	192.8375	193.2706
87	84	129.279	128.3832	134.6306	133.7016	143.9693	142.972	158.3319	157.3459	178.5614	179.506	186.2691	187.1753	190.3133	190.5181	192.8139	193.2505
88	85	128.9216	128.0346	134.2902	133.3582	143.6773	142.6864	158.1322	157.1458	178.4704	179.4252	186.234	187.1469	190.2711	190.4777	192.7908	193.2309
89	86	128.5767	127.6864	133.9505	133.0265	143.3947	142.4018	157.9338	156.9517	178.3815	179.3457	186.1996	187.1191	190.2297	190.438	192.7681	193.2116
90	87	128.2322	127.3502	133.6222	132.6954	143.1133	142.1265	157.7413	156.7589	178.294	179.2678	186.1658	187.0918	190.1891	190.3991	192.7459	193.1927
91	88	127.8996	127.0145	133.2948	132.3755	142.8408	141.8522	157.5502	156.5719	178.2083	179.1913	186.1328	187.0651	190.1492	190.361	192.7242	193.1742
92	89	127.5675	126.6903	132.9782	132.0564	142.5695	141.5868	157.3646	156.3861	178.124	179.1164	186.1003	187.0389	190.1101	190.3235	192.7028	193.1561
93	90	127.2467	126.3666	132.6626	131.7478	142.3068	141.3224	157.1804	156.2058	178.0415	179.0427	186.0685	187.0132	190.0717	190.2868	192.6819	193.1383
94	91	126.9265	126.0539	132.3573	131.4402	142.0454	141.0665	157.0016	156.0268	177.9603	178.9705	186.0374	186.9881	190.0341	190.2508	192.6614	193.1209
95	92	126.6171	125.7419	132.053	131.1427	141.7921	140.8117	156.8241	155.853	177.8809	178.8995	186.0068	186.9634	189.9971	190.2155	192.6413	193.1038
96	93	126.3085	125.4404	131.7586	130.8462	141.5401	140.5649	156.6518	155.6805	177.8027	178.83	185.9768	186.9392	189.9609	190.1808	192.6216	193.0871
97	94	126.0101	125.1396	131.4653	130.5592	141.2959	140.3193	156.4807	155.513	177.7262	178.7617	185.9474	186.9155	189.9253	190.1468	192.6022	193.0707
98	95	125.7126	124.8488	131.1814	130.2734	141.053	140.0814	156.3146	155.3468	177.6509	178.6947	185.9186	186.8922	189.8905	190.1134	192.5833	193.0546
99	96	125.4249	124.5589	130.8987	129.9967	140.8175	139.8447	156.1498	155.1853	177.5772	178.6289	185.8903	186.8694	189.8562	190.0807	192.5647	193.0388
100	97	125.1381	124.2785	130.6249	129.7212	140.5834	139.6153	155.9896	155.0251	177.5046	178.5644	185.8625	186.8471	189.8227	190.0485	192.5464	193.0234
101	98	124.8606	123.999	130.3524	129.4544	140.3564	139.3871	155.8308	154.8695	177.4336	178.501	185.8353	186.8251	189.7897	190.017	192.5286	193.0082
102	99	124.5842	123.7286	130.0884	129.1888	140.1307	139.166	155.6765	154.7151	177.3637	178.4389	185.8086	186.8036	189.7573	189.986	192.511	192.9933
103	100	124.3166	123.4592	129.8257	128.9316	139.9119	138.9461	155.5234	154.5651	177.2953	178.3779	185.7824	186.7825	189.7256	189.9557	192.4938	192.9787

4-95

	$\Delta\tau_{max}$
$C_1 = 37,500$	6912
$C_2 = 75,000$	6912
$C_3 = 75,000$	6912
$C_4 = 75,000$	16,304
$C_5 = 150,000$	16,304
$C_6 = 150,000$	16,304
$C_7 = 75,000$	16,304
$C_8 = 150,000$	16,304
$C_9 = 150,000$	16,304

The Equations

	A	B	C	D
1	T1=			=((5*(5-C1)+1.15*(C2-C1)+1.15*(C4-C1))*$C$11)/37500+C1
2	T2=			=((1.15*(C1-C2)+1.15*(C3-C2)+6.25*(5-C2)+2.3*(C5-C2))*$C$11/75000+C2)
3	T3=			=((1.15*(C2-C3)+1.15*(100-C3)+6.25*(5-C3)+2.3*(C6-C3))*$C$11/75000+C3)
4	T4=			=((1.15*(C1-C4)+1.15*(C7-C4)+2.3*(C5-C4))*$C$11/75000+C4)
5	T5=			=((2.3*(C2+C4+C6+C8-4*C5))*$C$11)/150000+C5
6	T6=			=((2.3*(C3+C5+100+C9-4*C6))*$C$11)/150000+C6
7	T7=			=((1.15*(C4-C7)+1.15*(100-C7)+2.3*(C8-C7))*$C$11/75000+C7)
8	T8=			=((2.3*(C5+C7+100+C9-4*C8))*$C$11)/150000+C8
9	T9=			=((2.3*(C6+C8+200-4*C9))*$C$11)/150000+C9
10				
11	Dt=		6912	

The Solution

	G	H	I	J	K	L	M	N	O	P
1		T1=	T2=	T3=	T4=	T5=	T6=	T7=	T8=	T9=
2	Time									
3	incr	100	100	100	100	100	100	100	100	100
4	0	12.448	45.28	45.28	100	100	100	100	100	100
5	1	31.10148	30.19794	39.47705	90.72089	94.20056	94.20056	100	100	100
6	2	19.49224	30.32963	36.64893	86.1232	87.66316	89.63004	99.01656	99.38535	99.38535
7	3	22.557	27.41379	35.6939	80.7543	82.87434	85.93939	97.83247	98.16904	98.48173
8	4	19.74192	26.62213	34.60251	76.84571	78.71756	83.10881	96.32353	96.73957	97.44113
9	5	19.71833	25.32695	33.91854	73.25473	75.37334	80.81171	94.73702	95.20534	96.39018
10	6	18.69077	24.54301	33.29432	70.30658	72.52294	78.95012	93.1173	93.68756	95.3787
11	7	18.25474	23.7637	32.8166	67.7235	70.12722	77.40228	91.55006	92.23226	94.43787
12	8	17.69269	23.15899	32.40588	65.51535	68.0725	76.10637	90.06499	90.87419	93.5776
13	9	17.29066	22.62032	32.06707	63.59082	66.30944	75.00737	88.6876	89.62552	92.80076
14	10	16.90746	22.16806	31.77701	61.91986	64.78394	74.06918	87.42549	88.49104	92.10443
15	11	16.58981	21.77331	31.53019	60.45955	63.46045	73.2625	86.28086	87.46826	91.48363
16	12	16.30636	21.43293	31.31735	59.1828	62.30754	72.56557	85.24991	86.5517	90.93211
17	13	16.06152	21.13593	31.13353	58.06362	61.30099	71.9609	84.32643	85.7338	90.4434
18	14	15.84594	20.87712	30.97387	57.08173	60.42029	71.43462	83.50245	85.00628	90.0111
19	15	15.65744	20.65066	30.83488	56.21924	59.64858	70.97537	82.76952	84.3607	89.62919
20	16	15.49175	20.45235	30.71352	55.46115	58.97152	70.57381	82.11905	83.78886	89.29209
21	17	15.34628	20.2784	30.60738	54.79443	58.37696	70.22214	81.54278	83.28302	88.99473
22	18	15.21835	20.1257	30.51439	54.20784	57.85448	69.91378	81.03293	82.83602	88.73255
23	19	15.10584	19.99152	30.43284	53.69158	57.39508	69.64313	80.5823	82.44132	88.50146
24	20	15.00685	19.87357	30.36124	53.23712	56.99099	69.40539	80.18433	82.09301	88.29783

4-96

	Node	$\Delta\tau_{max}$
$C_1 = 70,312$	1	11,719
$C_2 = 70,312$	2	11,719
$C_3 = 140,625$	3	23,438
$C_4 = 140,625$	4	23,438
$C_5 = 140,625$	5	23,438
$C_6 = 140,625$	6	23,438

The Equations

	A	B	C	D
1	T1=			=((0.75*(C2-C1)+3*(15-C1)+0.75*(50-C1)+1.5*(C3-C1))*$C$8)/70312+C1
2	T2=			=((0.75*(C1-C2)+3*(15-C2)+0.75*(50-C2)+1.5*(C4-C2))*$C$8)/70312+C2
3	T3=			=(1.5*(C1+C4+C5+50-4*C3)*$C$8)/140625+C3
4	T4=			=(1.5*(C2+C3+C6+50-4*C4)*$C$8)/140625+C4
5	T5=			=(1.5*(C3+C6+100-4*C5)*$C$8)/140625+C5
6	T6=			=(1.5*(C4+C5+100-4*C6)*$C$8)/140625+C6
7				
8	Dt=		11719	

The Solution

	G	H	I	J	K	L	M
1		T1=	T2=	T3=	T4=	T5=	T6=
2	Time						
3	incr	50	50	50	50	50	50
4	1	32.4995	32.4995	50	50	50	50
5	2	30.31238	30.31238	47.81239	47.81239	50	50
6	3	29.49212	29.49212	46.17176	46.17176	49.72654	49.72654
7	4	28.97944	28.97944	45.00966	45.00966	49.35055	49.35055
8	5	28.62483	28.62483	44.17227	44.17227	48.97029	48.97029
9	6	28.37116	28.37116	43.55704	43.55704	48.62796	48.62796
10	7	28.18565	28.18565	43.09803	43.09803	48.3371	48.3371
11	8	28.04771	28.04771	42.75161	42.75161	48.09794	48.09794
12	9	27.94386	27.94386	42.48795	42.48795	47.90516	47.90516
13	10	27.86497	27.86497	42.28609	42.28609	47.75171	47.75171

4-98

$\rho = 7600 \qquad C = 450 \qquad k = 35$

$C_1 = (7600)(450)\pi(0.01)^2(0.02) = 21.488 = C_2 = C_3 = C_4$

$C_5 = 10.744$

$$\Delta\tau_{\max,\,1} = \frac{21.488}{1.1499} = 18.687 \text{ sec}$$

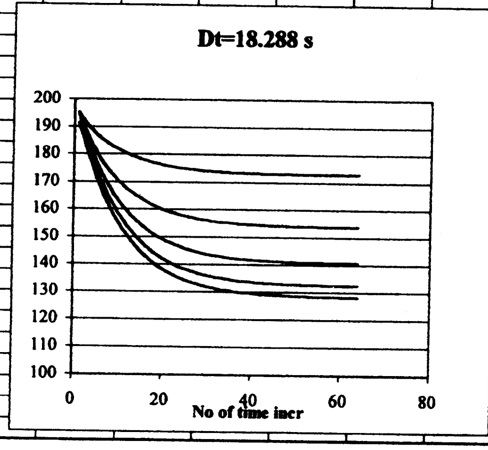
$$\Delta\tau_{\max,\,5} = \frac{10.744}{0.5875} = 18.288 \text{ sec}$$

The Equations

	A	B	C	D
1	T1			=(0.5498*(200-C1)+0.5498*(C2-C1)+0.05027*(25-C1)+3.142)*$C$7/21.488+C1
2	T2			=(0.5498*(C1-C2)+0.5498*(C3-C2)+0.05027*(25-C2)+3.142)*$C$7/21.488+C2
3	T3			=(0.5498*(C2-C3)+0.5498*(C4-C3)+0.05027*(25-C3)+3.142)*$C$7/21.488+C3
4	T4			=(0.5498*(C3-C4)+0.5498*(C5-C4)+0.05027*(25-C4)+3.142)*$C$7/21.488+C4
5	T5			=(0.5498*(C4-C5)+0.0377*(25-C5)+1.571)*$C$7/10.744+C5
6				
7	Dt			

The Solution

	G	H	I	J	K	L	M	N	O	P	Q	R	S
1		T1	T2	T3	T4	T5							
2	Time												
3	Incr	200	200	200	200	200							
4	1	195.1869	195.1869	195.1869	195.1869	191.4441							
5	2	192.8319	190.5798	190.5798	188.8284	186.94							
6	3	190.6258	187.2236	185.3502	184.4292	180.9895							
7	4	189.0082	183.6725	181.6095	179.1037	176.8725							
8	5	187.312	181.0894	177.3761	175.3132	171.8888							
9	6	186.0671	178.2596	174.3032	170.9193	168.3415							
10	7	184.7163	176.1787	170.8574	167.7276	164.2296							
11	8	183.7138	173.8898	168.3166	164.123	161.2427							
12	9	182.6213	172.1829	165.5046	161.4594	157.8694							
13	10	181.7993	170.3194	163.3995	158.5082	155.3768							
14	11	180.9097	168.9099	161.1016	156.2938	152.615							
15	12	180.2312	167.3883	159.3567	153.8789	150.5426							
16	13	179.5047	166.2218	157.4775	152.0412	148.2827							
17	14	178.9434	164.9776	156.0316	150.0651	146.5629							
18	15	178.3492	164.0118	154.4938	148.5415	144.7136							
19	16	177.8845	162.9936	153.2961	146.9241	143.2879							
20	17	177.3982	162.194	152.0373	145.662	141.7742							
21	18	177.0136	161.3602	151.0456	144.3377	140.5931							
22	19	176.6153	160.6985	150.0146	143.2927	139.3538							
23	20	176.2971	160.0155	149.194	142.208	138.3758							
24	21	175.9707	159.468	148.3493	141.3432	137.3608							
25	22	175.7076	158.9084	147.6705	140.4546	136.5515							



4–94

	Node	$\Delta\tau_{max}$
$C_1 = 600$	1	600
$C_2 = 1200$	2	1200
$C_3 = 1800$	3	1500
$C_4 = 2400$	4	1714
$C_5 = 2400$	5	1714
$C_6 = 1200$	6	800
$C_7 = 2400$	7	2400
$C_8 = 3600$	8	4000
$C_9 = 4800$	9	6000
$C_{10} = 4800$	10	6000
$C_{11} = 1200$	11	800
$C_{12} = 2400$	12	2400
$C_{13} = 1200$	13	800
$C_{14} = 2400$	14	2400

The Equations

	A	B	C	D
1	T1=			=((0.2*(C2-C1)+0.05*(C6-C1)+0.75*(20-C1))*$C$16)/600+C1
2	T2=			=((0.2*(C1-C2)+0.2*(C4-C2)+0.1*(C7-C2)+0.5*(20-C2))*$C$16)/1200+C2
3	T3=			=((0.2*(C2-C3)+0.1*(C4-C3)+0.15*(C8-C3)+0.75*(20-C3))*$C$16)/1800+C3
4	T4=			=((0.1*(C3-C4)+0.1*(C5-C4)+0.2*(C9-C4)+(20-C4))*$C$16)/2400+C4
5	T5=			=((0.2*(C4-C5)+0.2*(C10-C5)+(20-C5))*$C$16)/2400+C5
6	T6=			=((0.05*(C1-C6)+0.05*(C11-C6)+0.4*(C7-C6)+(20-C6))*$C$16)/1200+C6
7	T7=			=((0.4*(C6-C7)+0.4*(C8-C7)+0.1*(C2-C7)+0.1*(C12-C7))*$C$16)/2400+C7
8	T8=			=((0.4*(C7-C8)+0.15*(C3-C8)+0.2*(C9-C8)+0.15*(500-C8))*$C$16)/3600+C8
9	T9=			=((0.2*(C8-C9)+0.2*(C4-C9)+0.2*(C10-C9)+0.2*(500-C9))*$C$16)/4800+C9
10	T10=			=((0.4*(C9-C10)+0.2*(C5-C10)+0.2*(500-C10))*$C$16)/4800+C10
11	T11=			=((0.4*(C12-C11)+0.05*(C6-C11)+0.05*(C13-C11)+(20-C11))*$C$16)/1200+C11
12	T12=			=((0.4*(C11-C12)+0.1*(C7-C12)+0.1*(C14-C12)+0.4*(500-C12))*$C$16)/2400+C12
13	T13=			=((0.8*(C11-C13)+0.4*(C14-C13)+(20-C13))*$C$16)/1200+C13
14	T14=			=((0.4*(C13-C14)+0.2*(C12-C14)+0.4*(500-C14))*$C$16)/2400+C14
15				
16	Dt=		600	

The Solution

	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1		T1=	T2=	T3=	T4=	T5=	T6=	T7=	T8=	T9=	T10=	T11=	T12=	T13=	T14=
2	Time														
3	incr	500	500	500	500	500	500	500	500	500	500	500	500	500	500
4	1	140	380	380	380	380	260	500	500	500	500	500	500	500	500
5	2	104	272	296	296	296	185	500	500	500	500	260	500	260	500
6	3	78.65	204.65	235.45	237.05	237.05	158.15	473	497	497	497	188	476	188	476
7	4	63.8375	160.9775	192.3375	195.4975	195.5375	143.8719	441.65	490.45	492.05	492.05	161.525	449.525	161.6	449.6
8	5	54.38909	132.0448	161.7066	166.0619	166.1679	133.7613	412.4519	481.1138	485.8338	485.8738	148.28	425.5775	148.37	425.8363
9	6	48.09703	112.3926	139.8597	145.0777	145.2606	125.5876	386.5013	469.9464	478.8125	478.9665	139.4915	404.9683	139.6423	405.4931
10	7	43.7579	98.69738	124.1308	129.9938	130.2565	118.6513	363.6721	457.7243	471.3056	471.6647	32.7016	387.4753	132.931	388.3324
11	8	40.67204	88.90296	112.6423	119.0325	119.3762	112.6515	343.582	445.0171	463.5367	464.195	127.1334	372.6767	127.454	373.9162
12	9	38.41317	81.71382	104.0912	110.9547	111.3816	107.409	325.8377	432.2245	455.6632	456.7088	122.4713	360.1583	122.8912	361.8164
13	10	36.71321	76.29832	97.58171	104.8972	105.411	102.7945	310.0924	419.6182	447.796	449.3054	118.5381	349.5572	119.0627	351.6593
14	11	35.39939	72.11289	92.50101	100.2587	100.8645	98.70741	296.0538	407.3771	440.0134	442.0492	115.2078	340.5655	115.8408	343.1286
15	12	34.35795	68.79621	88.43079	96.62078	97.32387	95.06718	283.4791	395.6141	432.3701	434.9803	112.3809	332.9245	113.1247	335.9588
16	13	33.5126	66.10435	85.08535	93.6926	94.49767	91.80862	272.1674	384.3954	424.9044	428.1224	109.9759	326.4174	110.8317	329.9278
17	14	32.8113	63.8703	82.269	91.27189	92.18254	88.87852	261.9522	373.7549	417.6425	421.4884	107.9249	320.863	108.8928	324.8499
18	15	32.21799	61.9782	79.847	89.2181	90.23645	86.23313	252.6946	363.7037	410.6016	415.0842	106.1714	316.1098	107.2507	320.5699
19	16	31.7073	60.34664	77.72591	87.4335	88.56012	83.83617	244.2787	354.2379	403.7929	408.9104	104.668	312.0311	105.8575	316.9579
20	17	31.26114	58.91772	75.84027	85.85006	87.084	81.65739	236.6064	345.3434	397.2128	402.9649	103.3755	308.5211	104.6731	313.9058
21	18	30.86641	57.6497	74.14366	84.42035	85.75928	79.67128	229.5944	336.9995	390.5941	397.2436	102.2608	305.4911	103.664	311.3227
22	19	30.5135	56.51211	72.60261	83.11115	84.5516	77.85624	223.1717	329.1817	384.807	391.741	101.2965	302.8674	102.8025	309.133
23	20	30.19523	55.48237	71.19256	81.8991	83.43666	76.19379	217.277	321.8631	378.9599	386.4512	100.4594	300.5878	102.0649	307.2733
								211.8572	315.0158	373.3495	381.3679	99.73045	298.6005	101.4319	305.6909

4-101

	Node	$\Delta\tau_{max}$
$C_1 = 412.5$	1	23.23
$C_2 = 412.5$	2	23.23
$C_3 = 825$	3	24.26
$C_4 = 825$	4	24.26
$C_5 = 412.5$	5	24.26
$C_6 = 412.5$	6	24.26

The Equations

	A	B	C	D
1	T1=			=((0.5*(C2-C1)+16*(C3-C1)+0.5*(100-C1)+0.75*(0-C1))*$C$8)/412.5+C1
2	T2=			=((0.5*(C1-C2)+16*(C4-C2)+0.5*(100-C2)+0.75*(0-C2))*$C$8)/412.5+C2
3	T3=			=((C4-C3+100-C3+16*(C5-C3)+16*(C1-C3))*$C$8)/825+C3
4	T4=			=((C3-C4+100-C4+16*(C6-C4)+16*(C2-C4))*$C$8)/825+C4
5	T5=			=((0.5*(C6-C5)+0.5*(100-C5)+16*(C3-C5))*$C$8)/412.5+C5
6	T6=			=((0.5*(C5-C6)+0.5*(100-C6)+16*(C4-C6))*$C$8)/412.5+C6
7				
8	Dt=		23.23	

The Solution

	G	H	I	J	K	L	M
1		T1=	T2=	T3=	T4=	T5=	T6=
2	Time						
3	incr	100	100	100	100	100	100
4	1	95.77636	95.77636	100	100	100	100
5	2	95.65572	95.65572	98.09716	98.09716	100	100
6	3	93.93774	93.93774	97.90809	97.90809	98.28546	98.28546
7	4	93.7183	93.7183	96.34828	96.34828	97.99371	97.99371
8	5	92.30658	92.30658	96.00754	96.00754	96.5676	96.5676
9	6	91.95924	91.95924	94.70492	94.70492	96.15961	96.15961
10	7	90.7756	90.7756	94.2724	94.2724	94.95701	94.95701
11	8	90.35207	90.35207	93.16672	93.16672	94.48215	94.48215
12	9	89.34371	89.34371	92.6837	92.6837	93.45226	93.45226
13	10	88.87968	88.87968	91.73122	91.73122	92.94412	92.94412

4-102

	Node	$\Delta\tau_{max}$
$C_1 = 87.4$	1	4.263
$C_2 = 174.8$	2	4.263
$C_3 = 174.8$	3	4.263
$C_4 = 174.8$	4	4.37
$C_5 = 349.6$	5	4.37
$C_6 = 349.6$	6	4.37
$C_7 = 174.8$	7	4.37
$C_8 = 349.6$	8	4.37
$C_9 = 349.6$	9	4.37
$C_{10} = 87.4$	10	4.37
$C_{11} = 174.8$	11	4.37
$C_{12} = 174.8$	12	4.37

The Equations

	A	B	C	D
1	T1=			`=((10*(C2-C1)+10*(C4-C1)+0.5*(20-C1)+2250)*$C$14)/87.4+C1`
2	T2=			`=((10*(C1-C2)+10*(C3-C2)+20*(C5-C2)+(20-C2)+4500)*$C$14)/174.8+C2`
3	T3=			`=((10*(C2-C3)+10*(100-C3)+20*(C6-C3)+(20-C3)+4500)*$C$14)/174.8+C3`
4	T4=			`=((20*(C5-C4)+10*(C1-C4)+10*(C7-C4)+4500)*$C$14)/174.8+C4`
5	T5=			`=(20*(C2+C4+C6+C8-4*C5+450)*$C$14)/349.6+C5`
6	T6=			`=(20*(C3+C5+100+C9-4*C6+450)*$C$14)/349.6+C6`
7	T7=			`=((20*(C8-C7)+10*(C4-C7)+10*(C10-C7)+4500)*$C$14)/174.8+C7`
8	T8=			`=(20*(C5+C7+C9+C11-4*C8+450)*$C$14)/349.6+C8`
9	T9=			`=(20*(C6+C8+100+C12-4*C9+450)*$C$14)/349.6+C9`
10	T10=			`=((10*(C11-C10)+10*(C7-C10)+2250)*$C$14)/87.4+C10`
11	T11=			`=((10*(C10-C11)+10*(C12-C11)+4500)*$C$14)/174.8+C11`
12	T12=			`=((10*(C11-C12)+10*(100-C12)+4500)*$C$14)/174.8+C12`
13				
14	Dt=		4.263	

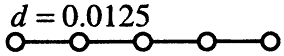
The Solution

	G	H	I	J	K	L	M	N	O	P	Q	R	S
1		T1=	T2=	T3=	T4=	T5=	T6=	T7=	T8=	T9=	T10=	T11=	T12=
2	Time												
3	incr	100	100	100	100	100	100	100	100	100	100	100	100
4	1	207.7944	207.7944	207.7944	209.7454	209.7454	209.7454	209.7454	209.7454	209.7454	209.7454	209.7454	209.7454
5	2	313.9115	313.9115	287.6228	319.015	319.015	292.2505	319.4908	319.4908	292.7263	319.4908	319.4908	292.7263
6	3	418.9784	412.5671	353.7527	427.6319	421.1046	360.6248	429.1202	422.5929	361.8811	429.2363	422.709	361.9972
7	4	520.0872	504.1228	411.1692	532.4461	515.973	420.1895	535.3472	518.7892	422.2876	535.7413	519.2399	422.6534
8	5	615.878	589.0655	462.5564	631.8501	604.1731	473.5189	636.405	608.4612	476.5462	637.2459	609.4543	477.2659
9	6	705.8037	667.9876	509.2889	725.3115	686.1659	522.0996	731.6148	692.0467	526.0685	733.0256	693.7396	527.2421
10	7	789.8938	741.3159	552.2365	812.7431	762.4467	566.7599	820.8674	769.9421	571.7017	822.9209	772.4608	573.3975
11	8	868.3139	809.5113	591.9073	894.365	833.4091	608.0596	904.2931	842.5468	613.9641	907.0524	845.9651	616.2386
12	9	941.396	872.9303	628.6868	970.4467	899.4628	646.3588	982.173	910.2095	653.2258	985.6561	914.583	656.1098
13	10	1009.446	931.9475	662.8376	1041.344	960.9432	681.9505	1054.807	973.2781	689.7527	1059.036	978.6255	693.268
14	11	1072.819	986.8653	694.5941	1107.377	1018.193	715.0525	1122.534	1032.057	723.7703	1127.498	1038.389	727.9206
15	12	1131.82	1037.995	724.1362	1168.891	1071.5	745.866	1185.663	1086.846	755.4621	1191.359	1094.149	760.2461
16	13	1186.768	1085.594	751.638	1226.183	1121.153	774.5545	1244.509	1137.912	784.9983	1250.911	1146.17	790.4035
17	14	1237.935	1129.926	777.2423	1279.556	1167.4	801.2767	1299.353	1185.513	812.5265	1306.446	1194.696	818.5381
18	15	1285.596	1171.21	801.0901	1329.274	1210.486	826.1675	1350.474	1229.88	838.1878	1358.225	1239.958	844.7843
19	16	1329.988	1209.669	823.3015	1375.598	1250.625	849.359	1398.119	1271.238	862.1076	1406.504	1282.172	869.2671
20	17	1371.346	1245.494	843.9949	1418.756	1288.027	870.9663	1442.53	1309.788	884.4064	1451.515	1321.54	892.1032
21	18	1409.874	1278.874	863.273	1458.973	1322.877	891.1018	1483.923	1345.724	905.1927	1493.482	1358.253	913.4021
22	19	1445.775	1309.973	881.2366	1496.447	1355.353	909.8646	1522.507	1379.221	924.5706	1532.606	1392.488	933.2658
23	20	1479.226	1338.953	897.9745	1531.371	1385.617	927.3511	1558.471	1410.446	942.6344	1569.082	1424.411	951.7901

4-103

$k = 43 \qquad \rho = 7800 \qquad C = 470 \qquad \Delta x = 5 \text{ cm} \qquad h = 35 \ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

$d = 0.0125$



$$\frac{1}{R_{12}} = \frac{kA}{\Delta x} = \frac{(43)\pi(0.0125)^2}{(4)(0.05)} = 0.1055$$

$$\frac{1}{R_{1-\infty}} = hA = (35)\pi(0.0125)(0.05) = 0.0687$$

$$\frac{1}{R_{4-\infty}} = (35)[\pi(0.00625)^2 + \pi(0.025)(0.0125)] = 0.0387$$

$$C_1 = \frac{(7800)(470)\pi(0.0125)^2(0.05)}{4} = 22.494$$

$$C_4 = \frac{C_1}{2} = 11.247$$

Node	$\sum \frac{1}{R}$	$\frac{C}{\sum \frac{1}{R}}$
1	0.2797	80.42
2	0.2797	80.42
3	0.2797	80.42
4	0.1442	78.00

Excel solution for $\Delta\tau = 25$ sec and 75 sec shown below
$T_2 = 190°C$ occurs at six, 25 sec time increments.
Time $= (25)(6) = 150$ sec
Steady state reached at about $(30)(75) = 2250$ sec

The Equations

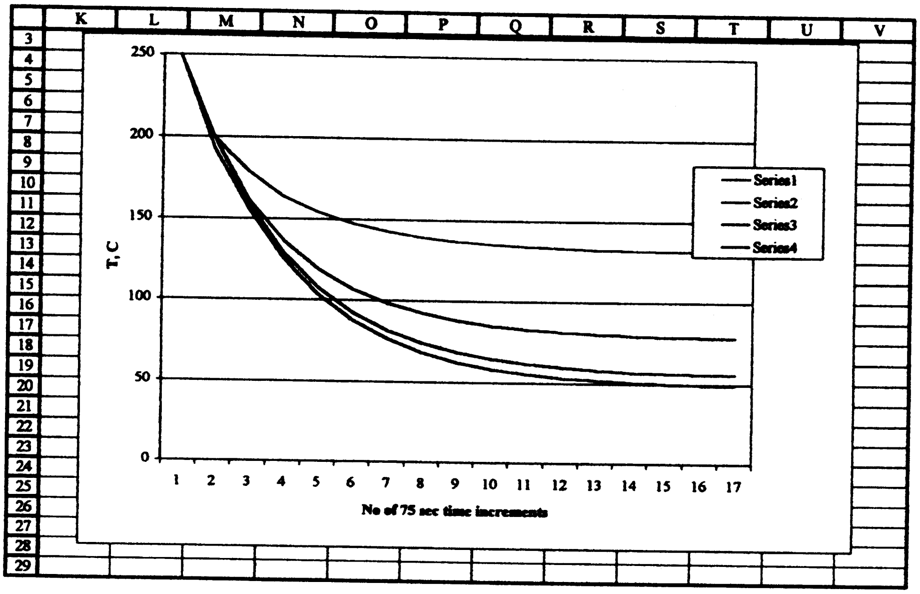
	A	B	C
1	T1=	250	=(0.1055*(250-B1)+0.1055*(B2-B1)+0.0687*(30-B1))*$E$1/22.494+B1
2	T2=	250	=(0.1055*(B1-B2)+0.1055*(B3-B2)+0.0687*(30-B2))*$E$1/22.494+B2
3	T3=	250	=(0.1055*(B2-B3)+0.1055*(B4-B3)+0.0687*(30-B3))*$E$1/22.494+B3
4	T4=	250	=(0.1055*(B3-B4)+0.0387*(30-B4))*$E$1/11.247+B4

The Solution $\Delta\tau = 25$ sec

	E	F	G	H	I	J
1	25	T1=	T2=	T3=	T4=	NO. time i
2		250	250	250	250	0
3		233.2021872	233.202	233.202	231.075	1
4		219.6565517	217.687	217.438	214.277	2
5		208.5025058	203.558	202.785	199.166	3
6		199.1591519	190.795	189.258	185.462	4
7		191.2238042	179.318	176.833	172.979	5
8		184.4095428	169.022	165.462	161.584	6
9		178.5062724	159.794	155.081	151.174	7
10		173.3560731	151.525	145.625	141.666	8
11		168.8373313	144.114	137.025	132.989	9
12		164.8543305	137.469	129.211	125.076	10
13		161.3302817	131.506	122.119	117.867	11
14		158.2025554	126.152	115.688	111.306	12
15		155.4193442	121.341	109.858	105.339	13
16		152.9372761	117.017	104.577	99.9179	14
17		150.7196685	113.126	99.7953	94.996	15
18		148.7352253	109.624	95.4665	90.5303	16
19		146.9570465	106.47	91.5491	86.4809	17
20		145.3618616	103.629	88.0049	82.8108	18
21		143.9294278	101.069	84.7989	79.4859	19
22		142.6420506	98.7601	81.8995	76.4749	20
23		141.4841976	96.6784	79.2777	73.7491	21
24		140.4421843	94.8006	76.9072	71.2822	22
25		139.5039146	93.1064	74.7642	69.0501	23
26		138.6586669	91.5776	72.827	67.0309	24
27		137.8969134	90.1977	71.0759	65.2046	25

The Solution $\Delta\tau = 75$ sec

	F	G	H	I	J
1	T1=	T2=	T3=	T4=	NO. time i
2	250	250	250	250	0
3	199.6065617	199.607	199.607	193.225	1
4	178.4827185	160.756	158.511	155.591	2
5	163.3926051	136.251	128.837	125.235	3
6	153.7552402	118.852	107.538	103.192	4
7	146.9853904	106.797	92.2282	87.3609	5
8	142.2884591	98.2178	81.3868	75.9821	6
9	138.9538804	92.1736	73.6354	67.9179	7
10	136.60297	87.8665	68.15	62.1549	8
11	134.9294051	84.8196	64.2379	58.0744	9
12	133.7448107	82.6494	61.4671	55.1655	10
13	132.9015472	81.1117	59.4936	53.1044	11
14	132.3038016	80.0172	58.0947	51.6368	12
15	131.8785075	79.2411	57.0992	50.5963	13
16	131.5768177	78.689	56.393	49.8559	14
17	131.3622656	78.2972	55.8908	49.3307	15
18	131.2100006	78.0187	55.5343	48.9572	16
19	131.1017517	77.821	55.2809	48.6921	17
20	131.0249059	77.6804	55.1011	48.5036	18
21	130.9702875	77.5806	54.9732	48.3698	19
22	130.931506	77.5097	54.8824	48.2748	20
23	130.9039466	77.4594	54.8179	48.2072	21
24	130.8843754	77.4236	54.7721	48.1593	22
25	130.8704692	77.3982	54.7395	48.1252	23
26	130.8605927	77.3801	54.7164	48.101	24
27	130.8535756	77.3673	54.7	48.0838	25
28	130.8485916	77.3582	54.6883	48.0716	26
29	130.8450507	77.3517	54.68	48.0629	27
30	130.8425356	77.3471	54.6742	48.0567	28
31	130.8407489	77.3439	54.67	48.0523	29
32	130.8394797	77.3415	54.667	48.0492	30
33	130.8385781	77.3399	54.6649	48.047	31
34	130.8379376	77.3387	54.6634	48.0455	32
35	130.8374826	77.3379	54.6623	48.0443	33
36	130.8371594	77.3373	54.6616	48.0435	34
37	130.8369298	77.3369	54.661	48.043	35
38	130.8367668	77.3366	54.6606	48.0426	36
39	130.8366509	77.3364	54.6604	48.0423	37



4-105

$$\frac{\alpha\Delta\tau}{(\Delta x)^2}=\frac{1}{4} \qquad \Delta\tau_{max}=\frac{\frac{(0.15)^2}{4}}{1.29\times10^{-5}}=436\text{ sec}$$

$$C_1=C_2=C_3=C_4=78{,}488$$

<u>The Equations</u>

	A	B	C	D
1	T1=			=(45*(C2+C3+100+30-4*C1)*$C$6)/78488+C1
2	T2=			=(45*(C1+C4+30+100-4*C2)*$C$6)/78488+C2
3	T3=			=(45*(C1+C4+200-4*C3)*$C$6)/78488+C3
4	T4=			=(45*(C2+C3+200-4*C4)*$C$6)/78488+C4
5				
6	Dt=		436	

The Solution

	G	H	I	J	K
1		T1=	T2=	T3=	T4=
2	Time				
3	intr	500	500	500	500
4	1	282.5222	282.5222	300.0204	300.0204
5	2	178.1463	178.1463	195.6463	195.6463
6	3	125.9535	125.9535	143.4535	143.4535
7	4	99.85439	99.85439	117.3544	117.3544
8	5	86.80353	86.80353	104.3035	104.3035
9	6	80.27743	80.27743	97.77743	97.77743
10	7	77.01405	77.01405	94.51405	94.51405
11	8	75.38219	75.38219	92.88219	92.88219
12	9	74.56618	74.56618	92.06618	92.06618
13	10	74.15813	74.15813	91.65813	91.65813

4-106

	Node	$\Delta\tau_{max}$
$C_1 = 350$	1	16.83
$C_2 = 700$	2	17.16
$C_3 = 700$	3	17.16
$C_4 = 350$	4	16.83
$C_5 = 700$	5	17.16
$C_6 = 1400$	6	17.16
$C_7 = 1400$	7	17.5
$C_8 = 700$	8	17.5

The Equations

	A	B	C	D
1	T1=			=((10*(C2-C1)+10*(C5-C1)+0.8*(300-C1))*$C$8)/350+C1
2	T2=			=((10*(C1-C2)+20*(C5-C2)+0.8*(300-C2))*$C$8)/700+C2
3				
4				
5	T5=			=((10*(C1-C5)+20*(C6-C5)+10*(50-C5))*$C$8)/700+C5
6	T6=			=((20*(C2-C5)+50-3*C6)*$C$8)/1400+C6
7				
8	Dt=			

The Solution 10 sec

	G	H	I	J	K	L	M
1	**Dt= 0.25**	T1=	T2=			T5=	T6=
2	Time						
3	incr	50	50			50	50
4	1	50.14286	50.07143			50	49.98214
5	2	50.2841	50.14258			50.00038	49.96455
6	3	50.42376	50.21346			50.00114	49.94722
7	4	50.56185	50.28406			50.00226	49.93015
8	5	50.69841	50.35439			50.00373	49.91334
9	6	50.83345	50.42444			50.00556	49.89678
10	7	50.96699	50.49422			50.00772	49.88047
11	8	51.09907	50.56372			50.01021	49.86442
12	9	51.2297	50.63294			50.01302	49.84861
13	10	51.3589	50.70189			50.01614	49.83305
14	11	51.48669	50.77057			50.01957	49.81773
15	12	51.61311	50.83897			50.0233	49.80265
16	13	51.73816	50.9071			50.02732	49.78781
17	14	51.86186	50.97495			50.03162	49.77321
18	15	51.98425	51.04253			50.0362	49.75884
19	16	52.10533	51.10984			50.04104	49.74471
20	17	52.22513	51.17687			50.04615	49.73081
21	18	52.34366	51.24363			50.05152	49.71713
22	19	52.46095	51.31012			50.05713	49.70368
23	20	52.57701	51.37633			50.06299	49.69046
24	21	52.69186	51.44227			50.06908	49.67746
25	22	52.80552	51.50794			50.0754	49.66468
26	23	52.91801	51.57334			50.08195	49.65212
27	24	53.02933	51.63847			50.08872	49.63977
28	25	53.13952	51.70333			50.0957	49.62765
29	26	53.24858	51.76792			50.10288	49.61573
30	27	53.35654	51.83224			50.11027	49.60402
31	28	53.4634	51.89629			50.11785	49.59253
32	29	53.56919	51.96007			50.12563	49.58124
33	30	53.67392	52.02358			50.13359	49.57016
34	31	53.7776	52.08682			50.14173	49.55928
35	32	53.88025	52.1498			50.15005	49.54861
36	33	53.98189	52.21251			50.15854	49.53814
37	34	54.08252	52.27496			50.1672	49.52786
38	35	54.18217	52.33713			50.17602	49.51779
39	36	54.28084	52.39905			50.185	49.5079
40	37	54.37855	52.4607			50.19413	49.49822
41	38	54.47532	52.52208			50.20341	49.48873
42	39	54.57116	52.5832			50.21283	49.47942
43	40	54.66607	52.64406			50.2224	49.47031

The Solution 1 min

	G	H	I	J	K	L	M
1	**Dt= 5**	T1=	T2=			T5=	T6=
2	Time						
3	incr	50	50			50	50
4	1	52.85714	51.42857			50	49.64286
5	2	55.06939	52.74694			50.15306	49.39158
6	3	56.83448	53.95515			50.38451	49.22623
7	4	58.28076	55.0567			50.65229	49.13243
8	5	59.4929	56.05746			50.93347	49.09918
9	6	60.528	56.96481			51.21614	49.11769
10	7	61.42553	57.78686			51.49463	49.18062
11	8	62.21358	58.53195			51.76664	49.2817
12	9	62.91278	59.20827			52.03167	49.41549
13	10	63.53868	59.8236			52.29003	49.57722
14	11	64.10342	60.38518			52.54239	49.76272
15	12	64.61663	60.89959			52.78948	49.96832

The Solution 1 sec

	G	H	I	J	K	L	M
1	**Dt= 0.25**	T1=	T2=			T5=	T6=
2	Time						
3	incr	50	50			50	50
4	1	50.14286	50.07143			50	49.98214
5	2	50.2841	50.14258			50.00038	49.96455
6	3	50.42376	50.21346			50.00114	49.94722
7	4	50.56185	50.28406			50.00226	49.93015

The Solution Steady state

	G	H	I	J	K	L	M
1	D0= 15	T1=	T2=			T5=	T6=
2	Time						
3	incr	50	50			50	50
4	1	58.57143	54.28571			50	48.92857
5	2	61.33878	57.57959			51.37755	48.80995
6	3	63.64127	59.88289			52.11651	49.10578
7	4	65.19508	61.4761			52.84225	49.72732
8	5	66.35762	62.66179			53.54526	50.51477
9	6	67.29328	63.61532			54.23228	51.38033
10	7	68.09796	64.43446			54.90189	52.27519
11	8	68.82336	65.17237			55.55348	53.17332
12	9	69.49762	65.85796			56.18693	54.06109
13	10	70.13612	66.50702			56.80237	54.93149
14	11	70.74738	67.12829			57.40015	55.78111
15	12	71.33619	67.72669			57.98065	56.60847
16	13	71.90536	68.30511			58.54433	57.41306
17	14	72.45663	68.86531			59.09165	58.19495
18	15	72.99113	69.40848			59.62306	58.95447
19	16	73.5097	69.93544			60.13902	59.69209
20	17	74.01296	70.44685			60.63998	60.40836
21	18	74.50148	70.94327			61.12636	61.10385
22	19	74.97572	71.42518			61.59859	61.77914
23	20	75.43612	71.89304			62.05708	62.43479
24	21	75.88312	72.34727			62.50223	63.07138
25	22	76.3171	72.78827			62.93444	63.68945
26	23	76.73844	73.21643			63.35406	64.28954
27	24	77.14753	73.63214			63.76148	64.87217
28	25	77.54471	74.03574			64.15704	65.43785
29	26	77.93033	74.42761			64.54109	65.98707
30	27	78.30474	74.80807			64.91397	66.52031
31	28	78.66825	75.17746			65.276	67.03803
32	29	79.02118	75.5361			65.6275	67.54069
33	30	79.36384	75.88431			65.96876	68.02873
34	31	79.69653	76.22239			66.3001	68.50257
35	32	80.01955	76.55063			66.6218	68.96262
36	33	80.33316	76.86932			66.93414	69.40928
37	34	80.63765	77.17873			67.23739	69.84295
38	35	80.93328	77.47915			67.53182	70.264
39	36	81.22031	77.77082			67.81768	70.6728
40	37	81.49899	78.05401			68.09522	71.06971
41	38	81.76956	78.32896			68.36469	71.45506
42	39	82.03226	78.5959			68.62632	71.82921
43	40	82.28731	78.85508			68.88033	72.19246
44	41	82.53494	79.10672			69.12696	72.54515
45	42	82.77537	79.35104			69.3664	72.88758
46	43	83.0088	79.58825			69.59889	73.22005
47	44	83.23544	79.81856			69.8246	73.54284
48	45	83.45549	80.04216			70.04375	73.85623
49	46	83.66913	80.25926			70.25653	74.16051
50	47	83.87656	80.47005			70.46311	74.45594
51	48	84.07795	80.6747			70.66368	74.74277
52	49	84.27348	80.87339			70.85842	75.02126
53	50	84.46333	81.06631			71.04749	75.29164
54	51	84.64764	81.25361			71.23106	75.55416
55	52	84.8266	81.43546			71.40928	75.80903
56	53	85.00035	81.61202			71.58233	76.0565
57	54	85.16904	81.78345			71.75033	76.29676
58	55	85.33283	81.94988			71.91345	76.53003
59	56	85.49185	82.11147			72.07183	76.75651
60	57	85.64624	82.26837			72.22559	76.97641
61	58	85.79615	82.42069			72.37488	77.1899
62	59	85.94169	82.56859			72.51983	77.39719
63	60	86.08299	82.71218			72.66056	77.59844
64	61	86.22018	82.85159			72.79719	77.79384
65	62	86.35339	82.98695			72.92985	77.98355
66	63	86.48271	83.11837			73.05865	78.16774
67	64	86.60828	83.24596			73.18371	78.34657
68	65	86.73019	83.36985			73.30512	78.5202
69	66	86.84855	83.49012			73.423	78.68878
70	67	86.96347	83.6069			73.53745	78.85245
71	68	87.07504	83.72028			73.64857	79.01136
72	69	87.18337	83.83037			73.75646	79.16565
73	70	87.28855	83.93724			73.86121	79.31545
74	71	87.39067	84.04101			73.96291	79.46089
75	72	87.48981	84.14176			74.06165	79.6021
76	73	87.58607	84.23958			74.15752	79.7392
77	74	87.67953	84.33455			74.2506	79.87231
78	75	87.77027	84.42676			74.34097	80.00154
79	76	87.85837	84.51629			74.42872	80.12702
80	77	87.94391	84.60321			74.51391	80.24885
81	78	88.02696	84.6876			74.59662	80.36713
82	79	88.10759	84.76954			74.67692	80.48197
83	80	88.18588	84.84909			74.75489	80.59346
84	81	88.26189	84.92633			74.83059	80.70172
85	82	88.33568	85.00132			74.90408	80.80682
86	83	88.40733	85.07413			74.97544	80.90887
87	84	88.4769	85.14482			75.04472	81.00795
88	85	88.54444	85.21346			75.11199	81.10414
89	86	88.61001	85.28009			75.1773	81.19754
90	87	88.67368	85.34479			75.2407	81.28822
91	88	88.7355	85.40761			75.30227	81.37626
92	89	88.79551	85.46859			75.36204	81.46173
93	90	88.85378	85.52781			75.42007	81.54473
94	91	88.91036	85.5853			75.47642	81.6253
95	92	88.96529	85.64111			75.53112	81.70353
96	93	89.01862	85.69531			75.58424	81.77949
97	94	89.0704	85.74792			75.63581	81.85324
98	95	89.12067	85.79901			75.68587	81.92484
99	96	89.16948	85.84861			75.73448	81.99435
100	97	89.21687	85.89677			75.78168	82.06185
101	98	89.26288	85.94352			75.8275	82.12738
102	99	89.30755	85.98892			75.87199	82.191
103	100	89.35092	86.03299			75.91519	82.25277

4-112

Wall thickness = 0.25 m $\qquad T_\infty = 600°\text{C} \qquad h = 100\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$k = 0.16\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \qquad \alpha = 3.5\times10^{-7}\ \text{m}^2/\text{s}$

Approximate as inf. plate with $2L = 0.5$
Center plane is insulated

$$\frac{k}{hL} = \frac{0.16}{(100)(0.25)} = 6.4\times10^{-3} \qquad A = (6)(1.0) = 6.0\ \text{m}^2$$

$$V = 1.0 - (0.5)^3 = 0.875\ \text{m}^3$$

$$\frac{h(V/A)}{k} = 546.9 \qquad \underline{\text{Not}}\ \text{lumped capacity}$$

$$\frac{\theta_0}{\theta_i} = \frac{150-600}{30-600} = 0.789 \qquad \frac{\alpha\tau}{L^2} = 0.2$$

$$\tau = \frac{(0.25)^2(0.2)}{3.5\times10^{-7}} = 3.57\times10^4\ \text{sec} = 9.92\ \text{hr}$$

4-114

$k = 1.07 \qquad \alpha = 5.4\times10^{-7} \qquad T_i = 20°\text{C} \qquad x = 2.0\ \text{cm}$

$$\frac{q}{A} = 4500\ \text{W/m}^2 \qquad T_0 = T_i + \frac{\left(2\frac{q_0}{A}\right)\sqrt{\frac{\alpha\tau}{\pi}}}{k}$$

τ	T_0
0	20
100	54.87
300	80.4
900	124.61
1500	155.05
1400	150.47
1390	150
2000	175.94

at $x = 2$ cm $\quad \tau = 1390$

$$\frac{q}{A} = -k\frac{\partial T}{\partial x}$$

$$= \frac{2q_0}{A}\sqrt{\frac{\alpha\tau}{\pi}}\left(\frac{-x^2}{4\alpha\tau}\right)\exp\left(\frac{-x^2}{4\alpha\tau}\right)\left(\frac{-2x}{4\alpha\tau}\right) - \frac{q_0}{A}x\left[-\exp\left(\frac{-x^2}{4\alpha\tau}\right)\left(\frac{1}{2\sqrt{\alpha\tau}}\right)\right]$$

$$- \frac{q_0}{A}\left[1-\operatorname{erf}\left(\frac{x}{2\sqrt{\alpha\tau}}\right)\right]$$

$$= 1180 \ \mathrm{W/m^2}$$

4-117

$$\frac{k}{hr_0} = \frac{3.2}{(350)(0.075)} = 0.122 \qquad \frac{\alpha\tau}{r_0^2} = \frac{(13\times10^{-7})(21)(60)}{(0.075)^2} = 0.291$$

$$\frac{r}{r_0} = \frac{4.5}{7.5} = 0.6 \qquad \frac{\theta_0}{\theta_i} = 0.65 \qquad \frac{\theta}{\theta_0} = 0.59$$

$$T = (0.65)(0.59)(120-30)+30 = 64.5°\mathrm{C}$$

$$\frac{hr_0}{k} = 8.2 \qquad \frac{h^2\alpha\tau}{k^2} = \frac{(350)^2(13\times10^{-7})(21)(60)}{(3.2)^2} = 19.6$$

$$\frac{Q}{Q_0} = 0.92 \qquad \rho c = \frac{k}{\alpha}$$

$$Q = \frac{(0.92)(3.2)}{13\times10^{-7}}(120-30)\frac{4}{3}\pi(0.075)^3 = 3.6\times10^5 \ \mathrm{J}$$

4-127

$$\frac{1}{R}=\frac{kA}{\Delta x}=\frac{(20)(0.01)}{0.01}=20 \qquad \alpha=\frac{k}{\rho c}=5\times10^{-6}$$

$$\sum\frac{1}{R}=(4)(20)=80 \qquad C=\rho c\Delta V=\frac{k}{\alpha}(\Delta x)^2=\frac{(20)(0.01)^2}{5\times10^{-6}}=400$$

$$\Delta\tau_{max}=\frac{C}{\sum\frac{1}{R}}=\frac{400}{80}=5\text{ sec}$$

$$T_i=100°C \qquad \frac{\Delta\tau}{C}=\frac{5}{400}=\frac{1}{80}$$

Excel solution shown for $\Delta\tau = 5$ sec
1 minute = 12 time increments
The Equations

	A	B	C	D	E	F
1	T1=	T2=	T3=	T4=	T5=	T6=
2	100	100	100	100	100	100
3	=(140+B2+C2)/4	=(40+A2+D2)/4	=(100+A2+E2+D2)/4	=(B2+C2+F2)/4	=(200+C2+F2)/4	=(100+D2+E2)/4
4	=(140+B3+C3)/4	=(40+A3+D3)/4	=(100+A3+E3+D3)/4	=(B3+C3+F3)/4	=(200+C3+F3)/4	=(100+D3+E3)/4
5	=(140+B4+C4)/4	=(40+A4+D4)/4	=(100+A4+E4+D4)/4	=(B4+C4+F4)/4	=(200+C4+F4)/4	=(100+D4+E4)/4
6	=(140+B5+C5)/4	=(40+A5+D5)/4	=(100+A5+E5+D5)/4	=(B5+C5+F5)/4	=(200+C5+F5)/4	=(100+D5+E5)/4
7	=(140+B6+C6)/4	=(40+A6+D6)/4	=(100+A6+E6+D6)/4	=(B6+C6+F6)/4	=(200+C6+F6)/4	=(100+D6+E6)/4
8	=(140+B7+C7)/4	=(40+A7+D7)/4	=(100+A7+E7+D7)/4	=(B7+C7+F7)/4	=(200+C7+F7)/4	=(100+D7+E7)/4
9	=(140+B8+C8)/4	=(40+A8+D8)/4	=(100+A8+E8+D8)/4	=(B8+C8+F8)/4	=(200+C8+F8)/4	=(100+D8+E8)/4
10	=(140+B9+C9)/4	=(40+A9+D9)/4	=(100+A9+E9+D9)/4	=(B9+C9+F9)/4	=(200+C9+F9)/4	=(100+D9+E9)/4
11	=(140+B10+C10)/4	=(40+A10+D10)/4	=(100+A10+E10+D1	=(B10+C10+F10)/4	=(200+C10+F10)/4	=(100+D10+E10)/4
12	=(140+B11+C11)/4	=(40+A11+D11)/4	=(100+A11+E11+D1	=(B11+C11+F11)/4	=(200+C11+F11)/4	=(100+D11+E11)/4
13	=(140+B12+C12)/4	=(40+A12+D12)/4	=(100+A12+E12+D1	=(B12+C12+F12)/4	=(200+C12+F12)/4	=(100+D12+E12)/4
14	=(140+B13+C13)/4	=(40+A13+D13)/4	=(100+A13+E13+D1	=(B13+C13+F13)/4	=(200+C13+F13)/4	=(100+D13+E13)/4
15	=(140+B14+C14)/4	=(40+A14+D14)/4	=(100+A14+E14+D1	=(B14+C14+F14)/4	=(200+C14+F14)/4	=(100+D14+E14)/4
16	=(140+B15+C15)/4	=(40+A15+D15)/4	=(100+A15+E15+D1	=(B15+C15+F15)/4	=(200+C15+F15)/4	=(100+D15+E15)/4
17	=(140+B16+C16)/4	=(40+A16+D16)/4	=(100+A16+E16+D1	=(B16+C16+F16)/4	=(200+C16+F16)/4	=(100+D16+E16)/4
18	=(140+B17+C17)/4	=(40+A17+D17)/4	=(100+A17+E17+D1	=(B17+C17+F17)/4	=(200+C17+F17)/4	=(100+D17+E17)/4
19	=(140+B18+C18)/4	=(40+A18+D18)/4	=(100+A18+E18+D1	=(B18+C18+F18)/4	=(200+C18+F18)/4	=(100+D18+E18)/4
20	=(140+B19+C19)/4	=(40+A19+D19)/4	=(100+A19+E19+D1	=(B19+C19+F19)/4	=(200+C19+F19)/4	=(100+D19+E19)/4
21	=(140+B20+C20)/4	=(40+A20+D20)/4	=(100+A20+E20+D2	=(B20+C20+F20)/4	=(200+C20+F20)/4	=(100+D20+E20)/4
22	=(140+B21+C21)/4	=(40+A21+D21)/4	=(100+A21+E21+D2	=(B21+C21+F21)/4	=(200+C21+F21)/4	=(100+D21+E21)/4
23	=(140+B22+C22)/4	=(40+A22+D22)/4	=(100+A22+E22+D2	=(B22+C22+F22)/4	=(200+C22+F22)/4	=(100+D22+E22)/4
24	=(140+B23+C23)/4	=(40+A23+D23)/4	=(100+A23+E23+D2	=(B23+C23+F23)/4	=(200+C23+F23)/4	=(100+D23+E23)/4
25	=(140+B24+C24)/4	=(40+A24+D24)/4	=(100+A24+E24+D2	=(B24+C24+F24)/4	=(200+C24+F24)/4	=(100+D24+E24)/4
26	=(140+B25+C25)/4	=(40+A25+D25)/4	=(100+A25+E25+D2	=(B25+C25+F25)/4	=(200+C25+F25)/4	=(100+D25+E25)/4
27	=(140+B26+C26)/4	=(40+A26+D26)/4	=(100+A26+E26+D2	=(B26+C26+F26)/4	=(200+C26+F26)/4	=(100+D26+E26)/4
28	=(140+B27+C27)/4	=(40+A27+D27)/4	=(100+A27+E27+D2	=(B27+C27+F27)/4	=(200+C27+F27)/4	=(100+D27+E27)/4
29	=(140+B28+C28)/4	=(40+A28+D28)/4	=(100+A28+E28+D2	=(B28+C28+F28)/4	=(200+C28+F28)/4	=(100+D28+E28)/4

The Solution

	A	B	C	D	E	F
1	T1=	T2=	T3=	T4=	T5=	T6=
2	100	100	100	100	100	100
3	85	60	100	75	100	75
4	75	50	90	58.75	93.75	68.75
5	70	43.4375	81.875	52.1875	89.6875	63.125
6	66.3281	40.5469	77.96875	47.1094	86.25	60.4688
7	64.6289	38.3594	74.92188	44.7461	84.6094	58.3398
8	63.3203	37.3438	73.49609	42.9053	83.3154	57.3389
9	62.71	36.5564	72.38525	42.0447	82.7087	56.5552
10	62.2354	36.1887	71.86584	41.3742	82.2351	56.1884
11	62.0136	35.9024	71.46118	41.0607	82.0135	55.9023
12	61.8409	35.7686	71.27197	40.8165	81.8409	55.7686
13	61.7601	35.6643	71.12456	40.7023	81.7601	55.6643
14	61.6972	35.6156	71.05564	40.6133	81.6972	55.6156
15	61.6678	35.5776	71.00194	40.5717	81.6678	55.5776
16	61.6449	35.5599	70.97683	40.5393	81.6449	55.5599
17	61.6342	35.546	70.95727	40.5241	81.6342	55.546
18	61.6258	35.5396	70.94813	40.5123	81.6258	55.5396
19	61.6219	35.5345	70.941	40.5068	81.6219	55.5345
20	61.6189	35.5322	70.93767	40.5025	81.6189	55.5322
21	61.6175	35.5304	70.93507	40.5005	81.6175	55.5304
22	61.6164	35.5295	70.93386	40.4989	81.6164	55.5295
23	61.6158	35.5288	70.93291	40.4982	81.6158	55.5288
24	61.6154	35.5285	70.93247	40.4976	81.6154	55.5285
25	61.6152	35.5283	70.93213	40.4974	81.6152	55.5283
26	61.6151	35.5282	70.93197	40.4972	81.6151	55.5282
27	61.615	35.5281	70.93184	40.4971	81.615	55.5281
28	61.615	35.528	70.93178	40.497	81.615	55.528
29	61.615	35.528	70.93174	40.497	81.615	55.528

4-134

Plate

$$\frac{hV}{kA}=\frac{h(2LA)}{kA}=\frac{2hL}{k}<0.1 \qquad \frac{hL}{k}<0.05$$

From figure $\dfrac{\theta}{\theta_0}=0.98$ for $\dfrac{k}{hL}=20$ and $\dfrac{x}{L}=1.0$ worst case

Cylinder

$$\frac{hV}{kA}=\frac{1}{2}\frac{hr_0}{k}<0.1 \qquad \frac{hr_0}{k}<0.2$$

From figure $\dfrac{\theta}{\theta_0}=0.91$ for $\dfrac{k}{hr_0}=5$ and $\dfrac{r}{r_0}=1.0$ worst case

Sphere

$$\frac{hV}{kA}=\frac{1}{3}\frac{hr_0}{k}<0.1 \qquad \frac{hr_0}{k}<0.3$$

From figure $\dfrac{\theta}{\theta_0}=0.85$ for $\dfrac{k}{hr_0}=3.333$ and $\dfrac{r}{r_0}=1.0$ worst case

4-135

Aluminum $k = 204$ $\alpha = 8.4 \times 10^{-5}\ \text{m}^2/\text{s}$ $T_i = 200°\text{C}$

$T_\infty = 25°\text{C}$ $h = 5000\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $T_0 = 90°\text{C}$ $L = 5\ \text{cm} = 0.05\ \text{m}$

$$\frac{\theta_0}{\theta_i} = \frac{90-25}{200-25} = 0.37 \qquad \frac{k}{hL} = \frac{204}{(5000)(0.05)} = 0.82$$

From chart $\dfrac{\alpha\tau}{L^2} = 1.3$ $\tau = \dfrac{(1.3)(0.05)^2}{8.4 \times 10^{-5}} = 38.7\ \text{sec}$

4-136

$$\frac{k}{hL} = 0 \text{ for } h \to \infty$$

$\dfrac{\theta_0}{\theta_i} = 0.37$ From chart $\dfrac{\alpha\tau}{L^2} = 0.5$ $\tau = \dfrac{(0.5)(0.05)^2}{8.4 \times 10^{-5}} = 14.9\ \text{sec}$

4-137

Lumped capacity $\rho = 2707\ \text{kg/m}^3$ $c = 896\ \dfrac{\text{J}}{\text{kg} \cdot °\text{C}}$

$$\frac{hA}{\rho c V} = \frac{(5000)A}{(2707)(896)A(0.05)} = 0.041$$

$$\frac{90-25}{200-25} = e^{-0.041\tau} = 0.37$$

$$\tau = 24.2\ \text{sec}$$

4-138

Suddenly exposed plate

$$\frac{90-25}{200-25} = 0.37 = \text{erf}\left(\frac{x}{2\sqrt{\alpha\tau}}\right) \qquad \frac{x}{2\sqrt{\alpha\tau}} = 0.34$$

$$\tau = \frac{\left[\frac{0.05}{(2)(0.34)}\right]^2}{8.4 \times 10^{-5}} = 64.4\ \text{sec}$$

Convectively exposed plate

$$\frac{hx}{k} = \frac{(5000)(0.05)}{204} = 1.22 \qquad \frac{\alpha\tau}{L^2} \approx 7$$

$$\tau = \frac{(7)(0.05)^2}{8.4 \times 10^{-5}} = 208\ \text{sec}$$

4-139

$h = 23$ $\quad k = 1.37$ $\quad \alpha = 7.5\times10^{-7}$ $\quad L = 18$ cm
$T_i = 30°\text{C}$ $\quad T_\infty = 0°\text{C}$ $\quad T_{x=0} = 5°\text{C}$

a. Back side insulated

$$\frac{\theta_L}{\theta_i} = \frac{5-0}{30-0} = 0.167 \qquad \frac{k}{hL} = \frac{1.37}{(23)(0.18)} = 0.331$$

At $\frac{x}{L} = 1.0$, using infinite plate Heisler chart $\frac{\theta}{\theta_0} = 0.355$

By iteration, $F_0 = \frac{\alpha\tau}{L^2} = 0.7 \qquad \tau = 8.4$ h

b. Semi-infinite solid
Iterative solution of Eq. (4–15) yields $\tau = 13.7$ h

4-140

$T_1 = 70°\text{F}$ $\quad T_\infty = 350°\text{F}$ $\quad h = 2.5$ $\quad k_w = 0.395$ $\quad \rho = 59.6$
$c_p = 1.0$ $\quad \alpha = 0.00663$ $\quad V = 0.084\ \text{ft}^3$ (assume spherical roast)

$$r = 0.271\ \text{ft} = r_0 \qquad \frac{k}{hr_0} = 0.583 \qquad \frac{\theta_0}{\theta_i} = 0.536 \qquad \frac{\alpha\tau}{r_0^2} = 0.3$$

4-145

$r_0 = 1.5\ \text{in} = 3.81$ cm $\quad k = 0.585$ $\quad \alpha = 1.4\times10^{-7}$ $\quad \rho = 999$
$c = 4195$
Take $T = 3°\text{C}$ at outside of orange to prevent frostbite

$$\frac{k}{hr_0} = \frac{0.585}{(45)(0.0381)} = 0.34 \qquad \frac{\theta}{\theta_0} = 0.33$$

$$\frac{\theta}{\theta_i} = \frac{3-0}{25-0} = 0.12 = \frac{\theta_0}{\theta_i}(0.33)$$

$$\frac{\theta_0}{\theta_i} = 0.364 \rightarrow \frac{\alpha\tau}{r_0^2} = 0.35$$

$$\tau = \frac{(0.35)(0.0381)^2}{1.3\times10^{-7}} = 3888\ \text{sec}$$

$$\frac{hr_0}{k} = 2.93 \qquad \frac{h^2\alpha\tau}{k^2} = \frac{(45)^2(1.3\times10^{-7})(3888)}{(0.585)^2} = 2.99$$

$$\frac{Q}{Q_0} = 0.84$$

<u>For 100 oranges:</u>

$$Q = (100)(999)(4195)\frac{4}{3}\pi(0.0381)^3(0.84)(25-0) = 1.19\times10^6\ \text{J} = 1125\ \text{Btu}$$

4-147

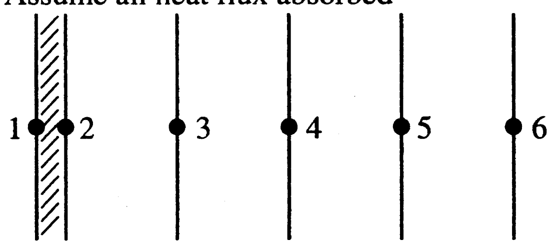
The thermal resistance between the slab in contact with the ground may be determined from the shape factors of table 3–1. This information may be used in conjunction with properties of the insulation to determine the steady state temperature distribution. For the transient analysis a numerical model must be formulated.

4-149

Polyethylene $k = 0.33$ $\rho = 960$ $c = 2100$ $\alpha = 1.64 \times 10^{-7}$

Particle board $k = 0.17$ $\rho = 1000$ $c = 1300$ $\alpha = 1.31 \times 10^{-7}$

Assume all heat flux absorbed



2 mm 2.5 cm

Because polyethylene is so thin it can be neglected in comparison to other material

<u>Node 2</u>

$$1300 + \frac{0.17}{0.025}(T_3{}^p - T_2{}^p) = (1000)(1300)(0.0125)\frac{T_2{}^{p+1} - T_2{}^p}{\Delta\tau}$$

$$= 16{,}250\frac{T_2{}^{p+1} - T_2{}^p}{\Delta\tau}$$

$\Delta\tau_{max} = 2390 \text{ sec}$, and the same value results for other nodes. Choosing $\Delta\tau = 2390 \text{ sec}$ for the time increment, the nodal equations are

$$T_2{}^{p+1} = 191.2 + T_3{}^p$$

$$T_3{}^{p+1} = \frac{(T_2{}^p + T_4{}^p)}{2}$$

$$T_4{}^{p+1} = \frac{(T_3{}^p + T_5{}^p)}{2}$$

$$T_5{}^{p+1} = \frac{(T_4{}^p + T_6{}^p)}{2}$$

$$T_6{}^{p+1} = T_5{}^p$$

Time to reach $50°\text{C} = 5.8$ time increments $= (5.8)(2390) = 13{,}860 \text{ sec} = 3.85 \text{ h}$

4-150

A_r = surface area for radiation
A_c = surface area for convection
m = mass
c = specific heat

$$h = A(T - T_\infty)^n$$

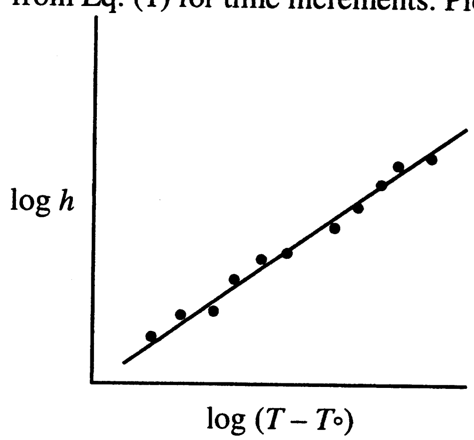
$$q_{\text{rad}} = \sigma\varepsilon A_r(T^4 - T_\infty{}^4)$$

$$q_{\text{conv}} = hA_c(T - T_\infty)$$

$$q_r + q_c = -mc\frac{dT}{d\tau}$$

$$\sigma\varepsilon A_r[(T^p)^4 - T_\infty{}^4] + hA_c(T^p - T_\infty) = \frac{T^{p+1} - T^p}{\Delta\tau} \qquad (1)$$

T^p measured as function of time during cooling process. Calculate values of h from Eq. (1) for time increments. Plot



Determine values of A and n from graph or from least squares analysis of
$\log h = \log A + n\log(T - T_\infty)$

Chapter 5

5–2

$$\frac{u}{u_\infty}=\frac{y}{\delta} \qquad \rho\frac{d}{dx}\int_0^\delta (u_\infty-u)u\,dy=\mu\frac{du}{dy}\bigg|_{y=0}$$

$$\rho\frac{d}{dx}\int_0^\delta\left(u_\infty-u_\infty\frac{y}{\delta}\right)u_\infty\frac{y}{\delta}dy=\mu\frac{u_\infty}{\delta}\bigg|_{y=0} \qquad \frac{d}{dx}\frac{1}{6}\rho u_\infty{}^2\delta=\frac{\mu u_\infty}{\delta}$$

$$\delta d\delta=\frac{6\nu}{u_\infty}dx \qquad \delta^2=\frac{12\nu x}{u_\infty} \qquad \delta=\frac{3.47x}{\sqrt{\mathrm{Re}_x}}$$

5–3

$$\delta=\frac{4.64x}{\sqrt{\mathrm{Re}_x}}=4.64x^{1/2}\left(\frac{\mu}{u_\infty\rho}\right)^{1/2} \qquad u=\frac{3u_\infty}{2}-\frac{u_\infty}{2}\left(\frac{y}{\delta}\right)^3$$

$$u=\frac{3u_\infty yx^{1/2}}{2\left[4.64\left(\frac{\mu}{u_\infty\rho}\right)^{1/2}\right]}-\frac{u_\infty y^3x^{1/2}}{2\left[4.64\left(\frac{\mu}{u_\infty\rho}\right)^{1/2}\right]^3} \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x}=-\frac{3u_\infty x^{-1}y}{4\delta}+\frac{3u_\infty x^{-1}y^3}{4\delta^3} \qquad v=\int\left[\frac{-3u_\infty 4.64}{4\sqrt{\mathrm{Re}_x}}\left(\frac{y}{\delta^2}\right)+\frac{3u_\infty 4.64}{4\delta\sqrt{\mathrm{Re}_x}}\left(\frac{y^3}{\delta}\right)\right]dy$$

$$v=\frac{2.32u_\infty}{\sqrt{\mathrm{Re}_x}}\left[\frac{3}{4}\left(\frac{y}{\delta}\right)^2-\frac{3}{8}\left(\frac{y}{\delta}\right)^4\right] \qquad \text{at } y=\delta \qquad v=\frac{2.32u_\infty\left(\frac{3}{8}\right)}{\sqrt{\mathrm{Re}_x}}$$

at $x=6\text{ in}=15.24\text{ cm}$

$$\mathrm{Re}=\frac{(0.99)(30)(0.1524)}{2.13\times10^{-5}}=213{,}000$$

at $x=12$ in. $\quad \mathrm{Re}=426{,}000$

$$v=\frac{(2.32)(30)\left(\frac{3}{8}\right)}{(213{,}000)^{1/2}}=0.0566\ \text{m/sec} \qquad v=\frac{(2.32)(30)\left(\frac{3}{8}\right)}{(426{,}000)^{1/2}}=0.0400\ \text{m/sec}$$

5–4

$$\frac{u}{u_\infty}=\frac{y}{\delta} \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \delta=\frac{3.47x}{\sqrt{\mathrm{Re}_x}}$$

$$u=u_\infty\frac{y}{3.47}\sqrt{\frac{\rho u_\infty}{\mu}}x^{-1/2} \qquad \frac{\partial u}{\partial x}=-u_\infty\frac{y}{2(3.47)}\sqrt{\mathrm{Re}_x}\,x^{-2}$$

$$\frac{\partial v}{\partial y}=u_\infty\frac{y}{2(3.47)}\frac{1}{x^2}\sqrt{\mathrm{Re}_x} \qquad v=u_\infty\frac{y^2}{4(3.47)}\frac{1}{x^2}\sqrt{\mathrm{Re}_x}$$

at $x=6$ in. $\quad v=\dfrac{(3.47)(30)}{(4)(213{,}000)^{1/2}}=0.0563$ m/sec

at $x=12$ in. $\quad v=\dfrac{(3.47)(30)}{(4)(426{,}000)^{1/2}}=0.040$ m/sec

5–5

$$\frac{u}{u_\infty}=\frac{y}{\delta} \qquad \frac{\theta}{\theta_\infty}=\frac{T-T_w}{T_\infty-T_w}=\frac{3}{2}\frac{y}{\delta_t}-\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

$$\frac{d}{dx}\int_0^H (T_\infty-T)u\,dy=\alpha\left(\frac{dT}{dy}\right)_w \qquad \frac{d}{dx}\int_0^H(\theta_\infty-\theta)u\,dy=\alpha\left(\frac{dT}{dy}\right)_w$$

$$\frac{d}{dx}\int_0^H\left[\theta_\infty-\theta_\infty\frac{3}{2}\frac{y}{\delta_t}+\frac{\theta_\infty}{2}\left(\frac{y}{\delta_t}\right)^3\right]u\,dy=\alpha\left(\frac{dT}{dy}\right)_w$$

$$\frac{\theta_\infty u_\infty}{10}\frac{d}{dx}\left(\frac{\delta_t{}^2}{\delta}\right)=\alpha\left(\frac{dT}{dy}\right)_w \qquad \zeta=\frac{\delta_t}{\delta}$$

$$\frac{\theta_\infty u_\infty}{10}\frac{d}{dx}(\delta\zeta^2)=\alpha\left(\frac{dT}{dy}\right)_w$$

$$\alpha\left(\frac{dT}{dy}\right)_w=\alpha\theta_0\left[\frac{3}{2\delta_t}-\frac{3y^2}{2\delta_t{}^3}\right]_0=\frac{3\alpha\theta_\infty}{2\delta_t}$$

$$\frac{\theta_\infty u_\infty}{10}\frac{d}{dx}(\delta\zeta^2)=\frac{3\alpha\theta_\infty}{2\delta_t}$$

$$\frac{1}{15}u_\infty\frac{d}{dx}(\delta\zeta^2)=\frac{\alpha}{\zeta\delta} \qquad \frac{1}{15}u_\infty\left(2\delta^2\zeta^2\frac{d\zeta}{dx}+\zeta^3\delta\frac{d\delta}{dx}\right)=\alpha$$

$$\rho\frac{d}{dx}\int_0^\delta(u_\infty-u)u\,dy=\mu\left.\frac{du}{dy}\right|_{y=0}$$

$$\rho\frac{d}{dx}\left(u_\infty{}^2\frac{\delta}{6}\right)=\mu\frac{d}{dy}\left(\frac{u_\infty y}{\delta}\right)_{y=0}$$

$$\delta\, d\delta = \frac{6\mu}{\rho u_\infty}dx \qquad \delta^2 = \frac{12\mu}{\rho u_\infty}x$$

$$\frac{1}{15}u_\infty\left(\frac{24\mu}{\rho u_\infty}x\zeta^2\frac{d\zeta}{dx} + \frac{6u}{\rho u_\infty}\zeta^3\right) = x$$

$$4x\zeta^2\frac{d\zeta}{dx} + \zeta^3 = \frac{15x}{6\nu} \qquad \zeta^2\frac{d\zeta}{dx} = \frac{1}{3}\frac{d}{dx}(\zeta^3)$$

$$\frac{d}{dx}(\zeta^3) + \frac{3}{4x}(\zeta^3) = \frac{15x}{6\nu}\frac{3}{4x}$$

$$x^{3/4}\zeta^3 = \frac{45}{24}\frac{\alpha}{\nu}\int x^{-1/4}dx + c \qquad \zeta^3 = \frac{15}{6}\frac{\alpha}{\nu} + cx^{-3/4}$$

$$\zeta = 0 \quad \text{at } x = x_0 \qquad c = -\frac{15}{6}\frac{\alpha}{\nu}x_0^{3/4}$$

$$\zeta = \frac{\delta_t}{\delta} = \left\{\frac{15}{6}\frac{\alpha}{\nu}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]\right\}^{1/3}$$

$$\delta = \left(\frac{12\mu x}{\rho u_\infty}\right)^{1/2} = \frac{3.47x}{\sqrt{\mathrm{Re}_x}}$$

$$h = \frac{3}{2}\frac{k}{\delta\zeta} = \frac{3}{2}\frac{k\sqrt{\mathrm{Re}_x}}{3.47x}\frac{\mathrm{Pr}^{1/3}}{\left\{\frac{15}{6}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]\right\}^{1/3}}$$

$$h_x = 0.319\frac{k}{x}\sqrt{\mathrm{Re}_x}\,\mathrm{Pr}^{1/3}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

5–6

$$p = 2\times10^4 \ \mathrm{N/m^2} \qquad T = 5°\mathrm{C} = 278°\mathrm{K} \qquad \text{Air}$$
$$\delta = 1.25 \text{ cm} = 0.0125 \text{ m} \qquad u = 1.5 \ \mathrm{m/sec}$$

$$\rho = \frac{2\times10^4}{(287)(278)} = 0.251 \ \mathrm{kg/m^3} \qquad \mu = 1.864\times10^{-5} \ \frac{\mathrm{kg}}{\mathrm{m\cdot sec}}$$

$$x = \frac{\rho u_\infty}{\mu}\left(\frac{\delta}{4.64}\right)^2 = \frac{(0.25)(1.5)}{1.864\times10^{-5}}\left(\frac{0.0125}{4.64}\right)^2 = 0.147 \text{ m}$$

5–7

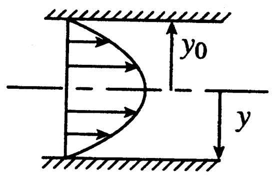
O_2 at 2 atm $\qquad T_f = 250\text{ K} \qquad \nu = \dfrac{20.8\times10^{-6}}{2} \qquad k = 0.0307$

$\Pr = 0.702 \qquad \text{Re} = \dfrac{(30)(0.5)(2)}{20.8\times10^{-6}} = 1.442\times10^6$

$$\bar{h} = \frac{0.0307}{0.5}[(0.037)(1.442\times10^6)^{0.8} - 871](0.702)^{1/3} = 123\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (123)(0.5)^2(127-27) = 3080\text{ W}$$

5–8



$$2yp - 2yp - 2yp = -2\tau\,dx = -2\left(\mu\frac{du}{dy}\right)dx$$

$$y\,dp = \mu\,dx\frac{du}{dy}$$

$u = \dfrac{1}{2\mu}\dfrac{dp}{dx}y^2 + c \qquad$ at $y = y_0 \qquad u = 0 \qquad c = -\dfrac{1}{2\mu}\dfrac{dp}{dx}{y_0}^2$

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - {y_0}^2)$$

at $y = 0 \qquad u = u_0 \qquad u_0 = \dfrac{1}{2\mu}\dfrac{dp}{dx}(-{y_0}^2)$

$$\frac{u}{u_0} = \left(1 - \frac{y^2}{{y_0}^2}\right)$$

5–9

$$\frac{\theta}{\theta_\infty} = \frac{T - T_w}{T_\infty - T_w} = \frac{y}{\delta_t} \qquad u = u_\infty = \text{const}$$

$$\left.\frac{\partial T}{\partial y}\right|_w = \frac{\theta_\infty}{\delta_t} \qquad \frac{q}{A} = -k\left.\frac{\partial T}{\partial y}\right|_w = h(T_w - T_\infty) \qquad h = \frac{k}{\delta_t}$$

$$\frac{d}{dx}\left[\int_0^H (T_\infty - T)u\,dy\right] = \frac{d}{dx}\left[\int_0^{\delta_t} \theta_\infty u_\infty \left(1 - \frac{y}{\delta_t}\right)dy\right] = \theta_\infty u_\infty \frac{d}{dx}\left(\frac{\delta_t}{2}\right) = \frac{\theta_\infty \alpha}{\delta_t}$$

$$\delta_t \frac{d\delta_t}{dx} = \frac{2\alpha}{u_\infty}$$

$$\delta_t = 0 \text{ at } x = 0$$

$$\delta_t{}^2 = \frac{4\alpha x}{u_\infty}$$

$$\delta_t = \left(\frac{4\alpha x}{u_\infty}\right)^{1/2} = 2\left(\frac{\alpha x}{u_\infty}\right)^{1/2}$$

$$h = \frac{k}{\delta_t} = \frac{k}{2}\left(\frac{u_\infty}{\alpha x}\right)^{1/2}$$

$$\text{Nu}_x = \frac{hx}{k} = \frac{x}{2}\left(\frac{u_\infty}{\alpha x}\right)^{1/2} = \frac{1}{2}\left(\frac{u_\infty x}{\alpha}\right)^{1/2} \qquad \alpha = \frac{\nu}{\text{Pr}}$$

$$\text{Nu}_x = \frac{1}{2}\left(\frac{u_\infty x}{\nu}\text{Pr}\right)^{1/2} = \frac{1}{2}(\text{Re}\,\text{Pr})^{1/2}$$

5–11

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\nu}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial^3 u}{\partial y^3} = \frac{1}{\nu}\left(u\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial x} + v\frac{\partial^2 u}{\partial y^2} + \frac{\partial v}{\partial y}\cdot\frac{\partial u}{\partial y}\right) - \frac{1}{\nu}\left[u\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial u}{\partial y}\left(\frac{\partial y}{\partial x} + \frac{\partial v}{\partial y}\right) + v\frac{\partial^2 u}{\partial y^2}\right]$$

$$u = v = 0 \text{ at } y = 0 \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \therefore \quad \frac{\partial^3 u}{\partial y^3} = 0 \text{ at } y = 0$$

5–13

$$\frac{\delta_t}{\delta} = \frac{1}{1.026\,\mathrm{Pr}^{1/3}}$$

	Pr	$\frac{\delta_t}{\delta}$
Air	0.709	1.093
H_2O	7	0.5095
He	0.705	1.095
NH_3	2.02	0.771
Glycerine	12.5	0.42

5–14

$T_\infty = 15°\mathrm{C}$ $\quad u_\infty = 3\ \mathrm{m/s}$ $\quad x = 5\ \mathrm{cm}$ $\quad \mu = 1.13 \times 10^{-3}\ \dfrac{\mathrm{kg}}{\mathrm{m \cdot s}}$

$\rho = 999.8\ \mathrm{kg/m^3}$ $\qquad \mathrm{Re} = \dfrac{(999.8)(3)(0.05)}{1.13 \times 10^{-3}} = 1.33 \times 10^5$

$$\delta = \frac{(4.64)(0.05)}{(1.33 \times 10^5)^{1/2}} = 6.37 \times 10^{-4}\ \mathrm{m}$$

mass flow $= \int_0^\delta \rho u\,dy = \frac{5}{8}\rho u_\infty \delta = 1.19$ kg/sec for 1 m depth.

