$\frac{80}{85}$

جامعة بيرزيت

BIRZEIT UNIVERSITY

Mathematics Department

CALCULUS FOR HEALTH SCIENCES – Math1431

Second Exam Time: 90 Minutes First Semester 2022 – 2023

Name: Malak Laham Number: 1221071 Section: 3D

- Write your full name (In Arabic) and your number
- Choose your section from table below
- Turn off your mobile
- Calculator is not allowed
- Answer all questions

Section	Instructor	Day	Time	Room
1	Mohammad Adarbeh	R	10:00 - 10:50	SCI215
2	Farah Omar	M	09:00 - 09:50	SCI213
3	Mohammad Adarbeh	T	10:00 - 10:50	SCI215
4	Farah Omar	T	10:00 - 10:50	SCI113

Question One (42 points) Circle the most correct answer:

39

1. $\lim\limits_{x\to\infty} \frac{(-2x+4)^2}{3x^2+4x+1}$

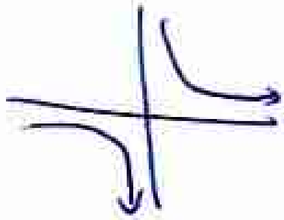
(a) $\frac{4}{3}$

(b) ∞

(c) 0

x (d) $-\frac{2}{3}$

2. If $\frac{1}{x} \le \frac{\ln x}{x} \le \frac{1}{\sqrt{x}}$, then $\lim\limits_{x\to\infty} \frac{\ln x}{x} =$ $\lim\limits_{x\to\infty} \frac{1}{x} = 0$



(a) 0

(b) ∞

(c) $-\infty$

(d) 1

3. The limit $\lim\limits_{h\to 0} \frac{4(a+h)^3-4a^3}{h}$ represents the derivative of $f(x)$ at the point $(a, f(a))$, then:

$f(x) = 4x^3$

(a) $f(x) = 12x^2$

(b) $f(x) = 4x^3$

(c) $f(x) = x^4$

(d) $f(x) = 4(x+1)^3$

4. $\lim\limits_{x\to -\infty} \frac{1-5x^2}{3x^4+1}$

(a) $-\frac{5}{3}$

(b) ∞

(c) $-\infty$

(d) 0

5. If $f(x) = 2^{\sqrt{x}}$, then $f'(1) =$

$f'(x) = \ln 2 \times 2^{\sqrt{x}} \times \frac{1}{2\sqrt{x}}$

$f'(1) = \ln 2 \times 2^{1} \times \frac{1}{2}$
$= \ln 2$

(a) $\ln 2$
(b) $\ln 4$
(c) 1
(d) 2

6. The horizontal tangent lines to the curve $y^2 = 8x^4 - x^2$ occur at:

$y' = 0$
$y^2 = 8x^4 - x^2$
$2y y' = 32x^3 - 2x$
$0 = 2x(16x^2 - 1)$
$2x = 0$
$x = 0$
$16x^2 = 1$
$x^2 = \frac{1}{16}$
$x = \pm\frac{1}{4}$

(a) $x = 0$
(b) $x = \frac{1}{4}$
(c) $x = -\frac{1}{4}$
(d) All of the above.
(e) There is no horizontal tangent.

7. The function $f(x) = x^3 - 3x^2 + 2x + 1$ has a root in the interval:

(a) $[0, 1]$
(b) $[-1, 0]$
(c) $[2, 3]$
(d) f has no roots.

