Contents

I

Mechanics

KINEMATICS

1

1.1 Problems

1.1 A dog chasing a ball starts at rest and accelerates uniformly over a distance of 5 meters. It takes the dog 1 s to cover that first 5 m. What is the dog's acceleration, and what speed is the dog travelling when it reaches the 5 m point? Answer: $a = 10\ \mathrm{m\,s^{-2}}$, $v_f = 10\ \mathrm{m\,s^{-1}}$

1.2 During a particular car crash, it takes just 0.18 s for the car to come to a complete stop from 50 km h^{-1}.

(a) At what rate is the car accelerating during the crash?

(b) How many times larger than the acceleration due to gravity is this?

Answer: (a) 77 m s^{-2} (b) $7.7 \times g$

1.3 A jogger starting their morning run accelerates from a standstill to their steady jogging pace of 8.0 km h^{-1}. They reach a speed of 8.0 km h^{-1}, 5 s after starting. How long does it take the jogger to reach the end of their 20 m driveway? Answer: 12 s

1.4 A driver in a blue car travelling at 50 km h^{-1} sees a red car approaching in his rear-view mirror. The red car is travelling at 60 km h^{-1} and is 30 m behind the blue car when first spotted.

(a) How many seconds from the time the driver of the blue car first noticed it until the red car passes the blue car?

(b) How much farther down the road will the blue car travel in this time?

Answer: $t = 11$ s, $d = 150$ m

1.5 You are abducted by aliens who transport you to their home world in a galaxy far far away. Oddly, the only thing you can think of doing is measuring the acceleration due to gravity on this strange new world. You drop an alien paperweight from a height of 12 m and use an alien stopwatch to measure the interval of 1.36 s it takes the paperweight to hit the ground below. What is the acceleration due to gravity on the alien home world? Answer: 13 m s^{-1}

1.6 In a bid to escape from your alien captors you hurl your paperweight straight up towards the door switch on a space ship above you. If the switch is 25 m above you how fast does the paperweight need to leave your hand? Answer: $v_i = 26\ \mathrm{m\,s^{-1}}$ (remember $g = 13\ \mathrm{m\,s^{-2}}$ on the alien planet)

1.7 An initially stationary hovercraft sits on a large lake. When a whistle blows the hovercraft accelerates due north at a rate of 1.2 m s^{-2} for 10 s, does not accelerate at all for the next 10 s, and then accelerates at a rate of 0.6 m s^{-2} due east for another 10 s. The hovercraft then coasts for another 10 s without any acceleration.

(a) What is the velocity of the hovercraft 40 s after the whistle blows?

(b) What is the displacement of the hovercraft 40 s after the whistle blows?

Answer: (a) $v = 13\ \mathrm{m\,s^{-1}}$, 27° east of north (b) $v = 430\ \mathrm{m\,s^{-1}}$, 12.1° east of north

1.8 A jogger takes the following route to the entrance of their local park: north 120 m, west 100 m, south 35 m, and finally west 50 m. It takes them 2 minutes 18 seconds to reach the park entrance.

(a) What distance did the jogger travel?

(b) What is the displacement of the jogger as she enters the park?

(c) What is the average speed of the jogger?

(d) What is the average velocity of the jogger?

Answer: (a) 305 m (b) 172 m, 60.5° west of north (c) 2.21 m s^{-1} (8 km hr^{-1}) (d) 1.25 m s^{-1}, 60.5° west of north (or alternatively 19.5° north of west)

1.9 A tennis ball is hit down at an angle of 30° below the horizontal from a height of 2 m. It is initially travelling at 5.0 m s^{-1}. What is the velocity of the ball when it hits the ground if we can neglect air resistance? Answer: $v = 8.0\ \mathrm{m\,s^{-1}}$ at an angle of 57.5° below horizontal

1.10 A stunt rider is propelled upward from his motorbike by a spring loaded ejector seat. The rider was travelling horizontally at 60 km h^{-1} when the ejector seat was triggered, and as they leave the seat they are travelling with a vertical velocity of 15 m s^{-1}.The seat is 1.0 m off the ground.

(a) What is the initial velocity of the stunt rider (in km h^{-1})?

(b) How high does the stunt rider reach?

(c) How far along the track does the stunt rider land on the ground?

(d) What is the velocity of the stunt rider when they hit the ground (in km h^{-1})?

Answer: (a) 81 km h^{-1}, 42° above the horizontal (b) 12 m (c) 51 m (d) 82 km h^{-1}, 43° below the horizontal

1.11 A bullet is fired horizontally from a gun that is 1.5 m from the ground. The bullet travels at 1000 m s^{-1} and strikes a tree 150 m away. How far up the tree from the ground does the bullet hit? [Neglect air resistance.] Answer: 1.39 m above the ground

2 Force and Newton's Laws of Motion

2.1 Problems

2.1 A courier is delivering a 5 kg package to an office high in a tall building.

(a) What upwards force does the courier apply to the package when carrying it horizontally at a constant velocity of $2\,\mathrm{m\,s^{-1}}$ into the building?

(b) The courier uses the elevator to reach the office. While the elevator (containing the courier who is holding the package) is accelerating upwards at $0.11\,\mathrm{m\,s^{-2}}$ what upwards force is the courier applying to the package?

(c) When the elevator is traveling upwards at a constant speed of $6\,\mathrm{m\,s^{-1}}$ what upwards force does the courier apply to the package?

(d) In order to stop at the correct floor the elevator accelerates downwards (decelerates) at a rate of $0.20\,\mathrm{m\,s^{-2}}$. What is the upwards force the courier applies to the package during the deceleration?

Answer: (a) Taking upwards as the positive direction, $F_{\mathrm{app}} = 50\,\mathrm{N}$, (b) $F_{\mathrm{app}} = 50.55\,\mathrm{N}$, (c) $F_{\mathrm{app}} = 50\,\mathrm{N}$, d) $F_{\mathrm{app}} = 49\,\mathrm{N}$

2.2 You live at the top of a steep (a slope of 15° above the horizontal) hill and must park your 2200 kg car on the street at night.

(a) You unwisely leave your car out of gear one night and your handbrake fails. Assuming no significant frictional forces are acting on the car, how quickly will it accelerate down the hill?

(b) The increase in insurance premiums due to the results of your mistake mean that you cannot afford to fix your handbrake properly. You resolve to always leave your car in gear when parked on a slope. If the rolling frictional force caused by leaving the drive-train connected to the wheels is 5000 N, at what rate will your car accelerate down the hill if the handbrake fails again?

Answer: (a) $a = 2.6\,\mathrm{m\,s^{-2}}$
(b) $a = 0.32\,\mathrm{m\,s^{-2}}$

2.3 You are pulling your younger sister along in a small wheeled cart. You weigh 65.0 kg and the combined mass of your sister and the cart is 35.0 kg. You are pulling the cart via a short rope which you pull horizontally. You hold one end of the rope and your sister holds the other end. If you are accelerating at a rate of $0.10\,\mathrm{m\,s^{-2}}$, the rope is inelastic, and the frictional force acting upon the cart is 30 N:

(a) What is the tension in the rope?

(b) What force are you applying to the ground in order to produce this acceleration?

Answer: (a) $T = 34\,\mathrm{N}$.
(b) You apply a force of 40 N to the ground.

2.4 Two flexible balls rolling along a frictionless horizontal surface collide with each other. The larger of the balls weighs 50 g and the smaller weighs 30 g. Immediately after the balls first touch each other (the beginning of the collision), the center of mass of the larger ball is accelerating at a rate of $5\,\mathrm{m\,s^{-2}}$ to the right. What is the acceleration of the center of mass of the smaller ball?
Answer: $8.3\,\mathrm{m\,s^{-2}}$ to the left.

2.5 The Earth has a mass of 5.97×10^{24} kg and the Moon has a mass of 7.36×10^{22} kg. The average distance between the center of the Earth and the center of the Moon is 3.84×10^{8} m.

(a) What is the gravitational force acting on the Moon due to the Earth?

(b) What is the gravitational force acting on the Earth due to the Moon?

(c) How far away would the Moon need to be for the magnitude of the gravitational force acting on it due to the Earth be the same as the magnitude of the gravitational force of a 72 kg student sitting at their desk on the surface of the earth?

(c) A supermassive black hole passes through the edge of the solar system 1.20×10^{13} m away. The gravitational force between an observant 79 kg astronomer and the black hole is 1 N. What is the mass of the black hole?

Answer: (a) 1.99×10^{20} N towards the Earth (b) 1.99×10^{20} N towards the Moon (c) 2.04×10^{17} m (this is 21.6 light years!) (d) 2.7×10^{34} kg (1400× more massive than the sun)

2.6 A 4 kg vase of flowers is placed directly in the middle of a glass table. The glass tabletop itself weighs 8 kg. With what force do each of the four legs of the table push on the glass after the vase has been placed on top? Answer: 30 N

2.7 In order to drink from your glass you first need to lift it to your mouth. As you begin to lift your 0.20 kg glass of water it is accelerating upwards at 0.090 m s^{-2}.

(a) What is the net force acting on the glass?

(b) What force are you applying to the glass?

(c) What force is the glass applying to you?

Answer: (a) 0.018 N upwards (b) 2.018 N upwards (c) 2.018 N downwards

2.8 During a car crash a 65 kg person's head goes from travelling at 50 km h^{-1} to stationary in 0.15 s.

(a) What is the magnitude of the average net force acting on the head of a person with a 4.5 kg head?

(b) How does this compare with the weight force acting on the person?

Answer: (a) 420; N (b) This is 64% of the weight force acting on this person.

2.9 A 10 kg box is being pushed up a slippery ramp as shown in Figure 2.1. The coefficient of friction between the box and the ramp is just $\mu = 0.1$.

(a) What force does the man need to apply to the box to keep it traveling up the ramp at a steady speed?

(b) What fraction of the weight force of the box is this?

If the angle of the ramp is raised to 45° then:

(c) What force does the man need to apply to the box to keep it traveling up the ramp at a steady speed now?

(d) What fraction of the weight force of the box is this?

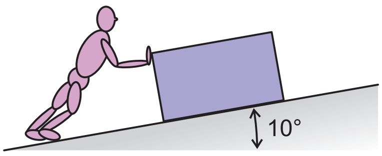

Figure 2.1 A box is pushed up a ramp

Answer: (a) 27 N (b) 0.27 × mg (c) 78 N (d) 0.78 × mg

2.10 A 1.6 kg chicken is blown into a wall by a strong gust of wind, and held there as shown in Figure 2.2. If the maximum coefficient of friction between the chicken and the wall is $\mu_{\text{max}} = 0.25$, what minimum force must the gust of wind be applying to the chicken?

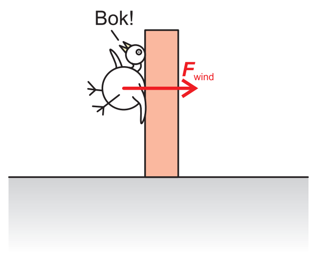

Figure 2.2 A chicken is blown into a wall.

Answer: $F_{\text{wind}} = 64$ N

MOTION IN A CIRCLE

3

3.1 Problems

3.1 (a) Convert the following values from radians to degrees: (i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{2}$ (iv) 0.1 (v) $\frac{3\pi}{4}$

(b) Convert the following values from degrees to radians: (i) 1° (ii) 45° (iii) 60° (iv) 180° (v) 360° Answer: (a) (i) 30° (ii) 45° (iii) 90° (iv) 5.7° (v) 135°

(b) (i) $\frac{\pi}{180}$ or 0.018 radians (ii) $\frac{\pi}{4}$ or 0.79 radians (iii) $\frac{\pi}{3}$ or 1.05 radians (iv) π or 3.14 radians (v) 2π or 6.28 radians.

3.2 In a particular rear-end car collision the driver's head rotates 45° backward before being stopped by the headrest. What is the average angular velocity of the driver's head if the duration of the collision was 0.1 s? Answer: $\omega_{av} = 7.9$ radians s^{-1}

3.3 When an athlete throws a javelin her forearm snaps through an angle of approximately π radians in 0.20 s. The athlete's hand moves with approximately constant speed, the length of her forearm is 45 cm, and the combined mass of her forearm and javelin is 2.0 kg. Assuming that the system is well approximated by a mass of 2.0 kg located 45 cm from the pivot, what force do the ligaments holding the forearm to the elbow need to exert? Answer: $F = 220$ N

3.4 (a) A 3800 kg car travels round an unbanked corner (i.e. a horizontal road) at the recommended speed of 65 km h^{-1}. The radius of curvature is 80 m. What is the force that the road exerts on the car to keep it in motion around the corner?

(b) What force would the road need to exert if the car was travelling at 100 km h^{-1}? Answer: (a) $F = 16 \times 10^3$ N (b) $F = 37 \times 10^3$ N

3.5 A car is traveling around a circular race track at 180 km h^{-1}. If a single lap of the track is 2.4 km long, what is the angular velocity of the car (in rad s^{-1})? Answer: 0.13 rad s^{-1}

3.6 If the car in problem 3.5 weighs 2500 kg, what is the centripetal force acting on the car as it travels around the track? Answer: 16×10^3 N

3.7 An adventurous ant finds herself at the end of a fan blade when it is switched on. It is a high speed fan with blades measuring 0.20 m long. If she has a mass of 0.20 g and can hold on to the fan blade with a maximum force of 0.0124 N. What is the maximum number of *revolutions per minute* the fan can run at before she will be flung off? Answer: 154 rpm

3.8 An 8.0 m radius merry-go-round completes one revolution every 7.0 s.

(a) What is the angular velocity of the merry-go-round?

(b) With what speed are children moving when they ride on the merry-go-round?

(c) What is the centripetal *acceleration* these children feel when riding on the merry-go-round?

(d) What is the average acceleration of each child over the course of half a revolution of the merry-go-round?

(e) What is the average acceleration of each child over the course of a full revolution turn of the merry-go-round?

Answer: (a) 0.90 rad s^{-1} (b) 7.2 m s^{-1} (c) 6.5 m s^{-2} (d) 4.1 m s^{-2} (e) 0 m s^{-2}

Statics

4

4.1 Problems

4.1 A waiter holds two plates of food in one hand. His forearm has a mass of 2.2 kg and the centre of mass of his forearm is located 13 cm from his elbow joint. The centre of mass of the two plates is located 37 cm from his elbow joint and the total mass of the two plates is 1.1 kg. His bicep is attached to the bones of his forearm 3.5 cm from his elbow joint.

(a) What is the torque produced by the mass of the two plates?

(b) What is the torque produced by the mass of the waiter's forearm?

(c) What force must be exerted by the waiter's biceps muscle to ensure that the plates and forearm are motionless? Answer: (a) $\tau_{\text{plates}} = 4.1$ N m (b) $\tau_{\text{arm}} = 2.9$ N m (c) $F_{\text{biceps}} = 200$ N

4.2 A second waiter works at the same restaurant as the waiter in Problem 4.1. She is exactly the same size as the first waiter except the distance between her elbow and the point at which her bicep is attached is 0.5 cm shorter than his. By what percentage must the forces her bicep muscle exerts be larger than that of the first waiter? Answer: 15% larger (230 N)

4.3 A toddler weighs 10 kg and raises herself onto tiptoe (on both feet). Her feet are 8 cm long with each ankle joint being located 4.5 cm from the point at which her feet contact the floor. While standing on tip toe:

(a) what is the upward normal force exerted by the floor at the point at which one of the toddler's feet contacts the floor?

(b) what is the tension force in one of her Achilles tendons?

(c) what is the downward force exerted on one of the toddler's ankle joints? Answer: (a) $F_{\text{normal}} = 50$ N (b) $F_{\text{tendon}} = 64$ N (c) $F_{\text{ankle}} = 114$ N

4.4 The waiter in Problem 4.1 has an argument with the sous-chef over his paycheck and proceeds to throw the plates of food he is carrying straight up into the air. If the plates are accelerating at a rate of 2.5 m s^{-2}, and the center of mass of his arm is accelerating upwards at 0.9 m s^{-2} what is the force being applied by the waiter's bicep? (Assume that the waiter's forearm also accelerates upwards at the same rate). Answer: 235 N

4.5 A diagram of a hypothetical 40 cm long arm is shown in Figure 4.1.

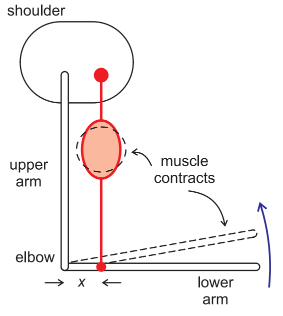

Figure 4.1 A simplified diagram of a arm showing the articulation of the lower arm by the biceps muscle.

For the purposes of answering this question assume that the arm itself is weightless. If the muscle attached to the arm can contract at a rate of 7.0 cm s^{-1} with a force of 15 000 N then:

(a) What is the maximum angular velocity of the arm if the muscle is attached 1 cm from the elbow?

(b) What is the maximum weight that can held in the hand if the muscle is attached 1 cm from the elbow?

(c) What is the maximum angular velocity of the arm if the muscle is attached 3 cm from the elbow?

(d) What is the maximum weight that can held in the hand if the muscle is attached 3 cm from the elbow?

Answer: (a) $\omega = 7.0$ rad s^{-1} (b) 38 kg (c) $\omega = 2.3$ rad s^{-1} (d) 110 kg

4.6 A 4 kg vase is placed on a shelf at the position shown in Figure 4.2. If the shelf itself has a negligible weight, what is the force F_{nail} with which the upper nail must hold to prevent being pulled out of the wall?

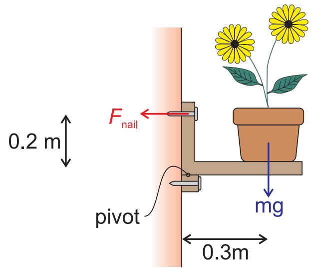

Figure 4.2 A vase sits on a shelf. The lower block acts like a pivot.

Answer: $F_{\text{nail}} = 60$ N

4.7 A uniform lever arm is used in conjunction with a pivot to weigh an object A. The mass of the lever arm is not negligible but is unknown. The pivot point may be moved relative to the lever arm.

(a) Where should the pivot be placed along the lever arm so that the mass of the lever arm does not appear in the calculation?

(b) If the unknown mass is balanced by a 0.05 kg mass hung from the lever arm 0.15 m from the pivot point, how much does the unknown mass weigh, given that it is hung from the lever arm at a point 0.3 m on the other side of the pivot point? Answer: (a)

The pivot point must be placed directly underneath the center of mass of the lever arm (b) 25 g

4.8 Which of the wheelbarrows in Figure 4.3 will require the smallest *upwards* force on the handles in order to lift?

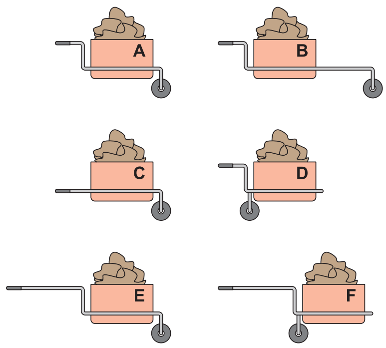

Figure 4.3 Six different ways of arranging a wheelbarrow.

Answer: wheelbarrow E

4.9 Two match officials carry an injured rugby player from a rugby field on a stretcher. The rugby player weighs 95 kg and the stretcher is 2.5 m long. One of the stretcher bearers (bearer A) is able to lift a weight of 60 kg; the other (bearer B) is unsure how much weight he is able to lift.
(a) How much weight must bearer B support?
(b) How far from bearer A must the injured rugby players center of mass be in order for the stretcher bearers to carry the load without tipping the injured player out? Answer: (a) 350 N (b) 0.92 m

Energy

5

5.1 Problems

5.1 In Figure **??**, the ball had an initial vertical velocity of 30 m s^{-1}. Prove that the maximum height reached is 45 m using energy arguments instead of kinematic equations. Answer: Kinetic energy

lost = potential energy gained. Therefore:

$$\Delta h = \frac{v_i^2}{2g}$$

5.2 A pulley system is used to raise a 30 kg load upwards at a steady rate. The energy input to the system is 1000 J every second and the efficiency of the system is 50% (that is, half the energy input is used to do work on the load). How fast is the load being raised? Answer: $v = 1.7\ \mathrm{m\,s^{-1}}$

5.3 Two 30 kg children in a 20 kg cart are stationary at the top of a hill. They start rolling down the 80 m tall hill and they are travelling at 30 km h^{-1} when they reach the bottom. (The cart had brakes!) How much work was done on the cart by friction during its travel down the hill? (Note the use of the word *on* and remember to specify the sign of the work done.) Answer: $W_{\text{friction}} = -61$ kJ

5.4 A car travelling at 50 km h^{-1} brakes as hard as it can and stops in a distance of 15 m. Suppose that the maximum braking force is not dependent on speed i.e. the coefficient of kinetic friction is constant. What is the shortest stopping distance when the car is travelling at 75 km h^{-1}? Answer: 34 m

5.5 A 20 kg box slides 45 m from the top down to the bottom of an incline which is at an angle of 10° to the horizontal. The box is stationary at the top of the slope and is accelerating down the slope at a rate of 1.0 m s^{-2}.

(a) How much work is done by the force of gravity on the box?

(b) How much work on the box is done by the frictional force?

(c) What is the kinetic energy of the box at the bottom of the slope?

(d) How much gravitational potential energy has the box lost while traveling down the slope?

Answer: (a) 1600 J (b) −660 J (c) 900 J (d) 1600 J

5.6 A 70 kg physicist is running up the stairs of the physics building and makes it up 48 m vertically in 1 minute.

(a) What is the physicist's power output while running up the stairs?

(b) If the physicist's work efficiency is just 3% at what rate were they using metabolic energy?

(b) If the physicist's metabolism can provide 5.6×10^6 J of energy before they need a rest, how long could they continue running up the stairs?

Answer: (a) 560 W (b) 19000 W (c) 5 minutes

5.7 An 85 kg sky diver is falling through the air at a constant speed of 195 km h^{-1}. At what rate does air resistance remove energy from the sky diver? Answer: $P = 46$ kW

5.8 The human body loses heat at a rate of 120 W when sitting quietly at rest. If a 65 kg student takes 100 hours to read *War and Peace* by Leo Tolstoy, how high could they have been lifted if all of the heat energy lost was utilised to lift them against gravity (assuming that the force of gravity on the student remains constant)? Answer: 67 km

5.9 An 8 g bullet leaves a gun at 700 m s^{-1}.

(a) What is the maximum height that this bullet could reach (ignoring air resistance)?

