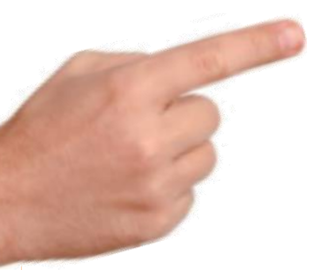
Mustafa Jarrar & Radi Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2021

Propositional Logic

2.1. Introduction and Basics

2.2 Conditional Statements



2.3 Inferencing

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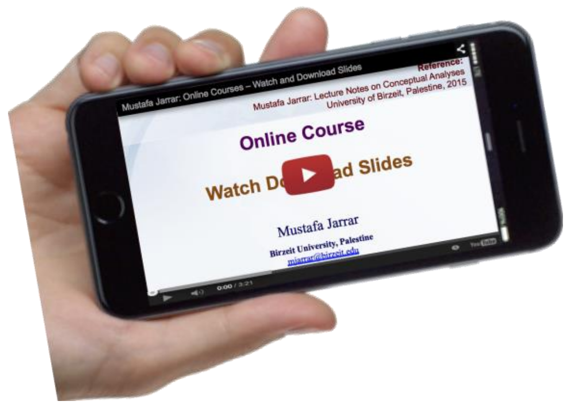

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 5 in “Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)”.

Propositional Logic

2.3 Logical Inferencing

In this lecture:

- ❑ Part 1: **Numeration Method**
- ❑ Part 2: **Rules of Inference**
- ❑ Part 3: **Examples**

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Numeration Method

Example 1

If today is Friday then today is holiday

Today is Friday

$\therefore$today is holiday?

$$p \rightarrow q$$
$$p$$
$$\therefore q?$$

		premises		conclusion	
p	q	$p \rightarrow q$	p	q	
T	T	T	T	T	← critical row
T	F	F	T		
F	T	T	F		
F	F	T	F		

Numeration Method

Example 1

If Socrates is a man, then Socrates is mortal.

Socrates is a man.

∴ Socrates is mortal

$p \rightarrow q$

p

$\therefore q?$

• Definition

An **argument** is a sequence of statements, and an **argument form** is a sequence of statement forms. All statements in an argument and all statement forms in an argument form, except for the final one, are called **premises** (or **assumptions** or **hypotheses**). The final statement or statement form is called the **conclusion.** The symbol ∴, which is read "therefore," is normally placed just before the conclusion.

To say that an *argument form* is **valid** means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true. To say that an *argument* is **valid** means that its form is valid.

Testing for validity

Testing an Argument Form for Validity

1. Identify the premises and conclusion of the argument form.
2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. A row of the truth table in which all the premises are true is called a **critical row**. If there is a critical row in which the conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid. If the conclusion in *every* critical row is true, then the argument form is valid.

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Numeration Method

Example 2

$$p \rightarrow q \vee \sim r$$
$$q \rightarrow p \wedge r$$
$$\therefore p \rightarrow r \ ?$$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

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Building a Valid Argument

- A valid argument is a sequence of statements where each statement is either a premise or follows from previous statements (called premises) by rules of inference. The last statement is called conclusion.

- A valid argument takes the following form:

Premise 1
Premise 2
.
.
.
Premise n
____
∴ Conclusion

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Rules of Inference

An argument form consisting of two premises and a conclusion is called a syllogism

- The first and second premises are called the major premise and minor premise, respectively

- A rule of inference is a form of argument that is valid

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Rules of Inference

1. Modus Ponens

Corresponding Tautology:
$((p \rightarrow q) \land p) \rightarrow q$

$$p \rightarrow q$$
$$p$$
$$\therefore \boldsymbol{q}$$

If today is Friday then today is holiday

Today is Friday

∴ Today is holiday

Modus Ponens = method of affirming (the conclusion is an affirmation)

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Rules of Inference

1. Modus Ponens

Example:

Let p be "It is snowing."

Let q be "I will study discrete math."

"If it is snowing, then I will study discrete math."

"It is snowing."

"Therefore , I will study discrete math."

Rules of Inference

2. Modus Tollens:

Corresponding Tautology:
$((p \to q) \wedge \neg q) \to \neg p$

$p \longrightarrow q$

$\sim q$

$\therefore \sim p$

If today is Friday then today is holiday

Today is not holiday

∴ **Today is not Friday**

Modus Tollens = method of denying (the conclusion is a denial)

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Rules of Inference

2. Modus Tollens

Example:

Let p be "it is snowing."
Let q be "I will study discrete math."
"If it is snowing, then I will study discrete math." "I will not study discrete math."
"Therefore , it is not snowing."

