

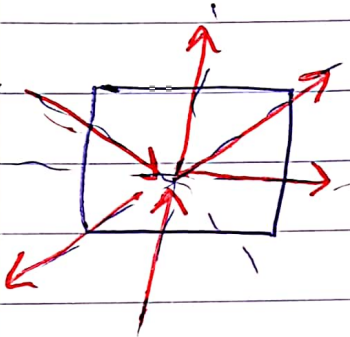
CHAPTER 4

in the equilibrium there is no Rotational motion and no translational motion

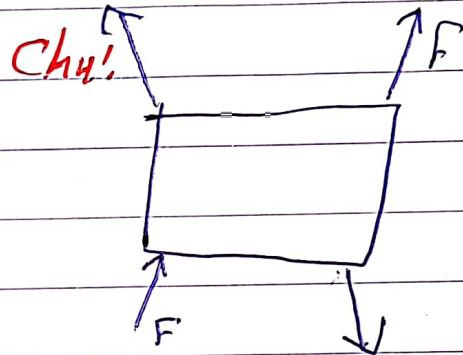
$$\sum F = 0$$
$$\sum M = 0$$

notice the difference

ch 2:



ch 4:



At concurrent forces

don't make moment around the point

so M is already

zero ~~but~~ F .

is

maybe $F=0$

but we should check M

Moment around any Axis

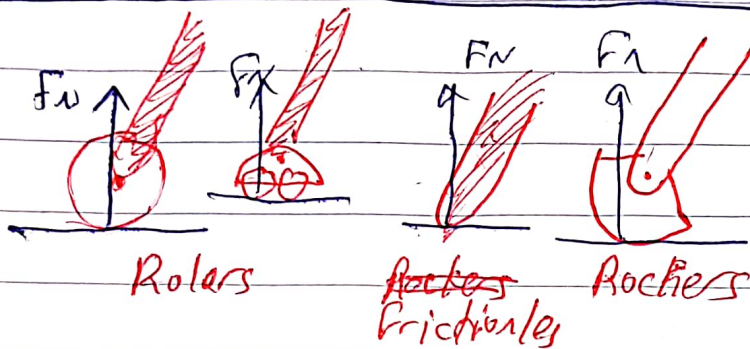
Any point

net force should equal zero

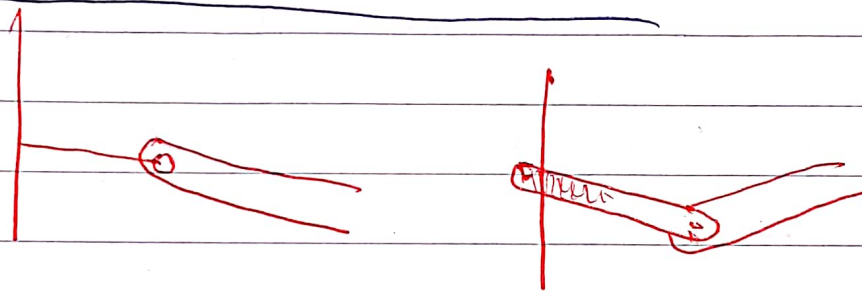
△ Any Axis even if not $x/y/z$

should equal to zero

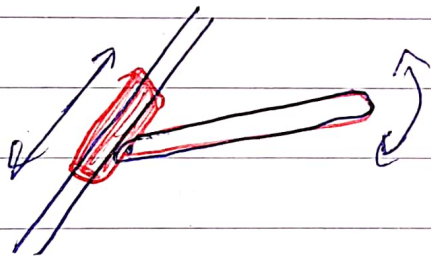
Some times choosing the axis of moment is a very big part of solution



Prevent moment in the direction force with known direction

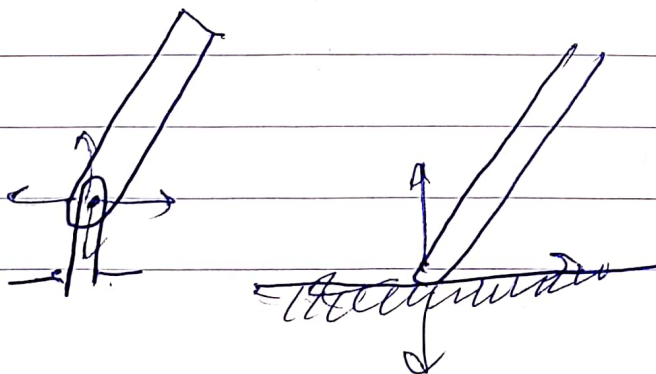


Force with known line of action

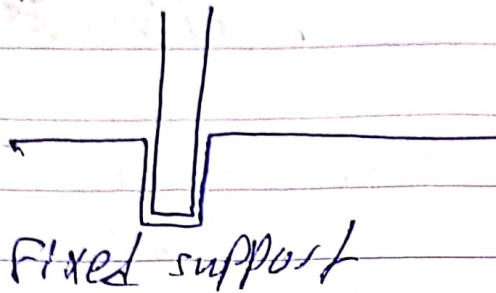


Force with known L.A

Frictionless pin rough surface



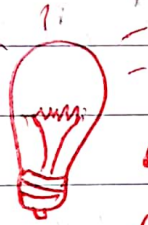
Force in 2 direction (unknown direction)



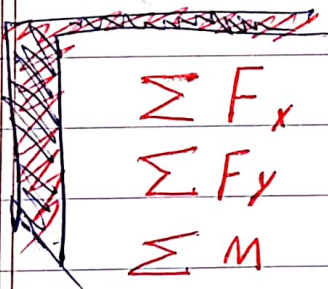
Force with
unknown
direction
and a couple
of unknown
magnitude

⚠ when we have a force with unknown direction we can use F_x/F_y or F/α

The max number of reaction that we can calculate is 3 in 2D



Equilibrium equations in 2D are 3 at most

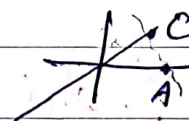


$$\sum F_x = 0$$

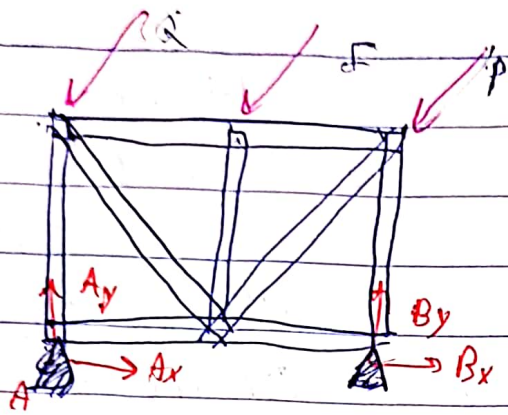
$$\sum F_y = 0$$

$$\sum M = 0$$

→ at any point or
any axis even ~~any~~
axis AC

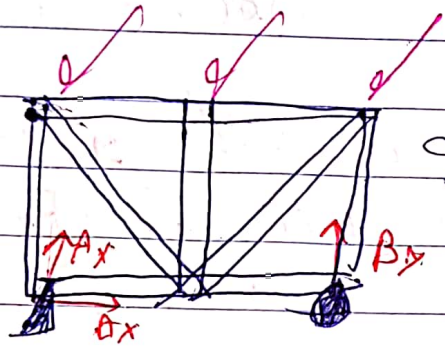


reactions \approx eq.



لا يمكن ان يكون
 في حالة توازن
 في هذه الحالة
 في الحالة

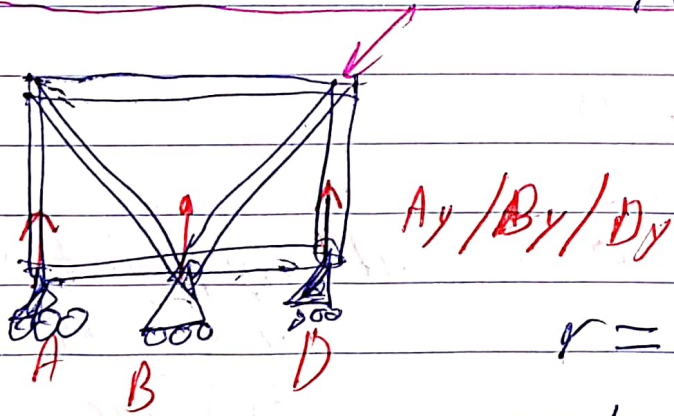
re. \approx eq.



لا يمكن ان يكون
 في حالة توازن
 في هذه الحالة
 Reactions \approx
 في حالة توازن
 المتعادلة

The reaction is the resistance ~~of the~~ force of moving the body

re. $<$ eq.

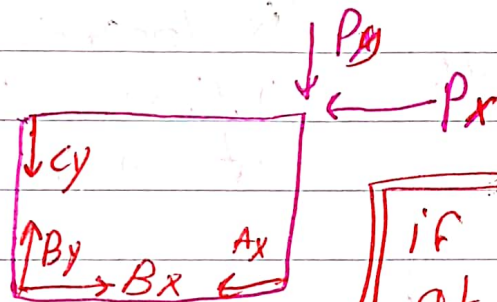
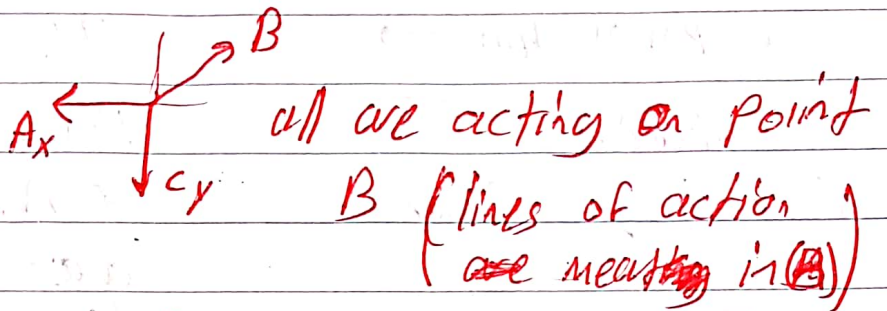
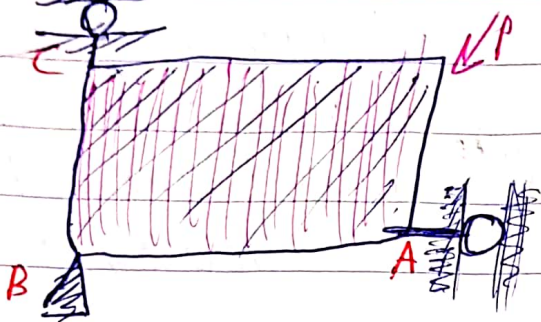


r = eq

but it is not stable

if reactions are parallel
 the body won't be
 stable

If reactions are concurrent !!!
~~watch~~ watch this!



