

# Statics

# ENCE

# 232

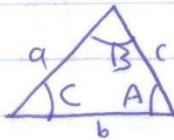
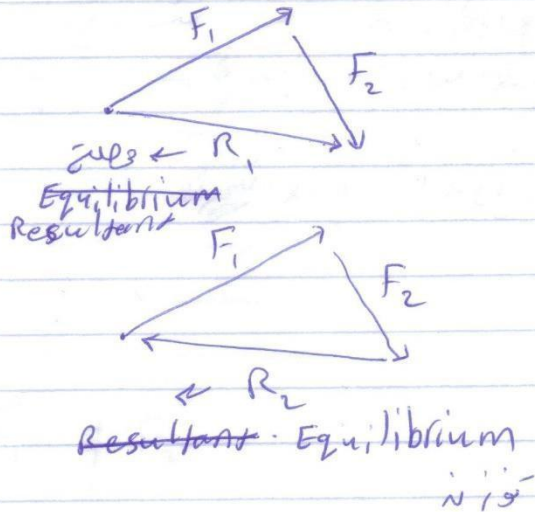
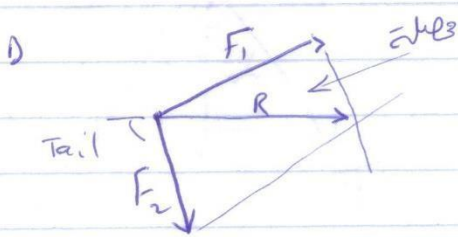
شرح صيفي

الدكتور جمال زلاطيمو

بشار عاصي

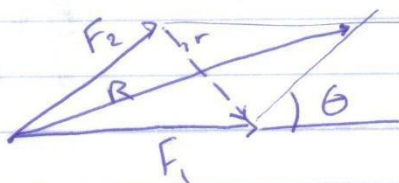
Ch2

# Force:



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\frac{\sin C}{c} = \frac{\sin B}{b} = \frac{\sin A}{a}$$



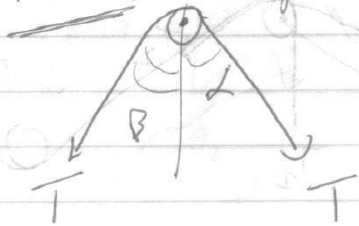
$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \theta \quad (\text{sin law})$$

$$r^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos \theta \quad (\text{cos law})$$

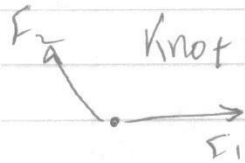
~~ΣF~~

$$\Sigma F_s = 0 \Rightarrow \Sigma F_x = 0, \Sigma F_y = 0$$

Frictionless Pulley

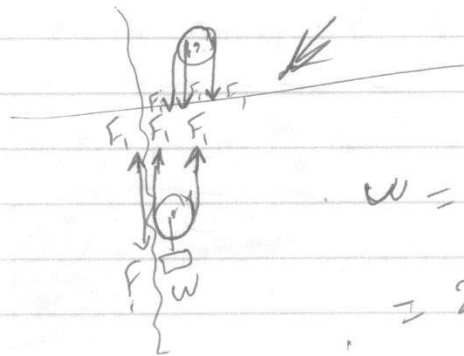


دائماً في البكرات يكون السلك متساوي في جميع الجبال حول البكرة  
لو القوتين غير متساويتين تتحرك



$$F_1 \neq F_2$$

Free-Body

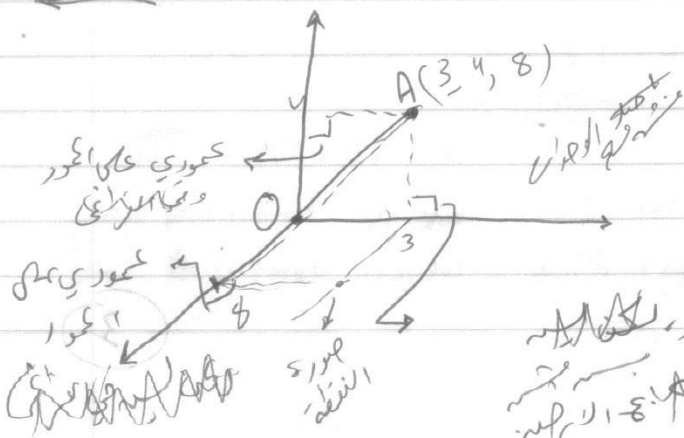


$$w = 3F_1$$

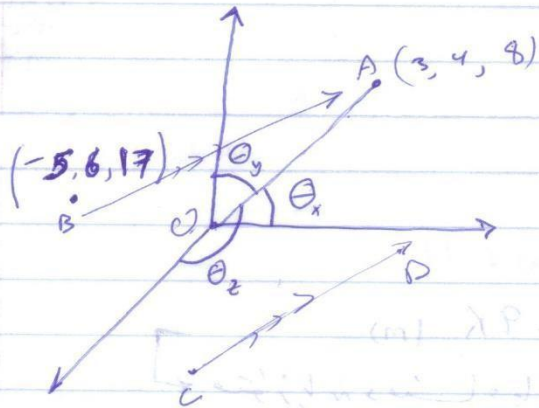
$$= 2F_1$$

$$w + F_1 = 3F_1$$

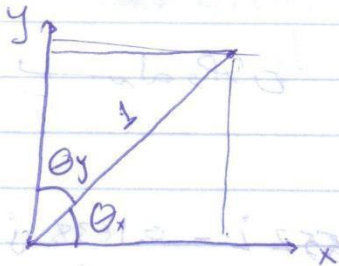
دائماً في البكرات



محور عمودي على محور السينات واليعنات  
محور السينات واليعنات



$$\theta_x, \theta_y, \theta_z \neq 90$$



$$\cos^2(\theta_x) + \cos^2(\theta_y) = 1$$

\* directional cosines : [the cosines of  $\theta_x, \theta_y, \theta_z$ ]

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

\* directional vector



directional vector

$\vec{BA}$  : from B to A

$\vec{A/B}$  : to A from B

$$\begin{aligned} \vec{BA} &= \vec{A} - \vec{B} = [3i + 4j + 8k] - [-5i + 6j + 17k] \\ &= 8i + -2j + -9k \quad (m) \\ &= \underline{\underline{8, -2, -9}} \end{aligned}$$

$$BA = 12.21 \text{ m}$$

- positional vector (with refs to the origin)  $\vec{OA}; \vec{rA}$
- Directional vector (with refs to another point)

$$\vec{OA} = +3\hat{i} + 4\hat{j} + 8\hat{k} \text{ (m)}$$

$$OA = 9.434 \text{ m}$$

$$\vec{CD} = \vec{BA} = +8\hat{i} - 2\hat{j} - 9\hat{k} \text{ (m)}$$

متوزن و متساوی

$$\hat{\lambda} \text{ (unit vector)} = \frac{\vec{AB}}{|\vec{AB}|} \text{ (unitless)}$$

$$\hat{\lambda}_{BA} = \frac{(+8\hat{i} - 2\hat{j} - 9\hat{k})}{12.21} = +0.6552\hat{i} - 0.1638\hat{j} - 0.7371\hat{k}$$

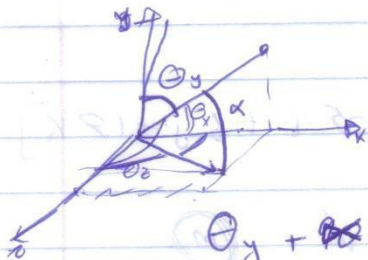
$$|\hat{\lambda}| = 1$$

$$\cos\theta_x \quad \cos\theta_y \quad \cos\theta_z$$

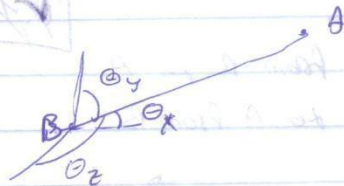
$$\theta_x = 49.06^\circ \quad \theta_y = 99.43^\circ \quad \theta_z = 137.48^\circ$$

$$\theta_x + \theta_y + \theta_z \neq 90 \neq 180$$

نیست

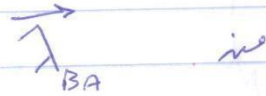


$$\theta_y + \theta_x = 90$$



برای زاویه بین بردار  $\vec{A}$  و محاوره  $\theta_x$ ،  $\theta_y$  و  $\theta_z$  از فرمول زیر استفاده می‌کنیم:

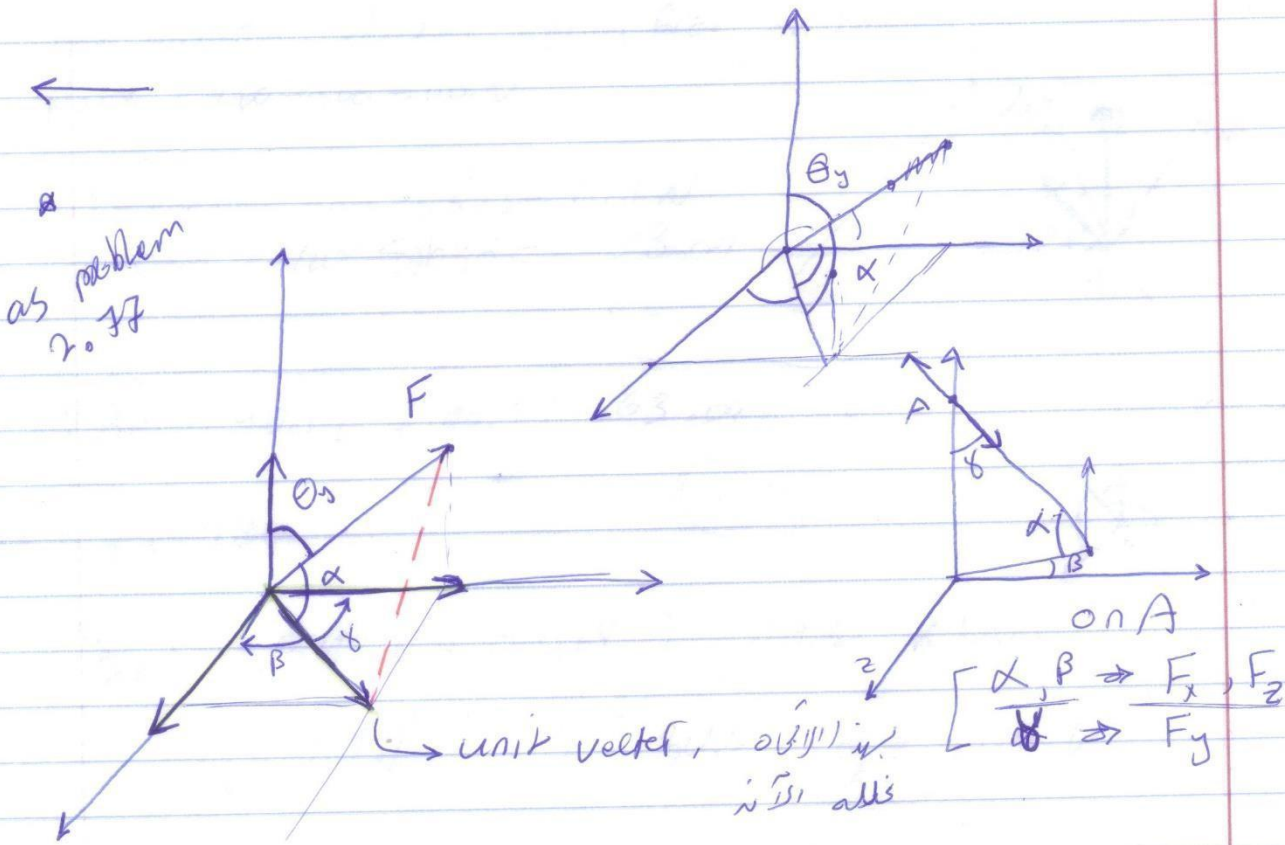
$\vec{F}_{CD} = +65.52\hat{i} - 16.38\hat{j} - 73.21\hat{k} \text{ N}$   
 $\vec{F}_{O/C}$   
 $\vec{F}_{CD} = 100 \text{ N}$



!! Vector لا يمكن أن يكون في أي اتجاه، فقط في اتجاه واحد.

the force is a sliding vector, can be moved anywhere on its line of action.

!! لا يمكن أن يكون في أي اتجاه، فقط في اتجاه واحد.



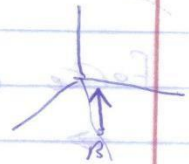
$\Rightarrow F_y = F \sin \theta_y$   
 $\Rightarrow F_x = F \cos \alpha \cos \beta = F \cos \alpha \sin \beta$   
 $\Rightarrow F_z = F \cos \alpha \sin \beta = F \cos \alpha \cos \beta$

2.75

$$F_x = F \cos 60 \cos 35 = -90.107$$

$$F_y = F \sin 60 = 190.526$$

$$F_z = F \cos 60 \sin 35 = -63.09$$



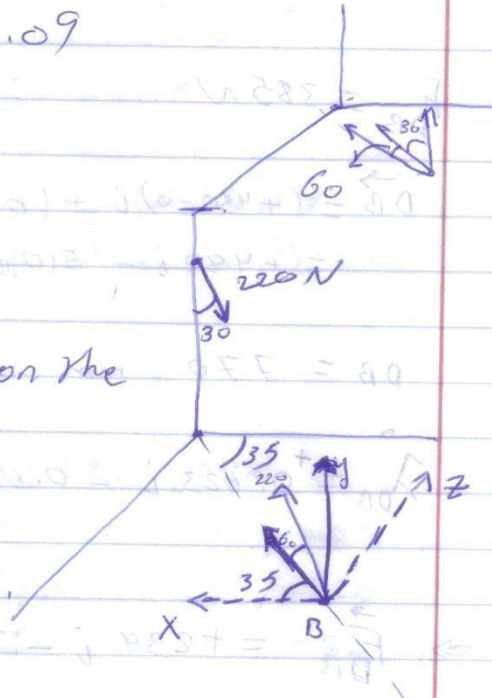
Sol

y-component  $220 \sin 60 = 190.5 \text{ N}$

The projection of the spring force on the plate  $= 220 \cos 60 = 110 \text{ N}$

x-component  $= 110 \cos 35 = 90.1 \text{ N}$

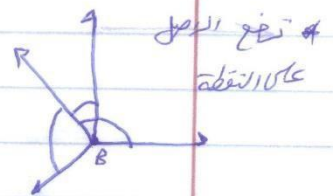
z-component  $= 110 \sin 35 = 63.09$



$$\vec{F}_{BA} = -90.1 \hat{i} + 190.5 \hat{j} - 63.09 \hat{k} \text{ N}$$

$F_{BA} = 220 = \sqrt{\dots}$

(5.11)



$$\hat{u}_{BA} = -0.410 \hat{i} + 0.864 \hat{j} - 0.287 \hat{k} \text{ (N)}$$

$$\theta_x = 117.2^\circ \quad \theta_y = 30.2^\circ \quad \theta_z = 106.6^\circ$$



2.89

Force on D



B

$F_{DB}$   
From D to B

D (0, +510, +280) mm

(+480, 0, +600)

B

$$F_{DB} = 385 \text{ N}$$

$$\vec{DB} = (+480 - 0)\mathbf{i} + (0 - 510)\mathbf{j} + (+600 - 280)\mathbf{k} \text{ mm}$$
$$= +480\mathbf{i} - 510\mathbf{j} + 320\mathbf{k} \text{ mm}$$

$$DB = 770 \text{ mm}$$

$$\hat{DB} = 0.623\mathbf{i} - 0.662\mathbf{j} + 0.416\mathbf{k}$$

$$\Rightarrow \vec{F}_{DB} = +239\mathbf{i} - 254.9\mathbf{j} + 160.2\mathbf{k} \text{ (N)}$$



(7)

(8)

supports ; نقاط ارتكاز

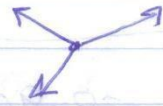
reaction ;

Equilibrium ;

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$



لا يتولد هنا عزم!! العزوم تتولد عندما يكون هناك مسافة بين نقاط تأثير القوى والحدود! أي ليس نقطة



2.107

$$A: (960, 240, 0) \text{ m}$$

$$B: (0, 0, 3.80) \text{ m}$$

$$C: (0, 0, 3.20) \text{ m}$$

$$D: (0, 960, -220) \text{ m}$$

$F_{AD} = 305 \text{ N}$   
 $\vec{AD} = (-960, 720, 0)$

$$\vec{AD} = -960 \mathbf{i} + 720 \mathbf{j} + 0 \mathbf{k} \text{ N}$$

$$|\vec{AD}| = 1220$$

$$\lambda = -0.787 \mathbf{i} + 0.590 \mathbf{j} + 0 \mathbf{k}$$

$$\vec{F}_{AD} = -244 \mathbf{i} + 183 \mathbf{j}$$

$$\vec{F}_{AD} = -240.0 \mathbf{i} + 179.9 \mathbf{j} + -54.9 \mathbf{k}$$

$$\vec{P} = 240 \mathbf{i} \text{ N}$$



$$\vec{A}_P = +1 \mathbf{i} + 0 \mathbf{j} + 0 \mathbf{k}$$

$$\vec{P} = \oplus P \mathbf{i}$$

9

$$\vec{AD} = -960i - 240j + 380k \text{ mm} \quad [1060] \text{ mm}$$

$$\vec{\lambda}_{AB} = \frac{-960i}{1060} - \frac{240j}{1060} + \frac{380k}{1060}$$

$$= -0.9057i - 0.2264j + 0.3585k$$

$$\vec{F}_{AB} = -0.9057 F_{AB} i - 0.2264 F_{AB} j + 0.3585 F_{AB} k$$

$$\vec{AC} = (-960, -240, -320)$$

$$AC = 1040$$

$$\vec{\lambda}_{AC} = -0.9231i - 0.2308j - 0.3077k$$

$$\vec{F}_{AC} = -0.9231 F_{AC} i - 0.2308 F_{AC} j - 0.3077 F_{AC} k$$

$$\text{From } \sum F_y = 0$$

$$\rightarrow -0$$

$$20108 \rightarrow$$

$$F_{AC} = 341.7 \text{ N}$$

$$F_{AB} = 447.5 \text{ N}$$

$$P = 960.8 \text{ N}$$

(P)

(17)

9.109

$$F_{AC} = 60 \text{ N}$$

the weight of the plate  
equals the y-component  
at point A.

$$A: (0, +480, 0)$$

$$B: (-320, 0, +360)$$

$$C: (+450, 0, +360)$$

$$D: (+250, 0, -360)$$

So we first find the unit components of  $F_{AC}$ ,  $F_{AB}$ ,  $F_{AD}$

$$\textcircled{a} \vec{AB} = -320 \hat{i} - 480 \hat{j} + 360 \hat{k}$$

$$AB = 680 \text{ m}$$

$$\hat{\lambda}_{AB} = -0.4706 \hat{i} - 0.7059 \hat{j} + 0.5294 \hat{k}$$

$$\leftarrow \vec{F}_{AB} = -0.4706 F_{AB} \hat{i} - 0.7059 F_{AB} \hat{j} + 0.5294 F_{AB} \hat{k}$$

$$\textcircled{b} \vec{AC} = 450 \hat{i} - 480 \hat{j} + 360 \hat{k}$$

$$AC = 750 \text{ m}$$

$$\hat{\lambda}_{AC} = 0.6 \hat{i} - 0.64 \hat{j} + 0.48 \hat{k}$$

$$\leftarrow \vec{F}_{AC} = 36 \hat{i} - 38.4 \hat{j} + 28.8 \hat{k}$$

$$\textcircled{c} \vec{AD} = 250 \hat{i} - 480 \hat{j} - 360 \hat{k}$$

$$AD = 650 \text{ m}$$

$$\hat{\lambda}_{AD} = 0.3846 \hat{i} - 0.7385 \hat{j} - 0.5538 \hat{k}$$

$$\leftarrow \vec{F}_{AD} = 0.3846 F_{AD} \hat{i} - 0.7385 F_{AD} \hat{j} - 0.5538 F_{AD} \hat{k}$$

Mo item  $\sum F_y = 0$ , and  $\sum F_z = 0 \Rightarrow$

(11)

$$\Sigma F_x = -0.4706 F_{AB} + 36 + 0.4 F_{AD} = 0$$

$$F_{AB} = \frac{36 + 0.4 F_{AD}}{0.4706} \quad \text{--- (1)}$$

$$\Sigma F_z = 0 = 0.5294 F_{AB} + 28.8 + 0.5538 F_{AD} = 0$$

$$\Rightarrow \frac{0.5294}{0.4706} [36 + 0.4 F_{AD}] + 28.8 + 0.5538 F_{AD} = 0$$

$$\Rightarrow 1.125 [36 + 0.4 F_{AD}] + 28.8 + 0.5538 F_{AD} = 0$$

$$40.5 + 28.8 + 0.4327 F_{AD} - 0.5538 F_{AD} = 0$$

$$F_{AD} = \frac{69.3}{0.1211} = 572.2 \text{ N}$$

$$\Rightarrow F_{AB} = \frac{36 + 0.4(572.2)}{0.4706} = 643.9 \text{ N}$$

Now,  $\Sigma F_y = 0$

$$-0.7059(643.9) - 38.4 - 0.7385(572.2) + P = 0$$

$$P(\text{weigh}) = 845$$

یہ وزن ہے جس سے سولہ کے ساتھ یو کے ساتھ  
 N میں وزن کے ساتھ

$$T_{AB} = -T_{BA}$$

2.125

First we work on point A

$$A(0, 155, 0)$$

$$B(200, 0, z)$$

But, at first we must find the coordinate (z) (460)

By Py,  $(525)^2 =$   $p = 341$

$$(525)^2 = (155)^2 + (OB)^2 \Rightarrow (OB) = 501.6$$

and then

$$(OB)^2 = (501.6)^2 = (200)^2 + z^2 \Rightarrow \boxed{z = 460}$$

$$\vec{p} = 0\ i + 341\ j + 0\ k$$

⊗ Now we work at point B

$$\vec{AB} = 200\ i - 155\ j + 460\ k$$

$$AB = 525$$

$$\lambda_{AB} = 0.3809\ i - 0.2952\ j + 0.8762\ k$$

$$\vec{T} = 0.3809T\ i - 0.2952T\ j + 0.8762T\ k \quad F$$

But  $\Sigma F_y = 0$ , so

$$341 = p = 0.2952T \Rightarrow T = 1155\ N$$

$$\textcircled{B} \quad \vec{BA} = -200\ i + 155\ j - 460\ k$$

$$AB = 525$$

$$T_{BA} = -T_{AB} = -0.3809T\ i + 0.2952T\ j - 0.8762T\ k$$

$$\Sigma F_x \neq 0$$

$$\Sigma F_z \neq 0$$

لكن في حال الحركة في  
الخط  $z$  لا تكون صفرًا.

$$\text{Now, } \sum F_z = 0 \quad (\text{on B})$$

$$Q = 0.8762 T$$

$$= 0.8762 (1155)$$

$$Q = 1012.011 \quad \checkmark$$

← 1140292

2.120

$$F_{DAx} = -T \cos 60 \cos 40 = 0.$$

$$F_{DAy} = -T \sin 60$$

$$F_{DAz} = T \cos 60 \sin 40$$

$$F_{DBx} = T \cos 60 \cos 40 =$$

$$F_{DBy} = -T \sin 60$$

$$F_{DBz} = T \cos 60 \sin 40$$

$$F_{DCx} = T \cos 60 \cos 60$$

$$F_{DCy} = -T \sin 60$$

$$F_{DCz} = -T \cos 60 \sin 60$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\boxed{\quad} + P = 0$$

$$\sum F_z = 0$$



(14)





2-1056

AD is taut  $\Rightarrow$  الكبل مشدود، أي التوتر فيه أكبر من صفر.

$$\vec{P} = +1200\hat{i} + 0 + 0$$

$$\vec{F}_{AB} = -0.9057 F_{AB} \hat{i} - 0.2264 F_{AB} \hat{j} + 0.3585 F_{AB} \hat{k}$$

$$\vec{Q} = 0 \hat{i} + 0 \hat{j} + 0$$

$$\vec{F}_{AD} = -0.7869 F_{AD} \hat{i} + 0.5902 F_{AD} \hat{j} - 0.1803 F_{AD} \hat{k}$$

$$\vec{F}_{AC} = -0.9231 F_{AC} \hat{i} - 0.2307 F_{AC} \hat{j} - 0.3077 F_{AC} \hat{k}$$

at first, let AD be not taut

على فرض أن الكبل غير مشدود، أي التوتر فيه = صفر.

$$\Rightarrow \text{let } \vec{F}_{AD} = 0 + 0 + 0$$

$$1200 - 0.9057 F_{AB} - 0.9231 F_{AC} = 0$$

$$\Rightarrow F_{AB} = \frac{1200 - 0.9231 F_{AC}}{0.9057} = \underline{605.8} \text{ N}$$

$$0.3585 F_{AB} - 0.3077 F_{AC} = 0$$

$$0.3959 \frac{0.3585 [1200 - 0.9231 F_{AC}]}{0.9057} - 0.3077 F_{AC} = 0$$

$$474.9 - 0.3654 F_{AC} - 0.3077 F_{AC} = 0$$

$$\underline{F_{AC} = 705.5} \text{ N}$$

(16)

$$-0.2264 F_{AB} + Q - 0.2307 F_{AC} = 0$$

$$Q = 300 \text{ N}$$

then  $Q \leq 300 \rightarrow$  tension force (الشد)

إذا شد AD ←

$$\leftarrow 2.112$$

$$AC = 3.68 \text{ kN}$$

$$\vec{AB} = -9 \mathbf{j} + 18 \mathbf{j} + 6 \mathbf{k}$$

$$AB = 21$$

$$\lambda = 0.4286 \mathbf{i} - 0.8571 \mathbf{j} + 0.2857 \mathbf{k}$$

$$\vec{F}_{AB} = -0.4286 F_{AB} \mathbf{i} + 0.8571 F_{AB} \mathbf{j} - 0.2857 F_{AB} \mathbf{k}$$

$$A (0, 0, 0)$$

$$B (-9, 0, 6)$$

$$C (6, 0, 13)$$

$$D (4, 0, -12)$$

$$\vec{AC} = 6 \mathbf{i} - 18 \mathbf{j} + 13 \mathbf{k}$$

$$AC = 23$$

$$\lambda_{AC} = 0.2608 \mathbf{i} - 0.7826 \mathbf{j} + 0.5652 \mathbf{k}$$

$$\vec{F}_{AC} = 0.2608 F_{AC} \mathbf{i} - 0.7826 F_{AC} \mathbf{j} + 0.5652 F_{AC} \mathbf{k}$$

2.112

$$AC = 3.68$$

$$A(0, 18, 0)$$

$$B(-9, 0, 6)$$

$$C(6, 0, 13)$$

$$D(4, 0, -12)$$

$$\vec{AB} = -9i + 18j + 6k$$

$$AB = 21$$



$$\lambda_{AB} = -0.4286i + 0.8571j + 0.2857k$$

$$\leftarrow \vec{F}_{AB} = -0.4286 F_{AB} i + 0.8571 F_{AB} j + 0.2857 F_{AB} k$$

$\leftarrow$

$$\vec{AC} = +8i - 18j + 13k$$

$$AC = 23$$

$$\vec{\lambda}_{AC} = +0.2609i - 0.7826j + 0.5652k$$

$$\leftarrow \vec{F}_{AC} = +0.9595i - 2.879j + 2.079k$$

$\leftarrow$

$$\vec{AD} = 4i - 18j - 12k$$

$$AD = 22$$

$$\lambda_{AD} = 0.1818i - 0.8181j - 0.5454k$$

$$\leftarrow \vec{F}_{AD} = 0.1818 F_{AD} i - 0.8181 F_{AD} j - 0.5454 F_{AD} k$$

$\leftarrow$

$$P = 0 + Pj + 0k$$



$$\sum F_x = 0$$

$$-0.4286 F_{AB} + 0.9595 + 0.1818 F_{AD} = 0$$

$$F_{AB} = \frac{0.9595 + 0.1818 F_{AD}}{0.4286}$$

$$\sum F_z = 0$$

$$0.2857 F_{AB} + 2.079 - 0.5454 F_{AD} = 0$$

$$\frac{0.2857}{0.4286} [0.9595 + 0.1818 F_{AD}] + 2.079 - 0.5454 F_{AD} = 0$$

$$0.6396 + 0.1212 F_{AD} + 2.079 - 0.5454 F_{AD} = 0$$

$$F_{AD} = \frac{2.719}{0.4242} = 6.409 \text{ N}$$

$$F_{AB} = \text{---}$$

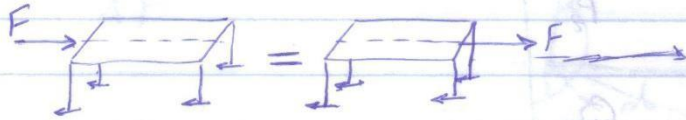
$$P = 123 \text{ ---}$$

(19)



# Ch3

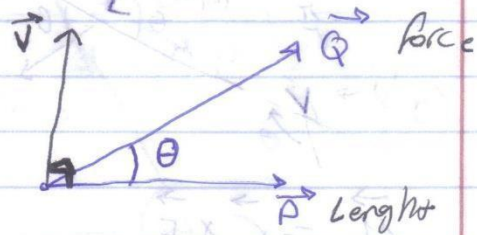
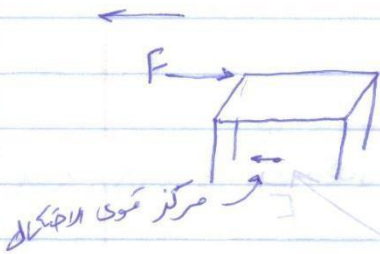
# Ch3



عند نقطة الانتقال  
النتيجة هي نفسها

A force is a sliding vector.

A principle of transmissibility.



