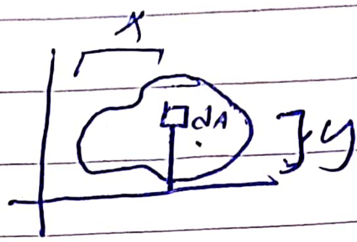
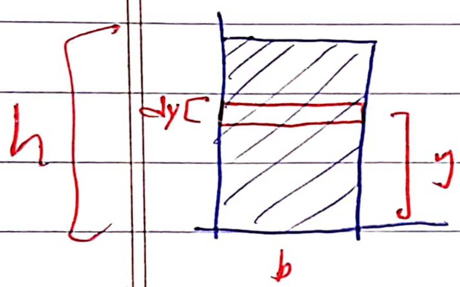
CHAPTER 9

$$I_x = \int y^2 \, dA \qquad m^4$$



$$I_y = \int x^2 \, dA \qquad m^4$$

عند حساب التكامل
نأخذ شريحة موازية للمحور

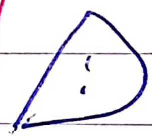


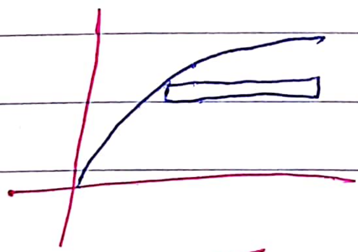
find I_x

$$\int_0^h y^2 \, b \, dy$$

$$= \frac{y^3 b}{3} \Big|_0^h = \frac{bh^3}{3}$$

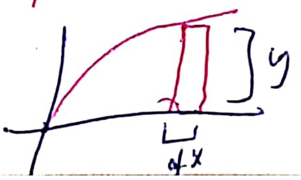
يمكن الاستفادة من $\frac{bh^3}{3}$
لإيجاد I_x حيث نأخذ مستطيل
عمودي على المحور
شرط انه يلامس المحور





