$$\frac{99}{100}$$

Mathematics Department
MATH1411-First Exam
First Semester 2022/2023

- Name (in Arabic)……رؤى علاء الدين سرور. •Number…[redacted]

- Circle your discussion's section number from the tables below:

#	Discussion Instructor	Time
1	Rasha Shadid	T:14:15-15:05
2	Amer Ja'far	S:8:00-08:50
3	Sundos Khalil	R:09:00-09:50
4	Salam Shriteh	R:11:25-12:15
5	Mohammad Adarbeh	S:11:25-12:15
6	Rasha Shadid	T:09:00-09:50
7	Farah Omar	T:14:15-15:05
[✓ 8]	Sondos Khalil	S:08:00-08:50
9	Aya Sharsheer	R:13:00-13:50
10	Mohammad Adarbeh	S:13:00-13:50
11	Aya Sharsheer	T:08:00-08:50

#	Discussion Instructor	Time
12	Sondos Khalil	S:09:00-09:50
13	Salam Shriteh	R:10:00-10:50
14	Rasha Shadid	S:08:00-08:50
15	Sundos Khalil	T:09:00-09:50
16	Mohammad Adarbeh	R:14:15-15:05
17	Muna Abu-Alhalawa	T:10:00-10:50
18	Aya Sharsheer	S:09:00-09:50
19	Aya Sharsheer	R:08:00-08:50
20	Salam Shriteh	S:13:00-13:50
21	Rasha Shadid	W:14:15-15:05
22	Salam Shriteh	T:13:00 - 13:50

- **Instructions**

1. Write your name and number.
2. Choose your section from the above table.
3. There are four questions in the next 6 pages.
4. Answer all questions.
5. Turn off your mobile phones.
6. Calculators are not allowed.

Q1) [60%] Circle the correct answer.

(1) The function $f(x) = x^3 - 3x^2 + 2x + 1$ has a root in the interval

- (a) $[0, 1]$
- [✓ (b) $[-1, 0]$]
- (c) $[2, 3]$
- (d) f has no roots

$1 - 3 - 2 + 1$ -5 $0 - 0 + 0 + 1$

$f(0) = 0 - 0 + 0 + 1 = 1$

$f(1) = 1 - 3 + 2 + 1$

$-1 - 3 - 2 + 1$

(2) $\lim_{x\to\infty} (\sqrt{x^2 + x} - x) =$

- (a) 0
- [✓ (b) $\frac{1}{2}$]
- (c) $-\frac{1}{2}$
- (d) does not exist

~~$\sqrt{x^2 - x} - x \times (\sqrt{x^2 - x} + x)$~~

~~$\frac{x^2 - x - x^2}{\sqrt{x^2 - x} + x}$~~ ~~$\frac{-x}{\sqrt{x^2 - x} + x}$~~

$\lim_{x\to\infty} (\sqrt{x^2 + x} - x)$

$\sqrt{x^2}\sqrt{1 + \frac{1}{x}} - \frac{1}{x})$

$\sqrt{x^2}$

$\frac{\sqrt{x^2 + x} - x}{\sqrt{x^2 + x} + x}$

$\frac{\cancel{x^2} + x - \cancel{x^2}}{\sqrt{x^2 + x} + x}$

$\frac{x}{x\sqrt{1 + \frac{1}{x}} - \frac{1}{x}}$

$\frac{1}{\sqrt{1 + 0} - 0}$

$\frac{1}{1}$

(3) $\lim_{x\to 0} \frac{\sin(2x)\sin(3x)}{x^2} =$ $\frac{2(\cos 2x)\sin 3x + 3\cos 3x \sin 2x}{2x}$

- [✓ (a) 6]
- (b) $\frac{1}{6}$
- (c) 0
- (d) Does not exist

$-4\sin 2x \sin 3x + (3\cos 3x)(2\cos 2x) +$

36

$2\cos 2x\ 36$

$\frac{6+6}{2}$ $\frac{12}{6}$

$\frac{6 \cdot 6}{2}$ $\frac{12}{2} = 6$

(4) $\boxed{\lim_{x\to 1^+} \frac{\sqrt{x-1}}{x^2-1}} =$ $\frac{\sqrt{1-1}}{1-1}$ $\frac{0}{0}$ → 0^+ → $\frac{\sqrt{x-1}}{(x-1)(x+1)}$ $= \frac{\sqrt{x-1}}{\sqrt{x-1}\sqrt{x-1}(x+1)}$ $\frac{1}{\sqrt{x-1}(x+1)}$ 0

- [✓ (a) 0]
- (b) $\frac{1}{2}$
- [✓ (c) $+\infty$]
- (d) does not exist

$\frac{\frac{1}{2\sqrt{x-1}}}{2x} = \frac{1}{4x\sqrt{x-1}}$ $\frac{6}{0}$

$\lim_{x\to 1^+} \frac{\sqrt{x-1}}{x^2-1} = \frac{0}{0}$

$x^4 - 1$ $x^2 + 1$

(5) The domain of the function $f(x) = \sqrt{x^2 - \frac{1}{x^2}}$ is

- (a) $(-\infty, \infty)$
- [✓ (b) $(-\infty, -1] \cup [1, \infty)$]
- (c) $[-1, 1]$
- (d) $(-\infty, 0) \cup [1, \infty)$

