CH 5

for earth gravity

center of mass = center of gravity

centroid = center of Area

$$\bar{x} = \frac{\int x\,dm}{m} \quad / \quad \bar{y} = \frac{\int y\,dm}{m}$$

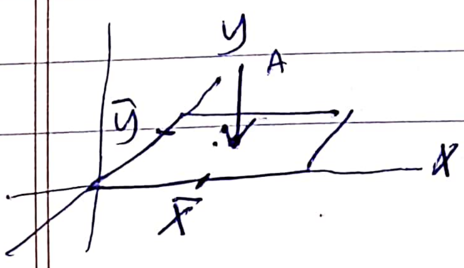
$$\boxed{\bar{X} = \frac{\int X\,dA}{A} \quad / \quad \bar{y} = \frac{\int y\,dA}{A}}$$ For 2d

$A\bar{x} = Q_y$ = first moment of Area with respect to y

$$\boxed{A\bar{x} = Q_y = \int x\,dA}$$

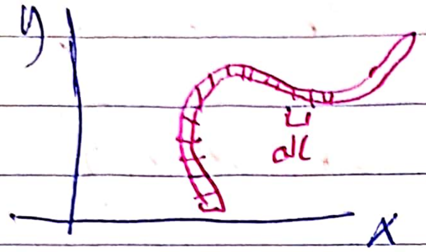
تاك بتوخد المومنت
حول y اذا "تضرب" القوه بـ $\bar{x}$ لانه القوة
في المركز لكن بدل القوه ناخد المساحه
لانها Moment of Area مش moment of force

$$\boxed{Q_x = A\bar{y} = \int y\,dA}$$ = first moment of Area with respect to X



$$Q_y = A\bar{x} = \int x\,dA$$
$$Q_x = A\bar{y} = \int y\,dA$$

For thin elements



A = مساحة المقطع

L = كل الطول

dL = طول المقطع

ρ = الكثافة

$$M_x = AL\rho g\,\bar{y} = \int y * A * \rho * g * dL$$

$$L\bar{y} = \int y\,dL$$

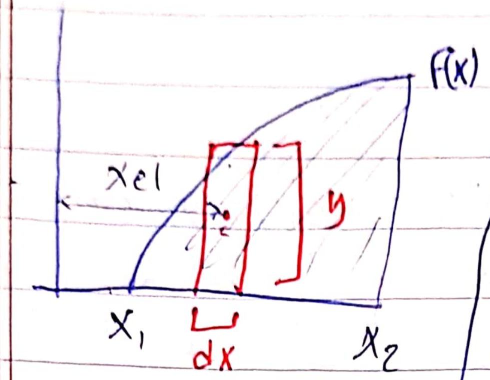
$$\boxed{\bar{y} = \frac{\int y\,dL}{L}}$$

$$\boxed{\bar{x} = \frac{\int x\,dL}{L}}$$

$\bar{x}L$ = first moment of length with respect to (y) Axis $= \int x\,dL$

the same for X-axis

Centroid by integration



$$\bar{X}A = \int X_{el}\, dA$$

$$\bar{X}A = \int X * y\, dx$$

$$\bar{X}A = \int_{X_1}^{X_2} X * f(x)\, dx$$

$$\bar{X} = \frac{\int_{X_1}^{X_2} X\, f(x)\, dx}{\int dA}$$

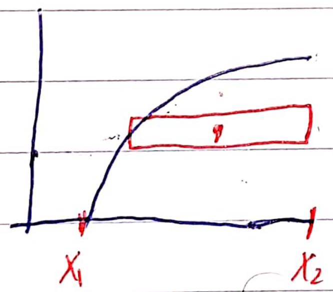
$$\bar{y}A = \int y_{el}\, dA$$

$$\bar{y}A = \int \frac{y}{2} * y * dx$$

$$\bar{y}\int dA = \int \frac{(f(x))^2}{2}\, dx$$

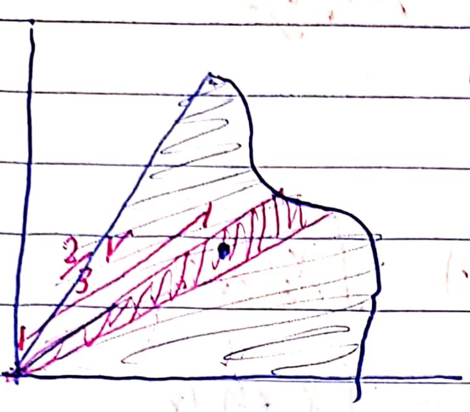
مهم جداً انك تعرف عند اخذ شريحة عمودية على المحور فإنه السنترويد في منتصف المسافة على بعد $y/2$ مثلاً

