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جامعة بيرزيت
BIRZEIT UNIVERSITY

Mathematics Department

CALCULUS FOR HEALTH SCIENCES – Math1431

First Exam First Semester 2022 – 2023

Name: Mulak Lahom Number: 1221071 Section: 30

- Write your full name and your number
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Section	Instructor	Day	Time	Room
1	Mohammad Adarbeh	R	10:00 - 10:50	SCI215
2	Farah Omar	M	09:00 - 09:50	SCI213
3	Mohammad Adarbeh	T	10:00 - 10:50	SCI215
4	Farah Omar	T	10:00 - 10:50	SCI113

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Question One (70 points) Choose the correct answer in each of the following parts.

1. The solution of the inequality $|1-2x| < 3$ is

(a) $(-\infty,-1)$
(b) $(-2,2)$
(c) $(-\infty,-1)\cup(2,\infty)$
(d) $(-1,2)$

$-3 < 1-2x < 3$
$-1 \quad -1 \quad -1$
$\frac{-4}{-2} < \frac{-2x}{-2} < \frac{2}{-2}$
$2 > x > -1$

2. The period of the trigonometric function $y = 3\sin(\frac{4x}{7})+2$ is

(a) $\frac{7\pi}{2}$
(b) $\frac{10\pi}{3}$
(c) $\frac{4\pi}{5}$
(d) 5π

$P = \frac{2\pi}{\frac{4}{7}}$

$\frac{2\pi \times 7}{4}$

$= \frac{7\pi}{2}$

3. The domain of the function $\frac{\sqrt{x^2-1}}{x-2}$ is

(a) $(-\infty,-1]\cup[1,2)\cup(2,\infty)$
(b) $(-\infty,-1]\cup[1,\infty)$
(c) $(-\infty,-2]\cup[2,\infty)$
(d) $(-\infty,-3)\cup(-3,-2]\cup[2,\infty)$

$x^2-1 \geq 0$
$x^2 = 1$
$x = \pm 1$
$-1 \quad 1 \quad 2$
$(-\infty,-1] \cup [1,2)$
$\cup (2,\infty)$

4. $\lim\limits_{x\to-8} \frac{x^2+10x+16}{x^2-64} =$

(a) $\frac{-3}{2}$
(b) $\frac{3}{8}$
(c) $\frac{-1}{8}$
(d) $\frac{1}{10}$

$\lim\limits_{x\to-8} \frac{x^2+10x+16}{x^2-64}$
$= \frac{(x+2)\cancel{(x+8)}}{(x-8)\cancel{(x+8)}}$
$= \frac{x+2}{x-8}$
$= \frac{-8+2}{-8-8}$
$= \frac{-6}{-16} = \frac{3}{8}$

5. The line L_1 has x-intercept 2 and y-intercept 6, the line L_2 passes through the point $(2,3)$ and perpendicular to L_1, then the equation of L_2 is

(a) $3y-x=13$
(b) $3y-x=9$
(c) $3y-x=7$
(d) $3y-x=11$

L_1 $(2,0)$
$(0,6)$
$m_1 = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{6-0}{0-2}$
$= \frac{6}{-2}$
$= -3$

L_2
$(2,3)$
$m_2 = \frac{1}{3}$
$y-y_1 = m(x-x_1)$
$y-3 = \frac{1}{3}(x-2)$
$y-3 = \frac{1}{3}x - \frac{2}{3}$
$+3 \qquad +3$
$3\left(y = \frac{1}{3}x + \frac{7}{3}\right)$
$3y = x+7$
$3y-x=7$

6. The solution of the equation:

$$\log_2(x^2-3x) - \log_2(-x+1) = 1$$

(a) $\{-1,2\}$
(b) $\{2\}$
(c) $\{3\}$
(d) $\{-1\}$
(e) $\{-4,2\}$

$\log_2 \frac{x^2-3x}{-x+1} = 1$
$2^1 = \frac{x^2-3x}{-x+1}$
$2(-x+1) = x^2-3x$
$-2x+2 = x^2-3x$
$+2x \qquad +2x$

