Math 243

Quiz 1

(1) Under the assumption $P \to Q$ is false give the truth values of

$Q \to P$, $P \leftrightarrow Q$, $P \vee Q$, $\neg P \wedge \neg Q$

Since $P \to Q$ is False $\Longrightarrow$ P is true & Q is false

$Q \to P$ is True T

$P \leftrightarrow Q$ is False F

$P \vee Q$ is True T

$\neg P \wedge \neg Q$ is False F

1 Point each

(2) Construct the truth table of the following expression

$P \to (Q \to R)$

P	Q	R	$Q \to R$	$P \to (Q \to R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F		T

4 points

(3) Negate the following statements

1) $\forall \epsilon > 0, \exists K \in \mathbb{N}$ s.t $\forall n \geq K, |x_n - x| < \epsilon$

2) $\forall K \in \mathbb{N}, \exists n \geq K$ s.t $x_n \geq K$

2 points

(1) $\exists \epsilon > 0, \forall K \in \mathbb{N}$ st $\exists n \geq K$ But $|x_n - x| \geq \epsilon$

2 points

(2) $\exists K \in \mathbb{N}, \forall n \geq K, \quad x_n \leq K$

(4) Let A, B be integers

Prove that if AB is odd

then $A+B$ is even.

method 1 Direct, let A, B integers

and AB is odd

Since AB is odd, Both A and B must be odd, because if one of them is ~~even~~ then even . odd is even

4 points

Since Both are odd, $A = 2n+1$, $B = 2m+1$

$A+B = 2n+1+2m+1 = 2(n+m+1)$ which is even

(5) let A, B, C be integers s.t $A^2+B^2=C^2$

Prove that at least one of A, B is even.

By contradiction: suppose A, B are odd & $A^2+B^2=C^2$

$A = 2k+1$, $B = 2m+1$

since A is odd, B is odd, A^2 is odd, B^2 is odd

$\Rightarrow A^2+B^2=C^2$, C^2 is even $\Rightarrow C$ is even, $C=2j$

$\Rightarrow (2k+1)^2+(2m+1)^2=(2j)^2$

4 points

$4k^2+4k+1+4m^2+4m+1 = 4j$

$k^2+k+\frac{1}{4}+m^2+m+\frac{1}{4} = j$

$\underbrace{k^2+k+m^2+m}_{\text{integer}}+\frac{1}{2} = \underbrace{j}_{\text{inte}}$ $\times$.