5–15

$\rho = \dfrac{p}{RT} = \dfrac{1.032 \times 10^5}{(287)(363)} = 0.99\ \mathrm{kg/m^3}$ $\qquad T_\infty = 363°\mathrm{K}$

$\mu = 2.13 \times 10^{-5}\ \dfrac{\mathrm{kg}}{\mathrm{m \cdot sec}}$

$\mathrm{Re} = \dfrac{(0.99)(30)(0.025)}{2.13 \times 10^{-5}} = 34{,}860$ $\qquad \delta = \dfrac{(4.64)(0.025)}{(34{,}860)^{1/2}} = 6.21 \times 10^{-4}\ \mathrm{m}$

5–16

$$T_f = \frac{75+20}{2} = 47.5°\text{C} = 320.5\ \text{K} \qquad \mu = 1.94\times10^{-5}$$

$$\rho = \frac{2\times10^4}{(287)(320.5)} = 0.217 \qquad k = 0.028 \qquad \text{Pr} = 0.7$$

$$\text{Re}_x = \frac{(0.217)(20)x}{1.94\times10^{-5}} = 2.24\times10^5 x$$

$$h_x = (0.332)(0.028)(0.7)^{1/3}(2.24\times10^5)^{1/2} x^{-1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

$$h_x = \frac{3.904}{x^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \qquad q = \int_{x_0}^{x} h_x\, dx (T_w - T_\infty)$$

x	h_x $\frac{\text{W}}{\text{m}^2\cdot°\text{C}}$
0.08	38.18
0.19	11.27
0.3	8.24

$$\int h_x\, dx = 3.79 \qquad q = (3.79)(75-20) = 208.6\ \text{W}$$

5–17

$$\delta = 7.5\ \text{mm} = 0.0075\ \text{m} \qquad \rho = 1000 \qquad \mu = 1.5\times10^{-3}\ \frac{\text{kg}}{\text{m}\cdot\text{sec}}$$

$$x = \frac{\rho u_\infty}{\mu}\left(\frac{\delta}{4.64}\right)^2 = \frac{(1000)(1.5)}{1.5\times10^{-3}}\left(\frac{0.0075}{4.64}\right)^2 = 2.61\ \text{m}$$

5–18

$$T_f = \frac{90+30}{2} = 60°\text{C} = 333\ \text{K} \qquad \nu = 19.09\times10^{-6} \qquad k = 0.0288$$

$$\text{Pr} = 0.7 \qquad \text{Re} = \frac{(20)(0.6)}{19.09\times10^{-6}} = 6.29\times10^5$$

$$\bar{h} = \frac{0.0288}{0.6}(0.7)^{1/3}[(0.037)(6.29\times10^5)^{0.8} - 871] = 31.5\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (31.5)(0.6)^2(90-30) = 681.3\ \text{W}$$

5–19

$p = 7\ \text{kN/m}^2$ $\quad T_\infty = 35°\text{C}$ $\quad L = 0.3\ \text{m}$ $\quad u_\infty = 7.5\ \text{m/sec}$

$T_w = 65°\text{C}$ $\quad T_f = \dfrac{65+35}{2} = 50°\text{C} = 323\ \text{K}$

$\rho = \dfrac{7000}{(287)(323)} = 0.0755\ \text{kg/m}^3$ $\quad \mu = 2.025\times10^{-5}\ \dfrac{\text{kg}}{\text{m}\cdot\text{sec}}$

$k = 0.02798\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}}$ $\quad \text{Pr} = 0.71$ $\quad \text{Re} = \dfrac{(0.0755)(7.5)(0.3)}{2.025\times10^{-5}} = 8390$

$\bar{h} = \dfrac{0.02798}{0.3}(0.71)^{1/3}(8390)^{1/2}(0.664) = 5.04\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$q = (5.04)(0.3)^2(65-35) = 13.6\ \text{W}$

5–20

$T_\infty = 90°\text{C}$ $\quad u_\infty = 60\ \text{m/sec}$ $\quad L = 60\ \text{cm}$ $\quad T_w = 10°\text{C}$

$T_f = 50°\text{C} = 323\ \text{K}$ $\quad \rho = \dfrac{1.0132\times10^5}{(287)(323)} = 1.093$ $\quad \mu = 1.716\times10^{-5}$

$k = 0.0241$ $\quad \text{Pr} = 0.71$

$\text{Re} = \dfrac{(1.093)(60)(0.6)}{1.716\times10^{-5}} = 2.292\times10^6$

$\bar{h} = \dfrac{0.0241}{0.6}(0.71)^{1/3}[(0.037)(2.29\times10^6)^{0.8} - 871]$

$\bar{h} = 131.1\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$ $\quad q = (131.1)(0.6)^2(10-90) = 3776$

5–21

N_2 at 2 atm $\quad T_f = 400\ \text{K}$ $\quad \nu = \dfrac{25.74\times10^{-6}}{2}$ $\quad k = 0.03335$

$\text{Pr} = 0.691$ $\quad \text{Re} = \dfrac{(0.4)(2)(25)}{25.74\times10^{-6}} = 7.78\times10^5$ turbulent

$\bar{h} = \dfrac{0.0335}{0.4}[0.037(7.78\times10^5)^{0.8} - 871](0.691)^{1/3} = 76.4\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$q = \bar{h}A(T_w - T_\infty) = (76.4)(0.4)^2(500-300) = 2446\ \text{W}$

5–23

$L = 0.2 \qquad T_\infty = 35°\text{C} \qquad p = 14{,}000\ \text{N/m}^2 \qquad T_w = 250°\text{C} \qquad u_\infty = 6\ \text{m/sec}$

$T_f = \dfrac{250+35}{2} = 142.5 = 416\ \text{K} \qquad \rho = \dfrac{14{,}000}{(287)(416)} = 0.117$

$\mu = 2.349\times10^{-5} \qquad k = 0.03474 \qquad \text{Pr} = 0.685$

$\text{Re} = \dfrac{(0.117)(6)(0.2)}{2.349\times10^{-5}} = 5977$

$h = \dfrac{(0.3474)(0.664)}{0.2}(5977)^{1/2}(0.685)^{1/3} = 7.86\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$q = (7.86)(0.2)^2(250-35) = 676\ \text{W}$

5–24

$T_w = 55°\text{C} \qquad T_\infty = 20°\text{C} \qquad p = 20{,}000\ \text{N/m}^2 \qquad x = 0.3\ \text{m}$

$u_\infty = 30\ \text{m/sec} \qquad u = 22.5\ \text{m/sec} \qquad T_f = \dfrac{55+20}{2} = 37.5°\text{C} = 310\ \text{K}$

$\rho = \dfrac{20{,}000}{(287)(310)} = 0.225\ \text{kg/m}^3 \qquad \mu = 2.001\times10^{-5}$

$\text{Re} = \dfrac{(0.225)(30)(0.3)}{2.001\times10^{-5}} = 1.01\times10^5 \qquad \delta = \dfrac{(0.3)(4.64)}{(1.01\times10^5)^{1/2}} = 4.38\times10^{-3}\ \text{m}$

$\dfrac{u}{u_\infty} = \dfrac{22.5}{30} = \dfrac{3}{2}\left(\dfrac{y}{\delta}\right) - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3 \qquad \dfrac{y}{\delta} = 0.56 \qquad y = 2.45\times10^{-3}\ \text{m}$

5–25

$\dfrac{C_{fx}}{2} = 0.332\,\text{Re}_x^{-1/2}$ at $x = 15$ cm $\quad \text{Re}_x = 0.5005\times10^5 \quad C_{fx} = 2.968\times10^{-3}$

5-26

T_f = °C = 315 K

$\nu = 17.2\times10^{-6}/2 = 8.6\times10^{-6}$

$k = 0.0274;\quad Pr = 0.7$

$L = 10^6(8.6\times10^{-6})/30 = 0.287$ m

$Nu = (0.7)^{1/3}[(0.037)(10^6)^{0.8} - 871] = 1299$

$h = (1299)(0.0274)/0.287 = 124.1$

$q = (124.1)(0.287)^2(57 - 27) = 307$ W

5-27

Eq. (5-95); $\delta = (0.287)[0.381(10^6)^{-0.2} - 10256/10^6]$

0.00396 m $= 3.96$ mm

5-28

$T_f = 308\text{ K}$ $\qquad \rho = \dfrac{70{,}000}{(287)(308)} = 0.792$ $\qquad \mu = 2\times10^{-5}$

$k = 0.0268$ $\qquad \Pr = 0.7$ $\qquad \mathrm{Re}_{x_0} = \dfrac{(0.792)(6)(0.15)}{2\times10^{-5}} = 35{,}640$

Use Eq. (5–43) and numerically integrate: $\mathrm{Re}_{16.5} = \dfrac{16.5}{15}(35{,}640) = 38{,}610$

$$h_{16.25} = \frac{0.0268}{0.1625}(0.332)(38{,}610)^{1/2}(0.7)^{1/3}\left[1-\left(\frac{15}{16.25}\right)^{3/4}\right]^{-1/3} = 24.64$$

$$h_{17.5} = 19.98\left(\frac{16.25}{17.5}\right)^{1/2} = 19.25$$

$$h_x = (0.332)k\Pr^{1/3}\left(\frac{400}{2}\right)^{1/2}\frac{1}{x^{1/2}}\frac{1}{\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}}$$

$h_{15.5} = 33.76$ $\qquad h_{15.25} = 42.68$

$$\bar{h} = \frac{\int h_x\,dx}{\Delta x} = 27.85\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$q = (27.85)(0.025)(65-5) = 41.8\text{ W}$

5-29

$T_f = \dfrac{55+27}{2} = 41^\circ\text{C} = 314\text{ K}$ $\qquad \nu = 17.94\times10^{-6}$ $\qquad k = 0.0273$

$\Pr = 0.7$ $\qquad \mathrm{Re}_L = \dfrac{(4.5)(15)}{17.94\times10^{-6}} = 3.76\times10^6$

$$\bar{h} = \frac{0.0273}{15}(0.7)^{1/3}[(0.037)(3.76\times10^6)^{0.8} - 871] = 9.48$$

$q = (9.48)(15)(55-27) = 3980\ \text{W/m}$

5-30

$T_f = \dfrac{300+400}{2} = 350\text{ K}$ $\qquad \rho = \dfrac{75{,}000}{(287)(350)} = 0.747\ \text{kg/m}^3$

$\mu = 2.075\times10^{-5}$ $\qquad k = 0.03003$ $\qquad \Pr = 0.697$

$$\mathrm{Re} = \frac{(0.747)(45)(1.0)}{2.075\times10^{-5}} = 1.62\times10^6$$

$$\bar{h} = \frac{0.03003}{1.0}(0.697)^{1/3}[(0.037)(1.62\times10^6)^{0.8} - 871] = 68.25\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (68.25)(1)^2(400-300) = 6825\text{ W}$

5-31

$$T_f = \frac{50+10}{2} = 30°\text{C} = 303\text{ K} \qquad \rho = \frac{50{,}000}{(287)(303)} = 0.575\ \text{kg/m}^3$$

$$\mu = 1.85\times10^{-5} \qquad k = 0.0263 \qquad \Pr = 0.7$$

$$\text{Re} = \frac{(0.575)(0.5)(20)}{1.85\times10^{-5}} = 3.11\times10^5$$

$$\bar{h} = \frac{0.0263}{0.5}(0.664)(3.11\times10^5)^{1/2}(0.7)^{1/3} = 17.3\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (17.3)(0.5)^2(50-10) = 172.9\ \text{W}$$

5-32

$$T_f = \frac{100+10}{2} = 55°\text{C} = 328\text{ K} \qquad \rho = \frac{1.01\times10^5}{(2)(287)(328)} = 0.538$$

$$\mu = 1.974\times10^{-5} \qquad k = 0.0284 \qquad \Pr = 0.7$$

$$\text{Re}_L = \frac{(0.538)(5)(0.2)}{1.974\times10^{-5}} = 27{,}254 \qquad \text{Re at } x = 10\text{ cm} = 13{,}627$$

$$\overline{T_w - T_\infty} = \frac{q_w L/k}{0.6795\,\text{Re}_L^{1/2}\Pr^{1/3}}$$

$$q_w = \frac{(100)(0.6795)(27{,}254)^{1/2}(0.7)^{1/3}(0.0284)}{0.2} = 1414\ \text{W/m}^2$$

$$\frac{h_x}{k} = 0.453\,\text{Re}_x^{1/2}\Pr^{1/3}$$

$$\text{at } x = 10\text{ cm} \quad h = \frac{0.0284}{0.1}(0.453)(13{,}627)^{1/2}(0.7)^{1/3} = 13.33\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

5-33

	ν	u (m/s)
H_2O	9.8×10^{-7}	9.8
Air	1.53×10^{-5}	153
F-12	0.198×10^{-6}	1.98
NH_3	0.359×10^{-6}	3.59
He	122.2×10^{-6}	1222

$$u = \frac{10^7\nu}{L} = 10^7\nu$$

5-34

$L = 1.0$ m $\qquad \text{Re}_L = 10^7$

	ν	k	Pr	$\bar{h}$
H_2O	9.8×10^{-7}	0.6	6.8	15,750
Air	1.53×10^{-5}	0.026	0.7	320
F-12	0.198×10^{-6}	0.073	3.5	1536
NH_3	0.359×10^{-6}	0.521	2.02	9127
He	122.2×10^{-6}	0.147	0.72	1825

$$\bar{h} = k[0.037(10^7)^{0.8} - 871]\text{Pr}^{1/3} = 13{,}859k\,\text{Pr}^{1/3}$$

5-35

$$L = 1.0 \text{ m} \qquad \frac{\delta}{L} = 0.381\,\text{Re}_L^{-1/5} - \frac{10{,}256}{\text{Re}_L}$$

$$\delta = (1.0)\left[(0.381)(10^7)^{-0.2} - \frac{10{,}256}{10^7}\right] = 0.0141 \text{ m} \quad \text{for all fluids}$$

5-36

Evaluate properties at 350 K $\qquad \nu = 20.76\times10^{-6} \qquad k = 0.03003$

$$\text{Pr} = 0.697 \qquad \text{Re}_L = \frac{(3)(0.25)}{20.76\times10^{-6}} = 36{,}127$$

$$\overline{T_w - T_\infty} = \frac{q_w L/k}{0.6795\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3}} = \frac{(800)(0.25)/0.03003}{(36{,}127)^{1/2}(0.697)^{1/3}(0.6795)} = 58.16°\text{C}$$

$$\overline{T_w} = 25 + 58.16 = 83.16°\text{C}$$

at $x = 25$ cm

$$h = \frac{k}{x}0.453\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3} = \frac{0.03003}{0.25}(0.453)(36{,}127)^{1/2}(0.697)^{1/3} = 9.17\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

or $\text{Nu}_x = 76.34$

$$T_w - T_\infty = \frac{q_w x}{k\,\text{Nu}_x} = \frac{(800)(0.25)}{(0.03003)(76.34)} = 87.24°\text{C}$$

$$T_w = 87.24 + 25 = 112.24°\text{C}$$

5-37

$u_\infty = 20$ m/s $\qquad$ Air at $\frac{1}{2}$ atm $\qquad T_\infty = 27°\text{C} \qquad T_w = 127°\text{C}$
$L = 34$ cm square

$T_f = 350$ K $\qquad \nu = \dfrac{20.76\times10^{-6}}{0.5} \qquad k = 0.03003 \qquad \text{Pr} = 0.697$

$$\text{Re} = \frac{(20)(0.34)(0.5)}{20.76\times10^{-6}} = 1.638\times10^5$$

$$\bar{h} = \frac{0.03003}{0.34}(0.664)(0.697)^{1/3}(1.638\times10^5)^{1/2} = 21.04\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (21.04)(0.34)^2(127-27) = 243.3\ \text{W}$$

5-38

$T_f = \dfrac{113+73}{2} = 93°\text{C} = 366\ \text{K} \qquad \nu = \dfrac{173.6\times10^{-6}}{3} \qquad k = 0.1691\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}}$
$\text{Pr} = 0.71$

$$\text{Re}_L = \frac{(50)(0.35)(3)}{173.6\times10^{-6}} = 3.02\times10^5$$

$$\bar{h} = \frac{0.1691}{0.35}(0.664)(3.02\times10^5)^{1/2}(0.71)^{1/3} = 157.4\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (157.4)(0.35)^2(113-73) = 771\ \text{W}$$

5-39

$T_f = 350$ K $\qquad \nu = 20.76\times10^{-6} \qquad k = 0.03003 \qquad \text{Pr} = 0.697$

$u_\infty = \dfrac{1.1\times10^5\nu}{L} = 4.57$ m/sec $\qquad x_0 = 25$ cm

$$h_x = \frac{k}{x}0.332\,\text{Pr}^{1/3}\,\text{Re}^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

x (cm)	$h_x\left(\dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}\right)$
26	36.7
35	13.81
44	9.50
50	7.92

$\displaystyle\int_{x_0}^{L} h_x dx = 3.844 \qquad \bar{h} = \frac{3.844}{0.25} = 15.38\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$$q = \bar{h}(L-x_0)L(T_w - T_\infty) = (15.38)(0.5-0.25)(0.5)(400-300) = 192.2\ \text{W}$$

5-40

$u_\infty = 150\ \text{m/sec} \qquad T_w = 150°\text{C} \qquad L = 1\ \text{m} \qquad T_\infty = 20°\text{C}$

$\mu = 2.11\times10^{-5}$

$$T_f = \frac{150+20}{2} = 85°\text{C} = 358\ \text{K} \qquad \rho = \frac{p}{RT} = \frac{14{,}000}{(287)(358)} = 0.136\ \text{kg/m}^3$$

$$k = 0.03060 \qquad \text{Pr} = 0.695 \qquad \text{Re} = \frac{(0.136)(150)(1)}{2.11\times10^{-5}} = 9.67\times10^5$$

$$\bar{h} = \frac{0.03060}{1}(0.695)^{1/3}[(0.037)(9.67\times10^5)^{0.8} - 871] = 38.00\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{A} = (38.00)(150-20) = 4939\ \text{W/m}^2$$

5-41

$$\overline{St}\,\text{Pr}^{2/3} = \frac{\overline{C_f}}{2} = 0.074\,\text{Re}_L^{-1/5} - \frac{A}{2}\text{Re}_L^{-1}$$

Re_{crit}	Eq. (5–85) becomes
3×10^5	$\text{Pr}^{1/3}(0.037\,\text{Re}_L^{0.8} - 528)$
5×10^5	$\text{Pr}^{1/3}(0.037\,\text{Re}_L^{0.8} - 871)$
10^6	$\text{Pr}^{1/3}(0.037\,\text{Re}_L^{0.8} - 1670)$
5×10^6	$\text{Pr}^{1/3}(0.037\,\text{Re}_L^{0.8} - 4420)$

5-42

$$h_x = cx^{-1/5}$$

$$\bar{h} = \frac{1}{L}\int_0^L cx^{-1/5}\,dx = 1.25cL^{-1/5}$$

$$\overline{\text{St}}\,\text{Pr}^{2/3} = (1.25)(0.0296)\text{Re}_L^{-1/5} = 0.037\,\text{Re}_L^{-1/5}$$

5-43

$$T_f = \frac{80+10}{2} = 45°\text{C} \qquad \rho = 990 \qquad \mu = 6\times10^{-4} \qquad k = 0.64$$

$$\text{Pr} = 4.81 \qquad \text{Max Temp. at } x = L \qquad \text{Re}_L = \frac{(990)(2)(0.1)}{6\times10^{-4}} = 3.3\times10^5$$

$$\text{Nu}_x = 0.453\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3} = (0.453)(3.3\times10^5)^{1/2}(4.81)^{1/3} = 439$$

$$q_w = \frac{\text{Nu}_x k}{x}(T_w - T_\infty) = \frac{(439)(0.64)}{0.1}(80-10) = 1.97\times10^5\ \text{W/m}^2$$

$$q = q_w A = (1.97\times10^5)(0.1)^2 = 1968\ \text{W}$$

5-44

Take properties at 350 K. $\quad \nu = 20.76 \times 10^{-6} \quad k = 0.03003$

$$\Pr = 0.697 \qquad \mathrm{Re}_L = \frac{(3)(0.1)}{20.76 \times 10^{-6}} = 14{,}451$$

$$\mathrm{Nu}_x = (0.453)(14{,}451)^{1/2}(0.697)^{1/3} = 48.28$$

$$q_w = \frac{\mathrm{Nu}_x}{x}(T_w - T_\infty)k = \frac{(48.28)(0.03003)}{0.1}(80 - 10) = 1015 \ \mathrm{W/m^2}$$

$$q = q_w A = (1015)(0.1)^2 = 10.15 \ \mathrm{W}$$

5-45

$$T_f = \frac{300 + 500}{2} = 400 \ \mathrm{K} \qquad \nu = 202.9 \times 10^{-6} \qquad k = 0.178 \qquad \Pr = 0.72$$

$$\mathrm{Re} = \frac{(50)(1)}{202.9 \times 10^{-6}} = 2.46 \times 10^5$$

$$\bar{h} = \frac{k}{L} 0.664 \, \mathrm{Re}^{1/2} \Pr^{1/3} = \frac{0.178}{1}(0.664)(2.46 \times 10^5)^{1/2}(0.72)^{1/3} = 52.59 \ \frac{\mathrm{W}}{\mathrm{m^2 \cdot {}^\circ C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (52.59)(1)^2(500 - 300) = 10.52 \ \mathrm{kW}$$

$$\frac{\delta}{L} = \frac{4.64}{\mathrm{Re}_L^{1/2}} \qquad \delta = \frac{(1)(4.64)}{(2.46 \times 10^5)^{1/2}} = 0.0094 \ \mathrm{m}$$

5-46

$$u_\infty = 10 \ \mathrm{mi/h} = 4.47 \ \mathrm{m/s} \qquad T_\infty = 27^\circ\mathrm{C} \qquad \left.\frac{q}{A}\right|_{\mathrm{sun}} = 347 \ \mathrm{W/m^2}$$

$$L = 6.1 \ \mathrm{m} \qquad k = 0.0262 \qquad \Pr = 0.71 \qquad \mu = 1.98 \times 10^{-5} \qquad \rho = 1.177$$

$$\mathrm{Re}_L = \frac{(1.177)(4.47)(6.1)}{1.98 \times 10^{-5}} = 1.62 \times 10^6$$

$$\bar{h} = \frac{0.0262}{6.1}(0.71)^{1/3}[(0.037)(1.62 \times 10^6)^{0.8} - 871] = 9.81 \ \frac{\mathrm{W}}{\mathrm{m^2 \cdot {}^\circ C}}$$

$$T_w - T_\infty = \frac{347}{9.81} = 35.4^\circ\mathrm{C}$$

$$T_w = 62.4^\circ\mathrm{C}$$

5-47

$T_\infty = 400°\text{C}$ $\quad T_w = 420°\text{F}$ $\quad u_\infty = 1 \text{ ft/sec} = 0.3048 \text{ m/sec}$

$L = 10 \text{ ft} = 3.048 \text{ m}$ $\quad \text{Re} = \dfrac{(0.3048)(3.048)}{2\times 10^{-6}} = 4.645\times 10^5$

$$\bar{h} = \frac{0.12}{3.048}(0.664)(4.645\times 10^5)^{1/2}(40)^{1/3} = 60.93 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (60.93)(3.048)(3)(0.3048)(420-400)\left(\frac{5}{9}\right) = 1887 \text{ W}$$

$$\delta = \frac{(3.048)(4.64)}{(4.645\times 10^5)^{1/2}} = 0.0208 \text{ m}$$

5-48

$T_f = \dfrac{27+77}{2} = 57°\text{C} = 325 \text{ K}$ $\quad \nu = 18.23\times 10^{-6}$ $\quad k = 0.0281$

$\text{Pr} = 0.7$ $\quad \text{Re}_L = \dfrac{(40)(4)}{18.23\times 10^{-6}} = 8.78\times 10^6$

$$\text{Nu}_L = \text{Pr}^{1/3}(0.037\,\text{Re}_L{}^{0.8} - 850)$$

$$\bar{h} = \frac{0.0281}{4}(0.7)^{1/3}[(0.037)(8.78\times 10^6)^{0.8} - 871] = 77.4 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (77.4)(4)^2(77-27) = 6.19\times 10^4 \text{ W}$$

5-49

$T_f = \dfrac{300+273}{2} = 287 \text{ K}$ $\quad \nu = 14.51\times 10^{-6}$ $\quad k = 0.0252$ $\quad \text{Pr} = 0.71$

$u_\infty = 5 \text{ mi/hr} = 7.33 \text{ ft/sec} = 2.235 \text{ m/sec}$

$$\text{Re}_L = \frac{(2.235)(30)}{14.51\times 10^{-6}} = 4.62\times 10^6$$

$$\text{Nu}_L = \text{Pr}^{1/3}(0.037\,\text{Re}_L{}^{0.8} - 871)$$

$$\bar{h} = \frac{0.0252}{30}(0.71)^{1/3}[(0.037)(4.62\times 10^6)^{0.8} - 871] = 5.30 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (5.30)(30)(60)(300-273) = 2.58\times 10^5 \text{ W}$$

5-50

at 300 K $\quad \nu = 15.69\times 10^{-6}$

$$\text{Re}_L = \frac{(10)(0.15)}{15.69\times 10^{-6}} = 9.56\times 10^4$$

$\dfrac{\delta}{L} = \dfrac{4.64}{\text{Re}^{1/2}}$ $\quad \delta = (0.15)(9.56\times 10^4)^{1/2}(4.64) = 0.00225 \text{ m}$

5-51

$T_f = 350\ \text{K} \qquad \nu = 20.69\times10^{-6} \qquad k = 0.0318 \qquad \Pr = 0.7$

$$\mathrm{Re}_L = \frac{(33)(0.6)}{20.69\times10^{-6}} = 9.57\times10^5 > 5\times10^5 \ \rightarrow \ \text{Turbulent}$$

$$\bar{h} = \frac{0.0318}{0.6}(0.7)^{1/3}[0.037(9.57\times10^5)^{0.8} - 871] = 65.08\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (65.08)(0.6)(0.3)(400 - 300) = 1171\ \text{W}$$

5-52

$T = 303$ K; $\nu = 128\times10^{-6}$; $u_\infty = 15$

$L = (250000)(128\times10^{-6})/15 = 2.13$ m

$\delta = (2.13)(5)/(250000)^{1/2} = 0.0213$ m $= 21.3$ mm

5-53

$T_f = 45°\text{C} = 318$ K; $k = 0.155$; $\Pr = 0.7$

$h = (0.155/2.13)(0.664)(250000)^{1/2}\ (0.7)^{1/3} = 21.45$

$q = (21.45)(2.13)(60 - 30) = 1371$ W/m depth

5-54

$$T_f = \frac{25+150}{2} = 87.5°\text{C} = 360.5 \text{ K} \qquad \mu = 2.119\times10^{-5} \qquad k = 0.0308$$

$$\text{Pr} = 0.693 \qquad \rho = \frac{0.2\times10^6}{(287)(360.5)} = 1.933 \ \text{kg/m}^3$$

$$\text{Re}_L = \frac{(1.933)(60)(0.5)}{2.119\times10^{-5}} = 2.74\times10^6$$

$$\text{Nu} = \text{Pr}^{1/3}(0.037\,\text{Re}^{0.8} - 871)$$

$$\bar{h} = \frac{(0.693)^{1/3}}{0.5}(0.0308)[(0.037)(2.74\times10^6)^{0.8} - 871] = 238.6 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (238.6)(0.5)^2(150-25) = 7424 \text{ W}$$

5-55

$$T_w = 100°\text{C} \qquad T_\infty = 20°\text{C} \qquad T_f = \frac{20+100}{2} = 60°\text{C} = 333 \text{ K}$$

$$p = 150 \text{ kPa} \qquad u_\infty = 50 \ \text{m/sec} \qquad \mu = 216\times10^{-7} \qquad k = 0.15$$

$$\text{Pr} = 0.7$$

$$\rho = \frac{150\times10^3}{(2078)(333)} = 0.217 \ \text{kg/m}^3$$

$$\text{Re}_L = \frac{(50)(1)(0.217)}{216\times10^{-7}} = 4.98\times10^5$$

$$\bar{h} = 0.664\frac{k}{L}\text{Re}_L{}^{1/2}\,\text{Pr}^{1/3} = (0.664)\frac{0.15}{1.0}(4.98\times10^5)^{1/2}(0.7)^{1/3} = 62.4 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (62.4)(1)^2(100-20) = 4993 \text{ W}$$

5-56

$T_f = 300\text{ K} \qquad \mu = 1.8462\times10^{-5} \qquad k = 0.02624 \qquad \Pr = 0.71$

$\rho = \dfrac{40{,}000}{(287)(300)} = 0.465\ \text{kg/m}^3 \qquad \text{Re}_L = \dfrac{(0.465)(20)(2)}{1.8462\times10^{-5}} = 1.01\times10^6$

$$\bar{h} = \frac{k}{L}\Pr^{1/3}(0.037\,\text{Re}_L{}^{0.8} - 871) = \frac{0.02624}{2}(0.71)^{1/3}[(0.037)(1.01\times10^6)^{0.8} - 871]$$
$$= 17.3\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (17.3)(2)^2(350-250) = 693\text{ W}$

5-57

$T_f = 350\text{ K} \qquad \mu = 19.91\times10^{-6} \qquad k = 0.0298 \qquad \Pr = 0.7$

$\rho = \dfrac{50{,}000}{(287)(350)} = 0.481\ \text{kg/m}^3 \qquad \text{Re}_L = \dfrac{(0.481)(1.2)(100)}{19.91\times10^{-6}} = 2.9\times10^6$

$$\bar{h} = \frac{0.0298}{1.2}(0.7)^{1/3}[(0.037)(2.9\times10^6)^{0.8} - 871] = 101.4\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (101.4)(1.2)(400-300) = 1.22\times10^4\ \text{W/m}$

5-58

$T_f = \dfrac{15+139}{2} = 77^\circ\text{C} = 350\text{ K} \qquad \mu = 9.954\times10^{-6} \qquad k = 0.206$

$\Pr = 0.697 \qquad \rho = \dfrac{(2)(1.01\times10^5)}{(4157)(350)} = 9.139\ \text{kg/m}^3$

$\text{Re}_L = \dfrac{(0.139)(6)(1)}{9.954\times10^{-6}} = 83{,}952$

$$\bar{h} = \frac{k}{L}0.664\,\text{Re}_L{}^{1/2}\Pr^{1/3} = \frac{0.206}{1}(0.664)(83{,}592)^{1/2}(0.697)^{1/3} = 35.14\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (35.14)(1)^2(139-15) = 4357\text{ W}$

5-59

$T_f = \dfrac{10+50}{2} = 30^\circ\text{C} \qquad \nu = 0.349\times10^{-6} \qquad k = 0.507 \qquad \Pr = 2.01$

$\text{Re}_L = \dfrac{(5)(0.4)}{0.349\times10^{-6}} = 5.73\times10^6$

$$\bar{h} = \frac{k}{L}\Pr^{1/3}(0.037\,\text{Re}_L{}^{0.8} - 871) = \frac{0.507}{0.4}(2.01)^{1/3}[(0.037)(5.73\times10^6)^{0.8} - 871]$$
$$= 13{,}698\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (13{,}698)(0.4)^2(50-10) = 87{,}670\text{ W}$

5-60

$$T_f = \frac{50+136}{2} = 93°\text{C} = 366\ \text{K} \qquad \rho = \frac{45\times 10^3}{(2078)(366)} = 0.0592\ \text{kg/m}^3$$

$\mu = 230.5\times 10^{-7} \qquad k = 0.1691 \qquad \Pr = 0.71$

$$\text{Re}_L = \frac{(0.0592)(1)(50)}{230.5\times 10^{-7}} = 1.28\times 10^5$$

$$\bar{h} = \frac{k}{L}0.664\,\text{Re}_L^{1/2}\Pr^{1/3} = \frac{0.1691}{1}(0.664)(1.28\times 10^5)^{1/2}(0.71)^{1/3}$$

$$= 35.88\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (35.88)(1)^2(136-50) = 3086\ \text{W}$$

5-61

$$T_f = \frac{100+10}{2} = 55°\text{C} = 328\ \text{K} \qquad \nu = (18.53\times 10^{-6})(10) = 18.53\times 10^{-5}$$

$k = 0.0284 \qquad \Pr = 0.7$

$$\text{Re}_L = \frac{(300)(0.8)}{18.53\times 10^{-5}} = 1.295\times 10^6$$

$$\bar{h} = \frac{k}{L}\Pr^{1/3}(0.037\,\text{Re}_L^{0.8} - 871) = \frac{0.0284}{0.8}(0.7)^{1/3}[(0.037)(1.295\times 10^6)^{0.8} - 871]$$

$$= 63.04\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (63.04)(0.8)^2(100-10) = 3631\ \text{W}$$

5-62

$$T_f = \frac{21+54}{2} = 37.5°\text{C} \qquad \rho = 993 \qquad \mu = 6.82\times 10^{-4} \qquad k = 0.63$$

$\Pr = 4.53 \qquad L = 0.3 \qquad u_\infty = 6\ \text{m/s}$

$$\text{Re}_L = \frac{(993)(6)(0.3)}{6.82\times 10^{-4}} = 2.62\times 10^6$$

$$\bar{h} = \frac{0.63}{0.3}(4.53)^{1/3}[(0.037)(2.62\times 10^6)^{0.8} - 871] = 14{,}500\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (14{,}500)(0.3)^2(54-21) = 43{,}080\ \text{W}$$

5-64

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \qquad \dot{m} = \int_0^\delta \rho u\, dy = \int_0^\delta \rho u_\infty \left(\frac{y}{\delta}\right)^{1/7} dy = \rho \frac{u_\infty}{\delta^{1/7}} \frac{y^{8/7}}{8/7}\bigg|_0^\delta = \frac{7}{8}\rho u_\infty \delta$$

$\nu = 20.76 \times 10^{-6}\ \text{m}^2/\text{s}$ at 350 K $\qquad \rho = 0.998\ \text{kg/m}^3$

$\underline{\text{At } \text{Re}_x = 10^6}$ $\qquad x = \dfrac{(10^6)(20.76 \times 10^{-6})}{30} = 0.692\ \text{m}$

$\delta = x[(0.381)\text{Re}_x^{-1/5} - 10{,}256\,\text{Re}_x^{-1}] = 9.54 \times 10^{-3}\ \text{m}$

$\dot{m} = \dfrac{7}{8}(0.998)(6)(9.54 \times 10^{-3}) = 4.998 \times 10^{-2}\ \text{kg/s}$

$\underline{\text{At } \text{Re}_x = 10^7}$ $\qquad x = 6.92\ \text{m} \qquad \delta = 0.0979\ \text{m}$

$\dot{m} = \dfrac{7}{8}(0.998)(6)(0.0979) = 0.513\ \text{kg/s}$

5-65

$T_f = \dfrac{600+300}{2} = 450\ \text{K} \qquad \mu = 2.484 \times 10^{-5} \qquad k = 0.03707$

$\text{Pr} = 0.683 \qquad \rho = \dfrac{50{,}000}{(287)(450)} = 0.387\ \text{kg/m}^3$

$\text{Re}_x = \dfrac{(0.387)(6)(0.2)}{2.484 \times 10^{-5}} = 18{,}703$

$\text{Nu}_x = 0.453\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3} = (0.453)(18{,}703)^{1/2}(0.683)^{1/3} = 54.56$

$q_w = \dfrac{k\,\text{Nu}_x(T_w - T_\infty)}{x} = \dfrac{(0.03707)(54.56)(600-300)}{0.2} = 3034\ \text{W/m}^2$

$q = q_w A = (3034)(0.2)^2 = 121.3\ \text{W}$

5-66

$$\frac{1}{ur}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial x} \qquad \frac{\partial T}{\partial r}=\frac{u_0}{\alpha}\frac{\partial T}{\partial x}\frac{r}{2}+\frac{c_1}{r}$$

$$T=\frac{u_0}{\alpha}\frac{\partial T}{\partial x}\frac{r^2}{4}+c_1\ln r+c_2 \qquad \frac{c_1}{r}=f(x)=c \qquad c_1=0$$

at $r=0$ $\quad T=T_c \quad T_c=c_2 \quad$ then $T=\dfrac{u_0}{\alpha}\dfrac{\partial T}{dx}\dfrac{r^2}{4}+T_c$

$$T_b=\frac{\int_0^{r_0}\rho(2\pi r\,dr)uc_pT}{\int_0^{r_0}\rho(2\pi r\,dr)uc_p}=\frac{u_0}{8\alpha}r_0^2\frac{\partial T}{\partial x}+T_c \qquad T=T_w \text{ at } r=r_0$$

$$T_w=\frac{u_0}{4\alpha}\frac{\partial T}{\partial x}r_0^2+T_c \qquad h=\frac{-k\left(\frac{\partial T}{\partial r}\right)_{r=r_0}}{T_w-T_b}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_0}=\left.\frac{u_0}{\alpha}\frac{\partial T}{\partial x}\cdot\frac{2r}{4}\right|_{r=r_0}=\frac{u_0r_0}{2\alpha}\frac{\partial T}{\partial x}$$

$$h=\frac{-k\frac{u_0r_0}{2\alpha}\frac{\partial T}{\partial x}}{\frac{u_0}{4\alpha}\frac{\partial T}{\partial x}r_0^2+T_c-\left[\frac{u_0}{8\alpha}\frac{\partial T}{\partial x}r_0^2+T_c\right]}=\frac{-k\frac{u_0r_0}{2\alpha}\frac{\partial T}{\partial x}}{\frac{u_0}{8\alpha}r_0^2\frac{\partial T}{\partial x}}=-\frac{4k}{r_0}$$

$h=\dfrac{8k}{d_0}$ or $\mathrm{Nu}_d=8.0$

5-68

Air $T_f=\dfrac{27+127}{2}=102°\mathrm{C}=375\ \mathrm{K}$

$\nu=\dfrac{23.33\times10^{-6}}{1.2} \qquad k=0.03184 \qquad \mathrm{Pr}=0.698$

$\mathrm{Re}_L=\dfrac{(40)(0.6)(1.2)}{23.33\times10^{-6}}=1.234\times10^6$ Turbulent

$\bar{h}=\dfrac{0.03184}{0.6}(0.693)^{1/3}[0.037(1.234\times10^6)^{0.8}-871]=88.85\ \dfrac{\mathrm{W}}{\mathrm{m^2\cdot°C}}$

$q=\bar{h}A(T_w-T_\infty)=(88.85)(0.6)^2(179-27)=4798\ \mathrm{W}$

5-69

$p = \frac{1}{2}$ atm $\qquad T_f = \frac{77-23}{2} = 27°C = 300$ K

$\nu = (15.69\times10^{-6})(2) = 31.38\times10^{-6} \qquad k = 0.02624 \qquad \Pr = 0.71$

$\mathrm{Re}_L = \frac{(30)(0.5)}{31.38\times10^{-6}} = 4.78\times10^5 < 5\times10^5$ Laminar

$$\bar{h} = \frac{0.02624}{0.5}(0.664)(4.78\times10^5)^{1/2}(0.71)^{1/3} = 21.5\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot°\mathrm{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (21.5)(0.5)^2(77+23) = 537$ W

5-70

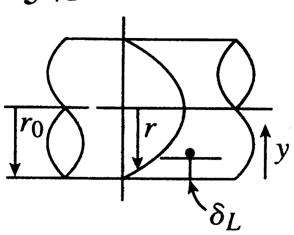
$\nu = \left(\frac{101}{200}\right)(134.6\times10^{-6}) = 6.8\times10^{-5} \qquad k = 0.152 \qquad \Pr = 0.7$

$\mathrm{Re}_L = \frac{(20)(0.3)}{6.8\times10^{-5}} = 88{,}235$ Laminar

$$\bar{h} = \frac{0.152}{0.3}(0.664)(88{,}235)^{1/2}(0.7)^{1/3} = 88.7\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot°\mathrm{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (88.7)(0.3)^2(93+18) = 886$ W

5-71



$\frac{u}{u_c} = \left(1 - \frac{r}{r_0}\right)^{1/7} \qquad f = \frac{0.316}{\mathrm{Re}_r^{1/4}} \qquad u_m = \frac{49}{60}u_c = 0.816u_c$

$\frac{dp}{dx} = f\frac{1}{d}\rho\frac{u_m^2}{2} \qquad \pi r_0^2 dp = \tau_w(2\pi r_0 dx) \qquad \frac{dp}{dx} = 2\frac{\tau_w}{r_0}$

$\tau_w = \mu\left.\frac{du}{dy}\right|_{r=r_0} \qquad \tau = \mu b = \frac{\mu u_c}{\delta_L^{6/7} r_0^{1/7}}$

Assume Linear profile in sublayer $\quad u = a + br$ at $r = r_0 \quad u = 0$

$a = -br \qquad u_c = \left(1 - \frac{r_0 - \delta_L}{y_0}\right)^{1/7} = -b\delta_L \qquad b = \frac{-u}{\delta_L^{6/7} r_0^{1/7}}$

$$\frac{dp}{dx} = \frac{0.316}{[\rho(0.816)u_c(2r_0)]^{1/4}}\left(\frac{1}{2r_0}\right)\rho\frac{(0.816)^2 u_c}{2}$$

$$\delta_t^{6/7} = \frac{2\mu u_c}{r_0^{8/7}(0.316)}\left[\frac{(0.816)\rho u_c(2r_0)}{\mu}\right]^{1/4}\frac{(2r_0)(2)}{(0.816)^2\rho u_c^2}$$

$$\delta_L = \left[43\left(\frac{\mu}{u_c\rho}\right)^{3/4} r_0^{3/28}\right]^{7/6}$$

$$\delta_L = 81\left(\frac{\mu}{u_c\rho}\right)^{7/8} r_0^{1/8} \quad \text{or} \quad \frac{\delta_L}{r_0} = 124\,\mathrm{Re}_d^{-7/8}$$

5-72

$$\Delta p \pi r^2 = \tau(2\pi r)\Delta x \qquad \tau = \frac{\Delta p}{\Delta x}\frac{r}{2} = \rho(\nu+\varepsilon_m)\frac{du}{dy} \qquad \frac{u}{u_0} = \left(1-\frac{r}{r_0}\right)^{1/7}$$

$$\frac{du}{dy} = -\frac{du}{dr} = -u_c\left(\frac{1}{7}\right)\left(1-\frac{r}{r_0}\right)^{-6/7}\left(-\frac{1}{r_0}\right) \qquad \frac{du}{dy} = \frac{u_c}{7r_0}\left(1-\frac{r}{r_0}\right)^{-6/7}$$

$$\varepsilon_m = \frac{\tau}{\rho\frac{du}{dy}} - \nu = \frac{\frac{dp}{dx}\frac{r}{2}}{\rho\frac{u_c}{7r_0}\left(1-\frac{r}{r_0}\right)^{-6/7}} - \nu \qquad u_m = \frac{1}{\pi r_0^2}\int_0^{r_0} 2\pi r u\,dr$$

$$u_m = \frac{1}{\pi r_0^2}\int_0^{r_0} 2\pi r u_c\left(1-\frac{r}{r_0}\right)^{1/7} dr = \frac{1}{\pi r_0^2}(2\pi u_c)\left(\frac{7}{8}\right)\left(\frac{7}{15}\right)r_0^2 = 0.816u_c$$

$$\frac{dp}{dx} = f\frac{1}{2r_0}\rho\frac{u_m^2}{2g_c} \qquad f = 0.316\left(\frac{2u_m r_0}{\nu}\right)^{-1/4} = 0.316\left(\frac{2(0.816)u_c r_0}{\nu}\right)^{-1/4}$$

$$\frac{dp}{dx} = 0.316\left(\frac{\nu}{2(0.816)u_c r_0}\right)^{1/4}\frac{1}{2r_0}\rho\frac{(0.816u_c)^2}{2g_c}$$

$$\varepsilon_m = 0.316\left(\frac{\nu}{2(0.816)u_c r_0}\right)^{1/4}\frac{\rho}{2r_0}\frac{(0.816u_c)^2}{2g_c}\frac{r}{2}\frac{\left(1-\frac{r}{r_0}\right)^{6/7}7r_0}{\rho u_c} - \nu$$

$$\varepsilon_m = 0.162\left(\frac{r}{g_c}\right)u_c^{3/4}\left(\frac{\nu}{r_0}\right)^{1/4}\left(1-\frac{r}{r_0}\right)^{6/7} - \nu$$

5-74

$$Re = 1500 = \frac{\rho u_m d}{\mu} \qquad T = 35°C \qquad d = 0.025\ \text{m}$$

$$\rho = 993\ \text{kg/m}^3 \qquad \mu = 7.24\times10^{-4} \qquad k = 0.627$$

$$u_m = \frac{(1500)(7.24\times10^{-4})}{(993)(0.025)} = 0.0437\ \text{m/sec}$$

$$u_0 = 2u_m = 0.0875\ \text{m/sec}$$

$$h = \frac{(4.364)(0.627)}{0.025} = 109.4\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

5-75

$$u = \text{const} \qquad \frac{\partial T}{\partial x} = \text{const}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{ur}{\alpha}\frac{\partial T}{\partial x}$$

$$r\frac{\partial T}{\partial r} = \frac{ur^2}{2\alpha}\frac{\partial T}{\partial x} + c_1$$

$$\frac{\partial T}{\partial r} = \frac{ur}{2\alpha}\frac{\partial T}{\partial x} + \frac{c_1}{r}$$

$$T = \frac{ur^2}{4\alpha}\frac{\partial T}{\partial x} + c_1 \ln r + c_2$$

$$T = T_w(x) \qquad \text{at } r = r_1 \text{ and } r = r_2$$

$$T_w(x) = \frac{ur_2^2}{4\alpha}\frac{\partial T}{\partial x} + c_1 \ln r_1 + c_2$$

$$T_w(x) = \frac{ur_1^2}{4\alpha}\frac{\partial T}{\partial x} + c_1 \ln r_2 + c_2$$

$$\frac{u(r_2^2 - r_1^2)}{4\alpha}\frac{\partial T}{\partial x} + c_1 \ln\left(\frac{r_2}{r_1}\right) = 0$$

$$c_1 = \frac{-u(r_2^2 - r_1^2)}{4\alpha}\frac{\partial T}{\partial x}\left[\frac{1}{\ln(r_2/r_1)}\right]$$

$$T - T_w(x) = \frac{u}{4\alpha}\frac{\partial T}{\partial x}\left[(r^2 - r_1^2) - \frac{\ln(r/r_1)}{\ln(r_2/r_1)}(r_2^2 - r_1^2)\right]$$

5-76

Air at $m_\infty = 4$ $\quad p = 3$ psia $\quad T_\infty = 0°F$ $\quad$ Plate: 18 in long

$T_w = 200°F$ $\quad \gamma = 1.402$ $\quad d = \sqrt{\gamma g_c RT} = 1052$ ft/sec

$u_\infty = m_\infty = 1.51 \times 10^7$ ft/hr $\quad \rho_\infty = 0.0176$ $\quad u_\infty = 0.0394$

$$\mathrm{Re}_{L_\infty} = \frac{u_\infty x}{v_\infty} = 9.98 \times 10^6$$

$$T_0 = T_\infty\left(1 + \frac{\gamma - 1}{2} M_\infty^{\ 2}\right) \text{ (laminar portion)} = 1940°R$$

$\mathrm{Pr} = 0.681$ (Assume) $\quad r = \mathrm{Pr}^{1/2} = 0.825$ $\quad r = \dfrac{T_{aw} - T_\infty}{T_0 - T_\infty}$

$T_{aw} = 1680°R$ $\quad T^* = T_\infty + 0.5(T_w - T_\infty) + 0.22(T_{aw} - T_\infty) = 829°R$

$\rho^* = 0.00977$ $\quad \mu^* = 0.061$ $\quad k^* = 0.0218$ $\quad c_p = 0.2444$

Turbulent portion: $\mathrm{Pr} = 0.682$ $\quad r = \mathrm{Pr}^{1/3} = 0.882$ $\quad T_{aw} = 1800°R$

$T^* = 855°R$ $\quad \therefore\ \mathrm{Pr} = 0.681$ (close enough)

$\rho^* = 0.00947$ $\quad \mu^* = 0.0626$ $\quad c_p^{\ *} = 0.245$

$u_\infty = 1.51 \times 10^7$ $\quad x = 1.5$ ft

Laminar Heat Transfer: $x_c = \dfrac{\mathrm{Re}_{crit}^{\ *} \mu^*}{\rho^* u_\infty^{\ *}} = 0.206$ ft $\qquad \overline{\mathrm{Nu}}^* = \dfrac{\bar{h} x_c}{k^*}$

$\overline{\mathrm{Nu}}^* = 0.664(\mathrm{Re}_{crit}^{\ *})^{1/2}(\mathrm{Pr}^*)^{1/3} = 416$

$q = \bar{h}A(T_w - T_\infty) = -9250$ Btu/hr

Turbulent Heat Transfer:

$$h_x = (\mathrm{Pr}^*)^{-2/3}(\rho^* c_p^{\ *} u_\infty)(0.0288)\left(\frac{\rho^* u_\infty x}{\mu^*}\right)^{-1/5} = 69x^{-1/5}$$

$$\bar{h} = \frac{\int_{0.206}^{1.5} h_x dx}{\int_{0.206}^{1.5} dx} = 73.3 \ \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot °\text{F}} \qquad q = \bar{h}A(T_w - T_\infty) = -108{,}000 \text{ Btu/hr}$$

Total Cooling $= -117{,}250$ Btu/hr

5-77

$T_w = 65°\text{C} \qquad u_\infty = 600 \text{ m/s} \qquad T_\infty = 15°\text{C} = 288 \text{ K}$

$p = 7000 \text{ N/m}^2 \qquad L = 1 \text{ m} \qquad a = [(1.4)(287)(288)]^{1/2} = 340.2 \text{ m/s}$

$$M = \frac{600}{340.2} = 1.764 \qquad T_0 = 288[1 + (0.2)(1.764)^2] = 467 \text{ K}$$

Assume $\Pr = 0.7$ Laminar: $r = (0.7)^{1/2} = \dfrac{T_{aw} - 288}{467 - 288}$ $\qquad T_{aw} = 438 \text{ K}$

$T^* = 288 + 0.5(467 - 288) + 0.22(438 - 288) = 346 \text{ K}$

$$\rho = \frac{7000}{(287)(346)} = 0.0705 \text{ kg/m}^3$$

$\mu = 2.07 \times 10^{-5} \qquad k = 0.02973 \qquad \Pr = 0.7$

$$x_c = \frac{(5 \times 10^5)(2.07 \times 10^{-5})}{(0.0705)(600)} = 0.245 \text{ m}$$

Turbulent: $r = \Pr^{1/3} = (0.7)^{1/3} = \dfrac{T_{aw} - 288}{467 - 288}$ $\qquad T_{aw} = 347 \text{ K}$

$$\rho = \frac{7000}{(287)(347)} = 0.0701 \qquad \mu = 2.07 \times 10^{-5} \qquad c_p = 1009 \qquad k = 0.0298$$

$\Pr = 0.7$

$$h_x = (0.0701)(600)(1009)(0.7)^{2/3}(0.0296)\left[\frac{2.07 \times 10^{-5}}{(0.0901)(600)}\right]^{1/5} x^{-1/5} = 87.24x^{-1/5}$$

$$\int_{x_c}^{L} h_x dx = (87.24)\left(\frac{5}{4}\right)[(1)^{4/5} - (0.245)^{4/5}] = 73.66$$

Laminar heat transfer coeff. $\bar{h} = \dfrac{(0.664)(0.02973)}{0.245}(5 \times 10^5)^{1/2}(0.7)^{1/3} = 50.59$

For entire plate: $\bar{h} = \displaystyle\int \frac{h_x dx}{L} = \frac{(50.59)(0.245) + 73.66}{1} = 86.05 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

5-78

$\Pr = 0.69 \qquad T_0 = 233[1 + (0.2)(4)^2] = 979 \text{ K}$

$r = (0.69)^{1/2} = \dfrac{T_{aw} - 233}{979 - 233}$ $\qquad T_{aw} = 853 \text{ K}$

5-79

$T_\infty = -40°\text{C} = 233\ \text{K}$ $\quad a = [(1.4)(1)(287)(233)]^{1/2} = 306\ \text{m/s}$

$u_\infty = (2.8)(306) = 856.7$ $\quad T_0 = (233)[1 + (0.2)(2.8)^2] = 598\ \text{K}$

$T^* \approx 450\ \text{K}$ $\quad \text{Pr} = 0.69$

Laminar: $\quad r = \text{Pr}^{1/2} = 0.83 \quad T_{aw} = (0.83)(598 - 233) + 233 = 536\ \text{K}$

Turbulent: $\quad r = \text{Pr}^{1/3} = 0.88 \quad T_{aw} = (0.88)(598 - 233) + 233 = 555\ \text{K}$

5-81

$T_\infty = 30°\text{C} = 303\ \text{K}$ $\quad \rho = 1258 \quad L = 0.3\ \text{m} \quad c_p = 2445$

$u_\infty = 1.5\ \text{m/sec}$ $\quad \text{Pr} = 5380 \quad D = 8.9N = \tau_w A$ (both sides)

$$\tau_w = \frac{8.9}{2(0.3)^2} = 49.44\ \text{N/m}^2 \qquad C_f = \frac{(2)(49.44)}{(1258)(1.5)^2} = 0.0349$$

$$\text{St}\,\text{Pr}^{2/3} = \frac{C_f}{2} \qquad \text{St} = \frac{0.0349}{2}(5380)^{-2/3} = 5.689 \times 10^{-5}$$

$$\bar{h} = (5.689 \times 10^{-5})(1258)(1.5)(2445) = 262\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

5-83

$T_f = 60°\text{C} = 333\ \text{K}$ $\quad \rho = 1.046 \quad c_p = 1042 \quad \mu = 19.22 \times 10^{-6}$

$$k = 0.02858 \qquad \text{Re} = \frac{(1.046)(3.0)(1.3)}{19.22 \times 10^{-6}} = 2.122 \times 10^{-5}$$

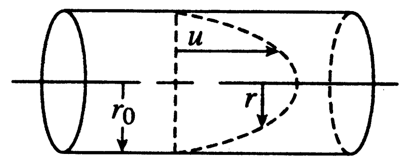
$$h = \frac{0.02858}{1.3}(0.664)(2.122 \times 10^5)^{1/2}(0.7)^{1/3} = 5.97$$

$$q = (5.97)(1.3)^2(100 - 20) = 807.2\ \text{W}$$

$$\overline{C}_f = (2)(0.664)(2.122 \times 10^5)^{-1/2} = 2.883 \times 10^{-3}$$

5-84

$$\frac{u}{u_0} = 1 - \frac{r^2}{r_0^2} \qquad \Delta p = f\frac{L}{D}\rho\mu_m^2$$



$$f = \frac{\Delta p d 2g_c}{L\rho u_m^2} = \left(\frac{\Delta p}{L}\right)\frac{d}{\rho u_m^2/2g_c}$$

$$\frac{dp}{dx} = \frac{2\mu}{r}\frac{du}{dr} = \frac{\Delta p}{L}$$

$$f = \frac{2\mu}{r}\left(\frac{du}{dr}\right)\frac{d}{\rho u_m^2/2g_c} \qquad du = -2r\,dr\left(\frac{u_0}{r_0^2}\right)$$

$$\frac{du}{dr} = -2r \qquad f = \left(\frac{2\mu}{r}\right)\left(\frac{-2ru_0}{r_0^2}\right)\frac{d}{\rho u_m^2/2g_c}$$

$$u_m = \frac{\int_0^{r_0} 2\pi r u\,dr}{\pi r_0^2} = \frac{2u_0\int_0^{r_0}\left(1-\frac{r^2}{r_0^2}\right)r\,dr}{r_0^2}$$

$$u_m = \frac{u_0}{2} \qquad \therefore\ f = \frac{64g_c}{\frac{\rho u_m d}{\mu}} = \frac{64}{\mathrm{Re}_d}$$

5-85

oil at 10°C 15 cm square plate

$q_w = 10\ \mathrm{kW/m^2}$

Oil at 20°C $\nu = 9\times10^{-4}\ \mathrm{m^2/s}$ $k = 0.145\ \frac{\mathrm{W}}{\mathrm{m\cdot°C}}$ $\mathrm{Pr} = 10{,}400$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{q_w x}{k(T_w - T_\infty)} = \frac{0.4637\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.0207}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}} = c\,\mathrm{Re}_x^{1/2}$$

$$q_w = h_x(T_w - T_\infty)$$

$$T_w - T_\infty = \frac{q_w x}{k\,\mathrm{Nu}_x} = \frac{q_w x}{kc\,\mathrm{Re}_x^{1/2}}$$

$$\overline{T_w - T_\infty} = \frac{1}{L}\int_0^L \frac{q_w x}{kc\left(\frac{\rho u_\infty x}{\mu}\right)^{1/2}}dx = \frac{q_w L/k}{(1/2)c\,\mathrm{Re}_L^{1/2}}$$

$$q_w = \frac{3}{2}h_{x=L}\overline{(T_w - T_\infty)}$$

At $T_f = 20°\mathrm{C}$ $\qquad \mathrm{Re}_L = \dfrac{(0.5)(0.15)}{0.0009} = 83.33$

$$h_{x=L} = \frac{(0.145)}{0.15}\frac{(0.4637)(83.33)^{1/2}(10{,}400)^{1/3}}{\left[1+\left(\frac{0.0207}{10{,}400}\right)^{2/3}\right]^{1/4}} = 92.76$$

$$\overline{T_w - T_\infty} = \frac{10{,}000}{(3/2)(92.36)} = 72.18°\text{C}$$

Recalculate $T_f = 10 + \frac{72}{2} = 46°\text{C}$

Evaluate properties at 40°C

$\nu = 0.00024 \qquad k = 0.144 \qquad \Pr = 2870$

$$\text{Re}_L = \frac{(0.5)(0.15)}{0.00024} = 312.5$$

$$h_{x=L} = \frac{0.144}{0.15}\frac{(0.4637)(312.5)^{1/2}(2870)^{1/3}}{\left[1+\left(\frac{0.0207}{2870}\right)^{2/3}\right]^{1/4}} = 112$$

$$\overline{T_w - T_\infty} = \frac{10{,}000}{\left(\frac{3}{2}\right)(112)} = 59.6°\text{C}$$

Check $T_f = 10 + \frac{59.6}{2} = 39.8°\text{C} \approx 40°\text{C}$

Close enough

$$\bar{h} = \left(\frac{3}{2}\right)(112) = 168 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}} \qquad (T_w - T_\infty)_{x=L} = \frac{10{,}000}{112} = 89.3°\text{C}$$

5-86

$T_\infty = 10°\text{C} \qquad T_w = 10 + 89.3 = 99.3°\text{C} \qquad T_f = 55°\text{C} \qquad \nu = 0.000123 \ \text{m}^2/\text{s}$

$$k = 0.141 \ \frac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \Pr = 1505 \qquad \text{Re}_L = \frac{(0.5)(0.15)}{0.000123} = 610$$

$$h_{x=L} = \frac{(0.141/0.15)(0.3387)(610)^{1/2}(1505)^{1/3}}{\left[1+\left(\frac{0.0468}{1505}\right)^{2/3}\right]^{1/4}} = 90.1 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\bar{h} = (2)(90.1) = 180.2 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (180.2)(0.15)^2(89.3) = 362 \text{ W}$$

5-87

$$\nu = 15.51\times10^{-6}\ \text{m}^2/\text{s}$$

$$\text{Re}_1 = 5\times10^5 = \frac{45x}{15.51\times10^{-6}} \qquad x = 0.172\ \text{m}$$

$$\text{Re}_2 = 10^8 = \frac{45x}{15.51\times10^{-6}} \qquad x = 34.467\ \text{m}$$

$$\delta_1 = \frac{(5.0)(0.172)}{(5\times10^5)^{1/2}} = 1.22\times10^{-3}\ \text{m}$$

$$\delta_2 \cong (34.467)\left[0.381(10^8)^{-1/5} - \frac{10{,}256}{10^8}\right] = 0.326\ \text{m}$$

5-88

$$x = \frac{5\times10^5\nu}{u_\infty} \qquad u_\infty = 20\ \text{m/s}$$

$$\delta = 5x(5\times10^5)^{-1/2} = \frac{5\nu}{u_\infty}(5\times10^5)^{1/2} = 3536\frac{\nu}{u_\infty} = 1768\nu$$

(a) $\nu = 14.2\times10^{-6}\ \text{m}^2/\text{s}$ $\qquad \delta = 0.025\ \text{m}$

(b) $\nu = 1.31\times10^{-6}$ $\qquad \delta = 0.0023\ \text{m}$

(c) $\nu = 99.69\times10^{-6}$ $\qquad \delta = 0.176\ \text{m}$

(d) $\nu = 0.368\times10^{-6}$ $\qquad \delta = 6.5\times10^{-4}\ \text{m}$

(e) $\nu = 0.203\times10^{-6}$ $\qquad \delta = 3.59\times10^{-4}\ \text{m}$

5-89

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L kC\left(\frac{\rho u_\infty}{\mu}\right)^n x^{n-1} f(\text{Pr})dx = \frac{1}{L}Ckf(\text{Pr})\left(\frac{\rho u_\infty}{\mu}\right)^n \frac{L^n}{n} = \frac{h_{x=L}}{n}$$

$$\frac{\bar{h}_L}{h_{x=L}} = \frac{1}{n}$$

5-90

$\nu = 0.0009\ \text{m}^2/\text{s}$ $\qquad$ $\text{Pr} = 10{,}400$

Eq. (5–44) $\quad \text{Nu}_x = (0.332)(10{,}000)^{1/2}(10{,}400)^{1/3} = 724.7$

Eq. (5–51) $\quad \text{Nu}_x = \dfrac{(0.3387)(10{,}000)^{1/2}(10{,}400)^{1/3}}{\left[1+\left(\frac{0.0468}{10{,}400}\right)^{2/3}\right]^{1/4}} = 739.3$

5-91

$T_f = 325\ \text{K}$ $\qquad$ $\nu = 18.23\times10^{-6}$ $\qquad$ $k = 0.02814$ $\qquad$ $\text{Pr} = 0.7$

$\rho = 1.086$

$$\text{Re} = \frac{(45)(0.75)}{18.23\times10^{-6}} = 1.85\times10^6$$

$$h = \frac{0.02814}{0.75}(0.7)^{1/3}[(0.037)(1.85\times10^6)^{0.8} - 871] = 98.3\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (98.3)(0.75)^2(350-300) = 2764\ \text{W}$$

$$\overline{C}_f = \frac{0.074}{(1.85\times10^6)^{0.2}} - \frac{1055}{1.85\times10^6} = 0.00356$$

$$D = \frac{(0.00356)(0.75)^2(1.086)(45)^2}{2} = 2.2\ \text{N}$$

<u>Laminar Portion:</u> $\qquad L = \dfrac{(5\times10^5)(18.23\times10^{-6})}{45} = 0.203\ \text{m}$

$$h = \frac{0.02814}{0.203}(0.7)^{1/3}(5\times10^5)^{1/2}(0.332) = 28.96\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (28.96)(0.203)(0.75)(350-300) = 220\ \text{W}$$

5-92

$x_c = 0.203\ \text{m}$

$$\delta_c = \frac{(5)(0.203)}{(5\times10^5)^{1/2}} = 0.0014\ \text{m}$$

$$\delta_L = (0.75)\left[(0.381)(1.85\times10^6)^{-1/5} - \frac{10{,}256}{1.85\times10^6}\right] = 0.0118\ \text{m}$$

All turbulent $\quad \delta = \dfrac{(0.75)(0.381)}{(1.85\times10^6)^{0.2}} = 0.0159\ \text{m}$

5-93

$T_w = 500$ K $\qquad T_f = 400$ K

(a) Propeties at $T_\infty = 300$ K: $\nu = 15.69\times10^{-6}$, $k = 0.02624$, Pr $= 0.708$

(b) Propeties at $T_f = 400$ K: $\nu = 25.9\times10^{-6}$, $k = 0.03365$, Pr $= 0.689$

(c) Propeties at $T_w = 500$ K: $\nu = 37.9\times10^{-6}$, $k = 0.04038$, Pr $= 0.68$

(a) $\mathrm{Re} = \dfrac{(45)(0.75)}{15.69\times10^{-6}} = 2.15\times10^6$

$h = \dfrac{0.02624}{0.78}(0.708)^{1/3}[(0.037)(2.15\times10^6)^{0.8} - 871] = 107.2$

$q = (107.2)(0.75)^2(500-300) = 12{,}053$ W

(b) $\mathrm{Re} = \dfrac{(45)(0.75)}{25.9\times10^{-6}} = 1.303\times10^6$

$h = \dfrac{0.03365}{0.75}(0.689)^{1/3}[(0.037)(1.303\times10^6)^{0.8} - 871] = 79.82$

$q = (79.82)(0.75)^2(500-300) = 8979$ W

(c) $\mathrm{Re} = \dfrac{(45)(0.75)}{37.9\times10^{-6}} = 8.9\times10^5$

$h = \dfrac{0.04038}{0.75}[(0.037)(8.9\times10^5)^{0.8} - 871](0.68)^{1/3} = 59.5$

$q = (59.5)(0.75)^2(500-300) = 6694$ W

Properties rather strongly dependent on temperature.

5-94

$q_w = 700\ \text{W/m}^2 \qquad L = 0.3\ \text{m}$

Properties at 300 K

$\nu = 15.69 \times 10^{-6} \qquad k = 0.02624 \qquad \text{Pr} = 0.708$

at $x = 15$ cm $\qquad \text{Re} = \dfrac{(10)(0.15)}{15.69 \times 10^{-6}} = 95{,}600$

$\text{Nu}_x = 0.453(95{,}600)^{1/2}(0.708)^{1/3} = 124.8$

$T_w - T_\infty = \dfrac{(700)(0.15)}{(124.8)(0.02624)} = 32°\text{C}$

300 K is close to average film temperature.

at $x = 1$ cm $\qquad \text{Re} = 6373 \qquad \text{Nu}_x = 32.2$

$T_w - T_\infty = 8.3°\text{C} \qquad T_w = 258.3\ \text{K}$

at $x = 5$ cm $\qquad \text{Re} = 31{,}865 \qquad \text{Nu}_x = 72.1$

$T_w - T_\infty = 18.5°\text{C} \qquad T_w = 268.5\ \text{K}$

at $x = 10$ cm $\qquad \text{Re} = 63{,}730 \qquad \text{Nu}_x = 101.5$

$T_w - T_\infty = 26.3°\text{C} \qquad T_w = 276\ \text{K}$

at $x = 20$ cm $\qquad \text{Re} = 128{,}670 \qquad \text{Nu}_x = 144.8$

$T_w - T_\infty = 36.8°\text{C} \qquad T_w = 286.8\ \text{K}$

at $x = 30$ cm $\qquad \text{Re} = 191{,}200 \qquad \text{Nu}_x = 176.5$

$T_w - T_\infty = 45.3°\text{C} \qquad T_w = 295.3\ \text{K}$

5-95

at $30°\text{C} = T_f \qquad \nu = 0.00057 \qquad k = 0.144 \qquad \text{Pr} = 6635$

$$\text{Re} = \frac{(10)(0.2)}{0.00057} = 3509$$

$$h = \frac{0.144}{0.2}(0.664)(3509)^{1/2}(6635)^{1/3} = 532\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$q = (532)(0.2)^2(40 - 20) = 426\ \text{W}$

at 20°C $\qquad \nu = 0.0009 \qquad \rho = 888.2$

$$\text{Re} = \frac{(10)(0.2)}{0.0009} = 2228$$

$$\frac{\overline{C}_f}{2} = 0.664(2228)^{-1/2} = 0.0141$$

$D = (0.0141)(0.2)^2(888.2)(10)^2 = 50.1\ \text{N}$

5-96

$$T_f = \frac{27+77}{2} = 52°\text{C} = 325\text{ K} \qquad \nu = 18.23\times10^{-6} \qquad k = 0.02814$$

$$\text{Pr} = 0.7 \qquad u_\infty = 44\ \text{ft/s} = 13.4\ \text{m/s}$$

$$\text{Re}_L = \frac{(13.4)(4)}{18.23\times10^{-6}} = 2.94\times10^6$$

$$\bar{h} = \frac{0.02814}{4}(0.7)^{1/3}[(0.037)(2.94\times10^6)^{0.8} - 871] = 29.1\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (29.1)(4)(1)(77-27) = 5820\text{ W}$$

$$h_{x=L} = \frac{(0.0296)(2.94\times10^6)^{0.8}(0.7)^{1/3}(0.02814)}{4} = 27.64\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{A} = (27.64)(50) = 1383\ \text{W/m}^2$$

at $x = 50$ cm $\qquad \text{Re}_L = 3.68\times10^5$

$$h_x = \frac{0.02814}{0.5}(0.332)(3.68\times10^5)^{1/2}(0.7)^{1/3} = 10.06$$

$$\frac{q}{A} = (10.06)(50) = 500\ \text{W/m}^2$$

5-97

at $T_f = 350$ K $\qquad \nu = 20.76\times10^{-6} \qquad k = 0.03003 \qquad \text{Pr} = 0.697$

$$\text{Re}_L = \frac{(30)(0.1)}{20.76\times10^{-6}} = 1.445\times10^5 \qquad x_0 = 0.05\text{ m}$$

x	Re_x	$\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$	Nu_x	h_x
0.06	0.867×10^5	1.985	171.8	85.98
0.075	1.084×10^5	1.562	151.2	60.54
0.1	1.445×10^5	1.351	151	45.34

$$\bar{h} = \left[\frac{85.98+60.54}{2}(0.015) + \frac{60.54+45.34}{2}(0.025)\right]\frac{1}{0.04} = 60.56\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (60.56)(0.1-0.05)(0.1)(400-300) = 30.3\text{ W}$$

5-98

at $x = 5$ cm $\qquad \mathrm{Re}_x = 72{,}250$

$$h_x = \frac{0.03003}{0.05}(0.332)(72{,}250)^{1/2}(0.697)^{1/3} = 47.52\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$q = (47.52)(0.1)(0.005)(400-300) = 2.38\ \mathrm{W}$$

5-99

$$\nu = 15.69\times10^{-6} \qquad \mathrm{Re}_L = \frac{(15)(0.2)}{15.69\times10^{-6}} = 1.91\times10^5$$

$$\delta = \frac{(5)(0.2)}{(1.91\times10^5)^{0.8}} = 0.00229\ \mathrm{m} = 0.23\ \mathrm{cm}$$

No interference

5-100

$T_f = 26.7^\circ\mathrm{C} \qquad u_\infty = 2\ \mathrm{m/s} \qquad x_0 = 0.1\ \mathrm{m} \qquad x_1 = 0.105\ \mathrm{m}$

$\mu = 8.6\times10^{-4} \qquad k = 0.614 \qquad \rho = 996 \qquad \mathrm{Pr} = 5.85$

$$\bar{h} = \frac{\int_{x_0}^{x_1} 0.332k\,\mathrm{Pr}^{1/3}\left(\frac{\rho u_\infty}{\mu x}\right)^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3} dx}{x_1 - x_0} = 40{,}450\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (40{,}450)(0.2)(0.005)(37.8-15.6) = 898\ \mathrm{W}$$

5-101

$$T_f = \frac{300+400}{2} = 350\ \mathrm{K} \qquad L = 0.1\ \mathrm{m} \qquad T_w = 400\ \mathrm{K} \qquad u_\infty = 35\ \mathrm{m/s}$$

$$\nu = \frac{20.76\times10^{-6}}{4} = 5.19\times10^{-6}\ \mathrm{m^2/s} \qquad k = 0.03003\ \frac{\mathrm{W}}{\mathrm{m}\cdot{}^\circ\mathrm{C}}$$

$\mathrm{Pr} = 0.697$

$$\mathrm{Re}_L = \frac{u_\infty L}{\nu} = \frac{(35)(0.1)}{5.19\times10^{-6}} = 6.74\times10^5$$

$$\bar{h} = \frac{k}{L}\mathrm{Pr}^{1/3}(0.037\,\mathrm{Re}_L{}^{0.8} - 871)$$

$$= \frac{(0.697)^{1/3}(0.03003)}{0.1}[(0.037)(6.74\times10^5)^{0.8} - 871]$$

$$= 222\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (222)(0.1)^2(400-300) = 222\ \mathrm{W}$$

5-102

For same flow properties

$Re_L = 6.74 \times 10^5$

$h(\text{const heat flux}) = 1.04 \ (h \text{ isothermal})$

At $x = L \ (T = c)$

$$Nu = Pr^{1/3}(0.0296)Re_L^{0.8} = (0.697)^{1/3}(0.0296)(6.74 \times 10^5)^{0.8} = 1208$$

$$h = \frac{(1208)(0.03003)}{0.1} = 363 \ \frac{W}{m^2 \cdot °C}$$

Minimum h and max ΔT will occur at $Re_{crit} = 5 \times 10^5$ and $L = 0.074$ m

$$h = \frac{0.03003}{0.074}(0.453)(5 \times 10^5)^{1/2}(0.697)^{1/3} = 115.3 \ \frac{W}{m^2 \cdot °C}$$

$$\Delta T_{max} = \frac{q/A}{h} = \frac{1000}{115.3} = 8.68°C$$

$$T_{max} = 300 + 8.68 = 308.68°C$$

5-103

$Re_{crit} = 10^6$ $\quad Re_L = 5 \times 10^6$ $\quad u_\infty = 10$ m/s $\quad T_f = 350$ K

$\nu = 20.76 \times 10^{-6}$ m²/s $\quad k = 0.03003$ $\quad Pr = 0.697$

$$5 \times 10^6 = \frac{(10)L}{20.76 \times 10^6} \qquad L = 10.38 \text{ m}$$

$$\overline{C}_f = \frac{0.074}{Re_L^{1/5}} - \frac{A}{Re_L} \qquad A = 3340$$

$$St\,Pr^{2/3} = \frac{\overline{C}_f}{2} = 0.037 Re_L^{-1/5} - 1670 Re_L^{-1}$$

$$\frac{\overline{h}L}{k} = Pr^{1/3}(0.037 Re_L^{0.8} - 1670)$$

At $Re_L = 5 \times 10^6$

$$\overline{h} = \frac{0.03003}{10.38}(0.697)^{1/3}[(0.037)(5 \times 10^6)^{0.8} - 1670] = 17.41 \ \frac{W}{m^2 \cdot °C}$$

$$q = \overline{h}A(T_w - T_\infty) = (17.41)(10.38)(400 - 300) = 18{,}080 \ \frac{W}{m \cdot \text{depth}}$$

5-104

$$\overline{h} = \frac{0.03003}{10.38}(0.697)^{1/3}[(0.037)(5 \times 10^6)^{0.8} - 871] = 19.47 \ \frac{W}{m^2 \cdot °C}$$

$$q = \overline{h}A(T_w - T_\infty) = (19.47)(10.38)(400 - 300) = 20{,}207 \ \frac{W}{m \cdot \text{depth}}$$

5-105

$T_f = \dfrac{10+30}{2} = 20°\text{C} \qquad L = 30 \text{ cm} \qquad u_\infty = 2 \text{ m/s} \qquad \nu = 0.00118 \text{ m}^2/\text{s}$

$k = 0.286 \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \text{Pr} = 12.5 \times 10^3 \qquad \text{Re}_L = \dfrac{(2)(0.3)}{0.00118} = 508.4$

$$\overline{\text{Nu}}_L = \frac{(2)(0.3387)\text{Re}_L^{1/2}\,\text{Pr}^{1/3}}{\left[1+\left(\frac{0.0468}{\text{Pr}}\right)^{2/3}\right]^{1/4}} = \frac{(2)(0.3387)(508.4)^{1/2}(12{,}500)^{1/3}}{\left[1+\left(\frac{0.0468}{12{,}500}\right)^{2/3}\right]^{1/4}} = 354.5$$

$\bar{h} = \dfrac{(354.5)(0.286)}{0.3} = 338 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q = \bar{h}A(T_w - T_\infty) = (338)(0.3)^2(30-10) = 608 \text{ W}$

5-106

$T_\infty = 20°\text{C} \qquad T_w = 0°\text{C} \qquad T_f = 10°\text{C} \qquad L = 0.2 \text{ m}$

$\text{Re}_L = 1 \times 10^5 \qquad k = 0.246 \dfrac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \text{Pr} = 410$

$$\overline{\text{Nu}}_L = \frac{(2)(0.3387)\text{Re}_L^{1/2}\,\text{Pr}^{1/3}}{\left[1+\left(\frac{0.0468}{\text{Pr}}\right)^{2/3}\right]^{1/4}} = \frac{(2)(0.3387)(1\times 10^5)^{1/2}(410)^{1/3}}{\left[1+\left(\frac{0.0468}{410}\right)^{2/3}\right]^{1/4}} = 1591$$

$\bar{h} = \dfrac{(1591)(0.246)}{0.2} = 1957 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q = \bar{h}A(T_w - T_\infty) = (1957)(0.2)^2(0-20) = -1566 \text{ W}$

5-107

$\nu = 15.7 \times 10^{-6} \qquad \text{Re} = 10^7 = \dfrac{u_\infty x}{\nu} \qquad x = \dfrac{(10^7)(15.7 \times 10^{-6})}{30} = 5.23 \text{ m}$

at $\quad u = 7 \text{ m/s} \qquad x = 22.4 \text{ m}$

$\quad\quad u = 12 \text{ m/s} \qquad x = 13.08 \text{ m}$

5-109

$$T_f = \frac{50+10}{2} = 30°\text{C} = 86°\text{F} \qquad \mu = 8.03\times10^{-4} \qquad k = 0.619$$

$\text{Pr} = 5.41 \qquad \rho = 995$

at $\text{Re} = 10^4 \quad u_\infty = \dfrac{(10^4)(8.03\times10^{-4})}{(995)(0.3)} = 0.027\ \text{m/sec}$

at $\text{Re} = 10^7 \quad u_\infty = 27\ \text{m/sec}$

at $\text{Re} = 10^4 \quad \bar{h} = \dfrac{0.619}{0.3}(0.664)(10^4)^{1/2}(5.41)^{1/3} = 240.5\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$q = (240.5)(0.3)(50-10) = 866\ \text{W}$

at $\text{Re} = 10^7 \quad \bar{h} = \dfrac{0.619}{0.3}(5.41)^{1/3}[(0.037)(10^7)^{0.8} - 871] = 49{,}956\ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$q = (49{,}956)(0.3)^2(50-10) = 1.80\times10^5\ \text{W}$

5-110

For laminar flow

$$\bar{h} = \left(\frac{k}{L}\right)(0.664)\left(\frac{\rho u_\infty L}{\mu}\right)^{1/2}\left(\frac{c_p\mu}{k}\right)^{1/3}$$

Pr ~ constant which implies $\mu \sim k$

For ideal gas $\rho \sim \dfrac{1}{T}$ (T = abs. temp.)

$$\mu \sim T^a \quad (a = \text{const})$$

Therefore, for constant u_∞ and L

$$\bar{h} = \text{const}\times T^a\left(\frac{1}{T}\right)^{1/2}\left(\frac{1}{T^a}\right)^{1/2} = \text{const}\times T^{\frac{a-1}{2}}$$

For air at 350 K $a = 0.74$ and $\bar{h} = \text{const}\times T^{-0.13}$
or, not a strong function of absolute temperature

Chapter 6

6–1

For $L = 20$ cm $\quad \dfrac{L}{d} = 40$ at 120°C $\quad \Pr = 175 \quad k = 0.135$

$c_p = 2.307 \quad \mu = (0.124\times10^{-4})(829)$

1st iteration

$$h = \frac{0.135}{0.005}(1.86)(175{,}000)^{1/3}\left(\frac{1}{40}\right)^{1/3} = 821.3$$

$$\mathrm{Re} = 1000 = \frac{\dot m(0.005)(4)}{\pi(0.005)^2(0.124\times10^{-4})(829)}$$

$\dot m = 0.0404$ kg/sec

$$(0.0404)(2307)(120 - T_2) = (821.3)\pi(0.005)(2)\left(60 + \frac{T_2}{2} - 50\right) \qquad \text{(a)}$$

$T_2 = 118.1$°C $\quad$ Small change in temp.

at $T_w = 50$°C $\quad \nu = 0.00057$

$$\bar h = (821.3)\left(\frac{0.124\times10^{-4}}{0.00057}\right)^{0.14} = 480.6$$

Recalculating T_2 from eq. (a) gives $T_2 = 118.9$°C

$q = (0.0404)(2307)(120 - 118.9) = 104.8$ W

6–2

$\rho = 996 \quad c_p = 4180 \quad k = 0.614 \quad \Pr = 5.85 \quad \mu = 8.6\times10^{-4}$

$d = 3$ mm $\quad u_m = 0.04$ m/s $\quad L = 2$ m

$$\dot m = \frac{(996)\pi(0.003)^2(0.04)}{4} = 2.82\times10^{-4} \text{ kg/s}$$

$$\mathrm{Re}_d = \frac{(996)(0.04)(0.003)}{8.6\times10^{-4}} = 139 \qquad \text{Laminar}$$

$$\mathrm{Gz}^{-1} = \frac{2}{(0.003)(139)(5.85)} = 0.82$$

For $T_w =$ const $\quad \overline{\mathrm{Nu}} \to 3.66$

$$\bar h = \frac{(3.66)(0.614)}{0.003} = 749 \ \frac{\mathrm{W}}{\mathrm{m^2\cdot°C}}$$

$$q = \bar h A(T_w - T_b) = (749)\pi(0.003)(2)(180 - 80)\left(\frac{5}{9}\right) = 784 \text{ W}$$

6-3

$d = 0.025$, $T = 300$ K

$\mu_{air} = 1.846\times 10^{-5}$

$\mu_{water} = 8.6 \times 10^{-4}$

$Re = 15000 = 4\dot{m}/\pi d\mu$

$\dot{m}_{air} = 0.0054$ kg/s $\quad \dot{m}_{water} = 0.253$ kg/s

6-4

$Nu_T = 2.47 \qquad k = 0.521$

$$D_H = \frac{4A}{p} = \frac{(4)(1)\left(\frac{1}{2}\right)(0.866)}{3} = 0.5744 \text{ cm}$$

$$h = \frac{(2.47)(0.521)}{0.005744} = 222.9 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\frac{q}{L} = (222.9)(0.03)(50-20) = 200.6 \ \text{W/m}$$

6-5

$$D_H = \frac{(4)(5)(10)}{30} = 6.667 \text{ mm} \qquad k = 0.6 \qquad Nu_T = 3.657$$

$$h = \frac{(3.657)(0.6)}{6.667\times 10^{-3}} = 329.1$$

$$\frac{q}{L} = (329.1)(30\times 10^{-3})(60-20) = 394.9 \ \text{W/m}$$

6-6

$q = (3)(4175)(15-5) = 125{,}850$ W $\quad$ at 10°C $\qquad \mu = 1.31\times 10^{-3}$

$$k = 0.585 \qquad Pr = 9.40 \qquad Re = \frac{(0.05)(3)(4)}{\pi(0.05)^2(1.31\times 10^{-3})} = 58{,}316$$

$$\bar{h} = \frac{0.585}{0.05}(0.023)(58{,}316)^{0.8}(9.4)^{0.4} = 4283 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = 125{,}850 = (4283)\pi(0.05)L(90-10) \qquad L = 2.338 \text{ m}$$

6-7

$q = (0.8)(4221)(40-35) = 16{,}884$ W $\qquad \mu = 6.82\times 10^{-4} \qquad \rho = 993$

$$k = 0.63 \qquad Pr = 4.53 \qquad Re = \frac{(0.025)(0.8)(4)}{\pi(0.025)^2(6.82\times 10^{-4})} = 59{,}741$$

$$h = \frac{(0.023)(0.63)}{0.025}(59{,}741)^{0.8}(4.53)^{0.4} = 7024 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = 16{,}884 = (7024)\pi(0.025)L(90-37.5) \qquad L = 0.583 \text{ m}$$

6-8

$$4D = \frac{1.5}{0.025} = 60 \qquad T_f \approx \frac{50+20}{2} = 35°\text{C} \qquad \text{at } 20°\text{C} \qquad \rho = 998$$

$$c = 4180 \qquad \text{at } 35°\text{C} \qquad \text{Pr} = 5.45 \qquad 1.0 = (998)\frac{\pi(0.025)^2 u_m}{4}$$

$$u_m = 2.04 \ \text{m/sec} \qquad 7000 = f(60)\frac{(998)(2.04)^2}{2} \qquad f = 0.0562$$

$$\text{St} = \frac{(0.0562)(5.45)^{-2/3}}{8} = 2.268 \times 10^{-3}$$

$$\bar{h} = (2.268 \times 10^{-3})(998)(4180)(2.04) = 19{,}297 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = (19{,}297)\pi(0.025)(1.5)\left(50 - \frac{T_{b_{\text{out}}} + 20}{2}\right) = 1.0(4180)(T_{b_{\text{out}}} - 20)$$

$$T_{b_{\text{out}}} = 32.83°\text{C}$$

6-9

$$\overline{T}_b = 80°\text{F} = 26.67°\text{C} \qquad k = 0.614 \qquad c_p = 4179 \qquad \mu = 8.6 \times 10^{-4}$$

$$\text{Pr} = 5.85 \qquad \text{Re}_d = \frac{(\dot{m}/A_c)_d}{\mu} = \frac{(1.3)(0.025)(4)}{\pi(0.025)^2(8.6 \times 10^{-4})} = 76{,}990 > 2300$$

$$\bar{h} = \frac{k}{d}0.023\text{Re}_d^{0.8}\text{Pr}^{0.4} = \frac{0.614}{0.025}(0.023)(76{,}990)^{0.8}(5.85)^{0.4} = 9289 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = \dot{m}c_p(T_{b_2} - T_{b_1}) = \bar{h}\pi dL(T_w - \overline{T}_b)$$

$$= (1.3)(4179)(100 - 60)\left(\frac{5}{9}\right) = (9289)\pi(0.025)L(40 - 26.67)$$

$$L = 12.42 \ \text{m}$$

6-10

$$T_{b_{\text{avg}}} = \frac{15+50}{2} = 32.5°\text{C} \qquad q = 1.0(4170)(50-15) = 145{,}950 \ \text{W}$$

$$\text{Pr} = 5.1 \qquad \mu = 7.7 \times 10^{-4} \qquad k = 0.623$$

$$\text{Re} = \frac{(0.025)(1.0)(4)}{\pi(0.025)^2(7.7 \times 10^{-4})} = 66{,}142$$

$$\bar{h} = \frac{(0.023)(0.623)}{0.025}(66{,}142)^{0.8}(5.1)^{0.4} = 7901 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = 145{,}950 = (7901)\pi(0.025)L(14) \qquad L = 16.8 \ \text{m}$$

6-11

Assume $T_{b_{avg}}$ about 50°C $\quad \rho = 870 \quad c_p = 2000 \quad u_m = 30$ cm/s

$k = 0.139 \quad \nu = 1.24\times10^{-4}$ m^2/sec $\quad \mathrm{Pr} = 1960$

$$\mathrm{Re} = \frac{(0.30)(0.0125)}{1.24\times10^{-4}} = 30.24$$

$$\mathrm{Nu} = 1.86\left[(30.24)(1960)\left(\frac{0.0125}{3}\right)\right]^{1/3}\left(\frac{1.24}{0.723}\right)^{0.14} = 12.59$$

$$h = \frac{(12.59)(0.139)}{0.0125} = 139.9 \ \frac{\mathrm{W}}{\mathrm{m^2\cdot °C}}$$

$$q = (139.9)\pi(0.0125)(3)\left(65 - \frac{T_c}{2} - \frac{38}{2}\right) = (870)\frac{\pi(0.0125)^2}{4}(0.3)(2000)(T_c - 38)$$

$49.839 = 1.287T_c \qquad T_c = 44.16°\mathrm{C} \qquad q = 394.6$ W

6-12

7.5×15 cm $\qquad L = 1.8$ m $\qquad$ air at 1 atm

$T_w = 120°\mathrm{C} \qquad T_b$ (inlet) $= 15°\mathrm{C} \qquad T_b$ (exit) $= 25°\mathrm{C}$

$$D_H = \frac{(4)(0.075)(0.15)}{(2)(0.075+0.15)} = 0.1 \text{ m} \qquad A_c = (0.075)(0.15) = 0.01125 \text{ m}^2$$

$$\overline{T_b} = \frac{15+25}{2} = 20°\mathrm{C} = 293 \text{ K}$$

$A = 2(0.15+0.075)(1.8) = 0.81$ m^2

$c_p = 1005 \qquad \mathrm{Pr} = 0.7 \qquad \mu = 1.83\times10^{-5} \qquad k = 0.026$

$$q = \dot{m}c_p(T_{b_2} - T_{b_1}) = \bar{h}A(T_w - \overline{T_b})$$

$$\bar{h} = \frac{k}{D_H}(0.023)\left(\frac{\dot{m}D_H}{A_c\mu}\right)^{0.8}\mathrm{Pr}^{0.4} \qquad \text{Assuming turbulent}$$

$$q = \dot{m}(1005)(25-15) = \frac{(0.026)(0.023)}{0.1}\left[\frac{\dot{m}(0.1)}{(0.01125)(1.83\times10^{-5})}\right]^{0.8}(0.7)^{0.4}$$

$\dot{m} = 0.141$ kg/s

$q = (0.141)(1005)(25-15) = 1417$ W

$\mathrm{Re}_{DH} = 68{,}500$ so turbulent assumption was correct.

6-13

$\mu = 1.31\times10^{-3}$ $\quad k = 0.585$ $\quad \Pr = 9.4$

$$\mathrm{Re} = \frac{(0.5)(0.025)(4)}{\pi(0.025)^2(1.31\times10^{-3})} = 19{,}439$$

$$h = \frac{(0.023)(0.585)}{0.025}(19{,}439)^{0.8}(9.4)^{0.4} = 3557\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$q = (3557)\pi(0.025)(15)(15) = (0.5)(4190)\Delta T_b \qquad \Delta T_b = 30.01^\circ\mathrm{C}$$

$$T_{\mathrm{out}} = 40.01^\circ\mathrm{C}$$

6-14

$$\Delta p = f\frac{L}{D}\rho\frac{u_m{}^2}{2g_c} \qquad \text{at } T_f = \frac{27+55}{2} = 41^\circ\mathrm{C} \qquad \Pr_f = 4.23 \qquad \rho = 996$$

$\mu = 8.6\times10^{-4}$ $\quad k = 0.614$ $\quad \Pr = 5.85$ $\quad c_p = 4179$

$$u_m = \frac{0.7}{(996)\pi(0.0125)^2} = 1.43\ \mathrm{m/s} \qquad f = \frac{(2000)(0.025)(2)}{(6)(996)(1.43)^2} = 8.16\times10^{-3}$$

$$\mathrm{St}_b\Pr_f{}^{2/3} = \frac{f}{8}$$

$$h = \frac{(8.16\times10^{-3})(996)(1.43)(4179)}{8}(4.23)^{-2/3} = 2321\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$q = hA(\overline{T_w - T_b}) = \dot{m}c_p\Delta T_b$$

$$\Delta T_b = \frac{(2321)\pi(0.025)(6)(55-27)}{(0.7)(4179)} = 10.47^\circ\mathrm{C}$$

$$T_{\mathrm{exit}} = 27 + \frac{10.47}{2} = 32.24^\circ\mathrm{C}$$

6-15

$$\mathrm{Re} = \frac{(0.3)(0.0025)}{1.6\times10^{-4}} = 4.69 \qquad \mathrm{Re}\Pr\frac{d}{L} = (4.69)(1960)\frac{0.25}{60} = 38.28$$

$$h = \frac{0.14}{0.0025}(1.86)(38.28)^{1/3} = 351\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}} \qquad c_p = \frac{\Pr k}{\mu} = \frac{(1960)(0.14)}{(1.6\times10^{-4})(860)}$$

$$c_p = 1994\ \frac{\mathrm{J}}{\mathrm{kg}\cdot{}^\circ\mathrm{C}} \qquad \dot{m} = \frac{(860)\pi(0.0025)^2(0.3)}{4} = 1.266\times10^{-3}\ \mathrm{kg/sec}$$

$$q = (1.266\times10^{-3})(1994)(T_e - 20) = (351)\pi(0.0025)(0.6)\left(120 - \frac{T_e - 20}{2}\right)$$

$$T_e = 69.36^\circ\mathrm{C} \qquad q = 120.67\ \mathrm{W}$$

6-16

$\dot{m} = 0.4\ \text{kg/sec} \qquad T_{b_{\text{avg}}} = \dfrac{10+38}{2} = 24°\text{C} \qquad \rho = 605.6$

$\nu = 0.355 \times 10^{-6} \qquad k = 0.515 \qquad c_p = 4840 \qquad \Pr = 2.02$

$$\text{Re} = \frac{(0.025)(0.4)(4)}{\pi(0.025)^2(0.355\times 10^{-6})(605.6)} = 1.0 \times 10^5$$

$$h = \frac{0.515}{0.025}(0.023)(1.0\times 10^5)^{0.8}(2.02)^{0.4} = 6277\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = (0.4)(4840)(38-10) = 54,200\ \text{W} = (6277)\pi(0.025)(2.5)(T_w - 24)$$

$$T_w = 68°\text{C}$$

6-17

<u>Freon 12</u> $\qquad \rho = 1364 \qquad k = 0.073 \qquad \nu = 0.203\times 10^{-6} \qquad \Pr = 3.6$

$$\text{Re} = \frac{(3)(0.0125)}{0.203\times 10^{-6}} = 184,700$$

$$h = \frac{0.073}{0.0125}(0.023)(184,700)^{0.8}(3.6)^{0.4} = 3663\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

<u>water</u> $\qquad \rho = 999 \qquad \mu = 1.31\times 10^{-3} \qquad k = 0.585 \qquad \Pr = 9.4$

$$\text{Re} = \frac{(999)(3)(0.0125)}{1.31\times 10^{-3}} = 57,195$$

$$h = \frac{0.585}{0.0125}(0.023)(57,195)^{0.8}(9.4)^{0.4} = 16,870\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

6-18

$T_f = \dfrac{50+10}{2} = 30°\text{C} \qquad L = 6\ \text{m} \qquad d = 2.5\ \text{cm} \qquad m = 0.4\ \text{kg/s}$

$\Delta p = 3000\ \text{N/m}^2 \qquad$ at 10°C $\qquad \rho = 999 \qquad c_p = 4195$

at 30°C $\qquad \Pr = 5.22 \qquad 0.4 = \dfrac{(999)\pi(0.025)^2 u_m}{4}$

$u_m = 0.816\ \text{m/s} \qquad \Delta p = f\dfrac{L}{D}\rho\dfrac{u_m{}^2}{2} \qquad f = \dfrac{(3000)(2)(0.025)}{(6)(0.816)^2(999)} = 0.0376$

$\text{St}_b \Pr_f{}^{2/3} = \dfrac{f}{8} \qquad \text{St}_b = \dfrac{(0.0376)(5.22)^{-2/3}}{8} = 1.56\times 10^{-3}$

$$h = (1.56\times 10^{-3})(999)(4195)(0.816) = 5338\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = (5338)\pi(0.025)(6)(50-10) = (0.4)(4195)\Delta T_b$$

$q = 100,600\ \text{W} \qquad \Delta T_b = 60°\text{C} \qquad T_{\text{out}} \approx \dfrac{60}{2} + 10 = 40°\text{C}$

6-19

$q = (0.4)(4175)(71-32) = 65{,}130\ \text{W} \qquad T_{b_{avg}} = 51.5°\text{C} = 125°\text{F}$

$\mu = 5.38\times10^{-4} \qquad k = 0.647 \qquad \text{Pr} = 3.47 \qquad \rho = 987$

(a) $12.5\ \text{mm} = d \qquad \text{Re} = \dfrac{(0.4)(0.0125)(4)}{\pi(0.0125)^2(5.38\times10^{-4})} = 75{,}731$

$$h = \frac{0.647}{0.0125}(0.023)(75{,}731)^{0.8}(3.47)^{0.3} = 13{,}843\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = 65{,}130 = (13{,}843)\pi(0.0125)L(51.5-4) \qquad L = 2.52\ \text{m}$

$u_m = \dfrac{(0.4)(4)}{\pi(0.0125)^2(987)} = 3.3\ \text{m/s} \qquad f = 0.019$

$$\Delta p = (0.019)\left(\frac{2.52}{0.0125}\right)\left(\frac{987}{2}\right)(3.3)^2 = 20.6\ \text{kPa}$$

(b) $d = 25\ \text{mm} \qquad T_w = 20°\text{C} \qquad \text{Re} = 37{,}866$

$$h = \frac{(0.64)(0.023)(37{,}866)^{0.8}(3.47)^{0.3}}{0.025} = 3932$$

$65{,}130 = (3932)\pi(0.025)L(51.5-20) \qquad L = 6.7\ \text{m}$

$f = 0.024 \qquad u = \dfrac{3.3}{4} = 0.825$

$$\Delta p = \frac{(0.024)(6.7)}{0.025}\left(\frac{987}{2}\right)(0.825)^2 = 2.16\ \text{kPa}$$

6-20

$\overline{T_b} = 550\ \text{K} \qquad \mu = 2.848\times10^{-5} \qquad k = 0.0436 \qquad \text{Pr} = 0.68$

$c_p = 1.039 \qquad \rho = \dfrac{1.4\times10^6}{(287)(550)} = 8.87\ \text{kg/m}^3$

$$\text{Re} = \frac{(0.075)(4)(0.5)}{\pi(0.075)^2(2.848\times10^{-5})} = 298{,}000$$

$$\text{Nu} = 0.036\,\text{Re}^{0.8}\,\text{Pr}^{1/3}\left(\frac{d}{L}\right)^{0.0555}$$

$$h = \frac{0.0436}{0.075}(0.036)(298{,}000)^{0.8}(0.68)^{1/3}\left(\frac{0.075}{6}\right)^{0.0555} = 346.5\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = (346.5)\pi(0.075)(6)(550-500) = (0.5)(1039)\Delta T_b = 24{,}493\ \text{W}$

$\Delta T_b = 47.15°\text{C}$

6-21

Neglect conduction resistance

At 100°C (373 K) k = 0.032; Pr = 0.693

$h_{inside} = (0.032/0.012)(0.023)(15000)^{0.8}(0.693)^{0.3} = 120$

At tube outside $T_f \approx 65°C = 338$ K; k = 0.029; $\nu = 20\times10^{-6}$; Pr = 0.7

$Re = (20)(0.014)/20\times10^{-6} = 14000$

$h_{outside} = (0.029/0.014)(0.193)(14000)^{0.618}(0.7)^{1/3} = 130$

$U_i = 1/[1/120 + (1/130)(0.012/0.014)] = 67$

$q/cm = (67)(\pi)(0.012)(0.01)(100 - 30) = 1.768W$

$= mc_p(\Delta T/cm)$

At 100°C $\mu = 2.2\times10^{-5}$; $m = (15000)\pi(0.012)(2.2\times10^{-5})/4 = 0.00311$ kg/s

$c_p = 1010$; $\Delta T/cm = 1.768/(0.00311)(1010) = 0.56°C/cm$

6-22

$T_b = 90°F \quad \mu = 7.65\times10^{-4} \quad k = 0.623 \quad Pr = 5.12$
$\rho = 995 \quad c_p = 4174$

$$Re_d = 100{,}000 = \frac{(\dot{m}/A_c)d}{\mu} \qquad A_c = \frac{\pi d^2}{4}$$

$$\frac{\dot{m}}{d} = \frac{\pi}{4}(100{,}000)(7.65\times10^{-4}) = 60.08 \qquad \dot{m} = 60.08(d) = 0.3$$

$$q = \dot{m}c_p\Delta T_b = \bar{h}\pi dL(T_w - \bar{T}_b)$$

$$(0.3)(4174)(120-60)\left(\frac{5}{9}\right) = \bar{h}\pi(0.005)L(30)\left(\frac{5}{9}\right)$$

$$\bar{h} = \frac{k}{d}0.023Re^{0.8}Pr^{0.4} = \frac{0.623}{0.005}(0.023)(100{,}000)^{0.8}(5.12)^{0.4} = 55{,}075\ \frac{W}{m^2\cdot °C}$$

$L = 2.894$ m

6-23

$\bar{T}_b = 30°C \quad \nu_b = 13.94\times10^{-6} \quad k = 0.252 \quad Pr = 148 \quad c_p = 2.428$
$\nu_w = 2.98\times10^{-6} \quad \rho = 1109 \quad D_H = 5 - 4 = 1\ \text{cm} = 0.01\ \text{m}$

$$\dot{m} = \rho A u_m = (1109)\frac{\pi}{4}(0.05^2 - 0.04^2)(6.9) = 5.409\ \text{kg/s}$$

$$\dot{q} = \dot{m}c_p\Delta T_b = (5.409)(2428)(40-20) = 2.627\times10^5\ \text{W}$$

$$Re_{D_H} = \frac{(6.9)(0.01)}{13.94\times10^{-6}} = 4950$$

$$\bar{h} = \frac{0.252}{0.01}(0.027)(4950)^{0.8}(148)^{1/3}\left(\frac{13.94}{2.98}\right)^{0.14} = 4033\ \frac{W}{m^2\cdot °C}$$

$$q = \bar{h}\pi d_i L(T_w - \bar{T}_b)$$

$2.627\times10^{-5} = (4033)\pi(0.04)L(80-30) \qquad L = 10.37$ m

6-24

At 300 K and 1 atm. $\nu = 15.69\times10^{-6} \quad \rho = 1.1774 \quad k = 0.02624$

$$Pr = 0.708 \qquad D_H = \frac{(4)(45)(90)}{(2)(45+90)} = 60\ \text{cm} = 0.6\ \text{m}$$

$$Re = \frac{(0.6)(7.5)}{15.69\times10^{-6}} = 2.87\times10^5 \qquad \bar{h} = \frac{0.02624}{0.6}(0.023)(2.87\times10^5)^{0.8}(0.708)^{0.4}$$

$$\bar{h} = 20.35\ \frac{W}{m^2\cdot °C} \qquad \Delta p = f\frac{L}{d}\rho\frac{u_m^2}{2} \qquad f = 0.0145$$

$$\Delta p = (0.0145)\left(\frac{1}{0.6}\right)\frac{(1.1774)(7.5)^2}{2} = 0.8\ \text{Pa}$$

6-25

Assume $\overline{T}_b$ about $38°\text{C} = 100°\text{F}$ $\quad \rho = 993 \quad c_p = 4.174$

$\mu = 6.82\times10^{-4} \quad k = 0.63 \quad \Pr_f = 2.9$

$$\text{Re} = \frac{\rho u_m d}{\mu} = 10^5 = \frac{(993)u(0.03)}{6.82\times10^{-4}} \qquad u = 2.29 \text{ m/sec} \qquad \rho u = 2274$$

$$f = 0.0255 \qquad \text{St}_b \Pr_f^{2/3} = \frac{f}{8}$$

$$h = \frac{0.0255}{8}(2274)(4174)(2.9)^{-2/3} = 14{,}878 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

6-26

$\overline{T}_b = 10°\text{C} \quad T_w = 40°\text{C} \quad c_p = 0.9345 \quad$ at $0°\text{C} \quad \nu = 0.214\times10^{-6}$

$$\rho = 1397 \qquad 700 = \frac{u(0.0035)}{0.214\times10^{-6}} \qquad u = 0.0428 \text{ m/sec}$$

$$q = (1397)(934.5)(0.0428)\frac{\pi}{4}(0.0035)^2(20-0) = 10.75 \text{ W}$$

At $10°\text{C} \quad k = 0.073 \quad \Pr = 3.6$

$$q = hA(T_w - \overline{T}_b) = h\pi(0.0035)L(40-10) = 10.75$$

$$\bar{h}L = 32.59 \qquad \text{Gz} = \text{Re}\Pr\frac{d}{L} = (700)(3.6)\frac{d}{L} = 2520\frac{d}{L}$$

$$\overline{\text{Nu}}_d = \frac{\bar{h}L}{k}\times\frac{d}{L} = \left(\frac{32.59}{0.073}\right)\frac{d}{L} = 446.4\frac{d}{L}$$

From Figure:

$\frac{1}{\text{Gz}}$	$\frac{d}{L}$	$\overline{\text{Nu}}_d$	$446.4\frac{d}{L}$
0.1	0.00397	4.3	1.771
0.01	0.0397	7.5	17.72
0.03	0.0132	5.4	5.89
0.035	0.0134	5.2	5.06

$$\frac{d}{L} \approx 0.035 \qquad L = \frac{0.0035}{0.035} = 0.1 \text{ m}$$

6-27

$\overline{T_b} = \dfrac{27+77}{2} = 52°\text{C} = 325\ \text{K}$ $\qquad \mu = 1.96 \times 10^{-5}$ $\qquad k = 0.0282$

$\text{Pr} = 0.7 \qquad c_p = 1.007 \qquad L = 30\ \text{cm} \qquad \rho = 1.088$

$$D_H = \frac{(4)(0.003)^2/2}{(3)(0.003)} = 0.002 \qquad \text{Re} = \frac{(0.002)(5\times10^{-5})(2)}{(0.003)^2(1.96\times10^{-5})} = 1134$$

$$q = \dot{m}c_p\Delta T_b = (5\times10^{-5})(1007)(77-27) = 2.518\ \text{W}$$

$$\text{Re}\,\text{Pr}\frac{d}{L} = (1134)(0.7)\left(\frac{0.002}{0.3}\right) = 5.292$$

From Figure for $\dfrac{1}{\text{Gz}} = \dfrac{1}{5.292} = 0.189$

$\overline{\text{Nu}}_d \cong 4.0$ and flow is nearly developed.