$x^2-x-2=0$
$(x+1)(x-2)=0$
$x=-1 \quad x=2$
✓ ✗

7. The center and the radius of the circle whose equation is

$$x^2 + 4x + 1 + y^2 - 2y = 0$$

$x^2 + 4x + 4 - 4 + y^2 - 2y + 1 - 1 = -1 + 4 + 1$

$x^2 + 4x + 4 + y^2 - 2y + 1 = 4$

$(x+2)^2 + (y-1)^2 = 2^2$

$x = -2 \quad y = 1 \quad r = 2$

$(-2, 1)$

(a) $(-2,-1), r=2$

(b) $(2,-1), r=2$

(c) $(2,1), r=2$

(d) $(-2,1), r=2$

(e) $(-2,-1), r=3$

8. $16^{-\log_{\frac{1}{2}} x}$

$16^{-\log_{\frac{1}{2}} x}$

$\left(\frac{1}{2}\right)^{4(-\log_{\frac{1}{2}} x)}$

$\frac{1}{2}^{\log_{\frac{1}{2}} x^4}$

$= x^4$

(a) x^2

(b) x^{-2}

(c) x^4

(d) x^{-4}

9. The range of the function: $f(x) = 5\cos(2x-3)+2$ is

$-1 \leq \cos\theta \leq 1$

$-5 \leq 5\cos\theta \leq 5$

$+2 \quad +2 \quad +2$

$-3 \leq 5\cos\theta \leq 7$

(a) $[-3,7]$

(b) $[-1,1]$

(c) $[0,3]$

(d) $\mathbb{R}$

10. Suppose that $N_t = 80.2^t$, $t = 0,1,2,\ldots$ and one unit of time is 8 hours. Determine the time it takes the population to double in size

$N_t = 80.2^t$

2^8

(a) 1 hour.

(b) 4 hours.

(c) 8 hours.

(d) 0.5 hour.

(e) 2 hours.

15. The inverse of the function $f(x) = e^{2x} - 1$ is

(a) $f^{-1}(x) = \ln \frac{x+1}{2}$
(b) $f^{-1}(x) = \frac{\ln(x+1)}{\ln 2}$
(c) $f^{-1}(x) = \ln\sqrt{x-1}, x > 1$
(d) $f^{-1}(x) = \ln\sqrt{x+1}, x > -1$

$f(x) = e^{2x} - 1$
$y = e^{2x} - 1$
$y + 1 = e^{2x}$
$\ln y+1 = \ln e^{2x}$
$\ln y+1 = 2x$
$x = \frac{1}{2}\ln y+1$
$x = \ln (y+1)^{\frac{1}{2}}$
$x = \ln\sqrt{y+1}$
$f^{-1}(x) = \ln\sqrt{x+1}$

16. The set of fixed points of the recursive sequence $a_{n+1} = \frac{4a_n}{2+a_n}$ is

(a) $\{0, -1\}$
(b) $\{0, 1\}$
(c) $\{0, 2\}$
(d) $\{0, -2\}$
(e) $\{0, 3\}$

$L = \frac{4L}{2+L}$
$L(2+L) = 4L$
$2L + L^2 = 4L$
$-4L \quad -4L$
$L^2 - 2L = 0$
$L(L-2) = 0$
$L = 0 \quad L = 2$

17. $\lim\limits_{n\to\infty} \frac{-2n^2+5n-7}{-n+7} =$

(a) 2
(b) ∞
(c) $-\infty$
(d) 0

$\lim\limits_{n\to\infty} \frac{\frac{-2n^2}{-n} + \frac{5n}{-n} - \frac{7}{-n}}{\frac{-n}{-n} + \frac{7}{-n}}$
$= \frac{2n - 5 + \frac{7}{n}}{1 + \frac{-7}{n}}$
$= \frac{\infty + 0}{1 + 0}$
$= \infty$

18. If the function

$$f(x) = \begin{cases} 2x+2 & \text{if } x \le 2, \\ x^2+bx & \text{if } 2 < x \le 4, \\ ax+4 & \text{if } x > 4. \end{cases}$$

is continuous for all x, then the values of a and b are

(a) $a = 1, b = 4$
(b) $a = 4, b = 1$
(c) $a = -1, b = -3$
(d) $a = -3, b = -1$

$\lim\limits_{x\to 2^-} = \lim\limits_{x\to 2^+}$
$4 + 2 = 4 + 2b$
$6 = 4 + 2b$
$-4 \quad -4$
$2 = 2b$
$b = 1$

$\lim\limits_{x\to 4^+} = \lim\limits_{x\to 4^-}$
$4a + 4 = 16 + 4x1$
$4a + 4 = 20$
$-4 \quad -4$
$4a = 16$
$\boxed{a = 4}$

$|x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$

19. $\lim_{x\to 0} \frac{|x-1|-1}{x} =$

(a) 0

(b) 1

(c) -1

(d) Does not exist.

$\frac{|x-1|-1}{x} = \begin{cases} \frac{x-1-1}{x}, & x \geq 1 \\ \frac{-x+1-1}{x}, & x < 1 \end{cases}$

$\begin{cases} \frac{x-2}{x}, & x \geq 1 \\ -1, & x < 1 \end{cases}$

20. If the graph of the the function $f(x) = x^2 - 6x + c$ intersects the x axis twice, then c must be

(a) $c > 9$

(b) $c > -9$

(c) $c < 9$

(d) $c < -9$

$f(x) = x^2 - 6x + c$

$b^2 - 4ac > 0$

$36 - 4 \times 1 \times c > 0$

$36 - 4c > 0$

$\frac{36}{4} > \frac{4c}{4}$

$9 > c$

21. If $\sin^2\theta - \sin\theta = 2$, where $\theta \in [0, 2\pi]$, then $6\theta =$

(a) 6π

(b) $\frac{3\pi}{2}$

(c) $\frac{\pi}{2}$

(d) 9π

$\sin^2\theta - \sin\theta = 2$

$\sin\theta = x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ $\quad x = -1$

$\sin\theta = 2$ $\quad \sin\theta = -1$

$\times$ $\quad \theta = \frac{3\pi}{2}$

$6\theta = \frac{6 \times 3\pi}{2}$

$= 9\pi$

22. The value of $\frac{1-i}{i+1} =$

(a) i

(b) -1

(c) 1

(d) None of the above.

