

CHAPTER

# 6

## *VECTOR MECHANICS FOR ENGINEERS:* **STATICS**

Ferdinand P. Beer  
E. Russell Johnston, Jr.

Lecture Notes:  
J. Walt Oler  
Texas Tech University

### Analysis of Structures

# Vector Mechanics for Engineers: Statics

## Contents

[Introduction](#)

[Definition of a Truss](#)

[Simple Trusses](#)

[Analysis of Trusses by the Method of Joints](#)

[Joints Under Special Loading Conditions](#)

[Space Trusses](#)

[Sample Problem 6.1](#)

[Analysis of Trusses by the Method of Sections](#)

[Trusses Made of Several Simple Trusses](#)

[Sample Problem 6.3](#)

[Analysis of Frames](#)

[Frames Which Cease to be Rigid When Detached From Their Supports](#)

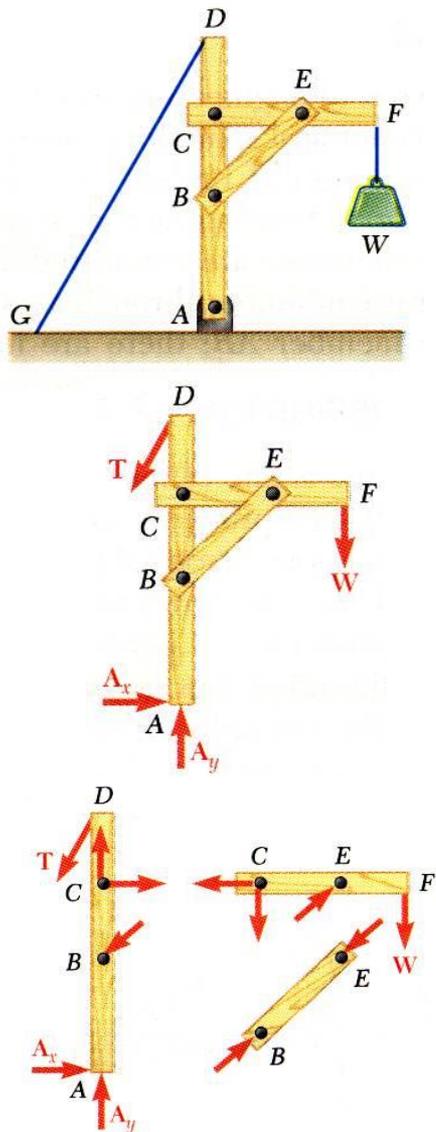
[Sample Problem 6.4](#)

[Machines](#)



# Vector Mechanics for Engineers: Statics

## Introduction

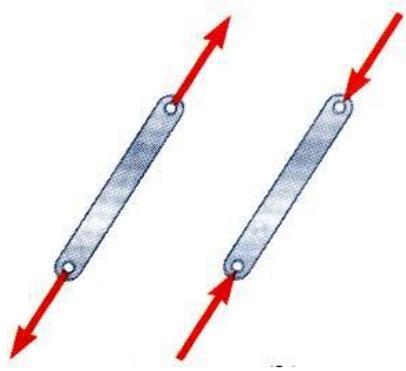
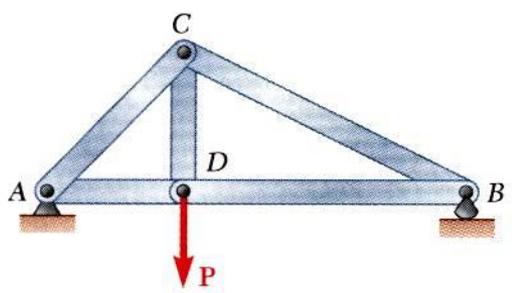
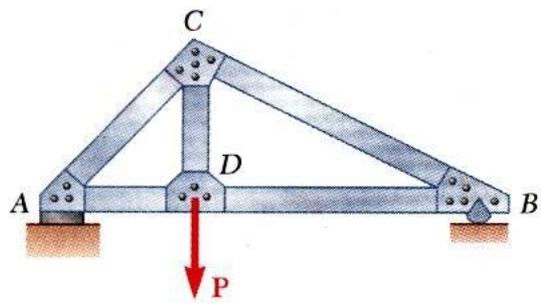


- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.
- In the interaction between connected parts, Newton's 3<sup>rd</sup> Law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite sense.
- Three categories of engineering structures are considered:
  - Frames*: contain at least one one multi-force member, i.e., member acted upon by 3 or more forces.
  - Trusses*: formed from *two-force members*, i.e., straight members with end point connections
  - Machines*: structures containing moving parts designed to transmit and modify forces.



# Vector Mechanics for Engineers: Statics

## Definition of a Truss

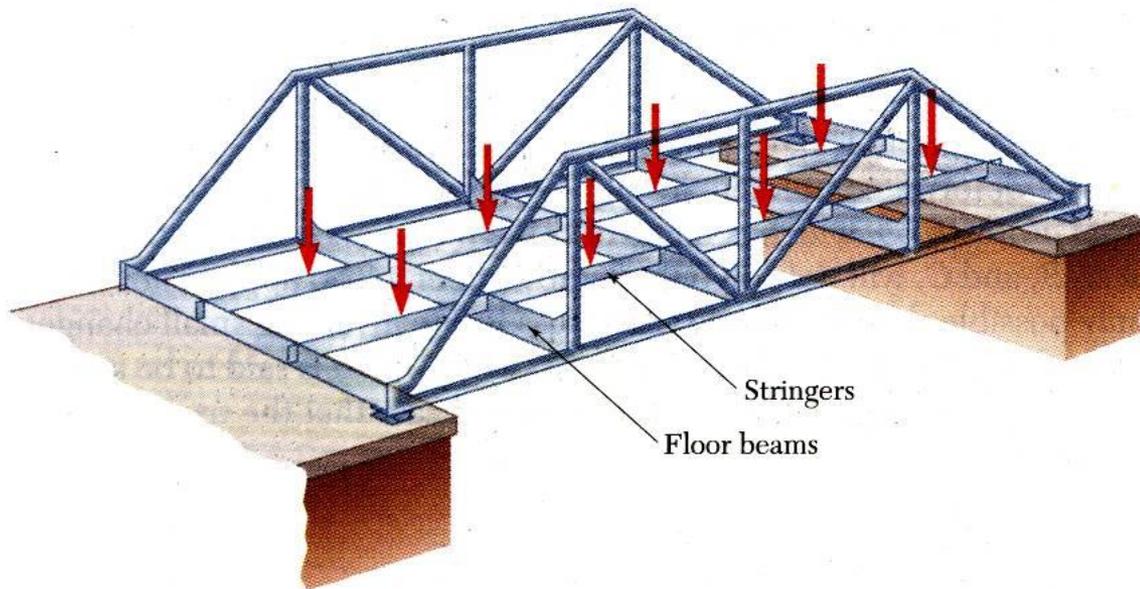


- A truss consists of straight members connected at joints. No member is continuous through a joint.
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.



