Eighth Edition

CHAPTER

7

VECTOR MECHANICS FOR ENGINEERS:
STATICS

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Forces in Beams and Cables

Contents

Introduction

- Preceding chapters dealt with:

  a) determining external forces acting on a structure and

  b) determining forces which hold together the various members of a structure.

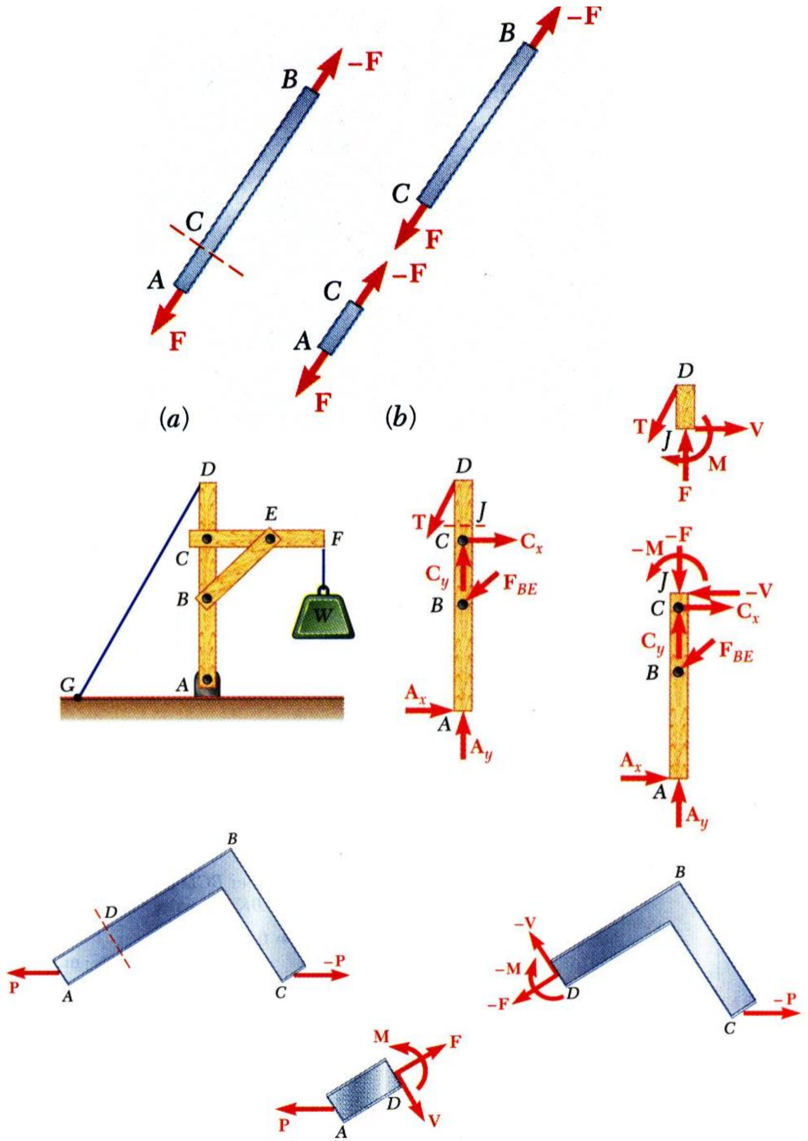
- The current chapter is concerned with determining the *internal forces* (i.e., tension/compression, shear, and bending) which hold together the various parts of a given member.

- Focus is on two important types of engineering structures:

  a) Beams - usually long, straight, prismatic members designed to support loads applied at various points along the member.

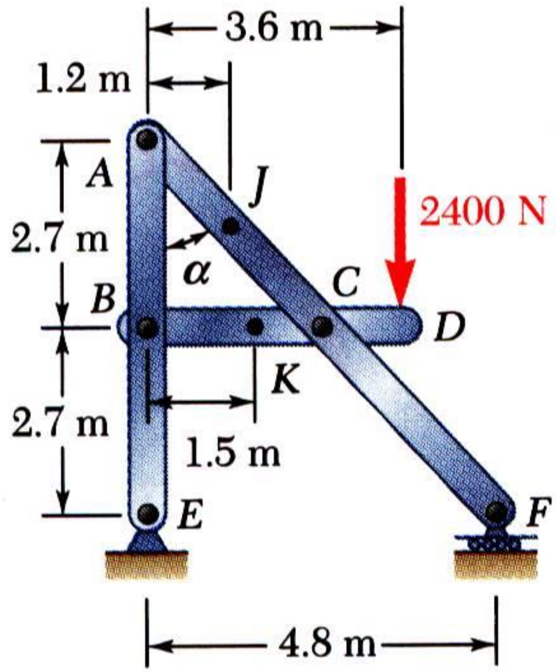
  b) Cables - flexible members capable of withstanding only tension, designed to support concentrated or distributed loads.

Internal Forces in Members



- Straight two-force member *AB* is in equilibrium under application of *F* and *-F*.
- *Internal forces* equivalent to *F* and *-F* are required for equilibrium of free-bodies *AC* and *CB*.
- Multiforce member *ABCD* is in equilibrium under application of cable and member contact forces.
- Internal forces equivalent to a force-couple system are necessary for equilibrium of free-bodies *JD* and *ABCJ*.
- An internal force-couple system is required for equilibrium of two-force members which are not straight.

Sample Problem 7.1



Determine the internal forces *(a)* in member *ACF* at point *J* and *(b)* in member *BCD* at *K*.

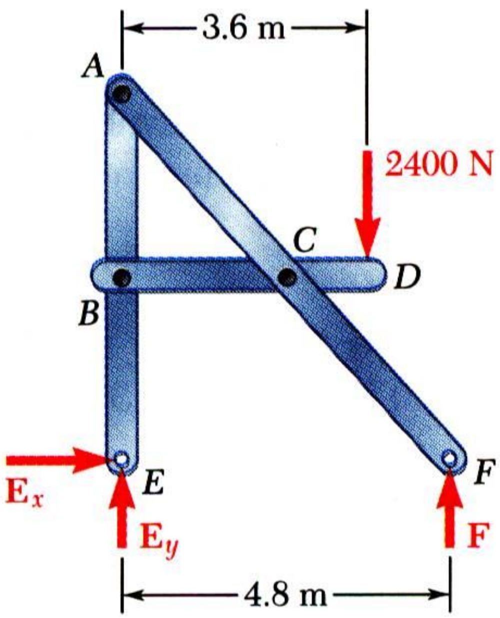
SOLUTION:

- Compute reactions and forces at connections for each member.

- Cut member *ACF* at *J*. The internal forces at *J* are represented by equivalent force-couple system which is determined by considering equilibrium of either part.

- Cut member *BCD* at *K*. Determine force-couple system equivalent to internal forces at *K* by applying equilibrium conditions to either part.

Sample Problem 7.1



SOLUTION:

- Compute reactions and connection forces.

Consider entire frame as a free-body:

$$\sum M_E = 0:$$

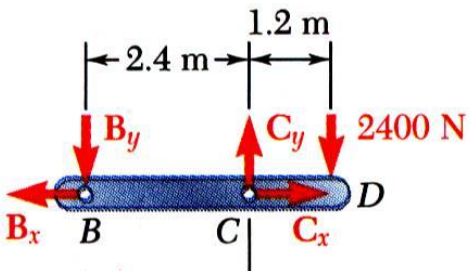
$$-(2400\,\mathrm{N})(3.6\,\mathrm{m}) + F(4.8\,\mathrm{m}) = 0 \qquad F = 1800\,\mathrm{N}$$

$$\sum F_y = 0:$$

$$-2400\,\mathrm{N} + 1800\,\mathrm{N} + E_y = 0 \qquad E_y = 600\,N$$

$$\sum F_x = 0: \qquad E_x = 0$$

Sample Problem 7.1

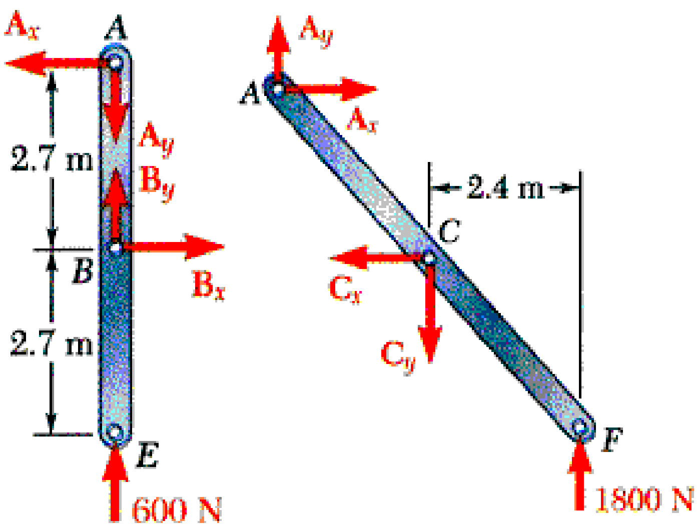


Consider member *BCD* as free-body:

$$\sum M_B = 0:$$
$$-(2400\,\mathrm{N})(3.6\,\mathrm{m}) + C_y(2.4\,\mathrm{m}) = 0 \qquad C_y = 3600\,\mathrm{N}$$

$$\sum M_C = 0:$$
$$-(2400\,\mathrm{N})(1.2\,\mathrm{m}) + B_y(2.4\,\mathrm{m}) = 0 \qquad B_y = 1200\,\mathrm{N}$$

$$\sum F_x = 0: \qquad -B_x + C_x = 0$$



Consider member *ABE* as free-body:

$$\sum M_A = 0: \qquad B_x(2.4\,\mathrm{m}) = 0 \qquad B_x = 0$$

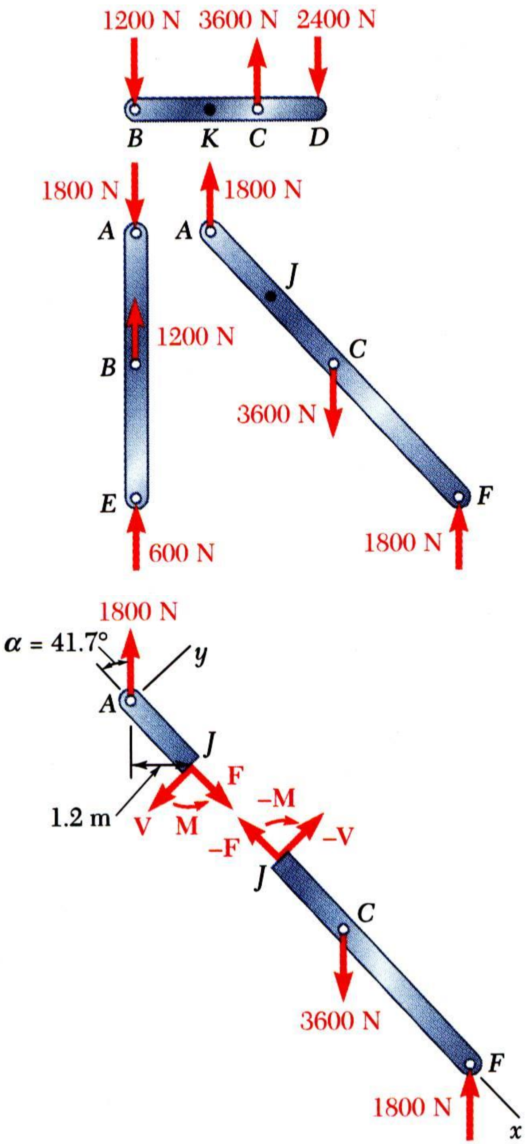
$$\sum F_x = 0: \qquad B_x - A_x = 0 \qquad A_x = 0$$

$$\sum F_y = 0: \qquad -A_y + B_y + 600\,\mathrm{N} = 0 \qquad A_y = 1800\,\mathrm{N}$$

From member *BCD*,

$$\sum F_x = 0: \qquad -B_x + C_x = 0 \qquad C_x = 0$$

Sample Problem 7.1



- Cut member ACF at J. The internal forces at J are represented by equivalent force-couple system.