Assuming ratio for Nu_T from table

$$\overline{\text{Nu}}_{D_H} = (2.47)\frac{4.0}{3.66} = 2.7$$

$$h = \frac{(2.7)(0.0282)}{0.002} = 38.1\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - \overline{T}_b) \qquad T_w - \overline{T}_b = \frac{2.518}{(38.1)(3)(0.3)(0.003)} = 24.5°\text{C}$$

$$T_w = 24.5 + 52 = 76.5°\text{C}$$

6-28

At $27°\text{C} = 300\ \text{K}$ $\qquad \mu = 1.8462\times10^{-5}$ $\qquad k = 0.02624$ $\qquad \text{Pr} = 0.708$

$$\rho = \frac{9000}{(287)(300)} = 1.045\ \text{kg/m}^3 \qquad \text{Re} = \frac{(0.004)(7\times10^{-5})(4)}{\pi(0.004)^2(1.8462\times10^{-5})} = 1207$$

$$\text{Gz} = (1207)(0.708)\left(\frac{0.004}{0.12}\right) = 28.48 > 10$$

At $T_w = 27 + 70 = 97°\text{C} = 390\ \text{K} \approx 400\ \text{K}$ $\qquad \mu_w = 2.286\times10^{-5}$

$$h = \frac{k}{d}(1.86)\text{Gz}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} = \frac{0.02624}{0.004}(1.86)(28.48)^{1/3}\left(\frac{1.8462}{2.286}\right)^{0.14}$$

$$= 36.16\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = hA(T_w - T_b) = \dot{m}c_p\Delta T_b$$

$$\Delta T_b = \frac{(36.16)\pi(0.004)(0.12)(70)}{(7\times10^{-5})(1007)} = 54.15°\text{C}$$

$$T_{b_e} = 54.15 + 27 = 81.15°\text{C}$$

6-29

At $40°C = 313\text{ K}$ $\quad \mu = 1.906\times10^{-5}$ $\quad k = 0.0272$ $\quad \Pr = 0.7$

$$\rho = \frac{110\times10^3}{(287)(313)} = 1.225\ \text{kg/m}^3 \qquad \text{Re} = \frac{(0.006)(8\times10^{-5})(4)}{\pi(0.006)^2(1.906\times10^{-5})} = 891$$

$$\text{Gz} = (891)(0.7)\left(\frac{0.006}{0.14}\right) = 26.73 \qquad \frac{1}{\text{Gz}} = 0.0374 \qquad \overline{\text{Nu}}_d = 5.2$$

$$h = \frac{(5.2)(0.0272)}{0.006} = 23.57\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - \overline{T_b}) = \dot{m}c_p(T_{b_e} - T_{b_i})$$

$$(23.57)\pi(0.006)(0.14)\left(140 - 20 - \frac{T_{b_e}}{2}\right) = (8\times10^{-5})(1005)(T_{b_e} - 40)$$

$$T_{b_e} = 95.8°\text{C}$$

6-30

At $40°C$ $\quad \rho = 876\ \text{kg/m}^3$ $\quad c_p = 1.964\ \frac{\text{kJ}}{\text{kg}\cdot°\text{C}}$ $\quad \nu = 0.00024$

$k = 0.144$ $\quad \Pr = 2870$ $\quad \text{Re} = 50 = \frac{u(0.01)}{0.00024}$ $\quad u = 1.2\ \text{m/s}$

$$\dot{m} = \frac{(876)(1.2)\pi(0.01)^2}{4} = 0.0826\ \text{kg/sec}$$

$$\text{Gz} = (50)(2870)\left(\frac{0.01}{0.08}\right) = 1.794\times10^4$$

at $T_w = 80°C$ $\quad \nu_w = 0.375\times10^{-4}$

$$\text{Nu}_d = (1.86)(1.794\times10^4)^{1/3}\left(\frac{2.4}{0.375}\right)^{0.15} = 64.3$$

$$h = \frac{(64.3)(0.144)}{0.01} = 926$$

$$q = (926)\pi(0.01)(0.08)\left(80 - 20 - \frac{T_e}{2}\right) = (0.0826)(1964)(T_e - 40)$$

$$T_e = 40.57°\text{C}$$

6-31

$\overline{T_b} = 25°\text{C} = 77°\text{F} \qquad \rho = 996 \qquad \mu = 8.96 \times 10^{-4} \qquad k = 0.611$

$\text{Pr} = 6.13 \qquad c_p = 4180 \qquad \text{Re} = \dfrac{(996)(8)(0.02)}{8.96 \times 10^{-4}} = 1.78 \times 10^5$

$h = (0.023)\dfrac{0.611}{0.02}(1.78 \times 10^5)^{0.8}(6.13)^{0.4} = 23{,}003 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q = hA(\overline{T_w} - \overline{T_b}) = \dot{m}c_p\Delta T_b$

$(23{,}003)\pi(0.02)(10)(\overline{T_w} - 25) = (996)(8)\pi\dfrac{(0.02)^2}{4}(4180)(10)$

$T_w = 32.2°\text{C}$

6-32

$\text{At } 20°\text{C} \qquad \rho = 888 \qquad c_p = 1880 \ \dfrac{\text{J}}{\text{kg} \cdot °\text{C}} \qquad \nu = 0.0009 \qquad k = 0.145$

$\text{Pr} = 10{,}400 \qquad \text{Re} = \dfrac{(1.2)(0.002)}{0.0009} = 2.67$

$\text{Gz} = (2.666)(10{,}400)\left(\dfrac{0.002}{1}\right) = 55.47 > 10 \qquad \dfrac{1}{\text{Gz}} = 0.018$

$\overline{\text{Nu}}_d = 6.1 \qquad h = \dfrac{(6.1)(0.145)}{0.002} = 442$

$q = (442)\pi(0.002)(1.0)\left(60 - 10 - \dfrac{T_e}{2}\right) = (888)(1.2)\dfrac{\pi}{4}(0.002)^2(1880)(T_e - 20)$

$T_e = 34.46°\text{C}$

6-33

$\overline{T}_b = 40°\text{C} = 104°\text{F} \qquad \rho = 993 \qquad \mu = 6.55 \times 10^{-4} \qquad k = 0.633$

$\text{Pr} = 4.33 \qquad c_p = 4175 \qquad \text{Re} = \dfrac{(993)(1)(0.025)(3)}{6.55 \times 10^{-4}} = 1.14 \times 10^5$

$h = \dfrac{0.633}{0.025}(0.023)(1.14 \times 10^5)^{0.8}(4.33)^{0.4} = 11{,}600 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q = \dot{m}c_p\Delta T_b = hA(T_w - T_b)$

$(993)\pi\dfrac{(0.025)^2}{4}(3.0)(4175)(50 - 30) = (11{,}600)\pi(0.025)L(20)$

$L = 6.7 \text{ m}$

6-34

Liquid NH_3 $T_\infty = 10°C$ $T_w = 30°C$ $u_\infty = 5$ m/s $d = 0.025$

$L = 1.25$ $T_f = 20°C$ $\nu = 0.359 \times 10^{-6}$ $k = 0.521$ $\Pr = 2.02$

$$\mathrm{Re}_d = \frac{(5)(0.025)}{0.359 \times 10^{-6}} = 3.88 \times 10^5 \qquad C = 0.0266 \qquad n = 0.805$$

$$\bar{h} = \frac{0.521}{0.025}(0.0266)(3.48 \times 10^5)^{0.805}(2.02)^{1/3} = 20{,}653\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot °\mathrm{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (20{,}653)\pi(0.025)(1.25)(30-10) = 39{,}788\ \mathrm{W}$$

6-35

$$\bar{T}_b = \frac{21+32}{2} = 26.5°C \qquad \rho = 996 \qquad \mu = 8.6 \times 10^{-4}$$

$$\mathrm{Re} = 600 = \frac{\rho u d}{\mu} \qquad u = \frac{(600)(8.6 \times 10^{-4})}{(996)(0.003)} = 0.173\ \mathrm{m/s}$$

$$\dot{m} = \rho A u = (996)\pi\frac{(0.003)^2}{4}(0.173) = 0.00122\ \mathrm{kg/s}$$

6-36

$\bar{T}_b = 26.5°C$ $\rho = 996$ $\mu = 8.6 \times 10^{-4}$ $k = 0.614$

$$\Pr = 5.85 \qquad c_p = 4180 \qquad \mathrm{Re} = \frac{(0.03)(1.0)(4)}{\pi(0.03)^2(8.6 \times 10^{-4})} = 49{,}350$$

$$h = \frac{(0.023)(0.614)}{0.03}(49{,}350)^{0.8}(5.85)^{0.4} = 5423\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot °\mathrm{C}}$$

$$q = \dot{m}c_p\Delta T_b = hA(T_w - \bar{T}_b)$$

$$(1.0)(4180)(38-15) = (5423)\pi(0.03)L(60-26.5)$$

$$L = 5.61\ \mathrm{m}$$

6-37

At $\overline{T}_b = 20°\text{C}$ $\rho = 1264$ $c_p = 2386$ $k = 0.286$

$\Pr = 12.5 \times 10^3$ $\nu = 0.003$ $\text{Re} = 10 = \dfrac{ud}{\nu}$

$$u = \frac{(10)(0.003)}{0.005} = 6 \text{ m/sec}$$

$$\dot{m} = \rho A u = (1264)\pi \frac{(0.005)^2}{4}(6) = 0.149 \text{ kg/sec}$$

$$q = \dot{m} c_p \Delta T_b = (0.149)(2386)(30 - 10) = 7106 \text{ W}$$

$$= hA(T_w - \overline{T}_b) = h\pi(0.005)L(40 - 20)$$

$hL = 22{,}619$ $h = \dfrac{22{,}619}{L}$

$$\frac{hd}{k} = 1.86\left(\text{Re}\Pr\frac{d}{L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

At $T_w = 40°\text{C}$ $\nu_w = 0.00022$

$$\frac{(22{,}619)(0.005)}{0.286L} = 1.86\left[\frac{(10)(12{,}500)(0.005)}{L}\right]^{1/3}\left(\frac{0.003}{0.00022}\right)^{0.14}$$

$L^{2/3} = 17.25$ $L = 71.66 \text{ m}$

$$\text{Gz} = (10)(12{,}500)\left(\frac{0.005}{71.66}\right) = 8.72 < 10$$

Below range of equations: $\dfrac{1}{\text{Gz}} \approx \dfrac{1}{8.72} = 0.115$

From Figure: $\overline{\text{Nu}}_d \approx 4.2$

$h = \dfrac{(4.2)(0.286)}{0.005} = 240.2$ $L = \dfrac{22{,}619}{240.2} = 94.2 \text{ m}$

New $\dfrac{1}{\text{Gz}} = (0.115)\left(\dfrac{94.2}{71.66}\right) = 0.15$

New iteration: $\overline{\text{Nu}}_d \approx 4.0$ $h = 228.8$ $L = 98.8 \text{ m}$

Close enough to fully developed tube, could take $\overline{\text{Nu}}_d = 3.66$.

6-38

$$T_f = \frac{100+10}{2} = 55°\text{C} = 328\ \text{K} \qquad \mu = 19\times10^{-6} \qquad k = 0.0282$$

$$\text{Pr} = 0.7 \qquad \rho = \frac{(2)(1.01\times10^5)}{(297)(328)} = 2.08\ \text{kg/m}^3$$

$$\text{Re} = \frac{(2.08)(5)(0.05)}{19.0\times10^{-6}} = 27{,}377 \qquad C = 0.193 \qquad n = 0.618$$

$$h = \frac{k}{d} C\,\text{Re}^n\,\text{Pr}^{1/3} = \frac{0.0282}{0.05}(0.193)(27{,}377)^{0.618}(0.7)^{1/3} = 53.4\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (53.4)\pi(0.05)(100-10) = 755\ \text{W/m}$$

6-39

$$T_f = \frac{54+0}{2} = 27°\text{C} = 300\ \text{K} \qquad \nu = 15.69\times10^{-6} \qquad k = 0.02624$$

$$\text{Pr} = 0.708 \qquad \text{Re} = \frac{\rho u d}{\mu} = \frac{(25)(0.04)}{15.69\times10^{-6}} = 63{,}735 \qquad C = 0.0266$$

$n = 0.805$

$$h = \frac{k}{d} C\,\text{Re}^n\,\text{Pr}^{1/3} = \frac{0.02624}{0.04}(0.0266)(63{,}735)^{0.805}(0.708)^{1/3} = 114.6\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (114.6)\pi(0.04)(54-0) = 778\ \text{W/m}$$

6-40

$$T_f = \frac{80+10}{2} = 45°\text{C} = 318\ \text{K} \qquad \mu = 1.929\times10^{-5} \qquad k = 0.0276$$

$$\text{Pr} = 0.7 \qquad \rho = \frac{200\times10^3}{(287)(318)} = 2.191\ \text{kg/m}^3$$

$$\text{Re} = \frac{\rho u d}{\mu} = \frac{(2.191)(20)(0.2)}{1.929\times10^{-5}} = 5.68\times10^5$$

Churchill Equation:

$$\text{Nu}_d = 0.3 + \frac{(0.62)(5.68\times10^5)^{1/2}(0.7)^{1/3}}{\left[1+\left(\frac{0.4}{0.7}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{5.68\times10^5}{282{,}000}\right)^{5/8}\right]^{4/5} = 769.7$$

$$h = \frac{k}{d}\text{Nu} = \frac{(0.0276)(769.7)}{0.2} = 106.2\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (106.2)\pi(0.2)(80-10) = 4672\ \text{W/m}$$

From Figure $\quad C_D = 0.3$

$$F_D = C_D A \frac{\rho u_\infty^2}{2g_c} = \frac{(0.3)(0.2)(2.191)(25)^2}{2} = 41.1 \text{ N/m Length}$$

6-41

$$T_f = \frac{71+43}{2} = 57°\text{C} = 135°\text{F} \qquad \Pr_f = 3.15 \qquad \rho_b = 991 \qquad c = 4174$$

$$\mu = 6.16\times10^{-4} \qquad \text{Re} = \frac{(0.05)(6)(4)}{\pi(0.05)^2(6.16\times10^{-4})} = 2.48\times10^5 \qquad f = 0.014$$

$$\dot{m} = \rho A u_m \qquad u_m = \frac{(6)(4)}{(991)\pi(0.05)^2} = 3.08 \text{ m/sec} \qquad \text{St}_b \Pr_f^{2/3} = \frac{f}{8}$$

$$h = \frac{(0.014)(991)(3.08)(4174)}{(8)(3.15)^{2/3}} = 10{,}388 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = hA(T_w - \overline{T}_b) = \dot{m}c_p\Delta T_b$$

$$(10{,}388)\pi(0.05)(9)\left(71 - 21.5 - \frac{T_e}{2}\right) = (6)(4174)(T_e - 43)$$

$$T_e = 55.7°\text{C}$$

6-42

$$\text{At } 38°\text{C} \qquad \rho = 993 \qquad \mu = 6.82\times10^{-4} \qquad k = 0.63 \qquad \Pr = 4.53$$

$$c_p = 4180 \qquad \text{Re} = \frac{(993)(1.5)(0.0064)}{6.82\times10^{-4}} = 13{,}978$$

$$h = \frac{0.63}{0.0064}(0.036)(13{,}978)^{0.8}(453)^{1/3}\left(\frac{0.0064}{0.15}\right)^{0.055} = 10{,}213 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (10{,}213)\pi(0.0064)(0.15)(28) = (993)\frac{\pi}{4}(0.0064)^2(1.5)(4180)(T_e - 38)$$

$$= 862.5 \text{ W}$$

$$T_e = 42.31°\text{C}$$

6-43

$\rho = 1094 \qquad c_p = 2518 \qquad k = 0.258 \qquad \Pr = 72 \qquad \nu = 6.72 \times 10^{-6}$

$$\dot{m} = \frac{(1094)(10)\pi(0.03)^2}{4} = 7.733 \text{ kg/sec}$$

$$q = (7.733)(2518)(65 - 40) = 4.867 \times 10^5 \text{ W}$$

$$\text{Re} = \frac{(10)(0.03)}{6.72 \times 10^{-6}} = 4.464 \times 10^4 \qquad \nu_w = 19.18 \times 10^{-6}$$

$$\bar{h} = \frac{0.258}{0.03}(0.027)(4.464 \times 10^4)^{0.8}(72)^{1/3}\left(\frac{6.72}{19.18}\right)^{0.14} = 4375 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$(4375)\pi(0.03)L(52.5 - 20) = 4.867 \times 10^5 \qquad L = 36.3 \text{ m}$$

6-44

$$T = 20^\circ\text{C} = 293 \text{ K} \qquad \mu = 1.91 \times 10^{-5} \qquad \rho = \frac{7000}{(287)(293)} = 0.832$$

$$\text{Re} = \frac{(0.832)(15)(0.05)}{1.91 \times 10^{-5}} = 32{,}687 \qquad C_D = 1.2$$

$$F_D = (1.2)(0.05)(0.832)\frac{(15)^2}{(2)(1.0)} = 5.62 \text{ N/m length}$$

6-45

$$T_f = \frac{450 + 325}{2} = 387.5 \text{ K} \qquad \nu = 24.62 \times 10^{-6} \qquad k = 0.0327$$

$$\Pr = 0.69 \qquad \text{Re} = \frac{(30)(0.025)}{24.62 \times 10^{-6}} = 30{,}463 \qquad C = 0.193 \qquad n = 0.618$$

$$\bar{h} = \frac{0.0327}{0.025}(0.193)(30{,}463)^{0.618}(0.69)^{1/3} = 131.6 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$\frac{q}{L} = \bar{h}\pi d(T_w - T_\infty) = (131.6)\pi(0.025)(450 - 325) = 1293 \text{ W/m}$$

6-46

$$T_w = 24^\circ\text{C} \qquad T_\infty = 0^\circ\text{C} \qquad u_\infty = 30 \text{ mi/h} = 13.4 \text{ m/s} \qquad T_f = 285 \text{ K}$$

$$d = 0.3 \text{ m} \qquad L = 1.8 \text{ m} \qquad \nu = 14.3 \times 10^{-6} \text{ m}^2/\text{s} \qquad k = 0.025 \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}}$$

$$\Pr = 0.71$$

$$\text{Re}_d = \frac{(13.4)(0.3)}{14.3 \times 10^{-6}} = 2.82 \times 10^5 \qquad C = 0.0266 \qquad n = 0.805$$

$$\bar{h} = \frac{k}{d} C \text{Re}^n \Pr^{1/3} = \frac{0.025}{0.3}(0.0266)(282{,}000)^{0.805}(0.71)^{1/3} = 48.3 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (48.3)\pi(0.3)(1.8)(24 - 0) = 1967 \text{ W}$$

6-48

$\overline{T}_b = 21.11°\text{C}$ $\rho = 997$ $c_p = 4179$ $k = 0.604$ $\text{Pr} = 6.78$

$\mu = 9.8 \times 10^{-4}$

$$\text{Re}_d = \frac{(997)(0.3)(0.002)}{9.8 \times 10^{-4}} = 610 < 2300$$

Try fully developed $\text{Nu} = 3.66$ $\quad \overline{h} = \frac{(3.66)(0.604)}{0.002} = 1105$

$$q = \dot{m} c_p \Delta T_b = (997)(0.3)\pi(0.001)^2(4179)(26.67 - 15.56) = 43.6 \text{ W}$$

$$= \overline{h}\pi d L(T_w - T_b)$$

$$43.6 = (1105)\pi(0.002)L(48.99 - 21.11) \;\rightarrow\; L = 0.225 \text{ m}$$

$$\text{Gz}^{-1} = \frac{0.225}{(610)(6.78)(0.002)} = 0.0272$$

Fig. 6–5, $\overline{\text{Nu}} \sim 5.8 \rightarrow \overline{h} = 1751 \rightarrow L = 0.142$

$\text{Gz}^{-1} = 0.017 \rightarrow \overline{\text{Nu}} = 6.3$

$$L = \frac{3.66}{6.3}(0.225) = 0.131 \;\rightarrow\; \text{Gz}^{-1} = 0.16$$

$L \approx 0.13$ m

6-50

$$T_f = \frac{175 + (-30)}{2} = 72.5°\text{C} = 345.5 \text{ K} \qquad \rho = \frac{54{,}000}{(287)(345.5)} = 0.544$$

$\mu = 2.07 \times 10^{-5}$ $k = 0.03$ $\text{Pr} = 0.7$

$$\text{Re} = \frac{(0.544)(0.00013)(230)}{2.07 \times 10^{-5}} = 786.6$$

$$\overline{h} = \frac{0.03}{0.00013}(0.683)(786.6)^{0.466}(0.7)^{1/3} = 3129 \; \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = (3129)\pi(0.00013)(0.0125)(175 + 30) = 3.275 \text{ W}$$

6-51

$$T_f = \frac{90 + 150}{2} = 120°\text{C} = 393 \text{ K} \qquad \rho = 0.899 \qquad \mu = 2.256 \times 10^{-5}$$

$k = 0.0331$ $\quad \text{Pr} = 0.69$ $\quad \text{Re} = \frac{(0.899)(0.0015)(6)}{2.256 \times 10^{-5}} = 359$

$$\overline{h} = \frac{0.0331}{0.0015}(0.683)(359)^{0.466}(0.69)^{1/3} = 206.5 \; \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\frac{q}{L} = (200.3)\pi(0.0015)(150 - 90) = 58.4 \text{ W/m}$$

6-52

$d = 0.025$ mm $\quad L = 0.15$ m $\quad \rho_e = 70\times10^{-6}\ \Omega\cdot\text{cm}$

$a = 0.006°\text{C}^{-1} \quad R = R_0[1+a(T-T_0)] \quad R = \dfrac{E}{I}$

$u_\infty = 10$ m/s $\quad T_\infty = 20°\text{C} \quad T_w = 40°\text{C}$

$$R_0 = \frac{\rho_e L}{A} = \frac{(70\times10^{-6})(15)}{\pi(0.0125\times10^{-3})^2} = 2.14\times10^6\ \Omega$$

At $T = 40°\text{C}$ $\quad R = (2.14\times10^6)[1+0.006(40-20)] = 2.4\times10^6\ \Omega$

At $T_f = 30°\text{C}$ $\quad \nu = 15.7\times10^{-6} \quad k = 0.027 \quad \text{Pr} = 0.7$

$$\text{Re} = \frac{(10)(0.025\times10^{-3})}{15.7\times10^{-6}} = 15.92$$

$\text{Nu} = C\,\text{Re}^n\,\text{Pr}^{1/3} \quad C = 0.911 \quad n = 0.385$

$$\bar{h} = \frac{0.027}{0.025\times10^{-3}}(0.911)(15.92)^{0.385}(0.7)^{1/3} = 2536\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = I^2R$$

$$I = \left[\frac{(2536)\pi(0.025\times10^{-3})(0.15)(40-20)}{2.4\times10^6}\right]^{1/2} = 4.99\times10^{-4}\ \text{Amp}$$

$$E = IR = (4.99\times10^{-4})(2.4\times10^6) = 1197\ \text{Volts}$$

6-53

$T_f = \dfrac{425+325}{2} = 375$ K $\quad \nu = 181.4\times10^{-6} \quad k = 0.192 \quad \text{Pr} = 0.71$

$\text{Re} = \dfrac{(0.003)(9)}{181.4\times10^{-6}} = 148.8 \quad C = 0.683 \quad n = 0.618$

$$\bar{h} = \frac{0.192}{0.003}(0.683)(148.8)^{0.618}(0.71)^{1/3} = 85.86\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = \bar{h}\pi d(T_w - T_\infty) = (85.86)\pi(0.003)(425-325) = 809\ \text{W/m}$$

6-54

$$T_f = \frac{65+20}{2} = 42.5°\text{C} = 315.5\text{ K}$$

Air: $\rho = \dfrac{1.0132\times10^5}{(287)(315.5)} = 1.119$ $\quad \mu = 2.012\times10^{-5}$ $\quad k = 0.0274$

$\text{Pr} = 0.7 \quad C = 0.911 \quad n = 0.385$

$$\text{Re} = \frac{(1.119)(6)(0.025\times10^{-3})}{2.012\times10^{-5}} = 8.342$$

$$h = \frac{0.0274}{0.025\times10^{-3}}(0.911)(8.342)^{0.385}(0.7)^{1/3} = 2006 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (2006)\pi(0.025\times10^{-3})(65-20) = 7.091 \ \text{W/m}$$

Water: $\rho = 991$ $\quad \mu = 6.2\times10^{-4}$ $\quad k = 0.635$

$\text{Pr} = 4.1 \quad C = 0.683 \quad n = 0.466$

$$\text{Re} = \frac{(991)(6)(0.025\times10^{-3})}{6.2\times10^{-4}} = 239.8$$

$$h = \frac{0.635}{0.025\times10^{-3}}(0.683)(239.8)^{0.466}(4.1)^{1/3} = 3.57\times10^5 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (3.57\times10^5)\pi(0.025\times10^{-3})(65-20) = 1261 \ \text{W/m}$$

6-57

At 90°C $\quad \text{Pr} = 1.978$

Eq. (6–17) $\quad \text{Nu} = C\,\text{Re}^n\,\text{Pr}^{1/3} \quad \text{Pr}^{1/3} = 1.255$

Eq. (6–18) $\quad \text{Nu} = (0.35 + 0.56\,\text{Re}^{0.52})\text{Pr}^{0.3} \quad \text{Pr}^{0.3} = 1.227$

Re	C	n	Nu [Eq. (6–17)]	Nu [Eq. (6–18)]
10^3	0.683	0.466	21.44	25.38
10^4	0.193	0.618	71.81	83.04
10^5	0.0266	0.805	353.6	274.0

6-56

$$T_f = \frac{50-35}{2} = 7.5°\text{C} = 280.5 \text{ K} \qquad \text{Pr} = 0.71 \qquad \mu = 1.79\times10^{-5}$$

$$\rho = \frac{1.0132\times10^5}{(287)(280.5)} = 1.259 \qquad k = 0.0247 \qquad \text{Re} = \frac{(1.259)(13)(0.5)}{1.79\times10^{-5}} = 457{,}000$$

$$C = 0.266 \qquad n = 0.805$$

$$h = \frac{0.0247}{0.5}(0.0266)(457{,}000)^{0.805}(0.71)^{1/3} = 42.21 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (42.21)\pi(0.5)(50+35) = 5636 \ \text{W/m}$$

6-57

$$T_f = \frac{50+27}{2} = 38.5°\text{C} = 311.5 \text{ K} \qquad \nu = 17.74\times10^{-6}$$

$$k = 0.02711 \qquad \text{Pr} = 0.7 \qquad \text{Re} = \frac{(20)(0.04)}{17.74\times10^{-6}} = 4.51\times10^4$$

circ. tube: $h = \frac{0.02711}{0.04}(0.0266)(4.51\times10^4)^{0.805}(0.7)^{1/3} = 89.32 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

sq. tube: $h = \frac{0.02711}{0.04}(0.102)(4.51\times10^4)^{0.675}(0.7)^{1/3} = 85.04 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$$\frac{q}{L}(\text{circ.}) = (89.32)\pi(0.04)(50-27) = 258.1 \ \text{W/m}$$

$$\frac{q}{L}(\text{sq.}) = (85.04)(4)(0.04)(50-27) = 312.9 \ \text{W/m}$$

6-58

$$T_f = \frac{200+50}{2} = 125°\text{C} = 398 \text{ K} \qquad \nu = 14.35\times10^{-6}$$

$$k = 0.0246 \qquad \text{Pr} = 0.74 \qquad \text{Re} = \frac{u_\infty d}{\nu} = \frac{(40)(0.03)}{14.35\times10^{-6}} = 83{,}624$$

$$C = 0.0266 \qquad n = 0.805$$

$$h = \frac{0.0246}{0.03}(0.0266)(83{,}624)^{0.805}(0.74)^{1/3} = 181 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (181)\pi(0.03)(200-50) = 814 \ \text{W/m}$$

Compare Churchill Equation

$$\mathrm{Nu}_d = 0.3 + \frac{(0.62)(83{,}624)^{1/2}(0.74)^{1/3}}{\left[1+\left(\frac{0.4}{0.74}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{83{,}624}{282{,}000}\right)^{1/2}\right] = 220.8$$

$$h = \frac{(220.8)(0.0246)}{0.03} = 181 \ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot {}^\circ\mathrm{C}}$$

Very close check. No need to use more complicated relation.

6-59

$\mu = 6.82\times10^{-4}$ $\quad k = 0.63$ $\quad \mathrm{Pr} = 4.53$ $\quad c_p = 4174$ $\quad \overline{T}_b = 37.8^\circ\mathrm{C}$

$$\mathrm{Re}_d = 100{,}000 = \frac{\rho u_m(0.0125)}{6.82\times10^{-4}} \qquad \rho u_m = 5456$$

$$\dot{m} = \rho u_m A = \frac{(5456)\pi(0.0125)^2}{4} = 0.67 \ \mathrm{kg/sec}$$

$$\dot{m}c_p\Delta T_b = hA(T_w - \overline{T}_b)$$

$$\bar{h} = \frac{0.63}{0.0125}(0.023)(100{,}000)^{0.8}(4.53)^{0.4} = 21{,}200$$

$$(0.67)(4174)(\Delta T_b) = (21{,}200)\pi(0.0125)(1.5)(160-100)\left(\frac{5}{9}\right)$$

$$\Delta T_b = 14.89^\circ\mathrm{C}$$

$$T_b(\mathrm{exit}) = 37.8 + \left(\frac{1}{2}\right)(14.89) = 45.25^\circ\mathrm{C}$$

6-61

$T_f = \dfrac{100+20}{2} = 60^\circ\mathrm{C} = 333\ \mathrm{K}$ $\quad \mu = 216\times10^{-7}$ $\quad k = 0.159$

$\mathrm{Pr} = 0.7$ $\quad \rho = \dfrac{150\times10^3}{(2078)(333)} = 0.217$ $\quad \mathrm{Re} = \dfrac{(0.217)(50)(0.3)}{216\times10^{-7}} = 1.5\times10^5$

$C = 0.0266$ $\quad n = 0.805$

$$h = \frac{0.159}{0.3}(0.0266)(1.5\times10^5)^{0.805}(0.7)^{1/3} = 184.3 \ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$q = h\pi dL(T_w - T_\infty) = (184.2)\pi(0.3)(6)(100-20) = 83{,}360 \ \mathrm{W}$$

Churchill Equation

$$\mathrm{Nu}_d = 0.3 + \frac{(0.62)(1.5\times10^5)^{1/2}(0.7)^{1/3}}{\left[1+\left(\frac{0.4}{0.7}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{1.5\times10^5}{282{,}000}\right)^{5/8}\right]^{4/5} = 323.3$$

$$h = \frac{(323.3)(0.159)}{0.3} = 171.4 \ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}} \ \text{or about 7\% less.}$$

6-62

$$T_f = \frac{300+30}{2} = 115°\text{C} = 388\ \text{K} \qquad \nu = 13.62\times10^{-6} \qquad k = 0.0236$$

$$\text{Pr} = 0.742 \qquad \text{Re} = \frac{(0.25)(0.0254)(35)}{13.62\times10^{-6}} = 16{,}318 \qquad C = 0.193 \qquad n = 0.618$$

$$h = \frac{0.0236}{(0.25)(0.0254)}(0.193)(16{,}318)^{0.618}(0.742)^{1/3} = 260.6\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = h\pi dL(T_w - T_\infty) = (260.5)\pi(0.0254)(0.25)(4.5)(300-30) = 6316\ \text{W}$$

6-63

$$T_f = \frac{300+400}{2} = 350\ \text{K} \qquad \nu = 11.19\times10^{-6} \qquad k = 0.02047$$

$$\text{Pr} = 0.755 \qquad \text{Re} = \frac{(50)(0.2)}{11.19\times10^{-6}} = 8.94\times10^{-5}$$

Churchill Equation:

$$\text{Nu}_d = 0.3 + \frac{(0.62)(8.94\times10^{-5})^{1/2}(0.755)^{1/3}}{\left[1+\left(\frac{0.4}{0.755}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{8.94\times10^5}{282{,}000}\right)^{5/8}\right]^{4/5} = 1151$$

$$h = \frac{(1151)(0.0247)}{0.2} = 117.8\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (117.8)\pi(0.2)(400-300) = 7401\ \text{W/m}$$

6-64

$$T_f = \frac{20+85}{2} = 52.5°\text{C} = 325.5\ \text{K} \qquad \mu = 1.96\times10^{-5} \qquad k = 0.0281$$

$$\text{Pr} = 0.7 \qquad \rho = \frac{(0.6)(1.01\times10^5)}{(287)(325.5)} = 0.651\ \text{kg/m}^3 \qquad C = 0.102$$

$$n = 0.675 \qquad \text{Re} = \frac{(0.651)(12)(0.04)}{1.96\times10^{-5}} = 15{,}937$$

$$h = \frac{0.0281}{0.04}(0.102)(15{,}937)^{0.675}(0.7)^{1/3} = 43.68\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (43.68)\pi(0.04)(85-20) = 356.7\ \text{W/m}$$

6-65

At 38°C $\rho = 993$ $\mu = 6.82 \times 10^{-4}$ $k = 0.63$ $\Pr = 4.53$

At 93°C $\mu = 3.06 \times 10^{-4}$ $\mathrm{Re}_\infty = \dfrac{(993)(0.003)(5)}{6.82 \times 10^{-4}} = 21{,}840$

Eq. 6–20 $h = \dfrac{0.63}{0.003}[1.2 + 0.53(21{,}840)^{0.54}]\left(\dfrac{6.82}{3.06}\right)^{0.25}(4.53)^{0.3}$

$h = 47{,}640 \ \dfrac{\mathrm{W}}{\mathrm{m}^2 \cdot {}^\circ\mathrm{C}}$

$q = (47{,}640)4\pi(0.0015)^2(93 - 38) = 74.08 \ \mathrm{W}$

6-66

$T_\infty = 293 \ \mathrm{K}$ $\nu = 15.96$ $k = 0.026$ $\Pr = 0.71$ $T_w = 313 \ \mathrm{K}$

$\nu = 17.86$ $\mathrm{Re} = \dfrac{(6)(4)}{15.96 \times 10^{-6}} = 1.5 \times 10^6$

$$h = \frac{0.026}{4.0}\left\{2 + [(0.4)(1.5 \times 10^6)^{1/2} + (0.06)(1.5 \times 10^6)^{2/3}](0.71)^{0.4}\left(\frac{15.96}{17.86}\right)^{1/4}\right\}$$

$= 7.045 \ \dfrac{\mathrm{W}}{\mathrm{m}^2 \cdot {}^\circ\mathrm{C}}$

$q = (7.045)4\pi(2)^2(40 - 20) = 7082 \ \mathrm{W}$

6-68

At $T_\infty = 20°\mathrm{C}$ $\mu = 9.75 \times 10^{-4}$ $k = 0.6$ $\Pr = 6.7$

$\rho = 997$

At $T_w = 90°\mathrm{C}$ $\mu_w = 3.2 \times 10^{-4}$

$$\mathrm{Nu}\,\Pr^{-0.3}\left(\frac{\mu_w}{\mu}\right)^{0.25} = 1.2 + 0.53\,\mathrm{Re}_d^{\,0.54}$$

$$\mathrm{Re} = \frac{(997)(3.5)(0.03)}{9.75 \times 10^{-4}} = 1.073 \times 10^5$$

$$h = \frac{0.6}{0.03}[1.2 + 0.53(1.073 \times 10^5)^{0.54}]\left(\frac{9.75}{3.2}\right)^{0.25}(6.7)^{0.3} = 12{,}957 \ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot {}^\circ\mathrm{C}}$$

$q = h4\pi r^2(T_w - T_\infty) = (12{,}957)(4)\pi(0.015)^2(90 - 20) = 2564 \ \mathrm{W}$

6-69

$$T_f = \frac{220+20}{2} = 120°\text{C} = 393 \text{ K} \qquad \nu = 2.256\times10^{-5} \qquad k = 0.0331$$

$$\text{Pr} = 0.69 \qquad \text{Re} = \frac{u_\infty d}{\nu} = \frac{(20)(0.006)}{2.256\times10^{-5}} = 5319 \qquad \frac{hd}{k} = 0.37\text{Re}^{0.6}$$

$$h = \frac{0.0331}{0.006}(0.37)(5319)^{0.6} = 351.1 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = h4\pi r^2(T_w - T_\infty) = (351.1)(4)\pi(0.003)^2(220-20) = 7.94 \text{ W}$$

6-70

$$T_f = \frac{200+30}{2} = 115°\text{C} = 388 \text{ K} \qquad \mu = 2.235\times10^{-5} \qquad k = 0.0328$$

$$\text{Pr} = 0.69 \qquad \rho = \frac{(3)(1.01\times10^5)}{(287)(388)} = 2.73 \ \text{kg/m}^3$$

$$\text{Re} = \frac{(2.73)(75)(1)}{2.235\times10^{-5}} = 9.15\times10^6$$

$$\bar{h} = \frac{k}{L}\text{Pr}^{1/3}(0.037\text{Re}^{0.8} - 871) = \frac{0.0328}{1}(0.69)^{1/3}[(0.037)(9.15\times10^6)^{0.8} - 871]$$

$$= 372 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (372)(1)^2(200-30) = 6.33\times10^4 \text{ W}$$

6-71

$$\text{At } 38°\text{C} \qquad \rho = \frac{3.5\times10^6}{(287)(311)} = 39.2 \ \text{kg/m}^3$$

$$\dot{m} = (39.2)(9)(1.5)(20)(0.025) = 264.7 \ \text{kg/sec}$$

$$T_f \approx \frac{200+38}{2} = 119°\text{C} = 392 \text{ K} \qquad \rho_f = \frac{3.5\times10^6}{(287)(392)} = 31.1 \ \text{kg/m}^3$$

$$\mu_f = 2.25\times10^{-5} \qquad k_f = 0.0331 \qquad \text{Pr}_f = 0.69 \qquad c_p \approx 1010 \ \frac{\text{J}}{\text{kg}\cdot°\text{C}}$$

$$u_{\max} = u_\infty\left(\frac{S_n}{S_n - d}\right) = 9\left(\frac{2.5}{2.5-1.25}\right) = 18 \ \text{m/sec} \qquad \frac{S_n}{d} = 2 \qquad \frac{S_p}{d} = 3$$

$$\text{Re}_{\max} = \frac{(31.1)(18)(0.0125)}{2.25\times10^{-5}} = 311{,}000 \qquad C = 0.488 \qquad n = 0.562$$

$$h = \frac{0.331}{0.0125}(0.488)(311{,}000)^{0.562}(0.69)^{1/3} = 1395 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \dot{m}c_p\Delta T = hA(T_w - T_{\text{avg}})$$

$$q = (264.7)(1010)(T_e - 38) = 1395(400)\pi(0.0125)(1.5)\left(200 - \frac{T_e - 38}{2}\right)$$

$T_e = 56.76°\text{C}$ $\quad q = 5.016\ \text{MW}$

6-72

$\frac{S_n}{d} = \frac{1.9}{0.633} = 3$ $\quad \frac{S_p}{d} = 3$ $\quad u_\infty = 4.5\ \text{m/sec}$ $\quad T_\infty = 293\ \text{K}$

$p = 1\ \text{atm}$ $\quad \rho_\infty = 1.18\ \text{kg/m}^3$ $\quad T_f = \frac{90+20}{2} = 55°\text{C} = 328\ \text{K}$

$\rho = 1.077$ $\quad \mu = 2.034 \times 10^{-5}$ $\quad k = 0.0284$ $\quad \Pr = 0.7$ $\quad c_p = 1007$

$u_{\max} = u_\infty\left(\frac{S_n}{S_n - d}\right)$ $\quad u_{\max} = (4.5)(1.5) = 6.75\ \text{m/sec}$

$\text{Re} = \frac{(1.077)(6.75)(0.00633)}{2.034 \times 10^{-5}} = 2262$ $\quad C = 0.317$ $\quad n = 0.608$

$$h = \frac{0.0284}{0.00633}(0.317)(2262)^{0.608}(0.7)^{1/3}(0.94) = 131.4\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\frac{q}{L} = (131.4)(6)(50)\pi(0.00633)\left(90 - \frac{T_e}{2} - \frac{20}{2}\right)$$
$$= (1.18)(50)(0.019)(4.5)(1007)(T_e - 20)$$

$T_e = 30.03°\text{C}$ $\quad \frac{q}{L} = 50{,}944\ \text{W/m}$

$$\text{Re}_{\max} = \frac{(7.27)(0.00633)}{2.034 \times 10^{-5}} = 2262$$

$$G_{\max} = (1.077)(6.75) = 7.27\ \frac{\text{kg}}{\text{m}^2 \cdot \text{sec}} = 5361\ \frac{\text{lbm}}{\text{ft}^2 \cdot \text{hr}}$$

$\rho = 0.0737\ \text{lbm/ft}^3$

$$f' = \left[0.044 + \frac{(0.08)(3)}{2^{0.807}}\right](2262)^{-0.15} = 0.0569$$

At 90°C $\quad \mu_w = 2.17 \times 10^{-5}$

At 20°C $\quad \mu = 1.98 \times 10^{-5}$

$$\Delta p = \frac{(0.0569)(5361)^2(6)}{(0.0737)(2.09 \times 10^8)}\left(\frac{2.17}{1.98}\right)^{0.14} = 0.645\ \text{lbf/ft}^2$$

$\Delta p = 30.89\ \text{N/m}^2$

6-73

$\nu = 15.69 \times 10^{-4}$ $\quad k = 0.02624 \quad \Pr = 0.71 \quad u_{\max} = (10)\left(\frac{4}{2}\right) = 20 \text{ m/s}$

$\mathrm{Re}_d = \dfrac{(20)(0.02)}{15.69 \times 10^{-6}} = 25{,}494 \quad \dfrac{S_n}{d} = \dfrac{S_p}{d} = 2.0 \quad C = 0.254$

$n = 0.632$

$$\bar{h} = \frac{0.02624}{0.02}(0.254)(25{,}494)^{0.632}(0.71)^{1/3} = 181 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

6-75

$\dfrac{S_n}{d} = \dfrac{S_p}{d} = 1.5 \quad T_f = \dfrac{150+35}{2} = 92.5^\circ\text{C} = 365.5 \text{ K} \quad \rho = 5.226$

$\rho_f = \dfrac{(3)(1.0132 \times 10^5)}{(189)(365.5)} = 4.404 \quad \text{At } 35^\circ\text{C} = 308 \text{ K} \quad c_p = 921$

$\mu_f = 17.82 \times 10^{-6} \quad \Pr_f = 0.75 \quad k_f = 0.0218$

$u_{\max} = (5)\left(\dfrac{1.875}{1.875 - 1.25}\right) = 15 \text{ m/s} \quad C = 0.278$

$\mathrm{Re}_{\max} = \dfrac{(4.404)(15)(0.0125)}{17.82 \times 10^{-6}} = 46{,}339 \quad n = 0.620$

$$h = \frac{0.0218}{0.0125}(0.278)(46{,}339)^{0.620}(0.75)^{1/3} = 344.2 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$q = (344.2)(100)\pi(0.0125)(0.6)\left(150 - \frac{T_e - 35}{2}\right)$$
$$= (5.226)(10)(0.6)(0.01875)(5)(921)(T_e - 35)$$

$T_e = 62.35^\circ\text{C} \quad q = 74{,}059 \text{ W}$

6-76

$T_f = 55°\text{C} = 328\ \text{K} \qquad u_\infty = 12\ \text{m/s} \qquad \text{Pr} = 0.7$

$k = 0.0284 \qquad \nu = 19.04\times10^{-6} \qquad d = 2.5\ \text{cm}$

$$u_{\max} = (12)\frac{5}{5-2.5} = 24\ \text{m/s} \qquad \text{Re}_{\max} = \frac{(24)(0.025)}{19.04\times10^{-6}} = 3.15\times10^4$$

$$c = 0.112 \qquad n = 0.702 \qquad \bar{h} = \frac{0.0284}{0.025}(0.112)(3.15\times10^4)^{0.702}(0.7)^{1/3}(0.96)$$

$$h = 156\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}} \qquad \frac{q}{L} = (156)\pi(0.025)(15)(7)(90-20) = 9\times10^4\ \text{W/m}$$

$$f' = \left[0.044 + \frac{(0.08)(1.5)}{1}\right](3.15\times10^4)^{-0.15} = 3.468\times10^{-2}$$

$$p = \frac{1.013\times10^5}{(287)(328)} = 1.076 \qquad G_{\max} = (1.076)(24) = 25.83 \qquad \rho_\infty = 1.205$$

$$\Delta p = \frac{(2)(3.468\times10^{-2})(25.83)^2(7)}{1.205}\left(\frac{2.12}{1.96}\right)^{0.14} = 272\ \text{N/m}^2 = 0.033\ \text{psi}$$

6-77

$$T_f = \frac{350+300}{2} = 325\ \text{K} \qquad \nu = 18.23\times10^{-6} \qquad k = 0.0281 \qquad \text{Pr} = 0.7$$

$$u_{\max} = (10)\left(\frac{5}{2.5}\right) = 20 \qquad \text{Re}_{\max} = \frac{(20)(0.025)}{18.23\times10^{-6}} = 27{,}427$$

$$\frac{S_p}{d} = \frac{S_n}{d} = \frac{5}{2.5} = 2.0 \qquad C = 0.254 \qquad n = 0.632$$

$$h = \frac{0.0281}{0.025}(0.254)(27{,}427)^{0.632}(0.7)^{1/3} = 161.8\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

correction for 5 tubes = 0.92

$$h = (161.8)(0.92) = 148.8\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

<u>Zukauskas</u> $\quad C = 0.27 \quad n = 0.63$ and $h = 166.9$ correction for 5 tubes = 0.92

$$h = (166.9)(0.92) = 153.5\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$ only 3% difference

6-78

$$T_f = \frac{90+20}{2} = 55°C = 328 \text{ K} \qquad \rho = \frac{1.01\times10^5}{(287)(328)} = 1.076$$

$\mu = 2.035\times10^{-5}$ $\qquad k = 0.0284$ $\qquad \Pr = 0.7$

$$\text{Re} = \frac{(1.076)(0.05)(15)}{2.035\times10^{-5}} = 39{,}656 \qquad C = 0.102 \qquad n = 0.675$$

$$h = \frac{0.0284}{0.05}(0.102)(39{,}656)^{0.675}(0.7)^{1/3} = 65.34 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (65.34)(4)(0.05)(90-20) = 915 \ \text{W/m}$$

6-79

$$T_f = \frac{870+20}{2} = 445°C = 718 \text{ K} \qquad \Pr = 0.685 \qquad \mu = 3.386\times10^{-5}$$

$$\rho = \frac{1.01\times10^5}{(287)(718)} = 0.492 \qquad k = 0.053 \qquad \text{Re} = \frac{(0.492)(2)(0.006)}{3.386\times10^{-5}} = 174.37$$

$C = 0.228$ $\qquad n = 0.731$ (Approximate values)

$$h = \frac{0.053}{0.006}(0.228)(174.37)^{0.731}(0.685)^{1/3} = 77.2 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = (77.2)(7)(2)(0.006)(0.35)(870-20) = 1929$ W

This value is probably low because of the range of Table 6–2.

6-80

$$T_f = \frac{50+30}{2} = 40°C = 313 \text{ K} \qquad \mu = 2.007\times10^{-5} \qquad k = 0.0272$$

$$\Pr = 0.7 \qquad \rho = \frac{1.01\times10^5}{(287)(313)} = 1.128 \qquad \text{Re} = \frac{(1.128)(6)(0.3)}{2.007\times10^{-5}} = 101{,}165$$

$C = 0.102$ $\qquad n = 0.675$

$$h = \frac{0.0272}{0.3}(0.102)(101{,}165)^{0.675}(0.7)^{1/3} = 19.63 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (19.63)(4)(0.3)(50-30) = 471 \ \text{W/m}$$

If velocity halved: $h_{1/2} = (19.63)\left(\frac{1}{2}\right)^{0.675} = 12.29$ $\quad q$ reduced by 37.3%

6-81

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

$u=u_\infty=\text{const}$ $\qquad T_\infty=\text{const}$

$$u_\infty\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial^2 T}{\partial y^2}=\frac{u_\infty}{\alpha}\frac{\partial T}{\partial x}$$

Analog to semi-infinite plate

$y \sim x$

$x \sim \tau$

$\frac{u_\infty}{\alpha} \sim \frac{1}{\alpha}$

Boundary Conditions

Solid

$T(x,\ 0)=T_i$

$T(0,\ \tau)=T_0$ for $\tau>0$

Boundary Layer

$T(y,\ 0)=T_\infty$

$T(0,\ x)=T_w$

Solutions

Solid

$$\frac{T(x,\ \tau)-T_0}{T_i-T_0}=\operatorname{erf}\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$$

Boundary Layer

$$\frac{T(y,\ x)-T_w}{T_\infty-T_w}=\operatorname{erf}\left(\frac{y}{2\sqrt{\frac{\alpha\tau}{u_\infty}}}\right)$$

6-82

Bismuth at 400°C $\qquad \rho=9950\ \text{kg/m}^3 \qquad \mu=1.47\times10^{-3}\ \frac{\text{kg}}{\text{m}\cdot\text{sec}}$

$c_p=0.15\ \frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}} \qquad k=16.3 \qquad \Pr=0.013 \qquad \operatorname{Re}\Pr=450.4$

$$\operatorname{Re}=\frac{(0.025)(1)(4)}{\pi(0.025)^2(1.47\times10^{-3})}=34{,}645$$

$$h=\frac{16.3}{0.025}[5.0+0.025(450.4)^{0.8}]=5423\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q=(1)(150)(T_e-400)=(5423)\pi(0.025)(0.6)\left(450-\frac{T_e-400}{2}\right)$$

$T_e=445.98^\circ\text{C} \qquad q=6898\ \text{W}$

6-83

Sodium at 134.5°C $\rho = 900$ $\mu = 0.45 \times 10^{-3}$ $c_p = 1345$

$k = 82$ $\Pr = 0.08$ $q = (2.3)(1345)(149 - 120) = 151.6\ \text{kW}$

$$\text{Re} = \frac{(0.025)(2.3)(4)}{(0.025)^2(0.45 \times 10^{-3})} = 260{,}300 \qquad \text{Re}\Pr = 2082$$

$$h = \frac{8.2}{0.025}[4.82 + 0.01825(2082)^{0.8277}] = 49{,}500\ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$151{,}600 = (49{,}500)\pi(0.025)L(200 - 149) \qquad L = 0.765\ \text{m}$$

6-84

$$h = \frac{k}{x} 0.53 \text{Re}_x{}^{1/2} \Pr^{1/2} = Cx^{-1/2}$$

$$\bar{h} = 2h_{x=L} \qquad \frac{\bar{h}L}{k} = \overline{\text{Nu}}_L = 1.06 \text{Re}_L{}^{1/2} \Pr^{1/2}$$

6-85

<u>Air</u> $T_f = \frac{93 + 15}{2} = 54^\circ\text{C} = 327\ \text{K}$ $\mu = 2.03 \times 10^{-5}$ $k = 0.028$

$\Pr = 0.7$ $\rho = \frac{1.01 \times 10^5}{(287)(327)} = 1.08\ \text{kg/m}^3$

$$\text{Re} = \frac{(1.08)(15)(0.0516)}{2.03 \times 10^{-5}} = 41{,}163$$

$$h_a = \frac{0.028}{0.0516}(0.0266)(41{,}163)^{0.805}(0.7)^{1/3} = 66.44\ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

<u>Water</u> at 93°C $\mu = 3.06 \times 10^{-4}$ $k = 0.678$ $\Pr = 1.90$

$$\text{Re} = \frac{(0.05)(0.8)(4)}{\pi(0.05)^2(3.06 \times 10^{-4})} = 66{,}574$$

$$h_w = \frac{(0.678)(0.023)(66{,}574)^{0.8}(1.9)^{0.4}}{0.5} = 2911\ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$\frac{q}{L} = \frac{(93 - 15)\pi(1)}{\frac{1}{(66.44)(0.0516)} + \frac{1}{(2911)(0.05)}} = 821\ \text{W/m}$$

6-86

$$G = \frac{\dot{m}}{A} = \frac{0.035}{\pi(0.0625)^2} = 2.852$$

Properties at 325 K $\quad \mu = 1.96 \times 10^{-5} \ \frac{\text{kg}}{\text{m}\cdot\text{s}}$

$$k = 0.02813 \ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

$$\Pr = 0.7$$

$$\text{Re} = \frac{dG}{\mu} = \frac{(0.0125)(2.852)}{1.96 \times 10^{-5}} = 1819 \quad \text{Laminar}$$

$$\text{Re}\Pr\frac{d}{L} = (1819)(0.7)\left(\frac{0.0125}{12}\right) = 1.33$$

$$h = \frac{0.02813}{0.0125}\left[3.66 + \frac{(0.0668)(1.33)}{1+(0.04)(1.33)^{2/3}}\right] = 8.32 \ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = \dot{m}c_p\Delta T_b = hA(T_w - \overline{T}_b) = (0.035)(1005)(T_e - 300)$$

$$= (8.32)\pi(0.0125)(12)\left(350 - 150 - \frac{T_e}{2}\right)$$

$$T_e = 305.3 \text{ K}$$

$$q = (0.035)(1005)(305.3 - 300) = 186 \text{ W}$$

6-87

$$T_f = \frac{77+20}{2} = 48.5^\circ\text{C} = 321.5 \text{ K}$$

Properties $\nu = 17.87 \times 10^{-6} \qquad k = 0.028 \qquad \Pr = 0.7$

$$\text{Re} = \frac{u_\infty d}{\nu} = \frac{(20)(0.05)}{17.87 \times 10^{-6}} = 55{,}960$$

$C = 0.0266 \qquad n = 0.805$

$$h = \frac{0.028}{0.05}(0.0266)(55{,}960)^{0.805}(0.7)^{1/3} = 87.8 \ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (87.8)\pi(0.05)(77-20) = 786 \ \text{W/m}$$

6-88

Properties at 20°C $\quad \nu = 0.0009 \ \mathrm{m^2/s} \quad \Pr = 10{,}400$

$$k = 0.145 \ \frac{\mathrm{W}}{\mathrm{m \cdot °C}} \qquad \rho = 888 \ \mathrm{kg/m^3} \qquad c_p = 1880 \ \frac{\mathrm{J}}{\mathrm{kg \cdot °C}}$$

$$G = \frac{(0.4)(4)}{\pi(0.02)^2} = 1273$$

$$\mathrm{Re} = \frac{dG}{\mu} = \frac{(0.02)(1273)}{(888)(0.0009)} = 31.9$$

$$\mathrm{Re}\,\Pr\frac{d}{L} = (31.9)(10{,}400)\frac{0.02}{8} = 829 \qquad \mathrm{Gz}^{-1} = 1.21 \times 10^{-3}$$

From Fig. 6–5 $\quad \overline{\mathrm{Nu}} = 16 \quad h = \dfrac{(16)(0.145)}{0.02} = 116 \ \dfrac{\mathrm{W}}{\mathrm{m^2 \cdot °C}}$

$$q = \pi dLh(T_w - \overline{T}_b) = \dot{m}c_p\Delta T_b$$

$$= \pi(0.02)(8)(116)\left(80 - 10 - \frac{T_e}{2}\right) = (0.4)(1880)(T_e - 20)$$

$$19{,}120 = 781T_e$$

$$T_e = 24.5°\mathrm{C}$$

6-89

at 325 K, properties $\quad k = 0.02814 \ \dfrac{\mathrm{W}}{\mathrm{m \cdot °C}} \quad \nu = 18.23 \times 10^{-6} \ \mathrm{m^2/s}$

$$\Pr = 0.7 \qquad \mu = 1.96 \times 10^{-5} \qquad \rho = 1.09 \ \mathrm{kg/m^3}$$

$$G = \frac{0.2}{(0.12)(0.2)} = 8.333 \qquad D_H = \frac{(4)(0.12)(0.2)}{(2)(0.12 + 0.2)} = 0.15$$

$$\mathrm{Re} = \frac{D_H G}{\mu} = \frac{(0.15)(8.333)}{1.96 \times 10^{-5}} = 63{,}770$$

$$h = \frac{0.028}{0.15}(0.023)(63{,}770)^{0.8}(0.7)^{0.4} = 26 \ \frac{\mathrm{W}}{\mathrm{m^2 \cdot °C}}$$

$$q = \dot{m}c_p\Delta T_b = hA(T_w - \overline{T}_b)$$

$$= (0.2)(1005)(T_e - 300) = 26(2)(0.12 + 0.2)(2.5)\left(400 - 150 - \frac{T_e}{2}\right)$$

$$5.382T_e = 1700$$

$$T_e = 315.8 \ \mathrm{K}$$

$$q = 3175.8 \ \mathrm{W}$$

6-90

constant temperature tube

$\Pr = 0.7 \qquad d = 1.5\text{ mm} \qquad k = 0.02624\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$

$\text{Re}\Pr = (1200)(0.7) = 840 \qquad \text{Re}\Pr d = 1.26$

Use Fig. 6–5

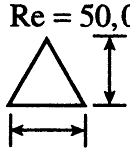
x, m	$\text{Gz}^{-1} = \dfrac{x}{d}(\text{Re}\Pr)$	$\overline{\text{Nu}}_d$	$\bar{h}$, $\dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$
0.01	7.94×10^{-3}	7.9	138
0.1	7.94×10^{-2}	4.5	78.8
0.2	0.159	4.0	70
1.0	0.793	3.66	64.1

6-91

at 15°C $\qquad c_p = 4180 \qquad \rho = 999 \qquad \mu = 1.12\times10^{-3}$

$k = 0.595\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad \Pr = 7.88$

$\text{Re} = 50{,}000$



$$A_c = \left(\frac{1}{2}\right)(2.165)(2.5) = 2.71\text{ cm}^2$$

$$D_H = \frac{(4)(2.71)}{(3)(2.5)} = 1.443\text{ cm} = 0.01443\text{ m}$$

$$h = \frac{0.595}{0.01443}(0.023)(50{,}000)^{0.8}(7.88)^{0.4} = 12{,}438\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = \dot{m}c_p\Delta T_b = hA(T_w - \overline{T}_b) \qquad \text{Re} = \frac{D_H \frac{\dot{m}}{A_c}}{\mu}$$

$$\dot{m} = \frac{(50{,}000)(1.12\times10^{-3})(2.71)(10^{-4})}{0.01443} = 1.052\ \text{kg/sec}$$

$q = (1.052)(4180)(10) = (12{,}438)(3)(0.025)L(15) = 43{,}961$ W

$L = 3.14$ m

Checking Fig. 6–6 fully developed flow is present

6-92

$$\text{Re} = 10^4 \qquad k = 0.026\ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}} \qquad \text{Pr} = 0.7$$

Geom	C	n	Nu_d	h	A	$q \sim hA$
○	0.193	0.618	50.8	$50.8\frac{k}{d}$	πd	$159.6k$
□	0.102	0.675	45.4	$45.4\frac{k}{d}$	$4d$	$181.6k$

6-93

$\text{Pr} = 0.7 \qquad k = 0.026$

Eq. (6–25) $\quad \text{Nu} = (0.37)(50{,}000)^{0.6} = 244$

Eq. (6–26) $\quad \text{Nu} = 2 + [0.25 + (3 \times 10^{-4})(50{,}000)^{1.6}]^{1/2} = 101.5$

Eq. (6–30) $\quad \text{Nu} = 2 + [0.4(50{,}000)^{1/2} + 0.06(50{,}000)^{2/3}](0.7)^{0.4} = 150.1$

6-94

at 10°C $\quad \text{Pr} = 9.4 \qquad \mu = 1.31 \times 10^{-3} \qquad k = 0.585 \qquad \rho = 999$

at 60°C $\quad \mu_w = 0.471 \times 10^{-3} \qquad \text{Re} = \dfrac{(999)(4)(0.025)}{1.31 \times 10^{-3}} = 76{,}260$

Use Eq. (6–29)

$$\text{Nu} = (9.4)^{0.3}\left(\frac{1.31}{0.471}\right)^{0.25}[1.2 + (0.53)(76{,}260)^{0.54}] = 584$$

$$h = \frac{(584)(0.585)}{0.025} = 13{,}647\ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$q = hA(T_w - T_\infty) = (13{,}647)4\pi\left(\frac{0.025}{2}\right)^2(60 - 10) = 1340\ \text{W}$$

6-95

$$u_{max} = (6)\left(\frac{2}{2-1.5}\right) = 24 \text{ m/s}$$

Take $T_f = 325$ K $\rho = 1.09$ $\mu = 1.96\times10^{-5}$ $\rho_{inlet} = 1.18$

$k = 0.02814$ $\Pr = 0.7$ $c_p = 1005$

$$\text{Re} = \frac{\rho u_\infty d}{\mu} = \frac{(1.09)(24)(0.015)}{1.96\times10^{-5}} = 20{,}020$$

$$\frac{S_n}{d} = \frac{S_p}{d} = \frac{2}{1.5} = 1.33$$

From Table 6.4, by interpolation

$n = 0.597$ $C = 0.364$

$$h = \frac{0.02814}{0.015}(0.364)(20{,}020)^{0.597}(0.7)^{1/3} = 224 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = hA(T_w - \overline{T}_\infty) = \dot{m}c_p\Delta T_\infty$$

$$A = (144)\pi(0.015)(1.0) = 6.786 \text{ m}^2$$

$$\dot{m} = (1.18)(6)(12)(1.0)(0.02) = 1.69 \text{ kg/s}$$

$$(224)(6.786)\left(350 - 150 - \frac{T_e}{2}\right) = (1.69)(1005)(T_e - 300)$$

$$8.14\times10^5 = 2458T_e$$

$$T_e = 330.9 \text{ K}$$

$$q = 52{,}500 \text{ W}$$

6-96

$k = 0.026 \ \frac{\text{W}}{\text{m}\cdot°\text{C}}$ $\Pr = 0.7$

(a) $\bar{h} = \frac{0.026}{0.1}(0.0266)(50{,}000)^{0.805}(0.7)^{1/3} = 37.2 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

(b) $\bar{h} = \frac{0.026}{0.1}(0.023)(50{,}000)^{0.8}(0.7)^{0.4} = 29.8 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

(c) $\bar{h} = \frac{0.026}{0.1}(0.664)(50{,}000)^{1/2}(0.7)^{1/3} = 34.3 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

6-97

$\rho = 999 \qquad c_p = 4186 \qquad \mu = 1.12\times10^{-3} \qquad k = 0.595 \qquad \Pr = 7.88$

$$\mathrm{Re}_d = \frac{(999)(10)(0.3048)(0.025)}{1.12\times10^{-3}} = 67{,}967$$

$$\bar{h} = \frac{0.595}{0.025}(0.023)(67{,}967)^{0.8}(7.88)^{0.4} = 9178\ \frac{\mathrm{W}}{\mathrm{m^2\cdot{}^\circ C}}$$

$$\dot{m} = \rho u_m A_c = (999)(10)(0.3048)\frac{\pi}{4}(0.025)^2 = 1.495\ \mathrm{kg/s}$$

$$q = \dot{m}c_p\Delta T_b = \bar{h}A(T_w - \bar{T}_b)$$

$$(1.495)(4181)(10) = (9178)\pi(0.025)L(150-60)\left(\frac{5}{9}\right)$$

$$L = 1.74\ \mathrm{m}$$

6-98

$$h = \frac{k}{d}(0.023)\left(\frac{mxd}{ud^2\mu}\right)^{0.8}\Pr^{0.4}$$

$\Pr \sim \text{const} = 0.7$

$h = ck\mu^{-0.8}$

T (K)	k	$\mu\times10^5$	$k\mu^{-0.8}$
300	0.02624	1.8462	161
400	0.03365	2.286	174
500	0.04038	2.671	184
800	0.05779	3.625	206

Comment:

h varies approximately as $[T(\mathrm{K})]^{0.25}$ for constant mass flow.

6-99

Helium–Same relation as Prob. 6–92

T (K)	k	$\mu \times 10^5$	$k\mu^{-0.8}$
255	0.1357	1.817	842
477	0.197	2.75	877
700	0.251	3.475	927

Comment:
h varies approximately as $[T(\mathrm{K})]^{0.1}$ for constant mass flow, i.e., not strongly dependent on temperature.

6-100

$d = 5\ \mathrm{mm} \qquad \overline{T}_b = 300\ \mathrm{K}$

$\mathrm{Re}_d = 50{,}000 \qquad L = 50\ \mathrm{mm} \qquad \dfrac{L}{d} = \dfrac{50}{5} = 10 \qquad \mathrm{Pr} = 0.708$

$k = 0.02624\ \dfrac{\mathrm{W}}{\mathrm{m}\cdot{}^\circ\mathrm{C}}$

$$\mathrm{Nu} = 0.036\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{d}{L}\right)^{0.055} = (0.036)(50{,}000)^{0.8}(0.708)^{1/3}(0.1)^{0.055} = 162.4$$

$$\bar{h} = \frac{(162.4)(0.02624)}{0.005} = 852\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

6-101

Water $\qquad d = 5\ \mathrm{mm} \qquad L = 50\ \mathrm{mm} \qquad \mathrm{Pe} = 1000 = \mathrm{Re}\,\mathrm{Pr}$

$T_w = 49^\circ\mathrm{C} = \mathrm{const} \qquad \overline{T}_b = 15.6^\circ\mathrm{C} \qquad \rho = 999 \qquad c_p = 4186$

$k = 0.595 \qquad \mu = 1.12 \times 10^{-3} \qquad \mathrm{Pr} = 7.88$

$\mathrm{Gz} = \mathrm{Re}\,\mathrm{Pr}\dfrac{d}{L} = (1000)\left(\dfrac{5}{50}\right) = 100 \qquad \mathrm{Gz}^{-1} = 0.01 \qquad \overline{\mathrm{Nu}}_d = 7.35$

$\bar{h} = \dfrac{(7.35)(0.595)}{0.005} = 874\ \dfrac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}} \qquad \mathrm{Re} = \dfrac{\mathrm{Pe}}{\mathrm{Pr}} = \dfrac{1000}{7.88} = 128.2 = \dfrac{dG}{\mu}$

$$G = \frac{(128.2)(0.00112)}{0.005} = 28.7\ \frac{\mathrm{kg}}{\mathrm{m}^2\cdot\mathrm{s}}$$

$$\dot{m} = \frac{(28.7)\pi(0.005)^2}{4} = 5.64 \times 10^{-4}\ \mathrm{kg/s}$$

$$q = \bar{h}\pi d L(T_w - \overline{T}_b) = \dot{m}c_p\Delta T_b$$

$$\Delta T_b = \frac{(874)\pi(0.005)(0.05)(49 - 15.6)}{(5.64 \times 10^{-4})(4186)} = 9.71^\circ\mathrm{C}$$

6-102

At 300 K: $u_m = 7.5$ m/s $L = 30$ m $T_w = 325$ K
$k = 0.02624$ $\rho = 1.006$ $\mathrm{Pr} = 0.706$ $c_p = 1006$
$\mu = 1.8462 \times 10^{-5}$

30×60 cm; $D_H = \dfrac{(4)(30)(60)}{(2)(30+60)} = 40 \text{ cm} = 0.4 \text{ m}$

$$\mathrm{Re}_{D_H} = \frac{(1.006)(0.4)(7.5)}{1.846 \times 10^{-5}} = 1.63 \times 10^5 \text{ (turbulent)}$$

$\dot{m} = \rho u_m A = (1.006)(7.5)(0.3)(0.6) = 1.358$ kg/s

$\mathrm{Nu} = 0.023 \mathrm{Re}^{0.8} \mathrm{Pr}^{0.4} = (0.023)(1.63 \times 10^5)^{0.8}(0.706)^{0.4} = 296$

$\bar{h} = \dfrac{(296)(0.02624)}{0.4} = 19.4 \ \dfrac{\mathrm{W}}{\mathrm{m}^2 \cdot {}^\circ\mathrm{C}}$ $\qquad q = \bar{h}A(T_w - \bar{T}_b) = \dot{m}c_p \Delta T_b$

$$\Delta T_b = \frac{(19.4)(30)(2)(0.3+0.6)(325-300)}{(1.358)(1006)} = 19.2^\circ\mathrm{C}$$

6-103

Glycerine at 10°C $L = 1$ m
1×8 cm duct $\mathrm{Re} = 250$ $T_w = \text{const}$

$D_H = \dfrac{(4)(1)(8)}{2(1+8)} = 1.78$ cm $\qquad \mathrm{Pr} = 31.0 \times 10^3 \qquad k = 0.284 \ \dfrac{\mathrm{W}}{\mathrm{m} \cdot {}^\circ\mathrm{C}}$

$$\frac{L}{D_H} = 56.2$$

$$\mathrm{Gz} = \mathrm{Re}\,\mathrm{Pr}\frac{d}{L} = (250)(31{,}000)\left(\frac{0.0178}{1}\right) = 1.38 \times 10^5$$

$$\mathrm{Nu} = 3.66 + \frac{(0.0668)(1.38 \times 10^5)}{1 + 0.04(1.38 \times 10^5)^{2/3}} = 88.8$$

$$\bar{h} = \frac{(88.8)(0.284)}{0.0178} = 1417 \ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot {}^\circ\mathrm{C}}$$

6-104

Air $T_\infty = 300$ k $T_w = 600$ K $d = 6$ mm $L = 50$ cm $\mathrm{Re} = 15{,}000$

$T_f = \dfrac{300+600}{2} = 450$ K $\qquad \mathrm{Pr} = 0.683 \qquad k = 0.03707$

$\mathrm{Nu} = C\,\mathrm{Re}^n \mathrm{Pr}^{1/3}$ $\qquad C = 0.228 \qquad n = 0.731$
$\mathrm{Nu} = (0.228)(15{,}000)^{0.731}(0.683)^{1/3} = 227$

$$\bar{h} = \frac{(227)(0.03707)}{0.006} = 1401 \ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot {}^\circ\mathrm{C}}$$

$q = \bar{h}A(T_w - T_\infty) = (1401)(2)(0.006)(0.5)(600-300) = 2521$ W

6-105

$C = 0.102 \qquad n = 0.675$

$\text{Nu} = (0.102)(15{,}000)^{0.675}(0.683)^{1/3} = 59.2$

$\bar{h} = \dfrac{(59.2)(0.03707)}{0.006} = 366 \ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

$q = \bar{h}A(T_w - T_\infty) = (366)(4)(0.006)(0.5)(600 - 300) = 1316 \text{ W}$

6-106

$\overline{\text{Nu}} = 0.664\,\text{Re}^{1/2}\,\text{Pr}^{1/3} = 0.664(15{,}000)^{1/2}(0.683)^{1/3} = 71.6$

$\bar{h} = \dfrac{(71.6)(0.03707)}{0.006} = 442 \ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

$q = \bar{h}A(T_w - T_\infty) = (442)(2)(0.006)(0.5)(600 - 300) = 796 \text{ W}$

6-107

$d = 6$ mm in-line tube bank

$L = 50$ cm 20×20 tubes

$S_n = S_d = 9$ mm T_∞ enter at 300 K

R_{max} at inlet = 50,000 $T_w = 400$ K $T_f = 350$ K $k = 0.03003$

$\text{Pr} = 0.697 \qquad \rho = 0.998 \qquad \mu = 2.075 \times 10^{-5} \qquad c_p = 1009$

$\dfrac{S_p}{d} = \dfrac{S_n}{d} = \dfrac{9}{6} = 1.50 \qquad C = 0.278 \qquad n = 0.62$

$\text{Nu} = (0.278)(50{,}000)^{0.62}(0.697)^{1/3} = 201.9$

$\bar{h} = \dfrac{(201.9)(0.03003)}{0.006} = 1011 \ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

$\text{Re}_{\text{max}} = \dfrac{\rho u_{\text{max}} d}{\mu}$

$u_{\text{max}} = \dfrac{(50{,}000)(2.075 \times 10^{-5})}{(0.998)(0.006)} = 173 \text{ m/s}$

$u_\infty = \left(\dfrac{9-6}{9}\right)(173) = 57.7 \text{ m/s}$

For 1-m depth, $\dot{m} = \rho u_\infty A = (0.998)(57.7)(20)(0.009) = 10.76 \text{ kg/s}$

Tube area $= (400)\pi(0.006)(1) = 7.54 \text{ m}^2$

$q = \bar{h}A\left[T_w - \left(300 + \dfrac{\Delta T}{2}\right)\right] = (10.36)(1009)\Delta T$

$\Delta T = 53.4^\circ\text{C}$

6-108

$u_\infty = 57.7\ \text{m/s}$ from Prob. 6–107

$$\text{Diagonal dimension} = \left[\left(\frac{9}{2}\right)^2 + 9^2\right]^{1/2} - 6 = 4.06\ \text{mm}$$

Normal space is still minimum flow area so $\text{Re}_{max} = 50{,}000$

For staggered arrangement $C = 0.511$ $n = 0.562$

$$\text{Nu} = (0.511)(50{,}000)^{0.562}(0.697)^{1/3} = 198$$

$$\bar{h} = \frac{(198)(0.03003)}{0.006} = 992$$

Mass flow is same as before so energy balance is

$$(992)(7.54)\left[400 - \left(300 + \frac{\Delta T}{2}\right)\right] = (10.36)(1009)\Delta T$$

$\Delta T = 52.8°\text{C}$

6-109

Air $\text{Pr} = 0.7$ at 300 K

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} \quad \text{(a)}$$

$$\text{Nu} = 0.021(\text{Re}^{0.8} - 100)\,\text{Pr}^{0.4} \quad \text{(b)}$$

$$\text{Nu} = 0.012(\text{Re}^{0.87} - 280)\,\text{Pr}^{0.4} \quad \text{(c)}$$

Re	Nu
50,000	114.5 (a)
50,000	102.8 (b)
50,000	124.5 (c)
100,000	199.4 (a)
100,000	180.3 (b)
100,000	230.0 (c)

6-110

Water $\quad$ $Pr = 6.78$ at 21°C

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (a)$$

$$Nu = 0.021(Re^{0.8} - 100) Pr^{0.4} \quad (b)$$

$$Nu = 0.012(Re^{0.87} - 280) Pr^{0.4} \quad (c)$$

Re	Nu
50,000	284.0 (a)
50,000	254.9 (b)
50,000	308.8 (c)
100,000	494.5 (a)
100,000	447.1 (b)
100,000	570.4 (c)

6-111

$T_f = 350$ K $\quad$ $Pr = 0.697$

$$Nu = 0.0266 Re^{0.805} Pr^{1/3} \quad (a)$$

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5} \quad (b)$$

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282{,}000}\right)^{1/2}\right] \quad (c)$$

$$Nu = (0.4 Re^{0.5} + 0.06 Re^{2/3}) Pr_\infty^{0.4} \left(\frac{\mu_\infty}{\mu_w}\right)^{1/4} \quad (d)$$

Calculated values of Nu

Equation	Re = 50,000	Re = 100,000
a	143	249.8
b	136.5	213.8
c	153.5	243.6
d	141.5	211.1

Agreement within ±7%

6-112

water $\quad Pr_f = 5.842 \quad Pr_\infty = 6.772 \quad \mu_\infty = 9.789 \times 10^{-4}$

$\mu_w = 7.629 \times 10^{-4}$

Same equations as Problem 6–~~107~~ ١١٨

Calculated values of Nu

Equation	Re = 50,000	Re = 100,000
a	290.4	507.4
b	303.7	475.9
c	341.6	542.3
d	390.9	585.0

Agreement within ±12%

6-113

0.5×0.5 m plate $\quad L = 0.5 \quad T_f = 42.5°C = 315.5\text{ K} \quad \rho = 1.12$

$k = 0.028 \quad \mu = 1.91 \times 10^{-5} \quad Pr = 0.7 \quad q = 850\text{ W}$

$$q_w = \frac{850}{0.5^2} = 3400\ \text{W/m}^2 = \frac{3}{2} h_{x=L}(\overline{T_w - T_\infty})$$

$$h_{x=L} = \frac{(3400)(2)}{(3)(65-20)} = 50.4\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\text{Nu} = \frac{hx}{k} = \frac{(50.4)(0.5)}{0.028} = 900$$

For $Re = 5 \times 10^5$ max Nu will be $Nu = 0.453(5 \times 10^5)^{1/2}(0.7)^{1/3} = 284$
so turbulent flow must be used and would be about same (+4%) as for const wall temperature

$$Nu = Pr^{1/3}(0.037 Re_L^{0.8} - 871) = 900, \text{ and } Re_L = 7.65 \times 10^5 = \frac{\rho u_\infty L}{\mu}$$

$$u_\infty = \frac{(7.65 \times 10^5)(1.91 \times 10^{-5})}{(1.12)(0.5)} = 26.1\ \text{m/s}$$

Radiation $\quad T_w = 338\text{ K} \quad T_\infty = 293\text{ K}$

$$\frac{q}{A} = \sigma\varepsilon(T_w^4 - T_\infty^4) = (5.668 \times 10^{-8})(1.0)(338^4 - 293^4) = 322\ \text{W/m}^2$$

$$q_{\text{conv}} = 3400 - 322 = 3078\ \text{W/m}^2$$

Required $\quad \text{Nu} = (900)\left(\frac{3078}{3400}\right) = 815 = (0.7)^{1/3}(0.037 Re_L^{0.8} - 871)$

$$Re_L = 7.17 \times 10^5 = \frac{\rho u_\infty L}{\mu} \qquad u_\infty = \frac{(7.17 \times 10^5)(1.91 \times 10^{-5})}{(1.12)(0.5)} = 24.5\ \text{m/s}$$

6-114

Gas at 700 K $\rho = 0.503\ \text{kg/m}^3$ $\mu = 3.332\times 10^{-5}$ $\text{Pr} = 0.684$

$k = 0.0523$ $\text{Re} = \dfrac{(0.025)(0.8)(4)}{\pi(0.025)^2(3.332\times 10^{-5})} = 1.22\times 10^6$

$$h_g = \frac{(0.0523)(0.023)}{0.025} = (1.22\times 10^6)^{0.8}(0.684)^{0.3} = 3176\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

Water at $150^\circ\text{C} = 423\ \text{K}$ $\mu = 1.86\times 10^{-4}$ $\rho = 918$ $k = 0.684$

$\text{Pr} = 1.17$ $D_H = 5 - 2.5 - 2(0.16) = 2.18\ \text{cm}$

$$\text{Re} = \frac{(4)(1.5)}{\pi(0.05 + 0.0282)(1.86\times 10^{-4})} = 1.313\times 10^5$$

$$h_w = \frac{(0.684)(0.023)}{0.0218}(1.313\times 10^5)^{0.8}(1.17)^{0.4} = 9555\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

Neglect conduction resistance $q = 17.5\ \text{kW}$

$$175{,}000 = \frac{700-423}{\frac{1}{h_g A_i} + \frac{1}{h_w A_0}} = \frac{(700-423)L}{\frac{1}{(3176)\pi(0.025)} + \frac{1}{(9555)\pi(0.0282)}}$$

$L = 0.328\ \text{m}$

6-118

$$\frac{\bar{h}d}{k} = \text{const}\left(\frac{\rho u_m d}{\mu}\right)^{0.8}\text{Pr}^{0.4}$$

For constant mass flow and diameter

$\rho u_m = \text{const}$

For air as ideal gas $\text{Pr} \sim \text{const}$ which implies $\mu \sim k$ over modest temperature range

$\mu \sim T^a$

$a = \text{const}$, $T = \text{abs. temp.}$

Thus, $\bar{h} = \text{const}\times T^a\left(\dfrac{1}{T^a}\right)^{0.8} = \text{const}\times T^{0.2a}$

For air at 300 K, $a = 0.74$ and $\bar{h} \sim \text{const}\times T^{0.148}$ or modest variation with temperature

6-119

One must recognize that significant amounts of energy are transmitted by radiation. For the radiation calculation assume that Eq. (1–12) is applicable and that the food has $\varepsilon = 1.0$. The area for radiation transfer is the surface area of the food, the same as for convection. Although there are times when the radiant heaters in the oven are substantially above the control temperature for the oven, a reasonable assumption for the radiation temperature of the oven is probably at the control temperature because that is the temperature maintained by the walls which have the large area for radiation. A simple geometry to consider would be that of a sphere having the same mass as the turkey, for that part of the problem. For cooking pies or other "flat" objects some other assumption must be made. To make an assumption for the flow velocity, one might visit the oven display section of a local appliance store. In performing an analysis it is prudent to consider only a steady state operation of the oven, after it has reached the set point temperature.

Chapter 7

7–3

Show that $\beta = \frac{1}{T}$ for an ideal gas.

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \qquad pV = nRT \qquad V = \frac{nRT}{p} \qquad \left(\frac{\partial V}{\partial T}\right)_p = \frac{nR}{p}$$

$$\therefore \quad \beta = \frac{1}{\frac{nRT}{p}}\left(\frac{nR}{p}\right) = \frac{1}{T}$$

7–4

$T_f = 40°\text{C} = 313\ \text{K} \qquad \rho = 1.128 \qquad L = 0.3048\ \text{m}$

$\mu = 1.906\times10^{-5} \qquad \text{Pr} = 0.905 \qquad \rho = 1.13 \qquad k = 0.027$

$\beta = 0.00319 \qquad \text{Gr} = 1.558\times10^{8}$

$\delta = L[0.393\,\text{Pr}^{-1/2}(0.952+\text{Pr})^{1/4}\,\text{Gr}^{-1/4}] = 0.014\ \text{m}$

$u = u_{max}$ at $\frac{y}{\delta} = \frac{1}{3} = \frac{4}{27}u_x = 0.722\ \text{m/s}$

$\text{Nu} = 0.508\,\text{Pr}^{1/2}(0.952+\text{Pr})^{-1/4}\,\text{Gr}^{1/4} = 42.0$

$\bar{h} = \frac{4}{3}\text{Nu}\frac{k}{L} = 5.0\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$ (Free convection)

Forced Convection

$\text{Re}_L = \frac{\rho u_{max} L}{\mu} = 13{,}000$ Laminar

$\bar{h} = 0.664\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3}\left(\frac{k}{L}\right) = 6.03\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$$\frac{\bar{h}\ \text{(Forced)}}{\bar{h}\ \text{(Free)}} = \frac{6.03}{5.0} = 1.206$$

Conclusion

Heat transfer is strongly influenced by velocity of fluid over the heated surface

7-5

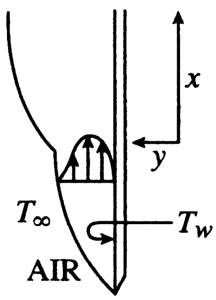
$T_f = 325$ K, $\text{Pr} = 0.7$; $\nu = 18.2\times10^{-6}$; $\delta = 0.02\text{m}$

$\text{Gr}_x = (9.8)(1/3250950)x^3/(18.2\times10^{-6})^2 = 4.455\times10^9 x^3$

Eq. (7-20a): $\quad 0.02 = (4.55\times10^9)^{-1/4}(3.93)(0.7)^{-1/2}(0.952+0.7)^{1/4}x^{1/4}$

$x = 0.9$ m

7–6



$$\frac{u}{u_x} = \left(\frac{y}{\delta}\right)\left(1 - \frac{y}{\delta}\right)^2$$

$$\frac{du}{dy} = u_x\left[\frac{y}{\delta}\left(-\frac{2}{\delta}\right)\left(1 - \frac{y}{\delta}\right) + \frac{1}{\delta}\left(1 - \frac{y}{\delta}\right)^2\right] = 0$$

$$y = \frac{\frac{4}{\delta} \pm \sqrt{\left(\frac{16}{\delta^2}\right) - 4\left(\frac{3}{\delta^2}\right)}}{\frac{6}{\delta^2}}$$

$$y = \delta \qquad\qquad y = \frac{\delta}{3} \quad \text{max } u \text{ occurs at } \frac{\delta}{3}$$

$$u_{\max} = u_x\left(\frac{y}{3y}\right)\left(1 - \frac{y}{3y}\right)^2 = \frac{4}{27}u_x = \frac{4}{27}C_1x^m$$

7–7

Spacing $\sim 2\delta$

$$\frac{\delta}{L} = 3.93\,\mathrm{Pr}^{-1/2}(0.952 + \mathrm{Pr})^{1/4}\,\mathrm{Gr}_L^{-1/4}$$

$$T_f = \frac{65 + 25}{2} = 45°\mathrm{C} \qquad L = 0.3\text{ m} \qquad \mathrm{Pr} = 3.9$$

$$\mathrm{Gr}_L = \frac{(4.5 \times 10^{10})(0.3)^3(65 - 25)}{3.9} = 1.25 \times 10^{10}$$

Outside of laminar range but use above equation for estimate of B.L. thickness

$$\delta = 0.3\left(\frac{3.93}{3.9^{1/2}}\right)(0.952 + 3.9)^{1/4}(1.25 \times 10^{10})^{-1/4} = 0.0026\text{ m}$$

Spacing should be at least 1 cm.

7–8

$T_f = 100°\text{F} \qquad k = 0.63$

$$\text{Gr}_d\,\text{Pr} = (3.3 \times 10^{10})(0.02)^3(130-70)\left(\frac{5}{9}\right) = 8.8 \times 10^6$$

$$\text{Nu} = C(\text{Gr}\,\text{Pr})^n \qquad C = 0.53 \qquad n = \frac{1}{4}$$

$$h = \frac{(0.53)(8.8 \times 10^{10})^{1/4}}{0.02} = 909 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\frac{q}{L} = \bar{h}\pi d(T_w - T_\infty) = (909)\pi(0.02)(130-70)\left(\frac{5}{9}\right) = 1904 \ \text{W/m}$$

7–9

$L = 30 \text{ cm} \qquad T_f = 57.5°\text{C} = 330.5 \text{ K} \qquad \nu = 19.23 \times 10^{-6}$

$$\text{Gr}_L = \frac{(9.8)\left(\frac{1}{330.5}\right)(100-15)(0.3)^3}{19.23 \times 10^{-6}} = 1.84 \times 10^8$$

$$D = \frac{(0.3)(35)}{(1.84 \times 10^8)^{1/4}} = 9.01 \times 10^{-2} \ \text{m} = 9.01 \text{ cm}$$

7–10

$$T_f = \frac{300+25}{2} = 162.5°\text{C} = 436 \text{ K} \qquad \nu = 30.1 \times 10^{-6} \qquad k = 0.036$$

$$\text{Pr} = 0.68 \qquad \text{Gr}\,\text{Pr} = \frac{(9.8)\left(\frac{1}{436}\right)(300-25)(1)^3(0.68)}{(30.1 \times 10^{-6})^2} = 4.64 \times 10^9$$

$$h = \frac{0.036}{1}(0.1)(4.64 \times 10^9)^{1/3} = 6.0 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = hA(T_w - T_\infty) = (6.0)(1)^2(300-25) = 1650 \text{ W}$$

7–11

$$\delta \sim x^{1/4} \qquad \frac{\delta_{24}}{\delta_{14}} = \left(\frac{24}{14}\right)^{1/4} \qquad \delta_{24} = (1.0)\left(\frac{24}{14}\right)^{1/4} = 1.14 \text{ in.}$$

7-12

$H = 0.2$; $W = 3.0$ $T_f = 60°C = 333\ K$

$\nu = 23.3\times10^{-6}$; $Pr = 0.7$; $k = 0.028$; $\rho = 1.06$

$Gr_x = 3.47\times10^7$; Eq. (7-20a) $\delta = 0.0139$

Eq. (7-20b); $u_x = 2.74\ m/s$

$u_{max} = (0.148)(2.74) = 0.41\ m/s$

$\dot{m} = (9/16)(1.06)(o.41)(0.0139) = 0.0034\ kg/s$

$GrPr = 2.43\times10^7$; $h = (0.028)(0.59/0.2)(2.43\times10^7)^{1/4} = 5.8$

$q = (6.8)((0.2)(3.0)(100 - 20) = 278\ W$

7-13

$T_f = 300\ K$; $H = 0.1$; $W = 2.0$; $k = 0.61$; $Pr = 5.85$; $\rho = 996$

$\Delta T = 20°C$; $GrPr = (1.91\times10^{10})(0.1)^3(20) = 3.82\times10^8$

$Gr_x = 6.53\times10^7$

$h = (0.61/0.1)(0.59)(3.82\times10^8)^{1/4} = 503$

$q = (503)(0.1)(1.0)(20) = 1006\ W$

Eq. (7-20a); $\delta = 0.00292\ m$

Eq. (7-20b); $\nu = 8.63\times10^{-7}$

$u_{max} = (0.148)(8.63\times10^{-7}/0.1)(5.17)(0.952 + 5.85)^{-1/2}(6.53\times10^7)^{1/2}$

$= 0.02\ m/s$

$\dot{m} = (9/16)((996)(0.02)(0.00292) = 0.0033\ kg/s$

7-14

$$T_f = \frac{93+30}{2} = 61.5°\text{C} = 334.5 \text{ K} \qquad \beta = \frac{1}{334.5} = 2.99\times10^{-3}$$

$$\nu = 19.19\times10^{-6} \qquad k = 0.0289 \qquad \Pr = 0.7$$

$$\text{Gr}\Pr = \frac{(9.806)(2.99\times10^{-3})(93-30)(1.8)^3}{(19.19\times10^{-6})^2}(0.7) = 2.05\times10^{10}$$

$$h = \frac{0.0289}{1.8}(0.1)(2.05\times10^{10})^{1/3} = 4.394 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (4.394)\pi(0.075)(1.8)(93-30) = 117.4 \text{ W}$$

7-15

$$\frac{q}{A} = 1100 - 95 = 1005 \ \text{W/m}^2 \text{ at } 300 \text{ K} \qquad \beta = 3.33\times10^{-3}$$

$$\nu = 15.68\times10^{-6} \qquad \Pr = 0.7 \qquad k = 0.02624$$

$$\text{Gr}^* = \frac{(9.806)(3.33\times10^{-3})(1005)(6)^4}{(15.68\times10^{-6})^2(0.02624)} = 6.59\times10^{15}$$

$$h = \frac{0.02624}{6}(0.17)(6.59\times10^{15})^{1/3} = 6.7 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\Delta T = \frac{1005}{6.7} = 150°\text{C} \qquad T_{\text{wall}} \approx 150 + 20 = 170°\text{C}$$

This does not take into account radiation which would lower the temperature substantially.