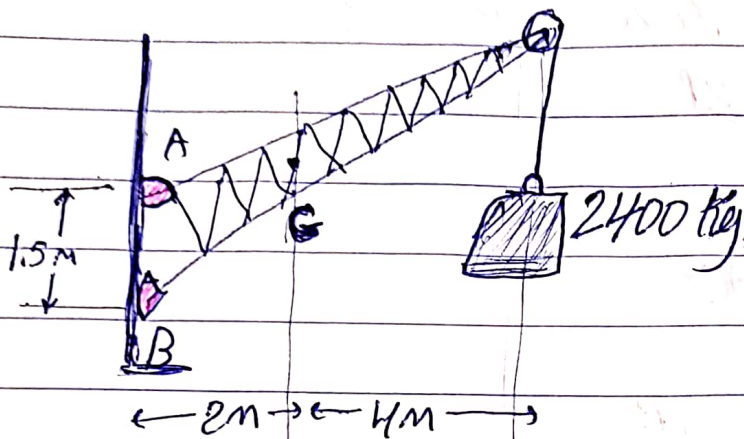
if P_x made moment M_p
 at point B

A_x can't make moment
 because its line of action
 go through B

and the same for
 reaction B_y

- 1) 3 reactions to be able to determine
 for building a structure you
 can have more
- 2) not parallel reactions
- 3) 4 concurrent

Sample Problem



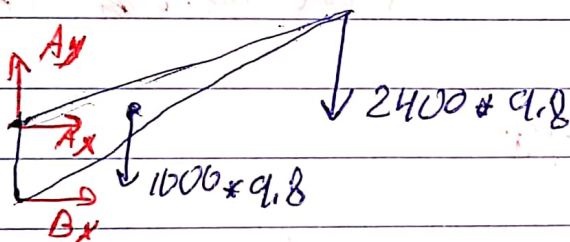
a fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate.

it's held in place by pin at A and a rocker at B. the center of gravity of the crane is located at G.

Determine the reactions at A & B

Solution

Free Body Diagram



$F_x \rightarrow 2m$ known

$F_y \rightarrow$ one is known

$M_B = 2$ unknown / $M_A =$ one unknown / $M_G = 3$ unknown

always use the point that reduce the unknowns

$$\sum F_y = 0 \rightarrow \boxed{A_y} = (2400 + 1000) * 9.8 = \boxed{+33320 \text{ J}}$$

$$M_A = 0 \rightarrow 2400 * 9.8 * 6 + 1000 * 9.8 * 2 = B_x * 1.5$$

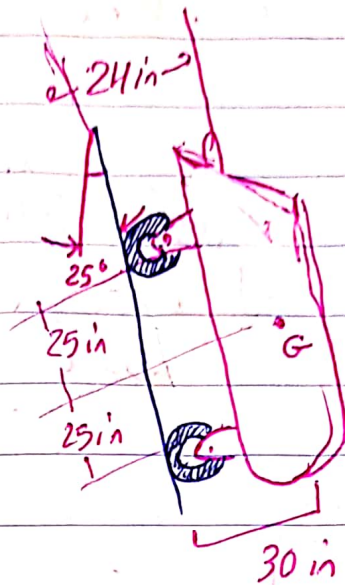
$$\boxed{B_x = 107146.67 \text{ N}}$$

$$\sum F_x = 0$$

$$\boxed{A_x} = -B_x = -107146.67$$

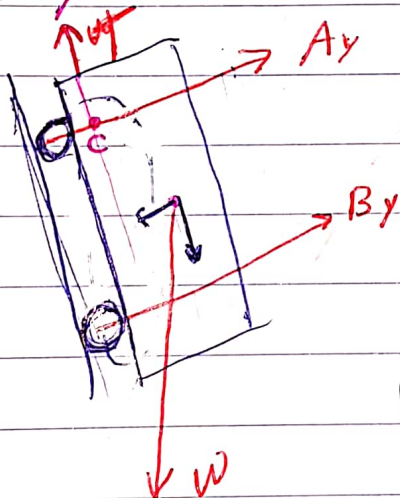
Sample Problem

A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb and it's ~~applied~~ applied at G. The cart is held in position by the cable.



Determine the tension in the cable and the reaction at each pair of wheels.

My solution



$$\sum F_x = 0$$

$$W \cos 25 = T$$

$$T = 4984.7 \text{ lb}$$

$$M_c = 0$$

(C) نقطة التقاء
خطي ثقل ما
اننا نتخلص
عن قدرته

$$W \sin 25 * 25$$

$$+ W \cos 25 * 6$$

$$- B_y * 50 = 0$$

$$B_y = \frac{25W \sin 25 + W \cos 25 * 6}{50}$$

$$B_y = 1760.4 \text{ lb}$$

$$A_y = ?$$

$$\sum F_y = 0$$

$$1760.4 + A_y - W \sin 25 = 0$$

$$A_y = 564$$

من
طريقة

اختيار

المحاور

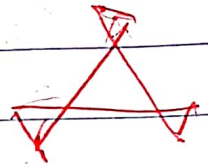
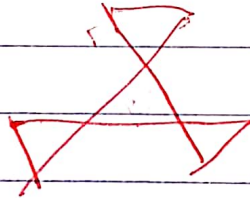
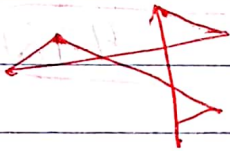
اختار الدور او النقلة

التي

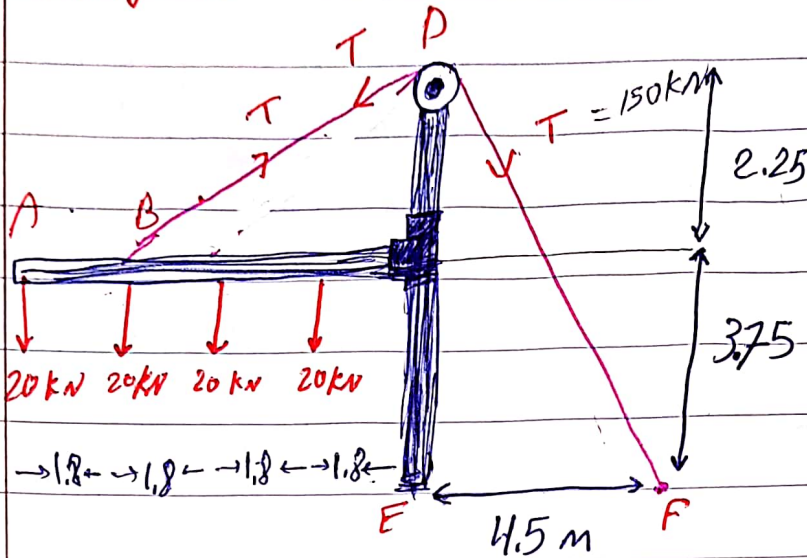
اختار النقلة التي تتقاطع فيها
ظروف انطباق القوى الى حولة
ثم اصبحت الوقت عنها
لتجاهل هذه القوى

او اختار الدور - غير بالكثر
من support لتجاهل
هذه ال reactions
الغير ملزمة لاجار

a circular reaction

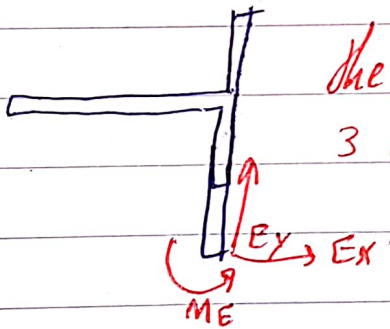


Sample



The frame supports part of the roof of a small building. The tension in the cable is 150 kN. Determine the reaction at the fixed end E.