Consider free-body AJ:

$$\sum M_J = 0:$$

$$-(1800\,\mathrm{N})(1.2\,\mathrm{m}) + M = 0 \qquad \boxed{M = 2160\,\mathrm{N\cdot m}}$$

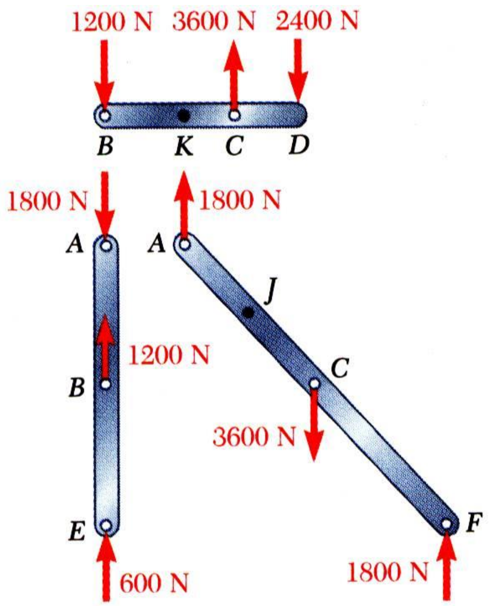
$$\sum F_x = 0:$$

$$F - (1800\,\mathrm{N})\cos 41.7^\circ = 0 \qquad \boxed{F = 1344\,\mathrm{N}}$$

$$\sum F_y = 0:$$

$$-V + (1800\,\mathrm{N})\sin 41.7^\circ = 0 \qquad \boxed{V = 1197\,\mathrm{N}}$$

Sample Problem 7.1



- Cut member *BCD* at *K*. Determine a force-couple system equivalent to internal forces at *K*.

Consider free-body *BK*:

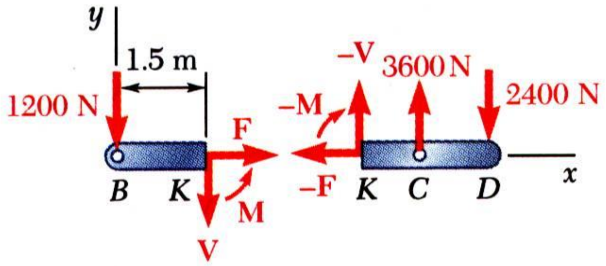
$$\sum M_K = 0:$$

$$(1200\,\mathrm{N})(1.5\,\mathrm{m}) + M = 0 \qquad \boxed{M = -1800\,\mathrm{N\cdot m}}$$

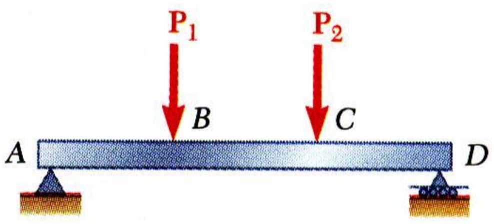
$$\sum F_x = 0: \qquad \boxed{F = 0}$$

$$\sum F_y = 0:$$

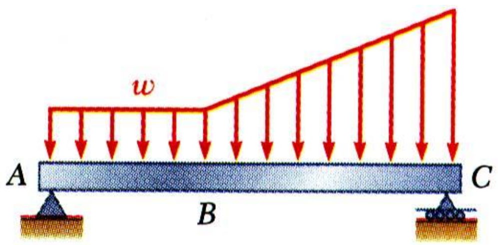
$$-1200\,\mathrm{N} - V = 0 \qquad \boxed{V = -1200\,\mathrm{N}}$$



Various Types of Beam Loading and Support



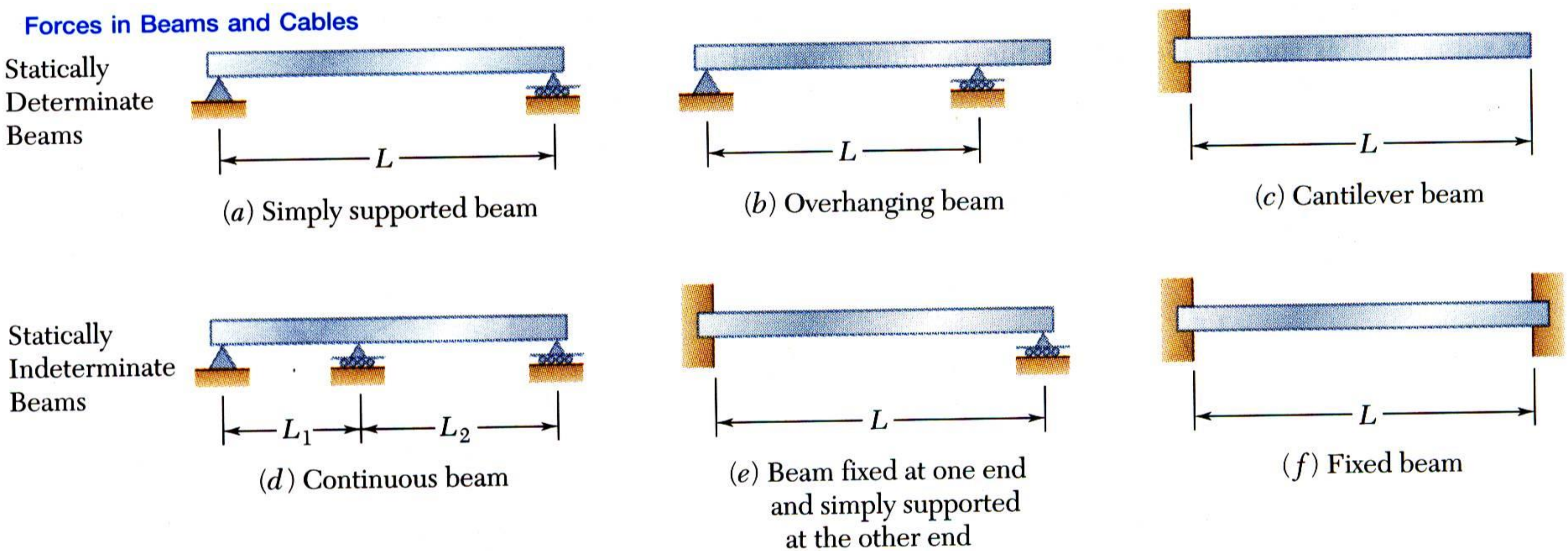
(a) Concentrated loads



(b) Distributed load

- *Beam* - structural member designed to support loads applied at various points along its length.
- Beam can be subjected to *concentrated* loads or *distributed* loads or combination of both.
- *Beam design* is two-step process:
  1) determine shearing forces and bending moments produced by applied loads
  2) select cross-section best suited to resist shearing forces and bending moments

Various Types of Beam Loading and Support



- Beams are classified according to way in which they are supported.
- Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.