7-16

$$\Delta T = 24 - 20 = 4°\text{C} \qquad h = (0.95)(4)^{1/3} = 1.508 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (1.508)\pi(0.3)(2.0)(4.0) = 11.3 \text{ W}$$

7-17

$$q_w = \frac{20}{(0.3)^2} = 222.2\ \text{W/m}^2$$ Take properties at 300 K

$k = 0.02624$ $\beta = 3.33\times10^{-3}$ $\nu = 15.68\times10^{-6}$ $\text{Pr} = 0.71$

$$\text{Gr}^* = \frac{(9.806)(3.33\times10^{-3})(222.2)(0.15)^4}{(0.02624)(15.68\times10^{-6})^2} = 5.694\times10^8 \text{ at 15 cm}$$

$\text{Gr}^* = 9.11\times10^9$ at 30 cm

at 15 cm $$h = \frac{0.02624}{0.15}(0.6)[(5.694\times10^8)(0.7)]^{1/5} = 5.51\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

at 30 cm $$h = \frac{0.02624}{0.3}(0.6)[(9.11\times10^9)(0.7)]^{1/5} = 4.796\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$\bar{h} = \frac{5}{4}h_{x=L} = (1.25)(4.796) = 6.0\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

7-18

$L = 0.3$ $T_w = 55^\circ\text{C}$ $T_\infty = 20^\circ\text{C}$ $T_f = 37.5^\circ\text{C} = 310.5\text{ K}$

$\nu = 16.7\times10^{-6}$ $k = 0.027$ $\text{Pr} = 0.7$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{310.5}\right)(55-20)(0.3)^3(0.7)}{(16.7\times10^{-6})^2} = 7.48\times10^7$$

$C = 0.59$ $m = \frac{1}{4}$

$$\bar{h} = \frac{0.027}{0.3}(0.59)(7.48\times10^7)^{1/4} = 4.94\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = 2hA(T_w - T_\infty) = (2)(4.94)(0.3)^2(55-20) = 31.1\text{ W}$$

7-19

$$T_f = \frac{100+20}{2} = 60^\circ\text{C} = 333\text{ K}$$ $\mu = 2.16\times10^{-7}$

$$\rho = \frac{(2)(1.01\times10^5)}{(333)(2078)} = 0.293\ \text{kg/m}^3$$ $k = 0.159$ $\text{Pr} = 0.7$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{333}\right)(0.293)^2(100-20)(0.61)^3(0.7)}{(2.16\times10^{-7})^2} = 6.87\times10^7$$

$$h = \frac{(0.159)(0.59)(6.87\times10^7)^{1/4}}{0.61} = 14\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = hA(T_w - T_\infty) = (14)(0.61)^2(100-20) = 417\text{ W}$$

7-20

$$T_f = \frac{57+4}{2} = 30.5°\text{C} = 303.5 \text{ K} \qquad \nu = 16.1\times10^{-6}$$

$k = 0.0265 \qquad \text{Pr} = 0.7$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{303.5}\right)(57-4)(6.1)^3(0.7)}{(16.1\times10^{-6})^2} = 1.04\times10^{12}$$

$$\overline{\text{Nu}}^{1/2} = 0.825 + \frac{0.387\,\text{Ra}^{1/4}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}} = 33.44$$

$$\text{Nu} = 1118 \qquad h = \frac{(1118)(0.0265)}{6.1} = 4.86 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = hA(T_w - T_\infty) = (4.86)(6.1)(1.22)(57-4) = 1907 \text{ W}$$

Using $C = 0.1$ and $m = \frac{1}{3}$

$\text{Nu} = 0.1(1.04\times10^{12})^{1/3} = 1012$ 9.5% lower

7-21

$$T_f = \frac{49+21}{2} = 35°\text{C} = 308 \text{ K} \qquad \nu = 16.5\times10^{-6} \qquad k = 0.0268$$

$\text{Pr} = 0.7$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{308}\right)(49-21)(1)^3(0.7)}{(16.5\times10^{-6})^2} = 2.27\times10^9$$

$$\overline{\text{Nu}}^{1/2} = 0.825 + \frac{0.387\,\text{Ra}^{1/4}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}} = 12.59 \qquad \overline{\text{Nu}} = 158.1$$

$$h = \frac{(158.1)(0.0268)}{1} = 4.24 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = hA(T_w - T_\infty) = (4.24)(1)^2(49-21) = 118 \text{ W}$$

Using $C = 0.1$ and $m = \frac{1}{3}$

$\text{Nu} = (0.1)(2.27\times10^9)^{1/3} = 131$ 17% lower

7-22

Take properties at 300 K

$\nu = 15.69\times10^{-6} \qquad k = 0.02624 \qquad \text{Pr} = 0.7$

$$10^{12} = \frac{(9.8)\left(\frac{1}{300}\right)(10)L^3(0.7)}{(15.69\times10^{-6})^2}$$

$L = 10.35$ m

7-23

Take properties at 15.56°C $\quad k = 0.595 \quad \dfrac{g\beta\rho^2 c_p}{\mu k} = 1.08\times 10^{10}$

$$\mathrm{Gr}^*\,\mathrm{Pr} = \frac{(1.08\times 10^{10})(1000)(0.25)^4}{0.595} = 7.09\times 10^{10}$$

$$h_x = \frac{0.595}{0.25}(0.6)(7.09\times 10^{10})^{1/5} = 211$$

$$\bar{h} = 1.25h_x = 264\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$\overline{\Delta T} = \frac{q_w}{\bar{h}} = \frac{1000}{264} = 3.79^\circ\mathrm{C}$$

$\bar{T}_w = 3.79 + 15 = 18.79^\circ\mathrm{C}$ $\quad$ it <u>is</u> very nearly isothermal.

7-24

<u>cylinder</u>: $\quad h = 1.32\left(\dfrac{\Delta T}{d}\right)^{1/4}$

<u>Vert. plate</u>: $\quad h = 1.42\left(\dfrac{\Delta T}{L}\right)^{1/4}$

<u>$\frac{1}{2}$ cyl</u>: $\quad q = \dfrac{1}{2}(1.32)\left(\dfrac{\Delta T}{d}\right)^{1/4}\pi d\Delta T = 2.073d^{3/4}\Delta T^{5/4}$

<u>Plate</u>: $\quad q = (1.42)\left(\dfrac{\Delta T}{L}\right)^{1/4}\dfrac{\pi d}{2}\Delta T = 1.992d^{3/4}\Delta T^{5/4}$

Very close

7-25

$$T_f = \frac{200+120}{2} = 160°\text{C} \qquad \text{NaK, 22\% Na}$$

$k = 24.4 \qquad \Pr = 0.019 \qquad d = 0.02 \text{ m} \qquad \mu = 0.4\times10^{-3} \qquad L = 0.4 \text{ m}$

$\rho = 830$

$$\beta \approx 690\left(\frac{\frac{1}{690}-\frac{1}{830}}{760-93}\right) = 2.5\times10^{-4}$$

$$\mathrm{Gr}_d \Pr = \frac{(9.8)(2.5\times10^{-4})(830)^2(200-120)(0.02)^3(0.019)}{(0.4\times10^{-3})^2} = 1.28\times10^5$$

$$\mathrm{Nu} = 0.36 + \frac{0.518(\mathrm{Gr}\Pr)^{1/4}}{\left[1+\left(\frac{0.559}{\Pr}\right)^{9/16}\right]^{4/9}} = 0.36 + \frac{0.518(1.28\times10^5)^{1/4}}{\left[1+\left(\frac{0.559}{0.019}\right)^{9/16}\right]^{4/9}} = 4.31$$

$$\bar{h} = \frac{(4.31)(24.4)}{0.02} = 5264 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (5264)\pi(0.02)(0.4)(200-120) = 10{,}580 \text{ W}$$

7-26

$T_f = 299\text{K}$; $L = 0.15$; $W = 0.5$; $p = 2.2$ atm; $\rho = 0.36$

$\nu = 128\times10^{-6}/2.2 = 58\times10^{-6}$; $k = 0.149$; $\Pr = 0.7$; $\Delta T = 52°\text{C}$

$\mathrm{Gr} = (9.8)(1/299)(52)(0.15)^3/(58\times10^{-6})^2 = 1\times10^6$

$\mathrm{Gr}\Pr = 7\times10^5$

$h = (0.149/0.15)(0.59)(7\times10^5)^{1/4} = 17$

$q = (17)(0.15)(0.5)(52) = 66.2$ W

Eq. (7-20b); $u_{max} = (58\times10^{-6}/0.15)(0.148)(0.952+0.7)^{-1/2}(10^6)^{1/2}(5.17)$

$= 0.23$ m/s

Eq. (7-20a); $\delta = (0.15)(3.93)(0.7)^{-1/2}(0.952+0.7)^{1/4}(10^6)^{-1/4}$

$= 0.025$ m

$\dot{m} = (9/16)(0.23)(0.025)(0.36) = 0.0016$ kg/s

7-27

$$T_f = \frac{70+20}{2} = 45°\text{C} = 318\ \text{K} \qquad \nu = 0.335\times10^{-6} \qquad k = 0.485$$

$$\text{Pr} = 2.0 \qquad \beta = 2.45\times10^{-3}$$

$$\text{Gr}\,\text{Pr} = \frac{(9.8)(2.45\times10^{-3})(70-20)(0.03)^3(2)}{(0.335\times10^{-6})^2} = 5.78\times10^8$$

$$C = 0.53 \qquad m = \frac{1}{4}$$

$$h = \frac{0.485}{0.03}(0.53)(5.78\times10^8)^{1/4} = 1328\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = h\pi dL(T_w - T_\infty) = (1328)\pi(0.03)(1)(70-20) = 6260\ \text{W}$$

7-28

$$h = (1.32)\left(\frac{120-20}{0.075}\right)^{1/4} = 7.98\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = 29{,}300\ \text{W} = 7.98\pi(0.075)L(120-20)$$

$$L = 155.9\ \text{m}$$

7-29

$$T_f = \frac{93+38}{2} = 65.5°\text{C} \qquad k = 0.659$$

$\text{Gr Pr} = (7.62\times10^{10})(93-38)(0.0004)^3 = 268.2$

From Fig. 7–8 $\quad \log \text{Nu} = 0.41 \quad \text{Nu} = 2.57$

$$h = \frac{(2.57)(0.659)}{0.0004} = 4235 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = (4235)\pi(0.0004)(0.1)(93-38) = 29.27 \text{ W}$

7-30

Laminar:

$$h = 1.32\left(\frac{85-20}{0.03}\right)^{1/4} = 9.006 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = (9.006)\pi(0.03)(15)(85-20) = 827.6 \text{ W}$

7-31

$$T_f = \frac{140+25}{2} = 82.5°\text{C} = 356 \text{ K} \qquad \nu = 2.54\times10^{-6} \qquad k = 0.031$$

$\text{Pr} = 0.697$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{356}\right)(140-25)(0.08)^3(0.697)}{(21.1\times10^{-6})^2} = 2.54\times10^6$$

$$C = 0.53 \qquad m = \frac{1}{4}$$

$$h = \frac{0.031}{0.08}(0.53)(2.54\times10^6)^{1/4} = 8.2 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (8.2)\pi(0.08)(140-25) = 237 \ \text{W/m}$$

7-32

$$h = 1.32\left(\frac{250-20}{0.0125}\right)^{1/4} = 15.38 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (15.38)\pi(0.0125)(250-20) = 138.9 \ \text{W/m}$$

7-33

$$T_f = \frac{150+93}{2} = 121.5°\text{C} \qquad \nu = 0.124\times10^{-4} \qquad k = 0.135 \qquad \Pr = 175$$

$$\beta = 0.7\times10^{-3} \qquad \text{Gr}\Pr = \frac{(9.8)(0.7\times10^{-3})(150-93)(0.025)^3}{(0.124\times10^{-4})^2} = 6.96\times10^6$$

$$h = \frac{0.135}{0.025}(0.53)(6.96\times10^6)^{1/4} = 147 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (147)\pi(0.025)(150-93) = 658 \ \text{W/m} = 200.6 \ \text{W/ft}$$

7-34

0.3 m square duct $\quad T_w = 15.6°\text{C} \quad T_\infty = 27°\text{C} \quad \Delta T = 11.4°\text{C}$

<u>Bottom</u> $\quad h = 1.32\left(\frac{\Delta T}{L}\right)^{1/4} = (1.32)\left(\frac{11.4}{0.3}\right)^{1/4} = 3.28 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

<u>Sides</u> $\quad h = 1.42\left(\frac{\Delta T}{L}\right)^{1/4} = (1.42)\left(\frac{11.4}{0.3}\right)^{1/4} = 3.53 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

<u>Top</u> $\quad h = 0.59\left(\frac{\Delta T}{L}\right)^{1/4} = (0.59)\left(\frac{11.4}{0.3}\right)^{1/4} = 1.46 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$$q = \sum hA\Delta T = (0.3)(1)(11.4)[3.28+(2)(3.53)+1.46] = 40.4 \ \text{W/m}\cdot\text{length}$$

7-35

$$T_f = \frac{240+10}{2} = 125°\text{C} = 398 \text{ K} \qquad \nu = 6.76\times10^{-5} \qquad k = 0.178$$

$\Pr = 0.71$

$$\text{Gr}\Pr = \frac{(9.8)\left(\frac{1}{398}\right)(240-10)(2.54\times10^{-5})^3}{(6.76\times10^{-5})^2}(0.71) = 1.42\times10^{-5}$$

$$\text{Nu} = 0.36 + \frac{(0.518)(1.42\times10^{-5})^{1/4}}{\left[1+\left(\frac{0.559}{0.7}\right)^{9/16}\right]^{4/9}} = 0.384$$

$$h = \frac{(0.384)(0.178)}{2.54\times10^{-5}} = 2691$$

$$\frac{q}{L} = (2691)\pi(2.54\times10^{-5})(240-10) = 49.4 \ \text{W/m}$$

7-36

He at 1 atm $\quad L = 2$ m $\quad d = 0.1$ m

$T_w = 93°\text{C} = 366$ K $\quad T_\infty = -18°\text{C} = 255$ K $\quad T_f = 310.5$ K

$\nu = 134.6 \times 10^{-6}\ \text{m}^2/\text{s} \quad k = 0.1524\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \quad \text{Pr} = 0.7$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{310.5}\right)(366-255)(0.1)^3(0.7)}{(134.6\times10^{-6})^2} = 1.356\times10^5$$

$C = 0.53 \quad m = \dfrac{1}{4}$

$$\bar{h} = \frac{0.1524}{0.1}(0.53)(1.356\times10^5)^{1/4} = 15.5\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (15.5)\pi(0.1)(2)(366-255) = 1081\ \text{W}$$

7-37

$T_f = 135°\text{C} = 408$ K $\quad \nu = 26.83\times10^{-6} \quad k = 0.0342 \quad \text{Pr} = 0.688$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{408}\right)(250-20)(3)^3(0.688)}{(26.83\times10^{-6})^2} = 1.43\times10^{11}$$

$$\text{Nu}^{1/2} = 0.6 + 0.387\left\{\frac{1.43\times10^{11}}{\left[1+\left(\frac{0.559}{0.688}\right)^{9/16}\right]^{16/9}}\right\}^{1/6} = 23.78$$

$\text{Nu} = 565.3 \quad h = \dfrac{(565.3)(0.0342)}{3.0} = 6.44$

$$\frac{q}{L} = (6.44)\pi(3.0)(250-20) = 13.97\ \text{kW/m}$$

7-38

$T_f = 40°\text{C} = 313$ K $\quad \nu = 0.00022 \quad k = 0.286$

$\beta = 0.5\times10^{-3} \quad \text{Pr} = 2.45$

$$\text{Gr Pr} = \frac{(9.8)(0.5\times10^{-3})(60-20)(0.02)^3}{(0.00022)^2}(2.45) = 79.37$$

$$\text{Nu} = 0.36 + \frac{(0.518)(79.37)^{1/4}}{\left[1+\left(\frac{0.559}{2.45}\right)^{9/16}\right]^{4/9}} = 1.677$$

$$h = \frac{(1.677)(0.286)}{0.02} = 23.98$$

$$q = (23.98)\pi(0.02)(0.6)(60-20) = 36.15\ \text{W}$$

7-39

$\frac{q}{A} = 1500 \ \mathrm{W/m^2}$ $\beta = \frac{1}{293}$ $\frac{q}{L} = (1500)\pi(0.035) = 165 \ \mathrm{W/m}$

Properties at 350 K $\Delta T \approx 100°\mathrm{C}$ $\nu = 20.76 \times 10^{-6}$ $k = 0.03003$

$\mathrm{Pr} = 0.7$ $\theta = 65°$ Take $L = 1$ m

$$\mathrm{Gr\,Pr} = \frac{(9.8)\left(\frac{1}{293}\right)(100)(1.0)^3}{(20.76 \times 10^{-6})^2}(0.7) = 5.4 \times 10^9$$

$$\frac{1}{4} + \frac{1}{12}(\sin\theta)^{1.75} = 0.32$$

$$\mathrm{Nu}_L = [0.6 - (0.488)(\sin 65°)^{1.03}](5.4 \times 10^9)^{0.32} = 206.9$$

$h = \frac{(206.9)(0.03003)}{1} = 6.21$ h is insensitive to L because of 0.32 exponent.

$$\Delta T = \frac{1500}{6.21} = 242°\mathrm{C}$$

With properties at 400 K and $\Delta T = 242$

$\nu = 25.9 \times 10^{-6}$ $k = 0.03365$ $\mathrm{Nu}_L = 237.6$

$\mathrm{Gr\,Pr} = 8.32 \times 10^9$ $h = 8.0$ $\Delta T = \frac{1500}{8} = 187.5°\mathrm{C}$

$T_w = 20 + 187.5 = 207.5°\mathrm{C} = 480.5\ \mathrm{K}$

$T_f = 386$ K close enough

7-40

$T_f = \frac{25 + 20}{2} = 22.5°\mathrm{C} = 295.5\ \mathrm{K}$ $\nu = 15.3 \times 10^{-6}$ $k = 0.0259$

$\mathrm{Pr} = 0.7$ $\mathrm{Gr\,Pr} = \frac{(9.8)\left(\frac{1}{295.5}\right)(25 - 20)(0.3)^3(0.7)}{(15.3 \times 10^{-6})^2} = 1.34 \times 10^7$

$C = 0.53$ $m = \frac{1}{4}$

$$h = \frac{0.0259}{0.3}(0.53)(1.34 \times 10^7)^{1/4} = 2.77 \ \frac{\mathrm{W}}{\mathrm{m^2 \cdot °C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (2.77)\pi(0.3)(25 - 20) = 13.04 \ \mathrm{W/m}$$

7-41

$T_f = \dfrac{260+20}{2} = 140°\text{C} = 413\ \text{K} \qquad d = 0.125\ \text{m} \qquad \nu = 27.2\times10^{-6}$

$k = 0.035 \qquad \Pr = 0.687$

$$\text{Gr}\,\Pr = \frac{(9.8)\left(\frac{1}{413}\right)(260-20)(0.125)^3(0.687)}{(27.2\times10^{-6})^2} = 1.03\times10^7$$

$C = 0.53 \qquad m = \dfrac{1}{4}$

$$\bar{h} = \frac{0.035}{0.125}(0.53)(1.03\times10^7)^{1/4} = 8.41\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = 37{,}000\ \text{W} = (8.71)\pi(0.125)L(260-20)$

$L = 46.7\ \text{m}$

There will also be substantial radiation heating.

7-42

$T_f = \dfrac{180+60}{2} = 120°\text{F} \qquad k = 0.644 \qquad \dfrac{g\beta\rho^2 c_p}{\mu k} = 4.89\times10^{10}$

$$\text{Gr}\,\Pr = (4.89\times10^{10})(0.05)^3(180-60)\left(\frac{5}{9}\right) = 4.075\times10^8$$

$C = 0.53 \qquad m = \dfrac{1}{4}$

$$h = \frac{0.644}{0.05}(0.53)(4.078\times10^8)^{1/4} = 970\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = h\pi dL(T_w - T_\infty) = (970)\pi(0.05)(3)(180-60)\left(\frac{5}{9}\right) = 3.047\times10^4\ \text{W}$$

7-43

$$T_f = \frac{77+27}{2} = 52°\text{C} = 325\text{ K} \qquad \nu = 12.83\times10^{-6} \qquad k = 0.0281$$

$$\Pr = 0.7 \qquad \text{Gr}\Pr = \frac{(9.8)\left(\frac{1}{325}\right)(77-27)(2)^3(0.7)}{(18.23\times10^{-6})^2} = 2.54\times10^{10}$$

$$\overline{\text{Nu}}^{1/2} = 0.6 + 0.387\left\{\frac{\text{Gr}\Pr}{\left[1+\left(\frac{0.559}{\Pr}\right)^{9/16}\right]^{16/9}}\right\}^{1/6} = 17.998$$

$\text{Nu} = 323.9$

$$h = \frac{(323.9)(0.0281)}{2} = 4.55\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = h\pi dL(T_w - T_\infty) = (4.55)\pi(2)(20)(77-27) = 2.86\times10^4\text{ W}$$

Using $C = 0.13$ and $m = \frac{1}{3}$

$$\text{Nu} = (0.13)(2.54\times10^{10})^{1/3} = 293.7 \ \Rightarrow\ 9\%\text{ Lower}$$

7-44

$$T_f = \frac{90+20}{2} = 55°\text{C} = 328\text{ K} \qquad \beta = 3.049\times10^{-3} \qquad \nu = 18.52\times10^{-6}$$

$k = 0.0284 \qquad \Pr = 0.7$

$$\text{Gr}\Pr = \frac{(9.806)(3.049\times10^{-3})(90-20)(0.3)^3}{(18.52\times10^{-6})^2}(0.7) = 1.153\times10^8$$

$$h = \frac{0.0284}{0.3}[2 + 0.43(1.153\times10^8)^{1/4}] = 4.407\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (4.407)\pi(4)(0.15)^2(90-20) = 87.23\text{ W}$$

7-45

$$T_f = \frac{35+10}{2} = 22.5°\text{C} \qquad k = 0.604$$

$$\text{Gr}\Pr = (1.46\times10^{10})(35-10)(0.025)^3 = 5.703\times10^6$$

$$h = \frac{0.604}{0.025}[2 + 0.43(5.703\times10^6)^{1/4}] = 556\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (556)4\pi(0.0125)^2(35-10) = 27.3$$

7-46

$T_f = -25°\text{C} = 248 \text{ K}$ $\qquad \rho = \dfrac{1400}{(287)(248)} = 1.967 \times 10^{-2}$ $\qquad \text{Pr} = 0.72$

$\mu = 1.488 \times 10^{-5}$ $\qquad \beta = 4.032 \times 10^{-3}$ $\qquad k = 0.0223$

$$\text{Gr Pr} = \frac{(9.8)(4.032 \times 10^{-3})(1.967 \times 10^{-2})(50)(2.4)^3}{(1.488 \times 10^{-5})^2}(0.72) = 3.44 \times 10^7$$

$$h = \frac{0.0223}{2.4}[2 + 0.43(3.44 \times 10^7)^{1/4}] = 0.325 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$q = (0.325)\pi(4)(1.2)^2(50) = 294 \text{ W}$

Forced Convection:

$$\text{Re} = \frac{(1.967 \times 10^{-2})(0.3)(2.4)}{1.488 \times 10^{-5}} = 952$$

$$h = \frac{0.0223}{2.4}(0.37)(952)^{0.6} = 0.211 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$q = (0.211)(4\pi)(1.2)^2(50) = 191 \text{ W}$

7-47

$T_f = \dfrac{38 + 15}{2} = 26.5°\text{C}$ $\qquad k = 0.614$

$\text{Gr Pr} = (1.91 \times 10^{10})(38 - 15)(0.04)^3 = 2.81 \times 10^7$

$$h = \frac{0.614}{0.04}[2 + 0.43(2.81 \times 10^7)^{1/4}] = 511 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$q = (511)(4\pi)(0.02)^2(38 - 15) = 59.1 \text{ W}$

7-48

$$L=\frac{\pi d}{2} \qquad C=0.52 \qquad m=\frac{1}{4}$$

$$\frac{h\left(\frac{\pi d}{2}\right)}{k}=0.52\left[\text{const}\times\left(\frac{\pi d}{2}\right)^3\right]$$

$$\frac{hd}{k}=\text{Nu}_d=(0.52)\left(\frac{2}{\pi}\right)\left[\left(\frac{\pi}{2}\right)^3\right]^{1/4}(\text{Gr}_d\,\text{Pr})^{1/4}=0.464(\text{Gr}_d\,\text{Pr})^{1/4} \qquad \text{(a)}$$

Compare to

$$\text{Nu}=2+0.43(\text{Gr}_d\,\text{Pr})^{1/4} \qquad \text{(b)}$$

GrPr	Nu (a)	Nu (b)
10^4	4.64	6.3
10^5	8.25	9.64
10^6	14.67	15.6

7-49

Flat plate

$$h=\frac{k}{L}(0.59)(\text{Gr}\,\text{Pr})^{1/4}=1.42\left(\frac{\Delta T}{L}\right)^{1/4}$$

Eq. (7–50)

$$h=2\frac{k}{d}+0.43\frac{k}{d}(\text{Gr}\,\text{Pr})^{1/4}$$

$$0.43\frac{k}{d}(\text{Gr}\,\text{Pr})^{1/4}=(1.42)\frac{0.43}{0.59}\left(\frac{\Delta T}{d}\right)^{1/4}=1.03\left(\frac{\Delta T}{d}\right)^{1/4}$$

$$h=2\frac{k}{d}+1.03\left(\frac{\Delta T}{d}\right)^{1/4}$$

7-50

$$T_f = \frac{204+93}{2} = 148.5°\text{C} = 421.5\ \text{K} \qquad \text{He at 3 atm}$$

$$\nu = \frac{221.5\times10^{-6}}{3} = 73.8\times10^{-6} \qquad k = 0.183 \qquad \text{Pr} = 0.71$$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{421.5}\right)(204-93)(0.3)^3(0.71)}{(73.8\times10^{-6})^2} = 9.08\times10^6$$

$$C = 0.53 \qquad m = \frac{1}{4}$$

$$\bar{h} = \frac{0.183}{0.3}(9.08\times10^6)^{1/4} = 33.5\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}\pi dL(T_w - T_\infty) = (33.5)\pi(0.3)(10.4)(204-93) = 36{,}435\ \text{W}$$

7-51

$$T_f = \frac{20+120}{2} = 70°\text{C} = 343\ \text{K} \qquad \nu = 20.05\times10^{-6} \qquad k = 0.0295$$

$$\text{Pr} = 0.7$$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{343}\right)(120-20)(0.3)^3(0.7)}{(20.05\times10^{-6})^2} = 1.34\times10^8 < 10^9$$

$$C = 0.53 \qquad m = \frac{1}{4}$$

$$\bar{h} = \frac{0.0295}{0.3}(0.53)(1.34\times10^8)^{1/4} = 5.61\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (5.61)\pi(0.3)(100)(120-20) = 52{,}847\ \text{W}$$

7-52

$$T_f = \frac{150+20}{2} = 85°\text{C} = 358\ \text{K} \qquad \beta = 2.793\times10^{-3} \qquad \nu = 21.58\times10^{-6}$$

$$k = 0.0306 \qquad \text{Pr} = 0.7$$

$$\text{Gr Pr} = \frac{(9.806)(2.793\times10^{-3})(150-20)(0.15)^3}{(21.58\times10^{-6})^2}(0.7) = 1.806\times10^7$$

$$h = \frac{0.0306}{0.15}(0.15)(1.806\times10^7)^{1/3} = 8.03\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (8.03)\pi(0.075)^2(150-20) = 18.4\ \text{W}$$

7-53

$$T_f = \frac{60+20}{2} = 40°\text{C} \qquad \nu = 0.00024 \qquad k = 0.144$$

$$\text{Pr} = 2870 \qquad \beta = 0.7\times10^{-3}$$

$$\text{Gr Pr} = \frac{(9.8)(0.7\times10^{-3})(60-20)(0.3)^3(2870)}{(0.00024)^2} = 3.69\times10^8$$

$$h = \frac{0.144}{0.3}(0.15)(3.69\times10^8)^{1/3} = 51.6\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (51.6)(0.3)^2(60-20) = 185.8\ \text{W}$$

7-54

laminar: $L = 6$ mm $\quad \Delta T = 500 - 20 \quad q = 2$ kW

Top: $$h = 1.32\left(\frac{480}{0.006}\right)^{1/4} = 22.2\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

Bottom: $$h = 0.61\left(\frac{480}{(0.006)^2}\right)^{1/5} = 16.23\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = 2000 = (16.23+22.2)(0.006)L(480) \qquad L = 18.07\ \text{m}$$

7-55

$$T_f = \frac{25+28}{2} = 26.5°\text{C} = 299.5\ \text{K} \qquad \nu = 16.84\times10^{-6} \qquad k = 0.02624$$

$$\text{Pr} = 0.71$$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{299.5}\right)(3)(10)^3(0.71)}{(16.84\times10^{-6})^2} = 2.42\times10^{11}$$

$$h = \frac{0.02624}{10}(0.15)(2.42\times10^{11})^{1/3} = 2.454$$

$$q = (2.454)(10)^2(3) = 736\ \text{W}$$

7-56

$$T_f = \frac{15+50}{2} = 32.5°\text{C} = 305.5 \text{ K} \qquad \nu = 16.25\times10^{-6} \qquad k = 0.0267$$

$\Pr = 0.7$

$$\text{Gr}\,\text{Pr} = \frac{(9.8)\left(\frac{1}{305.5}\right)(50-15)(4)^3(0.7)}{(16.25\times10^{-6})^2} = 1.9\times10^{11}$$

<u>Upper</u>: $$h_u = \frac{0.0267}{4}(0.15)(1.9\times10^{11})^{1/3} = 5.75 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

<u>Lower</u>: $$h_l = \frac{0.0267}{4}(0.27)(1.9\times10^{11})^{1/4} = 1.19 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (5.75+1.19)(4)^2(50-15) = 3886 \text{ W}$$

7-57

$$T_f = \frac{400+300}{2} = 350 \text{ K} \qquad \nu = 2.075\times10^{-6} \qquad k = 0.03003$$

$$\Pr = 0.697 \qquad x = \frac{A}{P} = \frac{L\left(\frac{L}{2}\right)}{3L} = \frac{L}{6} = \frac{45}{6} = 7.5 \text{ cm}$$

$$\text{Gr}\,\text{Pr} = \frac{(9.8)\left(\frac{1}{350}\right)(400-300)(0.075)^3(0.697)}{(2.075\times10^{-6})^2} = 1.91\times10^8$$

$$h = \frac{0.03003}{0.075}(0.15)(1.91\times10^8)^{1/3} = 34.58 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (34.58)\frac{(0.45)^2}{2}(400-300) = 350 \text{ W}$$

7–58

$$T_w = 40°\text{C} \qquad T_\infty = 20°\text{C} \qquad T_e = 40-(0.25)(40-20) = 35°\text{C} \qquad k = 0.626$$

$$\text{Gr}\,\text{Pr} = (2.89\times10^{10})(0.2)^3(40-20) = 4.624\times10^9$$

$$\theta = 30° \qquad \bar{h} = \frac{0.626}{0.2}(0.56)[(4.624\times10^9)\cos 30°]^{1/4} = 440 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (440)(0.2)^2(40-20) = 352.7 \text{ W}$$

7-59

$\theta = -30°$

$$\overline{\text{Nu}}_e = (0.14)[(4.624\times10^9)^{1/3} - (2\times10^9)^{1/3}] + 0.56[(4.624\times10^9)^{1/3}\cos 30°]^{1/4}$$

$$\overline{\text{Nu}}_e = 56.85 + 140.87 = 197.72$$

$$h = \frac{(0.626)(197.72)}{0.2} = 618.9 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (168.9)(0.2)^2(40-20) = 495.1 \text{ W}$$

7-60

Assume $T = 20°C$ inside

$T_m = 20 + \frac{30}{2} = 35°C = 308\text{ K} \qquad \beta = 3.247 \times 10^{-3}$

$\nu = 16.49 \times 10^{-6} \qquad k = 0.0268 \qquad \Pr = 0.7$

$$\mathrm{Gr}_\delta \Pr = \frac{(9.8)(3.247 \times 10^{-3})(30)(0.0125)^3}{(16.49 \times 10^{-6})^2}(0.7) = 4803$$

$$k_e \approx k \qquad q = \frac{(0.0268)(1.8)(1.2)(30)}{0.0125} = 138.9\text{ W}$$

7-61

$$T_m = \frac{160 + 40}{2} = 100°C = 373\text{ K} \qquad \beta = 2.68 \times 10^{-3} \qquad k = 0.0317$$

$$\Pr = 0.69 \qquad \rho = \frac{1.01 \times 10^4}{(287)(373)} = 0.0946 \qquad \mu = 2.172 \times 10^{-5}$$

$$\mathrm{Gr}_\delta \Pr = \frac{(9.8)(2.68 \times 10^{-3})(0.946)^2(160 - 40)(0.08)^3(0.69)}{(2.172 \times 10^{-5})^2} = 2.11 \times 10^4$$

$$\theta = 20° \qquad C = 0.212 \qquad n = \frac{1}{4}$$

$$\frac{k_e}{k} = (0.212)[(2.11 \times 10^4)\cos 20°]^{1/4} = 2.516$$

$$q = (2.516)(0.0317)(1)^2(160 - 40) = 9.57\text{ W}$$

7-62

$q_w = 700\ \text{W/m}^2 \qquad L = 1\text{ m} \qquad \theta = 60° \quad \text{at } 30°C \qquad \nu = 15.98 \times 10^{-6}$

$\Pr = 0.7 \qquad k = 0.027 \qquad \beta = 3.3 \times 10^{-3}$

$$\mathrm{Gr}^* = \frac{(9.8)(3.3 \times 10^{-3})(700)(1)^4}{(0.027)(15.98 \times 10^{-6})^2} = 3.286 \times 10^{12}$$

$$h = \frac{0.027}{1}(0.17)[(3.286 \times 10^{12})(0.7)]^{1/4} = 5.65\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\Delta T = \frac{700}{5.65} = 123.8°C \qquad T_w \approx 123.8 + 30 = 153.8°C$$

7-63

$$\sigma \varepsilon A(T_w{}^4 - T_s{}^4) = hA(T_s - T_a)$$

$$(5.669 \times 10^{-8})(0.5)(308^4 - 303^4) = (6.5)(303 - T_a)$$

$$T_a = 300.5\text{ K} = 27.5°C$$

7-64

$T_e = 80 - \frac{1}{4}(80 - 20) = 65°\text{C} = 338\ \text{K}$ $\qquad \beta = \frac{1}{308}$ $\qquad \nu = 19.54 \times 10^{-6}$

$k = 0.0291$ $\qquad \Pr = 0.7$

Downward facing $\qquad \theta = 45°$

$$\text{Gr}\Pr = \frac{(9.8)\left(\frac{1}{308}\right)(80 - 20)(0.1)^3(0.7)}{(19.54 \times 10^{-6})^2} = 3.5 \times 10^6$$

$$h_d = \frac{0.0291}{0.1}(0.56)(3.5 \times 10^6 \times 0.707)^{1/4} = 6.46\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

Upward facing

$\text{Gr}_c = 1.05 \times 10^9 \qquad \text{Gr}_c \Pr = 7.35 \times 10^8$

$\text{Gr}_L < \text{Gr}_c$ so that $h_u = \frac{0.0291}{0.1}(0.56)(3.5 \times 10^6 \times 0.707)^{1/4} = 6.46\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q = (2)(6.46)(0.1)^2(80 - 20) = 7.752\ \text{W}$

7-65

$T_e = 50 - \frac{1}{4}(50 - 20) = 42.5°\text{C}$ $\qquad \rho = 990$ $\qquad \mu = 6.2 \times 10^{-4}$

$k = 0.635$ $\qquad \Pr = 0.41$ $\qquad \theta = 30°$ $\qquad \cos\theta = 0.866$

$$\text{Gr}\Pr = \frac{(9.8)(2.07 \times 10^{-4})(50 - 20)(0.05)^3(4.1)(990)^2}{(6.2 \times 10^{-4})^2} = 7.95 \times 10^7$$

$$h_d = \frac{0.635}{0.05}(0.56)(7.95 \times 10^7 \times 0.866)^{1/4} = 647.8\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

Since $\text{Gr}_L < \text{Gr}_c \qquad h_u = h_d$

$q = (2)(647.8)(0.05)^2(50 - 20) = 97.2\ \text{W}$

7-66

$T_b = 38°\text{C} = 311\ \text{K}$ $\qquad T_w = 540°\text{C}$ $\qquad L = 30\ \text{cm}$

$d = 6.5\ \text{mm}$ $\qquad \frac{L}{d} = 46$ $\qquad$ fully developed

Air $u_m = 30\ \text{m/s}$ $\qquad \nu = 16.6 \times 10^{-6}$ $\qquad k = 0.028$ $\qquad \Pr = 0.7$

$$\text{Re}_d = \frac{(30)(0.0065)}{16.6 \times 10^{-6}} = 11{,}750 \qquad \text{turbulent}$$

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\Pr^{0.4} = (0.023)(11{,}750)^{0.8}(0.7)^{0.4} = 35.95$$

$$\bar{h} = \frac{(35.95)(0.028)}{0.0065} = 155\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

7-67

$$T_f = \frac{100+15}{2} = 55°\text{C} = 328\text{ K} \qquad \nu = 18.7\times10^{-6} \qquad k = 0.0285$$

$$\text{Pr} = 0.7 \qquad L = (2)\left(\frac{2.5}{2}\right) + 5 = 7.5\text{ cm} = 0.075\text{ m}$$

$$\text{Gr Pr} = \frac{(9.8)(328)(100-15)(0.075)^3(0.7)}{(18.7\times10^{-6})^2} = 2.15\times10^6$$

$$C = 0.52 \qquad m = \frac{1}{4}$$

$$\bar{h} = \frac{0.0285}{0.075}(0.52)(2.15\times10^6)^{1/4} = 7.56\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$A = (2.5)^2(2) + (4)(2.5)(5) = 62.5\text{ cm}^2$$

$$q = hA(T_w - T_\infty) = (7.56)(62.5\times10^{-4})(100-15) = 4.02\text{ W}$$

7-68

$$A = \frac{1}{2}(40)^2\sin 60° = 692.8\text{ cm}^2 \qquad p = (3)(40) = 120\text{ cm}$$

$$L = \frac{A}{p} = \frac{692.8}{120} = 5.77\text{ cm} \qquad T_f = \frac{55+25}{2} = 40°\text{C} = 313\text{ K}$$

$$\beta = 3.195\times10^{-3} \qquad \nu = 17\times10^{-6} \qquad k = 0.0272 \qquad \text{Pr} = 0.7$$

$$\text{Gr Pr} = \frac{(9.8)(3.195\times10^{-3})(55-25)(0.0577)^3}{(17\times10^{-6})^2}(0.7) = 4.37\times10^5$$

$$h = \frac{0.0272}{0.0577}(0.54)(4.37\times10^5)^{1/4} = 6.546\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (6.546)(0.0693)(55-25) = 13.61\text{ W}$$

7-69

$$T_f = 40°\text{C} = 313\text{ K} \qquad \beta = 3.195\times10^{-3} \qquad \nu = 17\times10^{-6}$$

$$k = 0.0272$$

$$\text{Characteristic length} = \frac{A}{P} = \frac{d}{4} = 0.02\text{ m}$$

$$\text{Pr} = 0.7 \qquad \text{Gr Pr} = \frac{(9.8)(3.195\times10^{-3})(50-30)(0.02)^3(0.7)}{(17\times10^{-6})^2} = 1.21\times10^4$$

$$h = \frac{0.0272}{0.02}(0.54)(1.21\times10^4)^{1/4} = 7.71\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (7.71)\pi(0.04)^2(50-30) = 0.775\text{ W}$$

7-70

$$T_f = \frac{400+27}{2} = 213.5°\text{C} = 486.5\text{ K} \qquad \nu = 36.23\times10^{-6} \qquad k = 0.03948$$

$$\text{Pr} = 0.681 \qquad \beta = \frac{1}{486.5} \qquad \frac{1}{L} = \frac{1}{15} + \frac{1}{8} \qquad L = 5.22\text{ cm}$$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{486.5}\right)(400-27)(0.0522)^3(0.681)}{(36.23\times10^{-6})^2} = 5.54\times10^5$$

$$h = \frac{0.03948}{0.0522}(0.6)(5.54\times10^5)^{1/4} = 12.38\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$A = (2)(15)^2 + (4)(15)(8) = 930\text{ cm}^2$$

$$q = (12.38)(930\times10^{-4})(400-27) = 429\text{ W}$$

7-71

$$\frac{1}{L} = \frac{1}{0.15} + \frac{1}{0.15} \qquad L = 0.075\text{ m} \qquad \nu = 17\times10^{-6} \qquad k = 0.027$$

$$\text{Pr} = 0.7$$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{300}\right)\Delta T(0.075)^3(0.7)}{(17\times10^{-6})^2} = 3.34\times10^4\,\Delta T$$

$$A = 6(0.15)^2 = 0.135\text{ m}^2$$

$$h = \frac{0.027}{0.075}(0.6)(3.34\times10^4)^{1/4}\Delta T^{1/4} = 2.92\Delta T^{1/4}$$

$$50 = (2.92)(0.135)(\Delta T)^{5/4} \qquad \Delta T = 48.2°\text{C}$$

$$T_w = 48.2 + 20 = 68.2°\text{C}$$

7-72

$$\sigma\varepsilon(T_t^{\,4} - T_w^{\,4}) = h(T_a - T_t)$$

$$(5.669\times10^{-8})(0.95)(303^4 - 283^4) = 5(T_a - 303)$$

$$T_a = 325\text{ K} = 52°\text{C}$$

7-73

$$L = 30 + 15 = 45 \text{ cm} \qquad T_f = \frac{45+20}{2} = 32.5°\text{C} = 306 \text{ K}$$

$$\nu = 16.5 \times 10^{-6} \qquad k = 0.0268 \qquad \text{Pr} = 0.7$$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{306}\right)(45-20)(0.45)^3(0.7)}{(16.5\times10^{-6})^2} = 1.88 \times 10^8$$

$$C = 0.52 \qquad m = \frac{1}{4}$$

$$h = \frac{0.0268}{0.45}(0.52)(1.88\times10^8)^{1/4} = 3.62 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\frac{A}{L}(T_w - T_\infty) = (3.62)(2)(0.3+0.15)(45-20) = 81.5 \ \text{W/m}$$

7-74

$$T_m = \frac{38+60}{2} = \cdot \qquad k = 0.644 \qquad \text{Pr} = 3.64$$

$$\text{Gr}_\delta \text{ Pr} = (4.89\times10^{١٠})(60-38)(0.0125)^3 = 2.1\times10^6$$

$$\frac{k_e}{k} = (0.42)(2.1\times10^6)^{1/4}(3.64)^{0.012}\left(\frac{30}{1.25}\right)^{-0.3} = 6.26$$

$$q = \frac{(6.26)(0.644)(60-38)(0.3)^2}{0.0125} = 638.5 \text{ W}$$

7-75

$$T_m = \frac{80+20}{2} = 50°\text{C} = 323 \text{ K} \qquad k = 0.156$$

$$\nu = \frac{143.3\times10^{-6}}{1.3} = 1.102\times10^{-4} \qquad \text{Pr} = 0.7$$

$$\text{Gr}_\delta \text{ Pr} = \frac{(9.8)\left(\frac{1}{323}\right)(80-20)(0.02)^3(0.7)}{(1.102\times10^{-4})^2} = 839$$

$$k_e = k$$

$$q = \frac{(0.156)(0.4)^2(80-20)}{0.02} = 74.88 \text{ W}$$

7-76

$T_m = 30°\text{C}$ $\qquad k = 0.62$

$\text{Gr}_\delta \Pr = (2.25\times10^{10})(40-20)(0.02)^3 = 3.6\times10^6$

$C = 0.40 \qquad n = 0.20 \qquad \frac{k_e}{k} = (0.40)(3.6\times10^6)^{0.2} = 8.191$

$$q = \frac{(2\pi)(8.191)(0.62)(1)(40-20)}{\ln\left(\frac{10}{8}\right)} = 2860 \text{ W}$$

7-77

$T_m = \frac{30-10}{2} = 10°\text{C} = 283 \text{ K} \qquad \rho = \frac{(0.05)(1.01\times10^5)}{(287)(283)} = 0.0624$

$k = 0.0249 \qquad \beta = 3.53\times10^{-3} \qquad \mu = 1.814\times10^{-5} \qquad \Pr = 0.71$

$$\text{Gr}_\delta \Pr = \frac{(9.8)(0.0624)^2(3.53\times10^{-3})(40)(0.05)^3}{(1.814\times10^{-5})^2}(0.71) = 1454$$

$$\frac{k_e}{k} = (0.228)(1454)^{0.226} = 1.182$$

$$q = \frac{4\pi(1.182)(0.0249)(1)(1.05)(40)}{1.05-1} = 310.7 \text{ W}$$

7-78

$T_m = \frac{400+140}{2} = 270°\text{F} = 132.2°\text{C} = 405 \text{ K} \qquad \nu = 26.2\times10^{-6}$

$k = 0.034 \qquad \beta = 2.47\times10^{-3} \qquad \Pr = 0.7$

$$\text{Gr}_\delta \Pr = \frac{(9.8)(2.47\times10^{-3})(400-140)\left(\frac{5}{9}\right)(0.03)^3}{(26.2\times10^{-6})^2}(0.7) = 9.63\times10^4$$

$C = 0.197 \qquad n = \frac{1}{4} \qquad m = -\frac{1}{9}$

$$\frac{k_e}{k} = (0.197)(9.63\times10^4)^{1/4}\left(\frac{35}{3}\right)^{-1/9} = 2.64$$

$$\frac{q}{A} = \frac{(2.64)(0.034)(400-140)\left(\frac{5}{9}\right)}{0.03} = 432 \text{ W/m}^2$$

7-79

$$T_m = \frac{200+90}{2} = 145°\text{C} = 418\ \text{K} \qquad \nu = 26.97\times10^{-6} \qquad k = 0.0349$$

$$\beta = 2.39\times10^{-3} \qquad \Pr = 0.685$$

$$\text{Gr}_\delta \Pr = \frac{(9.8)(2.39\times10^{-3})(200-90)(0.025)^3(0.685)}{(26.97\times10^{-6})^2} = 3.79\times10^4$$

$$C = 0.197 \qquad n = \frac{1}{4} \qquad m = -\frac{1}{9}$$

$$\frac{k_e}{k} = (0.197)(3.79\times10^4)^{1/4}\left(\frac{30}{2.5}\right)^{-1/9} = 2.086$$

$$q = (2.086)(0.0349)(0.3)^2\left(\frac{200-90}{0.025}\right) = 28.83\ \text{W}$$

7-80

$$T_m = 90 + \frac{165}{2} = 172.5°\text{C} = 445.5\ \text{K} \qquad \Pr = 0.684$$

$$\beta = 2.245\times10^{-3} \qquad \nu = 28.59\times10^{-6} \qquad k = 0.0368$$

$$\text{Gr}_\delta \Pr = \frac{(9.8)(2.245\times10^{-3})(165)(0.0016)^3}{(28.59\times10^{-6})^2}(0.684) = 12.45$$

$$k_e = k \qquad \frac{q}{A} = \frac{(0.0368)(165)}{0.0016} = 3795\ \text{W/m}^2$$

7-81

at 172.5°C $\quad k = 0.669 \quad \text{Gr}_\delta \Pr = (10.11\times10^{10})(165)(0.0016)^3$

$$\text{Gr}_\delta \Pr = 6.83\times10^4 \qquad C = 0.13 \qquad n = 0.3 \qquad m = 0$$

$$\frac{k_e}{k} = (0.13)(6.83\times10^4)^{0.3} = 3.67$$

$$k_e = (3.67)(0.669) = 2.453 \qquad \frac{q}{A} = \frac{(2.453)(165)}{0.0016} = 2.53\times10^5\ \text{W/m}^2$$

7-83

$$T_m = \frac{50+20}{2} = 35°\text{C} \qquad \rho = 994 \qquad \mu = 7.2\times10^{-4} \qquad k = 0.626$$

$$\text{Pr} = 4.82 \qquad \frac{g\beta\rho^2 c_p}{\mu k} = 2.89\times10^{10}$$

$$\text{Gr}_\delta \text{Pr} = (2.89\times10^{10})(0.04)^3(30) = 5.55\times10^7$$

$$\frac{k_e}{k} = (0.046)(5.55\times10^7)^{1/3} = 17.55$$

$$q = (17.55)(0.626)(0.5)^2\left(\frac{50-20}{0.04}\right) = 2060 \text{ W}$$

7-84

$$\frac{k_e}{k} = (0.057)(5.55\times10^7)^{1/3} = 21.74$$

$$q = (21.74)(0.626)(0.5)^2\left(\frac{50-20}{0.04}\right) = 2552 \text{ W}$$

7-87

Assume 20°C inside

$$T_m = 20 + \frac{17}{2} = 28.5°\text{C} = 302 \text{ K} \qquad \beta = 3.32\times10^{-3} \qquad \text{Pr} = 0.71$$

$$\nu = 15.68\times10^{-6} \qquad k = 0.0262$$

$$\text{Gr}_\delta \text{Pr} = \frac{(9.8)(3.32\times10^{-3})(17)(0.1)^3(0.71)}{(15.68\times10^{-6})^2} = 1.598\times10^6$$

$$C = 0.073 \qquad n = \frac{1}{3} \qquad m = -\frac{1}{9}$$

$$\frac{k_e}{k} = (0.073)(1.598\times10^6)^{1/3}\left(\frac{2}{0.1}\right)^{-1/9} = 6.118$$

$$\frac{q}{A} = \frac{(6.118)(0.0262)(17)}{0.1} = 27.25 \text{ W/m}^2$$

7-88

$$T_m = 350\ \text{K} \qquad \nu = \frac{20.76\times10^{-6}}{2} \qquad k = 0.03003 \qquad p = 2\ \text{atm} \qquad \text{Pr} = 0.7$$

$$\text{Gr}_\delta\,\text{Pr} = \frac{(9.8)\left(\frac{1}{350}\right)(400-300)(0.06)^3(0.7)(2)^2}{(20.76\times10^{-6})^2} = 3.93\times10^6$$

$$C = 0.073 \qquad m = \frac{1}{3} \qquad n = -\frac{1}{9}$$

$$k_e = (0.03003)(0.073)(3.93\times10^6)^{1/3}\left(\frac{3}{0.06}\right)^{-1/9} = 0.224\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

$$q = k_e A\frac{\Delta T}{\delta} = (0.224)(3)(2)\left(\frac{400-300}{0.06}\right) = 2240\ \text{W}$$

7-90

$$T_m = \frac{100+20}{2} = 60^\circ\text{C} = 333\ \text{K} \qquad \nu = 19.04\times10^{-6} \qquad k = 0.0287$$

$$\text{Pr} = 0.7$$

$$\text{Gr}_\delta\,\text{Pr} = \frac{(9.8)\left(\frac{1}{333}\right)(100-20)(0.08)^3(0.7)}{(19.04\times10^{-6})^2} = 2.33\times10^6$$

$$C = 0.073 \qquad n = \frac{1}{3} \qquad m = -\frac{1}{9}$$

$$\frac{k_e}{k} = (0.073)(2.33\times10^6)^{1/3}\left(\frac{1}{0.08}\right)^{-1/9} = 7.304$$

$$q = \frac{(0.0287)(7.304)(1)^2(100-20)}{0.08} = 209.6\ \text{W}$$

7-91

Take properties at 300 K $\quad \nu = 15.69\times10^{-6} \quad k = 0.02624 \quad \Pr = 0.7$

$$\mathrm{Gr}_\delta \Pr = \frac{(9.8)\left(\frac{1}{300}\right)(30)(0.01)^3(0.7)}{(15.69\times10^{-6})^2} = 2787$$

For horizontal plate $\quad C = 0.059 \quad n = 0.4$

$$\frac{k_e}{k} = (0.059)(2787)^{0.4} = 1.41$$

$$q = \frac{(1.41)(0.02624)(0.3)^2(30)}{0.01} = 9.98\ \mathrm{W}$$

For 30 cm vertical plate

$$\mathrm{Gr}\Pr = (2787)\left(\frac{30}{1}\right)^3 = 7.52\times10^7 \quad C = 0.59 \quad m = \frac{1}{4}$$

$$h = \frac{0.02624}{0.3}(0.59)(7.52\times10^7)^{1/4} = 4.806\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot{}^\circ\mathrm{C}}$$

$$q = hA(T_w - T_\infty)(4.806)(0.3)^2(30) = 129.8\ \mathrm{W}$$

the 9.98 W is a reduction of 93%

7-92

$$T_m = \frac{120+20}{2} = 70^\circ\mathrm{C} = 343\ \mathrm{K} \qquad \mu = 2.04\times10^{-5} \qquad k = 0.0295$$

$\Pr = 0.7 \quad \delta = 1\ \mathrm{cm}$

$$\mathrm{Gr}_\delta = 1700 = \frac{\rho^2(9.8)\left(\frac{1}{343}\right)(120-20)(0.01)^3(0.7)}{(2.04\times10^{-5})^2}$$

$\rho^2 = 0.3537 \quad \rho = 0.595$

At 1 atm $\quad \rho = 1.023$

Thus at $\delta = 1$ cm $\quad p = \dfrac{0.595}{1.023} = 0.582$ atm (58.96 kPa)

For the other spacings:

δ (cm)	ρ^2	ρ	p (atm, kPa)
2	0.0442	0.2103	0.2056, 20.83
5	0.00283	0.0532	0.052, 5.27
10	3.537×10^{-3}	0.0188	0.0184, 1.86

7-94

$q = h\pi dL\Delta T \qquad h = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} \qquad q \sim \Delta T^{5/4}$

$\frac{q_{83}}{175} = \left(\frac{83-27}{55-27}\right)^{5/4} \qquad q_{83} = 416 \text{ W}$

7-95

$$T_f = \frac{140+70}{2} = 105°\text{F} = 40.6°\text{C} = 314 \text{ K}$$

$\nu = 17.11 \times 10^{-6} \text{ m}^2/\text{s} \qquad k = 0.0273 \qquad \Pr = 0.7$

$$\text{Gr}\,\text{Pr} = 10^9 = \frac{(9.8)\left(\frac{1}{314}\right)(140-70)\left(\frac{5}{9}\right)x^3}{(17.11\times 10^{-6})^2}(0.7)$$

$x = 0.701 \text{ m}$

$$h = \frac{0.0273}{0.701}(0.59)(10^9)^{1/4} = 4.09 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\frac{q}{A} = h\Delta T = (4.09)(140-70)\left(\frac{5}{9}\right) = 159 \ \text{W/m}^2$$

$$u = u_{\max} \text{ at } y = \frac{\delta}{3} = 0.148 u_x$$

$u_x = C_1 x^{1/2}$

$$C_1 = (5.17)(17.11\times 10^{-6})\left(\frac{20}{21}+0.7\right)^{-1/2}\left[\frac{(9.8)\left(\frac{1}{314}\right)(70)\left(\frac{5}{9}\right)}{(17.11\times 10^{-6})^2}\right]^{1/2} = 4.43$$

$u_x = (4.43)(0.701)^{1/2} = 3.71$

$u_{\max} = (0.148)(3.71) = 0.55 \text{ m/s}$

7–96

$\delta = 0.025$ m $\quad T_f = 50°C = 323$ K $\quad \nu = 18.02 \times 10^{-6}$ m^2/s

$k = 0.02798 \quad \Pr = 0.7$

$$\mathrm{Gr}_\delta \Pr = \frac{(9.8)\left(\frac{1}{323}\right)(70-30)(0.025)^3(0.7)}{(18.02\times 10^{-6})^2} = 40{,}878$$

$$\frac{k_e}{k} = (0.212)(40{,}878)^{1/4} = 3.014$$

$$k_e = 0.0843 \ \frac{\mathrm{W}}{\mathrm{m}\cdot °\mathrm{C}}$$

$$\frac{q}{A} = (0.0843)\left(\frac{70-30}{0.025}\right) = 135 \ \mathrm{W/m^2}$$

For $\delta = 0.01$ m $\quad \mathrm{Gr}_\delta \Pr = 2616$

$$\frac{k_e}{k} = 0.059(2616)^{0.4} = 1.374$$

$$\frac{q}{A} = (1.374)(0.02798)\left(\frac{70-30}{0.01}\right) = 154 \ \mathrm{W/m^2}$$

7-97

$$\mathrm{Gr}_\delta \Pr = 1700 = 40{,}878\left(\frac{p}{1\ \mathrm{atm}}\right)^2 \ \text{since}\ \rho = \frac{p}{RT} \qquad p = 0.204\ \mathrm{atm}$$

7-98

$$T_f = \frac{120+80}{2} = 100°\mathrm{F} = 37.8°\mathrm{C} = 311\ \mathrm{K}$$

(a) Air $\quad \nu = 16.81\times 10^{-6} \quad k = 0.02707 \quad \Pr = 0.7$

$$\mathrm{Gr}\Pr = \frac{(9.8)\left(\frac{1}{311}\right)(120-80)\left(\frac{5}{9}\right)(0.025)^3(0.7)}{(16.81\times 10^{-6})^2} = 27{,}100$$

$$h = \frac{0.02707}{0.025}[2 + 0.43(27{,}100)^{1/4}] = 8.14 \ \frac{\mathrm{W}}{\mathrm{m}^2\cdot °\mathrm{C}}$$

$$q = hA\Delta T = (8.14)4\pi(0.0125)^2(40)\left(\frac{5}{9}\right) = 0.36\ \mathrm{W}$$

(b) Water $\dfrac{g\beta\rho^2 c_p}{\mu k} = 3.3\times10^{10}$ $k \doteq 0.630$

$$\mathrm{Gr\,Pr} = (3.3\times10^{10})(0.025)^3(120-80)\left(\frac{5}{9}\right) = 1.146\times10^7$$

$$h = \frac{0.63}{0.025}[2+0.50(1.146\times10^7)^{1/4}] = 783\ \frac{\mathrm{W}}{\mathrm{m^2\cdot{}^\circ C}}$$

$$q = hA\Delta T = (783)4\pi(0.0125)^2(40)\left(\frac{5}{9}\right) = 34.2\ \mathrm{W}$$

7-99

$T_f = 350$ K $\nu = 20.76\times10^{-6}$ $k = 0.03003$ $\mathrm{Pr} = 0.7$

$r_i = 0.04$ $r_0 = 0.05$ $\delta = 0.01$

$$\mathrm{Gr}_\delta\,\mathrm{Pr} = \frac{(9.8)\left(\frac{1}{350}\right)(400-300)(0.01)^3(0.7)}{(20.76\times10^{-6})^2} = 4548$$

$$\frac{k_e}{k} = (0.228)(4548)^{0.226} = 1.53$$

$$k_e = (0.03003)(1.53) = 0.0459$$

$$q = \frac{4\pi(0.0459)(0.04)(0.05)(100)}{0.05-0.04} = 11.54\ \mathrm{W}$$

7-100

$d = L = 0.08$ m $C = 0.775$ $m = 0.21$ $T_f = 287.5$ K

$\nu = 14.6\times10^{-6}$ $k = 0.0252$ $\mathrm{Pr} = 0.71$

$$\mathrm{Gr\,Pr} = \frac{(9.8)\left(\frac{1}{287.5}\right)(300-275)(0.08)^3(0.71)}{(14.6\times10^{-6})^2} = 1.45\times10^6$$

$$h = \frac{0.0252}{0.08}(0.775)(1.45\times10^6)^{0.21} = 4.81\ \frac{\mathrm{W}}{\mathrm{m^2\cdot{}^\circ C}}$$

$$hA = (4.81)[2\pi(0.04)^2 + \pi(0.08)(0.08)] = 0.1452$$

$$\rho c V = (999)(4180)\pi(0.04)^2(0.08) = 1679$$

$$\frac{hA}{\rho c V} = \frac{0.1452}{1679} = 8.65\times10^{-5}$$

$$\frac{290-275}{300-275} = \exp[-8.65\times10^{-5}\tau]$$

$$\tau = 5905\ \mathrm{sec} = 1.64\ \mathrm{hr}$$

7-101

$$L = \frac{A}{\rho} = \frac{\pi d^2}{4\pi d} = \frac{d}{4}$$

$$T_f = \frac{120+80}{2} = 100°\mathrm{F} = 37.8°\mathrm{C}$$

$$\frac{g\beta\rho^2 c_p}{\mu k} = 3.3\times 10^{10} \qquad k = 0.63$$

$$\mathrm{Gr\,Pr} = (3.3\times10^{10})\left(\frac{0.05}{4}\right)^3(120-80)\left(\frac{5}{9}\right) = 1.43\times10^6$$

Upper surface $\qquad C = 0.54 \qquad m = \frac{1}{4}$

$$h = \frac{0.63}{0.05/4}(0.54)(1.43\times10^6)^{1/4} = 942\ \frac{\mathrm{W}}{\mathrm{m^2\cdot°C}}$$

$$q = (942)\pi(0.025)^2(40)\left(\frac{5}{9}\right) = 41.1\ \mathrm{W}$$

Lower surface $\qquad C = 0.27 \qquad m = \frac{1}{4}$

$$h = 471\ \frac{\mathrm{W}}{\mathrm{m^2\cdot°C}}$$

$$q = 20.6\ \mathrm{W}$$

7-102

$$\theta = 45° \qquad L = 0.1\ \mathrm{m} \qquad T_f = 350\ \mathrm{K}$$

$$T_e = 400 - 0.25(400-300) = 375\ \mathrm{K}$$

$$\nu = 23.33\times10^{-6} \qquad k = 0.0318 \qquad \mathrm{Pr} = 0.69$$

$$\mathrm{Gr}_e\,\mathrm{Pr}_e\cos\theta = \frac{(9.8)\left(\frac{1}{350}\right)(400-300)(0.1)^3(0.7)(0.707)}{(23.33\times10^{-6})^2} = 2.54\times10^6$$

Eq. (7–43)

$$h = \frac{0.0318}{0.1}(0.56)(2.54\times10^6)^{1/4} = 7.11\ \frac{\mathrm{W}}{\mathrm{m^2\cdot°C}}$$

$$q = hA\Delta T = (7.11)(0.1)^2(400-300) = 7.11\ \mathrm{W}$$

7-103

Upward facing surface

Because $\mathrm{Gr}_e < \mathrm{Gr}_c$ 1st term of Eq. (7–46) drops out and the result is the same as in Prob. (7–98).

7-104

$$\frac{D}{L} \geq \frac{35}{\mathrm{Gr}_L{}^{1/4}} \qquad L = 50 \text{ cm} = 0.5 \text{ m} \qquad T_f = 350 \text{ K}$$

$$\nu = 20.76 \times 10^{-6} \qquad k = 0.03003 \qquad \mathrm{Pr} = 0.7$$

$$\mathrm{Gr}_L = \frac{(9.8)\left(\frac{1}{350}\right)(400-300)(0.5)^3}{(20.76 \times 10^{-6})^2} = 8.12 \times 10^8$$

$$d_{\min} = (0.5)(35)(8.12 \times 10^8)^{-1/4} = 0.104 \text{ m}$$

7-108

Const heat flux, $T_w = 65°\text{C} = 338 \text{ K}$ $\qquad T_\infty = 20°\text{C} = 293 \text{ K}$

$L = 0.5 \text{ m}$

$$T_f = \frac{65+20}{2} = 42.5°\text{C} = 316 \text{ K}$$

$$\nu = 17.3 \times 10^{-6} \qquad \mathrm{Pr} = 0.7 \qquad k = 0.0275$$

For laminar $\quad \bar{h} = \frac{5}{4} h_{x=L}$

$$\mathrm{Gr}\,\mathrm{Pr} = \frac{(9.8)\left(\frac{1}{316}\right)(65-20)(0.5)^3(0.7)}{(17.3 \times 10^{-6})^2} = 4.08 \times 10^8$$

$$\mathrm{Nu}_x = 0.6(\mathrm{Gr}_x \mathrm{Nu}_x \mathrm{Pr})^{1/5} = 75.05$$

$$h_{x=L} = \frac{(75.05)(0.0275)}{0.5} = 4.13 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$\bar{h} = \left(\frac{5}{4}\right)(4.13) = 5.16 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q_{\text{conv}} = \bar{h}A(T_w - T_\infty) = (5.16)(2)(0.5)^2(65-20) = 116.1 \text{ W}$$

$$q_{\text{rad}} = \sigma\varepsilon A(T_w{}^4 - T_\infty{}^4) = (5.668 \times 10^{-8})(2)(0.5)^2(338^4 - 293^4) = 161 \text{ W}$$

$$q_{\text{total}} = 116.1 + 161 = 277.1 \text{ W}$$

7-109

$$L = 0.2 \text{ m} \qquad T_w = 30°\text{C} \qquad T_w = 10°\text{C} \qquad T_f = \frac{30+10}{2} = 20°\text{C}$$

$$\nu = 0.00118 \qquad k = 0.286 \qquad \mathrm{Pr} = 12.5 \times 10^3 \qquad \beta = 0.5 \times 10^{-3}$$

$$\mathrm{Gr}\,\mathrm{Pr} = \frac{g\beta\Delta T L^3}{\nu^2}\mathrm{Pr} = \frac{(9.8)(0.5 \times 10^{-3})(30-10)(0.2)^3(12.5 \times 10^3)}{(0.00118)^2} = 7.038 \times 10^6$$

$$\mathrm{Nu} = C(\mathrm{Gr}\,\mathrm{Pr})^m = (0.59)(7.038 \times 10^6)^{1/4} = 30.39$$

$$\bar{h} = \frac{(30.39)(0.286)}{0.2} = 43.46 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (43.46)(2)(0.2)^2(30-10) = 69.53 \text{ W}$$

7-110

$T_w = 43.3°C$ $\quad T_\infty = 10°C$ $\quad T_f = 26.65°C$ $\quad L = d = 0.3$ m

$\dfrac{g\beta\rho^2 c_p}{\mu k} = 1.91\times10^{10}$ $\quad \Pr = 5.85$ $\quad k = 0.614$

$$\mathrm{Gr}_L \Pr = (1.91\times10^{10})(0.3)^2(43.3-10) = 1.72\times10^{10}$$

$$\mathrm{Gr}_L = \frac{1.72\times10^{10}}{5.85} = 2.94\times10^9 = \mathrm{Gr}_d$$

$\dfrac{35}{\mathrm{Gr}_L{}^{1/4}} = 0.15$ $\quad \dfrac{d}{L} = 1.0 > 0.15$

Treat as vertical plate

Sides $\quad C = 0.1 \quad m = \dfrac{1}{3}$

$$\bar{h} = \frac{0.614}{0.3}(0.1)(1.72\times10^{10})^{1/3} = 528\ \frac{\mathrm{W}}{\mathrm{m^2\cdot °C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (528)\pi(0.3)(0.3)(43.3-10) = 4974\ \mathrm{W}$$

Top and Bottom $\quad L = \dfrac{A}{P} = \dfrac{d}{4} = 0.075$ m

$$\mathrm{Gr}_L \Pr = \frac{1.72\times10^{10}}{(4)^3} = 2.69\times10^8$$

Top $\quad C = 0.15 \quad m = \dfrac{1}{3}$

Bottom $\quad C = 0.27 \quad m = \dfrac{1}{4} \quad A = \dfrac{\pi d^2}{4}$

$$h(\mathrm{Top}) = \frac{0.614}{0.075}(0.15)(2.69\times10^8)^{1/3} = 792\ \frac{\mathrm{W}}{\mathrm{m^2\cdot °C}}$$

$$h(\mathrm{Bottom}) = \frac{0.614}{0.075}(0.27)(2.69\times10^8)^{1/4} = 283\ \frac{\mathrm{W}}{\mathrm{m^2\cdot °C}}$$

$$q(\mathrm{Top+Bottom}) = (792+283)\frac{\pi(0.3)^2}{4}(43.3-10) = 2530\ \mathrm{W}$$

$$q(\mathrm{Total}) = 4974 + 2530 = 7504\ \mathrm{W}$$

<u>Second Method</u>

Treat solid as vertical plate with characteristic dimension $= L + (2)\left(\dfrac{d}{2}\right) = 0.6$ m

$$\text{Gr Pr} = (1.72\times10^{10})\left(\frac{0.6}{0.3}\right)^3 = 1.38\times10^{11}$$

$$C = 0.1 \qquad m = \frac{1}{3}$$

$$\text{Nu} = (0.1)(1.38\times10^{11})^{1/3} = 516$$

$$\bar{h} = \frac{(516)(0.614)}{0.6} = 528\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$A(\text{total}) = \pi dL + 2\left(\frac{\pi d^2}{4}\right) = \pi\left[(0.3)(0.3) + \frac{(0.3)^2}{2}\right] = 0.424\ \text{m}^2$$

$$q(\text{total}) = \bar{h}A_{\text{total}}(T_w - T_\infty) = (528)(0.424)(43.3 - 10) = 7457\ \text{W}$$

Good agreement between the two methods

7-111

$$T_f = 350\ \text{K} \qquad \nu = \frac{20.8\times10^{-6}}{1.5}$$

O_2 $\quad k = 0.0307 \qquad \text{Pr} = 0.702$

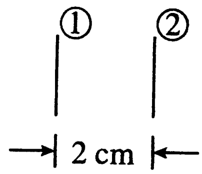
$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{350}\right)(400-300)(0.01)^3(0.702)}{(20.8\times10^{-6})^2} = 10{,}222$$

$$C = 0.53 \qquad m = \frac{1}{4}$$

$$\bar{h} = \frac{0.0307}{0.01}(0.53)(10{,}222)^{1/4} = 16.36\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (16.36)\pi(0.01)(1.25)(400-300) = 64.25\ \text{W}$$

7-112



$\lambda = 2\text{ cm} = 0.02\text{ m} = L$
$T_1 = 400\text{ K} \qquad \varepsilon_1 = 0.1$
$T_2 = 300\text{ K} \qquad \varepsilon_2 = 0.15$
$\alpha_1 = \alpha_2 = 0.9$

$$k = 0.03003\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad \text{Pr} = 0.7 \qquad \gamma = 1.4$$

$$\Delta T = \frac{2-\alpha}{\alpha}\,\frac{2\gamma}{\gamma+1}\,\frac{\lambda}{\text{Pr}}\,\frac{T_1 - T_2 - 2\Delta T}{L} = \left(\frac{1.1}{0.9}\right)\left(\frac{2.8}{2.4}\right)\left(\frac{0.02}{0.7}\right)\left(\frac{400-300-2\Delta T}{0.02}\right)$$
$$= 40.15^\circ\text{C}$$

$$\left.\frac{q}{A}\right|_{\text{cond}} = k\left(\frac{T_1 - T_2 - 2\Delta T}{L}\right) = (0.03003)\left[\frac{400-300-(2)(40.15)}{0.02}\right] = 29.59\ \text{W/m}^2$$

$$\left.\frac{q}{A}\right|_{\text{rad}} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(5.668\times10^{-8})(400^4 - 300^4)}{\frac{1}{0.1}+\frac{1}{0.15}-1} = 63.31\ \text{W/m}^2$$

$$\left.\frac{q}{A}\right|_{\text{Total}} = 29.59 + 63.31 = 92.89\ \text{W/m}^2$$

7-113

$$T_w = 400\text{ K} \qquad T_\infty = 20^\circ\text{C} = 293\text{ K} \qquad d = 0.4\text{ m}$$

$$T_f = \frac{400+293}{2} = 347\text{ K}$$

$$\nu = 20.5\times10^{-6} \qquad k = 0.03 \qquad \text{Pr} = 0.7$$

$$\text{Gr Pr} = \frac{(9.8)\left(\frac{1}{347}\right)(400-293)(0.4)^3(0.7)}{(20.5\times10^{-6})^2} = 3.23\times10^8$$

$$\text{Nu} = 2 + 0.5(\text{Gr Pr})^{1/4} = 2 + (0.5)(3.23\times10^8)^{1/4} = 69.0$$

$$\bar{h} = \frac{(69)(0.03)}{0.4} = 5.18\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (5.18)4\pi(0.2)^2(400-293) = 279\text{ W}$$

$$q_{\text{rad}} = \sigma\varepsilon A(T_w{}^4 - T_\infty{}^4) = (5.668\times10^{-8})(0.9)(4\pi)(0.2)^2(400^4 - 293^4) = 467\text{ W}$$

7-114

$L = 0.2$ m $\quad T_1 = 350$ K $\quad T_2 = 400$ K $\quad \delta = 0.02$ m

$$\bar{T} = \frac{350 + 400}{2} = 375 \text{ K}$$

Helium at $p = 2$ atm

$$\nu = \frac{181 \times 10^{-6}}{2} = 90.5 \times 10^{-6} \qquad k = 0.171 \qquad \Pr = 0.71$$

$$\mathrm{Gr}_\delta \Pr = \frac{g\beta\Delta T\delta^3}{\nu^2}\Pr = \frac{(9.8)\left(\frac{1}{375}\right)(400-350)(0.02)^3(0.71)}{(90.5 \times 10^{-6})^2} = 906 < 2000$$

Therefore $\quad \dfrac{k_e}{k} = 1.0 \qquad q = k_e A\dfrac{\Delta T}{\delta} = (0.171)(0.2)^2\left(\dfrac{400-350}{0.02}\right) = 17.1 \text{ W}$

7-115

$T_w = 125°\text{C} = 398$ K $\quad T_\infty = 25°\text{C} = 298$ K $\quad T_f = 348$ K

$\nu = 20.7 \times 10^{-6} \quad k = 0.03 \quad \Pr = 0.7$

$$\text{Characteristic} = \frac{A}{P} = \frac{L^2}{4L} = \frac{L}{4} = 0.075 \text{ m}$$

$$\mathrm{Gr}\Pr = \frac{(9.8)\left(\frac{1}{348}\right)(125-25)(0.075)^3(0.7)}{(20.7 \times 10^{-6})^2} = 1.94 \times 10^6$$

Top $\quad C = 0.54 \quad m = \dfrac{1}{4}$

Bottom $\quad C = 0.27 \quad m = \dfrac{1}{4}$

Top $\quad \bar{h} = \dfrac{0.03}{0.075}(0.54)(1.94 \times 10^6)^{1/4} = 8.06 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

Bottom $\quad \bar{h} = \dfrac{0.27}{0.54}(8.06) = 4.03 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$q(\text{total}) = (8.06 + 4.03)(0.3)^2(125 - 25) = 108.8$ W

7-116

$T_w = 134°\text{C} = 407\text{ K}$ $\quad T_\infty = 20°\text{C} = 293\text{ K}$ $\quad d = 0.001\text{ m}$

$T_f = 350\text{ K}$ $\quad k = 0.03003$ $\quad \Pr = 0.697$ $\quad \nu = 20.76\times10^{-6}$

$$\text{Gr}\,\text{Pr} = \frac{(9.8)\left(\frac{1}{350}\right)(134-20)(0.001)^3(0.697)}{(20.76\times10^{-6})^2} = 5.16$$

$C = 1.02$ $\quad m = 0.148$ $\quad \text{Nu} = C(\text{Gr}\,\text{Pr})^m = (1.02)(5.16)^{0.148} = 1.3$

$$\bar{h} = \frac{(1.3)(0.03003)}{0.001} = 39.05\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (39.05)\pi(0.001)(1)(134-20) = 15.21\text{ W} = \dot{q}V = \dot{q}\left(\frac{\pi d^2}{4}\right)L$$

$$\dot{q} = \frac{(15.21)(4)}{\pi(0.001)^2(1)} = 1.94\times10^7\ \text{W/m}^3$$

From chap. 2 $\quad T_0 - T_\infty = \dfrac{\dot{q}r^2}{4k} = \dfrac{(1.94\times10^7)(0.0005)^2}{(4)(16)} = 0.076°\text{C}$

$T_0 = 134.076°\text{C}$

$$R_{\text{elec}} = \rho_e\frac{L}{A} = \frac{(70\times10^{-6})(100)(4)}{\pi(0.1)^2} = 0.89\ \Omega$$

$$q = \frac{E^2}{R} = 15.21\text{ W}$$

$E = [(15.21)(0.89)]^{1/2} = 3.68\text{ Volts}$

7-117

$L = 0.3\text{ m}$ $\quad d = 0.105\text{ m}$ $\quad T_\infty = 15°\text{C} = 288\text{ K}$

$T_w = 100°\text{C} = 373\text{ K}$ $\quad T_f = 330.5\text{ K}$ $\quad \nu = 18.7\times10^{-6}$

$k = 0.029$ $\quad \Pr = 0.7$

$$\text{Gr}_L = \frac{(9.8)\left(\frac{1}{330.5}\right)(100-15)(0.3)^3}{(18.7\times10^{-6})^2} = 1.95\times10^8$$

$\dfrac{d}{L} = \dfrac{0.105}{0.3} = 0.35$ $\quad \dfrac{35}{\text{Gr}_L^{1/4}} = 0.296 < 0.35$ $\quad$ treat as vertical plate

$\text{Gr}_L\,\Pr = (1.95\times10^8)(0.7) = 1.365\times10^8$

$C = 0.59$ $\quad m = \dfrac{1}{4}$ $\quad \text{Nu} = C(\text{Gr}\,\text{Pr})^m = (0.59)(1.365\times10^8)^{1/4} = 63.77$

$$\bar{h} = \frac{(63.77)(0.029)}{0.3} = 6.16\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (6.16)\pi(0.105)(0.3)(100-15) = 51.9\text{ W}$$

7-118

$T_w = 865\ \text{K}$ $\quad T_\infty = 300\ \text{K}$ $\quad d = 0.025\ \text{mm}$ $\quad T_f = 582.5\ \text{K}$

$\nu = 48.9 \times 10^{-6}$ $\quad k = 0.0455$ $\quad \Pr = 0.683$

$$\text{Gr}\Pr = \frac{g\beta\Delta T d^3 \Pr}{\nu^2} = \frac{(9.8)\left(\frac{1}{582.5}\right)(865-300)(0.025\times10^{-3})^3(0.683)}{(48.9\times10^{-6})^2} = 4.24\times10^{-5}$$

$C = 0.675$ $\quad m = 0.058$ $\quad \text{Nu} = C(\text{Gr}\Pr)^m = (0.675)(4.24\times10^{-5})^{0.058} = 0.376$

$$\bar{h} = \frac{(0.376)(0.0455)}{0.025\times10^{-3}} = 685\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q_{\text{conv}} = \bar{h}A(T_w - T_\infty) = (685)\pi(0.025\times10^{-3})(865-300) = 30.4\ \text{W/m}\cdot\text{length}$$

$$q_{\text{rad}} = \sigma A\varepsilon(T_w^{\ 4} - T_\infty^{\ 4}) = (5.668\times10^{-8})\pi(0.025\times10^{-3})(0.9)(865^4 - 300^4)$$
$$= 2.21\ \text{W/m}\cdot\text{length}$$

7-119

$T_w = 400\ \text{K}$ $\quad T_\infty = 300\ \text{K}$ $\quad T_f = 350\ \text{K}$ $\quad \nu = 20.76\times10^{-6}$ $\quad k = 0.03003$

L

$\Pr = 0.697$ $\quad A = \frac{1}{2}L\left(\frac{\sqrt{3}}{2}L\right) = \frac{\sqrt{3}}{4}L^2$ $\quad L = 0.2\ \text{m}$

$$\text{characteristic dimension} = \frac{A}{P} = \frac{\frac{\sqrt{3}}{4}L^2}{3L} = \frac{L}{4\sqrt{3}} = 0.144L = 0.0288\ \text{m}$$

$$\text{Gr}\Pr = \frac{(9.8)\left(\frac{1}{350}\right)(400-300)(0.0288)^3(0.697)}{(20.76\times10^{-6})^2} = 1.09\times10^5$$

$C = 0.54$ $\quad m = \frac{1}{4}$ $\quad \text{Nu} = C(\text{Gr}\Pr)^m = (0.54)(1.09\times10^5)^{1/4} = 9.81$

$$\bar{h} = \frac{(9.81)(0.03003)}{0.0288} = 10.23\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (10.23)(0.2)^2\frac{\sqrt{3}}{4}(400-300) = 17.7\ \text{W}$$

7-120

Water $\quad d = 10$ cm $\quad T_w = 49°C \quad T_\infty = 10°C \quad T_f = 29.5°C$

$$\text{characteristic dim} = \frac{A}{P} = \frac{d}{4} = 0.025 \text{ m}$$

$$\frac{g\beta\rho^2 c_p}{\mu k} = 2.2\times 10^{10} \qquad k = 0.618$$

$$\text{Gr Pr} = (2.2\times 10^{10})(0.025)^3(49-10) = 1.34\times 10^7$$

$$C = 0.15 \qquad m = \frac{1}{3} \qquad \text{Nu} = C(\text{Gr Pr})^m = (0.15)(1.34\times 10^7)^{1/3} = 35.6$$

$$\bar{h} = \frac{(35.6)(0.618)}{0.025} = 881 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \bar{h}A(T_w - T_\infty) = (881)\pi(0.05)^2(49-10) = 270 \text{ W}$$

7-125

$$T_f = \frac{15-10}{2} = 2.5°\text{C} = 276 \text{ K} \qquad \nu = 13.5\times 10^{-6} \ \text{m}^2/\text{s} \qquad k = 0.0243$$

$$\text{Pr} = 0.71 \qquad \delta = 0.025 \text{ m} \qquad L = 1.0 \text{ m}$$

$$\text{Gr}_\delta \text{ Pr} = \frac{(9.8)\left(\frac{1}{276}\right)(0.025)^3(0.71)(15+10)}{(13.5\times 10^{-6})^2} = 54{,}231$$

$$C = 0.197 \qquad n = \frac{1}{4} \qquad m = -\frac{1}{9}$$

$$\frac{k_e}{k} = (0.197)(54{,}231)^{1/4}\left(\frac{1}{0.025}\right)^{-1/9} = 1.995$$

$$k_e = 0.0485 \ \frac{\text{W}}{\text{m}\cdot°\text{C}}$$

$$\frac{q}{A} = \frac{\Delta T}{R} = \frac{\Delta T}{\delta/k_e}$$

$$R = \frac{\delta}{k_e} = \frac{0.025}{0.0485} = 0.516$$

For fiberglass blanket, $k = 0.04 \ \frac{\text{W}}{\text{m}\cdot°\text{C}}$

$$R = \frac{0.025}{0.04} = 0.625$$

7-126
.2

For $\frac{k_e}{k} = 1.0$ $\quad$ $Gr_\delta \Pr < 2000$

Worst condition $\quad$ $\Delta T = 200°C$

$\nu \sim 20\times10^{-6}$ $\quad$ $\beta = \frac{1}{373}$ $\quad$ $\Pr = 0.7$

$$Gr_\delta \Pr = \frac{g\beta\Delta T\delta^3 \Pr}{\nu^2} = 2000$$

$$\frac{(9.8)(\frac{1}{373})(200)\delta^3(0.7)}{(20\times10^{-6})^2} = 2000$$

$\delta = 0.006$ m

For smaller values of ΔT (multiple layers) the spacing would be larger.

7-127

The apparent "leakage" of the window in the winter is in reality the downward free convection flow of cool air from the vertical window surface. In the summer, the free convection flow is upward so Aunt Maude feels no draft. An estimate of the draft velocity may be made by calculating the free convection velocity at the bottom of a vertical plate under typical winter conditions. The window might be about 3 ft high and a typical temperature difference might be 20°C, although it would obviously depend on weather conditions.

7-128

Neglect resistance of steel duct. Also, forced convection h is much larger than free convection h so that $T_w \approx 5°C$

$T_f = \frac{5+20}{2} = 12.5°C = 286$ K $\quad$ $d = 0.18$ m $\quad$ $\nu = 14.5\times10^{-6}$

$\Pr = 0.71$ $\quad$ $k = 0.0251$ $\quad$ $L = 30$ m

$$Gr \Pr = \frac{(9.8)(\frac{1}{286})(20-5)(0.18)^3(0.71)}{(14.5\times10^{-6})^2} = 9.98\times10^6$$

$C = 0.53$ $\quad$ $m = \frac{1}{4}$ $\quad$ $Nu = C(Gr\Pr)^m = (0.53)(9.98\times10^6)^{1/4} = 29.79$

$$\bar{h} = \frac{(29.79)(0.0251)}{0.18} = 4.15 \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = \dot{m}c_p(T_2 - T_1) = \bar{h}A\left(T_\infty - \frac{T_1+T_2}{2}\right)$$

$$\dot{m} = \rho A_c u_m (1.27)\pi(0.09)^2(7.5) = 0.242 \text{ kg/s}$$

$$(0.242)(1005)(T_2 - 5) = (4.15)\pi(0.18)(30)\left(20 - \frac{5+T_2}{2}\right)$$

$T_2 = 8.8°C$

7-129

$d = 5$ mm and 50 mm range
$\Delta T \sim 30$ to 210°C

$$\beta \approx \frac{1}{373} \qquad \Pr = 0.7 \qquad \nu \approx 20 \times 10^{-6}$$

Low limit

$$\mathrm{Gr}\,\Pr = \frac{g\beta\Delta T d^3 \Pr}{\nu^2} = \frac{(9.8)(20)\left(\frac{1}{373}\right)(0.005)^3(0.7)}{(20 \times 10^{-6})^2} = 115$$

High limit

$$\mathrm{Gr}\,\Pr = \frac{(9.8)(210)\left(\frac{1}{373}\right)(0.05)^3(0.7)}{(20 \times 10^{-6})^2} = 1.21 \times 10^6$$

7-130

The general idea in this problem is to determine the surface temperature necessary to dissipate the same amount of energy in forced convection as in free convection. Because the forced convection value of h is larger than the value for free convection a lower surface temperature is required, and thereby produces the wind chill effect. The key assumption required is that of the surface temperature under still air conditions, and that obviously depends on the clothing the person is wearing! Although body temperature is about 98.6°F, the skin temperature can be much cooler; certainly when exposed to winter weather. Take 75°F as a first assumption for skin temperature, and then work through for other assumed values.