$$I_x = \int y \, dA$$

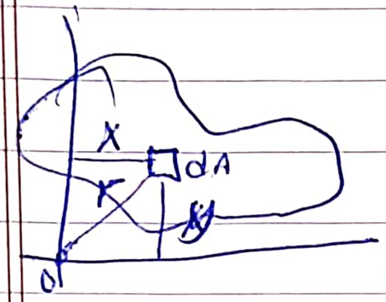
or



$$dI_x = \frac{dx \, y^3}{3} \Rightarrow I_x = \int \frac{y^3}{3} dx$$

Polar moment of inertia

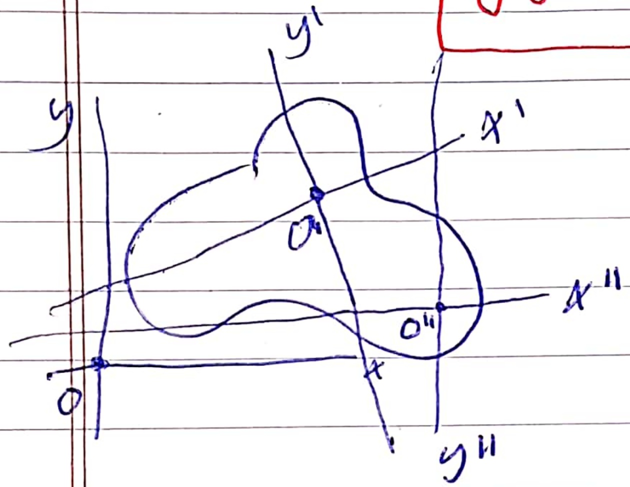
$$\boxed{I_z = J_o = \int r^2\,dA}$$



$$= \int x^2 + y^2\,dA$$

$$= \int x^2\,dA + \int y^2\,dA$$

$$\boxed{J_o = I_x + I_y}$$



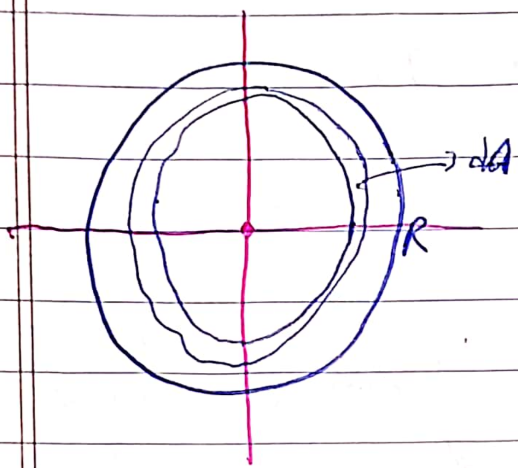
for any 2 perpendicular axes

$J_o = I_{axis1} + I_{axis2}$

I_x & I_y are against bending moment

I_z / J_o = against twisting

Find J_o / I_x / I_y for the circle



$$J_o = \int_0^R r^2 \, dA$$

$$= \int_0^R r^2 \, 2\pi R \, dr$$

$$= \int_0^R r^3 \, 2\pi \, dr$$

$$= \frac{R^4}{4} \times 2\pi$$

~~J_o~~

$$\boxed{J_o = \frac{R^4 \pi}{2}}$$

but $J_o = I_x + I_y$

and area is symetric around x and y and distributed samely

then $I_x = I_y$

$$2I_x = \frac{R^4 \pi}{2}$$

$$\boxed{I_x = I_y = \frac{R^4}{4} \pi}$$

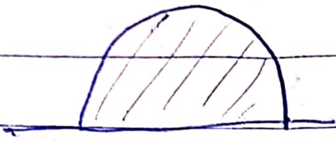
تنساش ان I_x / I_y

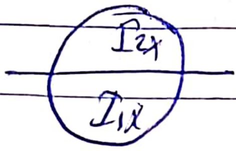
كميات قياسية

يعني تجمع تجمع عادي

I_x circle $= \frac{R^4 \pi}{4}$

find I_x

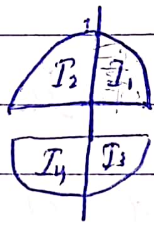




$I_1 = I_2$ because they have the same distribution

$\boxed{I_x = \frac{R^4 \pi}{8}}$

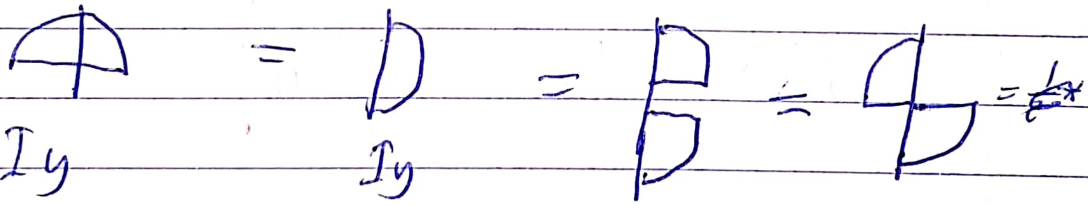
$I_y = ?$



$I_1 \neq I_2 \neq I_3 \neq I_4 = 2$

the same distribution

$I_1 = \frac{1}{2} * (I_1 + I_3)$ notice the distribution

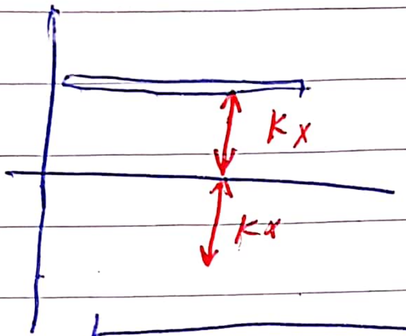


$= \frac{R^4 \pi}{8}$

Distribution [illegible]

بالنسبة لـ y

Radius of Gyration of an Area



كأنني اضغط كل المساحة
في شريحة تبعد مسافه
K_x عن المحور

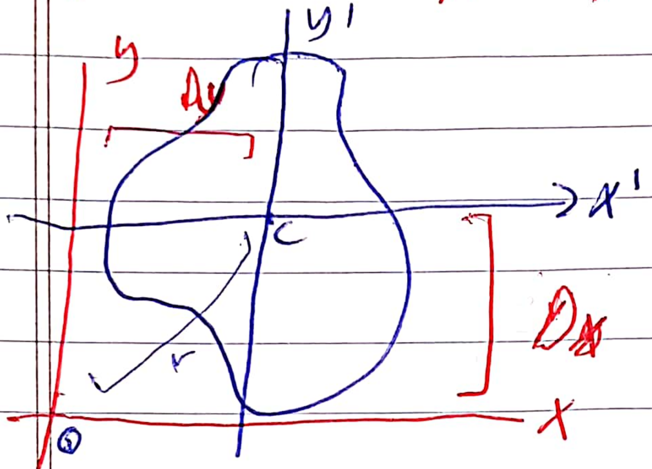
$$\boxed{K_x = \sqrt{\frac{I_x}{A}}} \qquad \boxed{K_y = \sqrt{\frac{I_y}{A}}}$$

