Birzeit University
Mathematics Department
First Semester 2022/2023
Math234-First Exam
Time: 70 minutes
December 17, 2022

Key
Form A

Name:.. Number:..............

Sections	Instructor Name
(1) and (5)	Dr. Ala Talahmeh
(2)	Dr. Alaeddin Elayyan
(3) and (4)	Dr. Hasan Yousef
(6)	Dr. Mohammad Saleh

10x2.5

Exercise#1 [25 marks]. Answer the following statements as **True** or **False.**

1. (....T....) Let A, B and C be $n \times n$ matrices. If A is row equivalent to I and $AB = AC$, then B must equal C.

2. (....F....) If $AB = O$ then either A or B is a zero matrix.

3. (....T....) If A is a 3×3 matrix such that the system $Ax = 0$ has only the trivial solution, then the system $Ax = b$ is consistent for every $b \in \mathbb{R}^3$.

4. (....F....) If A is invertible, then $A + I$ is also invertible.

5. (....F....) If A is a 3×3 matrix with $a_1 + a_2 + a_3 = 0$, then A is row equivalent to I_3.

6. (....F....) If A is a 3×3 matrix such that $\det(A) = 7$, then $\det\left(2A^T A^{-1}\right) = 2$.

7. (....T....) If u and v are both solutions to $Ax = b$, then $w = u - v$ is a solution to $Ax = 0$.

8. (....T....) If $A^3 - I = O$, then A is nonsingular.

9. (....T....) If A is an $n \times n$ nonsingular matrix, then AA^T is also nonsingular.

10. (....F....) Let A and B be any 3×3 matrices. If $\det(A) = 0$ or $\det(B) = 0$, then $\det(A + B) = 0$.

15×3

Exercise#2 [45 marks]. Circle the correct answer.

(1) A linear system of two equations in three unknowns has

- (a) exactly one solution
- (b) infinitely many solutions
- (c) no solution
- (d) either (b) or (c)

(2) A matrix A that can be obtained from an identity matrix by performing a single elementary row operation is

- (a) equivalent to a zero matrix
- (b) in row echelon form
- (c) an elementary matrix
- (d) in reduced row echelon form

(3) The value of α that make the system with augmented matrix $\left[\begin{array}{ccc|c} 4\alpha-1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 4\alpha-1 & 0 \end{array}\right]$

has a non-trivial solution is:

- (a) $\frac{1}{2}$
- (b) $\frac{1}{4}$
- (c) $\frac{3}{4}$
- (d) 1

(4) A matrix that is both symmetric and upper triangular must be a

- (a) diagonal matrix
- (b) non-diagonal but symmetric
- (c) both (a) and (b)
- (d) none of the above

(5) A homogeneous linear system in **n** unknowns whose corresponding augmented matrix has a reduced row echelon form with **r** leading 1's has

(a) **r-n** free variables

(b) **n-r** free variables

(c) **r** free variables

(d) cannot be determined

(6) Which of the following matrices is in **reduced row echelon form?**

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) both (a) and (b)

(d) none

(7) If A is an $n \times n$ matrix, then the linear system $Ax = 4x$ has a unique solution if and only if

(a) A is an invertible matrix.

(b) $A + 4I$ is an invertible matrix.

(c) $A - 4I$ is an invertible matrix.

(d) $4A$ is an invertible matrix.

(8) The value of $\boldsymbol{m}$ that make the system with augmented matrix $\left[\begin{array}{cc|c} -2 & 2 & m \\ 2 & 1 & 7 \\ 1 & -2 & -4 \end{array}\right]$ has a unique solution is:

(a) 2

(b) -2

(c) -10

(d) 10

(9) For which value of x, will the matrix $\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$ become singular?

(a) 4
(b) 6
(c) 3
(d) 12

(10) Let $A = \begin{bmatrix} 1 & 2 & 4 & 6 \\ 5 & 0 & 3 & 1 \\ 4 & 0 & 2 & 6 \\ 0 & 3 & 8 & 1 \end{bmatrix}$. The entry $(4, 1)$ of $\mathrm{adj}(A)$ is

(a) 32
(b) -32
(c) -6
(d) 6

(11) If $\begin{vmatrix} x & y \\ z & w \end{vmatrix} = 4$, then $\begin{vmatrix} 3z & 3w \\ 3z - 2x & 3w - 2y \end{vmatrix} =$

(a) 24
(b) -24
(c) 12
(d) -12

(12) If A is a 3×3 matrix, then one of the following statements is **true**:

(a) $\det(-A) = -\det(A)$
(b) $\det(A) = 0$
(c) $\det(A + I) = 1 + \det(A)$
(d) $\det(2A) = 2\det(A)$

(13) If $A^2 - A + I = O$, then $A^{-1} =$

(a) A^{-2}

(b) $A + I$

(c) $I - A$

(d) $A - I$

(14) If A is a skew-symmetric matrix, then

(a) A is nonsingular

(b) A is singular

(c) A is symmetric

(d) all entries on the main diagonal are zeros.

(15) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}$. Then an **LU-factorization** of the matrix A is

(a) $L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & -\frac{2}{3} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{2}{3} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{2}{3} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$

(d) None of the above

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Form B

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2. (....F....) If $A^2 = I$ then either $A = I$ or $A = -I$.