Solution :-



The support has 3 reactions

هذا من مبدأ التوازن
نقاسي بيننا المحاور
ويبقى صول كونه
الشئ BD ليس
نظم انه لا يكون
لا يمنع عنم لانه
هناك قوة محالة
للرئيسه
وهي الشئ DB

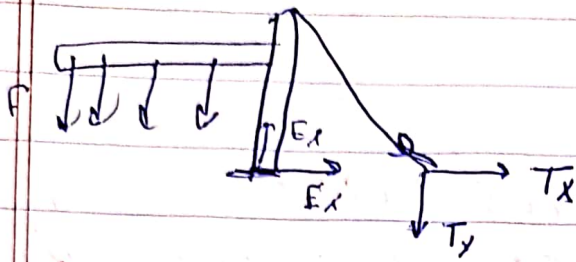
$$\sum F_y = 0 \Rightarrow T \cdot \frac{(2.25 + 3.75)}{\sqrt{(4.5)^2 + (6)^2}} + 80 \text{ kN} - E_y = 0$$

$$E_y = 120 + 80 = 200 \text{ kN up}$$

$$\sum F_x = E_x + T \cdot \frac{4.5}{\sqrt{4.5^2 + 6^2}}$$

$$E_x = -90 \text{ kN} = 90 \text{ kN left}$$

نظم التوازن
افتبار النظام هو موقفه جداً لاجاب
ذلك لانه القوى Ex Ey والشئ DB



$$M_E = ?$$



$$M_E + M(\text{all other forces}) = 0 \quad \text{at any point}$$

because M is free vector
 لأن العزم لا يتأثر بالمكان الذي نختاره

$$M_E + -T_y * 4.5 + 20 * 1.8 + 20 * 3.6 + 20 * 5.4 + 20 * 7.2$$

$$M_E + -120 * 4.5 + 20(18)$$

$$M_E - 120 * 4.5 + 360 = 0$$

$$M_E = 120 * 4.5 - 360$$

$$M_E = +180 \text{ kN.m counter clockwise}$$

مثال جديد في هذا المثال :-

1- لدينا حبل يتكئ على القوس بالعزم مثل ادو المثال
 نلاحظ ان هناك عزم نمر معلوم عند ال support

Principle of transability²

3- العزم Free vector يمكن M_E ان يكون عند D
 (4) القوى الداخلية لا تؤثر على ال support

Equilibrium of Two Force member

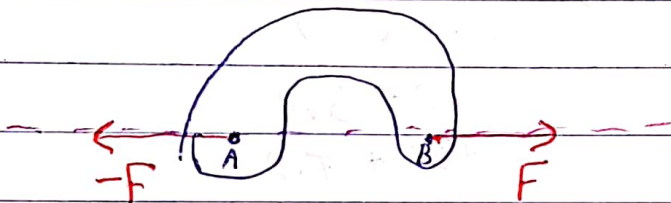
when two forces act on a member ~~whatever~~ what ever its shape look like

and these forces are (A) equal in magnitude

(B) and opposite in direction ~~the~~

(C) on the same line of action ~~then~~

→ they make no moment / ~~the~~ $\Sigma F = 0$



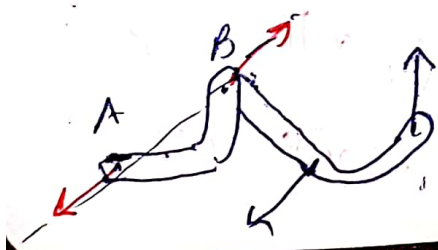
$$\Sigma F = 0$$

$$\Sigma M = 0$$

and if you saw a member such that

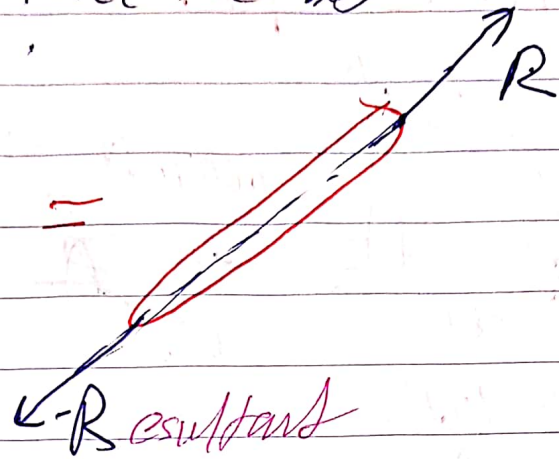
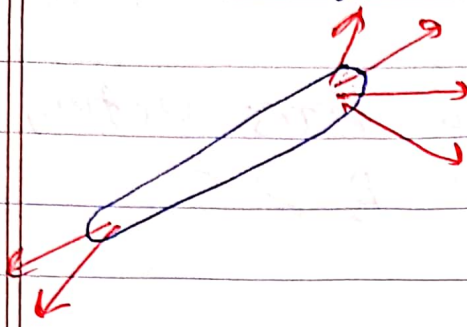
- 1) the member is acted on at two points ^{only}
- 2) the member is under equilibrium
- 3) there is no external force or moment or couple

then the member is
2 Force member



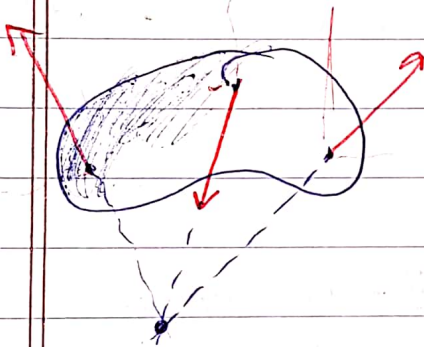
by this way you know the direction and line of action

look at this 2 force member



not exactly or let's say 2 forces

3 force Body



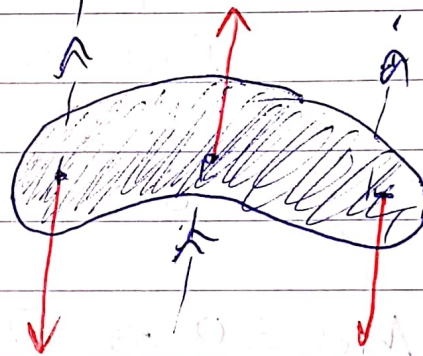
concurrent forces

then $M = 0$



يمكن إيجاد موقع نقطة التقاطع وبالتالي معرفة السراوية

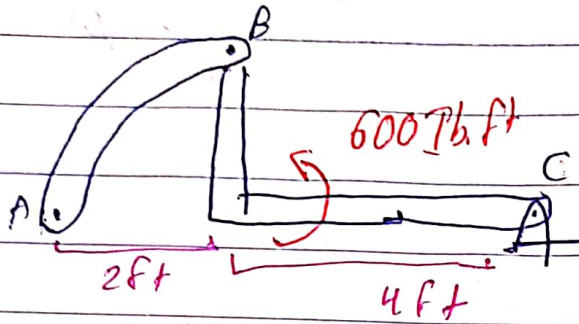
او يتحلل الى F_x/F_y كما هي



collinear forces

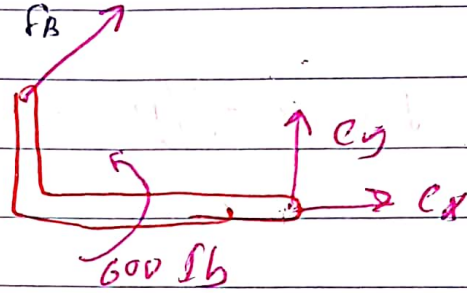
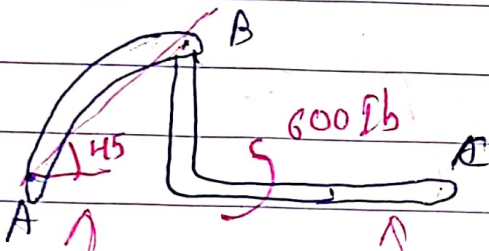
تنتهي نفس الاتجاه والى نفس الجسم

External questions From (Pr. Nizar)



Find forces acting on Pins B & C

Solution



2 force member

can't be 2 force member

hard to determine directions it's OK

$$\sum F_y = 0$$

Free vector

$$\sum M_C = 0 \Rightarrow F_B \sin 45^\circ \cdot 4 + F_B \cos 45^\circ \cdot 2 - 600 = 0$$

$$\frac{F_B}{\sqrt{2}} (6) = 600$$

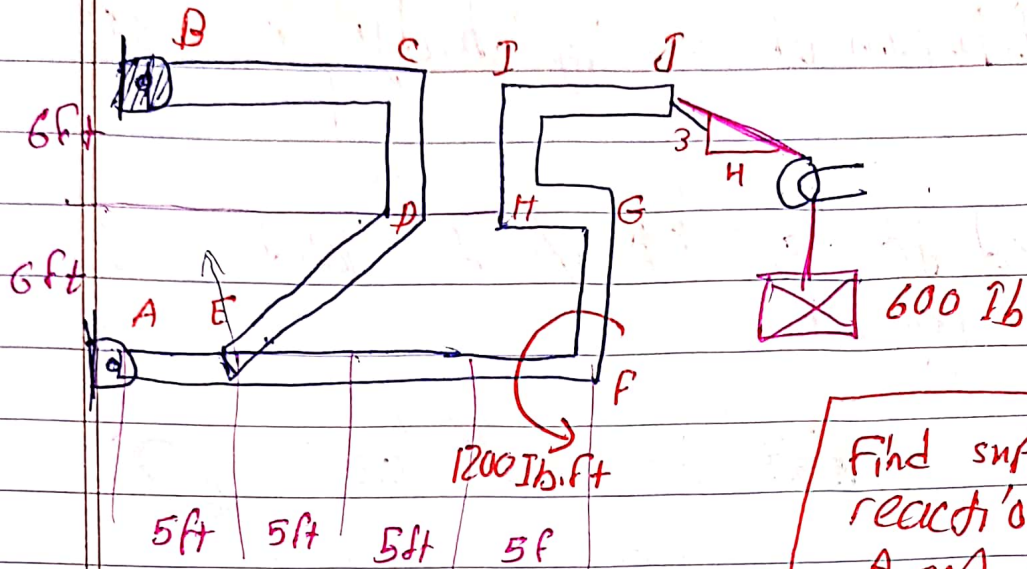
$$F_B = 100\sqrt{2} \text{ lb with angle } 45^\circ \text{ over } x^+$$