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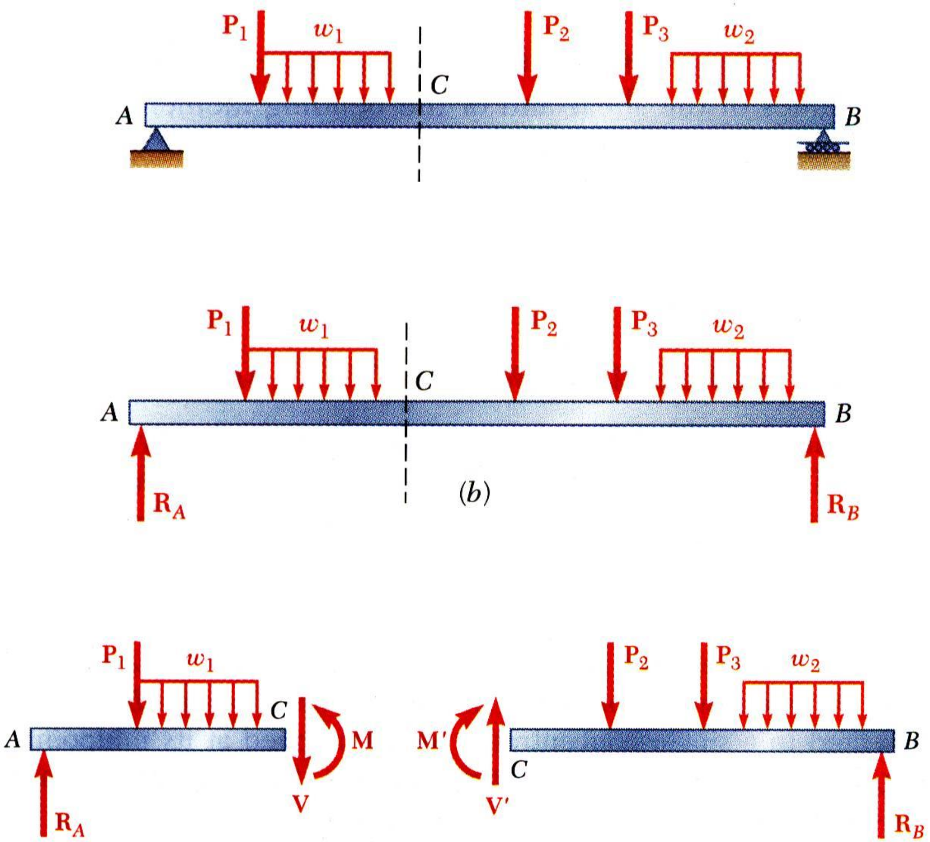
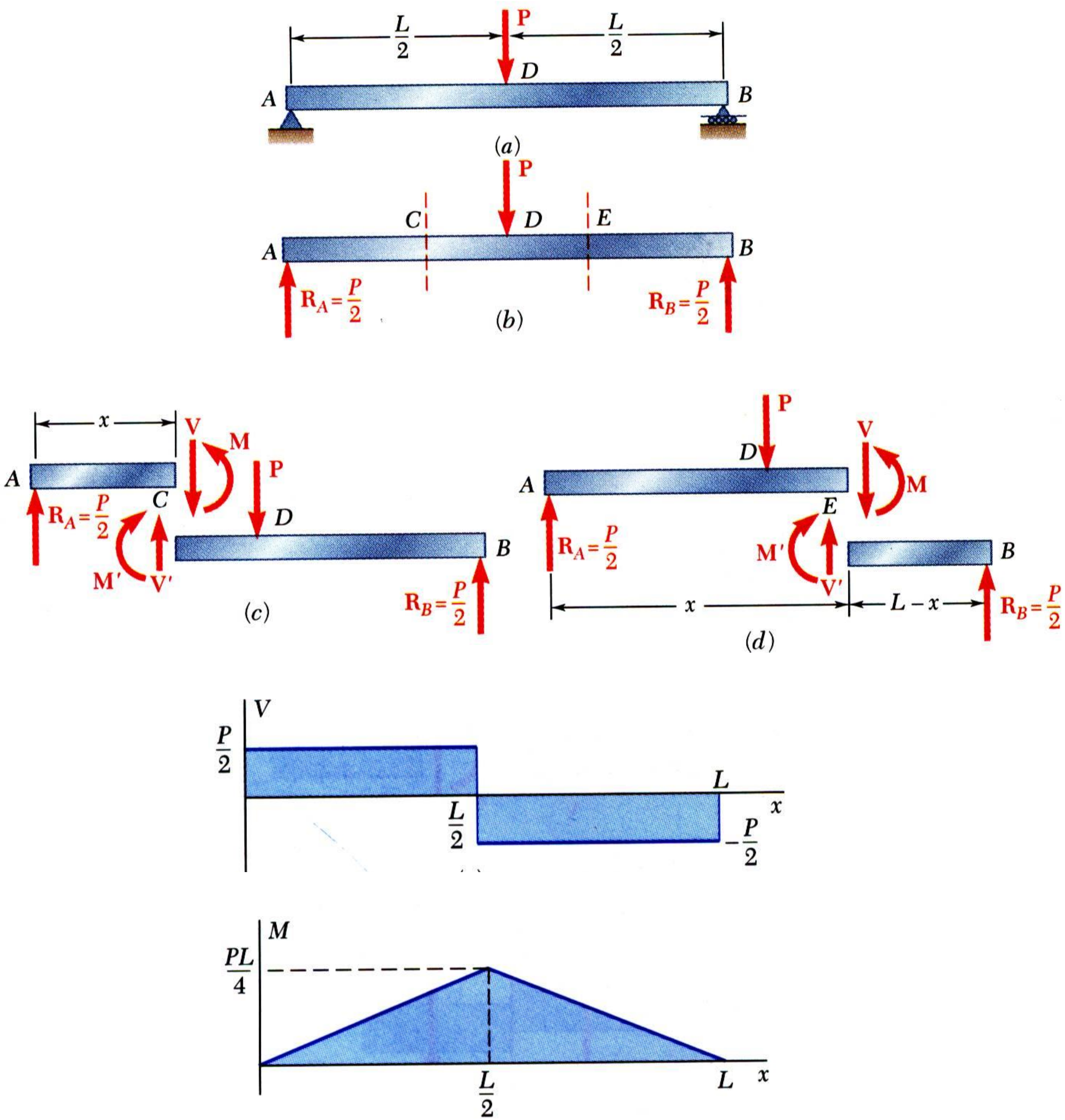
Shear and Bending Moment in a Beam



- Wish to determine bending moment and shearing force at any point in a beam subjected to concentrated and distributed loads.
- Determine reactions at supports by treating whole beam as free-body.
- Cut beam at C and draw free-body diagrams for AC and CB. By definition, positive sense for internal force-couple systems are as shown.
- From equilibrium considerations, determine M and V or M' and V'.

Shear and Bending Moment Diagrams

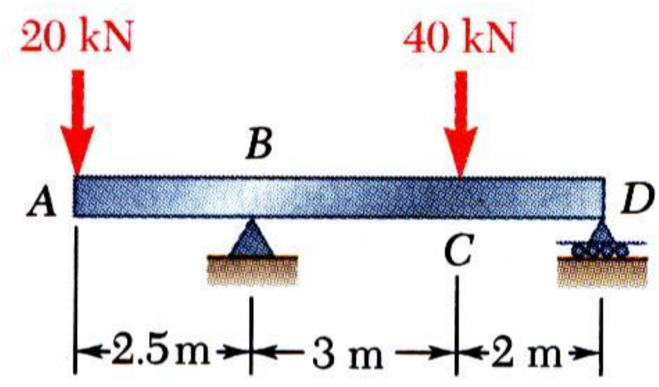


- Variation of shear and bending moment along beam may be plotted.
- Determine reactions at supports.
- Cut beam at C and consider member AC,

  $V = +P/2 \quad M = +Px/2$
- Cut beam at E and consider member EB,

  $V = -P/2 \quad M = +P(L-x)/2$
- For a beam subjected to <u>concentrated loads</u>, shear is constant between loading points and moment varies linearly.

Sample Problem 7.2

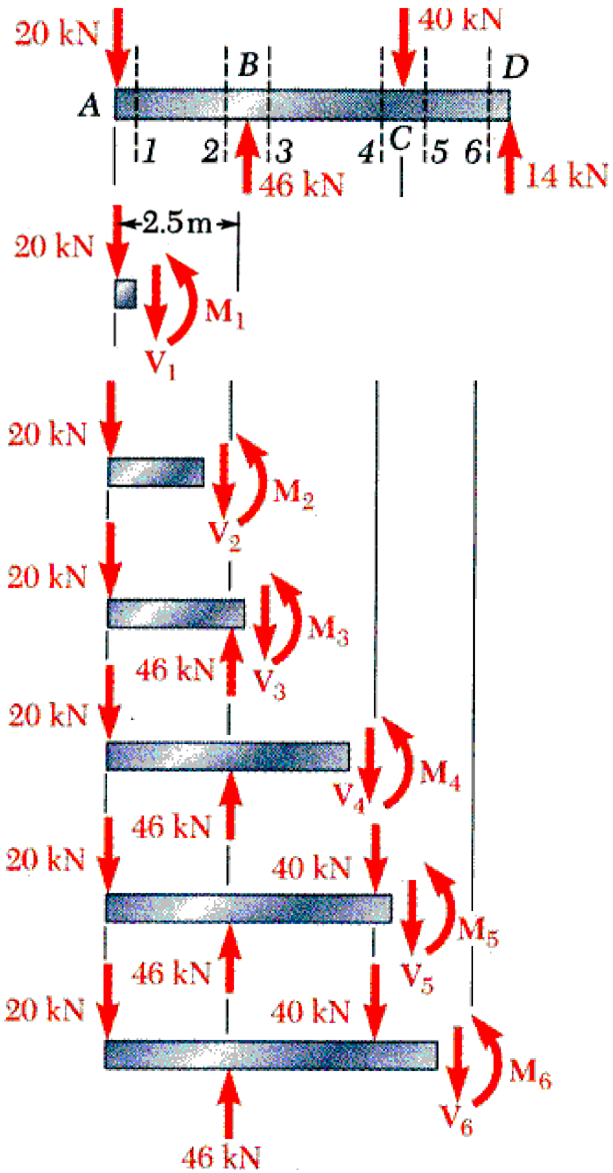


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.
- Plot results.

Sample Problem 7.2



SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems at sections on either side of load application points.

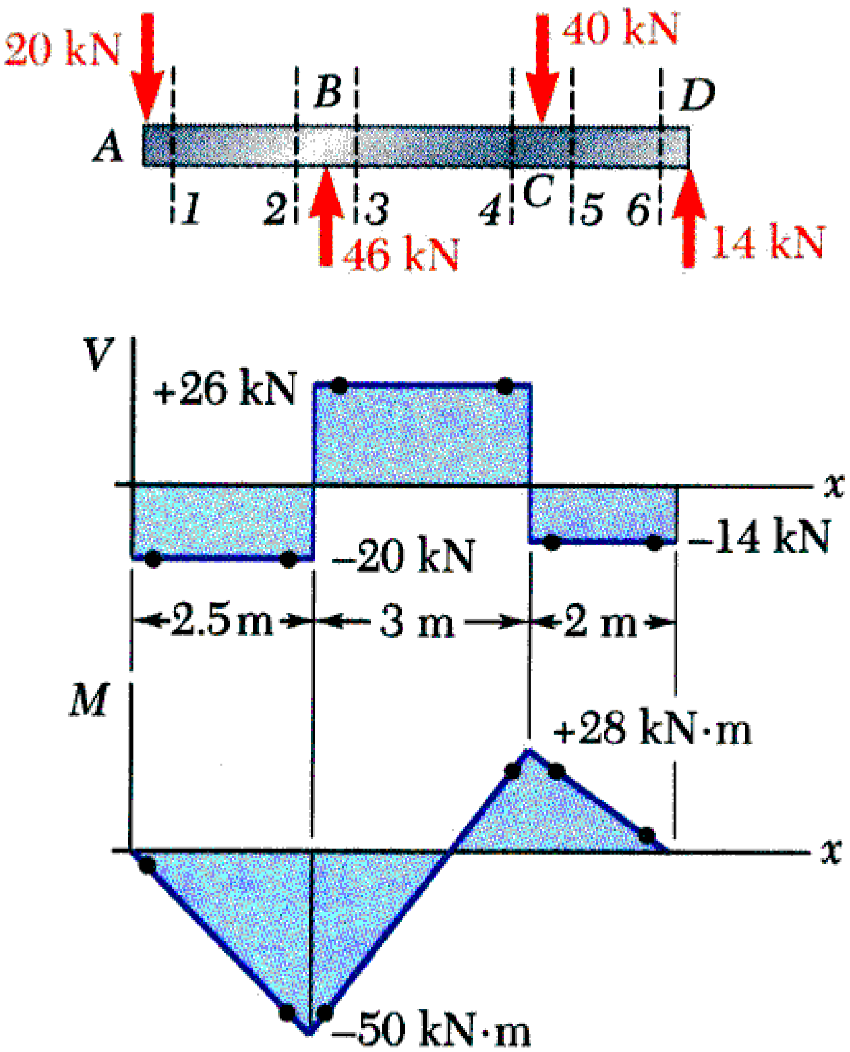
$$\sum F_y = 0: \quad -20\,\text{kN} - V_1 = 0 \qquad \boxed{V_1 = -20\,\text{kN}}$$

$$\sum M_2 = 0: \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \qquad \boxed{M_1 = 0}$$

