Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2021

Set Theory

6.1. Basics of Set Theory

6.2 Properties of Sets and Element Argument

6.3 Algebraic Proofs

6.4 Boolean Algebras

Watch this lecture and download the slides

http://jarrar-courses.blogspot.com/2014/03/discrete-mathematics-course.html

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 5 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

Set Theory

6.3 Algebraic Proofs

In this lecture:

- ❑ **Part 1: Disapproving and Problem-Solving**
- ❑ Part 2: Algebraic Proofs of Sets

(Dis)proving

Prove that: For all sets A, B, and C, **(A - B) U (B - C)** ≠ ***A - C* ?**

Example: All people except who are Palestinians with the set of Palestinians except who are female, are the same set as all people except who are female **?**

Counterexample 1: Let $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$, and $C = \{4, 5, 6, 7\}$. Then

$$A - B = \{1, 4\}, \quad B - C = \{2, 3\}, \quad \text{and} \quad A - C = \{1, 2\}.$$

Hence

$(A - B) \cup (B - C) = \{1, 4\} \cup \{2, 3\} = \{1, 2, 3, 4\}$, whereas $A - C = \{1, 2\}$.

Since $\{1, 2, 3, 4\} \neq \{1, 2\}$, we have that $(A - B) \cup (B - C) \neq A - C$.

Counterexample 2: Let $A = \emptyset$, $B = \{3\}$, and $C = \emptyset$. Then

$$A - B = \emptyset, \quad B - C = \{3\}, \quad \text{and} \quad A - C = \emptyset.$$

Hence $(A - B) \cup (B - C) = \emptyset \cup \{3\} = \{3\}$, whereas $A - C = \emptyset$.

Since $\{3\} \neq \emptyset$, we have that $(A - B) \cup (B - C) \neq A - C$.

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Problem-Solving Strategy

How can you discover whether a given universal statement about sets is true or false?

حاول قليلا ان تثبت الصحة،
وان احسست عدم الصحة
حاول ايجاد مثال داحض،
ولكن ان احسست الصحة حاول الاثبات،
وهكذا ...

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Set Theory

6.3 Algebraic Proofs

In this lecture:

- ❑ Part 1: Disapproving and problem-Solving
- ❑ **Part 2: Algebraic Proofs of Sets**

Remember the following

$$\underbrace{A_1}_{A} \cap (\underbrace{A_2}_{B} \cup \underbrace{A_3}_{C}) = (\underbrace{A_1}_{A} \cap \underbrace{A_2}_{B}) \cup (\underbrace{A_1}_{A} \cap \underbrace{A_3}_{C}),$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(\underbrace{W \cap X}) \cap (Y \cup Z) = ((\underbrace{W \cap X}) \cap Y) \cup ((\underbrace{W \cap X}) \cap Z),$$

$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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Algebraic Proofs

Deriving a Set Difference Property

Construct an algebraic proof that for all sets A, B, and C,

$(A \cup B) - C = (A - C) \cup (B - C)$.

$$
\begin{aligned}
(A \cup B) - C &= (A \cup B) \cap C^c && \text{by the set difference law} \\
&= C^c \cap (A \cup B) && \text{by the commutative law for } \cap \\
&= (C^c \cap A) \cup (C^c \cap B) && \text{by the distributive law} \\
&= (A \cap C^c) \cup (B \cap C^c) && \text{by the commutative law for } \cap \\
&= (A - C) \cup (B - C) && \text{by the set difference law.}
\end{aligned}
$$

Cite a property from Theorem 6.2.2 for every step of the proof.

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Algebraic Proofs

Deriving a Set Identity Using Properties of ∅

Construct an algebraic proof that for all sets A and B,

$$\boldsymbol{A - (A \cap B) = A - B.}$$

$$
\begin{aligned}
A - (A \cap B) &= A \cap (A \cap B)^c && \text{by the set difference law} \\
&= A \cap (A^c \cup B^c) && \text{by De Morgan's laws} \\
&= (A \cap A^c) \cup (A \cap B^c) && \text{by the distributive law} \\
&= \emptyset \cup (A \cap B^c) && \text{by the complement law} \\
&= (A \cap B^c) \cup \emptyset && \text{by the commutative law for } \cup \\
&= A \cap B^c && \text{by the identity law for } \cup \\
&= A - B && \text{by the set difference law.}
\end{aligned}
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