(b) If the gun is aimed at an angle of 30° above the horizontal what height will the bullet reach (again ignoring air resistance)?

Answer: (a) 24 km (air resistance has a very large effect!) (b) 6.1 km

5.10 A crane is lifting a 500 kg payload straight up at a constant speed of 0.7 m s^{-1}.

(a) What is the power output of the crane (ignoring losses)?

(b) If it takes the crane 2 minutes to raise the payload to its final height, how far above ground is this?

(c) If the cable were to break as the payload reaches its final height, how fast would it be travelling as it hit the ground?

Answer: (a) 3500 W (b) 84 m (c) 41 m s^{-1} (148 km hr^{-1})

MOMENTUM

6.1 Problems

6.1 How fast do the following need to be travelling in order to have a momentum of 10 kg m s^{-1} ?

(a) An Airbus A380 aeroplane carrying 853 belly dancers, total weight 560×10^3 kg.

(b) An ordinary land-based bus carrying 40 belly dancers, total weight 6000 kg.

(c) A single 50 kg belly dancer.

(d) A 3.5 kg pet cat (perhaps owned by a belly dancer).

(e) A 5 g moth, being chased around by a belly dancer.

Answer: (a) $1.8 \times 1-^{-5}$ m s^{-1} (0.018 mm s^{-1}) (b) $1.7 \times 1-^{-3}$ m s^{-1} (1.7 mm s^{-1}) (c) 0.2 m s^{-1} (d) 2.9 m s^{-1} (e) 2000 m s^{-1}

6.2 Car manufacturers conduct crash tests on their cars in order to improve crash safety. In the event of a crash the head of any child travelling in the front seat can strike the glove compartment at considerable (relative) speed, even if the child is wearing a seat-belt.

(a) The manufacturers of a particular brand of car conduct head-on collision tests and find that in the absence of a passenger side air bag, a child's head (which has a mass of 3.5 kg) goes from a speed of 40 km h^{-1} relative to the dashboard just before its collision to rebounding from the dash board at 15 km h^{-1} just after the collision. This collision lasts just 0.08 seconds. What is the average force exerted on the child's head during this collision?

(b) The manufacturer wishes to reduce the average force involved in such a collision to 200 N. In order to achieve this they install a passenger airbag on the front of the glove compartment which quickly inflates in the event of a crash and deflates as the child's head pushes into it, effectively increasing the amount of time it takes to slow the child's head (i.e., the collision lasts longer). How long would the collision between the child's head and the airbag need to last to reduce the speed of the head relative to the dashboard from 40 km h^{-1} to 0 km h^{-1} without exceeding the average force quoted above? Answer: (a) $F = 670$ N (b) $t = 0.19$ s

6.3 A 3500 kg car hits an 80 kg pedestrian who is standing in the middle of the road. Before the collision the car was travelling at 30 km h^{-1} and the pedestrian was stationary. After the collision the car was travelling at 28.5 km h^{-1}. At what speed will the pedestrian be flung down the road (in km h^{-1})? Answer: 65 km h^{-1}

6.4 Two basketball players collide head-on. Player A weighs 80 kg and is travelling 2.5 m s^{-1} to the right while Player B weighs 68 kg and is travelling 1.2 m s^{-1} to the left. After the collision Player A is travelling at 1.0 m s^{-1} to the right.

(a) What is the change in momentum of Player A?

(b) If the collision lasted 0.1 s, what is the average force Player B must have exerted on Player A during the collision?

(c) What is the average force that Player A must have exerted on Player B during the collision?

(d) What is the change in momentum of Player B?

(e) What is the final velocity of Player B? Answer: (a) 120 N s to the left (b) 1200 N to the left (c) 1200 N to the right (d) 120 N s to the right (e) 0.56 m s^{-1} to the right

6.5 An enthusiastic kitten collides with a ball of string that is rolled towards it. The 0.5 kg kitten is travelling at 0.5 m s^{-1} due east before the collision and the 0.6 kg ball of string was travelling at 0.7 m s^{-1} due west. If the kitten grabs the string during the collision and does not let go, what is the final speed of the kitten+string? Answer: 0.16 m s^{-1} due west

6.6 An 85 kg runner is accelerating at a rate of 2 m s^{-2} and a 65 kg runner at a rate of 3 m s^{-2}. If the heavier runner started at a speed of 1 m s^{-1} while the lighter runner started off stationary then how long is it before the runners have the same momentum? Answer: 3.4 s

6.7 A 40×10^3 kg train is travelling at 8.3 m s^{-1} when the engineer sees a sheep on the tracks. She throws the emergency brakes on. The emergency brakes can apply a maximum force of 11×10^3 N.

(a) How long is it before the train will come to a complete stop?

(b) The sheep, startled by the sparks and noise of the emergency brakes, trots off the tracks. The engineer is able to release the brakes. If the brakes had been applied for 11 s, how fast is the train travelling now?

Answer: (a) 30 s (b) 5.3 m s^{-1}

6.8 A kererū (New Zealand wood pigeon) travelling at 1.5 m s^{-1} due west has an elastic collision with an Airbus A380 aeroplane. Fortunately for the kererū, the A380 was only travelling at 0.1 m s^{-1} in an easterly direction at the time. With what speed is the kererū travelling after the collision? Answer: 1.7 m s^{-1} east

6.9 The kererū from Problem 6.8, having proven itself somewhat careless, has a totally inelastic collision with a cat. If, just before the collision, the 0.65 kg kererū was travelling 2.0 m s^{-1} north and the 3.0 kg cat was traveling 5.5 m s^{-1} south, what is the velocity of the ball of fur and feathers just after the collision? Answer: 4.2 m s^{-1} south

Simple Harmonic Motion

7

7.1 Problems

7.1 Problems

7.1 A spring is pressed against a wall so that it is compressed by 0.25 m (i.e. it is 0.25 m shorter than its equilibrium length). The spring is then released. The spring constant is $k = 35\,\mathrm{N\,m^{-1}}$, and the spring weighs 50 g. What is the speed at which the spring leaves the wall? Answer: $6.6\,\mathrm{m\,s^{-1}}$

7.2 How long is a simple pendulum with a period of 5 seconds? How long would this pendulum have to be if it were to operate on the moon with the same period? ($g_{\mathrm{moon}} = 1.62\,\mathrm{m\,s^{-2}}$) Answer: $L_{\mathrm{earth}} = 6.3\,\mathrm{m}$, $L_{\mathrm{moon}} = 1.0\,\mathrm{m}$

7.3 During an earthquake a skyscraper is designed to sway back and forth with simple harmonic motion with a period of 8 seconds. The amplitude at the top floor for a particular earthquake is 70 cm. With respect to the simple harmonic motion of the top floor, calculate the following quantities:

(a) The radius of the circle used to represent the SHM.
(b) The speed of the object moving round the circle.
(c) The angular velocity.
(d) The maximum speed of the top floor. Answer: (a)

$r = 70\,\mathrm{cm}$ (b) $v = 0.55\,\mathrm{m\,s^{-1}}$ (c) $\omega = 0.79\,\mathrm{radians\,s^{-1}}$ (d) $v_{\mathrm{max}} 0.55\,\mathrm{m\,s^{-1}}$

7.4 When a sound wave with a certain intensity is detected by the tympanic membrane (eardrum) the amplitude of the resultant motion is 1.0 nm (1.0×10^{-9} m). If the frequency of the sound is 600 Hz what is the maximum speed of the membrane oscillation? Answer: $v_{\mathrm{max}} = 3.8 \times 10^{-6}\,\mathrm{m\,s^{-1}}$

7.5 A fly beats its wings at a frequency of 1200 Hz. If the expansion and contraction of the wing muscles of the fly exhibits simple harmonic motion, the angular displacement of each wing of the fly also exhibits simple harmonic motion, the length of the wing muscles varies from 750 µm to 600 µm over the course of one beat of its 1.00 cm long wings, and the wing tips move through an arc of 150° then:

(a) What is the time period of a single beat of the fly's wings?
(b) What is the maximum velocity of the mobile end of the wing muscle?
(c) What is the maximum angular velocity of the wing?
(d) What is the maximum speed of the wingtip?

Answer: (a) 0.83 ms (b) $0.565\,\mathrm{m\,s^{-1}}$ (c) $9870\,\mathrm{rad\,s^{-1}}$ (d) $98.7\,\mathrm{m\,s^{-1}}$

7.6 Four identical springs used as part of a car's suspension system, one on each wheel. The springs compress by 6 cm when the weight of the 1900 kg car is applied to them.

(a) What is the spring constant of each of the springs?
(b) With what frequency will the suspension bounce if given a jolt?
(c) If the springs are chopped in half (which will double the spring constant) with what frequency will the suspension bounce now?

Answer: (a) $k = 79 \times 10^{3}\,\mathrm{N\,m^{-1}}$ (b) $T = 0.49$ s (c) $T' = 0.34$ s

7.7 A 5000 kg floating pier is moved up and down by the changing tide. If the period of this motion is 12 hours and the amplitude is 2.5 m and we treat the motion as being simple harmonic then:

(a) What is the frequency of the motion?
(b) What is the maximum vertical velocity of the pier?
(c) What is the maximum vertical acceleration of the pier (hint: use circular motion as an analogue)?

Answer: (a) 23×10^{-6} Hz (b) $0.36\,\mathrm{mm\,s^{-1}}$ (c) $5.2 \times 10^{-8}\,\mathrm{m\,s^{-2}}$

7.8 A small 5 g fly is buzzing along and hits a spider web. The spider web catches the fly and proceeds to oscillate with a time period of 0.09 s and an initial amplitude of 1.9 cm.

(a) With what velocity was the fly flying before it hit the web?
(b) What is the maximum force that the web exerts on the fly (hint: use circular motion as an analogue to find the maximum acceleration of the fly)?
(c) What is the spring constant of the web?

Answer: (a) $1.3\,\mathrm{m\,s^{-1}}$ (b) 0.47 N (c) $24\,\mathrm{N\,m^{-1}}$

7.9 A particular person's lower leg is 56 cm long and weighs 9.5 kg. If allowed to swing freely, with what time period would the leg swing? Is this close to what you would estimate such a person's walking pace is? (Treat the leg as a simple pendulum with length equal to half the leg length and all the leg's mass concentrated at this point.) Answer: $T = 1.05$ s

8 Waves

8.1 Problems

8.1 A transverse wave propagates in a system of mass-springs as shown in Figure **??**. The masses and springs are all identical: the masses are all 50 g weights and the springs all have the same spring constant. The separation of points P and Q (see Figure **??**) is 0.5 m and the wave propagates through the mass-spring system with a velocity of 30 m s^{-1}.

(a) What is the frequency of the oscillator at point P?

(b) Suppose that the oscillator at point P has an amplitude of 25 cm. What is the spring constant of this oscillator?

(c) Suppose that we halve the amplitude with which each of the masses is oscillating. How does the velocity of propagation of the wave change?

(d) Suppose that we double the spring constant of each oscillator in the system (without changing anything else). How would this change the velocity of propagation of the wave?

(e) Given the number of oscillators shown in Figure 8.2, what is the phase shift (in degrees) between two adjacent oscillators?

(f) If this phase shift is halved (and the oscillator positions and frequency remain the same), what is the velocity of propagation of the wave? Answer: (a) $f = 60$ Hz (b) $k = 7100$ N m^{-1} (c) no change (d) v increases by $\sqrt{2}$ (e) phase shift = 45 deg (f) v is doubled

8.2 A piano tuner wishes to use the beat frequency generated when two different notes are sounded together to tune one of the keys on a piano keyboard. She uses a tuning fork to tune the note named 'A' to 440 Hz. The next note higher than this should have a frequency of (approximately) 466 Hz.

(a) If the speed of sound in air is 340 m s^{-1}, what is the wavelength of the note named 'A'?

(b) When these two notes are played together how many beats per second will she hear when these two notes have the frequencies indicated? Answer: (a) $\lambda = 0.77$ m (b) $f_{\text{beat}} = 26$ Hz

8.3 A transverse wave propagates in the positive x-direction with a wavelength of 0.3 m and a period of 10^{-3} s.

(a) Use Eq.(8.2) to write down an expression for this wave.

(b) What are the frequency and angular frequency of this wave?

(c) What is the propagation velocity of this wave? Answer: (a) $y = A\cos(20.9x - 6280t)$ (b) 6280 radians s^{-1} (c) $v = 300$ m s^{-1}

8.4 A transverse wave propagates in the negative x-direction with a frequency of 20 Hz and a propagation velocity of 25 m s^{-1}.

(a) What are the wavelength, period and angular frequency of this wave?

(b) Use Eq.(8.2) to write down an expression for this wave. Answer: (a) $\lambda = 1.25$ m, $T = 0.05$ s, and $\omega = 126$ radians s^{-1} (b) $y = A\cos(5.0x + 126t)$

8.5 A transverse wave is described by the expression

$$y = 0.05\cos(4.19x - 1260t)$$

. You may assume all measurements are in the correct SI units.

(a) What is the amplitude of this wave?
(b) What is the wavelength of this wave?
(c) What is the frequency of this wave?
(d) How fast is this wave traveling?
(e) What is the maximum transverse velocity of this wave?

Answer: (a) 5 cm (b) 1.50 m (c) 200 Hz (d) 300 m s^{-1} (e) 63 m s^{-1}

8.6 A boat is bobbing up and down on the water as waves pass underneath it. The depth of the water under the boat oscillates between 3 m and 4 m. The boat is stationary with respect to the shore and it is 2.9 s between the crests of successive waves. A person on shore sees the crests of the wave passing by at 2 m s^{-1}. What is the distance between crests of this wave? Answer: $\lambda = 5.8$ m

8.7 Two ducks are floating close together on the water near the boat in Problem 8.6. When the first duck is on the peak of a passing wave the second duck is 30 cm below it and moving upwards. How far apart are the ducks? Answer: $d = 1.1$ m or $d = 4.7$ m

8.8 A violin is playing a note at 1200 Hz when a second violin starts playing. There is a distinct pulse in the resultant mix which repeats 16 times over the course of 5 seconds. What are the possible frequencies of the second violin? Answer: 1197 Hz or 1203 Hz

Sound and Hearing

9

9.1 Problems

9.1 What is the speed of sound through ice? ($B_{ice} = 8.8 \times 10^9$ Pa, $\rho_{ice} = 920$ kg m^{-3}) Answer: $c_{ice} = 3100$ m s^{-1}

9.2 A 0.500 m guitar string is placed under a tension of 270 N and the fundamental mode of vibration is at 150 Hz.

(a) What is the weight of the string per unit length (μ in kg m^{-1})?

(b) What tension would need to applied to the string such that its fundamental mode of vibration is a middle C (440 Hz)?

Answer: (a) $\mu = 0.012$ kg m^{-1} (b) $T = 2320$ N

9.3 A wave travels from air ($Z_{air} = 413$ kg m^{-2} s^{-1}) into a liquid and with a density of 950 kg m^{-3} and in which the speed of sound is 750 m s^{-1}.

(a) What proportion of the wave is reflected from the boundary?

(b) What proportion of the wave is transmitted through the boundary?

Answer: (a) 0.998 (99.8%) (b) 0.002 (0.2%)

9.4 During an extremely loud sound the amplitude of the pressure in the sound wave in air is 1.0 kPa ($Z_{air} = 413$ kg m^{-2} s^{-1}).

(a) What is the intensity of the sound (in W m^{-2})?

(b) What is the intensity level of the sound (in dB)?

(c) What is the sound pressure level (in dB)?

Answer: (a) $I = 1.2 \times 10^3$ W m^{-2} (b) $L_I = 151$ dB (c) $L_p = 154$ dB

9.5 A quiet whisper is measured at 30 dB and a loud shout at 110 dB. ($A_{ear\ canal} = 1.54 \times 10^{-4}$ m^2, $Z_{air} = 413$ kg m^{-2} s^{-1})

(a) What power, in watts, is delivered to the opening of the ear canal during the whisper?

(b) How many times larger is the power (in watts) delivered to the ear canal by the shout?

(c) What is the pressure variation during the whisper?

(d) How many times larger is the pressure variation during the shout?

Answer: (a) 1.54×10^{-13} W (b) 10 million times larger (10^8) (c) $p = 1.13 \times 10^{-5}$ Pa (d) ten thousand times larger (10^4)

9.6 An oboe and a double bass are playing the same note, a ‘G’ at 392 Hz. The speed of sound in air is 343 m s^{-1} and the speed of propagation of a wave on the string of the double bass is 500 m s^{-1}. The oboe can be modeled as a pipe, open at one end, and the bass as a string fixed at both ends. What are the lengths of the bass string and the oboe cavity if the note being produced is a result of the fundamental mode in each case? Answer: $L_{oboe\ cavity} = 22$ cm,

$L_{double\ bass} = 64$ cm

9.7 After giving an intense performance, a confused and disoriented flautist has wandered onto the motorway! They are playing a constant 300 Hz tone on their flute and are essentially stationary. If you are driving along the motorway at 100 km h^{-1} (27.8 m s^{-1}), what is the frequency you hear from the flautist's instrument before you pass them, and after you pass them? ($c_{air} = 343$ m s^{-1}.) Answer: $f_{approaching} = 324$ Hz, $f_{passed} = 276$ Hz

9.8 Coincidentally the horn on your car, which you sound as you narrowly miss the flautist in Problem 9.7, also gives a constant 300 Hz tone. What frequency does the flautist hear before and after you pass them? Answer: $f_{approaching} = 326$ Hz, $f_{passed} = 278$ Hz

II

Bulk Materials

Elasticity: Stress and Strain

10

10.1 Problems

10.1 The elasticity of a cylindrical sample of an unknown material is to be tested. The sample is 40 cm long and has a cross sectional area of 2.5 cm^2. The sample is hung vertically and a 50 kg weight is attached to its free end. It is found that the sample stretches to a length of 40.1 cm.

(a) What is the tensile stress on the sample?
(b) What is the tensile strain?
(c) What is Young's modulus for this material? Answer: (a) $stress = 2 \times 10^6$ Pa (b) $strain = 2.5 \times 10^{-3}$ (c) $\gamma = 8 \times 10^8$ Pa

10.2 (a) Bone has a tensile Young's modulus of $\gamma_{\text{tensile}} = 16 \times 10^9$ N m^{-2}. If the sample of unknown material in Problem 10.1 was replaced with an identical sample of bone, what would the length increase of the bone be?

(b) Bone has a compressive Young's modulus of $\gamma_{\text{compressive}} = 9 \times 10^9$ N m^{-2}. If the 50 kg weight was used to compress the sample rather than stretch it, how much would the length of the sample change? Answer: (a) $\Delta L = 5 \times 10^{-5}$ m (or 0.05 mm) (b) $\Delta L = 5 \times 10^{-5}$ m (or 0.05 mm)

10.3 Which of the following is most likely to be the Young's modulus of a rubber band? (Hint: What effect will a force of 1 N might have on a piece of rubber band 1 mm square. What is the strain found for such an applied stress for each value of the Young's modulus) (a) 5×10^{10} Pa, (b) 5×10^5 Pa, (c) 5×10^{-5} Pa. Answer: (b) 5×10^5 Pa.

10.4 A brand new type of rubber is discovered that can be manufactured from a combination of air and wishful thinking. A solid cylindrical rod made of this new type of rubber is fixed to the ceiling and a 1 kg weight is hung from the lower end. The rod was originally 20 cm long, and when it is hung it is 35.9 cm long. The radius of the rod is 0.37 cm. What is the Young's modulus of the rubber used to make the rod? Answer: $\gamma = 9.3 \times 10^5$ Pa

10.5 The type of rubber featured in Problem 10.4 is understandably very cheap to produce and as a result the manufacturers of all sorts of devices wish to incorporate it into their products. The makers of a car suspension system wish to see if a 0.3 m solid cylindrical length of this new rubber can be used in a car's suspension system. The rubber cylinder must be compressed by just 2 cm under a load of 5500 N. What radius must the cylinder be? (Is this practical?) Answer: (a) $r = 0.300$ m, so not very practical ($\gamma = 2.92 \times 10^5$ Pa).

10.6 The unknown material in Problem 10.1 is now tested for resistance to shear. One end of the cylinder is clamped to a heavy table and a horizontal force of 300 N is applied to the free end. The top of the sample is found to deflect by 5 cm. What is the shear modulus of the sample? Answer: $G = 9.6 \times 10^6$ Pa

10.7 The bulk modulus of water is 2.2×10^9 N m^{-2}. What increase in pressure needs to be applied to the surfaces of a cube of water 1 m on a side to reduce its volume by 1%? Answer: $\Delta P = 2.2 \times 10^7$ Pa

(This is a force of 2.2×10^7 N per side of the cube, which is equivalent to the weight force of a 2200 tonne object acting on each face of the cube.)

10.8 Refer to Figure 10.1 to answer the following questions:

(a) As shown in Figure 10.1 a cylinder is completely filled with water and a force F applied to the airtight movable piston which forms one end of the cylinder. What force will need to be applied to the piston for the volume of the cylinder to *increase* by 1 cubic millimetre (which equals 10^{-9} m^3)? ($B_{\text{water}} = 2.2 \times 10^9$ Pa)

(b) If the cylinder was filled with air instead ($B_{\text{air}} = 1.42 \times 10^5$ Pa) what force would be required to increase the volume of air by 1 cubic millimetre?

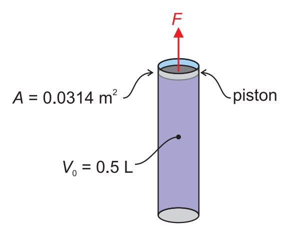

Figure 10.1 A force is applied to a piston attached to a cylinder filled with water. This causes the volume of the water to change.

Answer: (a) $F = 138$ N (b) 8.9×10^{-3} N

PRESSURE

11.1 Problems

11.1 Steve has a mass of 85 kg and wears size 11 shoes. The bottom of each of Steve's shoes has an area of 0.03 m^2 (approximately 30 cm by 10 cm). What pressure do the soles of Steve's shoes exert on the ground when

(a) he is standing still?

(b) he is standing on one leg?