Similarly,

$$\boxed{\begin{array}{ll} V_3 = 26\,\text{kN} & M_3 = -50\,\text{kN}\cdot\text{m} \\ V_4 = 26\,\text{kN} & M_4 = -50\,\text{kN}\cdot\text{m} \\ V_5 = 26\,\text{kN} & M_5 = -50\,\text{kN}\cdot\text{m} \\ V_6 = 26\,\text{kN} & M_6 = -50\,\text{kN}\cdot\text{m} \end{array}}$$

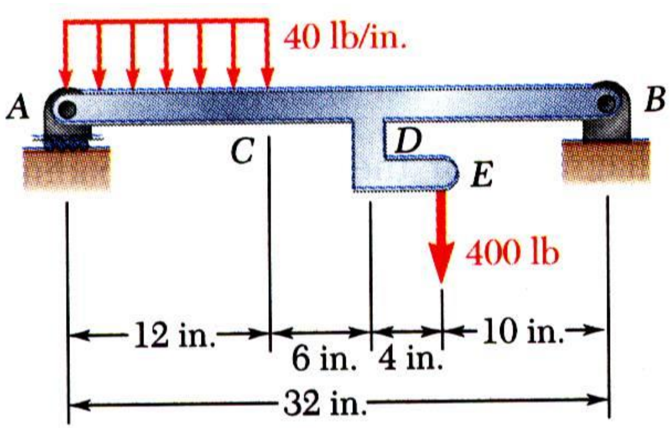
Sample Problem 7.2



- Plot results.

  Note that shear is of constant value between concentrated loads and bending moment varies linearly.

Sample Problem 7.3

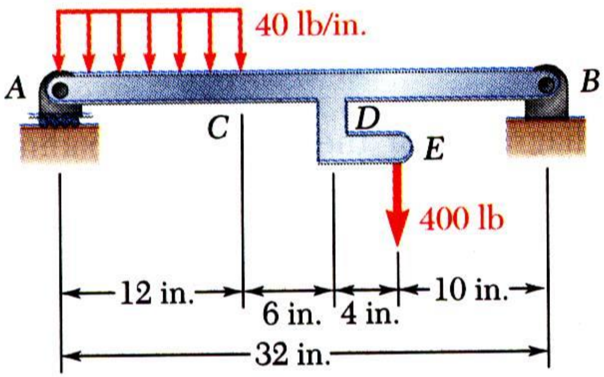


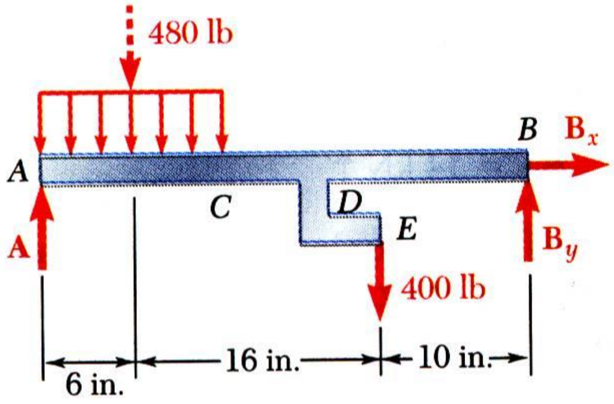
Draw the shear and bending moment diagrams for the beam *AB*. The distributed load of 40 lb/in. extends over 12 in. of the beam, from *A* to *C*, and the 400 lb load is applied at *E*.

SOLUTION:

- Taking entire beam as free-body, calculate reactions at *A* and *B*.
- Determine equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.
- Plot results.

Sample Problem 7.3





SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *A* and *B*.

$$\sum M_A = 0:$$

$$B_y(32\,\text{in.}) - (480\,\text{lb})(6\,\text{in.}) - (400\,\text{lb})(22\,\text{in.}) = 0$$

$$\boxed{B_y = 365\,\text{lb}}$$

$$\sum M_B = 0:$$

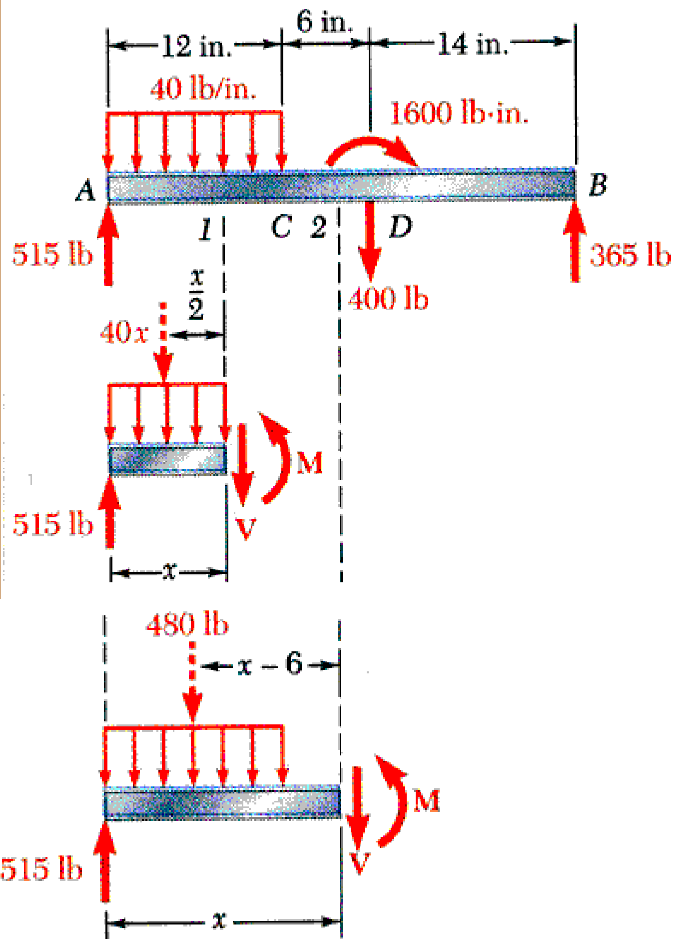
$$(480\,\text{lb})(26\,\text{in.}) + (400\,\text{lb})(10\,\text{in.}) - A(32\,\text{in.}) = 0$$

$$\boxed{A = 515\,\text{lb}}$$

$$\sum F_x = 0: \qquad \boxed{B_x = 0}$$

- Note: The 400 lb load at *E* may be replaced by a 400 lb force and 1600 lb-in. couple at *D*.

Sample Problem 7.3



- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From A to *C*:

$$\sum F_y = 0: \quad 515 - 40x - V = 0$$

$$\boxed{V = 515 - 40x}$$

$$\sum M_1 = 0: \quad -515x - 40x\left(\tfrac{1}{2}x\right) + M = 0$$

$$\boxed{M = 515x - 20x^2}$$

From *C* to *D*:

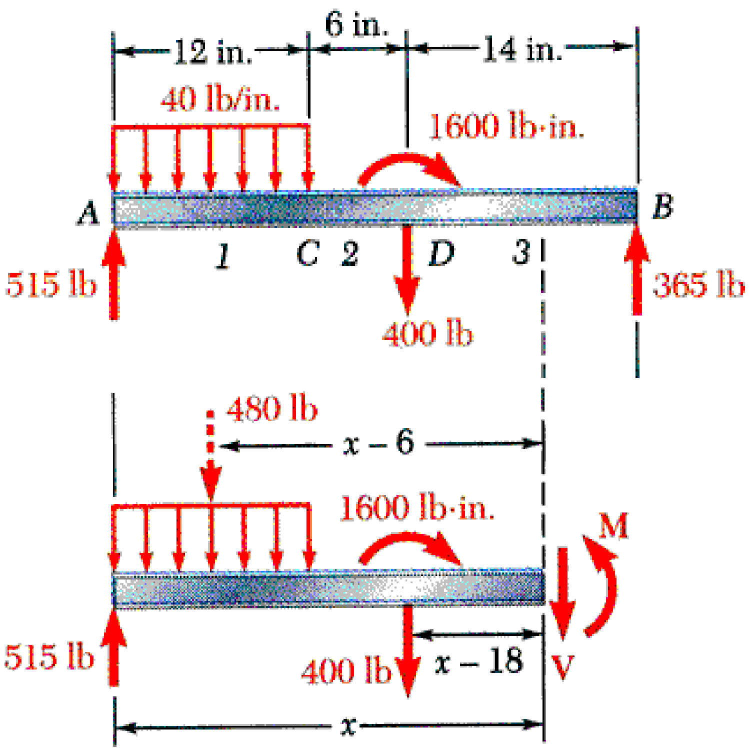
$$\sum F_y = 0: \quad 515 - 480 - V = 0$$

$$\boxed{V = 35\,\text{lb}}$$

$$\sum M_2 = 0: \quad -515x + 480(x - 6) + M = 0$$

$$\boxed{M = (2880 + 35x)\,\text{lb}\cdot\text{in.}}$$

Sample Problem 7.3



- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From *D* to *B*:

$$\sum F_y = 0: \qquad 515 - 480 - 400 - V = 0$$

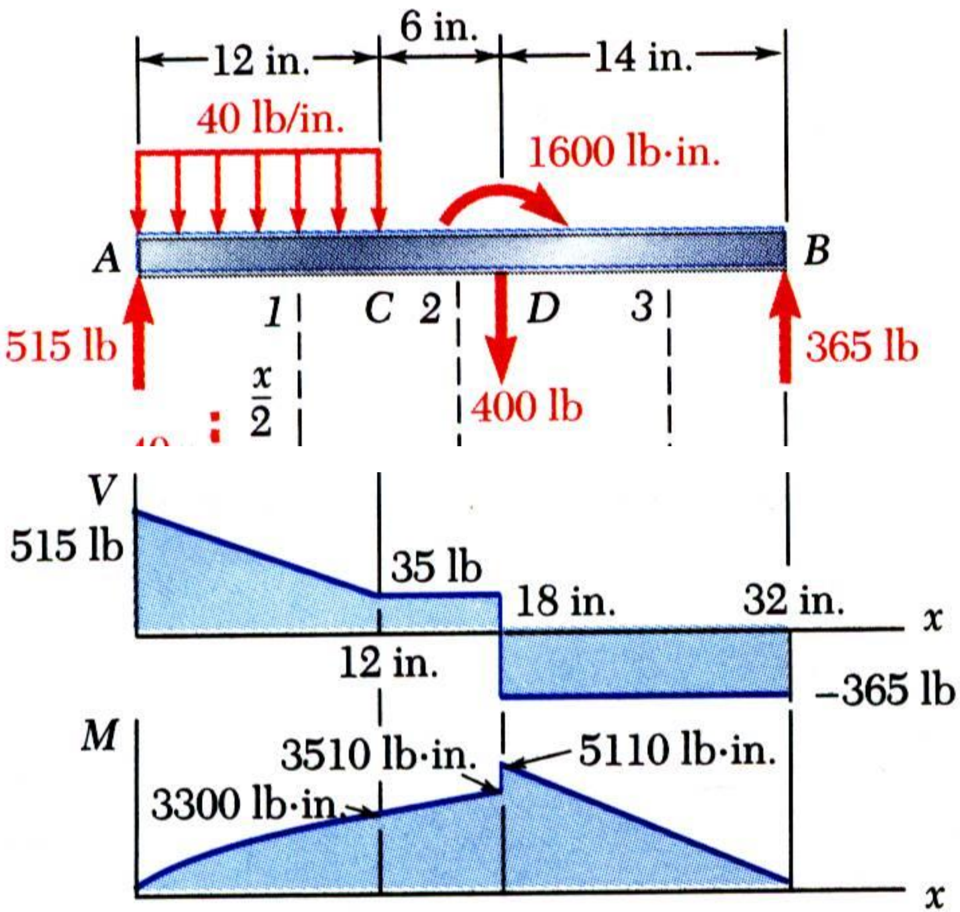
$$\boxed{V = -365\,\text{lb}}$$

$$\sum M_2 = 0:$$

$$-515x + 480(x-6) - 1600 + 400(x-18) + M = 0$$

$$\boxed{M = (11{,}680 - 365x)\,\text{lb}\cdot\text{in.}}$$

Sample Problem 7.3



- Plot results.

From A to C:

$$\boxed{V = 515 - 40x}$$

$$\boxed{M = 515x - 20x^2}$$

From C to D:

$$\boxed{V = 35\ \text{lb}}$$

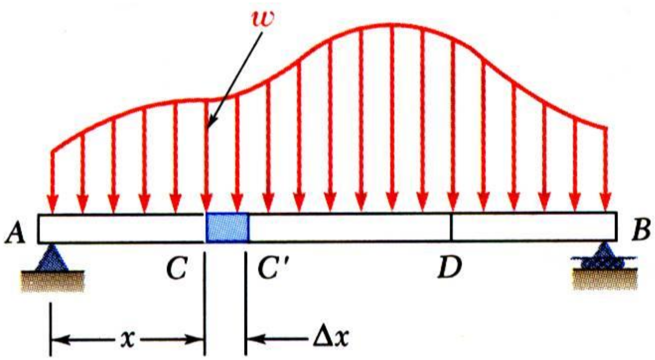
$$\boxed{M = (2880 + 35x)\ \text{lb} \cdot \text{in.}}$$

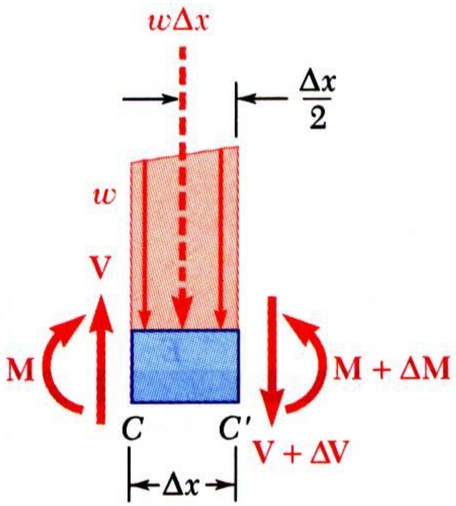
From D to B:

$$\boxed{V = -365\ \text{lb}}$$

$$\boxed{M = (11{,}680 - 365x)\ \text{lb} \cdot \text{in.}}$$

Relations Among Load, Shear, and Bending Moment





- Relations between load and shear:

$$V-(V+\Delta V)-w\Delta x=0$$

$$\frac{dV}{dx}=\lim_{\Delta x\to 0}\frac{\Delta V}{\Delta x}=-w$$

$$V_D-V_C=-\int_{x_C}^{x_D} w\,dx=-(\text{area under load curve})$$

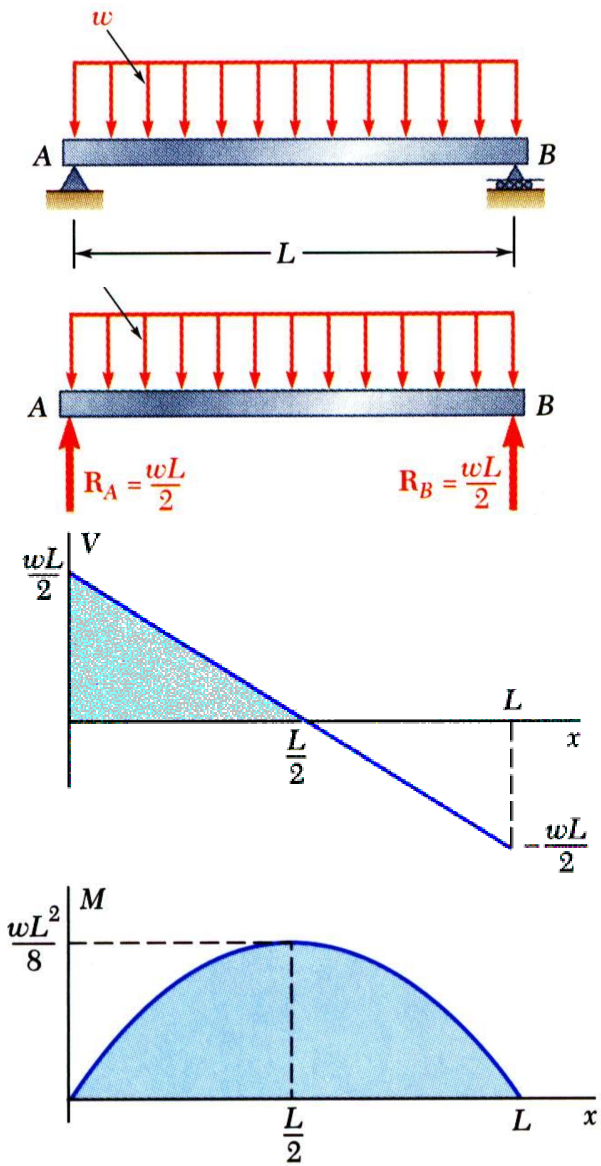
- Relations between shear and bending moment:

$$(M+\Delta M)-M-V\Delta x+w\Delta x\frac{\Delta x}{2}=0$$

$$\frac{dM}{dx}=\lim_{\Delta x\to 0}\frac{\Delta M}{\Delta x}=\lim_{\Delta x\to 0}\left(V-\tfrac{1}{2}w\Delta x\right)=V$$

$$M_D-M_C=\int_{x_C}^{x_D} V\,dx=(\text{area under shear curve})$$

Relations Among Load, Shear, and Bending Moment



- Reactions at supports, $R_A = R_B = \dfrac{wL}{2}$

- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

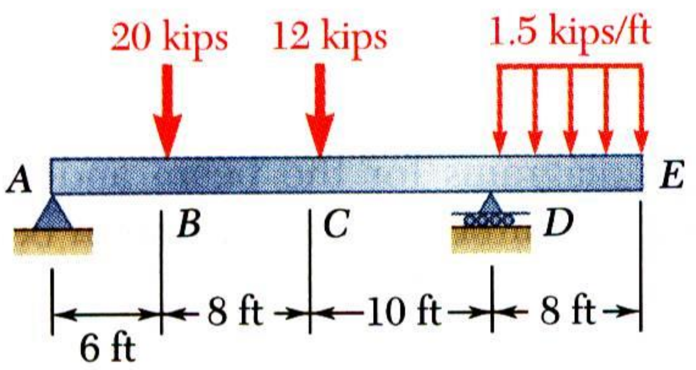
- Moment curve,

$$M - M_A = \int_0^x V dx$$

$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

Sample Problem 7.4

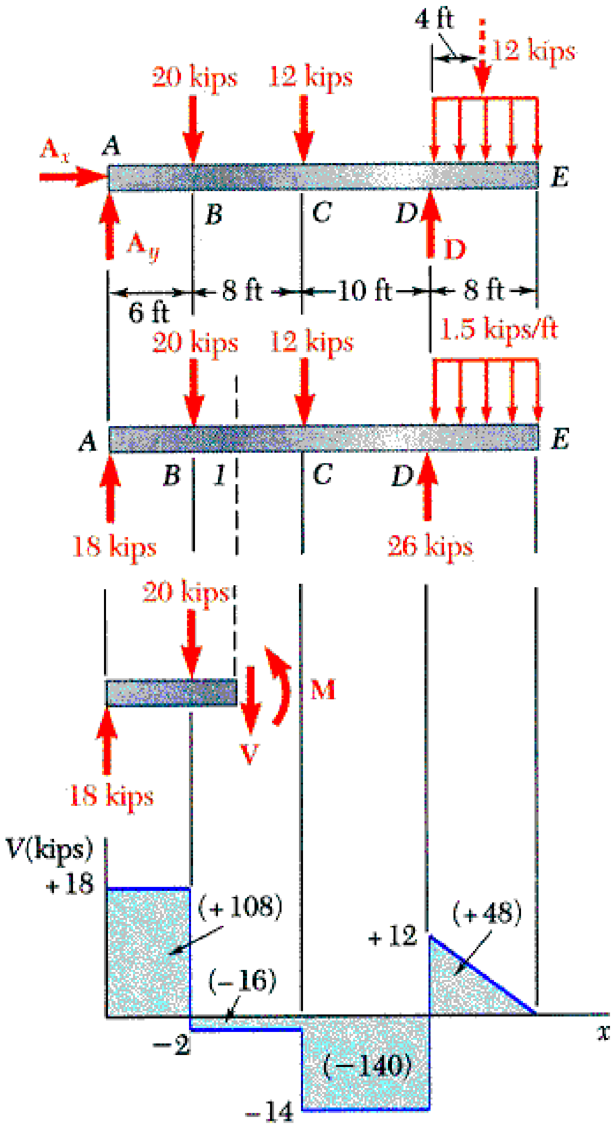


Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.
- Between concentrated load application points, $dV/dx = -w = 0$ and shear is constant.
- With uniform loading between *D* and *E*, the shear variation is linear.
- Between concentrated load application points, $dM/dx = V = \text{constant}$. The change in moment between load application points is equal to area under shear curve between points.
- With a linear shear variation between *D* and *E*, the bending moment diagram is a parabola.