$$\sum F_y = 0 \rightarrow C_y + \frac{F_B}{\sqrt{2}} = 0$$

$$C_y = -100 \text{ lb}$$

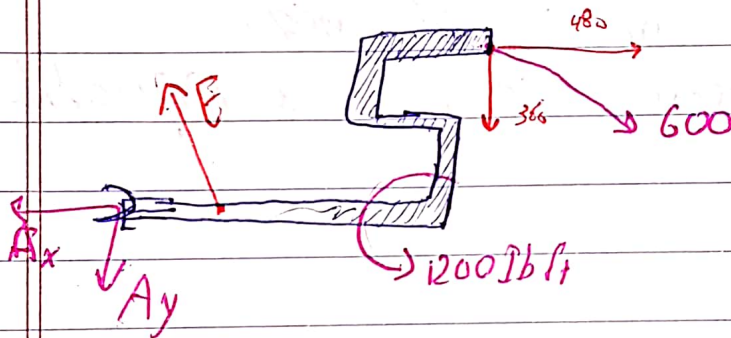
$$\sum F_x = 0 \rightarrow C_x + \frac{100\sqrt{2}}{\sqrt{2}} = 0$$

$$C_x = -100$$



solution

BCDE = Two force member
 then we now draw FBD for (AEFGHIJ)



first find moment around A

$$= 1200 + E * \frac{42}{13} * 5 - 360 * 20 - 480 * 12 = 0$$

$E = 2548$ ← دل هات زي
 # قبل البحث
 دونه ما طلب E طرفا فرق توري

moment at E / $A_y * 5 + 1200 - 360 * 15 - 480 * 12$

$A_y = -1992$ J 70

طلب الي قبل مش زيات العجب لك رفع نتاجه لو
 واصل طلب طلب E 880 لانك كمان

$A_x = E_x - T_x = 500 \text{ lb}$

Equilibrium of a Rigid body in 3D

no need to speak about kinds of supports

But: ~~9~~ Here we have 6

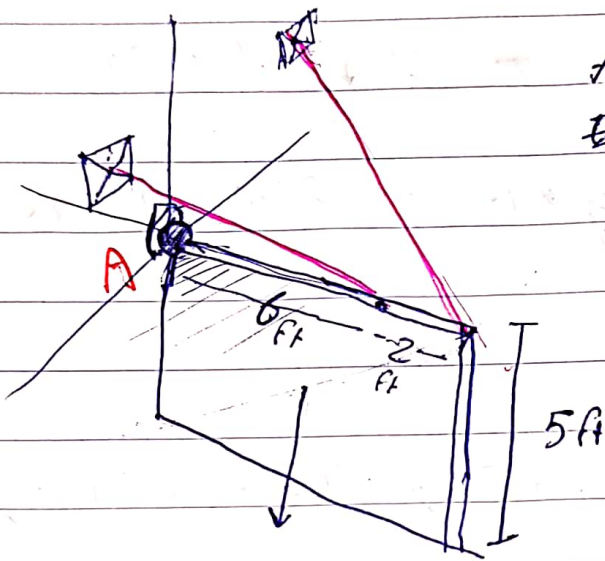
Equations. And ability for

6 reactions

but that does not mean
that it's necessary to find 6 equations

Sample problem

3D

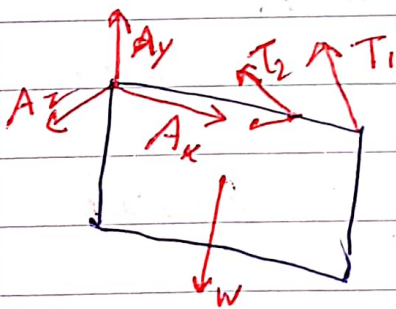


A sign of uniform density ~~the~~ weighs 270 lb & is supported by a ball-and socket joint at A and 2 cables

Determine Tension in each cable and reaction at A

Thinking:- Tension in each cable are known in direction / weight is in centroid

equations $\sum F_x = 0 / \sum F_y = 0 / \sum F_z = 0$
 $\sum M_x = 0 / \sum M_y = 0 / \sum M_z = 0$



$$\vec{T}_2 = T_2 \frac{-6\hat{i} + 3\hat{j} + 2\hat{k}}{7}$$

$$\vec{T}_1 = T_1 \frac{-8\hat{i} + 4\hat{j} - 8\hat{k}}{12}$$

my solution: find Moment around (AD) must be zero

$$\lambda_{AD} = \frac{4\hat{j} - 8\hat{k}}{\sqrt{80}}$$

$$r_w = (4\hat{i} - 2.5\hat{j})$$

$$\frac{T_2}{\sqrt{80} \cdot 7} \begin{vmatrix} 0 & 4 & -8 \\ 6 & 0 & 0 \\ -6 & +3 & +2 \end{vmatrix} + \begin{vmatrix} 0 & 4 & -8 \\ 4 & -2.5 & 0 \\ 0 & -270 & 0 \end{vmatrix} \times \frac{1}{\sqrt{80}} \rightarrow T_2 = 315.9$$

$$\sum A_x = 0 \quad \text{FRYMANIAW}$$

$$T_{1y} \times 8 + T_{2y} \times 6 - W \times 4 = 0$$

$$\frac{1}{3} T_1 \times 8 + 315 \times \frac{3}{4} \times 6 - 270 \times 4 = 0$$

$$T_1 = 101.25$$

$$\sum F_y = 0 \rightarrow T_{1y} + T_{2y} - W + A_y = 0$$

$$A_y = 101.25$$

$$\sum F_x = 0 \rightarrow A_x - T_{1x} - T_{2x} = 0$$

$$A_x = 337.5$$

$$\sum F_z = 0 \rightarrow T_{2z} - T_{1z} + A_z = 0$$

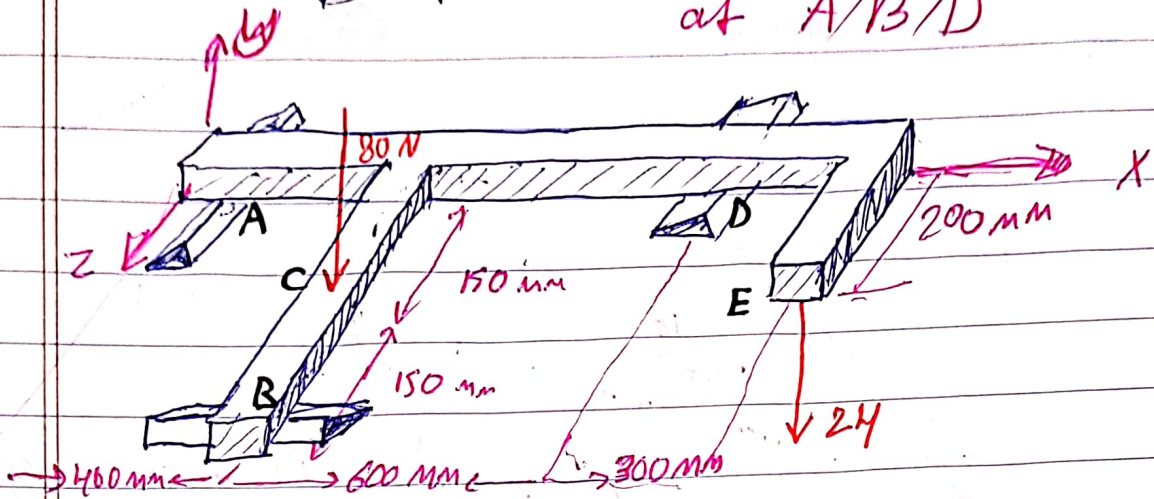
$$\frac{2}{7} \times 315 - \frac{2}{3} \times 101.25 + A_z = 0$$

$$A_z = -22.5$$

ممكن اننا احبان هذه المعادلات
في مسائلنا

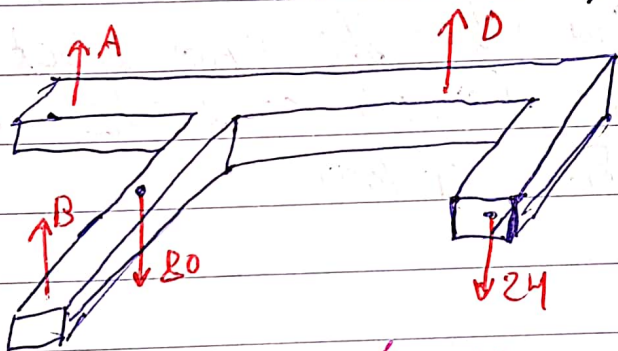
External ~~force~~

determine reactions at A/B/D



First notice you don't need all the 6 eq.

all forces are in y direction so there is no moment around y (also reactions in y)



no forces in other direction even if reaction can be

$$\sum F_y = 0 \quad / \quad \sum M_x = 0 \quad / \quad \sum M_z = 0$$

$$\sum F_x = 0 \quad \sum F_z = 0 \quad / \quad \sum M_y = 0$$

— 3 eqs & 6 unknowns

$$\sum M_x = 0 \quad 24 \times 200 + 80 \times 150 - B_y \times 300 \rightarrow B_y = 56$$