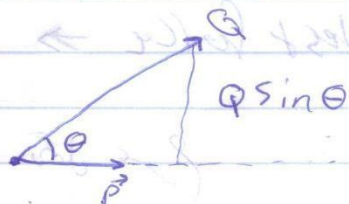
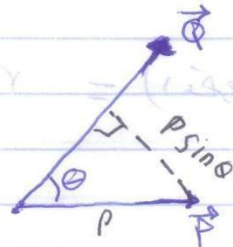
$$\vec{V} = \vec{P} \times \vec{Q}$$

$$V = PQ \sin \theta$$

$$\vec{P} \times \vec{Q} = -\vec{Q} \times \vec{P}$$

$$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

$$\vec{Q} = Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}$$







$$\vec{v} = \vec{r} \times \vec{\omega}$$

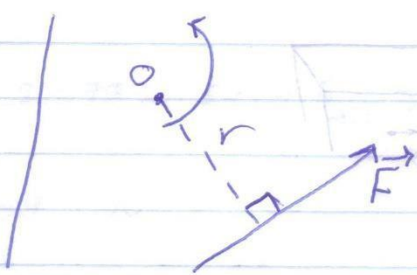
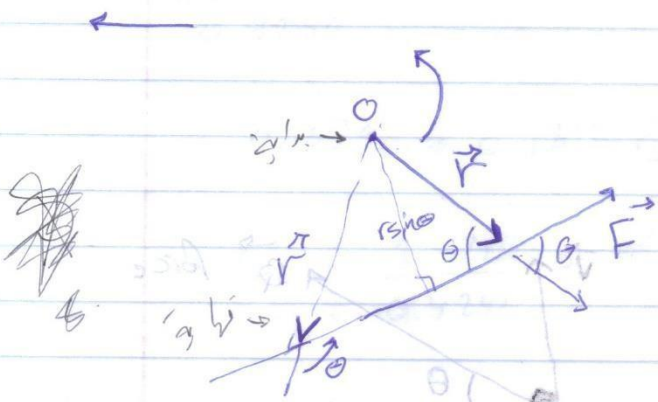
$$\vec{\omega} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

$$= \hat{i}(P_y Q_z - P_z Q_y) - \hat{j}(P_x Q_z - P_z Q_x) + \hat{k}(P_x Q_y - P_y Q_x)$$

$v_x$

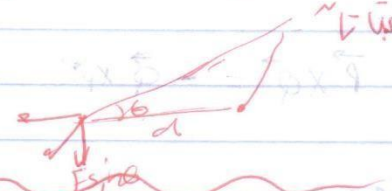
$v_y$   $v_z$



$$\vec{M}_0 = \vec{r} \times \vec{F}$$

$$(\text{المقدار}) = r \sin \theta$$

ليس بالضرورة التقابل مع القوة عند نقطة تأثيرها على الخط، ما يهم هو خط تاثير القوة ونقطة الدوران، انا امانه فيه غير مهمة

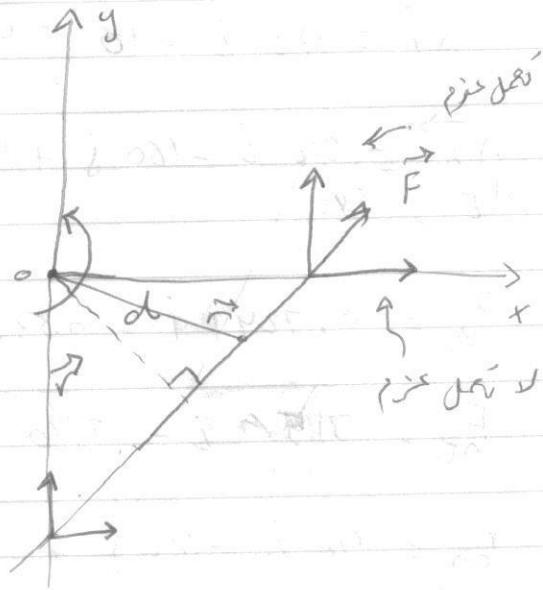


smallest possible  $\rightarrow r$  is maximum

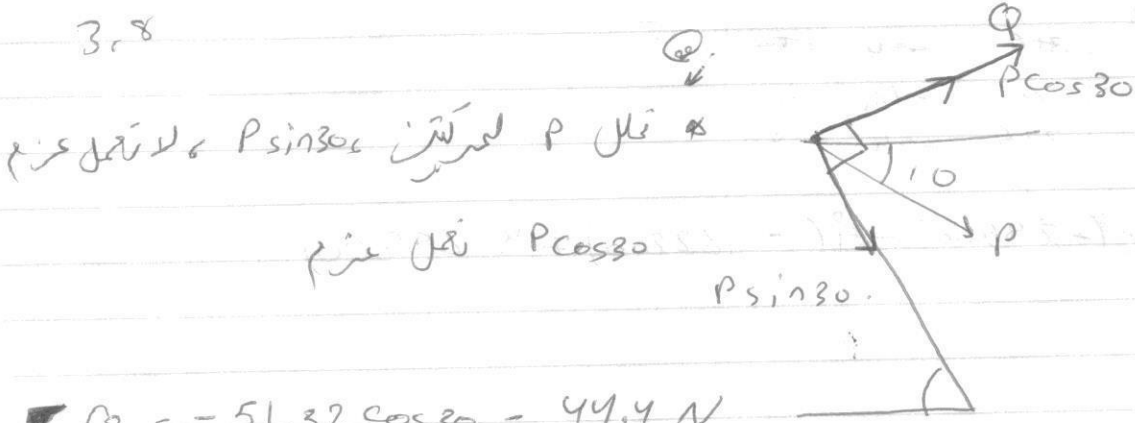
انزل كالتالي اسفود تدر بالسرعة، لا تقبل عرض

Varignon's Theorem:

$$M_o^F = \vec{r} \times \vec{F}$$



← 3.8



Q = 51.32 cos 30 = 44.4 N

M<sub>B</sub> = Q (4.50)

⇒ Q = \_\_\_\_\_

← 3.21 on A, F<sub>AB</sub>

▮

3.21

$\vec{F}_{AE}$



(X)  $\vec{AE} = 216\hat{i} - 160\hat{j} + 120\hat{k}$

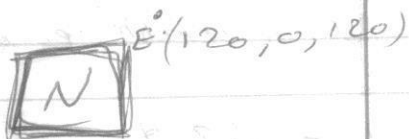
$(-90, +160, 0)$

$|\vec{AE}| = 240$

$\vec{O} (0, 0, 0)$

$\vec{\lambda}_{AE} = 0.7241\hat{i} - 0.5517\hat{j} + 0.4138\hat{k}$

$\vec{F}_{AE} = 315\hat{i} - 240\hat{j} + 180\hat{k}$



$\vec{r}_{OA} = -90\hat{i} + 160\hat{j} + 0\hat{k}$

$M_{AE}^F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -90 & 160 & 0 \\ 315 & -240 & 180 \end{vmatrix}$

$= \hat{i}(+28800) - \hat{j}(-16200) + \hat{k}(-28800) \text{ N}\cdot\text{mm}$

$\vec{F}_{AE} = +315\hat{i} + 240\hat{j} + 180\hat{k}$

$\vec{r}_{OE} = 120\hat{i} + 0\hat{j} + 120\hat{k}$

$M_{EA}^F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 120 & 0 & 120 \\ +315 & +240 & +180 \end{vmatrix} \text{ mm}$

$= \hat{i}(+28800) - \hat{j}(-16200) + \hat{k}(-28800) \text{ N}\cdot\text{mm}$

$\vec{M} = 28800\hat{i} + 16200\hat{j} - 28800\hat{k} \text{ N}\cdot\text{mm}$

$$\textcircled{B} \quad \vec{M}_O^{\vec{F}_{EA}} = -28.8 \hat{i} - 18.2 \hat{j} + 28.8 \hat{k} \text{ (N.m)}$$

الزاوية بين المحاور في الخلية.

27 Now, determine the perpendicular distance from  $O \rightarrow AG$

$$M_o = 43.8$$

$$d = \frac{M_o}{F} = \frac{43.83}{485} = 0.0903 \text{ m}$$

3.25

$$F_{force} = 200 \text{ N} \quad \theta_1 = 60^\circ$$

$$\vec{F}_{CA}$$

$$\theta_2 = 30^\circ$$

$$A(0, -50, 0)$$

$$B(0, 0, 0)$$

$$C(60, 25, 0)$$

$$\vec{CA} = -60 \hat{i} - 75 \hat{j} + 0 \hat{k}$$

$$F = 200 \cos 60^\circ \hat{i} = 100$$

$$F = 0 \hat{i} - 200 \cos 30^\circ \hat{j} + 200 \cos 60^\circ \hat{k}$$

$$\vec{F}_C = 0 \hat{i} - 173.2 \hat{j} + 100 \hat{k}$$

$$\textcircled{2} \quad \vec{r}_{AC} = 60 \hat{i} + 75 \hat{j} + 0 \hat{k}$$

$$\vec{M}_A^{\vec{F}_C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 60 & 75 & 0 \\ 0 & -173.2 & 100 \end{vmatrix} = \hat{i}(7500) - \hat{j}(6000) + \hat{k}(-10392)$$

(24)



3.34

Assume  $F_{AB} = 100 \text{ N}$

$$\vec{AB} = 7\hat{i} + 4\hat{j} - 32\hat{k}$$

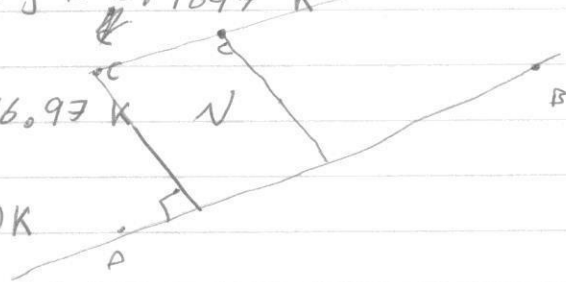
$AB > 32$

$$\vec{F}_{AB} = 0.2121\hat{i} + 0.1212\hat{j} - 0.9697\hat{k}$$

$$\vec{F}_{AB} = 21.21\hat{i} + 12.12\hat{j} - 96.97\hat{k}$$

$$\vec{r}_{CA} = 14\hat{i} - 5\hat{j} + (22 - L)\hat{k}$$

$$\begin{aligned} A & (14, -7, 22) \\ B & (21, -3, -10) \\ C & (0, -2, 2) \\ D & (0, 0, 30) \end{aligned}$$



$N_0$	$\hat{i}$	$\hat{j}$	$\hat{k}$
	14	-5	$(22 - L)$
	21.21	12.12	-96.97

$$= \hat{i}(218.2 + 22.2L) - \hat{j}(1826 + 21.21L) + \hat{k}(275.7)$$

$$\sqrt{(218.2 + 12.2L)^2 + (1826 + 21.21L)^2 + (275.7)^2} = d \times F$$

$$= (218.2 + 12.2L)\hat{i} + (1826 - 21.21L)\hat{j} + (275.8)\hat{k} \quad \text{N.m}$$

$$= \sqrt{(218.2 + 12.2L)^2 + (1826 - 21.21L)^2 + (275.8)^2} \quad \frac{1}{2}$$

$$\frac{d}{dt} \left[ (218.2 + 12.2L) \cdot 12.12 + (1826 - 21.21L) \cdot (-21.21) + 0 \right] = 0$$

$$-38499.14$$

$$\vec{P} \times (\vec{Q} \times \vec{S}) \neq (\vec{P} \times \vec{Q}) \times \vec{S}$$

$$\vec{P} \times (\vec{Q}_1 + \vec{Q}_2) = \vec{P} \times \vec{Q}_1 + \vec{P} \times \vec{Q}_2$$

\* to find Area, we do cross product and between to vector, and then find the magnitude.

\* Assume force (100N)<sub>5.5</sub>

3.23

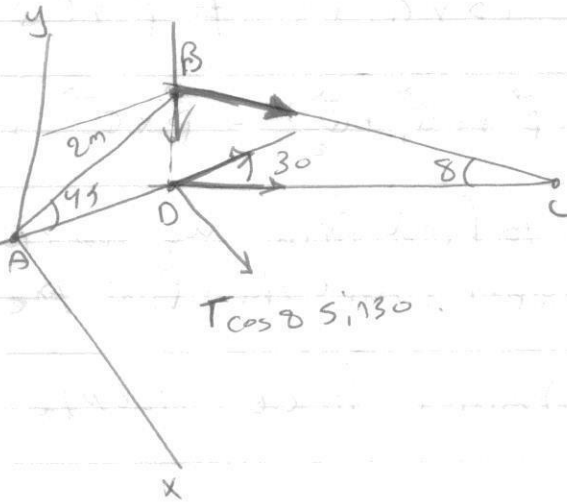
$T = 30$

$$F_y = T \sin 45 = -4.175 \hat{j}$$

$$F_x = T \cos 45 = 29.71$$

$$F_x = T \sin 30 = 14.86 \hat{i}$$

$$F_z = -T \cos 30 = -25.73 \text{ k}$$



$$\vec{F} = 14.86 \hat{i} - 4.175 \hat{j} - 25.73 \text{ k}$$

$$\vec{r}_{AB} = 0 \hat{i} + 2 \sin 45 \hat{j} - 2 \cos 45 \text{ k}$$

$$\vec{r}_{AD} = 0 \hat{i} + 1.414 \hat{j} - 1.414 \text{ k}$$

$$M_A = \begin{vmatrix} +\hat{i} & -\hat{j} & +\text{k} \\ 0 & 1.414 & -1.414 \\ 14.86 & -4.175 & -25.73 \end{vmatrix}$$

$$= \hat{i}(-42.28) - \hat{j}(21) + \text{k}(-21)$$

ans

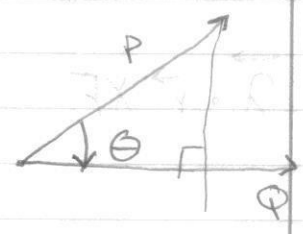


Dot Product:

$$S = \vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}$$

$$= PQ \cos \theta$$

prob

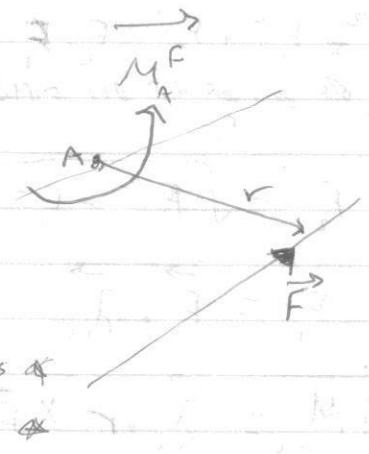


Projection ( $\lambda_Q$ )

P on Q  $\Rightarrow \lambda_Q$   
 Q on P  $\Rightarrow \lambda_P$

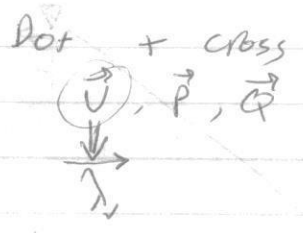
← ←  
 → A force may be moved with respect to the original location to a new point of application if

a moment is added



فيسرع في مادي من اهل لكنا في اهل المثل ان اهل

Mixed Triple Product:



محور الدوران عمله

$$M_{OL} \text{ moment about an axis} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$\vec{\lambda}_{OL}$  of the axis  
 $\vec{r}$  of the point  
 $\vec{F}$  of the force

$$\vec{\lambda} \cdot \vec{r} \times \vec{F}$$

$\vec{r}$ : a vector connecting the two lines between any two random points

المتجهين أي نقطتين عشوائيتين

An imaginary force vector acting along the direction of the "other" line

لدينا قوتين = نريد إيجاد المحاور المحورية  $F_1$  و  $F_2$  ، ويكون لدينا  $F$  ،  $F^2 = F_1^2 + F_2^2$  ،  $F_1$  ،  $F_2$  ،  $F$  لها متجهين

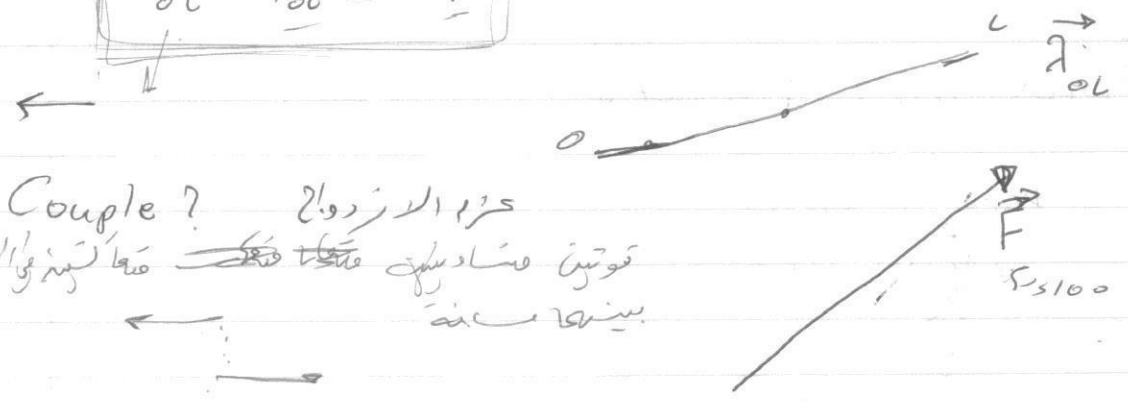
$\vec{F}$  الخط الذي نحده  
 $\vec{F}_1$  الخط الثاني نحده  
 $\vec{F}_2$  والمماس بيننا وبين  $r$

$$F_2 = \sqrt{F^2 - F_1^2}$$

$$F_1 = \vec{F} \cdot \vec{\lambda}_{OL}$$

$$M_{OL} = \vec{\lambda}_{OL} \cdot \vec{r} \times \vec{F}$$

$$M_{OL} \propto d(F_2)$$



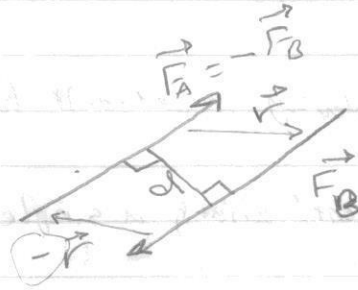
Couple? عزز الازدواج

توتين متساويتين متعاكستين متوازيتين

بيننا وبيننا

\* الكتل المتوازية / بين الكتل المتوازية

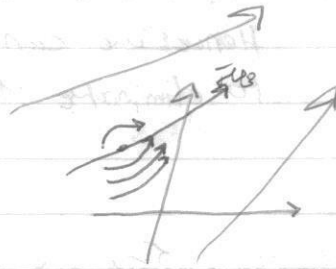
moment of a couple



$$M = F_A d = F_B d$$

لا يمكن إيجاد لحظة لزوجين لا يلتقيان

لجميع التوزيعات في نقطة واحدة  
يكون له عزيم



• يوجد قوة قوى  
• يوجد لحظة عزيم

3-D

• the resultant force " $\vec{R}$ " and the resultant moment " $\vec{M}$ " are perpendicular to each other in the cases only:

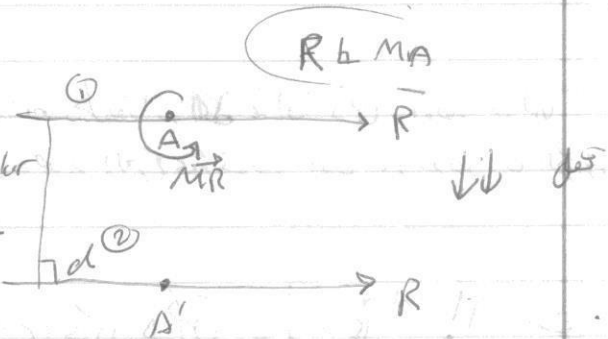
(1) Concurrent (التوازي) متقاطعات

$\vec{F} \perp \vec{M}$

(2) Coplanar (القوى المتوازية في نفس المستوى)  $\vec{F} \perp \vec{M}$

(3) Parallel (القوى المتوازية)  $\vec{F} \perp \vec{M}$

System: one force and one moment, perpendicular to each other, can replace it with a single force



force +  $\vec{M}$   $\rightarrow$  force  $1 \rightarrow 2$   
force  $\rightarrow$  force +  $\vec{M}$   $2 \rightarrow 1$

30

Particularly important for two-dimensional problem -

concurrent: already a single force

parallel } 1 resultant force and 1 couple that are  
coplaner } perpendicular to each other,

Hence: we can move the force to one point  
to eliminate the moment.

في سؤال رقم 3.76 ، بالنسبة للقوتين 50k ، يمكن تحليل كل منهما  
إما على المستوى ABCD وإيجاد العزم لـ لا يكون عمودي على نفسه المستوي  
ثم نجد مرتبات العزم  
✓ إذا تم تحليل كل المتويات المتبادلة وبتحليل العزم على مركزه.

### Equilibrium System

$$\sum F_x = 0$$

$$\sum F_y = 0$$

3.91 \* منه يتم ايجاد نسق القوى والعزم بفعل في نقطة بالكم

\* في سؤال 3.114 ، هناك خطيقتان للذات ، اما نقطة التماس  
وتكون على العزم عنها صفر ، او نجد محصلة العزم عند نقطة اخرى ، ونفرض ان القوة  
توجد عند تلك النقطة (المحطة)

\* ان نسبة اعساس للدائرة عمود على كل من التماسين !!

$$\frac{3.120}{3.124}$$

\* اذا طبق عزم عند نقطة معينة ، بالامكان إيجاد العزم والحللة عند نقطة + العزم الناتج عند  
المحطة عند النقطة المثل

(3)

3.47

$$\vec{M}_O^{F_{AB}} = \vec{r} \times \vec{F}$$

المبدأ والعزم حول المحاور، نجد العزم حول نقطة الأصل  
 العزم حول محور  $x$  هي نفسها المثلثة المربعة للعزم حول نقطة الأصل

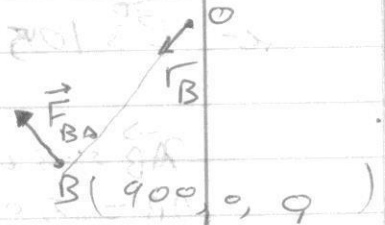
570

$$\vec{r}_{BA} = -900\hat{i} + 600\hat{j} + 360\hat{k} \quad A(0, 600, 360)$$

$$r_{BA} = 1140$$

$$\hat{\lambda}_{BA} = -0.7895\hat{i} + 0.0526\hat{j} + 0.3158\hat{k}$$

$$\vec{F}_{BA} = -78.95\hat{i}$$



$$\vec{M}_O^{F_{AB}} = \vec{r}_B \times \vec{F}_{BA}$$

القوة تقع على المحور  $x$  والعزم

$$= +\hat{i}(0) - \hat{j}(162) + \hat{k}(270)$$

this is the component of the moment created by force  $\vec{F}_{BA}$  that acts about the  $x$ -axis.

if only the component about the axis is required

$l=1$	0	0	$\vec{r}$	$\vec{r}$	$\vec{F}$
$r_x$	$r_y$	$r_z$	$F_x$	$F_y$	$F_z$

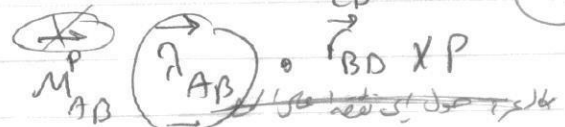
66

$$\frac{34}{56} = \frac{y}{25} \Rightarrow y = 15$$

235

$\frac{30}{2}$

3057



- C: (16, 15, 12)
- D: (16, 38, 12)
- G: (37, 30, 30)

to find  $\vec{P}$

$$\vec{DG} = 21\mathbf{i} - 38\mathbf{j} + 18\mathbf{k}$$

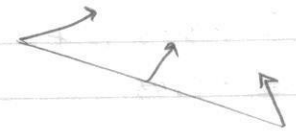
$$\vec{DG} = 25.4747$$

$$\vec{r}_{DG} = 534.8\mathbf{i} - 203.7\mathbf{j} - 305.6\mathbf{k}$$

$$\vec{r}_{DG} = 0.8245\mathbf{i} - 0.3141\mathbf{j} - 0.4711\mathbf{k}$$

$$\vec{r}_{DG} = 0.4468\mathbf{i} - 0.8085\mathbf{j} + 0.3829\mathbf{k}$$

$$\vec{P} = 1045\mathbf{i} - 190\mathbf{j} + 90\mathbf{k} \text{ N}$$



$$\vec{AB} = 32\mathbf{i} - 30\mathbf{j} - 24\mathbf{k}$$

$$AB = 50$$

$$\vec{r}_{AB} = 0.64\mathbf{i} - 0.6\mathbf{j} - 0.48\mathbf{k}$$

to find  $\vec{P}$

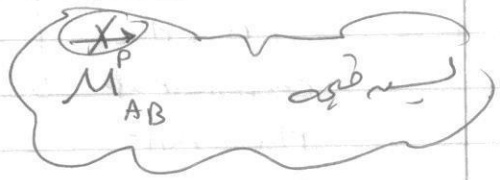
$$\vec{DG}$$

$$\vec{r}_{DG}$$

$$\vec{P} = 325$$

$$\vec{BD} = -16\mathbf{i} + 38\mathbf{j} + 12\mathbf{k} \text{ cm}$$

$$M_{AB}^P = \begin{vmatrix} 0.64 & -0.6 & -0.48 \\ -16 & +38 & +12 \\ +105 & -190 & +90 \end{vmatrix}$$



- $\vec{r}_{AB}$
- $\vec{r}_{BD}$
- $\vec{P}$

$$= + (0.64)(+3648) + (-0.6)(-1620) - 0.48(456)$$

$$= +2484$$

83

$$\Sigma F_x = 4.15 \text{ N}$$

$$\Sigma F_y = 6.073 \text{ N}$$

$$\Sigma M_B = (25)(375) - (15)(50 + 150) - (10)(150) - 6.025 \times 1000$$

$$= 2901 \text{ N}\cdot\text{mm}$$

$$2.9 \text{ N}\cdot\text{m}$$

$$F = 7.35 \text{ N} \quad \Delta = 55.6$$

← between B and C

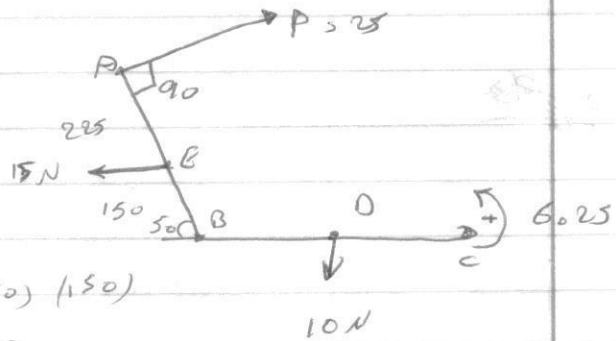
$$2.9 = (7.35 \times \sin 55.6) d$$

$$d = 0.178 \text{ m}$$

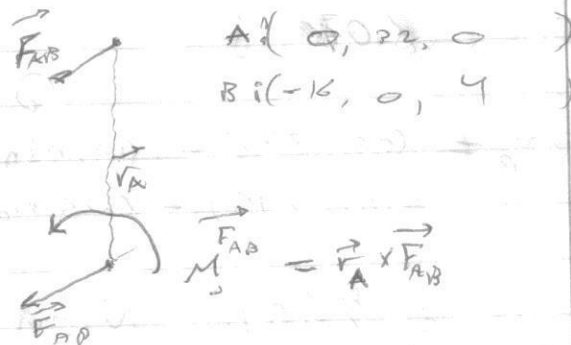
between A and B

$$2.9 = (7.35) \sin(55.6 + 50) d$$

$$d = 0.4096$$



~~30/3~~

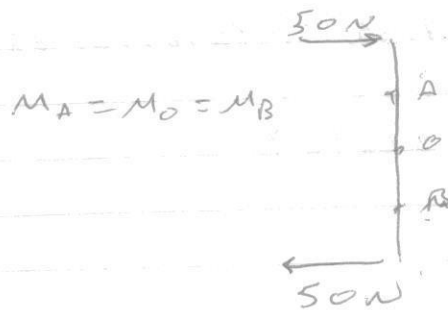


Force-couple system that replaces the original force applied at D.

