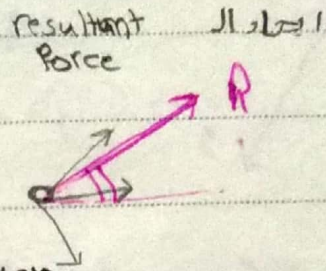


L2: Statics of particles

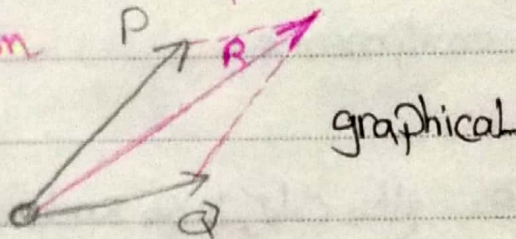
→ equilibrium → $\sum F = 0$



$F =$ Vector → magnitude [XN], direction

المقدار magnitude

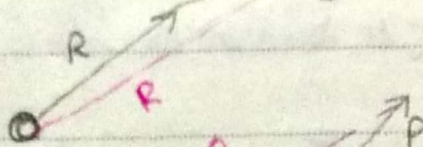
الاتجاه direction



Triangle rule + Trapezoid ⇒ graphical.

Law of cosines :-

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$

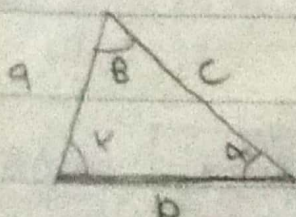


الزاوية بين P و Q الزاوية بين الرأسين
head to tail

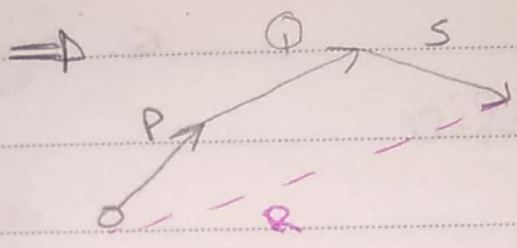
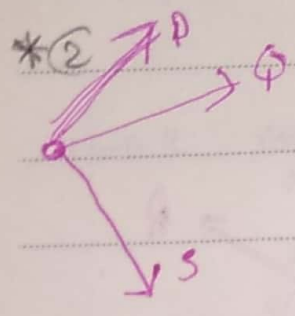
Law of sines :-

$$\frac{\sin A}{Q} = \frac{\sin B}{P} = \frac{\sin C}{R}$$

• $\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$, $\vec{P} - \vec{Q} = \vec{Q} - \vec{P}$



$$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$$\vec{P} + \vec{Q} + \vec{S} \quad (1) *$$

$$\downarrow$$

$$\vec{P} + \vec{R}_1$$

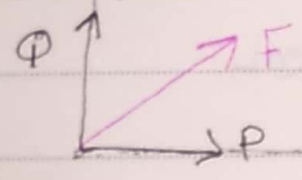
$$\downarrow$$

$$\vec{R}$$

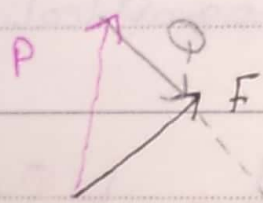
* Concurrent forces → Point of application
 يتطابقوا على نفس الـ Point of application.

Vector Force components

تحليل القوة إلى مركبات



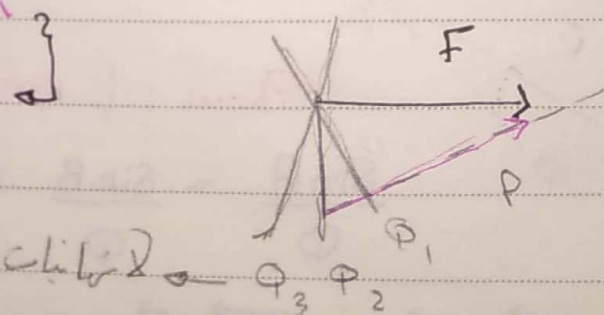
إذا عرف F و عرف واحدة من المركبات يتطلع المركبة التامة



إذا عرف واحد من direction للمركبات و F

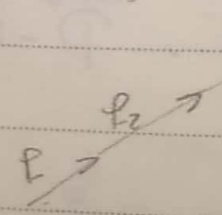
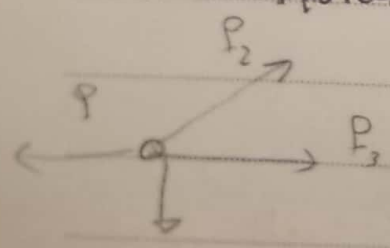
* minimum

$Q \perp P$



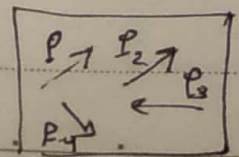
Concurrent Forces

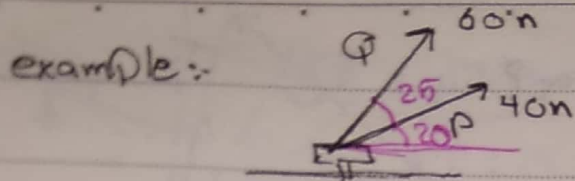
Collinear Force



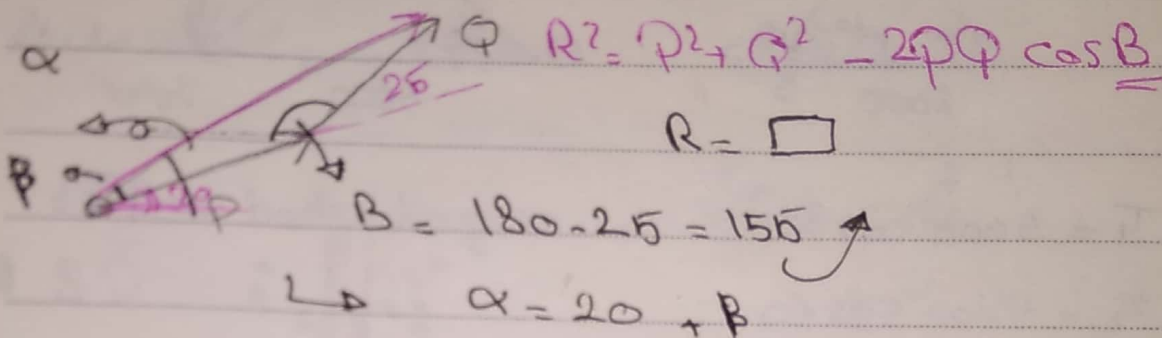
Line of action يصبح أو يتطابق

Coplanar Forces





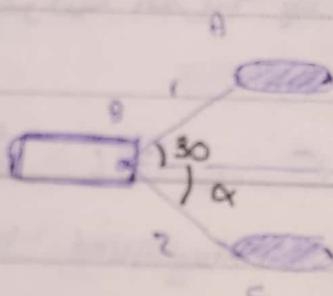
determine the R



$$\frac{60}{\sin B} = \frac{R}{\sin 155} = B = \square$$

L3:

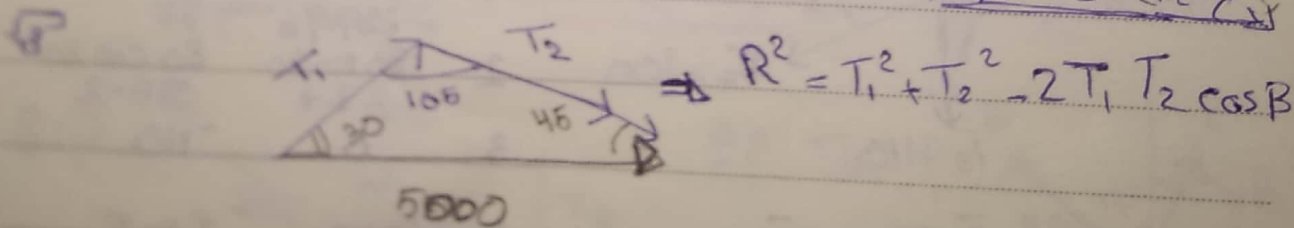
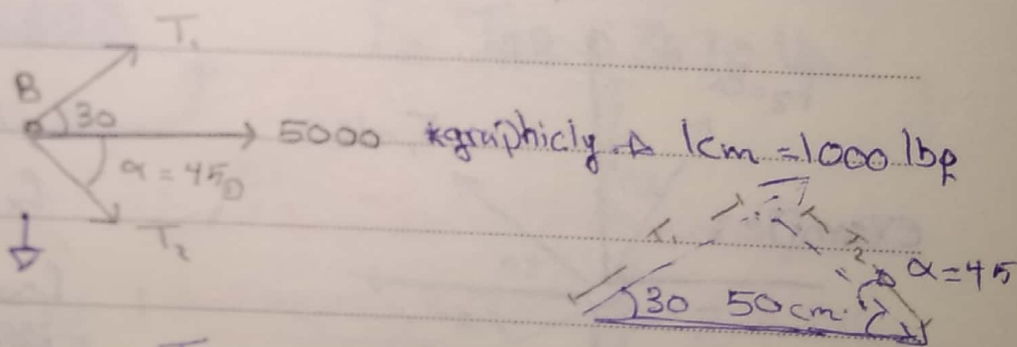
example:



$R = 5000 \text{ lbf}$

$T_1, T_2? : \alpha = 45^\circ$

2) $\alpha? : T_2$ minimum



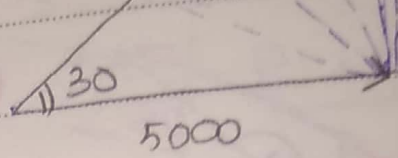
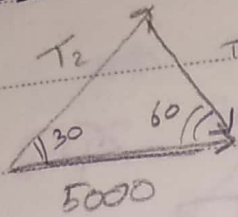
$$\frac{5000}{\sin 105} = \frac{T_1}{\sin 45} = \frac{T_2}{\sin 30} \Rightarrow T_1 = 3660.2 \text{ lb}$$

$$\Rightarrow T_2 = 2588.1 \text{ lb}$$

NO.

DATE / /

2) ~~F_2 ? $\alpha = \text{min}$~~ T_2 minimum : α ?

