First Order Logic

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&

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3.1 Predicates and Quantified Statements I

3.2 Predicates and Quantified Statements II

3.3 Statements with Multiple Quantifiers

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This lecture is based on, but not limited to, chapter 3 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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In this Lecture

We will learn

- ❑ **Part1: Multiple and Order of Quantifiers**
- ❑ Part 2: Verbalization of Formal Statements
- ❑ Part 3: Formalization of informal Statements
- ❑ Part 4: Negations of Multiply-Quantified Statements
- ❑ Part 5: Example: Using FOL to formalize text (optional)

Keywords: Predicates, FOL, First Order Logic, Universal Quantifier, Existential Quantifier, Negation, Truth of Universal Statements, Formalization, verbalization, Order of quatifiers

Multiple and Order of Quantifiers

$\forall x\ \exists y\ .\ \text{Loves}(x,y)$ Everything loves something Each thing loves one or more things	$\exists x\ \forall y\ .\ \text{Loves}(x,y)$ Something loves everything
$\forall y\ \exists x\ .\ \text{Loves}(x,y)$ Everything is loved by something Everything has something that loves it	$\forall y\ \exists x\ .\ \text{Loves}(y,x)$ Everything loves something
$\exists y\ \forall x\ .\ \text{Loves}(x,y)$ Something is loved by everything Everyone love the same thing There exists something that everything loves it	$\forall x\ \exists y\ .\ \text{Loves}(x,y)\ ,\ x \neq y$ Everything loves something but not itself
$\forall x\ \forall y\ .\ \text{Loves}(x,y)$ $\forall x, y\ .\ \text{Loves}(x,y)$ Everything loves everything	$\exists x\ \exists y\ .\ \text{Loves}(x,y)$ $\exists x, y\ .\ \text{Loves}(x,y)$ something loves something

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Multiple and Order of Quantifiers

Everyone loves all movies كل شخص يحب كل الافلام $\forall p_{\in Person}\ \forall m_{\in Movie} \cdot Loves(p,m)$	Some people loves some movies بعض الناس يحبون بعض الافلام $\exists p_{\in Person}\ \exists m_{\in Movie} \cdot Loves(p,m)$
There is a movie that everyone loves فلم يحبه كل الناس $\exists m_{\in Movie}\ \forall p_{\in Person} \cdot Lovedby(m,p)$	Some people love all movies بعض الناس يحب كل الافلام $\exists p_{\in Person}\ \forall m_{\in Movie} \cdot Loves(p,m)$
Everyone loves some movies كل شخص يحب بعض الافلام $\forall p_{\in Person}\ \exists m_{\in Movie} \cdot Loves(p,m)$	All movies are loved by someone كل فلم له بعض المحبين $\forall m_{\in Movie}\ \exists p_{\in Person} \cdot Lovedby(m,p)$

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- ❑ Part1: Multiple and Order of Quantifiers

- ❑ **Part 2: Formalise/Verbalization of Formal Statements**
- ❑ Part 3: Negations of Multiply-Quantified Statements
- ❑ Part 4: Example: Using FOL to formalize text (optional)

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Multiple Quantifiers with Negated Predicates

$\exists x \ \exists y \ . \ \sim Love(x,y)$

Somebody does not love somebody

بعض اشخاص لا يحبون بعض الاشخاص

$\forall x \ \forall y \ . \ \sim Love(x,y)$

Everyone does not love anyone

كل شخص لا يحب أي شخص

No one love any one.