\* قوتان متساويتان في الاتجاه لكنهما ليسا في نفس المكان عزيم  
 العزم = احداهما في اتجاه سببي (اعانة المحاور)

\* في مثال 3.6 ، القوتان 150 ن ، يقعا في equivalent couple ، كما في العزم  
 بينما ال 100 ن في مركزها مختلفين ،

\* القوة مترابطة ، او عزيم  $\alpha$  لا لزوم







3.99

$$\vec{EH} = 60 \hat{i} + 60 \hat{j} - 70 \hat{k}$$

$$EH = 110$$

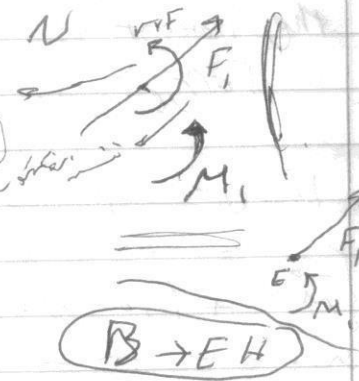
$$\vec{r}_{BH} = 0.5454 \hat{i} + 0.5454 \hat{j} - 0.8363 \hat{k}$$

- E (250, 0, 70)
- H (310, 60, 0)
- J (250-d, 30, 0)
- B (0, 833, 0)

$$\vec{F}_{EH} = 42 \hat{i} + 42 \hat{j} - 49 \hat{k} \quad N$$

$$\vec{M}_B = -31 \hat{i} \quad N \cdot m$$

$$\vec{M}_B = \vec{r}_{BH} \times \vec{F}_i$$



$$\vec{r}_{EJ} = -d \hat{i} + 30 \hat{j} - 70 \hat{k} \quad (mm)$$

$$r_{EH} = \sqrt{d^2 + 5800}$$

$$\vec{r}_{EH} = \frac{-d \hat{i} + 30 \hat{j} - 70 \hat{k}}{\sqrt{d^2 + 5800}}$$

$$\vec{M}_B = \frac{-31d \hat{i} + 930 \hat{j} - 2170 \hat{k}}{\sqrt{d^2 + 5800}} \quad (N \cdot m)$$

$$\vec{r}_{BH} = 310 \hat{i} - 23.3 \hat{j} + 0 \hat{k}$$

direction is the same as the direction of the force

$$\vec{M}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 310 & -23.3 & 0 \\ 42 & 42 & -49 \end{vmatrix}$$

direction of the force and moment

$$= \hat{i}(11,42) - \hat{j}(-15,190) + 14 \hat{k}$$

$$= 11,42 \hat{i} + 1519 \hat{j} + 14 \hat{k} \quad (N \cdot m)$$

37

$$\vec{F}_1 = 42i + 42j - 49k \text{ N}$$

$$\vec{M}_1 = -31d i + 930j - 2170k$$

$$\vec{M}_B = 1.142i + 15.19j + 14.00k \text{ (N.m)}$$

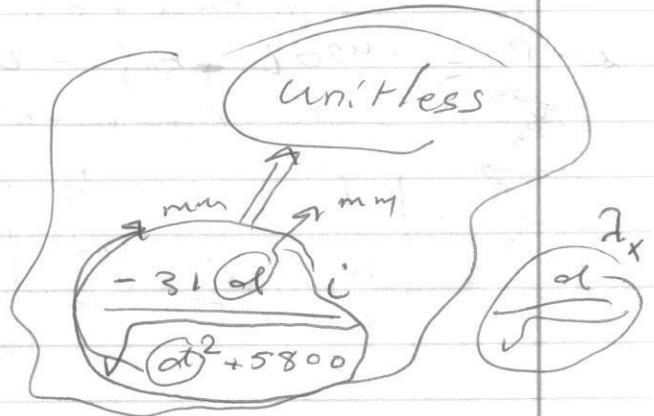
$$\vec{M}_2 = \frac{-2170}{\sqrt{d^2 + 5800}} + 14 = 0$$

$$\sqrt{d^2 + 5800} = \frac{2170}{14}$$

$$d = 135 \text{ mm}$$

$$F_2 = F_1$$

$\vec{M}_1$



$$M_1 = -37 \left( \frac{135}{155} \right) i + \frac{930}{155} j - \frac{2170}{155} k \text{ (N.m)}$$

$$M_1 = -27i + 6j - 14k \text{ N.m}$$

$$M_B = 1.142i + 15.19j + 14.00k \text{ N.m}$$

$$\vec{M}_2 = -25.86i + 21.19j + 0k \text{ (N.m)}$$

$$\vec{F}_2 = 42i + 42j - 49k \text{ N}$$

3.12

A (0, 0, 0)

$$\vec{F}_{50} = 0i - 50j + 0k \text{ N}$$

B (200, 0, 0)

$$\vec{F}_{300} = -300i + 0j + 0k \text{ N}$$

$$\vec{F}_{250} = 0i + 0j - 250k \text{ N}$$

E (200, -100, 160)

$$F_{120} = -120i + 0j + 0k \text{ N}$$

$$\leftarrow R = -420i + 50j - 250k \text{ N}$$

$$\vec{F}_{AB} = 200i + 0j + 0k$$

$$M_A^{F_{50}} = \begin{vmatrix} i & j & k \\ 200 & 0 & 160 \\ 0 & -50 & 0 \end{vmatrix} = (0)i - (0)j + (-10)k \text{ N}\cdot\text{m}$$

$$M_A^{F_{300}} = 0i + 0j + 0k \text{ N}\cdot\text{m}$$

$$M_A^{F_{250}} = \begin{vmatrix} i & j & k \\ 200 & 0 & 0 \\ 0 & 0 & -250 \end{vmatrix} = 0i + j(50) + k(0) \text{ N}\cdot\text{m}$$

$$M_A^{F_{120}} = \begin{vmatrix} i & j & k \\ 0 & -100 & 160 \\ -120 & 0 & 0 \end{vmatrix} = i(0) - j(192) + k(12) \text{ N}\cdot\text{m}$$

$$\leftarrow M_A = 0i + 30.8j - 22k \text{ N}\cdot\text{m}$$

3.152

$$F_x = F \cos \theta \cos \phi$$

$$A(-4, 11, -2)$$

$$F_y = -F \sin \theta$$

$$F_z = F \cos \theta \sin \phi$$

$$M = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 11 & -2 \\ F \cos \theta \cos \phi & -F \sin \theta & F \cos \theta \sin \phi \end{vmatrix}$$

$$= \hat{i} (11 F \cos \theta \sin \phi - 2 F \sin \theta) - \hat{j} (-4 F \cos \theta \sin \phi + 2 F \cos \theta \cos \phi) + \hat{k} (-4 F \sin \theta - 11 F \cos \theta \cos \phi)$$

$$\frac{11 \times 70 \cos 25 \sin \phi - 2 \times 70 \sin 25}{100} = -7.32$$

$$\frac{697.8 \sin \phi - 29.58 \cos \phi}{100} = -7.32 \quad (1)$$

$$\frac{4 \times 70 \cos 25 \sin \phi - 2 \times 70 \cos 25 \cos \phi}{100} = M_y$$

$$\frac{253.8 \sin \phi - 63.44 \cos \phi}{100} = M_y \quad (2)$$

$$\frac{118.3}{100} + \frac{697.8 \cos \phi}{100} = 75.16 \quad (3)$$

$$\cos \phi = \frac{-6.87}{697.8} = -0.00955$$

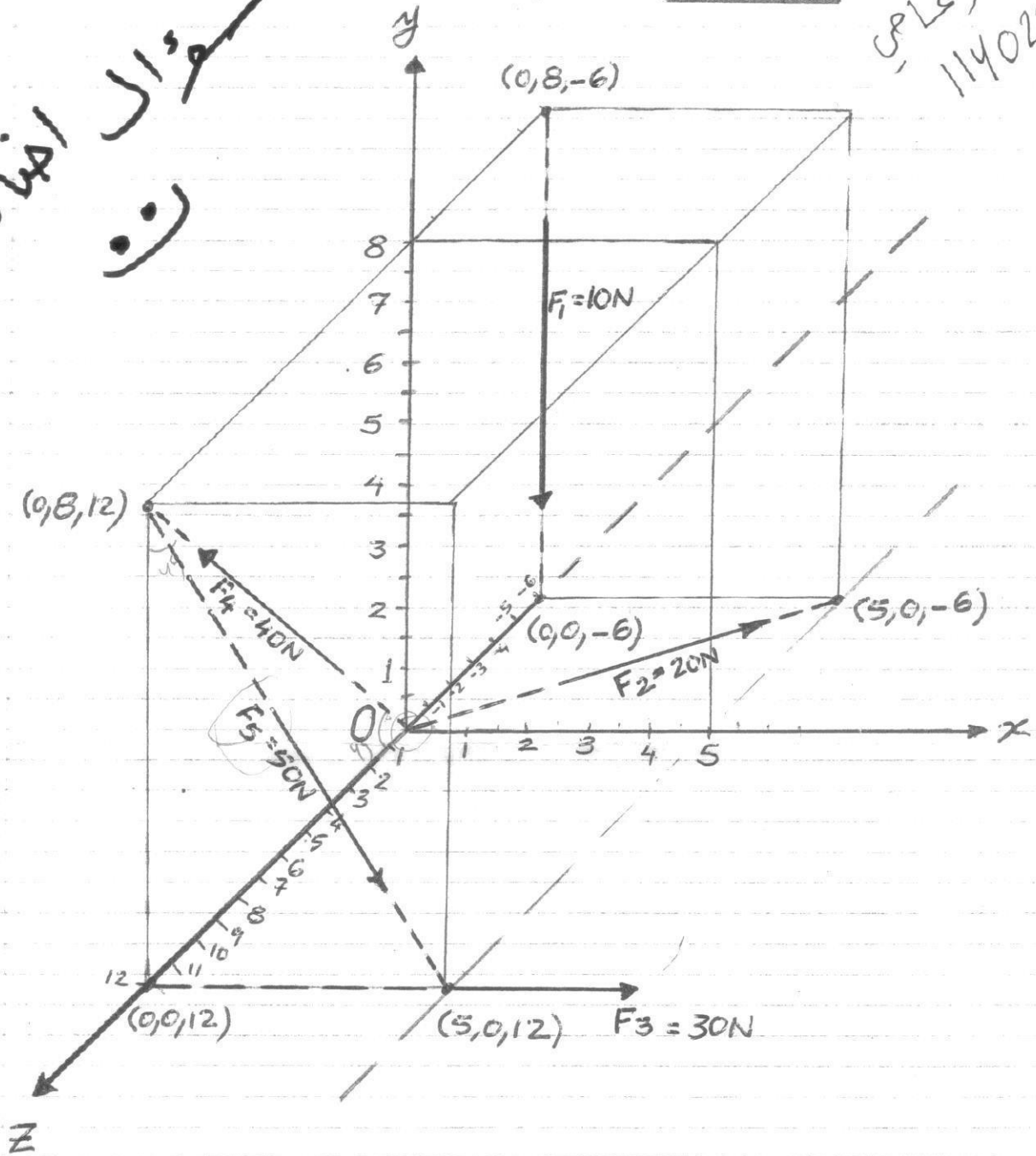
$$\phi = 24.63^\circ$$

$$d = 0.3958 \text{ m}$$

$$= 39.58 \text{ cm}$$

(40)

سوال اخلاص



Determine:

- $\vec{R}$  at  $O$  and  $\vec{M}_O^R$  to replace the force system.
- Perpendicular distance from  $O$  to  $\vec{F}_5$ .
- Perpendicular distance between lines of action of  $\vec{F}_1$  and  $\vec{F}_2$ .
- Perpendicular distance between lines of action of  $\vec{F}_1$  and  $\vec{F}_4$ .
- Perpendicular distance between line of action of  $\vec{F}_2$  and  $\vec{F}_5$ .
- Angle between lines of action of  $\vec{F}_1$  and  $\vec{F}_4$ , and  $\vec{F}_2$  and  $\vec{F}_4$ .

$$\vec{DE} = 0\mathbf{i} - 8\mathbf{j} + 0\mathbf{k}, \quad DE = 8$$

$$\vec{T}_{DE} = 0\mathbf{i} - 1\mathbf{j} + 0\mathbf{k}$$

$$F_1 = 0\mathbf{i} - 10\mathbf{j} + 0\mathbf{k}$$

$$\vec{OC} = 5\mathbf{i} + 0\mathbf{j} - 6\mathbf{k}, \quad OC = 7.8$$

$$\vec{T}_{OC} = 0.6410\mathbf{i} + 0\mathbf{j} - 0.7692\mathbf{k}$$

$$F_2 = 12.82\mathbf{i} + 0\mathbf{j} - 15.38\mathbf{k}$$

$$F_3 = 30\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

$$\vec{OA} = 0\mathbf{i} + 8\mathbf{j} + 12\mathbf{k}$$

$$OA = 14.42$$

$$\vec{T}_A = 0\mathbf{i} + 0.5556\mathbf{j} + 0.8333\mathbf{k}$$

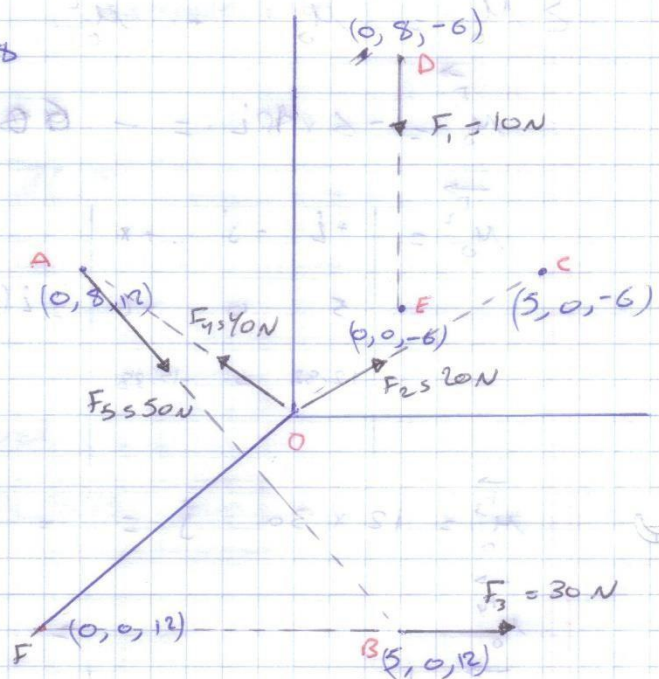
$$F_4 = 0\mathbf{i} + 22.22\mathbf{j} + 33.33\mathbf{k}$$

$$\vec{AB} = 5\mathbf{i} - 8\mathbf{j} + 0\mathbf{k}$$

$$AB = 9.434$$

$$\vec{T}_{AB} = 0.53\mathbf{i} - 0.848\mathbf{j} + 0\mathbf{k}$$

$$F_5 = 26.5\mathbf{i} - 42.4\mathbf{j} + 0\mathbf{k}$$



①  $\vec{R}_O$  and  $\vec{M}_O^R$

$$\vec{R}_x = (0 + 12.82 + 30 + 0 + 26.5)\mathbf{i} = 69.32\mathbf{i}$$

$$\vec{R}_y = (-10 + 0 + 0 + 22.22 - 42.4)\mathbf{j} = -20.18\mathbf{j}$$

$$\vec{R}_z = (0 + 0 - 15.38 + 33.33 + 0)\mathbf{k} = 17.95\mathbf{k}$$

$$\vec{R}_O = 69.32\mathbf{i} - 20.18\mathbf{j} + 17.95\mathbf{k}$$

$$\sum M_0 = \vec{M}_0^{F_1} + \vec{M}_0^{F_2} + \vec{M}_0^{F_3} + \vec{M}_0^{F_4} + \vec{M}_0^{F_5}$$

$$\vec{M}_0^{F_1} = -6 \times 10 \hat{i} = -60 \hat{i}$$

$$\vec{M}_0^{F_2} = \begin{vmatrix} +\hat{i} & -\hat{j} & +\hat{k} \\ 5 & 0 & -6 \\ 12.82 & 0 & -15.38 \end{vmatrix} = \hat{i}(0 \cdot 0 - 0) - \hat{j}(0 \cdot 0 - 0) + \hat{k}(0 \cdot 0 - 0)$$

$$\vec{M}_0^{F_3} = 12 \times 30 \hat{j} = 0 \hat{i} + 360 \hat{j} + 0 \hat{k}$$

$$\vec{M}_0^{F_4} = 0 \hat{i} + 0 \hat{j} + 0 \hat{k}$$

$$\vec{M}_0^{F_5} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 12 \\ 26.5 & -42.4 & 0 \end{vmatrix} = (5 \cdot 0 \cdot 0) \hat{i} + (318) \hat{j} + (-212) \hat{k}$$

$$= 4468 \hat{i} + 678 \hat{j} - 212 \hat{k} \quad \text{Nom}$$

$\sum M_0$

② perpendicular distance from 0 to  $\vec{F}_5$

$$M_0^{F_5} = \sqrt{(508.8)^2 + (318)^2 + (212)^2} = 636.4 \quad \text{Nom}$$

$$M_0^{F_5} = d F = 50 d = 636.4$$

$$d = 12.73$$

③ perpendicular distance between  $\vec{F}_1$  and  $\vec{F}_2$

$$\vec{r}_{F_1} = 0 \hat{i} - 1 \hat{j} + 0 \hat{k}$$

$$\vec{F}_{21} = \lambda \cdot \vec{F}_2 = (0 \hat{i} - 1 \hat{j} + 0 \hat{k}) \cdot (12.82 \hat{i} + \hat{j} - 15.38 \hat{k}) \quad (\text{parallel force})$$

$$= 0 + 0 + 0$$



$$F_{22} = \sqrt{F_2^2 - F_{21}^2} = 20 \text{ N} \quad (\text{perpendicular force})$$

$$r_2 = 5i + 0j + 0k$$

$$M_{F_1} = \begin{vmatrix} 0 & -1 & 0 \\ 5 & 0 & 0 \\ 12.82 & 0 & -15.38 \end{vmatrix} = 76.9$$

$$M_{F_1} = d F_{22}, \quad 76.9 = 20d$$

$$d = 3.845 \text{ mm}$$

$$F_1 = 0i - 1j + 0k$$

$$F_4 = 0i + 22.22j + 33.33k$$

$$F_{1 \rightarrow 2} = 0i + 0j + 6k$$

$$F_{41} = \lambda_{F_1} F_4 = (0i - 1j + 0k)(0i + 22.22j + 33.33k) \quad \text{parallel}$$

$$= -22.22 \text{ N}$$

$$F_{42} = \sqrt{F_4^2 - F_{41}^2} = 33.26 \text{ N}$$

$$M_{F_1} = \begin{vmatrix} 0 & -1 & 0 \\ 0 & 0 & 6 \\ 0 & 22.22 & 33.33 \end{vmatrix} = \text{Zero}$$

there is no perpendicular distance between  $F_1$  and  $F_4$

⊙ perpendicular distance between lines of action of  $\vec{F}_2$  and  $\vec{F}_5$

$$\vec{r}_{F_2} = 0.641 \hat{i} + 0 \hat{j} - 0.7692 \hat{k}$$

$$\vec{r} = 0 \hat{i} + 0 \hat{j} + 18 \hat{k}$$

$$\vec{F}_5 = 26.5 \hat{i} - 42.4 \hat{j} + 0 \hat{k}$$

$$M_{F_2}^{F_5} = \begin{vmatrix} 0.641 & 0 & -0.7692 \\ 0 & 0 & 18 \\ 26.5 & -42.4 & 0 \end{vmatrix} = 489.2 \text{ N}\cdot\text{m}$$

$$F_{51} = \frac{\vec{r}_{F_2} \cdot \vec{F}_5}{F_5} = (0.641 \hat{i} + 0 \hat{j} - 0.7692 \hat{k}) \cdot (26.5 \hat{i} - 42.4 \hat{j} + 0 \hat{k})$$

$$= 16.98 \quad \text{Parallel}$$

$$F_{52} = \sqrt{53^2 - (16.98)^2} = 47 \text{ N} \quad \text{perpendicular}$$

$$M_{F_2}^{F_5} = 489.2 = 47 d$$

$$d = 10.41 \text{ m}$$

⊙  $\vec{F}_1 \cdot \vec{F}_4 = (0\hat{i} - 10\hat{j} + 0\hat{k}) \cdot (0\hat{i} + 22.22\hat{j} + 33.33\hat{k}) = 10 \times 40 \cos \theta$

$$-222.2 = 400 \cos \theta$$

$$\theta = 123.07^\circ$$

$$\vec{F}_2 \cdot \vec{F}_4 = (12.82\hat{i} + 0\hat{j} - 15.038\hat{k}) \cdot (0\hat{i} + 22.22\hat{j} + 33.33\hat{k}) = 20 \times 40 \cos \alpha$$

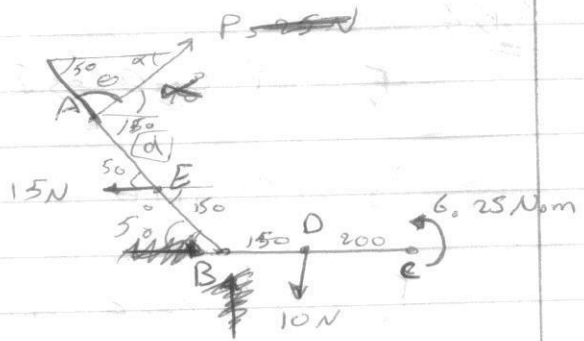
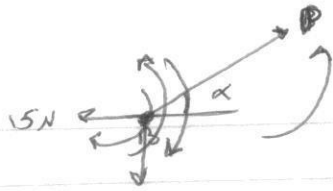
$$-512.06 = 800 \cos \alpha$$

$$\alpha = 129.8^\circ$$





3.115, Page



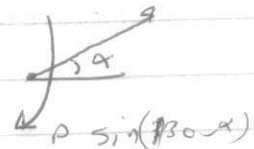
$$\Sigma F_x = 25 \cos 40 - 15 = 0$$

$$\Sigma M_B = 0 \quad (+)$$

$$= -(150)(15)(\sin 50) + (25)(\sin 90)(d + 150) + (150)(10) - 6.25$$

$$- 3520 - 25d = 0$$

$d =$



$\theta = (130 - \alpha)$

$$\Sigma F_x = P \cos \alpha - 15 = 0$$

$$P \cos \alpha = 15$$

$$\Sigma F_y = P \sin \alpha - 10 = 0$$

$$P \sin \alpha = 10$$

$$\alpha = 33.69^\circ$$

$$P = 18.03$$

$$\Sigma M_B = P \sin(130 - \alpha)(d + 150) - (15)(150) \sin 50 + (10)(150) - 6.25$$

$$P \sin(130 - \alpha)(d + 150) - 3026 = 0$$

6474

$$P (\sin 130 \cos \alpha - \sin \alpha \cos 130)(d + 150) - 3026 = 0$$

$$(15 \sin 130 - 10 \sin 130)(d + 150) = 3026$$

6474

$$(17.9)(d + 150) = 3026$$

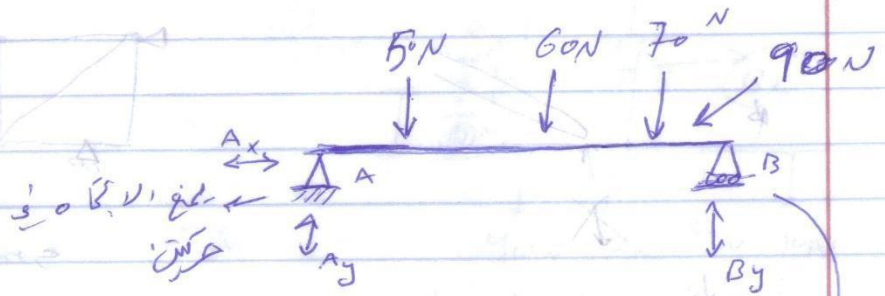
6474

$$d = 17.9 \text{ mm}$$

(4)

Ch4

~~CHM~~



$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M &= 0 \end{aligned}$$

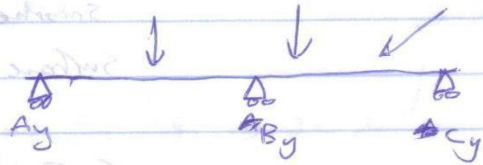
3-equations

3 unknowns

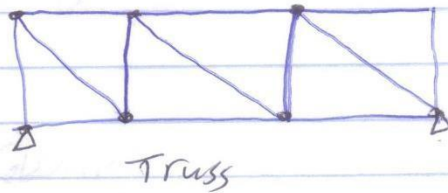
Statically determinate عدد المتغيرات

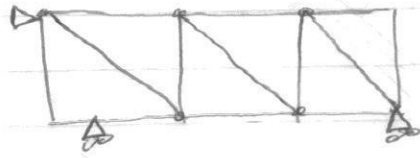
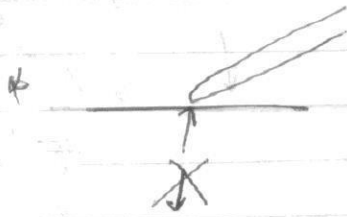
Rigid Body : الجسم الصلب

unstable , 3 unknown  
3 eqn  
statically indeterminate

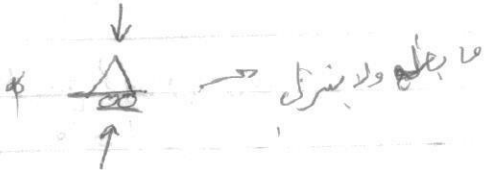


statically indeterminate  
عدد المتغيرات  
المتغيرات

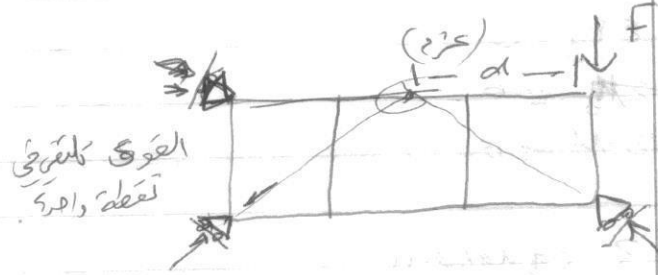




البحر كترنك stable

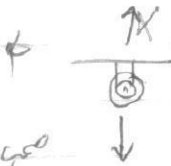


ما بطلع ولا ينزل

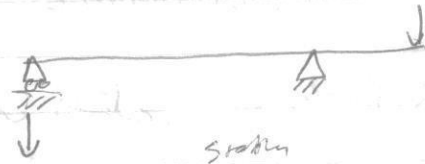


القوى كترنك  
تعلقه وادرس

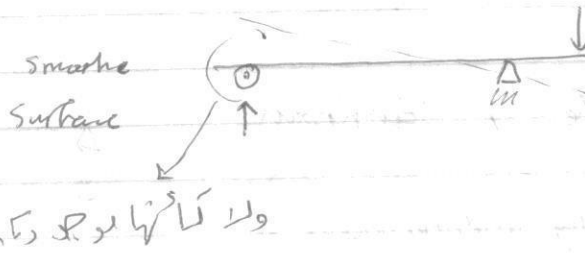
un stable



صحتي كترنك  
ما بطلع



stable



Smathe  
Subranc

ولا تأنها لو ردت

Cable & Ten

per ? T 10.80

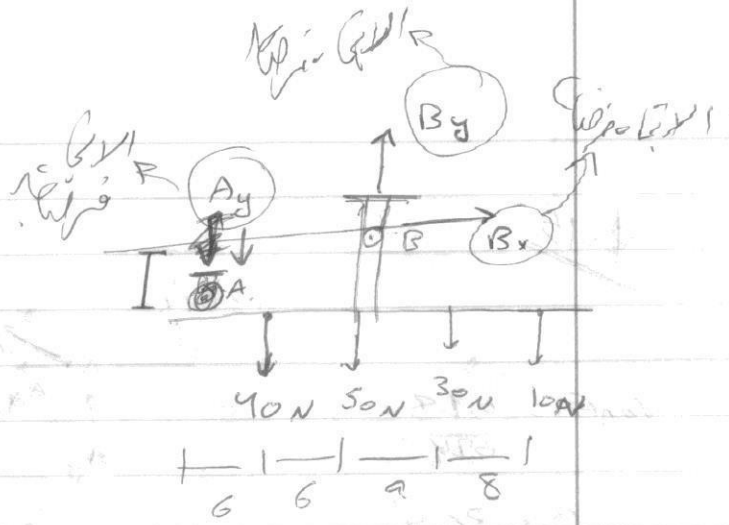


4.3

Ⓒ  $a = 10$

Pin: 2 reactions  
 ⚠️ reaction at on  
 unknown angle.

Roll or: 1 reaction  
 ⊥ to the surface.



$\sum F_x = 0$   $\rightarrow$   
 $B_x = 50$

لا في مجموع العزم ان يكون نقطة الارتكاز نقطة التقاطع  
 ذلك نقطة التقاطع في الجواب ان  $B_x = A_x$

$\sum M \text{ on A} = 0$   $\rightarrow$   $\rightarrow$   $(+B_x)(10)$   
 $(40)(6) + 50(12) - B_y(12) + 30(22) - 10(30) = 0$

$B_y = +150 \text{ N}$   $\uparrow$

$\sum F_y = 0$

$-A_y - 40 - 50 + 150 - 30 - 10 = 0$

$A_y = +20 \text{ N}$

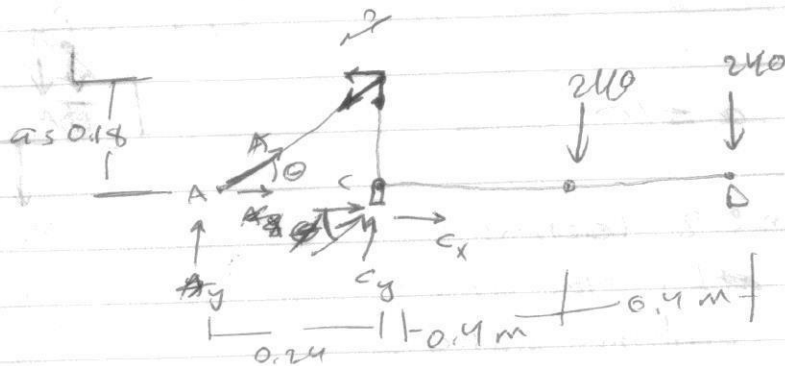
الجواب في الاتجاه الصحيح

Assumed direction is correct

Q.15

$$\tan \theta = \frac{0.18}{0.24}$$

$$\theta = 36.8^\circ$$



$$\sum F_x = 0 = A_x + C_x = 0 \quad \rightarrow$$

$$T \cos 36.8 + C_x = 0 \quad \uparrow +$$

$$-0.8T + C_x = 0$$

$$\sum F_y = 0 = T \sin 36.8 + C_y - 240 - 240 = 0$$

$$= T \sin 36.8 + C_y = 480 \quad \text{--- (2)}$$

$$-0.6T + C_y = 480$$

$$M_{\text{about C}} = 240(0.4) + 240(0.8) - T(0.18) = 0 \quad \text{--- (1)}$$

$$T_x = \frac{288}{0.18} = 1600 \text{ N} = T \cos 36.8$$

all the way to the top is  $T = 1998 \text{ N}$

$$C_x = 0.8 \times 1998 = 1598.4 \text{ N}$$

$$C_y = 480 + 0.6 \times 1998 = 1679$$

$$\phi = 46.4 \quad \tan \phi = \frac{1640}{1600}$$

14.17

$$\sum M_c = 0$$

$$= +100(7.5) - F_{rod}(5) = 0$$

$$F_{rod} = +150 \text{ N}$$

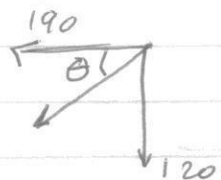
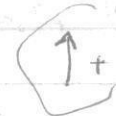
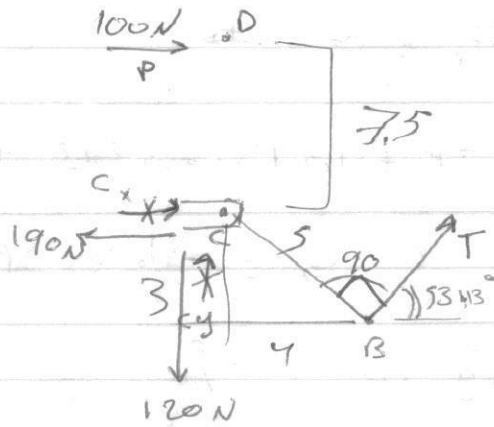
$$\sum F_x = 0 = 100 + C_x + T \cos 53 = 0$$

$$= C_x = -190 \text{ N}$$

$$\sum F_y = 0$$

$$T \sin 53 + C_y = 0$$

$$C_y = -120 \text{ N}$$



$\theta = 32.27^\circ$   
 $R = 224.7 \text{ N}$

14.18

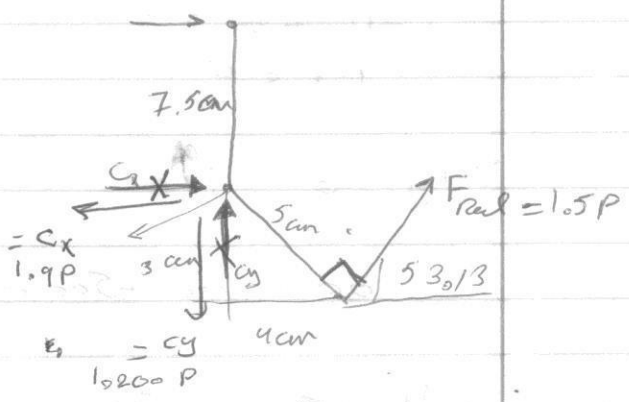
$$\text{Max } F_c = 250 \text{ N}$$

$$C_x = C_x$$

$$\sum M \text{ about } C = 0$$

$$+P(7.5) - F_{rod}(5) = 0$$

$$F_{rod} = 1.5P$$



46

11.4.3a H.W

$$\Sigma F_x = 0 \quad \rightarrow$$

$$+P + 1.5P \cos 30.13 + C_x = 0$$

$$C_x = -1.900 P$$

$$\Sigma F_y = 0 \quad \uparrow$$

$$+C_y + 1.5P \sin 30.13 = 0$$

$$C_y = -1.200 P$$

$$\sqrt{(1.20P)^2 + (1.90P)^2} \leq 250 N$$

$$P \leq 111.3 N$$

11.4.3

$$\Sigma F_x = 0 \quad \rightarrow$$

$$-D_x + 0 = 0$$

$$D_x = 0$$

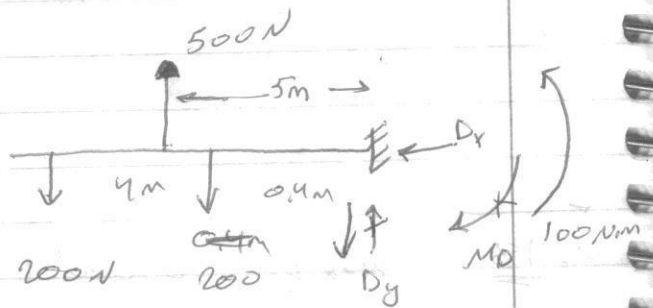
$$\Sigma F_y = 0 \quad \uparrow$$

$$-200 - 200 + 500 + D_y = 0 \quad D_y = -100 N$$

$$\Sigma M_D = 0 \quad \curvearrowright$$

$$200(8) + 500(5) - 200(4) + M_D = 0 \quad M_D = -100 N$$

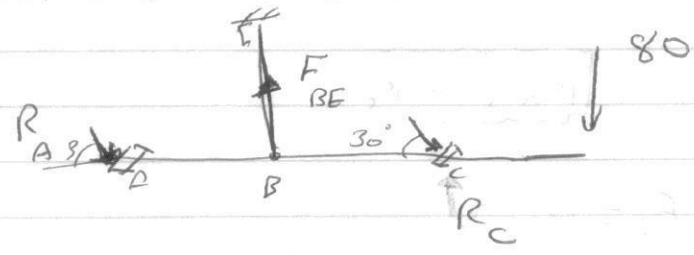
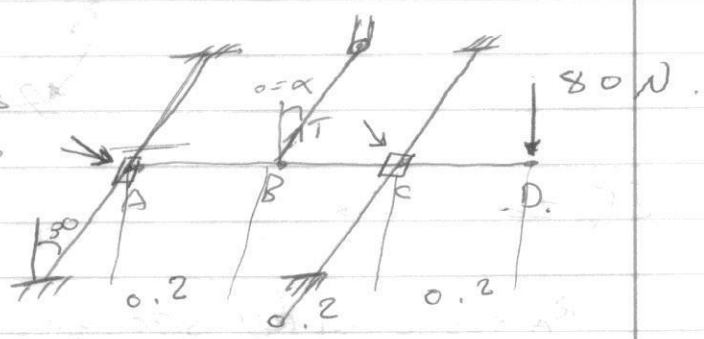
$$M_D = -100 N$$



