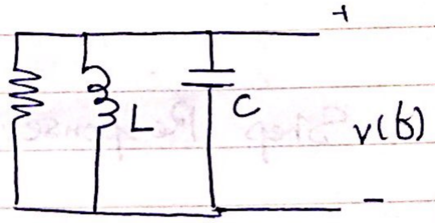
Chapter 8: RLC - Circuits

- Second order Differential Equation:-

Natural Response:

$$C\frac{d^2v(t)}{dt} + \frac{dv(t)}{R\,dt} + \frac{1}{L}v(t) = 0$$



Knowing that $v(t) = Ae^{st}$

$$Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0$$ ← Characteristic Eq

$$s_1, s_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

or $$s_1, s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$\underbrace{\omega_0 = \frac{1}{\sqrt{LC}}, \quad \alpha^2 = \frac{1}{2RC}}$$

Depending on Both There are 3 Cases:-

$\alpha > \omega_0$
- s_1, s_2 real, unequal
- over damped

$$v(t) = A_1e^{s_1t} + A_2e^{s_2t}$$

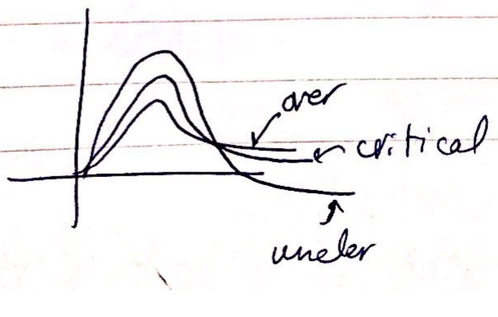
$\alpha < \omega_0$
- s_1, s_2 Complex Conjugate
- under damped

$$v(t) = e^{-\alpha t}[B_1\cos\omega_d t + B_2\sin\omega_d t]$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$\alpha = \omega_0$
- s_1, s_2 real equal
- Critically damped

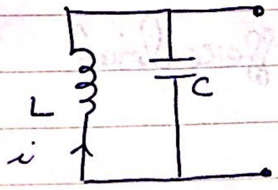
$$v(t) = A_1te^{-\alpha t} + A_2e^{-\alpha t}$$



loseless LC - Circuit:

$R = \infty$

$\alpha = 0$



Step Response:

$$\frac{d^2 i_L(t)}{dt^2} + \frac{1}{RC}\frac{d i_L(t)}{dt} + \frac{1}{LC} i_L(t) = \frac{I}{LC}$$

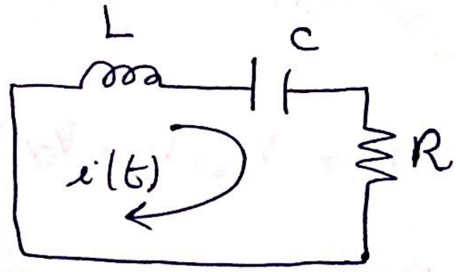
$$i = i_n + i_f$$

K

Series RLC Circuits

The equation Used:-

$$L\frac{di(t)}{dt} + R\,i(t) + V_c(0^-) + \frac{1}{C}\int_0^t i(t)\,dt = 0$$



↓

• $L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \alpha = \frac{R}{2L}$$

$$S_{1,2} = -\alpha \mp \sqrt{|\omega_0^2 - \alpha^2|}$$

* Cases:
  - underdamped
  - overdamped
  - critical Damping

• Same Equations as parallel But $i(t) = i_n$

Responses: Step Response: $v(t) = v_n + v_f$

$v(0^+) = V_c(0^-) = 0$, $i_L(0^-) = 0 = i_L(0^+)$

$V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt$ → from Solution

To find $V_c(t)$

$$V_s = RC\frac{dV_c(t)}{dt} + LC\frac{d^2V_c(t)}{dt^2} + V_c(t)$$

$$V_c(t) = V_{cn}(t) + V_{cf}(t)$$

$$K = V_s$$

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