$\frac{x^4 - 1}{x^2}$ $\frac{1}{4}$

~~$x^4 - 1 = 0$~~ ~~$x^4 = 1$~~ ~~$x = 1$~~

$- \quad +$

1

$x^4 - 1 \geq 0$

$\sqrt[4]{x^4} \geq \sqrt[4]{1}$

$|x| \geq 1$

$\frac{x^4 - 1}{x^2}$

$0 \quad 1$

$2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$-1 \quad 1$

$2x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = -1, 1$

(6) The area enclosed between the curves $y = 2 - x^2$ and $y = x^2$ is

(a) $\frac{4}{3}$

(b) $\frac{2}{3}$

[✓ (c) $\frac{8}{3}$]

(d) $\frac{9}{2}$

$\int_{-1}^{1} 2 - x^2 - x^2 = 2 - 2x^2 \Big|_{-1}^{1}$

$2x - \frac{2x^3}{3} \Big|_{-1}^{1}$

$2 - \frac{2}{3} - (-2 - -\frac{2}{3})$

$2 - \frac{2}{3} + 2 - \frac{2}{3}$

$4 - \frac{4}{3}$

$\frac{8}{3}$

$\int_{-1}^{1} 2 - x^2 - x^2$

$2 - 2x^2$

$2x - \frac{2x^3}{3} \Big|_{-1}^{1}$

$2 - \frac{2}{3}$

$-(+2 - \frac{2}{3})$

$4 - \frac{4}{3}$

$\boxed{\frac{8}{3}}$

(7) The function $f(x) = \frac{x^3+1}{x^2+1}$ has an oblique asymptote

[✓ (a) $y = x$]

(b) $y = x + 1$

(c) $y = x - 1$

(d) $y = -x$

$x^2 + 1 \overline{) x^3 + 1}$ quotient x

$x^3 + x$

$0 \quad 1 - x$

$x \cdot$ ~~$(1 - x)(x^2 + 1)$~~

(8) The linearization of the function $f(x) = \tan^2 x$ at $x = \frac{\pi}{4}$ is

(a) $L(x) = 4x - \pi$

(b) $L(x) = 2x - \frac{\pi}{2} + 1$

[✓ (c) $L(x) = 4x - \pi + 1$]

(d) $L(x) = 4x - \frac{\pi}{4} + 1$

①

$f(\frac{\pi}{4}) = 1$

$f' = 2\tan x(\sec^2 x)$

$2 \times 1 \times 2 = 4$

$1 + 4(x - \frac{\pi}{4})$

$1 + 4x - \pi$

$2\tan \sec^2$

$2 \times 1 \times 2 \; (4)$

$1 + 4(x - \frac{\pi}{4})$

$1 + 4x - \pi$

$4x - \pi + 1$

(9) $\int_{-1}^{0} x\sqrt{x+1}dx =$

(a) $\frac{2}{15}$

[✓ (b) $-\frac{4}{15}$]

(c) $\frac{4}{15}$

(d) $-\frac{2}{15}$

$u = x + 1$

$du = 1 dx$

$\int_0^1 (x) \sqrt{u} \, du$

$\int_0^1 (u-1)\sqrt{x}$

$u^{3/2} - \sqrt{u}$

$\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3} \Big|_0^1$

$\frac{2}{5} - \frac{2}{3}$

$\frac{6 - 10}{15}$

$\frac{-4}{15}$

(10) If $\frac{1}{x} + \frac{1}{y} = 1$, then $\frac{dy}{dx} =$

(a) $\frac{y^2}{x^2}$

[✓ (b) $-\frac{y^2}{x^2}$]

(c) $\frac{x^2}{y^2}$

(d) $-\frac{x^2}{y^2}$

$\frac{1}{x} + \frac{1}{y} = 1$

$\frac{-1}{x^2}$ ~~$+$~~ $\frac{-1 \times y'}{y^2} = 0$

$\frac{-y'}{y^2} = \frac{1}{x^2}$

$y' = -\frac{y^2}{x^2}$

$\frac{-1}{x^2} - \frac{y'}{y^2} = 0$

$\frac{-1}{x^2} - \frac{y'}{y^2} = 0$

$\frac{-y'}{y^2} = \frac{1}{x^3}$

$\frac{8}{3} - 8 + 6 + 1$

$\frac{8}{3} - 8 + 6 + 1$ $\frac{5}{3}$ $x^2 - 4x + 3$

$\frac{8}{3} - \frac{30}{3} \sim x^2 - 4x + 3 \,|$ $\frac{5}{3}$ $(x-3)(x-1)$ $\boxed{x=3}$ $\boxed{x=1}$

+ − + (1, 3)

(11) The function $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$ has

(a) local maximum at $x = 1$

(b) local minimum at $x = 3$

(c) inflection point at $(2, \frac{5}{3})$

[✓ (d) all of the above]

$f' = x^2 - 4x + 3 = 0$

$(x-3)(x-1)$

$x = 3 \quad x = 1$

+ − + (1, 3)

$2x - 4 = 0$ $2x - 4 = 0$

$2x = 4$ $2x = 4$

$x = 2$ $\boxed{x = 2}$

− + (2)

(12) The function $g(x) = 3x^4 - 8x^3 + 1$ is concave down on

[✓ (a) $[0, \frac{4}{3}]$]

(b) $(-\infty, 0]$

(c) $(-\infty, 0] \cup [\frac{4}{3}, \infty)$

(d) $[0, \infty)$

$g' = 12x^3 - 24x^2$

$g'' = 36x^2 - 48x$

$0 = 36x^2 - 48x$

$3x^2 - 4x$

$x(3x - 4) = 0$

$x = 0$ $x = \frac{4}{3}$ $x = 0$ $(x = \frac{4}{3})$

$4 \times 36 - 48 \times 2$ 12

$2 \times 2 \times 2 \times 2 \times 3 \times 3 - 2 \times 2 \times 2 \times 2 \times 3 \times 2$

+ − + (0, $\frac{4}{3}$)

(13) Let $f(x) = \int_0^{\tan x} \frac{dt}{t^2+1}$ then $f'(\pi) =$

[✓ (a) 1]

(b) 2

(c) 0

(d) Does not exist.