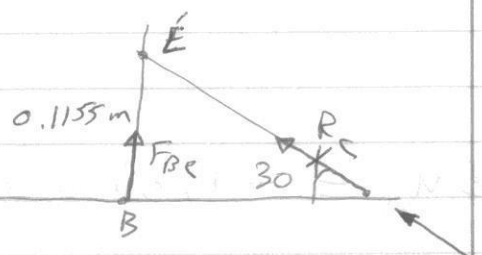
(47)

4.39

\* افترض ان العزم عند نقطة A  
 قويا اقل من العزم عند النقطة B  
 \* بيت توتين E  
 \* لو نجد العزم حول نقطة E



$\sum F_x =$



Moment E =  $(R_A \cos 30)(0.1155) - R_A (\sin 30)(0.2) + 80(0.4) = 0$  160 N

$R_A = +160$  المتجه الى اليمين

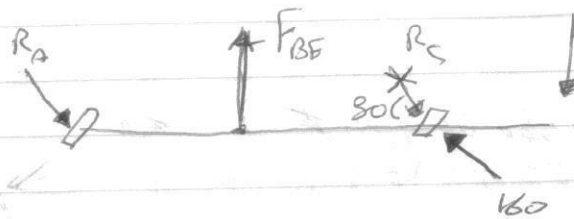
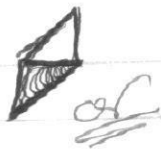
$\sum F_x = 0 = +160 \cos 30 + R_C \cos 30 = 0$

$R_C = -160 \text{ N}$

$\sum F_y = 0 \uparrow$

$-160 \sin 30 + 160 \sin 30 + F_{BE} - 80 = 0$

$F_{BE} = +80 \text{ N}$



$$\Sigma F_x = 0 \rightarrow$$

$$+R_A \cos 30 + R_C \cos 30 = 0$$

$$R_A \sin 30 - R_C$$

$$\Sigma F_y = 0 \uparrow$$

$$-R_A \sin 30 - R_C \sin 30 + F_{BE} - 80 = 0$$

$$\Sigma M_B = -R_A (\sin 30) (0.2) + R_C \sin 30 (0.2) + 80 (0.4) \uparrow$$

$$-R_A \sin 30 (0.2) - R_C \sin 30 (0.2) + 80 (0.4)$$

$$R_A \sin 30 + 160$$



40



$$\Sigma F_x = R_A \cos 30 + F_{BE} \cos 60 + R_C \cos 30 = 0$$

$$\Sigma F_y = -R_A \sin 30 + F_{BE} \sin 60 - R_C \sin 30 - 80 = 0$$

80

49

$$\Sigma F_x = R_A + T_{BE} (0.577) + R_C = 0$$

$$\Sigma F_y = -R_A + T_{BE} (1.73) - R_C = 80$$

$\frac{5,130}{5,130}$

---

$$2.307 T_{BE} = \frac{80}{5,130}$$

$$T_{BE} = \boxed{34.67} \text{ N}$$

$$\Sigma M =$$

Particle

2-dimensions

$$\sum F_x = 0$$

$$\sum F_y = 0$$

2

3-dimensions

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

3

Rigid Body

2-dimensions

$$\sum F_x$$

$$\sum F_y$$

$$\sum M$$

$$\sum F_x$$

$$\sum M$$

$$\sum M$$

3



3-dimensions

$$\sum F_x$$

$$\sum F_y$$

$$\sum F_z$$

$$\sum M_x$$

$$\sum M_y$$

$$\sum M_z$$

6

### Two-force member (body)

Pin

is

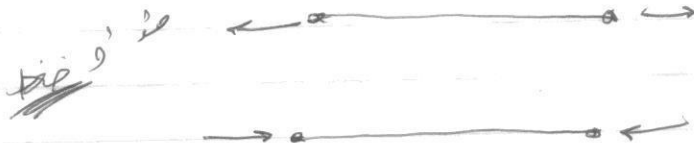
pins at the ends of the member

Pin

pin

No transverse loads along its

length



Cables

Chords (chords)

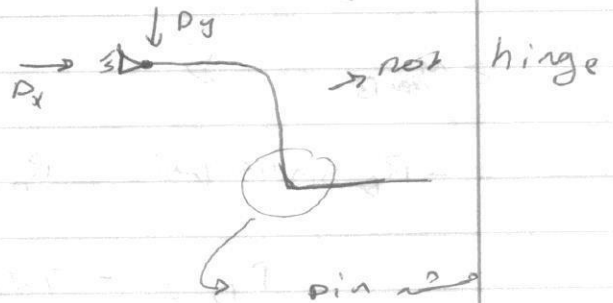
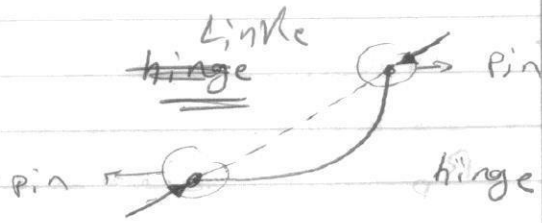
LINKS

2401

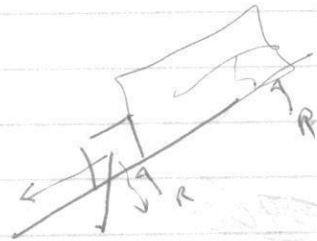
(51)



How - free member

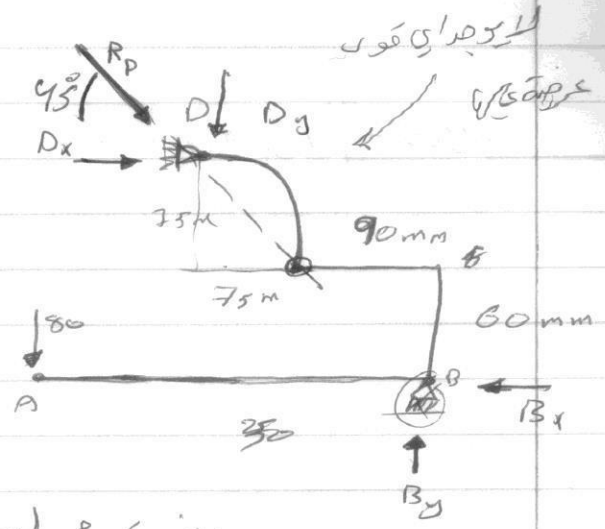


انسی وہی اسکی (ما) 9.45 ، حدیثوں سے معلوم ہے کہ عند B سے التوتیر ہے  
 لیکن لاشی عند A اور التوتیر الیہ من التوتیر



4.07

$R_D$  (جهت منفی)

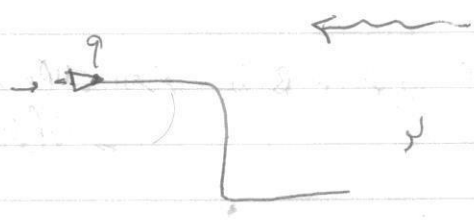


$\sum M_{\text{at } B} = 0$  (نشان می‌دهد که سیستم در تعادل است)

$$-R_D \sin 45 (165) + R_D \cos 45 (75) - 80 (250) = 0$$

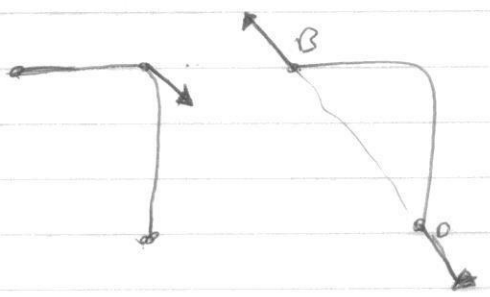
$$R_D = -942.8 \text{ N}$$

لاستیک در جهت مثبت



←

4.0766



(53)

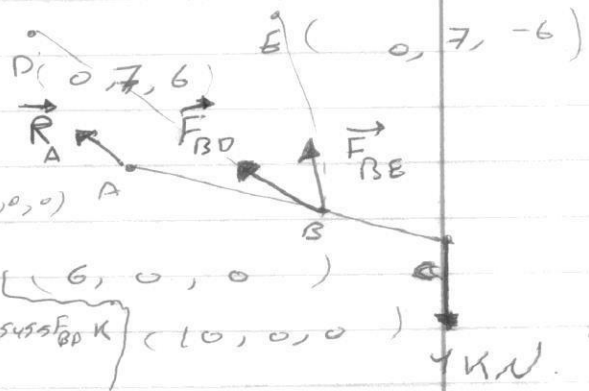
4.107

مطلوب التزم عند A ← ينشئ ثلاث اتجاهات  
 ومطلوب التزم عند B ← DA ← ينشئ اتجاه واحد

$$\vec{F}_y = 0 \hat{i} - 4 \hat{j} + 0 \hat{k} \text{ KN}$$

$$\vec{R}_{BD} =$$

$$BD =$$



$$\vec{r}_{BD} = -0.5455 \hat{i} + 0.6364 \hat{j} + 0.5455 \hat{k}$$

$$\vec{F}_{BD} = -0.5455 F_{BD} \hat{i} + 0.6364 F_{BD} \hat{j} + 0.5455 F_{BD} \hat{k}$$

$$\vec{R}_A = R_{Ax} \hat{i} + R_{Ay} \hat{j} + R_{Az} \hat{k}$$

$$\vec{F}_{BE} = -6 \hat{j}$$

$$\vec{F}_{BE} = -0.5455 F_{BE} \hat{i} + 0.6364 F_{BE} \hat{j} - 0.5455 F_{BE} \hat{k}$$

لا توجد معادلات ثلاث اتجاهات ← ينشئ ثلاث اتجاهات

لنكتب: التزم عند A ← ينشئ ثلاث اتجاهات

$$\vec{r}_{AD} \cdot (\vec{r}_B \times \vec{F}_{BE}) =$$

$$+ \vec{r}_{AD} \cdot (\vec{r}_C \times \vec{F}_y)$$

$R_A$  لا تدخل عن  
 معادلات التزم

مطلوب التزم عند B ← ينشئ اتجاه واحد



43

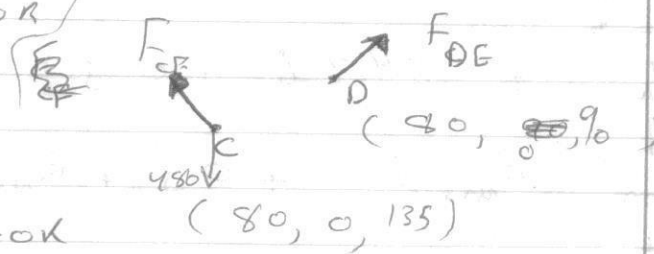
55

4.12)

$\vec{R}_{MA} = R_{MAx} \hat{i} + 0 \hat{j} + R_{MAz} \hat{k}$      $N_{omm}$      $M_y$   
 $\vec{F}_{CF} = -80 \hat{i} + 60 \hat{j} + 0 \hat{k}$      $(0, 60, 135)$      $A_x$      $A_z$      $E$   
 $CF = 100$      $(0, 0, 0)$      $(80, 120, 0)$

$\vec{r}_{CF} = -0.8 \hat{i} + 0.6 \hat{j} + 0 \hat{k}$

$\vec{F}_{CF} = \vec{F}_{CF}$



$\vec{F}_{CF} = -0.8 F_{CF} \hat{i} + 0.6 F_{CF} \hat{j} + 0 \hat{k}$      $(80, 0, 135)$

$\vec{DE} = 0 \hat{i} + 120 \hat{j} - 135 \hat{k}$   
 $DE = 180$

$\vec{r}_{DE} = 0 \hat{i} + 0.8 \hat{j} - 0.6 \hat{k}$

$\vec{F}_{DE} = 0 \hat{i} + 0.8 F_{DE} \hat{j} - 0.6 F_{DE} \hat{k}$

$F_{480} = 0 \hat{i}$

$\sum F_x = -0.8 F_{CF} + 0 \hat{i} + A_x = 0$     (1)

$\sum F_y = 0.6 F_{CF} + 0.6644 F_{DE} - 480 = 0$     (2)

$\sum F_z = 0 - 0.7475 F_{DE} + A_z = 0$     (3)

$-F_{CF} + 0 + 1.25 A_x$

$\vec{R}_{FA}$	$\vec{F}_{CF}$	$= -0.8 F_{CF} \hat{i} + 0.6 F_{CF} \hat{j} + 0 \hat{k}$	N
$F_{480}$	$\vec{F}_{DE}$	$= 0 \hat{i} + 0.8 F_{DE} \hat{j} - 0.6 F_{DE} \hat{k}$	N
$\vec{R}_{FA}$	$F_{480}$	$= 0 \hat{i} - 480 \hat{j} + 0 \hat{k}$	N
	$\vec{R}_{FA}$	$= R_{FAx} \hat{i} + 0 \hat{j} + R_{FAz} \hat{k}$	N
			N

$$\vec{M}_A^{F_{CF}} = \vec{r}_F \times \vec{F}_{CF} = -81 F_{CF} \hat{i} - 108 \hat{j} + 48 \hat{k}$$

$$\vec{M}_A^{F_{DE}} = -72 F_{DE} \hat{i} - 108 F_{DE} \hat{j} + 48 F_{DE} \hat{k}$$

~~$$\vec{M}_A^{F_{DE}}$$~~

$$\vec{M}_A^{480} = 64.8 \hat{i} + 0 \hat{j} - 38.4 \hat{k}$$

$$\vec{R}_{MA} = R_{MAX} \hat{i} + 0 \hat{j} + R_{MAZ} \hat{k}$$

$$\sum M_y = 0 = -108 F_{CF} + 48 F_{DE} = 0$$

$$F_{DE} = +2.25 F_{CF}$$

$$\sum F_y = 0 = 0.6 F_{CF} + 0.8 F_{DE} - 480 = 0$$

$$= +0.6 F_{CF} + 0.8 (2.25 F_{CF}) - 480 = 0$$

$$F_{CF} = 200 \text{ N}$$

$$F_{DE} = +450 \text{ N}$$

$$R_{FAX} = +160 \text{ N}$$

$$R_{FAB} = +270 \text{ N}$$

$$R_{MAX} = -16.2 \text{ N}\cdot\text{m}$$

$$R_{MAZ} = 0 \text{ N}\cdot\text{m}$$

4.133

$$\vec{CE} = -240\hat{i} + 600\hat{j} - 400\hat{k}$$

$$|\vec{CE}| = 760$$

$$\hat{r}_{CE} = -0.3158\hat{i} + 0.7895\hat{j} - 0.5263\hat{k}$$

$$\vec{F}_{CE} = -0.3158 F_{CE} \hat{i} + 0.7895 F_{CE} \hat{j} - 0.5263 F_{CE} \hat{k}$$

$$\vec{R}_F = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$$

$$\vec{F}_w = 0\hat{i} - 490.5\hat{j} + 0\hat{k}$$

$$\vec{AB} = 480\hat{i} - 200\hat{j} + 0\hat{k}$$

$$|\vec{AB}| = 520$$

$$\hat{r}_{AB} = 0.9231\hat{i} - 0.3846\hat{j} + 0\hat{k}$$

$$A (0, 0, 0)$$

$$B (480, -200, 0)$$

$$C (480, -200, 400)$$

$$D (0, 0, 400)$$

$$E (240, 400, 0)$$

$$w (240, 400, 200)$$

AB ليس حبل في الزاوية بين A و B و C في الزاوية بين A و B و C  
 في الزاوية بين A و B و C في الزاوية بين A و B و C  
 في الزاوية بين A و B و C في الزاوية بين A و B و C

$$M_{AB} = \begin{vmatrix} 0.9231 & -0.3846 & 0 \\ 240 & -100 & 200 \\ 0 & -490.5 & 0 \end{vmatrix} = \underline{190.586} + \text{Nm.}$$

$$\begin{matrix} \vec{F}_{CE} \\ M_{AB} \end{matrix} = \begin{vmatrix} 0.9231 & -0.3846 & 0 \\ 240 & 400 & 0 \\ -0.3158 F_{CE} & 0.7895 F_{CE} & -0.5263 F_{CE} \end{vmatrix} = -0.1943 F_{CE} - 0.0486 F_{CE} = \underline{-0.2429 F_{CE}}$$

(32)

$$F_{CE} = 372.8$$

(58)





$$\vec{F}_P = -x_0 \hat{i} - \underline{P} \hat{j} + 0 \hat{k}$$

$$\vec{F}_D = -F_D \cos 45 \hat{i} + F_D \sin 45 \hat{j} + 0 \hat{k}$$

$$\vec{F}_B = F_B \cos 45 \hat{i} - F_B \sin 45 \hat{j} + 0 \hat{k}$$

$$\vec{F}_C = 0 - F_C \sin 45 \hat{j} + F_C \cos 45 \hat{k}$$

$$\underline{R_F} = R_{Fx} \hat{i} + R_{Fy} \hat{j} + R_{Fz} \hat{k}$$

$$\sum \vec{M}_P = \vec{M}_P + \vec{F}_D + \vec{F}_B + \vec{F}_C$$

$$\vec{M}_P = 0 \hat{i} + 0 \hat{j} + 200 \hat{k}$$

$$\vec{M}_P^{F_D} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 80 & 200 \\ 0.7071 F_D & -0.7071 F_D & 0 \end{vmatrix} = \hat{i} (141.4 F_D) - \hat{j} (141.4 F_D) + \hat{k} (56.57 F_D)$$

$$\vec{M}_P^{F_B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 80 & -200 \\ 0.7071 F_B & -0.7071 F_B & 0 \end{vmatrix} = \hat{i} (-141.4 F_B) - \hat{j} (141.4 F_B) + \hat{k} (56.57 F_B)$$

$$\vec{M}_P^{F_C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 200 & 80 & 0 \\ 0 & -0.7071 F_C & 0.7071 F_C \end{vmatrix} = \hat{i} (56.57 F_C) - \hat{j} (141.4 F_C) + \hat{k} (141.4 F_C)$$

$$A (0, 80, 0)$$

$$B (0, 80, -200)$$

$$C (200, 80, 0)$$

$$D (0, 80, 200)$$

$$E (-200, 80, 0)$$

Ch5

center of gravity  
Centroid, Area

CAG

total weight =  $\sum \Delta w$

total moment about the x-axis =  $\sum z_i \Delta w_i$

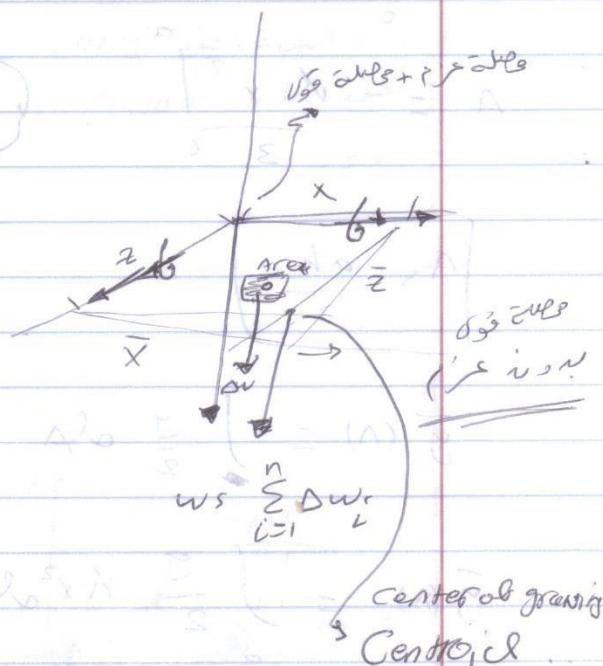
n = total number of pieces

$\bar{z} \cdot W = \int z \, dW$

$\bar{x} \cdot W = \int x \, dW$

$\bar{y} \cdot W = \int y \, dW$

$\frac{N}{S} = N$



the thickness is constant  
and the material is uniform

$\Delta w = (\Delta A)(t)(\rho)$

نجد المساحة للجزء والتكثيف إلى نقطة الأصل  
ثم نضرب القوة بجمع تأثير الأجزاء

$\bar{x}(A \cdot t \cdot \rho) = t \cdot \rho \int x \, dA$

$\bar{x} \cdot A \cdot t \cdot \rho = \int x \, dA$

نضرب العلاقة بالثقل عند المرجع  
والمعادلة الناتجة لو اني محور آخر

Determine Line Datum

Find the centroid

Page 225

$$dA = y \, dx$$

$$dA = Kx^2 \, dx$$

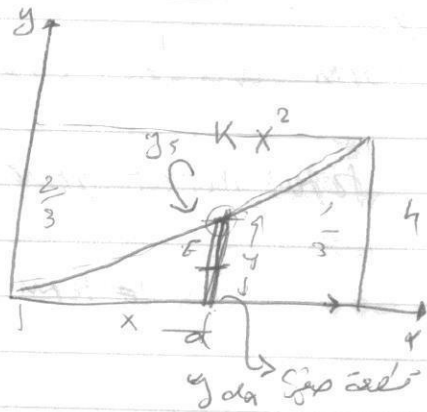
$$A = K \int_0^a x^2 \, dx$$

$$A = \frac{Kx^3}{3} \Big|_0^a$$

$$K = \frac{h}{a^2}$$

E: ...

...



$$A = \frac{ah}{3}$$

$$\bar{y}(A) = \int \frac{y}{2} \, dA$$

$$\bar{y}(A) = \int \frac{y}{2} Kx^2 \, dx$$

$$= \int_0^a \frac{Kx^2}{2} Kx^2 \, dx$$

$$\bar{y}(A) = \frac{K^2}{2} \int_0^a x^4 \, dx$$

$$\bar{y}(A) = \frac{K^2}{2} \frac{x^5}{5} \Big|_0^a$$

$$\bar{y}(A) = \frac{K^2 a^5}{2 \times 5}$$

$$\bar{y}(A) = \frac{h^2}{2a^4} \frac{a^5}{5}$$

$$\bar{y}(A) = \frac{h^2 a}{10}$$

$$\Rightarrow \bar{y} = \frac{ah}{3} \quad \bar{y} \frac{ah}{3} = \frac{h^2 a}{10}$$

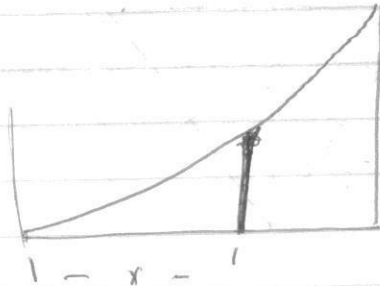
$$\bar{y} = \frac{3h}{10}$$

(61)

$$\bar{x} (A) = \int x \, dA$$

$$= \int x K x^2 \, dx$$

$$= K \int_0^a x^3 \, dx$$



$$\bar{x} \frac{ah}{3} = K \frac{x^4}{4} \Big|_0^a$$

(origin) (0) a'asili ip li q  
 2a result value is  
 1/2 a

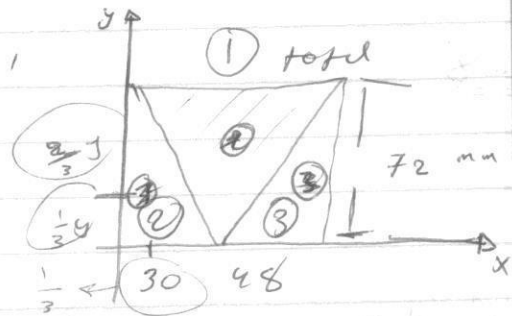
$$\bar{x} \frac{ah}{3} = \frac{K h^3 a^4}{4}$$

$$\bar{x} = \frac{3}{4} a$$

5.2

3. 200 a'asili

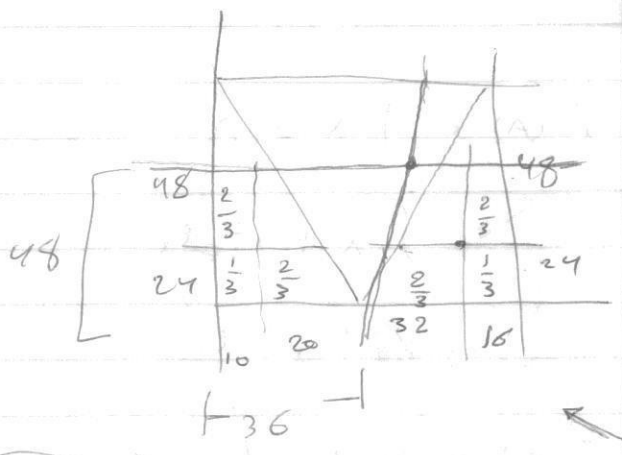
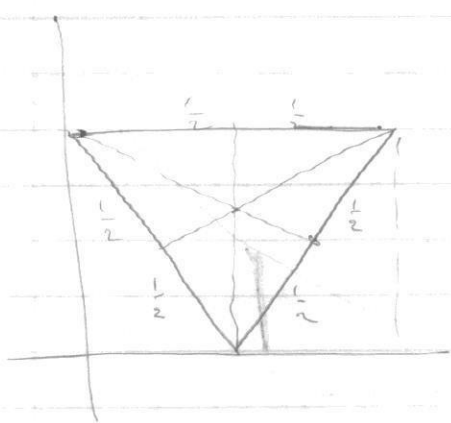
$$A = \frac{1}{2} \times y$$



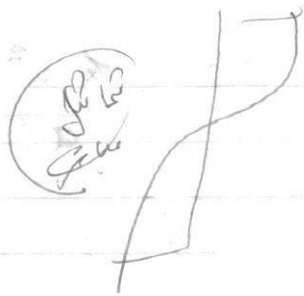
200

shape	Area (mm) <sup>2</sup> (element)	mm $\bar{x}$ element	mm <sup>3</sup> $\bar{x}$ element Area element	$\bar{y}$ mm	$\bar{y}$ x Area element (mm) <sup>3</sup>
①	+(78 x 72)	+39	39 x 78 x 72 219024	+36	36 x 78 x 72 +202176
②	-1080	10	$\frac{1}{3} \times 30$ -10800	$\frac{1}{3} \times 72$ +24	-25920
③	-1728	62	$30 + \frac{2}{3} \times 48$ -107136	+24	-41472
Σ	2808	36	101088	48	134784
		$\bar{x} = \frac{101088}{2808}$		$\bar{y} = \frac{134784}{2808}$	

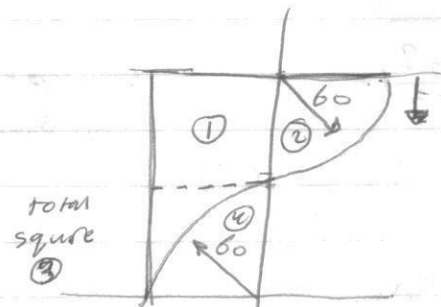
62



+ مساحتہ غورہ انکور علیا  
 + تحت انکور علیا  
 - غورہ انکور منقطعہ  
 - تحت انکور منقطعہ  
 طول فوق انکور موصیہ  
 تحت انکور موصیہ



5.8



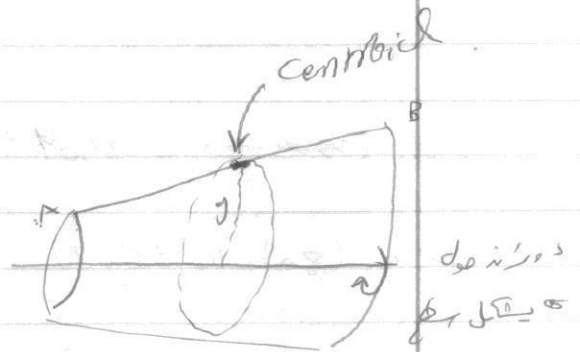
Sluque	Area (el)	$\bar{x}$ elm	$(\bar{x}d)(A el)$	$\bar{y}_{el}$	$\bar{y}_{el}(A el)$
1	+ 3600	-30	-108000	+90	
2	+ 2827	+25.46	+71995	(120 - 25.46)	
3	+ 3600	-30		130	
$\Sigma$	-2827	-25.46		+25.46	

$\bar{x} = \frac{0}{0} = -10 \text{ mm}$

$\bar{y} = +87.5$

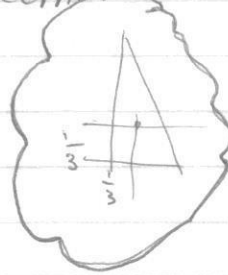
~~Pappus - Guldinus~~

Pappus Guldinus



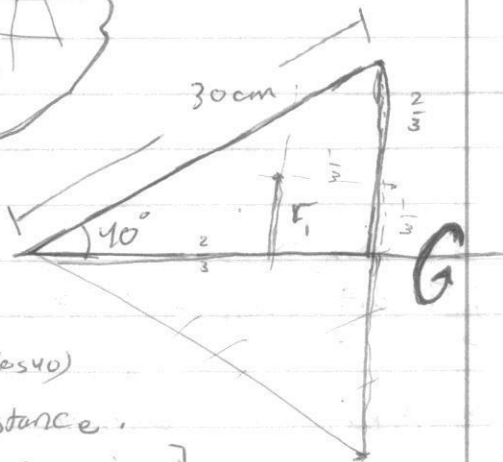
[2] the volume = Area of the shape  
being rotated to generate the volume  
X Distance traveled by the centroid.