Chapter 8

8–1

(a) $T = 800°C = 1073\ K$ $\qquad \lambda_1 T = (0.2)(1073) = 214.6$

$\lambda_2 T = (4)(1073) = 4292 \qquad E_{bo-\lambda_1} = 0 \qquad E_{bo-\lambda_2} = 0.53131$

$E_b = (5.669 \times 10^{-8})(1073)^4 = 7.515 \times 10^4\ \ W/m^2$

$q_{trans} = (0.53131)(0.9)(7.515 \times 10^4) = 35{,}933\ \ W/m^2$

(b) $T = 550°C = 823\ K \qquad \lambda_2 T = (4)(823) = 3292$

$E_b = (5.669 \times 10^{-8})(823)^4 = 26{,}007 \qquad E_{bo-\lambda_2} = 0.33834$

$E_{bo-\lambda_1} = 0$

$q_{trans} = (0.33834)(0.9)(26{,}007) = 7919\ \ W/m^2$

(c) $T = 250°C = 523\ K \qquad \lambda_2 T = (4)(523) = 2092$

$E_b = (5.669 \times 10^{-8})(523)^4 = 4241 \qquad E_{bo-\lambda_2} = 0.08182$

$q_{trans} = (0.08182)(0.9)(4241) = 312.3\ \ W/m^2$

(d) $T = 70°C = 343\ K \qquad \lambda_2 T = (4)(343) = 1372$

$E_b = (5.669 \times 10^{-8})(343)^4 = 784.7 \qquad E_{bo-\lambda_2} = 0.0069$

$q_{trans} = (0.0069)(0.9)(784.7) = 4.87\ \ W/m^2$

8–2

(a) $\lambda_2 T = (5.5)(1073) = 5901.5 \qquad E_{bo-\lambda_2} = 0.72921$

$q_{trans} = (0.72921)(0.85)(75{,}150) = 46{,}580\ \ W/m^2$

(b) $\lambda_2 T = (5.5)(823) = 4527 \qquad E_{bo-\lambda_2} = 0.5684$

$q_{trans} = (0.5684)(0.85)(26{,}007) = 12{,}565\ \ W/m^2$

(c) $\lambda_2 T = (5.5)(523) = 2877 \qquad E_{bo-\lambda_2} = 0.24497$

$q_{trans} = (0.24497)(0.85)(4241) = 883\ \ W/m^2$

(d) $\lambda_2 T = (5.5)(343) = 1887 \qquad E_{bo-\lambda_2} = 0.05059$

$q_{trans} = (0.05059)(0.85)(784.7) = 33.74\ \ W/m^2$

8–3

(a) $\lambda_2 T = (52)(1073) = 55{,}796 \qquad E_{bo-\lambda_2} = 1.0$

$q_{\text{trans}} = (1.0)(0.92)(75{,}150) = 69{,}138 \ \text{W/m}^2$

(b) $\lambda_2 T = (52)(823) = 42{,}796 \qquad E_{bo-\lambda_2} = 0.998$

$q_{\text{trans}} = (0.998)(0.92)(26{,}007) = 23{,}880 \ \text{W/m}^2$

(c) $\lambda_2 T = (52)(523) = 27{,}196 \qquad E_{bo-\lambda_2} = 0.993$

$q_{\text{trans}} = (0.993)(0.92)(4241) = 3874 \ \text{W/m}^2$

(d) $\lambda_2 T = (52)(343) = 17{,}836 \qquad E_{bo-\lambda_2} = 0.97912$

$q_{\text{trans}} = (0.97912)(0.92)(784.7) = 706.8 \ \text{W/m}^2$

8–4

$\lambda_1 T = (4)(560) = 2240 \ \mu \cdot {}^\circ\text{R} \qquad \lambda_2 T = (15)(560) = 8400 \ \mu \cdot {}^\circ\text{R}$

$\dfrac{E_{b\,(0-\lambda_1)}}{E_{b\,(0-\infty)}} = 0.00306 \qquad \dfrac{E_{b\,(0-\lambda_2)}}{E_{b\,(0-\infty)}} = 0.5890$

$$E_{b(\lambda_1-\lambda_2)} = (1.714\times10^{-9})(560)^4(0.5890-0.00306) = 98.9 \ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2}$$

$$E_{(\lambda_1-\lambda_2)} = (0.6)(98.9) = 59.3 \ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2} = 187 \ \text{W/m}^2$$

8–6

		$T = 300$ K		$T = 1000$ K		$T = 5000$ K	
λ	α	λT	frac	λT	frac	λT	frac
0	0.05	0	0	0	0	0	0
1.2	0.5	360	0	1200	0.0021	6000	0.7377
3	0.4	900	0.74×10^{-4}	3000	0.273	15,000	0.969
6	0.2	1800	0.039	6000	0.7377	30,000	0.995
20	0	6000	0.7377	20,000	0.9855	100,000	1.0
∞	0	—	—	—	—	—	—

		$T = 300$ K	$T = 1000$ K	$T = 5000$ K
λ	α	Frac $\alpha \times$ frac		
0–1.2	0.05	0	0.0021	0.7377
1.2–3	0.5	0.74×10^{-4}	0.271	0.231
3–6	0.4	0.039	0.465	0.026
6–20	0.2	0.699	0.248	0.005
20–∞	0	0	0	0
$\sum \alpha$ Frac =		0.155	0.371	0.164

8–7

$T = 5795$ K

λ	λT	Frac	λ	Frac contained
0	0		0–0.2	0.0015
0.2	1159	0.0015	0.2–0.4	0.1235
0.4	2318	0.125	0.4–1.0	0.596
1.0	5795	0.721	1.0–2.0	0.219
2.0	11,590	0.94	over 2.0	0.06
∞		1.0		1.0

8-10

$T = 1073$ K

λ	$\lambda T(\mu - k)$	fraction $0 - \lambda T$	fraction $\lambda_1 T - \lambda_2 T$
1	1073	7.74×10^{-4}	—
2	2146	9.13×10^{-2}	8.35×10^{-3}
3	3219	0.32233	0.23103
4	4292	0.53131	0.20898
5	5365	0.67651	0.1452
6	6348	0.76538	0.08887

8-11

$T = 1373$ K

fraction	λT	$\lambda(\mu)$
0.25	2897	2.11
0.5	4100	2.99
0.75	6149	4.48
0.98	18,222	13.27

8-12

$$E_b = \sigma(1400)^4 = 2.177 \times 10^5 \ \ \mathrm{W/m^2}$$

λ	λT	Fraction
0.6	840	0.00008
5	7000	0.80807

$$E = (2.177 \times 10^5)[(0.08)(0.00008) + (0.4)(0.80807 - 0.00008)$$
$$+ (0.7)(1 - 0.80807)]$$
$$= 1.000 \times 10^5 \ \ \mathrm{W/m^2}$$

8-13

$E = \int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda$ $\qquad T = 2000$ K $\qquad E_b = 9.07 \times 10^5$ $\qquad T^5 = 3.2 \times 10^{16}$

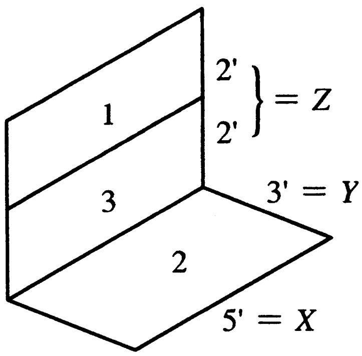
λ	λT	ε_λ	$E_{b\lambda}$	$\varepsilon_\lambda E_{b\lambda}$
0	0	0	0	0
0.5	1000	0.25	6752	1688
1.0	2000	0.5	281,145	140,573
1.5	3000	0.5	410,384	205,192
2.0	4000	0.5	329,366	164,683
2.5	5000	0.5	228,371	114,186
3.0	6000	0.4	153,907	61,563
3.5	7000	0.3	104,573	31,372
4.0	8000	0.2	22,467	14,493
4.5	9000	0.1	19,363	1936
5.0	10,000	0		0

$$E = \int_0^{10} \varepsilon_\lambda E_{b\lambda} d\lambda = \Delta\lambda \sum \varepsilon_\lambda E_{b\lambda} = (0.5)(735{,}686) = 367{,}843 \ \text{W/m}^2$$

$$\varepsilon \ (\text{avg.}) = \frac{E}{E_b} = \frac{3.678}{9.07} = 0.406$$

8-14

(a)



Find F_{1-2}

$$\frac{Y}{X} = \frac{3}{5} = 0.6 \qquad \frac{Z}{X} = \frac{4}{5} = 0.8$$

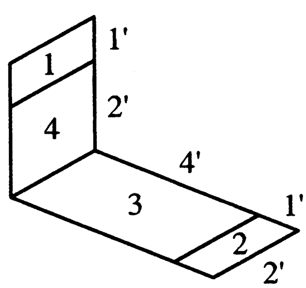
$$F_{2-3,1} = F_{2-3} + F_{2-1} = 0.25$$

$$F_{2-3} = 0.17$$

$$F_{2-1} = F_{2-3,1} - F_{2-3} = 0.08$$

$$A_1 F_{1-2} = A_2 F_{2-1} \quad \therefore \quad F_{1-2} = 0.105$$

(b)



Find F_{1-2}

$$F_{1-2} = \frac{1}{A_1}[A_{1,4}(F_{1,4-3,2} - F_{1,4-3}) + A_4(F_{4-3} - F_{4-3,2})]$$

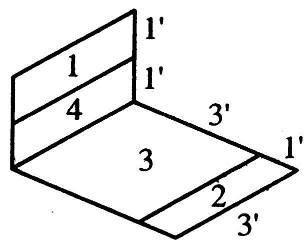
$$\frac{Y_{4-1}}{X} = \frac{3}{2} = 1.5 \qquad \frac{Z_{32}}{X} = \frac{5}{2} = 2.25 \qquad \frac{Y_{14}}{X} = 1.5$$

$$\frac{Z_3}{X} = 2 \qquad \frac{Y_4}{X} = 1 \qquad \frac{Z_3}{X} = 2 \qquad \frac{Z_{32}}{X} = \frac{5}{2} = 2.25$$

$$F_{1,4} = 0.19 \qquad F_{1,4-3} = 0.18 \qquad F_{4-3} = 0.23 \qquad F_{4-3,2} = 0.24$$

$$F_{1-2} = \frac{1}{2}[6(0.19 - 0.18) + 4(0.23 - 0.24)] = 0.01$$

(c)



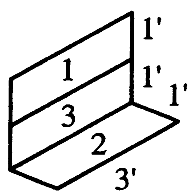
$$F_{1-2} = \frac{1}{A_1}[A_{1,4}(F_{1,4-3,2} - F_{1,4-3}) + A_4(F_{4-3} - F_{4-3,2})]$$

$\frac{Y_{3,2}}{X} = 1.33$ $\quad \frac{Z_{1,4}}{X} = 0.666$ $\quad \frac{Y_3}{X} = 1$ $\quad \frac{Z_4}{X} = \frac{1}{3}$

$F_{1,4-3} = 0.26$ $\quad F_{4-3} = 0.35$ $\quad F_{4-3,2} = 0.36$ $\quad F_{1,4-3,2} = 0.27$

$$F_{1-2} = \frac{1}{3}[6(0.27 - 0.26) + 3(0.36 - 0.35)] = 0.03$$

(d)



$F_{2-3,1} = F_{2-3} + F_{2-1}$ $\quad \frac{Y_2}{X} = \frac{1}{3}$ $\quad \frac{Z_{3,1}}{X} = 0.666$ $\quad \frac{Z_3}{X} = \frac{1}{3}$

$F_{2-3,1} = 0.32$ $\quad F_{2-3} = 0.26$ $\quad F_{2-1} = 0.06$

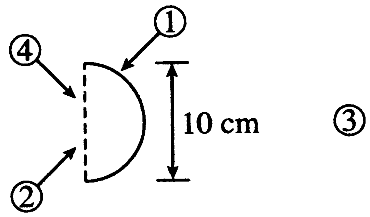
$A_2 F_{2-1} = A_1 F_{1-2}$ $\quad F_{1-2} = 0.06$

8-15

$r_1 = 5$ cm $\quad r_2 = 2.5$ cm $\quad L = 10$ cm $\quad \frac{r_2}{L} = 0.25$ $\quad \frac{L}{r_1} = 2.0$

$\frac{r_1}{r_2} = 2.0$ $\quad F_{12} = 0.06$ $\quad F_{21} = F_{12}\frac{r_1}{r_2} = 0.12$

8-16



$F_{21} = 1.0 \qquad A_1 = 2\pi r^2 = (2)\pi(12.5)^2 = 981.7 \qquad A_2 = \pi r^2 = \pi(5)^2 = 78.5$

$F_{12} = (1.0)\left(\frac{78.5}{981.7}\right) = 0.08 \qquad A_4 = \pi(12.5^2 - 5^2) = 412.3$

$F_{41} = 1.0 \qquad F_{14} = F_{13} = 1.0\left(\frac{412.3}{981.7}\right) = 0.42$

$F_{12} + F_{11} + F_{13} = 1.0 \qquad F_{11} = 1.0 - 0.08 - 0.42 = 0.50$

Summary

$F_{11} = 0.50 \qquad F_{12} = 0.08 \qquad F_{13} = 0.42 \qquad F_{21} = 1.0 \qquad F_{23} = 0$

$F_{31} = F_{32} = 0 \qquad F_{33} = 1.0$

8-17

$r_1 = 5.0$ cm $\qquad r_2 = 12.5$ cm $\qquad L = 7$ cm

open ends = 3 & 4

$\frac{L}{r_2} = 0.56 \qquad \frac{r_1}{r_2} = 0.4 \qquad F_{22} = 0.18 \qquad F_{21} = 0.19$

$A_3 = A_4 = \pi(12.5^2 - 5.0^2) = 412.3\ \text{cm}^2$

$A_2 = \pi(25)(7) = 549.8\ \text{cm}^2$

$A_1 = \pi(10)(7) = 219.9\ \text{cm}^2$

$F_{21} + F_{22} + F_{23} + F_{24} = 1.0 \qquad F_{23} = F_{24}$

$$F_{23} = \frac{1 - 0.19 - 0.18}{2} = 0.315$$

$$F_{12} = 0.19\left(\frac{25}{10}\right) = 0.475$$

$F_{12} + F_{13} + F_{14} = 1.0 \qquad F_{13} = F_{14} \qquad F_{13} = \frac{1 - 0.475}{2} = 0.263$

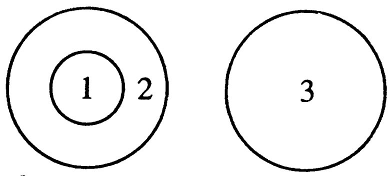
$$F_{31} = (0.263)\left(\frac{A_1}{A_3}\right) = (0.263)\left(\frac{220}{412}\right) = 0.140$$

$$F_{32} = (0.315)\left(\frac{A_2}{A_1}\right) = (0.315)\left(\frac{550}{412}\right) = 0.421$$

$F_{31} + F_{32} + F_{34} = 1.0$

$F_{34} = 1 - 0.140 - 0.421 = 0.439$

8-19



$d_1 = 20$ $\qquad d_2 = d_3 = 50$ $\qquad L = 10$

Two large disks $\dfrac{d}{x} = \dfrac{50}{10} = 5.0$

$F_{3-21} = 0.65 = F_{21-3}$

Large disk to hole $\dfrac{r_1}{L} = \dfrac{10}{10} = 1.0 \qquad \dfrac{L}{r_3} = \dfrac{10}{25} = 0.4$

$A_2 = \pi(25^2 - 10^2) \qquad A_3 = \pi(25^2)$

$F_{31} = 0.15 \qquad F_{31} + F_{32} = F_{3-21} \qquad F_{32} = 0.65 - 0.15 = 0.5$

$$F_{23} = F_{32}\left(\frac{A_3}{A_2}\right) = (0.5)\left(\frac{25^2}{25^2 - 10^2}\right) = 0.595$$

8-20

(a) $F_{21} = 1.0$

$$F_{12} = \frac{A_2}{A_1} = \frac{d}{\pi \frac{d}{2}} = \frac{2}{\pi} = 0.637$$

$$F_{11} = 1 - \frac{A_2}{A_1} = 1 - \frac{2}{\pi} = 0.363$$

(b) $F_{12} = \dfrac{A_1}{A_{opening}} = 0.637$

$F_{11} = 1 - 0.637 = 0.363$

$F_{21} \approx 0$ because $A_2 \to \infty$

(c) Parallel disk $\qquad \dfrac{d}{x} = \dfrac{2}{1} = 2 \qquad$ bottom = 2

$F_{23} = 0.37 \qquad F_{21} = 1 - 0.37 = 0.63 \qquad A_2 F_{21} = A_1 F_{12}$

$$F_{12} = \frac{\pi(1)^2(0.63)}{\pi(2)(1)} = 0.315 = F_{13}$$

$F_{11} + F_{12} + F_{13} = 1.0 \qquad F_{11} = 1 - 0.63 = 0.37$

(d) $F_{12} = 1.0$ $\quad F_{13} = 0$

$A_1 = \pi(0.5)^2 = \frac{\pi}{4}$ $\quad A_2 = 2\pi(1.5)^2 = 4.5\pi$ $\quad A_2F_{21} = A_1F_{12}$

$$F_{21} = \frac{\left(\frac{\pi}{4}\right)(1.0)}{4.5\pi} = 0.0555$$

$A_{open} = \pi(1.5)^2 = 2.25\pi$ $\quad F_{open\text{-}2} = 1.0$

$A_{open}F_{open\text{-}2} = A_2F_2 - \text{open}$

$$F_2\text{ open} = \frac{(2.25)(1)}{4.5} = 0.5 = F_{21} + F_{23}$$

$F_{23} = 0.5 - 0.0555 = 0.4444$

$F_{21} + F_{22} + F_{23} = 1.0$

$F_{22} = 0.5$

(e) <u>Parallel Disks</u>

$r_1 = 0.5$ $\quad r_3 = 1.5$ $\quad L = 2.0$ $\quad \frac{r_3}{L} = 0.75$

$\frac{L}{r_1} = 4.0$

$F_{13} = 0.35$ $\quad F_{12} = 1 - 0.35 = 0.65$ $\quad A_1 = \pi(0.5)^2 = \frac{\pi}{4}$

$A_2 = \pi(3)(2) = 6\pi$ $\quad A_{3'} = \pi(1.5)^2 = 2.25\pi$

$F_{33'} = 0.28$ $\quad$ Top = surface 3' $\quad \frac{d}{x} = \frac{3}{2}$

$F_{3'2} = 1 - 0.28 = 0.72$

$A_{3'} - F_{3'2} = A_2F_{23'}$

$$F_{23'} = \left(\frac{2.25}{6}\right)(0.72) = 0.27 = F_{2-\text{bottom}}$$

$A_1F_{12} = A_2F_{21}$

$$F_{21} = \frac{\pi/4}{6\pi}(0.65) = 0.0271$$

$F_{21} + F_{22} + F_{23} = 1.0$

$F_{23} = 0.27 + 0.27 - 0.0271 = 0.513$

$F_{22} = 1 - 0.0271 - 0.513 = 0.46$

(f) $F_{12} = 0.5$

$F_{21} \to 0$ because $A_2 \to \infty$

(g) $A_1 = \sqrt{2}(1.5) = 2.121$ $\qquad$ A_3 for opening $= 2.121$ $\qquad$ $A_2 = 3.0$

By symmetry $\qquad F_{21} = F_{23} = 0.5$ $\qquad A_1F_{12} = A_2F_{21}$

$$F_{12} = \frac{3}{2.121}(0.5) = 0.707$$

(h) Perpendicular Rectangles

$$\frac{Y}{X} = \frac{Z}{X}$$

$\frac{Y}{X}$	F_{12}
0.6	0.23
0.4	0.25
0.2	0.27
0.1	0.285

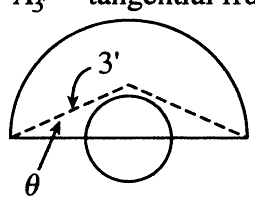
$F_{12} = 0.293 = F_{21}$

$F_{13} = F_{23} = 1 - 0.293 = 0.707$

(i) $A_2 = 4\pi(0.5)^2 = \pi$ $\qquad$ $A_1 = 2\pi(1.5)^2 = 4.5\pi$

$F_{21} = 0.5$ $\qquad A_1F_{12} = A_2F_{21}$ $\qquad F_{12} = \frac{0.5}{4.5} = 0.1111$

$A_{3'}$ = tangential frustrum of cone plus portion of sphere top



$\sin\theta = \frac{0.5}{1.5}$ $\qquad \theta = 19.47°$ $\qquad$ tangent $= 1.5\cos\theta = 1.414$

r at tangent $= 0.5\sin\theta = 0.1667$

cone area $= 2\pi\left(\frac{1.5 + 0.1667}{2}\right)(1.414) = 7.404$

Portion of sphere area $= 4\pi(0.5)^2(2)\left(\frac{19.47}{360}\right) = 0.3398$

$A_{3'} = 7.404 + 0.3398 = 7.744$

$F_{3'-1} = 1.0$ $\qquad A_{3'}F_{3'-1} = A_1F_{1-3'}$

$$F_{1-3'} = \frac{(1.0)(7.744)}{4.5\pi} = 0.548 = F_{12} + F_{13}$$

$F_{13} = 0.548 - 0.1111 = 0.4369$

$F_{11} + F_{12} + F_{13} = 1.0$ $\qquad F_{11} = 1 - 0.1111 - 0.4369 = 0.452$

8-21

$F_{ii} = 0$

Perpendicular sides $F_{ij} = 0.2$

Parallel sides $F_{ij} = 0.2$

8-22

$F_{ii} = 0 \qquad F_{ij} = \frac{1}{3}$

8-23

$T_1 = 260°C = 533\ K \qquad E_{b_1} = 4575\ \mathrm{W/m^2}$

$T_2 = 90°C = 363\ K \qquad E_{b_2} = 984$

$A_1 = A_2 = 16 \qquad F_{12} = 0.27 \qquad \varepsilon_1 = \varepsilon_2 = 1.0$

$$q = \frac{(16)(4575 - 984)}{\frac{(16+6)-(2)(16)(0.27)}{16-(16)(0.27)^2}} = 36,485\ \mathrm{W}$$

8-24

$A_1 = A_2 = (1.2)^2 = 1.44\ \mathrm{m^2} \qquad T_1 = 800\ K \qquad T_2 = 500\ K$

$E_{b_1} = 23,220\ \mathrm{W/m^2} \qquad E_{b_2} = 3543\ \mathrm{W/m^2} \qquad F_{12} = 0.2$

$q_{12} = (1.44)(0.2)(23,220 - 3543) = 5667\ \mathrm{W}$

8-25

$T_1 = 540°C = 813\ K \qquad T_2 = 300°C = 573\ K \qquad \varepsilon_1 = 0.7$

$\varepsilon_2 = 0.5 \qquad T_3 = 100°C = 373\ K \qquad A_1 = A_2 = 0.2827\ \mathrm{m^2}$

$F_{12} = 0.6 \qquad F_{13} = F_{23} = 0.4 \qquad E_{b_1} = 24,767\ \mathrm{W/m^2} \qquad E_{b_2} = 6111$

$E_{b_3} = 1097 = J_3$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 1.516 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 3.537 \qquad \frac{1}{A_1 F_{12}} = 5.896$$

$$\frac{1}{A_1 F_{13}} = \frac{1}{A_2 F_{23}} = 8.843 \qquad \frac{24,767 - J_1}{1.516} + \frac{J_2 - J_1}{5.896} + \frac{1097 - J_1}{8.843} = 0$$

$$\frac{J_1 - J_2}{5.896} + \frac{1097 - J_2}{8.843} + \frac{6111 - J_2}{3.537} = 0$$

$J_1 = 19,089\ \mathrm{W/m^2} \qquad J_2 = 9002\ \mathrm{W/m^2}$

$$q_1 = \frac{24,767 - 19,089}{1.516} = 3745\ \mathrm{W} \qquad q_2 = \frac{6111 - 9001}{3.537} = -817\ \mathrm{W}$$

8-26

$J_{2D} = \varepsilon_2 E_{b_2} = 3.056$ $\qquad \rho_{23} = 0.5 \qquad \dfrac{1}{A_1F_{12}(1-\rho_{23})} = 11.79$

$$\frac{1}{A_2F_{23}(1-\rho_{23})} = 17.69 \qquad \frac{24,767 - J_1}{1.516} + \frac{\frac{3.056}{0.5} - J_1}{11.79} + \frac{478 - J_1}{8.843} = 0$$

$J_1 = 19,719\ \text{W/m}^2 \qquad q_1 = \dfrac{24,767 - 19,719}{1.516} = 3330\ \text{W}$

$$q_2 = \frac{6111 - 19,719}{11.79} + \frac{6111 - 478}{17.69} = -836\ \text{W}$$

8-27

$E_{b_1} = 459\ \text{W/m}^2 \qquad E_{b_2} = 401\ \text{W/m}^2 \qquad F_{12} = 0.25 \qquad A_1 = A_2 = 9$

$\varepsilon_1 = 0.8 \qquad \varepsilon_2 = 0.8$

$$q = \frac{(9)(459-401)}{\frac{9+9-(2)(9)(0.25)}{9-(9)(0.25)^2} + (2)\left(\frac{1}{0.8} - 1\right)} = 248.6\ \text{W}$$

From symmetry $J_3 = \dfrac{1}{2}(E_{b_1} + E_{b_2}) = E_{b_3} = 430$

$$T_3 = \left(\frac{E_{b_3}}{\sigma}\right)^{1/4} = 295\ \text{K} = 22.1°\text{C}$$

8-28

$T_1 = 800°\text{C} = 1073\ \text{K} \qquad E_{b_1} = \sigma T_1^{\,4} = 75,146\ \text{W/m}^2$

$\varepsilon_1 = 0.8 \qquad \varepsilon_2 = 0.1 \qquad T_3 = 300\ \text{K} \qquad E_{b_3} = \sigma T_3^{\,4} = 459$

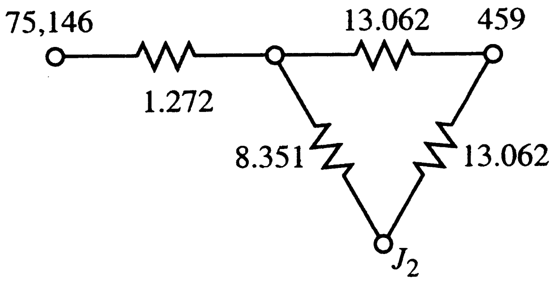
$d = 50\ \text{cm} \qquad x = 12.5\ \text{cm} \qquad \dfrac{d}{x} = 4$

$F_{12} = 0.61 = F_{21} \qquad F_{13} = 0.39 = F_{23}$

$A_1 = A_2 = \pi(0.25)^2 = 0.1963\ \text{m}^2$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{0.2}{(0.8)(0.1963)} = 1.273 \qquad \frac{1}{A_1F_{12}} = \frac{1}{(0.1963)(0.61)} = 8.351$$

$$\frac{1}{A_1F_{13}} = \frac{1}{(0.1963)(0.39)} = 13.062 = \frac{1}{A_2F_{23}} \qquad \frac{1-\varepsilon_3}{\varepsilon_3 A_3} \to 0$$



$$q = \frac{75{,}146 - 459}{1.272 + \frac{1}{\frac{1}{13.062} + \frac{1}{13.062 + 8.351}}} = \frac{74{,}687}{9.385} = 7958 \text{ W}$$

8-29

$T_f = 350$ K $\qquad T_r = 22°\text{C} = 295$ K $\qquad p = 1.7$ atm

$\nu = \dfrac{20.76 \times 10^{-6}}{1.7} \qquad k = 0.03003 \qquad \Pr = 0.697 \qquad \varepsilon = 1.0$

$$\text{Gr}\,\text{Pr} = \frac{(9.8)\left(\frac{1}{350}\right)(127 - 27)(0.5)^3(0.697)(1.7)^2}{(20.76 \times 10^{-6})^2} = 1.64 \times 10^9$$

$c = 0.1 \qquad m = \dfrac{1}{3}$

$$\bar{h} = \frac{0.03003}{0.5}(0.1)(1.64 \times 10^9)^{1/3} = 7.07 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$q_{\text{conv}} = hA(T_w - T_\infty) = (7.07)(2)(0.5)^2(127 - 27) = 353.6$ W

$q_{\text{rad}} = \sigma A \varepsilon (T_w{}^4 - T_r{}^4) = (5.669 \times 10^{-8})(2)(0.5)^2(1.0)(400^4 - 295^4) = 510.9$ W

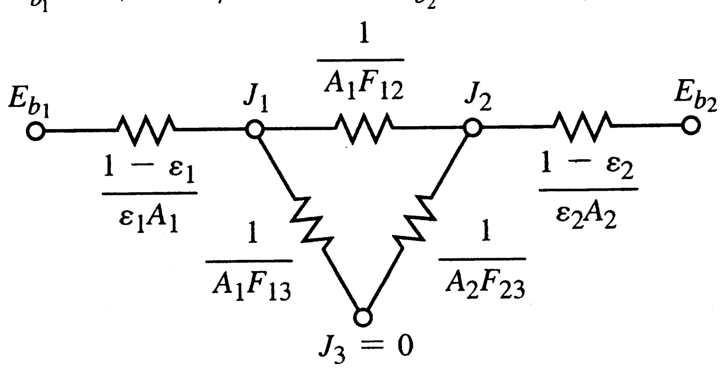
$q_{\text{total}} = 864.5$ W

8-30

$A_1 = A_2 = (1.5)^2 = 2.25 \text{ m}^2 \qquad T_1 = 800°\text{C} = 1073 \text{ K} \qquad T_2 = 553 \text{ K}$

$\varepsilon_1 = 0.5 \qquad \varepsilon_2 = 0.8 \qquad T_3 = 0 \text{ K} \qquad F_{12} = 0.7 \qquad F_{13} = F_{23} = 0.3$

$E_{b_1} = 75{,}146 \ \text{W/m}^2 \qquad E_{b_2} = 5302 \ \text{W/m}^2$



Node J_1

$$\frac{75{,}146 - J_1}{0.4444} + \frac{J_2 - J_1}{0.6349} + \frac{0 - J_1}{1.481} = 0$$

Node J_2

$$\frac{J_1 - J_2}{0.6349} + \frac{0 - J_2}{1.481} + \frac{5302 - J_2}{0.1111} = 0$$

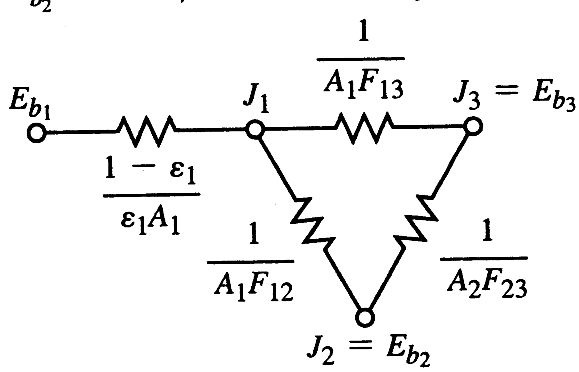
$J_1 = 41{,}070\ \text{W/m}^2 \qquad J_2 = 9992\ \text{W/m}^2$

$$q_1 = \frac{75{,}146 - 41{,}070}{0.4444} = 76{,}671\ \text{W} \qquad q_2 = \frac{5302 - 9992}{0.1111} = -42{,}210\ \text{W}$$

8-31

$A_1 = A_2 = (0.9)(0.6) = 0.54\ \text{m}^2 \qquad \varepsilon_1 = 0.6 \qquad E_{b_1} = 23{,}220\ \text{W/m}^2$

$E_{b_2} = 401\ \text{W/m}^2 \qquad A_3 \to \infty \qquad F_{12} = 0.25 \qquad F_{13} = 0.75 = F_{23}$



$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 1.235 \qquad \frac{1}{A_1F_{13}} = \frac{1}{A_2F_{23}} = 2.469 \qquad \frac{1}{A_1F_{12}} = 7.407$$

$$q = \frac{23{,}220 - 401}{1.235 + \dfrac{1}{\frac{1}{2.469} + \frac{1}{2.469 + 7.407}}} = 7108\ \text{W} = \frac{23{,}220 - J_1}{1.235}$$

$$J_1 = 14{,}441\ \text{W/m}^2 \qquad \frac{14{,}441 - J_2}{7.407} = \frac{J_2 - 401}{2.469}$$

$J_2 = 3911 = E_{b_2} = \sigma T_2^{\,4} \qquad T_2 = 512.5\ \text{K} = 239.5°\text{C}$

8-32

$$\frac{q}{L} = (5.669 \times 10^{-8})\pi(0.05)(0.6)(366^4 - 293^4) = 56.5\ \text{W/m} = 17.22\ \text{W/ft}$$

8-33

$\Delta T = 100 - 27 = 73°C$; $k = 0.04$; $W = 0.05$; $r = 0.0125$; $L = 2.0$

Table 3-1; $S = 2\pi(2)/\ln[0.54(0{,}05)/0.0125] = 1.632$

$R_{ins} = 1/kS = 15.318$

$R_{conv} = 1/hA = 1/(5.1)(4)(0.05)(2) = 0.49$

$h_{rad} = \sigma(T+300)(T^2+300^2)$

$h_{tot} = h_{rad} + 5.1$

$q = (373 - T)(0.04)(1.632) = (T - 300)(5.1 + h_r)(4)(0.05)(2)$

Solve by iteration for T. The result is: $T = 301$ K

$q = 4.69$ W

8-34

$$q = \sigma A\varepsilon(T_1^4 - T_2^4) = (5.669\times 10^{-8})(0.6)(0.3)(0.8)(368^4 - 293^4) = 89.55\ \text{W}$$

8-35

$T_w = 150°\text{C} = 423\text{ K}$ $\quad T_\infty = 20°\text{C} = 293\text{ K}$ $\quad T_r = 38°\text{C} = 311\text{ K}$

$d = 0.125\text{ m}$ $\quad L = 6\text{ m}$

Free convection

$$h = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} = (1.32)\left(\frac{150-20}{0.125}\right)^{1/4} = 7.5\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q(\text{conv}) = hA(T_w - T_\infty) = (7.5)\pi(0.125)(6)(150-20) = 2296\text{ W}$

Radiation

$q(\text{rad}) = \sigma\varepsilon A(T_w{}^4 - T_r{}^4) = (5.669\times10^{-8})\pi(0.125)(6)(423^4 - 311^4) = 3027\text{ W}$

$q(\text{total}) = 2296 + 3027 = 5323\text{ W}$

8-36

$A_1 = \text{heater}$ $\quad A_2 = \text{side walls}$ $\quad A_4 = \text{room} \to \infty$ $\quad A_3 = \text{belt}$

$J_2 = E_{b_2}$ $\quad T_1 = 698\text{ K}$ $\quad T_3 = 393\text{ K}$ $\quad T_4 = 298\text{ K}$

$E_{b_1} = 13{,}456\ \text{W/m}^2$ $\quad E_{b_3} = 1352$ $\quad E_{b_4} = 447$

$\varepsilon_1 = 0.7$ $\quad \varepsilon_3 = 0.8$ $\quad A_1 = A_3 = 3\text{ m}^2$ $\quad A_2 = 1.8\text{ m}^2$

$F_{12} = F_{32} = 0.28$ $\quad F_{31} = F_{13} = 0.67$ $\quad F_{14} = F_{34} = 1 - 0.67 - 0.28 = 0.05$

$F_{24} = (0.04)(2) = 0.08$ $\quad \dfrac{1}{A_3F_{32}} = 1.19$ $\quad \dfrac{1}{A_1F_{13}} = 0.4975$

$\dfrac{1}{A_1F_{12}} = 1.19$ $\quad \dfrac{1}{A_2F_{24}} = 6.944$ $\quad \dfrac{1}{A_1F_{14}} = 6.667$

$\dfrac{1}{A_3F_{34}} = 6.667$ $\quad \dfrac{1-\varepsilon_1}{\varepsilon_1A_1} = 0.1429$ $\quad \dfrac{1-\varepsilon_3}{A_3\varepsilon_3} = 0.0833$

$$\frac{13{,}456 - J_1}{0.1429} + \frac{J_2 - J_1}{1.19} + \frac{J_3 - J_1}{0.4975} + \frac{447 - J_1}{6.667} = 0$$

$$\frac{J_1 - J_2}{1.19} + \frac{J_3 - J_2}{1.19} + \frac{447 - J_2}{6.944} = 0$$

$$\frac{J_1 - J_3}{0.4975} + \frac{J_2 - J_3}{1.19} + \frac{1352 - J_3}{0.0833} + \frac{447 - J_3}{6.667} = 0$$

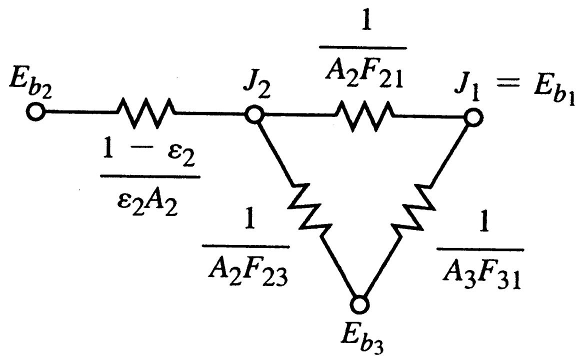
Solving: $\quad J_1 = 10{,}515$ $\quad J_2 = 6185$ $\quad J_3 = 2841$

$$q_1 = \frac{13{,}456 - 10{,}515}{0.1429} = 20{,}580\text{ W}$$

8-37

$T_2 = 1200$ K $\quad \varepsilon_2 = 0.75 \quad \varepsilon_3 = 0.3 \quad q_3 = 0 \quad T_1 = 293$ K

$E_{b_2} = \sigma T_2^{\,4} = 1.176 \times 10^5 \quad E_{b_1} = \sigma T_1^{\,4} = 417.8$



$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 42.44 \quad \frac{1}{A_2F_{23}} = 167.53 \quad \frac{1}{A_2F_{21}} = 530.52$

$\frac{1}{A_3F_{31}} = 361.84$

$$\text{Req.} = 42.44 + \frac{1}{\frac{1}{530.52} + \frac{1}{167.53+361.84}} = 307.44$$

$$q = \frac{1.176\times10^5 - 477.8}{307.4} = 382 \text{ W} = \frac{1.176\times10^5 - J_2}{42.44} \qquad J_2 = 1.014\times10^5$$

$$\frac{1.014\times10^5 - E_{b_3}}{167.53} = \frac{1.014\times10^5 - 477.8}{167.53+361.84} \qquad E_{b_3} = 6.946\times10^4$$

$T_3 = 1052$ K

8-38

$A_1 = \pi(0.02) = 0.06283 \quad A_2 = \pi(0.06) = 0.1884$

$E_{b_1} = \sigma T_1^{\,4} = 32{,}922 \text{ W/m}^2 \quad E_{b_3} = \sigma T_3^{\,4} = 459 \quad F_{12} = 1.0$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{0.6}{(0.4)(0.06283)} = 23.874 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{0.8}{(0.2)(0.1884)} = 21.231$$

$\frac{1}{A_1F_{12}} = 15.916 \quad \frac{1}{A_2F_{23}} = 5.308$

$$q_1 = \frac{32{,}992 - 459}{23.874 + 15.916 + (2)(21.231) + 5.308} = 371.6 \text{ W/m}$$

$$= \frac{32{,}992 - E_{b_2}}{23.874 + 15.916 + 21.231} = \frac{32{,}992 - E_{b_2}}{61.021}$$

$E_{b_2} = 10{,}317 \text{ W/m}^2 \quad T_2 = 653 \text{ K} = 380°\text{C}$

8-39

$L = 30$ cm $\quad r_1 = 4$ cm $\quad F_{12} = 0.8 \quad r_2 F_{21} = r_1 F_{12}$

<u>Iteration Solution</u>

r_2	$\frac{r_1}{r_2}$	$\frac{L}{r_2}$	F_{21}	$F_{12} = F_{21}\frac{r_2}{r_1}$	$0.8 - F_{12}$
8	0.5	3.75	0.45	0.9	–0.1
10	0.4	3.0	0.35	0.87	–0.07
12	0.33	2.5	0.27	0.81	–0.01
16	0.25	1.875	0.18	0.72	+0.08
12.44	0.3215	2.41	Interpolated		0

8-40

$E_{b1} = 117500$

$E_{b2} = 3543$

$E_{b3} = 459$; $\quad A_1 = A_2 = \pi(0.3)^2/4 = 0.0707$; $\quad d/x = 3.0$

Fig. 8-13 $F_{12} = 0.52$; $\quad F_{13} = F_{23} = 0.48$

$q_{12} = (0.0707)(0.52)(117500\text{-}3543) = 4190$ W

$q_{13} = (0.0707)(0.48)(117500\text{-}459) = 3971$

$q_1 = 8161$ W

$q_{21} = -4190$

$q_{23} = (0.0707)(0.48)(3543\text{-}459) = 105$W

$q_2 = 14085$ W; $\quad$ Check $\quad q_1 + q_2 + q_3 = 0$

8-41

$(E_{b1} - E_{b2})(0.52) = ().48)(E_{b2} - E_{b3})\ (1.0(E_{b2} - E_{b3})$

$E_{b2} = 30890$; $\quad T_2 = 859$K

$q_1 = 0.0707)[(0.52)(117500 - 30890) + (0.48)(117500 - 459)]$

$= 7456$ W

8-42

$d = 75 \text{ cm}$ $\quad x = 50 \text{ cm}$ $\quad \dfrac{d}{x} = 1.5$ $\quad \left.\dfrac{q}{A}\right|_1 = 7000$ $\quad \varepsilon_1 = 0.8$

$\varepsilon_2 = 0.6$ $\quad T_2 = 400 \text{ K}$ $\quad F_{12} = F_{21} = 0.3$ $\quad F_{13} = F_{23} = 0.7$

$E_{b_2} = 1451$ $\quad A_1 = A_2 = \pi\left(\dfrac{0.75}{2}\right)^2 = 0.4418$ $\quad A_3 = \pi(0.5)(0.75) = 1.178$

$F_{31} = F_{32} = 0.263$ $\quad F_{33} = 1-(0.2)(0.263) = 0.474$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0.5659$ $\quad \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 1.509$ $\quad \dfrac{1}{A_1 F_{12}} = 7.545$

$\dfrac{1}{A_1 F_{13}} = \dfrac{1}{A_2 F_{23}} = 3.234$

$J_1 - (F_{12}J_2 + F_{13}J_3) = 7000$ $\quad J_2 - 0.4(F_{21}J_1 + F_{23}J_3) = 871$

$J_3(1-0.474) - (F_{31}J_1 + F_{32}J_2) = 0$

$J_1 = 7000 + 0.3J_2 + 0.7J_3 = 16{,}795 \text{ W/m}^2$

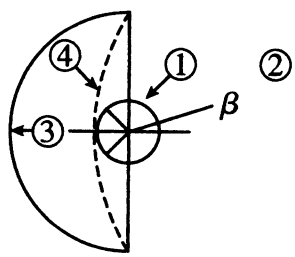
$J_2 = 871 + 0.12J_1 + 0.28J_2 = 6075 \text{ W/m}^2$

$J_3 = 0.498J_1 + 0.498J_2 = 11{,}389 = E_{b_3} = \sigma T_3{}^4$

$T_3 = 669 \text{ K}$ $\quad \dfrac{E_{b_1} - 16{,}795}{\frac{1-0.8}{0.8}} = 7000$

$E_{b_1} = 18{,}545 = \sigma T_1{}^4$ $\quad T_1 = 756 \text{ K}$

8-43



$T_1 = 1253\text{ K}$ $\quad T_2 = 288\text{ K}$ $\quad F_{13} = 0.5$ $\quad F_{12} = 0.5$

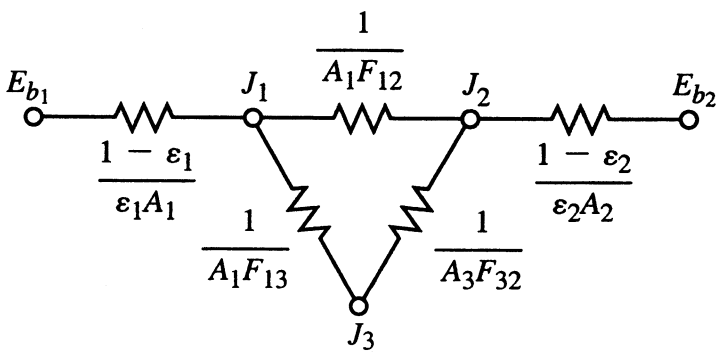
$\beta = 2\alpha$ $\quad \sin\alpha = \dfrac{3.75}{25}$ $\quad \alpha = 8.627°$ $\quad \beta = 17.25°$

$A_4 = 2(0.25\cos\alpha) + \pi(0.075)\left(\dfrac{2\alpha}{360}\right) = 0.5056$ $\quad F_{43} = 1.0$

$A_3 = \pi(0.25) = 0.7854$ $\quad F_{34} = F_{31} + F_{32} = \dfrac{(0.5056)(1.0)}{0.7854} = 0.6437$

$A_1 = \pi(0.025) = 0.2356$ $\quad F_{31} = \dfrac{(0.2356)(0.5)}{0.7854} = 0.15$

$F_{32} = 0.6437 - 0.15 = 0.4937$



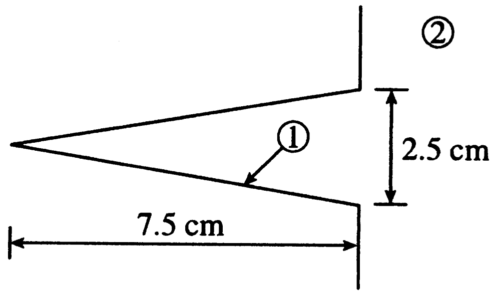
$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 1.061$ $\quad \dfrac{1}{A_1F_{12}} = 8.489$ $\quad \dfrac{1}{A_3F_{32}} = 2.579$ $\quad \dfrac{1}{A_1F_{13}} = 8.489$

$E_{b_1} = 139{,}736\ \text{W/m}^2$ $\quad E_{b_2} = 390\ \text{W/m}^2$

$$q = \frac{139{,}736 - 390}{1.061 + \frac{1}{\frac{1}{8.489} + \frac{1}{8.489+2.579}}} = 23{,}758\ \text{W/m}$$

q w/o reflector $= \varepsilon_1 A_1 (E_{b_1} - E_{b_2}) = 26{,}264\ \text{W/m}$

8–40



$T_1 = 823$ K $\quad \varepsilon_1 = 0.5 \quad T_2 = 0$ K

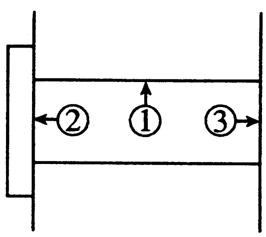
$A_1 = \pi(1.25)(1.25^2 + 7.5^2)^{1/2} = 29.86\ \text{cm}^2$

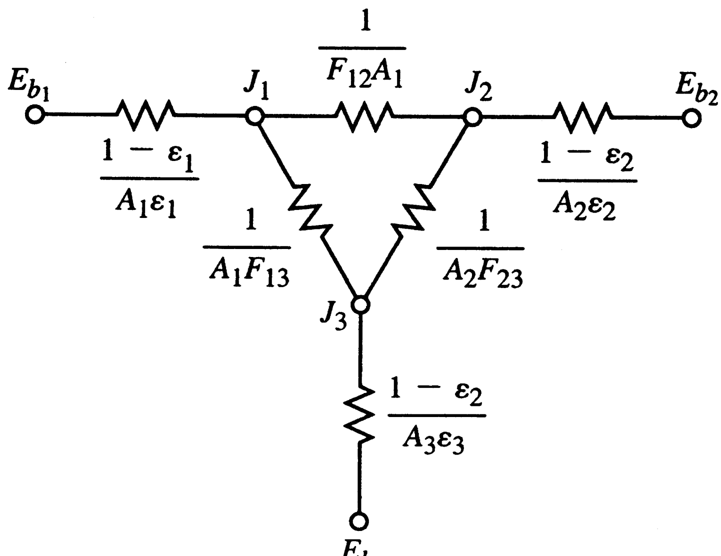
$A_2 = \pi(1.25)^2 = 4.91\ \text{cm}^2 \qquad F_{21} = 1.0$

$$q = \frac{(5.669 \times 10^{-8})(823)^4 - 0}{\frac{1-0.5}{(0.5)(0.002986)} + \frac{1}{4.91\times10^{-4}} + 0} = 10.97\ \text{W}$$

$$\varepsilon_{\text{app}} = \frac{10.97}{(5.669 \times 10^{-8})(823)^4(4.9 \times 10^{-4})} = 0.859$$

8-45





$T_1 = 260°\text{C} = 533$ K $\qquad T_2 = 425°\text{C} = 698$ K

$\varepsilon_3 = 1.0 \qquad \varepsilon_2 = 0.5 \qquad \varepsilon_1 = 0.07 \qquad T_3 = 0$ K $\qquad F_{23} = 0.04$

$F_{21} = 0.96 \qquad A_1 = \pi(2.5)(7.5) = 58.9\ \text{cm}^2$

$A_2 = A_3 = \pi(1.25)^2 = 4.91\ \text{cm}^2 \qquad F_{12} = F_{13} = \frac{A_2}{A_1}F_{21} = (0.96)\left(\frac{4.91}{58.9}\right) = 0.08$

$E_{b_1} = 4575 \qquad E_{b_2} = 13{,}456$

Node J_1

$$\frac{4575 - J_1}{2256} + \frac{J_2 - J_1}{2121} + \frac{0 - J_1}{2122} = 0$$

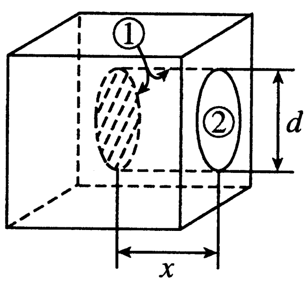
Node J_2

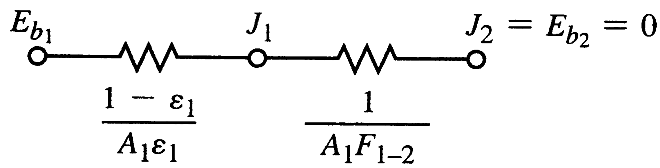
$$\frac{13{,}456 - J_2}{2037} + \frac{J_1 - J_2}{2121} + \frac{0 - J_2}{50{,}916} = 0 \qquad J_1 = 4484 \qquad J_2 = 8879$$

$$q_1 = \frac{4575 - 4484}{2256} = 0.0403 \qquad q_2 = \frac{13{,}456 - 8879}{2037} = 2.247$$

$q_3 = 2.287$ W

8-46

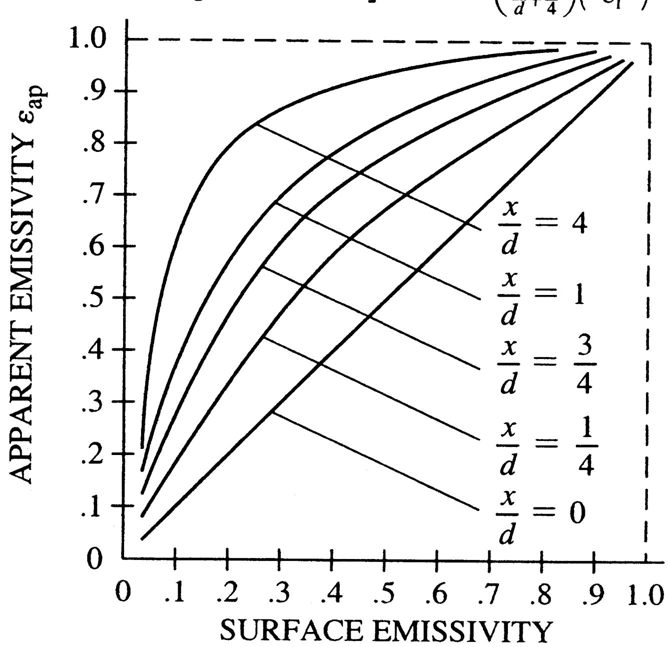




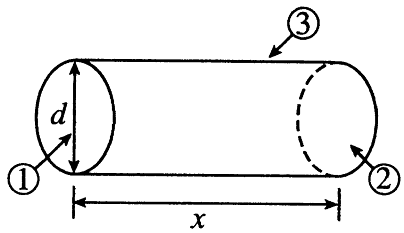
Plot apparent emissivity ε_{ap} vs $\frac{x}{d}$ and ε. $\qquad q_{1-2} = \dfrac{E_{b_1} - 0}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_2}}$

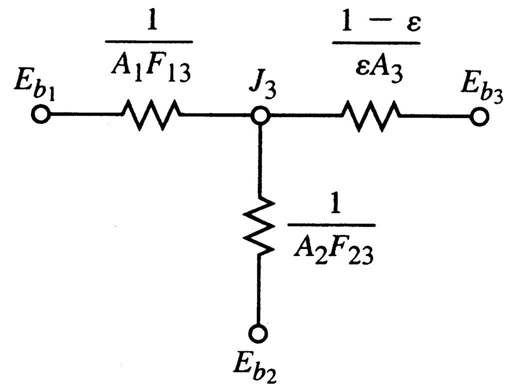
$A_1 F_{1-2} = A_2 F_{2-1} = A_2 \qquad F_{2-1} = 1$

$$\varepsilon_{ap} = \frac{q_{1-2}}{(q_2)_b} = \frac{E_{b_1}}{\left[\frac{(1-\varepsilon_1)A_2 + A_1\varepsilon_1}{A_1 A_2 \varepsilon_1}\right] E_{b_1} A_2} = \frac{4}{\left(\frac{1}{\frac{x}{d}+\frac{1}{4}}\right)\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + 4}$$



8-47





$\varepsilon_3 = \varepsilon$ $\quad E_{b_1} = E_{b_2} = 0$ $\quad \varepsilon_2 = \varepsilon_1 = 1.0$ $\quad F_{13} = F_{23} = 1 - F_{12}$

$A_2 = A_1 = \dfrac{\pi d^2}{4}$ $\quad A_3 = \pi x d$ $\quad q_1 = \dfrac{\frac{1}{2}(E_{b_3} - 0)}{\frac{1-\varepsilon_3}{\varepsilon_3 A_3} + \frac{1}{A_1(1-F_{12}) + A_2(1-F_{12})}}$

$\varepsilon_{\text{app1}} = \dfrac{q_1}{A_1 E_{b_3}}$ $\quad \varepsilon_{\text{app1}} = \varepsilon_{\text{app2}} = \dfrac{\frac{1}{2}}{\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{d}{4x}\right) + \frac{1}{2(1-F_{12})}}$

F_{12} is obtained from Fig. 8–13. Note that ε_{app} approaches 1.0 as $\dfrac{x}{d} \to \infty$.

As $\dfrac{x}{d} \to 0 \quad \varepsilon_{\text{app}} \to 0$

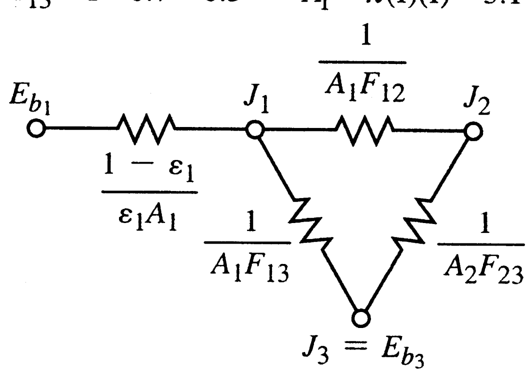
8-48

$r_1 = 0.5$ $\quad L = 1.0$ $\quad r_2 = 1.0$ $\quad q_2 = 0$ $\quad T_1 = 800\text{ K}$

$T_3 = 300\text{ K}$ $\quad \varepsilon_1 = 0.65$ $\quad \dfrac{r_1}{r_2} = 0.5$ $\quad \dfrac{L}{r_2} = 1.0$ $\quad F_{21} = 0.35$

$F_{22} = 0.23$ $\quad F_{23} = 1 - 0.35 - 0.23 = 0.42$ $\quad F_{12} = F_{21}\left(\dfrac{A_2}{A_1}\right) = 0.7$

$F_{13} = 1 - 0.7 = 0.3$ $\quad A_1 = \pi(1)(1) = 3.142$ $\quad A_2 = 2\pi = 6.284$



$E_{b_1} = \sigma T_1^{\,4} = 23{,}216\ \text{W/m}^2$ $\qquad E_{b_3} = \sigma T_3^{\,4} = 459\ \text{W/m}^2$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0.1714 \qquad \dfrac{1}{A_1 F_{13}} = 1.061 \qquad \dfrac{1}{A_1 F_{12}} = 0.455$

$\dfrac{1}{A_2 F_{23}} = 0.379$

$$q = \frac{23{,}216 - 459}{0.1714 + \frac{1}{\frac{1}{1.061} + \frac{1}{0.379+0.455}}} = 35{,}650\ \text{W}$$

8-49

L = 0.1 m $\qquad h_1 = (1.42)(100/0.1)^{1/4} = 7.99$

$h_2 = (1.42)[(T_2 - 300)/0.1]^{1/4}$

Fig. 8-12; $F_{12} = 0.4 \quad F_{13} = 0.6$

$(0.1)^2\sigma(400^4\text{-}T^4)(0.4) = (0.1)^2[\sigma(T^4 - 300^4)(0.6 + 1.0) + (1.42)(2)(T\text{-}300)^{1/4}/0.1^{1/4}]$

Solve for T by iteration; T = 327.5 K

$q_1 = (0.1)^2[\sigma(400^4\text{-}T^4)\,(0.4) + 0.6\sigma(400^4\text{-}300^4) + (7.99)(400\text{-}300)]$

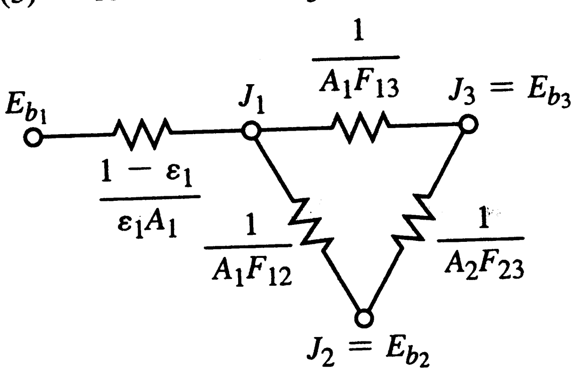
= 17.1 W

8-50

$T_1 = 973\text{ K} \quad E_{b_1} = 50{,}811\ \text{W/m}^2 \quad \varepsilon_1 = 0.8 \quad A_1 = (2)(3) = 6\ \text{m}^2$

(2) Surface = side walls $\quad A_2 = (10)(2) = 20\ \text{m}^2$

(3) room $\quad T_3 = 303\text{ K} \quad E_{b_3} = 478\ \text{W/m}^2$



$\frac{X}{D} = 1.0 \qquad \frac{Y}{D} = 1.5 \qquad F_{13} = 0.25 \qquad F_{12} = 0.75$

$F_{21} = \frac{6}{20}(0.75) = 0.225 \qquad F_{23} = 0.225$

$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 4.1666\times10^{-2} \qquad \frac{1}{A_1F_{13}} = 0.6667 \qquad \frac{1}{A_1F_{12}} = 0.2222$

$\frac{1}{A_2F_{23}} = 0.2222$

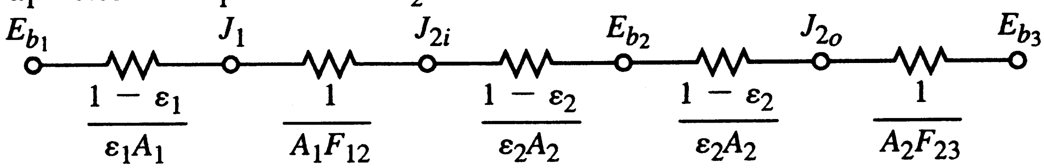
$$q = \frac{50{,}811 - 478}{0.0417 + \frac{1}{\frac{1}{0.6667} + \frac{1}{0.2222+0.2222}}} = 1.633\times10^5\ \text{W}$$

8-51

$T_1 = 1273\ \text{K} \quad E_{b_1} = 1.489 \times 10^5\ \text{W/m}^2 \qquad \varepsilon_1 = 0.6 \qquad T_3 = 303\ \text{K}$

Surface (2) is sphere in radiant balance. $\qquad E_{b_3} = 478\ \text{W/m}^2$

$d_1 = 0.03 \qquad d_1 = 0.09 \qquad \varepsilon_2 = 0.3$



$A_1 = 4\pi r_1^2 = 2.83 \times 10^{-3}\ \text{m}^2 \qquad A_2 = 4\pi r_2^2 = 0.0254\ \text{m}^2$

$F_{12} = 1.0 \qquad F_{23} = 1.0 \qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 235.6 \qquad \dfrac{1}{A_1 F_{12}} = 353.4$

$\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 91.86 \qquad \dfrac{1}{A_2 F_{23}} = 39.37$

$$q = \frac{A E_b}{\Sigma R} = \frac{148{,}900 - 478}{235.6 + 353.4 + (2)(91.86) + 39.37} = 182.7\ \text{W}$$

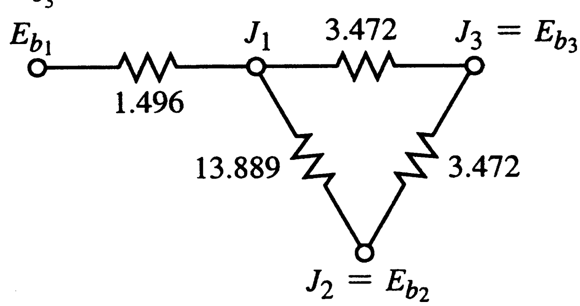
8-52

$T_1 = 873 \qquad \varepsilon_1 = 0.65 \qquad q_2 = 0 \qquad T_3 = 303\ \text{K} \qquad F_{12} = 0.2$

$F_{13} = F_{23} = 0.8 \qquad A_1 = A_2 = (0.6)^2 = 0.36 \qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 1.496$

$\dfrac{1}{A_1 F_{13}} = 3.472 = \dfrac{1}{A_2 F_{23}} \qquad \dfrac{1}{A_1 F_{12}} = 13.889 \qquad E_{b_1} = 32{,}928$

$E_{b_3} = 478$



$$q = \frac{32{,}928 - 478}{1.496 + \frac{1}{\frac{1}{3.472} + \frac{1}{13.889 + 3.472}}} = 7393\ \text{W} = \frac{32{,}928 - J_1}{1.496}$$

$J_1 = 21{,}868\ \text{W/m}^2 \qquad \dfrac{21{,}868 - J_2}{13.889} = \dfrac{21{,}868 - 478}{13.889 + 3.472}$

$J_2 = 4756 = E_{b_2} \qquad T_2 = 538\ \text{K}$

8-53

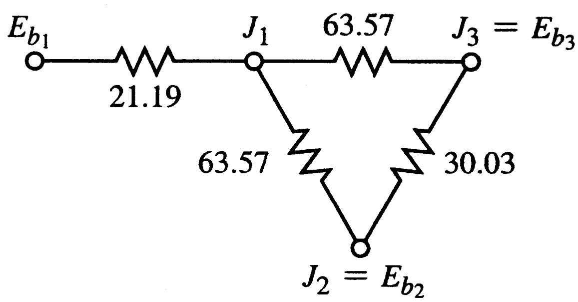
$r_1 = 5$ cm $\quad r_2 = 10$ cm $\quad L = 10$ cm $\quad T_1 = 973$ K $\quad T_3 = 303$ K

$\varepsilon_1 = 0.6 \quad \varepsilon_2 = 0.7 \quad q_2 = 0 \quad \dfrac{r_1}{r_2} = 0.5 \quad \dfrac{L}{r_2} = 1.0$

$F_{21} = 0.25 \quad F_{12} = 0.50 \quad F_{22} = 0.22 \quad F_{13} = 0.5 \quad F_{23} = 0.53$

$A_1 = \pi(0.1)^2 = 0.03146 \quad A_2 = \pi(0.2)(0.1) = 0.06283 \quad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 21.19$

$\dfrac{1}{A_1 F_{12}} = 63.57 \quad \dfrac{1}{A_1 F_{13}} = 63.57 \quad \dfrac{1}{A_2 F_{23}} = 30.03$



$$q = \frac{50{,}811 - 478}{21.19 + \frac{1}{\frac{1}{63.57} + \frac{1}{63.57 + 30.03}}} = 852 \text{ W}$$

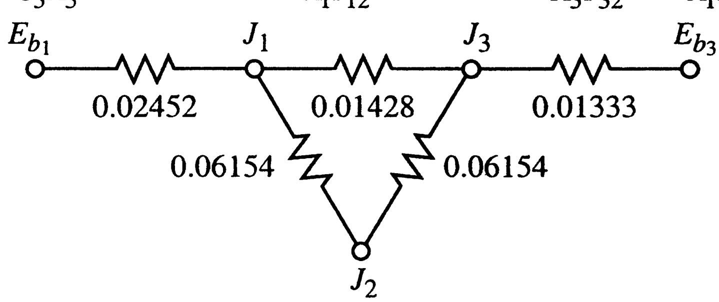
8-54

$A_1 = A_3 = 25 \text{ m}^2 \quad A_2 = (4)(5)(4) = 80 \text{ m}^2 \quad T_1 = 301 \text{ K} \quad T_3 = 293 \text{ K}$

$E_{b_1} = 465.3 \quad E_{b_3} = 417.8 \text{ W/m}^2 \quad \varepsilon_1 = 0.62 \quad \varepsilon_3 = 0.75$

$\dfrac{X}{D} = \dfrac{Y}{D} = 1.25 \quad F_{13} = 0.35 \quad F_{12} = 0.65 \quad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0.02452$

$\dfrac{1-\varepsilon_3}{\varepsilon_3 A_3} = 0.01333 \quad \dfrac{1}{A_1 F_{12}} = 0.06154 = \dfrac{1}{A_3 F_{32}} \quad \dfrac{1}{A_1 F_{13}} = 0.01428$



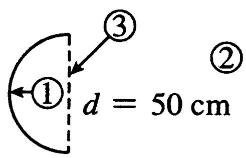
$$q = \frac{465.3 - 417.8}{0.02452 + \frac{1}{\frac{1}{0.01428} + \frac{1}{2(0.06154)}} + 0.01333} = 489.1 \text{ W} = \frac{E_{b_1} - J_1}{0.02452} = \frac{J_3 - E_{b_3}}{0.01333}$$

$J_1 = 453.3\ \text{W/m}^2 \qquad J_3 = 424.3\ \text{W/m}^2$

From Symmetry: $J_2 = \dfrac{J_1 + J_3}{2} = 438.8\ \text{W/m}^2 = \sigma T_2{}^4$

$T_2 = 296.6\ \text{K} = 23.6°\text{C}$

8-55



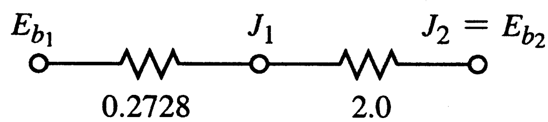
$T_1 = 1000\ \text{K} \qquad T_2 = 300\ \text{K} \qquad \varepsilon_1 = 0.7 \qquad F_{31} = 0.5$

$$F_{13} = F_{12} = 0.5\frac{A_3}{A_1} = 0.5\left(\frac{d}{\pi \frac{d}{2}}\right) = 0.3183$$

$E_{b_1} = 56{,}690\ \text{W/m}^2 \qquad E_{b_2} = 459\ \text{W/m}^2$

For unit length $A_1 = \pi d = 1.571$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0.2728 \qquad \frac{1}{A_1 F_{12}} = 2.0$$



$$\frac{q}{L} = \frac{56{,}690 - 459}{0.2728 + 2.0} = 2.47 \times 10^4\ \text{W}$$

8-56

$E_{b_1} = \sigma(673)^4 = 11{,}630 \qquad E_{b_3} = \sigma(873)^4 = 32{,}928\ \text{W/m}^2$

$$\frac{1-\varepsilon_1}{\varepsilon_1} = \frac{1-0.6}{0.6} = 0.6667 = \frac{1-\varepsilon_3}{\varepsilon_3} \qquad \frac{1-\varepsilon_2}{\varepsilon_2} = \frac{0.9}{0.1} = 9$$

$$\frac{1}{F_{12}} = \frac{1}{F_{23}} = 1.0$$

For unit area

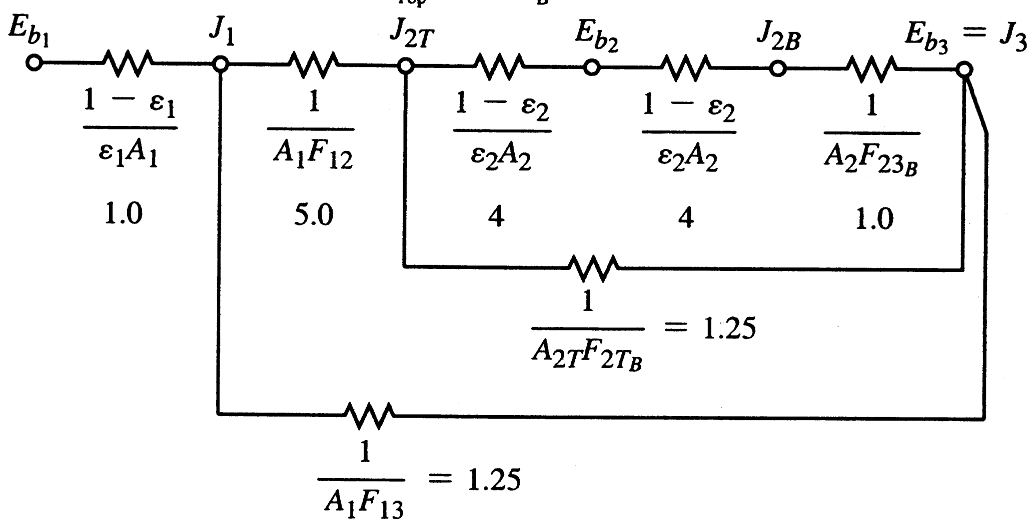
$$q = \frac{32{,}928 - 11{,}630}{0.6667 + 1 + (2)(9) + 1 + 0.6667} = 998.3\ \text{W/m}^2$$

8-57

$A_1 = A_2 = 1.0 \qquad A_3 \to \infty$

$E_{b_1} = 23{,}216 \ \text{W/m}^2 \qquad E_{b_3} = 459 \ \text{W/m}^2$

$F_{12} = 0.2 \qquad F_{13} = 0.8 = F_{23_{\text{Top}}} \qquad F_{23_B} = 1.0$



$$q = \frac{E_{b_1} - E_{b_3}}{\sum R} = \frac{23{,}216 - 459}{1.0373} = 11{,}170 \text{ W}$$

8-58

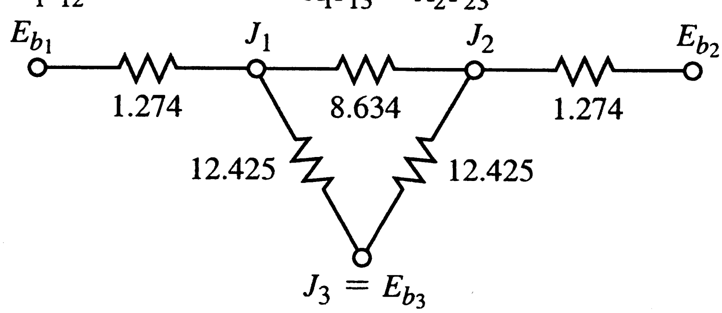
$\frac{d}{x} = \frac{50}{12.5} = 4.0 \qquad F_{12} = 0.59 \qquad F_{13} = F_{23} = 0.41$

$A_1 = A_2 = \pi(0.25)^2 = 0.1963 \text{ m}^2 \qquad \varepsilon_1 = \varepsilon_2 = 0.8 \qquad T_1 = 1000 \text{ K}$

$T_2 = 500 \text{ K} \qquad T_3 = 300 \text{ K} \qquad E_{b_1} = 56{,}690$

$E_{b_2} = 3543 \ \text{W/m}^2 \qquad E_{b_3} = 459 \qquad \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 1.274$

$\frac{1}{A_1F_{12}} = 8.634 \qquad \frac{1}{A_1F_{13}} = \frac{1}{A_2F_{23}} = 12.425$



$$\frac{56{,}690 - J_1}{1.274} + \frac{J_2 - J_1}{8.634} + \frac{459 - J_1}{12.425} = 0$$

$$\frac{J_1 - J_2}{8.634} + \frac{459 - J_2}{12.425} + \frac{3543 - J_2}{1.274} = 0$$

$J_1 = 46{,}373\ \text{W/m}^2$ $\qquad J_2 = 8346\ \text{W/m}^2$

$q_1 = \dfrac{56{,}690 - J_1}{1.274} = 8098\ \text{W}$ $\qquad q_2 = \dfrac{3543 - J_2}{1.274} = -3770\ \text{W}$

8-59

Disk = (1) $\qquad$ Shield = (2) $\qquad$ Room = (3) $\qquad \dfrac{d}{x} = \dfrac{50}{25} = 2.0$ $\qquad F_{13} = 0.28$

$F_{12} = 0.72$ $\qquad T_1 = 873\ \text{K}$ $\qquad \varepsilon_1 = 0.55$ $\qquad \varepsilon_2 = 0.1$ $\qquad T_3 = 30°\text{C}$

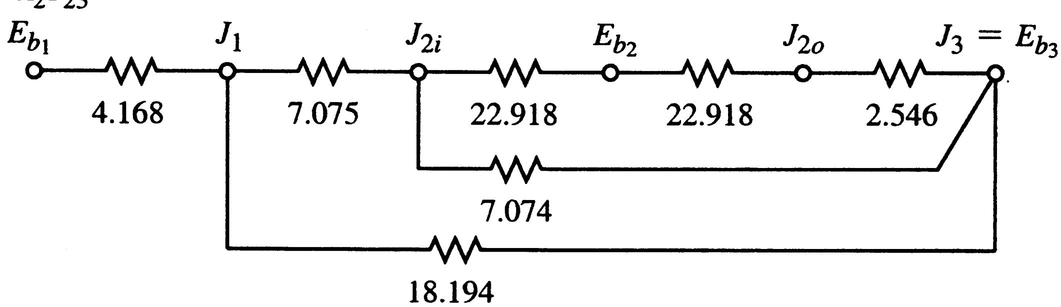
$E_{b_1} = 32{,}928\ \text{W/m}^2$ $\qquad E_{b_3} = 478\ \text{W/m}^2$ $\qquad A_1 = \pi(0.25)^2 = 0.1963\ \text{m}^2$

$A_2 = \pi(0.5)(0.25) = 0.3927\ \text{m}^2$ $\qquad F_{21} = F_{12}\dfrac{A_1}{A_2} = 0.36 = F_{23}$ (inside)

F_{23} (inside) $= 1.0$ $\qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 4.168$ $\qquad \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 22.918$

$\dfrac{1}{A_1 F_{12}} = 7.075$ $\qquad \dfrac{1}{A_1 F_{13}} = 18.194$ $\qquad \dfrac{1}{A_2 F_{23}} = 7.074$ (inside)

$\dfrac{1}{A_2 F_{23}} = 2.546$ (outside)



$q = \dfrac{E_{b_1} - E_{b_3}}{\sum R}$ $\qquad \dfrac{1}{R_1} = \dfrac{1}{(22.918)(2) + 2.546} + \dfrac{1}{7.074}$

$\dfrac{1}{R_2} = \dfrac{1}{R_1 + 7.025} + \dfrac{1}{18.194}$ $\qquad \sum R = 4.168 + R_2$ $\qquad R_1 = 6.1716$

$R_2 = 7.648$ $\qquad \sum R = 11.817$

$q = \dfrac{32{,}928 - 478}{11.817} = 2746\ \text{W} = \dfrac{32{,}928 - J_1}{4.168}$ $\qquad J_1 = 21{,}482\ \text{W/m}^2$

$\dfrac{21{,}482 - J_{2i}}{7.075} = \dfrac{J_{2i} - 478}{6.1716}$ $\qquad J_{2i} = 10{,}264\ \text{W/m}^2$ $\qquad J_{2o} = 993$

$\dfrac{J_{2i} - J_{2o}}{(2)(22.918)} = \dfrac{J_{2i} - 478}{48.382}$ $\qquad E_{b_2} = \dfrac{J_{2i} + J_{2o}}{2} = 5628\ \text{W/m}^2 = \sigma T_2{}^4$

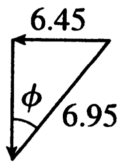
$T_2 = 561\ \text{K} = 288°\text{C}$

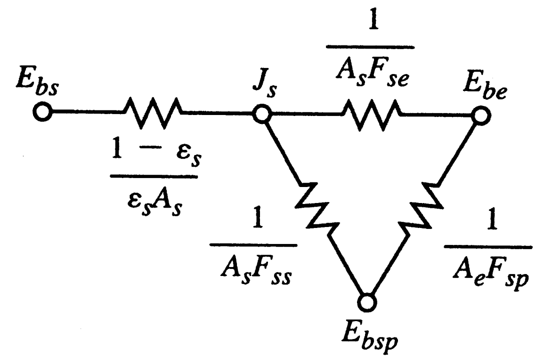
8-61

$1500\alpha = \varepsilon\sigma T^4 \qquad \varepsilon = \alpha \qquad T = 403.3\ \text{K} = 130.3°\text{C}$

8-62

(a) Dark Side





at equilibrium $J_s = E_{bs}$

$$\sin\phi = \frac{6.45}{6.95}$$
$$\phi = 70.3°$$
$$\cos\phi = 0.3367$$

$$\text{solid angle} = \frac{1-\cos\phi}{2} = 0.3316 = F_{se} \qquad F_{ss} = 0.6684 \qquad F_{esp} \approx 1.0$$

$$A_s = \pi(1)^2 = \pi\ \text{m}^2 \qquad E_{be} = (5.669\times10^{-8})(288)^4 = 390\ \text{W/m}^2$$

$$E_{bsp} = 0 \qquad \frac{390 - J_s}{1/0.3316} + \frac{0 - J_s}{1/0.6684} = 0 \quad J_s = 129.3 = E_s = \sigma T_s^{\,4}$$

$$T_s = 218.5\ \text{K} = -54.5°\text{C}$$

(b) Bright Side

$$A_{\text{view}} = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \text{ for sun}$$

For polished Aluminum $t = 0.048$

$$q \text{ absorbed} = (0.048)\left(\frac{\pi}{4}\right)(1400) = 52.779\ \text{W}$$

$$\frac{E_{bs} - J_s}{\frac{1-\varepsilon_s}{\varepsilon_s A_s}} = 52.779 \qquad 52.779 + \frac{390 - J_s}{\frac{1}{\pi(0.3316)}} + \frac{0 - J_s}{\frac{1}{\pi(0.6684)}} = 0$$

$$J_s = 146.1 \qquad E_{bs} = 146.1 + \frac{(52.779)(1-0.048)}{(0.048)\pi} = 479.3\ \text{W/m}^2$$

$$T_s = \left(\frac{479.3}{5.669\times10^{-8}}\right)^{1/4} = 303.2\ \text{K} = 30.2°\text{C}$$

8-63

$d = 0.02$; $x = 0.03$

$A_1 = \pi[().02)(0.03) + (0.02)^2/4]$

$A_o = \pi(0.02)^2/4$; $A_o/A_1 = 0.143$

Table A-10; $\varepsilon = 0.44$ freshly turned

Fig. 8-32; $\varepsilon_a = 0.88$

Heat to 930°C = 1700°F, $\varepsilon_i \approx 0.65$

Fig 8-32 $\varepsilon_a = 0.94$

8-64

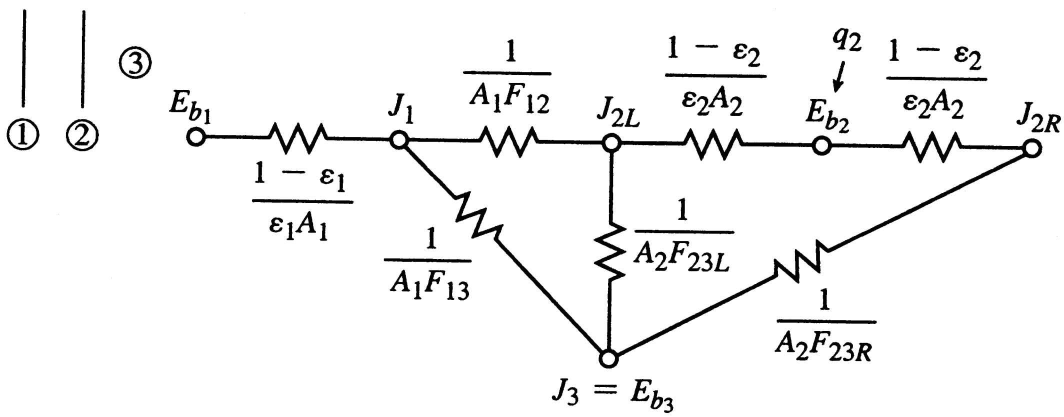
Black shiny lacquer, $\varepsilon_i \approx 0.88$; $\varepsilon_a = 0.99$

8-65

$A_1 = A_2 = 0.09\ \text{m}^2$ $\quad T_1 = 423\ \text{K}$ $\quad \varepsilon_1 = \varepsilon_2 = 0.8$

$T_3 = 293\ \text{K}$ $\quad h = 1.42\left(\dfrac{\Delta T}{L}\right)^{1/4}$ $\quad F_{12} = 0.55$ $\quad F_{13} = F_{23L} = 0.45$

$F_{23R} = 1.0$



$-q_2$ net rad $= q_2$ conv

$$\frac{J_{2L} - E_{b_2} - J_{2R} - E_{b_2}}{\frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = 2hA_2(T_2 - 293)$$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 2.778 \qquad \frac{1}{A_1F_{12}} = 20.20 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 2.778$$

$$\frac{1}{A_1F_{13}} = 24.69 = \frac{1}{A_2F_{23L}} \qquad \frac{1}{A_2F_{23R}} = 11.11 \qquad E_{b_1} = 1815\ \text{W/m}^2$$

$$E_{b_3} = 418\ \text{W/m}^2 \qquad \frac{J_{2R} - E_{b_2}}{\frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = \varepsilon_2 A_2(418 - E_{b_2})$$

$$(J_1) \qquad \frac{1815 - J_1}{2.778} + \frac{J_{2L} - J_1}{20.20} + \frac{418 - J_1}{24.69} = 0$$

$$(J_2) \qquad \frac{J_1 - J_{2L}}{20.20} + \frac{418 - J_{2L}}{24.69} + \frac{E_{b_2} - J_{2L}}{2.778} = 0$$

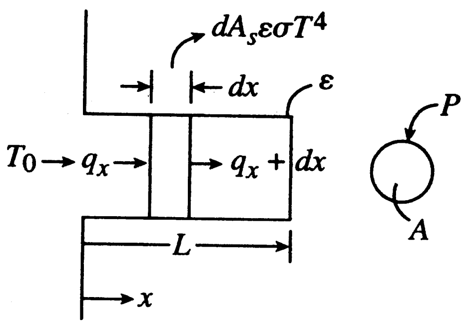
$$J_1 = 1489.5 + 0.11 J_{2L}$$

$$J_{2L} = 0.11 J_1 + 334.4 + \frac{E_{b_2}}{2.778} = 504.34 + 0.3644 E_{b_2}$$

$$\frac{504.34 - 0.6356 E_{b_2}}{2.778} + (0.8)(0.09)(418 - E_{b_2}) = (2)(1.42)\left(\frac{1}{0.3}\right)^{1/4}(T - 293)^{5/4}$$

Solution by iteration: $E_{b_2} = \sigma T_2^{\,4}$ $\qquad T_2 = 304.8\ \text{K} = 31.8°\text{C}$

8-66



$$q_x = q_{x+dx} + dA_s\varepsilon\sigma T^4 \qquad -kA\frac{dT}{dx} = -kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right) + Pdx\varepsilon\sigma T^4$$

$$kA\frac{d^2T}{dx^2} - P\varepsilon\sigma T^4 = 0 \qquad \frac{d^2T}{dx^2} - \frac{P\varepsilon\sigma}{kA}T^4 = 0$$

Boundary conditions

(1) at $x = 0$ $\quad T = T_0$

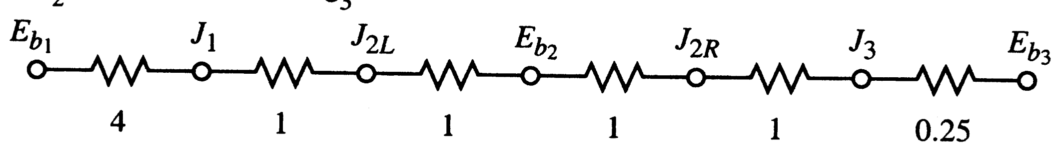
(2) at $x = L$ $\quad -kA\frac{dT}{dx} = A\varepsilon\sigma T^4$

8-67

$T_1 = 1200$ K $\quad T_3 = 300$ K $\quad E_{b_1} = 1.1755\times10^5 \quad E_{b_3} = 459$

$\varepsilon_1 = 0.2 \quad \varepsilon_2 = 0.5 \quad \varepsilon_3 = 0.8 \quad F_{12} = F_{23} = 1.0 \quad \frac{1-\varepsilon_1}{\varepsilon_1} = 4.0$

$\frac{1-\varepsilon_2}{\varepsilon_2} = 1.0 \quad \frac{1-\varepsilon_3}{\varepsilon_3} = 0.25$



$$\frac{117{,}550 - E_{b_2}}{4+1+1} = \frac{117{,}550 - 459}{4+1+1+1+1+0.25}$$

$E_{b_2} = 32{,}393\ \text{W/m}^2 = \sigma T_2^4 \qquad T_2 = 869$ K

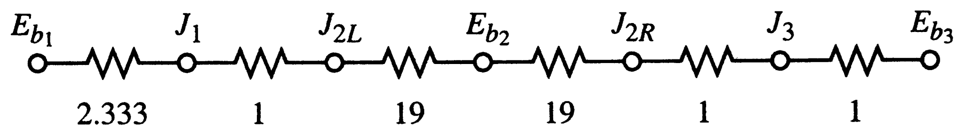
8-68

$T_1 = 900\text{ K}$ $\varepsilon_1 = 0.3$ $T_3 = 400\text{ K}$ $\varepsilon_3 = 0.5$ $\varepsilon_2 = 0.05$

$E_{b_1} = 37{,}194$ $E_{b_3} = 1451\ \text{W/m}^2$

(a) $\dfrac{q}{A}\ (\text{w/o shield}) = \dfrac{37{,}194 - 1451}{\frac{1}{0.3} + \frac{1}{0.5} - 1} = 8248\ \text{W/m}^2$

$\dfrac{1-\varepsilon_1}{\varepsilon_1} = 2.333$ $\dfrac{1-\varepsilon_2}{\varepsilon_2} = 19$ $\dfrac{1-\varepsilon_3}{\varepsilon_3} = 1.0$

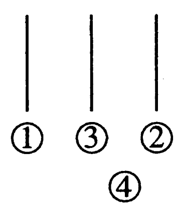


(b) $q = \dfrac{37{,}194 - 1451}{42.333} = 844.3\ \text{W/m}^2$ a reduction of 90%

(c) $q = \dfrac{37{,}194 - E_{b_2}}{2.333 + 1 + 19} = 844.3$

$E_{b_2} = 18{,}338\ \text{W/m}^2 = \sigma T_2^{\,4}$ $T_2 = 754\text{ K}$

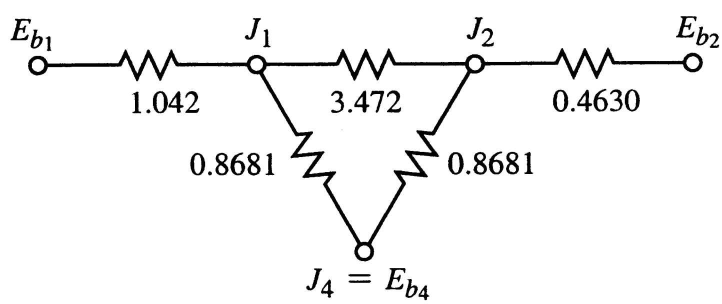
8-69



$A_1 = A_2 = A_3 = 1.44\ \text{m}^2$ $\varepsilon_1 = 0.4$ $\varepsilon_2 = 0.6$ $A_4 \to \infty$

$T_1 = 1033\text{ K}$ $T_2 = 573\text{ K}$ $\varepsilon_3 = 0.05$ $T_4 = 313\text{ K}$

(a)



$E_{b_1} = 64{,}552$ $E_{b_2} = 6111$ $E_{b_4} = 544$

$F_{12} = 0.2$ $F_{14} = F_{24} = 0.8$

Node J_1

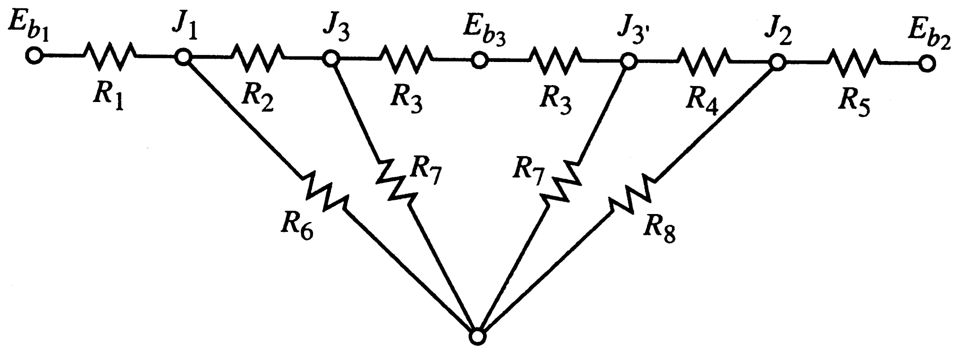
$$\frac{64{,}552 - J_1}{1.042} + \frac{J_2 - J_1}{3.472} + \frac{544 - J_1}{0.8681} = 0 \qquad J_1 = 26{,}791 \text{ W/m}^2$$

Node J_2

$$\frac{J_1 - J_2}{3.472} + \frac{544 - J_2}{0.8681} + \frac{6111 - J_2}{0.4630} = 0 \qquad J_2 = 5984 \text{ W/m}^2$$

$$q_1 = \frac{64{,}552 - 26{,}791}{1.042} = 36{,}239 \text{ W} \qquad q_2 = \frac{6111 - 5984}{0.463} = 274 \text{ W}$$

(b)



$F_{12} = 0.42$ $\quad F_{23} = 0.42$ $\quad F_{14} = F_{24} = 0.58$

$R_1 = 1.042$ $\quad R_2 = 1.653$ $\quad R_4 = 1.653$

$R_6 = R_7 = R_8 = 1.197$ $\quad R_3 = 13.194$ $\quad R_5 = 0.4630$

$$J_1: \ \frac{64{,}552 - J_1}{1.042} + \frac{J_3 - J_1}{1.653} + \frac{544 - J_1}{1.197} = 0 \qquad J_1 = 28{,}895$$

$$J_3: \ \frac{J_1 - J_3}{1.653} + \frac{J_{3'} - J_3}{(2)(13.194)} + \frac{544 - J_3}{1.197} = 0 \qquad J_3 = 11{,}602$$

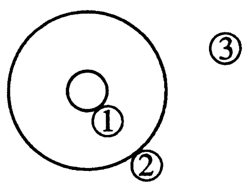
$$J_{3'}: \ \frac{J_3 - J_{3'}}{(2)(13.194)} + \frac{J_2 - J_{3'}}{1.653} + \frac{544 - J_{3'}}{1.197} = 0 \qquad J_{3'} = 1855$$

$$J_2: \ \frac{J_{3'} - J_2}{1.653} + \frac{6111 - J_2}{0.4630} + \frac{544 - J_2}{1.197} = 0 \qquad J_2 = 4195$$

$$q_1 = \frac{64{,}552 - 28{,}895}{1.042} = 34{,}220 \text{ W} \qquad q_2 = \frac{6111 - 4195}{0.463} = 4138 \text{ W}$$

$$E_{b_3} = \frac{J_3 + J_{3'}}{2} = 6729 = \sigma T_3^{\,4} \qquad T_3 = 587 \text{ K} = 314°\text{C}$$

8-70



$T_1 = 773\text{ K}$ $\quad T_1 = 298\text{ K}$ $\quad \varepsilon_1 = 0.8$ $\quad \varepsilon_2 = 0.2$ $\quad \dfrac{A_1}{L} = 0.07854$

$\dfrac{A_2}{L} = 0.9425$ $\quad E_{b_1} = 20{,}241\ \text{W/m}^2$ $\quad E_{b_3} = 447$ $\quad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 3.183$

$\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 4.244$ $\quad \dfrac{1}{A_1 F_{12}} = 12.732$ $\quad \dfrac{1}{A_2 F_{23}} = 1.061$

$\dfrac{1-\varepsilon_3}{\varepsilon_3 A_3} \to 0$ $\quad$ w/o shield $\quad \dfrac{q}{L} = \dfrac{20{,}241 - 447}{3.183 + 12.732} = 1244\ \text{W/m}$

w/ shield: $\quad \dfrac{q}{L} = \dfrac{20{,}241 - 447}{3.183 + 12.732 + (2)(4.244) + 1.061} = 777\ \text{W/m}$

$\dfrac{20{,}241 - E_{b_2}}{20{,}241 - 447} = \dfrac{3.183 + 12.732 + 4.244}{3.183 + 12.732 + (2)(4.244) + 1.061}$ $\quad E_{b_2} = 4571$

$4571 = \sigma T_2^{\,4}$ $\quad T_2 = 533\text{ K} = 260°\text{C}$

8-71

$T_1 = 1073\text{ K}$ $\quad T_3 = 373\text{ K}$ $\quad \varepsilon_1 = 0.8$ $\quad \varepsilon_3 = 0.4$

$d_1 = 4\text{ cm}$ $\quad d_3 = 8\text{ cm}$ $\quad d_2 = 6\text{ cm}$ $\quad \varepsilon_2 = 0.3$

$E_{b_1} = 75{,}146\ \text{W/m}^2$ $\quad E_{b_3} = 1097\ \text{W/m}^2$

<u>w/o shield</u>

$A_1 = \pi d_1 = 0.1257$ (per unit length)

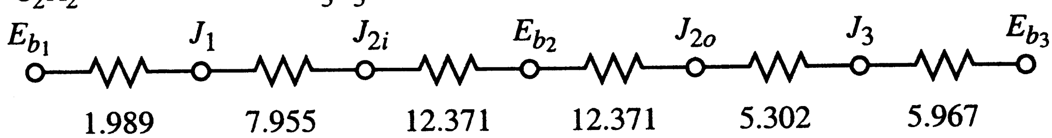
$q = \dfrac{0.1257(75{,}146 - 1097)}{\frac{1}{0.8} + 0.5\left(\frac{1}{0.4} - 1\right)} = 4654\ \text{W/m}$

<u>with shield</u>

$A_2 = 0.1886$ $\quad A_3 = 0.2514\ \text{m}^2/\text{m}$ $\quad F_{12} = F_{23} = 1.0$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 1.989$ $\quad \dfrac{1}{A_1 F_{12}} = 7.955$ $\quad \dfrac{1}{A_2 F_{23}} = 5.302$

$\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 12.371$ $\quad \dfrac{1-\varepsilon_3}{\varepsilon_3 A_3} = 5.967$



$\dfrac{q}{L} = \dfrac{E_{b_1} - E_{b_3}}{\sum R} = \dfrac{75{,}146 - 1097}{45.953} = 1611\ \text{W/m}$ $\quad$ a reduction of 65%

8-73

$E_{b_1} = \sigma(1073)^4 = 75{,}133$ $\quad E_{b_2} = \sigma(673)^4 = 11{,}628$ $\quad E_{b_3} = \sigma(23)^4 \sim 0$

$A_1 = 0.1963$ $\quad A_2 = \dfrac{A_1}{4} = 0.0491$ $\quad A_3 \to \infty$ $\quad \varepsilon_1 = 0.6$ $\quad \varepsilon_2 = 0.4$

$r_1 = 0.25$ $\quad r_2 = 0.125$ $\quad L = 0.25$ $\quad \dfrac{L}{r_1} = 1.0$ $\quad \dfrac{r_2}{L} = 0.5$

$F_{12} = 0.12$ $\quad F_{13} = 0.88$ $\quad F_{21} = (4)F_{12} = 0.48$ $\quad F_{23} = 1 - 0.48 = 0.52$

$J_1 - (1-0.6)[0.12J_2 + 0.88(0)] = 0.6(75{,}133)$

$J_2 - (1-0.4)[0.48J_1 + 0.52(0)] = 0.4(11{,}628)$

$J_1 = 45{,}938$ $\quad J_2 = 17{,}881\ \mathrm{W/m^2}$ $\quad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 3.396$

$$q = \frac{E_{b_1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{75{,}133 - 45{,}938}{3.396} = 8597\ \mathrm{W}$$

8-75

d = 0.03; x = 0.06

d/x = 0.5, F=12+ = 0.021; $T_1 = 373$ K; $T_2 = 273$ K; $T_3 = 264$ K

$A = \pi(0.03)^2/4 = 0.000707$

$q_{21} = (0.000707)(0.021)\sigma(273^4 - 373^4) = -0.01162$ W

$q_{23} = (0.000707)(0.979)\sigma(273^4 - 263^4) = 0.03022$

$q_{2\,tot} = 0.0186$ W; 1 percent = 0.186 mW

8-77

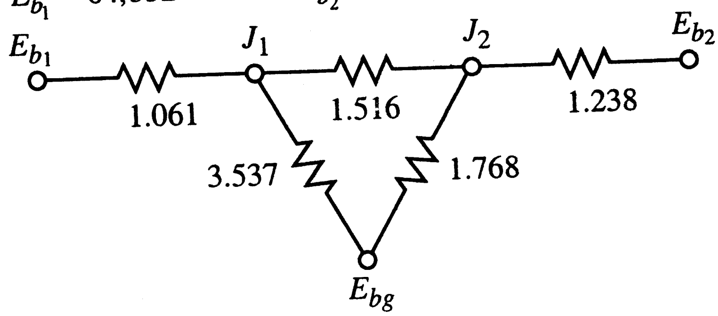
$d_1 = 0.3$ $\quad d_2 = 0.6$ $\quad \varepsilon_1 = 0.5$ $\quad \varepsilon_2 = 0.3$ $\quad T_1 = 1033\ \text{K}$

$T_2 = 643\ \text{K}$ $\quad A_1 = \pi(0.3) = 0.9425$ $\quad \tau_g = 0.7$

$A_2 = \pi(0.6) = 1.885$ $\quad \alpha_g = \varepsilon_g = 0.3$ $\quad F_{12} = 1.0$

$F_{1g} = F_{2g} = 1.0$ $\quad$ Resistance shown on figure

$E_{b_1} = 64{,}552$ $\quad E_{b_2} = 9691$



$$q = \frac{64{,}552 - 9691}{1.061 + \dfrac{1}{\frac{1}{5.16} + \frac{1}{3.537 + 1.768}} + 1.238} = 15{,}773\ \text{W/m}$$

$$15{,}773 = \frac{64{,}552 - J_1}{1.061} = \frac{J_2 - 9691}{1.238} \qquad J_1 = 47{,}816 \qquad J_2 = 29{,}218$$

$$\frac{47{,}816 - E_{bg}}{47{,}816 - 29{,}218} = \frac{3.537}{3.537 + 1.768} \qquad E_{bg} = 35{,}416 = \sigma T_g^{\,4}$$

$T_g = 889\ \text{K} = 616°\text{C}$

8-79

$T_1 = 1033\text{ K}$ $\quad T_2 = 643\text{ K}$ $\quad E_{b_1} = 64{,}550$ $\quad E_{b_2} = 9690$

$\varepsilon_1 = 0.5$ $\quad \varepsilon_2 = 0.3$ $\quad F_{12} = 1.0$ $\quad F_{1g} = 1.0$ $\quad F_{2g} = 1.0$

$\alpha_g = \varepsilon_g = 0.3$ $\quad \tau_g = 0.7$

Consider per unit area, $A_1 = A_g = A_2 = 1.0$

$$\frac{1-\varepsilon_1}{\varepsilon_1} = 1.0 \qquad \frac{1-\varepsilon_2}{\varepsilon_2} = 2.333 \qquad \frac{1}{F_{12}(1-\varepsilon_g)} = \frac{1}{0.7} = 1.429$$

$$\frac{1}{F_{1g}\varepsilon_g} = \frac{1}{F_{2g}\varepsilon_g} = \frac{1}{0.3} = 3.333$$

$$q = \frac{E_{b_1} - E_{b_2}}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2} + \cfrac{1}{\cfrac{1}{\frac{1}{F_{1g}\varepsilon_g} + \frac{1}{F_{2g}\varepsilon_g}} + F_{12}(1-\varepsilon_g)}}$$

$$= \frac{64{,}550 - 9690}{1 + 2.333 + \cfrac{1}{\frac{1}{(2)(3.333)} + 0.7}}$$

$$= \frac{54{,}860}{4.5097} = 12{,}160\text{ W}$$

$$q = 12{,}160 = \frac{64{,}550 - J_1}{1.0} = \frac{J_2 - 9690}{2.333} \qquad J_1 = 52{,}385 \qquad J_2 = 38{,}063$$

$$E_{bg} = \frac{J_1 + J_2}{2} = 45{,}224 = \sigma T_g^{\,4} \qquad T_g = 945\text{ K} = 672°\text{C}$$