$$\boxed{K_o = \sqrt{\frac{J_o}{A}}} = \sqrt{\frac{I_x + I_y}{A}} = \boxed{\sqrt{K_x^2 + K_y^2}}$$

$$I_x = K_x^2 A$$
$$I_x = K_y^2 A$$

Parallel axis Theorem

The axis x' pass through centroid



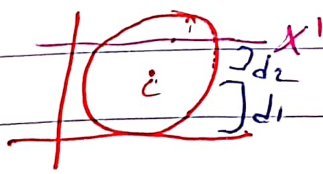
- $I_x = I_{x'} + D_x^2 A$
- $I_y = I_{y'} + D_y^2 A$
- $J_O = J_c + D_r^2 A$

المومنت حول المحور = المومنت الـ 2^{nd} الذي يمر بالسنترويد + $A \times$ (بعد)^2 بين المحورين

- $K_x^2 = K_{x'}^2 + D_x^2$
- $K_y^2 = K_{y'}^2 + D_y^2$
- $K_O^2 = K_c^2 + D_r^2$

easy to know, if you know the ~~up~~ rules up

إذا لم يكن الـ I_x المعطى عند السنترويد



$I_x = L$

$I_{x'} = ?$

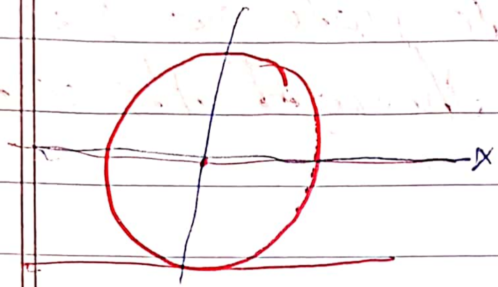
$$I_x = I_c + d_1^2 A$$

$$I_c = I_x - d_1^2 A \rightarrow I_{x'} = I_c + d_2^2 A$$

$$I_{x'} = I_x - d_1^2 A + d_2^2 A$$

تطلع مرتين

Example : find I_T of the circle
$I_{Tangential}$



$$I_x = \frac{\pi R^4}{4}$$

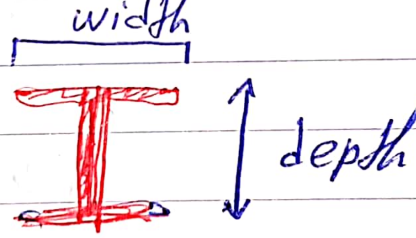
$$I_T = \frac{\pi R^4}{4} + AR^2$$

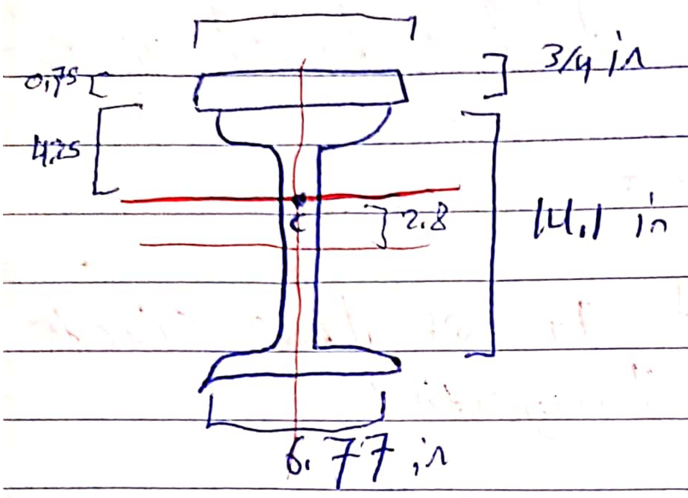
$$= \frac{\pi R^4}{4} + \pi R^4$$

$$= \pi R^4 * \frac{5}{4}$$

beams have standard shapes

W shape
S shape
C shape





W 14*38

Find the moment of inertia and radius of gyration with to any axis that is parallel to plate and pass through the centroid

