Key (Incomplete)

Birzeit University
Mathematics Department
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Math234-First Exam (Incomplete)
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Name:... Number:...............

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Exercise#1 [25 marks]. Answer the following statements as **True** or **False.**

1. (....T....) If A is an $n \times n$ matrix and $A^5 = I$, then $\det(A) = 1$.

2. (....T....) If u and v are both solutions to $Ax = b$, then $\frac{1}{3}u + \frac{2}{3}v$ is a solution to $Ax = b$.

3. (....T....) The adjoint of a matrix $\begin{bmatrix} 4 & 1 \\ 0 & -1 \end{bmatrix}$ is $\begin{bmatrix} -1 & -1 \\ 0 & 4 \end{bmatrix}$.

4. (....T....) If A is a 3×3 matrix with $a_1 = a_2 + 3a_3 = 0$, then $(-1, 1, 3)^T$ is a solution of the linear system $Ax = 0$.

5. (....F....) If A and B are 2×2 matrices such that $\det(A) = -2$ and $\det(B) = 4$, then $\det\big(-3A^{-1}B^T\big) = 18$.

6. (....T....) If A is an $n \times n$ matrix, then AA^T is symmetric.

7. (....F....) If $A = LU$ is the LU-factorization of a matrix A and A is singular, then L and U are both singular.

8. (....T....) If A is singular and B is nonsingular $n \times n$ matrices, then AB is singular.

9. (....T....) If A is a 4×5 matrix, then the linear system $Ax = 0$ has infinitly many solutions.

10. (....T....) Let A and B be any $n \times n$ matrices. If $AB = BA$, then $A^2 - B^2 = (A - B)(A + B)$.

Exercise#2 [45 marks]. Circle the correct answer.

(1) If E is an elementary matrix of type III, then E^T is

(a) not an elementary matrix

(b) an elementary matrix of type II

(c) an elementary matrix of type I

(d) an elementary matrix of type III

(2) If A is a 3×3 matrix such that $A^T A = A$. One of the following ia always true

(a) $A^2 = I$

(b) $A = I$

(c) A is symmetric

(d) A is singular

(3) The value(s) of $\boldsymbol{k}$ that make the system with augmented matrix $\left[\begin{array}{cc|c} 4 & k & 4 \\ k & 1 & -2 \end{array}\right]$ has infinitly many solutions is (are):

(a) $k = 2$

(b) $k \neq 2$

(c) $k = -2$

(d) $k \neq \pm 2$

(4) For which value(s) of α, will the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 0 & \alpha & 1 \\ 0 & 1 & \alpha \end{bmatrix}$ become singular?

(a) 0

(b) 1

(c) ± 1

(d) any real number

(5) If A is a 4×4 matrix with $\det(A) = 2$, then $\det\big(\operatorname{adj}(A)\big) =$

(a) 2

(b) 4

(c) 8

(d) 16

(6) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$, then $\begin{vmatrix} a & b & c \\ a+g & b+h & c+i \\ 4d & 4e & 4f \end{vmatrix} =$

(a) -16

(b) 16

(c) 4

(d) 8

(7) If A is a 3×3 skew-symmetric matrix, then one of the following statements is **true**:

(a) $\det(-A) = \det(A)$

(b) $\det(A) = 0$

(c) $\det(A + I) = 1 + \det(A)$

(d) $\det(2A) = 2\det(A)$

(8) If $(A - I)^2 = O$, then $A^{-1} =$

(a) I

(b) $A + I$

(c) $A - 2I$

(d) $2I - A$

(9) If A and B are $n \times n$ nonsingular matrices, then

(a) $A + B$ is nonsingular

(b) ABA^{-1} is nonsingular

(c) $B^T A^{-1}$ is nonsingular

(d) both (b) and (c)

(10) If A is a 3×5 matrix, then the number of free variables in the RREF of A is at least

(a) 4

(b) 2

(c) 5

(d) 3

(11) The values of α and β that make the system with augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & \alpha+3 & \beta-1 \end{array}\right]$ has infinitely many solution are:

(a) $\alpha = -3$ and $\beta \neq 1$

(b) $\alpha \neq -3$ and β is any real number.

(c) α and β are any real numbers.

(d) $\alpha = -3$ and $\beta = 1$

(12) Let A, B and C be $n \times n$ matrices. If $AB = AC$ and A is invertible, then

(a) $B \neq C$

(b) $B = C$

(c) $C = A$

(d) none of the above

(13) If A is a 3×3 matrix and the linear system $Ax = 0$ has only zero solution. If $b = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, then the system $Ax = b$

(a) has at most one solution.

(b) has an infinite number of solutions.

(c) has exactly one solution.

(d) is either inconsistent or has an infinite number of solutions.

(14) Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 0 & 3 \\ 4 & 0 & 2 \end{bmatrix}$. The entry $(1,3)$ of $\operatorname{adj}(A)$ is

(a) 0

(b) 4

(c) 6

(d) -6

(15) Which of the following matrices is in **reduced row echelon form?**

(a) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Exercise#3 [10 marks]. Consider the linear system whose augmented matrix is given by

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & \alpha & \beta \end{array}\right].$$

(a) For what values of α and β will the system have **infinitely many solutions**?

(b) For what values of α and β will the system be **inconsistent**?

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & \alpha & \beta \end{array}\right] \to$$

$$\begin{array}{l} \\ -R_1+R_2 \\ -R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & \alpha-3 & \beta-2 \end{array}\right] \quad \text{(4 pts}$$

$$\to \begin{array}{l} \\ \\ -2R_2+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \alpha-5 & \beta-4 \end{array}\right] \quad \text{(2 pts}$$

(a) $\alpha = 5$, $\beta = 4$. (2 pts

(b) $\alpha = 5$, $\beta \neq 4$. (2 pts

Exercise#4 [10 marks]. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$.

(a) Find the **inverse** of A

5 pts

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[-2R_1+R_3]{-R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 2 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[-2R_2+R_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -2 & 1 \end{array}\right]$$

$$\xrightarrow[-R_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -1 \end{array}\right] \xrightarrow{-R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

(b) Find an **LU**-factorization of A.

5 pts

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} \xrightarrow[-2R_1+R_3]{-R_1+R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\xrightarrow[-2R_2+R_3]{} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

Exercise#5 [15 marks].

(a) Let A, B, C be an $n \times n$ matrices such that C is nonsingular and $A = CBC^{-1}$. Prove that $\det(A) = \det(B)$.

5 pts

$$\det(A) = \det(CBC^{-1})$$
$$= \det(C)\det(B)\det(C^{-1})$$
$$= \cancel{\det(C)}\det(B)\frac{1}{\cancel{\det(C)}} = \det(B). \quad \square$$

(b) Let A and B be an $n \times n$ symmetric matrices. Prove that AB is symmetric if and only if $AB = BA$.

5 pts

AB is symmetric $\Leftrightarrow (AB)^T = AB$

$\Leftrightarrow B^T A^T = AB$

$\Leftrightarrow BA = AB$ (since A and B are symmetric

$\square$

(c) Prove that if A and B are $n \times n$ invertible matrices, then $\operatorname{adj}(AB) = \operatorname{adj}(B)\operatorname{adj}(A)$.

5 pts

$$\operatorname{adj}(AB) = |AB|(AB)^{-1}$$
$$= |A||B|B^{-1}A^{-1}$$
$$= (|B|B^{-1})(|A|A^{-1})$$
$$= \operatorname{adj}(B)\operatorname{adj}(A). \quad \square$$

Good Luck