ممكن اخذ شريحة عمودية او موازية للمحور !! مش مشكلة



$$\bar{X}A = \int_{y_1}^{y_2} \frac{(X_2 + X)}{2} * (X_2 - X)\, dy$$

$$\bar{y}A = \int y\, (X_2 - X)\, dy$$



Polar coordinates

$r = \boxed{}\ \theta$

$\boxed{x_{el} = \frac{2}{3} r * \cos\theta}$

$\boxed{y_{el} = \frac{2}{3} r * \sin\theta}$

$\boxed{dA = \frac{1}{2} r^2 \sin d\theta}$

but $\sin d\theta = d\theta$

$\bar{X}A = \int x_{el}\, dA$

$= \int \frac{2}{3} r \cos\theta * \frac{1}{2} r^2 * . d\theta$

$$\boxed{\bar{X} = \frac{\int \frac{1}{3} r^3 \cos\theta\, d\theta}{A}}$$

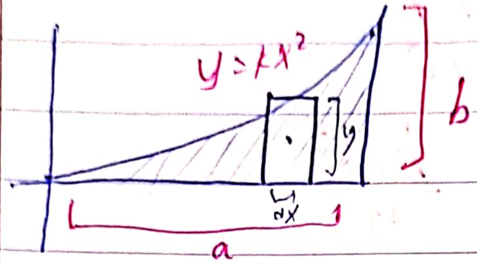
فهم جيداً

$\bar{y}A = \int y_{el}\, dA$

$= \int \frac{2}{3} r \sin\theta * \frac{1}{2} r^2 d\theta$

$$\boxed{\bar{y} = \frac{\int \frac{1}{3} r^3 \sin\theta\, d\theta}{A}}$$

sample 1



Find: $\bar{x}$, $\bar{y}$ of the centroid

Find: Q_x / Q_y

1) $k = ?$

at $x = a \rightarrow y = b$

$b = a^2 k$

$$\boxed{\frac{b}{a^2} = k}$$

2) $A = \int dA = \int y\,dx = \int_0^a kx^2\,dx = \frac{kx^3}{3} = \frac{ka^2 * a}{3}$

$= \frac{ab}{3}$

3) $\bar{x} = \frac{\int x\,dA}{A} = \frac{\int x * kx^2\,dx}{A} = \frac{\int kx^3\,dx}{A}$

$= \frac{kx^4}{4}\Big|_0^a = \frac{ka^2a^2}{4A} = \frac{\cancel{b}\,a^2 * 3}{4\,\cancel{a}\cancel{b}}$

$$\boxed{\bar{x} = \frac{3}{4}a}$$

4) $\bar{y} = \frac{\int y\,dA}{A} = \frac{3\int \frac{kx^2}{2}\,kx^2\,dx}{ab}\ \frac{\int \frac{k^2x^4}{2}\,dx}{ab} * 3$

$= \frac{k^2x^5 * 3}{10(ab)} = \frac{\cancel{ka^2} * \cancel{ka^2} * \cancel{a}}{10\,\cancel{a}\cancel{b}} * 3$

$= \frac{b * 3}{10} = \boxed{\frac{3b}{10} = \bar{y}}$

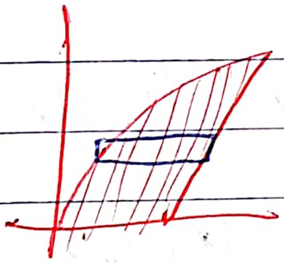
$Q_x = A\bar{y} = \frac{Ab^2}{10}$

$Q_y = A\bar{x} = \frac{ba^2}{4}$

Horizontal

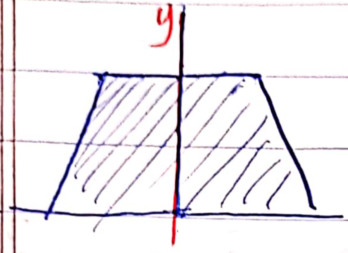
or

Vertical



in vertical it's hard because we'll need 2 integrals

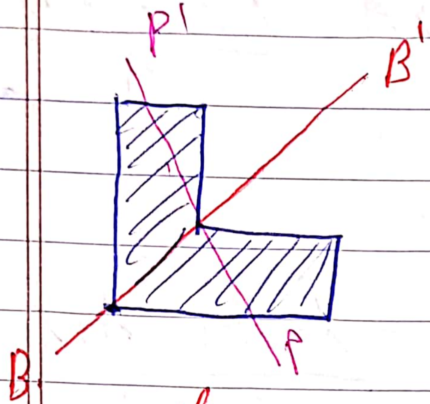
* symetry



$Q_y = 0$

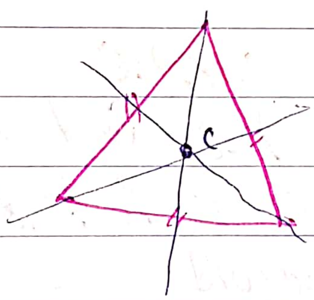
because the centroid is on y axis because of symetry
so the distance between centroid and axis $= 0$

