CH 1

Mechanics : The science whic describes and predicts the conditions of rest or motion of bodies under action of forces

* الميكانيكا ليست علم مجرد ... لكنه ليست تجريبية الى حد كبير كباقي علوم الهندسة

* لا غنى عن الميكانيكا في كل فروع الهندسة

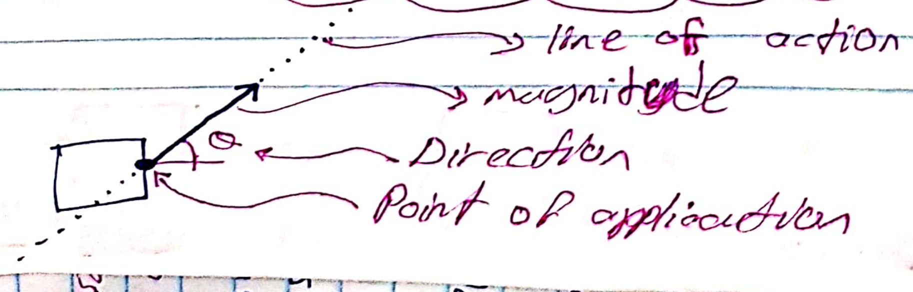
Statics : is the branch of mechanics that studies loads ((Forse and torque = "moment")) acting on a system that does not experience an acceleration (a=0) & system is in equilibrium with environment

Fundamental concepts

1 Force : Phenomenon (ظاهرة) that causes or prevents Deformation (تشوه) or movment

natural Forces : Gravity / Electromagnatic Force strong / weak nuclear Force

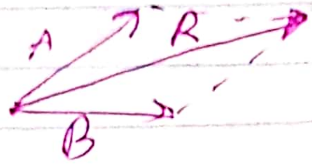
لتعريف اي قوة نحتاج هذه الخصائص



4 charactaristics

2 Parallelogram law

قانون متوازي الأضلاع

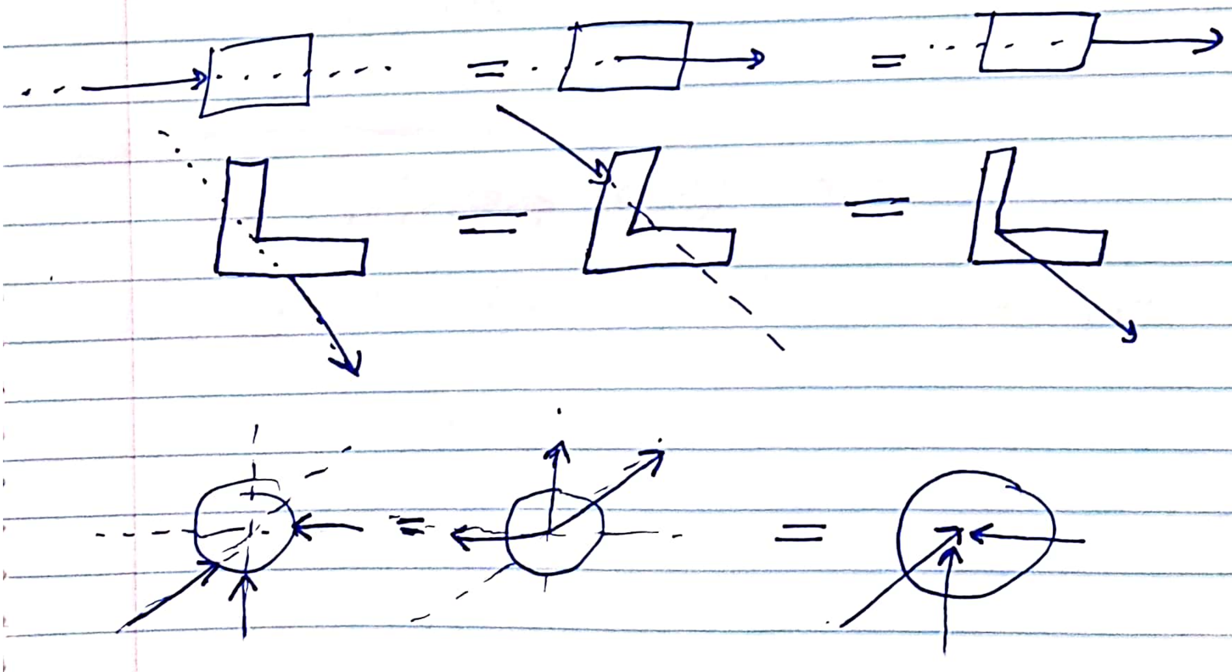


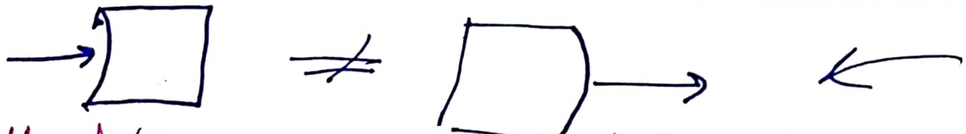
$$\vec{R} = \vec{A} + \vec{B}$$

3 Principle of Transmissibility

The state of rest or motion of a RIGID body is unaltered if a Force acting on the body is replaced by another Force of the same magnitude and direction but acting any where on body along the line of action

الأجسام الصلبة والقاسية يمكنه انه نؤثر عليها بقوة مساوية بالاتجاه والمقدار ونقطة التأثير تكون في موضع مختلف وذلك سيعطي نفس النتيجة





مش RIGID

الاستاتيكا فقط تتعامل مع Rigid غير قابل للتشوه

4] Newton's law of gravity

$$F = \frac{G\, m_1 m_2}{R^2}$$

m_1 = mass 1
m_2 = mass 2
R = distance
$G = 6.67 * 10^{-11}\ \frac{m^3}{kg \cdot s^2}$

5 Newtons 3 lows

1st الجسم الساكن يبقى ساكنة والمتحرك بسرعة ثابتة
يبقى على سرعته ما لم تؤثر عليه محصلة قوى

2nd $F_{net} = m\vec{a}$

3rd الفعل ورد الفعل

6 Free body diagram

مهم جداً وانت تعرفه من Phy 141

System of units

Kinematic units :	Length	time	mass	Force
	basic	basic	basic	derived From Newtons' second law
SI	m	s	kg	$kg\,m/s^2$
U.S	basic	basic	derived From $m = \frac{F}{a}$	basic
	foot (ft)	s	$slug = \frac{1\ lb}{ft/s^2}$	lb pound

Method of Problem solution

1 ——— : المعطيات

2- Free body diagram

3- Fundamental Principles : look in the previous pages

4- solution check

1 units can help to know if the needed result is the one you got

2 ~~[illegible]~~

2 تأكد من الحل والارقام
(عنوان ~~المو~~ الناتج يطابق اسم المطلوب)

3 النتيجة تكون منطقية
يعني القوة على شحنة
صعب تكون بالمئات
مساحة مسبح مستحيل
1000 دونم
وهكذا

Chapter 2

Statics of particles

1 Resultant of 2 Forces

2 vectors

3 Addition of vectors

4 Resultant of several concurrent Forces

5 rectangular components of a Force: unit vectors $\hat{i}\ \hat{j}\ \hat{k}$

6 Adition by summing components

7 Equilibrium of a particle

8 Free-Body Diagrams

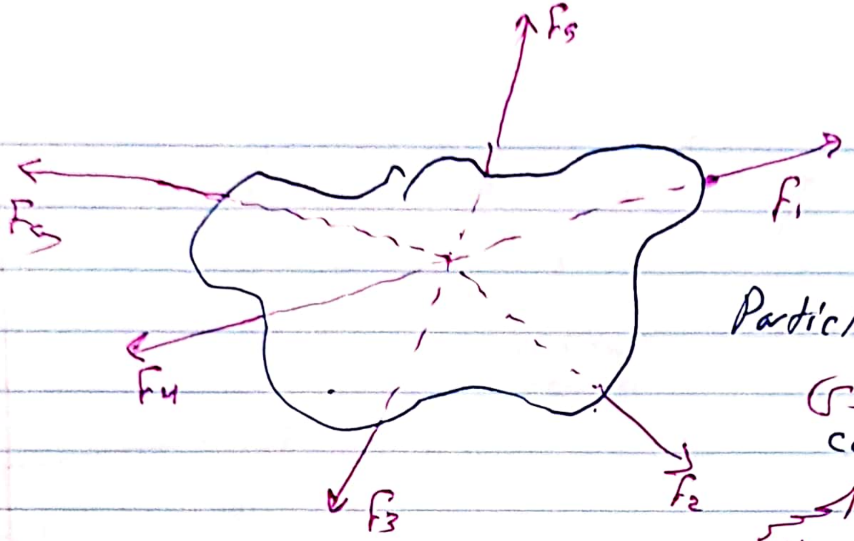
9 rectangular components in space

الاهداف من الشابتر:

→ replacing multiple forces with one resultant force

→ relations between forces acting on a particle that is in a state of equilibrium

What is a particle



Particle ممكن اعتبار الجسم
اذا كانت القوى
concurrent forces
بغض النظر عن الجسم
ولا الشكل ولو الاتجاه

concurrent forces :- Forces that it's line of action intersect in one point

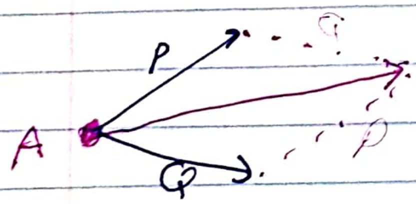
خط امتداد القوة يؤثر في نفس النقطة

اذا بدلت هاي القوى بقوة واحدة
رح تأثر نفس التأثير بشرط تكون
بنفس النقطة الخاصة بالتقاطع

خصائص اي قوة لتكون معرفة

1- magnitude
2- line of action
3- angle
4- Point of aplication

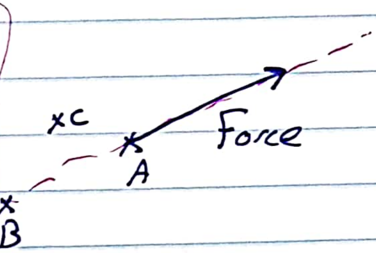
Resultant of 2 vectors



بالرسم

Vectors and scaler quantities

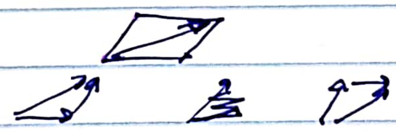
- **Vectors** — magnitude, direction
  - Fixed or bound
    - one point of aplication that cannot be changed without affecting an analysis
  - Free
    - can change the point of application any where freely with out affecting the analisis
    - like couple
  - Sliding vectors
    - like force
- **Scaler** — magnitude



Force can be slided on the line of action so the poin of action can be any point on the line of action (A/B) but not (C)

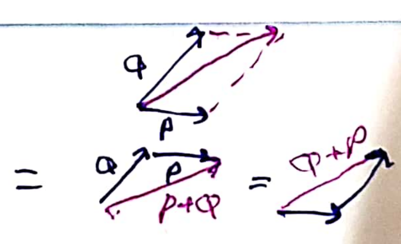
~~Add~~ ~~Addit~~ Addition of 2 vectors

* Trapezoid rule
* Triangle rule ~~for~~
* low of cosines $R^2 = P^2 + Q^2 - 2PQ\cos Q$, $\vec{R} = \vec{P} + \vec{Q}$
* low of sines



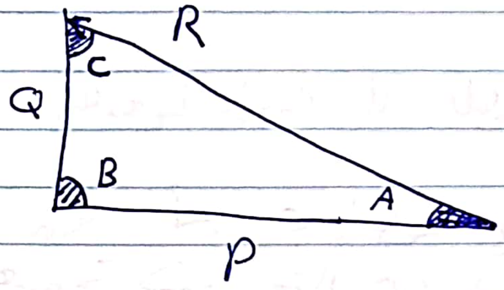
* Vector Addition is commutative

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

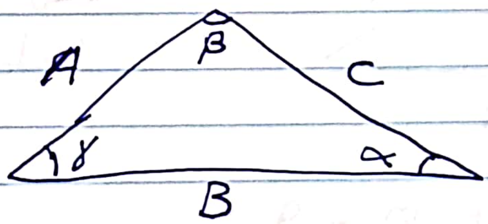


sin law

$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{P}$$



or in Phisycal Typing :-



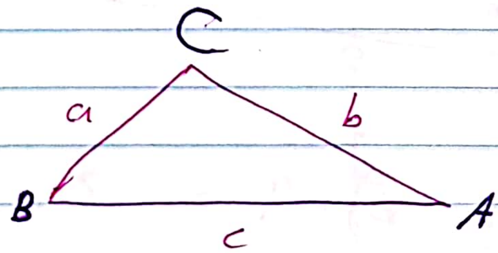
$$\frac{C}{\sin \gamma} = \frac{A}{\sin \alpha} = \frac{B}{\sin \beta}$$ ✓