لا احد يحب احد

$\exists x \ \forall y \ . \sim Love(x,y)$

Someone does not love anyone

يوجد شخص لا يحب أي الاشخاص

$\forall x \ \exists y \ . \ \sim Love(y,x)$

Everyone is not loved by someone

كل شخص يوجد اخرين لا يحبونه

Everyone has some people who do not love him

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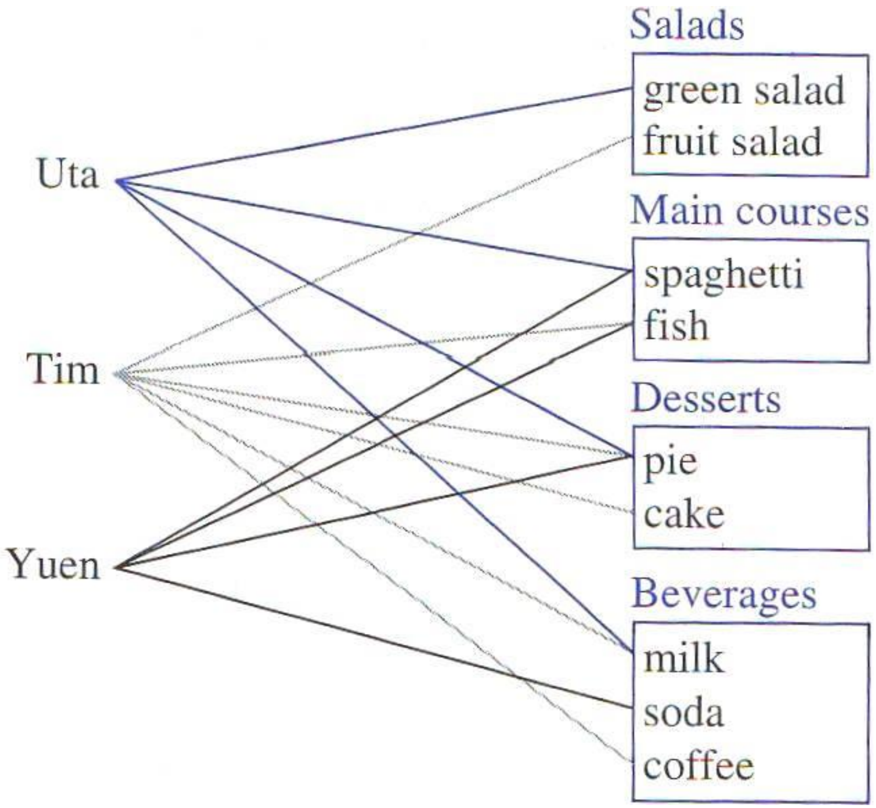
Verbalize and Test Statements



a. ∃ an item I such that ∀ students S, S chose I.

b. ∃ a student S such that ∀ items I, S chose I.

c. ∃ a student S such that ∀ stations Z, ∃ an item I in Z such that S chose I.

d. ∀ students S and ∀ stations Z, ∃ an item I in Z such that S chose I.

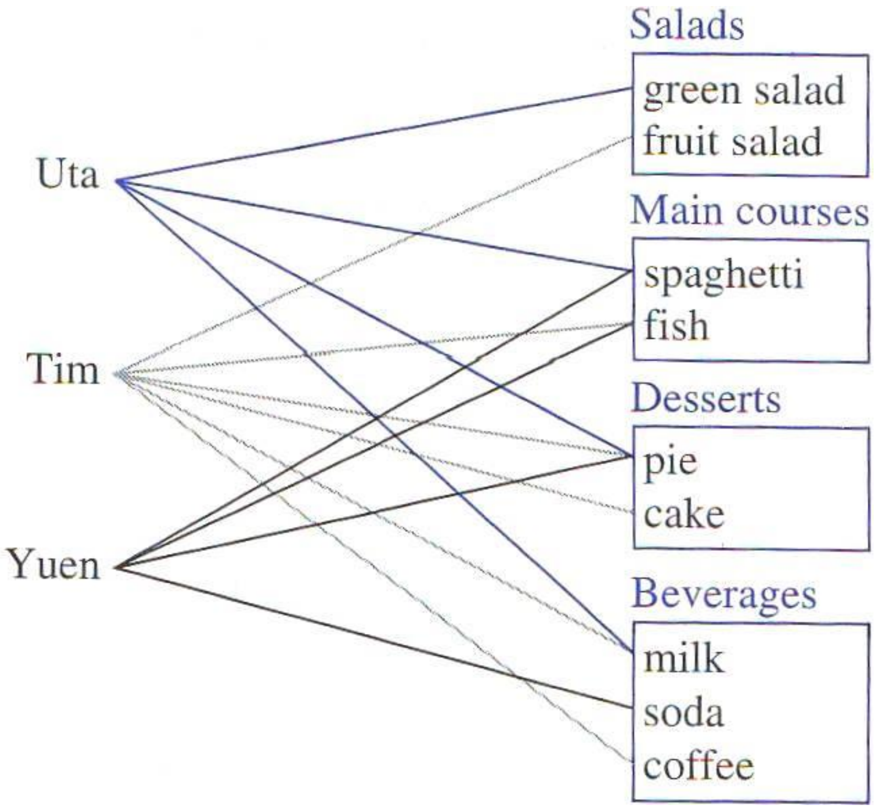
There is an item that was chosen by every student.

There is a student who chose every available item.

There is a student who chose at least one item from every station.

