Counting & Probability

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9.1 Basics of Probability and Counting

9.2 Possibility Trees and the Multiplication Rule

9.3 Counting Elements of Disjoint Sets: Addition Rule

9.4 Counting Subsets of a Set: Combinations

6.5 r-Combinations with Repetition Allowed

Watch this lecture and download the slides

http://jarrar-courses.blogspot.com/2014/03/discrete-mathematics-course.html

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Acknowledgement:
This lecture is based on, but not limited to, chapter 5 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

In this Lecture

<u>We will learn:</u>

- ❑ Part 1: **Probability and Sample Space**
- ❑ Part 2: **Counting in Sub lists**

Tossing Coins

Tossing two coins and observing whether 0, 1, or 2 heads are obtained.

What are the chances of having 0,1,2 heads?

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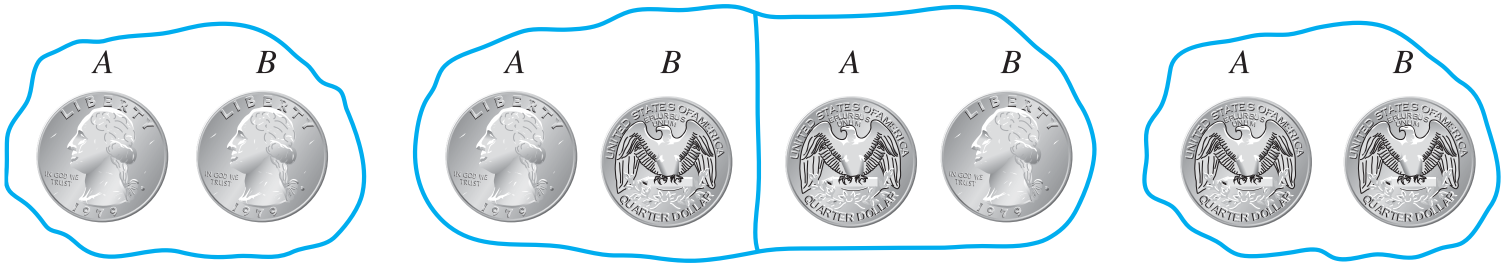

2 heads obtained 1 head obtained 0 heads obtained

Event	Tally	Frequency (Number of times the event occurred)	Relative Frequency (Fraction of times the event occurred)
2 heads obtained	𝍸 𝍸 I	11	22%
1 head obtained	𝍸 𝍸 𝍸 𝍸 𝍸 II	27	54%
0 heads obtained	𝍸 𝍸 II	12	24%

Sample Space

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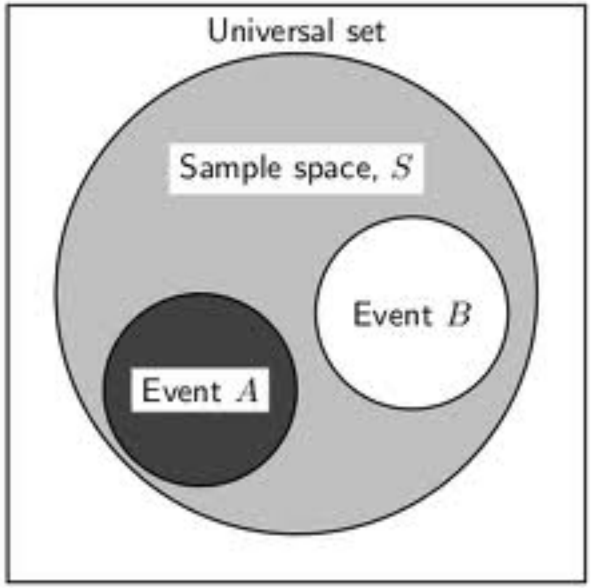
• Definition

A **sample space** is the set of all possible outcomes of a random process or experiment. An **event** is a subset of a sample space.