~~C B sin~~

cos law

$a^2 = b^2 + c^2 - 2bc \cos A$

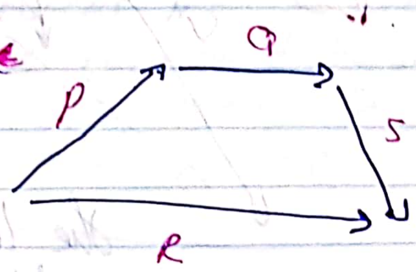
الزاوية المقابلة لـ الضلع a



$b^2 = a^2 + c^2 - 2ac \cos B$

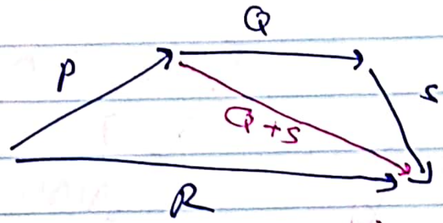
* Vector addition is Associative.

so we can make more than one triangle



1) نجد محصلة كل اثنين لوحدهم

2) نجمع المحصلات معاً لأن عملية جمع المتجهات تجميعية Associative

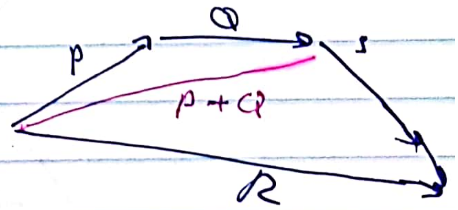


3) لا ننسى الاتجاهات اتجاه المحصلة مختلف

$$P + Q + S = R = P + (Q + S)$$

or $(P + Q) + S$

such that

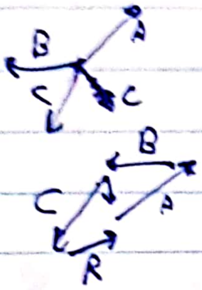


$$R = P + Q + S = (P + Q) + S = P + (Q + S)$$

~~Mult~~

مراجعة

* concurrent Forces : set of forces which all pass through the same point



⚠ concurrent forces can be replaced by a single resultant Force which is the vector sum of the concurrent forces

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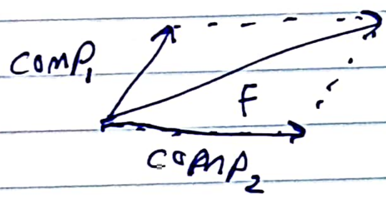
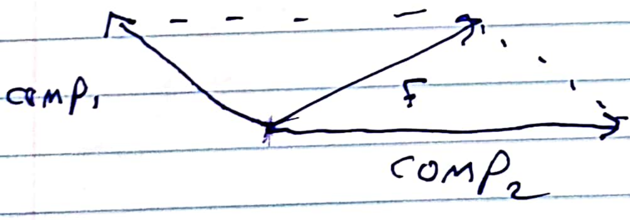
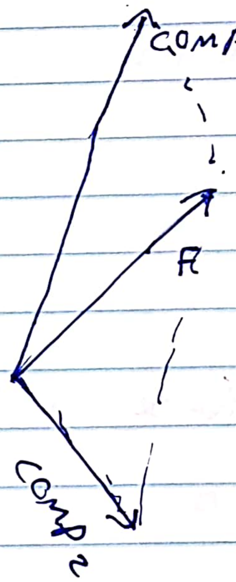
* Vector force components :- two or more forces that are together have the same effect as a single force vector

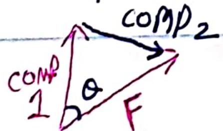
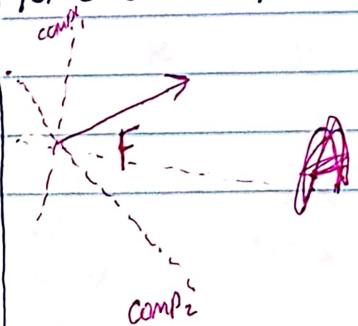
⚠ components are not always perpendicular

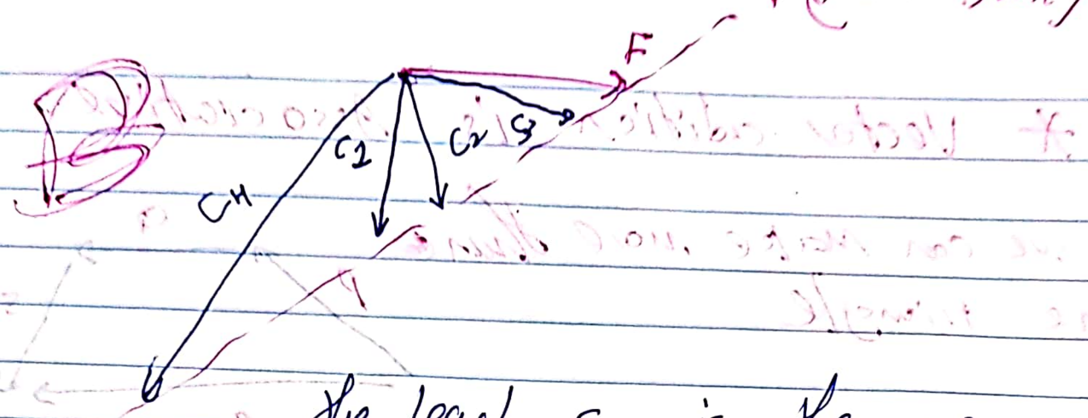




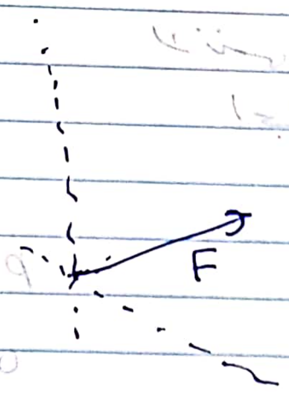


اذا اعطاني القوة وطلب مركبات

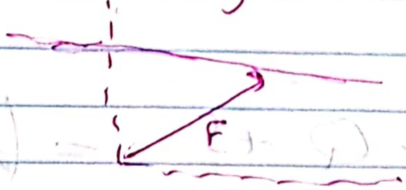
اذا عرفت مركبة واحدة مقداراً واتجاهاً بتجيب الثانية بقانون المثلث ثم جمع متجهات عادي  طبق قانون الجتا " الجيب	line Action is known for each component  How to do it	the direction and line of action of one comp. is known and we need comp2 to be as small as possible look on the left page B



the least C is the one which makes 90° angle with the line of Action OG P

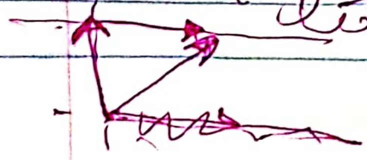


نرسم خط موازي ال (line of Action) لا مع المرتبطة حيث يبدأ من رأس العمود



③ نرسم المركبة الاولى

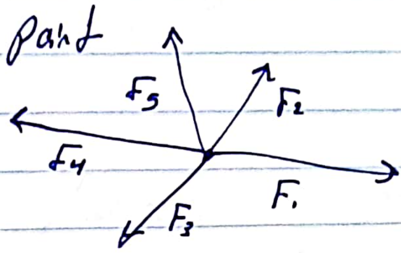
④ نقيس قانون الجيوب



~~خط موازي للمركبة~~ ~~الثانية~~ نرسم المركبة الثانية ؛ خط من نقطة الوصل إلى [illegible] للمركبة الاولى

Cocuncurent Forces

Forces acting on the same paint

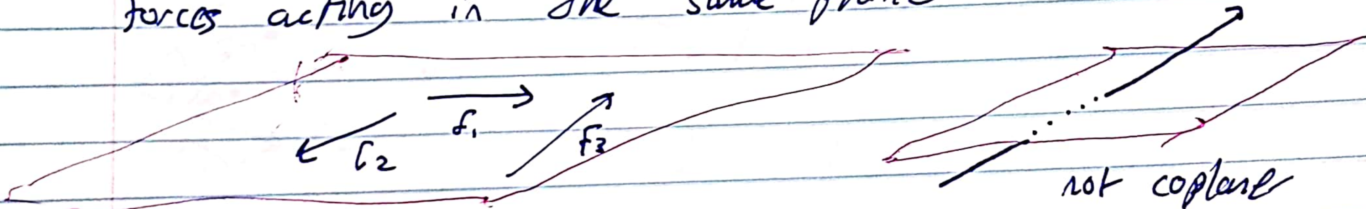


colinear forces

Forces acting on the same line

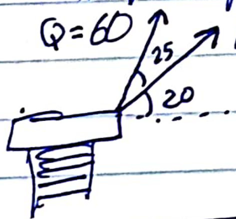
co planer Forces

Forces acting in the same plane



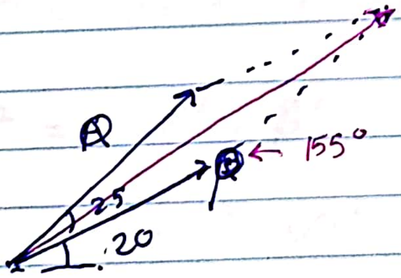
sample Problem : Find the resultant

~~sample problem~~



1 Graphical No need

2 Trigonometric solution

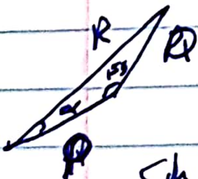


$R^2 = P^2 + Q^2 - 2PQ\cos 155$

$= 40^2 + 60^2 - 2 * 40 * 60 * \cos 155$

$=$

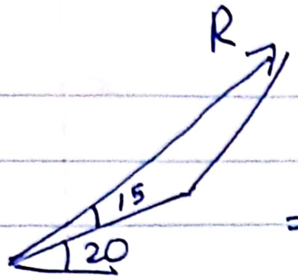
$R = \sqrt{\quad}$ $= 97.7$



sin law → $\frac{R}{\sin 155} = \frac{Q}{\sin\alpha}$ $\frac{97.7}{\sin 155} = \frac{60}{\sin\alpha}$

$= 231.12 = \frac{Q}{\sin\alpha} \rightarrow \frac{Q}{231.12} = \sin\alpha$

$\alpha = 15.04$



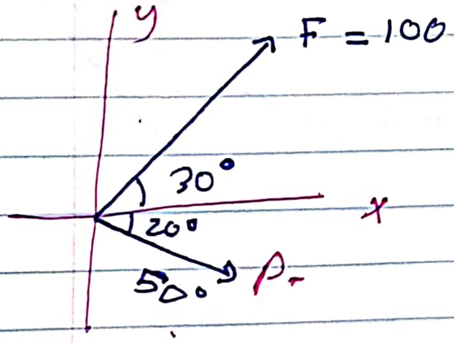
$\Rightarrow \quad 15 + 20 = 35^\circ$

then $R = 97.7$ / direction $= 35^\circ$ counter clock wise

~~97.7 , 35°~~

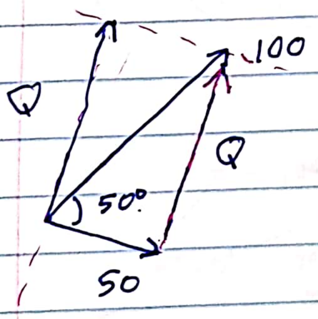
Problem from Dr. Nizar

سؤال : جد المركبة Q للقوة F



$\vec{F} = \vec{P} + \vec{Q}$

Find Q



فقط : توصيل بين رأس F ورأس P

ثم نعمل متوازي اضلاع او نحل على مثلث

$Q^2 = F^2 + P^2 - 2FP\cos 50$

$= 100^2 + 50^2 - 2 * 50 * 100 * \cos 50$

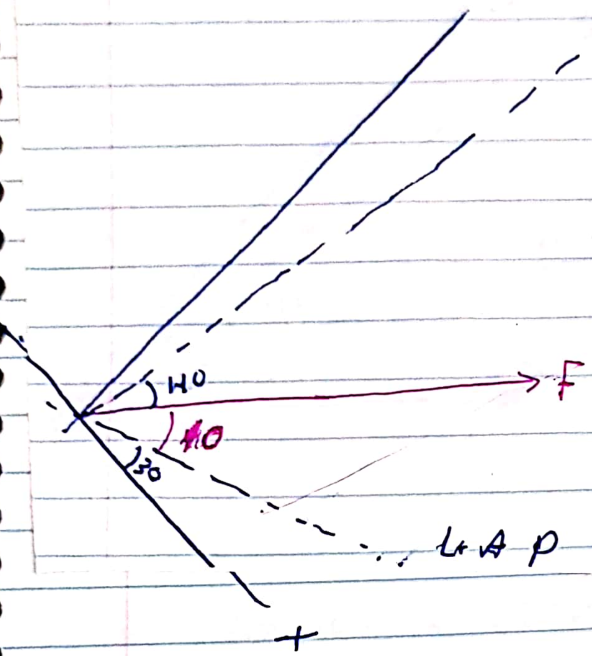
$Q = 77.9$

Find direction of Q ?

