

# Chapter 7: RL and RC Circuits

## Capacitors →

- $i_c(t) = C \frac{dv_c(t)}{dt}$
- $v_c(t) = v_c(0) + \frac{1}{C} \int_0^t i_c(t) dt$  for  $t \geq 0$
- open circuit for dc voltage
- at  $t=0^+$  :-  $v_c(0^+) = v_c(0^-)$



- at  $t=0^-$  and  $t > 0$
- 
- A circuit diagram showing a capacitor being replaced by an open circuit.

Note:  
 RL circuit to find initial Energy stored in:  $L: W = \frac{1}{2} L I^2$

Total Energy of a Resistor  
 $W = \int_0^t P dt$   
 $i^2 R$  or  $\frac{v^2}{R}$  or  $v i$

## Inductors:

- $v_L(t) = L \frac{di_L(t)}{dt}$
- $i_L(t) = i_L(0) + \frac{1}{L} \int_0^t v_L(t) dt$  for  $t > 0$
- short circuit for dc
- at  $t=0^+$   $i_L(0^+) = i_L(0^-)$



RC circuit to find initial Energy stored in C:  
 $W = \frac{1}{2} C V^2$

Total Energy of a Resistor (same as in RL circuit)

- at  $t=0^-$  and  $t > 0$
- 
- A circuit diagram showing an inductor being replaced by a short circuit.

## First order Circuit:

• Natural Response •  $(i_L, v_C) = 0$

• first order differential equation (Non Homogenous or Homogenous)

•  $\tau = \frac{L}{R}$  (for RL circuit)

•  $i(t) = \frac{V_s}{R} e^{-t/\tau} \quad t > 0$

•  $i_L(0^+) = i_L(0^-) = i_L(0^-) = \frac{V_s}{R}$

RL Circuit

•  $v_C(0^-) = V_s = V_{open\ circuit}$

•  $\tau = RC$

•  $i(t) = -\frac{V}{R} e^{-t/\tau}$

RC Circuit

• Note: unbounded response when the circuit is closed at  $t=0$

## Step Response

RL Circuit

$$V_s = Ri(t) + L \frac{di(t)}{dt}$$

$$i(t) = i_n(t) + i_f(t)$$

natural response

forced response

$$= \frac{V_s}{R} (1 - e^{-t/\tau})$$

$$\tau = \frac{L}{R}$$

use General form

$$r(t) = r(\infty) + [r(0^+) - r(\infty)] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}} \quad \text{or} \quad \tau = R_{eq} C$$

as seen from cor L

RC - Circuit

$$v_C(t) = v_{on}(t) + v_{off}(t)$$

$$v_C(0^+) = A e^{-t/\tau} + K (1 - e^{-t/\tau})$$

$$v_C(\infty) = V_s$$

Revise Sequential Response switching