# Vector Mechanics for Engineers: Statics

## Definition of a Truss

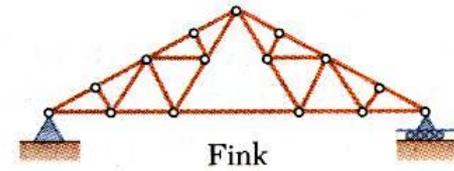
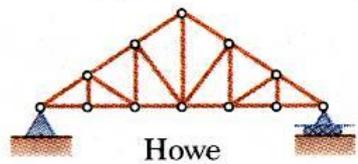
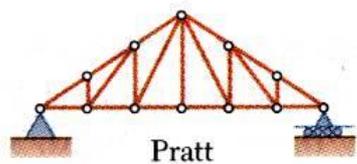


Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.

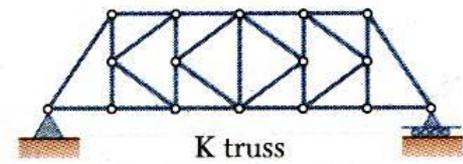
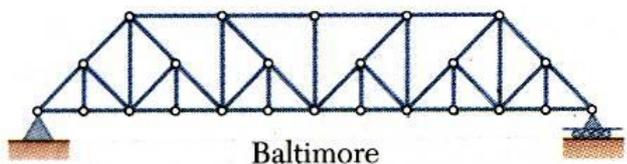
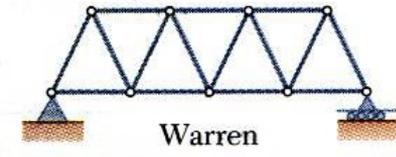
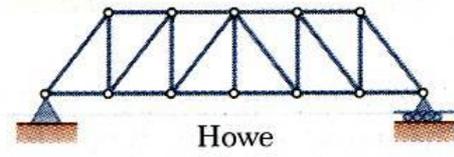
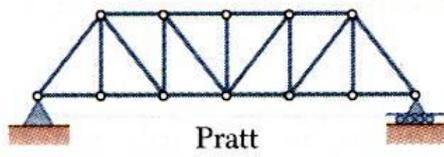


# Vector Mechanics for Engineers: Statics

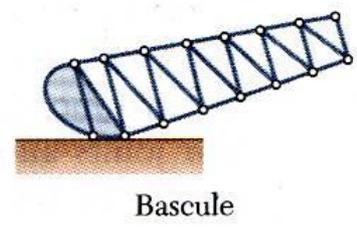
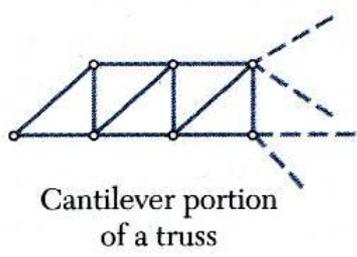
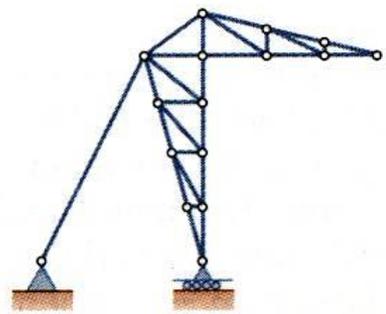
## Definition of a Truss



Typical Roof Trusses



Typical Bridge Trusses

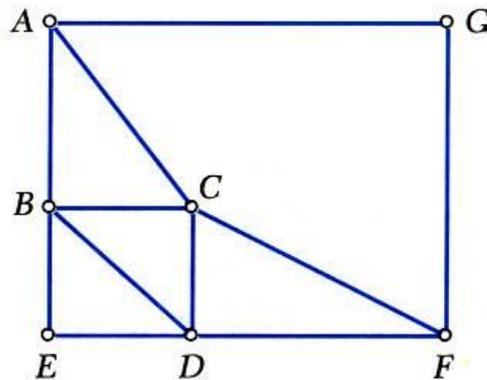
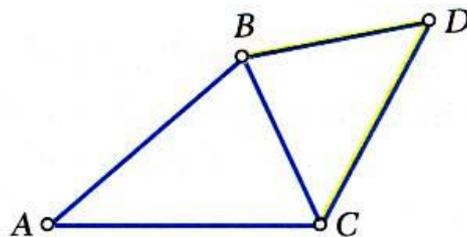
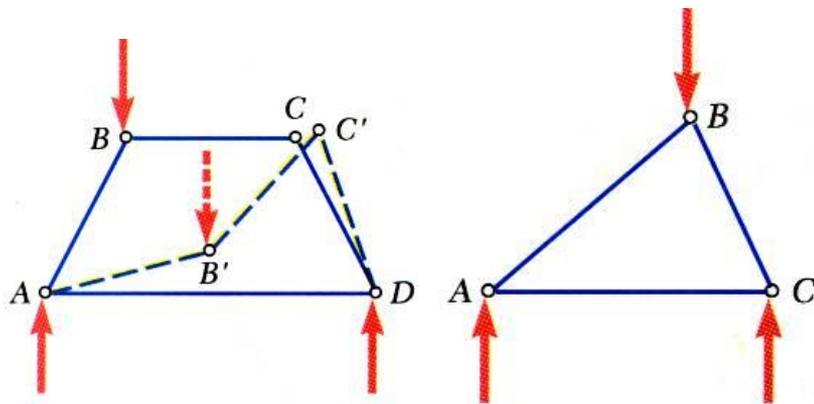


Other Types of Trusses



# Vector Mechanics for Engineers: Statics

## Simple Trusses

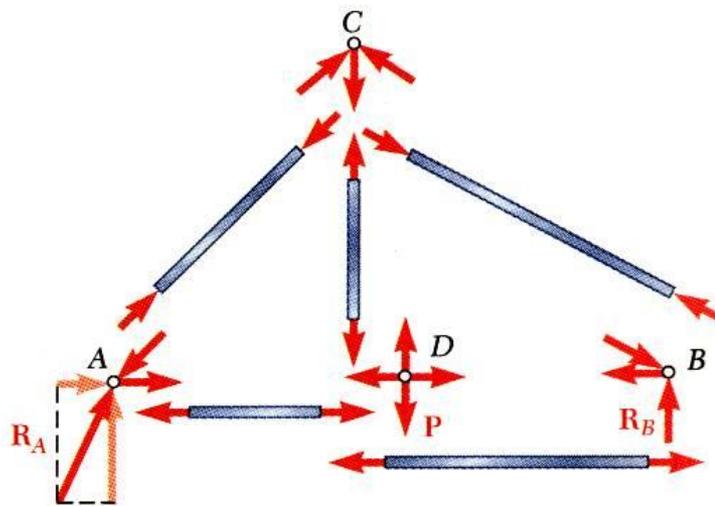
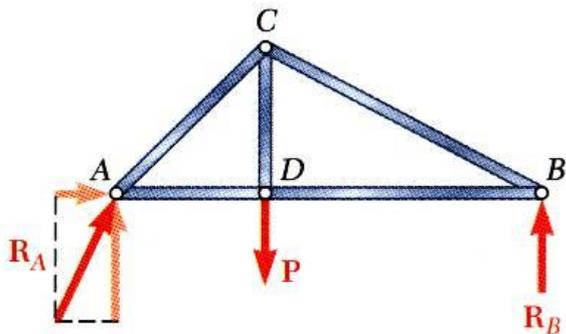


- A *rigid truss* will not collapse under the application of a load.
- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.
- In a simple truss,  $m = 2n - 3$  where  $m$  is the total number of members and  $n$  is the number of joints.