II

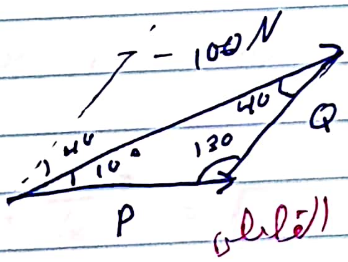
حلل القوة $F = 100N$

جد المركبات



① نكمل المتوازي الاضلاع عند طرف رسم شعاعين موازيين لـ L.A 1 و L.A2 ويتقاطعان عند رأس F

ينتج المثلث



② نجد الزوايا بالتناظر وهاي القابلي

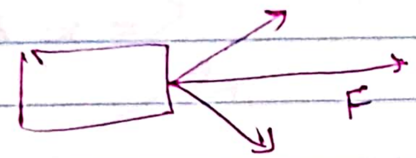
③ قانون جيوب فقط

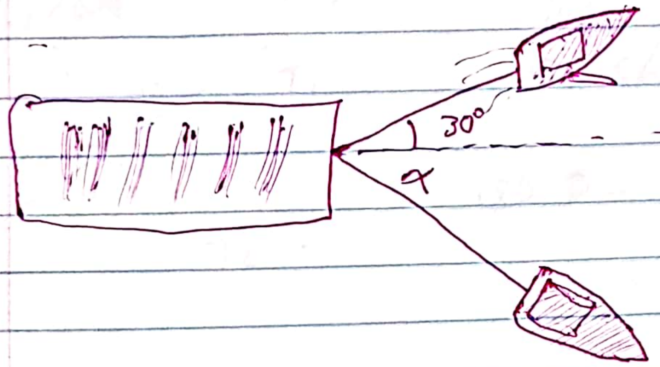
$$\frac{100}{\sin 130} = \frac{Q}{\sin 10} = \frac{P}{\sin 40}$$

$$Q = \frac{100 \sin 10}{\sin 130} = 22.7$$

$$P = \frac{100 \sin 40}{\sin 130} = 83.91$$

solve sample problem





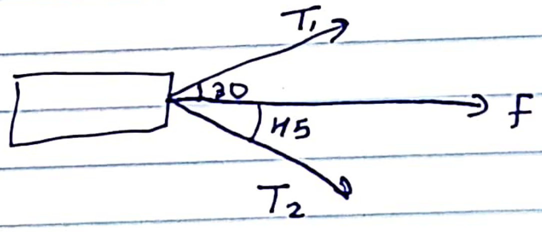
A barge is pulled by two tugboats

if the ~~resultan~~ resultant of the Forces is 5000 Ibf directed along the axis of the barge.

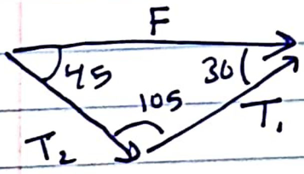
a) Find tension in each ropes For $\alpha = 45°$

b) the value of α for which the tension in rope 2 is minimum

solution :- a)



$$\vec{T_1} + \vec{T_2} = F$$



you have 3 Angles And one side length use the law of sines

$$\frac{F}{\sin 105} = \frac{T_2}{\sin 30} = \frac{T_1}{\sin 45}$$

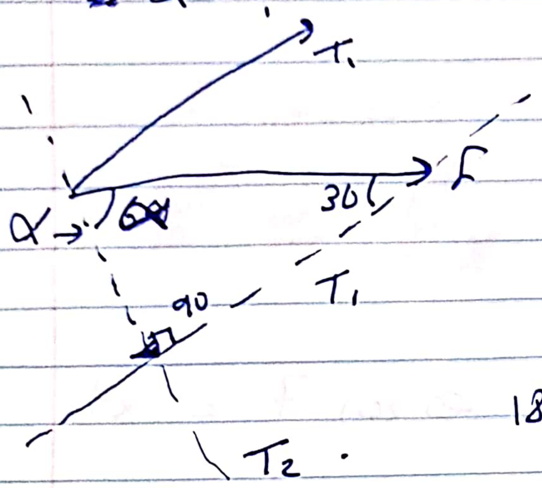
$$\frac{5000}{\sin 105} = 5176.4$$

$$T_2 = 5176.4 \sin 30$$
$$= 2588.19 \text{ Ibf}$$

$$T_1 = 3660.25$$

(b) $T_2 = min$

$\alpha = ?$ → L.A of T_2 should be preberdecular on L.A T_1



Then the value of $\alpha = 60$ as shown in the sketch

$180 - (90 + 30)$

$= 90 - 30 = 60°$

lecture 4 //

Rectangular components of a Force — unit vectors

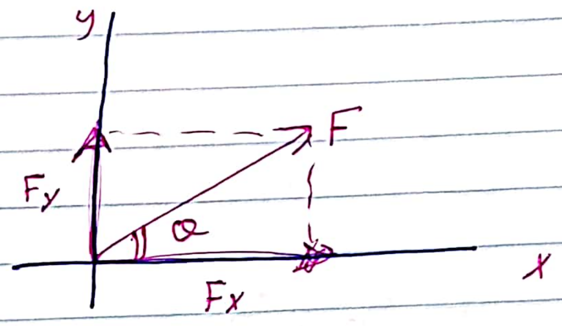
حالة خاصة من ال components حيث تكون المركبات متعامدة

$$\vec{F} = \vec{F_x} + \vec{F_y}$$

$F_x = F \cos\theta$

$F_y = F \sin\theta$

θ: الزاوية القريبة على F_x



unit vectors $\hat{i}\ \hat{j}\ \hat{k}$?

Vectors having magnitude 1 and direction +x for $\hat{i}$ / +y for $\hat{j}$ / +z for $\hat{k}$

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

F_x or F_y are not vectors but are the magnitude of $|\vec{F_x}|$ & $|\vec{F_y}|$

$\vec{F} = F\cos\theta\, \hat{i} + F\sin\theta\, \hat{j}$

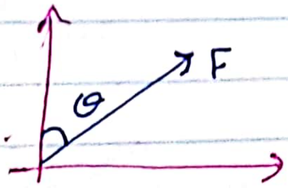
$\vec{F} = F(\cos\theta\, \hat{i} + \sin\theta\, \hat{j})$

$\vec{F} = |F|\,(\lambda)$ (where $\lambda = \hat{F}$)

متجه الوحدة باتجاه F ($= \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$)

unit vector باتجاه F — the direction of F

⚠ be carful F_x not always $F\cos\theta$

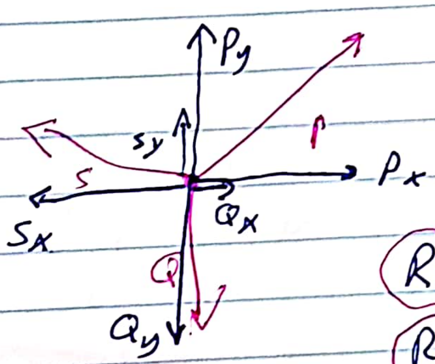


$F_x = F\sin\theta$ / $F_y = F\cos\theta$ in this situation

مثل فيزيا 1 هسا استخدم المركبات العمودية لإيجاد محصلة اكثر من قوة

أسهل تحليل القوة من عمل قانون الجمع التجميعي

$A + B + C = (A+B) + C$



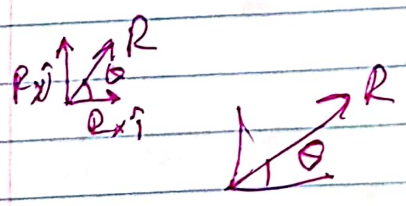
$$\vec{P} + \vec{S} + \vec{Q} = R$$

$$= (P_x + S_x + Q_x)\hat{i} + (P_y + S_y + Q_y)\hat{j}$$

$(P_x + S_x + Q_x)$ → R_x

$(P_y + S_y + Q_y)$ → R_y

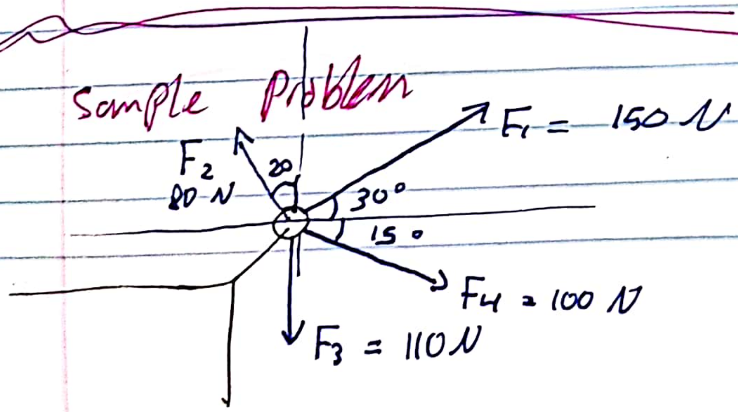
$$|R| = \sqrt{R_x^2 + R_y^2}$$



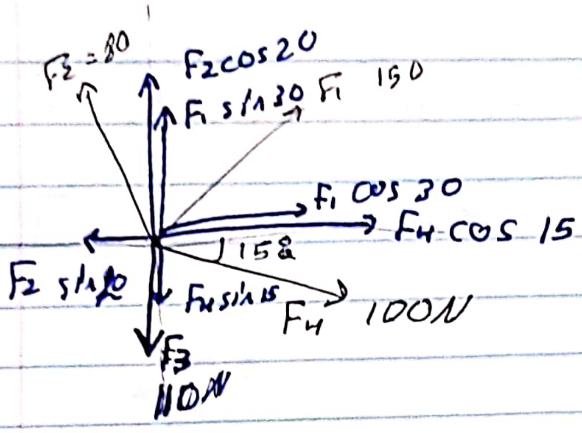
$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

Sample Problem



Find the resultant of the Four Forces



$\sum F_x = F_1 \cos 3 + F_4 \cos 15 - F_2 \sin 30$

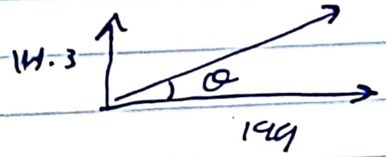
$= 130 + 96.6 - 27.4$

$= 199.2\ N\ x^+$

$\sum F_y = F_2 \cos 20 + F_1 \sin 30 - F_4 \sin 15 - F_3$

$= 75.2 + 75 - 25.9 - 110$

$= 14.3\ N\ y^+$

R = 

$= \sqrt{14.3^2 + 199.2^2} = 199.5\ N$

$\theta = 4.106°$

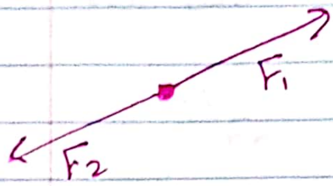
some times when there are so much forces we'll need to make a table for Fx and Fy to remember all of them

Equilibrium

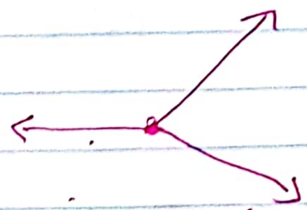
- when all forse's resultant is zero then the body it in equilibrium

- Newton's 1st law when $F_{net} = 0$ the particle will remain at rest or will continue at constant speed if the particle in Equilibrium :-

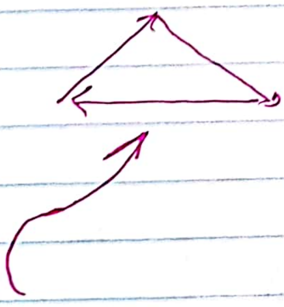
$F_{net} = 0 \rightarrow \sum F_x = 0 = \sum F_y = 0$



if only 2 forces



=

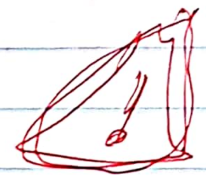


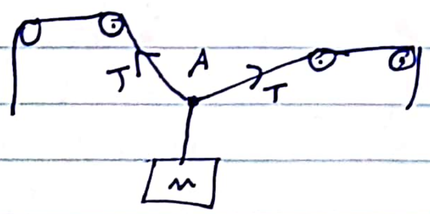
Fnet should be zero

المتجهات تنتج
مضلع مغلق
حيث تكون المحصلة صفر

Free body diagram

يجب أن الحذر في اختيار ال particle
مش شرط تكون دائما هي ال
حيث لازم يكون مكان تقاطع القوى