$$V = A \times 2\pi r = 2\pi r A$$



$$V = \frac{1}{3} (19,284)$$

Distance traveled by the centroid  
=  $2\pi r = 2\pi \left(\frac{1}{3} \times 30 \times \sin 40^\circ\right)$



Area of the triangle =  $\frac{1}{2} (35,740 / 30 (\cos 40^\circ))$

Area Distance  
Volume =  $\left[\frac{1}{2} (30 \sin 40^\circ) (30 \cos 40^\circ)\right] \left[2\pi \frac{1}{3} \times 30 \times \sin 40^\circ\right]$

Volume =  $8949 \text{ cm}^3$

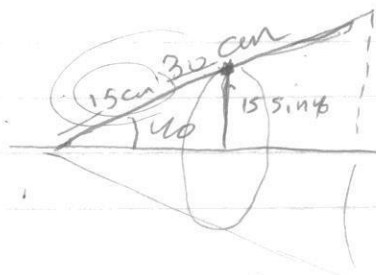
=  $8949.14$  !!

surface area =  $30(2\pi)(15 \sin 40^\circ) =$

= Length of the line rotated

[1] to create the shape X Distance by the centroid

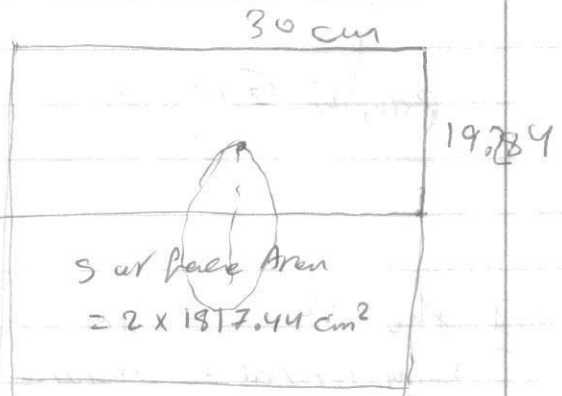
~~$\frac{1}{3} \pi r^2 h$~~   
 ~~$\frac{2}{3} h \pi r$~~



مساحة السطح =  $35048 \text{ cm}^2 = \xi$  ←

$$\text{radius} (30) (19.284) (2\pi) \frac{19.284}{2}$$

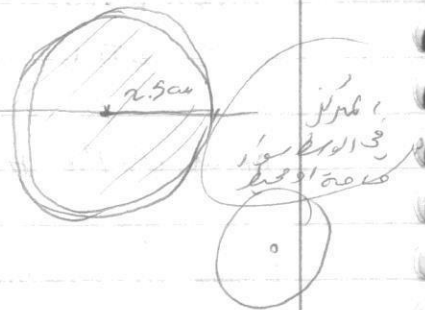
$$= 35048 \text{ cm}^2$$



$$\frac{2\pi r \times \text{height}}{2}$$

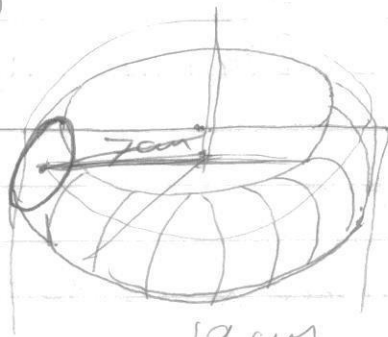
$$[\pi(2.5)^2] [2\pi \times 7] = 862.7 \text{ cm}^2$$

Area
distance by center

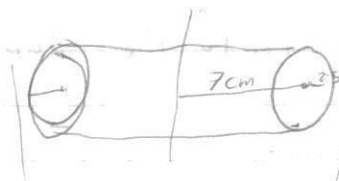


$$\text{Surface Area} = (2\pi \times 2.5) \times (2\pi \times 7)$$

$$= 690.87 \text{ cm}^2$$



19 cm



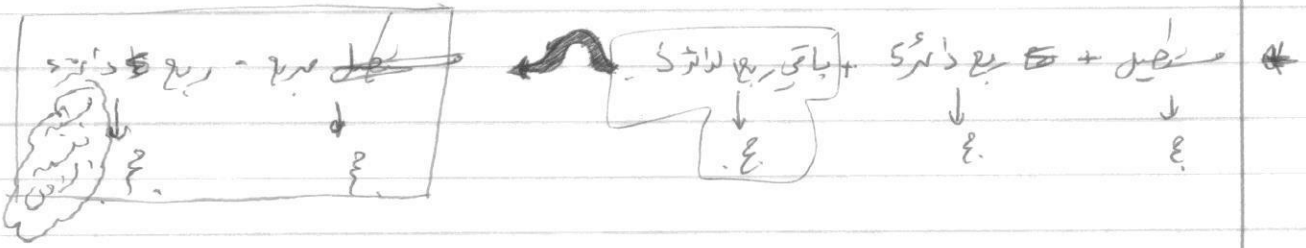
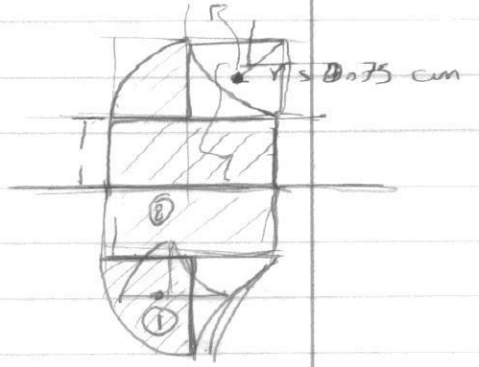
19 cm

(65)

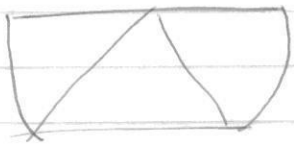
(10)



5.56



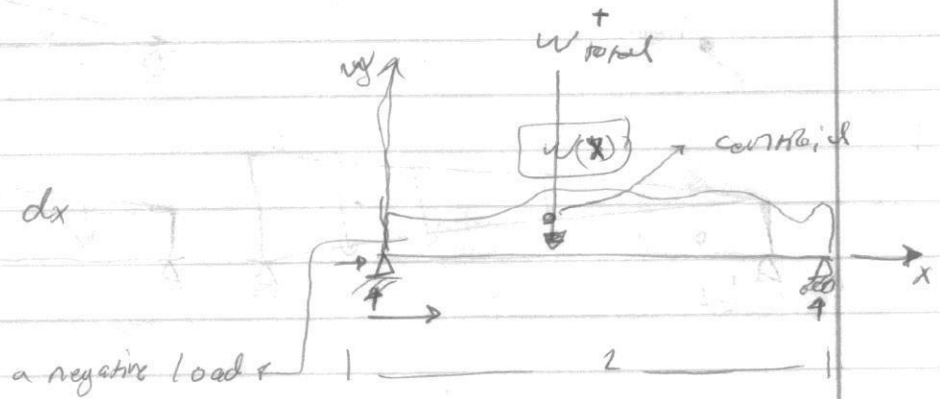
5.64



# Distributed Loads

## Beams

$$w_{total} = \int_0^L w(x) dx$$



$w_{total}$  + point of application

may be applied for equilibrium purposes only to determine the unknown forces (Reactions)

$\Sigma F_x = 0$  uniformly distributed

$$\Sigma F_x = 0 \quad A_x = 0$$

$$\Sigma M_A = 0 \quad \uparrow$$

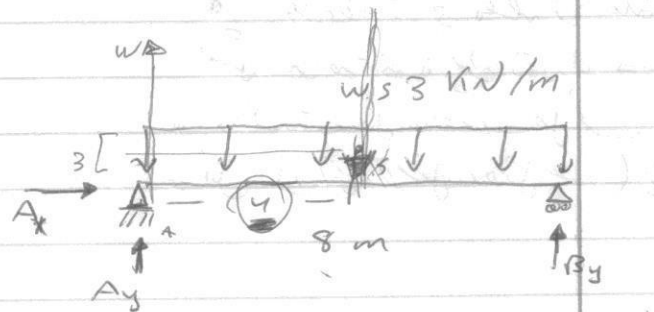
$$+(24 \times 4) - (B_y \times 8) = 0$$

$$B_y = +12 \text{ kN}$$

$$\Sigma F_y = 0 \quad \uparrow$$

$$+A_y - 24 + 12 = 0 \quad A_y$$

$$A_y = +12 \text{ kN}$$

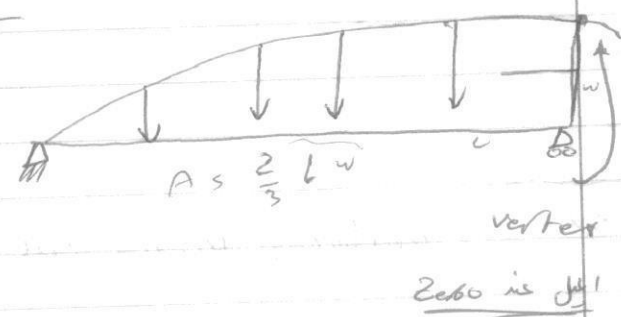
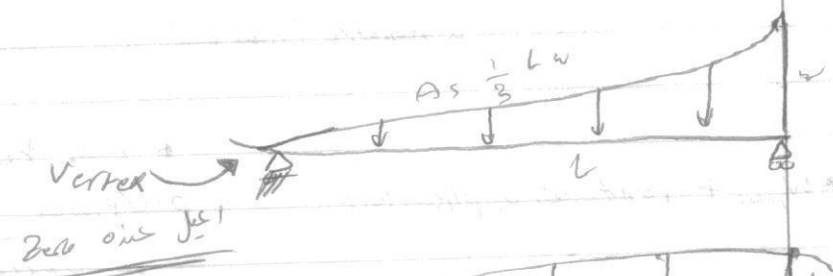
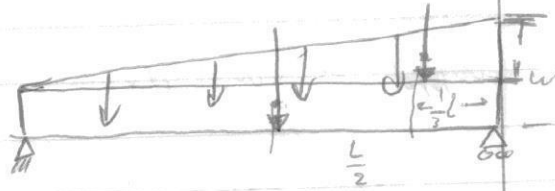
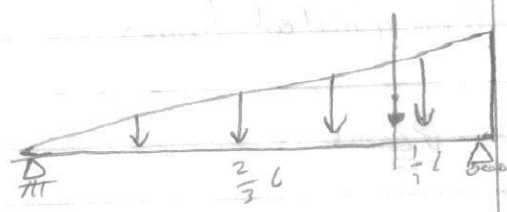
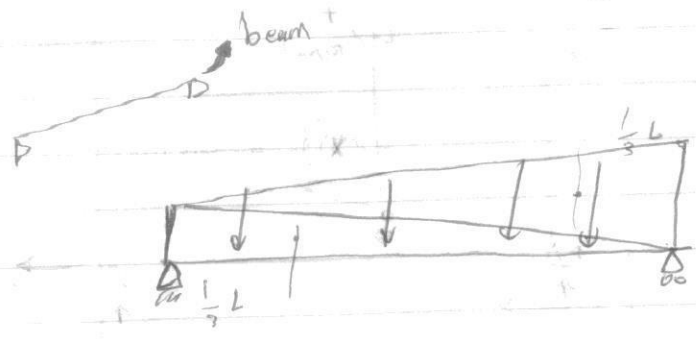
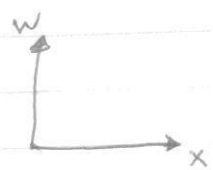


$$w_{total} = 3 \times 8 = 24 \text{ kN}$$

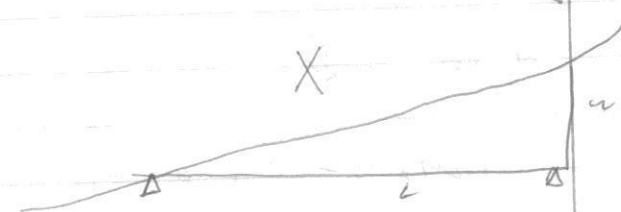
(65)

(67)

Ex



ملاحظة: في بعض الحالات  
 يمكن إضافة قوة محلي لتكون لها قوة إضافية  
 قوة معاكسة، مثل إضافة مثلث في  
 قوسين (مثل صورة) 5.72



~~Area =  $\frac{1}{3}Lw$~~

5.74

$$C_x = 50$$

$$w_1 = \frac{2}{3 \times 5} \times \frac{1}{2} \times 9$$

$$= 9 \times 15.75 \text{ KN}$$

$$\text{Centroid} = 3$$

$$w_{2s} = 105 \times 9 \times$$
  
$$= 13.5 \text{ KN}$$

$$\text{Centroid} = 4.5$$

$$\Sigma M_B = = \uparrow$$

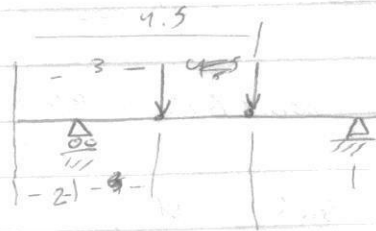
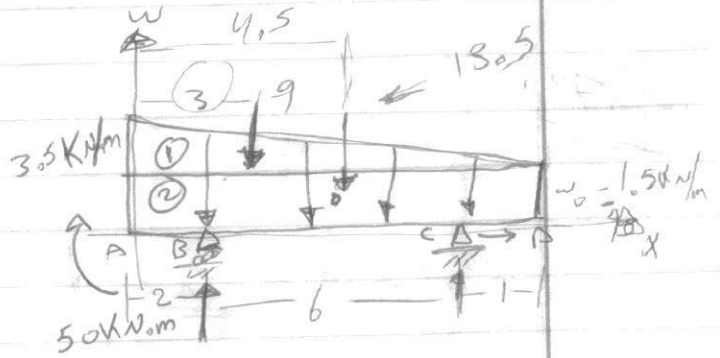
$$50 + 9 + 13.5(2.5) - C_y(6) = 0$$

$$C_y = 15.416$$

$$\Sigma F_y = 0 \uparrow$$

$$7B_y - 9 - 13.5 + 15.416 = 0$$

$$B_y = 7.0416$$



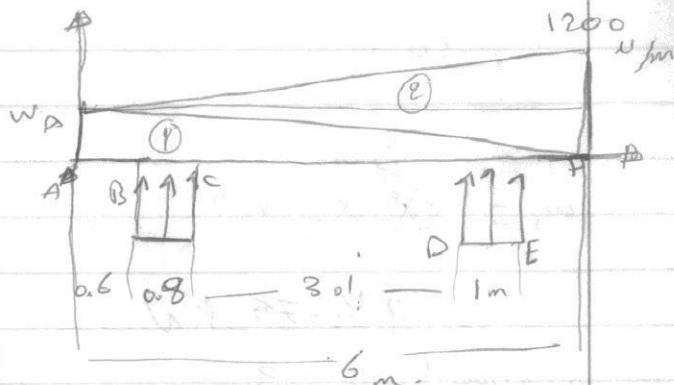
5079

$$W_1 = w_A \times \frac{1}{2} \times 6 = 3w_A \text{ N} \downarrow$$

Centroid = 2 m

$$w_2 = 1200 \times 6 \times \frac{1}{2} = 3600 \text{ N} \downarrow$$

centroid = 4 m

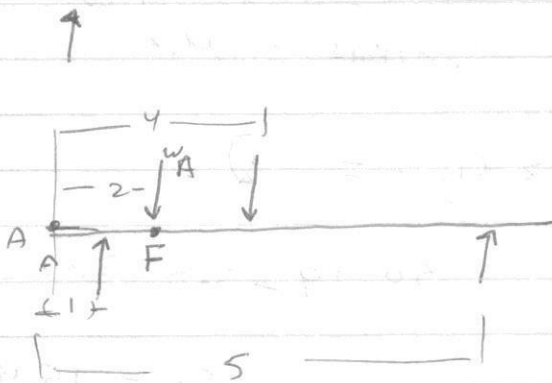


$$w_{BC} = 0.8 w_{BC} = 0.8 w_{BC} \uparrow$$

centroid = 1 m

$$w_{DE} = w_{DE} \uparrow$$

centroid = 5 m



$$\sum F_y = 0 = -3w_A + 3600 + 0.8w_{BC} + w_{DE} = 0 \uparrow$$

$$-3w_A - 3600 + 0.8w_{BC} = 0$$

$$\sum M_F = 0 \quad \uparrow$$

$$\Rightarrow w_A = 763.6 \text{ N/m}$$

$$= +0.8 w_{BC} + 3600 + 2(3600) - 3(w_{DE})$$

$$0.8 w_{BC} + 7200 - 3w_{BC}$$

$$w_{BC} = \frac{3272.7}{3} \text{ N/m}$$

$$w_{DE} = 3600 \text{ N/m}$$

70

## Boundary Conditions

5.72

$$w = ax^2 + bx + c$$

$$*** w_s = ax^2 + bx - 300$$

$$-300 = a(0) + b(0) + c$$

$$\boxed{c = -300}$$

$$900 = a(6)^2 + b(6) - 300$$

$$\frac{dw}{dx} = 2ax + b$$

$$0 = 2a(6) + b$$

$$\boxed{b = -12a}$$

$$36a + (-12a)(6) - 300 - 900 = 0$$

$$\boxed{a = -33.33}$$

$$\boxed{b = +400}$$

$$w = -33.33x^2 + 400x - 300$$

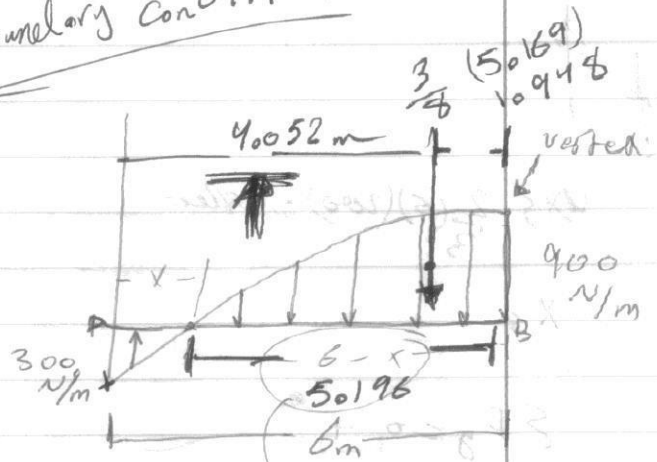
for  $w_s = 0 \Rightarrow x =$

$$0 = -33.33x^2 + 400x - 300$$

$$x = \frac{-400 \pm \sqrt{400^2 - 4(-33.33)(-300)}}{2(-33.33)}$$

$$\frac{-400 \pm 346.4}{-66.66} =$$

$$\boxed{0.804 \text{ m}} \checkmark$$

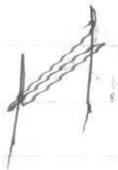


5.196

دیکھو دیکھو

(71)





$$W \times \frac{2}{3} (6)(200) = 4800$$

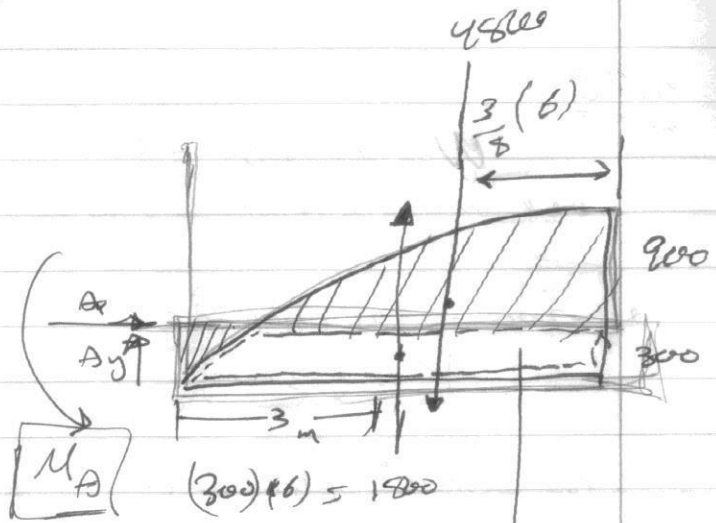
$X_s$

$$\sum F_y = 0 + \uparrow$$

$$+ A_y + 1800 - 4800 = 0$$

$$A_y = 3000 \text{ N}$$

هذا هو الوزن في هذا الحيز  
 هذا هو الوزن في هذا الحيز



$$\sum M_A = 0 + \curvearrowright$$

$$-M_A - 1800(3) + 4800(6 - 2.25) = 0$$

$$M_A = + 21600 \text{ Nm}$$



Ch9

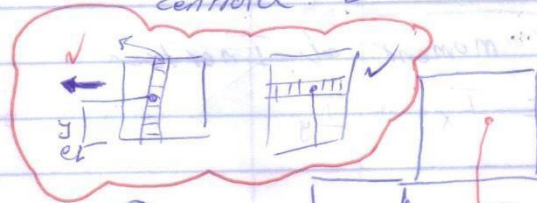
ch9

First moment of the area

(الذراع المرفوع)

مركز الثقل

Q



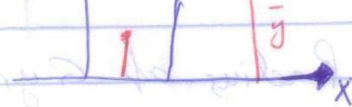
$$Q_x = \bar{y} A$$

$$= \int y dA$$

↳ element

محور x

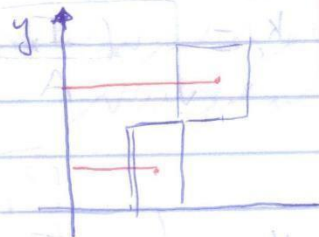
Q<sub>x</sub>



$$Q_y = \bar{x} A$$

Q<sub>y</sub>

محور y

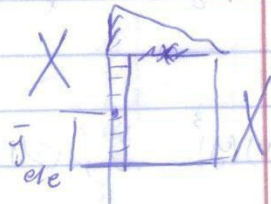


Second moment of the Area

Moment of inertia

$$I_x = \int y_{el}^2 dA$$

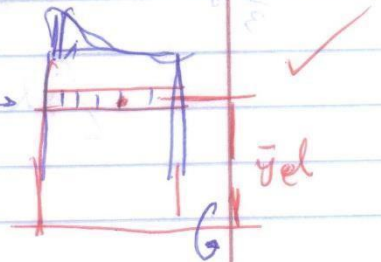
$$I_y = \int x_{el}^2 dA$$



المoment of inertia هو المقياس الذي يحدد مدى صلابة الجسم تحت تأثير الحمل. كلما زاد عزم القصور الذاتي، كلما كان الجسم أكثر صلابة.

from the axis about which the moment of inertia is to be determine

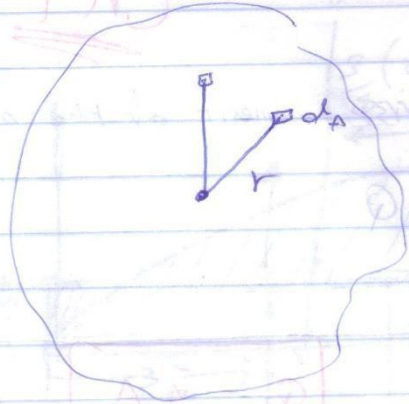
محور y



$$* J_o = \int r^2 dA$$

\* polar moment of Inertia

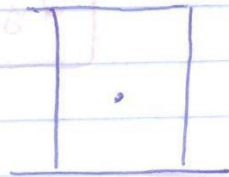
$$= I_x + I_y$$



\* Radius of Gyration (k) حول محور عمودي على الوجه (نقطة)

$$* k_x = \sqrt{\left(\frac{I_x}{A}\right)}$$

$$* k_y = \sqrt{\left(\frac{I_y}{A}\right)}$$



$$* k_o^2 = k_x^2 + k_y^2 = \left(\frac{J_o}{A}\right)$$

يمكن افتراض حركة عمودية على المحور، يلعب القانون  $\frac{1}{3}(x)(y)^3$

$$\frac{1}{3} y^3 dx$$

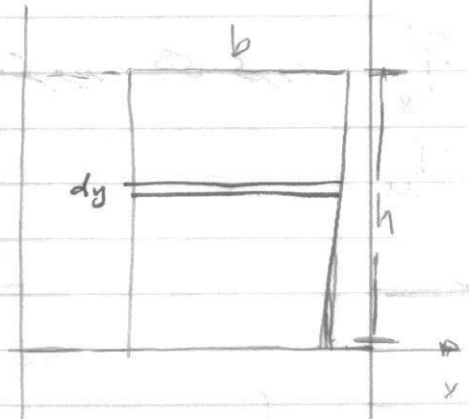
25

$$I_x$$

حول محور x

$$I_x, I_{xy}$$

moment of inertia about the centroidal axis



$$I_x = \int y^2 dA$$

$$dA = b dy$$

كأن يكون ان تكون اجزاء  
اجزاء الشريط متساوية  
عنه محور الدوران

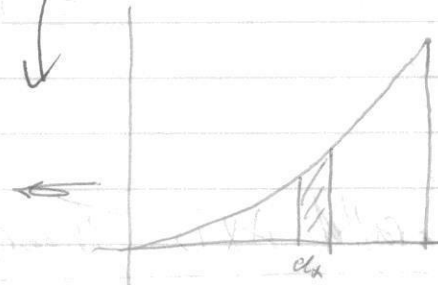
$$= \int y^2 (b) dy$$

$$= b \int_0^h y^2 dy = b \left[ \frac{y^3}{3} \right]_0^h = \frac{bh^3}{3}$$

بسط ان الشريط متساوية

$$I_y = \int x^2 dA$$

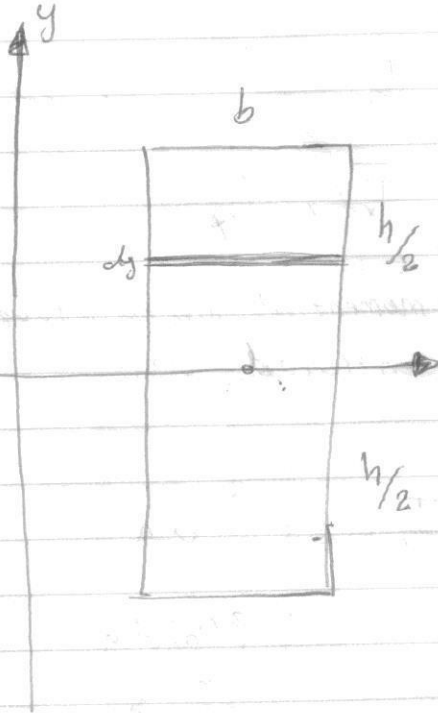
$$= \int x^2 (y dx)$$



$$dI_x = \frac{dx (y^3)}{3}$$

$$dI_x = \left[ \frac{y^3}{3} dx \right]$$

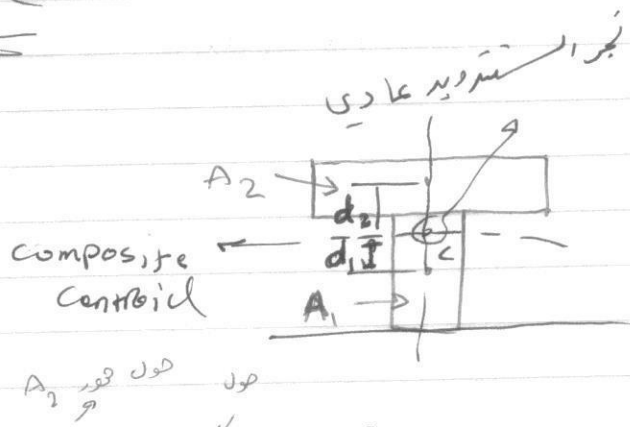
$\bar{I}_x$  بیان استرویید کسی محور  
 $I_{xx}$



$$\begin{aligned}
 \bar{I}_x &= \int y^2 dA \quad \text{(dA is body)} \\
 &= \int_{-h/2}^{h/2} y^2 (b) dy \\
 &= \frac{by^3}{3} \Big|_{-h/2}^{h/2} = \frac{b}{3} \left[ \frac{h^3}{8} + \frac{h^3}{8} \right] \\
 &= \frac{bh^3}{12}
 \end{aligned}$$

### Parallel Axis Theorem

$$\bar{I}_{Composite} = (\bar{I}_x)_{A_1} + A_1 d_1^2$$



$$\bar{I}_{Composite} = (\bar{I}_x)_{A_1} + (A_1)(d_1)^2 + (\bar{I}_x)_{A_2} + (A_2)(d_2)^2$$

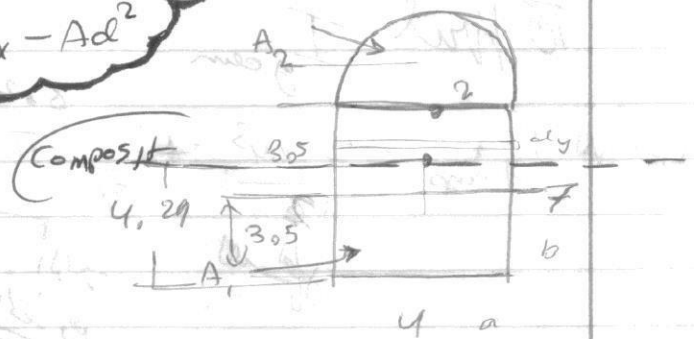
with respect to the centroidal axis

$$\bar{I}_x = Ad^2$$

$$\bar{I}_x = \bar{I}_x + A d^2$$

$$\bar{I}_x = I_x - A d^2$$

Prin  $I_{\text{composit}}$



$$(\bar{I}_x)_{A_1} = \int_{-3.5}^{3.5} y^2 dA$$

$$= \int_{-3.5}^{3.5} 4 y^2 dy$$

$$C_1 = 3.5 \quad A_1 = 28$$

$$C_2 = \frac{4r}{3\pi} = 0.849 \quad A_2 = 6.28$$

50/100 (blue)

$$\frac{4 y^3}{3} \Big|_{-3.5}^{3.5} = \frac{4}{3} [3.5^3 + 3.5^3]$$

$$= 114.03$$

$$(\bar{I}_x)_{A_2} = \frac{1}{8} \pi r^4 - (A d^2)$$

$$= \frac{\pi (2)^4}{8} - (6.28)(0.849)^2$$

$$= 1.757 \text{ cm}^4$$

Composite Centroid :

$$\bar{y}_{\text{com}} = \frac{(3.5)(28) + (7.849)(6.28)}{28 + 6.28} = 4.296$$

$$d_1 = 0.796, \quad d_2 = 3.553$$

Area	$I_x d$	$d$	$A d^2$
			17.79
			79.27

$$\bar{I}_{x \text{ com}} = 114.03 + 1.757 + 17.79 + 79.27$$

$$= 213.01 \text{ cm}^4$$

$$I_{\text{composit}} = 114.03 + 1.757 + 0.796^2 (28) + 3.553^2 (6.28) = 160.6$$

77

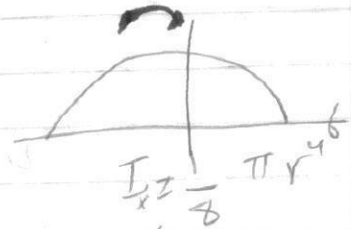
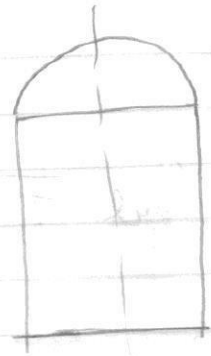
To find  $I_{y_{cm}}$