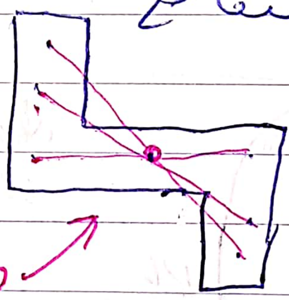
$\bar{X}A = 0 * A = 0$



the same for $Q_{BB'} = \overline{PP'}A = 0$

اذا وجدنا * خطي تماثل متقاطعانه
فانه ال centroid هو مكانه
التقاطع

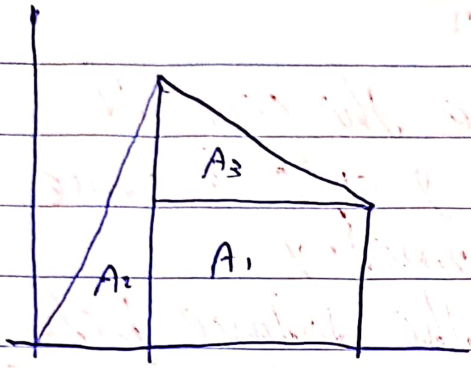




متماثل حول
نقطة

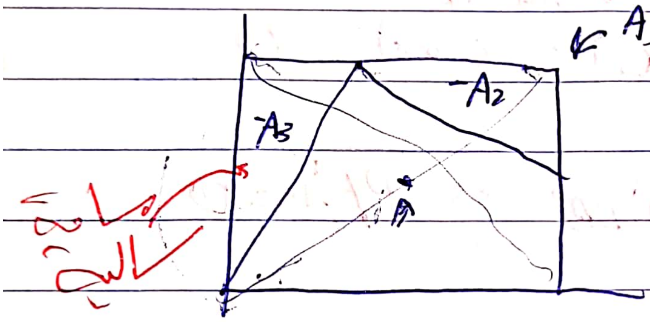
هذه النقطة هي ال centroid

Composite plates and Areas

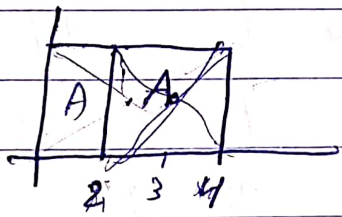


$$\bar{X} \cancel{A} = \frac{\bar{X}_1 A_1 + A_2 \bar{X}_2 + \bar{X}_3 A_3}{A}$$

Or



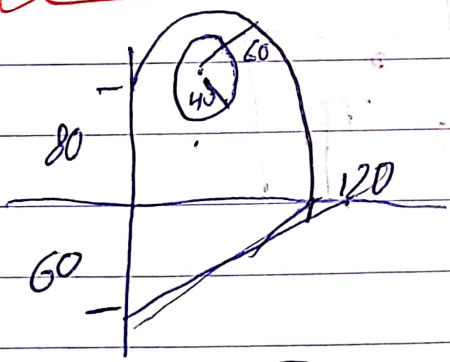
$$= \frac{\bar{X}_{sq} A_{sq} - A_3 \bar{X}_3 - A_2 \bar{X}_2}{A - A_3 - A_2}$$

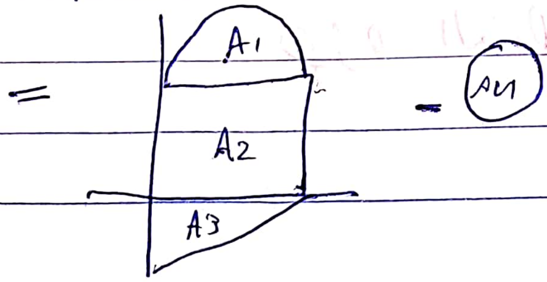


$$\frac{2A * 2 - A * 1}{A} =$$

sample

determine the first moment with respect to X & Y and centroid





$$A = \overset{A_3}{3600} + \overset{A_2}{9600} + \overset{A_1}{5655} - 5026.5$$

$$= 13828$$

$$\bar{X}A = 180 * 3600 + 9600 * 60 + 60 * 5655 - 5026 * 60$$

$$\boxed{\bar{X}A = Q_y = 757740}$$

$$Q_x = \bar{X}y = 3600 * -20 + 9600 * 40 + 5655 * 105.5 - 80 * 5026$$

$$\boxed{Q_x = 506522.5}$$

$$\boxed{\bar{X} = 54.8}$$

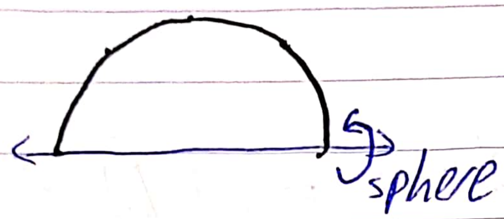
$$\boxed{\bar{y} = 36.6}$$

تأكد من
منطقية
الحل دوما

Areas and Volumes of Revolution

in general

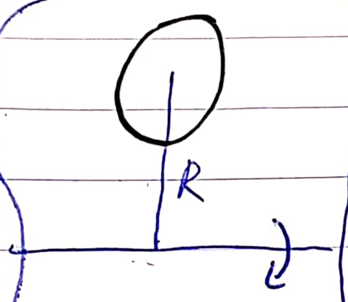
$$\boxed{A = 2\pi \bar{y} L}$$



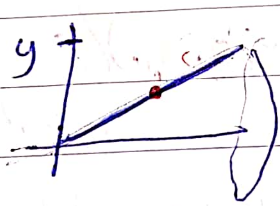
$$\bar{y} = \frac{2r}{\pi} =$$

$$2\pi * \frac{2r}{\pi} * \pi r$$

$$= 4\pi r^2$$

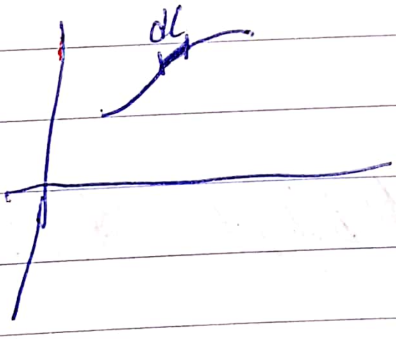


$$= R * 2\pi L$$



$$2\pi * \frac{y}{2} * L$$

more general



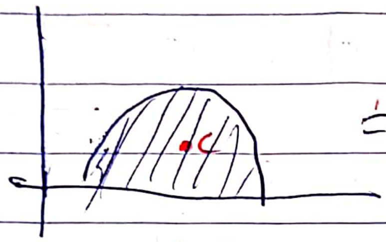
$$\int 2\pi y \, dL$$

$$= 2\pi \int y \, dL$$ ← first moment

$$= 2\pi * \bar{y} * L$$

CamScanner المسوحة ضوئيا بـ

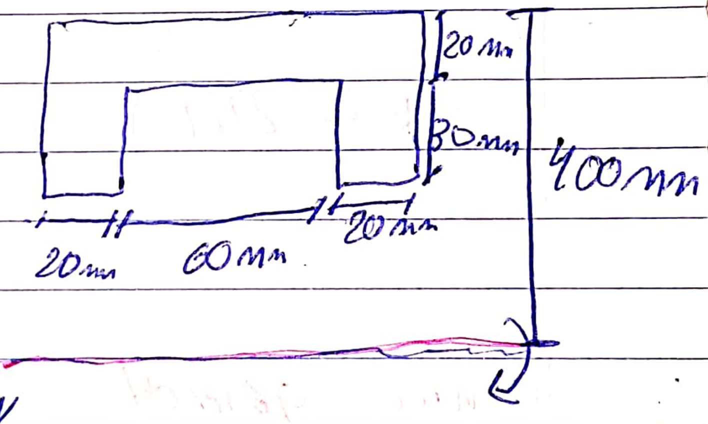
Volumes $2\pi \bar{y} A$



$\div$ sphere $= 2\pi \bar{y} A$

كأنّ المساحة
تتركز في
centroyd

example



the out side ~~of~~ diameter of pulley is 0.8 m and the crossection of its rim is shown
Knowing that the pulley is made of steel and that the density of steel is $\rho = 7.85 * 10^3$ kg/m^3
determine the mass and weight of the rim

first find volum $= A * 2\pi \bar{y}$

$A = 1200 + 2000 = 3200$ mm^2 $= 3.2 * 10^{-3}$

$\bar{y} = \dfrac{A_1 y_1 + A_2 y_2 + A_3 y_3}{3.2 * 10^3} = 1200 * 365 + 2000 * 390$