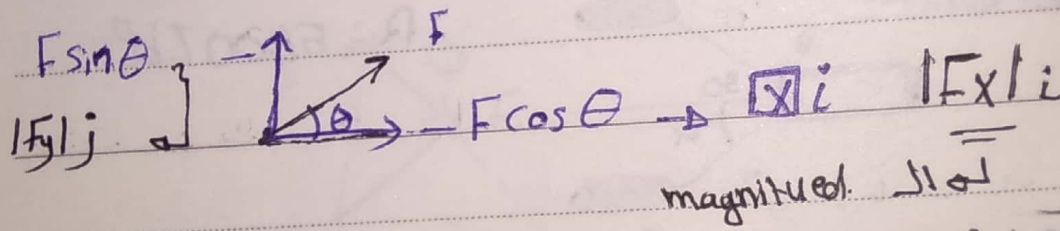


$$T_1 = 5000 \cos 30$$

$$T_2 = 5000 \cos 60$$

3 Rectangle component of force

* ينحلل القوة الى x_{com} و y_{com} ويكون مساحة \sin و \cos بالمخيل : الحركة القريبة يتأخذ \cos والبعيدة يتأخذ \sin



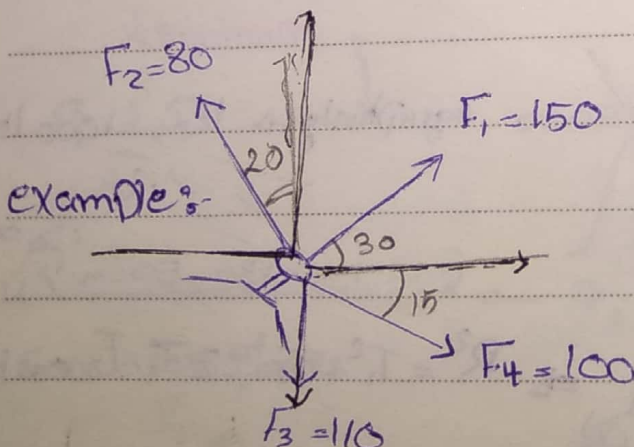
magnitude: $|F|$

$$* F = F_x \bar{i} + F_y \bar{j}$$

$$(\sum F_x) \bar{i} + (\sum F_y) \bar{j}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1} \frac{R_y}{R_x}$$



	F_x	F_y
1	$F_1 \cos 30 (N)$ 129.9	$F_1 \sin 30 (N)$ 75
2	$-80 \sin 20$ -27.4	$80 \cos 20$ 75.2
3	0	-110

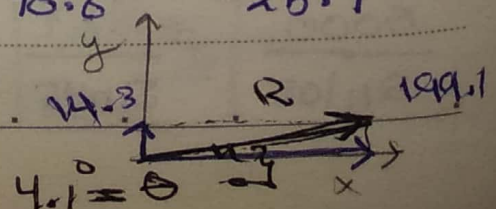
$$R = 199.1 \bar{i} + 14.3 \bar{j}$$

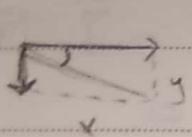
	F_x	F_y
4	$F \cos 15$ 96.6	$-F \sin 15$ -25.9

$$R = \sqrt{(199.1)^2 + (14.3)^2}$$

$$= 199.6 N$$

$$\theta = \tan^{-1} \frac{14.3}{199.1}$$





equilibrium of a particle

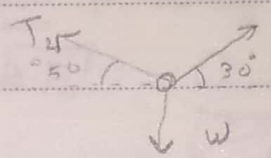
$\Sigma F = 0$

بإحداثيات الزاوية

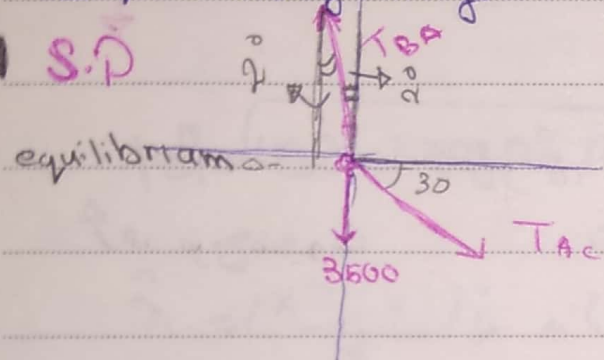
$\Sigma F_x = 0$

$\Sigma F_y = 0$

free body diagram



S.P



TAC, TAB ?

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$T_{AC} \sin 30 - T_{AB} \sin 2 = 0$

$0 = T_{AB} \cos 2 - 35000 - T_{AC} \sin 30$

$T_{AC} \cos 30 = T_{AB} \sin 2$

$25.7 T_{AC} \cos 2 - 3500 - T_{AC} \sin 30$

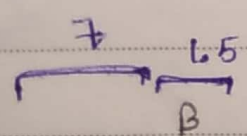
$0.86 T_{AC} = 0.034 T_{AB}$

$T_{AC} = 141.18 \text{ lb}$

$T_{AB} = 25.8 T_{AC}$

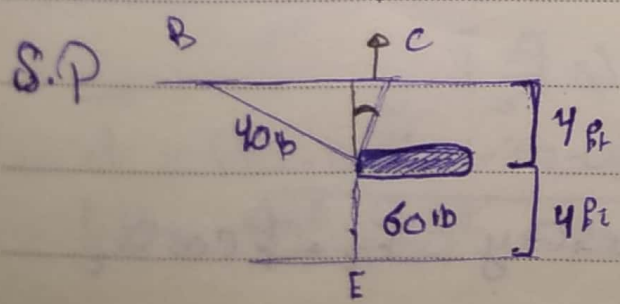
$T_{AB} = 3570 \text{ lb}$

3500 = TAB و TAC * الفرق بين الزاوية



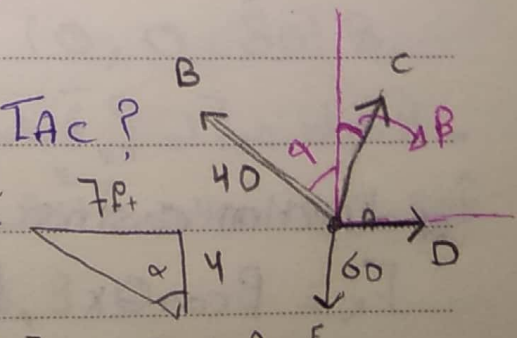
(solution check)

الزاوية من الـ 7

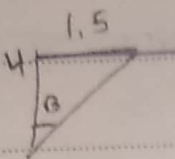


PP TAC ?

$\alpha =$



$\tan^{-1} \frac{7}{4} = 60.25^\circ$

$$B = \tan^{-1} \frac{1.5}{4} = 20.5$$


$$\sum F_x = 0$$

$$-40 \sin 40.25 + T_{AC} \sin 20.55 + D = 0$$

$$D = \boxed{19.6} \text{ lb}$$

$$\sum F_y = 0$$

$$40 \cos 40.25 + T_{AC} \cos 20.55 - 60 = 0$$

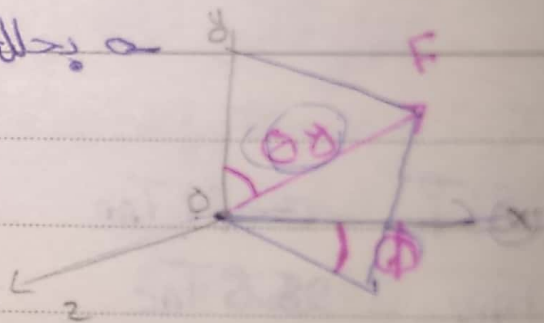
$$T_{AC} = \boxed{43.8} \text{ lb}$$

* Rectangular component of a force in space.

Plan XZ is θ_y angle of the force with the Y-axis

$$F_y = F \cos \theta_y$$

$$F_{xz} = F \sin \theta_y$$



$$F_x = F_{xz} \cos \phi, \quad F_z = F_{xz} \sin \phi$$

$$F \sin \theta_y$$

$$F = F_x \bar{i} + F_y \bar{j} + F_z \bar{k}$$

* direction cosines

$$F_x = F \cos \theta_x, \quad F_y = F \cos \theta_y, \quad F_z = F \cos \theta_z$$

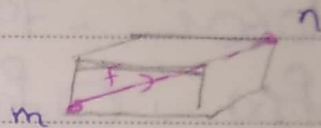
$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$= F \hat{n} \quad \hat{n} = \cos \theta_x \vec{i} + \cos \theta_y \vec{j} + \cos \theta_z \vec{k}$$

magnitud \vec{r}

magnitud $\hat{n} = 1 \rightarrow \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$

$$\vec{d} = dx \vec{i} + dy \vec{j} + dz \vec{k}$$



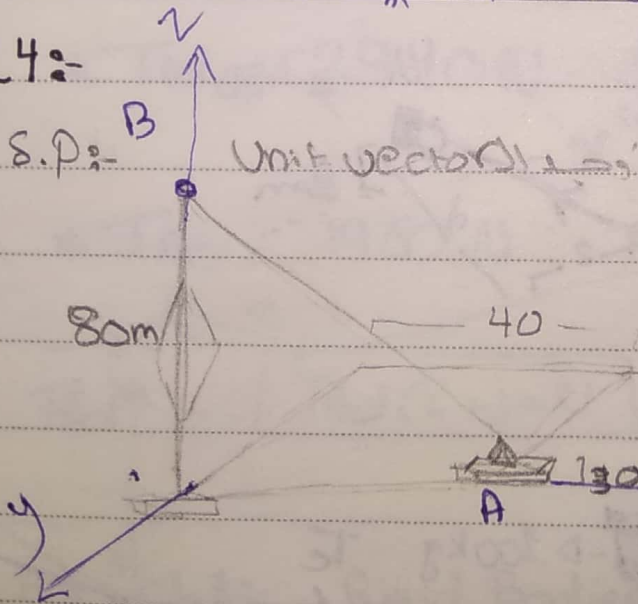
$$|\vec{d}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\hat{n} = \frac{(x_2 - x_1)}{d} \vec{i} + \frac{(y_2 - y_1)}{d} \vec{j} + \frac{(z_2 - z_1)}{d} \vec{k}$$

$$\cos \theta_x + \cos \theta_y + \cos \theta_z$$

L4:-

S.P:-



Unit vector \hat{n} direction of \vec{AB}

أوجد متجه الوحدة \hat{n} باتجاه \vec{AB} .

$$(40, -30, 0) \text{ A}$$

$$(0, 0, 80) \text{ B}$$

المتجه من B إلى A \vec{BA} \hat{n} \vec{BA} \hat{n}

$$\vec{d} = (40 - 0) \vec{i} + (-30 - 0) \vec{j} + (0 - 80) \vec{k} = 40\vec{i} - 30\vec{j} - 80\vec{k}$$

$$|\vec{d}| = \sqrt{40^2 + 30^2 + 80^2} = 94.3$$

$$\hat{\lambda} = \frac{40}{94.3} \hat{i} - 0.318 \hat{j} - 0.848 \hat{k}$$

$$0.424$$

$$* T = 2500 * \hat{\lambda} \rightarrow 2500 * (0.424 \hat{j} - 0.318 \hat{j} - 0.848 \hat{k})$$

$$\Rightarrow 1060 \hat{i} - 795 \hat{j} - 2120 \hat{k}$$

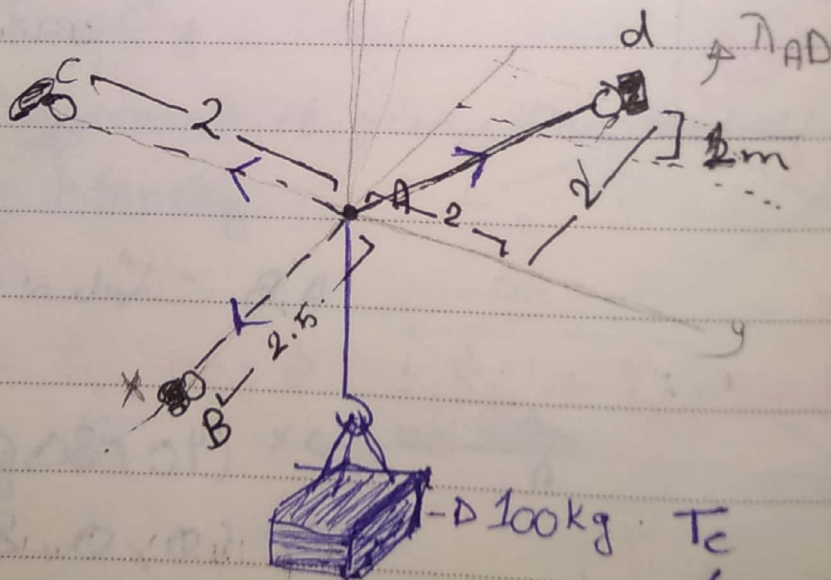
$$P_x \quad P_y \quad P_z$$

$$1060 \text{ N} \quad -795 \text{ N} \quad -2120 \text{ N}$$

$$\Rightarrow \cos^{-1} 0.424 = \theta_x, \quad \cos^{-1} 0.318 = \theta_y$$

$$\cos^{-1} 0.848 = \theta_z$$

S.P.:-



T AB, AC, AD P

$$* \vec{r}_{AD} = (-2\hat{i} + 2\hat{j} + 1\hat{k})$$

$$\hat{\lambda}_{AD} = \frac{-2}{3} \hat{i} + \frac{2}{3} \hat{j} + \frac{1}{3} \hat{k}$$

$$* T_{AD} = -2 T_{AD} \left(\frac{-2}{3} \hat{i} + \frac{2}{3} \hat{j} + \frac{1}{3} \hat{k} \right)$$