# Vector Mechanics for Engineers: Statics

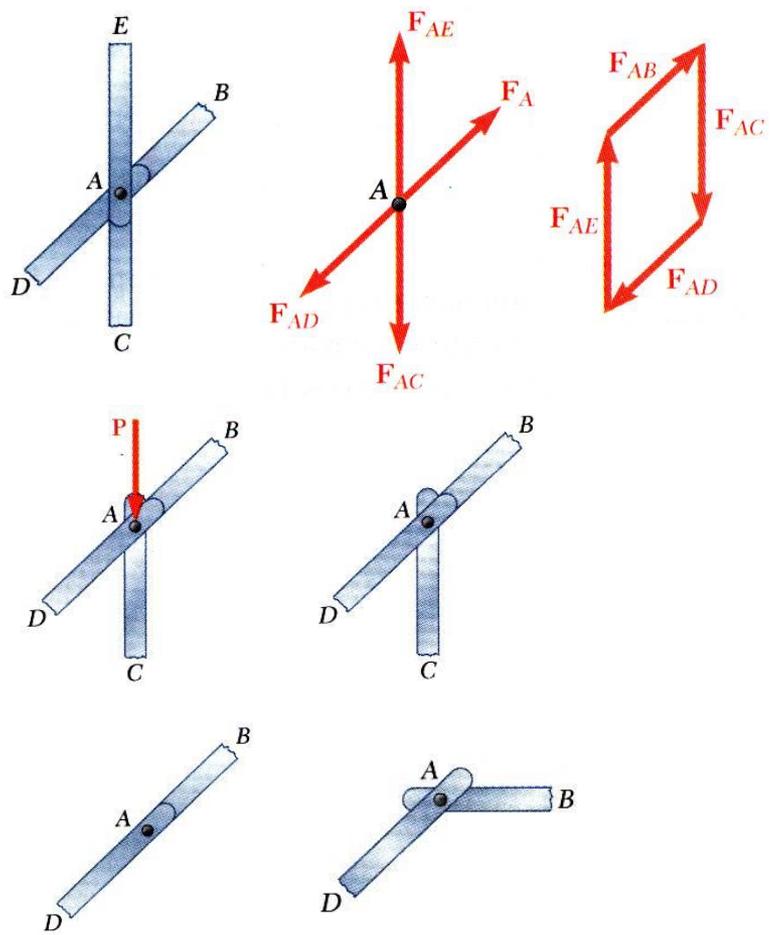
## Analysis of Trusses by the Method of Joints



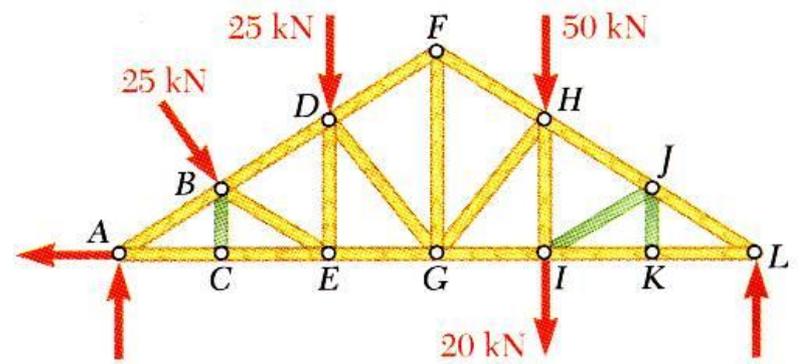
- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide  $2n$  equations for  $2n$  unknowns. For a simple truss,  $2n = m + 3$ . May solve for  $m$  member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

# Vector Mechanics for Engineers: Statics

## Joints Under Special Loading Conditions

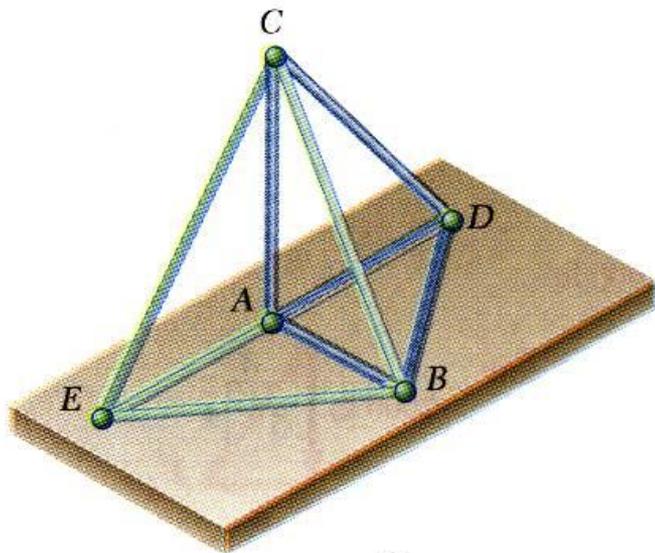
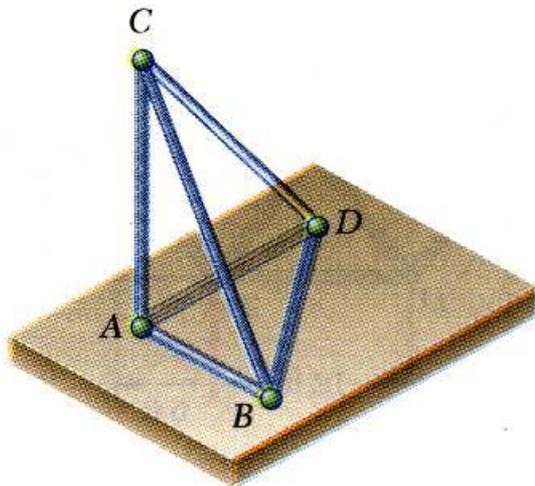


- Forces in opposite members intersecting in two straight lines at a joint are equal.
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.
- Recognition of joints under special loading conditions simplifies a truss analysis.