Sample Problem 7.4



SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.

$$\sum M_A = 0:$$

$$D(24\text{ ft}) - (20\text{ kips})(6\text{ ft}) - (12\text{ kips})(14\text{ ft}) - (12\text{ kips})(28\text{ ft}) = 0$$

$$\boxed{D = 26\text{ kips}}$$

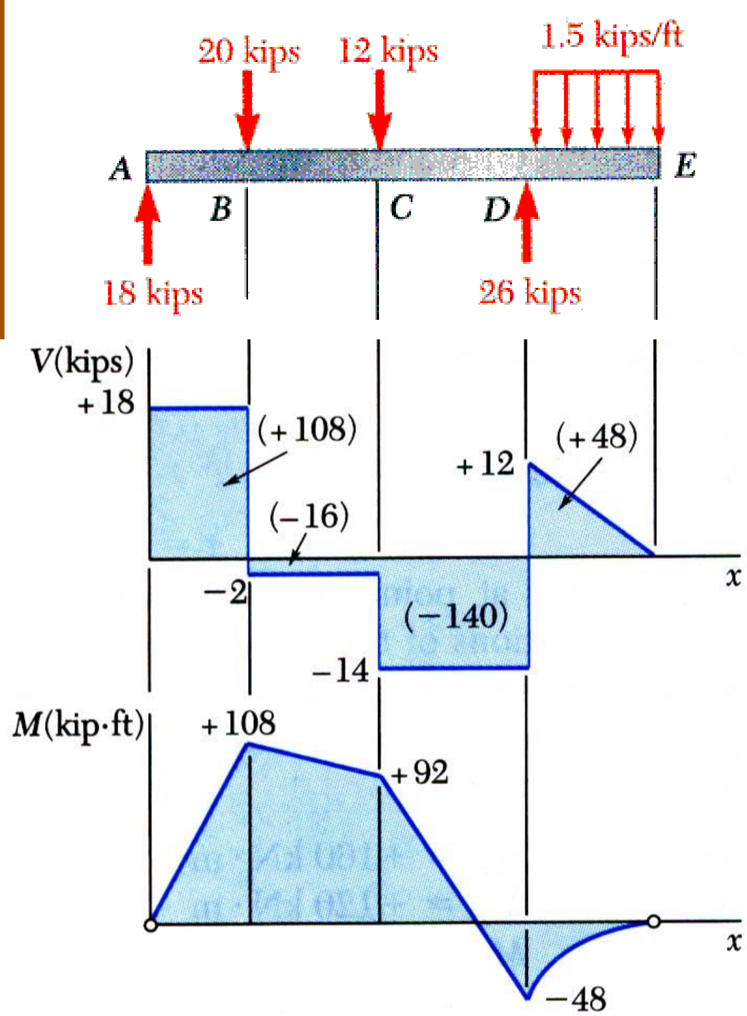
$$\sum F_y = 0:$$

$$A_y - 20\text{ kips} - 12\text{ kips} + 26\text{ kips} - 12\text{ kips} = 0$$

$$\boxed{A_y = 18\text{ kips}}$$

- Between concentrated load application points, $dV/dx = -w = 0$ and shear is constant.
- With uniform loading between D and E, the shear variation is linear.

Sample Problem 7.4

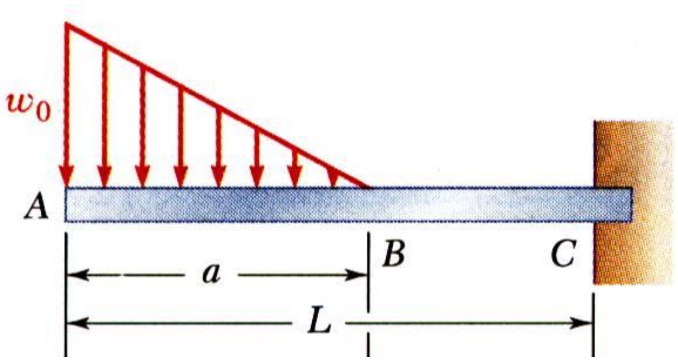


- Between concentrated load application points, $dM/dx = V =$ constant. The change in moment between load application points is equal to area under the shear curve between points.

$$M_B - M_A = +108 \qquad M_B = +108\,\text{kip}\cdot\text{ft}$$
$$M_C - M_B = -16 \qquad M_C = +92\,\text{kip}\cdot\text{ft}$$
$$M_D - M_C = -140 \qquad M_D = -48\,\text{kip}\cdot\text{ft}$$
$$M_E - M_D = +48 \qquad M_E = 0$$

- With a linear shear variation between D and E, the bending moment diagram is a parabola.

Sample Problem 7.6

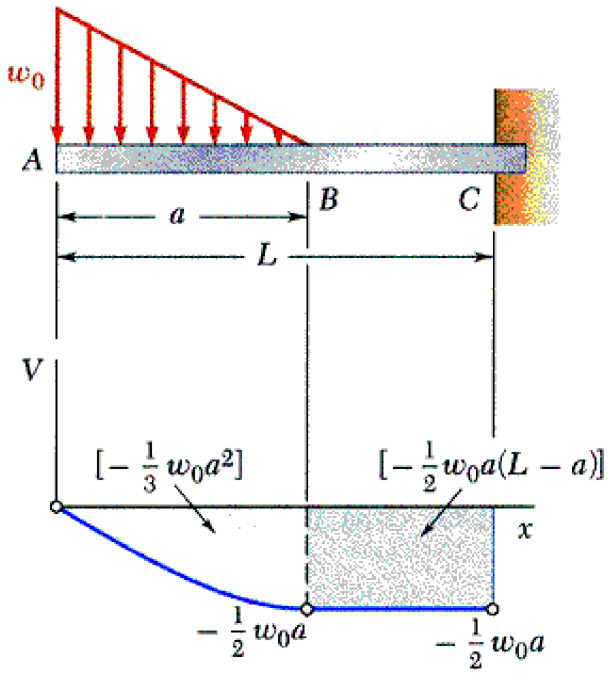


Sketch the shear and bending-moment diagrams for the cantilever beam and loading shown.

SOLUTION:

- The change in shear between A and B is equal to the negative of area under load curve between points. The linear load curve results in a parabolic shear curve.
- With zero load, change in shear between B and C is zero.
- The change in moment between A and B is equal to area under shear curve between points. The parabolic shear curve results in a cubic moment curve.
- The change in moment between B and C is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.

Sample Problem 7.6



SOLUTION:

- The change in shear between A and B is equal to negative of area under load curve between points. The linear load curve results in a parabolic shear curve.

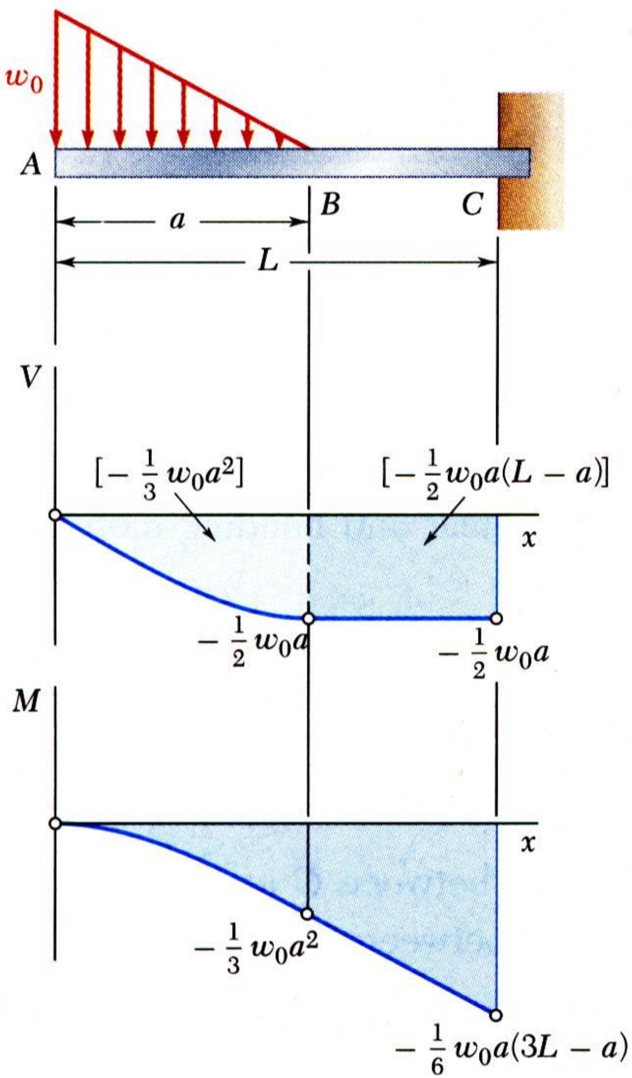
$$\text{at } A, \quad V_A = 0, \quad \frac{dV}{dx} = -w = -w_0$$

$$V_B - V_A = -\tfrac{1}{2}w_0 a \qquad\qquad V_B = -\tfrac{1}{2}w_0 a$$

$$\text{at } B, \quad \frac{dV}{dx} = -w = 0$$

- With zero load, change in shear between B and C is zero.

Sample Problem 7.6



- The change in moment between A and B is equal to area under shear curve between the points. The parabolic shear curve results in a cubic moment curve.