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Rules of Inference

3. Generalization:

p	Today is Saturday
$\therefore p \vee q$	**∴ Today is Saturday or today is Sunday**

q
$\therefore p \vee q$

According to the first, if p is true, then, more generally, "p or q" is true for any other statement q

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Rules of Inference

4. Specialization: aka Conjunction Elimination

$p \wedge q$

$\therefore \boldsymbol{p}$

Today is Friday and today is holiday

∴ Today is Friday

$p \wedge q$

$\therefore \boldsymbol{q}$

∴ Today is Holiday

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Rules of Inference

5. Conjunction:

$$p$$

$$q$$

$$\therefore p \wedge q$$

Today is Friday

Today is Holiday

∴ **Today is Friday and today is holiday**

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Rules of Inference

6. Elimination: also known as Disjunctive Elimination

$p \vee q$

$\sim q$

$\therefore p$

Today is Saturday or today is Sunday

Today is not Saturday

∴ Today is Sunday

These argument forms say that when you have only two possibilities and you can rule one out, the other must be the case. For instance, suppose you know that for a particular number x,

$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0.$$

If you also know that x is not negative, then $x \neq -2$, so

$$x + 2 \neq 0.$$

By elimination, you can then conclude that

$$\therefore x - 3 = 0.$$

$p \vee q$

$\sim p$

$\therefore q$

Rules of Inference

6. Elimination: also known as Disjunctive Elimination

$$p \vee q$$
$$\sim q$$
$$\therefore \boldsymbol{p}$$

$$p \vee q$$
$$\sim p$$
$$\therefore \boldsymbol{q}$$

Example:

Let p be “I will study discrete math.” Let q be “I will study English literature.”
“I will study discrete math or I will study English literature.” “I will not study discrete math.”
“Therefore , I will study English literature.”

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Rules of Inference

7. Transitivity: also known as Chain Argument

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

If today is Friday then Today is holiday

If today is holiday then I am happy

∴ If today is Friday then I am happy

,

Rules of Inference

7. Transitivity: also known as Chain Argument

$$p \to q$$
$$q \to r$$
$$\therefore p \to r$$

Example:
Let p be “it snows.”
Let q be “I will study discrete math.” Let r be “I will get an A.”
“If it snows, then I will study discrete math.”
“If I study discrete math, I will get an A.”
“Therefore , If it snows, I will get an A.”

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Rules of Inference

8. Division into Cases:

$$p \vee q$$
$$p \rightarrow r$$
$$q \rightarrow r$$

$$\therefore \boldsymbol{r}$$

Today is Friday or today is Sunday

If today is Friday then I am happy

If today is Sunday then I am happy

∴ **I am happy**

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Rules of Inference

8. Division into Cases:

$$p \lor q$$
$$p \rightarrow r$$
$$q \rightarrow r$$

$$\therefore r$$

Example:

Let p be "I will study discrete math."
Let q be "I will study Computer Science."
Let r be "I will study databases."

"If I will study discrete math, then I will study Computer Science."
"If I will study databases, then I will study Computer Science."
"I will study discrete math or I will study databases."

"Therefore, I will study Computer Science."

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Rules of Inference

9. Contradiction Rule:

$$\sim p \rightarrow c$$

$$\therefore p$$

If "Today is not Friday" is false

∴ Today is Friday

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Rules of Inference
Summary

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

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Inferencing Example

Formalize the following text in propositional logic and use the inference rules find the glasses.

If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.	$RK \rightarrow GK$
If my glasses are on the kitchen table, then I saw them at breakfast.	$GK \rightarrow SB$
I did not see my glasses at breakfast.	$\sim SB$
I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.	$RL \vee RK$
If I was reading the newspaper in the living room then my glasses are on the coffee table.	$RL \rightarrow GC$

, **Where are the glasses?**

Inferencing Example

Let
RK = I was reading the newspaper in the kitchen.
GK = My glasses are on the kitchen table.
SB = I saw my glasses at breakfast.
RL = I was reading the newspaper in the living room.
GC = My glasses are on the coffee table.

$RK \rightarrow GK$
$GK \rightarrow SB$
$\sim SB$
$RL \vee RK$
$RL \rightarrow GC$

$RK \rightarrow GK$
$GK \rightarrow SB$
$\therefore RK \rightarrow SB$ by transitivity

$RL \vee RK$
$\sim RK$
$\therefore RL$ by elimination

$RK \rightarrow SB$
$\sim SB$
$\therefore \sim RK$ by modus tollens

$RL \rightarrow GC$
RL
$\therefore GC$ by modus ponenes

Thus the glasses are on the coffee table.

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