# Vector Mechanics for Engineers: Statics

## Space Trusses

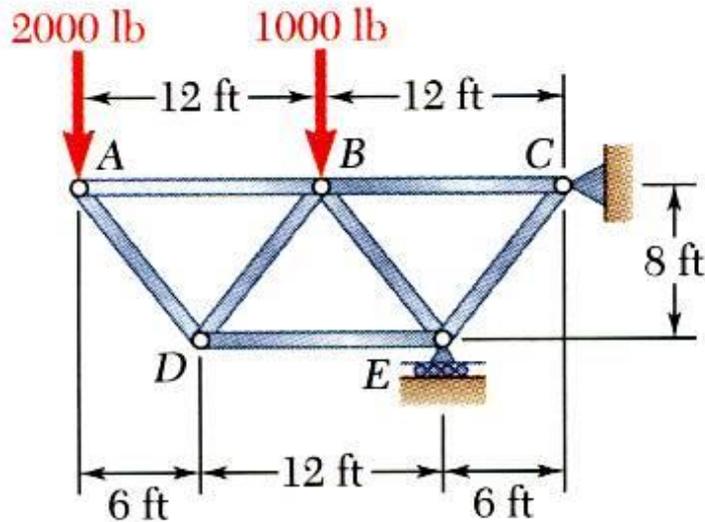


- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss,  $m = 3n - 6$  where  $m$  is the number of members and  $n$  is the number of joints.
- Conditions of equilibrium for the joints provide  $3n$  equations. For a simple truss,  $3n = m + 6$  and the equations can be solved for  $m$  member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.1



Using the method of joints, determine the force in each member of the truss.

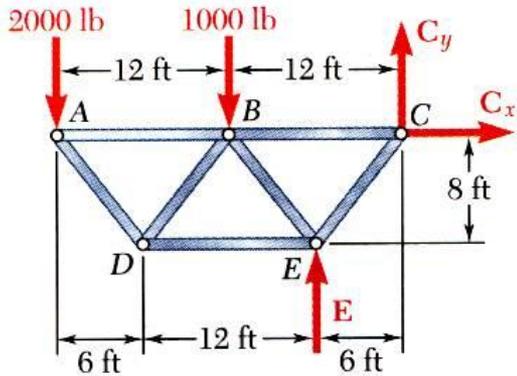
### SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at  $E$  and  $C$ .
- Joint  $A$  is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints  $D$ ,  $B$ , and  $E$  from joint equilibrium requirements.
- All member forces and support reactions are known at joint  $C$ . However, the joint equilibrium requirements may be applied to check the results.



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.1



### SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at  $E$  and  $C$ .

$$\begin{aligned} \sum M_C &= 0 \\ &= (2000 \text{ lb})(24 \text{ ft}) + (1000 \text{ lb})(12 \text{ ft}) - E(6 \text{ ft}) \end{aligned}$$

$$E = 10,000 \text{ lb } \uparrow$$

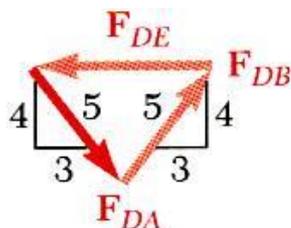
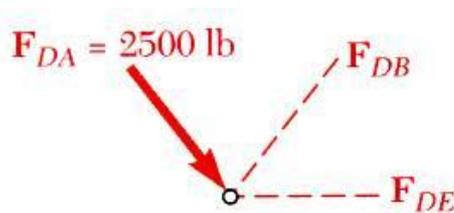
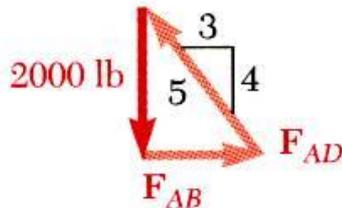
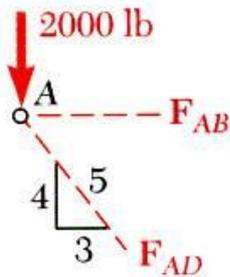
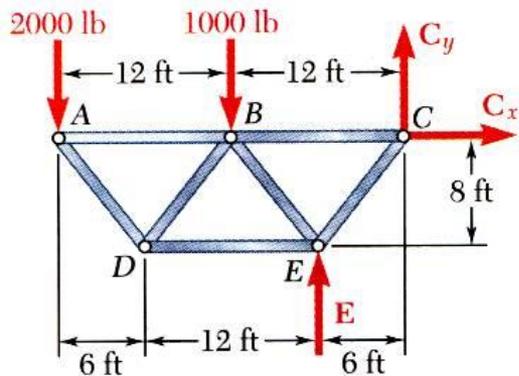
$$\sum F_x = 0 = C_x \qquad C_x = 0$$

$$\begin{aligned} \sum F_y = 0 &= -2000 \text{ lb} - 1000 \text{ lb} + 10,000 \text{ lb} + C_y \\ C_y &= 7000 \text{ lb } \downarrow \end{aligned}$$



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.1



- Joint  $A$  is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{2000 \text{ lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$F_{AB} = 1500 \text{ lb } T$$

$$F_{AD} = 2500 \text{ lb } C$$

- There are now only two unknown member forces at joint  $D$ .

$$F_{DB} = F_{DA}$$

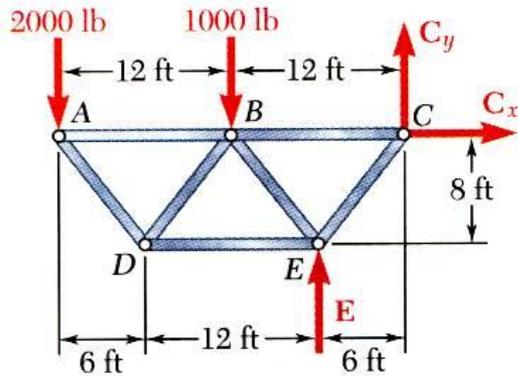
$$F_{DE} = 2\left(\frac{3}{5}\right)F_{DA}$$

$$F_{DB} = 2500 \text{ lb } T$$

$$F_{DE} = 3000 \text{ lb } C$$

# Vector Mechanics for Engineers: Statics

## Sample Problem 6.1



- There are now only two unknown member forces at joint B. Assume both are in tension.

$$\sum F_y = 0 = -1000 - \frac{4}{5}(2500) - \frac{4}{5}F_{BE}$$

$$F_{BE} = -3750 \text{ lb}$$

$$F_{BE} = 3750 \text{ lb } C$$

$$\sum F_x = 0 = F_{BC} - 1500 - \frac{3}{5}(2500) - \frac{3}{5}(3750)$$

$$F_{BC} = +5250 \text{ lb}$$

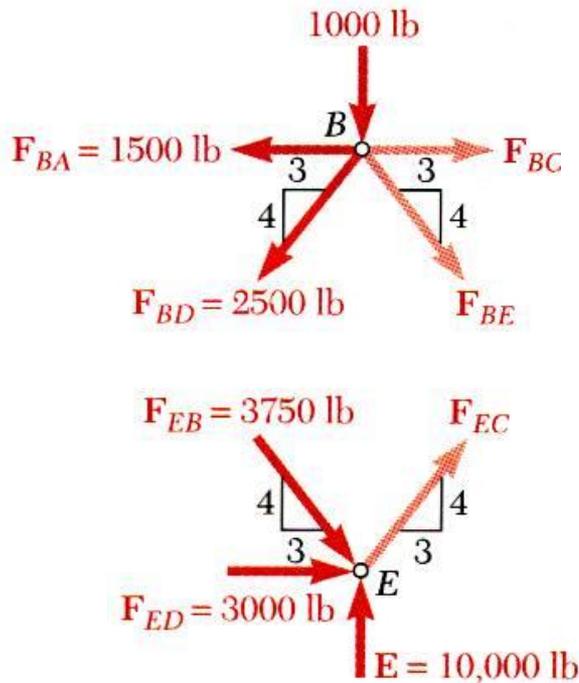
$$F_{BC} = 5250 \text{ lb } T$$

- There is one unknown member force at joint E. Assume the member is in tension.