$\vec{T}_{AC} = -T_{AC} \hat{j}$, $T_{AB} = T_{AB} \hat{i}$, $\vec{W} = 980 \hat{k}$
 . هذا يعني اننا نكتب (T) في اتجاه \hat{i}

$\sum \epsilon P_x = 0$, $\epsilon P_y = 0$, $\epsilon P_z = 0$



$* -\frac{2}{3} T_{Ad} + T_{AB} = 0 \rightarrow \textcircled{1} \quad (x)$

$* \frac{2}{3} T_{Ad} - T_{AC} = 0 \rightarrow \textcircled{2} \quad (y)$

$* \frac{1}{3} T_{Ad} - 980 = 0 \rightarrow \textcircled{3}$

$* T_{Ad} = 2940 \text{ N}$ عوضه مع المعادله
الاصغوفه (R)
عليه

$* T_{AC} = 1960 \text{ N}$, $T_{AB} = 1960 \text{ N}$

$\vec{T}_{Ad} = T_{Ad} (\hat{i} + \hat{j} + \hat{k})$

L5: ch8: Rigid Bodies "equivalent system of force"

* deformable body

* Rigid body
 $\left\{ \begin{array}{l} \text{Rotation} \rightarrow \text{moment} \\ \text{Translation} \end{array} \right.$

حركة ابد دون اثناء



NO. _____

DATE / /

SUN.

MON.

TUES.

WED.

THUR.

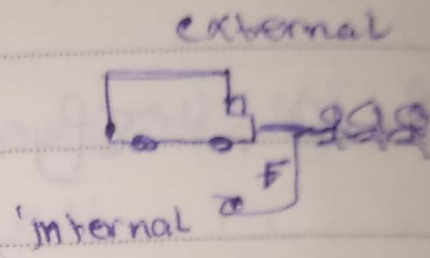
FRI.

SAT.

المركب القوي ك rigidbody لقوة واحدة و moment.

External and internal forces

section cut
القطع الداخلي

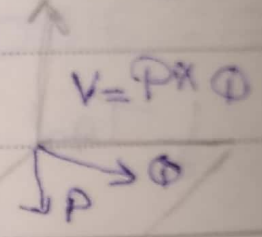


يقدر أحرار Face Force ال Line of action

ويؤثر بقية الأثر

منه دائما صحيح اذا لم يوجد مع قوتها

* Vector Product of two vectors.



مستوى المستوى الى المستوى المتجهين

(P, Q) magnitude و line of action

$|V| = P Q \sin \theta$, direction قاعدة اليد اليمنى

خارج المستوى

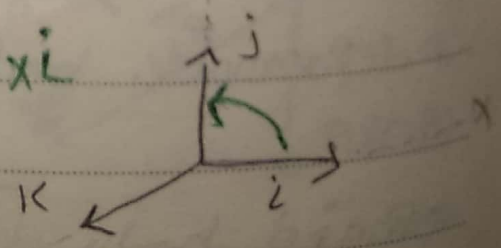
$\bullet Q \times P \neq (P \times Q) \rightarrow -(P \times Q)$ مع magnitude بالمثل متعاكس

$\bullet P(Q_1 + Q_2) = P \times Q_1 + P \times Q_2$

$\bullet (P \times Q) \times S \neq P \times (Q \times S)$

$K = j \times i$

Rectangular component



$i \times i = 0$

$G \cdot P \times Q$

$$P \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \Rightarrow \vec{V} = i(P_y Q_z - P_z Q_y) - j(P_x Q_z - P_z Q_x) + k(P_x Q_y - P_y Q_x)$$

Moment of a force about a point

لا يوجد قوة بدلية يوجد عن القوة الأصلية

$M_o = \vec{r} \times \vec{F}$

direction ↑

$r F \sin \theta$

$\rightarrow Fd$

المسافة العمودية بين القوى ونقطة التأثير

$\vec{r} \times (\vec{F}_1 + \vec{F}_2 + \dots) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \dots$

∴ vector

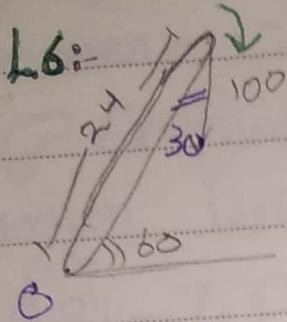
The moment of the group of forces about a point, equal the moment of the resultant about the point.

$$\begin{vmatrix} i & j & k \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$

$$= (yF_z - zP_y)i - (xP_y - yP_x)j + (yF_z - zP_y)k$$

3d $\rightarrow F(i, j), r(i, j)$

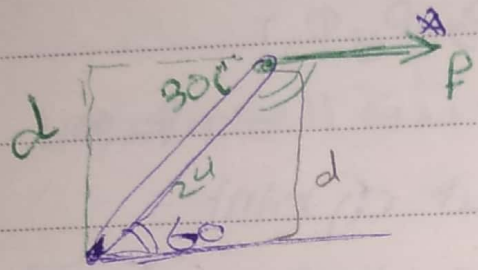
$M \rightarrow k [+ \dots - \dots]$



$M = r \times F = 24 \times 100 \times \sin 30 = 1200 \text{ lb in}$

$d = 24$

قوة انحناء في نقطة B

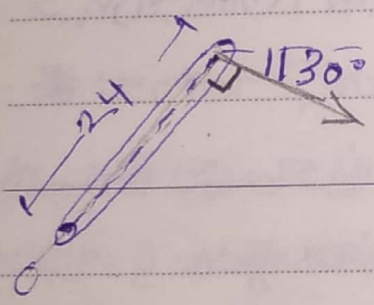


Moment

$M = Pd \Rightarrow 1200 = P \times 24 \cos 30$

$P = 57.7 \text{ lb}$

* Smallest force to produce same M



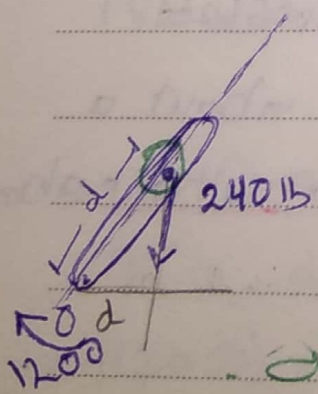
$M = F \times d$

Min. Max

direction $\theta = 30^\circ \perp OA$

$1200 = F \times 24 \Rightarrow F = 50 \text{ lb}$

* Location of vertical force 240 lb

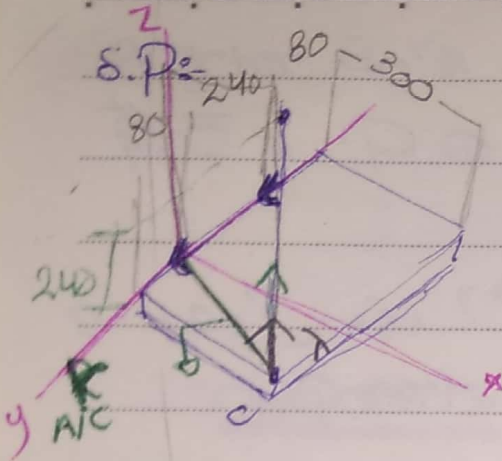


$M = F \times d \Rightarrow 1200 = 240 \times d$

$d = 5$

المسافة $d = OB \cos 60$

$OB = \frac{5}{\cos 60} = 10 \text{ in}$



$$M = \vec{r}_{CA} \times \vec{F}$$

$$A = (0, 0, 0), C = (300, 80, 0)$$

$$\vec{r}_{AC} = 300i + 80j$$

$$\vec{F}$$

الخط القوة \vec{F}

$$\vec{F} = 200 \vec{n} \rightarrow \vec{n} = \frac{d_{CD}}{|d_{CD}|}$$

$$D = (0, -240, 240)$$

$$D \vec{d}_{CD} = -300i - 320j + 240k$$

$$|d_{CD}| = 500 \text{ mm}$$

$$\vec{n} = -0.6i - 0.64j + 0.48k$$

$$\vec{F} = \sqrt{3} 200(\vec{n}) = -120i - 128j + 96k$$

$$\vec{M} = \vec{r}_{CA} \times \vec{F} = \begin{vmatrix} i & j & k \\ 300 & 80 & 0 \\ -120 & -128 & 96 \end{vmatrix} = \begin{matrix} i (80 \times 96 - 0) \\ -j (28800) \\ +k (-28800) \end{matrix}$$

$$\vec{M} = 7680i + 28800j - 28800k$$

$$|M| = 141447 \text{ Nmm}$$

$$\theta_x = \frac{\cos^{-1} 7680}{41497}$$

$$\theta_y = \frac{\cos^{-1} 22800}{41447}$$

$$\theta_z = \frac{-28800}{41447}$$

17:- Scalar product of two vectors

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta$$

أستخدمه غالباً كـ "مخرج زاوية"

$$\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}, \quad P(\vec{Q}_1 + \vec{Q}_2) = \vec{Q}_1 P + \vec{Q}_2 P$$

$$(\vec{P} \cdot \vec{Q}) \cdot \vec{S} \rightarrow \text{كمية قياسية}$$

↳ Scalar quantity

$$\vec{i} \cdot \vec{i} = 1, \quad \vec{j} \cdot \vec{j} = 1, \quad \vec{k} \cdot \vec{k} = 1$$

$$0 = \vec{k} \cdot \vec{j} \quad \text{أو} \quad \vec{i} \cdot \vec{j} = 0$$

أولئك

$$\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\vec{P} \cdot \vec{P} = P^2 \rightarrow PP \cos 0 = P^2$$

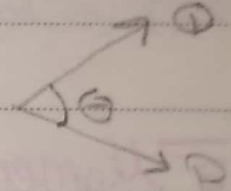
2 vector \vec{P} و \vec{Q} unknown.

$$\vec{P} \cdot \vec{Q} = P Q \cos \theta = P_x Q_x + P_y Q_y + P_z Q_z$$

بواسطة $(\vec{P} \cdot \vec{Q})$ يعرف \vec{P} و \vec{Q} في

components axis حول

3d



Mixed Triple Product of 3 vector

وهو axis $\vec{S} \cdot (\vec{P} \times \vec{Q})$ له اتجاه \vec{S} و \vec{P} و \vec{Q} في

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = \text{scalar result.}$$

حاصل ضرب

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = \vec{P} \cdot (\vec{S} \times \vec{Q}) = \vec{Q} \cdot (\vec{P} \times \vec{S}) = (-1) \vec{Q} \cdot (\vec{S} \times \vec{P})$$