First find $\bar{y}$

$$= \frac{y_w * A_w + y_p * A_p}{\Sigma A} = \frac{0 + \frac{27}{4} * (7.05 + 3/8)}{\frac{27}{4} + 11.2}$$

$$\bar{y} = 2.8 \text{ in}$$

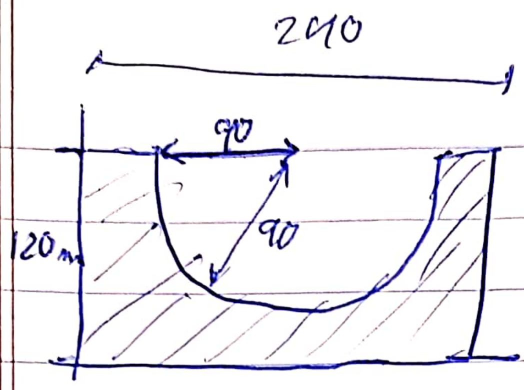
$$I_{xw} = I_{w'x} + 2.8^2 * 11.2 = \cancel{472}.8 \text{ in}^4$$

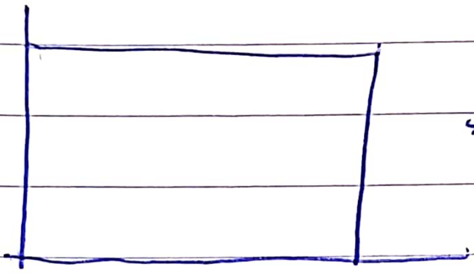
$$I_{xp} = \frac{5^3 * 9}{3} - \frac{4.25^3 * 9}{3}$$

$$= 3(5^3 - 4.25^3) = 144.7$$

$$\boxed{I_{x\,total} = \cancel{342.5} \quad 617.5}$$

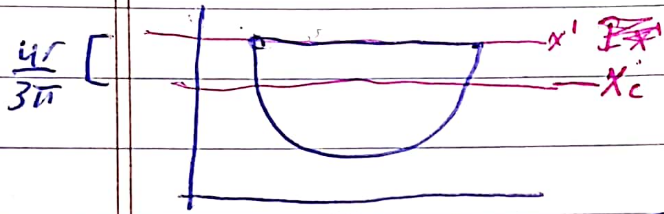
$$K_x = \sqrt{\frac{I_x}{A}} = 5.86$$





$$I_x = \frac{1}{3} * 240 * 120^3$$

$$= 13824 * 10^4$$



$$I_{x'} = \frac{1}{8}\pi r^4$$

$$I_{x_c} = \frac{1}{8}\pi r^4 - \left(\frac{4r}{3\pi}\right)^2 * \frac{\pi r^2}{2}$$

$$= \left(\frac{1}{8}\pi - \frac{16}{9\pi^2} * \frac{\pi}{2}\right) r^4$$

$$= \left(\frac{1}{8}\pi - \frac{8}{9\pi}\right) r^4$$

$$I_x = I_{x_c} + \left(120 - \frac{4r}{3\pi}\right)^2 \frac{\pi r^2}{2}$$

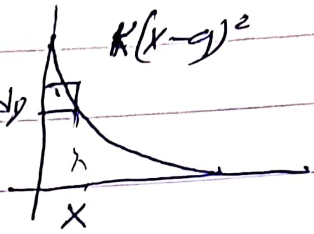
$$= 92.3 * 10^6$$

Home work 5

b5 $I_x = \int y^2 \, dA$

$dA = dy * x$

$x = ?$



$a^2 k = b \rightarrow k = b/a^2$

$y = k(x-a)^2$

$\sqrt{\frac{y}{k}} = |x-a|$

$\frac{a\sqrt{y}}{\sqrt{b}} = a - x$

$x = a - \frac{a\sqrt{y}}{\sqrt{b}}$

$x = a\left(1 - \frac{\sqrt{y}}{\sqrt{b}}\right)$

$I_x = \int y^2 x \, dy$

$= \int_0^b y^2 * a\left(1 - \frac{\sqrt{y}}{\sqrt{b}}\right) dy$

$= a\int_0^b y^2 - \frac{y^{\frac{5}{2}}}{\sqrt{b}} \, dy$

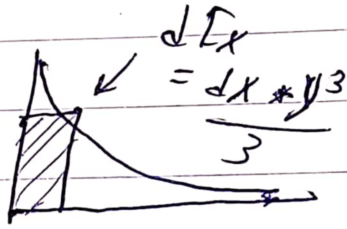
$= a \left. \frac{y^3}{3} - \frac{2 y^{\frac{7}{2}}}{7\sqrt{b}} \right|_0^b$