$$\sum F_x = 0 = \frac{3}{5}F_{EC} + 3000 + \frac{3}{5}(3750)$$

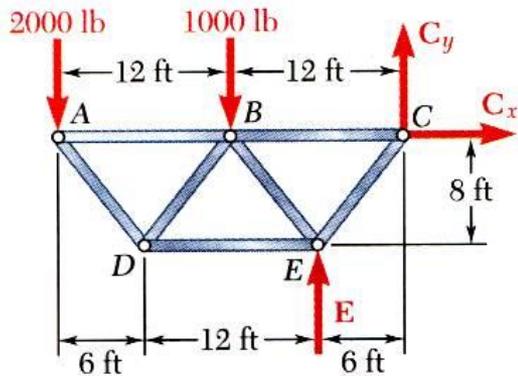
$$F_{EC} = -8750 \text{ lb}$$

$$F_{EC} = 8750 \text{ lb } C$$



# Vector Mechanics for Engineers: Statics

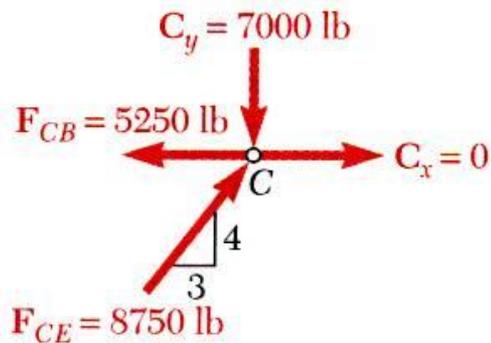
## Sample Problem 6.1



- All member forces and support reactions are known at joint  $C$ . However, the joint equilibrium requirements may be applied to check the results.

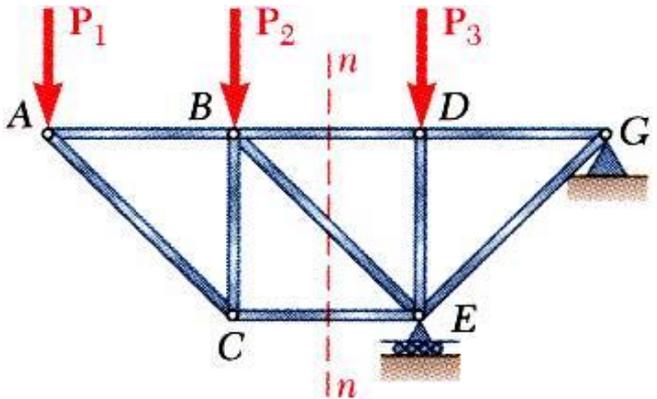
$$\sum F_x = -5250 + \frac{3}{5}(8750) = 0 \quad (\text{checks})$$

$$\sum F_y = -7000 + \frac{4}{5}(8750) = 0 \quad (\text{checks})$$

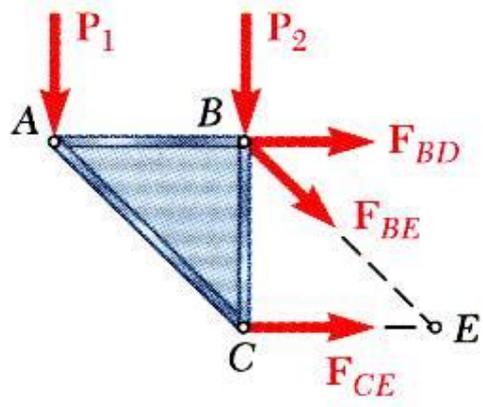


# Vector Mechanics for Engineers: Statics

## Analysis of Trusses by the Method of Sections

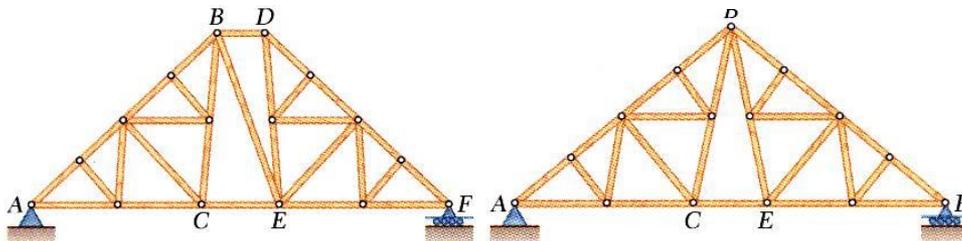


- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member  $BD$ , *pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including  $F_{BD}$ .



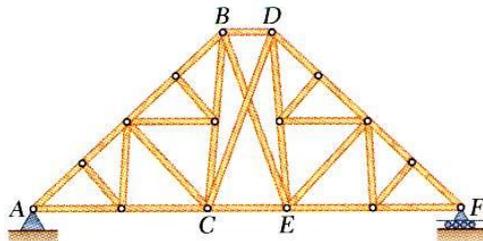
# Vector Mechanics for Engineers: Statics

## Trusses Made of Several Simple Trusses



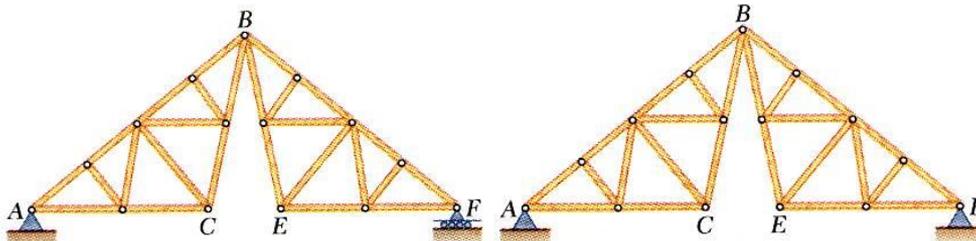
- *Compound trusses* are statically determinant, rigid, and completely constrained.

$$m = 2n - 3$$



- Truss contains a *redundant member* and is *statically indeterminate*.