(c) he is in the process of jumping (with both feet on the ground), and thus accelerating upwards at a rate of 5 m s^{-2}?

Answer: (a) 14.2 kPa (b) 28.3 kPa (c) 21.3 kPa

11.2 Water has a density of 1×10^3 kg m^{-3}. How many litres of water would weigh as much as an 80 kg man? Answer: 80 litres

11.3 Blood has a density of 1060 kg m^{-3} whereas the density of the cerebrospinal fluid is 1007 kg m^{-3}. What *mass* of cerebrospinal fluid will have the same *volume* as 50.0 g of blood? Answer: m_{csf} = 47.5 g

11.4 Given that the density of water is 1×10^3 kg m^{-3}, how deep would you have to dive to experience an absolute pressure of 2 atm? How deep would you have to dive to experience an absolute pressure of 5 atm? (Note that 1 atm = 101.3 kPa.) Answer: 10 m and 40 m respectively

11.5 What would you estimate that the difference in average blood pressure between the top of 1.7 m tall person's head and the bottom of their feet is? (ρ_{blood} = 1060 kg m^{-3}) Answer: 18 kPa

11.6 Blood pressure is generally quoted in mmHg and is a *gauge pressure*. In this question we will convert this into SI units. Suppose that the systolic pressure of a particular patient is 120 mmHg and the diastolic pressure is 80 mmHg. Given that 760 mmHg is equivalent to 101.3 kPa, what is the systolic and diastolic blood pressure of this patient in SI units (Pa, or kPa)? Answer: $P_{systolic}$ = 16000 Pa, and $P_{diastolic}$ = 10700 Pa (Note: these are gauge pressures.)

11.7 You wish to measure the blood pressure at the top of a patient's head but the patient is unable to lie flat. You measure the blood pressure at the bicep (i.e. level with the heart) and find that it is 140 mmHg systolic and 80 mmHg diastolic. What would you expect the blood pressure to be at the top of the head given that this point is 45 cm above the measurement point? Remember this is a gauge pressure and give your answers in both mmHg and kPa. (The density of blood is about 1060 kg m^{-3}.) Answer: $P_{systolic}$ = 14 kPa, and $P_{diastolic}$ = 6 kPa ($P_{systolic}$ = 105 mmHg, and $P_{diastolic}$ = 45.0 mmHg)

11.8 The aortic valve is located at the base of the aorta and controls the flow of blood from the left ventricle of the heart into the aorta. The valve has an area of about 3.5 cm^2 when closed (there are variations in this area from person to person). The aortic valve closes just before the diastolic phase of the cardiac cycle at which point the blood pressure is at about 90 mmHg. Using the figure just quoted calculate the force exerted on the aortic valve by the blood in the left ventricle (remember, the blood pressure shown above is a gauge pressure, not an absolute pressure.). Answer: Net force on valve using gauge pressure: F_{net} = 4.2 N. Force exerted on valve by blood in left ventricle: F_{net} = 39.7 N

11.9 The systolic pressure in a major artery is measured at 115 mmHg. What is the net force on a 1 cm^2 section of the arterial wall if the (absolute) pressure in the tissue outside the arterial wall is 109 kPa? (P_{atm} = 101.3 kPa) Answer: 0.76 N

11.10 What is the density of the unknown fluid in Figure 11.1? (You many assume that the unknown liquid is prevented from mixing with the water or displacing it in the event it is the more dense. ρ_{water} = 1000 kg m^{-3}.)

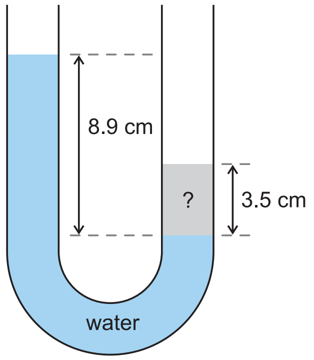

Figure 11.1 Water and an unknown fluid are placed in an open u-tube.

Answer: $\rho_{unknown}$ = 2500 kg m^{-3}

Buoyancy

12

12.1 Problems

12.1 A swimmer finds that she just floats in water. If she weighs 70 kg what is her volume ($\rho_{\text{water}} = 1 \times 10^3$ kg m^{-3})? Answer: $V =$ 0.07 m^3

12.2 Law 2 of the game of soccer specifies that the ball is an air-filled sphere with a circumference of 68–70 cm, and a mass of 410–450 g. A particular ball has a circumference of 69 cm, and a mass of 430 g. Calculate the fraction of the volume of this ball that floats above the surface of water. Answer: 0.92 (or 92%) of the balls volume will be above the surface

12.3 A product designer for a range of nautical safety devices conceives of a floating container in which emergency supplies can be stored, which would automatically become detached from a boat if it sunk. The container is to be a cylinder with a radius of 20 cm and a height of 1 m. When in the water 20% of the volume of the container must be above the surface. The container is attached to the deck of the boat by a cord which should break as the boat sinks and the container is submerged and pushed upwards by it's own buoyancy.

(a) What is the maximum mass of the container and it's contents?

(b) At what tension should the cord attaching the container to the boat break?

Answer: (a) 100 kg (b) 260 N

12.4 A small 0.5 m radius weather balloon is filled with helium ($\rho_{\text{He}} = 0.164$ kg m^{-3}). What is the maximum payload (including the balloon mass) of this weather balloon? ($\rho_{\text{air}} = 1.18$ kg m^{-3}) Answer: $m_{\text{payload}} = 0.532$ kg

12.5 A helium shortage forces some under-funded meteorologists to investigate alternative gases to use in their weather balloons. They settle on methane ($\rho_{CH_4} = 0.657$ kg m^{-3}). What is the minimum radius of a methane filled weather balloon that will allow the same minimum payload as the helium filled balloon in Problem 12.4? Answer: $r = 0.624$ m

12.6 A piece of polystyrene packaging material (density = 25 kg m^{-3}) that has a mass of 0.2 kg is tethered to the bottom of a container of water (density = 1×10^3 kg m^{-3}) with a piece of string. What is the tension in the string? Answer: $T = 78$ N

12.7 In an experiment to determine the density of an unknown material, its apparent weight when fully submerged in water is measured. The apparent weight in water is 17.5 N and the weight in air is 27.5 N. What is the density of the material? Answer: $\rho_{\text{unknown}} = 2750$ kg m^{-3}

12.8 Some salvage divers are raising a rectangular box of treasure measuring 1 m × 1.3 m × 1.9 m from the bottom of the sea. They are using a cable which can handle a maximum force of 10 000 N without breaking. They raise the treasure from the bottom of the ocean at a constant speed. All goes well while the treasure is rising through the water but much to the despair of the salvors as soon as the treasure is clear of the water the cable breaks, dropping the treasure back into the briny deep. What are the maximum and minimum possible weights of the lost treasure? Answer: $m_{\text{max}} =$ 3400 kg, $m_{\text{max}} = 1000$ kg

12.9 A large air-filled rubber ball is tethered to the bottom of a swimming pool. The tension in the tether is 100 N. The mass of the rubber in the ball itself is 2 kg while $\rho_{\text{water}} = 1000$ kg m^{-3} and $\rho_{\text{air}} = 1.2$ kg m^{-3}. What is the volume of the ball? Answer: $V =$ 0.012 m^3

12.10 A wooden cube 3 cm on a side floats level on water with just 1.5 mm of the cube showing above the surface. What is the density of the wood? ($\rho_{\text{water}} = 1000$ kg m^{-3}) Answer: $\rho_{\text{wood}} = 950$ kg m^{-3}

13 Surface Tension and Capillarity

13.1 Problems

13.1 Problems

13.1 A device such as that shown in Figure **??** is used to measure the surface tension, γ, of glycerol. The length of the moveable wire is 0.5 cm and a force of 0.63 N must be applied to this wire to maintain a constant area fluid film. what is the surface tension of glycerol? Answer: $\gamma = 63\ \mathrm{N\,m^{-1}}$

13.2 A soap bubble is formed using a mixture of detergent and water. The surface tension of the mixture is 0.030 N $\mathrm{m^{-1}}$. If the bubble has a radius of 2 cm and atmospheric pressure is 101.2 kPa, what is the gauge pressure inside the bubble? Answer: $P_{\mathrm{bubble}} =$ 6 Pa

13.3 A water bubble has radius 1 mm in air. Atmospheric pressure is 101.3 kPa and the surface tension of water at room temperature is 0.073 N $\mathrm{m^{-1}}$.

(a) What is the gauge pressure inside the bubble?

(b) If atmospheric pressure had been 101.2 kPa and the absolute internal pressure of the bubble was the same as in part (a), what would the radius of the bubble have been?

(c) If the surface tension of the water was lowered to 0.037 N $\mathrm{m^{-1}}$ (due to the addition of a surfactant for example), what would the radius of the bubble in part (b) be?

Answer: (a) $P_{\mathrm{gauge}} = +292$ Pa (b) $r = 0.75$ mm (c) $r = 0.38$ mm

13.4 An entertainer, when performing a bubble trick, forms one bubble inside another. The surface tension coefficient of the bubble liquid used was $\gamma = 0.04\ \mathrm{N\,m^{-1}}$. If the outer bubble has a radius of 4.5 cm and the inner bubble has a radius of 2 cm, what is the gauge pressure in the inner bubble? Answer: $P_{\mathrm{inner}} = +11.6$ Pa

13.5 The interface between blood and stainless steel makes a angle of 110°. Would you expect that capillary action would draw blood into a stainless steel needle, or expel it? Answer: Expel it

13.6 Fluid A is found to have a surface tension of 0.080 N $\mathrm{m^{-1}}$, a density of $1.2 \times 10^3\ \mathrm{kg\,m^{-3}}$ and a contact angle of 70° with dry glass.

Fluid B is found to have a surface tension of 0.100 N $\mathrm{m^{-1}}$, a density of $3.1 \times 10^3\ \mathrm{kg\,m^{-3}}$ and a contact angle of 110° with dry glass.

A glass capillary tube with inner radius 1 mm is lowered into a container of fluid A and an identical capillary tube is lowered into a flask of fluid B. To what height above (or below) the fluid surface will fluids A and B rise in their respective capillary tubes? Answer: $h_A = +4.6$ mm (i.e. fluid rises in the capillary tube) and $h_A = -2.2$ mm (i.e. fluid in the capillary tube is depressed)

13.7 Drops of two liquids are placed onto a glass slide. Liquid A remains a small rounded drop, sitting on the glass. Liquid B spreads out to form a thin film on the glass. If a narrow glass capillary tube is placed in a container of each liquid, would you expect the level of liquid in the capillary to be higher or lower than that in the container for each liquid? Answer: Liquid A : lower, Liquid B : higher

13.8 What is the minimum surface tension of a fluid that can sustain a gauge pressure of 0.1 kPa in a 0.5 cm radius bubble? Answer: 0.125 N $\mathrm{m^{-1}}$

14

Fluid Dynamics of Non-viscous Fluids

14.1 Problems

14.1 A large artery has a diameter of 7 mm. This artery divides into two identical smaller arteries, the velocity of the blood in the smaller arteries is the same as the velocity of the blood in the larger artery. What is the diameter of the smaller arteries? Answer: $d =$

4.95 mm

14.2 The diameter of a blood vessel narrows by 70% due to the presence of a plaque on the blood vessel walls.

(a) By what factor does the blood velocity increase?

(b) If the blood velocity in a normal blood vessel is 0.15 m s^{-1} and the systolic blood pressure is 130 mmHg, what is the systolic blood pressure in the narrowed vein (in mmHg)?

Answer: (a) $v_{\text{narrow}} = 11.1 v_{\text{normal}}$ (b) $P_{\text{systolic}} = 119$ mmHg

14.3 A small plastic pipe carries water horizontally at a speed of 10 m s^{-1}. A section of the pipe bulges out so that the radius is twice that of the rest of the pipe. If the gauge pressure in the pipe is ordinarily +90 kPa what is the gauge pressure in the bulge (in kPa) (the density of water is 1000 kg m^{-3})? Answer: $P_{\text{bulge}} = +137$ kPa

14.4 In this example we will construct a simple model of the circulatory system to investigate the rate at which cuts bleed. In this model we will assume that blood is a Newtonian fluid at all length scales so that the equations of fluid flow which we have been studying will apply. We will also assume that the effects of viscosity may be ignored. These will NOT be a good approximations for real blood in capillaries as the diameter of capillaries is about the same as the size of the red corpuscles and this has a major effect on blood flow in capillaries. However, our model will serve as an indication of the effects of Newtonian fluid flow in circulatory systems.

A large artery has a diameter of 7 mm and carries blood which flows with a peak velocity of 0.15 m s^{-1}. This vessel eventually feeds a network of capillaries which together have an area approximately 400 times that of the large artery which feeds into them. In this model, the capillaries are identical to each other and have a diameter of 7.5 μm.

(a) Suppose that the diastolic blood pressure is 130 mmHg at the level of the heart and the blood velocity in the large artery at the heart is 0.15 m s^{-1}. What is the blood velocity in the artery at a point 1 m below the heart? (The density of blood is 1050 kg m^{-3})

(b) If the artery is severed at a at a point 1 m blow the heart, what is the maximum velocity of blood flow from the artery?

(c) What is the blood velocity in a capillary in the capillary net at this point (1 m below the heart)?

(d) What is the blood pressure in the capillary net at this point (in mmHg and with the assumptions discussed above)?

(e) If a capillary is severed, what is the blood velocity leaving the wound? (again on the basis of this model)

Answer: (a) $v_{\text{artery}} = 0.15$ m s^{-1} (b) $v_{\text{wound}} = 7.28$ m s^{-1} (c) $v_{\text{capillary}} = 3.75 \times 10^{-4}$ m s^{-1} (d) $P_{\text{capillary}} = 209$ mmHg (e) $v_{\text{wound}} = 7.28$ m s^{-1}

14.5 At what rate (Pa m^{-1}) does the pressure need to change in a vertical pipe filled with water to keep the velocity of the water flowing through it constant? Answer: $\frac{\Delta P}{\Delta h} = \rho g =$

10 kPa m^{-1}$(increasinginthedownwardsdirection)$

14.6 A hole is punched in the side of a tank below the surface of the fluid in it. The fluid is coming out at a speed of 7 m s^{-1}.

(a) How far below the surface of the fluid was the hole punched?

(b) If the volume flow rate of the fluid coming out of the tank is 0.5 L min^{-1}, what is the radius of the hole?

Answer: (a) 2.45 m (b) 0.62 mm

14.7 A hydroelectric power plant draws water from a lake whose surface is 55 m above the turbines. It draws the water through a pipe with radius 1.2 m. ($P_{\text{atm}} = 100$ kPa, $\rho_{\text{water}} = 1000$ kg m^{-3})

(a) If, at point A, 55 m below the surface of the lake, the pipe is horizontal and the water is flowing through it at a rate of 9 m s^{-1}, at what pressure is the water? (Hint: compare this point to the surface of the lake using Bernoulli's Law.)

(b) In order for the turbines to work most efficiently the water should enter them at a speed of 20 m s^{-1}. In order to achieve this the pipe narrows to what radius just before it enters the turbines?

(c) What is the pressure just before the water enters the turbines?

(d) After passing through the turbines the water is now open to the atmosphere again. At what speed is it traveling (assuming that only a negligible fraction of the energy contained in the flow is removed by the turbines).

Answer: (a) $P_{\text{A}} = 560$ kPa (b) $R_{\text{B}} = 0.81$ m (c) $P_{\text{B}} = 450$ kPa (d) $v = 32$ m s^{-1}

15 FLUID DYNAMICS OF VISCOUS FLUIDS

15.1 Problems

15.1 Blood in large arteries and veins may be treated as a Newtonian fluid, which means we are able to ignore the effects of the cellular material in blood on blood flow. The viscosity of blood with a normal red blood cell count is 2.7×10^{-3} N s m^{-2}. Suppose we are considering blood flowing at a speed of 0.3 m s^{-1} in a large blood vessel with a radius of 2 mm.

(a) What is the pressure drop along a 2 cm length of this blood vessel?

(b) At approximately what velocity will the blood flow definitely be turbulent (take the density of blood to be 1050 kg m^{-3})? Answer: (a) $\Delta P = 32.4$ Pa (b) $v_{\text{turbulent}} = 1.9$ m s^{-1}

15.2 Saline solution is delivered into a patient's vein through a needle. The saline solution has a viscosity of 0.37×10^{-3} N s m^{-2}, a density of 1060 kg m^{-3}, and is delivered through a 7 cm long needle with an internal diameter of 0.24 mm directly into the patient's vein in which the blood pressure is 130 mmHg. The saline solution must be delivered at a flow rate of 0.04×10^{-6} m^{3}s^{-1}. How high must the saline solution be suspended in order to achieve this flow rate? (Ignore variations between systolic and diastolic blood pressure when doing this question. Also assume that the viscosity of the saline solution is low enough that it does not affect the flow of the solution through the IV tube, only the needle itself.) Answer: $h = 2.8$ m

15.3 The viscosity of cerebrospinal fluid is 0.8×10^{-3} N s m^{-2}. What pressure difference is required to produce a cerebrospinal fluid flow rate of 0.1 m s^{-1} in 1 cm long tubes of the following diameters: 1 mm, 5 mm, 1 cm? Answer: $\Delta P_{1\text{mm}} = 25.6$ Pa, $\Delta P_{5\text{mm}} = 1.02$ Pa, and $\Delta P_{1\text{cm}} = 0.256$ Pa

15.4 A small blood vessel near the skin surface has a radius of 10 μm, a length of 1 mm and the pressure drop along the blood vessel is 2.5 Pa (about 19 mmHg). The viscosity of blood is 2.7×10^{-3} N s m^{-2}.

(a) What is the volume flow rate of blood through this blood vessel? What is the velocity of blood flow?

(b) Vasodilation causes the radius of this blood vessel to increase to 12 μm, while leaving the pressure drop along the vessel unchanged. What is the volume flow rate through this blood vessel now? What is the velocity of blood flow? Answer: (a) $F = 3.64 \times 10^{-12}$ m^{3}s^{-1}, $v = 0.012$ m s^{-1} (b) $F = 7.55 \times 10^{-12}$ m^{3}s^{-1}, $v = 0.017$ m s^{-1}

15.5 Water flows via gravity from a high water tank to a point on the ground some distance away through a hose of diameter 1 cm and length 100 m. How high must the tank be for the flow rate to equal 1 L min^{-1}? ($\eta_{\text{water}} = 8.90 \times 10^{-4}$ Pa s, $\rho_{\text{water}} = 1000$ kg m^{-3}) Answer: 0.6 m

15.6 With what minimum speed would blood need to travel through a small blood vessel with a radius of 1 mm before the flow was turbulent? ($\eta_{\text{blood}} = 2.7 \times 10^{-3}$ Pa s, $\rho_{\text{blood}} = 1050$ kg m^{-3}) Answer: 10.3 m s^{-1}

15.7 If the blood vessel from Problem 15.6 had water flowing through it instead of blood, approximately what minimum speed would the flow become turbulent? Answer: 1.3 m s^{-1}

MOLECULAR TRANSPORT PHENOMENA

16

16.1 Problems

16.1 Problems

16.1 An experiment is performed to determine the diffusion constant of ants on a smooth tabletop. A handful of ants is placed in the centre of a large flat tabletop and a photograph of the tabletop is taken 1 minute after the ants are released. The number of ants within 5 cm concentric bands are counted and the numbers are recorded below. (Hint: use the inner radius of each zone to solve this problem and lookat the text box on page 146 entitled *'The rms subscript'*)

(a) What is the average displacement of the ants?

(b) What is the rms displacement of the ants?

(c) What is the diffusion constant for ants on a tabletop?

Number of Ants	*Inner radius of circular band* (in cm)
3	0
5	5
15	10
19	15
37	20
23	25
11	30
1	35
0	40
0	45
2	50

Answer: (a) 19 cm (b) 21 cm (c) 3.7×10^{-4} m^2 s^{-1}

16.2 The size of spherical aerobic bacteria is limited by the rate at which oxygen diffuses through water. A bacterium with a radius greater than about 10 μm is not able to obtain enough oxygen from the surrounding water to sustain itself. Given that the diffusion constant of oxygen in water is 8×10^{-10} m^2 s^{-1}, how long does it take oxygen in water to diffuse an rms distance of 10 μm? Answer: 6.35×10^{-2} s

16.3 The diffusion constant of ATP is 3×10^{-10} m^2 s^{-1}. How long would it take for ATP to diffuse across an average cell (about 20 μm across)? Answer: 0.67 s

16.4 A cylinder of water contains oxygen in solution. The cross-sectional area of the cylinder is 2 cm^2 and the length of the cylinder is 5 cm. At one end of the cylinder the concentration of oxygen is maintained at 0.2 mol m^{-3}, this concentration falls linearly to 0.05 mol m^{-3} at the other end of the cylinder. The diffusion constant of oxygen in water is 8×10^{-10} m^2 s^{-1}. How many moles of oxygen pass down this cylinder every second? What mass of oxygen passes down the cylinder each second? Answer: moles per second = 4.8×10^{-13} mol s^{-1}, mass of oxygen per second = 7.7×10^{-15} kg

III

Thermodynamics

17

17.1 Problems

Temperature and the Zeroth Law

17.1 Problems

17.1 The core temperature of the human body is about 37.0 °C and a temperature of 40.0 °C is regarded as a high fever. What are 37 °C and 40 °C in kelvin and degrees Fahrenheit? Answer: 310 K and 313 K, 98.6 °F and 104 °F

17.2 Which of the following statements are correct? (**Note**: more than one of the statements may be correct)

(a) The volume coefficient of thermal expansion of water is negative for temperatures in the range 0 to 4 °C.

(b) The triple point of water, 0.01 °C, and 611.73 Pa, is one of the primary fixed points on the Kelvin absolute temperature scale.

(c) The volume coefficient for the thermal expansion of a solid is twice the linear coefficient for thermal expansion.

(d) A temperature of 26.85 °C is the same as 300 K.

(e) The surface temperature of the sun (3142 °C) is the highest temperature found in nature.