Mixed Triple Product

\vec{S}	S_x	S_y	S_z
\vec{P}	P_x	P_y	P_z
\vec{Q}	Q_x	Q_y	Q_z

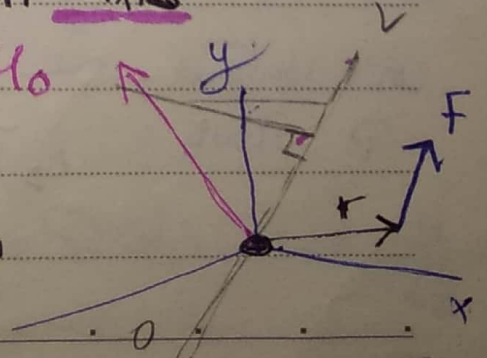
Moment of a force about a given axis

يوجد M_0 حول \vec{F} حول نقطة O

عند M_0 بوجهه حول

بعده النقطتين بين لفتية البقعة O

2. \vec{r} \vec{F} \vec{M}_0 \vec{S} \vec{P} \vec{Q}



$$1) * M_o = \vec{r} \times \vec{F}$$

$$2) * M_{OL} = \bar{\lambda} \cdot M_o = \bar{\lambda} (\vec{r} \times \vec{F})$$

Moment of F about coordinate axes.

$$M_x = yF_z - zF_y$$

$$M_y = zF_x - xF_z$$

$$M_z = xF_y - yF_x$$

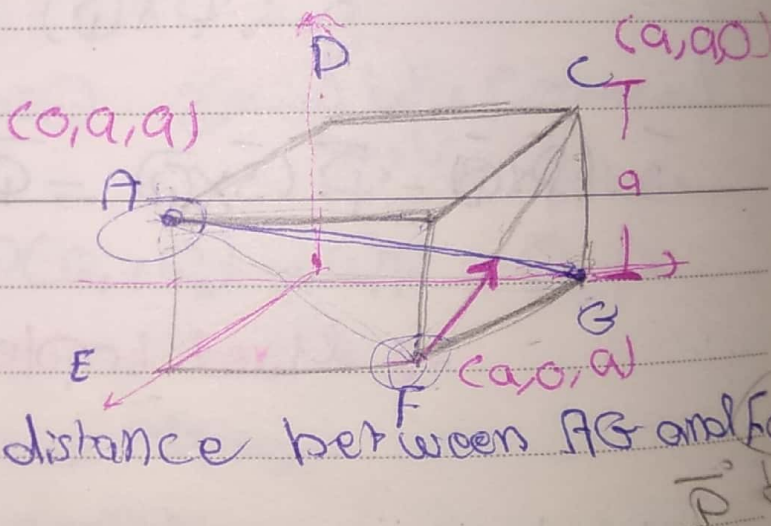
Example:-

a) about A (u)

b) about AB

c) about AG

d) perpendicular distance between AG and FC



$$a) M_A = \vec{r}_{AF} \times \vec{P}$$

$$* \vec{P} = \vec{P} \times \bar{\lambda} \quad \bar{\lambda} = \frac{FC}{|FC|} = \frac{0i + aj - ak}{\sqrt{a^2 + a^2}} = \frac{aj - ak}{\sqrt{2}a}$$

Unit vector $\bar{\lambda}$

P \perp FC

$$\vec{P} = \frac{P}{\sqrt{2}} j - \frac{P}{\sqrt{2}} k$$

$$r_{af} = a_i - a_j - OK$$

$$\Rightarrow \vec{r} \times \vec{p} = \begin{vmatrix} i & j & k \\ a & -a & 0 \\ 0 & \frac{p}{\sqrt{2}} & -\frac{p}{\sqrt{2}} \end{vmatrix} = \frac{ap}{\sqrt{2}} (i + j + k)$$

b) محال AB وحدة unit vector محال
X axis.

$$i \cdot \mu_A = \mu_{AB} = i \cdot \left(\frac{ap}{\sqrt{2}} (i + j + k) \right) = \frac{ap}{\sqrt{2}}$$

محال AB وحدة unit vector محال
reference point

c) AG وحدة unit vector

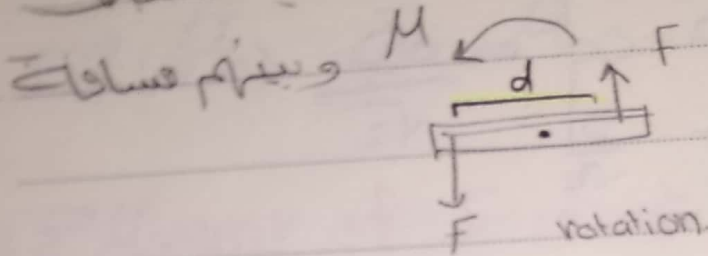
$$\vec{B} \cdot \mu_A = \mu_{AG} = \frac{1}{\sqrt{3}} (i - j - k)$$

$$\frac{1}{\sqrt{3}} (i - j - k) \cdot \left(\frac{ap}{\sqrt{2}} (i + j + k) \right) = \frac{-ap}{\sqrt{6}}$$

$$d) \vec{F} \times \vec{d} = \mu = \frac{-ap}{\sqrt{6}} = \mu d \Rightarrow d = \frac{a}{\sqrt{6}}$$

Moment of a couple

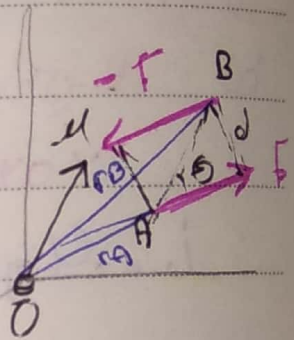
قوة متساويتان في المقدار ومتاكسبات في الاتجاه



$$M = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F})$$

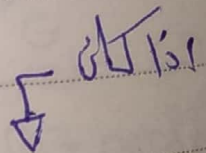
$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

$$= r F \sin \theta = [d]$$



Ps: Moment Vector \vec{M} origin \vec{r} و \vec{F}

couple forces (قوة متساوية)



in the same plane $F_1 d_1 = F_2 d_2$
rotation: $F_1 d_1 = F_2 d_2$

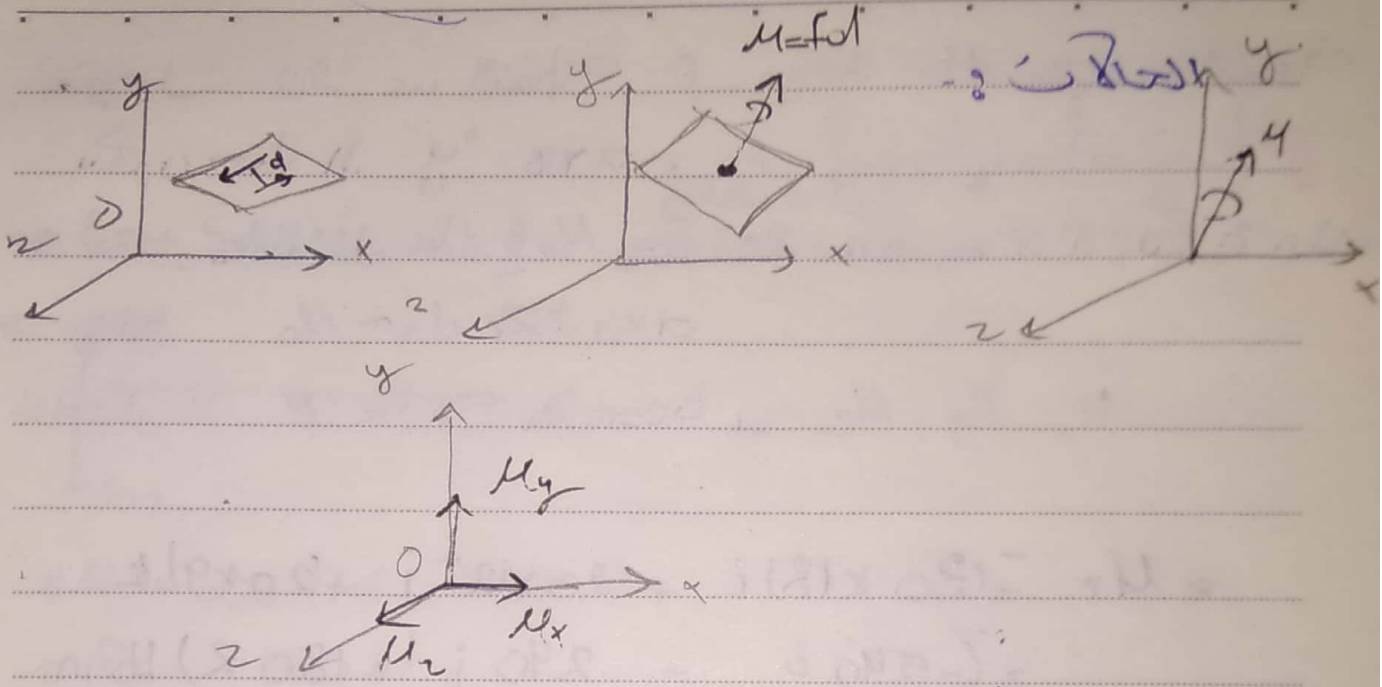
إذا كانا في نفس المستوى couple forces

$$M_1 = \vec{r} \times \vec{F}_1 \text{ in plan } P_1$$

$$M_2 = \vec{r} \times \vec{F}_2 \text{ in plan } P_2$$

لقد وجدنا أنهما في نفس المستوى وأوجدنا \vec{R}

$$M = \vec{r} \times \vec{R} = \vec{r} \cdot (\vec{F}_1 + \vec{F}_2)$$



L8:-

The Moment is the effect of a force to rotate a body about a point or axis. Effect تأثير تدوير القوة على نقطة أو محور في جسم صلب

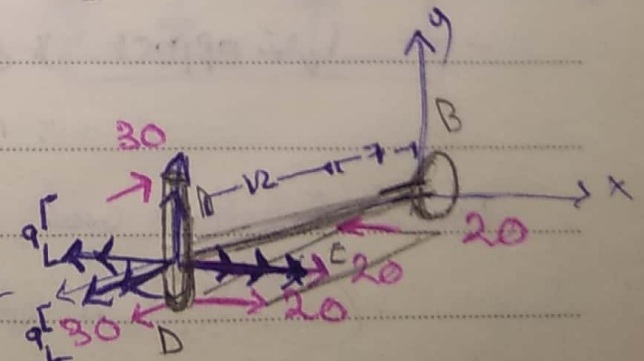
Point of application of the force is the point where the force acts on the body. نقطة تطبيق القوة هي النقطة التي تؤثر عليها القوة على الجسم صلب

$$\vec{M}_O = \vec{r} \times \vec{F}$$
 → free vector

Free vector is a vector that is not attached to any point on the body. متجه حُر هو متجه ليس مترابطاً بأي نقطة على الجسم صلب

SP:-

• The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force. العزم للقوة حول نقطة هو المنتج من القوة والمسافة العمودية من النقطة إلى خط تطبيق القوة



• The moment of a force about an axis is the product of the force and the perpendicular distance from the axis to the line of action of the force. العزم للقوة حول محور هو المنتج من القوة والمسافة العمودية من المحور إلى خط تطبيق القوة

* محور 20 عند الأصل A مع الـ Mo تبعها عند الأصل
يتكون حول الـ y axis

* محور 30 في الـ Mo مع الـ 20 السوية والـ 20 الـ Mo
بعلاوا حول الـ z axis

* مدار الـ M مع الـ 3 com to

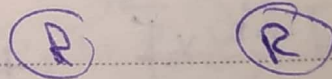
$$M_x = (30 \times 18)i + (20 \times 12)j + (20 \times 9)k$$

$$= (-540i + 240j + 180k) \text{ lb in}$$

System of forces to a force and couple.