6.283

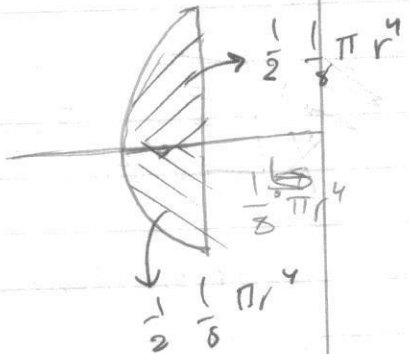
$$I_{y_{cm}} = \frac{7 r^4}{8} + \frac{1}{8} \pi r^4 =$$

للمساحة

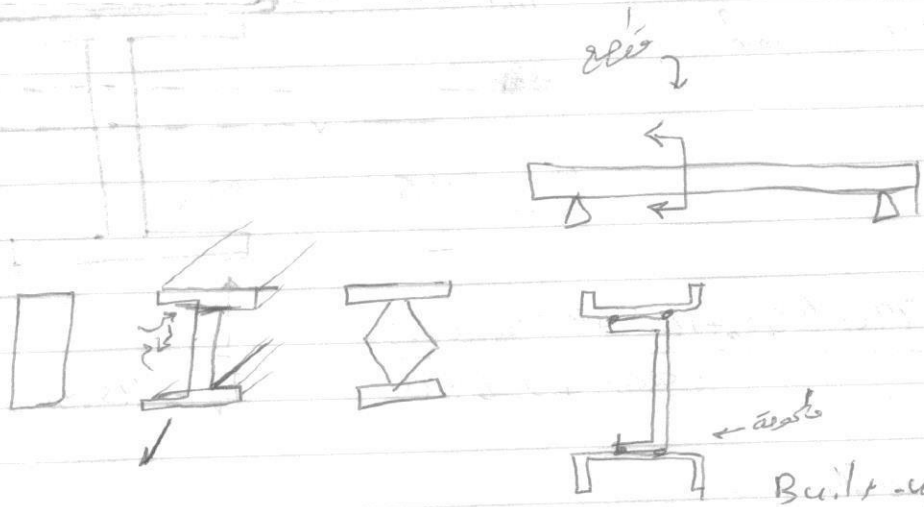
لنصف الكرة  
محول المحاور  
منه مستويين محور



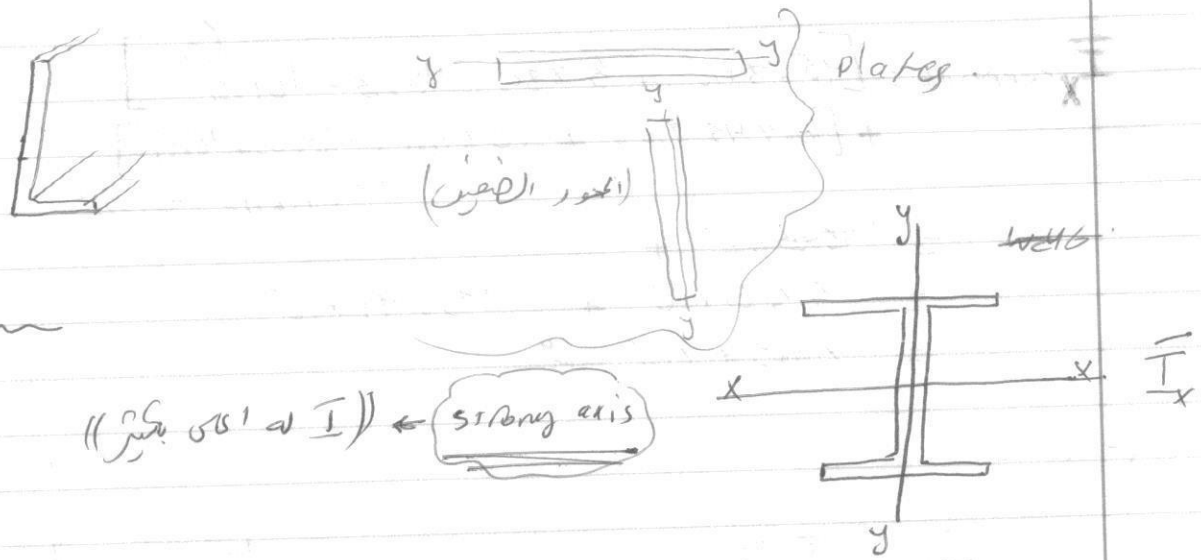
central axis



# Hot Rolled shapes



Built-up shapes



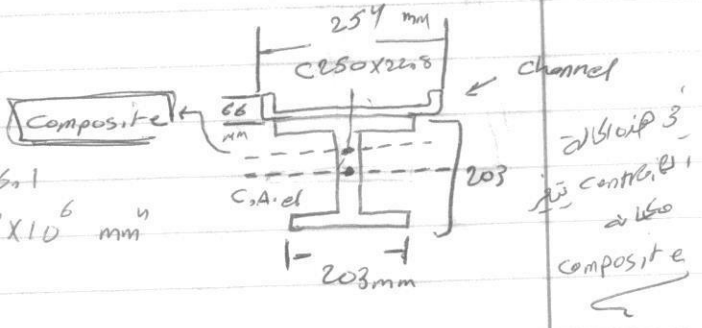
لكن هنا حول محور X  
لا نأخذها كالمعتاد

$$I_y = 0.945 \times 10^6 \text{ mm}^4$$

for channel

$$W_{200 \times 46.1}$$

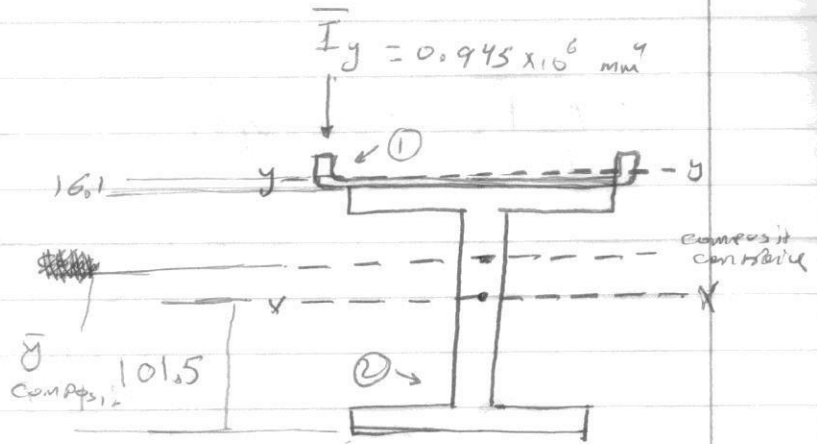
$$I_x = 45.8 \times 10^6 \text{ mm}^4$$





$$A_1 = 2890 \text{ mm}^2$$

$$A_2 = 5880 \text{ mm}^2$$



$$\bar{y}_{\text{composite}} = \frac{(2890)(203 + 16.1) + (5880)(101.5)}{2890 + 5880}$$

$$I_{\bar{y}} = 45.8 \times 10^6$$

$$= 140.25 \text{ mm}$$

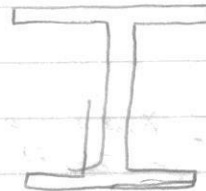
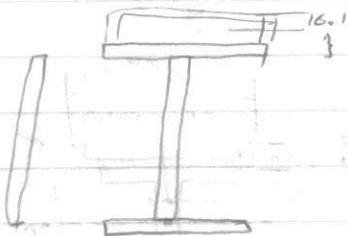
$$I_{\bar{x}}^{\text{composite}} = \left[ 45.8 \times 10^6 + 5880(140.25 - 101.5)^2 \right] + \left[ 0.945 \times 10^6 + (2890)(140.25 + 16.1)^2 \right]$$

$$= 45.8 \times 10^6 +$$

$$54.63 \times 10^6 + 18.91 \times 10^6$$

$$= 73.54 \times 10^6 \text{ mm}^4$$

W460 113



PE

80

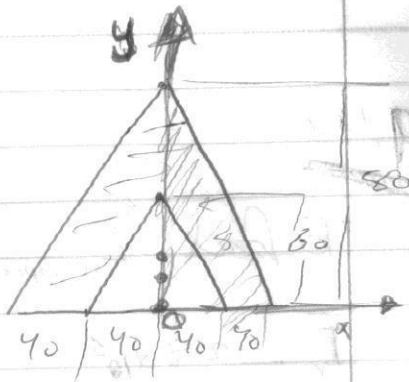
(I)

المساحة الكلية

9.47

→ المساحة الكلية  
 $J_o = I_x + I_y$

المساحة  
 $I_{Bx} = \frac{1}{12} bh^3 = \frac{1}{12} \times (80)^3 \times (60)$   
 $= 6.827 \times 10^6 \text{ mm}^4$



$I_{Sx} = \frac{1}{12} (80)(60)^3 = 1.440 \times 10^6 \text{ mm}^4$

$I_x = I_{Bx} - I_{Sx} = 5.387 \times 10^6$

$I_{By} = 2 \left( \frac{1}{12} \right) (80)(80) = 6.827 \times 10^6 \text{ mm}^4$

المساحة الكلية

$I_{Sy} = 2 \left( \frac{1}{12} \right) (60)(40)^3 = 6.40 \times 10^6 \text{ mm}^4$

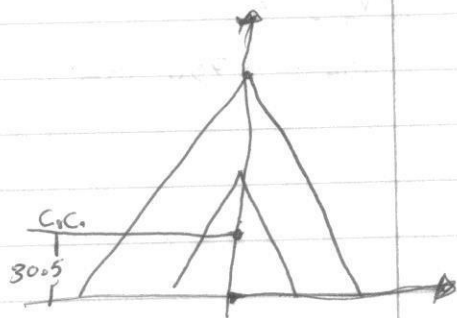
$I_y = I_{By} - I_{Sy} = 6.187 \times 10^6 \text{ mm}^4$

$J_o = I_x + I_y = 11.57 \times 10^6 \text{ mm}^4$

(B) at Centroid

#	mm <sup>2</sup> Area	$\bar{y}_d$	$\bar{y}_A$
B	+6400	$\frac{20}{3}$	$+1.7 \times 10^5$
S	-2400	$\frac{60}{3}$	-48000

$\bar{y} = 30.25$



(17)

$$\bar{I}_{Bx} = \frac{1}{36} \times 160 \times 80^3 = 2.275 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{Bx} \text{ (Composite Centre C.C.)} = 2.275 \times 10^6 + 6400 \left(30.5 - \frac{80}{2}\right)^2 = 2.384 \times 10^6$$

$$\left[ \begin{array}{l} \bar{I}_{Sy} = \frac{1}{36} \times 80 \times 60^3 = 0.48 \times 10^6 \text{ mm}^4 \\ \bar{I}_{Sx(C.C.)} = 0.48 \times 10^6 + 2400 \times (30.5 - 20)^2 = 0.745 \times 10^6 \end{array} \right.$$

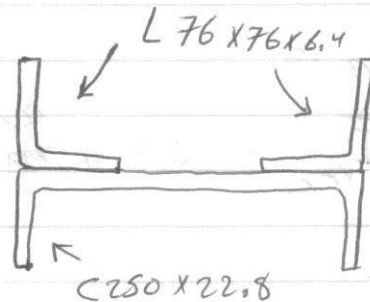
B-5

$$\bar{I}_{x(C.C.)} = 1.639 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{y(C.C.)} = 6.187 \times 10^6 \text{ mm}^4$$

$$\bar{J}_{(C.C.)} = \bar{I}_{x(C.C.)} + \bar{I}_{y(C.C.)} = 7.826 \times 10^6 \text{ mm}^4$$

← 9.55



← مانتني تعوض البعتر

$$\int (y_2 - y_1) dy$$

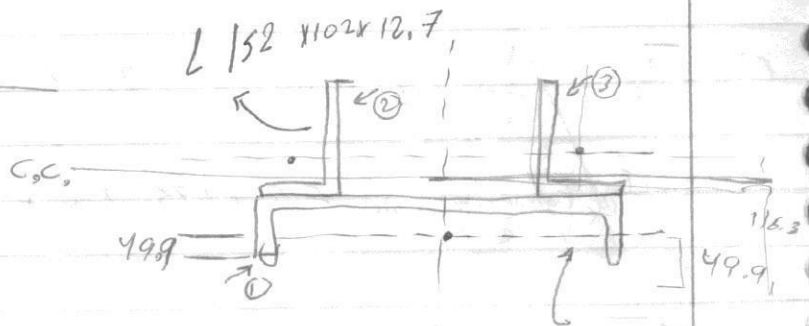
$$\int (\frac{1}{3} y_2^3 - \frac{1}{3} y_1^3) dy$$

انتبه كما يكونه صلب ، بتوخذ ال تعويض كالاته ، ال جزء هو جمع متغير ال ال  $I + Ad^2$

Add



A	$\bar{y}$	$\bar{y}A$
2890	49.9	
3060	<del>50.3</del> 116.3	
3060	116.3	



$$\bar{y}_{\text{composite}} = \frac{2890 \times 49.9 + 3060 \times 116.3 + 3060 \times 116.3}{2890 + 3060 + 3060}$$

$$= 95 \text{ mm}$$

$$\bar{I}_{x_1} = 0.945 \times 10^6$$

$$\bar{I}_{x_1, c.c.} = 0.945 \times 10^6 + 2890 \times (95 - 49.9)^2 = 6.8 \times 10^6$$

$$\bar{I}_{x_2} = 7.20 \times 10^6$$

$$\bar{I}_{x_2, c.c.} = 7.20 \times 10^6 + 3060 (116.3 - 95)^2 = 8.6 \times 10^6$$

$$\bar{I}_{x_3, c.c.} = 7.20 \times 10^6 + 3060 (116.3 - 95)^2 = 8.6 \times 10^6$$

$$\Rightarrow \bar{I}_{x_{c.c.}} = (6.8 + 8.6 + 8.6) \times 10^6 = 14.4 \times 10^6$$

$$\bar{I}_y = 28 \times 10^6$$

$$\bar{I}_{y_{c.c.}} =$$

$$\frac{254}{2} + 102 + 24.9$$

$$\bar{I}_{y_{c.c.}} = [(28 \times 10^6)] + 2 \left[ 2.59 \times 10^6 + 3060 \left( \frac{254}{2} + 102 + 24.9 \right)^2 \right]$$

$$= 174.4 \times 10^6 + 48.4 \times 10^6 \text{ mm}^4$$

83

Ch6

# oh 6

## Analysis of structures:

✓ Trusses

✓ Frames

X Machines

## Trusses

rods + bars

\* stability

statically indeterminate

\* Redundancy

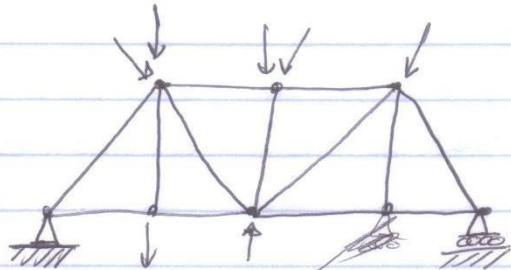
↓

Trusses members

(2-Force members)

① find Reactions

② Determine the force in some or all members

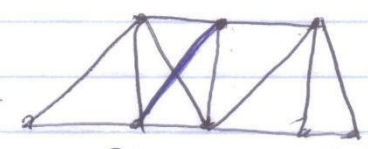


Redundant (extra supports)

\* statically Indeterminate

1- External indeterminacy

2- Internal redundancy

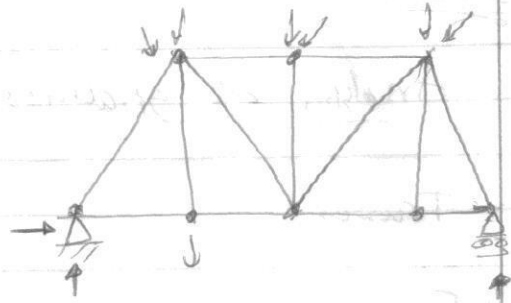


?! indeterminate statically



members  $m = 13$   
 unknowns  $\leq m + R$   
 $= 13 + 3 = 16$

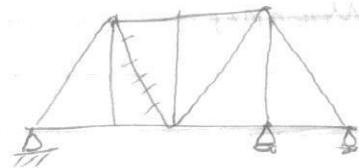
Reactions  $R = 3$



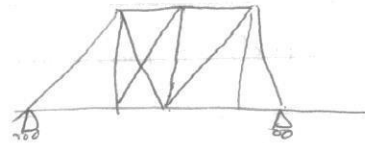
$\Delta$  joints  $= 8$   
 عدد المفاصل  $= 8$

عدد المفاصل  $= 8$

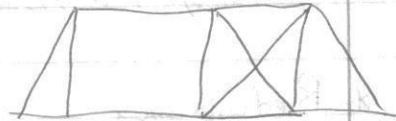
عدد نقاط معادلتها  $F_x$  و  $F_y$   $= 18$



unstable + indeterminate

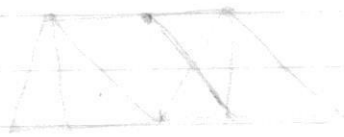


indeterminate



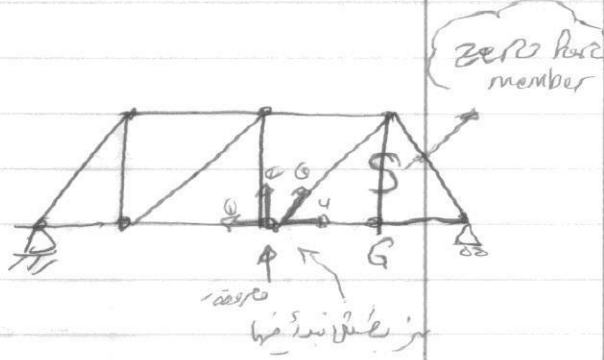
unstable + Indeterminate  
 16, 61 but

Method of joints  $\rightarrow$  لكل نقطة نقطة ، معادلتها  $F_x$  و  $F_y$   
 Method of sections  $\rightarrow$  قطع في مكان معين ، ومكانه انقطع يكون هناك قوى



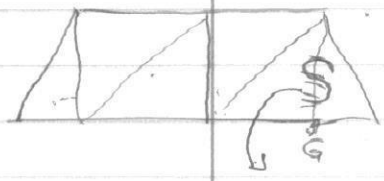
# Method of Joints

- ① Determine the Reaction  
usually  $\Sigma M$  about pin  
 $\Sigma F_x$   
 $\Sigma F_y$



- ② take 1 joint at a time:
  - a - at least one known force
  - b - No more than ~~two~~ unknown forces

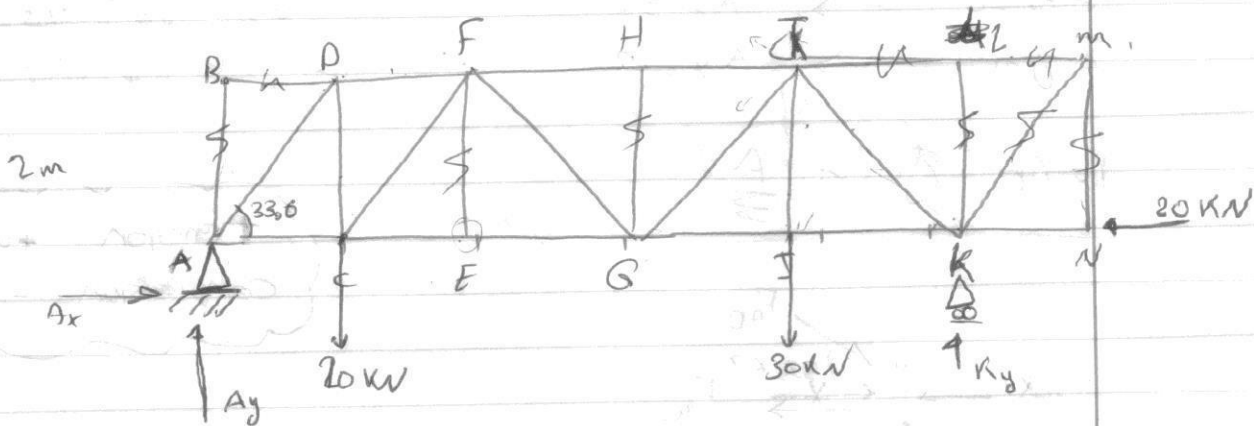
Identify all the zero-force members



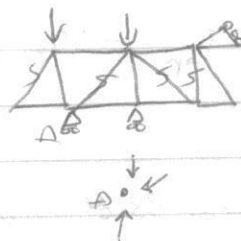
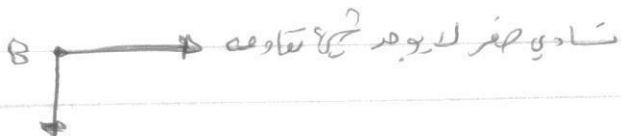
zero force member  
 $\Sigma F_x = 0$   
 $\Sigma F_y = 0$



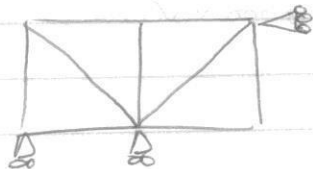
$\Sigma F_y = 0 \rightarrow +1$



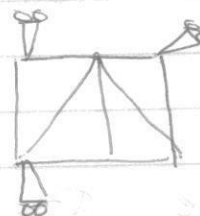
FH, FG, EC



إذا تقاطعت مقاومة العزم في نقطة تقاطع الدوران  
 وتكون Stable



Stable



البحر هنا هو unstable

عند تقاطعها إذا فوجئ حيد، وان الخط مائل، أما الجوانب تكون الدائرية  
 تدعى بالاقامة الزمنية

$$\sum M_A = 0 \quad (+)$$

$$+ 20(3) + 30(+2) - R_y(5) = 0 \quad \Rightarrow \quad R_y = 28$$

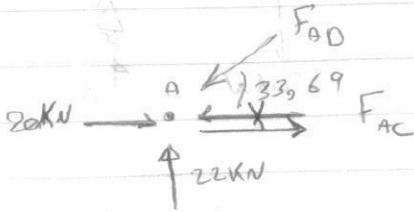
88

87

$$\begin{aligned} \Sigma F_x = 0 & \rightarrow A_x = 0 \\ \Sigma F_y = 0 & \rightarrow A_y = 22 \end{aligned}$$

0.99, 0.1 (0.01) are  
 $\rightarrow$  P (or) variables

Take joint A



Tension +ve  
 Compression -ve

$$\Sigma F_y = 0 \uparrow$$

$$-F_{AD} \sin 33.64 + 22 = 0$$

$$F_{AD} = +39.66 \text{ kN}$$

$$\Sigma F_x = 0 \rightarrow$$

$$20 - 39.66 \cos 33.64 = F_{AC}$$

$$F_{AC} = -13.00 \text{ kN}$$

Take joint D

$$\Sigma F_x = 0 \rightarrow$$

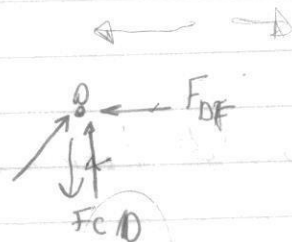
$$39.66 \cos 33.64 - F_{DF} = 0$$

$$F_{DF} = +33 \text{ kN}$$

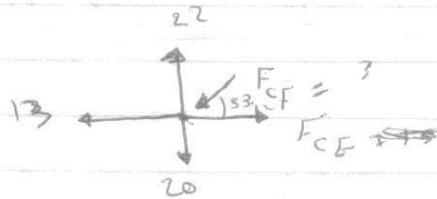
$$\Sigma F_y = 0 \uparrow$$

$$39.66 \sin 33.64 + F_{CD} = 0$$

$$F_{CD} = -22.0 \text{ kN}$$



Take Joint at C



$$\sum F_y = 0 \uparrow +$$

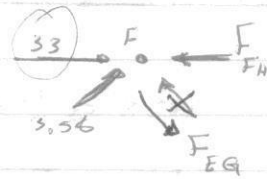
$$22 - 20 - F_{CF} \sin 33.64 = 0$$

$$F_{CF} = +3.5$$

$$\sum F_x = 0 \rightarrow$$

$$F_{CE} = +15.92$$

Take Joint at F



$$\sum F_y = 0 \uparrow +$$

$$3.58 \sin 33.64 + F_{FG} \sin 33.64 = 0$$

$$F_{FG} = -3.59 \text{ kN}$$

$$\sum F_x = 0 \rightarrow$$

$$33 + 3.58 \cos 33.64 + 3.58 \cos 33.64 - F_{FH} = 0$$

$$F_{FH} = 35.99$$

Take Joint at G

$$\sum F_y = 0 \uparrow$$

$$3.58 (\sin 33.64) - F_{GF} \sin 33.64 = 0$$

$$F_{GF} = 3.58$$

$$\sum F_x = 0 \rightarrow$$

$$-3.58 \cos 33.64 + 3.58 \cos 33.64 - 15.92 + F_{GI} = 0$$

$$F_{GI} = 21.9$$

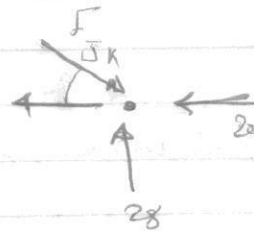
69

take joint at K

$$\sum F_y = 0 \uparrow$$

$$28 - F_{DK} \sin(33.04) = 0$$

$$F_{DK} = 150.47 \text{ kN}$$



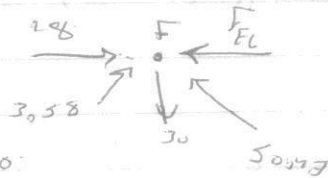
Take joint at F:

$$\sum F_x = 0 \rightarrow$$

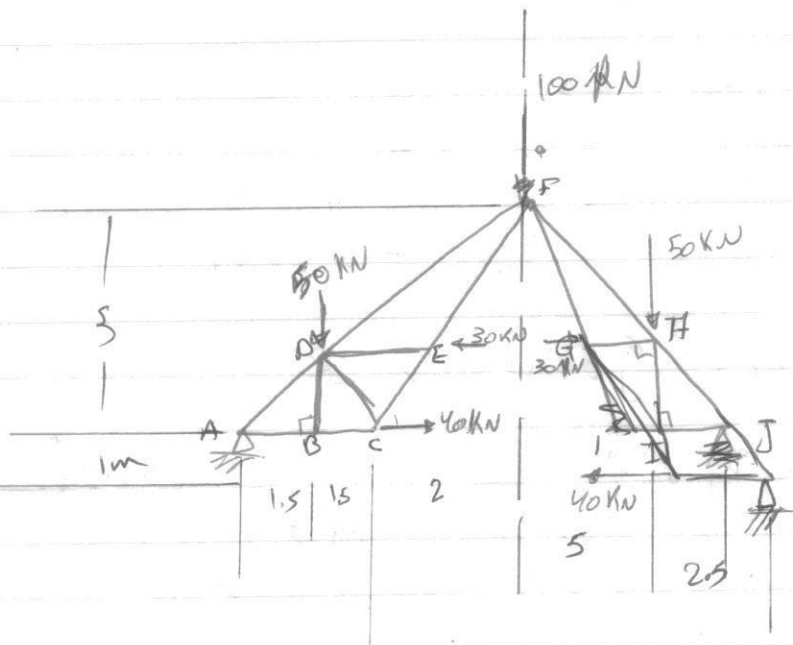
$$380.45 + 3.58 \cos 31.64 - 52 \cos 33.69 - F_{FI} = 0$$

$$F_{FI} = 0$$

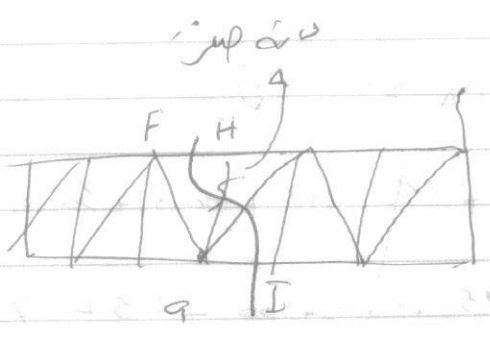
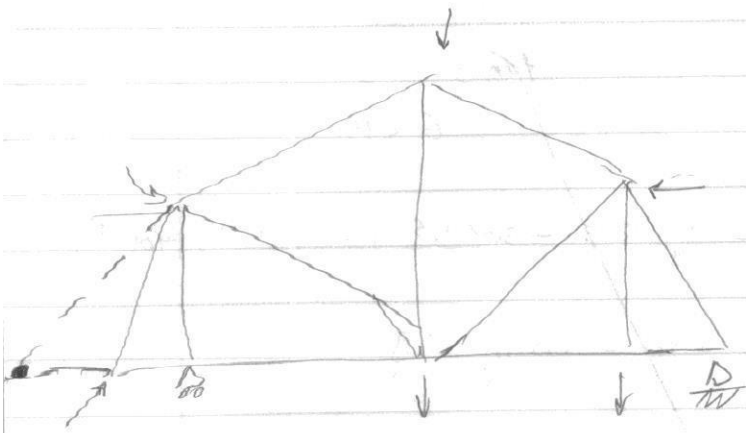
no force



How



90

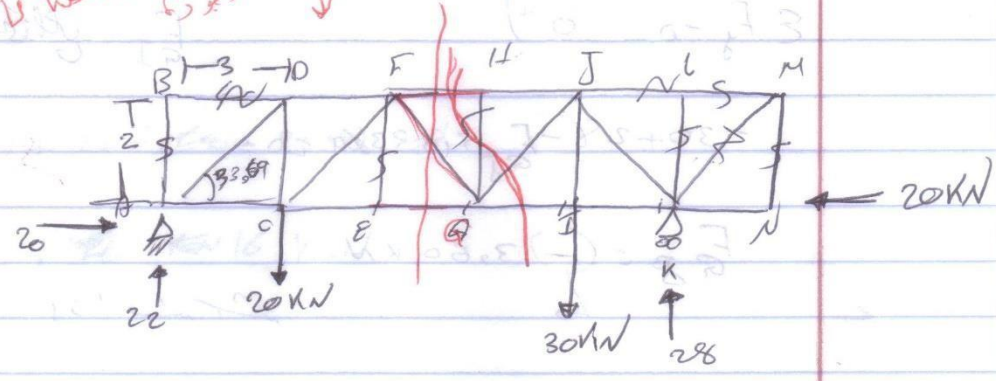


$F_{FH}$   
 $F_{GJ}$   
 $F_{GI}$

$F_{FH}$   
 $F_{FG}$   
 $F_{EG}$

إذا كانت القوة صفر  
 في هذه الحالة

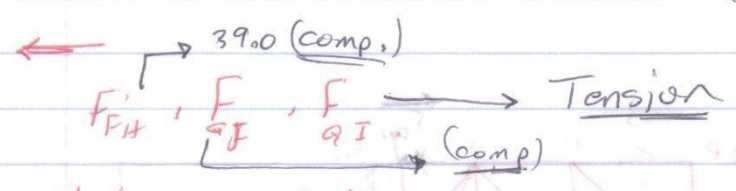
zero →  $\infty$



Method of sections:

- 1) Determine the reactions
  - 2) Identify the zero force members
  - 3) Take a section that passes through the members for which the internal forces to be determined.
- Do not cut through more than three members with unknown forces at one time.

