Key

جامعة بيرزيت

BIRZEIT UNIVERSITY

MATH DEPARTMENT **1st Semester 2022- 2023**

MATH243

Student Name :____________ **ID :** _________

1) True or False (15 points)

1) If 1=3 then $15 \geq 17$ T
2) 1=2 or $15 \geq 16$ F
3) $\phi \in \{1,2,3,\{\phi\}\}$ F
4) The converse of the statement $p \to q$ is $\neg q \to \neg p$ F
5) $p \wedge \neg p$ is a contradiction T
6) $p \vee \neg p$ is a tautology T
7) Every set has at least two subsets . F
8) For any two sets A, B , $A \cap B \subseteq B \subseteq A \cup B$ T
9) If $A \subseteq B$ and $C \subseteq B$ then $A \subseteq C$ for any sets A,B,C F
10) $\phi = \cap \{A_n, n \in N, n \geq 2\}$ where $A_n = (n, \infty)$. T

T
F
F
F
T
T
F
T
F
T

2) (15 points)

a) Write an equivalent statement to the statement

$(P \to Q) \to R$ using only $\wedge, \vee, \neg$

$\cong \neg(P \to Q) \vee R$

$\cong \neg(\neg P \vee Q) \vee R$

$\cong (P \wedge \neg Q) \vee R$

b) Negate the following statement

$\forall \in > 0$, $\exists \delta > 0$, such that if $0 < |x-a| < \delta$ then $|f(x) - L| < \epsilon$

$\exists \epsilon_0 > 0$, $\forall \delta > 0$ s.t $0 < |x-a| < \delta$ But $|f(x) - L| \geq \epsilon_0$

c) Is the following expression true, show your claim.

$\neg (P \leftrightarrow Q) \Leftrightarrow (\neg P \leftrightarrow \neg Q)$, NO

P	Q	$P \leftrightarrow Q$	$\neg P$	$\neg Q$	$\neg P \leftrightarrow \neg Q$	$\neg(P \to Q)$
T	T	T	F	F	T	F
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	F

3) (15 points)

a) prove that there are no smallest positive real number .

By contradiction, Suppose not,

i.e $\exists x_0 \in \mathbb{R}$, $x_0 > 0$ and x_0 is the smallest

(7) But $\frac{x_0}{2} < x_0$, $\frac{x_0}{2} \in \mathbb{R}$, $\frac{x_0}{2} > 0$

which contradicts that x_0 is the smallest.

b) Show $B' \subseteq A'$ if and only if $A \subseteq B$

$(\Rightarrow)$ let $B' \subseteq A'$, to show $A \subseteq B$.

let $x \in A \Rightarrow x \notin A'$

(4) $\Rightarrow x \notin B'$

$\Rightarrow x \in B$.

$(\Leftarrow)$ let $A \subseteq B$ to show $B' \subseteq A'$

(4) let $x \in B' \Rightarrow x \notin B$

$\Rightarrow x \notin A$

$\Rightarrow x \in A'$

4) (15 points)

a) Show that if $A \subseteq B$ then $\rho(A) \subseteq \rho(B)$

or let $H \in \wp(A)$
$\Rightarrow H \subseteq A$
$\Rightarrow H \subseteq B$ since $A \subseteq B$
$\Rightarrow H \in \wp(B)$

let $H \in \wp(A)$

either $H = \phi$

or H is a non-empty subset of A

If $H = \phi$ then $H \in \wp(B)$, since ϕ is a subset of any set.

otherwise $H \subseteq A$, since $H \subseteq A$ and $A \subseteq B$

then $H \subseteq B \Rightarrow H \in \wp(B)$

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b) Show that for any sets A, B

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

$\Rightarrow$

let $x \in (A \cup B) - (A \cap B)$

$\Rightarrow$ $x \in A \cup B$ But $x \notin A \cap B$

$\Rightarrow$ $x \in A$ or $x \in B$ But $x \notin A$ or $x \notin B$

$\Rightarrow$ $x \in A$ But $x \notin B$ or $x \in B$ But $x \notin A$

$\Rightarrow$ $x \in A - B$ or $x \in B - A$

$\Rightarrow$ $x \in (A - B) \cup B - A$

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$\Leftarrow$ Let $x \in (A - B) \cup (B - A)$

$x \in B - A$

$\Rightarrow$ $x \in A - B$ or $x \in B$ but $x \notin A$

$\Rightarrow$ $x \in A$ & $x \notin B$ But $x \notin A$ and $x \notin B$

$\Rightarrow$ $x \in A$ or $x \in B$

$\Rightarrow$ $x \in (A \cup B)$ and $x \notin (A \cap B) \Rightarrow x \in (A \cup B) - (A \cap B)$

