



Experiment 7

Damped Oscillations

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Abstract:

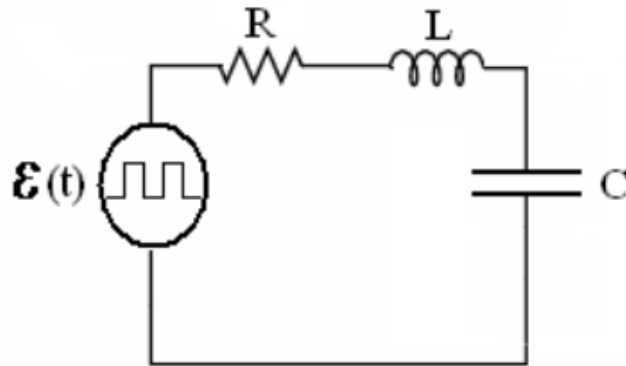
We will use the (CRO) and the signal generator in this experiment to create damping during the three different damping stages. The experiment's goal is to determine the highest level at which (R) can remain in the over-damping stage. We found $\frac{t_1}{2}$ for the over damping, critical damping, and under damping stages for voltage.

Introduction:

Apparatus:

Resistance decade box, 10mH inductor, 0.01 μ F capacitor, a signal generator and an oscilloscope.

In this experiment, we will build the following electrical circuits:



The charge equation for capacitors is: $Q(t) = A_1 e^{\lambda_+ t} + A_2 e^{\lambda_- t}$

Where A_1 and A_2 are constant, and:

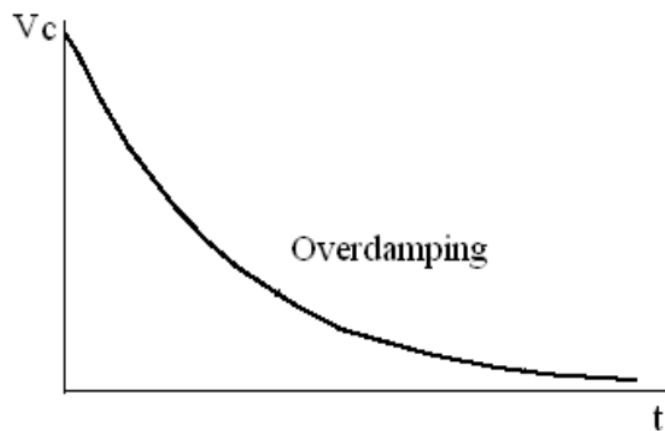
$$\lambda_+ = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}},$$
$$\lambda_- = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}.$$

For this solution three interesting cases emerge:

1- Over-damping:

$$\text{If } \left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

Both terms in charge equation decay exponentially with time and the voltage across the capacitor is said to be over-damped.

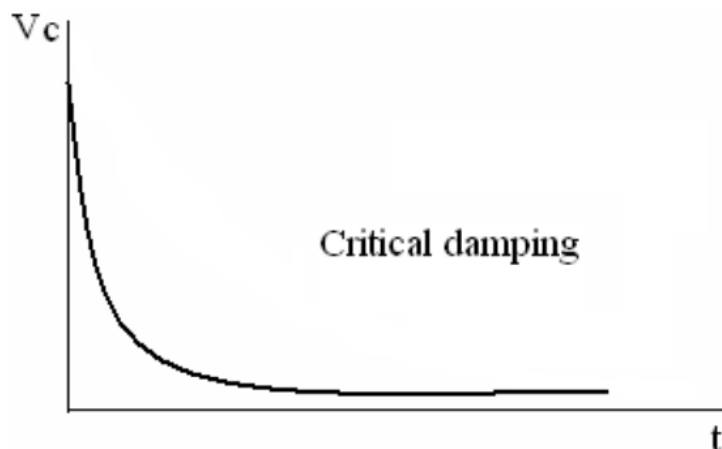


2- Critical-damping:

$$\text{If } \left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

The term under the square in equations (λ_+ , λ_-) root vanishes then :