8-80

Same network as 8–61 $\qquad \text{Re} = \dfrac{(1.6)(6)(0.6-0.3)}{5.4\times10^{-5}} = 53{,}330$

$$\text{Pr} = \frac{(1670)(5.4\times10^{-5})}{0.11} = 0.82$$

$$h_1 = \frac{0.11}{0.3}(0.023)(53{,}330)^{0.8}(0.82)^{0.4} = 47.11$$

$$h_2 = \frac{1}{2}h_1 = 23.55 \qquad T_g = 1373\text{ K} \qquad E_{bg} = 2.015\times10^5$$

$$(J_1) \quad \frac{64{,}552 - J_1}{1.061} + \frac{J_2 - J_1}{1.516} + \frac{2.015\times10^5 - J_1}{3.537} = 0 \qquad J_1 = 95{,}980$$

$$(J_2) \quad \frac{J_1 - J_2}{1.516} + \frac{2.015\times10^5 - J_2}{1.768} + \frac{9691 - J_2}{1.238} = 0 \qquad J_2 = 91{,}051$$

Inner Surface:

$$\text{cooling} = \frac{95{,}980 - 64{,}552}{1.061} + (47.11)\pi(0.3)(1373-1033) = 44{,}717\ \text{ W/m}$$

Outer Surface:

$$\text{cooling} = \frac{91{,}051 - 9691}{1.238} + (23.55)\pi(0.6)(1373 - 643) = 98{,}124 \ \text{W/m}$$

8-81

$E_{b_1} = 37{,}194$ $\qquad E_{b_2} = 14{,}513$ $\qquad \dfrac{A_1}{L} = 0.1571$ $\qquad \dfrac{A_2}{L} = 0.3142$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 9.548$ $\qquad \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 2.122$ $\qquad \dfrac{1}{A_1 F_{2g}\varepsilon_g} = 42.44$

$\dfrac{1}{A_2 F_{2g}\varepsilon_g} = 21.22$ $\qquad \dfrac{1}{A_1 F_{12}(1-\varepsilon_g)} = 7.489$

$$R = 9.548 + \frac{1}{\frac{1}{7.489} + \frac{1}{42.44+21.22}} = 18.37$$

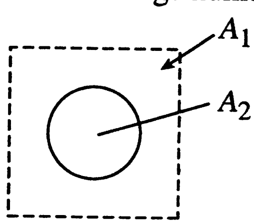
$$q = \frac{37{,}194 - 14{,}513}{18.37} = 1235 \ \text{W/m} \cdot \text{length} = \frac{37{,}194 - J_1}{9.548} = \frac{J_2 - 14{,}513}{21.22}$$

$J_1 = 25{,}406$ $\qquad J_2 = 17{,}134$

$E_{bg} = 17{,}134 + \dfrac{1}{3}(25{,}406 - 17{,}134) = 19{,}891$ $\qquad T_g = 770 \ \text{K}$

8-82

Assume large number of pins so that array behaves like single pin



cavity area $= A_3$ $\qquad A_2 = \pi(1)^2 = \pi \ \text{mm}^2 = 3.416$

$A_1 = (4)^2 - \pi = 16 - \pi = 12.858$ $\qquad A_3 = A_1 + (4)(4)(12) + \pi(2)(12) = 472.256$

$$\varepsilon_{a_1} = \frac{\varepsilon A_3}{A_1 + \varepsilon(A_3 - A_1)} = \frac{472.256\varepsilon}{12.858 + 459.398\varepsilon}$$

$$\text{Energy emitted / area} = \frac{\varepsilon_{a_1} A_1 + \varepsilon A_2}{A_1 + A_2} E_b$$

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_{a_1} A_1 + \varepsilon A_2}{A_1 + A_2} = \frac{\left[\frac{(472.256)(12.858)}{12.858+459.398(0.3)}\right](0.3)}{16} = 0.8145$$

8-83

Assume array behaves same as single cell

$A_o = (6)(6) = 36$

$A_i = 36 + (4)(6)(10) = 276$

$\varepsilon_i = 0.1$

$$\varepsilon_a = \frac{\varepsilon_i A_i}{A_o + \varepsilon_i(A_i - A_o)} = \frac{(0.1)(276)}{36 + (0.1)(240)} = 0.46$$

8-84

$A_o = \pi(12.5)^2 = 490.87\ \text{mm}^2 \qquad x = \text{depth}$

$A_i = \pi(3)^2 + \pi(3 + 12.5)[x^2 + (12.5 - 3)^2]^{1/2} = \pi[9 + 15.5(x^2 + 90.25)^{1/2}]$

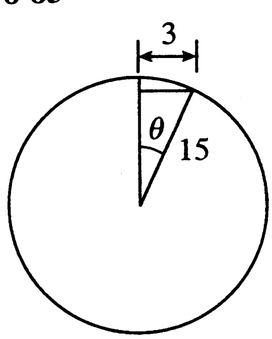
$$\varepsilon_a = \frac{\varepsilon_i A_i}{A_o + \varepsilon_i(A_i - A_o)} \qquad \varepsilon_a = 0.98 \qquad \varepsilon_i = 0.9$$

$$0.98 = \frac{0.9 A_i}{490.87 + (0.9)(A_i - 490.87)}$$

$A_i = 2672.52 = \pi[9 + 15.5(x^2 + 90.25)^{1/2}]$

$x = 53.46\ \text{mm}$

8-85



$$\sin\theta = \frac{3}{15} = 0.2 \qquad \theta = 11.537° \qquad A_o = \pi(3)^2 = 28.274$$

$\varepsilon_i = 0.5$

$A_i = 4\pi r^2 - 4\pi r^2(1 - \cos\theta) = 4\pi r^2\cos\theta = 4\pi(15)^2\cos(11.537°) = 2770$

$$\varepsilon_a = \frac{\varepsilon_i A_i}{A_o + \varepsilon_i(A_i - A_o)} = \frac{(0.5)(2770)}{28.274 + (0.5)(2770 - 28.274)} = 0.9899$$

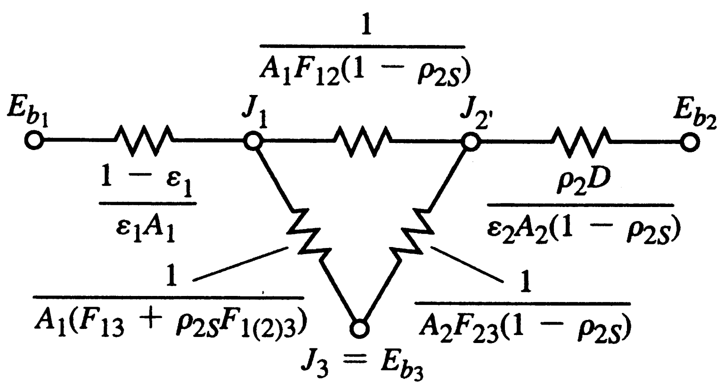
8-86

$\tau_o = 0.5 \qquad \varepsilon_o = 0.4 \qquad \rho_0 = 0.1 \qquad \varepsilon_i = 0.6 \qquad A_o = (25)^2 = 625 \text{ cm}^2$

$A_i = 625 + (4)(25)(10) = 1625 \qquad k = \dfrac{\varepsilon_i}{\tau_o + \frac{\varepsilon_o}{2}} = \dfrac{0.6}{0.5 + 0.2} = 0.85714$

$$\varepsilon_a = \frac{\left(\tau_o + \frac{\varepsilon_o}{2}\right)k}{\left(\frac{A_o}{A_i}\right)(1-\varepsilon_i) + k} = \frac{\varepsilon_i}{\left(\frac{A_o}{A_i}\right)(1-\varepsilon_i) + k} = \frac{0.6}{\left(\frac{625}{1625}\right)(1-0.6) + 0.85714} = 0.5934$$

8-87



$T_1 = 813 \text{ K} \qquad \varepsilon_1 = 0.3 \qquad T_2 = 533 \text{ K} \qquad T_3 = 293 \text{ K} \qquad \rho_{D2} = 0.2$

$F_{1(2)2} = 0.6 \qquad F_{1(2)1} = 0.37 \qquad F_{1(2)3} = 0.23 \qquad \rho_{S2} = 0.4 \qquad \varepsilon_2 = 1 - \rho_2 = 0.4$

$A_1 = A_2 = \pi(0.05)^2 = 7.854 \times 10^{-3} \text{ m}^2 \qquad F_{23} = 0.4 \qquad F_{13} = 0.4$

$E_{b_1} = 24{,}767 \text{ W/m}^2 \qquad E_{b_2} = 4575 \qquad E_{b_3} = 418 \qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 297.1$

$\dfrac{1}{A_1F_{12}(1-\rho_{2S})} = 3537 \qquad \dfrac{1}{A_1(F_{13} + \rho_{2S}F_{1(2)3})} = 258.8$

$\dfrac{\rho_2 D}{\varepsilon_2 A_2(1-\rho_{2S})} = 106.1 \qquad \dfrac{1}{A_2F_{23}(1-\rho_{2S})} = 530.5$

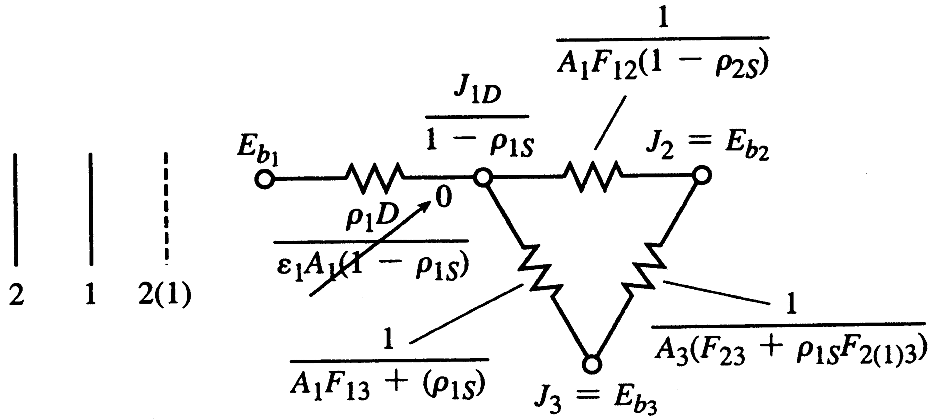
$J_{2'} = \dfrac{J_{D2}}{1-\rho_{2S}}$

$\dfrac{24{,}767 - J_1}{297.1} + \dfrac{J_{2'} - J_1}{353.7} + \dfrac{418 - J_1}{258.8} = 0 \qquad J_1 = 9878 \text{ W/m}^2$

$\dfrac{J_1 - J_{2'}}{353.7} + \dfrac{418 - J_{2'}}{530.5} + \dfrac{4575 - J_{2'}}{106.1} = 0 \qquad J_2 = 5081 \text{ W/m}^2$

$q_1 = \dfrac{24{,}767 - 9878}{297.1} = 50.11 \text{ W} \qquad q_2 = \dfrac{4575 - 5081}{106.1} = -4.76 \text{ W}$

8-88



$T_1 = 823\text{ K}$ $\quad \varepsilon_1 = 0.6$ $\quad T_3 = 283\text{ K}$ $\quad \rho_{1D} = 0$ $\quad \rho_{1S} = 0.4$

$A_1 = A_2 = 0.54\text{ m}^2$ $\quad F_{12} = F_{21} = 0.25$ $\quad F_{13} = 0.75$ $\quad J_{1D} = \varepsilon_1 E_{b_1} = 15{,}604$

$$\frac{J_{1D}}{1-\rho_{1S}} = 26{,}007\ \text{W/m}^2 \qquad \frac{1}{A_1F_{12}(1-\rho_{1S})} = 12.346 \qquad \frac{1}{A_1F_{13}(1-\rho_{1S})} = 4.15$$

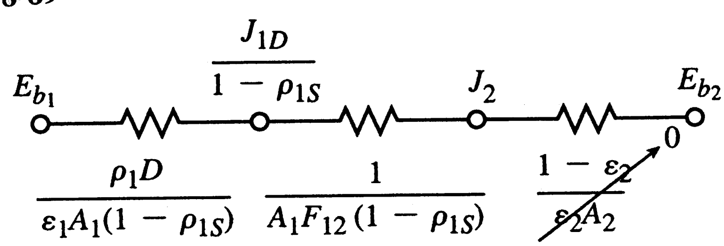
$$\frac{1}{A_2(F_{23}+\rho_{1S}F_{2(1)3})} = 2.281$$

$$q = \frac{26{,}007 - 363.6}{\frac{1}{\frac{1}{4.15}+\frac{1}{12.346+2.281}}} = 7985\text{ W}$$

$$\frac{26{,}007 - J_2}{26{,}007 - 363.6} = \frac{12.346}{12.346 + 2.281} \qquad J_2 = 4362 = \sigma T_2^{\,4}$$

$T_2 = 526.7\text{ K} = 253.7°\text{C}$

8-89



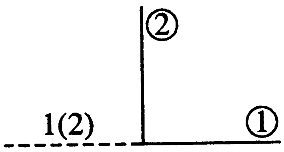
$F_{12} = 1$

$$q = \frac{E_{b_1} - E_{b_2}}{\sum R} = \varepsilon_1 A_1 (E_{b_1} - E_{b_2}) = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\frac{\rho_1 D}{\varepsilon_1 A_1(1-\rho_{1S})} + \frac{1}{A_1(1-\rho_{1S})}}$$

$$= \frac{A_1\sigma(1-\rho_{1S})(T_1^{\,4} - T_2^{\,4})}{\frac{\rho_{1D}}{\varepsilon_1} + 1}$$

Same answer with diffuse

8-90



$T_1 = 643\ \text{K} \qquad \varepsilon_1 = 0.6 \qquad A_1 = (0.3)(0.6) = 0.18\ \text{m}^2 \qquad T_3 = 363\ \text{K}$

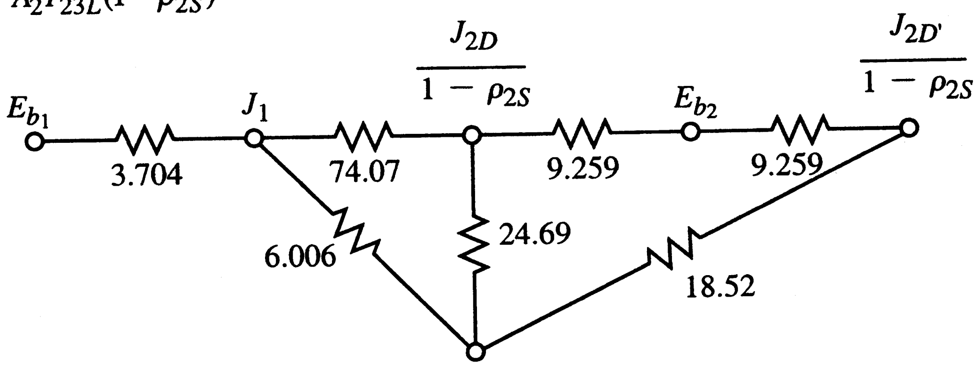
$\rho_{2S} = 0.7 \qquad \rho_{2D} = 0.1 \qquad \varepsilon_2 = 0.2 \qquad F_{12} = F_{21} = 0.25 \qquad F_{13} = 0.75$

$F_{23L} = 1.0 \qquad F_{23R} = 0.75 \qquad F_{1(2)3} = F_{12} = 0.25 \qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 3.704$

$\dfrac{1}{A_1 F_{12}(1-\rho_{2S})} = 74.07 \qquad \dfrac{\rho_{2D}}{\varepsilon_2 A_2 (1-\rho_{2S})} = 9.259$

$\dfrac{1}{A_1(F_{13} + \rho_{2S} F_{1(2)3})} = 6.006 \qquad \dfrac{1}{A_2 F_{23R}(1-\rho_{2S})} = 24.69$

$\dfrac{1}{A_2 F_{23L}(1-\rho_{2S})} = 18.52 \qquad E_{b_1} = 9691\ \text{W/m}^2 \qquad E_{b_3} = 984\ \text{W/m}^2$



$$q = \frac{E_{b_1} - E_{b_3}}{\sum R} = \frac{9691 - 984}{9.33} = 933\ \text{W}$$

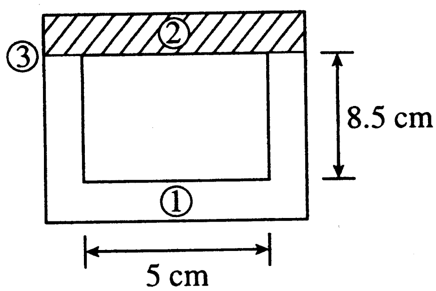
w/o Reflector: $q = (0.6)(0.18)(9691 - 984) = 940.4\ \text{W}$

$$\frac{9691 - J_1}{9691 - 984} = \frac{3.704}{9.33} \qquad J_1 = 6234\ \text{W/m}^2$$

$$\frac{6234 - \frac{J_{2D}}{1-\rho_{2S}}}{6234 - 984} = \frac{74.07}{74.07 + 14.84} \qquad \frac{J_{2D}}{1-\rho_{2S}} = 1859$$

$$\frac{1859 - E_{b_2}}{1859 - 984} = \frac{9.259}{37.038} \qquad E_{b_2} = 1640 = \sigma T_2^4 \qquad T_2 = 412\ \text{K} = 139°\text{C}$$

8-91



$T_1 = 533$ K $\quad \varepsilon_1 = 0.6 \quad \rho_{2S} = 0.1 \quad T_3 = 293$ K $\quad \rho_{2D} = 0.1$

$\tau_{2D} = 0.3 \quad \tau_{2S} = 0.3 \quad \varepsilon_2 = 0.2 \quad E_{b_1} = 4575\ \text{W/m}^2$

$E_{b_2} = 418\ \text{W/m}^2$

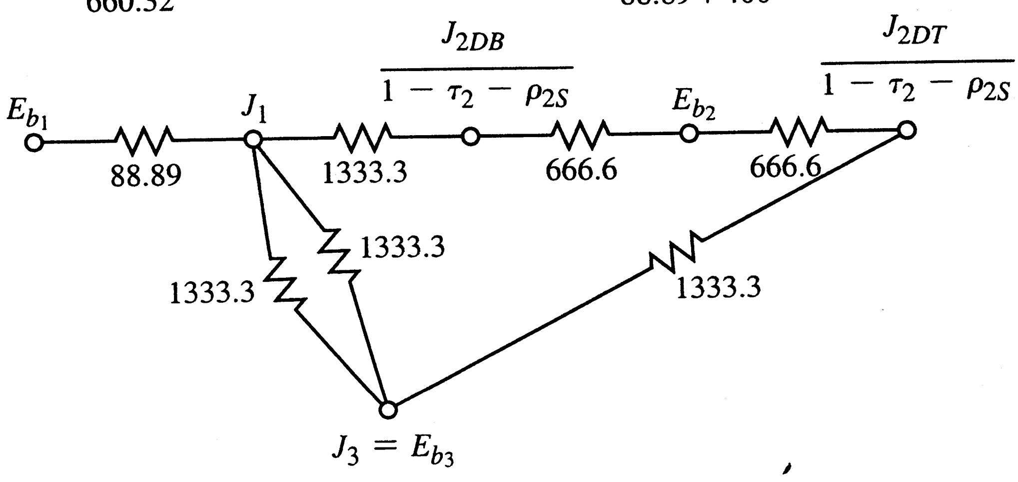
$A_1 = 75\ \text{cm}^2 = 7.5 \times 10^{-3}\ \text{m}^2 \qquad A_2 = 25\ \text{cm}^2 = 2.5 \times 10^{-3}\ \text{m}^2$

$A_3 \to \infty \qquad F_{21} = 1.0 \qquad F_{12} = \dfrac{25}{75} = 0.3333 = F_{13} \qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 88.89$

$$\frac{1}{A_2 F_{23T}(1-\tau_2-\rho_{2S})} = 1333.3 \qquad \frac{1}{A_1 F_{13}\tau_{2S}} = 1333.33 = \frac{1}{A_1 F_{12} F_{23T} \tau_{2D}}$$

$$\frac{1}{A_1 F_{12}(1-\tau_2-\rho_{2S})} = 1333.3 \qquad \frac{\rho_{2D}}{\varepsilon_2 A_2 (1-\tau_2-\rho_{2S})} = 666.6$$

$$q = \frac{4575-418}{660.32} = 6.295\ \text{W} \qquad \text{w/o window: } q = \frac{4575-418}{88.89+400} = 8.503\ \text{W}$$

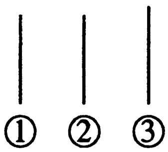


8-92

$$\frac{1}{A_1 F_{13}\tau_{2S}} = 666.6 \qquad \frac{1}{A_1 F_{12}(1-\tau_2-\rho_{2S})} = 1000 \qquad \frac{\rho_{2D}}{\varepsilon_2 A_2(1-\tau_2-\rho_{2S})} = 1000$$

$$\frac{1}{A_2 F_{23T}(1-\tau_2-\rho_{2S})} = 1000 \qquad q = \frac{4575-418}{660.32} = 6.295\ \text{W}$$

8-95

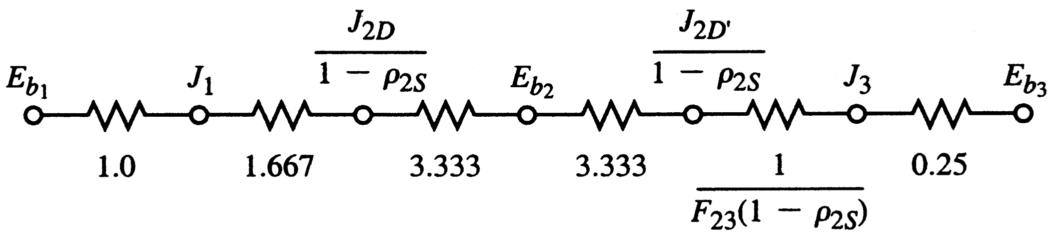


$T_1 = 1073\text{ K}$ $\quad T_3 = 308\text{ K}$ $\quad \varepsilon_1 = 0.5$ $\quad \varepsilon_3 = 0.8$ $\quad \rho_{2D} = 0.4$

$\varepsilon_2 = 0.2$ $\quad F_{12} = F_{23} = 1.0$ etc. $\quad \dfrac{1-\varepsilon_1}{\varepsilon_1} = 1.0$ $\quad \dfrac{1}{F_{12}(1-\rho_{2S})} = 1.67$

$\dfrac{\rho_{2D}}{\varepsilon_2(1-\rho_{2S})} = 3.333$ $\quad \dfrac{1-\varepsilon_3}{\varepsilon_3} = 0.25$ $\quad E_{b_1} = 75{,}146\ \text{W/m}^2$

$E_{b_3} = 510\ \text{W/m}^2$



with shield: $\dfrac{q}{A} = \dfrac{75{,}146 - 510}{1 + 1.667 + (2)(3.3333) + 1.667 + 0.25} = 6634\ \text{W/m}^2$

w/o shield: $\dfrac{q}{A} = \dfrac{75{,}146 - 510}{1 + 1 + 0.25} = 33{,}172\ \text{W/m}^2$

same for diffuse shield

8-96

$F_{12} = 1.0$ $\quad F_{21} = \dfrac{A_1}{A_2}$ $\quad F_{11} = 0$ $\quad F_{22} = 1 - \dfrac{A_1}{A_2}$

$J_1(1-0) - (1-\varepsilon_1)(F_{12}J_2) = \varepsilon_1 E_{b_1}$

$J_2\left[1 - \left(1 - \dfrac{A_1}{A_2}\right)(1-\varepsilon_2)\right] - (1-\varepsilon_1)(F_{21}J_1) = \varepsilon_2 E_{b_2}$

For $A_2 \to \infty$, $F_{22} \to 1.0$, and $F_{21} \to 0$, and

$J_1 - (1-\varepsilon_1)J_2 = \varepsilon_1 E_{b_1}$ $\quad J_2[1-(1-\varepsilon_2)] - (1-\varepsilon_2)(0)J_1 = \varepsilon_2 E_{b_2}$

Thus, $J_2 = E_{b_2}$

$J_1 = (1-\varepsilon_1)E_{b_2} + \varepsilon_1 E_{b_1}$

$q = \dfrac{\varepsilon_1 A_1}{1-\varepsilon_1}(E_{b_1} - J_1) = \dfrac{\varepsilon_1 A_1}{1-\varepsilon_1}[E_{b_1} - \varepsilon E_{b_1} - (1-\varepsilon_1)E_{b_2}] = \varepsilon_1 A_1 (E_{b_1} - E_{b_2})$

This agrees with Eq. (8–43a)

8-100

$d_1 = d_2 = 30$ cm $\qquad x = 5$ cm $\qquad \dfrac{d}{x} = 6 \qquad F_{12} = F_{21} = 0.7$

$F_{13} = F_{23} = 0.3 \qquad T_3 = 293$ K $\qquad \dfrac{q_1}{A} = 1 \times 10^5\ \mathrm{W/m^2}$

$\varepsilon_1 = 0.9 \qquad \varepsilon_2 = 0.5 \qquad J_1 - F_{12}F_{2T} = 10^5$

$J_{2T} - (1-0.5)(F_{23}E_{b_3} + F_{21}J_1) = 0.5E_{b_2}$

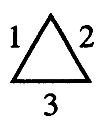
$J_{2B} - (1-0.5)(E_{b_3}) = 0.5E_{b_2} \qquad E_{b_2} = \dfrac{J_{2T} + J_{2B}}{2}$

Solution: $E_{b_2} = 4678 \qquad T_2 = 953$ K $\qquad J_{2T} = 69{,}961$

$J_1 = 1.49 \times 10^5 \qquad 10^5 = \dfrac{E_{b_1} - 1.49 \times 10^5}{\frac{1-0.9}{0.9}} \qquad E_{b_1} = 10.49 \times 10^5$

$T_1 = 2074$ K

8-101



$T_1 = 900$ K $\qquad \varepsilon_1 = 0.6 \qquad T_2 = 1500$ K $\qquad \varepsilon_2 = 0.8$

$\left.\dfrac{q}{A}\right|_3 = 1000\ \mathrm{W/m^2} \qquad \varepsilon_3 = 0.7$

$F_{12} = F_{13} = F_{21} = F_{23} = F_{31} = F_{32} = 0.5 \qquad E_{b_1} = 37{,}190 \qquad E_{b_2} = 2.87 \times 10^5$

$J_1 - 0.4(F_{12}J_2 + F_{13}J_3) = 0.6E_{b_1} \qquad J_1 = 111{,}479$

$J_2 - 0.2(F_{21}J_1 + F_{23}J_3) = 0.8E_{b_2} \qquad J_2 = 259{,}391$

$J_3 - (F_{31}J_1 + F_{32}J_2) = 1000 \qquad J_3 = 186{,}435$

$\left.\dfrac{q}{A}\right|_1 = \dfrac{37{,}190 - 111{,}479}{\frac{1-0.6}{0.6}} = -1.115 \times 10^5$

$\left.\dfrac{q}{A}\right|_2 = \dfrac{287{,}000 - 259{,}391}{\frac{1-0.8}{0.8}} = +1.105 \times 10^5$

$1000 = \dfrac{E_{b_3} - 186{,}435}{\frac{1-0.7}{0.7}} \qquad E_{b_3} = 1.888 \times 10^5 \qquad T_3 = 1350$ K

8-102

$A_1 = A_2 = 1\ \text{m}^2$ $T_1 = 573\ \text{K}$ $\varepsilon_1 = 0.5$ $E_{b_1} = 6111$

$q_2 = 0$ $\varepsilon_2 = 0.7$ $T_3 = 303\ \text{K}$ $E_{b_3} = 478$

$F_{12} = F_{21} = 0.2$ $F_{13} = F_{23} = 0.8$ $F_{11} = F_{22} = 0$ $F_{31} = F_{32} = 0$

$F_{33} \cong 1.0$ $E_{b_3} = J_3$

$J_1 - (1-0.5)(0.2J_2 + 0.8E_{b_3}) = 0.5E_{b_1}$

$J_2 - (0.2J_1 + 0.8E_{b_3}) = 0$

Solution:

$J_1 = 3352\ \text{W/m}^2$ $J_2 = 1053 = \sigma T_2{}^4$ $T_2 = 369\ \text{K}$

$$q_1 = \frac{E_{b_1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{(0.5)(6111-478)(1)}{1-0.5} = 5633\ \text{W}$$

8-103

$d_1 = d_2 = 60\ \text{cm}$ $x = 15\ \text{cm}$ $T_1 = 813\ \text{K}$ $T_2 = 573\ \text{K}$ $\varepsilon_1 = 0.7$

$\varepsilon_2 = 0.5$ $T_3 = 303\ \text{K}$ $A_1 = A_2 = 0.2827\ \text{m}^2$ $F_{12} = 0.6 = F_{21}$

$F_{13} = F_{31} = 0.4$ $E_{b_1} = 24{,}767\ \text{W/m}^2$ $E_{b_2} = 6111$

$F_{11} = F_{22} = 0$ $F_{31} = F_{32} = 0$ $F_{34} \to 1.0$ $E_{b_3} = J_3 = 478$

$J_1 = (1-0.7)(0.6J_2 + 0.4E_{b_3}) + 0.7E_{b_1}$ $J_1 = 18{,}980\ \text{W/m}^2$

$J_2 = (1-0.5)(0.6J_1 + 0.4E_{b_3}) + 0.5E_{b_2}$ $J_2 = 8840\ \text{W/m}^2$

$$q_1 = \frac{\varepsilon_1 A_1 (E_{b_1} - J_1)}{1-\varepsilon_1} = \frac{(0.7)(0.2827)(24{,}767 - 18{,}980)}{0.3} = 3817\ \text{W}$$

$$q_2 = \frac{\varepsilon_2 A_2 (E_{b_2} - J_2)}{1-\varepsilon_2} = \frac{(0.5)(0.2827)(6111 - 8840)}{0.5} = -771\ \text{W}$$

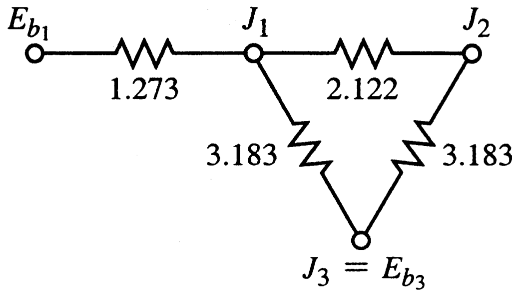
8-104

$d_1 = d_2 = 1\ \text{m} \qquad x = 0.25 \qquad \dfrac{d}{x} = 4 \qquad F_{12} = F_{21} = 0.6$

$F_{13} = F_{23} = 0.4 \qquad A_1 = A_2 = \pi(0.5)^2 = 0.7854 \qquad T_1 = 573\ \text{K}$

$T_3 = 303\ \text{K} \qquad \varepsilon_1 = 0.5 \qquad E_{b_1} = 6111 \qquad E_{b_3} = 478$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 1.273 \qquad \dfrac{1}{A_1 F_{12}} = 2.122 \qquad \dfrac{1}{A_1 F_{13}} = \dfrac{1}{A_2 F_{23}} = 3.183$



$$q = \frac{6111 - 478}{1.273 + \frac{1}{\frac{1}{3.183} + \frac{1}{2.122 + 3.183}}} = 1727\ \text{W}$$

8-105

$d_1 = d_2 = 50\ \text{cm} \qquad x = 10\ \text{cm} \qquad F_{12} = F_{21} = 0.65 \qquad F_{13} = F_{23} = 0.35$

$T_1 = 350\ \text{K} \qquad \varepsilon_1 = \varepsilon_2 = 0.6 \qquad E_{b_1} = 850.7$

$\left.\dfrac{q}{A}\right|_2 = 10{,}000\ \text{W/m}^2 \qquad \left.\dfrac{q}{A}\right|_3 = 0 \qquad F_{11} = F_{22} = 0 \qquad F_{33} \to 1.0$

$A_1 = A_2 = \pi(0.25)^2 = 0.1963\ \text{m}^2 \qquad A_3 = \pi(0.5)(0.1) = 0.1571\ \text{m}^2$

$F_{31} = F_{32} = 0.35\left(\dfrac{A_1}{A_3}\right) = 0.4373 \qquad F_{33} = 1 - (2)(0.4373) = 0.125$

$J_1 = (1-0.6)(0.65J_2 + 0.35J_3) + 0.6E_{b_1} \qquad J_1 = 7514.31\ \text{W/m}^2$

$J_2 = (0.65J_1 + 0.35J_3) + 10{,}000 \qquad J_2 = 19{,}632.83\ \text{W/m}^2$

$J_3 = \dfrac{1}{1-0.125}(0.4373J_1 + 0.4373J_2) \qquad J_3 = 13{,}566.78\ \text{W/m}^2$

$J_3 = E_{b_3} \qquad T_3 = \left(\dfrac{J_3}{5.669\times 10^{-8}}\right)^{1/4} = 699.4\ \text{K}$

$E_{b_2} = J_2 + \left(\dfrac{1-0.6}{0.6}\right)(10{,}000) = 26{,}299.5\ \text{W/m}^2$

$T_2 = \left(\dfrac{E_{b_2}}{5.669\times 10^{-8}}\right)^{1/4} = 825.3\ \text{K}$

8-106

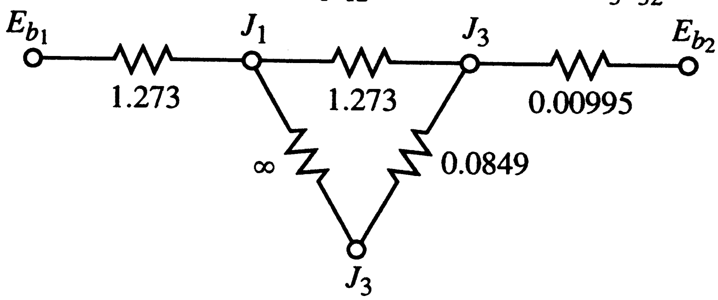
$T_1 = 900\text{ K} \qquad \varepsilon_1 = 0.5 \qquad E_{b_1} = 37{,}194 \qquad A_1 = \pi(0.5)^2 = 0.7854$

$A_2 = 2\pi(2)^2 = 25.13\text{ m}^2 \qquad T_2 = 300\text{ K} \qquad \varepsilon_2 = 0.8 \qquad E_{b_2} = 459$

$q_3 = 0 \qquad A_3 = \pi(2^2 - 0.5^2) = 11.781\text{ m}^2 \qquad F_{12} = F_{32} = 1.0$

$F_{21} = 1.0\left(\dfrac{A_1}{A_2}\right) = 0.0313 \qquad F_{23} = 1.0\left(\dfrac{A_3}{A_2}\right) = 0.4688 \qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 1.273$

$\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 0.00995 \qquad \dfrac{1}{A_1 F_{12}} = 1.273 \qquad \dfrac{1}{A_3 F_{32}} = 0.0849 \qquad \dfrac{1}{A_1 F_{13}} = \infty$



$$q = \frac{37{,}194 - 459}{1.273 + 1.273 + 0.00995} = 14{,}372\text{ W}$$

$$q = \frac{J_2 - E_{b_2}}{0.00995} \qquad J_2 = 602\ \text{W/m}^2$$

$$q = A_2(G_2 - J_2) \qquad G_2 = 1174\ \text{W/m}^2$$

8-107

$\dfrac{d}{x} = 1.0 \qquad F_{12} = 0.17 \qquad$ (1) disk $\qquad$ (2) room $\qquad$ (3) shield

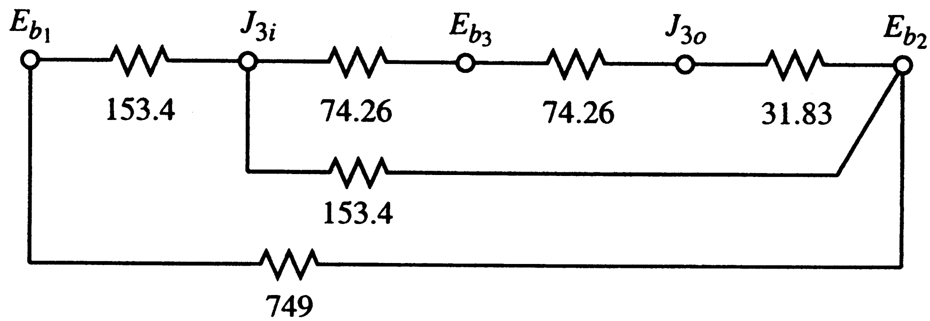
$F_{13} = 0.83 \qquad \varepsilon_1 = 1.0 \qquad \varepsilon_3 = 0.3 \qquad T_1 = 773\text{ K} \qquad E_{b_1} = 20{,}241\ \text{W/m}^2$

$T_2 = 313\text{ K} \qquad E_{b_2} = 544 \qquad A_1 = \pi(0.05)^2 = 0.007854\text{ m}^2$

$A_3 = \pi(0.1)(0.1) = 0.03142 \qquad F_{31} = 0.83\left(\dfrac{A_1}{A_3}\right) = 0.2075 = F_{32}$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0 \qquad \dfrac{1-\varepsilon_3}{\varepsilon_3 A_3} = 74.26 \qquad \dfrac{1}{A_1 F_{12}} = 749 \qquad \dfrac{1}{A_1 F_{13}} = 153.4$

$\dfrac{1}{A_3 F_{32}} = 153.4 \qquad F_{32}\text{ (out)} = 1.0 \qquad \dfrac{1}{A_3 F_{32}}\text{ (outside)} = 31.83$



$$\frac{1}{R_1} = \frac{1}{153.4} + \frac{1}{74.26 + 74.26 + 31.83} \qquad R_1 = 82.89 \qquad \frac{1}{R} = 5.5671 \times 10^{-3}$$

$$q = \frac{E_{b_1} - E_{b_2}}{\sum R} = (5.5671 \times 10^{-3})(20{,}241 - 544) = 109.7 \text{ W}$$

8-108

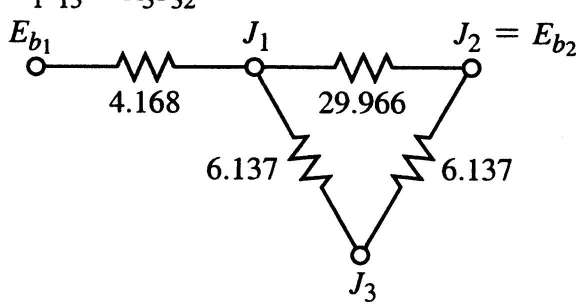
$\frac{d}{x} = 1.0 \qquad F_{12} = 0.17 \qquad$ (1) disk $\qquad$ (2) room $\qquad T_1 = 1273$ K

$T_2 = 293$ K $\qquad \varepsilon_1 = 0.55 \qquad E_{b_1} = 1.489 \times 10^5 \qquad E_{b_2} = 418 \ \text{W/m}^2$

$F_{13} = 0.83 \qquad A_1 = \pi(0.25)^2 = 0.1963 \qquad A_3 = \pi(0.5)^2 = 0.7854$

$F_{31} = F_{32} = \frac{0.83}{4} = 0.2075 \qquad \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 4.168 \qquad \frac{1}{A_1 F_{12}} = 29.966$

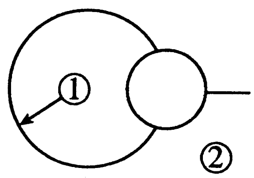
$\frac{1}{A_1 F_{13}} = \frac{1}{A_3 F_{32}} = 6.137$



$$q = \frac{148{,}900 - 418}{4.168 + \frac{1}{\frac{1}{29.966} + \frac{1}{2(6.137)}}} = 11{,}532 \text{ W}$$

8-109

$E_{b_1} = \sigma T_1^4 = 23{,}200 \ \text{W/m}^2 \qquad E_{b_2} = 459$



$$A_3 = \frac{\pi(7.1)^2}{4} = 39.6 \ \text{cm}^2$$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{0.55}{(0.45)(0.0156)} = 78.35 \qquad F_{13} = F_{12}$$

$$A_1 F_{13} = A_3 F_{31} = A_3$$

$$q = \frac{23{,}220 - 459}{78.35 + \frac{1}{0.00396}} = 68.79 \ \text{W}$$

8-110

Bottom = (1) Sides = (2) Room = (3) $A_1 = 7.854 \times 10^{-5}$

$A_2 = \pi(0.005 + 0.01)[(0.01^2 - 0.005^2) + 0.03^2]^{1/2} = 0.001433$

$T_1 = 773 \ \text{K} \qquad E_{b_1} = 20{,}241 \qquad E_{b_3} = 478 = J_3 \qquad T_3 = 303 \ \text{K}$

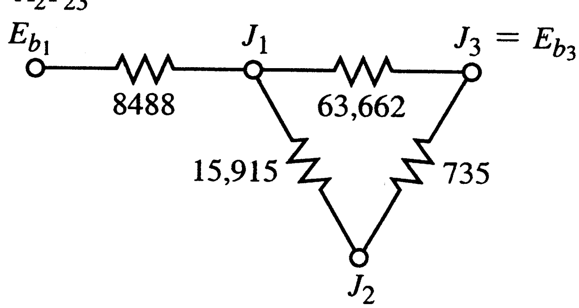
$\varepsilon_1 = 0.6$

For figure: $\dfrac{r_2}{L} = 0.167 \qquad \dfrac{L}{r_1} = 3 \qquad$ open hole $A_3 = 3.142 \times 10^{-4}$

$F_{13} = (0.05)(2)^2 = 0.8 \qquad \dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 8488 \qquad F_{23} = 1 - 0.05 = 0.95$

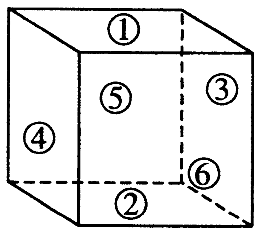
$F_{12} = 1 - 0.2 = 0.8 \qquad \dfrac{1}{A_1 F_{13}} = 63{,}662 \qquad \dfrac{1}{A_1 F_{12}} = 15{,}915$

$\dfrac{1}{A_2 F_{23}} = 735$



$$q = \frac{20{,}241 - 478}{8488 + \frac{1}{\frac{1}{63{,}662} + \frac{1}{15{,}915 + 735}}} = 0.911 \ \text{W}$$

8-111



$T_1 = 373\ \text{K}$ $\quad T_2 = 313\ \text{K}$ $\quad \varepsilon_1 = 0.7$ $\quad \varepsilon_2 = 0.5$

$\varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0.6$ $\quad T_3 = T_4 = 343\ \text{K}$ $\quad T_5 = T_6 = 60°\text{C} = 333\ \text{K}$

$E_{b_1} = 1097\ \text{W/m}^2$ $\quad E_{b_2} = 544$ $\quad E_{b_3} = E_{b_4} = 785$ $\quad E_{b_5} = E_{b_6} = 735$

$F_{12} = 0.2$ $\quad F_{13} = F_{14} = F_{15} = F_{16} = 0.2$ $\quad$ All shape factors are 0.2

$A_{ij} = 0$ $\quad$ From symmetry $\quad J_3 = J_4$ $\quad J_5 = J_6$

$$J_1 - (1-0.7)(0.2)(J_2 + 2J_3 + 2J_5) = (0.7)(1097)$$
$$J_2 - (1-0.5)(0.2)(J_1 + 2J_3 + 2J_5) = (0.5)(544)$$
$$J_3 - (1-0.6)(0.2)(J_1 + J_2 + J_3 + 2J_5) = (0.6)(785)$$
$$J_5 - (1-0.6)(0.2)(J_1 + J_2 + J_5 + 2J_3) = (0.6)(735)$$

Solving:

$J_1 = 995.215\ \text{W/m}^2$ $\quad J_2 = 682.195\ \text{W/m}^2$ $\quad J_3 = J_4 = 790.51$

$J_5 = J_6 = 762.68\ \text{W/m}^2$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0.4286$ $\quad \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 1.0$ $\quad \dfrac{1-\varepsilon_3}{\varepsilon_3 A_3}$, etc. $= 0.6667$

$q_1 = \dfrac{E_{b_1} - J_1}{0.4286} = 238\ \text{W}$ $\quad q_2 = \dfrac{E_{b_2} - J_1}{1.0} = -138\ \text{W}$

$q_3 = q_4 = \dfrac{E_{b_3} - J_3}{0.6667} = -8.3\ \text{W}$ $\quad q_5 = q_6 = \dfrac{E_{b_5} - J_5}{0.6667} = -41.5\ \text{W}$

8-112

$r_1 = 10\ \text{cm}$ $\quad r_2 = 5\ \text{cm}$ $\quad L = 10\ \text{cm}$ $\quad \dfrac{r_2}{L} = 0.5$ $\quad \dfrac{L}{r_1} = 1.0$

$F_{12} = 0.11$ $\quad$ (3) room $\quad F_{13} = 1 - 0.11 = 0.89$ $\quad F_{13_{\text{bottom}}} = 1.0$

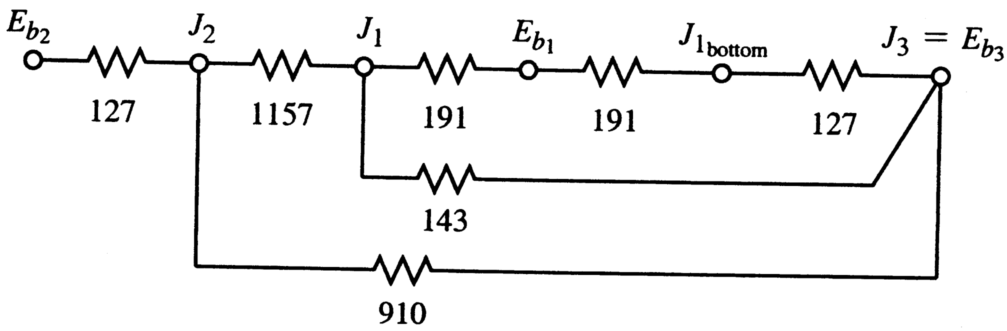
$T_2 = 700\ \text{K}$ $\quad \varepsilon_2 = 0.8$ $\quad \varepsilon_1 = 0.4$ $\quad T_3 = 298\ \text{K}$

$F_{21} = (0.11)(2)^2 = 0.44$ $\quad E_{b_2} = 13{,}611$ $\quad E_{b_3} = 447$

$F_{23} = 1 - 0.44 = 0.56$ $\quad A_1 = 0.007854$ $\quad A_2 = 0.001963\ \text{m}^2$

$\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} = 191$ $\quad \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2} = 127$ $\quad \dfrac{1}{A_3 F_{23}} = 910$ $\quad \dfrac{1}{A_1 F_{12}} = 1157$

$\dfrac{1}{A_1 F_{13}} = 143$ $\quad \dfrac{1}{A_1 F_{13_{\text{bot}}}} = 127$



$$\frac{1}{R_1} = \frac{1}{143} + \frac{1}{(2)(191)+127} = 0.008958 \qquad R_1 = 111.6$$

$$\frac{1}{R_2} = \frac{1}{910} + \frac{1}{1157 + R_1} \qquad R_2 = 529.9 \qquad R = 127 + R_2 = 656.9$$

$$q_2 = \frac{E_{b_2} - E_{b_3}}{R} = \frac{13{,}611 - 447}{656.9} = 20.04 \text{ W}$$

8-113

$T_1 = 873$ K $\quad \varepsilon_1 = 0.63 \quad r_1 = r_2 = 25$ cm $\quad q_2 = 80$ kW/m^2

$\varepsilon_2 = 0.75 \quad d = 12.5$ cm $\quad T_3 = 303$ K $\quad E_{b_1} = 32{,}928$ W/m^2

$E_{b_3} = 478 = J_3 \quad F_{12} = 0.6 \quad F_{13} = 0.4$

$A_1 = A_2 = \pi(0.25)^2 = 0.1963$ m$^2 \quad F_{21} = 0.6 \quad F_{23} = 0.4$

$J_1 - (1-0.63)[0.6J_2 + (0.4)(478)] = (0.63)(32{,}928) \qquad J_1 = 44{,}552$ W/m^2

$J_2 - [0.6J_2 + (0.4)(478)] = 80{,}000 \qquad J_2 = 106{,}900$

$$q_1 = \frac{\varepsilon_1 A_1}{1-\varepsilon_1}(E_{b_1} - J_1) = -3885 \text{ W} \qquad q_2 = \frac{\varepsilon_2 A_2}{1-\varepsilon_2}(E_{b_2} - J_2) = 15{,}700 \text{ W}$$

$E_{b_2} = 1.3356 \times 10^5 = \sigma T_2^{\,4} \qquad T_2 = 1239 \text{ K} = 966°\text{C}$

q (room) $= q_1 + q_2 = 11{,}815$ W

8-114

$J_1 - [0.6J_2 + (0.4)(478)] = 100{,}000 \qquad J_2 - [0.6J_1 + (0.4)(478)] = 80{,}000$

Solution

$J_1 = 2.317 \times 10^5$ W/m$^2 \qquad J_2 = 2.192 \times 10^5$ W/m^2

$$q_1 = \frac{\varepsilon_1 A_1}{1-\varepsilon_1}(E_{b_1} - J_1) = 19{,}630 \text{ W} \qquad E_{b_1} = 2.904 \times 10^5 \text{ W/m}^2 = \sigma T_1^{\,4}$$

$T_1 = 1504 \text{ K} = 1231°\text{C}$

$$q_2 = \frac{\varepsilon_2 A_2}{1-\varepsilon_2}(E_{b_2} - J_2) = 15{,}704 \text{ W} \qquad E_{b_2} = 2.459 \times 10^5 \text{ W/m}^2 = \sigma T_2^{\,4}$$

$T_1 = 1413 \text{ K} = 1170°\text{C}$

q (room) $= q_1 + q_2 = 35{,}064$ W

8-115

Heater surface (3) Room is (4) $d_1 = 10$ cm $\varepsilon_1 = 0.4$ $d_2 = 20$ cm

$\varepsilon_2 = 0.6$ $L = 10$ cm $\varepsilon_3 = 0.8$ $\left.\frac{q}{A}\right|_3 = 90\ \mathrm{kW/m^2}$ $T_4 = 303$ K

$A_1 = \pi(0.1)^2 = 0.03142\ \mathrm{m}^2$ $A_2 = 0.06283\ \mathrm{m}^2$

$A_3 = \pi(0.1^2 - 0.05^2) = 0.02356\ \mathrm{m}^2$ $F_{21} = 0.32$ $F_{22} = 0.23$

$F_{24} + F_{23} = 1 - 0.32 - 0.23 = 0.45$ $F_{24i} = F_{23} = 0.225$

$F_{12} = (2)(0.32) = 0.64$ $F_{13} = F_{14i} = \frac{1}{2}(1 - 0.64) = 0.18$ $F_{32} = 0.6$

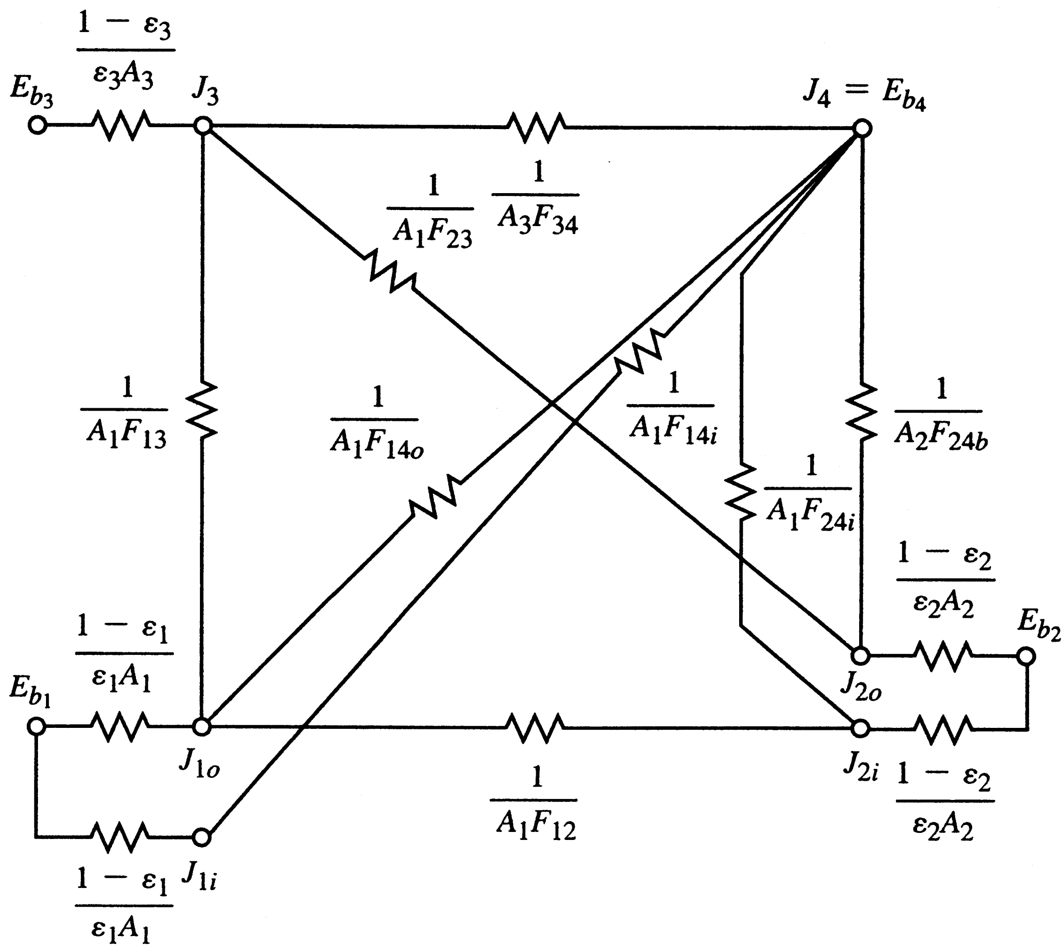
$F_{34} = 0.4$ $F_{31} = 0.24$

Both inside and outside of cylinder exchange heat with room and are in radiant balance

open ends of (1) $F = 0.175$ $A_{end} = 0.007854\ \mathrm{m}^2$

$F_{end-(1)} = 0.825$ $F_{(1)-end} = 0.2065$ $F_{14i} = (2)(0.206) = 0.413$

$E_{b_4} = 478\ \mathrm{W/m^2}$



$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 47.74 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 10.61 \qquad \frac{1}{A_1 F_{13}} = 176.8$$

$$\frac{1}{A_2 F_{23}} = 70.74 \qquad \frac{1}{A_1 F_{14o}} = 176.8 \qquad \frac{1}{A_1 F_{14i}} = 77.06 \qquad \frac{1}{A_1 F_{12}} = 49.73$$

$$\frac{1}{A_3 F_{34}} = 106.1 \qquad \frac{1}{A_2 F_{24i}} = 70.74 \qquad \frac{1}{A_2 F_{24o}} = 15.92$$

$$q_3 = \left(\frac{q}{A}\right) A_3 = (90{,}000)(0.02356) = 2120 \text{ W}$$

<u>Node J_3</u>

$$2120 + \frac{J_{1o} - J_3}{176.8} + \frac{J_{2i} - J_3}{70.74} + \frac{478 - J_3}{106.1} = 0$$

<u>Node J_{1o}</u>

$$\frac{J_{1i} - J_{1o}}{(2)(47.74)} + \frac{J_3 - J_{1o}}{176.8} + \frac{478 - J_{1o}}{176.8} + \frac{J_{2i} - J_{1o}}{49.73} = 0$$

<u>Node J_{1i}</u>

$$\frac{J_{1o} - J_{1i}}{(2)(47.74)} + \frac{478 - J_{1i}}{77.06} = 0$$

<u>Node J_{2o}</u>

$$\frac{J_{2i} - J_{2o}}{(2)(10.61)} + \frac{478 - J_{2o}}{15.92} = 0$$

<u>Node J_{2i}</u>

$$\frac{J_{2o} - J_{2i}}{(2)(10.61)} + \frac{J_{1o} - J_{2i}}{49.73} + \frac{J_3 - J_{2i}}{70.74} + \frac{478 - J_{2i}}{70.74} = 0$$

<u>Solution</u>

$J_{1o} = 81{,}792 \ \text{W/m}^2$ $\qquad J_{1i} = 36{,}584 \ \text{W/m}^2$ $\qquad J_{2o} = 47{,}927 \ \text{W/m}^2$

$J_{2i} = 111{,}173 \ \text{W/m}^2$ $\qquad J_3 = 142{,}337 \ \text{W/m}^2$

then $E_{b_1} = \dfrac{J_{1o} + J_{1i}}{2} = \sigma T_1^4 = 59{,}188 \ \text{W/m}^2$ $\qquad T_1 = 1011 \text{ K}$

$$E_{b_2} = \frac{J_{2o} + J_{2i}}{2} = \sigma T_2^4 = 79{,}550 \ \text{W/m}^2 \qquad T_2 = 1088 \text{ K}$$

$$q_3 = 2120 = \frac{E_{b_3} - 142{,}337}{\frac{1-0.8}{(0.8)(0.02356)}} \qquad E_{b_3} = 1.648 \times 10^5 = \sigma T_3^4 \qquad T_3 = 1306 \text{ K}$$

8-116

$$GA\alpha_{\text{solar}} = \varepsilon_{\text{low temp}} A\sigma(T^4 - T_\infty^4)$$

$$(1500)(0.15) = (0.04)(5.669 \times 10^{-8})T^4$$

$$T = 561 \text{ K} = 288°\text{C}$$

8-117

$T_\infty = -70°\text{C} = 203\ \text{K}$ $\alpha_{\text{solar}} = 0.46$ $\alpha_{\text{low temp}} = 0.95$

$(1070)(0.46) = (0.95)(5.669 \times 10^{-8})(T^4 - 203^4)$ $T = 322.6\ \text{K} = 49.6°\text{C}$

8-118

$\alpha_s(950) = \sigma\varepsilon(T^4 - 300^4) + 12(T - 300)$

$f(T) = 0 = -902.5 + (0.6)(5.669 \times 10^{-8})(T^4 - 300^4)$

T	$f(T)$
350	–67.6
360	+113.3
$T = 353.7$	0

8-119

$\alpha_{\text{solar}} = 0.94$ $\alpha_{\text{low temp}} = 0.21$

$(800)(0.94) = (0.21)(5.669 \times 10^{-8})(T^4 - 298^4)$

$T = 516\ \text{K} = 243°\text{C}$

8-120

$hA(T_\infty - T_t) + \sigma\varepsilon A(T_r{}^4 - T_t{}^4) = 0$

$T_\infty = 105°\text{C} = 378\ \text{K}$ $T_t = 98°\text{C} = 371\ \text{K}$

Solving $T_r = 305\ \text{K} = 32°\text{C}$

8-121

$T_t = 443\ \text{K}$ $T_w = 698\ \text{K}$ $\varepsilon_t = 0.43$ $h = 150\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$(5.669 \times 10^{-8})(0.43)(698^4 - 443^4) = (150)(443 - T_g)$

$T_g = 410.7\ \text{K} = 137.7°\text{C}$

8-122

$T_s = -40°\text{C} = 233\ \text{K}$ $\varepsilon = 0.8$

$hA(T_w - T_{aw})_{\text{lam}} + hA(T_w - T_{aw})_{\text{turb}} = -\sigma A\varepsilon(T_w{}^4 - T_s{}^4)$

laminar: $h = 56.26$ $x_c = 0.222\ \text{m}$ $T_{aw} = 584\ \text{K}$

turbulent: $h = 87.45$ 0.222 to $0.7\ \text{m}$ $T_{aw} = 605\ \text{K}$

$$(56.26)(0.222)(T_w - 584) + (87.45)(0.7 - 0.222)(T_w - 605)$$
$$= -(5.669 \times 10^{-8})(0.7)(0.8)(T_w{}^4 - 233^4)$$

$T_w = 549\ \text{K} = 276°\text{C}$

8-123

$(28)A(T_a - 273) = (5.669 \times 10^{-8})A(1.0)(273^4 - 203^4)$
$T_a = 280.8 \text{ K} = 7.8°\text{C}$

8-124

$h = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} \qquad T_f = 650°\text{C} = 923 \text{ K} \qquad T_a = 560°\text{C} = 833 \text{ K}$

$\left.\frac{q}{A}\right|_{\text{conv}} = 1.32(\Delta T)^{5/4}\left(\frac{1}{d}\right)^{1/4}$

$(5.669 \times 10^{-8})(0.6)(923^4 - T_t^4) = 1.32\left(\frac{1}{0.0032}\right)^{1/4}(T_t - 833)^{5/4}$

$3.4014 \times 10^{-8}T_t^4 + 5.55(T_t - 833)^{5/4} - 24{,}687 = 0 \qquad T_t = 911 \text{ K} = 638°\text{C}$

8-125

$T_\infty = 20°\text{C} = 293 \text{ K} \qquad T_r = 32°\text{C} = 305 \text{ K} \qquad d = 0.0032 \text{ m}$

$\varepsilon = 0.6 \qquad h = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} = 1.32\left(\frac{T_t - T_\infty}{d}\right)^{1/4}$

$\left.\frac{q}{A}\right|_{\text{conv}} = 1.32(\Delta T)^{5/4}\left(\frac{1}{d}\right)^{1/4} \qquad \left.\frac{q}{A}\right|_{\text{conv}} = \left.\frac{q}{A}\right|_{\text{rad}}$

$(5.669 \times 10^{-8})(0.6)(305^4 - T_t^4) = (1.32)(T_t - 293)^{5/4}\left(\frac{1}{0.0032}\right)^{1/4}$

$T_t = 297 \text{ K} = 24°\text{C}$

8-126

$T_w = 373 \text{ K} \qquad T_{\text{sun}} = 673 \text{ K} \qquad \varepsilon_w = 0.6 \qquad \text{assume } T_f = 350 \text{ K}$

$\rho = 0.998 \qquad \mu = 2.075 \times 10^{-5} \qquad k = 0.03003 \qquad \text{Pr} = 0.7$

$\text{Re} = \frac{(0.998)(7)(0.003)}{2.075 \times 10^{-5}} = 1010 \qquad C = 0.683 \qquad n = 0.466$

$h = \frac{0.03003}{0.003}(0.683)(1010)^{0.466}(0.7)^{1/3} = 152.5 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$(0.6)(5.669 \times 10^{-8})(673^4 - 373^4) = (152.5)(373 - T_g)$

$T_g = 331.5 \text{ K} = 58.5°\text{C} \qquad T_f = 352 \text{ K}$

8-127

$A_1F_{12} = A_2F_{21}$ $\quad F_{21} = (0.86)\left(\frac{1}{2}\right) = 0.43$ $\quad F_{22} = 1 - 0.43 - 0.24 = 0.33$

$J_1 - (1-\varepsilon_1)(F_{12}J_{2i} + F_{13}E_{b_3}) = \varepsilon_1 E_{b_1}$

$J_{2i}[1 - F_{22}(1-\varepsilon_2)] - (1-\varepsilon_2)(F_{21}J_1 + F_{23i}E_{b_3}) = \varepsilon_2 E_{b_2}$

$J_{2o} - (1-\varepsilon_2)(F_{23o}E_{b_3}) = \varepsilon_2 E_{b_2}$

$$hA_2(T_3 - T_2) = \frac{E_{b_2} - J_{2o}}{\frac{1-\varepsilon_2}{\varepsilon_2 A_2}} + \frac{E_{b_2} - J_{2i}}{\frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$$h(T_3 - T_2) = \frac{2\varepsilon_2}{1-\varepsilon_2}E_{b_2} - \frac{\varepsilon_2}{1-\varepsilon_2}(J_{2o} + J_{2i})$$

Solution yields:

$T_2 = 406$ K $\quad E_{b_2} = 1540$ $\quad J_{2i} = 23{,}644$ $\quad J_{2o} = 675$

$J_1 = 4943$

$$q_1 = \frac{56{,}690 - 4943}{\frac{1-0.8}{0.8(0.06283)}} = 1824 \text{ W}$$

8-128

$T_t = 973$ K $\quad h = 20 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$ $\quad T_\infty = 923$ K

$(5.669 \times 10^{-8})(0.7)(T_f{}^4 - 973^4) = (20)(973 - 923)$

$T_f = 980 \text{ K} = 707^\circ\text{C}$

8-129

$T_t = 328$ K $\quad h = 30 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$ $\quad T_w = 363$ K $\quad \varepsilon = 0.94$

$(5.669 \times 10^{-8})(0.94)(363^4 - 328^4) = (30)(328 - T_a)$

$T_a = 318 \text{ K} = 45^\circ\text{C}$

8-130

$$T_f = \frac{20+150}{2} = 85°\text{C} = 358 \text{ K} \qquad \nu = 21.58\times10^{-6} \qquad k = 0.0306$$

$$\Pr = 0.7 \qquad \text{Re} = \frac{(25)(0.5)}{21.58\times10^{-6}} = 5.792\times10^5$$

$$\text{Nu}_d = 0.3 + \frac{(0.62)(5.79\times10^5)^{1/2}(0.7)^{1/3}}{\left[1+\left(\frac{0.4}{0.7}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{5.79\times10^5}{282{,}000}\right)^{5/8}\right]^{4/5} = 601.7$$

$$h = \frac{(601.7)(0.0306)}{0.5} = 36.82 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q_c = (36.82)\pi(0.5)(150-20) = 7519 \ \text{W/m}$$

$$q_r = (5.669\times10^{-8})(0.7)\pi(0.5)(423^4 - 293^4) = 1536 \ \text{W/m}$$

$$q_{\text{total}} = 7519 + 1536 = 9055 \ \text{W/m}$$

8-131

$$\lambda_1 T = (0.4)(3400) = 1360\mu\cdot\text{K} = 2448\mu\cdot°\text{R}$$

$$\lambda_2 T = (0.7)(3400) = 2380\mu\cdot\text{K} = 4284\mu\cdot°\text{R}$$

$$E_b = (0-\lambda_1 T) = 0.00644$$

$$E_b = (0-\lambda_2 T) = 0.13626$$

Fraction between 0.4 and $0.7\mu = 0.12982$

8-132

$$E_b = 5.669\times10^{-8}(3400)^4 = 7.576\times10^6 \ \text{W/m}^2$$

Between 0.4 and $0.7\mu = (7.576\times10^6)(0.12982) = 9.835\times10^5$

$$400 \text{ W} = A(7.576\times10^6)$$

$$A = 5.286\times10^{-5} \ \text{m}^2 = 0.5286 \ \text{cm}^2$$

8-133

$$E_b = 5.669\times10^{-8}(3400)^4 = 4.592\times10^6 \ \text{W/m}^2$$

λ	ε	λT	Fraction
0		0	
2	0.6	6000	0.73777
8	0.2	24,000	0.99075

$$E = (4.592\times10^6)[(0.6)(0.73777) + (0.2)(0.99075 - 0.73777)]$$
$$= 2.265\times10^6 \ \text{W/m}^2$$

8-135

8–125

$T = 5800$ K

λ	λT	Fraction
0.25	1450	0.0099
0.5	2900	0.25055
1.5	8700	0.88066
2.5	14,500	0.96604

Fraction transmitted through plain glass $= (0.9)(0.96604 - 0.0099) = 0.86053$
Fraction transmitted through tinted glass $= (0.9)(0.88066 - 0.25055) = 0.56710$

8-136

$h = 12 \ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$ $\qquad T_\infty = 400$ K

For $T_w = 510$ K $\qquad E = 1100 \ \text{W/m}^2$

$E_{bw} = \sigma T_w{}^4 = 3835$ $\qquad \varepsilon = \dfrac{E}{E_b} = 0.287$

$A(G - J) = hA(T_w - 400)$ $\qquad G = 2200 \ \text{W/m}^2$ $\qquad J = 450 + \varepsilon\sigma T_w{}^4$

$2200 - 450 - (0.289)\sigma T_w{}^4 = 12(T_w - 400)$ $\qquad T_w = 475$ K

$J = 450 + (0.287)(5.669 \times 10^{-8})(475) = 1278 \ \text{W/m}^2$

8-137

$T_1 = 550$ K $\qquad \varepsilon_1 = 0.5$ $\qquad T_2 = 300$ K $\qquad \varepsilon_2 = 0.7$

$A_1 = A_2 = (3)^2 = 9$ $\qquad F_{12} = 0.2$

Eq. (8–41)

$$q = \frac{(5.669 \times 10^{-8})(9)(550^4 - 300^4)}{\frac{18(1-0.2)}{9(1-0.2^2)} + \left(\frac{1}{0.5} - 1\right)\left(\frac{1}{0.7} - 1\right)} = \frac{42,555}{3.095} = 13,750 \text{ W}$$

8-138

$T_1 = 700\ \text{K}$ $\quad \varepsilon_1 = 0.6$ $\quad A_1 = \pi(0.05)^2 = 7.85\times10^{-3}\ \text{m}^2$

$T_2 = 300\ \text{K}$ $\quad \varepsilon_2 = 0.3$ $\quad A_2 = 2\pi(0.05)^2 = 0.0157\ \text{m}^2$

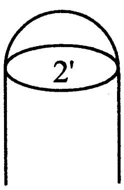
$E_{b_1} = \sigma T_1^4 = 13{,}611$ $\quad E_{b_2} = \sigma T_2^4 = 459\ \text{W/m}^2$

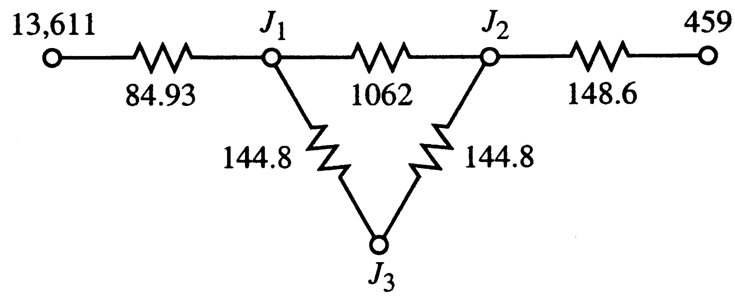
$\frac{d}{x} = \frac{10}{15} = \frac{2}{3}$ $\quad F_{12} = 0.12$ $\quad F_{13} = 1 - 0.12 = 0.88$

$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 84.93$ $\quad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 148.6$ $\quad \frac{1}{A_1F_{12}} = 1062$ $\quad \frac{1}{A_1F_{13}} = 144.8$

$\frac{1}{A_2F_{23}} = 144.8$ $\quad F_{2'-2} = 1.0$ $\quad F_{2-2'} = 0.5$ $\quad F_{22} = 0.5$

$F_{21} = \frac{1}{2}(0.12) = 0.06$ $\quad F_{23} = 1 - 0.5 - 0.06 = 0.44$





$$q = \frac{13{,}611 - 459}{84.93 + \frac{1}{\frac{1}{1062} + \frac{1}{(2)(144.8)}} + 148.6} = 28.5\ \text{W}$$

8-139

$E_{b_1} = 23{,}220$ $\quad E_{b_2} = 459\ \text{W/m}^2$

Per Unit Length:

$$\frac{q}{L} = \pi(0.05)(0.7)(23{,}220 - 459) = 2503\ \text{W/m}$$

with shield, as surface 3, $F_{13} = 1.0$ $\quad \varepsilon_3 = 0.2$

$\frac{1-\varepsilon_1}{\varepsilon_1\pi(0.05)} = 2.73$ $\quad \frac{1}{\pi(0.05)(1.0)} = 6.37$ $\quad \frac{1-\varepsilon_3}{\varepsilon_3\pi(0.1)} = 12.73$

$\frac{1}{\pi(0.1)} = 3.18$

q with shield per unit length

$$\frac{q}{L} = \frac{23{,}220 - 459}{2.73 + 6.37 + (2)(12.73) + 3.18} = 603\ \text{W}$$

Reduced by 76%

8-140

$T_1 = 450$ K $\qquad \varepsilon_1 = 0.5 \qquad A_1 = \pi(0.25)^2 = 0.1963\ \text{m}^2 = A_2$

$T_2 = 600$ K $\qquad \varepsilon_2 = 0.6$

$T_3 = 1000$ K $\qquad \varepsilon_3 = 0.7 \qquad A_3 = \pi(0.5)(0.5) = 0.7854\ \text{m}^2$

$\dfrac{d}{x} = 1.0 \qquad F_{12} = 0.17 = F_{21} \qquad F_{13} = 0.83 = F_{23}$

$$F_{31} = F_{32} = (0.83)\left(\frac{0.1963}{0.7854}\right) = 0.2075 \qquad F_{33} = 1 - (2)(0.2075) = 0.585$$

$E_{b_1} = 2325\ \text{W/m}^2 = E_{b_2} = 7347\ \text{W/m}^2 \qquad E_{b_3} = 56{,}690\ \text{W/m}^2$

$$\left\{\begin{array}{l} J_1 - 0.5[0.17J_2 + 0.83J_3] = 0.5(2325) \\ J_2 - 0.4[0.17J_1 + 0.83J_3] = 0.6(7347) \\ J_3[1 - 0.585(0.3)] - 0.3(0.2075J_1 + 0.2075J_2) = 0.7(56{,}690) \end{array}\right\}$$

$$\left\{\begin{array}{l} J_1 - 0.085J_2 - 0.415J_3 = 1162.5 \\ -0.068J_1 + J_2 - 0.332J_3 = 4408.2 \\ -0.06225J_1 - 0.06225J_2 + 0.8245J_3 = 39{,}683 \end{array}\right\}$$

Solving,

$J_1 = 24{,}614\ \text{W/m}^2 \qquad J_2 = 23{,}261 \qquad J_3 = 51{,}744$

$$q_1 = \frac{0.5}{0.5}(0.1963)(2325 - 24{,}614) = -4375\ \text{W} = 0.1963(J_1 - G_1)$$

$G_1 = 46{,}901\ \text{W/m}^2$

$$q_2 = \frac{0.6}{0.4}(0.1963)(7347 - 23{,}261) = -4686\ \text{W} = 0.1963(J_2 - G_2)$$

$G_2 = 47{,}133\ \text{W/m}^2$

$$q_3 = \frac{0.7}{0.3}(0.7854)(56{,}690 - 51{,}744) = 9064\ \text{W} = 0.7854(J_3 - G_3)$$

$G_3 = 40{,}203\ \text{W/m}^2$

8-141

Behaves as if $T_2 = 300$ K $\qquad G_2 = 1.0 \qquad E_{b_2} = 459 \text{ W/m}^2 = J_2$

Do not need Eqn. for surface 2. Inserting $J_2 = 459$ in Eqns. for 1 and 3 gives

$J_1 - 0.415 J_3 = 1201.5 \qquad -0.06225 J_1 + 0.8245 J_3 = 39{,}712$

Solving, $J_1 = 21{,}875 \qquad J_3 = 49{,}817$

$$q_1 = \frac{0.5}{0.5}(0.1963)(2325 - 21{,}875) = -3838 \text{ W} = 0.1963(J_1 - G_1)$$

$G_1 = 41{,}427 \text{ W/m}^2$

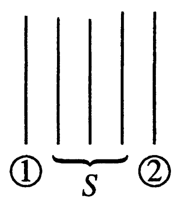
$$q_3 = \frac{0.7}{0.3}(0.7854)(56{,}690 - 49{,}817) = 12{,}595 \text{ W} = 0.7854(J_3 - G_3)$$

$G_3 = 33{,}781 \text{ W/m}^2$

$q_2 = -12{,}595 + 3838 = -8757$ W

$G_2 \to 0$ because $A_2 \to \infty$

8-142



$T_1 = 300$ K $\qquad T_2 = 85$ K $\qquad \varepsilon_1 = \varepsilon_2 = 0.11 \qquad \varepsilon_s = 0.04$

All F factors = 1.0

Work problem per unit area

$$\frac{1-\varepsilon_1}{\varepsilon_1} = \frac{1-\varepsilon_2}{\varepsilon_2} = 8.091 \qquad \frac{1-\varepsilon_s}{\varepsilon_s} = \frac{0.96}{0.04} = 24$$

4 space resistances = 1.0

Total resistance $= (2)(8.091) + (4)(1.0) + (6)(24) = 164.2$

$$q = \frac{(5.669 \times 10^{-8})(300^4 - 85^4)}{164.2} = 2.779 \text{ W/m}^2 = k_{\text{eff}} \frac{300 - 85}{0.008}$$

$$k_{\text{eff}} = 1.034 \times 10^{-4} \ \frac{\text{W}}{\text{m} \cdot {}^\circ\text{C}} \qquad R = 77.4 \ \frac{{}^\circ\text{C} \cdot \text{m}^2}{\text{W}}$$

8-143

$T_1 = 750\ \text{K}$ $\quad \varepsilon_1 = 0.75 \quad A_1 = \pi(0.2)^2 = 0.1256$

$T_2 = 600\ \text{K}$ $\quad \varepsilon_2 = 0.4 \quad A_2 = \pi(0.2^2 - 0.1^2) = 0.09425$

$q_3 = 0 \quad A_3 = \pi(0.4)(0.3) = 0.377\ \text{m}^2 \quad A_{\text{hole}} = \pi(0.1)^2 = 0.03146$

$\dfrac{d_1}{x} = \dfrac{40}{30} = 1.33 \quad F_{1-2'} = 0.25$

To hole $\dfrac{30}{20} = 1.5 \quad \dfrac{10}{30} = \dfrac{1}{3} \quad F_{1-\text{hole}} = 0.07 = F_{14}$

$F_{12} = 0.25 - 0.07 = 0.18 \quad F_{13} = 1 - 0.25 = 0.75$

$T_4 = \text{room} = 300\ \text{K}$

$F_{31} = \dfrac{(0.75)(0.1256)}{0.337} = 0.25 \quad F_{33} = 1 - (2)(0.25) = 0.5$

$F_{21} = \dfrac{(0.1256)(0.18)}{0.09425} = 0.24 \quad F_{\text{hole}-1} = \dfrac{(0.1256)(0.07)}{0.03146} = 0.28$

$F_{\text{hole}-3} = 1 - 0.28 = 0.72 \quad F_{3-\text{hole}} = F_{34} = \dfrac{(0.72)(0.03146)}{0.377} = 0.06$

$F_{32} = 0.25 - 0.06 = 0.19 \quad F_{23} = \dfrac{(0.19)(0.377)}{0.09425} = 0.76$

$E_{b_1} = 17{,}937\ \text{W/m}^2 \quad E_{b_2} = 7347 \quad E_{b_4} = 459 = J_4$

$$\begin{Bmatrix} J_1 - 0.25[0.18J_2 + (0.07)(459) + 0.75J_3] = 0.75(17{,}937) \\ J_3(1-0.5) - [0.25J_1 + 0.19J_2 + (0.06)(459)] = 0 \\ J_2 - 0.6(0.24J_1 + 0.76J_3) = 0.4(7347) \end{Bmatrix}$$

$$\begin{Bmatrix} J_1 - 0.045J_2 - 0.1875J_3 = 13{,}461 \\ -0.25J_1 - 0.19J_2 + 0.5J_3 = 27.54 \\ -0.144J_1 + J_2 - 0.456J_3 = 2939 \end{Bmatrix}$$

Solving,

$J_1 = 16{,}264\ \text{W/m}^2 \quad J_2 = 10{,}904\ \text{W/m}^2$

$J_3 = 12{,}330\ \text{W/m}^2 = G_3 \quad J_3 = E_{b_3} = \sigma T_3^{\,4} \quad T_3 = 683\ \text{K}$

$q_1 = \dfrac{(0.75)(0.1256)}{0.25}(17{,}937 - 16{,}264) = 630.4\ \text{W} = 0.1256(J_1 - G_1)$

$G_1 = 11{,}245\ \text{W/m}^2$

$q_2 = \dfrac{(0.4)(0.09425)}{0.6}(7347 - 10{,}904) = -223.5\ \text{W} = 0.09425(J_2 - G_2)$

$G_2 = 13{,}275\ \text{W/m}^2$

$q_4 = 630.4 - 223.5 = 406.9\ \text{W}$

8-144

$$\frac{q}{A} = E\sigma(T_1^4 - T_2^4) = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$E = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$\varepsilon_1 = \varepsilon_2$	E
0.9	0.818
0.8	0.667
0.5	0.333
0.2	0.111
0.1	0.0526
0.05	0.0256

8-145

For large plates, $\frac{A_2}{A_1} \to 1.0$ and $F_{12} \to 1.0$. Substituting these values gives the desired result.

8-146

$r_1 = 0.15$ $r_2 = 0.5$ $L = 2$ $\varepsilon_1 = 0.2$ $\varepsilon_2 = 0.4$

$T_1 = 800$ K $T_2 = 400$ K (3) is room $T_3 = 300$ K

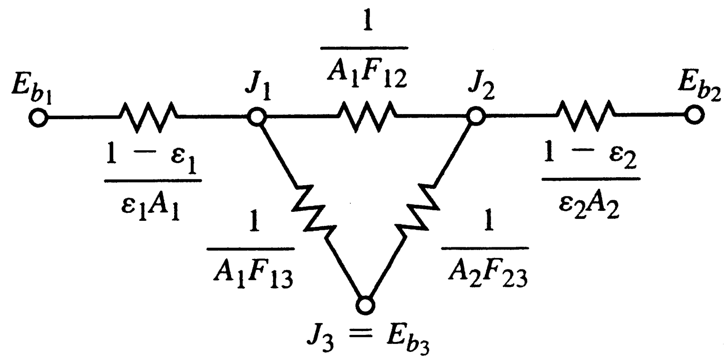
$\frac{r_1}{r_2} = 0.3$ $\frac{L}{r_2} = 4.0$ $F_{22} = 0.55$ $F_{21} = 0.265$

$F_{12} = \frac{0.5}{0.15}(0.265) = 0.883$ $F_{13} = 1 - 0.883 = 0.117$

$F_{23} = 1 - 0.55 - 0.265 = 0.185$ $F_{11} = 0$

Assume only inner surfaces exchange heat

(a)



$A_1 = \pi(0.3)(2) = 1.885 \qquad A_2 = \pi(1)(2) = 6.283$

$E_{b_1} = \sigma T_1{}^4 = 23{,}216\ \text{W/m}^2 \qquad E_{b_2} = \sigma T_2{}^4 = 1451\ \text{W/m}^2$

$E_{b_3} = \sigma T_3{}^4 = 459\ \text{W/m}^2$

$J_1 = (1-\varepsilon_1)(F_{12}J_2 + F_{13}J_3) + \varepsilon_1 E_{b_1}$

$$J_2 = \frac{1}{1-F_{22}(1-\varepsilon_2)}[(1-\varepsilon_1)(F_{12}J_1 + F_{23}J_3) + \varepsilon_2 E_{b_2}]$$

$J_3 = E_{b_3} = 459$

$J_1 = 0.7064 J_2 + 4686 \qquad J_2 = (1.4925)(0.159 J_1 + 631)$

Which have the solution:

$J_1 = 6428\ \text{W/m}^2 \qquad J_2 = 2467\ \text{W/m}^2$

$$q_1 = \frac{\varepsilon_1 A_1}{1-\varepsilon_1}(E_{b_1} - J_1) = \frac{(0.2)(1.885)}{1-0.2}(23{,}216 - 6428) = 7911\ \text{W}$$

$$q_2 = \frac{\varepsilon_2 A_2}{1-\varepsilon_2}(E_{b_2} - J_2) = \frac{(0.4)(6.283)}{1-0.4}(1451 - 2467) = -4256\ \text{W}$$

(b) Ends insulated, J_3 floats and $\neq E_{b_3}$

$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 2.122 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 0.2387$

$\frac{1}{A_1F_{12}} = 0.6008 \qquad \frac{1}{A_1F_{13}} = 4.534 \qquad \frac{1}{A_2F_{23}} = 0.8603$

$$q = \frac{23{,}216 - 1451}{2.122 + \frac{1}{\frac{1}{0.6008} + \frac{1}{4.534+0.8603}} + 0.2387} = 7502\ \text{W}$$

8-147

$d_1 = 20$ cm $\quad d_2 = 10$ cm $\quad T_1 = 500$ K $\quad \varepsilon_1 = 0.6 \quad T_2 = 700$ K

$\varepsilon_2 = 0.2$

Surface 3 = sides, $\varepsilon_3 = 0.8$

$A_1 = 0.03142 \quad A_2 = 0.007853 \quad A_3 = 0.05269$

$F_{12} = 0.12 \quad F_{11} = 0 \quad F_{13i} = 0.88 \quad F_{21} = 0.48 \quad F_{22} = 0$

$F_{23i} = 0.52 \quad F_{3i-1} = 0.525 \quad F_{3i-2} = 0.0775 \quad F_{3i-3i} = 0.397$

$F_{3o-4} = 1.0 \quad E_{b_1} = 3543 \quad E_{b_2} = 13{,}609 \quad E_{b_4} = 459$

$$J_1 = (1-\varepsilon_1)(F_{12}J_2 + F_{13i}J_{3i}) + \varepsilon_2 E_{b_2} \qquad \text{(a)}$$

$$J_2 = (1-\varepsilon_2)(F_{21}J_1 + F_{23i}J_{3i}) + \varepsilon_1 E_{b_1} \qquad \text{(b)}$$

<u>Energy balance on surface 3</u>

$$(A_1F_{1-3i})(J_1 - J_{3i}) + (A_2F_{23i})(J_2 - J_{3i}) = \frac{E_{b_4} - J_{3i}}{(2)\left(\frac{1-\varepsilon_3}{\varepsilon_3 A_3}\right) + \frac{1}{A_3 F_{34o}}} = 0 \qquad \text{(c)}$$

<u>Solution of equations is</u>

$J_1 = 2960 \quad J_2 = 4585 \quad J_{2i} = 1745\ \text{W/m}^2$

$$q_1 = \frac{\varepsilon_1 A_1}{1-\varepsilon_1}(E_{b_1} - J_1) = 27.48\ \text{W} \qquad q_2 = \frac{\varepsilon_2 A_2}{1-\varepsilon_2}(E_{b_2} - J_2) = 17.72\ \text{W}$$

q (total) $= 27.48 + 17.72 = 45.2$ W

8-148

See problem 8–137. Equations (a) and (b) remain the same, convection loss term must be added to Eq. (c) for surface $3o$. (outside of cone)

$$q_{\text{conv}} = hA_{3o}(T_4 - T_3) \qquad h = 25\ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

The energy balance on surface 3 is then

$$(A_1F_{13i})(J_1 - J_3) + (A_2F_{23i})(J_2 - J_{3i})$$
$$+ (A_{3o}F_{3o-4})(E_{b_4} - J_{3o}) + hA_{3o}(T_4 - T_3) = 0 \qquad \text{(c)}$$

In addition, we have $T_3 = \left(\dfrac{E_{b_3}}{\sigma}\right)^{1/4}$ (d)

and $hA_{3o}(T_4 - T_3) = \dfrac{\varepsilon_3 A_3}{1-\varepsilon_3}(E_{b_3} - J_{3o} - J_{3i})$ (e)

The original two equations are

$$J_1 - (1-\varepsilon_1)(F_{12}J_2 + F_{1-3i}J_{3i}) - \varepsilon_1 E_{b_1} = 0 \qquad \text{(a)}$$

$$J_2 - (1-\varepsilon_2)(F_{21}J_1 + F_{2-3i}J_{3i}) - \varepsilon_2 E_{b_2} = 0 \qquad \text{(b)}$$

The solution to the nonlinear set of equations may be obtained with Excel solver.

The results are $J_1 = 2677 \quad J_{3i} = 1009 \quad E_{b_3} = 727\ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

$J_2 = 4155 \qquad J_{3o} = 673 \qquad T_3 = \left(\dfrac{E_{b_3}}{\sigma}\right)^{1/4} = 337\ \text{K}$

The heat transfer rates are thus

$q_1 = \dfrac{\varepsilon_1 A_1}{1-\varepsilon_1}(E_{b_1} - J_1) = 40.81\ \text{W} \qquad q_2 = \dfrac{\varepsilon_2 A_2}{1-\varepsilon_2}(E_{b_2} - J_2) = 18.56\ \text{W}$

$q\ (\text{total}) = 59.37\ \text{W}$

As a check this value must equal the energy lost by convection and radiation from the outside of surface 3.

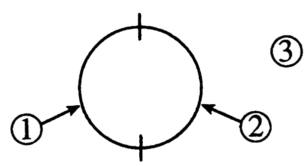
$q_{3o,\ \text{rad}} = \dfrac{\varepsilon_3 A_3}{1-\varepsilon_3}(E_{b_3} - J_{3o}) = 11.38\ \text{W}$

$q_{3o,\ \text{conv}} = hA_{3o}(T_3 - T_4) = (25)(0.05269)(337 - 300) = 48.74\ \text{W}$

$q\ (\text{total}) = 60.1\ \text{W}$

The difference in the two values for total heat transfer results from roundoff in the solution.

8-149



$T_1 = T_2 = 800°\text{C} = 1073\ \text{K} \qquad T_3 = 20°\text{C} = 293\ \text{K}$

$\varepsilon_1 = 0.5 \qquad \varepsilon_2 = 0.8 \qquad d = 10\ \text{cm}$

$A_1 = A_2 = \dfrac{4\pi r^2}{2} = 2\pi(0.05)^2 = 0.01571\ \text{m}^2$

$q = \sigma\varepsilon A(T^4 - T_3^{\ 4})$

$q_1 = (5.668\times 10^{-8})(0.5)(0.01571)(1073^4 - 293^4) = 587\ \text{W}$

$q_2 = (5.668\times 10^{-8})(0.8)(0.01571)(1073^4 - 293^4) = 939\ \text{W}$

$q_{\text{tot}} = 587 + 939 = 1526\ \text{W}$

8-150

$\alpha\ (\text{solar}) = 0.94 \qquad T_{\text{surr}} = 25°\text{C} = 298\ \text{K} \qquad \alpha\ (\text{low temp}) = 0.21 = \varepsilon$

$G\ (\text{solar}) = 700\ \text{W/m}^2$

$G\ (\text{solar})\alpha\ (\text{solar}) = \sigma\varepsilon(T^4 - 298^4)$

$(700)(0.94) = (5.668\times 10^{-8})(0.21)(T^4 - 298^4)$

$T = 501\ \text{K} = 228°\text{C}$

8-151

G (solar)α (solar) $= q_{\text{rad}} + q_{\text{conv}}$

$(700)(0.94) = (0.21)(5.668 \times 10^{-8})(T^4 - 298^4) + (7.5)(T - 298)$

Solving, $T = 369$ K $= 96°$C

8-152

copper sphere $\rho = 8954$ $c = 383$ $\varepsilon = 0.8$ $d = 5$ cm

$A = 4\pi(0.0025)^2 = 7.85 \times 10^{-3}$ m^2 $V = \frac{4}{3}\pi(0.025)^3 = 6.54 \times 10^{-5}$ m^3

$T_r = 0°\text{C} = 273$ K $\sigma\varepsilon A(T_r{}^4 - T^4) = \rho c V \dfrac{dT}{d\tau}$

$$T^{p+1} = \Delta\tau\left(\frac{\sigma\varepsilon A}{\rho c V}\right)(T_r{}^4 - T^{p^4}) + T^p$$

T(initial) $= 300°$C $= 573$ K T (final) $= 50°$C $= 323$ K

$$\frac{\sigma\varepsilon A}{\rho c V} = \frac{(5.668 \times 10^{-8})(0.8)(7.85 \times 10^{-3})}{(8954)(383)(6.54 \times 10^{-5})} = 1.586 \times 10^{-12}$$

Choose $\Delta\tau = 100$ sec

$T^{p+1} = 1.586 \times 10^{-10}(273^4 - T^{p^4}) + T^p$

At $T = 200°\text{C} = 473$ K 9 time increments = 900 sec
At $T = 100°\text{C} = 373$ K 34 time increments = 3400 sec
At $T = 50°\text{C} = 323$ K 70 time increments = 7000 sec

Excel printout follows on next page.