$f'(x) = \frac{1}{\tan^2 + 1} \times \sec^2 x = ①$

$f'(x)$

+ − (0, $\frac{4}{3}$)

$4 \times 3^2 6 - 48 \times 2$

$144 - 96$

(14) The volume of the solid whose cross sections perpendicular to the x−axis are squares with one side lying inside the unit circle $x^2 + y^2 = 1$

$y = \sqrt{1 - x^2}$

(a) $\frac{8}{3}$

[✓ (b) $\frac{16}{3}$]

(c) $\frac{4}{3}$

(d) $\frac{2}{3}$

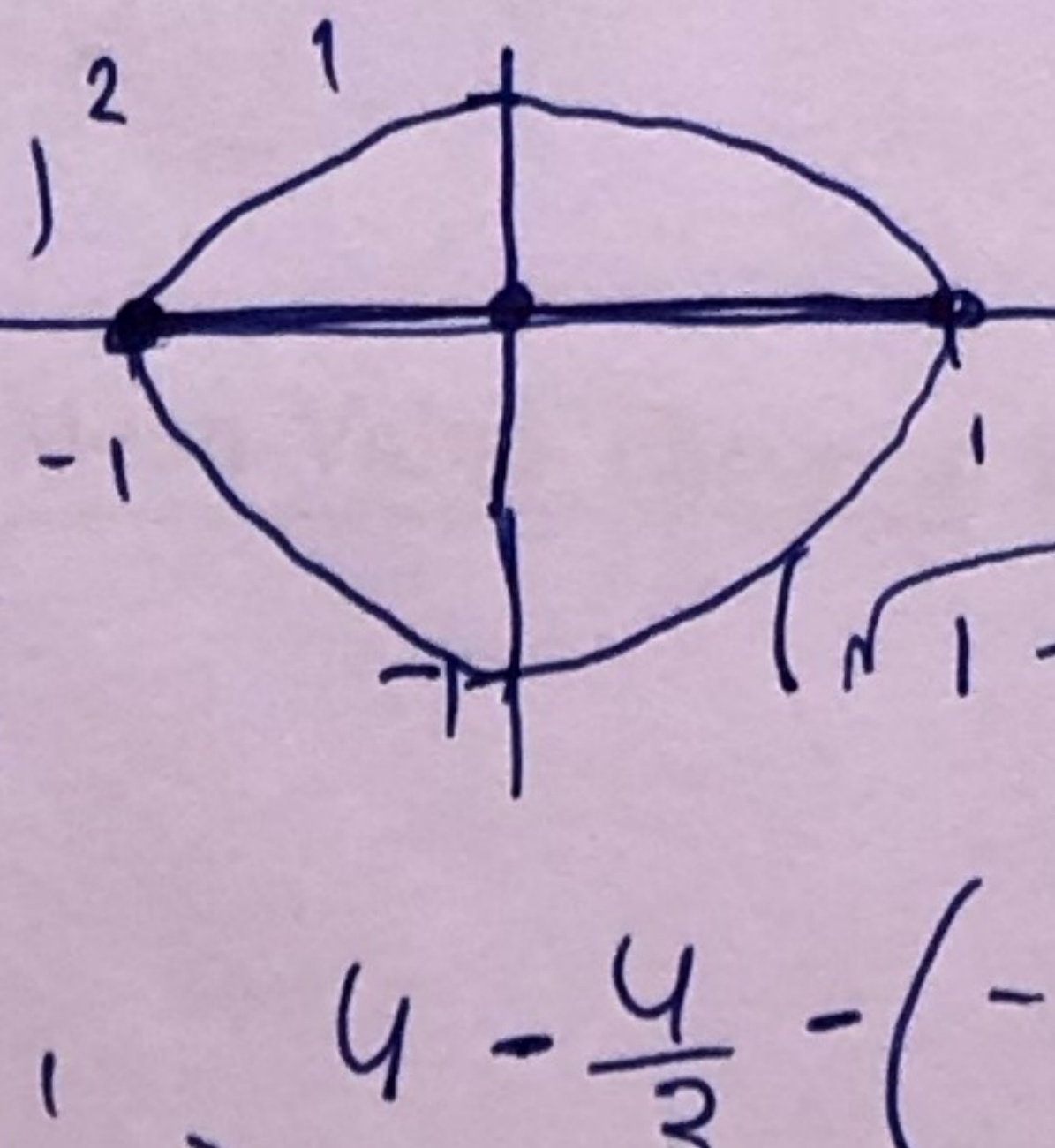
$\int_{-1}^{1} (\sqrt{1-x^2} - \sqrt{1-x^2})^2$