Resultant force و Resultant Moment

" Mo, Free vector, بحركو ونأخذ الـ عادي"
" F vector, كما بقوله الـ الـ تبعها"



في حالة الـ F والـ Moment عويان على يكون بقدر الـ
وأحرك القوة عن النقطة وننتج الـ effect تبعها

إذا كانت الـ Forces الـ P لها نفس الـ line of action
أو نفس الـ plane أو الـ موازيات

السافة الـ الـ الـ الـ
القوة فيها

$$M = r \times F \rightarrow r = \frac{M}{F}$$

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MON.



TUES.



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THUR.



FRI.



SAT.



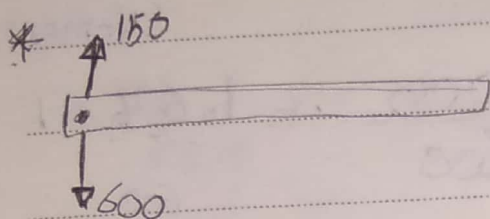
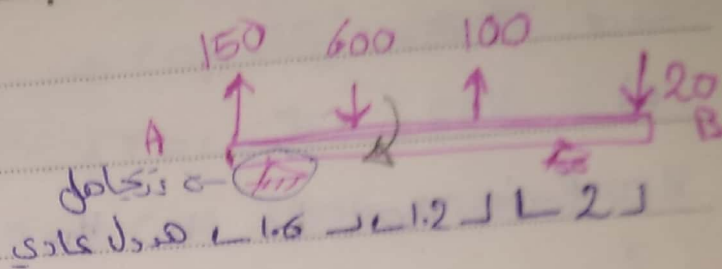
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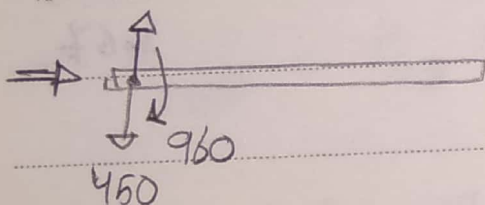
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S.P.:-

a) force and M_0 at A

$$* M_1 = 600 \times 1.6 = 960 \text{ Nm}$$

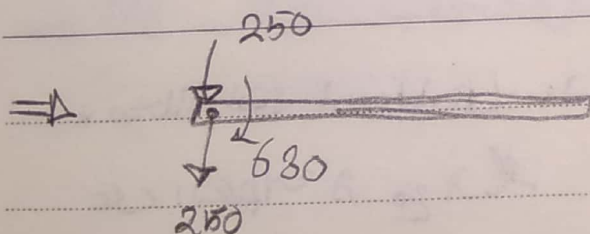
$$* R_1 = -450 \text{ N}$$



$$* M_2 = 100 \times 2.8 = 280 \text{ Nm}$$

$$M_R = -680 \text{ Nm}$$

$$F_R = -380 \text{ N}$$



$$M_3 = 250 \times 4.8 = 1200$$

$$M_P = -1880 \text{ Nm}$$

$$F_R = -600 \text{ N}$$

b) F and M at B

$$* R = 600$$

$$M_1 = -200 \text{ Nm}$$

$$M_3 = 720$$

$$M_2 = 1920$$

$$\rightarrow R = 600 \text{ N}, M = 1000 \text{ ccw}$$

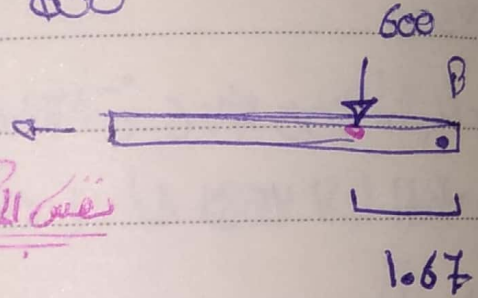
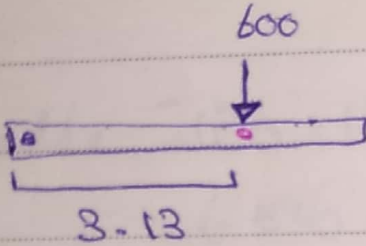


e)

1.67

force must be to the ~~right~~ to produce effect (Moment)

$$M = P \times d \rightarrow d = \frac{1000}{600} = 1.67$$



L9:

SiP → الرتبة والالات

vector form ...

على المساحة A مع الـ

$$P_0 = 700(\bar{\lambda})$$

$$\bar{r}_{B/E}$$

$$B = (75, 100, 100)$$

$$\frac{75i - 150j}{167.7}$$

$$\bar{r}_{B/E}$$

$$E = (150, -50, 100)$$

$$C = (75, 100, 0)$$

$$= 700(0.44i - 0.89j) \cdot D(100, 0, 0)$$

$$= (308i - 623j) N \cdot A(0, 100, 50)$$

$$P_c X = 1000 \cos 45 = 707.1 N$$

$$P_c Z = 1000 \cos 45 = 707.1 N$$

$$707.1i + 707.1j$$

$\bullet F_D x = 600 \rightarrow r_{AB} = 75j + 50k \text{ mm}$

$\bullet F_D y = 1039.2 \rightarrow r_{AC} = 75j - 50k \text{ mm}$

$600L + 1039.2 \rightarrow r_{AD} = 100j - 100i - 50k$

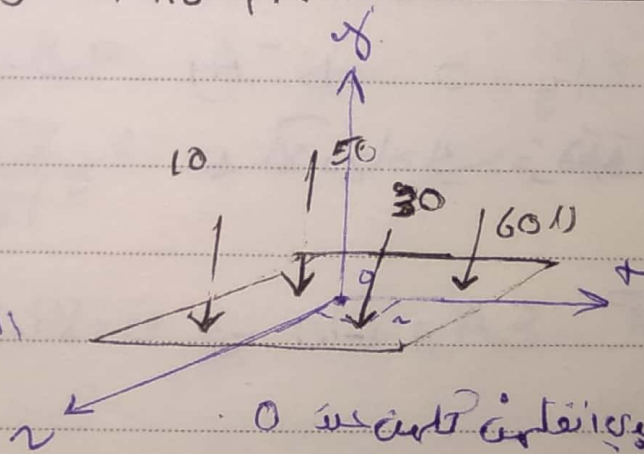
$\bullet M_{F_{AB}/A} = r_{AB} \times F_B = \begin{vmatrix} i & j & k \\ 75 & 50 & 0 \\ 300 & -600 & 0 \end{vmatrix} = 30i - 45k \text{ N}$

$\bullet M_{F_C/A} = r_{AC} \times F_C = 17.68j \text{ Nm}$

$\bullet M_{F_D/A} = r_{AD} \times F_D = 118.9k$

$\rightarrow M_A = 30i + 17.68j + 118.9k \text{ Nmm}$

$\rightarrow R_A = 1815.1 \text{ L}$



القوة 300 تنتقل لها 2 م وادخل

z، وادخل الـ y

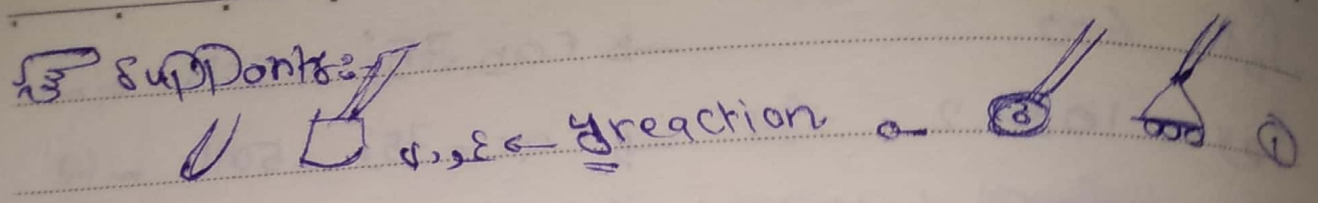
بحسب الـ M فالـ M فرق المسافة لا مخرج تحركها

ch4 :- equilibrium of rigid body.

له قانوني $\sum F = 0$ ، $\sum M = 0$

$M_x = 0, M_y = 0, M_z = 0$

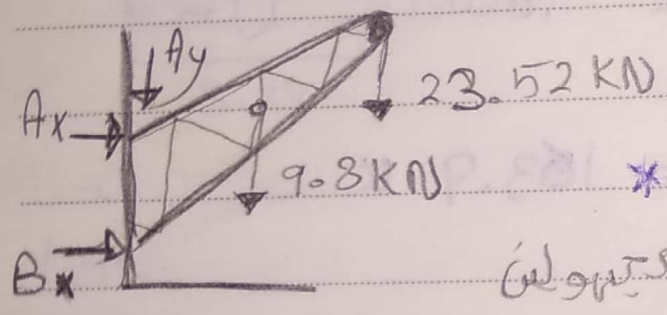
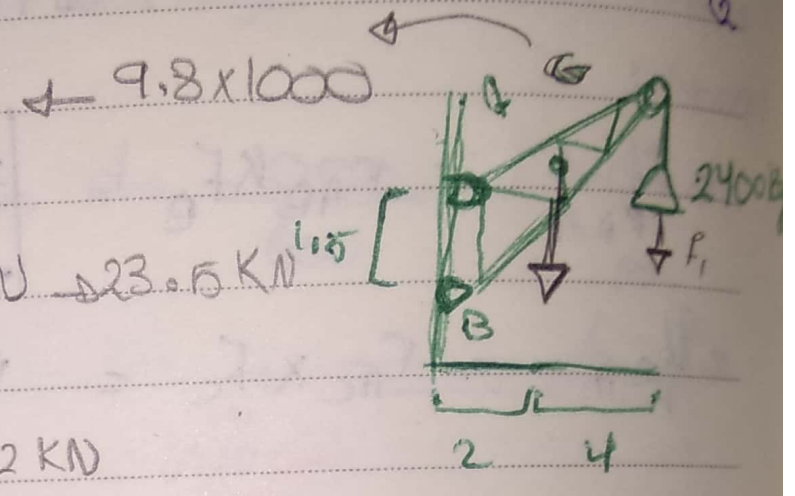
$\Rightarrow \sum F = 0, \sum M = 0, F_x = 0, F_y = 0, F_z = 0$