$= 380.625$ mm $= 0.380625$ m

$V = 2\pi \bar{y} A = 7.653 \times 10^{-3}\ m^3$

$= 60\ kg$ Mass

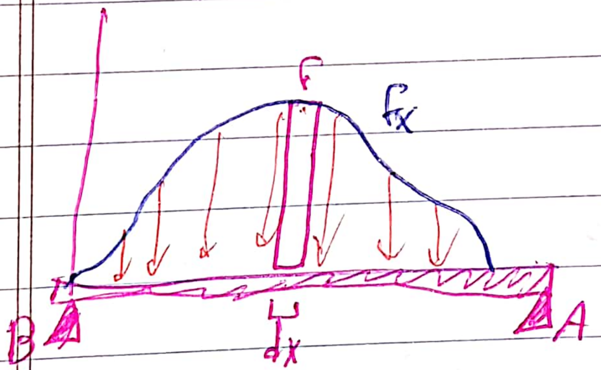
$= 588.7$ ~~N~~ weight

distribuded force

$\boxed{N/m \times length = force}$

$\boxed{or \int load \times dx}$

to find moment or reactions we need to find a single force at the centroid of the shape of distributed load

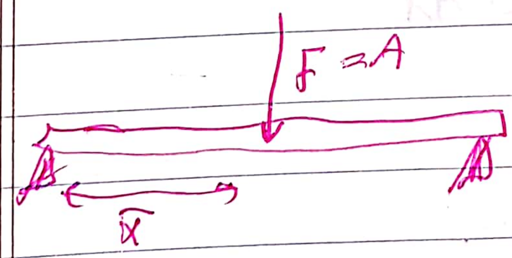


$load = \int dw$

$dw = F\,dx$

$load = \int F\,dx = Area$

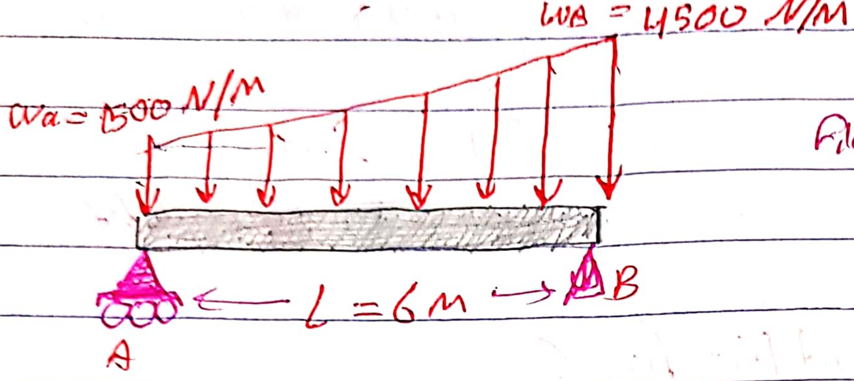
$= \int dA = A$



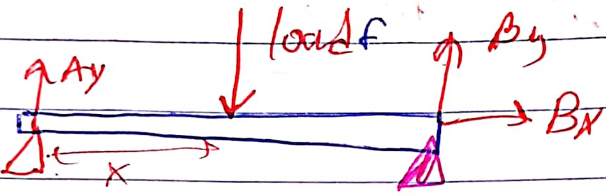
$A \times \bar{X} = \int x\,dA$

= moment of load at B

Sample Poblem



Find reactions



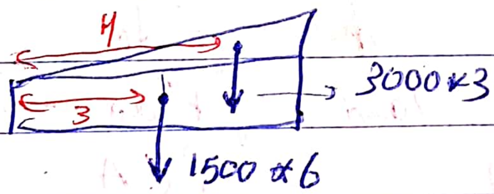
$\Sigma F_x = 0$

$B_x = 0$ ✓