$$m > 2n - 3$$



*non-rigid*

$$m < 2n - 3$$

*rigid*

$$m < 2n - 4$$

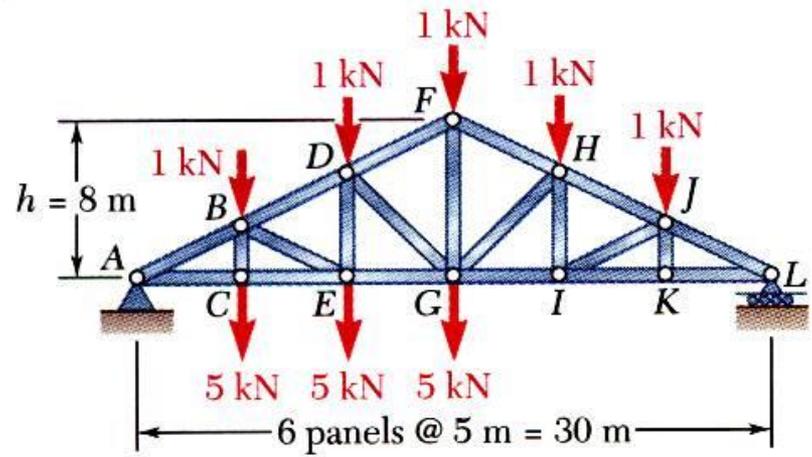
- Additional reaction forces may be necessary for a rigid truss.
- Necessary but insufficient condition for a compound truss to be statically determinant, rigid, and completely constrained,

$$m + r = 2n$$



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.3



### SOLUTION:

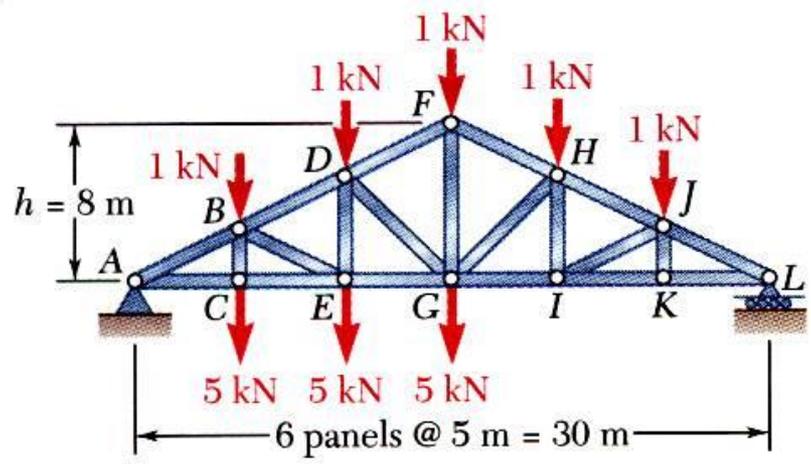
- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at  $A$  and  $L$ .
- Pass a section through members  $FH$ ,  $GH$ , and  $GI$  and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.

Determine the force in members  $FH$ ,  $GH$ , and  $GI$ .



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.3



### SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at  $A$  and  $L$ .

$$\sum M_A = 0 = -(5 \text{ m})(6 \text{ kN}) - (10 \text{ m})(6 \text{ kN}) - (15 \text{ m})(6 \text{ kN}) - (20 \text{ m})(1 \text{ kN}) - (25 \text{ m})(1 \text{ kN}) + (25 \text{ m})L$$

$$L = 7.5 \text{ kN } \uparrow$$

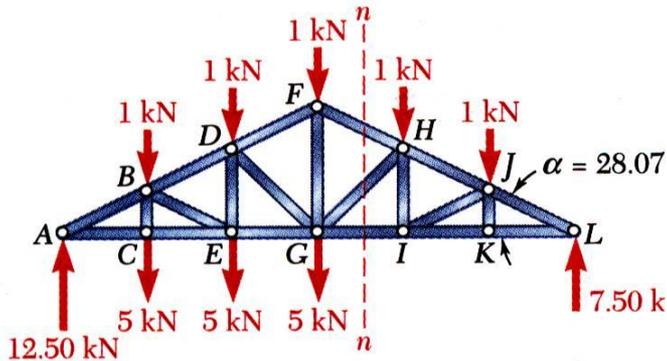
$$\sum F_y = 0 = -20 \text{ kN} + L + A$$

$$A = 12.5 \text{ kN } \uparrow$$



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.3



- Pass a section through members  $FH$ ,  $GH$ , and  $GI$  and take the right-hand section as a free body.

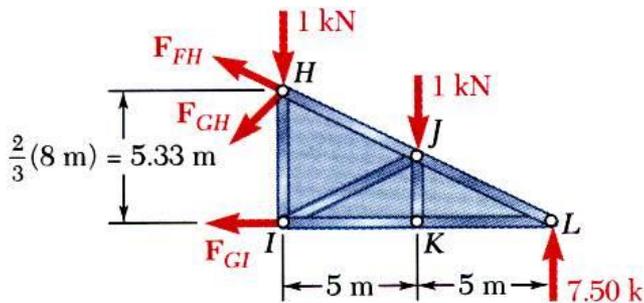
- Apply the conditions for static equilibrium to determine the desired member forces.

$$\sum M_H = 0$$

$$(7.50 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) - F_{GI}(5.33 \text{ m}) = 0$$

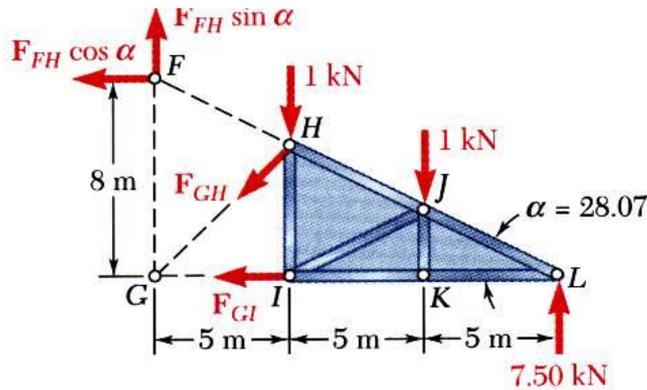
$$F_{GI} = +13.13 \text{ kN}$$

$$F_{GI} = 13.13 \text{ kN } T$$



# Vector Mechanics for Engineers: Statics

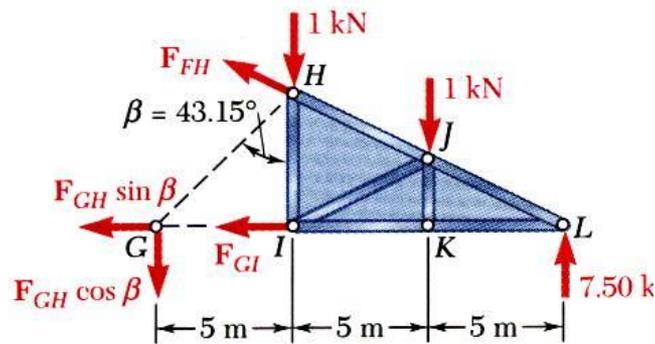
## Sample Problem 6.3



$$\tan \alpha = \frac{FG}{GL} = \frac{8 \text{ m}}{15 \text{ m}} = 0.5333 \quad \alpha = 28.07^\circ$$