$= a\left(\frac{b^3}{3} - \frac{2}{7}\frac{b^3\sqrt{b}}{\sqrt{b}}\right)$

$a\left(\frac{b^3}{3} - \frac{2}{7} b^3\right)$

$= a\left(\frac{b^3}{21}\right) = \frac{ab^3}{21}$

Way 2



$\int_0^a \frac{y^3}{3} dx = \frac{1}{3}\int k^3 (x-a)^6 \, dx$

$= \frac{1}{3}\left(k^3 \frac{(x-a)^7}{7}\right)\Big|_0^a$

$= k^3 \frac{a^7}{7} * \frac{1}{3}$

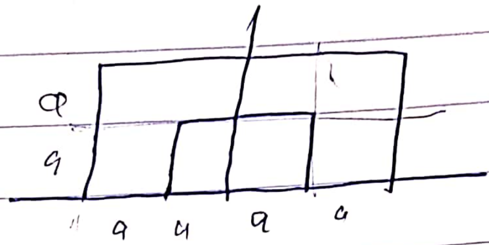
$\boxed{ka^2 = b \rightarrow}$

$\frac{b^3 a}{21}$

CamScanner الممسوحة ضوئيا بـ

P27 hard to use a polar equation

so $I_p = I_x + I_y$



$$I_x = \frac{1}{3} * 4a * (2a)^3 - \frac{1}{3} * 2a * a^3$$

$$I_x = \frac{1}{3} * 32a^4 - \frac{2}{3} * a^4 = \frac{30}{3} a^4 = 10a^4$$

$$I_y = 2 * \left(\frac{1}{3} * 2a * (2a)^3 - \frac{1}{3} * a * a^3\right)$$

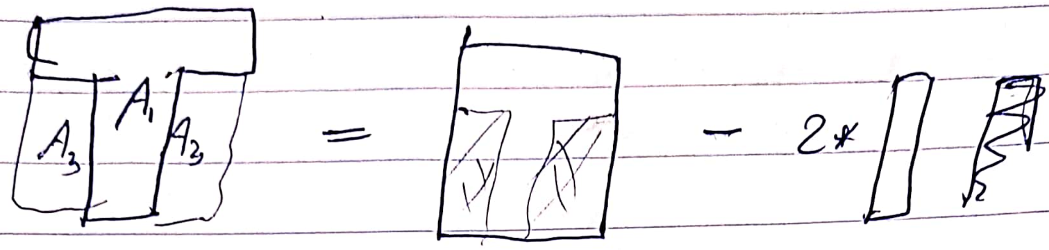
$$= 2\left(\frac{1}{3} * 2 * 8a^4 - \frac{1}{3}a^4\right)$$

$$I_y = 10a^4$$

$$I_p = 10a^4 + 10a^4 = 20a^4$$

$$K_p^2 = \frac{I_p}{A} = \frac{20a^4}{6a^2} = \frac{10}{3}a^2$$

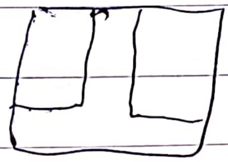
P31



$$I_{x_1} = \frac{1}{3} b h^3 - 2 * \frac{1}{3} b_3 h_3^3$$

$$\frac{1}{3}\left(24 * 30^3 - 2 * 8 * 24^3\right) = 142272 \text{ mm}^4$$

I_{x_2} =



the same Idea

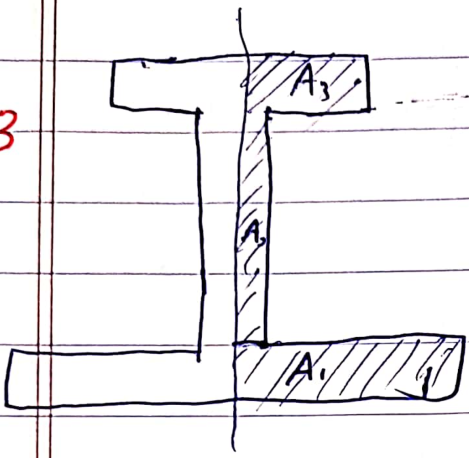
$$= 247680 \text{ mm}^4$$

$$\boxed{I_x = 389952}$$

$$K_x^2 = \frac{I_x}{A} = \frac{389952}{24*6 + 48*8 + 6*48} = \sqrt{477.88 \text{ mm}^2}$$