$M_A = F\bar{X} * B_y L$

$\oint A\bar{X} - B_x * 6$

Finding the $\bar{x}$



$6*3*1500 + 4*3000*3 = A\bar{X}$

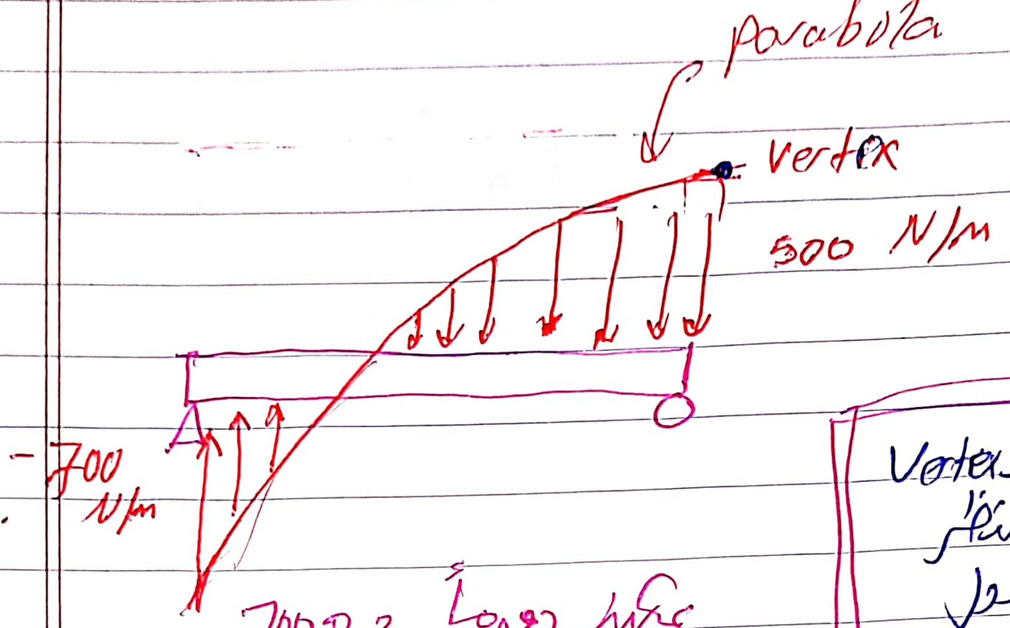
$63000 - 6B_x = 0$

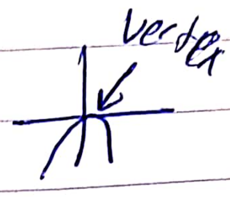
$\boxed{B_x = 10500 \uparrow N}$

$A_y + B_y - 18000 = 0$

$\boxed{A_y = 7500 \uparrow N}$

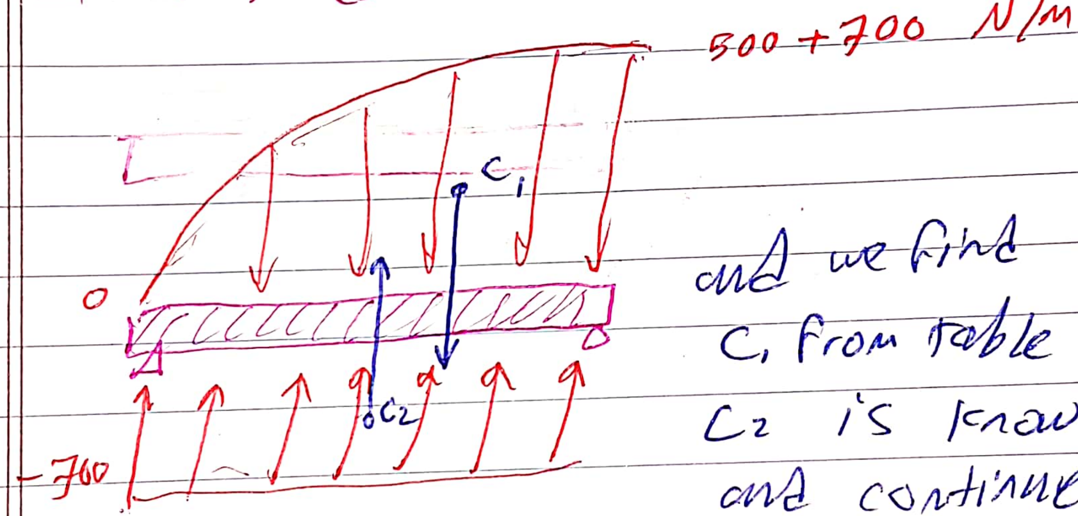
what if





انتبه لل Vertex
عند التكامل
للحدود

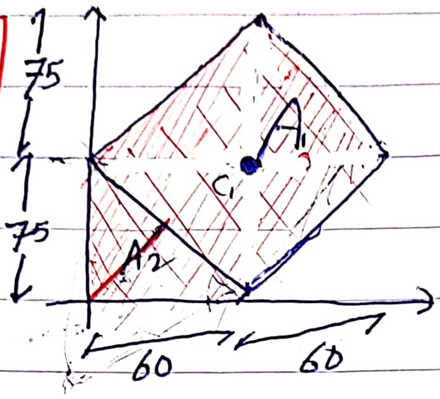
عملنا حدوداً ع 700
وطلع 700 صفراً



and we find c_1 from table
c_2 is known
and continue

Home work 4 ch5

Prob 4



=

A_1 is symetric around two perpendicular axes

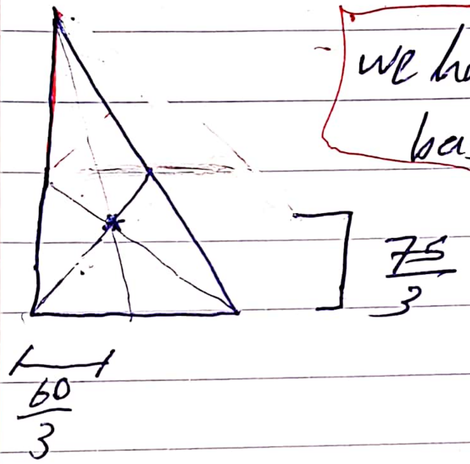
Axis 1: $y = 75$ mm

Axis 2: $x = 60$

so $C_1 = (60, 75)$

A_2 is a triangle.