$(2\sqrt{1-x^2})^2$

$\int 4(1-x^2)$

$4 - 4x^2$

$4x - \frac{4x^3}{3} \Big|_{-1}^{1}$



$(\sqrt{1-x^2} + \sqrt{1-x^2})$

$2\sqrt{1-x^2}$

$4 - \frac{4}{3} - (-4 + \frac{4}{3})$

$4 - \frac{4}{3} + 4 - \frac{4}{3}$ $\frac{8}{3} \cdot \frac{8}{3}$

$\frac{16}{3}$

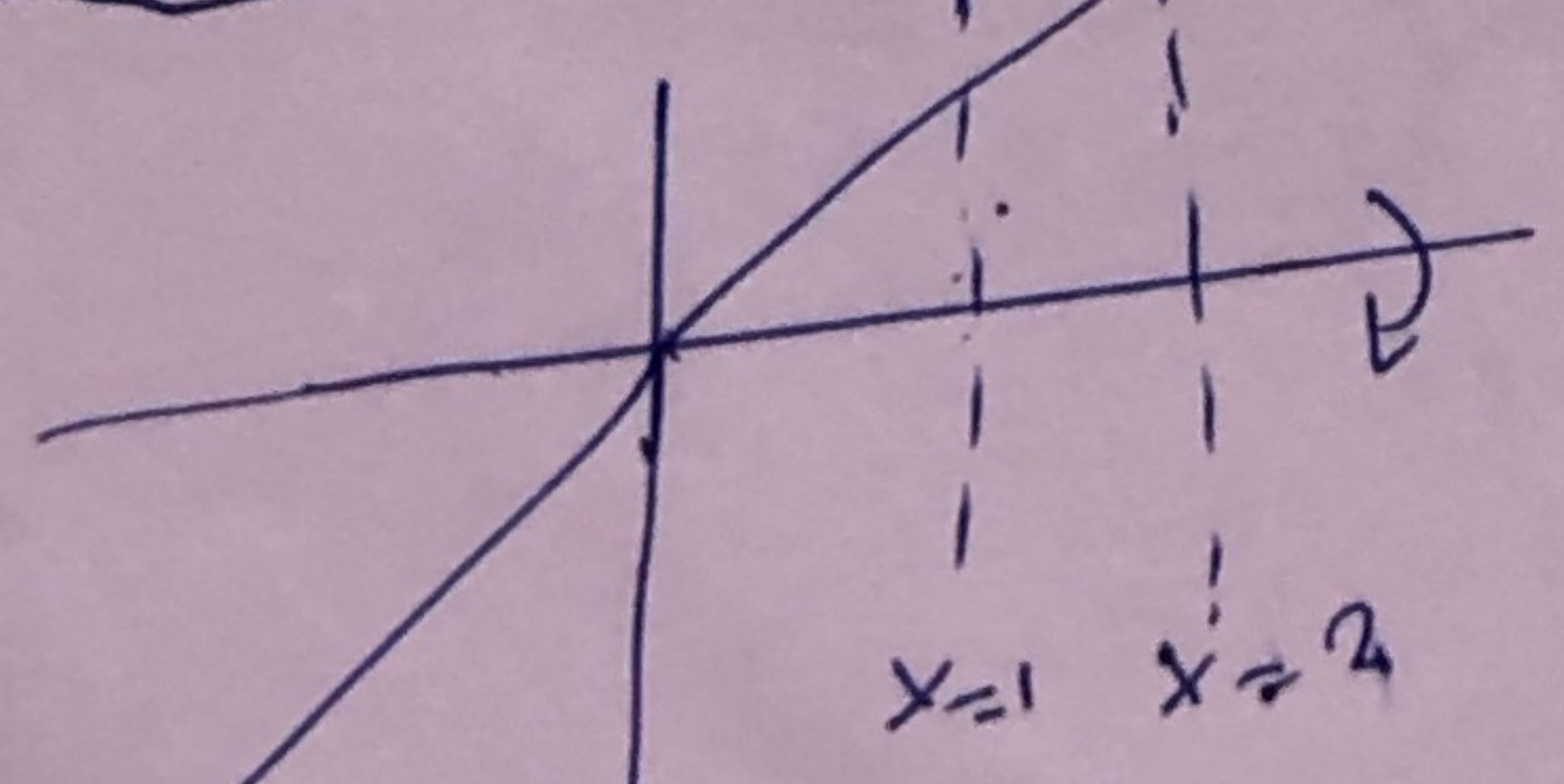
(15) The area of the surface generated by revolving the curve $y = x$ from $x = 1$ to $x = 2$ about the x−axis is

(a) 3π

(b) $\sqrt{\pi}$

(c) $2\sqrt{2}\pi$

[✓ (d) $3\sqrt{2}\pi$]