Answer: (a), (b), (d)

17.3 The Rankine temperature scale has the same temperature unit intervals, °R, as the Fahrenheit scale, °F, but it is an absolute scale, so 0 °R is the same as 0 K. At what temperature does water boil on the Rankine scale (at standard atmospheric pressure)? Answer: 672 °R

17.4 A titanium metal rod has been inserted into the tibia of an injured soccer player. The rod, which runs the length of the tibia, is 0.55 m long and is normally at a constant temperature of 37 °C. Suppose that the soccer player develops a severe fever and his core temperature rises to 40 °C. The linear coefficient for thermal expansion of titanium is 8.6×10^{-6} K^{-1}. By how much will the length of the titanium rod increase? Answer: 14 μm

17.5 You are scheduled to implant a metal brace into a fractured femur to secure the broken ends. You are concerned that the temperature variations in the body will cause the length of the brace to change, creating a risk that the break will not heal satisfactorily. A 2 m length of the metal from which the brace is made is available. You heat this rod by 10 °C and find that the length of the rod has increased by 0.2 mm. How much (in μm) would the length of a 10 cm brace change if the body temperature of your patient were to increase by 3 °C? (during a high fever for example) Answer: 3.0 μm

17.6 The temperature coefficient of linear expansion of steel is 12×10^{-6} K^{-1}. When the temperature increases from 5 °C to 25 °C, what is the increase in the length of a straight 25 m length of unclamped railway track? Answer: 6.0 mm

17.7 At 20 °C a steel ring has an inside diameter that is 0.5 mm smaller than the diameter of a steel rod, which is 0.2 m. The ring is heated until it fits over the rod, which remains at 20 °C. The temperature coefficient of linear expansion of steel is 12×10^{-6} K^{-1}. What is the temperature of the ring when it is just large enough to fit over the rod? Answer: 230 °C

17.8 If you drink 0.5 L of extremely cold water at 4 °C, how much will its volume increase (in mL) once the water has reached your core body temperature of 37 °C in your stomach? (Assume the average value for the volume coefficient of expansion of water is 207×10^{-6} K^{-1}.) Answer: 3.4 mL

17.9 A cube of aluminium measures 1 m along each side. In the centre of the cube there is a small spherical cavity with a diameter of 1 cm. When the cube is heated from 10 °C to 30 °C, what is the percentage increase in the volume of the central cavity? (The linear thermal expansion coefficient of aluminium is 23×10^{-6} K^{-1}.) Answer: 0.14%

17.10 A long hollow cylinder of aluminium, closed at one end, is held vertically and contains some water. The cylinder has an internal diameter of 1.5 cm and the height of the water in the cylinder is initially 20 cm. The linear coefficient of expansion for aluminium is $\alpha_{\mathrm{Al}} = 23 \times 10^{-6}$ K^{-1} while the volume coefficient of expansion of water is $\beta_{\mathrm{water}} = 207 \times 10^{-6}$ K^{-1}.

(a) How much will the height of the water increase if the temperature of both the cylinder and the water increase by 10 °C?

(b) What result do you get for part (a) if you ignore the expansion of the cylinder?

Answer: (a) 0.32 mm (b) 0.41 mm

Ideal Gases

18

18.1 Problems

18.1 Problems

18.1 A sample of an unknown gas (gas A) has a volume of 3.2 m^3 at a temperature of 10 °C. Another sample of an unknown gas (gas B) has a volume of 4.5 m^3 at a temperature of 250 °C. Assuming that both gases obey Charles' law at all temperatures and are at the same pressure,

(a) Could gas A and gas B be samples of the same gas at different temperatures?

(b) What is the volume of gas A at the following temperatures: -50 °C, 0 °C, 50 °C, 100 °C ? Answer: (a) No. (b) at –50 ° $V = 2.5$ m^3,

at 0 ° $V = 3.1$ m^3, at 50 ° $V = 3.7$ m^3, and at 100 ° $V = 4.2$ m^3

18.2 A sample of a hypothetical ideal gas has a volume of 0.5 m^3 at a temperature of 5 °C and a pressure of 250 kPa.

(a) How many molecules of gas are there in this sample?

(b) How many moles of gas are there in this sample? Answer: (a) 3.3×10^{25}. (b) 54 mol

18.3 A novice pearl diver takes a deep breath before diving. She fills her lungs to their maximum capacity, which is 3 litres. Before diving the pressure in her lungs is 101.3 kPa and the temperature is 37 °C. She then dives to a depth of 20 m.

(a) How many moles of air has she inhaled?

(b) If the temperature in her lungs does not change, she does not exhale at all, and the pressure in her lungs is the same as the surrounding water pressure, what is her lung volume at the bottom of her dive (in litres)? Answer: (a) 0.12 mol. (b) 1 L

18.4 Two identical industrial gas cylinders (cylinder A and B) each have a volume of 2.25×10^{-2} m^3 and are maintained at 20 °C ($M_{O_2} = 32\,g\,mol^{-1}$, $M_{N_2} = 28\,g\,mol^{-1}$, and $M_{CO} = 28\,g\,mol^{-1}$).

Cylinder A contains: 1 kg of O_2, 1 kg of N_2, and 1 kg of CO.

Cylinder B contains: 2 kg of O_2, 0.5 kg of N_2, and 0.5 kg of CO.

(a) What is the partial pressure of O_2, N_2, and CO in each cylinder?

(b) What is the total pressure in each cylinder?

(c) What is the total thermal energy in each cylinder?

(d) What is the rms velocity of O_2, N_2, and CO in each cylinder? Answer: (a) Cylinder A: O_2, 3.5 MPa; N_2, 3.9 MPa; CO, 3.9 MPa.

Cylinder B: O_2, 6.8 MPa; N_2, 1.9 MPa; CO, 1.9 MPa. (b) Cylinder A, 11.2 MPa. Cylinder B, 10.6 MPa. (c) Cylinder A, 625 kJ, Cylinder B, 598 kJ. (d) v_{rms} of $O_2 = 478\,m\,s^{-1}$, v_{rms} of N_2 and CO = $511\,m\,s^{-1}$.

18.5 A sealed cylinder (with fixed volume) contains one mole of He gas and is slowly heated until the temperature of the gas has increased by 50 K. Which of the following statements are correct? (Note: more than one statement may be correct)

(a) Rotation of the molecules contributes to the thermal energy of the gas as it is heated.

(b) The average kinetic energy of the individual atoms of He increases by 4.2×10^{-19} J.

(c) The total thermal energy of the gas increases by 620 J (to 2 s.f.).

(d) The density of the gas decreases as the gas is heated.

(e) The pressure of the gas decreases as the gas is heated.

Answer: (c) only

18.6 Container A has 1.0 mole of O_2 gas and container B has 1.0 mole of He gas. The containers, which have different volumes, are brought into thermal contact and reach thermal equilibrium. Which of the following statements are correct?

(a) The total thermal energy of the two gases is the same.

(b) The average kinetic energy of the atoms of He is the same as that of the molecules of O2.

(c) The temperatures of the two gases need not be the same.

(d) The gases in the two containers have the same mass.

(e) The gases in the two containers have the same pressures.

Answer: (b) only

18.7 Suppose air at 20 °C contains 10 g of water vapour per cubic meter of air. Given the molar mass of water is $18\,g\,mol^{-1}$, what is the partial pressure of water vapour (to 3 s.f.)? Answer: 1350 Pa

18.8 Which of the following statements are correct? (Note: more than one statement may be correct)

(a) The average kinetic energy of an atom of an ideal gas approaches zero at a temperature of 0 °C.

(b) Two moles of helium (He) has the same total thermal energy as one mole of nitrogen (N_2) when they are both at 10 °C, because nitrogen is a diatomic gas.

(c) The average translational kinetic energy of the molecules of air is $\frac{3}{2}kT$, where k is the Boltzmann constant and T is the temperature in kelvin.

(d) The average thermal energy of the molecules of air is $\frac{5}{2}kT$, where k is the Boltzmann constant and T is the temperature in degrees kelvin.

(e) The ideal gas law can be written as $PV = NkT$, where N is the number of molecules of the gas in the volume V and k is the Boltzmann constant.

Answer: (a), (c), (d), (e)

18.9 A marine mammal holds its breath and dives 200 m below the sea surface, where the total pressure is 2.061 MPa. The volume of the mammal's lungs when fully inflated at sea surface (air pressure = 101.3 kPa) is 7 L. Assuming that the mammal's core temperature remains constant at 310 K, what will the volume of its lungs be when it reaches a depth of 200 m? Answer: 0.344 litre

18.10 A gas mixture is contained in a sealed flask at atmospheric pressure, 101.33 kPa. When all the carbon dioxide is chemically removed from the sample, keeping the same temperature, the final pressure is 67.89 kPa. What percentage of the molecules of the original sample was carbon dioxide? Answer: 33%

18.11 In the "death-zone" on Mt Everest the atmospheric pressure is typically 34 kPa. The air at this elevation contains a negligible quantity of water vapour. On a molar basis the composition of the air is 20% O_2, 79% N_2 and 1% Ar. Determine the partial pressure of O_2 in the death-zone. Answer: 6.8 kPa

19 Phase and Temperature Change

19.1 Problems

19.1 Problems

19.1 Which of the following statements are correct? (Note: more than one statement may be correct)

(a) When the liquid and vapour phases of a substance co-exist the pressure depends only on the volume, not the temperature.

(b) When the temperature of a substance is less than the critical temperature there is a distinct phase change between the liquid and solid phases involving latent heat.

(c) The latent heat for the vaporisation of water is greater at 0 °C than at 100 °C.

(d) When two objects (1 and 2) exchange heat, the corresponding heat quantities, Q_1 and Q_2, satisfy the equation $Q_1 = Q_2$.

(e) The specific heat capacity of a substance is numerically the same as the amount of thermal energy required to increase the temperature of 2 kg of the substance by 0.5 °C.

Answer: (c), (e)

19.2 A 1 kg block of copper and a 2 kg block of wood each absorb the same amount of heat, and the temperature of the wood increases by 2 °C. The specific heat of wood is 1700 J kg^{-1} K^{-1}. The specific heat of copper is 387 J kg^{-1} K^{-1}. How much does the temperature of the block of copper increase? Answer: 17.6 K

19.3 To cool a hot bath containing 50 litres of water at 50 °C down to 20 °C,

(a) what volume of cold water at 5 °C would be needed (in litres)?

(b) what mass of ice at 0 °C would be needed?

Answer: (a) 100 litres (b) 15 kg

19.4 How much heat must be transferred into a 50.0 kg block of ice at -10 °C to raise its temperature to 0 °C, melt it into liquid water, heat it to 100 °C and then evaporate all of the water? Answer: 152 MJ

19.5 By mistake I run a bath using 60 kg of water from just the hot tap and the water temperature is 65 °C. I decide to cool the water to 40 °C with snow, which is at 0 °C. The specific heat capacity of the water is 4.19 kJ kg^{-1} K^{-1} and the latent heat of fusion of ice is 333 kJ kg^{-1}. What weight of snow should I use? Answer: 12.6 kg

19.6 A 65 kg patient suffers from hyperthermia, having a mean body temperature of 41 °C. The patient is placed in a bath containing 50 kg of water. The specific heat capacity of the water is 4.19 kJ kg^{-1} K^{-1} and the average specific heat capacity of the patient is 3.49 kJ kg^{-1} K^{-1}. In order for the final temperature of the water and patient to be 37 °C, what should the initial temperature of the water be? Answer: 33 °C

19.7 A hyperthermic male, weighing 104 kg, has a mean body temperature of 42 °C. He is to be cooled to 37 °C by placing him in a water bath, which is initially at 25 °C. What is the minimum amount of bath water required to achieve this result? The specific heat of the body is 3.5 kJ kg^{-1} k^{-1}. The specific heat of water is 4.19 kJ kg^{-1} k^{-1}. Answer: 36.2 litres

19.8 A beaker of water and a beaker of an unknown liquid are weighed and their temperatures measured. The unknown fluid has a mass of 1.2 kg and the water has a mass of 0.8 kg. They are both at an initial temperature of 20 °C. The beakers are then simultaneously heated on the same heating element for the same length of time and then they are weighed and their temperatures measured again. Their weights did not change when they were heated in this way, but the water now has a temperature of 28 °C whereas the unknown fluid has a temperature of 34 °C. What is the specific heat of the unknown fluid? Answer: 1.60 kJ kg^{-1} K^{-1}

19.9 Steam condenses on a 5 kg iron plate. The plate was initially at 15 °C and it is found that 10 g of steam has condensed onto the plate. What is the temperature of the plate after the steam has condensed? (The specific heat capacity of iron is 449.4 J kg^{-1} K^{-1}) Answer: 26.4 °C

19.10 A 70 kg runner loses 0.5 kg of water each hour through evaporation of perspiration in order to maintain a stable temperature. The latent heat of water at his skin temperature is 2440 kJ kg^{-1} and the average specific heat capacity of his body is 3.5 kJ kg^{-1} K^{-1}. If he stopped perspiring, how much would his temperature rise in the following 30 minutes? Answer: 2.5 °C

20 Water Vapour and the Atmosphere

20.1 Problems

20.1 You wish to measure the humidity in your flat. To do this you use two thermometers; one of which you wrap in a wet cloth and blow air over using a fan. The thermometer that is not wrapped in a wet cloth reads 18 °C, the one wrapped in a wet cloth reads 16 °C.

(a) What is the relative humidity in this room?

(b) You measure the temperature of the inner surface of one of the room's windows. At what window temperature would you expect water to condense on this window? Answer: (a) 80% (b) dew point temperature = 15 °C

20.2 At a dry bulb temperature of 14 °C and at an atmospheric pressure of 101.3 kPa the moisture content of the air is 4 g (kg-dry air)$^{-1}$ ($\rho_{air} = 1.2\,\mathrm{kg\,m^{-3}}$ and $M_{H_2O} = 18\,\mathrm{g\,mol^{-1}}$).

(a) What is the relative humidity?

(b) What is the partial pressure of water vapour? Answer: (a) 40% (b) 0.64 kPa

20.3 An infection control room in a paediatric ward has dimensions 3 × 3 × 4 m. The dry bulb temperature is 20 ° and the wet bulb temperature is 18 °. The temperature of the window surfaces drops to 15 °C overnight (the windows are not double glazed). What is the minimum volume of water that a dehumidifier must remove from this room such that condensation will not occur on the windows overnight? Answer: 43 ml

20.4 At a dry bulb temperature of 30 °C, an atmospheric pressure of 101.3 kPa, and a humidity of 90% what is the partial pressure of water vapour in the atmosphere ($M_{H_2O} = 18\,\mathrm{g\,mol^{-1}}$)? Answer: 4.2 kPa

20.5 The air dry-bulb temperature in a room is 22 °C and the relative humidity is 60%. What is the minimum temperature of the inside surface of the glass in order to avoid the windows of the room getting fogged-up? Answer: 14 °C

20.6 The bathroom mirror is all steamed up and the dry-bulb temperature is 28 °C. The mirror is slowly warmed from an initial cold temperature and it starts to clear when its temperature reaches 22 °C. What is the relative humidity in the bathroom? Answer: 70%

20.7 The wet-bulb temperature in the room is initially 20 °C and the dry-bulb temperature is 24 °C. The surface temperature of the windows falls to 12 °C, while the room temperature remains constant. When moisture finally stops collecting on the windows, what is the wet-bulb temperature? (Assume that the dry-bulb temperature remains constant) Answer: 16.5 °C

20.8 After taking a deep breath I exhale approximately 2.5 g of air. The intake air is at 20 °C at 40% relative humidity, and the air exhaled is at 100% relative humidity at a temperature of 34 °C. What is the net mass of water expelled during one breathing cycle? Answer: 70 mg

20.9 The temperature of the air in a room is 13 °C and the partial pressure of water vapour is 1.10 kPa. The volume of the room is 60 m^3 and the molar mass of water is 18 g mol^{-1}. What is the mass of water vapour in the room? Answer: 0.5 kg

20.10 The temperature of the air in a room is 14 °C and the relative humidity is 70%. The volume of the room is 60 m^3, the molar mass of water is 18 g mol^{-1}, and the density of air is 1.2 kg m^{-3}. What is the partial pressure of water vapour in the room? Answer: 1.11 kPa

20.11 On a cold evening, when the temperature of a particular room is 20 °C, condensation starts to form on the windows when the window surface temperature falls to 14 °C. Determine the relative humidity in the room. Answer: 70%

20.12 The dry-bulb temperature of a sample of air is 30 °C and the relative humidity is 40%. Which of the following statements are correct? (Note: more than one statement may be correct.)

(a) The moisture content of the air is 27 g kg^{-1}.

(b) The dew-point temperature is 15 °C.

(c) If the moisture content is increased while keeping the wet-bulb temperature fixed, the relative humidity will reach 100% when the moisture content is 22 g kg^{-1}.

(d) The wet-bulb temperature is 20 °C.

(e) If the moisture content is increased keeping the dry-bulb temperature fixed at 30 °C, the maximum moisture content will be 27 g kg^{-1}.

Answer: (b), (d), (e)

20.13 Which of the following statements are correct? (Note: more than one statement may be correct.)

(a) An air condition with high relative humidity normally makes people feel uncomfortable because they are unable to lose heat easily through evaporation of perspiration.

(b) Except at 100% relative humidity, the dew-point temperature for air is greater than the wet-bulb temperature.

(c) When air flows over a wet surface it can cool the surface right down to the dew-point temperature, but no further.

(d) When the dew-point temperature is defined the moisture content of the air is also specified.

(e) When the wet-bulb temperature is defined the moisture content of the air is also specified.

Answer: (a), (d)

20.14 On a certain day the dry-bulb temperature is 24 °C and the wet-bulb temperature is 20 °C. Which of the following statements is correct? (Note: more than one statement may be correct.)

(a) The dew-point temperature is 12 °C.

(b) The moisture content of the air is 6.5 g kg^{-1}.

(c) If the air is cooled to 6 °C at 100% relative humidity it will lose 9 g of moisture per kg of dry air.

(d) The relative humidity is 60%.

(e) At the same dry-bulb temperature the minimum wet-bulb temperature is approximately 8 °C.

Answer: (e)

HEAT TRANSFER 21

21.1 Problems

21.1 A penguin generates thermal energy through metabolic processes at a rate of 50 W, but loses 8 W due to respiration and moisture loss. Its surface area is 0.63 m^2 and the average thermal conductivity of its feather layer is 0.031 $W\,m^{-1}\,K^{-1}$. The penguin maintains a skin temperature of 35 °C when the outer surface of its feather layer is −10 °C. What is the thickness of the penguin's layer of feathers? Answer: 21 mm

21.2 The core temperature of an athlete is 37 °C and his surface area is 1.8 m^2. The thermal conductivity of his surface tissue, which is 6 mm thick on average, is 0.18 $W\,m^{-1}\,K^{-1}$. If heat is transferred from his core to his skin at a rate of 270 W, what is the average temperature of his skin surface? Answer: 32 °C

21.3 You have choice between wearing a dark woollen jersey ($\epsilon_{\text{jersey}} = 0.9$, $k_{\text{wool}} = 0.04\,W\,m^{-1}\,K^{-1}$, and $d_{\text{jersey}} = 5$ mm) and a top made with a new high-tech material ($\epsilon_{\text{htm}} = 0.1$, $k_{\text{htm}} = 0.01\,W\,m^{-1}\,K^{-1}$, and $d_{\text{htm}} = 1$ mm).

(a) Ignoring heat transfer due to radiative and convective processes, which top would you predict will keep you the warmest on a cold day?

(b) Which top would you predict will keep you warmest on a cold day if you take into account radiative and convective heat transfer (use $h_{\text{convection}} = 3.1\,W\,m^{-2}\,K^{-1}$ and $T_{\text{air}} = 2$ °C)? Answer: (a) $h_{\text{jersey}} = 8.0\,W\,m^{-2}\,K^{-1}$ and $h_{\text{htm}} = 10\,W\,m^{-2}\,K^{-1}$ so the jersey should keep you warmer. (b) $h_{\text{jersey}} = 3.83\,W\,m^{-2}\,K^{-1}$ and $h_{\text{htm}} = 3.33\,W\,m^{-2}\,K^{-1}$ so the high tech top should keep you warmer.

21.4 An Olympic swimmer is waiting to compete. Her average skin temperature is 33 °C and the average temperature of the pool hall is 23 °C. Her surface area is 1.6 m^2, her surface emissivity is 0.9 and where she stands the mean air speed is 1.5 $m\,s^{-1}$. Determine separately the rates at which she loses heat by radiative heat transfer and by convection. Answer: $\left(\frac{Q}{t}\right)_{rad} = 89$ W (calculated using the difference between emitted and absorbed radiation), $\left(\frac{Q}{t}\right)_{conv} = 169$ W

21.5 A penguin is standing in a sheltered spot somewhere near the coast of Antarctica. The penguin is losing heat from it's core, which is at 39 °C, through a layer of fatty tissue and also through a layer of insulating feathers. The layer of fatty tissue is 2 cm thick, the layer of feathers is also 2 cm thick, the total rate at which the penguin is loosing heat is 70 W, and the total surface area of the penguin is 0.7 m^2 ($k_{\text{tissue}} = 0.2\,W\,m^{-1}\,K^{-1}$, $k_{\text{feathers}} = 0.035\,W\,m^{-1}\,K^{-1}$).

(a) What is the rate of heat transfer across the layer of fatty tissue?

(b) What is the rate of heat transfer across the layer of feathers?

(c) What is the skin temperature of the penguin (in °C)?

(d) What is the temperature of the outside surface of the penguin (in °C)?

Answer: (a) $(\frac{Q}{t})_{\text{tissue}} = 70$ W (b) $(\frac{Q}{t})_{\text{feathers}} = 70$ W (c) $T_{\text{skin}} = 29\,°C$ (d) $T_{\text{surface}} = -28\,°C$

21.6 A person has a core body temperature of 37 °C, a tissue layer that is 5 mm thick with a thermal conductivity of 0.2 $W\,m^{-1}\,K^{-1}$ and a surface area of 1.4 m^2. The average thickness of her clothes is 9 mm, with a thermal conductivity of 0.04 $W\,m^{-1}\,K^{-1}$. The surface heat transfer coefficient of her clothes is 22 $W\,m^{-2}\,K^{-1}$. Determine her rate of heat loss when the external temperature is 12 °C. What will be her average skin temperature under this condition? Answer: $\left(\frac{Q}{t}\right)_{\text{net}} = 120$ W, $T_{\text{skin}} = 34.9$ °C

21.7 The core temperature of a naked male, standing in air at 8 °C, is 37 °C. His surface area is 1.6 m^2. The thermal conductivity of his surface tissue, which is 1.2 cm thick on average, is 0.20 $W\,m^{-1}\,K^{-1}$. If heat is transferred from his core to his skin at a rate of 195 W, what is his average skin temperature ? Answer: $T_{\text{skin}} = 29.7$ °C

21.8 You are working outside on a cold day ($T_{\text{air}} = 2$ °C). Your core body loses heat through a 2 cm thick layer of fatty tissue and a 1 cm thick layer of clothes. Your total surface area is 2 m^2 and the thermal conductivities of fatty tissue and your clothes are $k_{\text{tissue}} = 0.2\,W\,m^{-1}\,K^{-1}$ and $k_{\text{clothes}} = 0.01\,W\,m^{-1}\,K^{-1}$. Your clothes are a light color ($\epsilon = 0.4$) and it is a windy day ($v_{\text{air}} = 3.6\,m\,s^{-2}$).