$A_1 = 2*60*75 = 9*10^3$ mm



we have 2 perpendicular bases for tringle A_2 so we can apply the rule $\bar{y} = \frac{h}{3}$ once in x direction and another in y direction

$C_2 = (20, 25)$ / $A_2 = 2250$

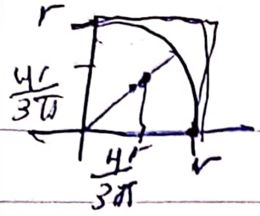
$$\bar{X} = \frac{X_1A_1 + X_2A_2}{A} = \frac{60*9*10^3 + 20*2250}{9*10^3 + 2250}$$

$$\bar{X} = 52$$

$$\bar{Y} = \frac{Y_1A_1 + Y_2A_2}{\Sigma A} = \frac{75*9*10^3 + 25*2250}{9*10^3 + 2250}$$

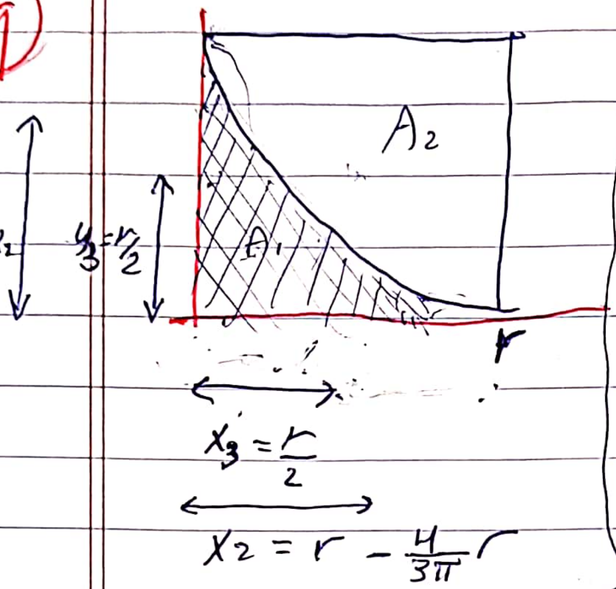
$$\bar{Y} = 65$$

$$C = (52, 65)$$



$A_3 = A_2 + A_1$ = square with side r

1



$$X_3 = \frac{A_1 X_1 + A_2 X_2}{A_1 + A_2}$$

$$(A_1 + A_2) X_3 = A_1 X_1 + A_2 X_2$$

$$A_1 X_1 = (A_1 + A_2) X_3 - A_2 X_2$$

$$\therefore X_1 = \frac{A_3 X_3 - A_2 X_2}{A_3 - A_2}$$

$$\bar{X}_1 = \frac{r^2 * r/2 - \left(r - \left(\frac{4}{3\pi}\right) r\right) \frac{\pi r^2}{4}}{r^2 - \frac{r^2 \pi}{4}}$$

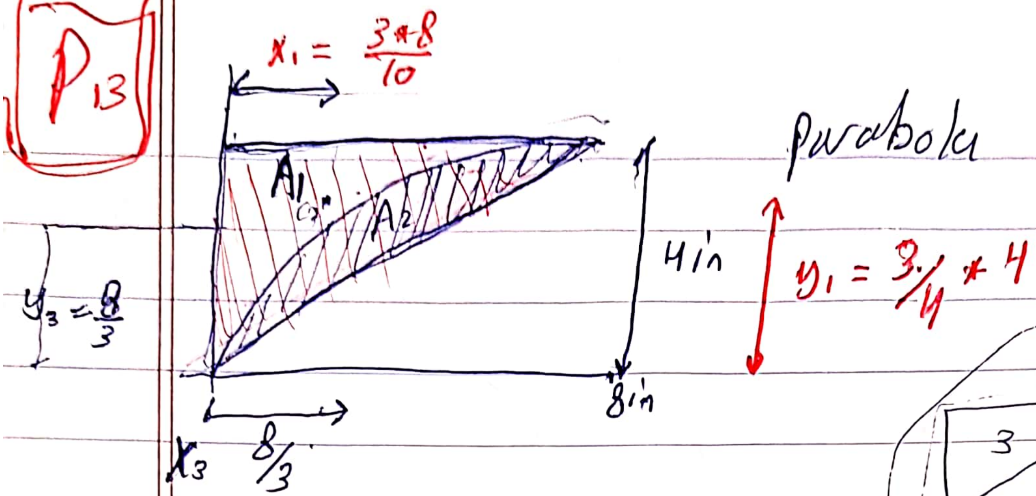
$$\bar{X}_1 = 16.753 \text{ mm}$$

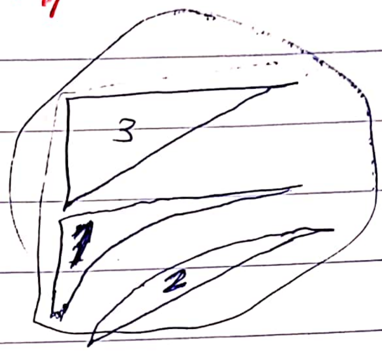
the shape is symetric Around Axis $(y = x)$ then the point is on that axis then

$$\bar{Y} = \bar{X} = 16.753 \text{ mm}$$

$$C = (16.75, 16.75)$$

P_{13}





$$A_2 = A_3 - A_1$$

$$= \frac{4*8}{2} - \frac{4*8}{3} = \frac{4*8}{6} = 5.333\ in^2$$

$$X_2 = \frac{X_3 A_3 - X_1 A_1}{A_3 - A_1} = \frac{8/3 * 16 - 3*0.8 * 4*8/3}{\frac{4*8}{6}}$$

$$X_2 = 3.2$$

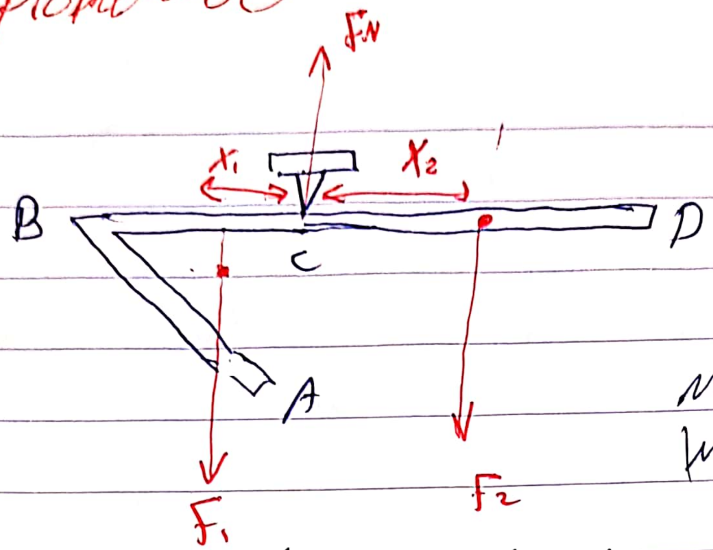
$$y_2 = \frac{y_3 A_3 - y_1 A_1}{A_3 - A_1} = \frac{\frac{8}{3} * 16 - 3 * \frac{4*8}{3}}{\frac{4*8}{6}}$$

