Key

Birzeit University
Mathematics Department
Math234-Section (1)
Quiz#5

Instructor: Dr. Ala Talahmeh — First Semester 2022/2023
Time: 10 minutes — Date: 13/2/2023
Name:... — Number:...............

Exercise#1 [10 marks]. Let $L : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by

$$L\big((x, y, z, w)^T\big) = (x + y - z, y - w, y - z)^T.$$

(a) Find a basis for $\ker(L)$ and its dimension.

$$x + y - z = 0$$
$$y - w = 0$$
$$y - z = 0$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$w = t$
$z = t$
$y = t$
$x = 0$

$$\ker(L) = \text{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$\Rightarrow \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ is a basis for $\ker(L)$, $\dim = 1$

(b) Find a basis for Imm(L) and its dimension.

$$L\big((x, y, z, w)^T\big) = x(1,0,0)^T + y(1,1,1)^T + z(-1,0,-1)^T + w(0,-1,0)^T.$$

$\Rightarrow \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \right\}$ is a basis for Imm(L)

$\dim = 3$.

(c) Find the **matrix representation** of L with respect to the standard bases
$E = [(1, 0, 0, 0)^T, (0, 1, 0, 0)^T, (0, 0, 1, 0)^T, (0, 0, 0, 1)^T]$ and $F = [(1, 0, 0)^T, (0, 1, 0)^T, (0, 0, 1)^T]$.

$$L\big((1,0,0,0)^T\big) = (1,0,0)^T = 1(1,0,0)^T + 0(0,1,0)^T + 0(0,0,1)^T$$

$$L\big((0,1,0,0)^T\big) = (1,1,1)^T = 1(1,0,0)^T + 1(0,1,0)^T + 1(0,0,1)^T$$

$$L\big((0,0,1,0)^T\big) = (-1,0,-1)^T = -1(1,0,0) + 0(0,1,0) - 1(0,0,1)^T$$

$$L\big((0,0,0,1)^T\big) = (0,-1,0)^T = 0(1,0,0) - 1(0,1,0) + 0(0,0,1)^T$$

$$\therefore A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Good Luck

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Exercise#1 [10 marks]. Let $L : P_2 \to \mathbb{R}^{2\times 2}$ be a linear transformation defined by

$$L(ax+b) = \begin{bmatrix} a-b & a \\ b & 0 \end{bmatrix}.$$

(a) Find a basis for $\ker(L)$ and its dimension.

$$\begin{bmatrix} a-b & a \\ b & 0 \end{bmatrix} \overset{1}{=} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow a=0, \ b=0 \quad 1$$

$$\therefore \ker(L) \overset{1}{=} \{0\}, \quad \dim \ker(L) \overset{1}{=} 0.$$

$$\text{basis} = \emptyset \quad 1$$

(b) Find a basis for $\mathrm{Imm}(L)$ and its dimension.

2 $$L(ax+b) = \begin{bmatrix} a-b & a \\ b & 0 \end{bmatrix} = a\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

2 $$\Rightarrow \Big\{ \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}}_{\text{lin. indep.}} \Big\} \text{ is a basis for } \mathrm{Imm}(L).$$

$$\dim(\mathrm{Imm}(L)) = 2. \quad 1$$

(c) Find the **matrix representation** of L with respect to the ordered bases $E = [1+x, 1-x]$ and $F = \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right]$.

2 $$L(1+x) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 0\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

2 $$L(1-x) = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} = -2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

1 $$\therefore A = \begin{bmatrix} 0 & -2 \\ 1 & -1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Good Luck