(a) What is the total *surface* heat transfer coefficient?

(b) What is the combined heat transfer coefficient for heat *conduction* through your fatty tissue and clothes?

(c) What is the total heat transfer coefficient taking into account all methods of heat transfer?

(d) If the rate at which your core body is losing heat is 56.6 W, what is your core body temperature? Should you stay outside? Answer: (a) $h_{\text{surface}} = 20.2\,W\,m^{-2}\,K^{-1}$ (b) $h_{\text{conduction}} = 0.909\,W\,m^{-2}\,K^{-1}$ (c) $h_{\text{total}} = 0.870\,W\,m^{-2}\,K^{-1}$ (d) $T_{\text{core}} = 34.5$ °C. This is very very low, you should go inside straight away!

21.9 A 110 kg fisherman with a surface area of 2.2 m^2 falls from his boat into the cold southern ocean (T_{ocean} = 2 °C). The fisherman has a 4 cm thick layer of fatty tissue (k_{tissue} = 0.2 W m^{-1} K^{-1}) and is wearing cold weather survival gear which is 1.5 cm thick and has a thermal conductivity of k_{sg} = 0.03 W m^{-1} K^{-1}.

(a) If the fisherman's core body temperature is initially a healthy 37 °C estimate how long will it be before his body temperature drops to 34 °C (specific heat of the human body - c_{body} = 3.5 kJ kg^{-1} K^{-1})?

(b) Suppose the fisherman was wearing ordinary clothes ($k_{clothes}$ = 0.1 W m^{-1} K^{-1} and $d_{clothes}$ = 5 mm) instead of the survival gear. Estimate how long would it take for their temperature to drop to 34 °C in this case. Answer: (a) 3 hours (b) 1 hour

21.10 For each square metre of the sun's visible surface the rate of energy transferred to outer space is 63 MW. The emissivity of the sun's surface is approximately 1.0. Which of the following statements is correct?

(a) The temperature of the sun's surface is approximately 5766 K.

(b) The solar energy that reaches the earth from the sun involves radiative heat transfer primarily.

(c) Because the sun's surface temperature is relatively stable it must be receiving energy from space at a rate of 63 MW m^{-2}.

(d) The energy transferred from the sun is carried mainly by the solar wind, so the process is primarily convective.

(e) The rate at which energy leaves the surface if the sun cannot be calculated using the Stefan–Boltzmann law, because the sun's surface is white, not black.

Answer: (a), (b)

22 THERMODYNAMICS AND THE BODY

22.1 Problems

22.1 A cyclist does 2 MJ of mechanical work over the course of a day. His work efficiency is 20% and the metabolic energy of the food consumed during the day is 8 MJ.

(a) What is the net metabolic energy used during the day?

(b) How much heat does the cyclist transfer to his surroundings during the day?

(c) How much extra food will the cyclist need to consume at the end of the day in order not to loose weight?

Answer: (a) 10 MJ (b) 8 MJ (c) 2 MJ

22.2 A rugby player uses metabolic energy at a rate of 430 W while playing. At the same time he loses energy at an average rate of 210 W, due to work being done, convection, radiation and respiration. Assume that his core temperature does not change during an 80 minute game, and that the latent heat of vaporisation of perspiration is 2440 kJ kg^{-1}. How much moisture will he lose as a result of the evaporation of perspiration during the game? Answer: 0.43 kg

22.3 During the course of a day a climber does 3.0 MJ of work with a mean work efficiency of 20%. How much heat must he lose during the day in order to avoid getting too hot? Answer: 12 MJ

22.4 On a long-distance polar trek an explorer has a nutritional intake of 15.6 MJ per day. She does mechanical work at an average rate of 100 W for 10 hours per day. Assuming she does not lose or gain weight, what is the average rate at which she loses heat to the environment (in W)? Answer: 139 W

22.5 During a tennis match the metabolic heat generation rate of a player is 450 W. She loses heat at a rate of 170 W by convection, radiation and respiration, how much moisture will she lose through evaporation of perspiration during a match lasting 3 hours? Assume that the core temperature of the tennis player does not change, and that the latent heat of vaporisation of water is 2440 kJ kg^{-1}. Answer: 1.24 kg

22.6 After eating a 150 g pottle of yoghurt, Bob decides to go for a run to burn off the extra energy. The yoghurt provides 4.4 MJ kg^{-1}, and Bob is able to raise his metabolic rate to 220 W while running. How long does he have to run? Answer: 50 minutes

22.7 An athlete runs 5000 m in 30 mins, with a mechanical power output of 130 W. If she runs with a work efficiency of 15%, what is the net metabolic energy used? Answer: 1.56 MJ

22.8 The following statements are about how humans can transfer thermal energy to their surroundings when the air temperature is greater than the core body temperature, 37 °C. Which statements are correct? (Note: more than one statement may be correct)

(a) The body temperature is simply allowed to go up by a few degrees, which is OK for a few hours.

(b) Sweat glands release liquid that is hotter than the air onto the surface of the skin, so that the skin can then cool by convective transfer of heat to the cooler surrounding air.

(c) Water-like body perspiration evaporates into the surrounding air, provided it is unsaturated, producing a cooling effect at the skin surface

(d) It is impossible for the body to transfer thermal energy to the surroundings under these conditions if the relative humidity is 100%

(e) They cannot reject heat under these conditions, whatever the relative humidity, so they should just drink cold liquids to get cooler.

Answer: (c), (d)

Activity	Metabolic Rate (W)
Standing relaxed	105 - 125
Walking 3 km h^{-1}	170 - 210
Walking 6 km h^{-1}	330 - 450
Running	500 - 800
Strenuous exercise	Exceeds 1000

Table 22.1 Metabolic rates associated with various levels of activity.

22.9 An endurance athletic event is conducted when the conditions are extremely stressful, the dry-bulb temperature being 45 °C. One of the participants in the event, a male, is able to absorb 1.1 litre of water per hour through the gut, although he can lose water through perspiration faster than this if necessary. The latent heat of vaporisation of water at the participant's skin temperature is 2440 kJ kg^{-1}.

(a) Determine the maximum rate at which he can reject heat by perspiration without becoming dehydrated.

(b) The athlete has a surface area of 1.8 m^2 and his surface heat transfer coefficient, including convection and radiation, is 22 W m^2 K^{-1}. During the endurance event his skin temperature is 37°C. Determine the rate of heat gain (or loss) due to convection and radiation alone, not including the effect of perspiration.

(c) The table above shows the metabolic rate of the athlete doing different activities. Assume this is the same as the rate of production of thermal energy in his body. Determine the maximum sustainable metabolic rate for the athlete during the endurance event without any temperature rise. What is the most vigorous activity that could be sustained under these conditions?

(d) Suppose the event is rescheduled for a cooler day. Again the athlete has sufficient water to perspire and evaporate 1.1 kg per hour and on this occasion he intends to run with a metabolic rate of 800 W. What is the maximum dry-bulb temperature at which he could do this sustainably, assuming his skin temperature is 37 °C?

Answer: (a) 746 W, (b) 317 W gained, (c) 429 W walking at 6 km h^{-1}, (d) 35.6 °C

IV

Electricity and DC Circuits

STATIC ELECTRICITY

23

23.1 Problems

23.1 An uncharged metal sphere, A, is on an insulated base. A second sphere, B, of the same shape, size and material carrying a charge $+Q$ is brought *into contact* with sphere A.

(a) Describe what happens to charges on spheres A and B as they are brought into contact.

(b) If we now remove sphere B and place it far away, what is the charge on sphere A?

(c) How is this charge (if any) distributed?

Answer: (a) Electrons will move from sphere A to sphere B so that the excess charge $+Q$ is spread evenly over both spheres. (b) $+\frac{1}{2}Q$ (c) Evenly over the surface of sphere A

23.2 An uncharged metal sphere, A, is on an insulated base. A second sphere, B, of the same shape, size and material carrying a charge $+Q$ is brought *close to, but not touching*, sphere A.

(a) Describe what happens to the charges on spheres A and B as they are brought close together *but not touching*.

(b) If we now remove sphere B, what is the charge on sphere A?

(c) How is this charge (if any) distributed?

Answer: (a) Sphere A becomes polarized with the side nearer sphere B becoming negatively charged and the far side becoming positively charged. (b) $Q = 0$ C (c) no charge to distribute

23.3 A positively charged metal sphere, sphere A, is held close to *but not touching* an identical uncharged sphere, sphere B. Sphere A is now removed. After sphere A has been removed Sphere B is touched to an initially uncharged sphere, sphere C. What is the sign of the charge (if any) on sphere C after it has been touched to sphere B? Answer: There is no charge on sphere C (nor is there a charge on sphere B).

23.4 A physicist traps an ionized atom in a magnetic trap. She performs an experiment and finds that the atom has a charge of $+3.2 \times 10^{-19}$ C. If the atom has 12 protons and 12 neutrons, now many electrons must it have at the time it was trapped by the physicist? Answer: $N_{e^-} = 10$

23.5 A small sheet of aluminium foil measuring 2×2 cm is charged by rubbing it on some plastic material. The charge on the small sheet of aluminium foil is then measured and found to be $+Q$. An *uncharged* sheet of gold foil measuring 4×4 cm is brought close to, but not touching, the sheet of aluminium.

(a) What is the total charge on the sheet of aluminium?

(b) What is the total charge on the sheet of gold?

The gold foil and aluminium foil are now allowed to touch before being separated again.

(c) What is the total charge on the sheet of aluminium now?

(d) What is the total charge on the sheet of gold now?

(Note: both aluminium and gold are good conductors) Answer: (a) $+Q$ (b) 0 C (c) $+\frac{1}{5}Q$ (d) $+\frac{4}{5}Q$

23.6 The following pairs of materials were rubbed together and the sign and approximate magnitude of the charge on each material noted. Use this information to rank these materials from *least electronegative* (most likely to lose electrons) to most electronegative. In other words construct a small triboelectric series.

Paper and synthetic rubber: paper, small +ve charge; rubber, small -ve charge.

Paper and polypropylene material: paper, medium +ve charge; polypropylene, medium -ve charge.

Rabbit fur and synthetic rubber: fur, medium +ve charge; rubber, medium -ve charge.

Rabbit fur and polypropylene material: fur, large +ve charge; polypropylene, large -ve charge. Answer: fur < paper < synthetic rubber < polypropylene material

23.7 To answer these questions use the small triboelectric series constructed in Problem 23.6.

(a) What would the sign of the charge on synthetic rubber be if it were rubbed against polypropylene material?

(b) What would the sign of the charge on paper be if it were rubbed against rabbit fur? Answer: (a) +ve (b) -ve

Electric Force and Electric Field

24

24.1 Problems

24.1 A $+3 \times 10^{-6}$ coulomb charge is placed 5 centimetres due west from a $+2 \times 10^{-6}$ coulomb charge.

(a) What is the force the $+2\,\mu C$ charge exerts on the $+3\,\mu C$ charge?

(b) What is the force the $+3\,\mu C$ charge exerts on the $+2\,\mu C$ charge?

Answer: (a) 21.6 N West (b) 21.6 N East

24.2 Two charges, one +4 C and the other +2 C, are separated by some distance R_0. If we increase the distance between the charges by a factor of 5, what happens to the magnitude of the force on the +2 C charge? Answer: $F = \frac{1}{25} F_0$

24.3 Two charges, Q_A and Q_B, are separated by a distance x. If we double the distance between the charges and triple the magnitude of charge A, what happens to the magnitude of the force that charge A exerts on charge B? What happens to the magnitude of the force that charge B exerts on charge A? Answer: $F = \frac{3}{4} F_0$ for both charges

24.4 Three charges, Q_A, Q_B, and Q_C are positioned as shown in Figure 24.1

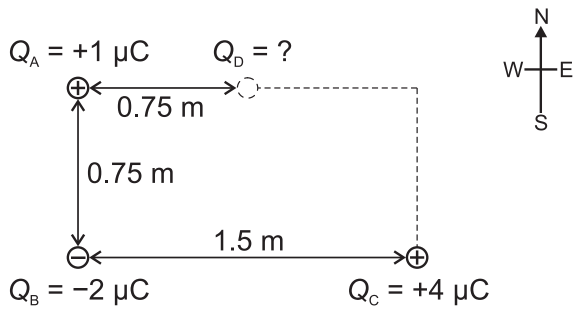

Figure 24.1 Three charges are arranged in 2 dimensions. What fourth charge will ensure that the net electrostatic force on charge B is zero?

(a) Determine the net force acting on charge Q_B due to the other two charges.

(b) A fourth charge, Q_D, is placed at position D shown in Figure 24.1 such that the net electrostatic force on charge Q_B is zero. What is the sign and magnitude of Q_D?

Answer: (a) $F_{\text{net, B}} = 0.045$ N in the North-East direction (b) $Q_D = -2.83\,\mu C$

24.5 A mad scientist invents a device that is able to teleport every electron in their body to the center of the Earth (6.37×10^6 m below). We can make the assumption that the human body is mostly water with a bit of carbon and as such there will be about 3.2×10^{26} electrons per kilogram of body. If the mad scientist, who weighs 65 kg is so unwise as to actually use this device on himself what will the magnitude of the attraction between his body (stripped of all electrons) and all the electrons newly deposited at the center of the Earth? (How does this compare with the gravitational attraction between the mad scientist and the Earth?) Answer: $F = 2.5 \times 10^{15}$ N

(This is some $3.8 \times 10^{12} \times$ larger than the gravitational force between the mad scientist and the Earth!)

24.6 A small charged particle of mass 9×10^{-6} kg and charge of magnitude -3×10^{-6} C is placed in a chamber in which there is a uniform electric field. If the charge accelerates due north at a rate of $250\ \mathrm{m\,s^{-2}}$ what is the magnitude and direction of the electric field inside the chamber? (ignore gravitational forces) Answer: $E = 750\ \mathrm{N\,C^{-1}}$ due south

24.7 Answer the following:

(a) What is the magnitude and direction of the electric field 10 m away from a +0.1 mC charge?

(b) What is the magnitude and direction of the electrostatic force on a +1.5 mC charge placed at this point (10 m from the charge in (a))?

(c) What is the magnitude and direction of the electrostatic force on a −3.5 mC charge placed at this point (10 m from the charge in (a))?

Answer: (a) $E = 9000\ \mathrm{N\,C^{-1}}$ pointing away from the charge (b) $F = 13.5$ N away from the original charge (c) $F = 31.5$ N towards the original charge

24.8 What is the magnitude and direction of the electric field at each of the three points, A, B, and C shown in Figure 24.2?

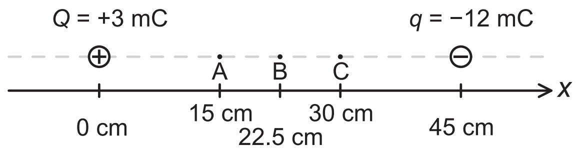

Figure 24.2 Two charges of opposite signs are placed 45 cm apart.

Answer: $E_A = +2.4 \times 10^9$ N C^{-1}, $E_B = +2.7 \times 10^9$ N C^{-1}, $E_C = +5.1 \times 10^9$ N C^{-1}

24.9 What is the magnitude of each of the charges q and Q in Figure 24.3?

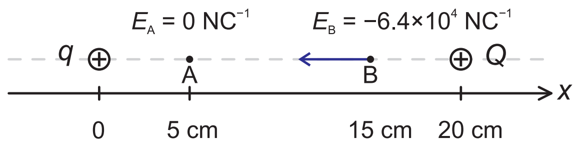

Figure 24.3 Two charges are placed 20 cm apart. The magnitude of the electric field at point A is zero, while the electric field at point B is non-zero.

Answer: $q = 2 \times 10^{-9}$ C, $q = 18 \times 10^{-9}$ C

24.10 An electric dipole shown in Figure 24.4.

(a) Calculate the electric field strength at points A, B, C, and D.

(b) What electrostatic force will a +0.2 μC charge experience when placed at each of the four points in part (a)?

(c) What electrostatic force will a −0.1 μC charge experience when placed at each of the four points in part (a)?

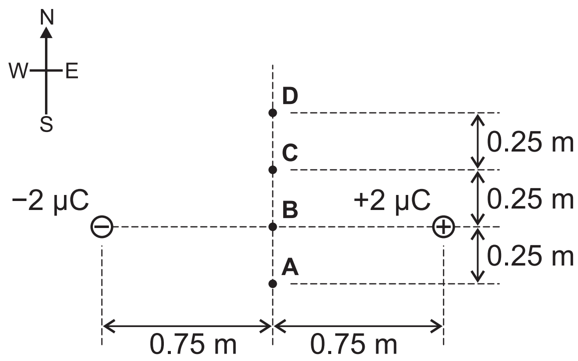

Figure 24.4 Find the electric force on charges placed at the points A to D.

Answer: (a) $E_A = 54.6 \times 10^3$ N C^{-1} due West, $E_B = 64.0 \times 10^3$ N C^{-1} due West, $E_C = 54.6 \times 10^3$ N C^{-1} due West, $E_D = 36.9 \times 10^3$ N C^{-1} due West (b) $F_A = 10.9 \times 10^{-3}$ N due West, $F_B = 12.8 \times 10^{-3}$ N due West, $F_C = 10.9 \times 10^{-3}$ N due West, $F_D = 7.38 \times 10^{-3}$ N due West (c) $F_A = 5.46 \times 10^{-3}$ N due East, $F_B = 6.40 \times 10^{-3}$ N due East, $F_C = 5.46 \times 10^{-3}$ N due East, $F_D = 3.69 \times 10^{-3}$ N due East

ELECTRICAL POTENTIAL AND ENERGY

25

25.1 Problems

25.1 Problems

25.1 A 20000 N C^{-1} uniform electric field does +5000 J of work on a +0.20 C charged object.

(a) Did the charged object move in the direction of the electric field or against it?

(b) How far did the object move?

(c) What was the change in electrical potential through which the object moved?

(d) If the object was initially at at point with an electrical potential of −2000 V, what was the electrical potential its end point?

Answer: (a) with the field (b) 1.25 m (c) −25000 V (d) −27000 V

25.2 A proton is moved at a constant velocity from a position at which the electrical potential is 100 V to one at which the electrical potential is −50 V.

(a) How much work was done on the proton by the electric field?

(b) How much work was done on the proton by the external force?

Answer: (a) $W_{\text{elec}} = +2.4 \times 10^{-17}$ J (b) $W_{\text{ext}} = -2.4 \times 10^{-17}$ J

25.3 In a region of space there is a uniform 6000 N C^{-1} electric field like that shown in Figure 25.1.

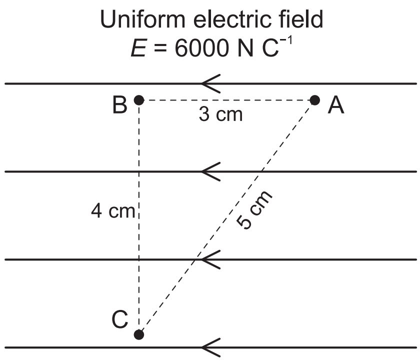

Figure 25.1 Three points in a region of uniform electric field.

(a) What is the potential difference between points A and B? Which point is at the lower electrical potential?

(b) What is the potential difference between points A and C? Which point is at the lower electrical potential?

(c) What is the potential difference between points B and C? Which point is at the lower electrical potential?

Answer: (a) ΔV_{AB} = 180 V, B is at the lower electrical potential (b) ΔV_{AC} = 180 V, C is at the lower electrical potential (c) ΔV_{BC} = 0 V, both points are at the same electrical potential

25.4 A charge of +0.1 μC is placed at point A in Figure 25.1.

(a) How much work is done on the charge by the electric field when moving the charge from point A to point B?

(b) How much work is done on the charge by the electric field when moving the charge from point B to point A?

(c) How much work is done on the charge by the electric field when moving the charge from point B to point C?

(d) How much work is done on the charge by the electric field when moving the charge from point C to point B?

Answer: (a) $+18 \times 10^{-6}$ J (b) -18×10^{-6} J (c) 0 J (d) 0 J

25.5 What is the potential difference between points A and D in Figure 24.4? Answer: ΔV = 0 V. A, B, C, and D all lie on an equipotential line.

25.6 Use Figure 25.2 to answer the following questions.

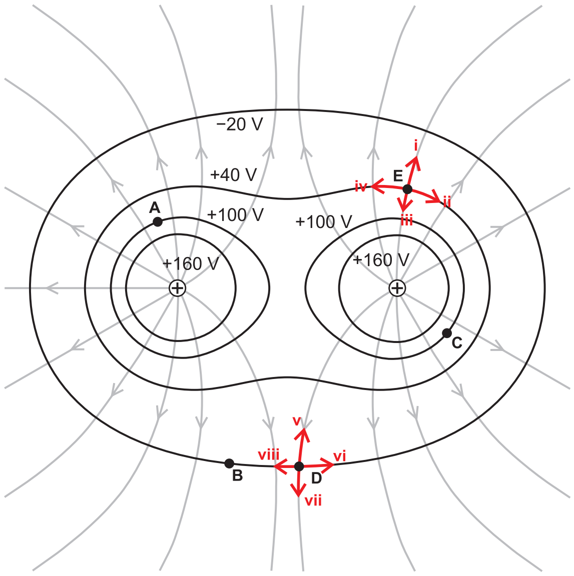

Figure 25.2 The electric field and some equipotentials in the region around two positive charges.

(a) What is the potential difference between points A and B?