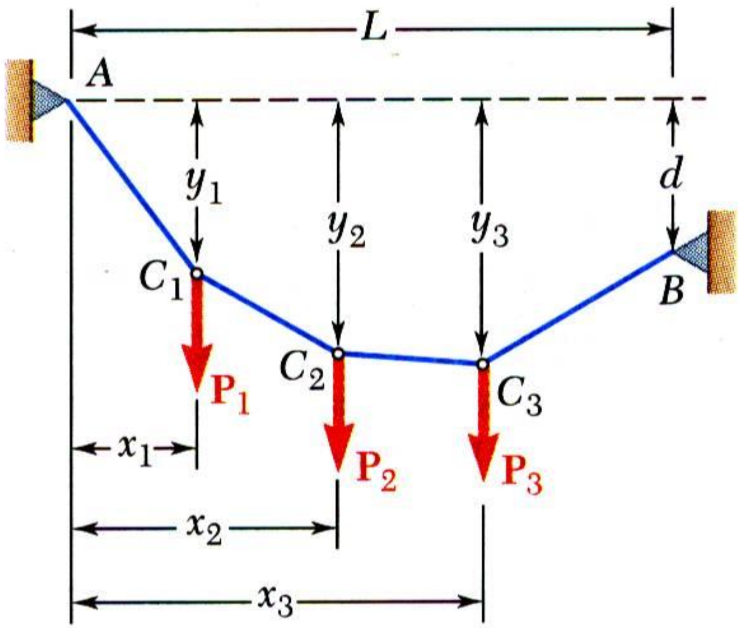
$$\text{at } A, \quad M_A = 0, \quad \frac{dM}{dx} = V = 0$$

$$M_B - M_A = -\tfrac{1}{3}w_0a^2 \qquad M_B = -\tfrac{1}{3}w_0a^2$$

$$M_C - M_B = -\tfrac{1}{2}w_0a(L-a) \qquad M_C = -\tfrac{1}{6}w_0a(3L-a)$$

- The change in moment between B and C is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.

Cables With Concentrated Loads



- Cables are applied as structural elements in suspension bridges, transmission lines, aerial tramways, guy wires for high towers, etc.
- For analysis, assume:
  a) concentrated vertical loads on given vertical lines,
  b) weight of cable is negligible,
  c) cable is flexible, i.e., resistance to bending is small,
  d) portions of cable between successive loads may be treated as two force members
- Wish to determine shape of cable, i.e., vertical distance from support *A* to each load point.

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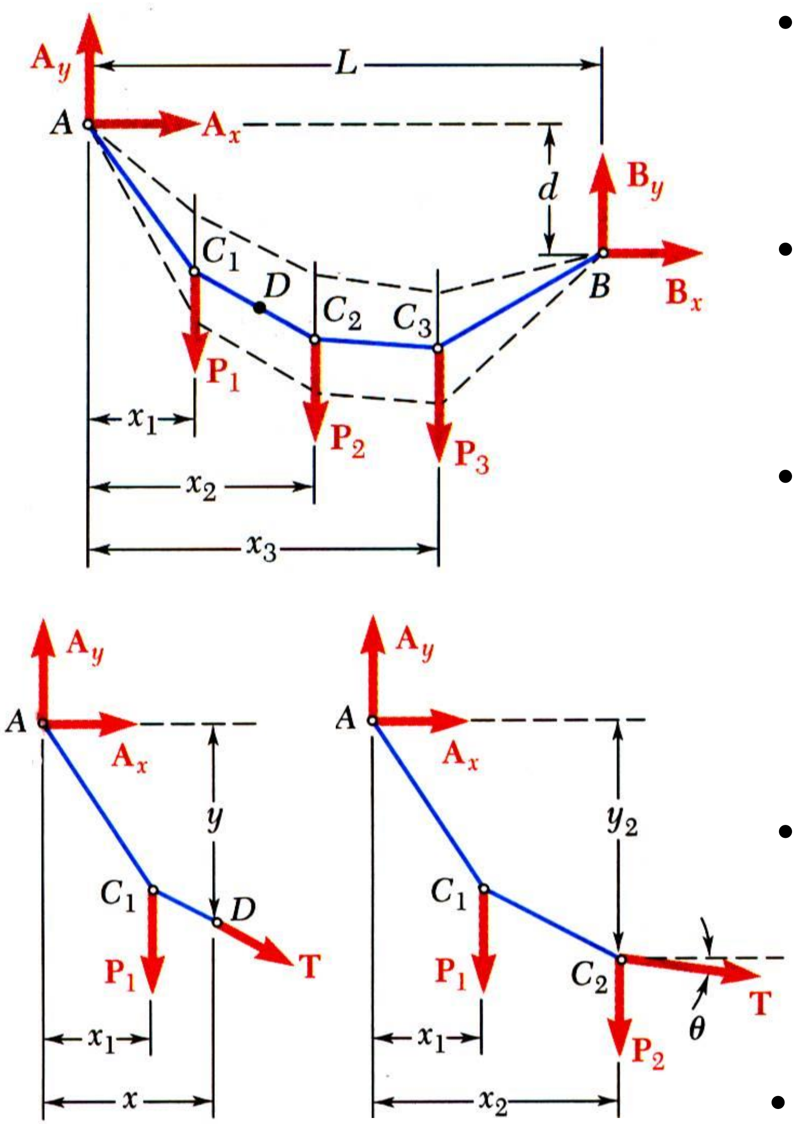
Cables With Concentrated Loads

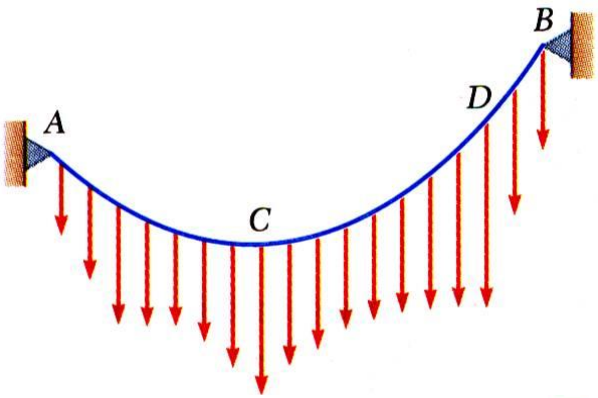


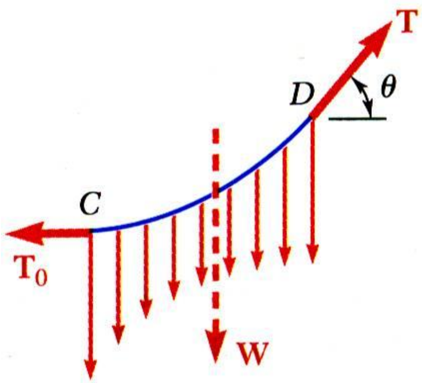
- Consider entire cable as free-body. Slopes of cable at A and B are not known - two reaction components required at each support.
- Four unknowns are involved and three equations of equilibrium are not sufficient to determine the reactions.
- Additional equation is obtained by considering equilibrium of portion of cable AD and assuming that coordinates of point D on the cable are known. The additional equation is $\sum M_D = 0$.
- For other points on cable,

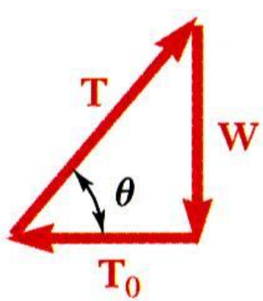
  $\sum M_{C_2} = 0$ yields y_2

  $\sum F_x = 0, \sum F_y = 0$ yield T_x, T_y
- $T_x = T\cos\theta = A_x = \text{constant}$

Cables With Distributed Loads







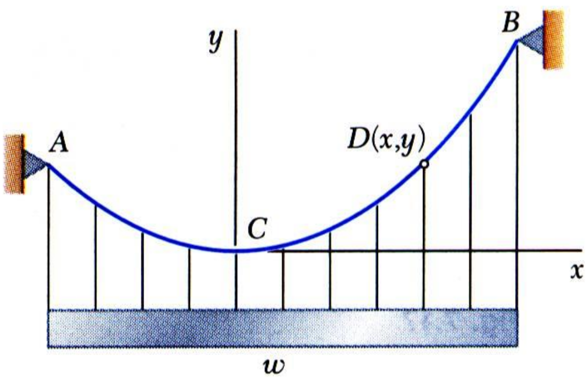
- For cable carrying a distributed load:
  a) cable hangs in shape of a curve
  b) internal force is a tension force directed along tangent to curve.
- Consider free-body for portion of cable extending from lowest point *C* to given point *D*. Forces are horizontal force $\boldsymbol{T_0}$ at C and tangential force $\boldsymbol{T}$ at *D*.
- From force triangle:

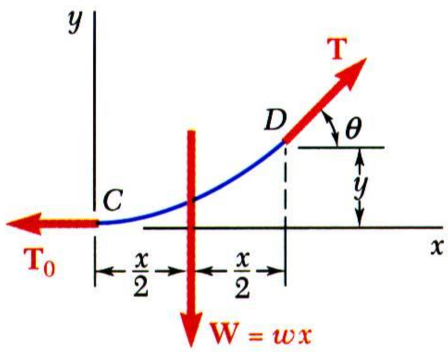
$$T\cos\theta = T_0 \qquad T\sin\theta = W$$

$$T = \sqrt{T_0^2 + W^2} \qquad \tan\theta = \frac{W}{T_0}$$

- Horizontal component of $\boldsymbol{T}$ is uniform over cable.
- Vertical component of $\boldsymbol{T}$ is equal to magnitude of W measured from lowest point.
- Tension is minimum at lowest point and maximum at *A* and *B*.

Parabolic Cable





- Consider a cable supporting a uniform, horizontally distributed load, e.g., support cables for a suspension bridge.

- With loading on cable from lowest point C to a point D given by $W = wx$, internal tension force magnitude and direction are

$$T = \sqrt{T_0^2 + w^2x^2} \qquad \tan\theta = \frac{wx}{T_0}$$

- Summing moments about D,

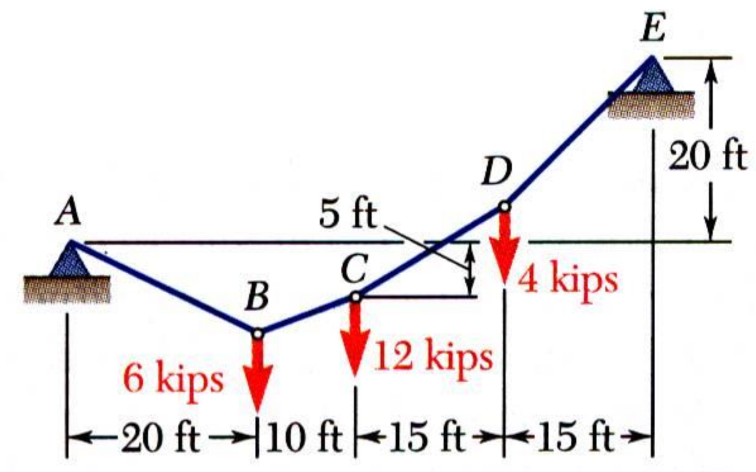
$$\sum M_D = 0: \qquad wx\frac{x}{2} - T_0 y = 0$$

or

$$y = \frac{wx^2}{2T_0}$$

The cable forms a parabolic curve.

