Mustafa Jarrar & Radi Jarrar: Lecture Notes in Discrete Mathematics.
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Propositional Logic

2.1. Introduction and Basics

2.2 Conditional Statements

2.3 Inferencing

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 2 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

If-Then Statements

If 9 is divisible by 6, then 9 is divisible by 3

hypothesis: 9 is divisible by 6

conclusion: 9 is divisible by 3

If you study, then you pass

hypothesis: you study

conclusion: you pass

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If-Then Statements

Remark that this if-then is a logical (not a causal) condition

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Your mum said {you study → you pass}, is it True?

you studied and you passed

you studied and you didn't passed

you didn't study and you passed

you didn't study and you didn't pass

If p and q are statement variables, the **conditional** of q by p is "If p then q" or "p implies q" and is denoted $p \to q$. It is false when p is true and q is false; otherwise it is true. We call p the **hypothesis** (or **antecedent**) of the conditional and q the **conclusion** (or **consequent**).

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Conditional Statement with a False Hypothesis

If 0 = 1 then 1 = 2.

The statement as whole is true.

Notice that we don’t test the correctness of the conclusion

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Truth Tables involving $\longrightarrow$

Construct a truth table for the statement form $p \vee \sim q \rightarrow \sim p$.

p	q	$\sim p$ (conclusion)	$\sim q$	$p \vee \sim q$ (hypothesis)	$p \vee \sim q \rightarrow \sim p$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

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Logical Equivalences involving $\rightarrow$

Division into Cases: Showing that $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$p \vee q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$p \vee q \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ always have the same truth values, so they are logically equivalent

Representation of If-Then As Or

$$p \rightarrow q \equiv {\sim}p \vee q$$

Examples?

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The Negation of a Conditional Statement

The negation of "if p then q" is logically equivalent to "p and not q."

$$\sim(p \rightarrow q) \equiv p \wedge \sim q$$

$\sim(p \rightarrow q) \equiv \sim(\sim p \vee q)$

$\equiv \sim(\sim p) \wedge (\sim q)$ by De Morgan's laws

$\equiv p \wedge \sim q$ by the double negative law

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The Negation of a Conditional Statement

Examples

If my lecture is at Masri109, then I cannot buy coffee

my lecture is at Masri109 and I can buy coffee

If Amjad loves Zatar, then Amjad is smart

Amjad loves Zatar and Amjad is not smart

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Contrapositive Statements

Conditional statement = its contrapositive.

If you don't pass then you didn't study

• Definition

The **contrapositive** of a conditional statement of the form "If p then q" is

$$\text{If } \sim q \text{ then } \sim p.$$

Symbolically,

$$\text{The contrapositive of } p \to q \text{ is } \sim q \to \sim p.$$

Try the truth table

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Contrapositive Statements

Examples

If my lecture is at Masri109, then I cannot buy coffee

If I can buy coffee then my lecture is not at Masri109

If Amjad loves Zatar, then Amjad is smart

If Amjad is not smart then Amjad does not love Zatar

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Converse and Inverse

• Definition

Suppose a conditional statement of the form "If p then q" is given.

1. The **converse** is "If q then p."
2. The **inverse** is "If $\sim p$ then $\sim q$."

Symbolically,

$$\text{The converse of } p \to q \text{ is } q \to p,$$

and

$$\text{The inverse of } p \to q \text{ is } \sim p \to \sim q.$$

مقلوب Converse
معكوس Inverse

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Converse and Inverse

Examples

If my lecture is at Masri109, then I cannot buy coffee

Converse: If I cannot buy coffee then my lecture is at Masri109

Inverse: If my lecture is not at Masri109 then I can buy coffee

If Amjad loves Zatar, then Amjad is smart

Converse: If Amjad is smart then Amjad loves Zatar

Inverse: If Amjad does not love Zatar, then Amjad is not smart

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Converse and Inverse

Caution! Many people believe that if a conditional statement is true, then its converse and inverse must also be true. This is not correct!

1. A conditional statement and its converse are *not* logically equivalent.
2. A conditional statement and its inverse are *not* logically equivalent.
3. The converse and the inverse of a conditional statement are logically equivalent to each other.

Biconditional

If and only if

iff

• Definition

Given statement variables p and q, the **biconditional of p and q** is "p if, and only if, q" and is denoted $p \leftrightarrow q$. It is true if both p and q have the same truth values and is false if p and q have opposite truth values. The words *if and only if* are sometimes abbreviated **iff.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Biconditional

Truth Table Showing that $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$

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Biconditional

Truth Table Showing that $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \land (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

$p \leftrightarrow q$ and $(p \to q) \land (q \to p)$
always have the same truth values,
so they are logically equivalent

Biconditional

Examples

This computer program is correct if, and only if,
it produces correct answers for all possible sets of input data.

If this program produces the correct answers for all possible sets of input data, then it is correct.

If this program is correct, then it produces the correct answers for all possible sets of input data;

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Order of Operations for Logical Operators

1.	$\sim$	Evaluate negations first.
2.	$\wedge$, $\vee$	Evaluate $\wedge$ and $\vee$ second. When both are present, parentheses may be needed.
3.	$\rightarrow$, $\leftrightarrow$	Evaluate $\rightarrow$ and $\leftrightarrow$ third. When both are present, parentheses may be needed.

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Necessary and Sufficient Conditions

• Definition

If r and s are statements:

r is a **sufficient condition** for s means "if r then s."
r is a **necessary condition** for s means "if not r then not s."

r is a necessary condition for s also means "if s then r."

r is a necessary and sufficient condition for s means "r if, and only if, s."

Examples

Studying is a sufficient condition for passing.	Study → Pass
In order to pass, it is sufficient to study.	Study → Pass
It is sufficient to study in order to pass.	Study → Pass
Studying is a necessary condition for passing.	~Study → ~Pass

Necessary and Sufficient Conditions

• Definition

If r and s are statements:

r is a **sufficient condition** for s means "if r then s."

r is a **necessary condition** for s means "if not r then not s."

r is a necessary condition for s also means "if s then r."

r is a necessary and sufficient condition for s means "r if, and only if, s."

Examples

Being above 16 is a sufficient condition for getting ID Card.

Above (16) → Get ID Card

Being above 16 is a necessary condition for getting ID Card

~Above (16) → ~Get ID Card

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Necessary and Sufficient Conditions

Examples

A is a sufficient condition for B

→ if A then B

→ The occurrence of A guarantees the occurrence of B

A is a necessary condition for B

→ If ~A then ~B

→ If A did not occur, then B did not occur either

>> It also means If B then A (if B occurred then A had also occurred)

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Examples

Examples

- Passing all exams is a sufficient condition for passing the course
  - If a person passes all exams, then the person will pass the course
- Passing all exams is a necessary condition for passing the course
  - If a person does not pass all exams, then the person will not pass the course
  - OR if a person passes the course, then the person will have passed all exams

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