Total Time Allowed = 1.5 hours, Total Number of Pages = 6

Name: Key Solution

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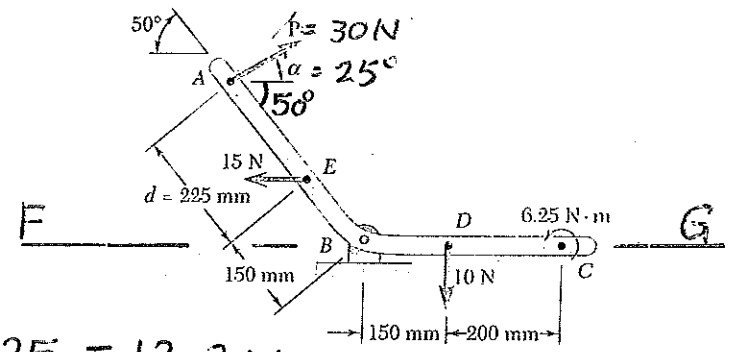
Statics ENCE 232

Mid-Term Exam **Sunday, July 30, 2017**

1. a (10 points) Replace the three applied forces and the moment with a single force and a single moment acting at point B.

b. (10 points) Determine the point of intersection of the force replacing the three applied forces and the moment with axis FG.



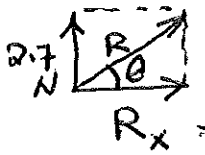
(a)

$\xrightarrow{+} \Sigma F_x = -15 + 30\cos 25 = 12.2\ N$

$+\uparrow \Sigma F_y = -10 + 30\sin 25 = 2.7\ N$

$R = \sqrt{12.2^2 + 2.7^2} = 12.5\ N$

$\theta = \tan^{-1}(2.7/12.2) = 12.5^\circ$

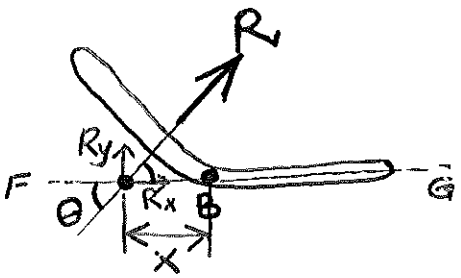


$\left(+\circlearrowleft \Sigma M_B = 6.25 - 10(0.15) + 15(0.15\sin 50) - (30\sin 75)(0.375)\right.$

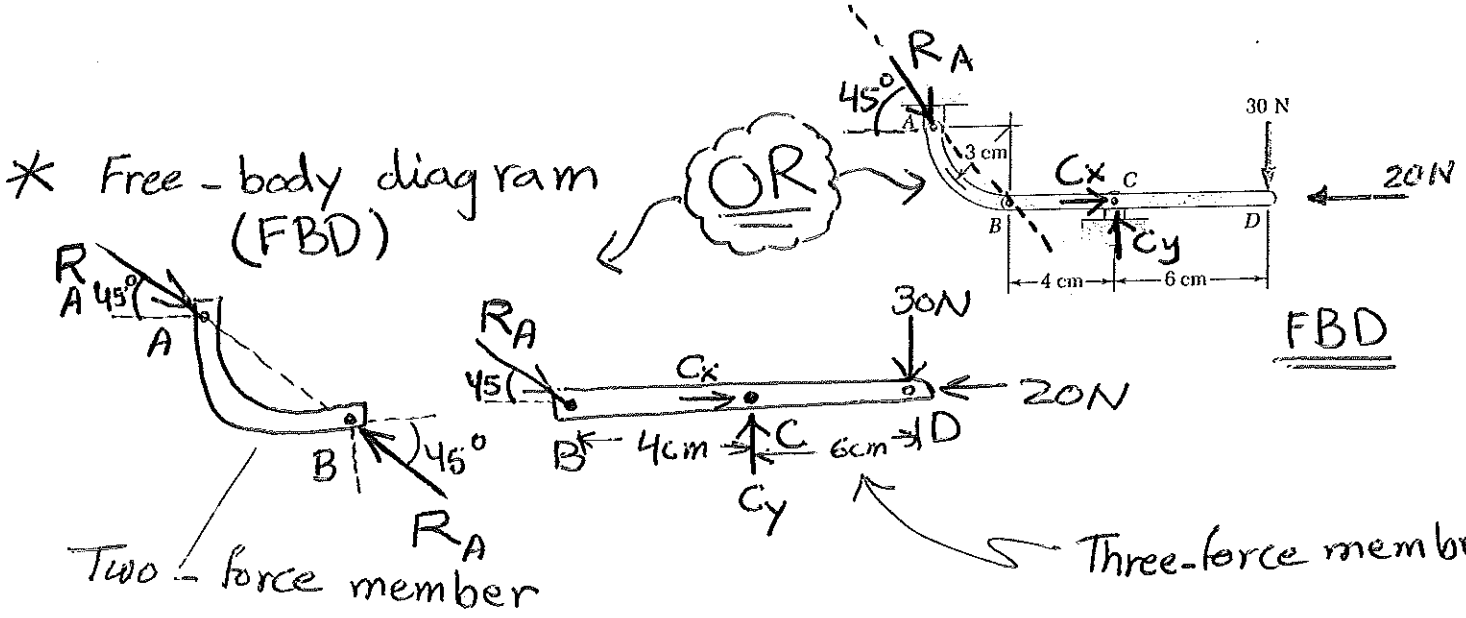
$= -4.4\ N.m$ ↷

(b) $M_B = 4.4\ N.m$ ↶ $= x\,(2.7)$ ↷

$x = 1.6\ m$ left of B



2. (20 points) Determine the reactions at A and C.