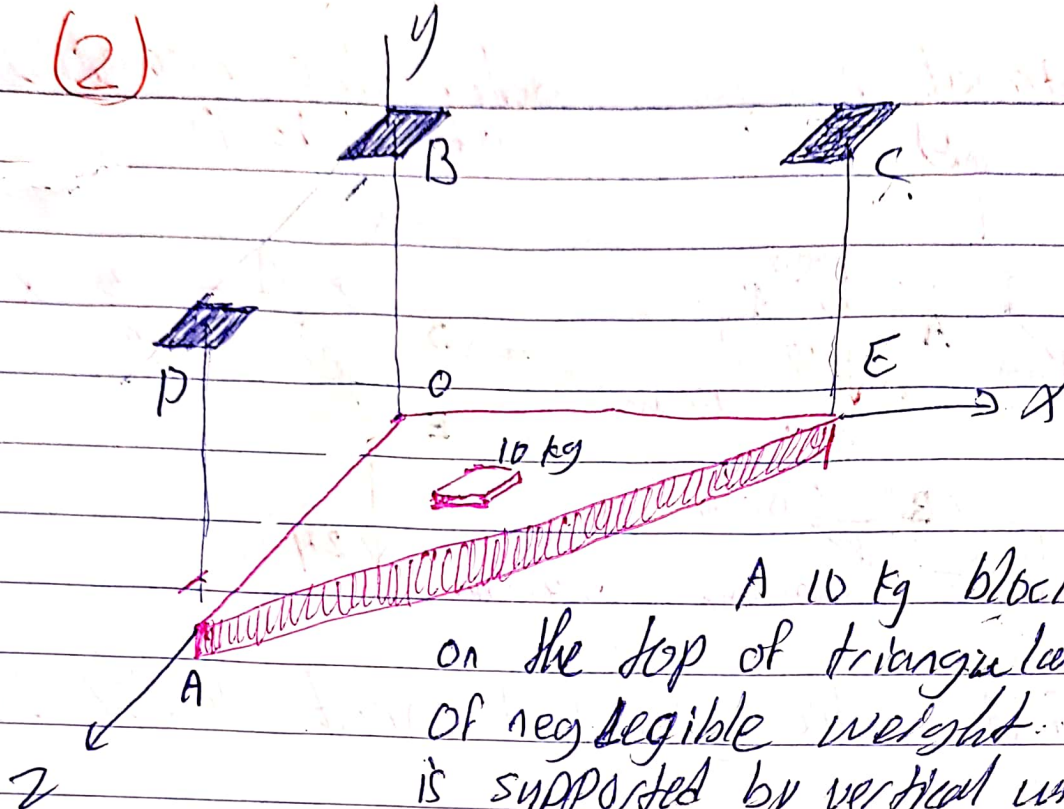
$$\sum M_z = 0 \quad -24 \times 1300 + D_y \times 1000 - 80 \times 400 + 56 \times 400$$

$$D_y = 408$$

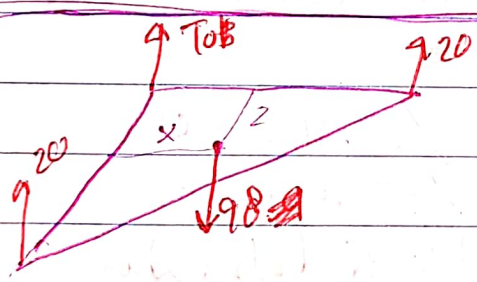
$$\sum F_y = 0 \rightarrow 408 + 56 + A_y - 24 - 80$$

$$A_y = 72$$

(2)



A 10 kg block rests on the top of triangular plate of negligible weight. The plate is supported by vertical wires DA, OB, CE. Both AD & CE are same tension = 20 N. Find where the 10 kg block should be (x, z) coordinates and find T(OB)



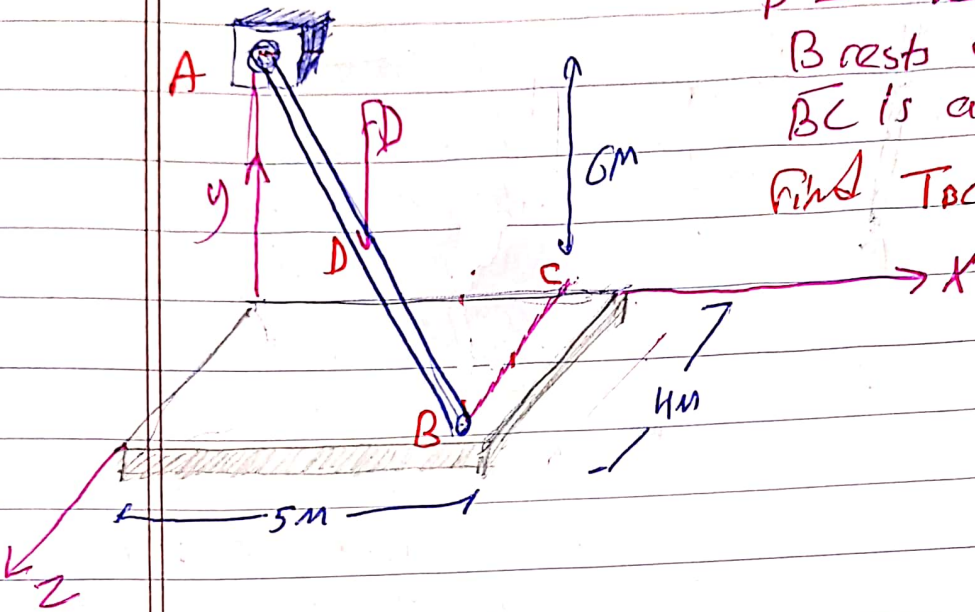
$$\sum F_y = 0 \rightarrow T_{OB} = 58 \checkmark$$

$$M_x = 0 \rightarrow 98 \times z = 20 \times 300$$
$$z = 61.2$$

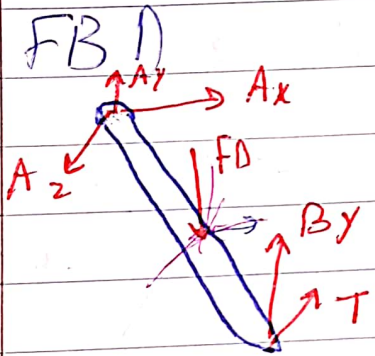
$$M_z = 0 \rightarrow 20 \times 400 = 98 \times x$$

$$x = 81.6$$

External



$F_D = 20\hat{i} - 40\hat{j} + 30\hat{k}$
 D = mid point
 B rests on smooth plate
 BC is a rod
 Find T_{BC} / reactions of A
 reactions of B



$$\sum M_{z(A)} = 0$$

$$-F_D y \times 2.5 + F_D x \times 3 + 5 \times B_y = 0$$

$$-40 \times 2.5 + 5 \times B_y + 60 = 0$$

$$\frac{40}{5} = B_y \rightarrow B_y = 8 \text{ N}$$

$$\sum M_y = 0$$

$$0 = T \times 5 + 20 \times 2 - 30 \times 2.5$$

$$T \times 5 = 35$$

$$T = 7 \text{ N}$$

$$\sum F_y = 0$$

$$8 + A_y - 40 = 0$$

$$40 - 8 = A_y$$

$$A_y = 32 \text{ N}$$

$$\sum F_x = 0$$

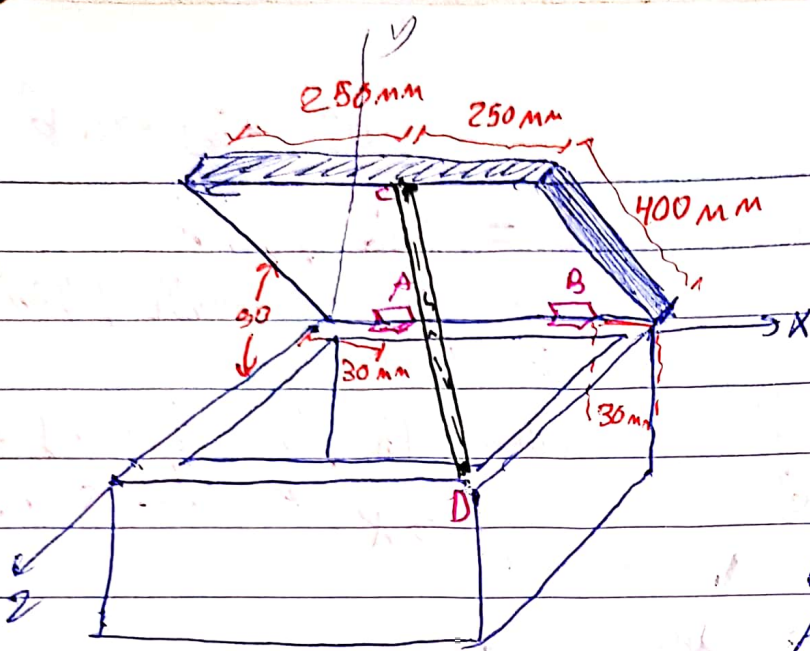
$$A_x + 20 = 0$$

$$A_x = -20 \text{ N}$$

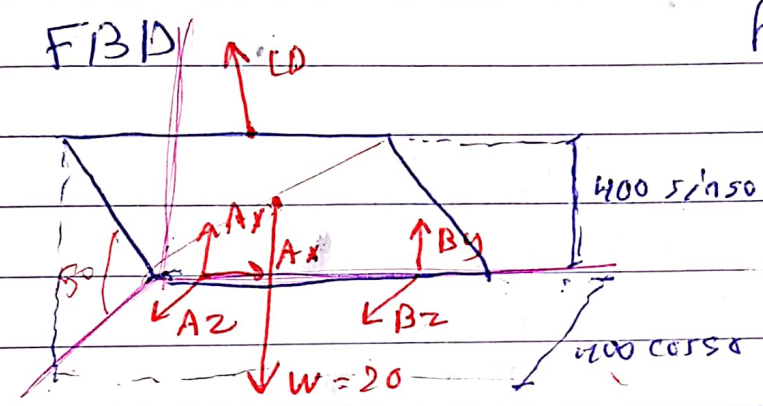
$$\sum F_z = 0$$

$$-T + 30 + A_z = 0$$

$$A_z = -23 \text{ N}$$



The uniform 20 N lid of the box is supported by a stick CD and short hinges at A & B. Assume that B transmits no axial force and reactions at A/B and force in the stick.