$$\lambda_+ = \lambda_- = -\frac{R}{2L}$$

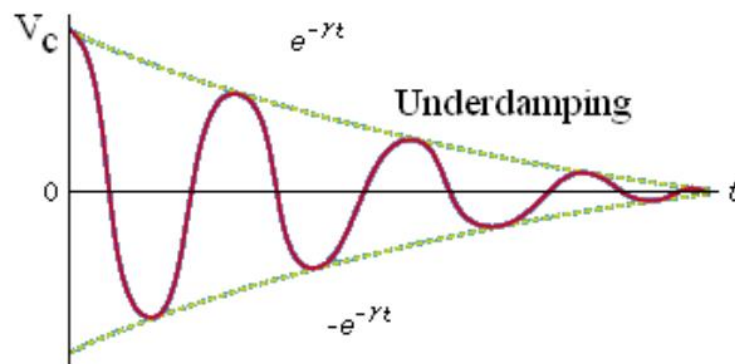


3- Under-damping:

$$\text{If } \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

The term under the square root in equation (λ_+, λ_-) becomes negative, then the solution will be :

$$Q(t) = Q_0 e^{-\delta t} \cos(\omega' t + \theta_0) \quad \text{where} \quad \delta = \frac{R}{2L}$$



Data & Analysis:

For Critical-damping:

$$\partial_{c,Theo} = \frac{R_{critical}}{2 * L}$$

$$\partial_{c,Theo} = \frac{1610}{2 * 10 * 10^{-3}}$$

$$\partial_{c,Theo} = 80.5 * 10^3 \quad \Omega/H$$

$$\partial_{c,Exp} = \frac{\ln(2)}{\frac{t_1}{2}}$$

$$\partial_{c,Exp} = \frac{\ln(2)}{15 * 10^{-6}}$$

$$\partial_{c,Exp} = 46.2 * 10^3 \quad s^{-1}$$

For Over-damping:

$$\partial_{c,Theo} = \frac{R_{over}}{2*L}$$

$$\partial_{c,Theo} = \frac{3610}{2*10*10^{-3}}$$

$$\partial_{c,Theo} = 180.5 * 10^3 \quad \Omega/H$$

$$\partial_{c,Exp} = \frac{\ln(2)}{t_{\frac{1}{2}}}$$

$$\partial_{c,Exp} = \frac{\ln(2)}{22 * 10^{-6}}$$

$$\partial_{c,Exp} = 31.5 * 10^3 \quad s^{-1}$$

For Under-damping:

$$t_{\frac{1}{2},theo} = \frac{(2*L) \ln(2)}{R}$$

$$t_{\frac{1}{2},theo} = \frac{(2*10*10^{-3}) \ln(2)}{5}$$

$$t_{\frac{1}{2},theo} = 2.77 \text{ ms}$$

$$\partial_{c,Theo} = \frac{R_{under}}{2*L}$$

$$\partial_{c,Theo} = \frac{5}{2*10*10^{-3}}$$

$$\partial_{c,Theo} = 0.25 * 10^3 \quad \Omega/H$$

$$\partial_{c,Exp} = \frac{\ln(2)}{\frac{t_1}{2}}$$

$$\partial_{c,Exp} = \frac{\ln(2)}{90*10^{-6}}$$

$$\partial_{c,Exp} = 7.7 * 10^3 \quad s^{-1}$$

$$\omega = \sqrt{\frac{1}{L*C} - \left(\frac{R}{2*L}\right)^2}$$

$$f = \frac{\omega}{2*\pi}$$

$$f = \sqrt{\frac{1}{L*C} - \left(\frac{R}{2*L}\right)^2} * \frac{1}{2*\pi}$$

$$f = \sqrt{\frac{1}{L*C} - \left(\frac{R}{2*L}\right)^2} * \frac{1}{2*\pi}$$

$$f = \sqrt{\frac{1}{20*0.01*10^{-9}} - \left(\frac{5}{2*10*10^{-3}}\right)^2} * \frac{1}{2*\pi}$$

$$f = 11.25 \text{ KHz}$$

Find R critical : $R_{critical} = \frac{2*L}{\sqrt{L*C}}$

$$R_{critical} = \frac{2*10^{-2}}{\sqrt{10^{-10}}}$$

$$R_{critical} = 2 * 10^3 \Omega$$

from the results the theoretical $R_{critical}$ is Approximately equal to the practical $R_{critical}$.

Conclusion:

after we calculate $t_{\frac{1}{2}}$ for all cases, As we can see from the results the theoretical $t_{\frac{1}{2}}$ is equal to the practical $t_{\frac{1}{2}}$. but there is a difference in some cases This difference is due to the resistances in the circuit (wires, capacitors and inductors). and the decay constant of the critical damping case is the largest so it decays faster.