$\int_1^2 (2\pi)(x)\sqrt{2}$

$\int 2\sqrt{2}\pi x$

$y = x$

$y' = 1$

$(y')^2 = 1$

$(y')^2 + 1 = 2$

$\int_1^2 2\pi x$

$2\sqrt{2}\pi \frac{x^2}{2} \Big|_1^2$

$4\sqrt{2}\pi - \sqrt{2}\pi$

$3\sqrt{2}\pi$

~~$\sqrt{2}\pi x^2 \Big|$~~

~~$2\sqrt{2}\pi 4$~~

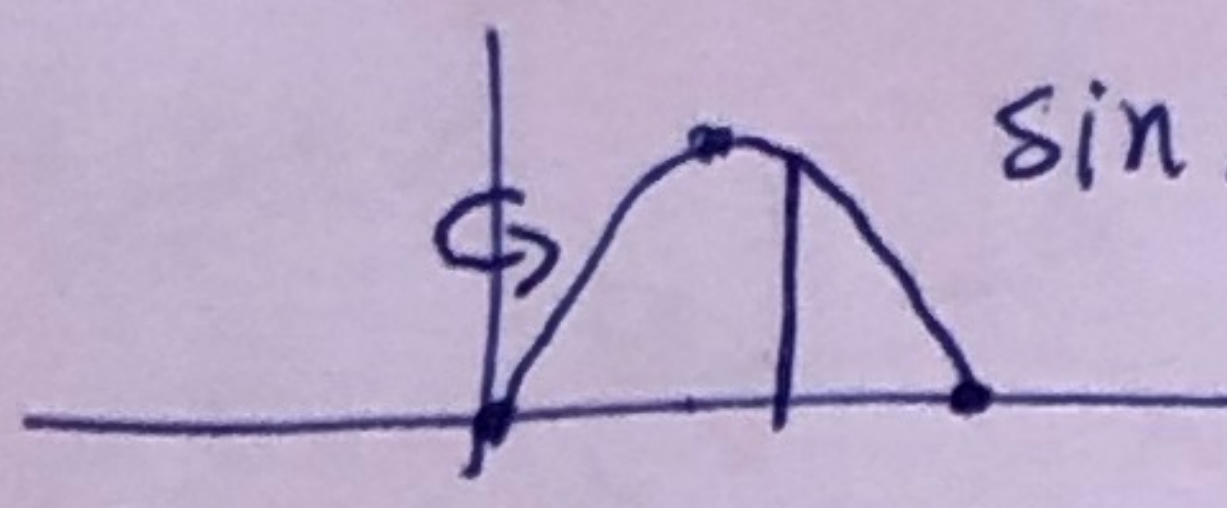
$\sqrt{2}\pi x^2 \Big|_1^2$

$4(\sqrt{2}\pi) - (\sqrt{2}\pi)$

$\boxed{3\sqrt{2}\pi}$

$\frac{x^2}{2} \Big|_1^2$

$2 - \frac{1}{2}$



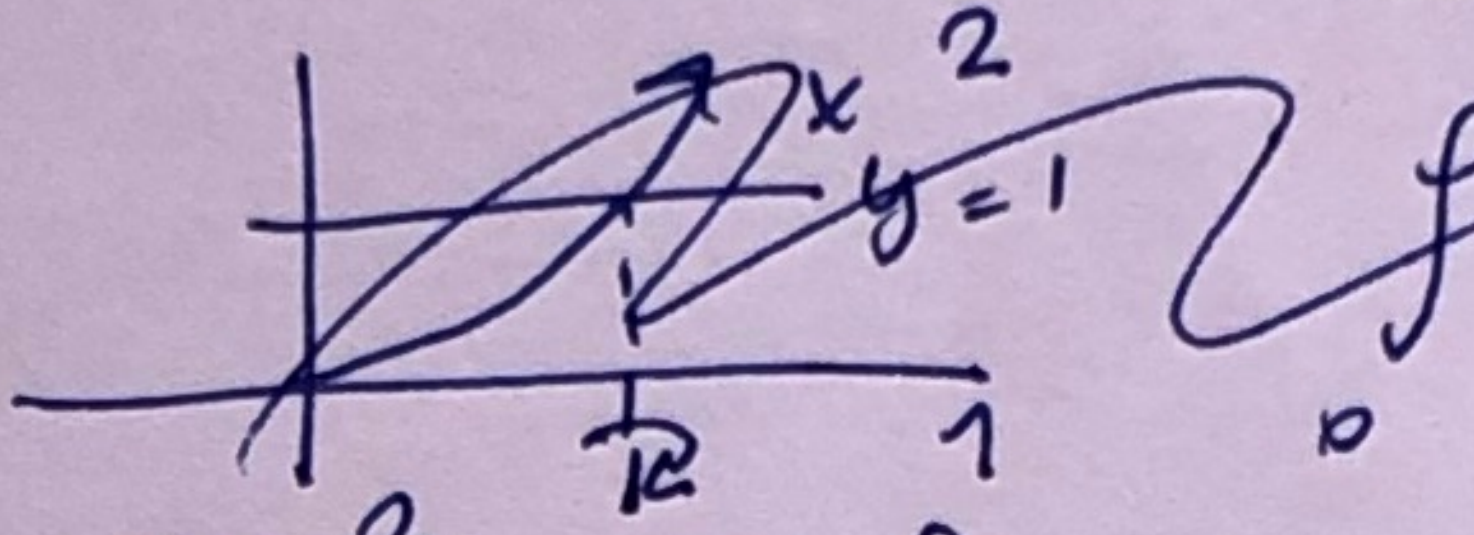
$\int_0^\pi 2\pi x(x)(\sin x)$

(16) The volume of the solid generated by revolving the area enclosed between the curve $y = \sin x$, $0 \le x \le \pi$ and the $x-$axis about the $y-$axis is

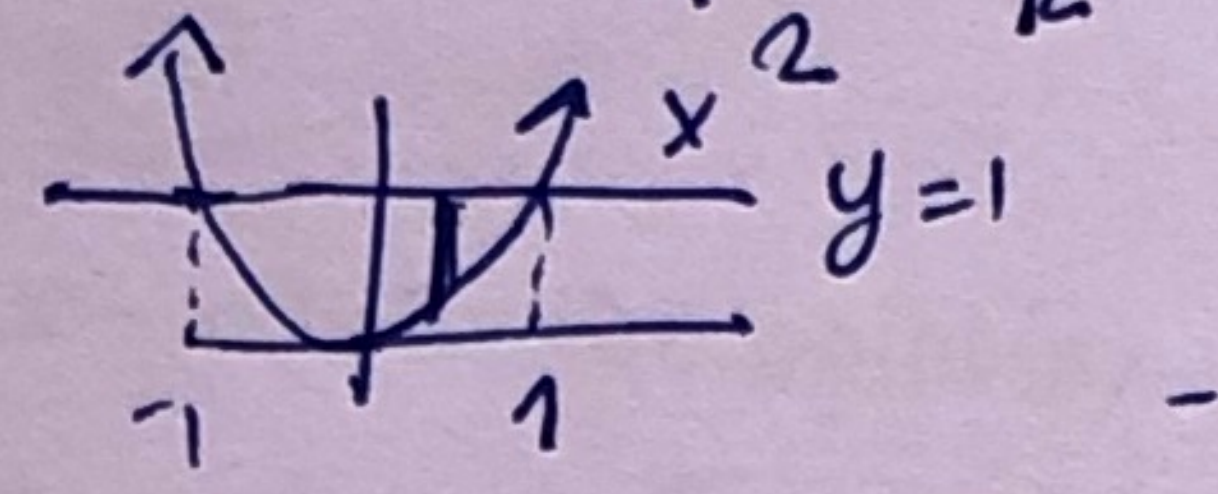
- (a) $V = 2\pi \int_0^\pi \sin^2 x dx$
- [✓ (b) $V = 2\pi \int_0^\pi x \sin x dx$]
- (c) $V = 2\pi \int_0^\pi x \sin^2 x dx$
- (d) $V = 2\pi \int_0^\pi \sin x dx$