3. (....F....) If A is a 3×3 matrix such that the system $Ax = 0$ has only the trivial solution, then the system $Ax = b$ is inconsistent for every $b \in \mathbb{R}^3$.

4. (....F....) If A is singular, then $A + I$ is also singular.

5. (....T....) If A is a 3×3 matrix with $a_1 + a_2 + a_3 = 0$, then A is singular.

6. (....T....) If A is a 3×3 matrix such that $\det(A) = 7$, then $\det\left(2A^T A^{-1}\right) = 8$.

7. (....F....) If u and v are both solutions to $Ax = b$, then $w = u - v$ is also a solution to $Ax = b$.

8. (....F....) If $A^3 - I = O$, then A is singular.

9. (....T....) If A is an $n \times n$ nonsingular matrix, then A^T is also nonsingular.

10. (....T....) Let A and B be any 3×3 matrices. If $\det(A) = 0$ or $\det(B) = 0$, then $\det(AB) = 0$.

15x3

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(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(c) both (a) and (b)

(d) none

(7) If A is an $n \times n$ matrix, then the linear system $Ax = -4x$ has a unique solution if and only if

(a) A is an invertible matrix.

(b) $A + 4I$ is an invertible matrix.

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(12) If A is a 4×4 matrix, then one of the following statements is **true**:

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(b) $\det(A) = 0$

(c) $\det(A + I) = 1 + \det(A)$

(d) $\det(-A) = \det(A)$

(13) If $A^2 + A - I = O$, then $A^{-1} =$

(a) A^{-2}

(b) $A + I$

(c) $I - A$

(d) $A - I$

(14) If A is a skew-symmetric matrix, then

(a) $a_{ii} = 0$, for all i

(b) A is singular

(c) A is symmetric

(d) A is nonsingular

(15) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}$. Then an **LU-factorization** of the matrix A is

(a) $L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & -\frac{2}{3} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{2}{3} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{2}{3} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$

(d) None of the above

Exercise#3 [10 marks]. Use the **Gauss reduction method** to solve the system:

$$\begin{aligned} x_1 - x_2 + 3x_3 + 2x_4 &= 1 \\ -x_1 + x_2 - 2x_3 + x_4 &= -2 \\ 2x_1 - 2x_2 + 7x_3 + 7x_4 &= 1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ -1 & 1 & -2 & 1 & -2 \\ 2 & -2 & 7 & 7 & 1 \end{array}\right]$$

4 pts

$$\xrightarrow[-2R_1+R_3]{R_1+R_2} \left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 3 & -1 \end{array}\right]$$

2 pts

$$\xrightarrow[-R_2+R_3]{} \left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_2 = t$, $x_4 = r$ be free variables.

$x_3 = -1 - 3r$

$x_1 = 1 + t - 3(-1 - 3r) - 2r$

4 pts

$\Rightarrow x_1 = 4 + t + 7r$

Solution $= \{ (4 + t + 7r, t, -1 - 3r, r)^T : r, t \in \mathbb{R} \}$.

Exercise#4 [10 marks]. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}$.

(a) Find det(A).

2 pts

$$\det(A) = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix}$$

$$= 9-8+6-6 = 1.$$

(b) Find adj(A).

6 pts

$$\operatorname{adj}(A) = \begin{bmatrix} \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix} \\ -\begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} \\ \begin{vmatrix} 0 & 1 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & -2 \\ -3 & -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}$$

(c) Use (a) and (b) to find A^{-1}.

2 pts

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}$$

Exercise#5 [15 marks].

(a) Let A be a 5×3 matrix. Suppose that $b = a_2 - a_3 = a_1 + 2a_2$. Show that the system $Ax = b$ has infinitely many solutions.

Pf. since $b = a_2 - a_3$, then $(0,1,-1)^T$ is a solution to $Ax=b$

5pts Also, since $b = a_1 + 2a_2$, then $(1,2,0)^T$ is another solution to $Ax=b$.

Since the system $Ax=b$ has more than one solution, it has infinite solutions. □

(b) Let A and B be an $n \times n$ matrices. If A, B and AB are symmetric, prove that $AB = BA$.

Pf. $AB = (AB)^T$ (since AB is symmetric)

5pts $= B^T A^T$

$= BA$ (since A and B are symmetric).

□

(c) Let A be an $n \times n$ matrix and let u and v be vectors in $\mathbb{R}^n$. Show that if $Au = Av$ and $u \neq v$, then A must be singular.

Pf. Suppose that $Au = Av$, $u \neq v$. Then $A(u-v) = 0, u \neq v$

5pts $\Rightarrow$ the homogeneous system $Ax=0$ has a nontrivial solutions

$\Rightarrow$ A is Singular. □

OR Suppose by contradiction that A is nonsingular, then $Au = Av \Rightarrow A^{-1}Au = A^{-1}Av \Rightarrow Iu = Iv \Rightarrow u = v$, a contradiction. □

OR Since $Au = Av$, $u \neq v$, then the system $Ax=b$ has two different solutions $\Rightarrow$ A must be singular.