$$y_2 = 2$$

$$C = (3.2, 2)$$

Problem 30

P30



$M_1 + M_2 = 0$

$|M_1| - |M_2| = 0$

~~when~~ $|M_1| = |M_2|$

$F_1 x_1 = F_2 x_2$

$m_1 g_1 x_1 = m_2 g_2 x_2$

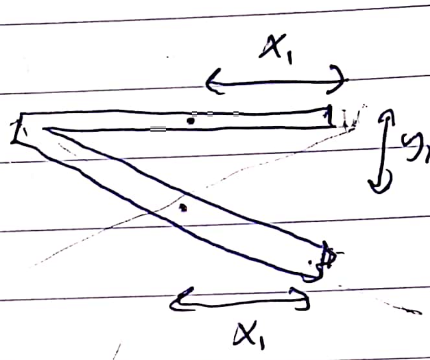
$\lambda L_1 x_1 = \lambda L_2 x_2$

$\boxed{L_1 x_1 = L_2 x_2}$

Portion ABC

Both BC And BA

have the same $\bar{x} = x_1$



$L(AB) + L(BC) = 80 + \sqrt{80^2 + 60^2}$

$\boxed{L_1 = 180}$

$\boxed{x_1 = 40}$ for the symitry

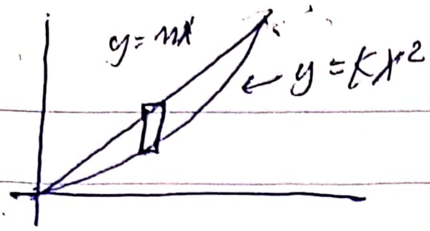
$\boxed{X_2 = \frac{L_2}{2}}$ because of symetry

$180 * 40 = \frac{L_2^2}{2}$

$\sqrt{180 * 40 * 2} = L_2$

$\boxed{L_2 = 120 \text{ mm}}$

P 35



$$\bar{x} = \frac{\int x\,dA}{A}$$

$$A = \int dA = \int_0^a (mx - kx^2)\,dx = \frac{mx^2}{2} - \frac{kx^3}{3}$$

$$= \frac{h}{2a}x^2\Big|_0^a - \frac{kx^3}{3}\Big|_0^a = \boxed{\frac{ma^2}{2} - \frac{ka^3}{3}}$$

$$\int x\,dA = \int_0^a x(mx - kx^2)\,dx$$

$$= \int_0^a (mx^2 - kx^3)\,dx = \left(\frac{mx^3}{3} - \frac{kx^4}{4}\right)\Big|_0^a$$

$$= \boxed{\frac{ma^3}{3} - \frac{ka^4}{4}}$$

$$\bar{x} = \frac{4ma^3 - 3ka^4}{2\,\cancel{12}} \div \frac{3ma^2 - 2ka^3}{\cancel{6}}$$

$$\boxed{\bar{x} = \frac{\cancel{a^3}(4m - 3ka)}{2\,\cancel{a^2}(3m - 2ka)}}$$

at Point (b,a) $y_1 = y_2 \rightarrow (mx = kx^2)\big|_a$

$m = kb$

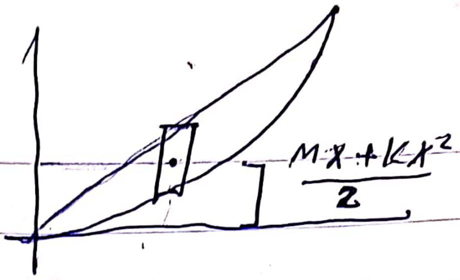
$\boxed{k = \frac{m}{a}}$

$\boxed{m = \frac{h}{a}} \rightarrow \boxed{k = \frac{h}{a^2}}$

$$\frac{a}{2} * \frac{4*\frac{h}{a} - 3*\frac{h}{a^2}a}{3\frac{h}{a} - 2*\frac{h}{a^2}a} = \frac{a}{2}\,\cancel{\frac{h/a}{h/a}}$$

$$\boxed{\bar{x} = \frac{a}{2}}$$

$$\bar{y} = \frac{\int y \, dA}{A}$$



$$\int y \, da = \int_0^a \frac{mx+kx^2}{2} * (mx-kx^2)\,dx$$

$$= \int \frac{(mx)^2 - k^2x^4}{2}\,dx$$

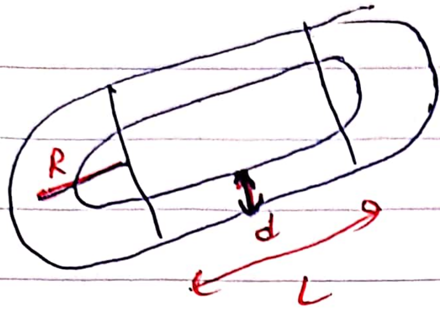
$$= \frac{1}{2}\int m^2x^2 - k^2x^4\,dx = \left(\left(\frac{m^2x^3}{3} - \frac{k^2x^5}{5}\right)\frac{1}{2}\right)\Big|_0^a$$