Sample Space

In case an experiment has finitely many outcomes and all outcomes are equally likely to occur, the *probability* of an event (set of outcomes) is just the ratio of the number of outcomes in the event to the total number of outcomes

Equally Likely Probability Formula

If S is a finite sample space in which all outcomes are equally likely and E is an event in S, then the **probability of *E***, denoted ***P*(*E*)**, is

$$P(E) = \frac{\text{the number of outcomes in } E}{\text{the total number of outcomes in } S}.$$

• Notation

For any finite set A, $N(A)$ denotes the number of elements in A.

$$P(E) = \frac{N(E)}{N(S)}.$$

Probabilities for a Deck of Cards

52 cards

diamonds (♦)

hearts (♥)

clubs (♣)

spades (♠)

a. What is the sample space of outcomes?

b. What is the event that the chosen card is a black face card?

c. What is the probability that the chosen card is a black face card?

Probabilities for a Deck of Cards

52 cards

diamonds (♦)

hearts (♥)

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spades (♠)

a. What is the sample space of outcomes?

➔ the 52 cards in the deck.

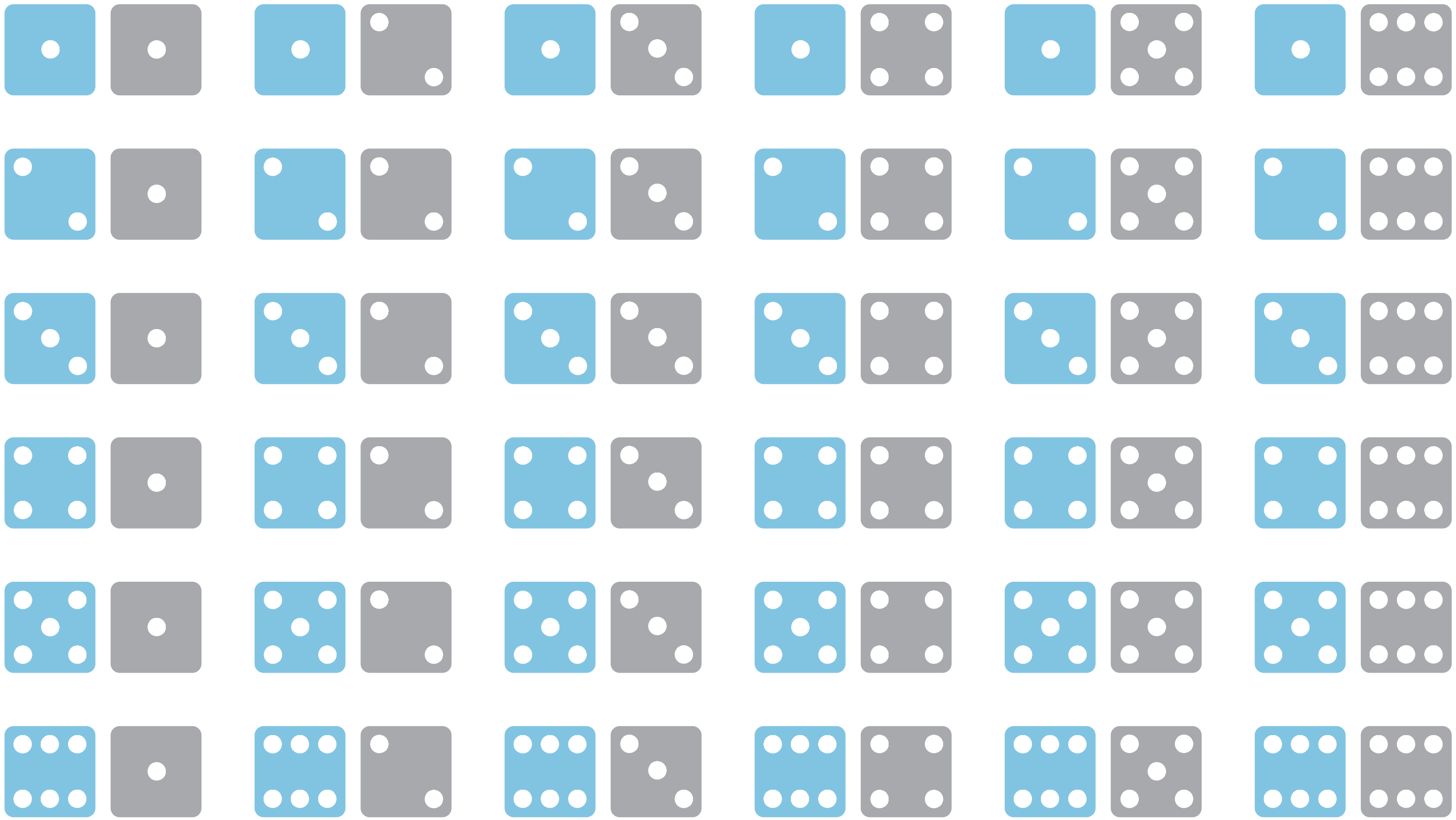
b. What is the event that the chosen card is a black face card?