$$\boxed{K_x = 21.86 \text{ mm}}$$

P33



because of the symitry and Areas are distributed samely around y on the left and right we count I_y for right part and multibly ~~with~~ by 2

$$I_{y1} = \frac{1}{3} * 6 * (24)^3 = 27648$$

$$I_{y2} = \frac{1}{3} * 48 * 4^3 = 1024$$

$$I_{y3} = \frac{1}{3} * 6 * 12^3 = 3456$$

$$I_{y(right)} = 32128$$

$$\boxed{I_y = 64256}$$

$$\sqrt{K_y^2} = \frac{64256}{24*6+48*8+6*48} = \sqrt{78.7 \ mm^2}$$

$$\rightarrow \boxed{K_y = 8.87 \ mm}$$

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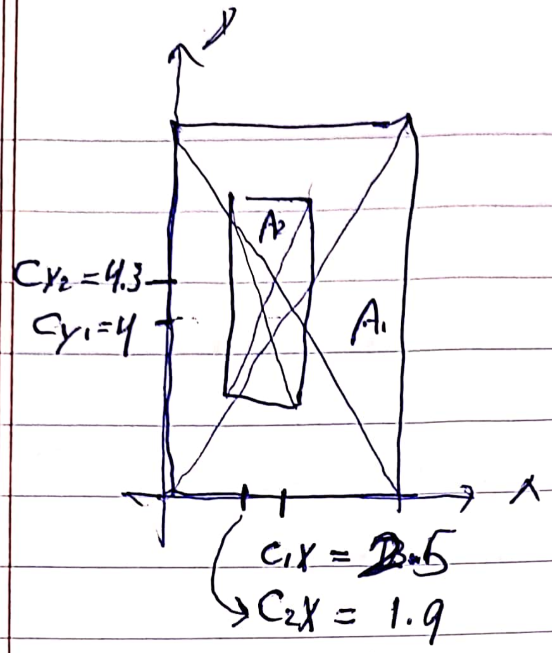


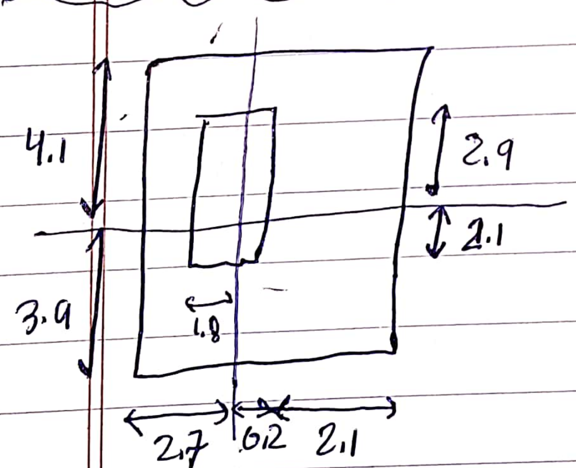
First find the centroid

$$C_X = \frac{A_1 C_1X - A_2 C_2X}{A_1 - A_2} = \frac{40 * 2.5 - 10 * 1.9}{40 - 10}$$

$$\boxed{C_X = \bar{X} = 2.7}$$

$$C_Y = \frac{A_1 C_{1y} - A_2 C_{2y}}{A_1 - A_2} = \frac{40 * 4 - 10 * 4.3}{30}$$

$$\boxed{\bar{Y} = C_Y = 3.9}$$



$$I_{y(r)} = \frac{1}{3} * 8 * 2.3^3 - \frac{1}{3} * 5 * 0.2^3$$

$$= 32.4 \ mm^4$$

$$I_{y(l)} = \frac{1}{3} * 8 * 2.7^3 - \frac{1}{3} * 5 * 1.8^3$$

$$= 42.768 \ mm^4$$

$$\boxed{I_X = 75.17 \ in^4}$$

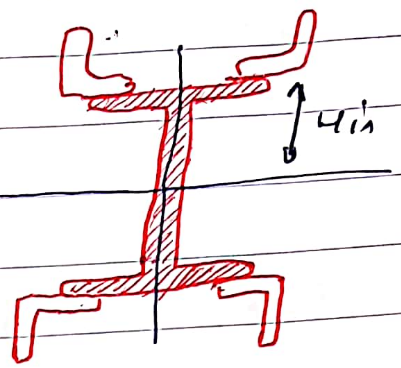
$$I_{X(up)} = \frac{1}{3}(5 * 4.1^3 - 2 * 2.9^3) = 98.6 \ mm^4$$