8-153

$T_\infty = 293$K; $T_{\text{wall}} = 313$ K; $u_\infty = 3$ m/s; Assume $\varepsilon_{\text{glass}} = 1.0$
Assume d = 0.006 m

$\nu = 15\times10^{-6}$; k = 0.026; Re = (3)(0.006)/15×10^{-6} = 1200

h = (0.026)/0.006)(0.683)(1200)$^{0.618}$ = 24

Assume half radiation to wall; half to surroundings

$(24)(T_t - 293) = 0.5\sigma(313^4 + 293^4 - 2T_t^4)$

Solve by iteration; $T_t = 295$ K $= 22°$C

Time incr. (100 sec)	Temp, K
0	573
1	556.7839
2	542.4226
3	529.574
4	517.9809
5	507.4447
6	497.8094
7	488.9505
8	480.7665
9	473.1744
10	466.105
11	459.5002
12	453.3107
13	447.4945
14	442.0155
15	436.8423
16	431.9476
17	427.3075
18	422.9007
19	418.7088
20	414.715
21	410.9046
22	407.2642
23	403.7819
24	400.4469
25	397.2496
26	394.1809
27	391.2328
28	388.3981
29	385.6698
30	383.0419
31	380.5087
32	378.0649
33	375.7056
34	373.4265
35	371.2234
36	369.0925
37	367.03
38	365.0329
39	363.0978
40	361.222
41	359.4028
42	357.6375
43	355.9238
44	354.2595
45	352.6425
46	351.0708
47	349.5425
48	348.0559
49	346.6093
50	345.2011
51	343.8299
52	342.4944
53	341.193
54	339.9246
55	338.688
56	337.4821
57	336.3057
58	335.1578
59	334.0375
60	332.9439
61	331.8759
62	330.8329
63	329.8139
64	328.8182
65	327.8451
66	326.8939
67	325.9638
68	325.0542
69	324.1645
70	323.2942
71	322.4425
72	321.6091
73	320.7933
74	319.9947
75	319.2127
76	318.4469
77	317.6969
78	316.9621
79	316.2423

8-159

$T_1 = 600$ K $\quad F_{12} = 1.0 \quad E_{b_1} = 7347\ \text{W/m}^2 \quad \varepsilon_1 = 0.75$

(3) room $\quad E_{b_3} = 447\ \text{W/m}^2 \quad \varepsilon_2 = 0.8 \quad T_3 = 298$ K

Per unit length: $A_1 = \pi(0.02) = 0.06283 \quad A_2 = 0.1571 \quad T_\infty = 308$ K

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 5.305 \qquad \frac{1}{A_1 F_{12}} = 15.916 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 1.591$$

$$\frac{1}{A_2 F_{23}} = 6.365$$

$$q = \frac{(5.669\times10^{-8})(600^4 - T_2{}^4)}{5.305 + 15.916 + 1.591}$$

$$= \frac{(5.669\times10^{-8})(T_2{}^4 - 298^4)}{1.591 + 6.365} + \pi(0.05)(180)(T_2 - 308)$$

By iteration: $T_2 = 318$ K $\qquad q = 297\ \text{W/m}$

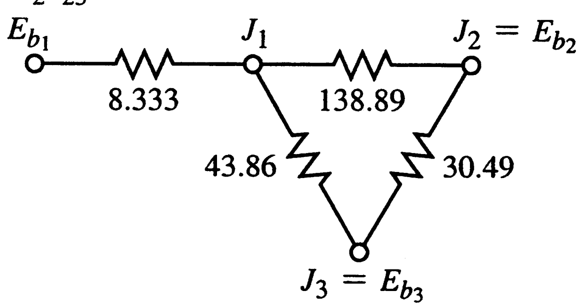
8-160

$T_1 = 500$ K $\quad T_3 = 300$ K $\quad \varepsilon_1 = 0.8 \quad \varepsilon_2 = 0.4 \quad E_{b_1} = 3543\ \text{W/m}^2$

$A_1 = 0.03 \quad A_2 = 0.04 \quad E_{b_3} = 459\ \text{W/m}^2 \quad F_{21} = 0.18 \quad F_{23} = 0.82$

$$F_{13} = 0.76 \qquad \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 8.333 \qquad \frac{1}{A_1 F_{12}} = 138.89 \qquad \frac{1}{A_1 F_{13}} = 43.86$$

$$\frac{1}{A_2 F_{23}} = 30.49$$



$$q = \frac{3543 - 459}{8.333 + \frac{1}{\frac{1}{43.86} + \frac{1}{138.89+30.49}}} = 71.44\ \text{W} = \frac{E_{b_1} - J_1}{8.333} \qquad J_1 = 2948\ \text{W/m}^2$$

$$\frac{2948 - J_2}{138.89} = \frac{2948 - 459}{138.89 + 30.49} \qquad J_2 = 907\ \text{W/m}^2 = E_{b_2} = \sigma T_2{}^4$$

$T_2 = 356$ K $= 83$°C

8-163

$\mathrm{Re} = 34{,}562 \qquad T_\infty = 27°\mathrm{C} = 300\ \mathrm{K}$

$$\bar{h} = \frac{(0.664)(0.02749)}{0.3}(34{,}562)^{1/2}(0.7)^{1/3} = 10.04\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot °\mathrm{C}}$$

$$(1100)(0.6) = (10.04)(T - 300) + (0.09)(5.669 \times 10^{-8})(T^4 - 300^4)$$

$$T = 316.5\ \mathrm{K} = 43.5°\mathrm{C}$$

8-167

The temperature of the walls obviously depends on the wall insulation. The radiation exchange also depends on sunlight through the windows if applicable. For comparison purposes any sunlight load should be neglected. To start the analysis some worst case conditions might be assumed with a winter wall temperature of 5°C and a summer wall temperature of 30°C. Regardless of the paint the walls will act as a blackbody enclosure. The people may be assumed to also radiate as black surfaces for worst case conditions. Assume a body surface temperature of 75°F for the radiant and convection exchange to compute the total heat loss/gain. Other assumptions may be made to examine the effects of different temperatures. The radiation shape factor from the people to the cool/hot walls depends on interception by other people, but for the worst case may be taken as 1.0.

8-169

Assume some convenient size of building wall (5 m high by 30 m long). The concrete slab might be assumed to be insulated or part of a semi infinite solid having the properties of concrete. The shape factor between the wall and concrete slab is that for a perpendicular rectangle with a common edge. Convection from the building might be assumed to have $h = 15\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot °\mathrm{C}}$ as a first approximation, with a like value from the concrete slab. Worst case conditions will be experienced with a black building and ε is about 0.6 for concrete. Solar irradiation might be taken as about $650\ \mathrm{W/m^2}$, although it could be higher. For the grass, assume that the temperature is substantially lower than for the concrete. Assume summer conditions for air temperature at about 33°C. Recall that the irradiation is calculated from the emissive power and the radiosity from $G = E_b - J$.

8-170

The open side of the box should be assumed to be a black surface with temperature equal to the room temperature. The maximum exchange per unit heater surface area is obtained with no walls at all and this fact may be used to set an upper limit to the size of the heater surface (area). As higher and higher walls are added the net heat transfer with the room will decrease and the surface area must be increased to maintain the net exchange value of 15 kW. The configuration of the heater surface (long and slim, square, etc.) may be a factor in a final design selection because it can influence the radiant coverage in the warehouse floor space. If time permits you may wish to influence the radiant coverage in the warehouse floor space. If time permits you may wish to examine several configurations with this fact in mind.

8-171

While there will be temperature gradients in the electronics package itself, there is no information given to determine these gradients. Therefore, one should just assume an outside temperature for the package as 80°C. The package will lose heat by both radiation and free convection to the surrounding enclosure space and containing box. The best case for radiation transfer is for black surfaces so that should be assumed (because they can be painted black). To start the design assume typical values of h for free convection from the outside of the package, to the inside of the enclosing box, and outside of the box. $h = 7 - 10 \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$ might be reasonable. From these assumptions an estimate may be made for the size (area) of the enclosing box. One can then work backward, refine the values for h using the relations of Chap. 7, and make a new determination of the area and shape of the enclosing box. The calculations may be refined further by assuming different values for ε for the electronics package and enclosing box in the event that black surfaces cannot be obtained. If interested, the calculation might also be made for forced convection dissipation from the outside of the box, but that is not asked for in the problem.

8-172

Designate the black panel as surface 1, the shield as surface 2, and the room as surface 3. It is reasonable to assume that the shields will occupy a view area equal to the projection parallel to the black surface. For the inside of the shield $F_{21} = 1.0$ and the value of F_{12} is obtained by reciprocity. Then $F_{13} = 1 - F_{12}$. Using either a network approach or numerical formulation the problem may be treated as one with four surfaces (1) the black plate, (2) the inside of the shield, (3) the outside of the shield and (4) the room which may be treated as black because $A_3 \to \infty$. Only two equations for the inside and outside shield surfaces are required because of the black conditions for surfaces 1 and 3. For the conditions where free convection must be considered, assume some average value of h which might be experienced with a vertical flat plate as calculated from the relations of chapter 7. Non-linear equations for the free convection case will result. The radiant heat transfer is calculated from the usual relations once values of the radiosity are determined. All of the radiant energy lost by the black panel is eventually delivered to the room, whether by direct radiation, radiation from the shield, or free convection from the shield. The convection loss from the black panel is dissipated directly, independent of the radiant loss.

8-173

Best conditions can be represented when all the solar energy is trapped inside the collector, i.e., no transmission out through the glass. The solar irradiation may be taken as about $650\ \mathrm{W/m^2}$ for an initial calculation. You will need to consult chapter 9 for condensation coefficients but a value of $h = 10{,}000\ \dfrac{\mathrm{W}}{\mathrm{m^2 \cdot °C}}$ might be reasonable. The energy to evaporate unit mass of water is the enthalpy of vaporization which may be taken as about $2.3 \times 10^6\ \mathrm{J/kg}$.

8-174

Because of the shadowing effect of the fins the assembly will probably radiate as if it had a surface area equal to that of a surface enclosing the outer periphery of the fins. The apparent emissivity of the assembly will then be given by the relation in Prob. 8–129, which will turn out to be close to that of a blackbody. The assembly may be assumed to convect with an area equal to the total area exposed to the air. The assembly may be heated and allowed to cool as a lumped capacity in room air. By a transient numerical procedure the value of h may be obtained as a function of surface temperature during the cooling process. The behavior will depend on orientation of the assembly but the convection might be expected to follow a relation like

$h = \mathrm{const} \times (T_{\mathrm{surface}} - T_{\mathrm{room}})^n$

where the constant and exponent n are obtained from a plot of $\log h$ vs ΔT. Correlation in terms of Grashof and Nusselt numbers is difficult because of the uncertain characteristic dimension of the fin assembly surface.

8-175

From the problem statement the radiation shape factor from the person to each of the walls is taken as 1/6. The outside wall temperatures (interior surface temperature) will probably be rather cold; about 10°C. The two interior walls and floor should be assumed at the room air temperature. A reasonable value for h from the person to air will be about $7-10\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$. Because the room area is large compared to the person, it will behave as a black surface. The objective is to obtain the same energy transfer to the person by radiation as would be obtained by convection in a typical forced air heating situation. Note that the net radiation to the person is that received from the ceiling less that dissipated to the floor and walls. Note that only the radiation exchange with the person is to be considered for comfort. The heating requirements for the space would also have to take account of losses to the outside from the exterior walls, etc.

8-176

The total energy dissipated to the room will be influenced by the free convection loss from the outside surface of the covering. The radiant loss to the room is also governed by the temperature of the surface cover. Minimum energy transfer will be obtained at the cavity low temperature limit of 350 K. A value of $\tau = 1$ might be selected for this condition with decreasing values as the cavity temperature is increased. No information is given as to the balance between α, ρ, and τ for the cover. The easiest design is one for which $\rho = 0$. Zero transmissivity then becomes that for $\alpha = 1$. Assume $\varepsilon = \alpha$. The calculation can then be performed for the minimum value of τ at the high temperature of 500 K. Variation of radiation properties between the temperature limits may then be determined. The calculation may be refined by assuming different values of surface reflectivity. It is suggested that the initial analysis be made neglecting the free convection losses because of the non linear factors introduced into the equations.

8-179

This problem obviously involves the assumption that $\varepsilon = 1.0$ for both heater surfaces. Otherwise the analysis is the same as Problem 8 – 161.

8-180

The total heat lost is obviously a strong function of the strip temperature. If a large value of the strip temperature is selected the major portion of heat loss will be by radiation. As a start choose a rather high temperature of 800°C and a strip width of 6 mm. Compute the convection loss using the relations of Chap. 5 (flow will be laminar). Be sure to compute convection for both sides of strip. Calculate radiation as if exchange is by openings between strips to large room. Select ε for strip as about 0.6. (Material with other value may be selected from appendix.) Large room behaves as a blackbody. These calculations will yield a heat transfer value per unit length of strip(s). Then choose a configuration of strips (height and length) based on a total length required to dissipate 3 kW. Refine the calculation by taking account of fact that outer surfaces of top and bottom strips will exchange more heat than inner surfaces. Various values of strip temperatures and material properties may be examined as desired, as well as different strip widths. The wider the strip, the less heat that will be dissipated by the inner surfaces by radiation with the room.

8-181

$T_m = 25°\text{C} = 298\ \text{K}$ $\qquad T_a = 20°\text{C} = 293\ \text{K}$ $\qquad h = 8\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$\varepsilon_m \sim 1.0$

(a) $T_r = 0°\text{C} = 273\ \text{K}$

$$\left.\frac{q}{A}\right|_{\text{conv}} = h\Delta T = (8)(25-20) = 40\ \text{W/m}^2$$

$$\left.\frac{q}{A}\right|_{\text{rad}} = \varepsilon\sigma(T_m{}^2 - T_r{}^4) = (5.668\times10^{-8})(298^4 - 273^4) = 132\ \text{W/m}^2$$

$$\left.\frac{q}{A}\right|_{\text{Total}} = 40 + 132 = 172\ \text{W/m}^2$$

(b) $T_r = 20°\text{C} = 293\ \text{K}$

$$\left.\frac{q}{A}\right|_{\text{conv}} = 40\ \text{W/m}^2$$

$$\left.\frac{q}{A}\right|_{\text{rad}} = (5.668\times10^{-8})(298^4 - 293^4) = 29.3\ \text{W/m}^2$$

$$\left.\frac{q}{A}\right|_{\text{Total}} = 40 + 29.3 = 69.3\ \text{W/m}^2$$

(c) Because wall is large $A \to \infty$ and $\dfrac{1-\varepsilon}{\varepsilon A} \to 0$ so same result as for large value of $\varepsilon_{\text{wall}}$.

(d) If emissivity of man reduced to $\varepsilon = 0.1$

At $T_r = 273$ K $\qquad \left.\frac{q}{A}\right|_{\text{rad}} = (0.1)(132) = 13.2 \ \text{W/m}^2$

$$\left.\frac{q}{A}\right|_{\text{Total}} = 40 + 13.2 = 53.2 \ \text{W/m}^2$$

At $T_r = 293$ K $\qquad \left.\frac{q}{A}\right|_{\text{rad}} = (0.1)(29.3) = 2.93 \ \text{W/m}^2$

$$\left.\frac{q}{A}\right|_{\text{Total}} = 40 + 2.93 = 42.93 \ \text{W/m}^2$$

Chapter 9

9–1

$$\bar{h}\left(\frac{\mu_f^2}{k_f^2\rho_f^2 g}\right)^{1/3} = 0.0077\,\mathrm{Re}_f^{0.4} \qquad \mathrm{Re}_f = \frac{4\bar{h}L(T_g - T_w)}{h_{fg}\mu_f}$$

$$\bar{h}\left(\frac{\mu_f^2}{k_f^2\rho_f^2 g}\right)^{1/3} = 0.0077\left[\frac{4\bar{h}L(T_g - T_w)}{h_{fg}\mu_f}\right]^{0.4}$$

$$\bar{h} = \frac{(0.0077)^{1/0.6}}{\left(\dfrac{\mu_f}{k_f^3\rho_f^2 g}\right)^{1/1.8}}\left[\frac{4L(T_g - T_w)}{h_{fg}\mu_f}\right]^{2/3}$$

$$\frac{\bar{h}L}{k_f} = \overline{\mathrm{Nu}} = \frac{\dfrac{(0.0077)^{1/0.6}L}{k_f}\left[\dfrac{4L(T_g - T_w)}{h_{fg}\mu_f}\right]^{2/3}}{\left(\dfrac{\mu_f}{k_f^3\rho_f^2 g}\right)^{1/1.8}}$$

$$\overline{\mathrm{Nu}} = \frac{(0.0077)^{1/0.6}L^{5/3}k_f^{2/3}[4(T_g - T_w)]^{2/3}(\rho_f^2 g)^{5/9}}{\mu_f^{1.22}h_{fg}^{2/3}}$$

9–2

Flat plate

$$\mathrm{Re}_f = \frac{4\dot{m}}{\mu_f} \qquad \dot{m} = \frac{\rho_f^2\delta^3}{3\mu_f} \qquad \delta^3 = \left[\frac{4\mu_f kL(T_g - T_w)}{gh_{fg}\rho_f^2}\right]^{3/4}$$

$$\mathrm{Re}_f = \frac{4\rho_f^2 g}{3\mu_f^2}\left[\frac{4\mu_f kL(T_g - T_w)}{gh_{fg}\rho_f^2}\right]^{3/4}$$

$$\mathrm{Re}_f = 3.77\left[\frac{\rho_f^2 g k_f^3 L^3(T_g - T_w)^3}{\mu_f^5 h_{fg}^3}\right]^{1/4}$$

9–3

Turbulent Film

$$\bar{h} = \frac{0.0077\,\mathrm{Re}_f^{0.4}}{\left(\dfrac{\mu_f^2}{k_f^3\rho_f^2 g}\right)^{1/3}} \qquad \mathrm{Re}_f = \frac{4\dot{m}}{W\mu_f}$$

$$\dot{m} = \frac{\bar{h}A(T_g - T_w)}{h_{fg}} = \frac{\bar{h}LW(T_g - T_w)}{h_{fg}} \qquad \bar{h} = \frac{\dot{m}h_{fg}}{LW(T_g - T_w)}$$

$$\frac{\dot{m}h_{fg}}{LW(T_g - T_w)} = \frac{0.0077\left(\dfrac{4\dot{m}}{W\mu_f}\right)^{0.4}}{\left(\dfrac{\mu_f^2}{k_f^3\rho_f^2 g}\right)^{1/3}}$$

$$\dot{m} = \left[\frac{0.0077(4)^{0.4}LW^{0.6}(T_g - T_w)k_f(\rho_f^2 g)^{1/3}}{\mu_f^{1.07}}\right]^{5/3}$$

9–5

$T_f = \dfrac{80+100}{2} = 90°\mathrm{C}$ $\quad L = 1.2$ $\quad \rho_f = 964$ $\quad \mu_f = 3.15\times10^{-4}$

$k_f = 0.676$ $\quad h_{fg} = 2255\ \mathrm{kJ/kg}$

$$\bar{h} = 1.13\left[\frac{(964)^2(9.806)(2.255\times10^6)(0.676)^3}{(1.2)(3.15\times10^{-4})(100-80)}\right]^{1/4} = 6082\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot°\mathrm{C}}$$

$q = (6082)(1.2)(0.3)(100-80) = 43{,}790\ \mathrm{W}$

$$\dot{m} = \frac{43{,}790}{2.255\times10^6} = 0.0195\ \mathrm{kg/sec} = 69.9\ \mathrm{kg/hr}$$

9–6

$L = 0.4\ \mathrm{m}$ $\quad \phi = 90 - 30 = 60°$ $\quad T_w = 98°\mathrm{C}$ $\quad T_g = 100°\mathrm{C}$ $\quad \rho = 960$

$h_{fg} = 2.255\times10^6$ $\quad k = 0.68$ $\quad \mu = 2.82\times10^{-4}$

$$\bar{h} = 1.13\left[\frac{(960)^2(9.806)(2.255\times10^6)(0.68)^3\sin 60°}{(0.4)(2.82\times10^{-4})(2)}\right]^{1/4} = 14{,}152\ \frac{\mathrm{W}}{\mathrm{m}^2\cdot°\mathrm{C}}$$

$q = (14{,}152)(0.4)^2(2) = 4528\ \mathrm{W}$ $\qquad \dot{m} = 2.008\times10^{-3}\ \mathrm{kg/sec} = 7.23\ \mathrm{kg/hr}$

9–7

$k = 0.684 \qquad \mu = 3.0 \times 10^{-4} \qquad \rho = 962$

$$\bar{h} = 1.13\left[\frac{(962)^2(9.8)(2.255\times10^6)(0.684)^3}{(0.5)(3\times10^{-4})(100-95)}\right]^{1/4} = 10{,}930\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (10{,}930)(0.5)^2(100-95) = 136{,}510\ \text{W}$$

$$\dot{m} = \frac{136{,}510}{2.255\times10^6} = 0.0606\ \text{kg/sec} = 217.9\ \text{kg/hr}$$

9–8

$T_f = 47.5°\text{F} \qquad k = 0.583\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad \mu = 1.37\times10^{-3} \qquad \rho = 999$

$$h = 1.13\left[\frac{(999)^2(9.8)(2.376\times10^6)(0.583)^3}{(1.5)(1.37\times10^{-3})(15)\left(\frac{5}{9}\right)}\right]^{1/4} = 4576\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (4576)(1.5)^2(15)\left(\frac{5}{9}\right) = 8.58\times10^4$$

$$\dot{m} = \frac{q}{h_{fg}} = 0.0361\ \text{kg/sec} = 130\ \text{kg/hr}$$

9–9

$T_g = 38°\text{C} \qquad T_f = \frac{38+30}{2} = 34°\text{C} \qquad \rho = 590 \qquad \nu = 0.345\times10^{-6}$

$h_{fg} = 1111\ \text{kJ/kg} \qquad k = 0.501$

$$h = 1.13\left[\frac{(590)(9.8)(1.111\times10^6)(0.501)^3}{(0.4)(0.345\times10^{-6})(8)}\right]^{1/4} = 5877\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = hA\Delta T = (5877)(0.4)^2(8.0) = 7522\ \text{W}$$

$$\dot{m} = \frac{q}{h_{fg}} = 0.00677\ \text{kg/sec} = 24.38\ \text{kg/hr}$$

9-11

Assume laminar condensation $T_{sat} = 328°F$ $h_{fg} = 889$ Btu/lbm

$p_{sat} = 100$ psia $T_w = 280°F$ $T_f = 304°F$ $\rho_p = 57.29 \text{ lbm/ft}^3$

$\mu = 0.44 \dfrac{\text{lbm}}{\text{hr} \cdot \text{ft}}$ $k = 0.395 \dfrac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot °\text{F}}$ $\text{Pr} = 1.15$

$\rho_f > \rho_r \therefore \rho_f(\rho_f - \rho_r) = \rho_p{}^2$

$$\bar{h} = 0.725\left[\frac{\rho_f{}^2 g h_{fg} k^3}{\mu d(T_g - T_w)}\right]^{1/4} = 1855 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot °\text{F}}$$

$$\dot{m} = \frac{q}{h_{fg}} = \frac{\bar{h}\pi d L(T_{sat} - T_w)}{h_{fg}} \qquad \frac{\dot{m}}{L} = \frac{(1855)\pi(1)(48)}{(12)(889)} = 26.31 \frac{\text{lbm}}{\text{hr} \cdot \text{ft}}$$

9-12

$35°C = 95°F$ $p_g = 0.8237$ psia $p_r = (0.8)(0.8237) = 0.659$ psia

$T_g = 88°F = 31.1°C$ $h_{fg} = 1043$ Btu/lbm $= 2426$ kJ/kg

$T_f = \dfrac{31.1 + 2}{2} = 16.55°C$ $\rho = 998$ $\mu_f = 1.1 \times 10^{-3}$ $k_f = 0.596$

$\text{Pr}_f = 7.8$

$$\bar{h} = 0.725\left[\frac{(998)^2(9.8)(2.426 \times 10^6)(0.596)^3}{(1.1 \times 10^{-3})(0.05)(31.1 - 2)}\right]^{1/4} = 5425 \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$q = (5425)\pi(0.05)(7.5)(31.1 - 2) = 185{,}970$

$$\dot{m} = \frac{185{,}970}{2.426 \times 10^6} = 0.0767 \text{ kg/sec} = 276 \text{ kg/hr}$$

9-13

Assume laminar film and heat transfer area $= 1 \text{ ft}^2$

$\bar{h}_1$: $T_f = 154°F$ $\rho_f = 61.1$ $\mu_f = 1.02$ $k_f = 0.382$

$$\bar{h}_1 = 873 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot °\text{F}}$$

$\bar{h}_2$: $T_f = 177°F$ $\rho_f = 60.6$ $h_{fg} = 992.1$ $\mu_f = 0.86$ $k_f = 0.388$

$$\bar{h}_2 = 838 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot °\text{F}}$$

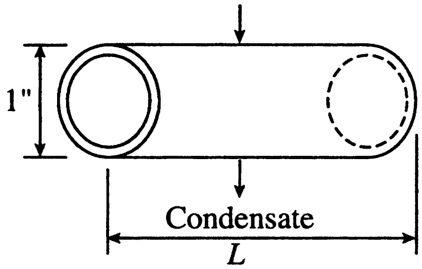
$q_1 = \bar{h}_1 A \Delta T = 1.02 \times 10^5$ Btu/hr $q_2 = \bar{h}_2 A \Delta T = 1.37 \times 10^5$ Btu/hr

$\dfrac{q_2 - q_1}{q_1} = 0.344$ 34.4% increase

9-14

$p_{sat} = 690$ kPa $\quad h_{fg} = 2.07\times10^6$ J/kg $\quad T_g = 164°C \quad T_w = 138°C$

$T_f = 151°C \quad \rho_f = 915 \quad \mu_f = 1.86\times10^{-4} \quad k_f = 0.683$



Assume laminar flow:

$$\bar{h} = 0.725\left[\frac{\rho^2 g h_{fg} k_f{}^3}{\mu_f d(T_g - T_w)}\right]^{1/4} = 10{,}450 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}} \qquad \text{Check } \mathrm{Re}_f = 73.6$$

$\therefore$ Laminar

$$q = \bar{h}A(T_g - T_w) = 22{,}231 \ \text{W/m} \qquad \dot{m} = \frac{q}{h_{fg}} = 38.7 \ \frac{\text{kg}}{\text{sec}\cdot\text{m length}}$$

9-15

$k = 0.684 \quad \mu = 3.0\times10^{-4} \quad \rho = 962$

$$\bar{h} = 0.725\left[\frac{(962)^2(9.8)(2.255\times10^6)(0.684)^3}{(3\times10^{-4})(0.3)(5)}\right]^{1/4} = 7962 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = h\pi dL(T_g - T_w) = (7962)\pi(0.3)(15)(5) = 5.628\times10^5 \ \text{W}$$

$$\dot{m} = \frac{q}{h_{fg}} = 0.2496 \ \text{kg/sec} = 898 \ \text{kg/hr}$$

9-16

CO_2 $\quad h_{fg} = 153.2\ \text{kJ/kg at } 20°\text{C}$ $\quad d = 0.1\ \text{m}$ $\quad L = 1.0\ \text{m}$

$T_w = 15°\text{C}$ $\quad T_g = 20°\text{C}$ $\quad T_f = 17.5°\text{C}$ $\quad \rho = 795\ \text{kg/m}^3$

$k = 0.0897\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}}$ $\quad \nu = 0.0935 \times 10^{-6}$

$\mu = (735)(0.0935 \times 10^{-6}) = 6.87 \times 10^{-5}$ $\quad \Pr = 3.78$

$$\bar{h} = 0.725\left[\frac{(795)^2(9.8)(153.2\times10^3)(0.0897)}{(6.87\times10^{-5})(0.1)(20-15)}\right]^{1/4} = 1532\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (1532)\pi(0.1)(1.0)(20-15) = 2406\ \text{W}$$
$$= \dot{m}h_{fg} = \dot{m}(153.2\times10^3)$$
$$\dot{m} = 0.0157\ \text{kg/sec} = 56.5\ \text{kg/hr}$$

9-17

$n = 20$ $\quad T_f = \dfrac{88+100}{2} = 94°\text{C}$ $\quad \rho_f = 963.2$

$d = 0.25\ \text{in.} = 0.00635\ \text{m}$ $\quad \mu_f = 3.06\times10^{-4}$ $\quad k_f = 0.627$

$h_{fg} = 2255\ \text{kJ/kg}$

$$\bar{h} = 0.725\left[\frac{(963)^2(9.8)(2.255\times10^6)(0.627)^3}{(3.06\times10^4)(20)(0.00635)(100-88)}\right]^{1/4} = 7845\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (400)\pi(0.00635)(7845)(100-88) = 7.51\times10^5\ \text{W/m}$$

$$\dot{m} = \frac{7.51\times10^5}{2.255\times10^6} = 0.333\ \text{kg/sec} = 1200\ \frac{\text{kg}}{\text{hr}\cdot\text{m}}$$

9-18

$d = 0.05$ $\quad T_g = 100°\text{C}$ $\quad T_w = 98°\text{C}$ $\quad L = 1.5\ \text{m}$ $\quad \rho = 960$

$k = 0.68$ $\quad \mu = 2.82\times10^{-4}$ $\quad h'_{fg} \approx h_{fg} = 2.255\times10^6$

$$\bar{h} = 0.555\left[\frac{(960)^2(9.8)(0.68)^3(2.255\times10^6)}{(2.82\times10^{-4})(0.05)(2)}\right]^{1/4} = 12{,}117\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (12{,}117)\pi(0.05)(1.5)(2) = 5710\ \text{W}$$

$$\dot{m} = \frac{5710}{2.255\times10^6} = 2.532\times10^{-3}\ \text{kg/sec} = 9.12\ \text{kg/hr}$$

9-19

$\rho = 962 \qquad \mu = 3.0 \times 10^{-4} \qquad k = 0.68 \qquad n = 10 \qquad d = 0.0254 \text{ m}$

$$\bar{h} = 0.725\left[\frac{(962)^2(9.8)(2.255\times10^6)(0.68)^3}{(10)(0.0254)(3.0\times10^{-4})(5)}\right]^{1/4} = 8264 \ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (8264)(100)\pi(0.0254)(2)(0.3048)(5) = 2.01\times10^5 \text{ W}$$

$$\dot{m} = \frac{q}{h_{fg}} = 0.0891 \text{ kg/sec} = 320.9 \text{ kg/hr}$$

9-20

$T_f = 98.5^\circ\text{C} \qquad \rho = 960 \qquad \mu = 2.82\times10^{-4} \qquad k = 0.68 \ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$

$n = 10 \qquad d = 0.0254 \text{ m}$

$$\bar{h} = 0.725\left[\frac{(960)^2(9.8)(2.255\times10^6)(0.68)^3}{(10)(0.0254)(2.82\times10^{-4})(3)}\right]^{1/4} = 9526 \ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (9526)(100)\pi(0.0254)(3)(0.3048)(3) = 2.085\times10^5 \text{ W}$$

$$\dot{m} = \frac{q}{h_{fg}} = 0.0925 \text{ kg/sec} = 332.9 \text{ kg/hr}$$

9-22

$h_{fg} = 1135 \text{ kJ/kg} \qquad T_f = \frac{90+82}{2} = 86^\circ\text{F} = 30^\circ\text{C} \qquad \rho = 596$

$\nu = 0.349\times10^{-6} \qquad k = 0.507 \qquad n = 20 \qquad d = 0.00635 \text{ m}$

$$\bar{h} = 0.725\left[\frac{(596)(9.8)(1.135\times10^6)(0.507)^3}{(20)(0.349\times10^{-6})(0.00635)(90-82)\left(\frac{5}{9}\right)}\right]^{1/4} = 5904 \ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (5904)(400)\pi(0.00635)(0.3048)(90-82)\left(\frac{5}{9}\right) = 63{,}817 \text{ W}$$

$$\dot{m} = \frac{q}{h_{fg}} = 0.0562 \text{ kg/sec} = 202 \text{ kg/hr}$$

9-23

$\dot{m} = 10{,}000 \text{ kg/hr} = 2.778 \text{ kg/sec}$ $\qquad h_{fg} = 130.09 \text{ kJ/kg}$

$q = \dot{m}h_{fg} = 361{,}400 \text{ W}$ $\qquad T_f = \dfrac{100+90}{2} = 95°\text{F} = 35°\text{C}$

$\rho = 1276$ $\qquad \nu = 0.193 \times 10^{-6}$ $\qquad k = 0.07 \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$ $\qquad n = 25$

$d = 0.012$

$$\bar{h} = 0.725\left[\frac{(1276)(9.8)(1.3\times10^5)(0.07)^3}{(25)(0.0125)(10)\left(\frac{5}{9}\right)(0.193\times10^{-6})}\right]^{1/4} = 823.4 \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$q = hA\Delta T = hn\pi dL\Delta T$

$$361{,}400 = (823.4)(625)\pi(0.0125)L(10)\left(\frac{5}{9}\right)$$

$L = 3.22 \text{ m}$

9-24

$d = 0.012 \text{ m}$ $\qquad T_f = \dfrac{32.2+26.7}{2} = 29.45°\text{C}$ $\qquad \rho = 1295$

$k = 0.07$ $\qquad \nu = 0.194 \times 10^{-6}$ $\qquad c = 984 \dfrac{\text{J}}{\text{kg} \cdot °\text{C}}$ $\qquad h_{fg} = 133.65 \text{ kJ/kg}$

$$h'_{fg} = 133.65 + (0.68)(0.984)(10)\left(\frac{5}{9}\right) = 137.4 \text{ kJ/kg}$$

$$\bar{h} = 0.555\left[\frac{(1295)(9.8)(0.137\times10^6)(0.07)^3}{(0.194\times10^{-6})(0.012)(5.5)}\right]^{1/4} = 1446 \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

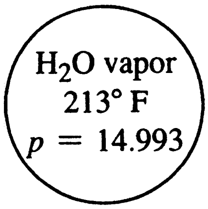
$$\frac{q}{L} = h\pi d\Delta T = (1446)\pi(0.012)(10)\left(\frac{5}{9}\right) = 303 \text{ W/m}$$

9-25

$\Delta T_x = 107 - 100 = 7°\text{C}$ $\qquad \dfrac{q}{A} = 7.96(7)^4 = 19.11 \text{ kW/m}^2$

$q = (19{,}110)(0.3) = 5734 \text{ W}$

9-26



Surface tension of H_2O at 212°F $= 58.8$ dyne/cm

$\pi r^2 (p_v - p_l) =$ pressure force $\qquad 2\pi r\sigma =$ surface tension force

H_2O at 212°F $\qquad p_l = 14.696 \qquad r = \dfrac{2\sigma}{p_v - p_l} = 0.00226$ in.

9-28

$\Delta T_x = 119 - 100 = 19°\text{C} \qquad c_{sf} = 0.013 \qquad \dfrac{q}{A} = 1.2\ \text{MW/m}^2$

$q = (1.2 \times 10^6)(0.3)^2 = 108{,}000$ W

9-32

$$\text{Re}_f = \frac{4\Gamma}{\mu_f} \qquad h_x = \left[\frac{\rho^2 g h_{fg} k^3}{4\mu(T_g - T_w)}\right]^{1/4} \qquad \bar{h} = \frac{1}{L}\int_0^L h_x dx$$

$$\bar{h} = \frac{1}{L}\int_0^L \left(\frac{\rho^2 g h_{fg} k^3}{4\mu_f \Delta T}\right) dx \qquad \bar{h} = \frac{4}{3}\left(\frac{L^3 \rho^2 g k^3 h_{fg}}{L^4 4\mu_f \Delta T}\right)^{1/4} = \frac{4}{3}(h_x)_{x=L}$$

$$\bar{h} = \Delta T L = q = h_{fg}\Gamma \qquad \Gamma = \frac{\bar{h}\Delta T L}{h_{fg}}$$

$$\text{Re}_f = \frac{4\Gamma}{\mu_f} = \frac{4\bar{h}\Delta T L}{\mu_f h_{fg}} \qquad \text{Re}^{-1/3} = \left(\frac{\mu_f h_{fg}}{4\bar{h}\Delta T L}\right)^{1/3} = \bar{h}\left(\frac{\mu_f h_{fg}}{4\bar{h}\Delta T L}\right)^{1/3}$$

$$\bar{h}^4 = \left(\frac{4}{3}\right)^4 \left(\frac{\rho_f^2 g h_{fg} k^3}{4L\mu_f \Delta T}\right) \qquad \left(\frac{4}{3}\right)^{4/3} \text{Re}_f^{-1/3} = \bar{h}\left(\frac{\mu_f}{\rho_f^2 k_f^3 g}\right)^{1/3}$$

$$1.466\,\text{Re}_f^{-1/3} = \bar{h}\left(\frac{\mu_f}{\rho_f^2 k_f^3 g}\right)^{1/3}$$

9-33

$\Delta T_x = 10°\text{C} \qquad \dfrac{q}{A} = 1.9\ \text{MW/m}^2$

9-34

$T_w = 110°C$ $\quad T_{b\,avg} = 96°C$ $\quad \Delta T_x = 10°C$ $\quad d = 1.25$ cm

$u_m = 1.2$ m/sec

Brass $C_{sf} = 0.006$ for plat $\dfrac{q}{A} = 2.5\times10^4 \ \dfrac{\text{Btu}}{\text{hr}\cdot\text{ft}^2} = 7.885\times10^4 \ \text{W/m}^2$

$$\left.\frac{q}{A}\right|_{\text{Brass}} = (7.885\times10^4)\left(\frac{0.013}{0.006}\right)^3 = 8.02\times10^5 \ \text{W/m}^2$$

$$\left.\frac{q}{L}\right|_{\text{boiling}} = (8.02\times10^5)\pi(0.0125) = 31{,}494 \ \text{W/m}$$

<u>Forced Convection</u>

At 96°C $\quad \rho = 960 \quad \mu = 2.96\times10^{-4} \quad k = 0.68 \quad \Pr = 1.83$

$$\text{Re} = \frac{(960)(1.2)(0.0125)}{2.96\times10^{-4}} = 48{,}650$$

$$h = \frac{0.68}{0.0125}(0.019)(48{,}650)^{0.8}(1.83)^{0.4} = 7396 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\left.\frac{q}{L}\right|_{\text{conv}} = (7396)\pi(0.0125)(14) = 4066 \ \text{W/m}$$

$$\left.\frac{q}{L}\right|_{\text{total}} = 31{,}494 + 4066 = 35{,}560 \ \text{W/m}$$

9-35

$q = (2.3)(2.255\times10^6) = 5.187\times10^6 \ \text{J/hr} = 1441 \ \text{W}$

$\dfrac{q}{A} = \dfrac{1441}{\pi(0.15)^2} = 20{,}382 \ \text{W/m}^2 \qquad \dfrac{q}{A} = 5.56(\Delta T_x)^4 \qquad \Delta T_x = 7.78°C$

$T_w = 107.8°C$

9-36

$Cu - H_2O$ $\quad p = 1$ atm $\quad d = 0.005$ m $\quad T_w - T_{sat} = 11°C$

$C_{sf} = 0.0128 \approx C_{sf}$ (plat)

From Fig. 9–8 at $\Delta T_x = 11°C$

$\dfrac{q}{A} = 0.11 \ \text{MW/m}^2$

For $L = 1.0$ m $\qquad A = \pi(0.005) = 0.01571 \ \text{m}^2$

$q = (0.11\times10^6)(0.01571) = 1728$ W/m length

9-37

$\Delta T_x = 11°\text{C} \qquad \left.\frac{q}{A}\right|_{\text{plat}} = \left.\frac{q}{A}\right|_{\text{Cu}} = 4000 \ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2} = 126{,}160 \ \text{W/m}^2$

$\frac{q}{L} = (126{,}160)\pi(0.005) = 1982 \ \text{W/m}$

9-38

$\Delta T_x = 10°\text{C} \qquad p = 3 \ \text{MN/m}^2 = 435 \text{ psia} \qquad d = 0.02 \qquad L = 1 \text{ m}$

$h = 2.54(10)^3 e^{3/1.551} = 17{,}576 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$q = (17{,}576)\pi(0.02)(1)(10) = 11{,}042 \text{ W}$

h_{fg} at 435 psia $= 771.3 \ \text{Btu/lbm} = 1.794\times10^6 \ \text{J/kg}$

$\dot{m} = \frac{11{,}042}{1.794\times10^6} = 6.15\times10^{-3} \ \text{kg/sec} = 22.16 \ \text{kg/hr}$

9-39

$\frac{q}{A} = 0.2 \ \text{MW/m}^2 = 200 \ \text{kW/m}^2 \qquad d = 0.003 \qquad L = 7.5 \text{ cm}$

$p = 1.6 \text{ atm} \qquad h = 5.56(\Delta T_x)^3 \qquad \frac{q}{A} = hAT_x$

$200\times10^3 = 5.56(\Delta T_x)^4(1.6)^{0.4}$

$\Delta T_x = 13.14°\text{C}$ at 1.16 atm $\qquad p = 23.52 \text{ psia} \qquad T_{\text{sat}} = 236.5°\text{F} = 113.6°\text{C}$

$T_w = 113.6 + 13.14 = 126.7°\text{C}$

9-40

$d = 2.5 \text{ cm} \qquad p = 101 + 34 = 135 \text{ kPa} \qquad \Delta T_x = 4°\text{C}$

Assume: horizontal $h = 5.56(4)^3 = 355.8 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$\frac{q}{A} = h\Delta T = (355.8)(4) = 1.423 \ \text{kW/m}^2$

above relation does not apply

$h = 1042(4)^{1/3} = 1654 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}} \qquad \frac{q}{A} = h\Delta T = 6.616 \ \text{kW/m}^2$

$\dot{m} = 908 \ \text{kg/hr} = 0.252 \ \text{kg/sec} \qquad q = (0.252)(2.255\times10^6) = 6616A$

$A = 85.96 \ \text{m}^2 = \pi dL \qquad L = 1095 \text{ m}$

9-41

$\Delta T_x = 15°\text{C} \qquad \dfrac{h}{h_w} = 0.83 \qquad \left(\dfrac{q}{A}\right)_{\text{water}} = 2\times10^5 \ \dfrac{\text{Btu}}{\text{hr}\cdot\text{ft}^2}$

$\left(\dfrac{q}{A}\right)_{\text{water}} = 6.308\times10^5 \ \text{W/m}^2 \qquad h_w = \dfrac{6.308\times10^5}{15} = 42{,}050 \ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$h_{\text{glycerine}} = (0.83)(42{,}050) = 34{,}900 \ \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$

9-43

$\Delta T_x = 30°\text{F} \qquad C_{sf} = 0.008 \qquad \text{For plat } \dfrac{q}{A} = 2.5\times10^5 \ \dfrac{\text{Btu}}{\text{hr}\cdot\text{ft}^2}$

$\left.\dfrac{q}{A}\right|_{ss} = (2.5\times10^5)\left(\dfrac{0.13}{0.008}\right)^3 = 1.073\times10^6 \ \dfrac{\text{Btu}}{\text{hr}\cdot\text{ft}^2} = 3.383 \ \text{MW/m}^2$

9-45

$\Delta T_x = 110 - 100 = 10°\text{C}$

From Figure $\dfrac{q}{A} = 0.7 \ \text{MW/m}^2$

$q = \left(\dfrac{q}{A}\right)\pi dL = (0.7\times10^6)\pi(0.001)(0.12) = 264 \ \text{W}$

9-46

$T_f = \dfrac{94+100}{2} = 97°\text{C} \qquad d = 0.04 \ \text{m} \qquad \rho = 962 \qquad k = 0.68$

$\mu = 3\times10^{-4} \qquad h_{fg} = 2.255\times10^6 \ \text{J/kg}$

$$\bar{h} = 0.725\left[\frac{(962)^2(9.8)(2.255\times10^6)(0.68)^3}{(3\times10^{-4})(0.04)(100-94)}\right]^{1/4} = 12{,}533 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$\dfrac{q}{L} = h\pi d\Delta T = (12{,}533)\pi(0.04)(100-94) = 9450 \ \text{W/m}$

9-47

$$\left.\frac{q}{A}\right|_{\text{total}} = \left.\frac{q}{A}\right|_{\text{FC}} + \left.\frac{q}{A}\right|_{\text{boiling}}$$

$$\Delta T_x = 110 - 98 = 12°\text{C} = 21.6°\text{F} \qquad \left.\frac{q}{A}\right|_{\text{plat}} = 4\times 10^4 \ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2} = 1.26\times 10^5 \ \text{W/m}^2$$

At 100°C $\quad k = 0.68 \quad \Pr = 1.7$

$$h = \frac{k}{d}(0.019)(40{,}000)^{0.8}(1.7)^{0.4} = 3070 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\left.\frac{q}{A}\right|_{\text{brass}} = (1.26\times 10^5)\left(\frac{c_{\text{plat}}}{c_{\text{brass}}}\right)^3 = 12.8\times 10^5 \ \text{W/m}^2$$

$$h_b = \frac{\left.\frac{q}{A}\right|_{\text{brass}}}{\Delta T} = \frac{12.8\times 10^5}{110-98} = 10{,}670 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$h\ (\text{total}) = 10{,}670 + 3070 = 13{,}740 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

9-49

$$h_{fg} = 2.255\times 10^6 \ \text{J/kg} \qquad \rho_v = 1.5 \ \text{kg/m}^3 \qquad \rho_l = 998 \ \text{kg/m}^3$$

$$\sigma = 70 \ \text{mN/m}$$

$$\left.\frac{q}{A}\right|_{\max} = \frac{\pi}{24}(2.255\times 10^6)(1.5)\left[\frac{(0.07)(9.8)(996)}{(1.5)^2}\right]^{1/4}\left(1+\frac{1.5}{998}\right)^{1/2} = 1.85 \ \text{MW/m}^2$$

9-51

$$h_{fg} = 2.255\times 10^6 \ \text{J/kg} \qquad T_w = 813 \ \text{K} \qquad T_{\text{sat}} = 373 \ \text{K}$$

(a) $T_f = \dfrac{540+100}{2} = 320°\text{C} = 593 \ \text{K} \qquad \rho_l = 960 \qquad \rho_v = 0.365$

$k_g = 0.042 \qquad \mu_v = 20.6\times 10^{-6} \qquad c_{p_v} = 2000$

$\Delta T_x = 440$

$$h_b = 0.62\left[\frac{(0.042)^3(0.365)(960)(9.8)(2.255\times 10^6 + 2000)(0.4)(4000)}{(0.0125)(20.6\times 10^{-6})(440)}\right]^{1/4}$$

$$= 171.5 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$h_r = \frac{(5.669\times 10^{-8})(0.8)(813^4 - 373^4)}{813-373} = 43.04$$

$$h = (171.5)^{4/3} h^{-1/3} + 43.04 = 204.7 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$\frac{q}{A} = h\Delta T_x = (204.7)(440) = 90{,}068 \ \text{W/m}^2$$

9-52

$T_w = 280°\text{C}$ $\quad$ $T_{sat} = 100°\text{C}$ $\quad$ $v = 3 \ \text{m/sec}$ $\quad$ $d = 0.0004 \ \text{m}$

at 25°C flow $= 1 \ \text{L/hr} = 1000 \times 10^{-6} \times 996 = 0.996 \ \text{kg/hr} = 2.767 \times 10^{-4} \ \text{kg/s}$

$T_f = \dfrac{280 + 100}{2} = 190°\text{C} = 463 \ \text{K}$ $\quad$ $h_{fg} = 2.255 \times 10^6$ $\quad$ $\rho_L = 996$

$\rho_{vf} = 0.478$ $\quad$ $c_{p_v} = 1980$ $\quad$ $\sigma = 58.8 \ \text{mN/m}$

$$\lambda = 2.255 \times 10^6 + (1980)\left(\frac{280 - 100}{2}\right) = 2.433 \times 10^6 \ \text{J/kg}$$

$$\frac{Q_{max}}{\text{drop}} = (1.83 \times 10^{-3})(996)(0.0004)^3(2.433 \times 10^6)\left[\frac{(996)^2(3)^2(0.0004)}{(0.478)(0.0588)(1)}\right]^{0.341}$$

$$= 0.0156 \ \text{J/drop}$$

$$\frac{\text{mass}}{\text{drop}} = (996)\left(\frac{4}{3}\pi\right)(0.0002)^3 = 3.338 \times 10^{-8} \ \text{kg}$$

$$\text{\# of drops} = \frac{2.767 \times 10^{-4} \ \text{kg/sec}}{3.338 \times 10^{-8} \ \text{kg/drop}} = 8290 \ \text{drops/sec}$$

$$Q_{max} = (0.0156)(8290) = 129.3 \ \text{W}$$

9-53

$T_f = \dfrac{100 + 92}{2} = 96°\text{C}$ $\quad$ $d = 0.0125$ $\quad$ $n = \sqrt{196} = 14$

$\rho = 961 \ \text{kg/m}^3$ $\quad$ $\mu = 2.96 \times 10^{-4}$ $\quad$ $k = 0.68$ $\quad$ $h_{fg} = 2255 \ \text{kJ/kg}$

$$\bar{h} = 0.725\left[\frac{(961)^2(9.8)(2255 \times 10^3)(0.68)^3}{(2.96 \times 10^{-4})(14)(0.0125)(100 - 92)}\right]^{1/4} = 8088 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$q = (8088)(196)\pi(0.0125)(2.0)(100 - 92) = 9.96 \times 10^5 \ \text{W} = \dot{m}(2255 \times 10^3)$$

$$\dot{m} = 0.442 \ \text{kg/s}$$

9-54

$$T_f = \frac{100+91}{2} = 95.5°\text{C} \qquad \rho = 961\ \text{kg/m}^3 \qquad \mu = 2.97\times10^{-4}$$

$$k = 0.68 \qquad h_{fg} = 2255\ \text{kJ/kg}$$

$$1800 = \frac{4\bar{h}L(100-91)}{2255\times10^3(2.97\times10^{-4})}$$

$$\bar{h}_L = 33{,}487 = 1.13\left[\frac{(961)^2(9.8)(2255\times10^3)(0.68)^3}{(2.97\times10^{-4})(100-91)}\right]^{1/4} L^{3/4}$$

$$L^{3/4} = 4.234 \qquad L = 6.85\ \text{m}$$

$$\frac{\dot{m}}{\rho} = \frac{(1800)(2.97\times10^{-4})}{4} = 0.134\ \text{kg/s}\cdot\text{m depth}$$

9-55

$$C_{sf} = 0.008 \qquad C_{sf}\ \text{plat} = 0.013 \qquad \Delta T_x = 15°\text{C}$$

$$\left.\frac{q}{A}\right|_{\text{plat}} = 2.7\times10^5\ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2} = 851\ \text{kW/m}^2$$

$$\left.\frac{q}{A}\right|_{ss} = (851)\left(\frac{0.013}{0.008}\right)^3 = 3.65\ \text{MW/m}^2$$

9-56

$$p = 5\ \text{atm} = 0.507\ \text{MPa}$$

$$\frac{q}{A} = 2.253(\Delta T_x)^{3.96} = 2.253(10)^{3.96} = 20{,}550\ \text{W/m}^2$$

$$h = \frac{\frac{q}{A}}{\Delta T} = 2055\ \text{W/m}^2$$

9-57

$$p = 1\ \text{atm} \qquad d = 0.3\ \text{m} \qquad T_w = 94°\text{C} \qquad T_f = 97°\text{C} \qquad \mu = 2.8\times10^{-4}$$

$$k = 0.68 \qquad \rho = 961 \qquad h_{fg} = 2.26\times10^6\ \text{J/kg}$$

$$\bar{h} = 0.725\left[\frac{(961)^2(9.8)(2.26\times10^6)(0.68)^3}{(2.8\times10^{-4})(0.3)(100-94)}\right]^{1/4} = 7706\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = (7706)\pi(0.3)(100-94) = 43{,}750\ \text{W/m}$$

9-58

$$T_f = \frac{100+93}{2} = 96.5°\text{C} \qquad h_{fg} = 2255\times 10^3 \text{ J/kg} \qquad \rho = 961 \text{ kg/m}^3$$

$$\mu = 2.96\times 10^{-4} \qquad k = 0.68$$

$$\bar{h} = 1.13\left[\frac{(961)^2(9.8)(2255\times 10^3)(0.68)^3}{(0.2)(2.96\times 10^{-4})(100-93)}\right]^{1/4} = 12{,}605 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$q = (12{,}605)(0.2)^2(100-93) = 3530 \text{ W} = \dot{m}(2255\times 10^3)$$

$$\dot{m} = 0.00156 \text{ kg/s}$$

$$\delta = \left[\frac{(4)(2.96\times 10^{-4})(0.68)(0.2)(100-93)}{(9.8)(2255\times 10^3)(961)^2}\right]^{1/4} = 8.6\times 10^{-5} \text{ m}$$

9-59

With $h = 5000 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$ $\qquad L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = 0.654 \qquad \eta_f = 0.76$

(No. of fins) × (Area of tube + fin) $= (5.9\times 10^{-4})(50) = 0.0295 \text{ m}^2$

Average $\Delta T \sim 0.76\Delta T$ (base)

$T_{\text{water}} = 100°\text{C}$

T (base)	ΔT (avg)	$\frac{q}{A}$ (Fig 9–8)	q
108	6.1	0.01×10^6	295 W
111	8.4	0.03×10^6	885 W
117	13	0.15×10^6	4425 W

9-60

$$\Delta T_x = 104 - 100 = 4 \qquad A = \pi\frac{(0.25)^2}{4} = 0.049 \text{ m}^2$$

$$\frac{q}{A} = 1042(\Delta T_x)^{1/3} = 1042(4)^{1/3} = 1654 \text{ W/m}^2 \qquad q = (1654)(0.049) = 81 \text{ W}$$

9-61

$$\Delta T_x = 104 - 100 = 4°\text{C} \qquad \frac{q}{A} = 1654 \text{ W/m}^2$$

$$A = \frac{\pi(0.125)^2}{4} + \pi(0.05)(0.125) = 0.0319 \text{ m}^2 \qquad q = (1654)(0.0319) = 52.8 \text{ W}$$

9-62

$\Delta T_x = 17°C$ Fig. 9–8 $\left.\frac{q}{A}\right|_{plat} \approx 0.95\times10^6 \text{ W/m}^2$

C_{sf} (plat) $= 0.013$ C_{sf} (teflon) $= 0.0058$

$$\frac{q}{A} \sim \left(\frac{1}{C_{sf}}\right)^3$$

$$\frac{q}{A} = (0.95\times10^6)\left(\frac{0.013}{0.0058}\right)^3 = 1.07\times10^7 \text{ W/m}^2$$

$$h = \frac{\frac{q}{A}}{\Delta T} = \frac{1.07\times10^7}{17} = 6.3\times10^5 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$A = \pi dL = \pi(0.01)(1) = 0.0314 \text{ m}^2$$

$$q = (1.07\times10^7)(0.0314) = 3.37\times10^5 \text{ W}$$

9-63

No of tubes $= (10)(20) = 200$ $d = 12$ mm $n = 15$

$T_w = 86°C$ $T_g = 100°C$ $T_f = 93°C$

$h_{fg} = 2.26\times10^6$ J/kg $k_f = 0.678 \ \frac{\text{W}}{\text{m}\cdot°\text{C}}$ $\mu_f = 3.06\times10^{-4}$

$\rho_f = 963$

$$\bar{h} = 0.725\left[\frac{\rho^2 g h_{fg} k^3}{n\mu d(T_g - T_w)}\right]^{1/4} = 0.725\left[\frac{(963)^2(9.8)(2.26\times10^6)(0.678)^3}{(15)(3.06\times10^{-4})(0.12)(100-86)}\right]^{1/4}$$

$$= 6920 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$A|_{meter} = 200\pi(0.012)(1) = 7.54 \text{ m}^2$$

$$q = \bar{h}A(T_g - T_w) = (6920)(7.54)(100-86) = 7.31\times10^5 \text{ W}$$

9-64

$T_w = 85°C \quad T_g = 100°C \quad L = 50 \text{ cm} \quad W = 25 \text{ cm} \quad T_f = 92.5°C$

$\rho_f = 964 \quad \mu_f = 3.1\times 10^{-4} \quad k_f = 0.677 \quad h_{fg} = 2.26\times 10^6 \text{ J/kg}$

<u>Assume laminar</u>

$$\bar{h} = 1.13\left[\frac{\rho^2 g h_{fg} k^3}{L\mu(T_g - T_w)}\right]^{1/4} = 1.13\left[\frac{(964)^2(9.8)(2.26\times 10^6)(0.627)^3}{(0.5)(3.1\times 10^{-4})(100-85)}\right]^{1/4}$$

$$= 7723 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = \bar{h}A\Delta T = (7723)(0.25)(0.5)(100-85) = 14,480 \text{ W} = \dot{m}h_{fg}$

$$\dot{m} = \frac{14,480}{2.26\times 10^6} = 0.0064 \text{ kg/sec} = 23.1 \text{ kg/hr}$$

$$\text{Re} = \frac{4\dot{m}}{p\mu_f} = \frac{(4)(0.0064)}{(0.25)(3.1\times 10^{-4})} = 330$$

Laminar assumption OK

$$\delta = \left[\frac{4\mu k L(T_g - T_w)}{g h_{fg}\rho^2}\right]^{1/4} = \left[\frac{(4)(3.1\times 10^{-4})(0.677)(0.5)(100-85)}{(9.8)(2.26\times 10^6)(964)^2}\right]^{1/4}$$

$$= 1.3\times 10^{-4} \text{ m} = 0.13 \text{ mm}$$

9-65

$\dot{m} = 1.3 \text{ kg/s} \quad d = 1.25 \text{ cm} \quad \text{Sp} = 1.9 \text{ cm} \quad T_w = 93°C$

$T_g = 100°C \quad n = \text{number of rows} \quad T_f = 96.5°C$

$h_{fg} = 2255 \text{ kJ/kg} \quad \rho = 962 \quad \mu = 2.96\times 10^{-4} \quad k = 0.68$

$$\bar{h} = 0.725\left[\frac{(962)^2(9.8)(2.255\times 10^6)(0.68)^3}{(2.96\times 10^{-4})(0.0125)(7)n}\right]^{1/4} = \frac{16,186}{n^{1/4}}$$

$q = (1.3)(2.255\times 10^6) = 2.932\times 10^6 \text{ W} = \bar{h}A(T_g - T_w)$

x = side of square array $\quad 3x$ = length $\quad x = n(0.0125) + (n-1)(0.019)$

$A = n^2\pi d(3x) = 0.00371n^3 - 0.00224n^2$

$$2.932\times 10^6 = \frac{113,302}{n^{1/4}}(0.00371n^3 - 0.00224n^2)$$

By iteration $n = 25 \quad x = 0.769 \quad 3x = 2.306$

number of tubes $= n^2 = 625$

9-66

$\dot{m} = 600\ \text{kg/hr} = 0.1667\ \text{kg/s} \qquad q = \dot{m}h_{fg} = (0.1667)(2.255\times10^6)$

$q = 3.75\times10^5\ \text{W} \qquad T_f = 98.5°\text{C} \qquad \rho = 960 \qquad \mu = 2.82\times10^{-4}$

$k = 0.68\ \dfrac{\text{W}}{\text{m}\cdot°\text{C}} \qquad n = 20 \qquad d = 0.01\ \text{m}$

$$\bar{h} = 0.725\left[\frac{(960)^2(9.8)(2.255\times10^6)(0.68)^3}{(20)(0.01)(2.82\times10^{-4})(3)}\right]^{1/4} = 10{,}123\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$q = hn\pi dL(T_g - T_w)$

$3.75\times10^5 = (10{,}123)(400)\pi(0.01)L(3) \qquad L = 0.983\ \text{m}$

Chapter 10

10–1

Pipe nearly constant temperature $\quad T_w = 82°\text{C} \quad T_\infty = 30°\text{C} \quad \varepsilon \approx 0.8$

$$h = 1.32\left(\frac{82-30}{0.0564}\right)^{1/4} = 7.27 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$q_{\text{conv}} = (7.27)\pi(0.0564)(15)(82-30) = 1005 \text{ W}$$

$$q_{\text{rad}} = (5.669 \times 10^{-8})\pi(0.0564)(15)(0.8)(355^4 - 303^4) = 898 \text{ W}$$

$$q_{\text{tot}} = 1903 = (0.6)(4175)\Delta T_w \qquad \Delta T_w = 0.76°\text{C}$$

$$T_w \text{ (exit)} = 82 - 0.76 = 81.2°\text{C}$$

10–2

$$q = m_w c_w \Delta T_w = (2)(4180)(90-60) = 2.508 \times 10^5 \text{ W}$$

$$\Delta T_m = \frac{40-50}{\ln\left(\frac{40}{50}\right)} = 44.81°\text{C} \qquad q = UA\Delta T_m$$

$$U = \frac{2.508 \times 10^5}{(20)(44.81)} = 279.8 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

<u>By NTU method</u>

$$\varepsilon = \frac{40}{80} = 0.5 \qquad \frac{C_{\min}}{C_{\max}} = \frac{30}{40} = 0.75$$

$$C_{\max} = C_w = (2)(4180) = 8360 \qquad C_{\min} = C_p = (0.75)(8360) = 6270$$

From Fig. 10–13 $\quad NTU = 0.9 = \dfrac{UA}{C_{\min}}$

$$U = \frac{(0.9)(6270)}{20} = 282 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

10–3

<u>Inside Tube</u>

$T = 473$ K $\quad d = 0.025 \quad u_m = 6$ m/sec $\quad \mu = 2.58 \times 10^{-5}$

$$\rho = \frac{2.07 \times 10^5}{(287)(473)} = 1.525 \qquad k = 0.0385 \qquad c_p = 1030 \qquad \text{Pr} = 0.681$$

$$\dot{m} = (1.525)\pi(0.0125)^2(6) = 4.491 \times 10^{-3} \text{ kg/sec}$$

$$\text{Re} = \frac{(1.525)(6)(0.025)}{2.58 \times 10^{-5}} = 8866$$

$$h_i = \frac{0.0385}{0.025}(8866)^{0.8}(0.681)^{0.3} = 45.4 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

Conduction resistance negligible.

Outside Tube

$T_f \approx \dfrac{200+20}{2} = 110°\text{C} = 383\text{ K} \qquad \nu_f = 25.15\times10^{-6} \qquad k_f = 0.0324$

$\Pr_f = 0.69 \qquad d_0 = 0.025 + 0.0016 = 0.0266 \qquad \text{Re} = \dfrac{(12)(0.0266)}{25.15\times10^{-6}} = 12{,}691$

$c = 0.193 \qquad n = 0.618 \qquad$ from Chap. 6

$$h_0 = \frac{(0.0324)(0.193)}{0.0266}(12{,}691)^{0.618}(0.69)^{1/3} = 71.36\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

Base U on A_i

$$U_i = \frac{1}{\frac{1}{45.4} + \frac{0.025}{(0.0266)(71.36)}} = 28.41\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}} \qquad A_i = \pi(0.025)(3) = 0.236\text{ m}^2$$

$C_{min} = (4.491\times10^{-3})(1030) = 4.626 \qquad \dfrac{C_{min}}{C_{max}} = 0$

$NTU = \dfrac{(28.41)(0.236)}{4.626} = 1.45 \qquad \varepsilon = 0.78$

$\Delta T_i = (0.78)(200-20) = 140.4°\text{C} \qquad T_{exit} = 200 - 140.4 = 59.6°\text{C}$

If reduce air flow in half, $NTU = (2)(1.45) = 2.90 \qquad \varepsilon = 0.95$

$\Delta T_i = (0.95)(200-20) = 171 \qquad T_{exit} = 200 - 171 = 29°\text{C}$

10–5

Water in tube

$u_m = 4\text{ m/s} \qquad T_b = 90°\text{C} \qquad d = 2.5\text{ cm} \qquad \rho = 965 \qquad \mu = 3.15\times10^{-4}$

$k = 0.676 \qquad \Pr = 1.98$

$$\text{Re}_d = \frac{(965)(4)(0.025)}{3.15\times10^{-4}} = 3.06\times10^5$$

$$h_i = \frac{0.676}{0.025}(0.023)(3.06\times10^5)^{0.8}(1.98)^{0.4} = 20{,}000\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

Oil across tube

$T = 20°\text{C} \qquad u = 7\text{ m/s} \qquad T_f \approx 40°\text{C} \qquad d = 0.0266\text{ m} \qquad \rho = 876$

$\nu = 0.00024\ \text{m}^2/\text{s} \qquad \Pr = 2870 \qquad k = 0.144 \qquad \text{Re}_d = \dfrac{(7)(0.0266)}{0.00024} = 776$

$C = 0.683 \qquad n = 0.466 \qquad \text{Nu} = C\,\text{Re}^n \Pr^{1/3}$

$$h_o = \frac{0.144}{0.0266}(0.683)(776)^{0.466}(2870)^{1/3} = 1167\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\underline{R-\text{conduction}} = \frac{\ln\left(\frac{2.66}{2.5}\right)}{2\pi(36)} = 2.74\times10^{-4}$$

$$\underline{R-\text{inside}} = \frac{1}{h_iA_i} = \frac{1}{(20{,}000)\pi(0.025)} = 16.36\times10^{-4}$$

$$\underline{R\text{-outside}} = \frac{1}{h_o A_o} = \frac{1}{(1167)\pi(0.0266)} = 0.0103$$

$$\sum R = 0.0112$$

$$\frac{UA}{\text{m length}} = \frac{1}{0.0112} = 89.2 \qquad \frac{A}{\text{m length}} = \pi(0.0266) = 0.0836$$

$$U_0 = \frac{89.2}{0.0836} = 1067 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

10–6

Water at 90°C: $\rho = 965$ $\quad \mu = 3.16 \times 10^{-4}$ $\quad k = 0.676$ $\quad \Pr = 1.96$

$$\text{Re} = \frac{(965)(4)(0.025)}{3.16 \times 10^{-4}} = 3.05 \times 10^5$$

$$h_i = \frac{0.676}{0.025}(0.023)(3.05 \times 10^5)^{0.8}(1.96)^{0.3} = 18{,}590 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

Engine oil at 20°C: $\nu = 0.0009$ $\quad k = 0.145$ $\quad \Pr = 10{,}400$

$$D_H = 0.0375 - 0.0266 = 0.0109 \text{ m} \qquad \text{Re} = \frac{(7)(0.0109)}{0.0009} = 84.78$$

$\text{Re}\Pr = (84.78)(10{,}400) = 8.82 \times 10^5$ $\quad h_o$ is smaller compared to h_i so approximately h_o can be obtained from const. Temp. eq.

$$\text{Re}\Pr\frac{d}{l} = \frac{(8.82 \times 10^5)(0.0109)}{6} = 1602.3 \qquad \text{at } 90°\text{C} \qquad \nu_w = 0.289 \times 10^{-4}$$

$$h_o = \frac{0.145}{0.0109}(1.86)(1602.3)^{1/3}\left(\frac{9}{0.289}\right)^{0.14} = 468.6 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

Based on A_i: $$U_i = \frac{1}{\frac{1}{18{,}590} + \frac{0.025}{(0.0266)(468.6)}} = 485.6 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

10–7

$d_o = 1.315$ in. $\quad d_i = 0.957$ in. $\quad k = 43 \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

For 1 m length: $\dfrac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} = 1.176 \times 10^{-3}$

$$\frac{1}{h_o A_o} = \frac{1}{(180)\pi(1.315)(0.0254)} = 0.05294$$

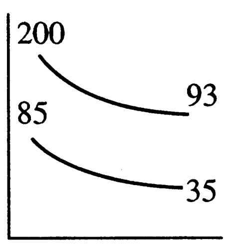
$$\frac{1}{h_i A_i} = \frac{1}{(65)\pi(0.957)(0.0254)} = 0.20146$$

$$UA = \frac{1}{0.05294 + 0.20146 + 1.176 \times 10^{-3}} = 3.9127$$

$$U_i = \frac{3.9127}{\pi(0.957)(0.0254)} = 51.24 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

10–8

(a)



$$\Delta T_m = \frac{(200-85)-(93-35)}{\ln\left(\frac{115}{58}\right)} = 83.3^\circ\text{C}$$

$p = 0.303 \qquad R = \dfrac{200-93}{85-35} = 2.14 \qquad F = 0.92$

$q = (2.5)(4175)(85-35) = (180)A(0.92)(83.3) = 521.8$ kW

$A = 37.8 \text{ m}^2$

(b) $m_g c_g(200-93) = (2.5)(4175)(85-35) \qquad m_g c_g = 4877 = C_{\min}$

$\dfrac{C_{\min}}{C_{\max}} = \dfrac{4877}{10{,}437} = 0.467 \qquad \varepsilon = \dfrac{200-93}{200-35} = 0.648 \qquad \dfrac{AU}{C_{\min}} = 1.4$

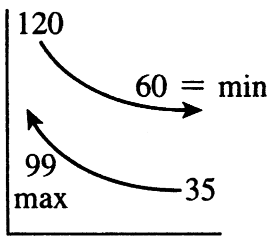
$$A = \frac{(1.4)(4877)}{180} = 37.9 \text{ m}^2$$

10–11

$m_w = 230$ kg/hr $\quad T_{in} = 35°C \quad A = 1.4\ m^2 \quad T_{0\ inlet} = 120°C$

$c_o = 2100\ \dfrac{J}{kg \cdot °C}$

$T_w(\text{exit max}) = 99°C \quad T_o(\text{exit min}) = 60°C \quad U = 280\ \dfrac{W}{m^2 \cdot °C}$



$$q_{max} = m_w c_w \Delta T_w = \frac{230}{3600}(4180)(99-35) = 17{,}090\ W$$

$$C_w = \frac{230}{3600}(4180) = 267$$

$\Delta T\ (\text{max, oil}) = 120 - 60 = 60 \qquad \Delta T\ (\text{max, water}) = 99 - 35 = 64$

Water is potentially minimum fluid, so that $NTU = \dfrac{UA}{C_{min}} = \dfrac{(280)(1.4)}{267} = 1.47$

$$\varepsilon = \frac{64}{120-35} = 0.753$$

For counterflow, double-pipe this can be accomplished with $\dfrac{C_{min}}{C_{max}} \approx 0.2$

$$C_o = \frac{C_w}{0.2} = 1335 = \dot{m}_o c_o = m_o(2100)$$

$m_o = 0.636$ kg/s $= 2290$ kg/hr

$T_{o,\ exit} = 120 - \dfrac{17{,}090}{1335} = 107.2°C$ which is greater than the allowed value of 60°C

10-12

$q = (2500)(2202)(1000)/3600 = 1.53$ MW

$\Delta T_m = (90\text{-}80)/\ln(90/80) = 84.9°C$

$A = (1.53\times10^6)/(84.9)(47) = 383\ m^2$

10-13

$C_{air} = 1.53\times10^6/(40\text{-}30) = 153000$
New $C_{air} = (0.6)(153000) = 91800$

$C_{min}/C_{max} = 0$; $NTU = (47)(383)/91800 = 0.196$

$\varepsilon = 1 - \exp(-0.196) = 0.178$

$q = (0.178)(120\text{-}30)(91800) = 1.49$ MW

Only 2.6 % drop in condensation rate for 40% drop in air flow rate.

10-14

$A = 4.64\ m^2 \qquad U = 280 \qquad \dot{m}_a = 0.45\ \text{kg/sec} \qquad c_a = 1009$

$c_w = 4175 \qquad q = (m_c)_w(93-20) = (0.45)(1009)(260 - T_e)$

$$q = (280)(4.64)\frac{(260-93)-(T_e-20)}{\ln\left(\frac{260-93}{T_e-20}\right)} \qquad \text{By iteration: } T_e = 45.4°C$$

$$\dot{m}_w = \frac{(0.45)(1009)(260-45.4)}{(4175)(93-20)} = 0.32\ \text{kg/sec}$$

10-15

$C_w = (5)(4180) = 20{,}900 \qquad C_o = (8)(2100) = 16{,}800 \qquad \dfrac{C_{\min}}{C_{\max}} = 0.8038$

$$N = \frac{UA}{C_{\min}} = \frac{(120)(100)}{16{,}800} = 0.714$$

From Fig. 10–13 $\qquad \varepsilon = 0.41 = \dfrac{\Delta T_o}{\Delta T_{\max\ \text{Hx}}}$

$\Delta T_o = (0.41)(100 - 50) = 20.5°\text{C} \qquad T_o\ (\text{exit}) = 100 - 20.5 = 79.5°\text{C}$

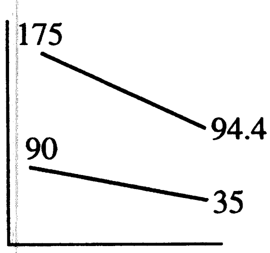
$q = C_o \Delta T_o = (16{,}800)(20.5) = 3.44 \times 10^5\ \text{W}$

10-16-

$q = (0.7)(4175)(90 - 35) = (0.95)(2100)(175 - T_e) = 160{,}740\ \text{W}$

$T_{e_o} = 94.4°\text{C} \qquad \Delta T_m = \dfrac{85 - 59.4}{\ln\left(\frac{85}{59.4}\right)} = 71.44 \qquad A = \dfrac{160{,}740}{(425)(71.44)} = 5.294\ \text{m}^2$

Oil is min fluid: $\varepsilon = \dfrac{175 - 94.4}{175 - 35} = 0.575$



10-18

Glycol $\qquad c_p = 2742 \qquad q = \dfrac{4500}{3600}(2742)(140 - 80) = 205{,}650\ \text{W}$

$q = \dot{m}_w(4175)(85 - 35) \qquad \dot{m}_w = 0.985\ \text{kg/sec} \qquad U = 850\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$C_w = 4112 \qquad C_g = \dfrac{(4500)(2742)}{3600} = 3428 \qquad \dfrac{C_{\min}}{C_{\max}} = 0.834$

$\varepsilon = \dfrac{140 - 80}{140 - 35} = 0.571 \qquad \left(\dfrac{U}{C_{\min}}\right)A = 1.3 \qquad A = 5.24\ \text{m}^2$

10-19

$\frac{UA}{C_{\min}} = 2.6 \qquad \frac{C_{\min}}{C_{\max}} = 0.417 \qquad \varepsilon = 0.82$

$\Delta T_g = (0.82)(140 - 35) = 86.1°\text{C} \qquad q = (86.1)\left(\frac{3428}{2}\right) = 147{,}575\ \text{W}$

reduction of 28.2% in q. $\quad \Delta T_w = \frac{147{,}575}{4112} = 35.9°\text{C} \qquad T_{w_e} = 70.9°\text{C}$

10-20

$C_{\min} = C_g = 4877 \qquad C_{\max} = C_w = (10{,}437)(0.7) = 7306 \qquad \frac{AU}{C_{\min}} = 1.4$

$\frac{C_{\min}}{C_{\max}} = \frac{4877}{7306} = 0.667 \qquad \varepsilon = 0.62 = \frac{\Delta T_g}{200 - 35} \qquad \Delta T_g = 102.3°\text{C}$

$q = (4877)(102.3) = 498.9\ \text{kW}$ $\quad$ 4.5% reduction

10-21

$\Delta T_m = 83.3°\text{C}$

(a) $P = 0.303 \qquad R = 2.14 \qquad F = 0.86$

$521.8 \times 10^3 = (180)A(0.86)(83.3) \qquad A = 40.5\ \text{m}^2$

(b) $\frac{C_{\min}}{C_{\max}} = 0.467 \qquad \varepsilon = 0.648 \qquad \frac{UA}{C_{\min}} = 1.55$

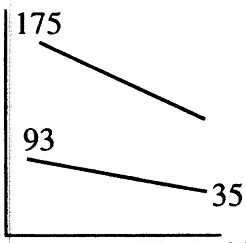
$A = \frac{(1.5)(4877)}{180} = 40.6\ \text{m}^2$

10-22

$\frac{AU}{C_{\min}} = 1.5 \qquad \frac{C_{\min}}{C_{\max}} = 0.934 \qquad \varepsilon = 0.53 \qquad \Delta T_g = 87.45$

$q = (4877)(87.45) = 426.5\ \text{kW} \qquad \%\ \text{reduction} = \frac{521.8 - 426.5}{521.8} = 18.3\%$

10-23



$m_o c_o = \dfrac{(230)(2100)}{3600} = 134.17$ $\qquad$ $m_w c_w = \dfrac{(230)(4175)}{3600} = 266.7$

$\Delta T_o = (93-35)\left(\dfrac{266.7}{134.17}\right) = 115.3$ $\qquad$ $T_{o\ \text{out}} = 175 - 115.3 = 59.7°\text{C}$

$\Delta T_m = \dfrac{82-24.7}{\ln\left(\frac{82}{24.7}\right)} = 47.75°\text{C}$ $\qquad$ $q = (266.7)(93-35) = 15{,}469\ \text{W}$

Ex. 1 $\quad A = \dfrac{15{,}469}{(570)(47.75)} = 0.568\ \text{m}^2$ $\qquad$ not large enough

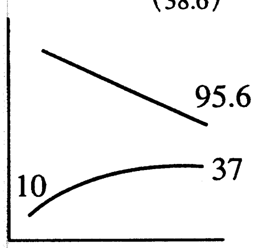
Ex. 2 $\quad A = \dfrac{15{,}469}{(370)(47.75)} = 0.876\ \text{m}^2$ $\qquad$ $0.94\ \text{m}^2$ is large enough

10-24

$p = 83\ \text{kN/m}^2 = 12\ \text{psia}$ $\qquad$ $T_{\text{sat}} = 204°\text{F} = 95.6°\text{C}$

$h_{fg} = 976\ \text{Btu/lbm} = 2.27\times10^6\ \text{J/kg}$ $\qquad$ $q = (2.27\times10^6)\left(\dfrac{0.76}{60}\right) = 28{,}753\ \text{W}$

$\Delta T_m = \dfrac{85.6-38.6}{\ln\left(\frac{85.6}{38.6}\right)} = 59°\text{C}$ $\qquad$ $A = \dfrac{q}{U\Delta T_m} = \dfrac{28{,}753}{(3400)(59)} = 0.143\ \text{m}^2$



10-25

$$28{,}753 = (3400)(0.143)\Delta T_m \qquad \Delta T_m = 59°\text{C}$$

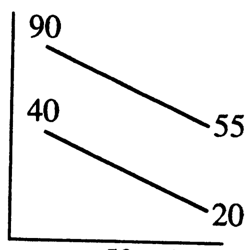
$$\Delta T_m = \frac{(95.6-30)-(95.6-T_{we})}{\ln\left(\frac{65.6}{95.6-T_{we}}\right)} = 59 \qquad \text{By iteration } T_{we} = 42.6°\text{C}$$

$$28{,}753 = q = \dot{m}_w(4175)(42.6-30) \qquad \dot{m}_w = 0.5466 \text{ kg/sec}$$

$$\dot{m}_w = \frac{28{,}753}{(4175)(57-10)} = 0.1465 \text{ kg/sec}$$

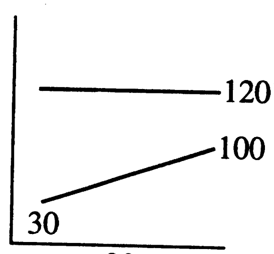
$$\%\text{ increase} = \frac{0.5466-0.1465}{0.1465} = 273\%$$

10-26



$$\Delta T_m = \frac{50-35}{\ln\left(\frac{50}{35}\right)} = 42.06°\text{C} \qquad q = UA\Delta T_m \qquad A = \frac{59{,}000}{(340)(42.06)} = 4.11 \text{ m}^2$$

10-27



$$\Delta T_m = \frac{90-20}{\ln\left(\frac{90}{20}\right)} = 46.54°\text{C}$$

$$q = m_w c_w \Delta T_w = (2.5)(4180)(100-30) = 7.315\times10^5 \text{ W}$$

$$A = \frac{q}{U\Delta T_m} = \frac{7.315\times10^5}{(2000)(46.54)} = 7.86 \text{ m}^2$$

10-28

q = (120-95)(1900)(3700)/3600 = 0.488 MW

C_{water} = 488000/30 = 16300; C_{oil} = 488000/25 = 19520

C_{min}/C_{max} = 25/30 = 0.833; ε = 30/(120-20) = 0.3

Fig. 10-15: NTU = 0.45

A = (0.45)(16300)/55 = 133 m^2

10-29

$\Delta T_{water} = 10°C$; $C_{water} = (3)(16300) = 48900$

$C_{oil} = C_{min} = 19520$; $C_{min}/C_{max} = 0.4$

$NTU = (55)(133)/19520 = 0.375$

Fig. 10-15: $\varepsilon \approx 0.3$

$q = (0.3)(120\text{-}40)(195200 = 0.468$ MW

Heat transfer has only dropped by 4.1 percent so no adjustments may be necessary

10-30

$$0.0002 = \frac{1}{U} - \frac{1}{2000} \qquad U = 143\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$C_w = C_{\min} = (2.5)(4180) = 10{,}450 \qquad \text{NTU} = \frac{UA}{C_{\min}} = \frac{(143)(7.86)}{10{,}450} = 0.1076$$

$$\varepsilon = 1 - e^{-0.1076} = 0.1020 = \frac{\Delta T_w}{120 - 30} \qquad \Delta T_w = 9.18°\text{C}$$

$$T_{w\,\text{exit}} = 30 + 9.18 = 39.18°\text{C}$$

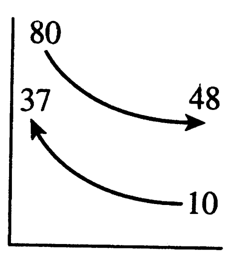
10-31

$$C = 1.0 \qquad C_{\min} = C_{\max} = (0.5)(1006) = 503 \qquad n = 0.903$$

$$N = \frac{(40)(15)}{503} = 1.19 \qquad \varepsilon = 0.50 \qquad \Delta T = (0.50)(400 - 20) = 190°\text{C}$$

$$T_{ce} = 190 + 20 = 210°\text{C} \qquad T_{he} = 400 - 190 = 210°\text{C}$$

10-32



$$c_a = 1005 \qquad \rho_a = \frac{1.013 \times 10^5}{(287)(283)} = 1.247\ \text{kg/m}^3$$

$$\dot{m}_A = (1500)(1.247) = 1871\ \text{kg/min} = 31.19\ \text{kg/sec}$$

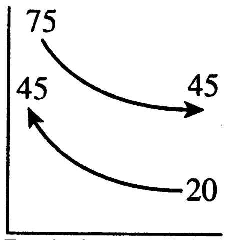
Both fluids unmixed

$$\Delta T_m = \frac{43 - 38}{\ln\left(\frac{43}{38}\right)} = 40.45°\text{C} \qquad R = \frac{80 - 48}{37 - 10} = 1.18 \qquad P = \frac{37 - 10}{80 - 10} = 0.385$$

$$F = 0.96 \qquad q = m_a c_a \Delta T_a = (31.19)(1005)(37 - 10) = 8.46 \times 10^5\ \text{W}$$

$$A = \frac{q}{UF\Delta T_m} = \frac{8.46 \times 10^5}{(50)(0.96)(40.45)} = 435.7\ \text{m}^2$$

10-33



Both fluids unmixed

$$\Delta T_m = \frac{30-25}{\ln\left(\frac{30}{25}\right)} = 27.42°\text{C} \qquad R = \frac{75-45}{45-20} = 1.2 \qquad P = \frac{45-20}{75-20} = 0.455$$

$$F = 0.91 \qquad A = \frac{q}{UF\Delta T_m} = \frac{35{,}000}{(50)(0.91)(27.42)} = 28.05 \text{ m}^2$$

10-34

$$c_o = 1920 \qquad c_w = 4180 \qquad C_o = \frac{(95)(1920)}{60} = 3040 \qquad \text{(min)}$$

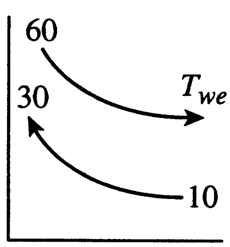
$$C_w = \frac{(55)(4180)}{60} = 3832 \qquad \frac{C_{\min}}{C_{\max}} = 0.793 \qquad NTU = \frac{(250)(14)}{3040} = 1.151$$

$$\varepsilon = 0.5 = \frac{\Delta T_o}{120-30} \qquad \Delta T_o = 45°\text{C} \qquad T_{oe} = 120-45 = 75°\text{C}$$

$$q = C_o \Delta T_o = (3040)(45) = 1.368 \times 10^5 \text{ W}$$

$$\Delta T_w = 45\left(\frac{3040}{3832}\right) = 35.7°\text{C} \qquad T_{we} = 30 + 35.7 = 65.7°\text{C}$$

10-35



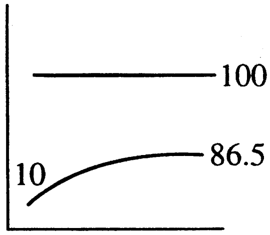
$$C_w = (5)(4180) = 20{,}900 \qquad C_a = 4800$$

$$q = 20{,}900(60 - T_{we}) = (4800)\dot{m}_a(30-10)$$

$$20{,}900(60 - T_{we}) = UA\Delta T_m = (800)(30)\frac{30 - T_{we} + 10}{\ln\left(\frac{30}{T_{we}-10}\right)}$$

Solve by iteration: $T_{we} = 31°\text{C}$ $\qquad \dot{m}_a = \dfrac{(20{,}900)(60-31)}{(4800)(20)} = 6.31$ kg/sec

10-36



$U = 150 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad A = 30 \text{ m}^2$ $\quad \varepsilon = \dfrac{\Delta T_a}{100 - 10} = 0.85$

$\Delta T_a = 76.5°\text{C}$ $\quad T_{a_e} = 76.5 + 10 = 86.5°\text{C}$ $\quad \Delta T_m = \dfrac{90 - 13.5}{\ln\left(\frac{90}{13.5}\right)} = 40.32°\text{C}$

$q = UA\Delta T_m = (150)(30)(40.32) = 1.81 \times 10^5 \text{ W}$

10-37

$N = (2)(1.59) = 3.18$ $\quad n = 0.775$ $\quad \varepsilon = 0.693$ $\quad T_{ce} = 283.3°\text{C}$

$T_{he} = 136.7°\text{C}$ $\quad \Delta T = (0.693)(400 - 20) = 263.3$

$N = \dfrac{1}{2}(1.59) = 0.795$ $\quad n = 1.052$ $\quad \varepsilon = 0.416$ $\quad \Delta T = 158.2$

$T_{ce} = 178.2°\text{C}$ $\quad T_{he} = 241.8°\text{C}$

10-38

$C_a = \dfrac{65}{60}(1006) = 1090 \text{ W/°C}$ $\quad q = (1090)(45 - 30) = 16{,}350 \text{ W}$

$U = 52$ $\quad A = 8.0$ $\quad T_{w_1} = 90°\text{C}$ $\quad$ Assume air min. fluid

$\varepsilon = \dfrac{45 - 30}{90 - 30} = 0.25$ $\quad N = \dfrac{(52)(8)}{1090} = 0.382$

Unable to match on graph so C_w is min. Iterative: $C = \dfrac{C_w}{C_a}$

C	N	n	ε_c	ΔT_w	$\Delta T_a = C\Delta T_w$	$\Delta T_a - 15$
0.6	0.636	1.104	0.405	24.3	14.6	–0.42
0.48	0.795	1.052	0.48	28.8	13.82	–1.17
0.50	0.764		0.4664	27.98	13.99	–1.008
0.7	0.546	1.143	0.3574	21.44	15.01	+0.01

$T_{we} = 90 - 21.44 = 68.6°\text{C}$

10-39

$C_{min} = (5)(4180) = 20{,}900$ $\quad n = 4 \quad$ $\varepsilon = \dfrac{90-70}{90-50} = 0.5$

$\varepsilon = \dfrac{n\varepsilon_p}{1+(n-1)\varepsilon_p} \quad \varepsilon_p = \dfrac{\varepsilon}{n(1-\varepsilon)+\varepsilon} = \dfrac{0.5}{4(0.5)+0.5} = 0.20 \quad C = 1.0$

$$N = -(1+1)^{-1/2} \times \ln\left[\frac{10-1-1-(2)^{1/2}}{10-1-1+(2)^{1/2}}\right] = 0.253 = \frac{UA}{20{,}900}$$

$A = \dfrac{(20{,}900)(0.253)}{800} = 6.6\ \mathrm{m}^2 \quad A_{Total} = (4)(6.6) = 26.4\ \mathrm{m}^2$

10-41

$\varepsilon = (120-90)/(120-30) = 0.333$

Same for parallel and counterflow

10-42

$C_{min}/C_{max} = 10/30 = 0.333$

$\varepsilon = 0.333$

Parallel flow : NTU $\approx$ 0.45

Counterflow: NTU$\approx$ 0.47

10-43

$C_{min}/C_{max} = 0$; $\quad \varepsilon = 10/(10-30) = 0.11111$

10-44

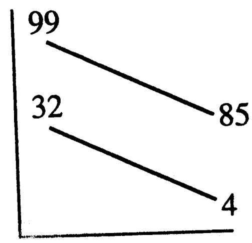
NTU = $-\ln(1 - \varepsilon) = 0.1178$ for both cases

10-45

$m_h = 2.6 \text{ kg/s} \qquad m_c = 1.3 \text{ kg/s} \qquad \Delta T_h = \frac{1}{2}\Delta T_c = \frac{1}{2}(32-4) = 14$

$$\Delta T_m = \frac{67-81}{\ln\left(\frac{67}{81}\right)} = 73.78 \qquad q = (1.3)(4175)(32-4) = (830)A(73.78)$$

$$A = 2.48 \text{ m}^2 \qquad \varepsilon = \frac{32-4}{99-4} = 29.5\%$$



10-46

$$\varepsilon_c = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_1}} \qquad dq = UdA(T_h - T_c) = \dot{m}_c c_c dT_c$$

$$\frac{dT_c}{T_h - T_c} = \frac{UdA}{\dot{m}_c c_c} \qquad -\ln(T_h - T_c)\Big|_1^2 = \frac{UA}{\dot{m}_c c_c}\Bigg|_0^A \qquad \frac{T_h - T_{c_2}}{T_h - T_{c_1}} = e^{-\frac{UA}{\dot{m}_c c_c}}$$

$$1 - \frac{T_{c_2} - T_{c_1}}{T_h - T_{c_1}} = e^{-\frac{UA}{\dot{m}_c c_c}} \qquad \varepsilon = \frac{T_{c_2} - T_{c_1}}{T_h - T_c} = 1 - e^{-\frac{UA}{\dot{m}_c c_c}}$$

10-47

$\Delta T_w = 75 - 30 = 45°\text{C} \qquad \Delta T_o = 48 - 25 = 23°\text{C}$

Water is minimum fluid. $\qquad \varepsilon = \dfrac{\Delta T_w}{\Delta T_{max}} = \dfrac{45}{75-25} = 0.9$

10-49

$$0.004 = \frac{1}{U} - \frac{1}{340} \qquad U = 144 \qquad A = (2.45)\left(\frac{340}{144}\right) = 5.78 \text{ m}^2$$

with original area, oil is min fluid $\quad c = 1900$

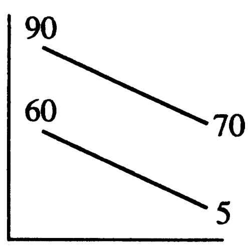
$$C_o = \frac{29{,}000}{90-55} = 828.6 \qquad C_w = \frac{29{,}000}{50-25} = 1160 \qquad \frac{C_{\min}}{C_{\max}} = 0.714$$

$$NTU = \frac{(144)(2.45)}{828.6} = 0.426 \qquad \varepsilon = 0.29 = \frac{\Delta T_o}{90-25}$$

$\Delta T_o = 18.85°\text{C} \qquad q = (828.6)(18.85) = 15{,}619 \text{ W}$

reduced by 46%

10-50



$q = 60 \text{ kW} \qquad U = 1100 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

Cold fluid is minimum fluid $\quad \varepsilon = \dfrac{60-5}{90-5} = 0.647 \qquad \dfrac{C_{\min}}{C_{\max}} = \dfrac{20}{55} = 0.364$

$$NTU = 1.5 \qquad C_{\min} = \frac{60{,}000}{60-5} = 1091 \qquad A = \frac{(1091)(1.5)}{1100} = 1.49 \text{ m}^2$$

10-51

$$NTU = 1.9 \qquad \frac{C_{\min}}{C_{\max}} = (2)(0.364) = 0.728 \qquad \varepsilon = 0.69 = \frac{\Delta T_{\min}}{80-5}$$

$\Delta T_{\min} = 51.75°\text{C} \qquad q = (51.75)(1091) = 56{,}460 \text{ W} \qquad$ only reduced by 6%

10-52

$$29{,}000 = C_a(25-15) = C_w(70-40) \qquad C_w = 966.7$$

$$\frac{C_w}{C_a} = \frac{C_{\min}}{C_{\max}} = \frac{10}{30} = 0.333 \qquad \varepsilon = \frac{30}{70-15} = 0.545 \qquad \frac{UA}{C_{\min}} = 1.0$$

$$A = \frac{(1.0)(966.7)}{45} = 21.5 \text{ m}^2$$

10-53

$$C_w = \frac{9667}{3} = 322.2 \qquad \frac{C_{\min}}{C_{\max}} = \frac{0.333}{3} = 0.111$$

$$NTU = \frac{UA}{C_{\min}} = \frac{(45)(21.5)}{322.2} = 3.0 \qquad \varepsilon = 0.92 = \frac{\Delta T_w}{70-15}$$

$$\Delta T_w = 50.6°\text{C} \qquad q = C_w \Delta T_w = 16{,}303 \text{ W}$$

10-54

For parallel flow assume $\dot{m}_c = \dot{m}_h$

$$q = \dot{m}_h c_{p_h}(T_{h_1} - T_{h_2}) = \dot{m}_c c_{p_c}(T_{c_1} - T_{c_2}) \qquad c_{p_h} = c_{p_c} \qquad \dot{m}_h = \dot{m}_c$$

$$\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \exp UA\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_h c_h}\right) = \exp\left[-\frac{UA}{\dot{m}_c c_{p_c}}\left(1 + \frac{\dot{m}_c c_{p_c}}{\dot{m}_h c_{p_h}}\right)\right]$$

$$\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \exp\left(-2\frac{UA}{\dot{m}_c c_{p_c}}\right) \qquad \text{consider cold fluid: } \varepsilon = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_1}}$$

$$\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \frac{T_{h_1} - 2T_{c_2} + T_{c_1}}{T_{h_1} - T_{c_1}} \qquad T_{h_2} = T_{h_1} + (T_{c_1} - T_{c_2})$$

$$\frac{(T_{h_1} - T_{c_1}) + (T_{c_1} - T_{c_2}) + (T_{c_1} - T_{c_2})}{T_{h_1} - T_{c_1}} = 1 - 2\varepsilon \qquad \varepsilon = \frac{1 - e^{-2UA}}{2}$$

For counter flow: $\dot{m}_h c_h dT_h = \dot{m}_c c_c dT_c = dq \qquad dq = UdA(T_h - T_c)$

$q = UA(T_h - T_c)$

$$\varepsilon = \frac{q_{\text{act}}}{q_{\max}} = \frac{UA(T_h - T_c)}{\dot{m}c(T_{h_1} - T_{h_2})} = NTU\left(\frac{T_{h_1} - T_{c_1}}{T_{h_1} - T_{c_2}}\right) = NTU\left(1 + \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_2}}\right) = NTU(1-\varepsilon)$$

$$= \frac{NTU}{1 + NTU}$$

10-55

$$C_w(90-55) = C_o(50-25) \qquad \frac{C_w}{C_o} = \frac{25}{35} \qquad C_w = C_{\min} \qquad \varepsilon = \frac{90-55}{90-25} = 53.8\%$$

10-56

$C_g = (3428)(0.6) = 2056.8 \qquad \dfrac{C_{\min}}{C_{\max}} = \dfrac{2057}{4112} = 0.500$

$\dfrac{UA}{C_{\min}} = \dfrac{(850)(5.24)}{2057} = 2.17 \qquad \varepsilon = 0.77$

$q = (2057)(0.77)(140 - T_{w_i}) = (4112)(85 - T_{w_i}) \qquad T_{w_i} = 50.54°\text{C}$

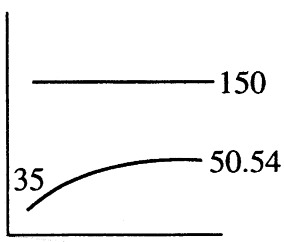
Preheater heats water from 35°C to 50.54°C

$q = (4112)(50.54 - 35) = 63{,}900 \text{ W}$

$\Delta T_m = \dfrac{115 - 99.5}{\ln\left(\frac{115}{99.5}\right)} = 107°\text{C} \qquad A = \dfrac{63{,}900}{(2000)(107)} = 0.299 \text{ m}^2$

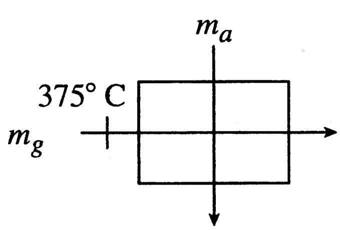
At $T = 150°\text{C} = 302°\text{F}$ $\qquad h_{fg} = 910 \text{ Btu/lbm}$

$h_{fg} = 2.117 \times 10^6 \text{ J/kg} \qquad m_s = \dfrac{63{,}900}{2.117 \times 10^6} = 0.03 \text{ kg/sec}$



10-58

$m_a = 5.0 \text{ kg/sec} \qquad A = 110 \text{ m}^2 \qquad U = 50 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \qquad m_g = 5 \text{ kg/s}$



(a) Both unmixed $\qquad \dfrac{C_{\min}}{C_{\max}} = 1.0 \qquad NTU = \dfrac{(110)(50)}{(5)(1005)} = 1.095$

$\varepsilon = 0.49 \qquad \Delta T = (0.49)(375 - 27) = 170.5$

$T_{a_2} = 170.5 + 27 = 197.5°\text{C} \qquad T_{g_2} = 375 - 170.5 = 204.5°\text{C}$

$q = (5025)(170.5) = 8.57 \times 10^5 \text{ W}$

(b) $\varepsilon = 0.48$ $\qquad$ one fluid mixed $\qquad$ about same results

10-59

$m_w = 30$ kg/s $\quad C_w = (30)(4174) = 125{,}220 \quad T_{w_1} = 20°C$

$T_{w_2} = 40°C \quad T_{o_i} = 200°C \quad U = 275$

$q = (125{,}220)(40 - 24) = 2.504 \times 10^6$ W $\quad N = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon - 1}{c\varepsilon - 1}\right)$

T_{be}	ΔT_o	C_o	C	ε	NTU
190	10	250,440	0.5	0.1111	0.1212
180	20	125,220	1.0	0.1111	0.125
140	60	41,740	0.333	0.333	0.432
80	120	20,870	0.167	0.667	1.177

10-60

$m_w = 50$ kg/s $\quad T_{w_1} = 60°C \quad T_{w_2} = 90°C \quad U = 4500$

$T_s = 200°C \quad C_w = (50)(4174) = 208{,}700 \quad C = 0$

$\varepsilon = \dfrac{30}{200 - 60} = 0.2143 \quad N = -\ln(1 - 0.2143) = 0.2412$

$A = \dfrac{(0.2412)(208{,}700)}{4500} = 11.18 \text{ m}^2 \quad \varepsilon = \dfrac{T_{w_2} - 60}{T_s - 60} = 0.2143$

$T_{w_2} = 0.2143T_s + 47.1$

10-61

(a) $q = (5.0)(4180)(80 - 30) = 1.045 \times 10^6 \quad \varepsilon = \dfrac{80 - 30}{100 - 30} = 0.7143$

$N = -\ln(1 - \varepsilon) = 1.253 \quad A = \dfrac{(1.253)(5)(4180)}{900} = 29.09 \text{ m}^2$

(b) $N = \dfrac{(900)(29.09)}{(1.3)(4180)} = 4.818 \quad \varepsilon = 1 - e^{-N} = 0.992$

$\Delta T_w = (0.992)(100 - 30) = 69.4°C \quad T_{we} = 69.4 + 30 = 99.4°C$

10-62

$C_w = (0.48)(4180) = 2006 \quad C_a = (1.0)(1005) = 1005$

$C = \dfrac{1005}{2006} = 0.5 \quad NTU = \dfrac{(50)(20)}{1005} = 1.0 \quad \varepsilon = 0.55$

$\Delta T_a = (0.55)(80 - 10) = 38.5°C \quad T_{a\,\text{exit}} = 10 + 38.5 = 48.5°C$

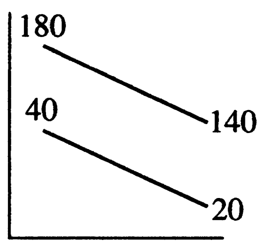
$\Delta T_w = (0.5)(38.5) = 19.25°C \quad T_{w\,\text{exit}} = 80 - 19.25 = 60.75°C$

10-63

$$\Delta T_m = \frac{60-40}{\ln\left(\frac{60}{40}\right)} = 49.33°\text{C} \qquad q = 600{,}000 = UA\Delta T_m = (300)A(49.33)$$

$$A = 40.55\ \text{m}^2 \qquad \varepsilon = \frac{160-90}{160-50} = \frac{70}{110} = 0.636$$

10-64



$$U = 130\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}} \qquad C_o = 2100\ \frac{\text{J}}{\text{kg}\cdot°\text{C}} \qquad \dot{m}_w = 3.0\ \text{kg/sec}$$

$$C_w = (3)(4180) = 12{,}540 \qquad q = C_w\Delta T_w = C_o\Delta T_o = 2.508\times10^5\ \text{W}$$

$$C_o = (12{,}540)\left(\frac{40-20}{180-140}\right) = 6270 \qquad \frac{C_{\min}}{C_{\max}} = 0.5 \qquad \varepsilon = \frac{180-140}{180-120} = 0.25$$

$$NTU = 0.35 \qquad A = \frac{(0.35)(6270)}{130} = 16.88\ \text{m}^2$$

if water flow cut in half $C_w = 6270$

$q = (2.508\times10^5)(0.5) = 1.254\times10^5$ W

Oil flow will be reduced so it will still be min. fluid.

$$q = c_o(180 - T_{o_2}) = (130)(16.88)\frac{(180-40)-(T_{o_2}-20)}{\ln\left(\frac{140}{T_{o_2}-20}\right)}$$

Solve by iteration: $T_{o_2} = 36°\text{C}$ $\quad C_o(180-36) = 1.254\times10^5$ $\quad C_o = 870.8$

Flow is reduced by 86%.