L10:

$$* P = 2400 \times 9.8$$

$$23520 \text{ N} \rightarrow 23.5 \text{ KN}$$



$$* \sum F_x = 0, \sum F_y = 0$$

$$\sum M = 0$$

$$\sum F_y = 0 \Rightarrow -A_y - 9.8 - 23.52 = 0$$

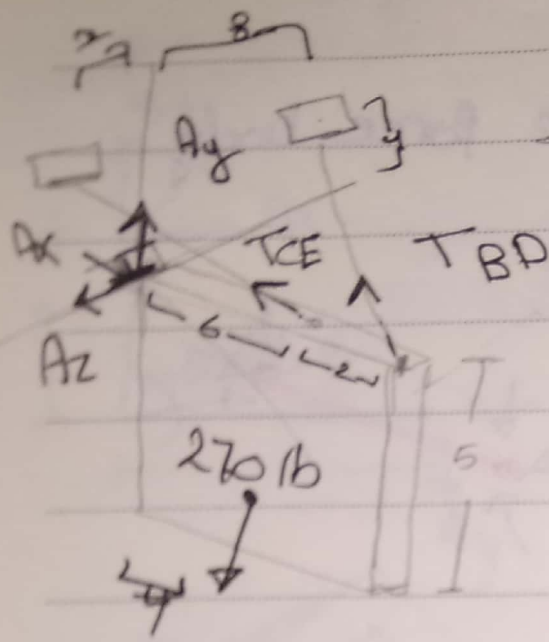
الانجاب في الارتفاع

$$A_y = 33.32 \text{ KN}$$

$$\sum M_A = 0 \Rightarrow B_x \times 1.5 - 9.8 \times 2 - 23.52 \times 6$$

$$B_x = 107.1 \text{ KN}$$

$$\sum F_x = 0 \Rightarrow A_x + B_x = 0 \rightarrow A_x = -107.1$$



$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0$$

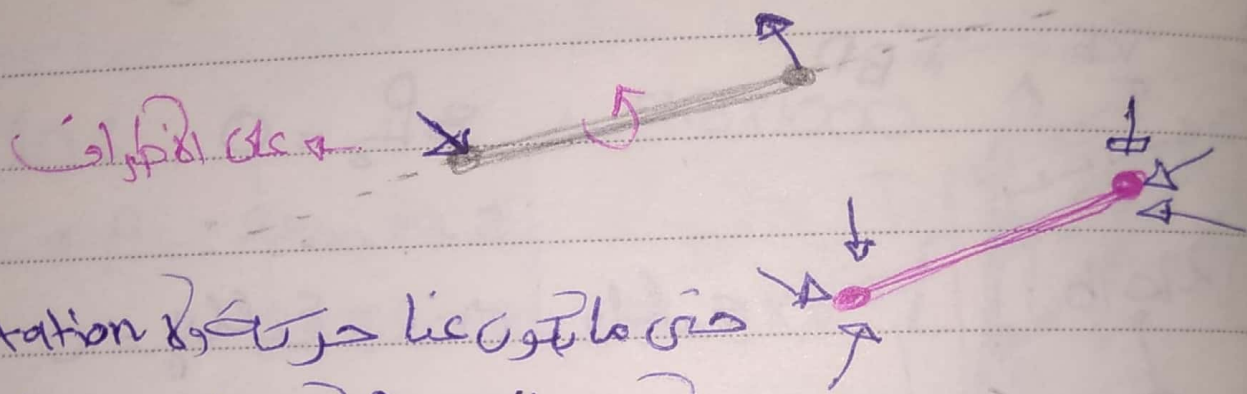
$$\sum M_z = 0$$

*reaction $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$$\vec{W} = -270 \hat{j}$$

$$\rightarrow T_{CE} \rightarrow \lambda_{CE} = \frac{d_{CE}}{|CE|} =$$

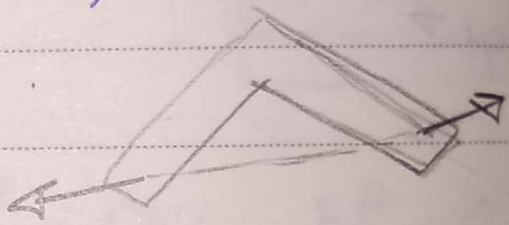
L11 :- Equilibrium of two force body



حتى لا يتحرك على حركته rotation
لا زيم ال R يتكونوا في نفس
member و Load

1. R F مساوية و متعاكسة و مساوية
بالا اتجاه

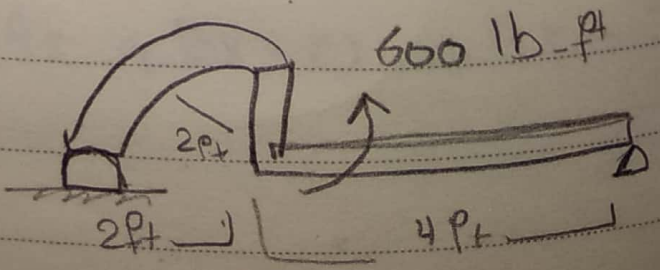
2. member لا مواز اتجاه
axis

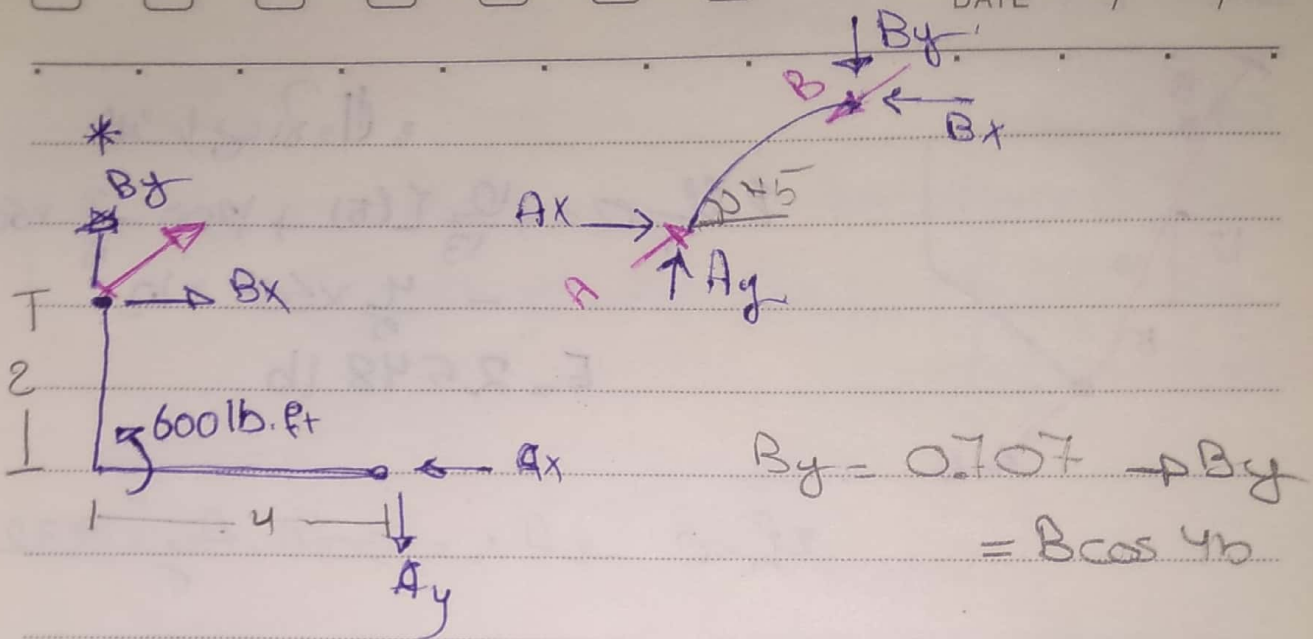


Equilibrium of a 3 force body :-

- 1) concurrent forces (متساوية في نقطة)
- 2) Parallel forces

* Example :-





* $\sum M_c = 0 \rightarrow 0.707B(4) - 0.707B(2) + 600$

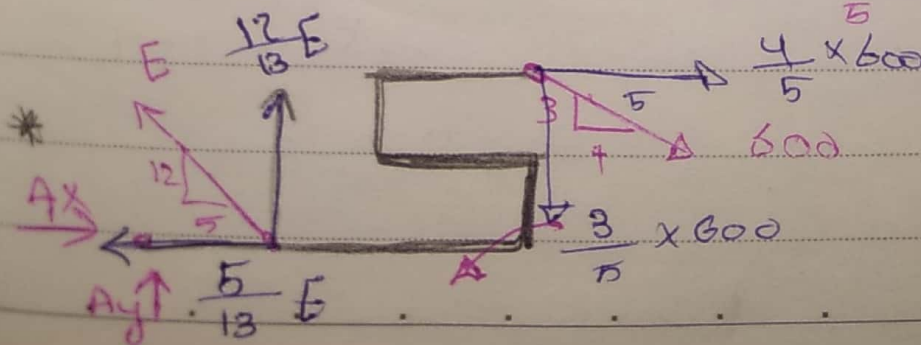
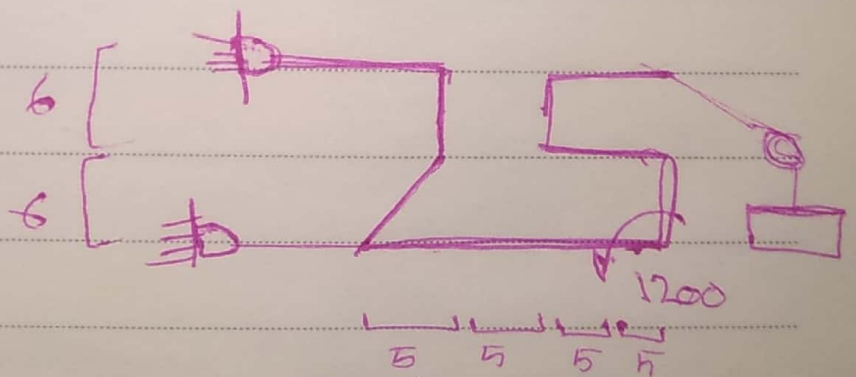
$B = 141.4 \text{ lb}$

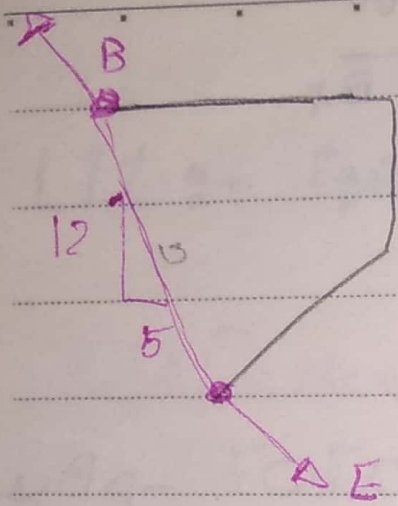
$\rightarrow Bx = 99.97 \text{ lb}$, $By = 99.97$

$\sum F_x = 99.97 - C_x = 0 \rightarrow C_x = 99.97 \text{ lb}$

$\sum F_y = 99.97 - C_y = 0 \rightarrow C_y = 99.97 \text{ lb}$

* Example





المساحة الكلية

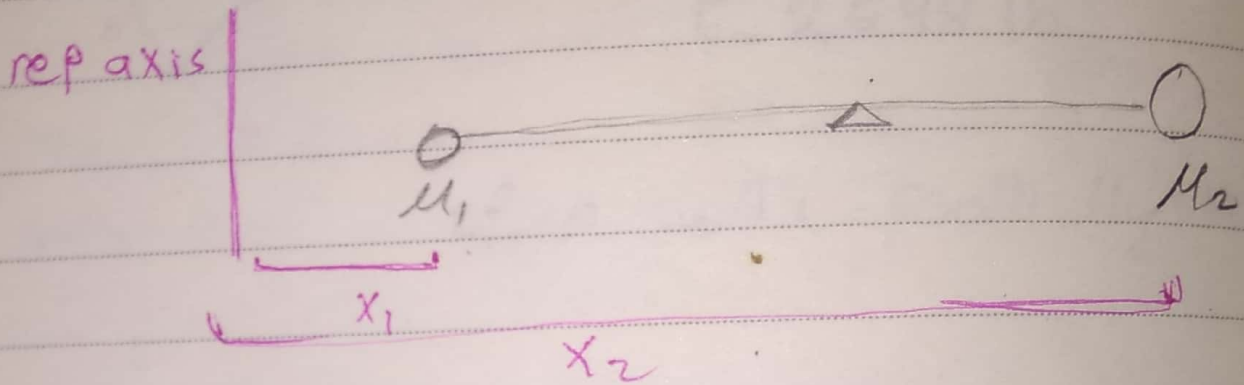
$$\sum M_A = 0 = +\frac{12}{13} E(5) + 1200 - \frac{3}{5} \times 600 - \frac{4}{5} \times 600 \times 12$$

$$E = 2548 \text{ lb}$$

$$\sum F_x = 0 \rightarrow A_x = 560 \text{ lb}, A_y = 1992 \text{ lb}$$

Ch 5:-

Distributed Forces centroid and center of gravity.