(17) The volume of the solid generated by revolving the area enclosed between the curves $y = x^2$ and $y = 1$ about the line $y = 1$ is

- (a) $V = \pi \int_{-1}^1 (1 - x^2) dx$
- [✓ (b) $V = \pi \int_{-1}^1 (1 - x^2)^2 dx$]
- (c) $V = \pi \int_{-1}^1 x^4 dx$
- (d) $V = \pi \int_{-1}^1 (1 - x^4) dx$



$\int_0^1 \pi x (1 - \sqrt{y})^2 dy$



$\int_{-1} \pi x (1 - x^2)^2$

$\sin^2 x =$

$\frac{\sin^2 x}{x} = \lim_{x \to} \frac{\sin^2 x}{x} = \lim$

(18) The function $f(x) = \begin{cases} \frac{\sin^2 x}{x} & , \ x < 0 \\ x + a & , \ x \ge 0 \end{cases}$ is continuous at $x = 0$ if $a =$

- (a) -1
- (b) 3
- (c) 1
- [✓ (d) 0]

$\lim_{x \to 0} \frac{\sin^2 x}{x} = \lim_{x \to 0} x + a$

$\lim = a$

$\lim_{x \to 0} \frac{\sin^2 x}{x}$ $\frac{2\sin x \cos x}{1}$ $\frac{2 \times 0 \times 1}{1}$ $\textcircled{0}$

$a = \lim_{x \to 0} \frac{\sin^2 x}{\textcircled{x}}$ $\frac{2\sin x \cos x}{1}$

0

$\frac{\sin x}{x} \times \frac{\sin x}{1 x}$

(19) Let $f(x) = x^3$, $x \in [0, 1]$, then the value of c in the conclusion of the Mean Value Theorem is

- (a) $c = \frac{1}{3}$
- (b) $c = 1$
- [✓ (c) $c = \frac{1}{\sqrt{3}}$]
- (d) $\pm\frac{1}{\sqrt{3}}$

$f'(c) = \frac{f(1) - f(0)}{1 - 0}$

$3x^2 = \frac{1 - 0}{1}$

$3x^2 = 1$

$\sqrt{x^2} = \sqrt{\frac{1}{3}}$

$x = \pm\sqrt{1}$

$\pm\frac{1}{\sqrt{3}}$

0.7

(20) If the radius of a sphere changes from 1 to 1.1, then the volume of the sphere, $\boxed{V = \frac{4}{3}\pi r^3,}$ changes approximately by

- (a) 0.1π
- (b) 0.2π
- (c) 0.3π
- [✓ (d) 0.4π]

$dr = 1,1 - 1 = \boxed{0,1}$

$r_1 = 1$

$dV = V'(x) dr$

$= \frac{4}{3}\pi 3 r^2 \times 0,1$

$4\pi \times 1 \times 0,1$

$0,4 11$

$V = \frac{4}{3}\pi r^3$

$dV = \frac{4}{3}\pi \times 3 r^2$

$\frac{dv}{V} = \frac{4\pi r^2 dr}{\frac{4}{3} r^3 \pi}$

$\frac{dv}{V} = \frac{3 \times dr}{r}$

$\frac{4-12}{9}$ $\frac{-8}{9}$ $4-$ $\frac{4-4\times 8}{}$ $\frac{4-8}{4}$ $\frac{-4}{4}$ $\frac{4}{x^2}$ $-\frac{4}{x}$ $\frac{x^4}{x}\left(\frac{1}{x}-1\right)$ $\frac{4-4x}{x^2}$ $\frac{+}{+}$ ~~$4(1-x)$~~ $4+$

Q2) [24 %] Consider the function $f(x)$ and its first and second derivatives

$$f(x) = \frac{4-4x}{x^2}, \quad f'(x) = \frac{4(x-2)}{x^3}, \quad f''(x) = \frac{8(3-x)}{x^4}$$

$4-4\times 0.1$

(1) The domain of f is $\mathbb{R}\setminus\{0\}$

(2) $\lim_{x\to 0^+} f(x) =$ $\lim_{x\to 0^+} \frac{4-4x}{x^2} = \frac{+}{+} \Rightarrow +\infty$

$4(x-2)=0$

$x-2=$

$\boxed{x=2}$

(3) $\lim_{x\to 0^-} f(x) =$ $+\infty$

(4) $\lim_{x\to +\infty} f(x) =$ 0

25

(5) $\lim_{x\to -\infty} f(x) =$ 0

$+ \quad 0 \quad - \quad 2 \quad +$

(6) Horizontal asymptote is $y=0$

(7) Vertical asymptote is $x=0$

(8) the graph of f crosses the x−axis at the point(s) $4-4x=0 \to 4=4x \to \boxed{1=x}$ $(1,0)$