$f(x) = 0$
$f(0) = 1$
$f(1) = 1 - 3 + 2 + 1$
$= 1$
$f(-1) = -1 - 3 - 2 + 1$
$= -5$
$f(2) = 8 - 12 + 4 + 1$
$= 1$
$f(3) = 27 - 27 + 6 + 1$
$= 2$

8. If $g(x) = 5x^2 + e^2$ then $g''(x) =$

$g'(x) = 10x + 0$
$g''(x) = 10$

(a) $10x + 2e$
(b) 12
(c) 10
(d) 0

9. The velocity of a moving object is given by: $v(t) = t^2 + 4t$, the acceleration of the object at time t is:

$a = v'(t) = 2t + 4$

(a) $2t + 4$
(b) $2t$
(c) $-2t + 4$
(d) $-2t$

10. Let $f(x) = \frac{(x+3)(x+5)}{(x-2)(x-6)}$, $f(0) = \frac{15}{12}$ and $f(3) = -16$, then the Intermediate value theorem (IMVT) can be used to conclude that $f(c) = 0$ for some c between 0 and 3

(a) True

(b) False

$[0, 3]$

(c) You need more information to determine whether you can use the IMVT or not.

11. The value of the slope of the normal line to the curve: $f(x) = 2x^3 - 3x^2 - 12x + 5$ at the point $(1, f(1))$ is:

(a) -12

(b) 12

(c) $\frac{1}{12}$

(d) $-\frac{1}{12}$

عمودي على المماس $M = ??$

$f'(x) = 6x^2 - 6x - 12$

$f'(1) = 6 - 6 - 12$

$f'(1) = -12$

$m = -12$

$m_n = \frac{1}{12}$

12. $\lim_{x \to 0} \frac{\sin(\pi x)}{x}$

(a) 1

(b) 0

(c) π

(d) $\frac{1}{\pi}$

$\lim_{x \to 0} \frac{\sin(\pi x)}{x \pi} \cdot \pi$

$= \pi \times 1 = \pi$

13. Consider the function $h(x) = e^{(g(x))^2}$. Assume that $g(2) = 2$ and $g'(2) = -3$. Then $h'(2) =$

(a) $4e^4$

(b) $-6e^4$

(c) e^4

(d) $-12e^4$

$h'(x) = 2\,g(x)\,g'(x)\,e^{(g(x))^2}$

$h'(2) = 2\,g(2)\,g'(2)\,e^{(g(2))^2}$

$= 2 \times 2 \times -3 \times e^4$

$= -12e^4$

14. If a function f is continuous at the point $(a, f(a))$, then it is differentiable there.

(a) True

(b) False

Question Two (10 points) Find the following limits:

1. $\lim_{x\to0} \frac{(1-\cos x)^2}{x\sin x}$

$\sin^2 x + \cos^2 x = 1$

$\lim_{x\to0} \frac{1 - 2\cos x + \cos^2 x}{x\sin x} = \lim_{x\to0} \frac{1 - 2\cos x + 1 - \sin^2 x}{x\sin x}$

$= \lim_{x\to0} \frac{2 - 2\cos x}{x\sin x} - \frac{\sin^2 x}{x\sin x}$

$= \lim_{x\to0} \frac{2(1-\cos x)}{x\sin x} - \frac{\sin x}{x}$

$= \lim_{x\to0} \frac{2}{\sin x} * \frac{1-\cos x}{x} - \frac{\sin x}{x}$

$=? \frac{2}{0} * 0 - 1$

$= 0 + -1$

$= -1$

X

2

2. $\lim_{x\to0} x^2 \sin\frac{1}{x}$

$-1 \leq \sin\frac{1}{x} \leq 1$

$-x^2 \leq x^2\sin\frac{1}{x} \leq x^2$

$\lim_{x\to0} -x^2 = 0 \qquad \lim_{x\to0} x^2 = 0$

by ST $\lim_{x\to0} x^2\sin\frac{1}{x} = 0$

5

Question Three (7 points) Given the function $f(x) = \frac{1}{3x}$. Using the definition of the derivative, find $f'(2)$. (You will receive NO credit for finding the derivative with rules).