$$x_1 m_1 + x_2 m_2 = (m_1 + m_2) \bar{x}$$

$$\bar{x}_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

* center of gravity

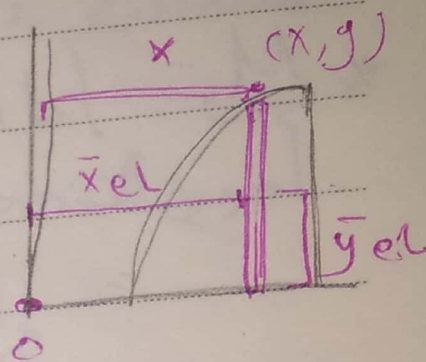
$$\bar{x} = \frac{\int dm x}{\int dm}$$

* center of gravity of a 2D Body

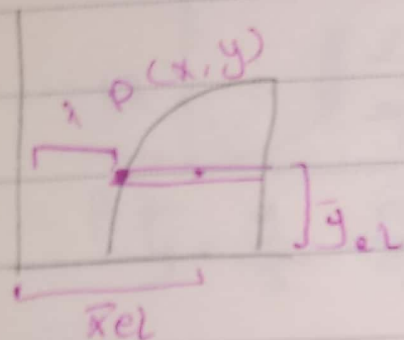
$$\sum m_y \bar{x} w = \sum x \Delta w \rightarrow \int x dw$$

$$\sum m_x = \bar{y} w = \sum y \Delta w \rightarrow \int y dw$$

$$\bar{x}_A = \int \bar{x}_{el} dA = \int x (y dx)$$



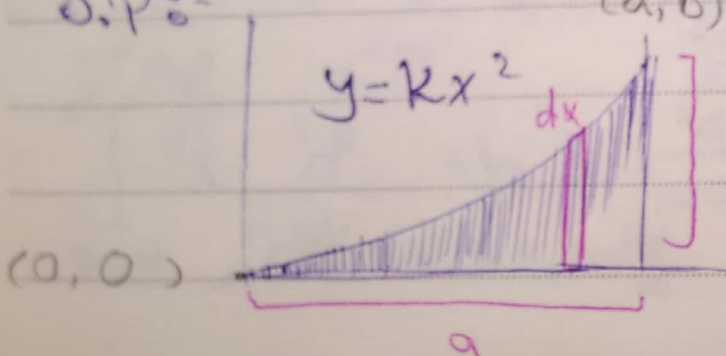
$$\bar{y}_A = \int \bar{y}_{el} dA = \int \frac{y}{2} (y dx)$$



$$\bar{x}_A = \int \bar{x}_{el} dA = \int \frac{a+x}{2}$$

S.P:-

(a, b) determine the centroid



b a, b a, k

$$x = a, y = b$$

$$y = \frac{b}{a^2} x^2$$

$$b = k a^2 \rightarrow k = \frac{b}{a^2}$$

vertical strip

$$dA = \left(\frac{b}{a} x^2\right) (dx)$$

dy " First M of A about y " = A * X

$$\rightarrow A = \int_0^a \frac{b}{a} x^2 dx = \frac{b}{a^2} \left[\frac{x^3}{3} \right]_0^a = \frac{ba}{3}$$

$$* M_y = \int_0^a dM_y = \int_0^a \left(\frac{b}{a^2} x^2 \right) dx \cdot x$$

$$= \int_0^a \frac{b}{a^2} x^3 dx = \frac{b}{a^2} \left[\frac{x^4}{4} \right]_0^a$$

$$= \frac{b a^2}{4}$$

$$* \text{Location } \bar{x} = \frac{\frac{b a^2}{4}}{\frac{b a}{3}} = \frac{\epsilon L}{\epsilon A}$$

$$= \frac{3a}{4} \rightarrow \bar{x}$$

\bar{y} :-

$$dM_x = dA \cdot x \cdot \frac{y}{2}$$

$$\frac{b}{a^2} x^2 dx \cdot \frac{b}{2a^2} x^2 = \frac{b^2}{2a^4} x^4 dx$$

$$M_x = \int_0^a \frac{b^2}{2a^4} x^4 dx = \frac{b^2}{10a^4} x^5 \Big|_0^a$$

$$= \frac{b^2 a}{10}$$

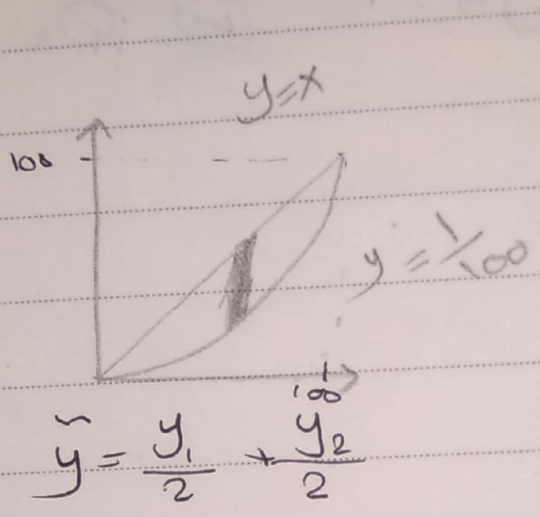
$$\bar{y} = \frac{M_x}{A} = \frac{\frac{b^2 a}{10}}{\frac{b a}{3}} = \frac{3b}{10}$$

مركز الكتلة

Ch 5 - center of mass

* center of area

لحساب مركز النقطه في بيانه عليها الجسم



$$A = \int dA$$

$$\bar{x} = \int \tilde{x} dA$$

$$\bar{y} = \int \tilde{y} dA$$

شريحة طولية

"بضول كل y تقسمها بال x"

$$\tilde{x} = x, \quad dA = (y_1 - y_2) dx$$

شريحة بالعرض

$$\tilde{x} = \frac{x_1}{2} + \frac{x_2}{2}$$

$$\bar{y} = y, \quad dA = (x_1 - x_2) dy$$

$$* A = \int dA, \quad \bar{x} = \frac{\int \tilde{x} dA}{A}, \quad \bar{y} = \frac{\int \tilde{y} dA}{A}$$

* $dA = y dx$, $dA = x dy$

$\tilde{x} = x, \tilde{y} = \frac{y}{2}$

بالعرض
 $\tilde{x} = \frac{x}{2}, \tilde{y} = y$

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wire :- $\bar{x} = \frac{\int x dl}{L}$, $\bar{y} = \frac{\int y dl}{L}$

$I_x = E \bar{x} A$, $I_y = E \bar{y} A$

(✓) ϕ_x , ϕ_y

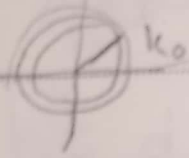
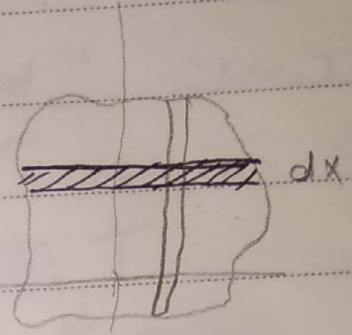
Ch6:-

Radius of gyration of an area :-

$$I_x = A(k_x)^2$$

↳ radius of gyration.

$$I_y = k_y^2 A \rightarrow k_y = \sqrt{\frac{I_y}{A}}$$



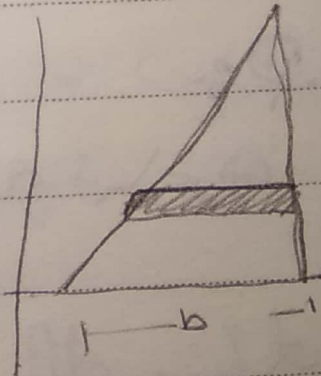
$$I_o = k_o^2 A \rightarrow k_o = \sqrt{\frac{I_o}{A}}$$

$$I_o = k_o^2 A \rightarrow k_o = \sqrt{\frac{I_o}{A}}$$

$$* k_o^2 = k_x^2 + k_y^2$$

S.P:-

determine the moment about the base.



بأخذ شريحة عرضها b وارتفاعها y من القاعدة

$$dI_x = dA \times y^2 \rightarrow$$

∫ dI_x = ∫ dA \times y^2 + I_{base}

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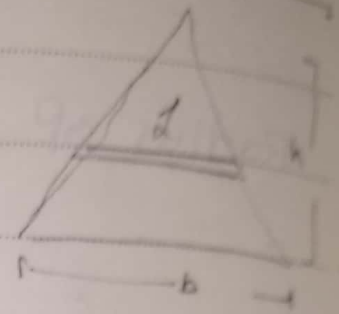
$$dI_x = dy \times \underbrace{h}_{\substack{\downarrow \\ \text{height}}}$$

المثلثين المتشابهين

$$\frac{b}{h} = \frac{l}{h-y}$$

المثلثين

$$\downarrow l = b - \frac{b}{h}y$$



$$dI_x = y^2 dy \left(b - \frac{b}{h}y \right) \Rightarrow by^2 - \frac{b}{h}y^3 dy$$

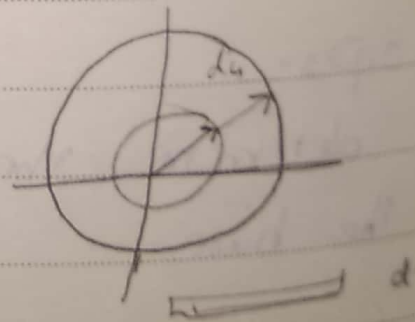
المثلثين

$$I_x = \int_0^h \left(by^2 - \frac{b}{h}y^3 \right) dy = \left[\frac{b}{3}y^3 - \frac{b}{4h}y^4 \right]_0^h$$

$$= \frac{bh^3}{3} - \frac{b}{4}h^3 = \boxed{\frac{bh^3}{12}}$$

8.P3

نأخذ شريحة دائرية لها عرض du

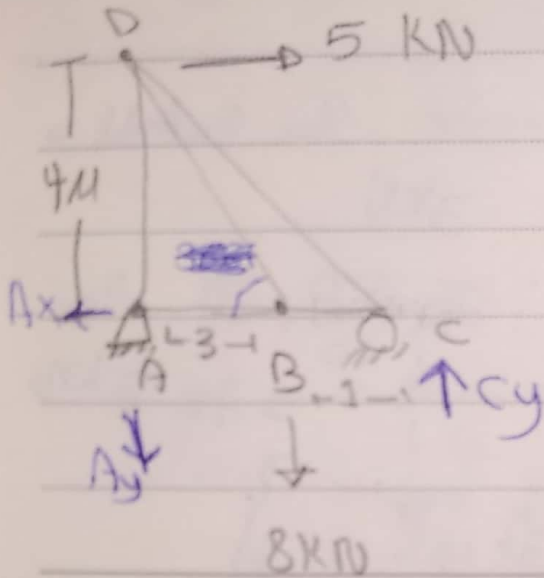


$$dI_o = \underbrace{dA}_{\substack{\downarrow \\ \text{area}}} \times u^2$$

$$2\pi u$$

Analysis of structure

Trusses "Method of joints"