(b) How much work does the electric field do on a −0.5 C charge that is moved from A to C?

(c) If a −5 C charge is released from point D which path would it take?

(d) If a +6 C charge is released from point E which path would it take?

Answer: (a) $\Delta V_{AB} = 120$ V (B is at the lower potential) (b) $W_{\text{e-field}} = 0$ J (c) v (d) i

25.7 A charge $Q_0 = -0.5\,\mu$C is placed in a region of space far from any other charges and is fixed so that it cannot move. Some equipotential lines around this charge are shown in Figure 25.3. A small object with a mass of $m_{\text{obj}} = 5$ mg and a charge $Q_{\text{obj}} = -0.8$ nC is placed at point A ($V_A = -800$ V) and released. The repulsive force between the two charges causes Q_{obj} to accelerate along the path shown towards point B. What is the velocity of the object when it reaches point B ($V_B = +100$ V)?

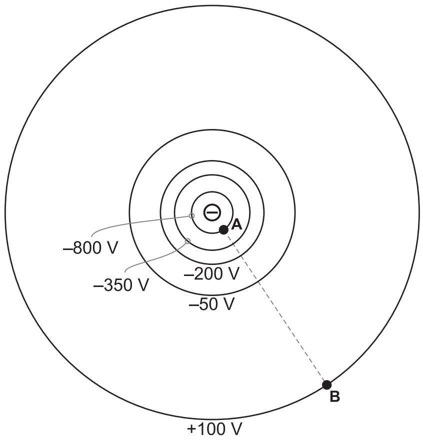

Figure 25.3 Equipotentials around an isolated point charge.

Answer: $v = 0.537$ m s^{-1}

25.8 At rest, the potential inside a nerve cell is lower than that of the extracellular fluid. The membrane potential, the potential difference between the inside and outside of the cell membrane, is 70 mV.

(a) What is the change in electrical potential energy of the sodium ion when moving from inside the cell to outside the cell?

(b) How much work must be done on a sodium ion (Na^+) to move it from inside the cell to outside the cell?

Answer: (a) $\Delta PE = +1.12 \times 10^{-20}$ J (b) $W = +1.12 \times 10^{-20}$ J

25.9 An electron at an initial electrical potential of 0 V is fired towards a second electron which is held *fixed* in space. The moving electron was fired at an initial speed of 1×10^4 m s^{-1}. When the moving electron is 5.11×10^{-6} m from the fixed electron it's speed has been reduced to 1×10^3 m s^{-1}. What is the electrical potential 5.11×10^{-6} m at the point 5.11×10^{-6} m away from an isolated electron? Answer: $V = 2.84 \times 10^{-4}$ V

26 Capacitance

26.1 Problems

26.1 Problems

26.1 A 9 V battery is connected to a capacitor which subsequently has a *magnitude* of 0.5 μC of charge on each plate, with the charge on each plate having an opposite sign. What is the capacitance of this capacitor? Answer: $C = 55.6$ nF

26.2 You have some metal shelving with two shelves, each of which measures 0.5 m × 0.2 m and the shelves are 0.3 m apart. (assume the shelves are electrically insulated from each other and any other objects)

(a) What is the capacitance of your shelves?

(b) How much charge would they hold if you connected each terminal of the shelves to a 1.25 V battery?

Answer: (a) 3.0×10^{-12} F (b) 3.7×10^{-12} C

26.3 You wish to construct a capacitor which has a capacitance of 1 F. You intend to construct your capacitor using two parallel sheets of tinfoil held 1 mm apart by placing plastic wrap between them (the plastic wrap fills the space between the tinfoil). Both tinfoil and plastic wrap come in rolls which measure 30 cm by 10 m. The relative permittivity of the plastic wrap is $\epsilon_r = 2.9$.

(a) How many rolls of tinfoil and plastic wrap would you need? Is your plan feasible?

(b) How much charge could you store on this capacitor if you connect it to a standard 9 V battery?

Answer: (a) You would need approximately 13 000 000 rolls of plastic wrap and 26 000 000 rolls of tinfoil so no, the plan is flawed (b) $Q = 9.0$ C

26.4 A large industrial capacitor with an air gap between the plates stores 140 kJ of energy at a potential difference of 1200 V. A dielectric is inserted into the space between the plates of this capacitor and it can now store 11200 kJ of energy at 600 V. What is the relative permittivity of the dielectric inserted between the plates of the capacitor? Answer: $\epsilon_r = 320$

26.5 A particular capacitor stores 1 J of energy when charged to a potential difference of 12 V.

(a) What is the capacitance of this capacitor?

(b) What is the charge stored on this capacitor?

Answer: (a) $C = 14$ mF (b) $Q = 0.17$ C

26.6 You measure the electric field between the plates of a 5 nC capacitor to be 2000 N C^{-1}. If the charge on this capacitor is 2 μC, how far apart are the plates? Answer: 0.2 m

26.7 You place books in the shelving described in Problem 26.2. The relative permittivity of paper is 2.4. How much charge does each of your bookshelves hold when connected to the same battery as before? Answer: 8.9×10^{-12} C

26.8 You have a sheet of paper which measures 15 cm × 20 cm and a piece of nylon sheeting which has the same dimensions. You charge these objects by rubbing them together and when you separate them and hold them parallel to each other and 1 cm apart you find that there is an electric field of magnitude 1500 V m^{-1} between them (you can assume that $\epsilon_r = 1$ for air).

(a) What is the potential difference between the paper and nylon sheets?

(b) What is the capacitance of these sheets in this position?

(c) What is the magnitude of the charge on each sheet?

(d) How much electrical energy is stored in the sheets?

The two sheets are now moved further apart to a separation of 5 cm.

(e) What is the capacitance of the sheets now?

(f) What is the potential difference between the sheets now?

(g) What is the magnitude of the electric field between the sheets now?

(h) How much electrical energy is stored in the sheets now?

Answer: (a) $V = 15$ V (b) $C = 2.7 \times 10^{-11}$ F (c) $Q = 4.0 \times 10^{-10}$ C (d) 3.0×10^{-9} J (e) $C = 5.3 \times 10^{-12}$ F (f) $V = 75$ V (g) $E = 1500$ V m^{-1} (h) 15.0×10^{-9} J

26.9 Nerve cells maintain a charge separation across their cell membrane. The cell membrane of a particular cell is 10 nm thick and the cell can be modeled as a cylinder with a diameter of 12 μm and a length of 80 μm. If the potential difference across the cell membrane is 90 mV, what is the charge stored on the cell? (you can assume that $\epsilon_r = 1$ for the cell membrane) Answer: 5.5×10^{-13} C

26.10 How much energy is stored in the form of charge separation in the cell in Problem 26.9? Answer: 2.5×10^{-14} J

27

Direct Currents and DC Circuits

27.1 Problems

27.1 Sodium ions (Na^+) are flowing through a cylindrical ion channel which has a diameter of 0.85 µm and is 5 µm long. There is a potential difference of 225 mV between the ends of the channel. The sodium ions have a drift velocity through the channel of $0.015\ \mathrm{m\,s^{-1}}$ and in a period of 1 ms a total of 15×10^6 ions exit the channel.

(a) What total charge exits the channel in a time period of 1 ms?

(b) What is the current in the ion channel?

(c) What is the 'resistance' of the channel to the flow of sodium ions?

(d) What is the number density, n, of ions in the channel?

(e) How many sodium ions are in the channel at any one time?

Answer: (a) $\Delta Q = 2.4 \times 10^{-12}$ C (b) $I = 2.4 \times 10^{-9}$ A (c) $R = 94 \times 10^6\ \Omega$ (d) $n = 1.8 \times 10^{24}\ \mathrm{ion\,m^{-3}}$ (e) $N = 5.0 \times 10^6$ ions

27.2 Given that the potential is +18 V at the point shown, what is the electrical potential at the points A, B, C, and D in the circuit shown in Figure 27.1?

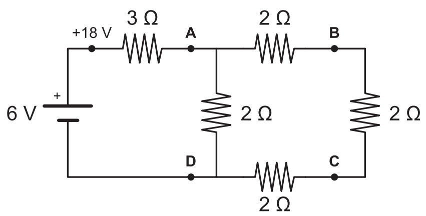

Figure 27.1 A circuit with 5 resistors and a single battery.

Answer: $V_A = +14$ V, $V_B = +13\frac{1}{3}$ V, $V_C = +12\frac{2}{3}$ V, and $V_D = +12$ V

27.3 A simple circuit is constructed in which a 1 Ω resistor is connected across a 1 V battery and so draws a 1 A current. What is the current drawn from the battery if:

(a) an extra 1 Ω resistor is connected in series with the existing one?

(b) four extra 1 Ω resistors are connected in series with the existing one?

Answer: (a) 0.5 A (b) 0.2 A

27.4 A simple circuit is constructed in which a 1 Ω resistor is connected across a 1 V battery and so draws a 1 A current. What is the current drawn from the battery if:

(a) an extra 1 Ω resistor is connected in parallel with the existing one?

(b) four extra 1 Ω resistors are connected parallel with the existing one?

Answer: (a) 2 A (b) 5 A

27.5 Circuit A shown in Figure 27.2 consists of a single light bulb ($R = 288\ \Omega$) and a 12 V battery.

(a) What current is drawn from the battery?

(b) What is the power dissipated in the light bulb?

An identical light bulb is now added to Circuit A in series with the first (see Circuit B in Figure 27.2).

(c) What is the current drawn from the battery in this case?

(d) What is the current passing through each light bulb?

(e) What is the potential difference across each light bulb?

(f) What is the power dissipated in each light bulb?

(g) What is the total power supplied by the battery?

An identical light bulb is added to Circuit A in parallel with the first (see Circuit C in Figure 27.2).

(h) What is the current drawn from the battery in this case?

(i) What is the current passing through each light bulb?

(j) What is the potential difference across each light bulb?

(k) What is the power dissipated in each light bulb?

(l) What is the total power supplied by the battery?

Answer: (a) $I_{batt} = 0.042$ A ($\frac{1}{24}$ A) (b) $P = 0.5$ W (c) $I_{batt} = 0.021$ A ($\frac{1}{48}$ A) (d) $I_{lb} = 0.021$ A ($\frac{1}{48}$ A) (e) $V_{lb} = 6$ V (f) $P_{lb} = 0.13$ W ($\frac{1}{8}$ W) (g) $P_{batt} = 0.25$ W ($\frac{1}{4}$ W) (h) $I_{batt} = 0.083$ A ($\frac{1}{12}$ A) (i) $I_{lb} = 0.042$ A ($\frac{1}{24}$ A) (j) $V_{lb} = 12$ V (k) $P_{lb} = 0.5$ W (l) $P_{batt} = 1.0$ W

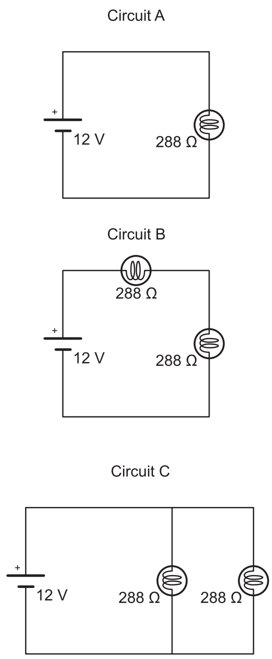

Figure 27.2 Three circuits showing different arrangements of light bulbs.

27.6 Five resistors are connected to a 24 V resistor as shown in Figure 27.3.

(a) What is the total resistance of the circuit?

(b) How much current flows from the battery?

(c) How much power is supplied by the battery?

(d) What is the potential difference across each resistor?

(e) What is the current through each resistor?

(f) What is the power dissipated in each resistor?

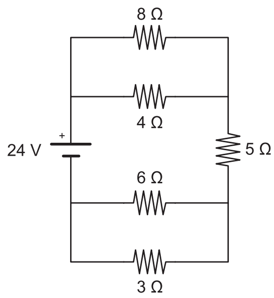

Figure 27.3 A circuit with 5 resistors in both series and parallel.

Answer: (a) $R_{\text{total}} = 9.7\,\Omega$ (b) $I_{\text{batt}} = 2.5\,\text{A}$ (c) $P_{\text{batt}} = 60\,\text{W}$ (d) $V_{5\,\Omega} = 12.4\,\text{V}$, $V_{8\,\Omega} = V_{4\,\Omega} = 6.62\,\text{V}$, and $V_{6\,\Omega} = V_{3\,\Omega} = 4.97\,\text{V}$ (e) $I_{8\,\Omega} = I_{6\,\Omega} = 0.828\,\text{A}$, $I_{5\,\Omega} = 2.48\,\text{A}$, and $I_{4\,\Omega} = I_{3\,\Omega} = 1.67\,\text{A}$ (f) $P_{8\,\Omega} = 5.48\,\text{W}$, $P_{6\,\Omega} = 4.11\,\text{W}$, $P_{5\,\Omega} = 30.8\,\text{W}$, $P_{4\,\Omega} = 11.0\,\text{W}$, and $P_{3\,\Omega} = 8.22\,\text{W}$

Time Behaviour of RC Circuits

28

28.1 Problems

28.1 Problems

28.1 An 18 µF capacitor has been charged to 100 V. A 15 kΩ resistor and a 5 kΩ resistor, are connected in series with the capacitor.

(a) What is the time constant of this circuit?

(b) Approximately how long (in terms of τ) will it take for the charge stored on the resistor to drop to 0.1% of its original charge?

Once the capacitor is fully discharged the 5 kΩ resistor is removed and replaced with a 12 V battery.

(c) What is the time constant of the circuit now?

(d) Approximately how long will it take for the charge stored on the resistor to rise to 95% of its maximum charge (in terms of τ)?

Answer: (a) $\tau = 0.36$ s (b) $7 \times \tau$ (c) $\tau = 0.27$ s (d) $3 \times \tau$

28.2 The circuit shown in Figure 28.1 is constructed using a 6 V battery, a 3 kΩ resistor, and a 6 kΩ resistor. Initially both switches are open and the capacitor is uncharged.

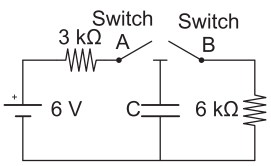

Figure 28.1 An RC circuit

Switch A is closed and the capacitor begins to charge.

(a) After $\frac{1}{2}$ second the potential difference across the capacitor is 3.78 V. What is the capacitance of the capacitor?

After several minutes switch A is opened.

(b) What is the approximate potential difference across the capacitor?

Switch B is now closed and the capacitor begins to discharge.

(c) What is the characteristic time for discharging this capacitor?

(d) What will the current through the 6 kΩ resistor be after 2 seconds?

(e) What will the charge on the capacitor be after 3 seconds?

Answer: (a) $C = 0.17 \times 10^{-3}$ F, (b) $V_{\text{cap}} = 6$ V, (c) $\tau_{\text{discharge}} = 1$ s, (d) $I = 0.13 \times 10^{-3}$ A (e) 4.7×10^{-5} C

28.3 A defibrillator can be modelled as a capacitor which discharges through the patient, inducing an electrical current in the chest. The resistance of the path which the electrical current takes through the chest of a typical adult is 50 kΩ. A particular defibrillator has a capacitance of 16.67 nF and is designed to be charged to an electrical potential of 900 V before discharging. The defibrillator includes a 'ballast' resistor of 40 kΩ which is connected in series with the patient.

(a) What is the maximum current I_{max} that will pass through the typical adult patient's chest?

(b) How long will it take the current passing through the typical adult patient's chest to drop below 1.37 mA?

You wish to redesign this defibrillator so it can be used on a child. The resistance of the typical child's chest is around 40 kΩ and the maximum current that be allowed through the chest is 8 mA. The current must drop to 0.15 mA in 4 ms. If the defibrillator is charged to a maximum potential of 900 V as before, then

(c) What is the required resistance that must be connected in series to the child patient's chest?

(d) What is the required capacitance of the defibrillator?

Answer: (a) $I_{\text{max}} = 10$ mA (b) $t = 3$ ms (or $2 \times \tau$) (c) 72.5 kΩ (d) 11.9 nF

28.4 A 10 µF capacitor has been charged to a potential difference of 12 V. A 100 kΩ resistor is connected in series with the capacitor.

(a) What are the potential difference across the resistor, current through the resistor, and charge on the capacitor just after the circuit has been closed?

(b) What are the potential difference across the resistor, current through the resistor, and charge on the capacitor 1 second after the circuit has been closed?

(c) What are the potential difference across the resistor, current through the resistor, and charge on the capacitor 5 minutes after the circuit has been closed?

Answer: (a) $V_{0\,\text{s}} = 12$ V, $I_{0\,\text{s}} = 0.12$ mA, and $Q_{0\,\text{s}} = 120$ mC (b) $V_{1\,\text{s}} = 4.32$ V, $I_{1\,\text{s}} = 0.0432$ mA, and $Q_{1\,\text{s}} = 43.2$ mC (c) $V_{5\,\text{min}} = 0$ V, $I_{5\,\text{min}} = 0$ mA, and $Q_{5\,\text{min}} = 0$ µC

28.5

(a) How much energy does the defibrillator in Problem 28.3 deposit in the patient (before it has been redesigned)?

(b) What fraction of this total energy does this defibrillator deposit in the patient before the current drops below 1.37 mA?

Answer: (a) 3.75 mJ (b) 98%

28.6 The circuit shown in Figure 28.2 is used to charge a capacitor from a potential of 0 V to a potential of 95 V in a period of 1 s. Furthermore I_1, the current through resistor R_1, and I_2, the current through resistor R_2 satisfy the relation $I_2 = 3I_1$ at all times. What is the capacitance of the capacitor?

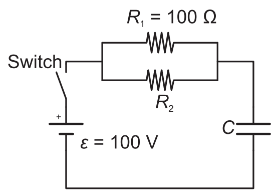

Figure 28.2 An RC circuit featuring a pair of resistors in parallel.

Answer: C = 13 mF

V

Optics

THE NATURE OF LIGHT

29

29.1 Problems

29.1 Problems

29.1 A diagnostic device uses a bright red laser light to illuminate structures just under the surface of the skin. Light from the laser passes first through the air, and then the skin, to scatter off the subcutaneous structures that are to be imaged. The scattered light passes back through the skin and into an optical device which forms an image of the scattered light on a CCD array. The laser light used has a wavelength of 633.0 nm in a vacuum. The refractive indices of air, the glass used in the imaging optics, and skin are 1.008, 1.700, and 1.381 respectively (use $c = 2.998 \times 10^8\ \mathrm{m\,s^{-1}}$ for this question).

(a) What is the frequency of the red light when it passes through each material?

(b) What is the wavelength of the red light as it passes through each material?

(c) How fast does the red light travel through each material?

Answer: (a) $f_{\mathrm{air}} = f_{\mathrm{skin}} = f_{\mathrm{glass}} = 4.736 \times 10^{14}$ Hz (b) $\lambda_{\mathrm{air}} = 628.0$ nm, $\lambda_{\mathrm{skin}} = 458.4$ nm, $\lambda_{\mathrm{glass}} = 372.4$ nm (c) $v_{\mathrm{air}} = 2.974 \times 10^8\ \mathrm{m\,s^{-1}}$, $v_{\mathrm{skin}} = 2.171 \times 10^8\ \mathrm{m\,s^{-1}}$, $v_{\mathrm{glass}} = 1.764 \times 10^8\ \mathrm{m\,s^{-1}}$

29.2 It is not possible to make images of, and therefore see, arbitrarily small objects using visible light. The minimum size of an object that can be 'seen' by light using conventional optics is roughly equal to a few times the wavelength of the light used. If a bacterium that is 1.2 µm across can *just* be seen using a particular optical system when the bacteria is floating in a watery solution ($n_{\mathrm{solution}} = 1.35$), what will be the minimum size of bacterium that this optical system could 'see' in air ($n_{\mathrm{air}} = 1.0$)? Answer: 1.6 µm

29.3 Light strikes a mirror as shown in Figure 29.1. This mirror has another mirror placed at right angles to it. Such an arrangement of mirrors is known as a corner reflector. At what angle does the light get reflected back (i.e., what angle is the outgoing light at when it crosses the dotted line)?

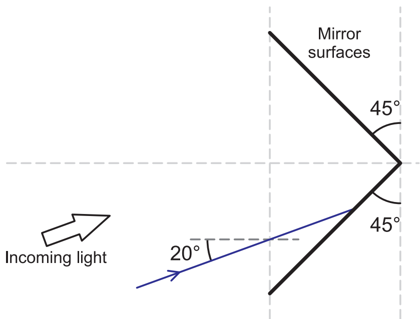

Figure 29.1 Two mirrors are placed a right angles to one and other. This arrangement of mirrors reflects light in a particular fashion, making them useful for a range of purposes.

Answer: 20° below the horizontal. i.e. parallel to the incoming light but in the opposite direction.

29.4 The glass half-cylinder prism shown in Figure 29.2 is used to measure the critical angle for light of various wavelengths. For red light the critical angle measured was 36.78°. For blue light the critical angle was 36.28° (The refractive index of air is $n = 1.0$).

(a) What is the refractive index of the glass for red light?

(b) What is the refractive index of the glass for blue light?

Answer: (a) 1.67 (b) 1.69

29.5 A beam of white light passes through a 1.5 cm thick pane of glass at an angle of 45° as shown in Figure 29.3. The refractive index of the glass for light of wavelength 470 nm (deep blue) is 1.66 while the refractive index of the glass for light of wavelength 630 nm (bright red) is 1.60.

(a) What is the spacing, *S*, between the red and blue components of a narrow beam after they have passed through the pane of glass?

(b) Use your answer in (a) to explain why we do we not ordinarily see the effects of dispersion when looking through flat panes of glass.

(c) How thick would the pane of glass need to be for the separation of the red and blue rays to be 1 cm?