$f'(x) = \lim_{h\to0} \frac{f(h+x) - f(x)}{h}$

$f'(2) = \lim_{h\to0} \frac{f(h+2) - f(2)}{h}$

$= \lim_{h\to0} \frac{\frac{6 \times 1}{6 \times 3(h+2)} - \frac{1}{6} \times \frac{3(h+2)}{3(h+2)}}{h}$

$= \lim_{h\to0} \frac{\frac{6 - 3(h+2)}{18(h+2)}}{h}$

$= \lim_{h\to0} \frac{\cancel{3}(2-(h+2))}{6\,\cancel{18}\,h(h+2)} = \lim_{h\to0} \frac{2-(h+2)}{6h(h+2)}$

$= \lim_{h\to0} \frac{\cancel{2} - h - \cancel{2}}{6h(h+2)} = \frac{-1}{6(h+2)}$

$= \frac{-1}{6(0+2)}$

$= \frac{-1}{12}$

4

Question Four (12 points) Find $f'(x)$ for the following functions.

2

1. $f(x) = x^3\, 5^{(-3x)}$

$f'(x) = 3x^2\, 5^{(-3x)} + x^3\ (\ln 5)\ (5^{(-3x)})\ -3$

$= 3x^2\, 5^{(-3x)} + -3\ln 5\ x^3\ 5^{-3x}$

3

2. $f(x) = \frac{sec^3 x}{x^2}$

$f'(x) = \frac{x^2(3\ sec^2 x\ \sec x \tan x) - sec^3 x\ (2x)}{(x^2)^2}$

3

$f'(x) = \frac{3x^2\ sec^3 x\ \tan x - 2x\ sec^3 x}{x^4}$

3. $f(x) = \tan^2(\sin x)$

$f(x) = (\tan(\sin x))^2$

$f'(x) = 2\,(\tan(\sin x))\ sec^2(\sin x)\ \cos x$

$= 2\ \cos x\ \tan(\sin x)\ sec^2(\sin x)$

3

4. $f(x) = (2x^3 - 12x^2)(18x + x^4)$

$f'(x) = (6x^2 - 24x)(18x + x^4) + (2x^3 - 12x^2)(18 + 4x^3)$

3

(a) Find $\frac{dy}{dx}$ by differentiating implicity $y^3 - 2y^2 + 1 = 2x^2 - 3x$.

$$y^3 - 2y^2 + 1 = 2x^2 - 3x$$

$$3y^2 \bar{y} - 4y\bar{y} + 0 = 4x - 3$$

$$\bar{y}(3y^2 - 4y) = 4x - 3$$

$$\bar{y} = \frac{dy}{dx} = \frac{4x-3}{3y^2 - 4y}$$

5

(b) Find the equation of the tangent line to the curve in (a) at the point (0, 1).

$$\left.\frac{dy}{dx}\right|_{\substack{x=0\\ y=1}} = \frac{4\times 0 - 3}{3\times 1^2 - 4\times 1} = \frac{0-3}{3-4} = \frac{-3}{-1} = 3$$

slope $= m = 3$

5

~~$x = 0$~~

~~$y^3 - 2y^2 + 1 = 20 - 3\times 0$~~

~~$y^3 - 2y^2 = -1$~~

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x - 0)$$

$$y - 1 = 3x$$

$$\boxed{y = 3x + 1}$$

(c) Air is being pumped into a spherical balloon at the rate of 7 cm^3/sec. What is the rate of change of the radius at the instant the volume equals $36\pi\ cm^3$? The volume of the sphere of radius r is $V = \frac{4\pi}{3}r^3$

$$\frac{dr}{dt} = ??$$

$$\frac{dV}{dt} = 7\ cm^3/sec \qquad V = 36\pi\ cm^3$$

5

$$V = \frac{4\pi}{3}r^3$$

$$36\pi = \frac{4\pi}{3}r^3$$

$$r^3 = 36 \times \frac{3}{4} = 9\times 3$$

$$r^3 = 27$$

$$r = 3$$

$$V = \frac{4\pi}{3}r^3$$

$$\frac{dV}{dt} = \frac{4\pi}{3}\times 3 r^2 \frac{dr}{dt}$$

$$7 = 4\pi \times 3^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{7}{4\pi\times 9}$$

$$\frac{dr}{dt} = \frac{7}{36\pi}$$