➔ E = {J♣, Q♣, K♣, J♠, Q♠, K♠}

c. What is the probability that the chosen card is a black face card?

$$P(E) = \frac{N(E)}{N(S)} = \frac{6}{52} \cong 11.5\%.$$

Rolling a Pair of Dice



Rolling a Pair of Dice

a. Write the sample space *S* of possible outcomes (using compact notion).

b. write the event *E* that the numbers showing face up have a sum of 6 and find the probability of this event.

Rolling a Pair of Dice

a. Write the sample space S of possible outcomes (using compact notion).

$$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}.$$

b. write the event E that the numbers showing face up have a sum of 6 and find the probability of this event.

$$E = \{15, 24, 33, 42, 51\}.$$

$$P(E) = \frac{N(E)}{N(S)} = \frac{5}{36}.$$

Counting the Elements of a List

Some counting problems are as simple as counting the elements of a list. E.g., how many integers are there from 5 through 12?

list:	5	6	7	8	9	10	11	12
	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
count:	1	2	3	4	5	6	7	8

list:	$m(= m + 0)$	$m + 1$	$m + 2$	$\ldots$	$n\ (= m + (n - m))$
	$\updownarrow$	$\updownarrow$	$\updownarrow$		$\updownarrow$
count:	1	2	3	$\ldots$	$(n - m) + 1$

Theorem 9.1.1 The Number of Elements in a List

If m and n are integers and $m \le n$, then there are $n - m + 1$ integers from m to n inclusive.

Counting the Elements of a Sublist

a. How many three-digit integers (integers from 100 to 999 inclusive) are divisible by 5?

b. What is the probability that a randomly chosen three-digit integer is divisible by 5?

Counting the Elements of a Sublist

a. How many three-digit integers (integers from 100 to 999 inclusive) are divisible by 5?

100	101	102	103	104	105	106	107	108	109	110	$\cdots$	994	995	996	997	998	999
$\updownarrow$					$\updownarrow$					$\updownarrow$			$\updownarrow$				
$5 \cdot 20$					$5 \cdot 21$					$5 \cdot 22$			$5 \cdot 199$				

From the sketch it is clear that there are as many three-digit integers that are multiples of 5 as there are integers from 20 to 199 inclusive. By Theorem 9.1.1, there are 199 – 20 + 1, or 180, such integers. Hence there are 180 three-digit integers that are divisible by 5.

b. What is the probability that a randomly chosen three-digit integer is divisible by 5?

$999 - 100 + 1 = 900.$

$180/900 = 1/5.$

By Theorem 9.1.1 the total number of integers from 100 through 999 is 999 – 100 + 1 = 900. By part (a), 180 of these are divisible by 5. Hence the probability that a randomly chosen three-digit integer is divisible by 5 is 180/900 = 1/5.