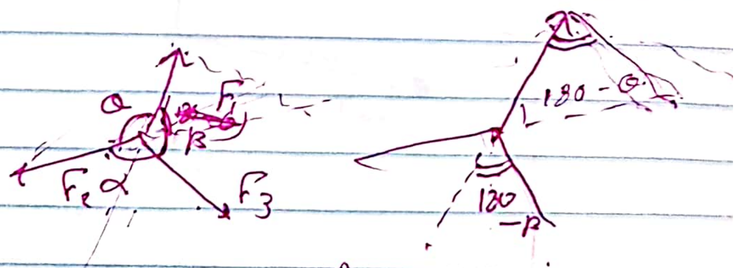




we chose the particle as Point A because all Forces (L.A) Affect that point

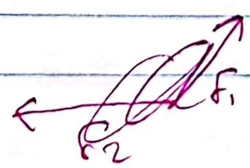
---

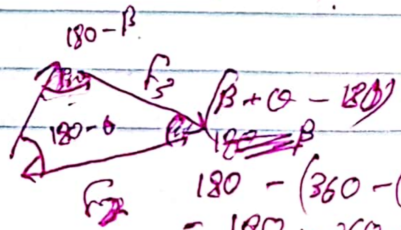
My thoughts



if $F_1 + F_2 + F_3 = 0$

then





$\beta + \theta - 180$

$180 - (360 - (\beta + \theta))$

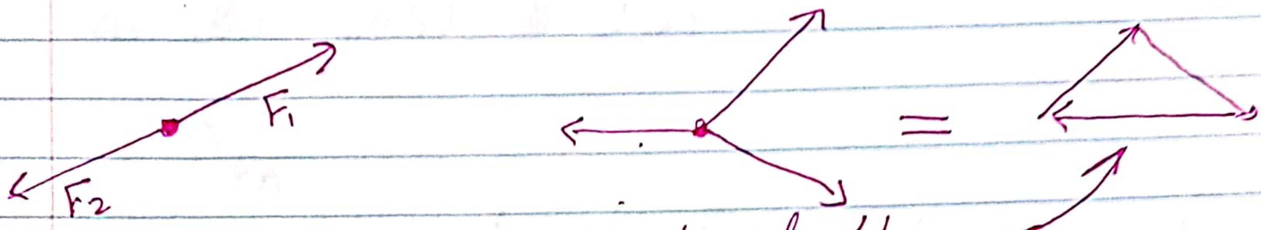
$= 180 - 360 + \beta + \theta$

$= \beta + \theta - 180$

$$\frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\theta)} = \frac{F_1}{(180-\alpha)}$$

If the particle in Equilibrium :-

$F_{net} = 0 \rightarrow \sum F_x = 0 = \sum F_y = 0$



If only 2 forces

Fnet should be zero

المتجهات أضلاع
مضلع مغلق
حيث تكون المحصلة صفر

Free body diagram

يجب أن الحذر اختيار ال particle
مش شرط تكون دايما هي ال
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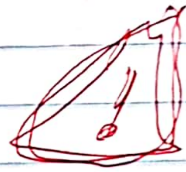
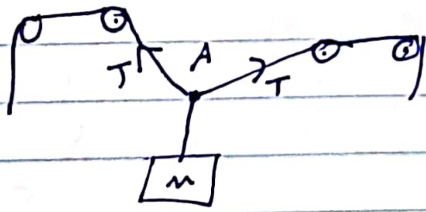
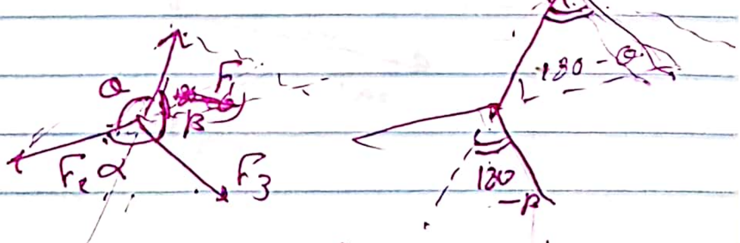




we chose the particle as Point A because all Forces (L.A) Affect that point

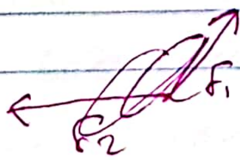
---

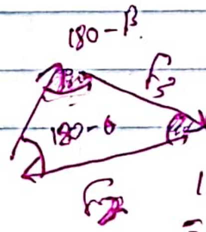
My thoughts



if $F_1 + F_2 + F_3 = 0$

then





$\beta + \theta - 180$

$180 - \beta$

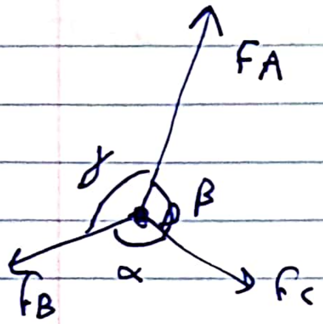
$180 - (360 - (\beta + \theta))$

$= 180 - 360 + \beta + \theta$

$= \beta + \theta - 180$

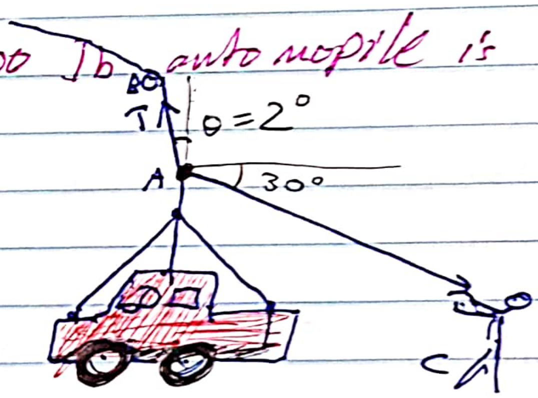
$$\frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\theta)} = \frac{F_1}{(180-\alpha)}$$

if ther was 3 forces acting on a point and the point is in Equilibrium as this point ~~A~~ then

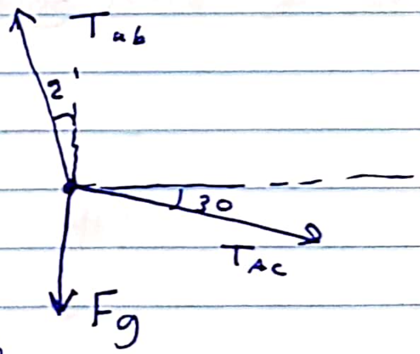


then $\frac{F_A}{\sin\alpha} = \frac{F_B}{\sin\beta} = \frac{F_C}{\sin\gamma}$

sample problem



in a ship operation, a 3500 Ib automobile is supported by a cable. A rope is tied to the cable and pulled to center the Automobile over its Intended position. what is the tension of the rope?



solution 1 -

$\Sigma F_x = \Sigma F_y = 0$

$T_{AB} \sin 2 = T_{AC} \cos 30$ — (1)

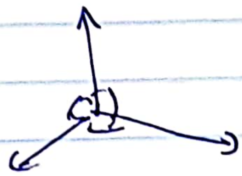
$T_{ab} \cos 2 - 3500 - T_{AC} \sin 30 = 0$

not very ~~hard~~ easy and not impossible but remember your Thoughts

~~3500~~

Way 2

$$\frac{T_{AB}}{\sin 60} = \frac{3500}{\sin 122} = \frac{T_{AC}}{\sin 178}$$



$T_{AB} = 3574.2$ Ib

$T_{AC} = 144$ Ib

Way 3

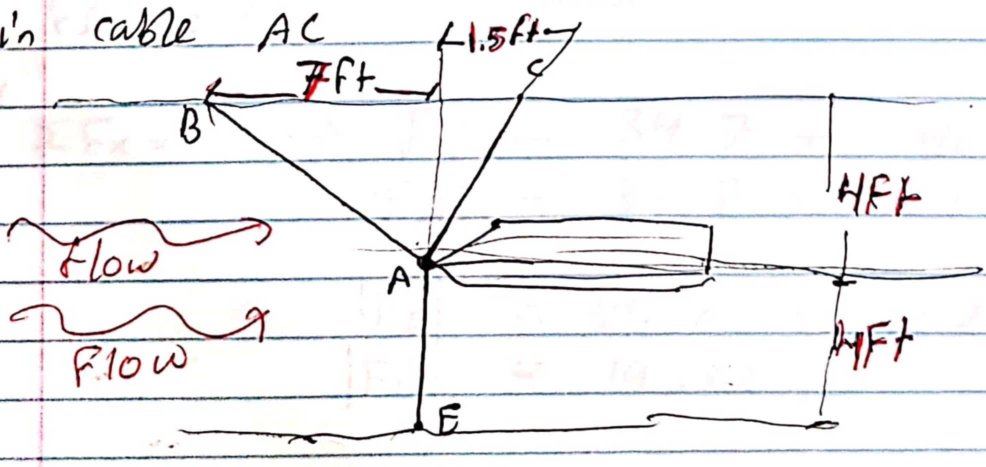
نحولها إلى مثلث

ونستخدم قانون الجيب

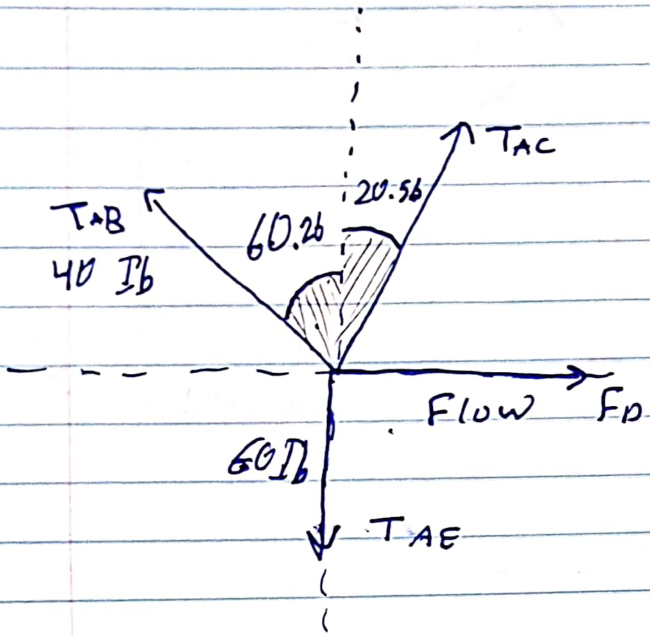
سهل جداً

---

it's desired to determine the drag Force at a given speed on a prototype sail boat hull. A model is Placed in test channel and ~~three~~ three cables are used to align its bow on the channel center line. For a given speed the tension is 40 Ib in cable AB and 60 Ib in cable AE Find: the drag Force & the tension in cable AC



solution



$\Sigma F_x = 0 \rightarrow F_D - 40 \sin 60.26 + T_{AC} \sin 20.56$

$\Sigma F_y = 0 \rightarrow 60 - 40 \cos 60.26 - T_{AC} \cos 20.56 = 0$

$\rightarrow 60 - 19.8 - 0.9363\, T_{AC} = 0$

$\frac{40.2}{0.936} = \frac{0.936\, T_{AC}}{0.936}$

$42.93\ Ib = T_{AC}$ ✓

~~TAC~~

$F_D = ?$

$\Sigma F_x = 0 \rightarrow |F_D| - 34.7 + 42.93 * 0.34 = 0$

$|F_D| - 34.7 + 14.866 = 0$

$|F_D| = 34.7 - 14.866$

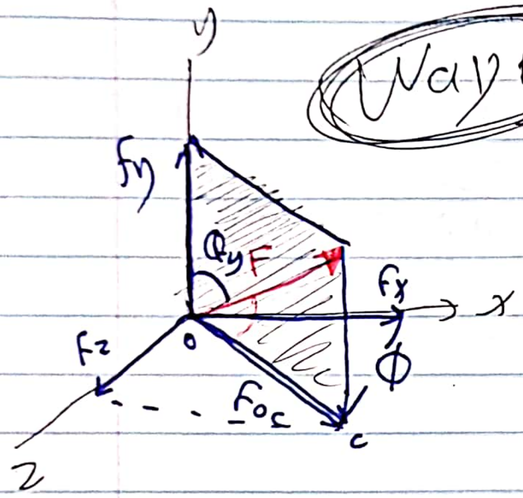
$|F_D| = 19.83\ Ib$

CamScanner الممسوحة ضوئيا بـ

New

Rectangular components of a Force in space

Way 1



$F_y = F\cos\theta_y$

$F_{oc} = F\sin\theta_y$

$F_x = F_{oc}\cos\phi = F\sin\theta_y\cos\phi$

$F_z = F_{oc}\sin\phi = F\sin\theta_y\sin\phi$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$\vec{F} = F\sin\theta_y\cos\phi\,\hat{i} + F\cos\theta_y\,\hat{j} + F\sin\theta_y\sin\phi\,\hat{k}$