الضغط (Compression)  
 الشد (Tension)

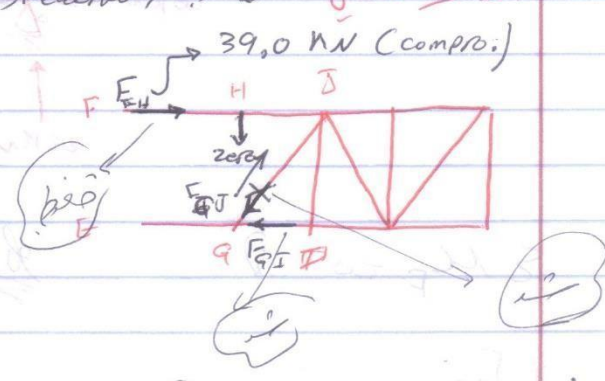


إذا كانت القوة صفر في هذه الحالة  
 إذا كانت القوة صفر في هذه الحالة

$\sum M_G = 0$

$$+F_{FH}(2) + 30(3) - 28(6) = 0$$

$$F_{FH} = +39.0 \text{ kN}$$





$$\sum F_y = 0 \quad \uparrow$$

$F_{GI}$  direction of force is to

$$-30 + 28 - F_{GI} \sin 33.69^\circ = 0$$

$$F_{GI} = -3.60 \text{ kN}$$

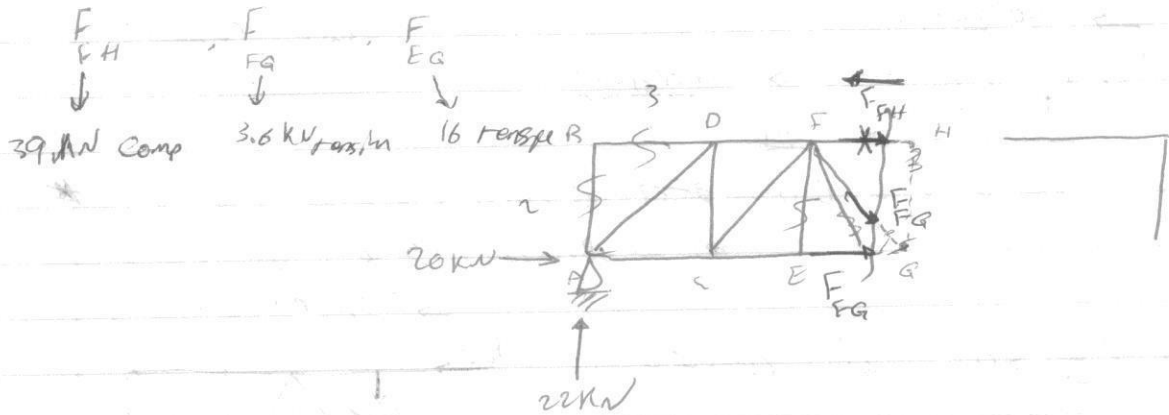


It is (direction of force)  $\sum F_x$  is to the right  $F_{GI}$  is to the left

$$\sum M_A = 0 \quad \uparrow$$

$$+F_{GI}(2) - 28(3) + 20(2) = 0$$

$$F_{GI} = +22.0 \text{ kN}$$



$$\sum M_F = 0 \quad \uparrow$$

$$F_{EG}$$



$$\sum M_G = 0 \quad \uparrow$$

$$F_{FH} \quad \sum F_y = 20 + \uparrow \rightarrow F_{FG}$$

16 يفرق منا قطعة الها وزنه ، وقتها جاء ~~من~~ القطعة الهفرية الهنا بده  
16 اله وزنه !! انسى 7.23

Aded

PP



$$\sum M_y = 0 \quad \uparrow$$

$$-F_{EG}(2) - (20)(2) + 22(6) - 20(3) = 0$$

$$F_{EG} = +16 \text{ kN}$$

$$F_{FH} = -39 \text{ kN}$$



المقصود بالاسم في اد Frame هو (يعني حمال) (تحت فوق)

\* في (س) 83 ، ما يزيد توفيق القوة بـ امية ، لا تو في عمل على المسير  
اما في (ص) 84 ، منه عمل قسرب عادى

لا ركن ايسه به ، (س) ، منه دائما بنبراً Research ، 6.139

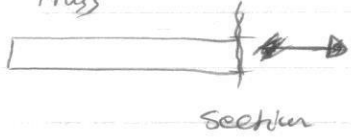
Dee

# Framed

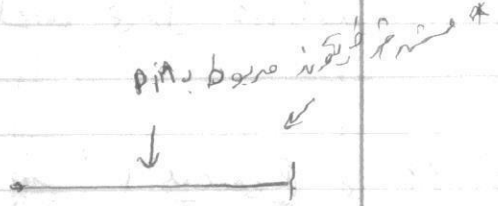
## \* Dismembering

axial } Compu  
           } tress

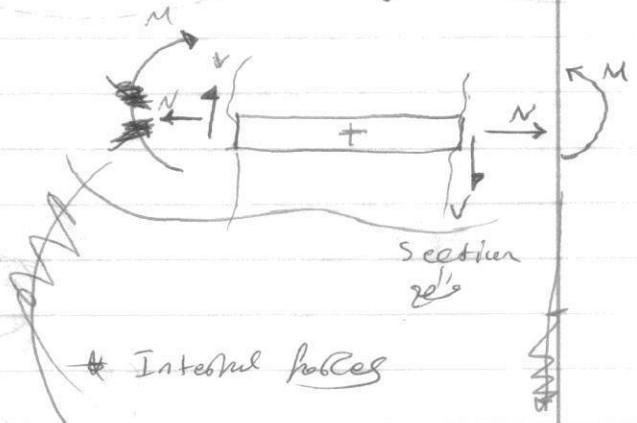
two half members → pins (hings)  
                           → No transverse load  
                           truss



Integrat force



N axial  
 V shear  
 M Bending moment



Sign convention  
 of sign of internal force

~~force~~

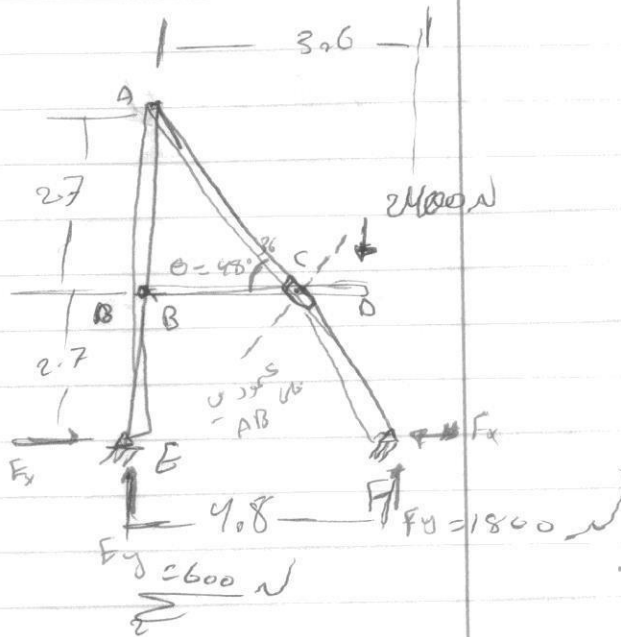
$\mu_{s0}$   
 $\frac{dM}{dx}$   
 $V_{s0}$



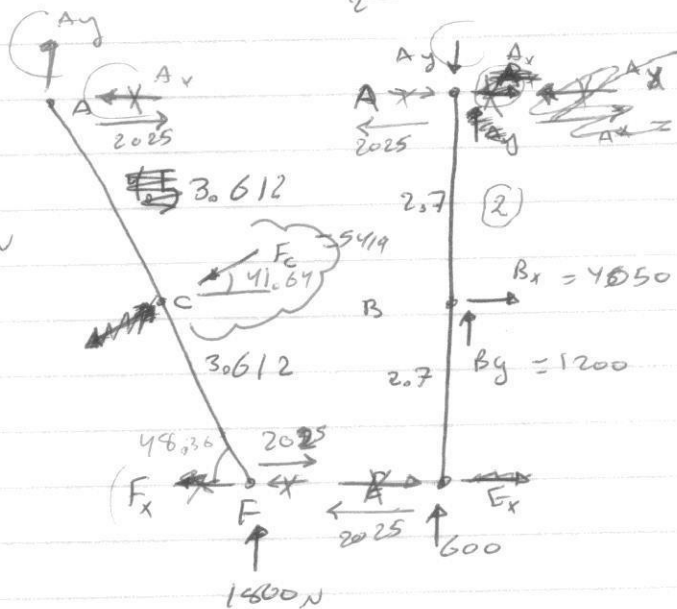
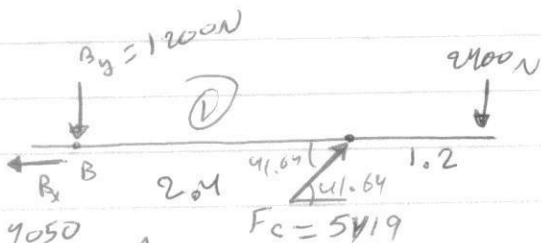
المسألة: عوارض الخشب

69/100

$\sum M_{at E} = 0$   
 $\downarrow 2400(3.6) - F_y(4.8) = 0$   
 $F_y = 1800$



$\sum F_y = 0$   
 $-2400 + F_y + 1800 = 0$   
 $F_y = 600$



$\sum M_{at B} = 0$

$-F_c \sin(41.63) \cdot 2.4 + 2400 \times 3.6 = 0$

$F_c = 5419 \text{ N}$

$\sum F_x = 0 \rightarrow B_x = 4050$

$\sum F_y = 0 = 5419 \sin(41.63) - 2400 - B_y = 0$

$B_y = 1200 \rightarrow$

(96)





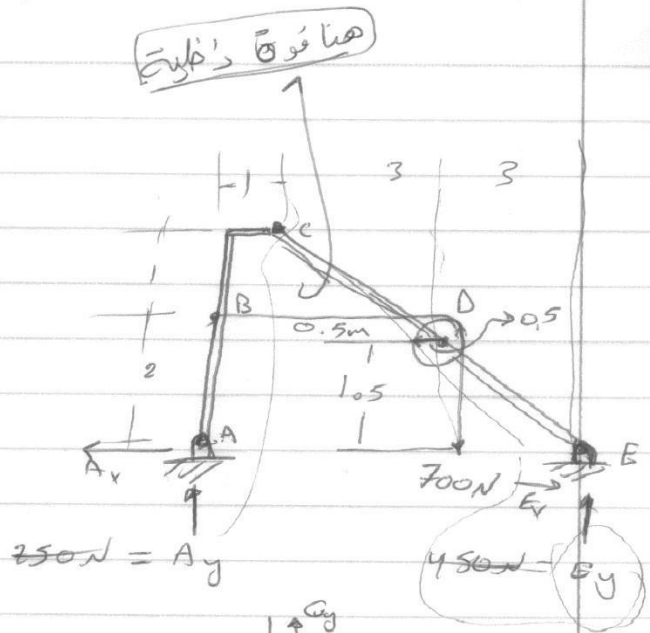
6.93

$\sum M_{at A} = 0 \rightarrow$

$$700(4.5) - E_y(7) = 0$$

$$E_y = 450 \text{ N}$$

$$A_y = 250 \text{ N}$$



لو ان  
قوة  
دافعة

$\sum M_{at B} = 0 \rightarrow$

$$+c_x(3) - c_y(1) + 700(2) = 0$$

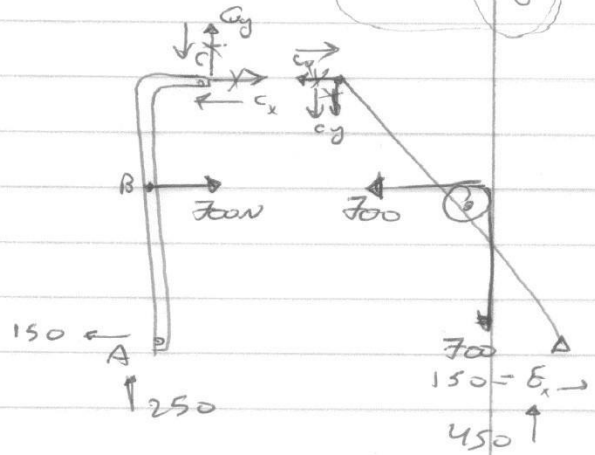
$\sum M_{at C} = 0 \rightarrow$

$$-3(c_x) - c_y(6) - 700(2) - 700(2.5) = 0$$

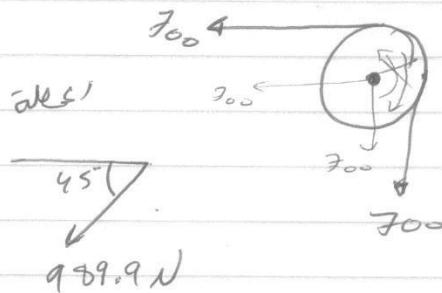
$$c_y = -250$$

$$c_x = \frac{+c_y - 1400}{3} =$$

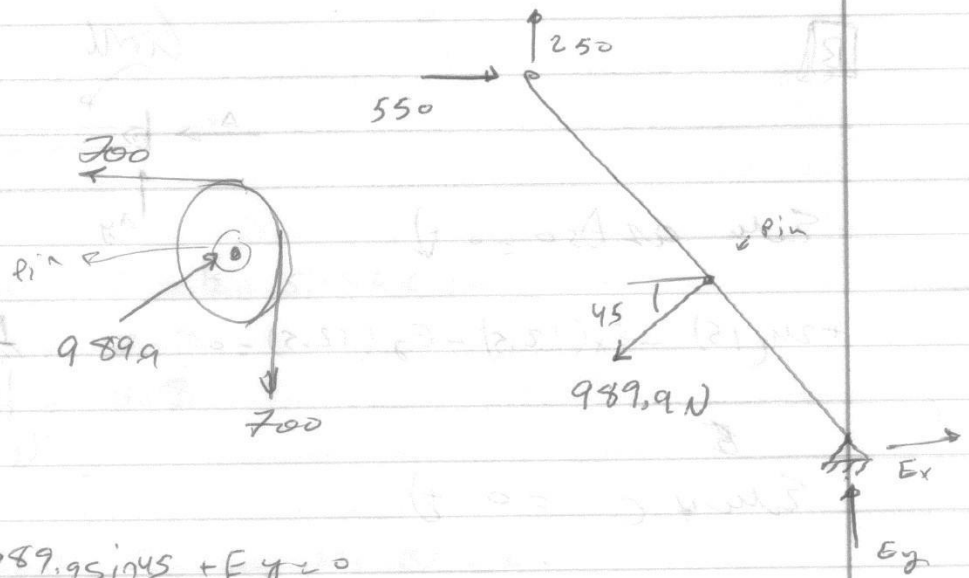
$$c_x = \frac{-250 - 1400}{3} = -550$$



و كما يحار القوس عند D



~~جواب~~



$$\sum F_y = 0 \uparrow$$

$$550 - 989.9 \sin 45 + E_y = 0$$

$$E_y = 450$$

$$\sum F_x = 0 \rightarrow$$

$$550 - 989.9 \cos 45 + E_x = 0$$

$$E_x = 150$$

~~6.87~~

(A)  $\sum M_A = 0 \quad \downarrow$  (Clockwise)

$$-24(15) - E_x(12.5) - E_y(7.5) = 0$$

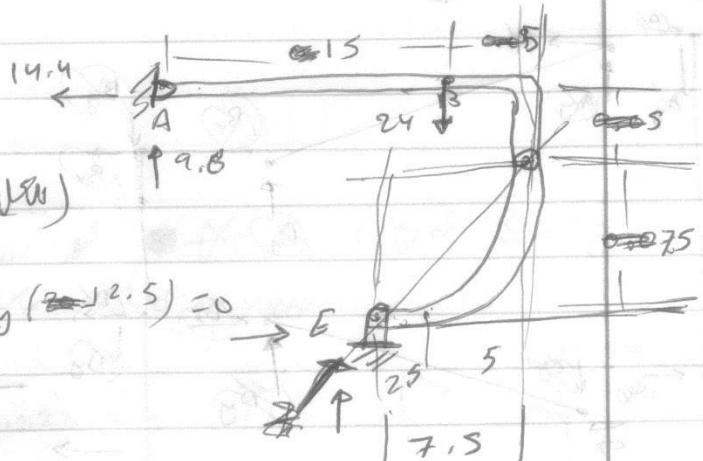
(B)  $\sum M_C = 0 \quad \downarrow$

$$-E_x(7.5) + E_y(7.5) = 0$$

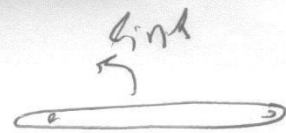
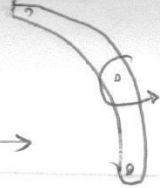
$$E_x = E_y$$

$$\Rightarrow E_x = +14.4$$

$$E_y = 14.4$$



link  $\rightarrow$



**B**

$$\sum M_{A \text{ or } D} = 0 \downarrow$$

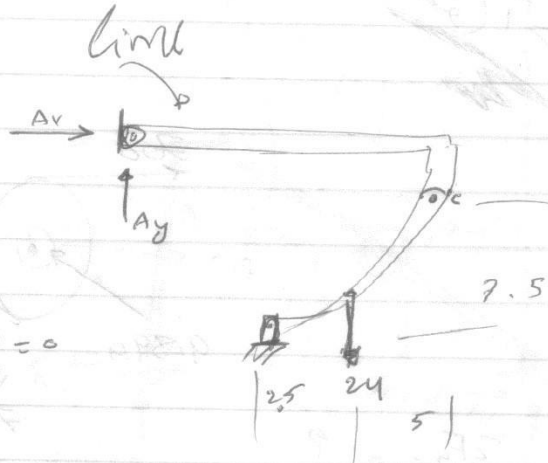
$$+24(15) - E_x(12.5) - E_y(12.5) = 0$$

$$\sum M_{C} = 0 \uparrow$$

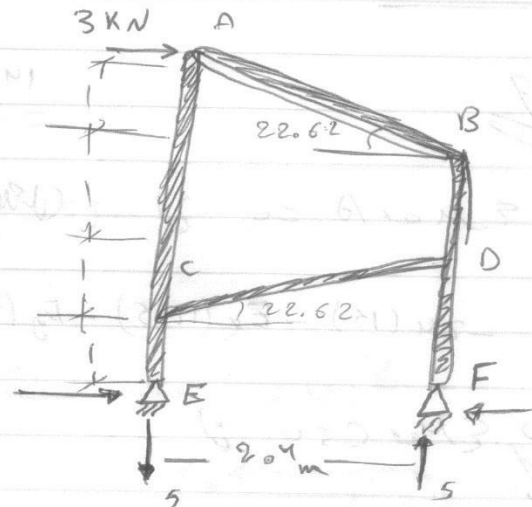
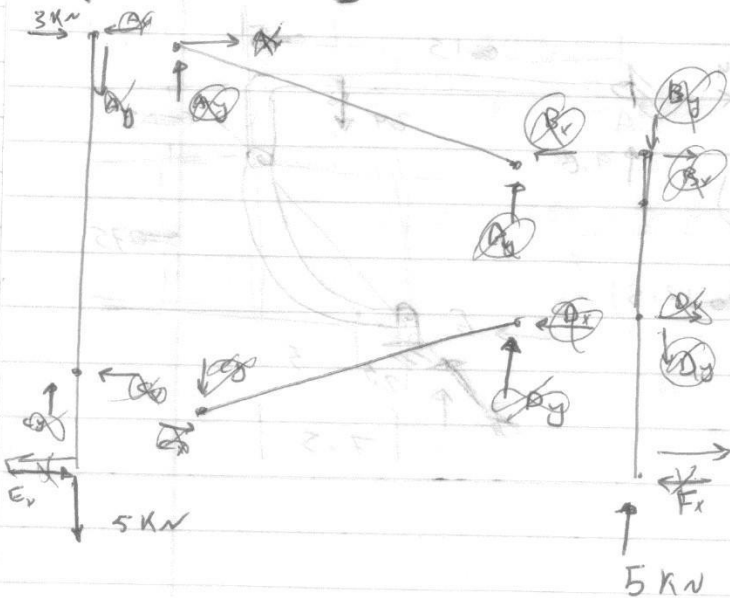
$$-24(5) - E_x(7.5) + E_y(7.5) = 0$$

$$E_x =$$

$$E_y =$$



**Example 6.6**



$A_x, A_y, B_x, B_y \rightarrow$



$$\frac{A_y}{A_x} = \tan 22.62$$

$$\sum F_y = 0 \quad \uparrow +$$

$$\textcircled{1} \Rightarrow -5 + F_{CD} \sin 22.6 + F_{AB} \sin 22.6 = 0$$

$$F_{CD} = 7.8$$

$$F_{AB} = 5.8$$

$$\sum M_{at F} = 0 \quad \uparrow +$$

$$-F_{CD} \cos 22(2) + F_{AB} \cos 22(3) = 0$$

$$F_{CD} = F_{AB}$$



$$\sum M_{at B} = 0 \quad \uparrow +$$

$$+ F_{CD} \cos 22(1) + 3(4) - F_{AB} \cos 22.6(4/2) = 0$$

$$\textcircled{1} \text{ solve } \& \quad F_{CD} =$$

$$F_{AB}$$

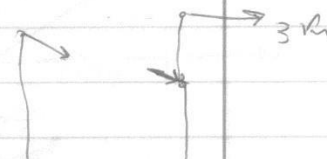
~~( $\omega = F_{AB}$ )~~ ~~use  $(\frac{3}{4})$  as 1 unit~~ ~~rigid body~~  
 Reaction at the sliding vertex ~~is 3kN~~

$$\Rightarrow -5 + F_{CD} \sin 22 + F_{AB} \sin 22 = 0 \quad \textcircled{1}'$$

$$\Rightarrow -F_{CD} \cos 22(2) + F_{AB} \cos 22(3) + 3(4) = 0$$

$$F_{CD} = 10.4 \text{ kN}$$

$$F_{AB} = 2.6 \text{ kN}$$





Reaction  $R_A$  ،  $R_B$  ،  $R_C$  في المرفق ،  $R_D$  في الطرف الأخرى ،  $R_E$  في الطرف الأخرى

$\sum M = 0$

Rigid Body  
 صلب 4 قواعد

$+ F_{CD} (\cos 22.6) (1) - F_{AB} \cos 22.6 (4) = 0$

①  $\sum M = 0$

$F_{CD} = 10.4$

$F_{AB} = 2.6$

$\sum F_x = 0$

$F_x = 2.4 \rightarrow E_x = 5.4 \leftarrow$  في المرفق ،  $R_A$  ،  $R_B$

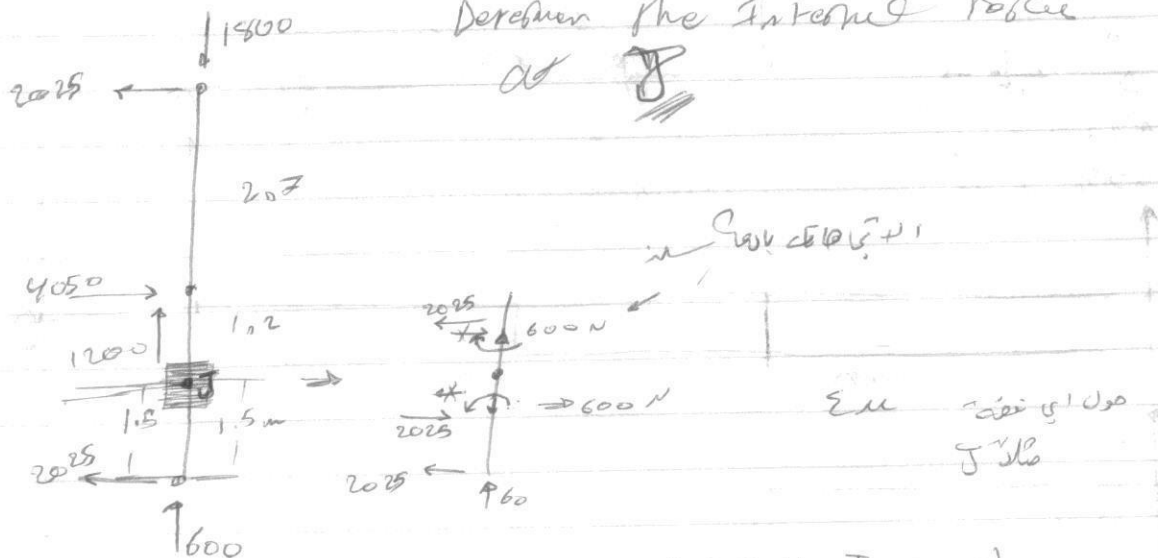
$F_x = 4.2 \rightarrow E_x = 7.2 \leftarrow$  في الطرف الأخرى

# Ch7

ملاحظة : آخر مثال بهاد التشابتر , مشروح كيف انحل خطوة بخطوة " بأخر صفحتين " , وتقريبا بلخص آلية الحل بهاد التشابتر

Qn 7

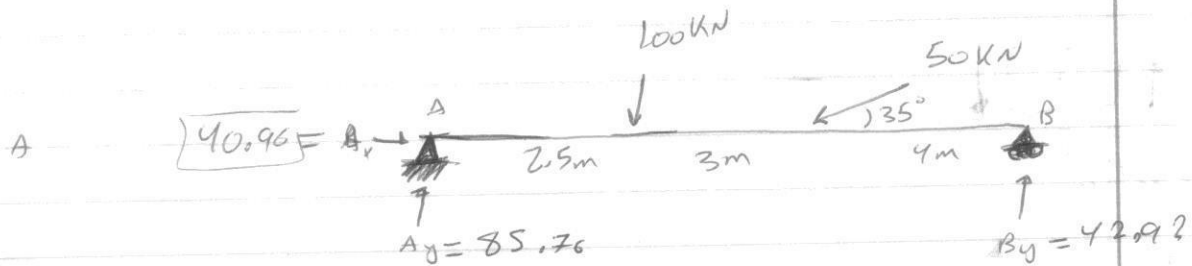
Determine the Internal Forces at J



$\sum M \text{ at } J = 0$   
 $+ 2025(1.5) - M_J = 0$   
 $M_J = +3037.5 \text{ Nm}$

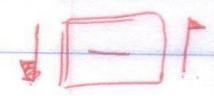
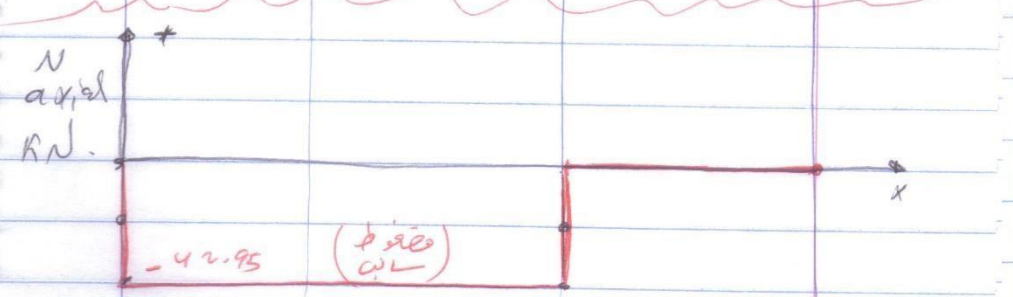
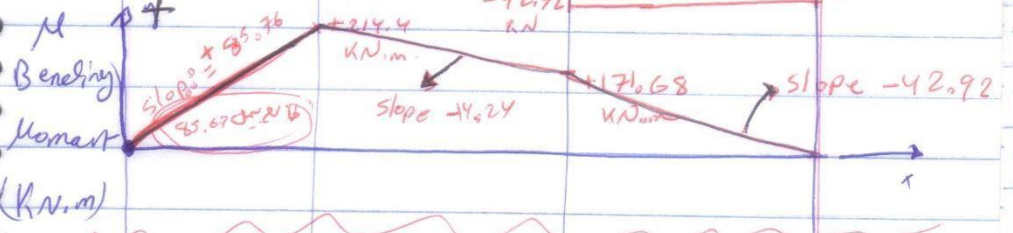
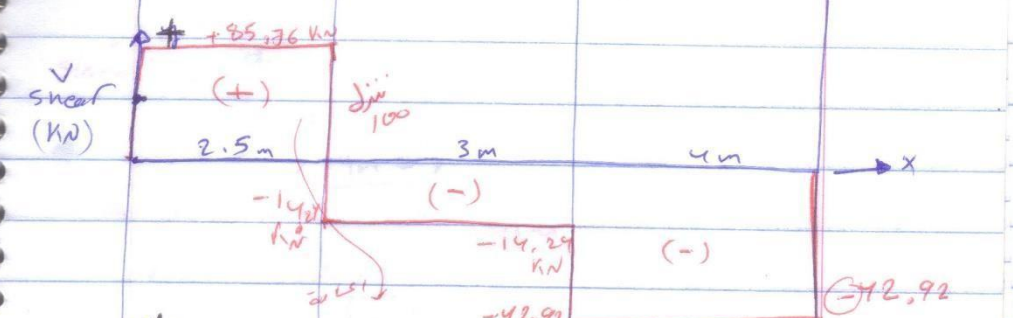
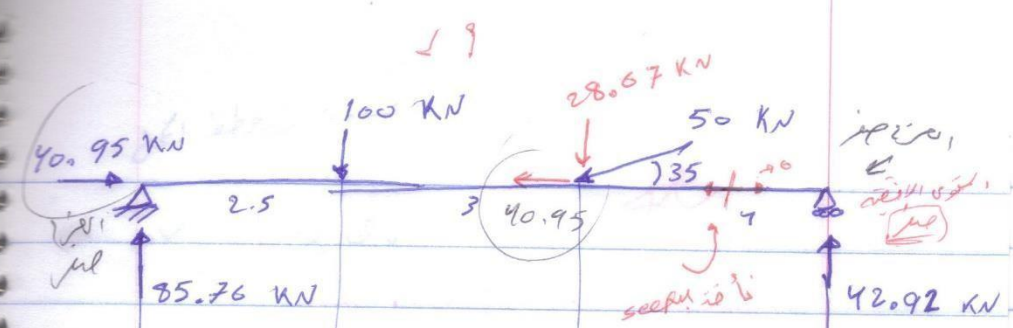
$\sum M \text{ at } C = 0$   
 $-M_J + 2025(1.5) = 0$   
 $M_J = 3037.5 \text{ Nm}$

$\sum M \text{ at } J = 0$   
 $+ 4050(1.2) - 2025(3.9) + M_J = 0$   
 $M_J = +3037.5 \text{ Nm}$

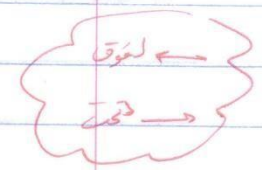


$\sum M \text{ at } B = 0$   
 $100(2.5) + 50(\sin 35)(5.5) - B_y(9.5) = 0$   
 $B_y = 42.92 \text{ N}$   
 $A_y = 85.76$





98



اعمال تحت نقطه الكمل

$$\Delta V = \int w(x) dx$$

↳ Distributed load.