$$C = \left(\frac{-250 \uparrow + 306.4 \downarrow}{420.5} - \frac{142.9 \downarrow}{420.5} \right) C$$

$$M_x = 0 = 20 * 128.6 - C_x * 257.1 - C_z * 306.4$$

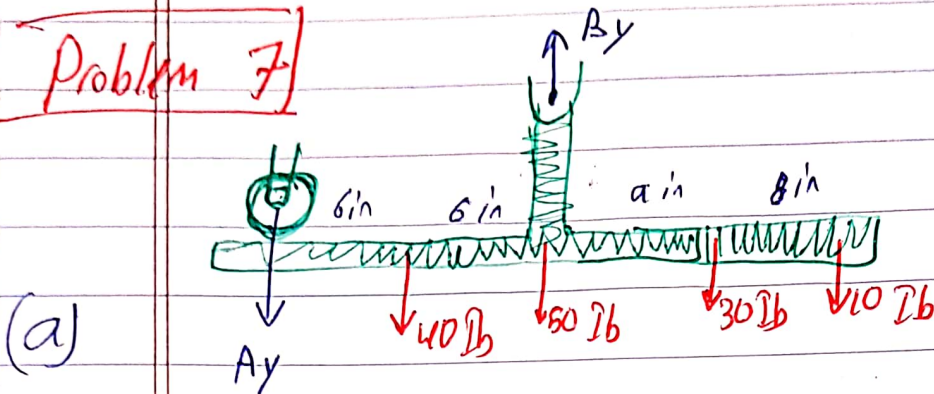
$$257.2 = C \left(\frac{306.4}{420} * 257.1 + \frac{142.9 * 306.4}{420.5} \right)$$

$$C = 8.824$$

$$\sum F_x = 0 \rightarrow A_x = C_x$$

Home Work 3

Problem 7



$$\begin{aligned}
 M_B = 0 &= A_y \times 12 + 40 \times 6 + 30 \times a + 10 \times (8+a) \\
 &= 12 A_y + 240 = 300 + 180 \\
 &\quad \boxed{A_y = -20 \text{ lb}} \quad \checkmark
 \end{aligned}$$

$$\sum F_y = 0$$

$$-(A_y + 40 + 50 + 30 + 10) + B_y = 0$$

$$\boxed{B_y = +150 \text{ lb}} \quad \checkmark$$

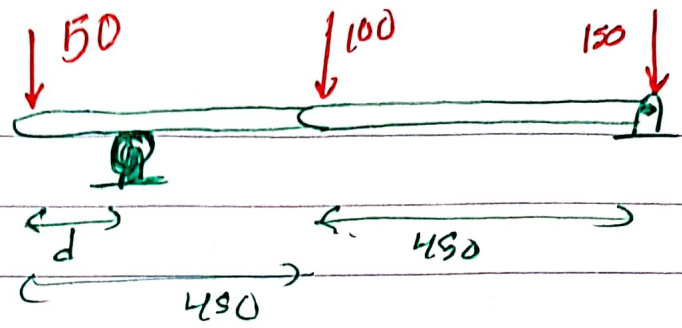
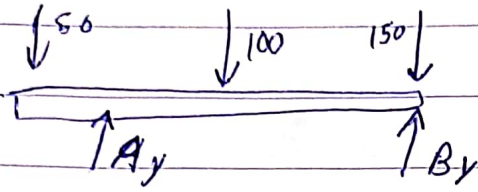
(b)

For $a = 7$

$$\begin{aligned}
 A_y &= 12 A_y + 240 = 210 + 180 \\
 &\quad \boxed{A_y = -10 \text{ lb}} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 B_y &= A_y + 40 + 50 + 30 + 10 \\
 &= +140 \text{ lb} \quad \checkmark
 \end{aligned}$$

Prob. 13



$$0 = M_B = 50 \times 0.9 + 100 \times 0.45 - A_y \times (0.9 - d)$$

$$A_y(0.9 - d) = 90$$

$$A_y = \frac{90}{0.9 - d}$$

Answer is $150 \leq d \leq 400$ in mm

$$A_y \leq 180 \rightarrow \frac{90}{0.9 - d} \leq 180$$

$$90 \leq 162 - 180d$$

$$72 \geq 180d$$

$$0.4 \geq d$$

$$400 \text{ mm} \geq d$$

this is for A

For B $M_A = 0$

$$50 \times d + B_y \times (0.9 - d) + 100 \times (0.45 - d) = 150(0.9 - d)$$

$$50d + 0.9B_y - B_yd - 45 + 100d = 135 + 150d$$

$$300d + (0.9 - d)B_y - 180 = 0$$

$$300d + (0.9 - d)B_y = 180 - 300d$$

$$\frac{180 - 300d}{0.9 - d} \leq 180$$

$$162 - 180d \geq 180 - 300d$$

$$120d \geq 18 \rightarrow d \geq 0.15 = 150 \text{ mm}$$

or easily we can say

$$A_x = \frac{90}{0.9 - d}$$

and $\sum F_y = 0$

$$B_y + A_y = 300$$

$$B_y + \frac{90}{0.9 - d} = 300$$

$$B_y = 300 - \frac{90}{0.9 - d}$$

$$\frac{300 - 90}{0.9 - d} \leq 180$$

$$120 \leq \frac{90}{0.9 - d}$$

$$108 - 120d \leq 90$$

$$120d \geq 18$$

$$d \geq 0.15$$

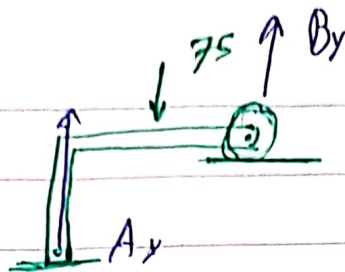
$$d \geq 150 \text{ mm}$$

and $d \leq 400 \text{ mm}$

then $150 \leq d \leq 400$

another way

Prob 37 (a)



$$M_B = 0 \rightarrow 75 \times 10 = 20 A_y$$

$$75 = 2 A_y$$

$$A_y = 37.5 \uparrow \text{ Ib } \checkmark$$

$$\Sigma F = 0 \rightarrow B_y + A_y = 75$$

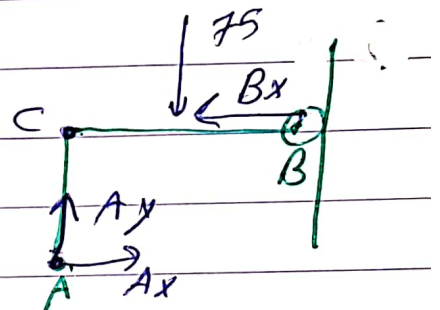
$$B_y = 75 - 37.5$$

$$B_y = 37.5 \uparrow \checkmark$$

(b) $\Sigma F_y = 0$

$$A_y \uparrow - 75 \uparrow = 0$$

$$A_y = +75 \uparrow \checkmark$$



$$\Sigma M_c = 0$$

$$12 A_x - 75 \times 10 = 0$$

$$A_x = \frac{750}{12} = 62.5 \text{ Ib } \checkmark$$

$$A_x = +62.5 \text{ Ib } \uparrow \checkmark$$

$$\Sigma F_x = 0 \rightarrow \vec{A}_x + \vec{B}_x = 0$$

$$|62.5| + |B_x| = 0$$

$$|B_x| = 62.5$$

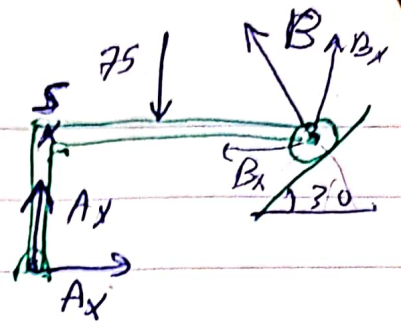
$$\vec{B}_x = -62.5 \uparrow \checkmark$$

c at $\alpha = 30$

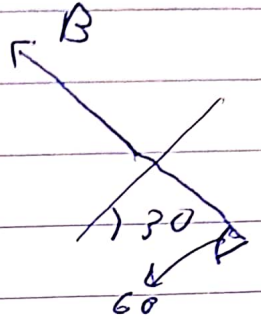
Next Page

(c)