$$\begin{aligned} \sum M_G = 0 \\ (7.5 \text{ kN})(15 \text{ m}) - (1 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) \\ + (F_{FH} \cos \alpha)(8 \text{ m}) = 0 \\ F_{FH} = -13.82 \text{ kN} \end{aligned}$$

$$F_{FH} = 13.82 \text{ kN } C$$



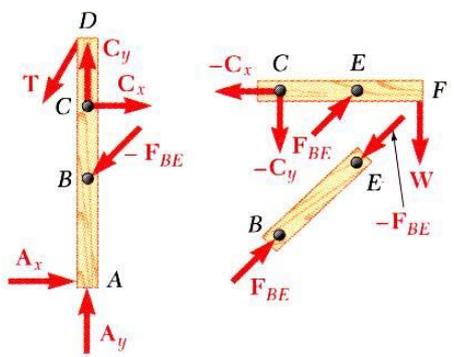
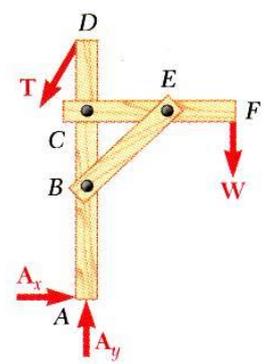
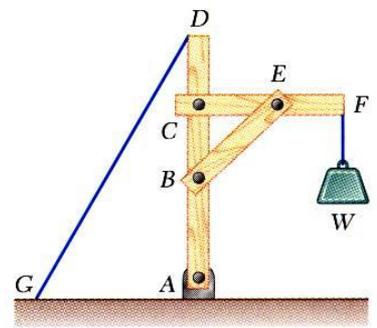
$$\tan \beta = \frac{GI}{HI} = \frac{5 \text{ m}}{\frac{2}{3}(8 \text{ m})} = 0.9375 \quad \beta = 43.15^\circ$$

$$\begin{aligned} \sum M_L = 0 \\ (1 \text{ kN})(10 \text{ m}) + (1 \text{ kN})(5 \text{ m}) + (F_{GH} \cos \beta)(10 \text{ m}) = 0 \\ F_{GH} = -1.371 \text{ kN} \end{aligned}$$

$$F_{GH} = 1.371 \text{ kN } C$$

# Vector Mechanics for Engineers: Statics

## Analysis of Frames

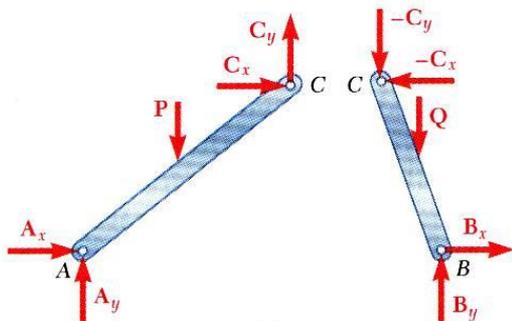
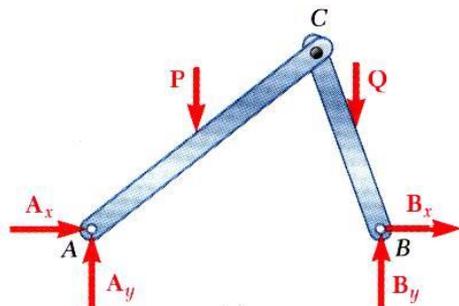
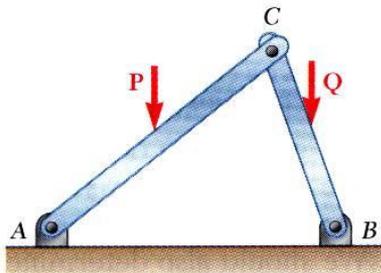


- *Frames* and *machines* are structures with at least one *multiforce* member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.
- Forces between connected components are equal, have the same line of action, and opposite sense.



# Vector Mechanics for Engineers: Statics

## Frames Which Cease To Be Rigid When Detached From Their Supports

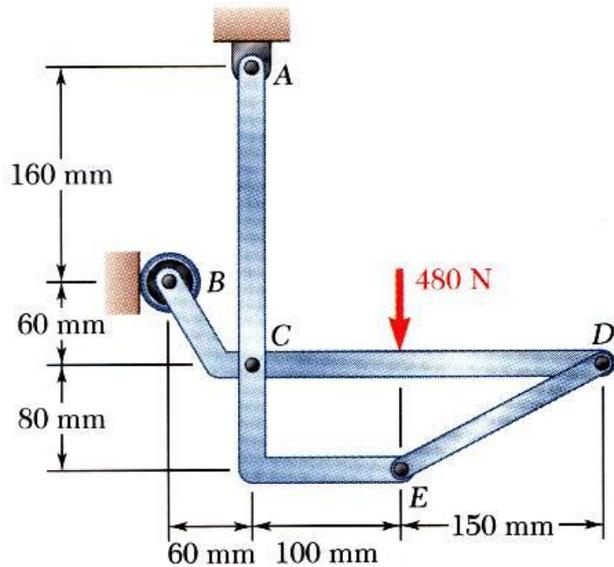


- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.
- A free-body diagram of the complete frame indicates four unknown force components which can not be determined from the three equilibrium conditions.
- The frame must be considered as two distinct, but related, rigid bodies.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams indicate 6 unknown force components.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations.



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.4



### SOLUTION:

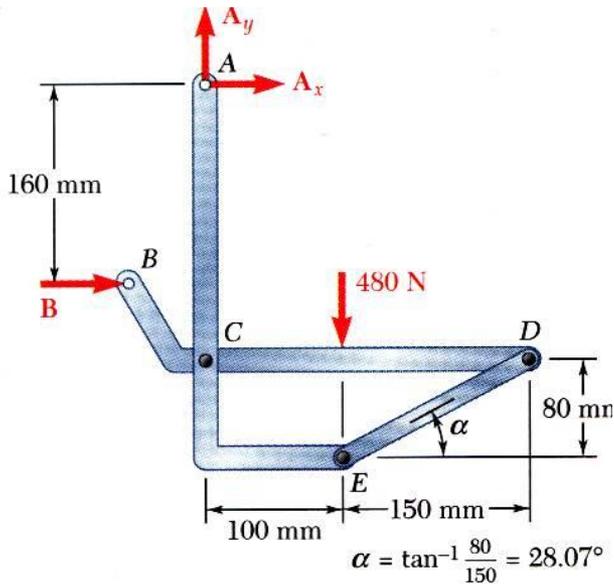
- Create a free-body diagram for the complete frame and solve for the support reactions.
- Define a free-body diagram for member  $BCD$ . The force exerted by the link  $DE$  has a known line of action but unknown magnitude. It is determined by summing moments about  $C$ .
- With the force on the link  $DE$  known, the sum of forces in the  $x$  and  $y$  directions may be used to find the force components at  $C$ .
- With member  $ACE$  as a free-body, check the solution by summing moments about  $A$ .