Every student chose at least one item from every station

Verbalize and Test Statements



a. $\exists$ an item I such that $\forall$ students S, S chose I.

b. $\exists$ a student S such that $\forall$ items I, S chose I.

c. $\exists$ a student S such that $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

d. $\forall$ students S and $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

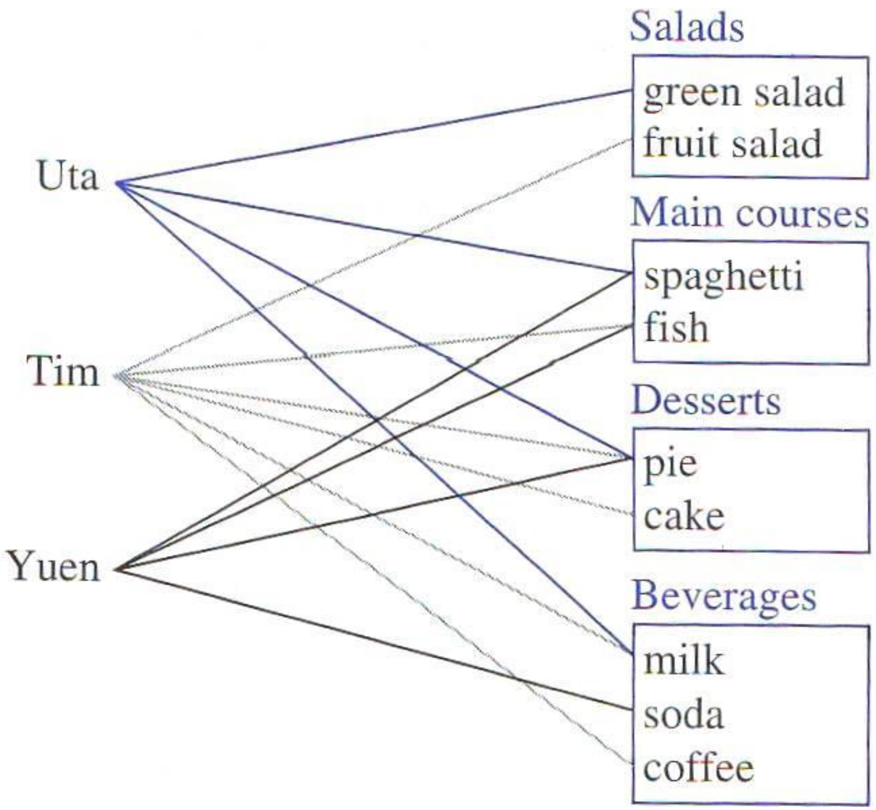
There is an item that was chosen by every student. → true

There is a student who chose every available item.

There is a student who chose at least one item from every station.

Every student chose at least one item from every station

Verbalize and Test Statements



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There is an item that was chosen by every student. → true

There is a student who chose every available item. →false

There is a student who chose at least one item from every station.

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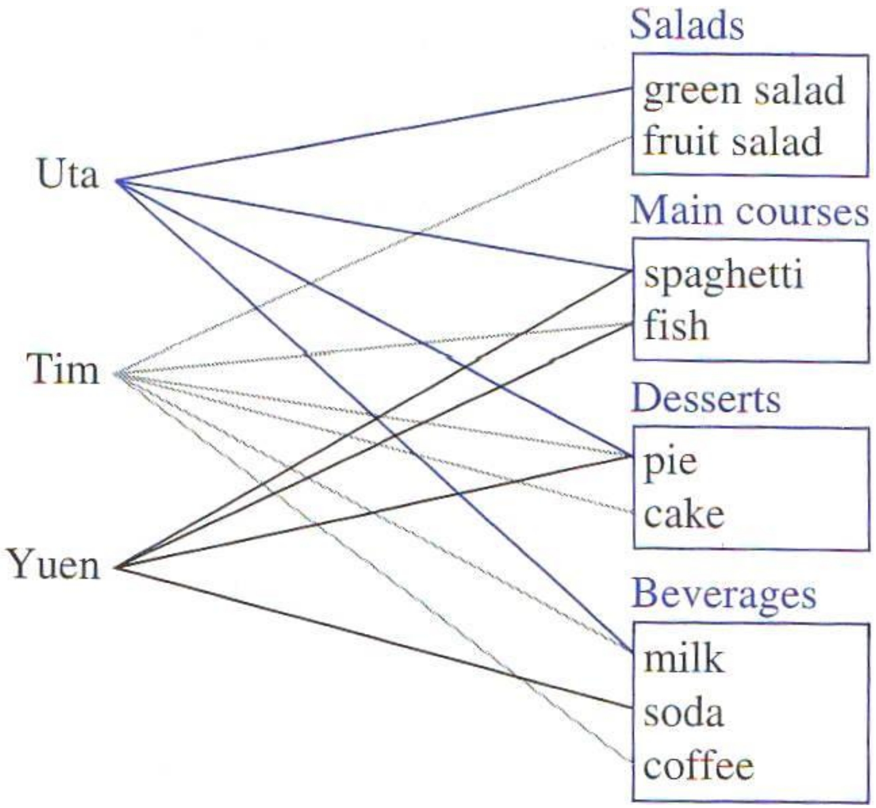
Verbalize and Test Statements