Answer: (a) 233 µm (b) The red and blue rays are too close together for our eye to resolve them into separate colors. (c) 65 cm

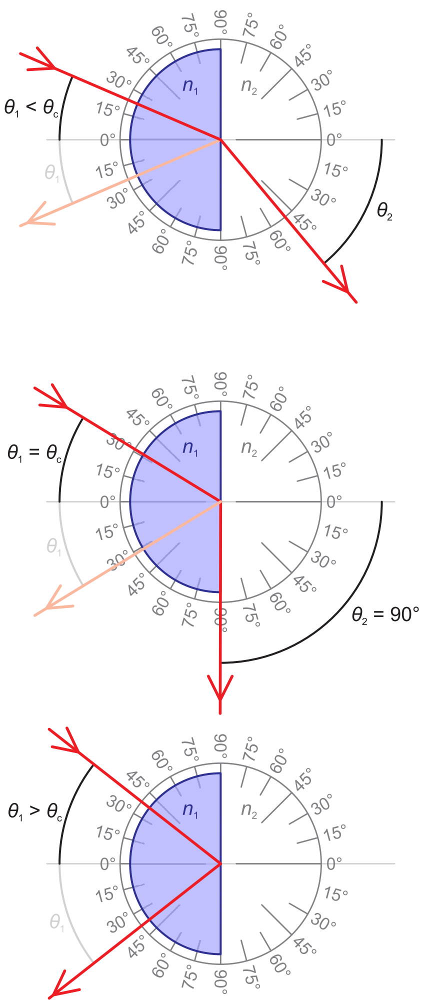

Figure 29.2 A glass half-cylinder prism is constructed from a section of a glass half cylinder. A beam of light aimed towards the center of the apparatus will not be refracted at the first air-light interface as the incident angle will be 0° and $\sin 0° = 0$.

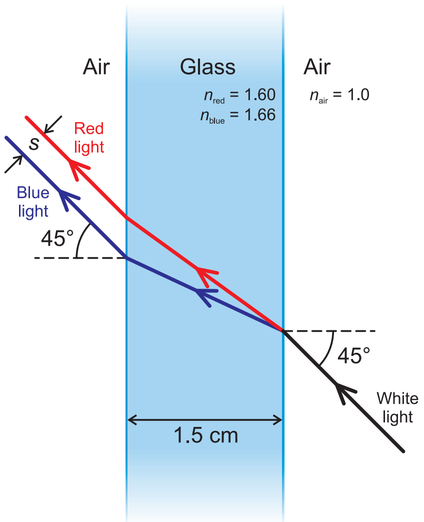

Figure 29.3 A beam of white light passes through a glass pane. Dispersion causes the red and blue components of the light to be bent at different angles. After passing through the pane of glass the red and blue components are slightly offset.

29.6 A beam of light of wavelength 550 nm strikes a water droplet as show in Figure 29.4. What are the angles θ_A and θ_B at which the reflected and refracted beams travel? Answer: $\theta_A = 60.0°$, $\theta_B =$

7.92°

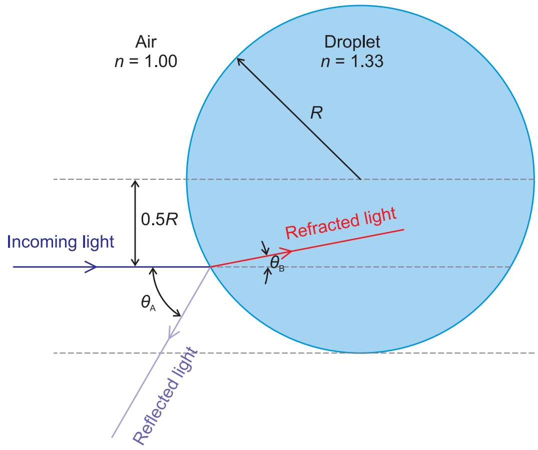

Figure 29.4 A beam of light hits a spherical water droplet.

29.7 A fish in a pond looks up and sees the light from a street lamp at an angle of 35° to the vertical. If the street light is 5.5 m tall and the fish is 30 cm below the surface of the pond and 3 m from its edge, how far from the *edge of the pond* is the street lamp? ($n_{\text{air}} = 1$, and $n_{\text{water}} = 1.33$) Answer: 3.7 m

29.8 Two divers jump out of their boat and swim straight down to a depth of 10 m. The water surface becomes calm again very quickly after the divers jump in. Once the divers reach their final depth they begin to swim in opposite directions at the same rate while periodically stopping to shine a flashlight back at the surface of the water where they had jumped in. After the divers have swum far enough apart they begin to notice a strong reflection from the other diver's flashlight that was not present before. How far apart are divers when this starts to happen? ($n_{\text{air}} = 1.0$, and $n_{\text{water}} = 1.33$) Answer: 23 m

29.9 By what angle (θ_{cornea}) is the beam of light shown in Figure 29.5 deviated as it passes from air to the cornea if the incident angle is $\theta_i = 23.6°$? The refractive index of air is $n_{\text{air}} = 1.00$, the refractive index of the cornea is $n_{\text{cornea}} = 1.38$. Ignore further deviation of light as it passes from the cornea into the aqueous humour, etc. Answer: $\theta_{\text{cornea}} = 6.74°$

29.10 The ability of your eyes to focus is impaired when you attempt to look around underwater (if you are not wearing a pair of swimming goggles). Recalculate your answer for Problem 29.9 for the case in which the eye is submerged in water ($n_{\text{water}} = 1.33$). Answer: $\theta_{\text{cornea}} = 0.90°$

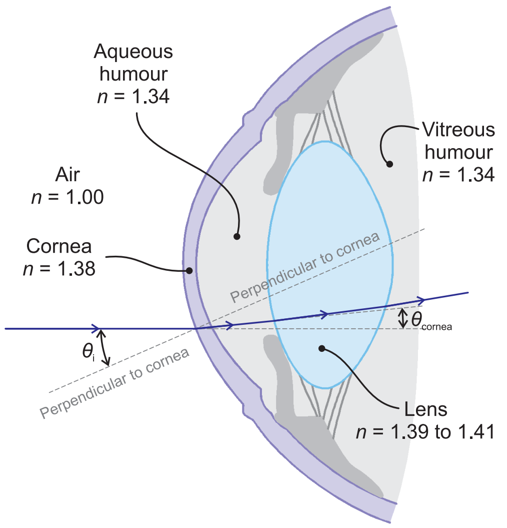

Figure 29.5 Most of the bending of light in the eye is done at the air-cornea interface. The lens is responsible for only a small amount of the bending, but of course is adjustable.

GEOMETRIC OPTICS

30

30.1 Problems

30.1 Is it possible for a converging lens to form a virtual image? If so, under what conditions is the image virtual? If not, why not? Answer: Yes it is possible. $d_o < f$

30.2 Is it possible for a diverging lens to form a real image of a physical object? If so, under what conditions is the image real? If not, why not? Answer: No it is not possible for an actual physical object. The light leaving the diverging lens will always be diverging, and thus cannot form a real image.

30.3 An object is placed 0.25 m away from a lens. The lens forms an image that is 0.167 m away from the lens, upright, and on the same side of the lens as the object.

(a) What is the focal length of the lens?

(b) What kind of lens is used? Answer: (a) $f = -0.50$ m (b) A diverging lens (-ve focal length)

30.4 You wish to produce inverted real images of an object with the given magnifications using a converging mirror. How far from the mirror must you place the object in each case (express your answer in terms of the focal length of the mirror)?

(a) $M = -0.5$

(b) $M = -1$

(c) $M = -2$

(d) $M = -4$ Answer: (a) $d_o = 3f$ (b) $d_o = 2f$ (c) $d_o = 1.5f$ (d) $d_o = 1.25f$

30.5 You wish to produce upright virtual images of an object with the given magnifications using a diverging lens. Where must you place the object in each case (express your answer in terms of the focal length of the lens)?

(a) $M = 0.1$

(b) $M = 0.25$

(c) $M = 0.5$

(d) $M = 0.75$ Answer: (a) $d_o = -9f$ (b) $d_o = -3f$ (c) $d_o = -f$ (d) $d_o = -0.33f$ As the lens is diverging, the focal length is negative giving a positive object distance.

30.6 What is the largest magnification attainable when imaging a real physical object using a diverging mirror, and how far from the mirror must the object be placed to attain this magnification? Answer: $M = 1$, $d_o = 0$ m (object touching mirror)

30.7 You are standing 5 m from the edge of a very large, 150 m diameter hemispherical building which is coated in a reflective material. You are carrying a small laser pointer which you hold 1.5 m above the ground.

(a) You point the laser pointer at the building and the reflected beam travels straight back at the pointer. At what angle below the horizontal are you holding the pointer?

(b) You point the laser pointer at the building and the reflected beam is traveling parallel to the ground. At What angle below the horizontal are you holding the pointer? Answer: (a) 1.07° (b) 2.02°

30.8 Light from a distant source enters a 0.5 dioptre lens parallel to the optical axis.

(a) How far from the first lens must a second, 1.2 D lens be placed such that the light leaving the second lens is also parallel to the optical axis?

(b) How far from the first lens must a second, −1.5 D lens be placed such that the light leaving the second lens is also parallel to the optical axis?

(c) A second 1.2 D lens is placed 1.2 m behind the first. Is the light leaving this lens, converging, diverging, or parallel to the optical axis? Answer: (a) 2.12 m (b) 1.33 m (c) converging

30.9 A converging lens with a focal length of 30 cm is used to create an image of a 2 mm long ant.

(a) If the lens is placed so that the image of the ant is 8 mm long, upright, and viewed by looking through the lens, how far away from the ant was the lens placed?

(b) If the lens is placed so that the image of the ant is 8 mm long, inverted, and viewed on a screen held some unspecified distance on the other side of the lens to the ant, how far away from the ant was the lens placed? Answer: (a) $d_o = 22.5$ cm (b) $d_o = 37.5$ cm

30.10 When you look at the back of a spoon you see an upright image of yourself. This is because the reflective curved surface of the metal acts as a diverging mirror. This image does tend to be distorted because spoons seldom have the spherical or parabolic curvature required for an undistorted image. Ignore these distortions when answering the following questions.

(a) If the image of your head is 3 cm tall, your head is 22 cm tall, and you are holding the spoon 16 cm away from your head, what is the focal length of the back of the spoon?

(b) When you flip the spoon around it now acts like a converging mirror and you see an inverted image. Assuming that the curvature of the inside of the spoon is the same as the curvature of the outside of the spoon how large is the image of your head?

Answer: (a) $f = -2.53$ cm (b) $h_i = -4.13$ cm (i.e. inverted and reduced)

THE EYE AND VISION

31

31.1 Problems

For the purposes of answering these questions you can assume that a normal human eye has a minimum optical power of 50 D, a maximum optical power of 54 D, and that the normal distance between the retina and the lens is 2 cm. **31.1** A person with axial hypermetropia has a lens-retina distance of 1.9 cm and the maximum optical power of their eye is the same as that for a normal person.

(a) What is the near point of this person?

(b) What is the range of accommodation this person needs to see objects from their near point all the way up to their far point (which is the same as for a normal eye)?

(c) What is the optical power of the contact lenses used to treat this person and give them a normal near point of 25 cm? Answer: (a) 73 cm (b) 1.37 D (c) 2.63 D

31.2 A person with a normal lens-to-retina distance wears contact lenses with an optical power of 1.2 D in order to be able to clearly see objects 25 cm in front of them.

(a) What kind of vision defect does this person have?

(b) What is this person's near point (without the contact lenses)? Answer: (a) Hypermetropia (converging lenses are required if the optical power of the eye is too weak) (b) 35.7 cm

31.3 A person has refractive myopia with a far point of only 5 m. They are to be prescribed a set of glasses that will enable them to see distant objects clearly and the person's glasses will typically sit 2 cm in front of their eyes.

(a) What is the minimum optical power of this persons eyes?

(b) What is the optical power of the glasses required? Answer: (a) 50.2 D (b) −0.2 D (diverging lenses are required))

31.4 A person has refractive hypermetropia with a near point at 3 m. They are to be prescribed a set of glasses that will enable them to have normal close-in vision and the person's glasses will typically sit 2 cm in front of their eyes.

(a) What is the maximum optical power of this person's eyes?

(b) What is the optical power of the glasses required? Answer: (a) 50.3 D (b) 3.33 D (converging lenses are required))

31.5 A person who had normal vision when they were younger now has age related presbyopia. They can still see distant objects clearly but have a reduced range of accommodation of just 1.0 D.

(a) What is this person's near point?

(b) What is the power of the contact lenses needed to correct this presbyopia (so that person has a normal near point of 25 cm)? Answer: (a) 1 m (b) +3.0 D

31.6 When driving you need to be able to clearly see road signs and traffic some distance ahead of you as well as the dashboard in your car. Assume that you have a normal lens-to-retina distance.

(a) What minimum optical power of the eye is needed to clearly see a road sign 200 m ahead?

(b) What maximum optical power is needed to clearly see the dashboard 40 cm away?

(c) What range of accommodation is needed?

(d) If you can accommodate at a maximum rate of 1.1 dioptres per second, how long does it take your eyes to adjust when looking up at the road from the dashboard?

(e) If you are traveling on the open road at 100 km h^{-1}, how far do you travel in the time it takes your eyes to accommodate between the dashboard and the road? Answer: (a) 50 D (b) 52.5 D (c) 2.5 D (d) 2.27 s (e) 63 m

31.7 Many automated industrial engineering plants use high definition cameras on the production line in order to monitor the quality of products on the assembly line. A particular plant manufactures small machined products which have a maximum depth of 5 cm. If the cameras used have a single lens which is around 3 cm from the CCD array on which the image is projected (and captured), and are placed such that the lens is 8 cm above the conveyor belt on which the circuit boards rest. What is the necessary range of accommodation of these cameras if they are required to clearly image details over the whole range of depth of the circuit boards? How does this compare to the accommodation range of the human eye?

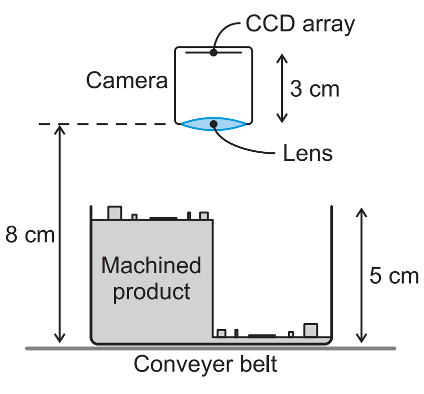

Figure 31.1 A machined product passing underneath an automated camera. The camera needs to be able to focus on all parts of the object (although not necessarily at the same time).

Answer: The range of accommodation of the camera is 20.8 D which is quite a bit larger than the typical human range of 4 D

31.8 A person with refractive myopia can see objects as close as 25 cm clearly, and objects as far away as 3 m clearly.

(a) What is the maximum optical power of this person's eye?

(b) What is the minimum optical power of this person's eye?

(c) What is the range of accommodation of this person's eye?

The person gets a set of contact lenses for their eyes to correct their far vision. When they are wearing the contact lenses they can see objects in the distance clearly.

(d) What is the optical power of the contact lenses?

(e) What is the person's new near point?

(f) What is the range of accommodation of this person when wearing their contact lenses?

Answer: (a) Pmax = 54 D (b) Pmin = 50.33 D (c) range of accommodation = 3.67 D (d) Pcl = −0.33 D (i.e. diverging lenses) (e) $n.p.$ = 27.2 cm (f) range of accommodation = 3.67 D (the same as before)

WAVE OPTICS

32

32.1 Problems

32.1 Problems

32.1 Light of wavelength 550 nm passes through a 10 μm wide slit on to a screen 1 m away from the slit.

(a) How far either side of the central maximum are the 1st, 2nd , and 3rd dark regions in the diffraction pattern?

(b) What is the maximum possible number of bright fringes that could be viewed either side of the central maximum in perfect conditions? Answer: (a) $y_1 = 0.0550$ m, $y_2 = 0.111$ m, $y_3 = 0.167$ m

(d) There are 17 bright fringes either side of the central maxima. Most of these fringes will be very dim however and in practice it will be difficult to detect the higher order fringes.

32.2 A high intensity source of microwaves used in a piece of medical diagnostic equipment is not adequately shielded. The microwaves produced by this equipment have a frequency of 150 GHz, and there is a gap in the shielding 1 cm wide. At what angles from the gap in shielding will the intensity of the microwave radiation be zero? (assume the microwaves are incident normally to the gap.) Answer: $\theta_1 = 11.53\,°$, $\theta_2 = 23.57\,°$, $\theta_3 = 36.85\,°$, $\theta_4 = 53.09\,°$, and $\theta_5 = 88.19\,°$

32.3 A concert is to be held in a large hall and two speakers are placed 5 m apart at the front of and in the middle of the hall. During sound testing these speakers are producing a steady 1000 Hz tone (the speed of sound in air is 340 m s^{-1}). The hall is 50 m long and 30m wide.

(a) The hall is 30 m wide and a person starts in the middle. As the person walks to their right they should notice the sound intensity vary. At what positions would you predict the person would be able to hear the 1000 Hz tone produced by the speakers *most clearly*?

(b) It is unlikely that the person will actually notice much difference in the sound intensity as they walk across the back of the hall. What is the single biggest reason for this difference between your prediction in (a) and the actual experience of the person? Answer: (a) 3.4 m, 6.8 m, and 10.4 m either side of the middle of the back of the hall. (b) The sound reflects off of the surfaces of the hall and so the net waveform at any given point is the sum of many waveforms with many different path lengths. As it is quite unlikely that all of the waveforms will be out of phase with each another, in general the sound intensity at a given point does not vary as we would expect it to based upon the superposition of just the two direct paths.

32.4 A particular toy telescope produces a magnified virtual image of distant objects. This virtual image has a magnification of +3 and is located at the same distance from the telescope as the object. The diameter of the objective lens of the telescope is 3 cm. When looking through the telescope, the pupil of the eye dilates to 6 mm. What is the minimum separation between two objects that can be resolved at a distance of 150 m? (Assume light of wavelength λ = 550 nm.) Answer: 5.5 cm (The limit is imposed by the eye viewing the magnified virtual image, not the telescope itself.)

32.5 The primary mirror of the Hubble Space Telescope is 2.4 m in diameter. Suppose it is most sensitive to light of wavelength of 820 nm.

(a) What is the diffraction limited angular resolution of the Hubble Space Telescope?

(b) If the HST were used to look at the surface of the Moon, what is the minimum distance between two distinguishable points? (The Moon orbits at a mean distance of 384000 km from the center of the Earth which has a radius of 6380 km, the HST orbits at an altitude of 569 km.)

(c) If the HST were used to look at the surface of Jupiter, what is the minimum distance between two distinguishable points? (The orbital radius of the Earth is 150×10^6 km, and the orbital radius of Jupiter is 779×10^6 km.)

(d) If the HST were used to try and find planets around a star that was only 20 light years away (very close in astronomical terms), what is the minimum distance between two distinguishable points? ($c = 3.00 \times 10^8$ m s^{-1}) Answer: (a) 4.17×10^{-7} rad (b) 157 m (c) 262 km (d) 78.8×10^6 km (around half the orbital radius of the earth)

32.6 A normal person's pupil diameter varies from a minimum of around 2 mm to a maximum of around 8 mm.

(a) What is the minimum and maximum diffraction limited angular resolution of a normal person's eye for light of wavelength 600 nm?

The light that falls on the retina, around 2 cm behind the pupil, is detected by photo-receptive cells. These cells are most closely spaced in a region called the fovea on which the image of objects directly in front of the eye is produced by the cornea and lens. The density of the photo-receptive cells in the fovea is approximately 3×10^5 cells per square millimeter and the cells are approximately circular which means that they have a diameter of around 2.06 µm.

(b) Is the theoretical maximum resolution of the eye for 600 nm light likely to be limited by the number of photo-receptive cells or the Rayleigh criterion for a wide open pupil?

(c) Given your answer in (b) what is the minimum distance between two objects that can be resolved by the unaided eye at distances of 1 m, 10 m, and 1 km? (Note that these are theoretical limits and the actual limits are somewhat worse than this.)

Answer: (a) $\theta_{2\,\text{mm}} = 3.66 \times 10^{-4}$ rad, and $\theta_{8\,\text{mm}} = 0.915 \times 10^{-4}$ rad (b) Limit is set by the size of the cells: $D_{\text{cell}} = 2.06\,\mu\text{m}$, $S_{\text{Rayleigh}} = 1.83\,\mu\text{m}$ (c) Using $\theta_{\text{Cell}} = 1.03 \times 10^{-4}$ rad based on the size of a photo-receptive cell, $S_{1\,\text{m}} = 0.103$ mm, $S_{10\,\text{m}} = 1.03$ mm, and $S_{1\,\text{km}} = 10.3$ cm

32.7 An eccentric physicist inexplicably makes his home in the bilge of a vodka delivery ship crossing the Atlantic ocean north of the arctic circle. As the physicist forgot to note down the wavelength of a new light source before throwing the packaging overboard he performs a single slit diffraction experiment using a slit of width 110.0 µm. The diffraction pattern is displayed on a screen 3.000 m from the slit and the 1st minima in the diffraction pattern are 1.420 cm either side of the central maximum. ($n_{\text{air}} = 1.008$.)

Disaster strikes at midnight in the form of an iceberg pushed into the path of the ship by a team of vengeful teetotaler polar bears. As the ship sinks and the lab becomes inundated the single slit diffraction experiment becomes submerged in the briny water. The very cold but excited physicist notices that the spacing between the 1st minima and the central maxima of the diffraction pattern changes to 1.066 cm.

With the air in his lungs rapidly running out and the chilling cries of triumphant polar bears filling his ears he hurriedly scrawls (to four significant figures) both the wavelength of the light source in air and the refractive index of salty water on the wall for the edification of future salvage divers. What did the dead physicist write on the wall? Answer: $\lambda = 524.8$ nm, and $n = 1.343$

VI

Radiation and Health

Atoms and Atomic Physics

33

33.1 Problems

33.1 Problems

33.1 A photon is emitted by an atom when one of the electrons orbiting the atom drops from an energy level of $E_i = -10.64$ eV to an energy level of $E_f = -12.70$ eV.