$\vec{F} = F\left[\sin\theta_y\cos\phi\,\hat{i} + \cos\theta_y\,\hat{j} + \sin\theta_y\sin\phi\,\hat{k}\right]$

$= F\lambda \quad : \lambda = \dfrac{\vec{F}}{F} = \hat{F}$ unit vector $= 1$ unit

الرموز ليست مقدسة

المهم انك فاهم التحليل ماتحفظ حاجة

θ can be defined between F and x axis θ_x
or y axis θ_y
or z " θ_z

Way 2

if we were given $\theta_x / \theta_y / \theta_z$ (only two are enough)
we can know
$F_x / F_y / F_z$ easily

$F_x = F\cos\theta_x \;/\; F_y = F\cos\theta_y \;/\; F_z = F\cos\theta_z$

$$|F| = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

مهم جداً F

$$\vec{F} = F\lambda = F(\cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k})$$

$1 = |\lambda|$ لأنه $\quad \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

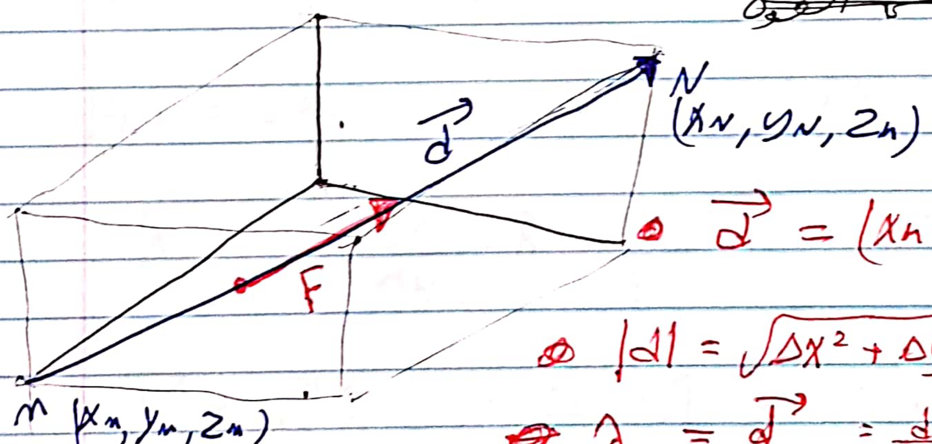
مهم جداً يمكن يعطيك زاويتين θ_x / θ_y / وانا اوجد الزاوية الاخيرة حسب العلاقة اعلاه

لأنه λ طولها وحدة واحدة

---

way 3

تحديد نقطتين على line of action للقوة



- $\vec{d} = (x_N - x_m)\hat{i} + (y_N - y_m)\hat{j} + (z_N - z_m)\hat{k}$
- $|d| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$
- $\lambda = \dfrac{\vec{d}}{|d|} = \dfrac{dx}{d}\hat{i} + \dfrac{dy}{d}\hat{j} + \dfrac{dz}{d}\hat{k}$

if we know a vector in the same direction and line of action of the force we should find the unit vector of $\vec{d}$ which is the same as λ unit vector of F

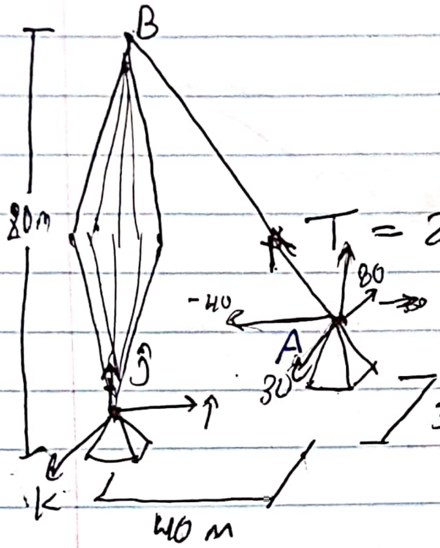
$$\vec{F} = F\lambda = \frac{F}{d}(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$F_x = \dfrac{F\,dx}{d}$ / $F_y = \dfrac{F\,dy}{d}$ / $F_z = \dfrac{F\,dz}{d}$

$\cos\theta_z = \frac{d_z}{|d|} / \frac{F_z}{|F|}$ $\left\{ \cos\theta_x = \frac{F_x}{F} / \frac{d_x}{d} \right.$ ...

etc

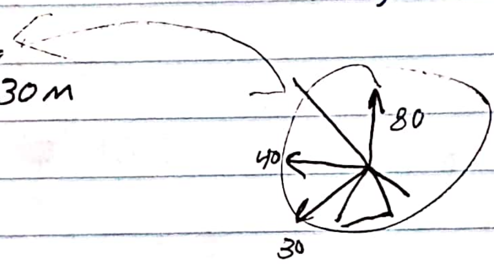
sample Problem



From these informations find :-

→ F_x F_y F_z on the Plot A

→ θ_x θ_y θ_z



$\vec{AB}$ = the length of wire = $30\hat{k} + 80\hat{j} - 40\hat{i}$

$|AB| = \sqrt{40^2+30^2+80^2} = \sqrt{8900} = 94$

$\lambda = \frac{30}{94.3}\hat{k} + \frac{80}{94.3}\hat{j} - \frac{40}{94.3}\hat{i}$ لماتك لازم 1 طلع

$\vec{F} = |F|\lambda = 2500\lambda = 2500 * \frac{30}{94.3}\hat{k} + 2500 * \frac{80}{94.3}\hat{j} + -2500 * \frac{40}{94.3}\hat{i}$

F_x = ~~...~~ 1060

F_y = 2120

F_z = 795

$\theta_x = \frac{F_x}{F} = \cos\theta_x \rightarrow \theta_x = 64.9°$

$\theta_y = 32$

$\theta_z = 71.4$

تأكد لما

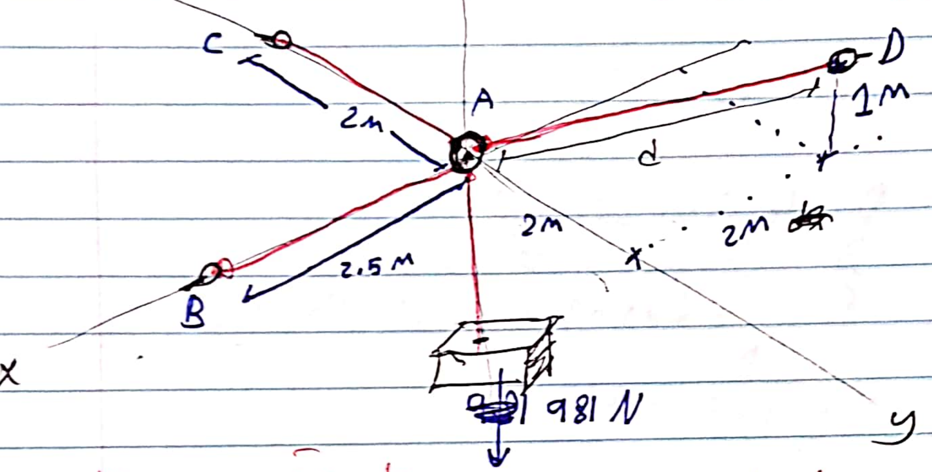
$\cos^2 + \cos^2 + \cos^2 = 1$

sample problem

100 kg crate is support by the cables as shown below Determine the Tension in cables AB, AC, AD

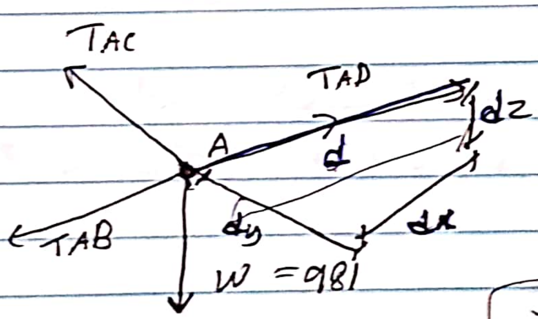
انتبه اذا كان الحبل مستمر (حبل واحد) تكون الشد نفسه

هنا في هذا السؤال الخيوط ليست نفسها 4 خيوط



انتبه ايما سالب وايما موجب حسب الورقة مش القلم تحلل المركبات

ايجاد مركبات T_{AD}



$$|d| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

$$\vec{d} = -2\hat{i} + 2\hat{j} + \hat{k}$$

$$\frac{\vec{d}}{|d|} = -\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$$

$$\lambda = \frac{\vec{d}}{d}$$

$$T_{AD} = \frac{-2}{3}\hat{i}\,T_{AD} + \frac{2}{3}\hat{j}\,T_{AD} + \frac{T_{AD}}{3}\hat{k}$$

$$W = -981\,\hat{k}$$

$$T_{AC} = T_{AC}\,\hat{i}$$

$$T_{AB} = T_{AB}\,\hat{i}$$

$$T_{AD} = \frac{-2}{3}\hat{i}\,T_{AD} + \frac{2}{3}T_{AD}\,\hat{j} + \frac{T_{AD}}{3}\hat{k}$$

$$\sum F_x = 0 \rightarrow T_{AB} - \frac{2}{3}T_{AD} = 0 \rightarrow \boxed{T_{AB} = \frac{2}{3}T_{AD}} \text{ — (1)}$$

$$\sum F_z = 0 \rightarrow -981\hat{k} + \frac{T_{AD}}{3}\hat{k} = 0$$

$$\boxed{T_{AD} = 3 * 981 = 2943} \text{ — (2)}$$

دائما استخدام k مش المعادلة افضل لانه يوجد مجهول واحد

$$\sum F_y = 0 \rightarrow \boxed{T_{AC} = \frac{2}{3}T_{AD}} \text{ — (3)}$$

بعد حل المعادلات

$T_{AC} = -1462.3\ N$

$T_{AD} = 2943.7\ N$ ~~2943 N~~

$T_{AB} = 1962.7\ N$

الخطوات
1) حدد النقطة التي عليها القوى
2) اعمل Free body D
3) حلل القوى باستخدام احد ال ٣ الآتية
أ) زاويتين $\theta_x / \theta_y / \theta_z$
ب) زاويتين θ_{axis} و θ_{plane}
ج) خط line of action غالباً للحبال والأجسام
4) $\Sigma F_x = 0 / \Sigma F_y = 0 / \Sigma F_z = 0$
5) استخدم أسهل معادلة تعطي مجهول واحد
6) عوضها وأوجد الحل

تذكر دوماً القوانين السابقة مثل

$\frac{F_x}{F} = \cos\theta_x$

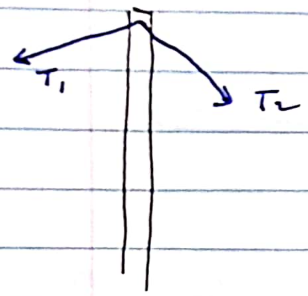
$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

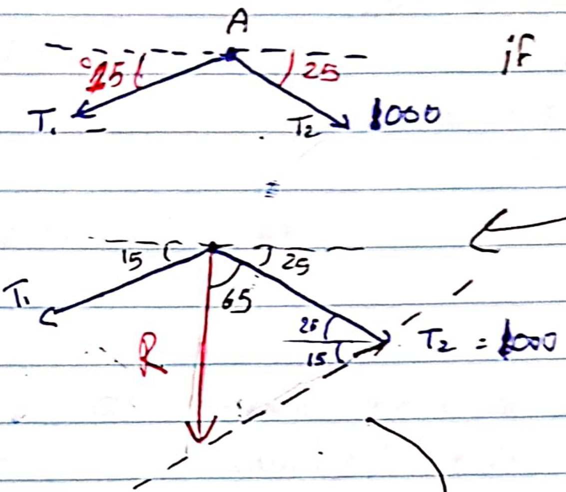
إلخ

Home work!