10-65

$\dot{m}_a = 0.8 \text{ kg/sec} \quad T_{a_1} = 30°\text{C} \quad T_{a_2} = 7°\text{C} \quad T_{w_1} = 3°\text{C}$

$U = 55 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \quad \dot{m}_w = 0.75 \text{ kg/sec} \quad C_a = (0.8)(1005) = 804$

$C_w = (0.75)(4180) = 3135 \quad \varepsilon = \dfrac{30-7}{30-3} = 0.85 \quad \dfrac{C_{\min}}{C_{\max}} = \dfrac{804}{3135} = 0.256$

$NTU = 2.4 \quad A = \dfrac{(2.4)(804)}{55} = 35.08 \text{ m}^2$

Water cut in half: $\dfrac{C_{\min}}{C_{\max}} = (2)(0.256) = 0.512 \quad NTU = 2.4$

$\varepsilon = 0.76 = \dfrac{\Delta T_a}{30-3} \quad \Delta T_a = 20.52°\text{C}$

% reduction $= \left(1 - \dfrac{20.52}{23}\right) \times 100 = 10.8\%$

10-66

$\varepsilon = \dfrac{30-7}{30-1.7} = 0.813 \quad NTU = -\ln(1-\varepsilon) = 1.675$

$A = \dfrac{(1.675)(804)}{125} = 10.77 \text{ m}^2$ Reduce air flow by $\dfrac{1}{3}$

$NTU = \dfrac{1.675}{\frac{2}{3}} = 2.512 \quad \varepsilon = 1 - e^{-N} = 0.919 = \dfrac{\Delta T_a}{30-1.7} \quad \Delta T_a = 26°\text{C}$

Initial $q = (804)(30-7) = 18{,}492 \text{ W}$

Reduced $q = (804)\left(\dfrac{2}{3}\right)(26) = 13{,}936 \text{ W}$

Reduced by 24%

10-67

$m_o = 4000 \text{ kg/hr} = 1.111 \text{ kg/sec}$ $\quad C_o = 2000 \dfrac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}$

$C_o = C_{\min} = (2000)(1.111) = 2222$ $\quad \varepsilon = \dfrac{\Delta T_o}{\Delta T_{\max}} = \dfrac{80-40}{100-40} = 0.667$

$N = -\ln(1-\varepsilon) = 1.099 = \dfrac{UA}{C_{\min}}$ $\quad A = \dfrac{(1.099)(2222)}{1200} = 2.034 \text{ m}^2$

When oil flow cut in half:

$NTU = (2)(1.099) = 2.198$ $\quad \varepsilon = 1 - e^{-N} = 0.889 = \dfrac{\Delta T_o}{100-40}$

$\Delta T_o = 53.34°\text{C}$ $\quad q = C_o\Delta T_o = \dfrac{2222}{2}(53.34) = 59{,}260 \text{ W} = \dot{m}_s h_{fg}$

$\dot{m}_s = \dfrac{59{,}260}{2.255\times 10^6} = 0.0262 \text{ kg/sec}$

10-68

$\varepsilon = 0.75 = \dfrac{\Delta T_w}{100-50}$ $\quad \Delta T_w = 37.5°\text{C}$ $\quad N = -\ln(1-\varepsilon) = 1.386$

$A = \dfrac{(1.386)(5)(4180)}{1200} = 24.14 \text{ m}^2$

10-69

Reduce water flow in half $\quad NTU = \dfrac{1.386}{0.5} = 2.772$

$\varepsilon = 1 - e^{-N} = 0.937 = \dfrac{\Delta T_w}{100-50}$ $\quad \Delta T_w = 46.9°\text{C}$

$T_{w_e} = 46.9 + 50 = 96.9°\text{C}$ $\quad q = C_w\Delta T_w = (5)(0.5)(4180)(46.9) = 4.901\times 10^5 \text{ W}$

10-70

$\Delta T_w = 80 - 52 = 28°\text{C}$ $\quad \Delta T_a = 40 - 7 = 33°\text{C}$

Air is minimum fluid: $\quad \varepsilon = \dfrac{33}{80-7} = 0.452$

10-71

$q = m_w C_w(28) = \dot{m}_a C_a(33)$ $\quad \dot{m}_w = 150 \text{ kg/min} = 2.5 \text{ kg/sec}$

$q = (2.5)(4180)(28) = 2.926\times 10^5 \text{ W}$ $\quad \dfrac{C_a}{C_w} = \dfrac{28}{33} = 0.848$

$NTU = 0.8 = \dfrac{UA}{C_a}$ $\quad A = \dfrac{(0.8)(2.5)(4180)(0.848)}{50} = 142 \text{ m}^2$

10-72

$\dot{m}_g = 10 \text{ kg/sec}$ $\quad \dot{m}_w = 15 \text{ kg/sec}$ $\quad C_g = 2420$ $\quad C_w = 4180 \dfrac{\text{kJ}}{\text{kg} \cdot °\text{C}}$

$C_g = 24,200$ $\quad C_w = 62,700$ $\quad \dfrac{C_{\min}}{C_{\max}} = 0.386$

Glycol is minimum fluid $\quad \varepsilon = \dfrac{40-20}{70-20} = 0.4$ $\quad NTU = 0.6 = \dfrac{UA}{C_{\min}}$

$$A = \frac{(0.6)(24,200)}{40} = 363 \text{ m}^2$$

10-73

$C_w = (10)(4180) = 41,800$ $\quad U = 35 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad \dfrac{C_{\min}}{C_{\max}} = \dfrac{24,200}{41,800} = 0.579$

$NTU = \dfrac{(35)(363)}{24,200} = 0.525$ $\quad \varepsilon = 0.38 = \dfrac{\Delta T_g}{70-20}$ $\quad \Delta T_g = 19°\text{C}$

$T_g = 20 + 19 = 39°\text{C}$

10-74

$C_w = (4)(4180) = 16,720 = C_{\min} = C_{\max}$ $\quad U = 800$ $\quad n = 3$

$\varepsilon = \dfrac{60-20}{80-20} = 0.66667$ $\quad \dfrac{C_{\min}}{C_{\max}} = 1.0$

From next to last entry of Table 10–3 for $C = 1$ $\quad \varepsilon = \dfrac{n\varepsilon_p}{1-(n-1)\varepsilon_p}$

Solving for ε_p, $\varepsilon_p = \dfrac{\varepsilon}{n-(n-1)\varepsilon} = 0.4$

From Fig. 10–17, at $C = 1$ $\quad \varepsilon = 0.4$ $\quad N_p = 0.8 = \dfrac{UA}{C_w}$

$A_p = \dfrac{(0.8)(16,720)}{800} = 16.72 \text{ m}^2$ $\quad A_{\text{total}} = (3)(16.72) = 50.16 \text{ m}^2$

10-75

Water is minimum fluid

$q = 800 \times 10^6 = C_w(30-25)$ $\quad C_w = 1.6 \times 10^8$ $\quad U = 2000 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$\varepsilon = \dfrac{\Delta T_w}{\Delta T_{\max}} = \dfrac{30-25}{100-25} = 0.06667$ $\quad N = -\ln(1-\varepsilon) = 0.069 = \dfrac{UA}{C_{\min}}$

$$A = \frac{(0.069)(1.6 \times 10^8)}{2000} = 5519 \text{ m}^2$$

10-76

Water flow in half $C_w = 0.8\times10^8$ $NTU = 0.138$

$\varepsilon = 1 - e^{-N} = 0.1289 = \dfrac{\Delta T_w}{100-25}$ $\Delta T_w = 9.67°\text{C}$

$q = C_w \Delta T_w = 7.73\times10^8 \text{ W} = \dot{m}_s h_{fg}$ $h_{fg} = 2.255\times10^6 \text{ J/kg}$

$$\dot{m}_s = \frac{7.73\times10^8}{2.25\times10^6} = 343 \text{ kg/sec} = 1.235\times10^6 \text{ kg/hr}$$

10-77

$T_{g_1} = 25°\text{C}$ $T_{g_2} = 65°\text{C}$ $C_g = 2474$ $\dot{m}_g = 1.2 \text{ kg/s}$ $T_{w_1} = 95°\text{C}$

$T_{w_2} = 55°\text{C}$ $U = 600 \dfrac{\text{W}}{\text{m}^2\cdot°\text{C}}$ $\Delta T_g = \Delta T_w$

$C_g = (1.2)(2474) = 2969 = C_w$ $\dfrac{C_{\min}}{C_{\max}} = 1.0$ $\varepsilon = \dfrac{40}{95-25} = 0.571$

$NTU = 1.4 = \dfrac{UA}{C_{\min}}$ $A = \dfrac{(1.4)(2969)}{600} = 6.93 \text{ m}^2$

10-78

$\rho = 1.1774 \text{ kg/m}^3$ $c_p = 1005 \dfrac{\text{J}}{\text{kg}\cdot°\text{C}}$ $\text{Pr} = 0.708$

$\mu = 1.846\times10^{-5}$ $\sigma = 0.697 = \dfrac{A_c}{A}$ $D_h = 0.0118 \text{ ft} = 0.0036 \text{ m}$

Frontal area $= (0.3)(0.6) = 0.18 \text{ m}^2$ $A_c = (0.697)(0.18) = 0.1254 \text{ m}^2$

$$G = \frac{\rho u_\infty A}{A_c} = \frac{(1.1774)(10)}{0.67} = 17.57 \frac{\text{kg}}{\text{m}^2\cdot\text{s}}$$

$\text{Re}\dfrac{D_h G}{\mu} = \dfrac{(0.0036)(12.57)}{1.846\times10^{-5}} = 3426$ $\text{St}\,\text{Pr}^{2/3} = 0.004 = \dfrac{h}{Gc_p}\text{Pr}^{2/3}$

$$h = (0.004)(17.57)(1005)(0.708)^{-2/3} = 88.09 \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

This coefficient is controlling

Volume $= (0.3)^2(0.6) = 0.054 \text{ m}^3$

Heat Transfer Area $= (0.054)\alpha = (0.054)\left(229 \dfrac{\text{ft}^2}{\text{ft}^3}\times 3.28 \dfrac{\text{ft}}{\text{m}}\right) = 40.56 \text{ m}^2$

$$q = hA\left(100 - \frac{27}{2} - \frac{T_2}{2}\right) = \dot{m}c_p(T_2 - 27)$$

$$(88.9)(40.56)\left(100 - \frac{27}{2} - \frac{T_2}{2}\right) = (1.1774)(10)(0.18)(1005)(T_2 - 27)$$

$T_2 = 93.93°\text{C}$

$q = 1.43 \times 10^5 \text{ W} = \dot{m}_s h_{fg}$ $\qquad\qquad h_{fg} = 2.255 \times 10^6 \text{ J/kg}$

$$\dot{m}_s = \frac{q}{h_{fg}} = 0.63 \text{ kg/s}$$

10-81

$$c_g = 2382 \frac{\text{J}}{\text{kg} \cdot °\text{C}}$$

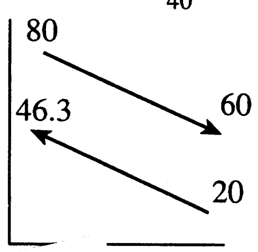
$$q = m_w c_w \Delta T_w = m_g c_g \Delta T_g = (0.6)(4180)(80-60) = (0.8)(2382)\Delta T_g = 50{,}160 \text{ W}$$
$$= UA\Delta T_w$$

$$\Delta T_g = 26.3°\text{C}$$

$$\varepsilon = \frac{26.3}{80-20} = 0.439$$

Assume counterflow

$$\Delta T_m = \frac{33.7-40}{\ln \frac{33.7}{40}} = 36.76°\text{C} \qquad\qquad A = \frac{50{,}160}{(1000)(36.76)} = 1.36 \text{ m}^2$$



10-82

$$q = m_w c_w \Delta T_w = m_o c_o \Delta T_o = (0.6)(4180)(50-20) = (1.2)(2100)(100 - T_{oe})$$
$$= 75{,}240 \text{ W}$$

$\Delta T_o = 29.86$ $\qquad\qquad T_{oe} = 70.14°\text{C}$

Water is min. fluid and mixed
Oil is max fluid and unmixed.

$$\varepsilon = \frac{30}{100-20} = 0.375 \qquad\qquad \frac{C_{\min}}{C_{\max}} = \frac{29.86}{30} = 0.995$$

$$N = \frac{-1}{0.995} \ln[1 + 0.995 \ln(1 - 0.375)] = 0.634 = \frac{UA}{C_{\min}}$$

$$A = \frac{(0.634)(0.6)(4180)}{250} = 0.636 \text{ m}^2$$

10-83

Water is min fluid and unmixed
Oil is max fluid and mixed

$$N = -\ln\left\{1 + \frac{1}{0.995} \ln[1 - 0.995(0.375)]\right\} = 0.469 \qquad\qquad A = 0.470 \text{ m}^2$$

10-84

$q = m_w c_w \Delta T_w = m_o c_o \Delta T_o = (2)(4180)(70-10) = (3)(2100)(120 - T_{o_e})$
$= 501{,}600$ N

$\Delta T_o = 79.62 \qquad T_{oe} = 40.38°\text{C}$

Oil is min fluid.

$C = \dfrac{60}{79.62} = 0.7536 \qquad \varepsilon = \dfrac{79.62}{120-10} = 0.7238 \qquad n = 3$

$$\varepsilon_p = \frac{\left(\frac{1-\varepsilon C}{1-\varepsilon}\right)^{1/n} - 1}{\left(\frac{1-\varepsilon C}{1-\varepsilon}\right)^{1/n} - C} \qquad \text{(See Prob. 10–83)}$$

$$= \frac{\left(\frac{1-(0.7278)(0.7536)}{1-0.7238}\right)^{1/3} - 1}{\left(\frac{1-(0.7278)(0.7536)}{1-0.7238}\right)^{1/3} - C} = \frac{1.1806 - 1}{1.1806 - 0.7536} = 0.423$$

From Fig. 10–16, $\quad N\,(1 \text{ shell pass}) = 0.75 \qquad N(\text{total}) = (3)(0.75) = 2.25$

$2.25 = \dfrac{UA}{(3)(2100)} \qquad A = \dfrac{(2.25)(3)(2100)}{300} = 47.25 \text{ m}^2$

10-85

$C_w = (1)(4180) = 4180 \qquad C_o = (3)(2100) = 6300 \qquad \dfrac{C_{\min}}{C_{\max}} = \dfrac{4180}{6300} = 0.663$

$NTU = \dfrac{(300)(47.25)}{4180} = 3.391$

$N\,(1 \text{ shell}) = 1.13 \qquad \varepsilon_p = 0.54 \qquad$ From Fig. 10–16

From Table 10–3

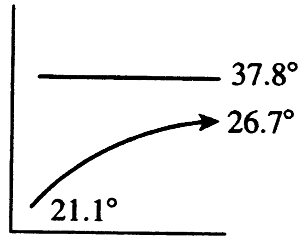
$$\varepsilon = \frac{\left[\frac{1-(0.54)(0.663)}{1-0.54}\right]^3 - 1}{\left[\frac{1-(0.54)(0.663)}{1-0.54}\right]^3 - 0.663} = 0.836 = \frac{\Delta T_w}{120-10}$$

$\Delta T_w = 91.97°\text{C} \qquad T_{we} = 10 + 91.97 = 101.97°\text{C}$

From energy balance: $\qquad \Delta T_o = 61.02°\text{C} \qquad T_{oe} = 120 - 61.02 = 58.98°\text{C}$

$q = 3.844 \times 10^5$ W

10-86



$$U = 700\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}} \qquad \dot{m}_F = 0.23\ \text{kg/s} \qquad h_{fg} = 120\ \text{kJ/kg}$$

$$q = \dot{m}_F h_{fg} = (0.23)(120) = 27{,}600\ \text{W} = \dot{m}_w c_w (26.7 - 21.1)$$

$$\dot{m}_w c_w = C_w = 4929 \qquad \Delta T_m = \frac{11.1 - 16.7}{\ln\left(\frac{11.1}{16.7}\right)} = 13.71^\circ\text{C} \qquad F = 1.0$$

$$q = UAF\Delta T_m \qquad A = \frac{27{,}600}{(700)(13.71)} = 2.88\ \text{m}^2 \qquad \varepsilon = \frac{5.6}{16.7} = 0.335$$

10-87

$$C_w = \left(\frac{1}{2}\right)(4929) = 2464 \qquad N = \frac{UA}{C_w} = \frac{(700)(2.88)}{2464} = 0.818$$

$$\varepsilon = 1 - e^{-N} = 0.559 = \frac{\Delta T_w}{37.8 - 21.1} \qquad \Delta T_w = (0.559)(16.7) = 9.33^\circ\text{C}$$

$$q = C_w \Delta T_w = (9.33)(2464) = 22{,}991\ \text{W}$$

$$\%\ \text{Reduction} = \frac{27{,}600 - 22{,}991}{27{,}600} \times 100 = 17\%$$

10-88

$$m_h c_h = m_c c_c = (3)(4180) = 12{,}540 \qquad C = 1.0 \qquad n = 4$$

$$\varepsilon = \frac{20}{80 - 10} = 0.2857$$

Using Eq. in Table 10–3 for $C = 1$

$$\varepsilon[1 + (n-1)\varepsilon_p] = n\varepsilon_p \qquad \varepsilon_p = \frac{\varepsilon}{n - \varepsilon(n-1)} = \frac{0.2857}{4 - (3)(0.2857)} = 0.0909$$

From Table 10–4

$$N = \frac{1}{\sqrt{2}}\ln\left(\frac{22 - 1 - 1 - \sqrt{2}}{22 - 1 - 1 + \sqrt{2}}\right) = 0.1002 = \frac{UA}{C_{min}} = \frac{1000A}{(3)(4180)}$$

$$A = 1.256\ \text{m}^2 \qquad A_{total} = (4)(1.256) = 5.024\ \text{m}^2$$

10-89

New $C_{\min} = (1.5)(4180) = 6270 \qquad C = 0.5$

One shell pass $\qquad N = (2)(0.1002) = 0.2004$

From Table 10–3 $\qquad \varepsilon_p = 2\left[1 + 0.5 + (1.118)\dfrac{1+e^{-0.224}}{1-e^{-0.224}}\right]^{-1} = 0.173$

Also from Table 10–3

$$\varepsilon = \frac{\left(\frac{1-0.0865}{1-0.173}\right)^4 - 1}{\left(\frac{1-0.0865}{1-0.173}\right)^4 - C} = 0.4943 = \frac{\Delta T_w}{80-10} \qquad \Delta T_w = 34.6°\text{C}$$

$q = (1.5)(4180)(34.6) = 2.156 \times 10^5$ W

For Prob. 10–80 $\qquad q = (3)(4180)(20) = 2.508 \times 10^5$ W

14% reduction in heat transfer

10-90

$C = 0.5$ and 1.0

Take N (1 shell) $= 1.0$

Fig. 10–16 $\qquad \varepsilon_p = 0.525$ at $C = 0.5$

$\qquad \varepsilon_p = 0.45$ at $C = 1.0$

Fig. 10–17 $\qquad N = 2 \qquad \varepsilon_p = 0.76$ at $C = 0.5$

$\qquad \varepsilon_p = 0.62$ at $C = 1.0$

at $C = 0.5 \qquad \varepsilon = \dfrac{\left(\frac{1-0.2625}{1-0.525}\right)^2 - 1}{\left(\frac{1-0.2625}{1-0.525}\right)^2 - 0.5} = 0.738$

at $C = 1.0 \qquad \varepsilon = \dfrac{(2)(0.45)}{1 + (2-1)(0.45)} = 0.621$

Good agreement, considering accuracy of reading figures.

10-91

Solve third from last equation of Table 10–3 for ε_p.

10-92

$C_o = (7.0)(2100) = 14,700$ $\quad T_{wi} = 20°\text{C}$

$C_{\text{mixed}} = C_w = (3.5)(4180) = 14,630$ $\quad T_{oi} = 100°\text{C}$

$\varepsilon = 0.6 = \dfrac{\Delta T_w}{100-20}$ $\quad \Delta T_w = 48°\text{C}$

$q = (14,630)(48) = (14,700)\Delta T_o$ $\quad \Delta T_o = 47.8°\text{C}$ $\quad T_{we} = 68°\text{C}$

$T_{oe} = 52.3°\text{C}$ $\quad \dfrac{C_{\text{mixed}}}{C_{\text{unmixed}}} = \dfrac{14,630}{14,700} = 0.995$ $\quad N = 2.5 = \dfrac{UA}{14,630}$

$UA = 36,575$

10-93

$C_w = (1.0)(4180) = 4180$ $\quad NTU = \dfrac{(2500)(0.8)}{4180} = 0.4785$

$\varepsilon = 1 - e^{-0.4785} = 0.38 = \dfrac{\Delta T_w}{100-20}$ $\quad \Delta T_w = 30.4°\text{C}$

$T_{we} = 20 + 30.4 = 50.4°\text{C}$

10-94

$C_w = (1.5)(4180) = 6270$ $\quad C_g = (3.0)(2474) = 7422$

$q = (6270)(50-20) = 7422\Delta T_g = 1.881 \times 10^5 \text{ W}$ $\quad \Delta T_g = 25.34°\text{C}$

$\varepsilon = \dfrac{\Delta T_w}{80-20} = \dfrac{30}{60} = 0.5$ $\quad C = \dfrac{6270}{7422} = 0.845$ $\quad n = 4$

From Prob. 10–77

$$\varepsilon_p = \frac{\left[\frac{1-0.5(0.845)}{1-0.5}\right]^{1/4} - 1}{\left[\frac{1-0.5(0.845)}{1-0.5}\right]^{1/4} - 0.845} = 0.1914$$

From Table 10–4 $\quad (1+C^2)^{1/2} = 1.3092$

$$N_p = \frac{-1}{1.3092}\ln\left(\frac{\frac{2}{0.1914} - 1 - 0.845 - 1.3092}{\frac{2}{0.1914} - 1 - 0.845 + 1.3092}\right) = 0.234$$

$N_{\text{total}} = (4)(0.234) = 0.937 = \dfrac{900A}{6270}$ $\quad A = 6.53 \text{ m}^2$

10-95

$C_g = \frac{7422}{2} = 3711 = C_{\min}$ $\qquad N_p = \frac{(900)(6.53)}{(371)(4)} = 0.3959$

$C = \frac{3711}{6270} = 0.592$ $\qquad (1+C^2)^{1/2} = 1.162$ $\qquad N(1+C^2)^{1/2} = 0.46$

From Table 10–3

$$\varepsilon_p = 2\left[1 + 0.592 + (1.162)\frac{1+e^{-0.46}}{1-e^{-0.46}}\right]^{-1} = 0.297$$

From Table 10–4

$$\varepsilon = \frac{\left[\frac{1-0.297(0.592)}{1-0.297}\right]^4 - 1}{\left[\frac{1-0.297(0.592)}{1-0.297}\right]^4 - 0.592} = 0.685 = \frac{\Delta T_g}{80-20}$$

$\Delta T_g = 41.13°\text{C}$ $\qquad \Delta T_w = \frac{(41.13)(3711)}{6270} = 24.34°\text{C}$

$T_{we} = 20 + 24.34 = 44.34°\text{C}$

10-96

$U = 57$ $\qquad C_w = (0.5)(4174) = 2087$ $\qquad C_a = (2)(1005) = 2010$

$\frac{C_{\min}}{C_{\max}} = \frac{2010}{2087} = 0.963$ $\qquad NTU = \frac{(57)(52)}{2010} = 1.474$

$\varepsilon = 0.55 = \frac{\Delta T_a}{130-75}$ $\qquad \Delta T_a = 30.25°\text{F} = 16.81°\text{C}$

$\Delta T_w = (16.81)\left(\frac{2010}{2087}\right) = 16.19°\text{C}$ $\qquad T_{w\,\text{exit}} = 100.9°\text{F} = 38.26°\text{C}$

$q = C_a \Delta T_a = (2010)(16.81) = 33{,}788\ \text{W}$

10-97

$C_a = (1005)(0.2) = 201$ $\qquad C_w = (4180)(0.2) = 836$ $\qquad \frac{C_{\min}}{C_{\max}} = \frac{201}{836} = 0.24$

$NTU = \frac{UA}{C_{\min}} = \frac{(40)(5)}{201} = 1.0$ $\qquad \varepsilon = 0.59 = \frac{\Delta T_a}{90-20}$ $\qquad \Delta T_a = 41.3°\text{C}$

$q = C_a \Delta T_a = (201)(41.3) = 8301\ \text{W}$ $\qquad \varepsilon = 1 - \exp\left[\frac{\exp(-0.24)-1}{0.24}\right] = 0.59$

10-98

Using equation from Problem 10–83 $\qquad n = 3$

$C = \frac{2}{3}$ $\qquad \varepsilon = \frac{60-30}{110-30} = 0.375$ $\qquad \varepsilon_p = 0.158$

10-99

$q = (3)(4180)(20) = C_o(30)$ $\quad C_o = C_{\min} = 8360$

Table 10–4 $\quad (1+c^2)^{1/2} = 1.202$

$$N_p = -\frac{1}{1.202} \times \ln\left(\frac{\frac{2}{0.158} - 1 - 0.667 - 1.202}{\frac{2}{0.158} - 1 - 0.667 + 1.202}\right) = 0.183$$

$N\ (\text{total}) = (3)(0.183) = 0.548 = \dfrac{230A}{8360}$ $\quad A\ (\text{total}) = 19.92$

Reduce water to 2 kg/s

$C_w = (2)(4180) = 8360 = C_o$ $\quad C = 1.0$ $\quad (1 + C^2)^{1/2} = 1.414$

$$\varepsilon_p = 2\left[1 + 1 + (1.414)\frac{1 + \exp(-0.183 \times 1.414)}{1 - \exp(-0.183 \times 1.414)}\right]^{-1} = 0.154 \quad \text{(From Table 10–3)}$$

Next to last Equation of Table 10–3 for $C = 1$

$$\varepsilon = \frac{n\varepsilon_p}{1 + (n-1)\varepsilon_p} = \frac{(3)(0.154)}{1 + (2)(0.154)} = 0.353 = \frac{\Delta T_w}{110 - 30} \qquad \Delta T_w = 28.2°\text{C} = \Delta T_o$$

$T_{we} = 110 - 28.2 = 81.8°\text{C}$ $\quad T_{oe} = 30 + 28.2 = 58.2°\text{C}$

$q = (8360)(28.2) = 2.36 \times 10^5$ W

10-100

$h_i = 3000$ $\quad h_o = 190\ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad d_o = 25$ mm $\quad r_o - r_i = 0.8$ mm

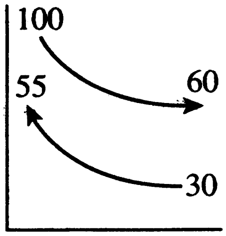
copper $k = 386\ \dfrac{\text{W}}{\text{m} \cdot °\text{C}}$

$r_o = 12.5$ mm $\quad r_i = 11.7$ mm $\quad d_i = 23.4$ mm

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi kL} + \frac{1}{h_o}\frac{A_i}{A_o}} = \frac{1}{\frac{1}{3000} + \frac{\pi(0.0234)\ln(12.5/11.7)}{2\pi(386)} + \frac{11.7}{(12.5)(190)}}$$

$$= 190\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

10-102



$\Delta T_h = 40°\text{C}$ $\qquad\qquad$ $\Delta T_c = 25°\text{C}$

Hot fluid is minimum fluid

$$\varepsilon = \frac{\Delta T \text{ (min fluid)}}{\Delta T \text{ (max Hx)}} = \frac{40}{100-30} = 0.571$$

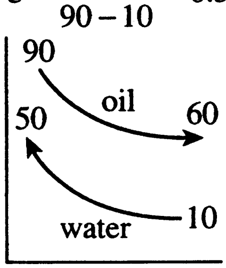
10-103

2 shell passes, 4 tube passes; oil in tubes, 90°C to 60°C;

water in shell, 10°C to 50°C $\qquad U = 53 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \qquad c_o = 2000 \dfrac{\text{J}}{\text{kg} \cdot °\text{C}}$

$q = 500 \text{ kW} \qquad \Delta T_h = 30 \qquad \Delta T_c = 40$ $\qquad$ Water is min fluid

$$\varepsilon = \frac{50-10}{90-10} = 0.5$$



$\dfrac{C_{\min}}{C_{\max}} = \dfrac{30}{40} = 0.75 \qquad NTU = 0.9 = \dfrac{UA}{C_{\min}}$

$q = C_o(30) = C_w(40) \qquad C_o = 16,667 \qquad C_w = 12,500 = C_{\min}$

$A = \dfrac{(0.9)(12,500)}{53} = 212 \text{ m}^2$ $\qquad$ ($\varepsilon - NTU$ method)

$\dot{m}_w = \dfrac{12,500}{4180} = 2.99 \text{ kg/s} \qquad \text{LMTD} = \Delta T_m = \dfrac{40-50}{\ln\left(\frac{40}{50}\right)} = 44.8°\text{C}$

$T_1 = 90 \qquad T_2 = 60 \qquad t_1 = 10 \qquad t_2 = 50 \qquad R = \dfrac{30}{40} = 0.75$

$P = \dfrac{40}{80} = 0.5 \qquad F = 0.98 \qquad q = UAF\Delta T_m$

$A = \dfrac{500,000}{(53)(0.98)(44.8)} = 215 \text{ m}^2$ $\qquad$ (LMTD method)

10-104

$q = 250{,}000 \text{ W}$ $\qquad$ $C_o = 16{,}667$ $\qquad$ $A = 212 \text{ m}^2$

$U = 53 \ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$

C_w	$\dfrac{C_w}{C_o}$	NTU	ε	ΔT_w	q
5000	0.3	2.25	0.82	65.6	328,000
2000	0.12	5.62	0.97	77.6	155,200
3000	0.18	3.75	0.94	75.2	225,600

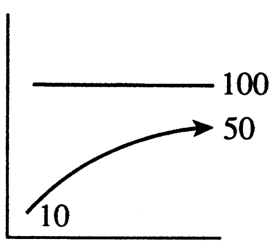
$C_w \sim 3500$

$$\% \text{ Reduction } = \frac{12{,}500 - 3500}{12{,}500} \times 100 = 72\%$$

10-105

Finned tube, steam at 100°C $\qquad$ $q = 44 \text{ kW}$

Air heated from 10°C to 50°C $\qquad$ $U = 25 \ \dfrac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$



$$\varepsilon = \frac{50 - 10}{100 - 10} = 0.444 \qquad \frac{C_{\min}}{C_{\max}} = 0$$

$$N = -\ln(1 - \varepsilon) = -\ln(1 - 0.444) = 0.588 \qquad C_{\min} = \frac{44{,}000}{50 - 10} = 1100$$

$$NTU = \frac{UA}{C_{\min}} \qquad A = \frac{(0.588)(1100)}{25} = 25.87 \text{ m}^2$$

10-106

$U \sim \dot{m}^{0.8} \sim C^{0.8}$ $\qquad \dfrac{C_{min}}{C_{max}} = 0$ $\qquad \varepsilon = 1 - e^{-N}$

$$q = \left(\frac{2}{3}\right)(44{,}000) = 29{,}350 \text{ W}$$

C_{min} (C_{air})	U	NTU	ε	ΔT_{air}	q
1000	23.16	0.599	0.451	40.57	40,570
600	15.39	0.664	0.485	43.66	26,194
700	17.41	0.644	0.474	42.71	29,899

$C_{air} = 690$ for $q = 29{,}350$ W $\qquad$ % Reduction $= \dfrac{1100 - 690}{1100} \times 100 = 37\%$

10-107

Steam in shell at 38°C $\qquad$ Water 20°C to 27°C $\qquad$ 1 shell, 2 tube passes

$q = 700$ MW $\qquad$ Assume $U = 2500 \ \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}}$

$\varepsilon = \dfrac{27 - 20}{38 - 20} = 0.38889$ $\qquad N = -\ln(1 - \varepsilon) = 0.492$ $\qquad q = C_w \Delta T_w$

$C_w = \dfrac{700 \times 10^6}{27 - 20} = 1 \times 10^8$ $\qquad m_w = \dfrac{1 \times 10^8}{4180} = 23{,}920$ kg/sec

$NTU = 0.492 = \dfrac{UA}{C_{min}}$ $\qquad A = \dfrac{(0.492)(1 \times 10^8)}{2500} = 19{,}680 \text{ m}^2$

10-108

$q = 7 \times 10^8 = C_w(34 - 20)$ $\qquad C_w = 5 \times 10^7$

$\varepsilon = \dfrac{34 - 20}{38 - 20} = 0.7778$ $\qquad N = -\ln(1 - \varepsilon) = 1.5 = \dfrac{UA}{C_{min}}$

$$A = \frac{(1.5)(5 \times 10^7)}{2500} = 30{,}080 \text{ m}^2$$

10-109

$A = 19{,}680\ \text{m}^2 \qquad \varepsilon = \dfrac{34-20}{38-20} = 0.7778$

$N = -\ln(1-0.7778) = 1.5 = \dfrac{UA}{C_{\min}} \qquad C_{\min} = \dfrac{(2500)(19{,}680)}{1.5} = 3.28\times10^7$

$q = C_w \Delta T_w = (3.28\times10^7)(34-20) = 459\ \text{MW}$

$\%\ \text{Reduction} = \dfrac{700-459}{700}\times100 = 34.4\%$

10-111

$d_o = 1.315 \qquad d_i = 1.049\ \text{in.} \qquad k = 43\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}} \qquad \bar{T}_b = \dfrac{80+60}{2} = 70^\circ\text{C}$

$\rho = 858 \qquad c_p = 2090 \qquad \nu = 0.6\times10^{-4} \qquad k = 0.139$

$\text{Pr} = 770 \qquad \text{Re} = \dfrac{(5)(1.049)(0.0254)}{0.6\times10^{-4}} = 2220$

Free convection $\qquad T_f = \dfrac{70+20}{2} = 45^\circ\text{C} \qquad \dfrac{g\beta\rho^2 c_p}{\mu k} = 4.2\times10^{10}$

$k = 0.64 \qquad \text{Gr Pr} = (4.2\times10^{10})(1.315)^3(0.0254)^3(70-20) = 7.82\times10^7$

$h_o = \dfrac{0.64}{(1.315)(0.0254)}(0.53)(7.82\times10^7)^{1/4} = 953\ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$

$q = \dot{m}c_p\Delta T_b = (858)(5)\dfrac{\pi(1.049)^2}{4}(0.0254)^2(2090)(80-60) = 99{,}970\ \text{W}$

Assume inside barely turbulent:

$h_i = \dfrac{0.139}{(1.049)(0.0254)}(0.023)(2220)^{0.8}(770)^{0.3} = 419\ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$

Neglect conduction resistance

$U = \dfrac{1}{\dfrac{1}{h_i}+\dfrac{1}{h_o}} = 291\ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}} \qquad q = U\pi d L \Delta T$

$d = \dfrac{1.049+1.315}{2} = 1.18\ \text{in.} \qquad L = \dfrac{99{,}970}{(291)\pi(1.18)(0.0254)(70-20)} = 72.8\ \text{m}$

10-113

At 40 mph $\quad q = 60{,}000\ \text{Btu/hr} \quad U = 35\ \dfrac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}} \quad \Delta T = 10^\circ\text{F}$

$U = kv^{0.7} = 0.0066v^{0.7} \qquad \dot{m}_{\text{air}} = k_2 v = 0.118v$

<u>At 40 mph</u> = 211,200 ft/hr

$G_a = \dfrac{\Delta T_a}{T_f - T_{a_1}} = 0.182 \qquad \dfrac{C_{\min}}{C_{\max}} = 0 \qquad NTU = 0.20 \qquad A = \dfrac{\frac{NTU}{C_{\min}}}{U} = 34.3\ \text{ft}^2$

<u>At 30 mph</u> = 158,000 ft/hr

$U = 28.5 \qquad \dot{m} = 18{,}700 \qquad C_{\min} = 4490 \qquad NTU_{\max} = 0.218$

$\varepsilon = 0.185 = \dfrac{T_{e_a} - 95}{55} \qquad T_{e_a} = 105.2^\circ\text{F}$

$q = \dot{m}_a c_{p_a} \Delta T_a = 58{,}000\ \text{Btu/hr}$ $\qquad$ 3.3% reduction

<u>At 20 mph</u> = 105,600 ft/hr

$U = 21.6 \qquad \dot{m}_a = 12{,}450 \qquad C_{\min} = 2290 \qquad NTU_{\max} = 0.248$

$\varepsilon = 0.20 = \dfrac{T_{e_a} - 95}{55} \qquad T_{e_a} = 106^\circ\text{F}$

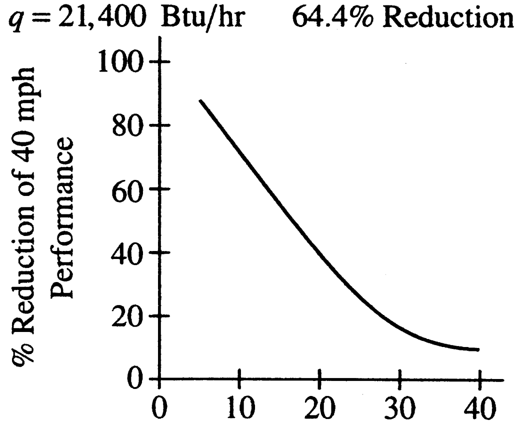
$q = 32{,}900\ \text{Btu/hr}$ $\qquad$ 45.2% reduction

<u>At 10 mph</u> = 52,800 ft/hr

$U = 13.3 \qquad \dot{m}_a = 6230 \qquad C_{\min} = 1495$

$NTU_{\max} = 0.305 \qquad \varepsilon = 0.26 \qquad T_{e_a} = 109.3^\circ\text{F}$

$q = 21{,}400\ \text{Btu/hr}$ $\qquad$ 64.4% Reduction



10-114

$p = 345\ \text{kN/m}^2 = 50\ \text{psia} \qquad T_{\text{sat}} = 281^\circ\text{F} = 138^\circ\text{C}$

$\Delta T_m = \dfrac{53 - 39}{\ln\left(\frac{53}{39}\right)} = 45.64 \qquad q = (7.5)(4175)(99 - 85) = 438{,}375\ \text{W}$

$q = (2800)A(45.64) \qquad A = 3.43\ \text{m}^2 = (30)\pi(0.025)(2L) \qquad L = 0.728\ \text{m}$

10-115

$$R_f = 0.0005 = \frac{1}{U_{\text{dirty}}} - \frac{1}{U_{\text{clean}}} \qquad U_{\text{dirty}} = 1167 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$q = (1167)(3.43)(45.64) = 182{,}636 \text{ W} \qquad \Delta T_w = \frac{182{,}636}{(7.5)(4175)} = 5.83^\circ\text{C}$$

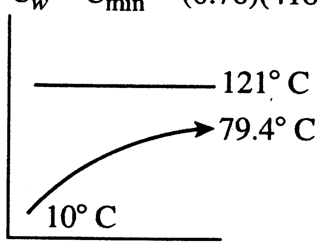
$$T_{w_e} = 99 - 5.83 = 93.2^\circ\text{C}$$

10-116

$$h \text{ (steam)} \sim 9000 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$U = \frac{1}{\frac{1}{h_o} + \frac{1}{h_i}} = \frac{1}{\frac{1}{9000} + \frac{1}{1420}} = 1226 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$C_w = C_{\min} = (0.76)(4180) = 3176$$



$$\varepsilon = \frac{69.4}{111} = 0.625 \qquad N = -\ln(1-\varepsilon) = 0.981 = \frac{UA}{C_w}$$

$$A = \frac{(0.981)(3176)}{1226} = 2.54 \text{ m}^2$$

Water flow reduced by 60% to $C_w = (0.4)(3176) = 1270$

$$N = \frac{UA}{C_w} = \frac{(1226)(2.54)}{1270} = 2.45 \qquad \varepsilon = 1 - e^{-N} = 0.914 = \frac{\Delta T_w}{111}$$

$$\Delta T_w = 101.4^\circ\text{C} \qquad T_{w\,\text{exit}} = 10 + 101.4 = 111.4^\circ\text{C}$$

10-117

$$0.0002 + 0.0004 = \frac{1}{U} - \frac{1}{150} \qquad U = 137.6 \ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}$$

$$\frac{A_{\text{dirty}}}{30} = \frac{150}{137.6} \qquad A_{\text{dirty}} = 32.7 \text{ m}^2 \qquad 9\% \text{ increase}$$

10-118

Sat steam: $p_s = 100$ psia $\quad T_s = 328°\text{F} \quad h = 1000 \dfrac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot°\text{F}}$

$h_{fg} = 888.8$ Btu/lbm $\quad$ neglect wall resistance.

CO_2: $\dot{m}_c = 3600$ lbm/hr $\quad p_c = 15$ psia $\quad T_{c_2} = 70°\text{F} \quad T_{c_1} = 200°\text{F}$

$c_{p_c} = 0.208 \dfrac{\text{Btu}}{\text{lbm}\cdot°\text{F}} \quad \rho_c = 0.1106\ \text{lbm/ft}^3$ at $90°\text{F} \quad \dfrac{S_o}{d} = \dfrac{S_u}{d} = \dfrac{3}{2}$

$C = 0.25 \quad n = 0.62 \quad \text{Nu} = \dfrac{hd}{k} = C\left(\dfrac{u_{\max} d}{\nu}\right)^n \quad T_f = 231°\text{F}$

$u_\infty = \dfrac{\dot{m}c}{\rho A} \quad \rho = 0.0874 \quad u_{\max} = u_\infty\left(\dfrac{S_n}{S_n - d}\right) \quad k = 0.0134$

$\text{Re}_d = \dfrac{\rho u_m d}{\mu} \quad h_c = \dfrac{k}{d}C(\text{Re})^n \quad q_c = UAF(\Delta T_m) \quad F = 1$

Assume $L = 3.25'' = 0.271$ ft $\quad UA = \dfrac{1}{\frac{1}{h_i A_i} + \frac{1}{h_o A_o}} \quad A = 0.1775\ \text{ft}^2$

$\Delta T_m = \dfrac{(T_{h_2} - T_{h_1}) - (T_{h_1} - T_{c_1})}{\ln\left(\frac{T_{h_2} - T_{h_1}}{T_{h_1} - T_{c_1}}\right)} \quad u_\infty = 199{,}000$ ft/hr

$u_{\max} = 597{,}000$ ft/hr $\quad \text{Re}_d = 24{,}200 \quad h = 83.5$

$UA(\Delta T_m) = \dot{m}c_p(T_2 - T_1)_{CO_2} \quad UA = \dfrac{4.98}{t} \quad L = 107.5$

$\dfrac{107.5}{400} = 0.268 = 3.22\ \text{in.} \approx 3.25''$ $\quad$ this assumption was o.k.

10-121

$m_o(2100)(138 - 93) = (2.5)(4175)(65 - 25)$

$m_o = 4.417$ kg/sec $\quad U = 450 \quad A_1 = A_2 \quad$ Assume oil is min fluid

$\varepsilon_1 = \dfrac{138 - T_1}{138 - 25} \quad \varepsilon_2 = \dfrac{138 - T_2}{138 - 50} \quad C_{\max} = (2.5)(4175) = 10{,}438$

$C_{\max_2} = (1.88)(4175) = 7849 \quad \dfrac{NTU_1}{NTU_2} = \dfrac{C_{\min_1}}{C_{\min_2}} = \dfrac{m_{o2}}{m_{o1}} = \dfrac{4.417}{m_{o1}} - 1$

$(10{,}438)(50 - 25) = C_{\min_1}(138 - T_1) = 260{,}950$

$(7849)(65 - 50) = C_{\min_2}(138 - T_2) = 117{,}735$

By iteration: $m_{o1} = 3.638 \quad T_1 = 103.8°\text{C} \quad \varepsilon_1 = 0.3023$

$C_1 = 0.732 \quad N_1 = 0.4099$

$A_1 = A_2 = \dfrac{(0.4099)(3.638)(2100)}{450} = 6.96\ \text{m}^2$

10-122

Water is min fluid $F = 1$ $A_1 = 1.535\ \text{m}^2$ $A_2 = 0.863\ \text{m}^2$

$$(\Delta T_m)_1 = \frac{(138-25)-(138-50)}{\ln\left(\frac{113}{88}\right)} = 99.98°\text{C}$$

$$(\Delta T_m)_2 = \frac{(138-50)-(138-65)}{\ln\left(\frac{88}{73}\right)} = 80.27°\text{C}$$

$$q_1 = (2.5)(4175)(50-25) = 1700A_1(99.98) = 260.9\ \text{kW}$$

$$q_2 = (1.88)(4175)(65-50) = 1700A_2(80.27) = 117.7\ \text{kW}$$

$$m_{s1} = \frac{q_1}{h_{fg}} = 0.116\ \text{kg/sec} \qquad m_{s2} = \frac{q_2}{h_{fg}} = 0.052\ \text{kg/sec}$$

10-126

Both unmixed $h_a = 174$ $h_s = 5000$ $U = 168$ $c = 0$

Volume $= (0.5)(0.4) = 0.2\ \text{m}^3 = 7.063\ \text{ft}^3$ Area $= (7.063)(229) = 1617\ \text{ft}^2$

$A = 150.3\ \text{m}^2$ $C_{\text{min}} = (1.1774)(0.5)(15)(1006) = 8883\ \text{W/°C}$

$$N = \frac{(168)(150.3)}{8883} = 2.843 \qquad \varepsilon = 1 - e^{-2.843} = 0.942$$

$\Delta T_a = (0.942)(373-300) = 68.7°\text{C}$ $T_{a2} = 368.7\ \text{K} = 95.7°\text{C}$

$q = (8883)(68.7) = 6.1 \times 10^5\ \text{W}$

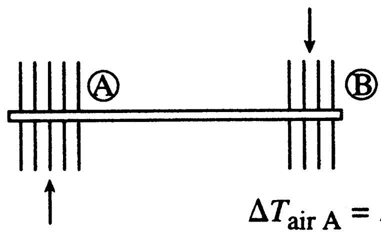
10-134

If the pair of heat exchangers had a very large mass flow of water circulating negligible temperature drop would occur in the water and there would be maximum transfer of heat from the outside to inside air. In addition, if the area of the exchangers is very large the limiting case will be experienced where the temperature of the air discharged to outside will be equal to the outside air temperature entering the system. These conditions represent the limiting case for operation of the system, and all other cases will involve somewhat degraded performance. Selection of an eventual design would have to balance system costs against energy savings achieved.

10-136

$T_{\text{dryer}} = 95°\text{C} = 368\ \text{K} \qquad T_{\text{air inlet}} = 10°\text{C}$

$$\dot{m}_a = \frac{p\dot{V}}{RT} = \frac{(1.0132\times 10^5)(34)}{(60)(287)(368)} = 0.544\ \text{kg/sec} \qquad U \sim 30\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$



$$\Delta T_{\text{air A}} = \Delta T_{\text{air B}}$$

Choose $\Delta T_{\text{air}} = 50°\text{C}$

$q = \dot{m}c_p\Delta T = (0.544)(1005)(50) = 27{,}340\ \text{W}$

Since heat pipe temp remains constant, air is min fluid

$$\varepsilon_A = \varepsilon_B = \frac{40}{95 - T_p} = \frac{40}{T_p - 10} \qquad T_p = \frac{105}{2} = 52.5°\text{C}$$

$$\varepsilon = \frac{40}{42.5} = 0.94 = 1 - e^{-N} \qquad N = 2.833 = \frac{UA}{C_{\min}}$$

$$A = \frac{(2.833)(0.544)(1005)}{30} = 51.6\ \text{m}^2$$

Cost savings with electricity $= (0.085)(27.34) = \$2.32/\text{hr}$

Cost savings with gas $= (27{,}340)(3600)\left(\dfrac{\$9.00}{10^9\ \text{J}}\right) = \$0.886/\text{hr}$

If operated 12 hr/day, 365 days/yr

savings (elec) $= (2.32)(12)(365) = \$10{,}161/\text{yr}$

savings (gas) $= (0.886)(12)(365) = \$3881/\text{yr}$

10-143

Installation of the pipes in the ground can be either in a vertical or horizontal configuration, which will influence the conduction shape factor between the pipe and soil. (Chapter 3) Actual installation conditions depend strongly on the locale. Both cases might be examined. For the calculations the thermal resistance due to the water side heat transfer coefficient can probably be neglected in comparison to the resistance of the pipe and conduction resistance to the soil.

Chapter 11

11–1

$$\dot{m} = -D\frac{dc}{dx} \qquad D = \frac{-\dot{m}}{dc/dx} \qquad \dot{m} \approx \bar{v} \sim T^{1/2} \qquad \frac{dc}{dx} \sim p \sim \frac{1}{T}$$

$$\therefore \; D \sim \frac{T^{1/2}}{1/T} \sim T^{3/2}$$

11–2

$v(C_6H_6) = 6(14.8 + 3.7) - 15 = 96 \qquad v_{air} = 29.9$

$T = 25°C = 298 \text{ K} \qquad M(C_6H_6) = 78 \qquad M(\text{air}) = 28.9$

$$D = \frac{(435.7)(298)^{3/2}}{(1.0132\times10^5)[(96)^{1/3} + (29.9)^{1/3}]^2}\left(\frac{1}{78} + \frac{1}{289}\right)^{1/2} = 0.082 \text{ cm}^2/\text{sec}$$

11–3

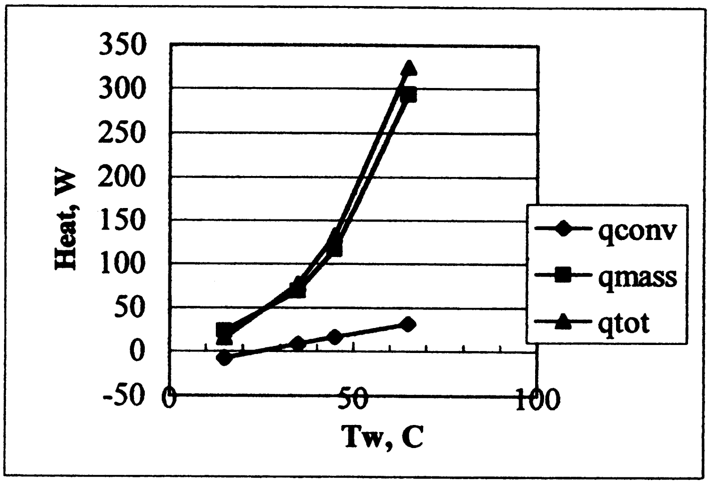
$T_\infty = 25°C \qquad u_\infty = 1.5 \text{ m/sec} \qquad L = 30 \text{ cm}$

°F	T_w °C	h_{fg} $\times10^{-6}$	C_w	T_f °K	ν $\times10^{-6}$	k	c_p	Pr	ρ_f
59	15	2.47	0.0128	293	14.81	0.026	1005	0.69	1.2
95	35	2.42	0.039	303	15.98	0.0265	1005	0.69	1.17
113	45	2.40	0.065	308	16.49	0.027	1005	0.69	1.15
149	65	2.34	0.164	318	17.51	0.0275	1005	0.69	1.11

$$\bar{h} = \frac{k}{L}(0.664)\text{Re}_L{}^{1/2}\,\text{Pr}^{1/3} \qquad h_D = \frac{\bar{h}}{\rho_f c_p}\left(\frac{\text{Pr}}{\delta_c}\right)^{2/3}$$

$$\left(\frac{\text{Pr}}{\delta_c}\right)^{2/3} = \left(\frac{0.69}{0.6}\right)^{2/3} = 1.098 \qquad \text{Pr}^{1/3} = 0.884$$

Tw	qconv	qmass	qtot
15	-7.983	22.99	15.01
35	7.83	68.97	76.8
45	15.71	116.39	132.1
65	31.07	293.23	324.3



$$q = hA(T_w - T_\infty) + h_D A(C_w - C_\infty)h_{fg} = q_{\text{conv}} + q_{\text{mass}}$$

11–4

$h_{fg} = 3.77\times10^5$ J/kg $\quad T_w = 26°\text{C} = 299$ K $\quad p = 13.3$ kN/m^2

$$T_\infty - T_w = \frac{h_D}{h}(C_w - C_\infty)h_{fg} \qquad C_w = \frac{(13{,}300)(78)}{(8316)(299)} = 0.417$$

$S_c = 1.76 \quad \text{Pr} = 0.7 \quad$ Assume $T_f \approx 70°\text{C} = 343$ K

$$\rho_f = \frac{1.0132\times10^5}{(287)(343)} = 1.029 \qquad \rho_f = \frac{1.0132\times10^5}{(287)(343)} = 1.029$$

$$\frac{h_D}{h} = \frac{1}{(1.029)(1009)}\left(\frac{0.7}{1.76}\right)^{2/3} = 5.209\times10^{-4}$$

$$T_\infty - T_w = (5.209\times10^{-4})(0.417)(3.77\times10^5) = 81.89°\text{C} \qquad T_\infty = 107.89°\text{C}$$

Repeat with ρ_f at $T_f = \dfrac{108+26}{2} = 67°\text{C} = 340\text{ K}$. Close enough.

11–5

$h_{fg} = 2.45\times10^6$ J/kg $\quad u_m = 3$ m/sec $\quad \rho = \dfrac{1.0132\times10^5}{(287)(298)} = 1.185$

$\mu = 1.98\times10^{-5} \quad k = 0.026 \quad \text{Pr} = 0.7 \quad \text{Sc} = 0.6$

$D = 0.256 \text{ cm}^2/\text{sec} = 2.56 \times 10^{-5} \text{ m}^2/\text{sec}$ $\qquad \text{Re} = \dfrac{(1.185)(0.05)(3)}{1.98 \times 10^{-5}} = 8977$

$$h_D = \frac{2.56 \times 10^{-5}}{0.05}(0.023)(8977)^{0.83}(0.6)^{0.44} = 0.018$$

At 25°C $C_w = 0.0229$

$$\dot{m}_w = (0.018)\pi(0.05)(3)\left(0.0229 - \frac{C_{w\,\text{exit}}}{2}\right) = (\text{air volume flow})C_{w\,\text{exit}}$$

$$\text{air volume flow} = \frac{\pi d^2}{4}u_m = \frac{\pi(0.05)^2}{4}(3) = 5.89 \times 10^{-3} \text{ m}^3/\text{sec}$$

Solving for $C_{w\,\text{exit}}$: $C_{w\,\text{exit}} = 0.0192 \text{ kg/m}^3$

11–6

$p_{w_1} = 0.4593$ psia $\qquad T = 25°\text{C} = 298$ K $\qquad p_{w_2} = \dfrac{0.4593}{2} = 0.2297$ psia

$D = 0.256 \text{ cm}^2/\text{sec} = 2.56 \times 10^{-5} \text{ m}^2/\text{sec}$

$p_{a_1} = 14.696 - 0.4593 = 14.237 \text{ psia} = 9.8155 \times 10^4 \text{ N/m}^2$

$p_{a_2} = 14.696 - 0.2297 = 14.466 \text{ psia} = 9.9736 \times 10^4 \text{ N/m}^2$

$$m_w = \frac{(2.56 \times 10^{-5})(1.0132 \times 10^5)\pi(0.1)^2(18)}{(8316)(298)(0.075)} \ln\left(\frac{9.9736}{9.8155}\right)$$
$$= 1.26 \times 10^{-7} \text{ kg/sec} = 0.454 \text{ g/hr}$$

11–7

$D = 0.088 \text{ cm}^2/\text{sec} = 8.8 \times 10^{-6} \text{ m}^2/\text{sec}$

$p_{a_1} = 101.32 - 13.3 = 88.02 \text{ kN/m}^2 \qquad p_{a_2} = 101.32 - 0 = 101.32 \text{ kN/m}^2$

$$m_B = \frac{(8.8 \times 10^{-6})(1.0132 \times 10^5)(78)\pi(0.00625)^2}{(8316)(299)(0.15)} \ln\left(\frac{101.32}{88.02}\right)$$
$$= 3.22 \times 10^{-9} \text{ kg/sec} = 0.0116 \text{ g/hr}$$

11–8

$T_f = \dfrac{25+0}{2} = 12.5°\text{C} = 285.5 \text{ K} \qquad \rho = 1.34 \qquad \mu = 1.64 \times 10^{-5}$

$k = 0.0235 \qquad \text{Pr} = 0.71 \qquad c_p = 1009 \qquad \text{Re} = \dfrac{(1.34)(1.5)(0.3)}{1.64 \times 10^{-5}} = 36{,}768$

$h = \dfrac{0.0235}{0.3}(0.664)(36{,}768)^{1/2}(0.71)^{1/3} = 8.9 \dfrac{\text{W}}{\text{m}^2 \cdot °\text{C}} \qquad \text{Sc} = 0.6$

$$h_D = \frac{8.9}{(1.34)(1009)}\left(\frac{0.71}{0.6}\right)^{2/3} = 7.36\times10^{-3} \quad \text{at } 0°\text{C} \quad C_w = 4.845\times10^{-3}$$

$$m_w = (7.36\times10^{-3})(0.3)^2(4.845\times10^{-3} - 0) = 3.21\times10^{-6}\ \text{kg/sec} = 0.0116\ \text{kg/hr}$$

11–9

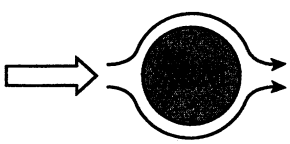
$T_w = 32°\text{C} = 305\ \text{K} = 89.6°\text{F}$ $\quad C_w = 0.0342$ $\quad h_{fg} = 2.42\times10^6\ \text{J/kg}$

$\text{Sc} = 0.6$ $\quad \text{Pr} = 0.7$ $\quad$ Assume $T_f \approx 350\ \text{K}$ $\quad \rho_f = 0.998$

$$c_p = 1009 \qquad T_\infty - T_w = \frac{(0.0342-0)(2.42\times10^6)}{(0.998)(1009)}\left(\frac{0.7}{0.6}\right)^{2/3} = 91°\text{C}$$

$$T_\infty = 32 + 91 = 123°\text{C} \qquad T_f = \frac{123+32}{2} = 77.5°\text{C} = 350.5\ \text{K}$$

11–10

Dry Air  $u_\infty = 10\ \text{mph}$

$T_\infty = 115°\text{F}$

$T_\infty = 115°\text{F}$ $\quad \mu_\infty = 10\ \text{mph} = 52{,}800\ \text{ft/hr}$ $\quad L = 1\ \text{ft}$

$q_{\text{sun}} = 350(A_{\text{new}})\varepsilon_1$ $\quad \varepsilon_1 = 1.0$

Neglect internal heat generation

$$q_{\text{rad}}\big|_{\text{sun}} + q_{\text{conv}}\big|_{\text{air}} = q_{\text{rad}}\big|_{\text{arm}} + q_{\text{evap}}\big|_{\text{arm}} \qquad q_{\text{rad}}\big|_{\text{sun}} = 350dL = 116.5\ \text{Btu/hr}$$

$$q_{\text{conv}} = hA(T_\infty - T_w) \qquad q_{\text{rad}}\big|_{\text{arm}} = \varepsilon\sigma A T_w{}^4 \qquad q_{\text{evap}} = h_D A(C_w - C_\infty)h_{fg}$$

$$C_\infty = 0 \qquad 350\frac{A}{\pi} + hA(T_\infty - T_w) - \varepsilon\sigma A T_w{}^4 - h_D A C_w h_{fg} = 0$$

$$h = \frac{k_f}{d}C\left(\frac{\rho u_\infty d}{\mu}\right)^n \qquad \text{assume } T_w = 58°\text{F}$$

$\therefore\ T_f = 86.5°\text{F}$

$\rho = 0.0728$ $\quad d = 0.333\ \text{ft}$ $\quad \mu = 0.045$ $\quad k = 0.0154$

$$\text{Re}_d = \frac{(0.0708)(52{,}800)(0.333)}{0.046} = 28{,}300 \qquad c = 0.174 \qquad m = 0.618$$

$$h = 4.53\ \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot°\text{F}} \qquad \frac{h}{h_0} = \rho c_p\left(\frac{\text{Sc}}{\text{Pr}}\right)^{2/3} \qquad \therefore\ h_D = 289$$

$$c_p = 0.24 \qquad \frac{\text{Sc}}{\text{Pr}} = 0.85$$

$$\frac{350}{\pi} + 4.53(T_\infty - T_w) - \varepsilon\sigma T_w{}^4 - 289C_w h_{fg} = 0$$

$111 + 256 - 123 - 237 = -3 \approx 0$

close enough

$T_w = 58°\text{F}$

The above calculation of the radiation from the arm is not strictly correct because the effective radiation temperature of the surroundings is not given. Certain view factors would also need to be known. When the calculation is repeated neglecting the radiation from the arm the result is:

$T_w = 67°\text{F}$

11–11

$h(T_\infty - T_w) = h_D C_w h_{fg} + \sigma(T_w{}^4 - T_s{}^4)$ $\qquad T_\infty = 43°\text{C} \qquad T_s = 10°\text{C}$

$L = 30\text{ cm} \qquad u_\infty = 12\text{ m/sec} \qquad \Pr = 0.71 \qquad \text{Sc} = 0.6$

Assume $T_f \approx 300\text{ K} \qquad \rho_f = 1.177 \qquad \mu_f = 1.983 \times 10^{-5}$

$k_f = 0.02624 \qquad c_p = 1006 \qquad h_{fg} = 2.42 \times 10^6$

$$\text{Re} = \frac{(1.177)(12)(0.3)}{1.983 \times 10^{-5}} = 2.14 \times 10^5$$

$$\bar{h} = \frac{(0.02624)(0.664)}{0.3}(2.14 \times 10^5)^{1/2}(0.71)^{1/3} = 23.95\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$h_D = \frac{23.95}{(1.177)(1.006)}\left(\frac{0.71}{0.6}\right)^{2/3} = 0.0226 \qquad \text{By iteration } T_w = 14.1°\text{C}$$

11–12

$$m_w = h_D C_w A = \frac{[h(T_\infty - T_w) - \sigma(T_w{}^4 - T_s{}^4)]A}{h_{fg}}$$

$$= \frac{(0.091)[(23.95)(43 - 14.1) - (5.669 \times 10^{-8})(287.1^4 - 283^4)]}{2.42 \times 10^6}$$

$$= 2.49 \times 10^{-5}\text{ kg/sec} = 0.09\text{ kg/hr}$$

11–13

$T_\infty = 115°\text{F} \qquad u_\infty = 10\text{ mph} = 52{,}800\text{ ft/hr}$

$$\dot{q}_{\text{gen}} V + q_{\text{rad}}\big|_{\text{sun}} + q_{\text{conv}}\big|_{\text{air}} = q_{\text{rad}}\big|_{\text{arm}} + q_{\text{evap}}\big|_{\text{arm}}$$

$$\dot{q} V = \frac{(180)\pi d^2 L}{4} = 15.7\text{ Btu/hr}$$

$$\frac{A15.7}{A} + 350\frac{A}{\pi} + hA(T_\infty - T_w) - \varepsilon\sigma A T_w{}^4 - h_D A C_w h_{fg} = 0$$

$$h = \frac{k_f}{d}C\left(\frac{\rho u_\infty d}{\mu}\right)^n \qquad \frac{h}{h_D} = \rho c_p\left(\frac{\text{Sc}}{\Pr}\right)^{2/3}$$

Assume value of T_w, evaluate properties at film temperature.

Solve for h, h_0, and Re_d.

$C = 0.174 \quad \dfrac{\text{Sc}}{\text{Pr}} = 0.85 \quad n = 0.618 \quad h_{fg} = 1057 \text{ Btu/lbm}$

$C_w = 0.000976 \text{ lbm/ft}^3$

By trial and error: $T_w = 51.785°\text{F}$

11–16

$T_\infty = 65°\text{C} \quad L = 0.3 \text{ m} \quad u_\infty = 6 \text{ m/sec} \quad T_w = 38°\text{C} = 311 \text{ K}$

$T_f = \dfrac{38+65}{2} = 51.5°\text{C} = 324.5 \text{ K} \quad \rho_f = 1.088 \quad \mu_f = 2.03\times10^{-5}$

$c_p = 1008 \quad k_f = 0.0281 \quad \text{Sc} = 0.6 \quad \text{Pr} = 0.7$

$$\text{Re} = \frac{(1088)(6)(0.3)}{2.03\times10^{-5}} = 96{,}472$$

$$\bar{h} = \frac{0.0281}{0.3}(0.664)(96{,}472)^{1/2}(0.7)^{1/3} = 17.5 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$h_D = \frac{17.15}{(1.088)(1008)}\left(\frac{0.7}{0.6}\right)^{2/3} = 0.0173$$

$$hA(T_\infty - T_w) + \sigma A(T_\infty{}^4 - T_w{}^4) + m_w i_w(25°\text{C}) = h_D C_w h_{fg}$$

i_w is neg. compared to h_{fg}. m_w is determined by energy balance:

$h_{fg} = 2.41\times10^6$ at 38°C

$$m_w = \frac{(0.09)[(17.15)(65-38)+(5.669\times10^{-8})(338^4-311^4)]}{2.41\times10^6}$$
$$= 2.51\times10^{-5} \text{ kg/sec} = 0.09 \text{ kg/hr}$$

11-20

$T_\infty = 35°C = 95°F$, $u = 250$ m/min = 224 mi/day

Evap rate = 350 g/m^2-hr = 32.52 g/ft^2-hr = 780 g/ft^2 –day

= 1.719 lbm/ft^2-day

$A = \pi(4)^2/4 = 12.57$ ft^2

1 inch depth = (12.57)(1/12)(62.4) = 65.35 lbm

Evap rate (in day) = (1.719)(12.57)/65.35 = 0.331

Eq. (11-36):

$0.331 = [0.37 + (0.0041)(224)](p_s - p_w)^{0.88}$

At 95°F $p_s = 1.662$ in. Hg

Solving; $p_w = 1.662 - 0.2135 = 1.448$ in Hg

Relative humidity = 1.448/1.662 = 87 %

11-21

$\bar{u} = 2.2 \text{ m/s} = 118 \text{ mi/day}$ $\quad T_\infty = 20°C$ $\quad RH = 40\%$

at 20°C $\quad p_s = 2.34 \text{ kPa} = 0.0691 \text{ in Hg}$

$p_w = (0.4)(0.0691) = 0.0276 \text{ in Hg}$

$$E_{lp} = [0.37 + (0.0041)(118)](0.0691 - 0.0276)^{0.88}$$

$$= 0.052 \text{ in./day} = 0.1313 \text{ cm/day} = 0.0055 \text{ cm/hr}$$

$$A = \pi(2)^2 = 12.57 \text{ ft}^2 = 1.1675 \text{ m}^2$$

$$\frac{E_{lp}}{A} = \frac{0.0055}{1.1675} = 0.00471 \frac{\text{cm}}{\text{m}^2 \cdot \text{hr}} = 47.1 \frac{\text{g}}{\text{m}^2 \cdot \text{hr}}$$

For $\bar{u} = 0$ $\quad \dfrac{E_{lp}}{A} = 20.4 \dfrac{\text{g}}{\text{m}^2 \cdot \text{hr}}$

11-22

$$E_{lp} = 0.3 \ \frac{\text{g}}{\text{s}\cdot\text{m}^2} = 1080 \ \frac{\text{g}}{\text{h}\cdot\text{m}^2} = (1080)(1.1675) = 1261 \ \text{g/h} = 1.02 \ \text{in./day}$$

for standard pan, 4 ft diameter

$\bar{u} = 4.5$ m/s $= 241.6$ mi/day $\qquad E_{lp} = (0.37 + 0.0041\bar{u})(p_s - p_w)^{0.88}$

$1.02 = [0.37 + (0.0041)(241.6)](p_s - p_w)^{0.88}$

$p_s - p_w = 0.721$ in Hg $= (1 - \text{RH})p_s$

at 40°C $\qquad p_s = 7.39$ kPa $= 2.18$ in Hg

$0.721 = (1 - \text{RH})(2.18)$

$\text{RH} = 67\%$

11-23

(a) Pasadena $\qquad \text{RH} = 30\% \qquad T_\infty = 80°\text{F}$

Assume breeze at 3 mi/hr $= 72$ mi/day $= \bar{u}$

$p_s = 0.5073$ psia $= 1.033$ in Hg

$p_w = (0.3)(0.5073) = 0.152$ psia $= 0.31$ in Hg

Floating Pan

$$E_{lp} = 0.8(0.37 + 0.0041\bar{u})(p_s - p_w)^{0.88}$$

$$= (0.8)[0.37 + (0.0041)(72)](1.033 - 0.31)^{0.88} = 0.4 \ \text{in./day}$$

(b) Phoenix $\qquad \text{RH} = 5\% \qquad T_\infty = 110°\text{F}$

$p_s = 1.276$ psia $= 2.6$ in Hg $\qquad p_w = 0.05p_s = 0.13$ in Hg

$E_{lp} = (0.8)[0.37 + 0.0041(72)](2.6 - 0.13)^{0.88} = 1.18$ in./day

11-24

$1 \ \text{mm/day} = \dfrac{\text{in./day}}{25.4} \qquad 1 \ \text{m/s} = 53.69 \ \text{mi/day}$

$1 \ \text{Pa} = 2.953 \times 10^{-4}$ in Hg

$E(\text{mm/day}) = (0.00736 + 0.00427\bar{u})(p_s - p_w)^{0.88}$

p_s and p_w in Pa $\qquad \bar{u}$ in m/s

Chapter 12

12-1

Air @ 300K k = 0.0262

Vertical plate $GrPr = 10^4$; $Nu = hL/k = 54.2(\Delta TL^3)^{1/4}$

Fig 12-8 $GrPr = 10^4$ $Nu = 6$

$GrPr = 10^9$ $Nu = 100$

$GrPr = 10^{12}$ $Nu = 1000$

$6 = 54.2\ (\Delta TL_1)^{1/4}$

$100 = 54.2\ (\Delta TL_2^3)^{1/4} = 50(\Delta TL_2^3)^{1/3}$

$1000 = 50(\Delta TL_3^3)^{1/3}$

For $\Delta T = 10°C$: $L_1 = 0.015 m$, $L_2 = 0.64$ m, $L_3 = 641$ m

For $\Delta T = 20°C$: $L_1 = 0.012$ m, $L_2 = 0.51$ m, $L_3 = 510$ m

12-2 Open ended problem

12-3

d = 15 mm $T_s = 800$ K, $T_a = 300K$ $A_1 = A_2 = \pi d/2 = 0.0236\ m^2 m$

$A_d = d = 0.015$ $\varepsilon = 0.7$

$0.7\sigma(0.0236)(800^4 - T_2^4) = 0.02136\ h(T_2 - 300) + \sigma(T_2^4 - T_1^4)?103$

$\sigma(T_2^4 - T_1^4)?103 = 0.0236\ h\ _1 - 300)$

In example 12-10 for d = 15 mm $Re_d = 6194$, and h = 37.9 W/m^2-°C

Substituting in above equations, $T_1 = 354$ K, $T_2 = 567$ K and q = 287 W/m

12-4

Water, H = 50 mm, W = 25 mm $T_f = 300$ K

Table A-9: $GrPr = (0.05)^3(20)(1.91\times10^{10}) = 4.78\times10^7$

$Pr = 5.85 \qquad k = 0.614$

Fig. 12-8 Nu = 60

$h = (60)(0.614)/0.05 = 737$

$q = (737)(0.05)(0.025)(20) = 18.4$ W

Fig. 12-12(a) $\delta = 2.5$ mm

Fig. 12-14 $u_{max} =$ 110 mm/s

12-5

Air   Fig. 12-11 $GrPr = 3.5\times10^5$

Fig. 12-8 Nu = 14

$h = (14)(0.02624)/0.05 = 7.35$

$q = 97.35)(0.05)(0.025)(20) = 0.184$ w

99.9% TOO LOW

12-6

L = 5 m   p = 1 atm, u = 300 mi/h = 134 m/s, $T_f = 300$ K

$\nu = 15.2\times10^{-6}$ k = 0.02624, Pr = 0.7

$Re = (134)(5)/15.2\times10^{-6} = 4.27 \times 10^7$

Fig. 12-3, Nu = 50000

$h = (50000)(0.02624)/5 = 262.4$

$q = (2)(5)(262)(20) = 52800$ W/ m length

12-7

$\lambda_1 T = 2900$ μ-K

$\lambda_2 T = 11600$ μ-K

Fig. 12-9, Fraction = 0.97 – 0.26 = 0.68

12-8

Vertical heater surface makes no sense!

12-9

Re = 10000, T = 300 K, k = 0.02624, Pr = 0.7

Take d = 1 cm = 10 mm

$h = (k/d)(0.023)Re^{0.8}\, Pr^{0.4} = 82.9$

At Re = 100, and taking d/L = 1/20, Gz = (100)(0.7)/20 = 3.5

Fig. 12-5, Nu = 3

h = (3)().02624)/0.01 = 7.87

12-10

k = 0.69, ρ = 1600, c = 800

$h = 1.42((400\text{-}300)/0.1)^{1/4} = 8$

Volume = (0.05)(0.1)(0.2) =0.001 m^3

Area = (2)[(0.05)(0.1) +(0.1)(0.2) + (0.05)(0.2)] = 0.07 m^2

h(V/A)/k = (8)(0.001/0.07)/0.69 = 0.127

If assume lumped capacity, $T_0 = 350$, $T_i = 400$, $T_\infty = 300$

$\theta_0/\theta_i = 0.5$

Fig. 4-13, BiFo = 0.7 = $(8)(0.07/0.001)\tau/(1600)(800)$

And $\tau = 1600$ sec

12-11

$T_f = 325$ K $\nu = 13\times10^{-6}$, $k = 0.024$, $Pr = 0.7$, $u_\infty = 50$, $L = 1.0$

$Re_L = (50)(1)/13\times10^{-6} = 3.85\times10^6$

Fig. 12-3, $Nu = (0.7)^{1/3}(7000) = 6215$

$h = (6215)(0.024)/1 = 149$

$q = (149)(1)(350\text{-}300) = 7458$ W/m depth

12-12

$Re_d = (50)(0.3)/13\times10^{-6} = 1.15\times10^6$

Fig. 12-6, $Nu_d = (1300)(0.7)^{1/3} = 1154$

$h = (1154)(0.024)/0.3 = 92.3$

$q/L = (92.3)(\pi)(0.3)(350\text{-}300) = 4352$ W/m

12-13

$Re_d = (50)(0.002)/15.7\times10^{-6} = 6400$; $k = 0.02624$

Fig. 12-7; $(Nu - 2)/Pr_\infty^{0.4}(\mu_\infty/\mu_w)^{1/4} = 55$

$\mu_\infty/\mu_w = 2.08/1.85 = 1.12$

$Nu_{d,\infty} = (55)(1.12)^{1/4}(0.7)^{0.4} + 2 = 51.1$

$h = (51.1)().02624)/0.002 = 670$

12-14 Open ended

12-15

$T_f = 325$ K, $\nu = 18\times10^{-6}$, k= 0.024, Pr = 0.7

$GrPr = (9.8)(1/325)(50)(0.025\times10^{-3})^3(0.7)/(18\times10^{-6})^2 = 5\times10^{-5}$

Fig. 12-9: Nu = 0.38

h = (0.38)(0.024)/0.000025 = 365

q = (365)π(0.000025)(50) = 1.43 W

12-16

$T_f = 325$ K

$GrPr = (9.8)(1/3250950090.002)^3(0.7)/(18\times10^{-6})^2 = 17.3$

Fig. 12-10 Nu = 2.9

h = (2.9)(0.024)/0.002 = 3.48

12-17

$GrPr = (9.8)((1/325)(50)(0.07)^3(0.7)/(18\times10^{-6})^2 = 1.1\times10^6$

Fig. 12-11; $\delta = (18)(20/50)^{1/4} = 14.3$ mm

Fig. 12-13; $u_{max} = 130$ mm/s

12-18

Fig. 12-12(a) $\delta = 2.7$ mm

Fig. 12-14(a) $u_{max} = 17$ mm/s

12-19

Liquid water $k = 0.61$

$Gr_\delta Pr = (1.91\times10^{10})(20)(0.01)^3 = 3.8\times10^5$

Fig. 12-15; $k_{eff}/k = 6.2$

$q/A = (6.2)(0.61)(20)/(0.01) = 7560 \text{ W/m}^2$

12-20

$Gr_\delta Pr = (9.8)(1/300)(20)(0.01)^3(.7)/(15.7\times10^{-6})^2 = 1865$

Fig 12-17; $k_{eff}/k = 1.3$

$q/A = (1.3)(0.026)(20)/0.01 = 67.6 \text{ W/m}^2$

12-21

$p_1 = 1$ atm, air at 300 K $\delta = 10$ cm, $\Delta T = 20°C$, $p_2 = 0.000001$ atm

Fig. 12-18(a) $q/A]_{1\text{ atm}} \approx 4.5 \text{ W/m}^2$

Fig. 12-18(b) $q/A]_{0.000001\text{ atm}} \approx 1.4$

Percent reduction = 69%

12-22

$\lambda T = (20)(300) = 6000\ \mu\text{-K}$

Fig. 12-19 Fraction below 20 μ = 0.735

12-23

$d = 4$ cm $T_1 = 475$ K $T_2 = 300$K, $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.7$

$A_1 = 4\pi(0.02)^2 = 0.005$

$A_2 = (6)(0.06)^2 = 0.022$

Fig. 12-20; $(1/\varepsilon_2 - 1)(A_1/A_2) = 0.0974$;

$q/A_1(E_{b1} - E_{b2}) = 0.48$

$E_{b1} = 2886$

$E_{b2} = 459$

$q = (0.48)(0.005)(2886 - 459) = 5.8$ W

12-24

$\varepsilon = 0.7$, $T = 273 + 200 = 473$ K, $d = 0.1$ m

Table 7-2, $h_{conv} = 1.32((473\text{-}300)/0.1)^{1/4} = 8.51$

Fig. 12-21; $h_{rad} = (0.7)(14) = 9.8$

$q = (8.51 + 9.8)(\pi)(0.1)(473 - 300) = 995$ W/m length

12-25

$p = 0.1$ MPa, $\Delta T_x = 10°C$

Fig. 12-23 ; $h \approx 0.0023$

Fig. 12-25; $h \approx 0.0045$

Not too bad considering equations upon which charts are based.

Also compare Figures 9-8 and 12-24 for $\Delta T_x = 10°C$ and $p = 2.6$ Mpa

Fig. 9-8; $h \approx 0.07$

Fig. 12-24 $h \approx 0.09$ agreement is rather good

Chapter 12

12-1

Air @ 300K $k = 0.0262$

Vertical plate $GrPr = 10^4$; $Nu = hL/k = 54.2(\Delta TL^3)^{1/4}$

Fig 12-8 $GrPr = 10^4$ $Nu = 6$

$GrPr = 10^9$ $Nu = 100$

$GrPr = 10^{12}$ $Nu = 1000$

$6 = 54.2\ (\Delta TL_1)^{1/4}$

$100 = 54.2\ (\Delta TL_2^3)^{1/4} = 50(\Delta TL_2^3)^{1/3}$

$1000 = 50(\Delta TL_3^3)^{1/3}$

For $\Delta T = 10°C$: $L_1 = 0.015m$, $L_2 = 0.64$ m, $L_3 = 641$ m

For $\Delta T = 20°C$: $L_1 = 0.012$ m, $L_2 = 0.51$ m, $L_3 = 510$ m

12-2 Open ended problem

12-3

$d = 15$ mm $T_s = 800$ K, $T_a = 300K$ $A_1 = A_2 = \pi d/2 = 0.0236\ m^2m$

$A_d = d = 0.015$ $\varepsilon = 0.7$

$0.7\sigma(0.0236)(800^4 - T_2^4) = 0.02136\ h(T_2 - 300) + \sigma(T_2^4\text{-}T_1^4)?103$

$\sigma(T_2^4 - T_1^4)?103 = 0.0236\ h\ _1 - 300)$

In example 12-10 for d = 15 mm $Re_d = 6194$, and $h = 37.9\ W/m^2\text{-}°C$

Substituting in above equations, $T_1 = 354$ K, $T_2 = 567$ K and $q = 287$ W/m

12-4

Water, H = 50 mm, W = 25 mm $T_f = 300$ K

Table A-9: $GrPr = (0.05)^3(20)(1.91\times10^{10}) = 4.78\times10^7$

$Pr = 5.85 \qquad k = 0.614$

Fig. 12-8 Nu = 60

$h = (60)(0.614)/0.05 = 737$

$q = (737)(0.05)(0.025)(20) = 18.4$ W

Fig. 12-12(a) $\delta = 2.5$ mm

Fig. 12-14 $u_{max} =$ 110 mm/s

12-5

Air Fig. 12-11 $GrPr = 3.5\times10^5$

Fig. 12-8 Nu = 14

$h = (14)(0.02624)/0.05 = 7.35$

$q = 97.35)(0.05)(0.025)(20) = 0.184$ w

99.9% TOO LOW

12-6

L = 5 m p = 1 atm, u = 300 mi/h = 134 m/s, $T_f = 300$ K

$\nu = 15.2\times10^{-6}$ k = 0.02624, Pr = 0.7

$Re = (134)(5)/15.2\times10^{-6} = 4.27\times10^7$

Fig. 12-3, Nu = 50000

$h = (50000)(0.02624)/5 = 262.4$

$q = (2)(5)(262)(20) = 52800$ W/ m length

12-7

$\lambda_1 T = 2900\ \mu\text{-K}$

$\lambda_2 T = 11600\ \mu\text{-K}$

Fig. 12-9, Fraction = 0.97 – 0.26 = 0.68

12-8

Vertical heater surface makes no sense!

12-9

Re = 10000, T = 300 K, k = 0.02624, Pr = 0.7

Take d = 1 cm = 10 mm

$h = (k/d)(0.023)Re^{0.8}\ Pr^{0.4} = 82.9$

At Re = 100, and taking d/L = 1/20, Gz = (100)(0.7)/20 = 3.5

Fig. 12-5, Nu = 3

h = (3)().02624)/0.01 = 7.87

12-10

$k = 0.69$, $\rho = 1600$, $c = 800$

$h = 1.42((400\text{-}300)/0.1)^{1/4} = 8$

Volume = (0.05)(0.1)(0.2) =0.001 m^3

Area = (2)[(0.05)(0.1) +(0.1)(0.2) + (0.05)(0.2)] = 0.07 m^2

h(V/A)/k = (8)(0.001/0.07)/0.69 = 0.127

If assume lumped capacity, $T_0 = 350$, $T_i = 400$, $T_\infty = 300$

$\theta_0/\theta_i = 0.5$

Fig. 4-13, $BiFo = 0.7 = (8)(0.07/0.001)\tau/(1600)(800)$

And $\tau = 1600$ sec

12-11

$T_f = 325$ K $\nu = 13\times10^{-6}$, $k = 0.024$, $Pr = 0.7$, $u_\infty = 50$, $L = 1.0$

$Re_L = (50)(1)/13\times10^{-6} = 3.85\times10^{6}$

Fig. 12-3, $Nu = (0.7)^{1/3}(7000) = 6215$

$h = (6215)(0.024)/1 = 149$

$q = (149)(1)(350\text{-}300) = 7458$ W/m depth

12-12

$Re_d = (50)(0.3)/13\times10^{-6} = 1.15\times10^{6}$

Fig. 12-6, $Nu_d = (1300)(0.7)^{1/3} = 1154$

$h = (1154)(0.024)/0.3 = 92.3$

$q/L = (92.3)(\pi)(0.3)(350\text{-}300) = 4352$ W/m

12-13

$Re_d = (50)(0.002)/15.7\times10^{-6} = 6400$; $k = 0.02624$

Fig. 12-7; $(Nu - 2)/Pr_\infty^{0.4}(\mu_\infty/\mu_w)^{1/4} = 55$

$\mu_\infty/\mu_w = 2.08/1.85 = 1.12$

$Nu_{d,\infty} = (55)(1.12)^{1/4}(0.7)^{0.4} + 2 = 51.1$

$h = (51.1)().02624)/0.002 = 670$

12-14 Open ended

12-15

$T_f = 325$ K, $\nu = 18\times10^{-6}$, k= 0.024, Pr = 0.7

$GrPr = (9.8)(1/325)(50)(0.025\times10^{-3})^3(0.7)/(18\times10^{-6})^2 = 5\times10^{-5}$

Fig. 12-9: Nu = 0.38

h = (0.38)(0.024)/0.000025 = 365

q = (365)π(0.000025)(50) = 1.43 W

12-16

$T_f = 325$ K

$GrPr = (9.8)(1/3250950090.002)^3(0.7)/(18\times10^{-6})^2 = 17.3$

Fig. 12-10 Nu = 2.9

h = (2.9)(0.024)/0.002 = 3.48

12-17

$GrPr = (9.8)((1/325)(50)(0.07)^3(0.7)/(18\times10^{-6})^2 = 1.1\times10^6$

Fig. 12-11; $\delta = (18)(20/50)^{1/4} = 14.3$ mm

Fig. 12-13; $u_{max} = 130$ mm/s

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$q/A = (6.2)(0.61)(20)/(0.01) = 7560\ W/m^2$

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