4 reactions!
but we already know
the angle of $B = 60$



its dot is to
3 concurrent forces



~~ΣFx=0~~

$$M_s = 0$$

$$0 = B \sin 60 \times 20 + 12 \times Ax - 750$$

$$17.32 B + 12 Ax = 750 \quad \text{--- (1)}$$

$$\Sigma F_x = 0 \rightarrow \vec{Ax} + \vec{Bx} = 0$$

$$|Ax| - |Bx| = 0$$

$$Ax = B \cos 60$$

$$2Ax = B \quad \text{--- (2)}$$

solve eq 1 and 2

$$17.32 \times 2Ax + 12 Ax = 750$$

$$\rightarrow Ax = +16.1 \text{ Ib} \quad Ax = 16.156 \uparrow$$

$$\rightarrow B = 32.16 \text{ Ib}$$

angle 90 on surface
= 120 with x^+

$$By = B \sin 60 = 27.85$$

$$\Sigma F_x = 0$$

$$\rightarrow \cancel{B} Ax + By - 75 = 0$$

$$Ax + 27.85 = 75$$

$$\rightarrow Ax = 47.15 \text{ Ib} \uparrow$$

Prob 33

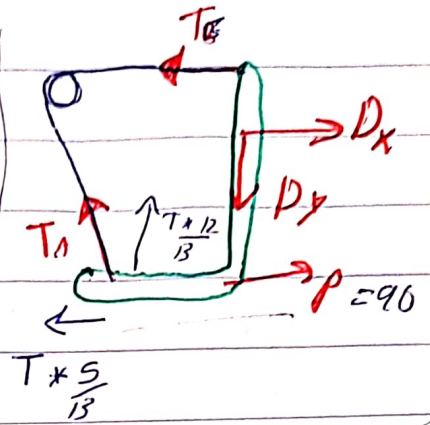
$$M_D = 0$$

$$T \times 3 + 90 \times 9 - T \times \frac{5}{13} \times 9$$

$$- T \times \frac{12}{13} \times 7$$

$$\approx 810 = 6.92 T$$

$$T = 117 \text{ Ib} \quad \checkmark$$



$$\sum F_y = 0$$

$$T \times \frac{12}{13} - D_y = 0$$

$$108 - D_y = 0$$

$$D_y = 108 \text{ Ib} \quad \checkmark$$

$$\sum F_x = 0$$

$$T \times \frac{5}{13} + T - P - D_x = 0$$

$$D_x = -72 \text{ Ib} \quad \checkmark$$

Prob 41

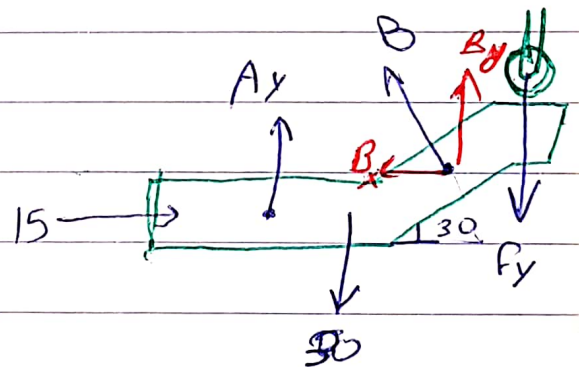
$$\sum F_x = 0$$

$$B_x - 15 = 0$$

$$B_x = 15 \quad \checkmark$$

$$B_x = B \cos 60$$

$$B = 30 \text{ Ib} \quad \theta = 60^\circ \text{ over } x \quad \checkmark$$



$$\sum M_A = 0$$

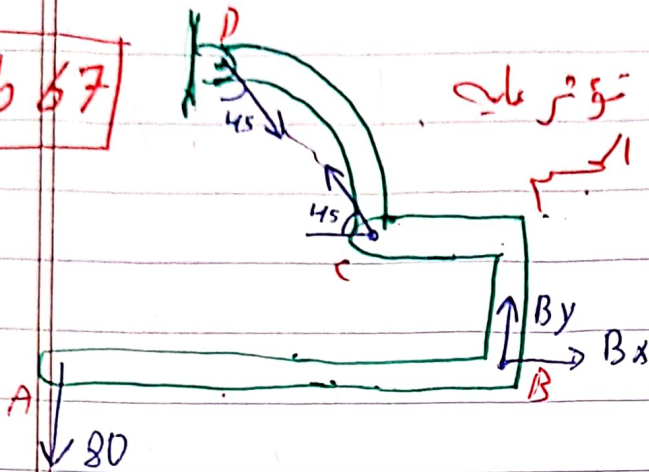
$$30 \times 4 + F_x \times 13 - B \sin 60 \times 11 - B \cos 60 \times 3 = 0$$

$$F_x \times 13 = 210.78 \rightarrow F_x = 16.2 \text{ Ib} \quad \checkmark$$

$$\sum F_y = 0$$

$$\rightarrow A_y = F_x + 30 - B_y \rightarrow A_y = 20.2 \text{ Ib} \quad \checkmark$$

Prob 67



لا بد ان القوس DC متوازن
توقع منه المائل وضوء منه الجسم
ABC قوتيه نقط وهو
متزنه اي ان هذه
القوات متساوية
وتقعان على خط
الواصل بين A و C

DC is under equilibrium we already know that

ABC is under equilibrium
then

$$M_B = 0 = 80 \times 250 - F_c \sin 45 \times 90 + F_c \sin 45 \times 60$$
$$= 20000 = 30 \times F_c \sin 45$$

$$F_c = 942.8 \text{ N}$$

$$\sum F_x = B_x - C_x$$
$$= B_x - F_c \sin 45$$

$$B_x = 666.7 \text{ N}$$

$$\sum F_y = C_y + B_y - 80 = 0$$

$$F_c \sin 45 + B_y - 80 = 0$$

$$666.6 - 80 + B_y = 0$$

$$B_y = -586.67$$

$$B_y = -586.67 \text{ N} \uparrow$$

reaction at D = $-F_c$

$$D = 942.8 \text{ N}$$

at angle $\sim 45^\circ$

لا تظن

اننا نعمل
لكم

الشمس
القاهرة

لوحده

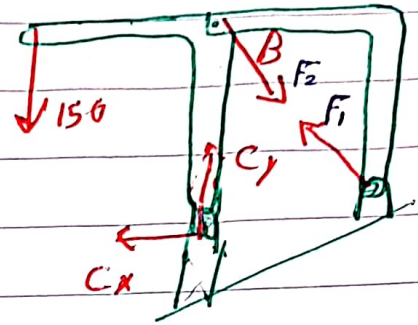
لا نعمل

لكم
الشمس

شمس
القاهرة

68

F_1 & F_2 are colinear
Two force ~~system~~ member



$$M_B = 0$$

$$150 \times 3 - C_x \times 3$$

$$C_x = 150 \text{ lb}$$

$$\sum F_x = 0$$

$$C_x - F_2 \times \frac{3}{\sqrt{3^2 + 1.5^2}} = 0$$

$$150 = F_2 \times 0.26666$$

$$F_2 = 167.57 \text{ lb}$$

$$\sum F_y = 0$$

$$C_y = 150 + F_2 \times \frac{1.5}{\sqrt{3^2 + 1.5^2}}$$

$$C_y = 225 \text{ lb}$$

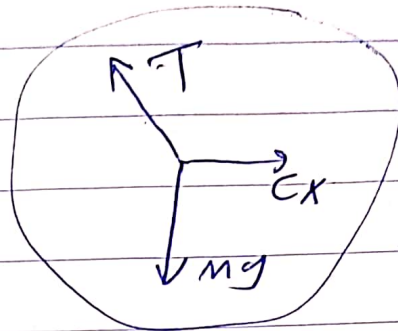
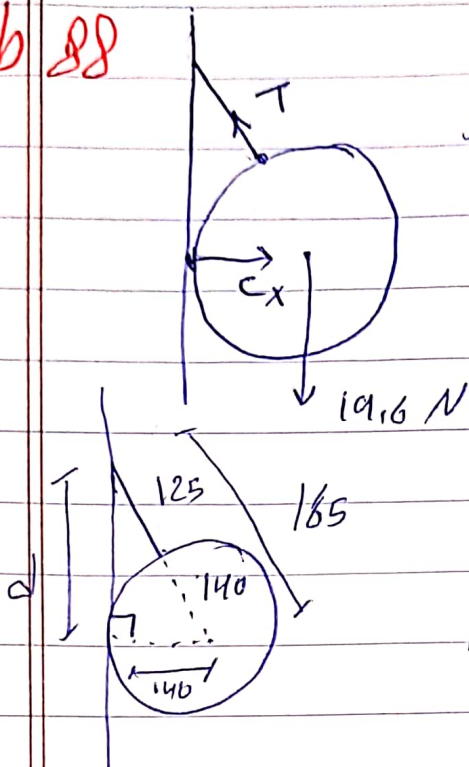
$$F_c = 3(150 \uparrow + 225 \uparrow) \text{ lb} \quad \checkmark$$

$$F_D = -F_2$$

$$= (-150.7 \uparrow + 75 \uparrow) \text{ lb} \quad \checkmark$$

Prob 88

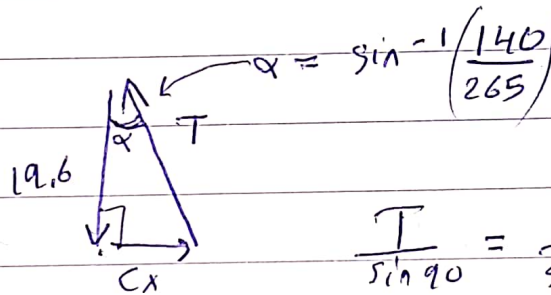
3 force \rightarrow concurrent



$$140^2 + d^2 = 165^2$$

$$d = \sqrt{165^2 - 140^2}$$

$$d = 225 \text{ mm}$$



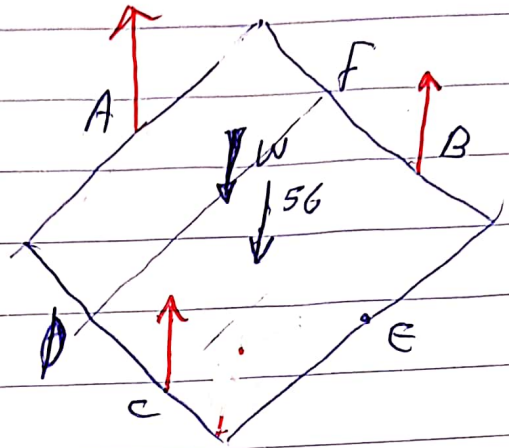
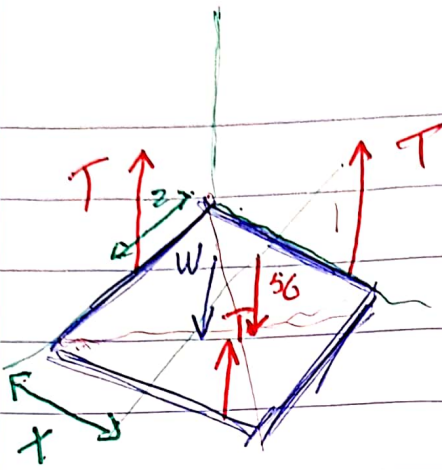
$$T = \frac{Cx}{\frac{140}{265}} = \frac{19.6}{\frac{225}{265}}$$

$$T \approx 23.1 \text{ N}$$

$$C_x = +12.2 \text{ N } \uparrow$$

3 un knowns

9.8



~~$M_{AB} = 0$~~

~~$-T \cdot 16 + W \cdot (10 - x) + 56 \cdot 6 = 0$~~
 ~~$16T = W(10 - x) + 336$~~