$$= \frac{\frac{m^2a^3}{\cancel{6}} - \frac{k^2a^5}{10}}{\frac{ma^2}{2} - \frac{ka^3}{3}} = \frac{\frac{h^2a}{6} - \frac{h^2a}{10}}{\frac{ha}{2} - \frac{ha}{3}}$$

$$= \frac{\frac{4\,h^{\cancel{2}}\cancel{a}}{\cancel{6a}\,10}}{\frac{\cancel{ha}}{\cancel{6}}} = \boxed{\frac{4h}{10} = \bar{y}}$$

$$C = \left(\frac{a}{2}, \frac{4h}{10}\right)$$

P 55



V = 2 * Volume of cylender + 2 * Volume of ends

$= (2A * L) + 2(A * \pi R)$

$= 2A(L + \pi R)$

$= 2 * 3^2 * \pi (30 + \pi * 10)$

$\boxed{= 3472.9 \quad mm^3}$

$\approx 3.473 \times 10^{-6} \quad m^3$

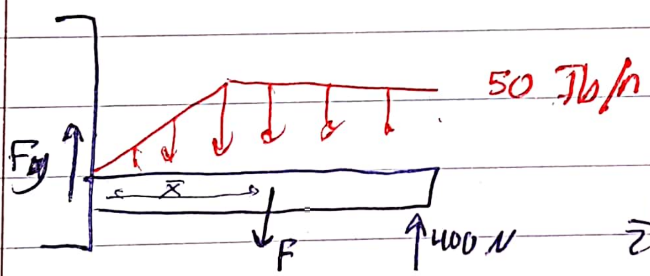
Surface area = 2 * surface area of cylender + 2 * Surface of ends

$= 2(2\pi r L) + 2(2\pi r) * \pi R$

$= 4\pi r(L + \pi R)$

$= 2315.3 \quad mm^2$

P 69



$F =$

$\frac{1}{2} * 12 * 50 + 50 * 20$

$= \boxed{1300 \text{ Ib}}$

$F \cdot \bar{X} = F_1 X_1 + F_2 X_2$

$$\bar{X} = \frac{300 * \frac{2}{3} * 12 + 1000 * 22}{1300} = 18.76 \text{ in}$$

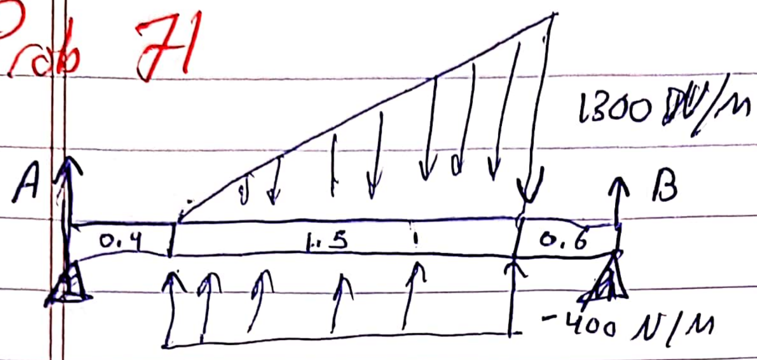
$\boxed{F_{Ay} = 1300 - 400 = 900 \text{ Ib} \uparrow}$ $\boxed{A_x = 0}$

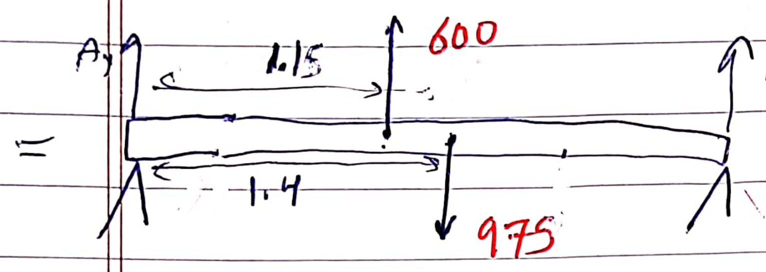
$M_A = 22000 + 2400 - 400 * 38$

$\boxed{M_A = +9200 \text{ Ib} \cdot \text{in} \quad \text{counter clockwise}}$

CamScanner الممسوحة ضوئيا بـ

Prob 71





$$\boxed{A_y + B_y = +375 \uparrow}$$

$$M_A = 0 = 600 * 1.15 - 975 * 1.4 + B_y * 2.5$$

$$0 = \quad - 675 + B_y * 2.5$$

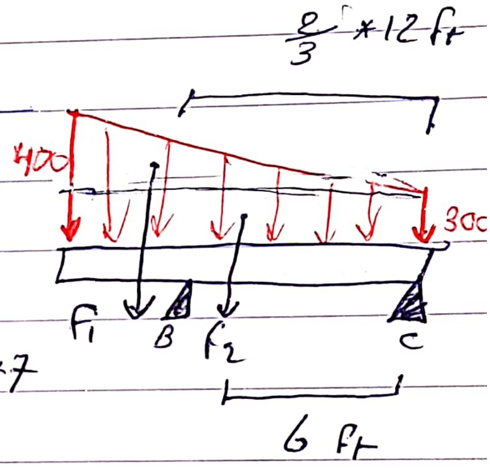
$$B_y = +270\ N \uparrow$$

$$A_y = +105\ N \uparrow$$

Prob 76

$$F_1 = 12 * 100 * \tfrac{1}{2} = 600\ lb$$

$$F_2 = 300 * 12 = 3600\ lb$$



$$M_c = F_1 * \tfrac{2}{3} * 12 + 6 * F_2 - B_y * 7$$

$$7B_y = 4800 + 21600$$

$$B_y = +3771.4\ lb \uparrow$$

$$C_y = ? \rightarrow F_{net} = 0 \quad \vec{F_1} + \vec{F_2} + \vec{B} + \vec{C} = 0$$

$$-4200 + 3771 + C = 0$$

$$\boxed{C = -428.6\ lb \downarrow}$$