P.2.7

when R is vertical then ~~[illegible]~~

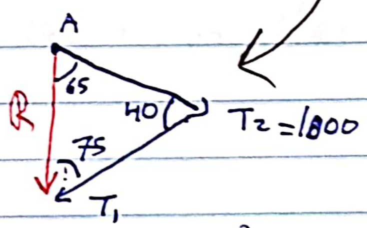




if we use parallelogram method

R should be vertical

now we got triangle with all the angles known and one side



by the sines low :- $\frac{R}{\sin 40} = \frac{T_2}{\sin 75} = \frac{T_1}{\sin 65}$

$$\frac{T_1}{0.906} = \frac{1000}{0.9659} \Rightarrow T_1 = 938.3 \text{ Ib}$$

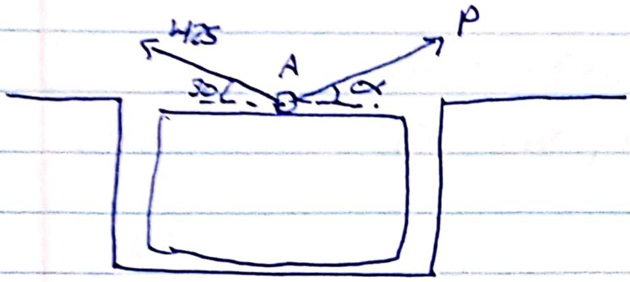
$$R = \frac{T_2 \sin 40}{\sin 75} = 665.46 \text{ Ib}$$

a) 938.3 Ib
b) 665.46 Ib

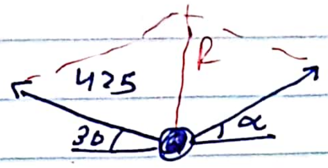
← answers

It can be also solved by rectangular components

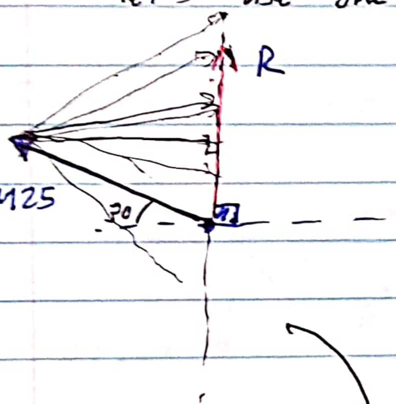
P 2.13



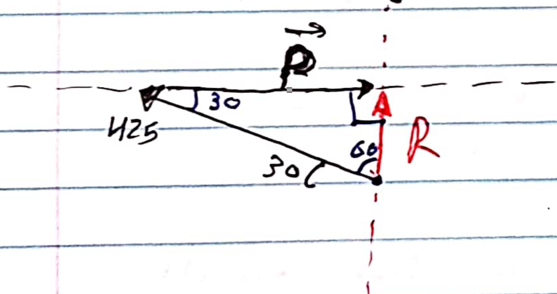
we need R to be vertical ! let's draw the free body diagram



now let's use the triangle way



as known from calculus and lectures the smalest vector between a point and a line is the prependecular vector on that line



by the sin law

$$\frac{425}{\sin 90} = \frac{P}{\sin 60} \rightarrow P = 425 \sin 60$$

$p = 368$

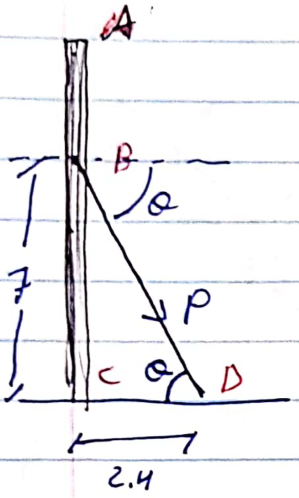
$R = 425 \sin 30 = 212.5$

it can also be solved by rectangular components

such that $P = 425 \cos 30$

$R = 425 \sin 30$

Problem 2.30

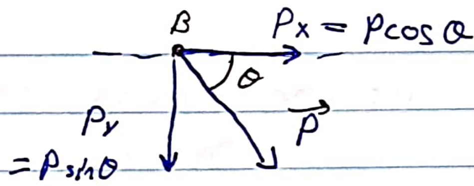


the component (P_x) that is perpendicular to the pole (AC) should be 720 N

First let's know the length of $\overline{BD}$

$= \sqrt{CD^2 + BC^2} = 7.4$ m

Free body diagram on point B :-



$P_x = 720 = P\cos\theta \Rightarrow P * \frac{2.4}{7.4} = 720$

$P = \frac{720 * 7.4}{2.4} = \boxed{2220\ N = |\vec{P}|}$

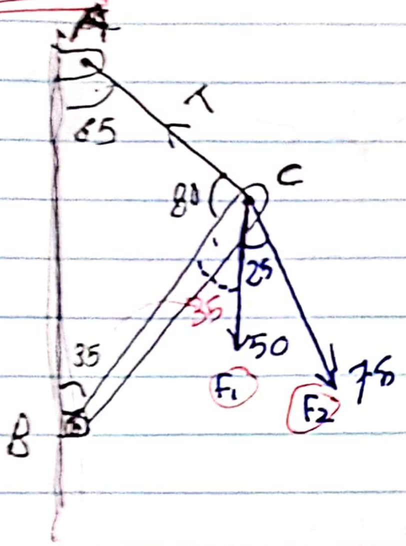
its component along AC $= P_y = P\sin\theta$

$= 2220 * \frac{7}{7.4} = \boxed{2100\ N = |P_y|}$

a 2.220 KN
b 2.160 KN → ~~Prob~~ Answer

I did not use the formula $\cos^{-1}$ because one day I may not have a good calculator so I should know how to deal with ~~that~~

41



R should be toward $\overrightarrow{CB}$!

First try to solve by Asociativ Law

$\vec{R} = \vec{T} + \vec{F_1} + \vec{F_2} = \vec{T} + (\vec{F_1} + \vec{F_2})$

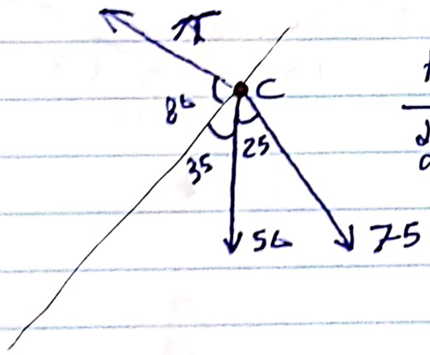
Find $\vec{F_1} + \vec{F_2} = \vec{R_i}$

$R_{i(x)} = F_2 \sin 25 = 31.7 \uparrow$

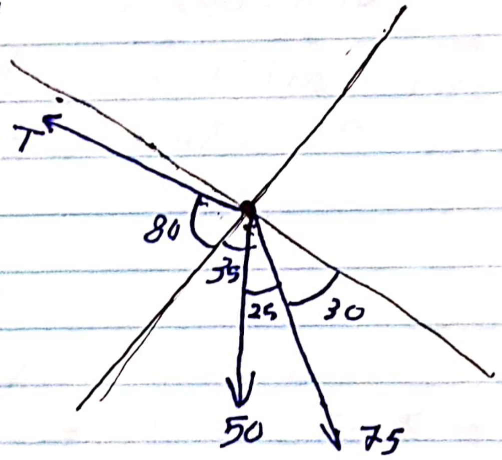
$R_{i(y)} = F_2 \cos 25 + \vec{F_1} =$

can be solved this way but it's better and easier to use Rectangular components

Free body diagram on point C



take $\overrightarrow{BC}$ direction as y+ axis
C = origin



when $\vec{R}$ is in y axis
then $R_x = 0$ the $\Sigma F_x = 0$

$|F_{2x}| = 75 \cos 30 = 64.95$

$|F_{1x}| = 50 \sin 35 = 28.68$

$|T_x| = T \sin 80 = 0.9848\ T$

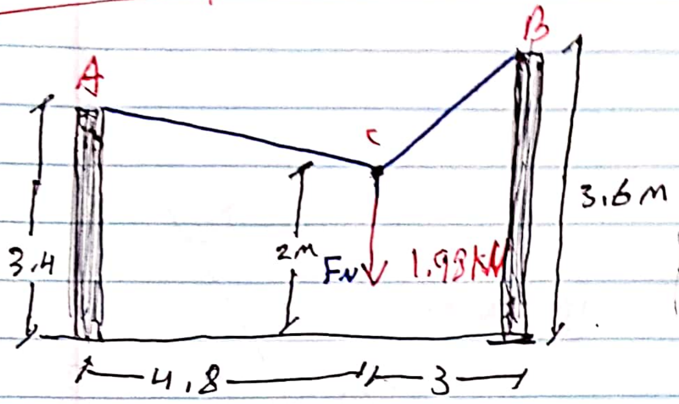
$\Sigma F_x = F_{2x} + F_{1x} - T_x \Rightarrow 64.95 + 28.68 - T\sin 80 = 0$

$\frac{93.63}{\sin 80} = T \rightarrow T = 95.1$

	comp y	
T	T cos 80	16.51
F1	50 cos 35	41
F2	75 sin 30	37.5

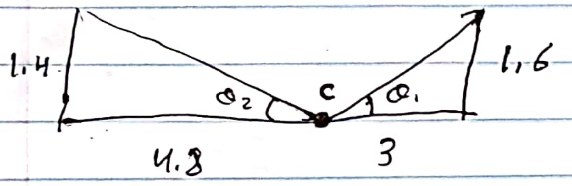
$|R| = 95$

45



need T_{AC} / T_{BC}

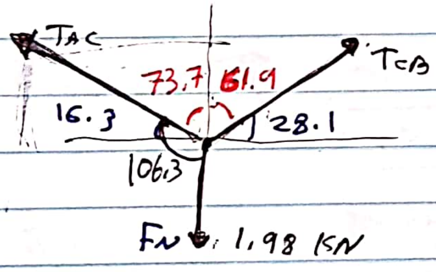
Ignore the aditional 2m



$\theta_1 = \tan^{-1} \frac{1.6}{3} = 28.1^\circ$

$\theta_2 = \tan^{-1} \frac{1.4}{4.8} = 16.3^\circ$

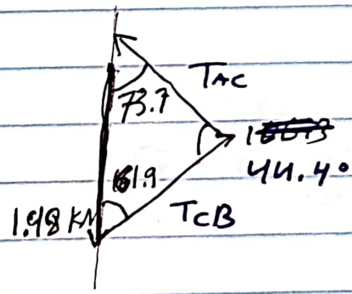
now Free body diagram on point c



c is in equilibrium

then $\vec{T}_{AC} + \vec{T}_{CB} + \vec{F}_N = 0$

they make a triangle



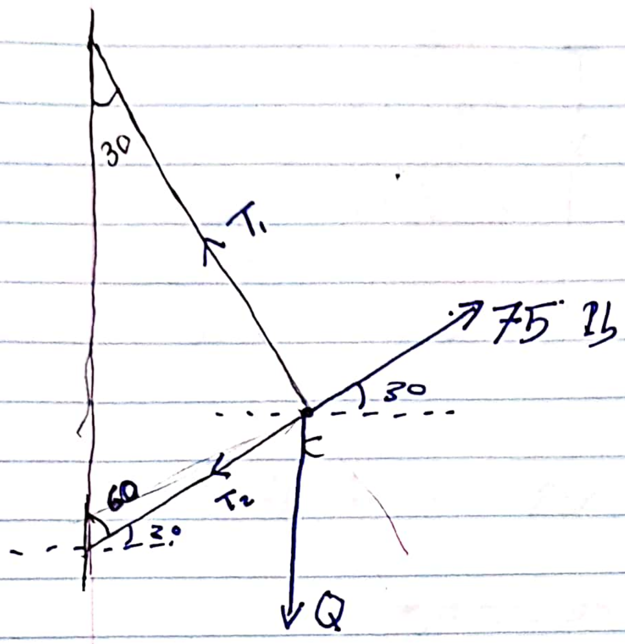
$\frac{F_N}{\sin 44.4} = \frac{T_{AC}}{\sin 61.9} = \frac{T_{CB}}{\sin 73.7}$