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5) (15 points) Let $f(x) = \frac{x}{x+1}$

show that if $x \neq y$ then $f(x) \neq f(y)$

a) Using the direct method

let $x \neq y$ to show $f(x) \neq f(y)$

since $x \neq y$ $\Longrightarrow$ $xy + x \neq xy + y$

$\Rightarrow$ $x+1 \neq y+1$

$\Rightarrow$ $\frac{1}{x+1} \neq \frac{1}{y+1}$

$\Rightarrow$ $\frac{x}{}$

$x(y+1) \neq y(x+1)$

$\frac{x}{x+1} \neq \frac{y}{y+1}$

$f(x) \neq f(y)$

b) Using the contradiction method

suppose $f(x) = f(y)$ and ~~to show~~ $x \neq y$

since $f(x) = f(y)$

$\frac{x}{x+1} = \frac{y}{y+1}$

$\Rightarrow$ $x(y+1) = y(x+1)$

$\cancel{xy} + x = \cancel{yx} + y$

$x = y$ $\times$ to $x \neq y$

6) (15 points)

Find $\cap\{A_n, n \in N, n \geq 1\}$ and $\cup\{A_n, n \in N, n \geq 1\}$ where $A_n = \left(1 - \frac{1}{n}, 2 + \frac{1}{n}\right)$ .Show your claim

$A_1 = (0,3)$

$A_2 = (\frac{1}{2}, 2\frac{1}{2})$

$A_3 = (\frac{2}{3}, 2\frac{1}{3})$

$\vdots$

3 $\cup A_n = (0,3)$

3 $\cap A_n = [1,2]$

To show $\bigcup_{n=1} A_n = (0,3)$

($\subseteq$) let $x \in (0,3) = A_1 \subseteq \bigcup_{n=1}^{\infty} A_n$

5 ($\Rightarrow$) let $x \in \bigcup_{n=1}^{\infty} A_n \Rightarrow x \in A_k$ for some k.

$\Rightarrow x \in (1-\frac{1}{k}, 2+\frac{1}{k})$

$\Rightarrow 0 < 1-\frac{1}{k} \leq x \leq 2+\frac{1}{k} \leq 3$ since $k \geq 1$

$\Rightarrow x \in (0,3)$

To show $\bigcap_{n=1}^{\infty} A_n = [1,2]$

($\Rightarrow$) let $x \in \bigcap_{n=1} A_n \Rightarrow x \in A_k$, for every k

5 $\Rightarrow x \in (1-\frac{1}{k}, 2+\frac{1}{k})$

$\Rightarrow 1-\frac{1}{k} \leq x \leq 2+\frac{1}{k} \Rightarrow \Rightarrow 1 \leq x \leq 2$

$\Rightarrow x \in [1,2]$

($\Leftarrow$) $x \in [1,2] \Rightarrow 1-\frac{1}{k} \leq x \leq 2+\frac{1}{k}$, for every k

$\Rightarrow x \in A_k$, for every k

$\Rightarrow x \in \bigcap_{n=1}^{\infty} A_n$

7) (15 points)

a)Show that $(\cup \{A_n, n \in N, n \geq 1\})' = \cap \{A'_n, n \in N, n \geq 1\}$

(2) let $x \in (\bigcup_{n\in\mathbb{N}} A_n)'$

$\Rightarrow x \notin \bigcup_{n\in\mathbb{N}} A_n$

$\Rightarrow x \notin A_n$ for $\forall n \in \mathbb{N}$

$\Rightarrow x \in A_n'$ for $\forall n \in \mathbb{N}$

(7) $\Rightarrow x \in \bigcap_{n\in\mathbb{N}} A_n'$

(2) let $x \in \bigcap_{n\in\mathbb{N}} A_n'$

$\Rightarrow x \in A_n'$ for all $n \in \mathbb{N}$

$\Rightarrow x \notin A_n$ for all $n \in \mathbb{N}$

$\Rightarrow x \notin \bigcup_{n\in\mathbb{N}} A_n$

$\Rightarrow x \in (\bigcup A_n)'$

b) Show that there are no rational number r such that $r^2 = 2$

proof suppose $\exists r \in \mathbb{Q}$ s.t $r^2 = 2$,

$r = \frac{p}{q}$, p and q has no common factor

(8) $\frac{p^2}{q^2} = 2 \Rightarrow p^2 = 2q^2$

$\Rightarrow p^2$ is even $\Rightarrow p$ is even $\Rightarrow p = 2m$

$\Rightarrow (2m)^2 = 2q^2$

$\Rightarrow 4m^2 = 2q^2 \Rightarrow q^2 = 2m^2 \Rightarrow q^2$ is even

$\Rightarrow q$ is even.

$\Rightarrow p, q$ are even X: to that they have no common factor