(a) What is the energy of this photon (in eV)?
(b) What is the energy of this photon (in J)?
(c) What is the frequency of this photon?
(d) What is the wavelength of this photon?
(e) What is the momentum of this photon? Answer: (a) $E_{photon} = 2.06$ eV (b) $E_{photon} = 3.30 \times 10^{-19}$ J (c) $f_{photon} = 4.99 \times 10^{14}$ Hz (d) $\lambda_{photon} = 600$ nm (e) $p_{photon} = 1.10 \times 10^{-27}$ N s

33.2 The electromagnetic radiation emitted from the Sun is most intense at around 502 nm.

(a) What is the energy per photon (in J) for light of this wavelength?
(b) What is the energy per photon in electron volts?
(c) What is the momentum per photon?
(d) How fast would an electron ($m_{electron} = 9.1 \times 10^{-31}$ kg) need to be traveling to have the same momentum as this photon?
(e) What would the de Broglie wavelength of such an electron be? Answer: (a) $E_{photon} = 3.94 \times 10^{-19}$ J (b) $E_{photon} = 2.47$ eV (c) $p_{photon} = 1.31 \times 10^{-27}$ N s (d) $v_{electron} = 1440$ m s^{-1} (e) $\lambda_{electron} = 502$ nm

33.3 What is the de Broglie wavelength of:

(a) an electron ($m_{electron} = 9.1 \times 10^{-31}$ kg) travelling at 15 km s^{-1}?
(b) an electron with a kinetic energy of 1 eV?
(c) a proton ($m_{proton} = 1.67 \times 10^{-27}$ kg) travelling at 15 km s^{-1}?
(d) a proton with a kinetic energy of 1 eV?
(e) an elephant ($m_{elephant}$ = 10 tonnes) travelling at 15 km h^{-1}?
(f) an elephant with a kinetic energy of 1 eV? Answer: (a) $\lambda = 48.4$ nm (b) $\lambda = 1.22$ nm (c) $\lambda = 0.0263$ nm (d) $\lambda = 0.0286$ nm (e) $\lambda = 1.58 \times 10^{-38}$ m (f) $\lambda = 1.17 \times 10^{-26}$ m

33.4 Which of the following atomic transitions in hydrogen (labelled (i) to (v)) will:

(a) release a photon of the highest energy?
(b) release a photon of the longest wavelength?
(c) release a photon of wavelength 433 nm
(d) release a photon of energy 0.661 eV

(i) $n = 2 \rightarrow n = 1$
(ii) $n = 5 \rightarrow n = 1$
(iii) $n = 5 \rightarrow n = 2$
(iv) $n = 4 \rightarrow n = 3$
(v) $n = 10 \rightarrow n = 5$

Answer: (a) ii (b) v (c) iii (d) iV

33.5 What are the wavelengths of the $n = 2 \rightarrow n = 1$, $n = 3 \rightarrow n = 1$, and $n = 4 \rightarrow n = 1$ transitions for a singly charged Helium nucleus? Answer: $\lambda_{2\rightarrow 1} = 30.3$ nm, $\lambda_{3\rightarrow 1} = 25.6$ nm, $\lambda_{2\rightarrow 1} = 24.2$ nm

33.6 A muon is a elementary particle whose properties are similar to those of an electron (a negative charge and a spin of 1/2) with the exception of its mass. Because of this it is possible to replace one or more electrons in an atom with muons. A muon is 207 times more massive than an electron (and so has mass 1.88×10^{-28} kg). If the electron in a hydrogen atom was replaced with a muon then an exotic 'muonic hydrogen' atom is created.

(a) By what factor would the Bohr radius of the 'muonic hydrogen' atom change?
(b) By what factor will the energy of a particular electronic energy level change for 'muonic hydrogen'?
(c) What will the wavelengths of the first three lines in the Balmer series ($n_i \rightarrow n_f = 2$) be for this exotic 'muonic hydrogen' atom?
(d) For ordinary hydrogen, the Balmer series falls in the visible region of the electromagnetic spectrum. In what region of electromagnetic spectrum would you search for the absorption/emission lines of the Balmer series of 'muonic hydrogen'? Answer: (a) $\frac{1}{207}$ or 207 times smaller (b) 207 or 207 times larger for a given energy level (c) $n = 3 \rightarrow n = 2$, $\lambda = 3.17$ nm; $n = 4 \rightarrow n = 2$, $\lambda = 2.35$ nm; $n = 5 \rightarrow n = 2$, $\lambda = 2.09$ nm (d) These wavelengths can be found in the X-ray/hard UV region of the EM spectrum

33.7 One of the most compelling demonstrations of wave-particle duality is the wave-like interference pattern displayed by electrons (which otherwise behave like a particle) when passing through a pair of double slits. If a beam of electrons is created by accelerating them from rest through a potential difference of just 500 V, and this beam is trained on a pair of slits 10 μm apart with a detector 1 m behind the slits, what is the separation between adjacent bright spots on the screen? Answer: $S = 5.47$ μm

33.8 List the possible states of an electron in the $n = 3$ shell of a hydrogen atom (i.e, reproduce Table **??** for $n = 3$)

Answer:

n	l	m_l	m_s
3	0	0	-1/2
3	0	0	+1/2
3	1	-1	-1/2
3	1	-1	+1/2
3	1	0	-1/2
3	1	0	+1/2
3	1	+1	-1/2
3	1	+1	+1/2
3	2	-2	-1/2
3	2	-2	+1/2
3	2	-1	-1/2
3	2	-1	+1/2
3	2	0	-1/2
3	2	0	+1/2
3	2	+1	-1/2
3	2	+1	+1/2
3	2	+2	-1/2
3	2	+2	+1/2

The Nucleus and Nuclear Physics

34

34.1 Problems

34.1 If a particular atom has a mass of 6.644×10^{-26} kg and has a total of 20 electrons when neutral (uncharged)what element is it (use a periodic table to answer this question)? Answer: Calcium 40 - ${}^{40}_{20}$Ca

34.2 (a) If all the mass of a carbon-12 atom was converted into energy, how much would this be in both joules and electron volts?

(b) How much energy (in J) would be released by the conversion of 1 kg of carbon to energy?

(c) A kilotonne of TNT releases 4.184×10^{12} J of energy. How much carbon would you need to convert to energy to create an explosion the size of the largest hydrogen bomb test at Bikini Atoll (equivalent to about 15,000 kilotonnes of TNT)? Answer: (a) $E_{\text{mass}} = 1.79 \times 10^{-9}$ J $= 1.12 \times 10^4$ MeV (b) 8.97×10^{16} J (c) $m = 0.697$ kg

34.3 An atom of uranium-235 (atomic mass: ${}^{235}_{92}$U $- m({}^{235}_{92}\text{U}) = 235.04392$ amu) decays to thorium-231 (atomic mass: ${}^{231}_{90}$Th $- m({}^{231}_{90}\text{Th}) = 231.03630$ amu) via the emission of an α particle (nuclear mass: ${}^{4}_{2}\alpha - m({}^{4}_{2}\alpha) = 4.00151$ amu). Use $c = 2.998 \times 10^8$ m s^{-1}. (Note: use the mass information on in Section **??** to solve this problem.)

(a) What is the binding energy of an ${}^{235}_{92}$U nucleus (in J)?

(b) What is the binding energy of a ${}^{231}_{90}$Th nucleus (in J)?

(c) What is the binding energy of an α particle (in J)?

(d) What is the maximum possible kinetic energy (in J) of the α particle emitted during this decay (Hint: what is the difference between the mass of ${}^{235}_{92}$U and the total mass of ${}^{231}_{90}$Th and an α particle)?

(e) What is the maximum velocity of the emitted α particle? Answer: (a) $\text{BE}_{{}^{235}_{92}\text{U}} = 2.8727 \times 10^{-10}$ J (b) $BE_{{}^{231}_{90}\text{Th}} = 2.8345 \times 10^{-10}$ J (c) $\text{BE}_{{}^{4}_{2}\text{He}} = 4.5323 \times 10^{-12}$ J (d) $\text{KE}_{\text{max}} = 7.1484 \times 10^{-13}$ J (e) $v_{\text{max}} = 14.7 \times 10^6$ m s^{-1}

34.4 There is a rare, but naturally occurring, isotope of helium called helium-3. A helium-3 nucleus has two protons and just one neutron and has an atomic mass of 3.01603 amu.

(a) What is the binding energy per nucleon of helium-3?

(b) How does this compare with the binding energy per nucleon of an alpha particle (helium nucleus)?

Answer: (a) 2.64 MeV per nucleon (b) 37% of that for helium ($\text{BE}_{{}^{4}_{2}\text{He}} = 7.07$ MeV)

34.5 What is the binding energy of each of the following nuclei?

(a) Silicon-28, $m({}^{28}\text{Si}) = 27.976927$ amu

(b) Iron-56, $m({}^{56}\text{Fe}) = 55.934939$ amu

(c) Selenium-80, $m({}^{80}\text{Se}) = 79.916520$ amu

Answer: (a) $\text{BE}_{{}^{28}_{14}\text{Si}} = 238.0$ MeV $= 3.808 \times 10^{-11}$ J (b) $\text{BE}_{{}^{56}_{26}\text{Fe}} = 494.9$ MeV $= 7.918 \times 10^{-11}$ J (c) $\text{BE}_{{}^{80}_{34}\text{Se}} = 700.3$ MeV $= 11.12 \times 10^{-11}$ J

Production of Ionising Radiation

35

35.1 Problems

35.1 Problems

You will need to use a periodic table to answer the three following questions. **35.1** What is the decay product ${}^A_Z$X in the following nuclear decay process?
${}^{238}_{92}\text{U} \rightarrow {}^A_Z\text{X} + {}^4_2\alpha$ Answer: Thorium-234, ${}^{234}_{90}$Th

35.2 What is the decay product ${}^A_Z$X in the following nuclear decay process?
${}^{237}_{92}\text{U} \rightarrow {}^A_Z\text{X} + e^- + \overline{\nu}_e$ Answer: Neptunium-237, ${}^{237}_{93}$Np

35.3 What is the decay product ${}^A_Z$X in the following nuclear decay process?
${}^{11}_{6}\text{C} \rightarrow {}^A_Z\text{X} + e^+ + \nu_e$ Answer: Boron-11, ${}^{11}_{5}$B

35.4 What is the decay product ${}^A_Z$X in the following nuclear decay process (gold-196 by electron capture)?
${}^{196}_{79}\text{Au} + e^- \rightarrow {}^A_Z\text{X} + \nu_e$ Answer: Platinum-196, ${}^{196}_{78}$Pt

35.5 Cadmium-107 has a half life of 6.52 hours. If you start off with sample which has an activity of 1.0×10^{10} Bq, what will the activity in Bq be after the following times (note: you can answer these questions without using Equation (**??**)?

(a) 6.52 hours;
(b) 19.6 hours;
(c) 3 days;
(d) 6 days. Answer: (a) $A = 0.50 \times 10^{10}$ Bq (b) $A = 0.13 \times 10^{10}$ Bq (c) $A = 4.9 \times 10^6$ Bq (d) $A = 2.4 \times 10^3$ Bq

35.6 Iodine-120 has a half life of 1.35 hours. If you start off with a sample which has an activity of 1.0 Ci, how long is it before the activity drops to the following values (note: you can answer these questions without using Equation (**??**)):

(a) 0.50 Ci;
(b) 0.125 Ci;
(c) 9.77×10^{-4} Ci;
(d) 9.31×10^{-10} Ci. Answer: (a) $t = T_{\frac{1}{2}} = 1.35$ hours (b) $t = 3T_{\frac{1}{2}} = 4.05$ hours (c) $t = 10T_{\frac{1}{2}} = 13.5$ hours (d) $t = 30T_{\frac{1}{2}} = 40.5$ hours

35.7 You find an old radioactive source in the back of the cupboard which is labeled *'Caesium-137 -* ${}^{137}Cs \rightarrow {}^{137}Ba + e^-$, $T_{\frac{1}{2}} =$ 30.2 years, $A = 1.4$ mCi *as of 01/01/1954'*. If the date is now 30/06/2009, what is the current activity of this sample (in mCi)? Answer: $A = 0.392$ mCi

35.8 Some X-rays are produced by accelerating a beam of electrons across a potential difference of 120 kV into a tungsten (${}^{184}_{74}$W) target.

(a) What are the lowest wavelength (highest energy) X-rays produced when the beam of electrons hits the target?

(b) What is the wavelength of the K_α characteristic X-rays produced in the target?

(c) What is the wavelength of the K_β characteristic X-rays produced in the target? Answer: (a) $\lambda_{\text{min}} = 0.0103$ nm (b) $\lambda_{K_\alpha} = 0.0228$ nm (c) $\lambda_{K_\beta} = 0.0192$ nm

35.9 Some X-rays are produced by accelerating a beam of electrons across a potential difference of 15 kV into a nickel (${}^{58}_{28}$Ni) target.

(a) What are the lowest wavelength (highest-energy) X-rays produced when the beam of electrons hits the target?

(b) What is the wavelength of the K_α characteristic X-rays produced in the target?

(c) What is the wavelength of the K_β characteristic X-rays produced in the target? Answer: (a) $\lambda_{\text{min}} = 0.0825$ nm (b) $\lambda_{K_\alpha} = 0.167$ nm (c) $\lambda_{K_\beta} = 0.141$ nm

35.10 You wish to design an X-ray tube that will produce a 1 W beam of 0.07217 nm X-rays.

(a) If your X-rays are produced by the K_α characteristic transition, what element is the target made out of? (Consult a periodic table.)

(b) You design your X-ray tube such that a beam of electrons is accelerated through some potential difference into a target and the maximum photon energies produced are twice that of the useful 0.07217 nm beam. Through what potential difference do you accelerate the electrons?

(c) If 0.5% of the energy deposited in the target is converted into X-rays, and only 3% of the X-rays are the useful 0.07217 nm X-rays, what is the power in the electron beam? Answer: (a) Molybdenum - ${}^{98}_{42}$Mo (b) $\Delta V = 34.4$ kV (c) $P = 6.67$ kW

36

Interactions of Ionising Radiation

Introduction to Biological Physics for the Health and Life Sciences Franklin, Muir, Scott, Wilcocks and Yates

Biological Effects of Ionising Radiation

37

37.1 Problems

37.1 A 65 kg person undergoing a series of X-rays receives a dose of 12 rem.

(a) What dose does he receive in sieverts and rad?

(b) How much energy was deposited in the person's body?

Answer: (a) $D_{Sv} = 0.12$ Sv, $D_{rad} = 12$ rad, $D_{Gy} = 0.12$ Gy (b) $E = 7.8$ J

37.2 A 14 g ovarian tumor is treated using a sodium phosphate solution in which the phosphorus atoms are the radioactive ^{32}P isotope with a half life of 14.3 days and which decays via beta emission with an energy of 1.71 MeV. Half of the sodium phosphate solution is absorbed by the tumor and deposits 9.00 J of energy into it. The other half of the solution is dispersed throughout the patients tissues, also depositing 9 J of energy into the 50.0 kg of body tissues.

(a) What is the dose (in Gy and rem) that the tumor receives?

(b) What is the dose (in Gy and rem) that the rest of the patient receives?

Answer: (a) $D_{tumor} = 643$ Gy, $D_{tumor} = 109000$ rem (b) $D_{patient} = 0.164$ Gy, $D_{patient} = 27.8$rem

37.3 A person is exposed to ionizing radiation which deposits 10 J of energy in their tissue.

(a) What dose (in Gy) would an 80 kg adult and a 15 kg child receive under these circumstances?

(b) What dose (in rem) would the adult and child each receive if the radiation were low energy (< 0.03 MeV) β radiation?

(c) What dose (in Sv) would the adult and child each receive if the radiation were low energy α radiation? (RBE)=10.)

Answer: (a) $D_{adult} = 0.125$ Gy, $D_{child} = 0.667$ Gy (b) $D_{adult} = 12.5$ rem, $D_{child} = 66.7$ rem (c) $D_{adult} = 1.25$ Sv, $D_{child} = 6.67$ Sv

37.4 A person with lymphoma receives a dose of 35 Gy in the form of γ radiation during a course of radiotherapy. Most of this dose is absorbed in 18 g of cancerous lymphatic tissue.

(a) How much energy is absorbed by the cancerous tissue?

(b) If this treatment consists of five 15 minute sessions per week over the course of 5 weeks and just 1% of the γ photons in the γ ray beam are absorbed, what is the power of the γ ray beam?

(c) If the γ ray beam consists of just 0.5% of the γ photons emitted by the γ source, each of which has an energy of 0.03 MeV, what is the activity (in Ci) of the γ ray source?

Answer: (a) $E = 0.63$ J (b) $P = 2.8$ mW (c) $A = 3150$ Ci

37.5 A 60 kg person accidentally ingests a small source of alpha particles (RBE=15). The activity of the source is 0.04 Ci, the half life of the source is 110 years, and each alpha particle emitted has an energy of 0.586 MeV. It takes 12 hours for the alpha source to pass through the persons digestive system and exit the body.

(a) How many alpha particles are absorbed by the person (you may assume that 100% of the alpha particles emitted by the source are absorbed by the person)?

(b) How much energy is deposited in the person by the source (in J)?

(c) What is the absorbed dose (in rad)?

(d) What is the absorbed dose (in rem)?

Answer: (a) $N_{abs} = 6.39 \times 10^{13}$ (b) $E = 5.99$ J (c) $D = 10.0$ rad (d) $D = 150$ rem (This would result in an immediate drop in blood cell count, nausea, and vomiting, although would likely prove non-fatal. This dose will also greatly increase the long term risk of cancer and genetic defects in offspring conceived after the event)

37.6 A radioactive contaminant gives an unfortunate 0.5 kg lab rat a dose of 1500 rem in just 1 minute. Assuming that the half life of the radioactive isotope in the contaminant is much longer than 1 minute, what would the activity (in Bq) of the contaminant be if ...

(a) the contaminant is a 5 MeV alpha emitter (RBE = 15)?

(b) the contaminant is a 1.1 MeV beta emitter?

(c) the contaminant is a 0.01 MeV gamma emitter?

Answer: (a) 1.0×10^{10} Bq (b) 4.2×10^{11} Bq (c) 7.8×10^{13} Bq

38

MEDICAL IMAGING

Introduction to Biological Physics for the Health and Life Sciences Franklin, Muir, Scott, Wilcocks and Yates

Magnetism and MRI

39

39.1 Problems

39.1 A β^+ particle is moving at 19 500 m s^{-1} parallel to the ground and due east through a region of space in which there is a uniform 0.05 T magnetic field. If the magnetic field lines point upwards in the vertical direction, what is the direction and magnitude of the magnetic force on the β^+ particle? Answer: $F = 1.56 \times 10^{-16}$ N due south

39.2 In a region of space there is a uniform magnetic field of magnitude 0.25 mT pointing vertically straight down.

(a) If an electron is moving in the horizontal plane at a speed of 550 km s^{-1}, what will the radius of the resultant circular path be?

(b) Will the electron be moving clockwise or counter-clockwise when viewed from above?

(c) If a positron (same mass as an electron but with a charge of $+q_e$) is moving in the horizontal plane at speed of 550 km s^{-1}, what will the radius of the resultant circular path be?

(d) Will the positron be moving clockwise or counter-clockwise when viewed from above?

(e) How fast would an α particle ($^4_2\alpha^{2+}$) need to be traveling to have a path of the same radius as the electron in part (a)?

Answer: (a) $r_{e^-} = 0.0125$ m (b) clockwise (c) $r_{p^+} = 0.0125$ m (d) counter-clockwise (e) $v_\alpha = 151$ m s^{-1}

39.3 An electron travelling parallel to the ground and due north enters a region of space in which there is uniform magnetic field of 0.5 μT pointing straight up. The electron is travelling at a speed of 1000 km s^{-1}.

(a) What is the magnitude of the magnetic force on the electron (in N)?

(b) In which direction is the magnetic force on the electron as it enters the region in which there is a magnetic field?

(c) As the electron curves in the magnetic field the direction of the magnetic force on the electron changes. How many seconds (after entering the region in which there is a magnetic field) before the magnetic force on the electron is pointing due south?

Answer: (a) $F = 8.0 \times 10^{-20}$ N (b) due west (c) $t = 1.8 \times 10^{-5}$ s

39.4 An electron enters a region in which there is a uniform electric field of 2.25 μT in the z–direction. The electron moves through the region in a 'corkscrew' pattern as shown in Figure 39.1. The radius of the corkscrew path is 0.15 m while the 'pitch' is 0.05 m. What is the velocity of the electron?

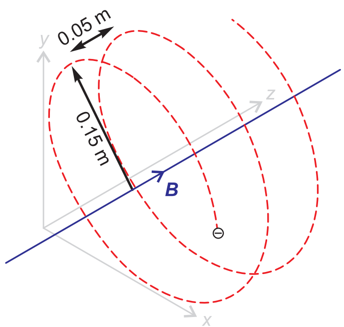

Figure 39.1 An electron moving in a region of uniform magnetic field.

Answer: $v = 59.4$ km s^{-1} at an angle of 87.0° to the magnetic field direction.

39.5 Earths magnetic field at the surface near the poles is around 60×10^{-6} T. What is the Larmor frequency of protons at this location? Answer: $f = 2550$ Hz

39.6 A patient is placed in an MRI machine and as the MRI is taken the magnetic field level with the patients eyes is 1.5 T. The magnetic field is largest at the persons head and the field gradient in the machine is 0.05 T m^{-1}. The patients heart is located 30 cm below their eyes, the patients liver is located 60 cm below their eyes, and the patients bladder is located 70 cm below their eyes. Assuming the magnetic field gradient remains constant across the whole body, what is the Larmor frequency at the patients eyes, their liver, and their bladder (in MHz)? Answer: $f_{\text{eyes}} = 63.75$ MHz, $f_{\text{liver}} = 62.48$ MHz, $f_{\text{bladder}} = 62.26$ MHz

39.7 The MRI apparatus in Problem 39.6 can distinguish between signals whose frequencies differ by just 0.005 MHz. What is the resolution of this machine along the persons body? Answer: Short Answer

39.8 The Larmor frequency of protons at the top of a patient's head is 54.30 MHz. The Larmor frequency of protons at the top of the spine 18.0 cm below is 55.15 MHz.

(a) What is the magnetic field strength in the MRI apparatus at the top of the head (in T)?

(b) What is the magnetic field gradient in the MRI apparatus (in $\mathrm{T\,m^{-1}}$)?

(c) The base of the patient's spine is 1.05 m below the top of their head. What will the Larmor frequency at the base of patients spine be assuming that the magnetic field gradient remains constant across the whole body (in MHz)?

Answer: (a) $B_{\mathrm{topofhead}} = 1.278\,\mathrm{T}$ (b) $B_{\mathrm{topofspine}} = 1.298\,\mathrm{T}$ (c) $f_{\mathrm{baseofspine}} = 59.27\,\mathrm{MHz}$