$\frac{1-i}{i+1} \times \frac{i-1}{i-1}$

$\frac{i + 1 + i}{-1-1}$

$\frac{2i}{-2} = -1i = -i$

23. The floor function $f(x) = \lfloor x \rfloor$

(a) is continuous everywhere

(b) is continuous everywhere from the right

(c) is continuous everywhere from the left

(d) None of the above

$0 \leq x < 1$

$\frac{1+\sqrt{-1}}{1+\sqrt{-1}} \times \frac{1-i}{i+1} = \frac{1-\sqrt{-1}}{\sqrt{-1}+1}$

$= \frac{1-i}{(i+1)}$

$= \frac{2}{1+2i+1}$

$= \frac{2i}{2i}$

$= \frac{2}{2}$

$=$

Question Two (5 points) Let $f(x) = \frac{x^2+4x+3}{x+1}$

Find a continuous extension $F(x)$ for $f(x)$ (if possible)

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$f(x)$ discontin of $x = -1$

$\lim_{x \to -1} f(x) = \lim_{x \to -1} \frac{x^2+4x+3}{x+1}$

$\lim_{x \to -1} \frac{(x+3)\cancel{(x+1)}}{\cancel{(x+1)}}$

$\lim_{x \to -1} x+3 = -1+3 = 2$ exist / removable discontin

$F(x) = \begin{cases} \frac{x^2+4x+3}{x+1}, & x \neq -1 \\ 2, & x = -1 \end{cases}$

Question Three (10 points) Use the graph of $f(x)$ defined on the closed interval $[-2, 4]$ to sketch the following functions:

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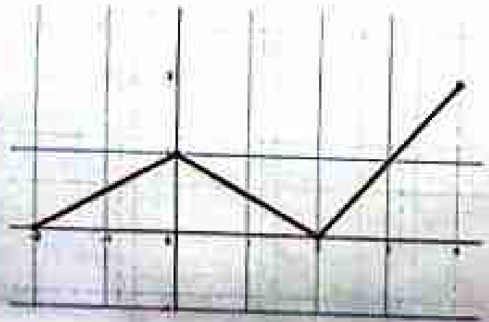
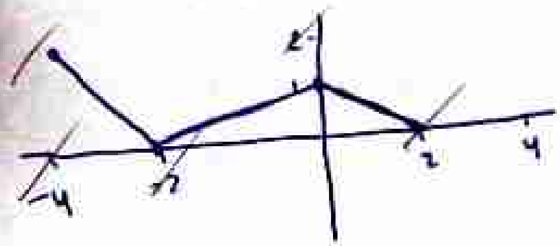
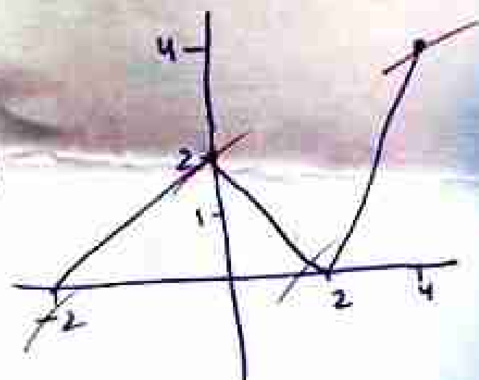


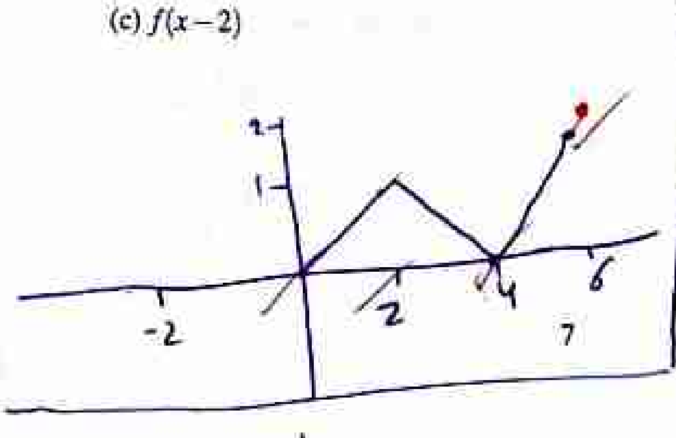
(a) $f(-x)$



(b) $2f(x)$



(c) $f(x-2)$



(d) $f(x+1)-2$