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There is a student who chose every available item. →false

There is a student who chose at least one item from every station. → true

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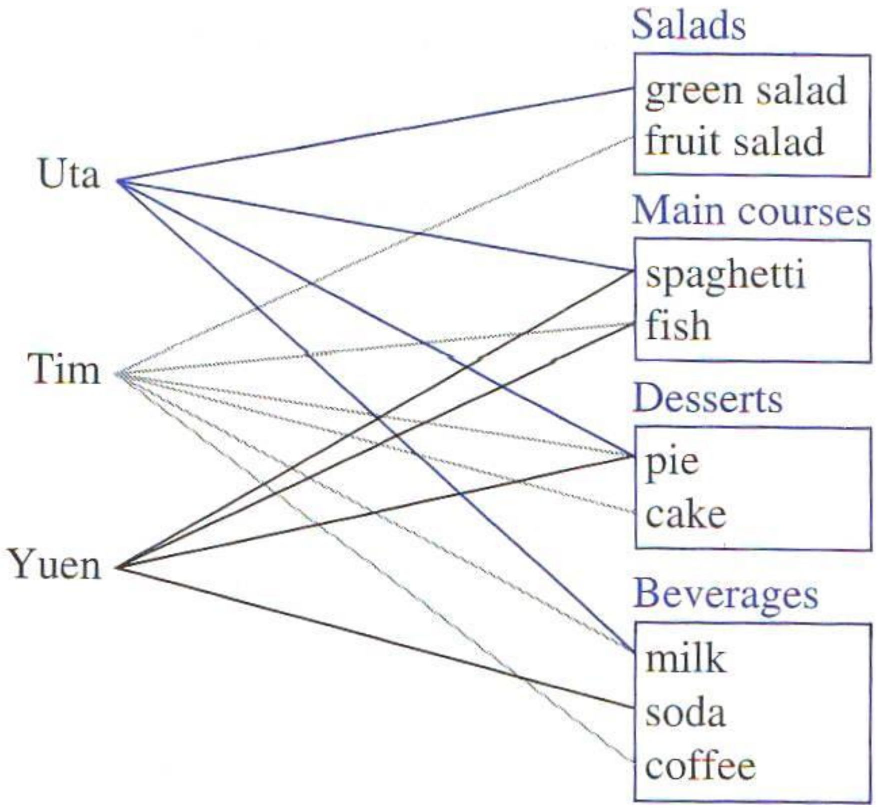
Verbalize and Test Statements



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There is a student who chose every available item. →false

There is a student who chose at least one item from every station. → true

Every student chose at least one item from every station → false.

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Tarski's world - Formalizing Statements

Describe Tarski's world using universal and external quantifiers

using Formal FOL Notation

a. For all circles x, x is above f.

$$\forall x(\text{Circle}(x) \rightarrow \text{Above}(x, f)).$$

b. There is a square x such that x is black.

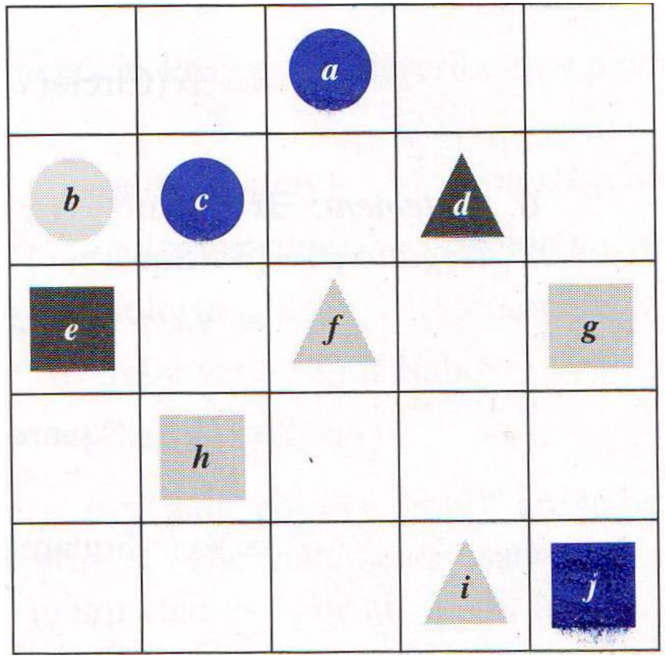
$$\exists x(\text{Square}(x) \land \text{Black}(x)).$$

c. For all circles x, there is a square y such that x and y have the same color.