Members  $ACE$  and  $BCD$  are connected by a pin at  $C$  and by the link  $DE$ . For the loading shown, determine the force in link  $DE$  and the components of the force exerted at  $C$  on member  $BCD$ .



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.4



### SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480 \text{ N}$$

$$A_y = 480 \text{ N } \uparrow$$

$$\sum M_A = 0 = -(480 \text{ N})(100 \text{ mm}) + B(160 \text{ mm})$$

$$B = 300 \text{ N } \rightarrow$$

$$\sum F_x = 0 = B + A_x$$

$$A_x = -300 \text{ N } \leftarrow$$

Note:

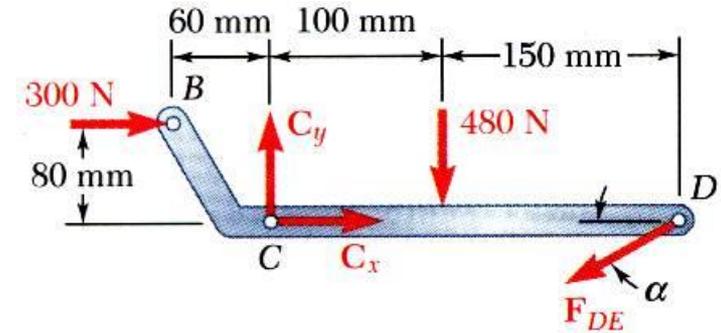
$$\alpha = \tan^{-1} \frac{80}{150} = 28.07^\circ$$



# Vector Mechanics for Engineers: Statics

## Sample Problem 6.4

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.



$$\sum M_C = 0 = (F_{DE} \sin \alpha)(250 \text{ mm}) + (300 \text{ N})(60 \text{ mm}) + (480 \text{ N})(100 \text{ mm})$$

$$F_{DE} = -561 \text{ N}$$

$$F_{DE} = 561 \text{ N } \quad C$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

$$\sum F_x = 0 = C_x - F_{DE} \cos \alpha + 300 \text{ N}$$

$$0 = C_x - (-561 \text{ N}) \cos \alpha + 300 \text{ N}$$

$$C_x = -795 \text{ N}$$

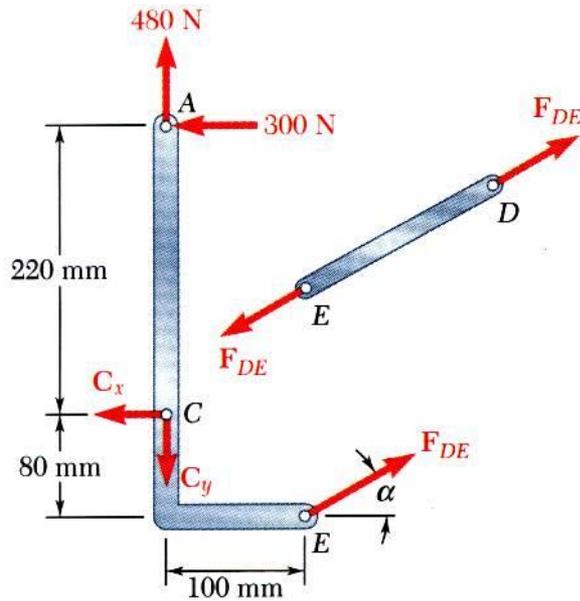
$$\sum F_y = 0 = C_y - F_{DE} \sin \alpha - 480 \text{ N}$$

$$0 = C_y - (-561 \text{ N}) \sin \alpha - 480 \text{ N}$$

$$C_y = 216 \text{ N}$$

# Vector Mechanics for Engineers: Statics

## Sample Problem 6.4



- With member  $ACE$  as a free-body, check the solution by summing moments about  $A$ .

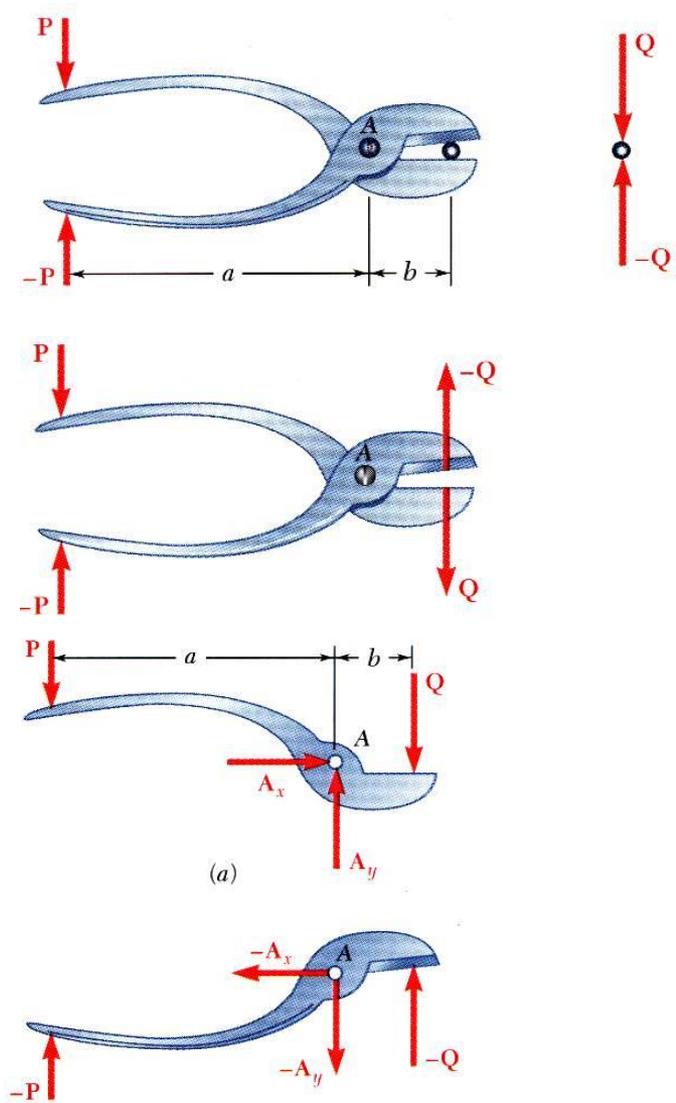
$$\begin{aligned} \sum M_A &= (F_{DE} \cos \alpha)(300 \text{ mm}) + (F_{DE} \sin \alpha)(100 \text{ mm}) - C_x(220 \text{ mm}) \\ &= (-561 \cos \alpha)(300 \text{ mm}) + (-561 \sin \alpha)(100 \text{ mm}) - (-795)(220 \text{ mm}) = 0 \end{aligned}$$

(checks)



# Vector Mechanics for Engineers: Statics

## Machines



- Machines are structures designed to transmit and modify forces. Their main purpose is to transform *input forces* into *output forces*.
- Given the magnitude of  $P$ , determine the magnitude of  $Q$ .
- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.
- The machine is a nonrigid structure. Use one of the components as a free-body.
- Taking moments about  $A$ ,

$$\sum M_A = 0 = aP - bQ \quad Q = \frac{a}{b}P$$