$\frac{1980}{\sin 44.4} = 2830$

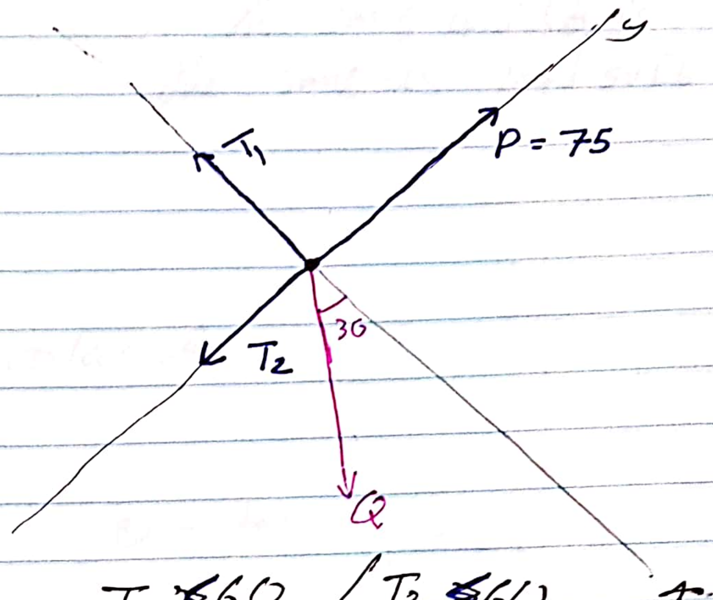
$T_{AC} = 2830 \sin 61.9 = 2496 \approx 2.49\ KN$

$T_{CB} = 2830 \sin 73.7 = 2716 \approx 2.71\ KN$

60



Free body diagram on point C



$\Sigma F_x = 0$ / $\Sigma F_y = 0$ $T_1 \leq 60$ / $T_2 \leq 60$

$\Sigma F_y = 75 - T_2 - Q\sin 30 = 0$

$\boxed{75 = T_2 + Q\sin 30}$

$\Sigma F_x = Q\cos 30 - T_1 = 0$

$\boxed{Q\cos 30 = T_1}$

then $Q\cos 30 < 60$

$Q \leq \frac{60}{\cos 30} \longrightarrow Q \leq 69.28$

this is for T_1
we should check
T_2 Too

$75 - Q\sin 30 = T_2$

$75 - Q\sin 30 \leq 60$

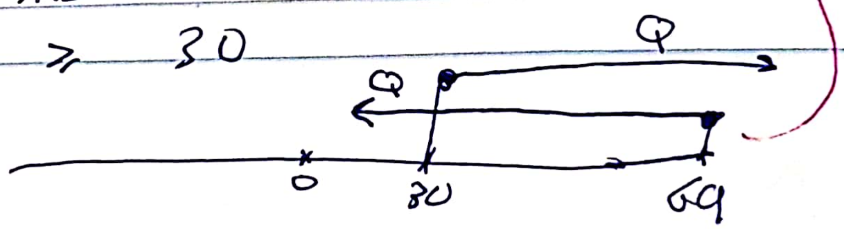
$-75 \qquad\qquad -75$

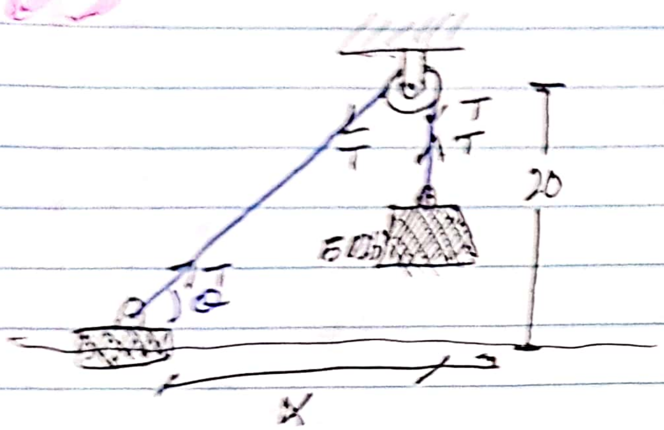
$\frac{-Q\sin 30}{-\sin 30} \leq \frac{-15}{-\sin 30}$

$Q \geq 30$

$Q \in [30, 69.28]$

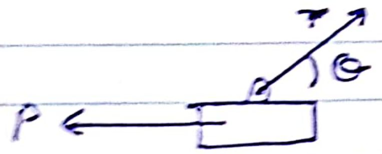
or $30 \leq Q \leq 69.28$





as the pulley is frictionless then Tension in the rope is the same which is the same as load 50Ib

Free body diagram for colar A



$\theta = \tan^{-1}\left(\frac{20}{x}\right)$

Colar A can't move vertically → ~~its~~ $\Sigma F_y = 0$

~~Fx~~

to be in Equilibrium then $\Sigma F_x = 0$

$$\Sigma F_x = T\cos\theta - P = 0$$

$$0 = T\cos\left(\tan^{-1}\left(\frac{20}{x}\right)\right) - P$$

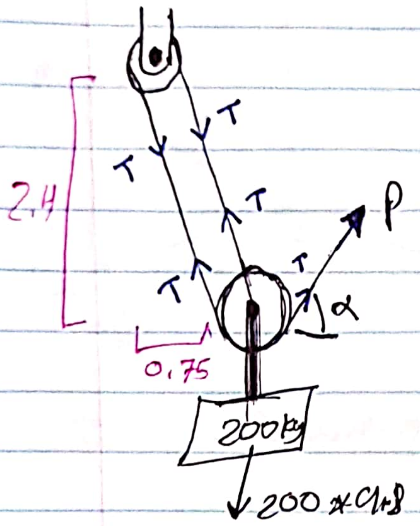
$$T\cos\left(\tan^{-1}\left(\frac{20}{x}\right)\right) = P$$

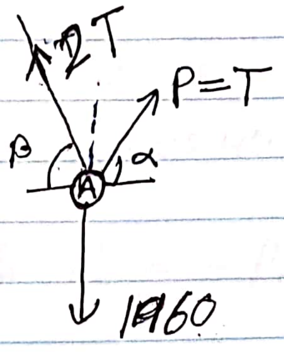
(a) $50\cos\left(\tan^{-1}\left(\frac{20}{4.5}\right)\right) = 10.975 \approx 11$ Ib

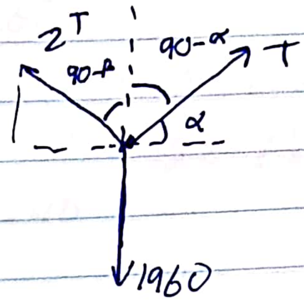
(b) $50\cos\left(\tan^{-1}\left(\frac{20}{15}\right)\right) = 30$ Ib

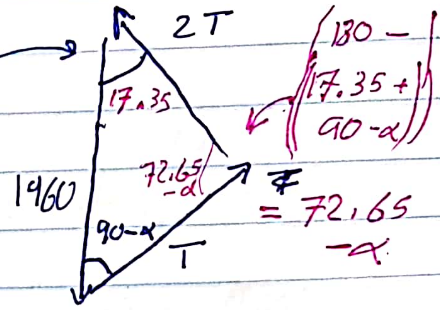
66

Free body diagram









$\beta = \tan^{-1} \frac{2.4}{0.75} = 72.65 \quad (90 - \beta = 17.35)$

Point A is in equilibrium

by sin rule

$$\frac{1960}{\sin(72.65 - \alpha)} = \frac{T}{\sin 17.35} = \frac{2T}{\sin(90 - \alpha)}$$

$$\frac{T}{\sin 17.35} = \frac{2T}{\sin(90-\alpha)} \rightarrow \sin 90-\alpha = 2\sin 17.35$$

$\cos\alpha = 2 \times 0.298$

$\cos^{-1}(2\sin 17.35) = \alpha$

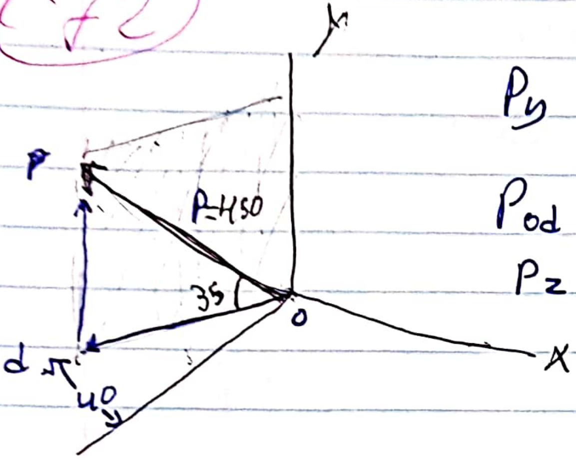
$\alpha = 53.38$

$$\frac{1960}{\sin 19.27} * \sin 17.35 = T$$

$P = 1771\ N$

$\alpha = 53.38$

72

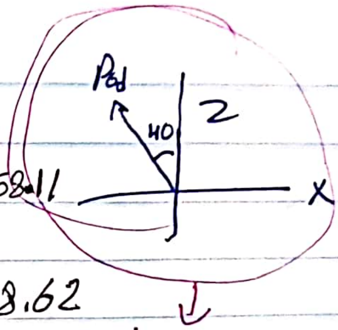


$P_y = P \sin 35 = 258.11$

$P_{od} = P \cos 35 = 368.62$

$P_z = P_{od} \cos 40$

$P_x = -P_{od} \sin 40$



imaginatic for XZ plane

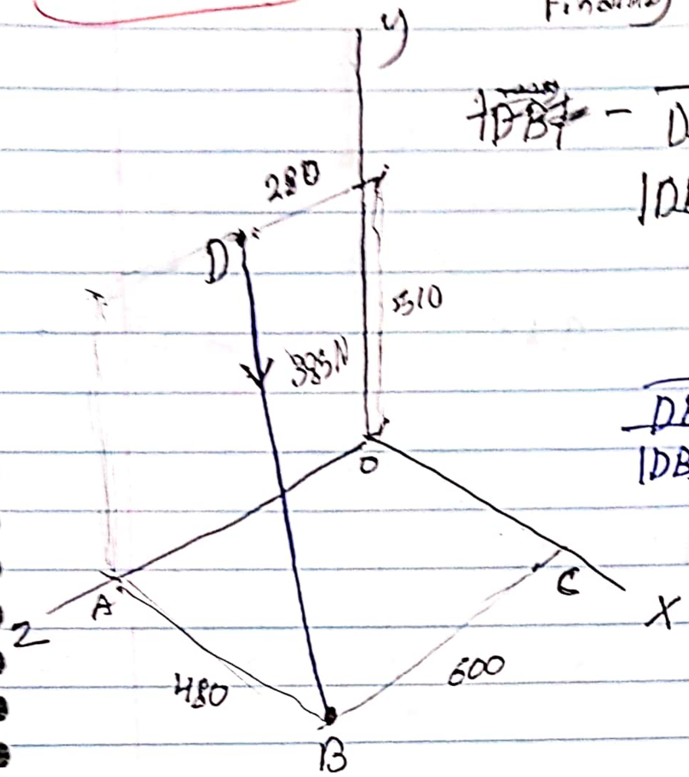
Answers :- $P_y = 258.11\,N$ $P_z = 282.37\,N$ $P_x = -236.9\,N$

making sure :- $\sqrt{P_y^2 + P_z^2 + P_x^2} =$ correct

$\theta_x = \cos^{-1} \frac{P_x}{P}$	$\theta_y = \cos^{-1} \frac{P_y}{P}$	$\theta_z = \cos^{-1} \frac{P_z}{P}$
$= 121.76°$	$= 55$	$= 51.13°$