Sample Problem 7.8

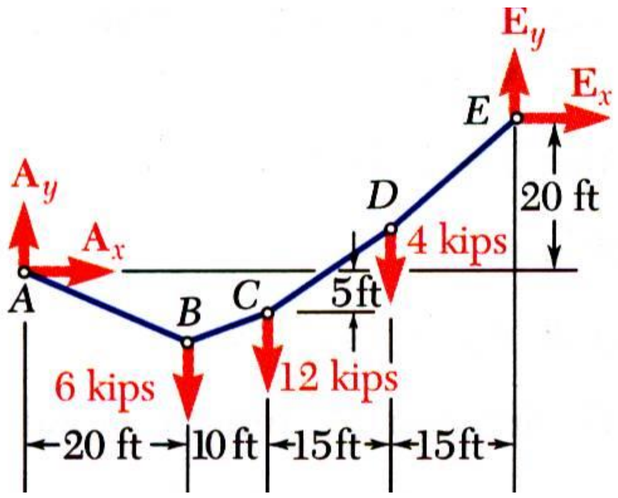


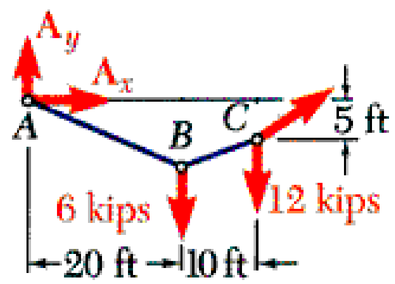
The cable *AE* supports three vertical loads from the points indicated. If point *C* is 5 ft below the left support, determine *(a)* the elevation of points *B* and *D*, and *(b)* the maximum slope and maximum tension in the cable.

SOLUTION:

- Determine reaction force components at *A* from solution of two equations formed from taking entire cable as free-body and summing moments about *E*, and from taking cable portion *ABC* as a free-body and summing moments about *C*.
- Calculate elevation of *B* by considering *AB* as a free-body and summing moments *B*. Similarly, calculate elevation of *D* using *ABCD* as a free-body.
- Evaluate maximum slope and maximum tension which occur in *DE*.

Sample Problem 7.8





SOLUTION:

- Determine two reaction force components at A from solution of two equations formed from taking entire cable as a free-body and summing moments about E,

$$\sum M_E = 0:$$

$$20A_x - 60A_y + 40(6) + 30(12) + 15(4) = 0$$

$$20A_x - 60A_y + 660 = 0$$

and from taking cable portion ABC as a free-body and summing moments about C.

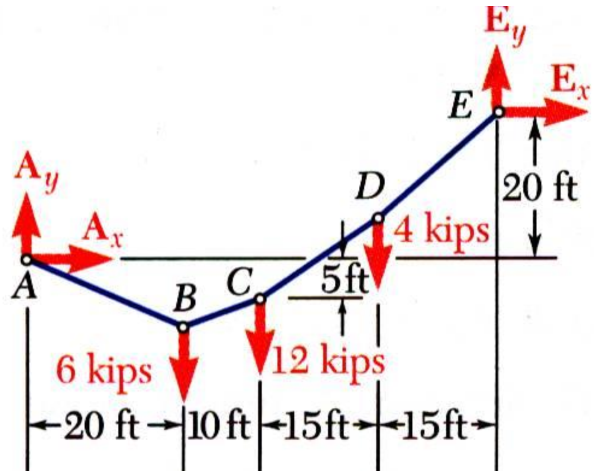
$$\sum M_C = 0:$$

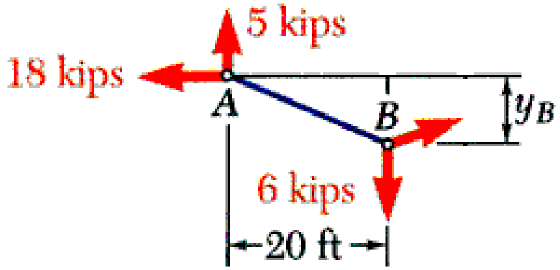
$$-5A_x - 30A_y + 10(6) = 0$$

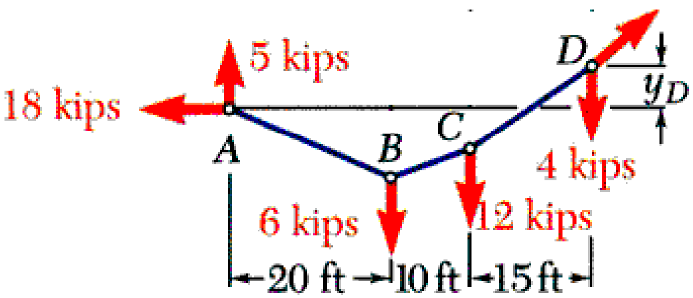
Solving simultaneously,

$$A_x = -18\text{ kips} \qquad A_y = 5\text{ kips}$$

Sample Problem 7.8







- Calculate elevation of B by considering AB as a free-body and summing moments B.

$$\sum M_B = 0: \qquad y_B(18) - 5(20) = 0$$

$$\boxed{y_B = -5.56\text{ ft}}$$

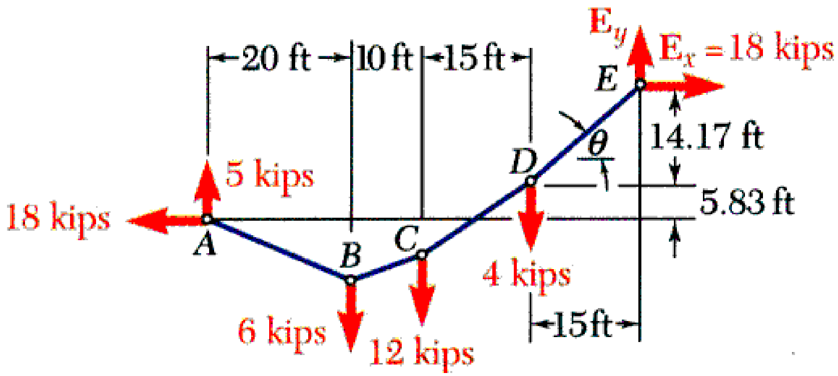
Similarly, calculate elevation of D using $ABCD$ as a free-body.

$$\sum M = 0:$$

$$-y_D(18) - 45(5) + 25(6) + 15(12) = 0$$

$$\boxed{y_D = 5.83\text{ ft}}$$

Sample Problem 7.8

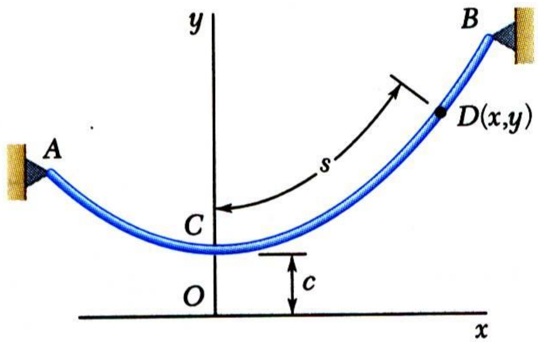


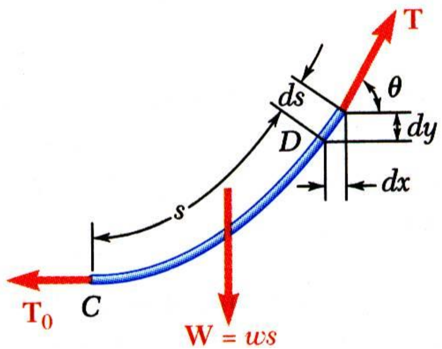
- Evaluate maximum slope and maximum tension which occur in *DE*.

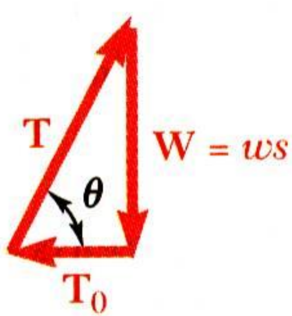
$$\tan\theta = \frac{14.7}{15} \qquad \boxed{\theta = 43.4^\circ}$$

$$T_{max} = \frac{18\,\text{kips}}{\cos\theta} \qquad \boxed{T_{max} = 24.8\,\text{kips}}$$

Catenary







- Consider a cable uniformly loaded along the cable itself, e.g., cables hanging under their own weight.

- With loading on the cable from lowest point C to a point D given by $W = ws$, the internal tension force magnitude is

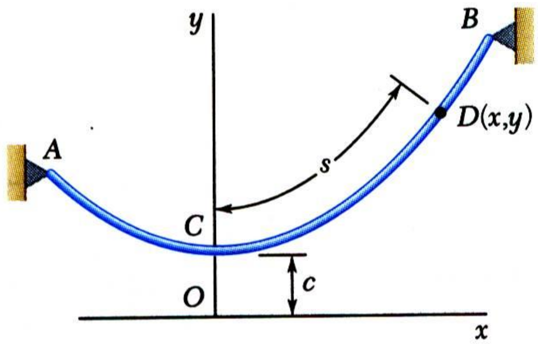
$$T = \sqrt{T_0^2 + w^2 s^2} = w\sqrt{c^2 + s^2} \qquad c = T_0 / w$$

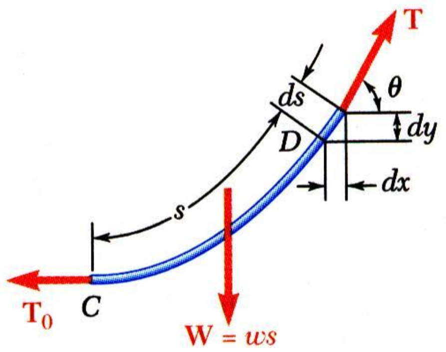
- To relate horizontal distance x to cable length s,

$$dx = ds \cos\theta = \frac{T_0}{T}\cos\theta = \frac{ds}{\sqrt{q + s^2/c^2}}$$

$$x = \int_0^s \frac{ds}{\sqrt{q + s^2/c^2}} = c\sinh^{-1}\frac{s}{c} \quad \text{and} \quad s = c\sinh\frac{x}{c}$$

Catenary





- To relate x and y cable coordinates,

$$dy = dx \tan\theta = \frac{W}{T_0} dx = \frac{s}{c} dx = \sinh\frac{x}{c}\, dx$$

$$y - c = \int_0^x \sinh\frac{x}{c}\, dx = c\cosh\frac{x}{c} - c$$

$$y = c\cosh\frac{x}{c}$$

which is the equation of a catenary.