* For member BCD:

① $\curvearrowleft +\sum M_C = 0$

$$-30(6) + (R_A \sin 45)(4) = 0$$

$$R_A = 63.6\,N$$

② $\overset{+}{\rightarrow} \sum F_x = 0$:

$$63.6\cos 45 + C_x - 20 = 0$$

$$C_x = -25.0\,N \ (\leftarrow)$$

③ $\overset{+}{\uparrow} \sum F_y = 0$

$$-63.6\sin 45 + C_y - 30 = 0$$

$$C_y = 75\,N \ (\uparrow)$$

3. (25 points) Replace the forces shown with a single force and a single moment acting at the origin.

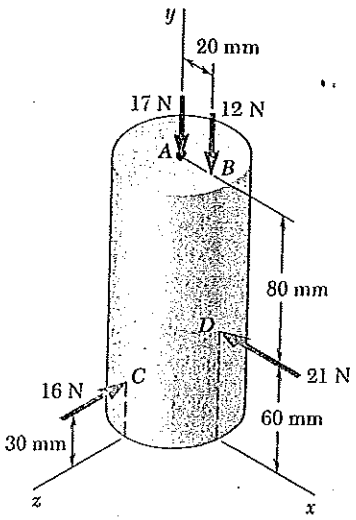
Diameter = 60 mm ← Not required

$\vec{F}_A = -17\,\mathbf{j}$ (N) $\quad \vec{r}_{OA} = 0$ (m)

$\vec{F}_B = -12\,\mathbf{j}$ (N) $\quad \vec{r}_{OB} = 0.02\,\mathbf{i}$ (m)

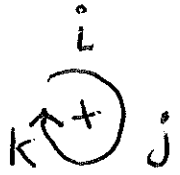
$\vec{F}_C = -16\,\mathbf{k}$ (N) $\quad \vec{r}_{OC} = 0.03\,\mathbf{j}$ (m)

$\vec{F}_D = -21\,\mathbf{i}$ (N) $\quad \vec{r}_{OD} = 0.06\,\mathbf{j}$ (m)



$$\vec{R} = \Sigma \vec{F} = -21\,\mathbf{i} - 29\,\mathbf{j} - 16\,\mathbf{k} \text{ (N)}$$

$$\Sigma \vec{M}_O^R = \Sigma (\vec{r} \times \vec{F}) = (0.02\,\mathbf{i} \times -12\,\mathbf{j}) + (0.03\,\mathbf{j} \times -16\,\mathbf{k}) + (0.06\,\mathbf{j} \times -21\,\mathbf{i})$$



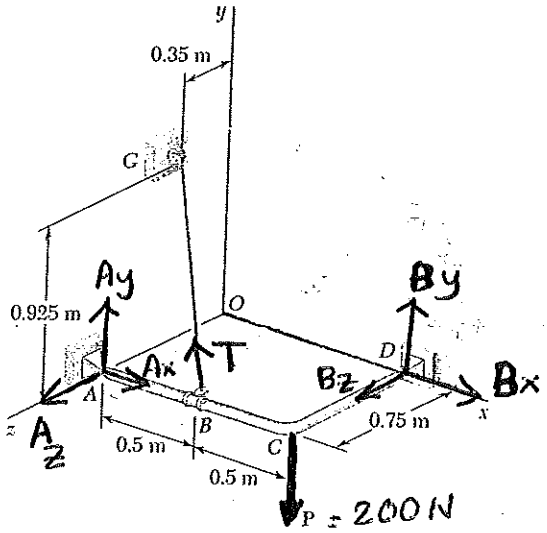
$$= -0.24\,\mathbf{k} - 0.48\,\mathbf{i} + 1.26\,\mathbf{k}$$

$$= -0.48\,\mathbf{i} + 1.02\,\mathbf{k} \text{ (N.m)}$$

4. (35 points) A and D are Ball-and-socket joints. Determine the tension in the cable BG.

Take the moment about line AD.

$$\Sigma \vec{M}_{AD} = \Sigma\left(\vec{\lambda}_{AD} \cdot [\vec{r} \times \vec{F}]\right)$$



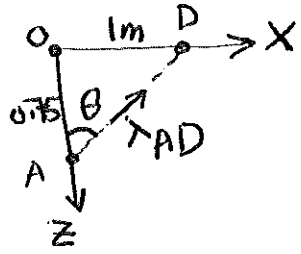
Free-body diagram (FBD)

* $\vec{P} = -200\,\hat{j}$ (N) $\quad \vec{r}_{AC} = \hat{i}$ (m)

* $\vec{T} = T\lambda_{BG} = -0.44T\,\hat{i} + 0.82T\,\hat{j} - 0.36T\,\hat{k}$

$\vec{r}_{AB} = 0.5\,\hat{i}$ (m)

* λ_{AD} :



$\theta = \tan^{-1}(1/0.75) = 53.1^\circ$

$\lambda_{AD} = \sin 53.1\,\hat{i} - \cos 53.1\,\hat{k}$

$\lambda_{AD} = 0.8\,\hat{i} - 0.6\,\hat{k}$

B(0.5, 0, 0.75)

G(0, 0.925, 0.35)

$\vec{BG} = -0.5\,\hat{i} + 0.925\,\hat{j} - 0.4\,\hat{k}$

$|\vec{BG}| = 1.125$ m

$\lambda_{BG} = \dfrac{\vec{BG}}{|\vec{BG}|} = -0.44\,\hat{i} + 0.82\,\hat{j} - 0.36\,\hat{k}$

$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -200 & 0 \end{vmatrix} + T \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.44 & 0.82 & -0.36 \end{vmatrix} = 0$$

$$-0.6(-200) + T(-0.5)(0.82 \times 0.6) = 0$$

$$T = 487.8 \text{ N}$$