$$I_{X(Down)} = \frac{1}{3}(5 * 3.9^3 - 2 * 2.1^3) = 92.7 \ mm^4$$

$$\boxed{I_Y = 191.3 \ in^4}$$

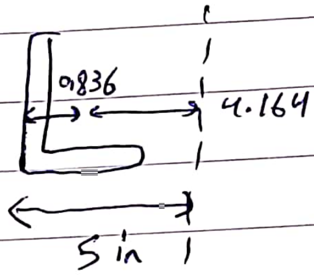
Prob 51

there is a symitry around Point c "centroid of (w shape)"
then Point c is the centroid



$I_x = I_x$ of wshape $+ 4 * I_x$ of ~~Shape~~ Angle

I_x of angle?



$I_y = I_{y'} + AX^2$

$= 1.23 + 1.44 * (4.164)^2$

$26.19 = I_{Ay}$ — (1)

I_x of angle = ?

$= I_{x'} + A * 4^2$

$= 1.23 + 1.44 * 4.836^2 = 34.9 = I_{Ax}$ — (2)

$I_x = I_{xw} + 4 * I_{Ax}$

$= 110 + 4 * 34.9 = \boxed{249.6 \text{ in}^4}$

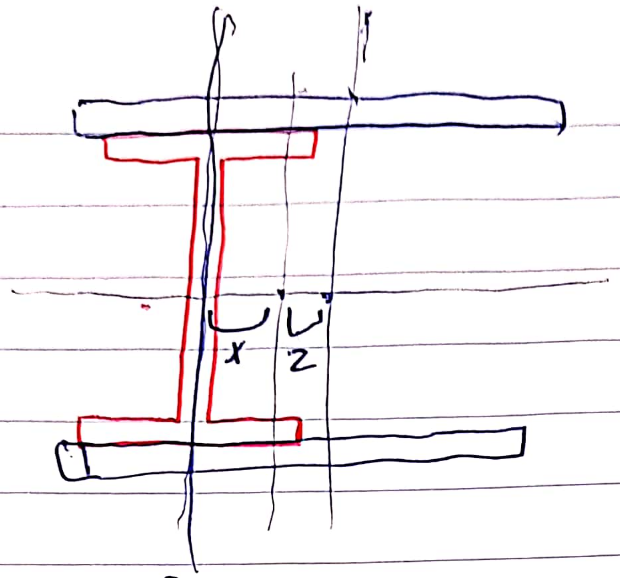
$r_x = \sqrt{\dfrac{I_x}{A}} = \sqrt{\dfrac{249}{4 * 1.44 + 9.12}} = 4.095$ in

$I_y = I_{yw} + 4 I_{Ay} =$

$= 37.1 + 4 * 26.19 = \boxed{141.86 \text{ in}^4}$

$r_y = \sqrt{\dfrac{141.86}{4 * 1.44 + 9.12}} = 3.09$ in

56



Iwx. remains the same because the centroid has shefted on X axi's not y axi's

$$I_x = I_{wx} + I_{px} = I_{wy} + I_{py}$$

$$= 385 + \frac{2}{3}\left(26 * (8.05^3 - 7.05^3)\right) = 3353.4 \text{ in}^4$$

$$I_{yw_c} = I_{wy}' + X^2 A_w$$

$$I_{py}' = 2929.3 \text{ in}^4$$

$$I_{Py_c} = I_{Py}' + A_p Z^2$$

$$I_y = I_x \Rightarrow$$

$$3353.4 = I_{Py}' + A_p Z^2 + I_{wy}' + X^2 A_w$$

$$3353.4 = 2929 + 52 * Z^2 + 26.7 + X^2 * 11.2$$

$$X = \bar{X} = \frac{0 * A_w + a * A_p}{\Sigma A} = \frac{52a}{63.2} = 0.823\,a$$

$$1473.6 = 2929 + 52 * (a - 0.823\,a)^2 + 26.7 + (0.823\,a)^2 * 11.2$$

$$424 = 0.03133\,a^2 * 52 + 11.2 * 0.6773\,a^2$$

$$424.2 = 9.2152\,a^2$$

$$a = \sqrt{46} \rightarrow \boxed{a = 6.7 \text{ in}}$$