(9) f is increasing on $(-\infty,0)\cup[2,\infty)$

(10) f is decreasing on $(0,2]$

(11) f has local maximum value(if any) No local max since $x\in D$

$8(3-x)$

(12) f has local minimum value (if any) $x=2,$ $\boxed{(2,-1)}$

$3=x$

(13) f is concave up on $(-\infty,0)\cup(0,3]$

$+ \quad 0 \quad + \quad 3 \quad -$

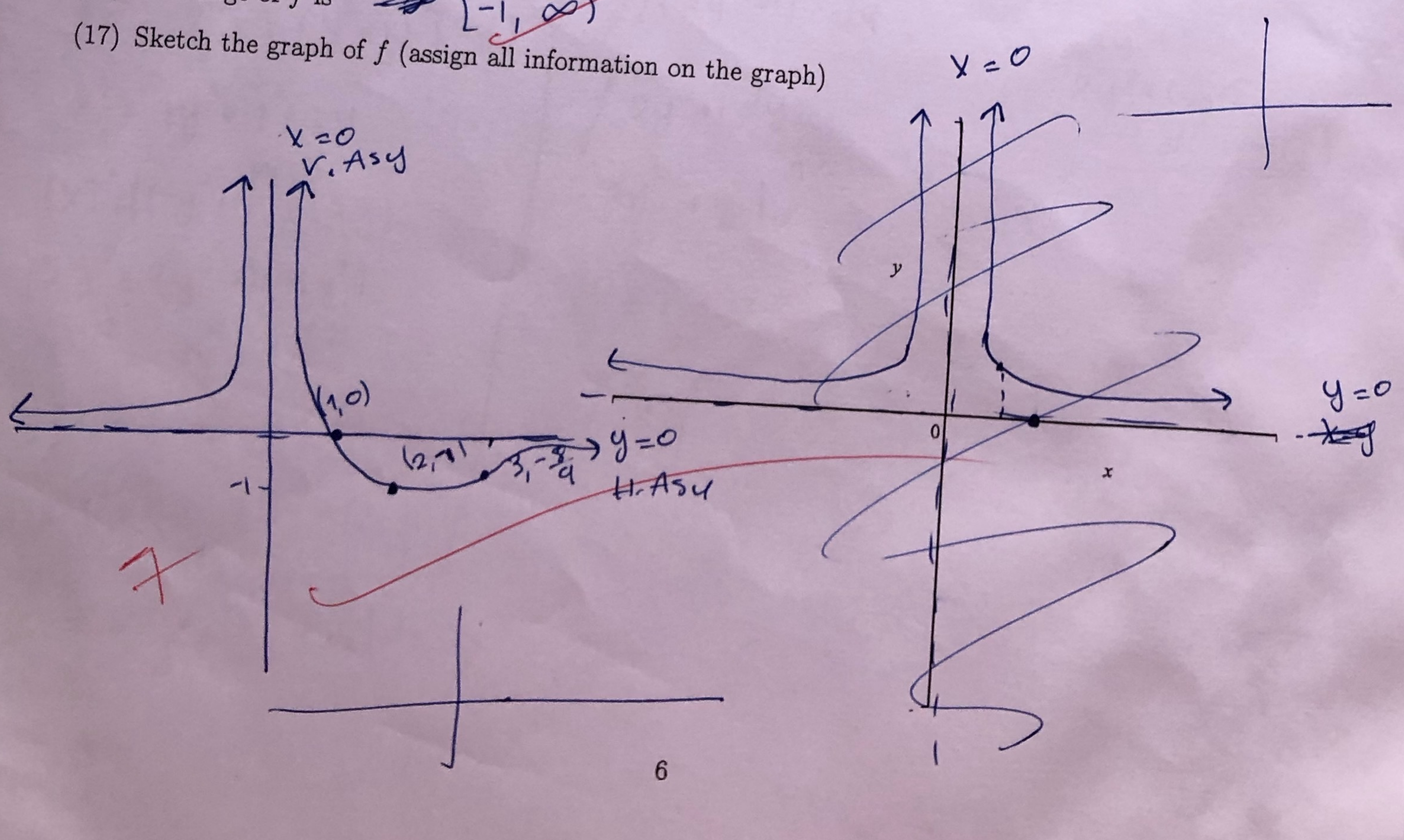
(14) f is concave down on $[3,\infty)$

(15) f has inflection point(s) $x=3 \to (3,-\frac{8}{9})$

(16) the range of f is $[-1,\infty)$

(17) Sketch the graph of f (assign all information on the graph)

$x=0$



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Q3) [10%] Find the volume of the solid generated by revolving the area enclosed between the curve $y = \sqrt{x}$ and the curve $y = x^3$ about (a) the x−axis using washers method and (b) about the y−axis using the shell method.