- AB = 5 kN T
- BC = 11 kN T
- AD = 3 kN T
- CD = 15.58 kN C
- BD = 10 kN T

- 1) Find global equilibrium -
- 2) Find the reaction of the supports.

$$\sum M_A = 0 = -5(4) - 8(3) + C_y(4)$$

$$20 + 24 = 4C_y$$

$$C_y = 11$$

$$\sum F_y = 0 = 11 - 8 - A_y$$

$$A_y = 3 \text{ kN}$$

$$A_x = 5 \text{ kN}$$

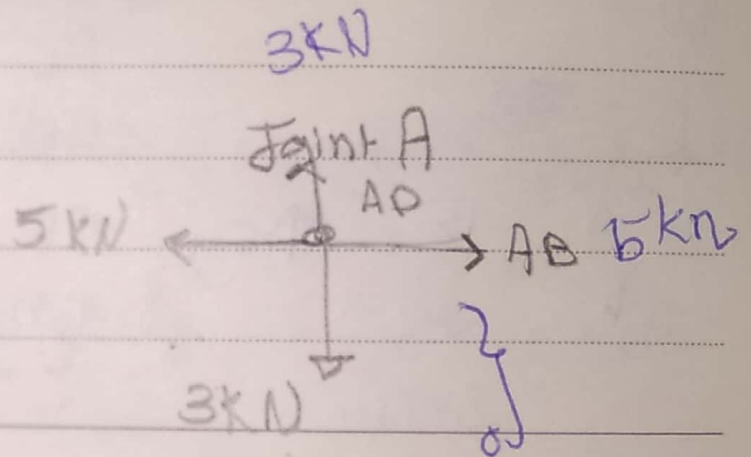
* select a joint

2 unknown \rightarrow 2 fields done, like $\left. \begin{matrix} \text{---} \\ \text{---} \end{matrix} \right\}$

* Draw FBD of joint

* solve

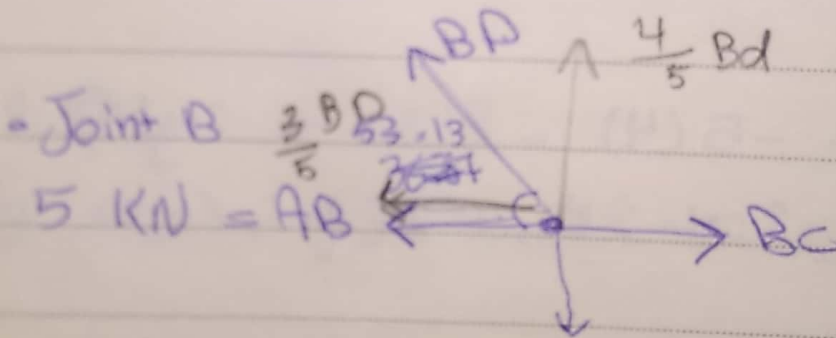
* Repeat



$$\sum P_x = 0$$

$$\sum P_y = 0$$

away is always tension.



$$\sum P_x = 0$$

$$\sum P_y = 0$$

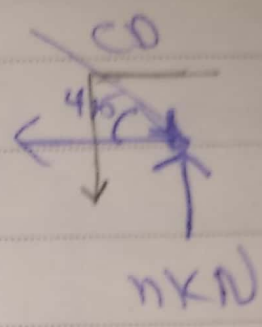
$$BC - 5 - \frac{3}{5} BD = 0 \quad \text{①}$$

$$BC = 11 \text{ kN}$$

$$\frac{4}{5} BD - 8 = 0 \rightarrow \text{②} \quad BD = 10 \text{ kN}$$

Joint C :-

$11 \text{ kN} = BC$

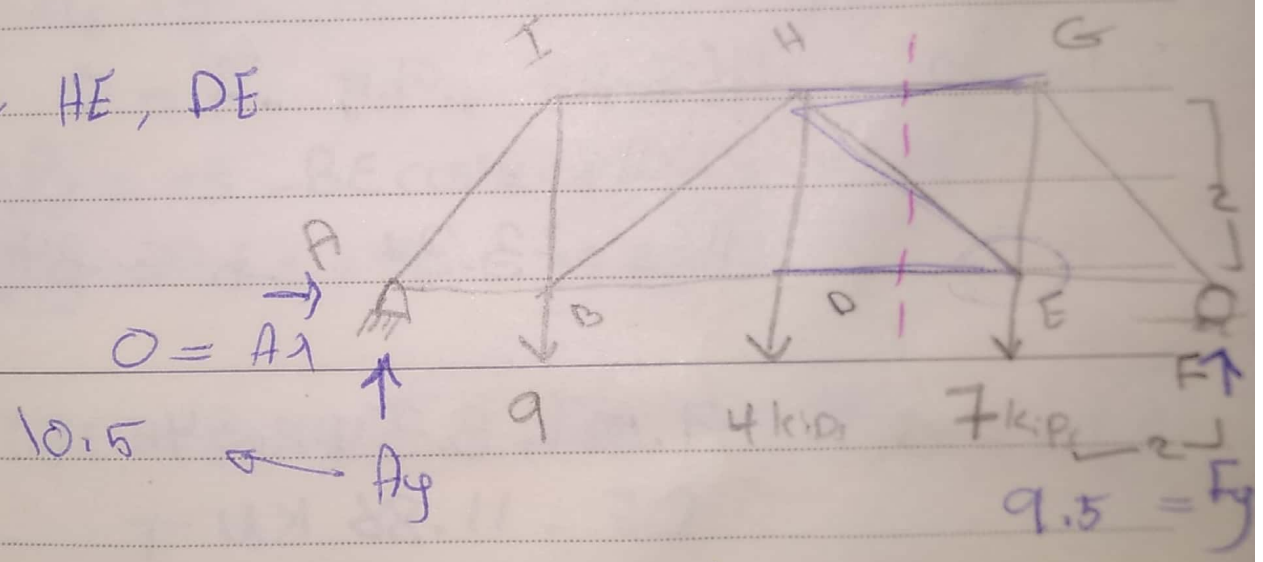


$\sum F_y = 11 - CD \sin 45 = 0$

$CD = 15.56 \text{ kN}$

* Method of sections

GH, HE, DE



$\sum M_A = 0 = 8F_y - 7(6) - 4(4) - 9(2)$

$F_y = 9.5 \text{ kips}$

$\sum F_y = 0, A_y = 10.5$

Members 3 *قاع* \leftarrow *مقاطع الجزء المطلوب*
مقاطع 3 من

$\sum F_x = 0, \sum F_y = 0, \sum M = 0$

ما رسم الجانب المطلوب

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$$9.5 = HG$$

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$$3.34$$

HE

DE

45

7

↑ 9.5

$$\sum M_E = 0 = -HG(2) + 9.5(2) = 0$$

$$HG = -9.5 \Rightarrow 9.5 \text{ C}$$

$$\sum F_y = 0 = HE \sin 45 + 9.5 - 7 = 0$$

$$HE =$$

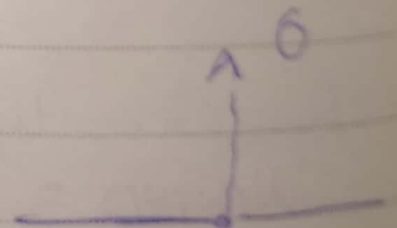
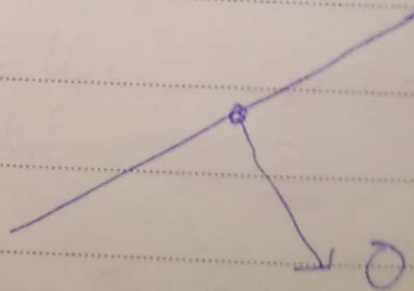
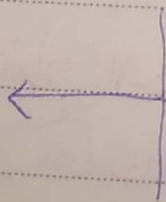
$$HE = -3.34 \Rightarrow 3.34 \text{ C}$$

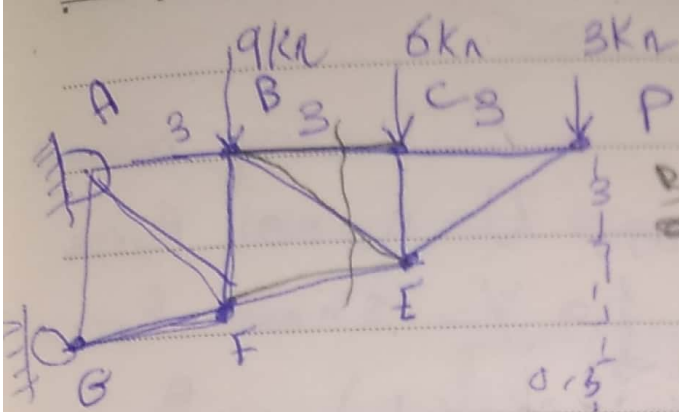
$$\sum F_x = 0 = 9.5 + 3.34 \cos 45 - DE = 0$$

$$DE = 11.88 \text{ KN T}$$

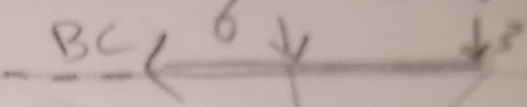
Zero force members

0





BC, BE, FE, P



BE, FE

45
18.42
FE

$$\sum M_C = 0 = BC(3) - 3(3) \Rightarrow BC = 3 \text{ kN/T}$$

$$\sum F_x = -3 - BE \cos 45 - FE \cos 18.42 = 0$$

$$\sum F_y = BE \sin 45 - FE \sin 18.42 - 6 - 3$$

$$BE = 8.49 \text{ T} \quad FE = -9.49 = 9.49 \text{ C}$$

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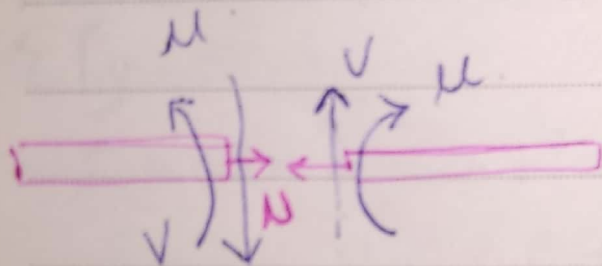
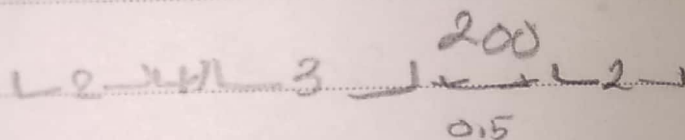
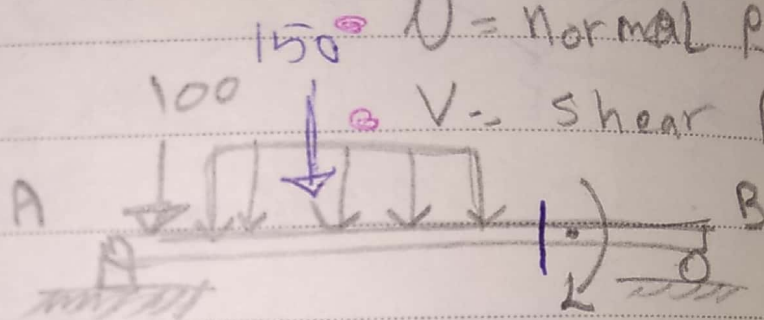
Internal forces

M = bending moment

N = Normal force

V = shear force

Positive sign convention



$$\sum M_A = 0 \rightarrow B_y =$$