~~$M_{AE} = 0$~~

$M_{AE} = 0$

$= 0 + T \cdot 10 - T \cdot 10 + W \cdot z - 10 + 0$

$W \cdot (z - 10) = 0 \rightarrow z = 10$

$\Sigma F_y = 0 \rightarrow 3T - 56 - W = 0$

$3T - 56 = W$

$M_{DE} = 0$

$0 = T \cdot x + 56 \cdot (10 - x) - 2T \cdot (16 - x)$

$2T \cdot (16 - x) - Tx = 56 \cdot (10 - x)$

$32T - 2Tx - Tx = 560 - 56x$

$T(32 - 3x) = 560 - 56x$

$T = \frac{560 - 56x}{32 - 3x}$

$\frac{1680 - 168x}{32 - 3x} - 56 = W$

T هو

Next Page

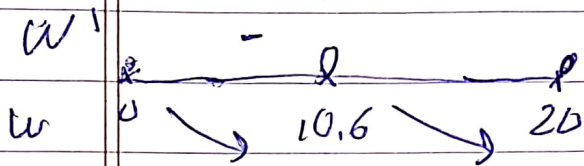
we need the smallest w

$$W' = \frac{-168(32-3x) + 3(1680-168x)}{(32-3x)^2}$$
$$= \frac{-6240 + 504x + 5040 - 504x}{(32-3x)^2}$$
$$= \frac{-1200}{(32-3x)^2}$$

DNE or look at the ends

W' can't be 0

even DNE so we check interval ends



check point 20 which is the only decision point

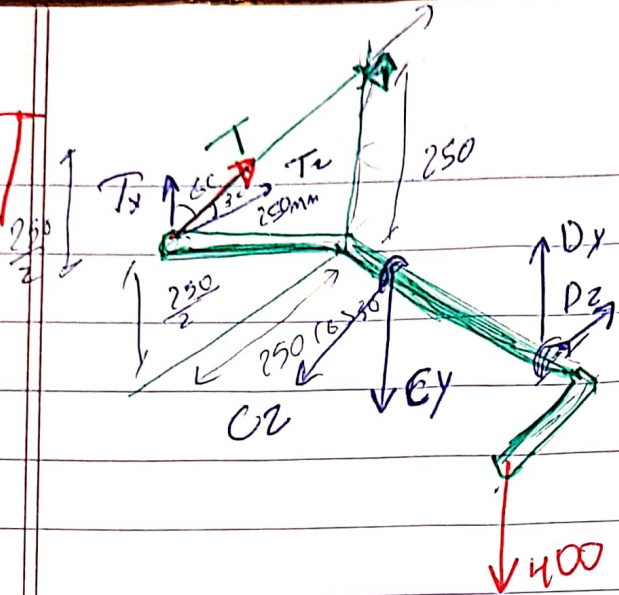
$$W(20) = \left(\frac{-560}{32-60}\right) \times 3 - 56$$
$$= 4N$$

$$W = 4 \text{ Ib/ft} \quad \text{at } x = 20 \text{ in/ft} \quad \text{at } z = 10 \text{ in}$$

$$T = \frac{W+56}{3} = \frac{4+56}{3} = 20 \text{ N}$$

نتيجة السؤال قوية !!

114



no forces at X axis
 $\Sigma F_x = 0$
 $\Sigma F_z = 0$
 $\Sigma M_x = 0$
 ~~$\Sigma M_y = 0$~~ $\Sigma M_y = 0$
 $\Sigma M_z = 0$

(a)

$\Sigma M_x = 0$

$400 \times 250 = T \times 250 \times \sin 60$

$T = \frac{400}{\sin 60} = 461.88 \text{ N}$

$\Sigma M_y = 0$ at Point C

$= D_z \times 300 - T_z \times 50 = 0$

$300 D_z \times 300 = 50 T_z \sin 60$

$D_z = \frac{5 \times 460}{300}$

$D_z = 66.67 \text{ N} \checkmark = 66.67 \text{ N } \hat{k}$

$\Sigma F_z = 0$

$C_z - D_z - T_z = 0$

$C_z = |D_z| + |T_z|$
 $= +66.67 + 400$

$C_z = +466.67 \text{ N } \hat{k} \checkmark$

ΣM_z at Point B = 0

$300 \times D_y - 400 \times 350 - T_y \times 50$

$300 D_y = 400 \times 350 + 50 \times 461.88 \times \sin 30$

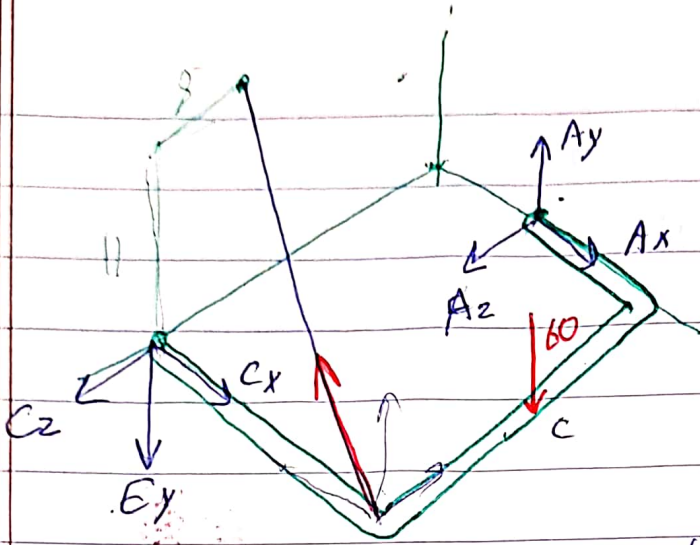
$D_y = 505.16 \text{ N } \checkmark$

$\Sigma F_y = 0 \rightarrow C_y = T_y + D_y - 400 \rightarrow C_y = -336 \text{ N}$

Answer: $T = 461.88 / C = (-336 \hat{j} + 466.7 \hat{k}) \text{ N} / \sqrt{(-336)^2 + (466.7)^2} = (505 \hat{j} - 66.7 \hat{k})$

كرة رصية

135



مسبب الأوزون
الذو في المحاور
موازى
لـ $x/y/z$

⚠️ **حاوره دائماً لاجاء المحاور الذي يحركها أكثر**
 عند هذه النقاط لتسهل الكه وبعين
 $M_{AB} \neq M_{BA}$ عند هذه النقاط
 ويستحسن ان تكونه النقاط مسبوقة القوى
 وابقاء النقاط مطلوبة القوى

$$\sum M_{AE} = 0$$

$$= F_1 \times r_{AC} \cdot \lambda + T_x \cdot r_2 \cdot \lambda$$

$$F_1 = 60 \hat{j}$$

$$r_{AC} = 9\hat{i} + 10\hat{k}$$

$$r_2 = r_{ED} = 16\hat{i}$$

$$T = -10\hat{i} + 11\hat{j} - 8\hat{k} \quad |T| = 21$$

$$-0.76|T|\hat{i} + 0.524|T|\hat{j} - 0.38|T|\hat{k}$$

$$\lambda_{EA} = \frac{\vec{EA}}{|EA|} = \frac{7\hat{i}}{25} + \frac{24\hat{k}}{25}$$

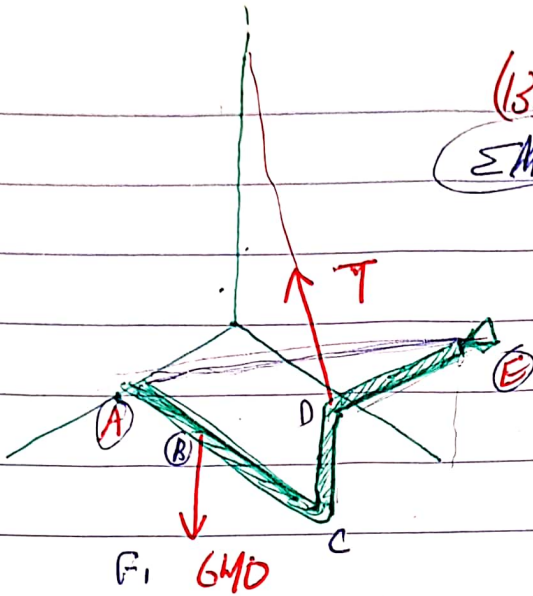
$$\begin{vmatrix} \frac{7}{25} & 0 & -\frac{24}{25} \\ 16 & 0 & 0 \\ -0.76T & 0.524T & -0.38T \end{vmatrix} + \begin{vmatrix} \frac{7}{25} & 0 & -\frac{24}{25} \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix} = 0$$

$$= -0.524 \left(\frac{24}{25} \times 16 \right) T + 60 \left(\frac{7}{25} \times 10 + \frac{9 \times 24}{25} \right)$$

$$\rightarrow T \approx 85$$

138

(135) ~~مسألة~~ مسألة



$\Sigma M_{AE} = 0$

$(\vec{r}_{A \rightarrow B} \times \vec{F}_i) \cdot \lambda_{AE} + ((\vec{r}_{E \rightarrow D}) \times \vec{T}) \cdot \lambda_{AE} = 0$

$\vec{F}_i = -640 \hat{j}$

$\vec{T} = T (-480 \hat{i} + 330 \hat{j} - 240 \hat{k})$
 630

$\vec{T} = T (-0.76 \hat{i} + 0.524 \hat{j} - 0.381 \hat{k})$

$\vec{r}_{A \rightarrow B} = +0.2 \hat{i}$
 $\vec{r}_{E \rightarrow D} = +0.24 \hat{i}$

$\lambda_{AE} = \frac{\vec{AE}}{|\vec{AE}|}$
 $= \frac{480 \hat{i} + 160 \hat{j} - 240 \hat{k}}{560}$

560	480	160	-240	T	480	160	-240	= 0
	-0.2	0	0		-0	+0	-0.24	
	0	-640	0		(630 * 560)	-480	330	

$\frac{0.2 * -153600}{560} + \frac{0.24 * 235200 * T}{630 * 560}$

$T = 342.86 \text{ N}$