(a) $V = \int_0^1 \pi x (R^2_{(x)} - r^2_{(x)})$

$= \int_0^1 \pi x (\sqrt{x})^2 - (x^3)^2 dx$

$\int_0^1 \pi (x - x^6) dx$

$\pi (\frac{x^2}{2} - \frac{x^7}{7} |_0^1 =$

$= \pi (\frac{1}{2} - \frac{1}{7} - 0)$

$= \pi (\frac{7-2}{14}) = \boxed{\frac{5\pi}{14}}$

5

(b) $V = \int_0^1 2\pi (\text{shell radius})(\text{shell length}) dx$

$V = \int_0^1 2\pi (x)(\sqrt{x} - x^3) dx$

$\int_0^1 2\pi (x^{3/2} - x^4) dx$

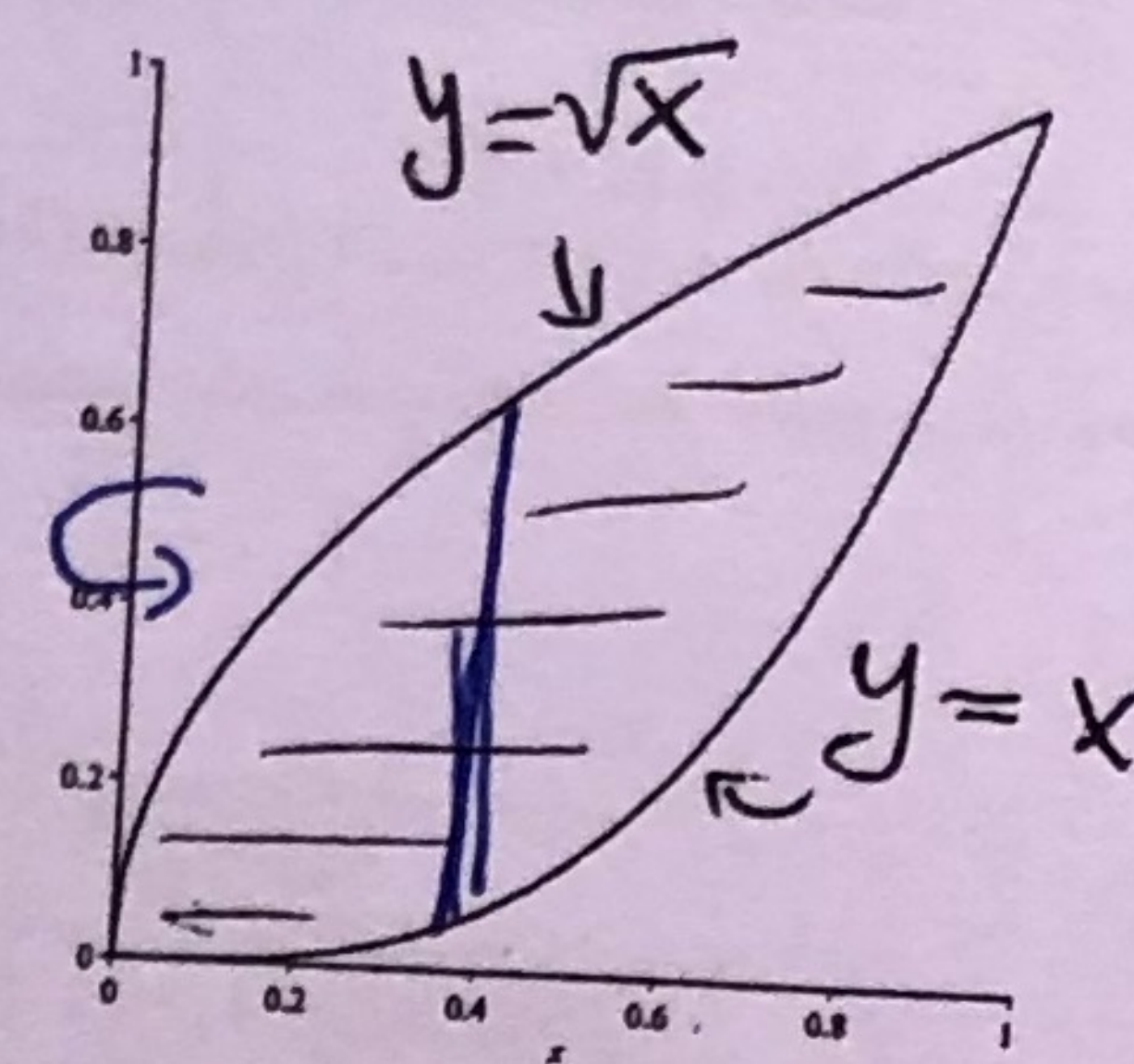
$\int_0^1 2\pi$

$2\pi (\frac{2x^{5/2}}{5} - \frac{x^5}{5}) |_0^1$

$2\pi (\frac{2}{5} - \frac{1}{5} - 0) =$

$\boxed{\frac{2\pi}{5}}$

5



$\sqrt{x} = x^3$

$\sqrt{x} - x^3 = 0$

$\sqrt{x}(1 - x^{5/2}$

$x = 0$

$x = 1$

Q4) [6%] Find the length of the curve $x = \frac{2}{3}(y^2 + 1)^{3/2}$, $0 \leq y \leq 1$.

length → $f'(x) > 0$ cont

→ x' is ≥ 0

$(2y^2+1)$

$4y^4 + 4y^2 + 1$

$x = \frac{2}{3}(y^2+1)^{3/2}$ $0 \leq y \leq 1$

$x' = \frac{2}{3} \times \frac{3}{2} (y^2+1)^{1/2} (2y)$

$x' = (y^2+1)^{1/2} (2y$

$(x')^2 = ((y^2+1)^{1/2})^2 (2y)^2$

$(x')^2 = (y^2+1)(4y^2)$

$(x')^2 = 4y^4 + 4y^2$

$(x')^2 + 1 = 4y^4 + 4y^2 + 1$

$(x')^2 + 1 = (2y^2+1)^2$

$L = \int_0^1 \sqrt{1 + (\frac{dy}{dx})^2} dy$

$= \int_0^1 \sqrt{(2y^2+1)^2} dy$

$\int_0^1 2y^2 + 1 dy$

$\frac{2y^3}{3} + y |_0^1$

$\frac{2}{3} + 1 - 0$

$\boxed{\frac{5}{3}}$

6

$2y^2 +$

$2y^2 + 1 \geq 0$

always