$$\forall x(\text{Circle}(x) \rightarrow \exists y(\text{Square}(y) \land \text{SameColor}(x, y))).$$

d. There is a square x such that for all triangles y, x is to right of y.

$$\exists x(\text{Square}(x) \land \forall y(\text{Triangle}(y) \rightarrow \text{RightOf}(x, y))).$$



Formalize these statements

The **reciprocal (نظير ضربي)** of a real number *a* is a real number *b* such that $a.b = 1$. The following two statements are true. Rewrite them formally using quantifiers and variables:

Every nonzero real number has a reciprocal.

There is a real number with no reciprocal.

The number 0 has no reciprocal.

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Formalize these statements

The **reciprocal (نظير ضربي)** of a real number *a* is a real number *b* such that $a.b = 1$. The following two statements are true. Rewrite them formally using quantifiers and variables:

Every nonzero real number has a reciprocal.

$\forall u \in \text{NonZeroR}, \exists v \in \text{R} \quad . \quad uv = 1.$

There is a real number with no reciprocal.

The number 0 has no reciprocal.

$\exists c \in \text{R} \;\; \forall d \in \text{R},. \quad cd \neq 1.$

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Formalize these statements

There Is a Smallest Positive Integer

There Is No Smallest Positive Real Number

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Formalize these statements

There Is a Smallest Positive Integer

$\exists m \in Z^+ \ \forall n \in Z^+ .\ \text{LessOrEqual}(m,n)$

In the book:

∃ a positive integer m such that ∀ positive integers n, $m \leq n$.

There Is No Smallest Positive Real Number

$\forall x \in R^+ \ \exists y \in R^+ \ .\ \text{Less}(y,x)$

In the book:

∀ positive real numbers x, ∃ a positive real number y such that $y < x$.

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Negations of Multiply-Quantified Statements

$$\sim(\forall x \text{ in } D, \exists y \text{ in } E \text{ such that } P(x, y)) \equiv \exists x \text{ in } D \text{ such that } \forall y \text{ in } E, \sim P(x, y).$$

$$\sim(\exists x \text{ in } D \text{ such that } \forall y \text{ in } E, P(x, y)) \equiv \forall x \text{ in } D, \exists y \text{ in } E \text{ such that } \sim P(x, y).$$

Examples:

$\sim (\forall x \; \exists y \; . \; \text{Loves}(x,y))$

$\sim(\exists x \; \forall y \; . \; \text{Loves}(x,y))$

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Negations of Multiply-Quantified Statements

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Examples:

$\sim (\forall x \exists y . \text{Loves}(x,y))$

$\exists x \forall y . \sim\text{Loves}(x,y)$

$\sim(\exists x \forall y . \text{Loves}(x,y))$

$\forall x \exists y . \sim\text{Loves}(x,y)$

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Negations of Multiply-Quantified Statements

Not all people love someone.

~ (all people love someone)

$\sim(\forall x\ \exists y\ .\ \text{Love}(x,y))$

$\exists x\ \ \forall y\ .\ \sim\text{Love}(x,y))$

Some people do not love everyone

Not all people love everyone.

~ (All people love everyone)

$\sim \forall x \forall y\ \text{Like}(x, y)$

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Example: Using FOL to formalize text

Example from: Russell & Norvig Book

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

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Example: Using FOL to formalize text

... it is a crime for an American to sell weapons to hostile nations:

$\forall x,y,z\ .\ American(x) \land Weapon(y) \land Sells(x,y,z) \land Hostile(z) \Rightarrow Criminal(x)$

Nono … has some missiles, i.e.,

$\exists x\ .\ Owns(Nono,x) \land Missile(x)$

… all of its missiles were sold to it by Colonel West

$\forall x\ .\ Missile(x) \land Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

Missiles are weapons:

$\forall x\ .\ Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile“:

$\forall x\ .\ Enemy(x,America) \Rightarrow Hostile(x)$

West, who is American …

$American(West)$

The country Nono, an enemy of America …

$Enemy(Nono,America)$