check was correct

P 85



Finding λ

$\vec{DB} = 480\hat{i} - 510\hat{j} + 320\hat{k}$

$|DB| = \sqrt{480^2 + 510^2 + 320^2}$

$= \boxed{770}$

$\frac{\vec{DB}}{|DB|}$ = unit vector λ

$= \frac{480}{770}\hat{i} - \frac{510}{770}\hat{j} + \frac{320}{77}k$

$= 0.62\hat{i} - 0.662\hat{j} + 0.416\hat{k}$

$\vec{F_D} = |F_D|\,\lambda$

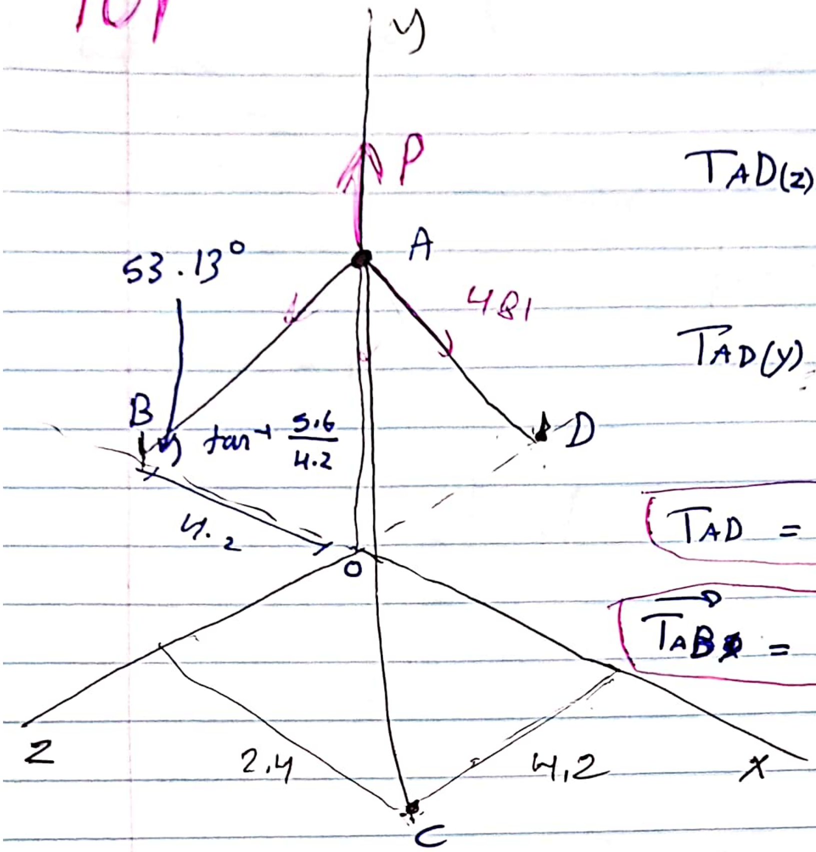
$= |T|\,\lambda$

$= 385 * \lambda$

$= 240\hat{i} - 255\hat{j} + 160\hat{k}$

$F_{Dx} = 240$ N
$F_{Dy} = -255$ N
$F_{Dz} = 160$ N

101



$T_{AD(z)} = -481 \cos \tan^{-1}\left(\frac{5.6}{3.3}\right)\hat{k}$
$= -244.2\,\hat{k}$

$T_{AD(y)} = -481 \sin \tan^{-1}\left(\frac{5.6}{3.3}\right)$
$= -414.4\,\hat{j}$

$T_{AD} = -244.2\,\hat{k} - 414.4\,\hat{j}$

$\vec{T}_{AB} = -T_{AB}\cos 53.13\,\hat{i} - T_{AB}\sin 53.13\,\hat{j}$

T_{AC} !
~~T_{AC}~~ findin λ $\vec{AC} =$

$\vec{AC} = -5.6\,\hat{j} + 2.4\,\hat{i} + 4.2\,\hat{k}$

$|AC| = 7.4 \rightarrow \lambda = -0.757\,\hat{j} + 0.324\,\hat{i} + 0.568\,\hat{k}$

$\lambda = \frac{\vec{AC}}{|AC|}$

$\vec{T}_{AC} = \lambda\,|T_{AC}|$

$\Sigma F_z = 0 \rightarrow |T_{AD}| = |T_{AC}|$

$244.2 = 0.568\,|T_{AC}|$

~~$-414.4 = -0.757\,|T_{AC}|$~~

~~$|T_{AC}| = 547.4$~~

$|T_{AC}| = 430$

$\Sigma F_x = 0$

$|T_{AB\,x}| = |T_{AC\,x}|$

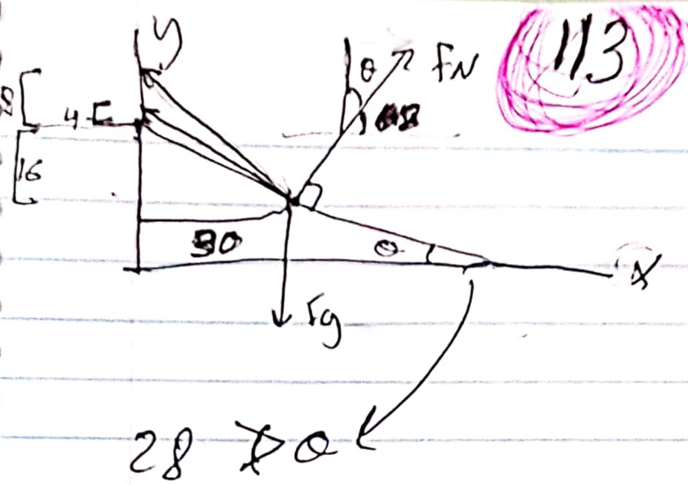
$|T_{AB}\cos 53.13| = |430 * 0.324|$

$T_{AB} = 232.16$

$\Sigma F_y = 0 \rightarrow |P| = |T_{AB\,y}| + |T_{AC\,y}| + |T_{AD\,y}|$

$185.6 + 325.5 + 414.4 = 925.5\,N = P$

113



FN is just in xy plane
fy " " " " "

$f_y = -175\,\hat{j}$

$\vec{F_N} = |F_N|\left(-\frac{16}{34}\hat{i} + \frac{30}{34}\hat{j}\right)$

$\vec{T_{AC}} = \lambda_{AC}|T_{AC}|$

$\lambda_{AC} = \frac{\vec{AC}}{|AC|} = \frac{-30\hat{i} + 20\hat{j} + 12\hat{k}}{38}$

$\vec{T_{AB}} = |T_{AB}|\lambda_{AB}$

$\lambda_{AB} = \frac{-30\hat{i} + 24\hat{j} + 32\hat{k}}{50}$

$\Sigma F_z = 0 \rightarrow |T_{ABz}| = |T_{ACz}|$

$\frac{12}{38}(T_{AC}) = \frac{32}{50}|T_{AB}|$ ... ①

$T_{AC} = 2.02667\, T_{AB}$ —— ①

~~$\Sigma F_y = 0$~~

~~$|F_g| = |F_{Ny}|$~~

~~$175 = \frac{30}{34}|F_N| \rightarrow F_N = 198.33$~~ ②

$\Sigma F_x = 0 \quad |T_{AC(x)}| + |T_{AB(x)}| = |F_{N(x)}|$

$\frac{30}{38}|T_{AC}| + \frac{30}{50}|T_{AB}| = \frac{16}{34}|F_N|$

نعوض قيمة T_{AC}

$\frac{30 * 2.02667}{38} T_{AB} + 0.6\, T_{AB} = 93.33$

$2.2\, T_{AB} = 93.33$

$T_{AB} = 42.42$ Ib /// $T_{AC} = 85.77 \simeq 86$ Ib

$\Sigma F_{N} = 0$

$F_{y} = F_{N(y)} + T_{AC(y)} + T_{AB(y)}$

$-175 = \frac{30}{34}|F_N| + \frac{20}{38}|T_{AC}| + \frac{24}{50}|T_{AB}|$ —— ②

$\Sigma F_x = 0 \rightarrow |F_{Nx}| = |T_{ACx}| + |T_{ABx}|$

$\frac{16}{34}|F_N| = \frac{30}{38}|T_{AC}| + \frac{30}{50}|T_{AB}|$ —— ③

TAB : بدلالة TAC نكتب قيمة

② → : $\frac{30}{34}|F_N| + \frac{20}{38} 2.02667\, T_{AB} + \frac{24}{50} T_{AB} = -175$

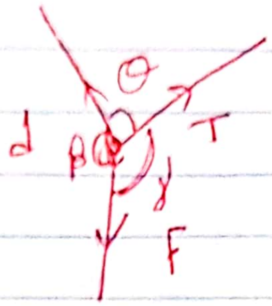
③ $* \frac{30}{16}$ = : $\frac{30}{34}|F_N| - \frac{30}{16} * \frac{30}{38} * 2.02667\, T_{AB} - \frac{30}{50} * \frac{30}{16} T_{AB} = 0$

② + $\frac{30}{16}$ ③ : $1.546667\, T_{AB} + 4.125\, T_{AB} = -175$

$T_{AB} = 37.35\, lb$

~~TAC = 637.4 lb~~

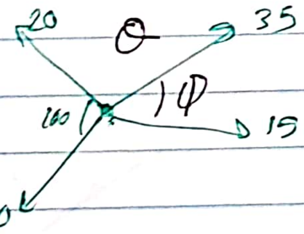
$T_{AC} = 63.54$



you can use Lami law

$$\frac{F}{\sin\theta} = \frac{T}{\sin\beta} = \frac{d}{\gamma}$$

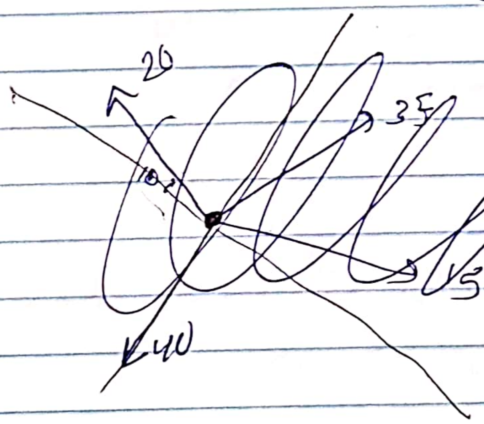
More questions



جد الزوايا θ ϕ
للوصول إلى اتزان
equilibrium

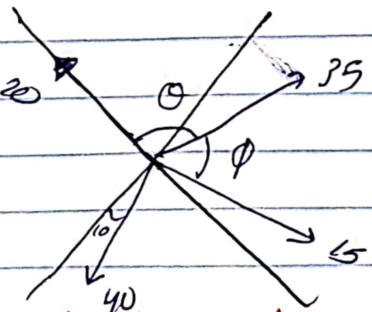
سؤال من آية يا بابا

~~$\Sigma F_x = 120 \cos 10 + 35 \cos(10+\theta) + 15 \cos(10+\theta+\phi) = 0$~~



~~$\Sigma F_x =$~~

$$\Sigma F_x = 20 + 35 \cos\theta + 15 \cos(\theta + \phi) - 40 \sin 10$$

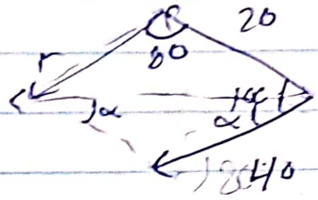


$$\Sigma F_y = 35 \sin\theta + 15 \sin\phi - 40 \cos 10 = 0$$

حل الأول
والصعب

قاعدة لامي والجيوب ليست دوماً الحل

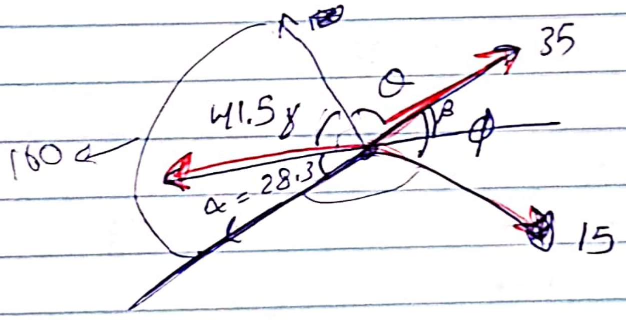
في التحليل ليست دوما الحل

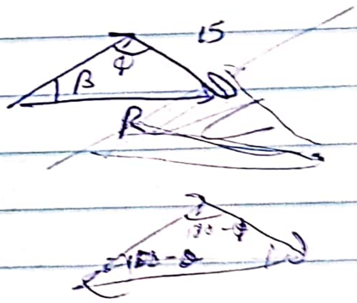


$$r = \sqrt{20^2 + 40^2 + 2 * 20 * 40 \cos 100}$$

$$= 41.5 \ N$$

$$\frac{R}{\sin 80} = \frac{20}{\sin \alpha} \Rightarrow \alpha = 28.3$$



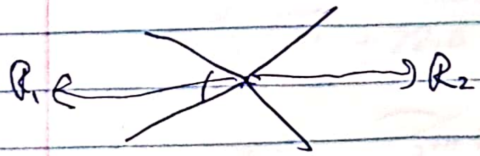


$$\frac{41.5}{\sin \phi} = \frac{15}{\sin(\theta + 71.7)} = \frac{35}{\sin(260 - \theta + \phi)}$$

مستحيل هاد حل طويل جداً وصعب

الحل الافضل — جد المحصلة لكل اثنتين على حدة

حيث تلغي المحصلتان بعضهما



$$\vec{R_1} + \vec{R_2} = 0$$

$$41.5 = \sqrt{35^2 + 15^2 + 2 * 15 * 35 \cos \phi}$$

$$\phi = 75^\circ$$

$$\frac{41.5}{\sin \phi} = \frac{15}{\sin \beta} \rightarrow \beta = 20.4, \ \gamma = 71.7 \rightarrow \theta = 88$$