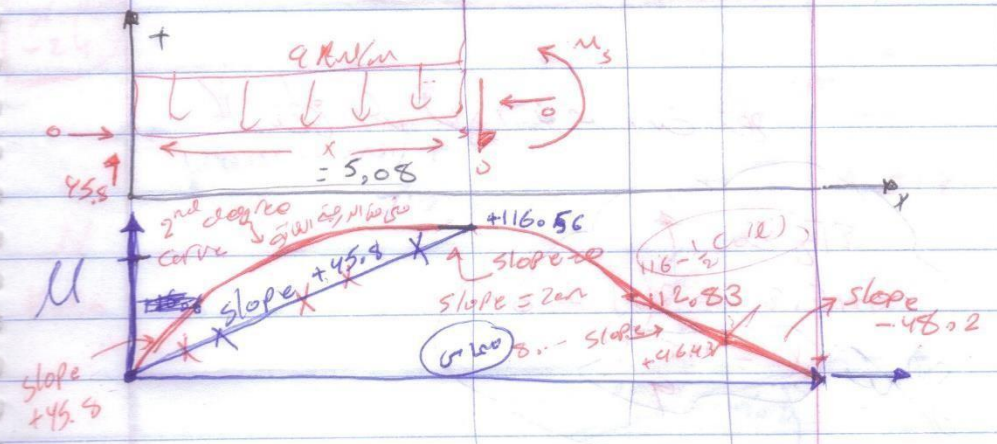
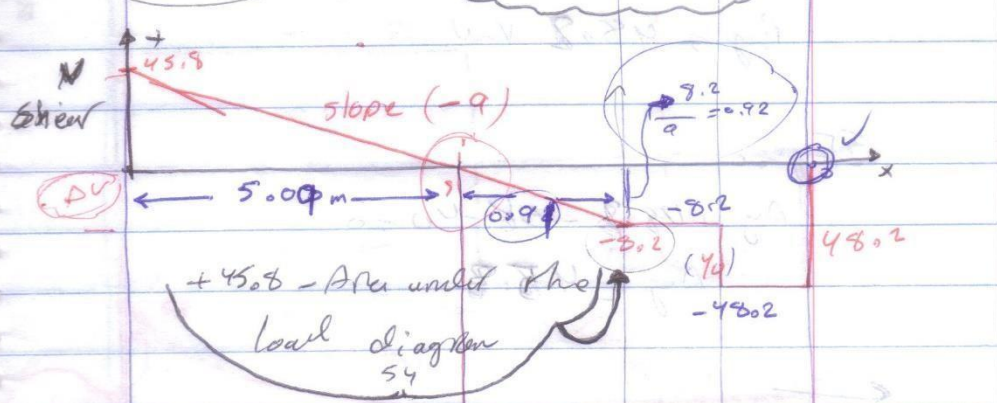
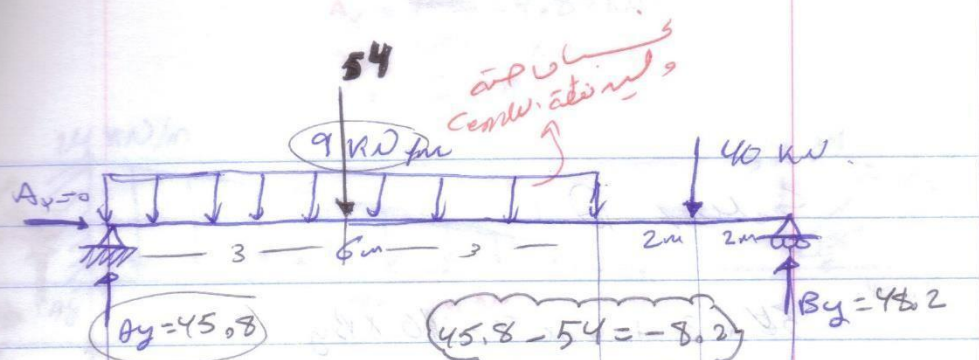
$$w(x) = \frac{dV}{dx}$$

slope. اعمال

$$\Delta M = \int v(x) dx$$

↳ change in M      ↳ Area under v

$$v(x) = \frac{dM}{dx} \rightarrow \text{السرعة}$$



Shear force → Bending moment → Slope

Section D

$$58 \times 3 + 40 \times 8 - 16 \times B_y$$

$$B_y = 45.8 \text{ kN}$$

$$\sum F_y = 0$$

$$A_y + 45.8 - 58 - 40 = 0$$

$$A_y = 45.8$$

و این مقدار را در جدول وارد کنید

$$\sum F_y = 0 \uparrow$$

$$+45.8 - 9(5) = 0$$

$$X = \frac{45.8}{9}$$

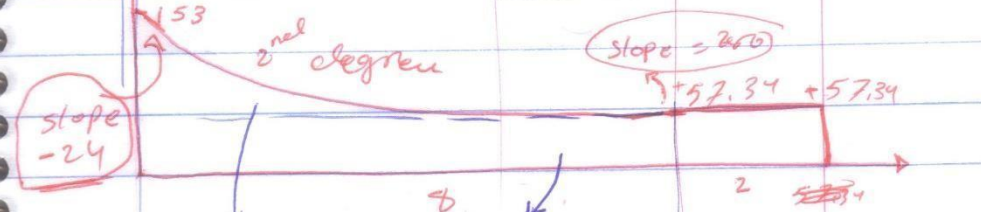
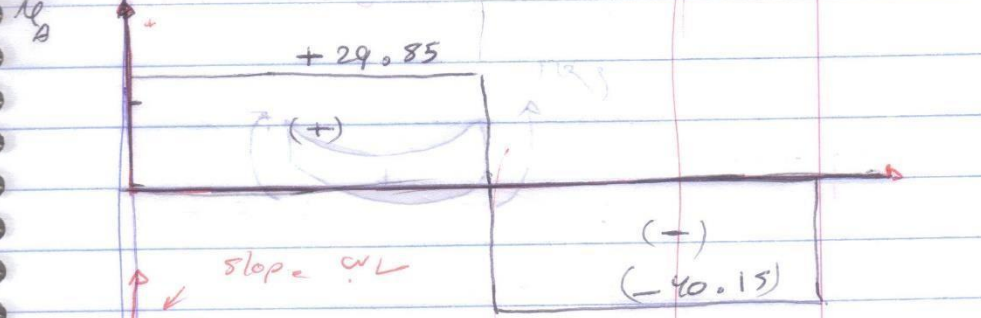
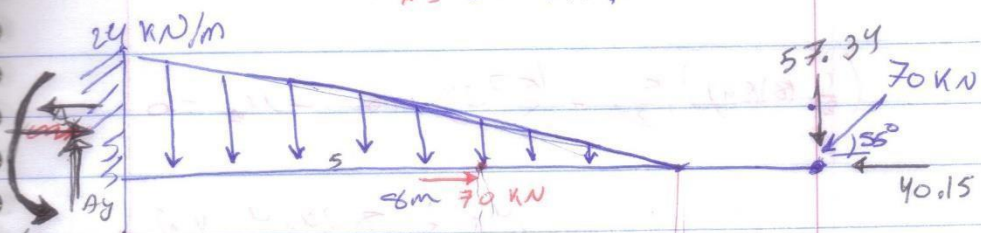
101

001

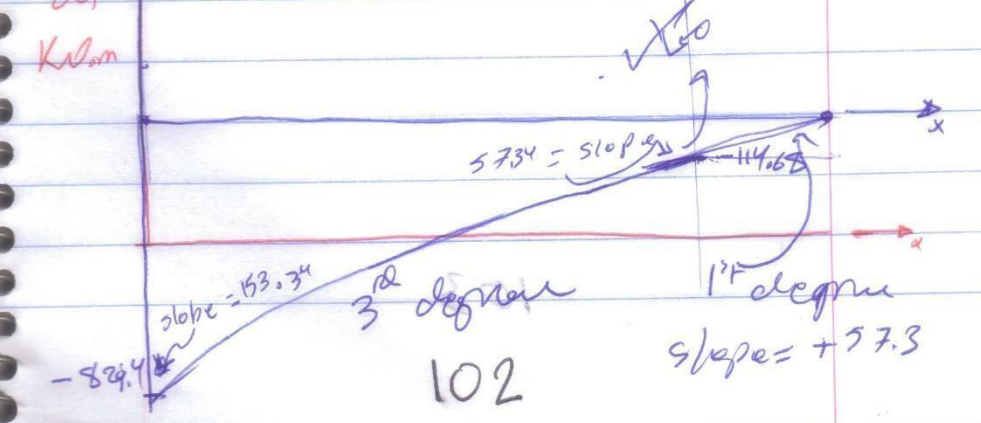
$A_x = 29.85 \text{ kN}$

$A_y = 153.34 \text{ kN}$

$M_A = 829.4 \text{ kNm}$



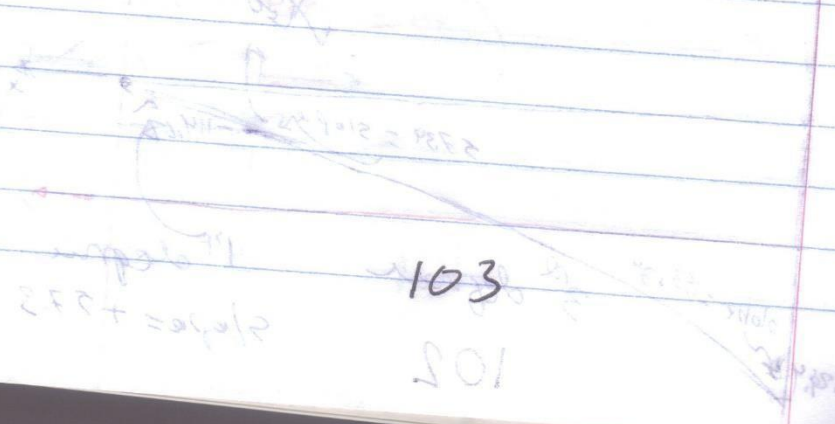
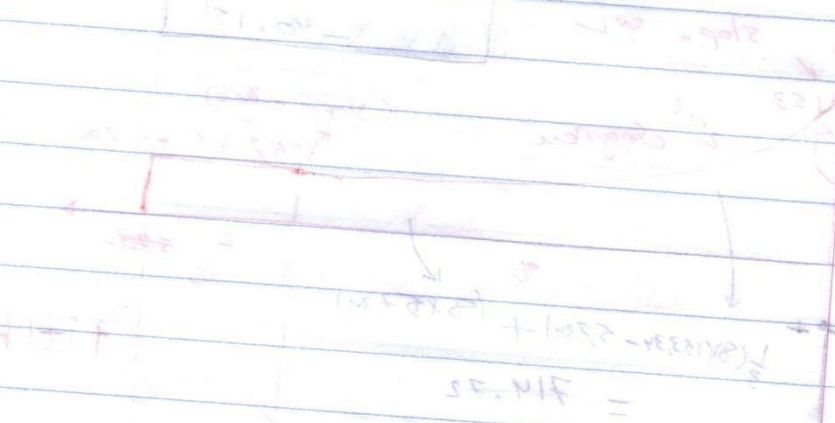
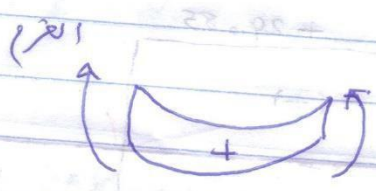
$\frac{1}{3}(8)(153.34 - 77.34) + (8 \times 77.34)$   
 $= 714.72$



$\sum M_A = 0$   
 $\sum F_y = 0$   
 $\sum F_x = 0$

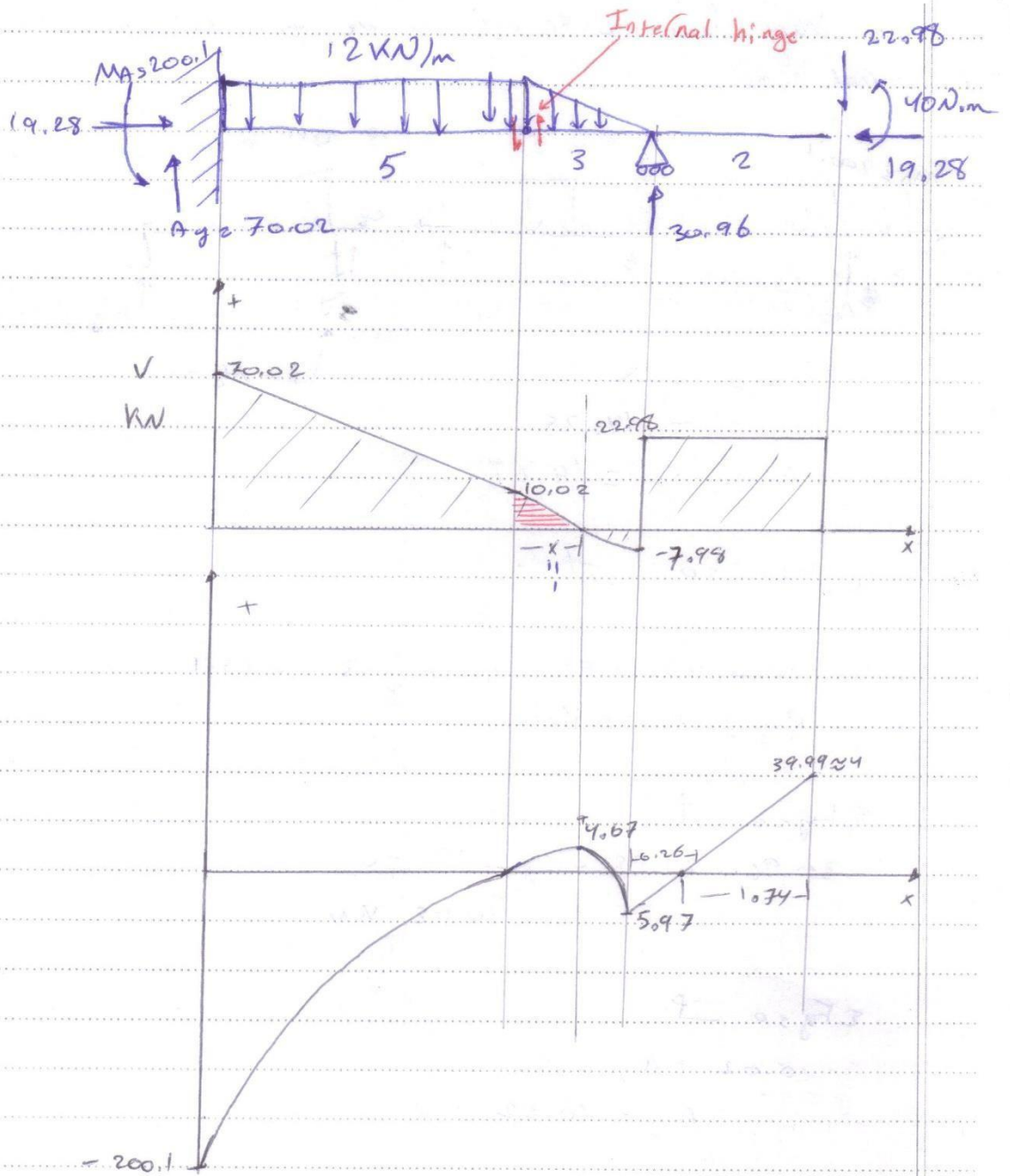
$$\left(\frac{1}{2} \cdot 8 \cdot 4\right) \left(\frac{8}{3}\right) + (57.34) \cdot 10 - M_A = 0$$

$$M_A = 829.4 \text{ kN}\cdot\text{m}$$

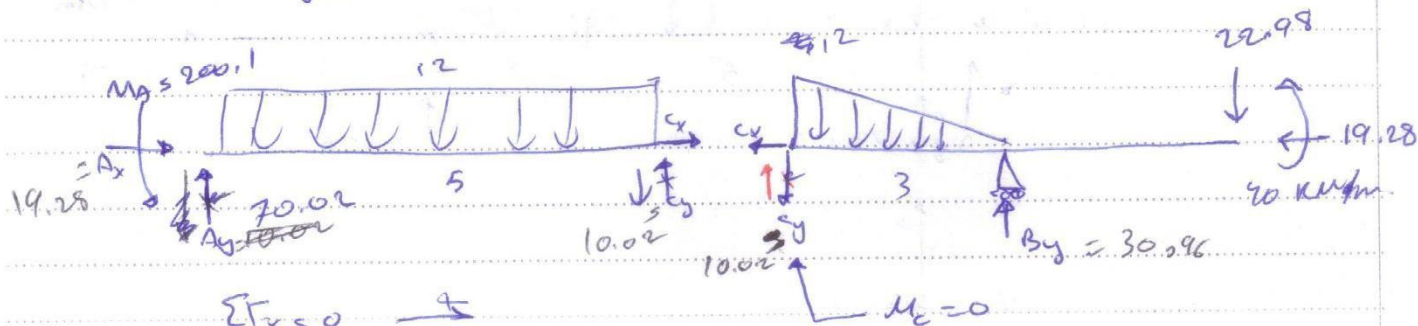


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 102

$\sum F_y = 0$   
 $\sum F_x = 0$



we have four unknowns, so we will take section at point C (Internal hinge),



$$\sum F_x = 0 \rightarrow$$

$$C_x = -19.28$$

$$A_x - C_x = 19.28$$

$$\sum M_{at C} = 0 \rightarrow$$

$$-B_y(3) + 22.98(5) - 40 + \frac{1}{3} \times 3 \times \frac{1}{2} \times 5 \times 12 = 0$$

$$B_y = 30.96 \text{ kN}$$

$$\sum F_y = 0 \uparrow$$

$$30.96 - 22.98 - C_y - 18 = 0$$

$$C_y = 10.02 \text{ kN}$$

$$\sum F_y = 0 \uparrow$$

$$-10.02 + A_y - 5 \times 12 = 0$$

$$A_y = 70.02 \text{ kN}$$

$$\sum M_{at C} = 0 \rightarrow$$

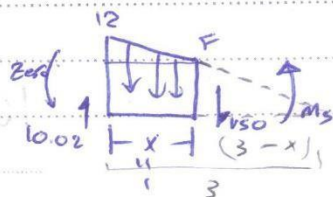
$$70.02 \times 5 - 12 \times 5 \times \frac{5}{2} - M_A = 0$$

$$M_A = 200.1 \text{ kN}\cdot\text{m}$$

To find  $x$ , we take section with some known

$$\frac{3-x}{3} = \frac{F}{12} \Rightarrow F_s \left( \frac{3-x}{3} \right) \cdot 12 = 8 \quad | \quad x=1$$

Check with



$$\sum F_y = 0 \quad x = 1$$



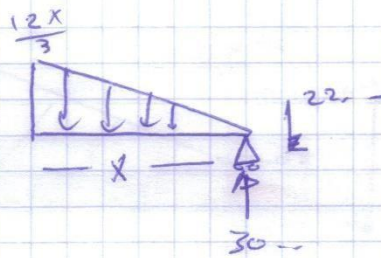


From Section :

$\Sigma \text{Mat c.s.o}$  ↓

$$\frac{1}{2} \times 1 \times 4 \frac{20}{3} \times \frac{1}{3} + M_s = 8(1) \left(\frac{1}{2}\right) - M_s = 0$$

$$M_s = 8 + 4.67$$



$$30.99 - 22.99 - \frac{12}{3} \times \frac{1}{2} \times x = 0$$

$$x = 1.99$$

هسا اذا بتشوف آخر مثال بالشرح , بتلاحظ تدرج بالمنحنيات المرسومة , يعني اذا كانت القوة على شكل مربع , برسمة ال shear بتصير مثلث وبرسمة ال moment بتصير منحنى وهيك , كيف كنا نرسمها ببساطة , كل رسمة بتعتمد ع كل اللي قبلها , رسمة ال shear بتعتمد ع قيم التغير بالفورس بالرسمة الاصلية , وميل الخطوط او المنحنيات بكون هو قيمة ال force اللي اضفناها , ورسمة المومنت بتعتمد ع شغلتين , قيم المومنت بالرسمة الاصلية , وكمان قيم ال shear عند عدة نقاط برسمة ال shear , قيم ال shear " اكتبها " هي بتمثل <<ميل>> المنحنى عند اسقاط هديك النقطة برسمة المومنت !

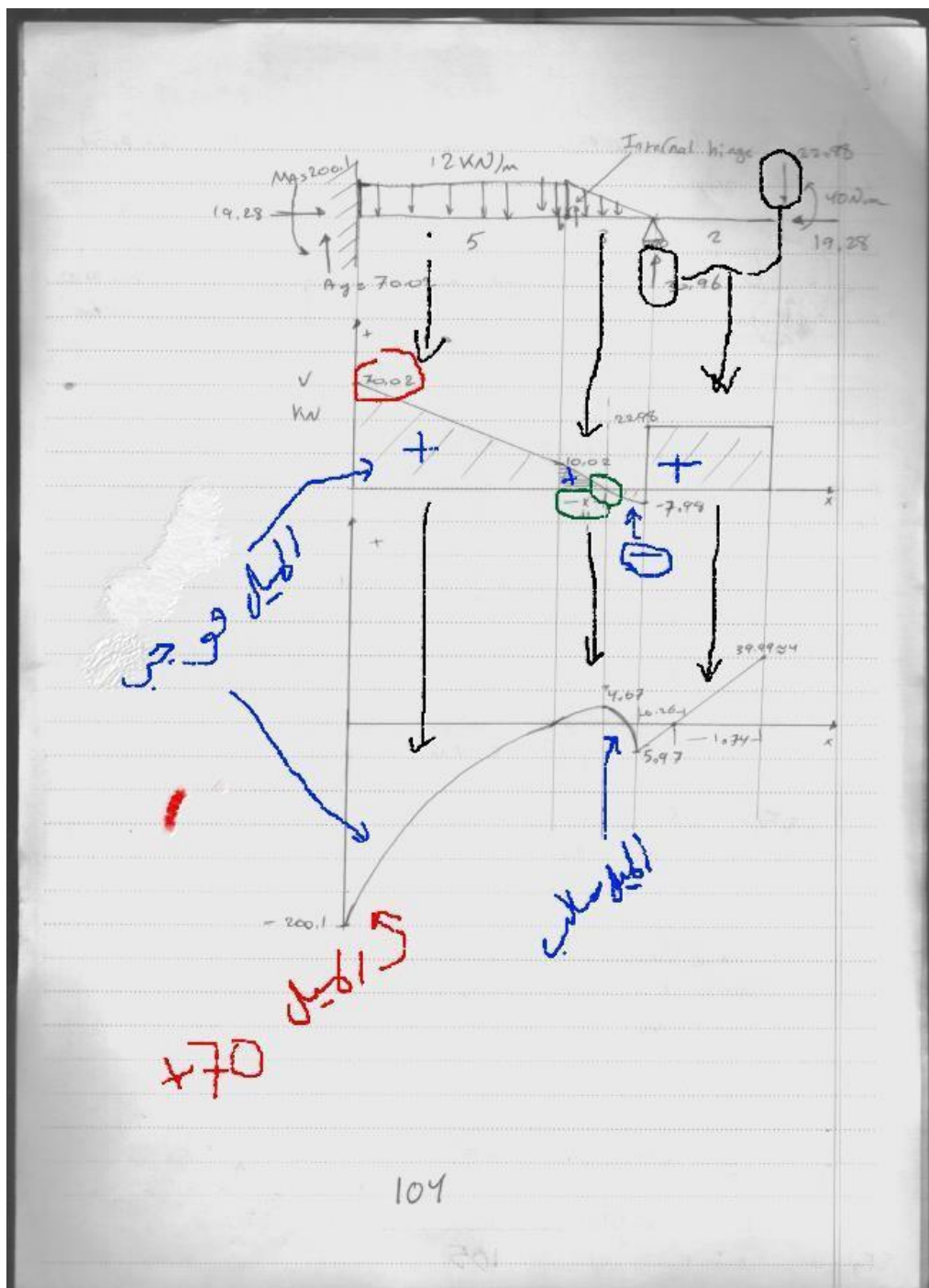
هسا كيف هون رسمنا ال shear مثلا , بدأنا ب + 70.02 , ولأنو عنا load مستطيل الشكل رح يصير خط مائل , القيمة الي رح ينتهي عندها هي

( + 70.02 - ) 12 \* 5 ( = + 10.2 ) وميله افتراضيا هو ) 12 \* 5 ( , هسا ال load اللي ع شكل مثلث , رسمه رح يكون منحنى , يبدأ بنفس الميل اللي قبله ) 12 \* 5 (

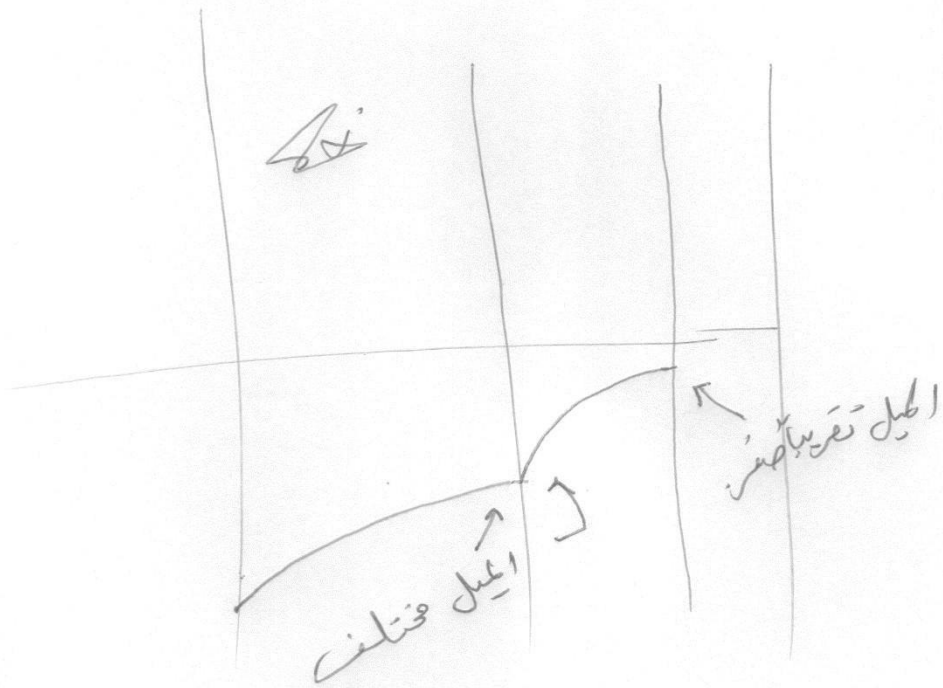
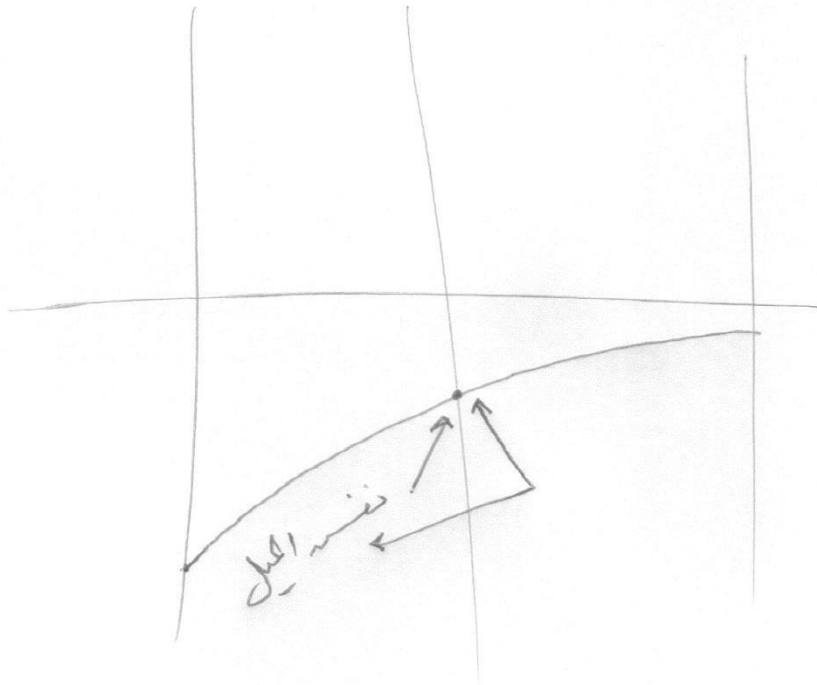
, والقيمة اللي رح ينتهي عندها هي + ) 10.02 - ( 12 \* 3 ( = - 7.98 ) بس رح تنتهي بميل صفر , بنهاية المثلث عندك قوة للاعلي , ع نفس نقطة نهاية المثلث ! ف رح نطلع لفوق ) - 7.98 + 30.96 ( = + 22.98 , وبعدها القوة الجديدة اللي رح تأثر مش load ف رح يكون خط افقي

أما رسمة المومنت , بدأنا بمومنت - 200.1 ( ( صحيح ضروري تعرف كيف تتعامل مع الاشارات متى الموجب ومتى السالب لل shear & moment ) بعدها منحنى , ينتهي عند ( - 200.1 + ) مساحة شبه المنحرف + ) 200.1 ( ( = صفر , يبدأ بميل 70.02 وينتهي بميل 10.02 , بعدها رح نوقف عند النقطة x اللي ميل منحنى المومنت عندها صفر ) صفر + مساحة المنطقة ( بس هالمنحنى مش معطى قانونه بالجدول ) مش قطع مكافئ ( , فهون بناخذ مقطع من الشكل الاصلى وبنوجد المومنت حسابيا ! , بعدها الجزء اللي تحت المحور قطع مكافئ , بنحسب مساحتها وبنقصها من 4.07 , بعدها خط مائل ينتهي ب - 5.97 + مساحة المستطيل ( والميل رح يكون + 22.98 )

الدائرة الخضرة لأنو قطع المحور , فلازم تلقي المسافة x عشان تقدري توجدي المومنت عندها ( مشروحة بال pdf )



دائما عند كل خط عمودي اوجد قيمة المومنت , والميل من الجهتين لأنو يمكن يختلف , هاي اجتنا بالفاينل وحسيتها مقصودة الشغلة



بالتوفيق  
فيا

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