

# Chapter 9:

## The Sinusoidal Source :-

$$V_s(t) = V_m \sin \omega t$$

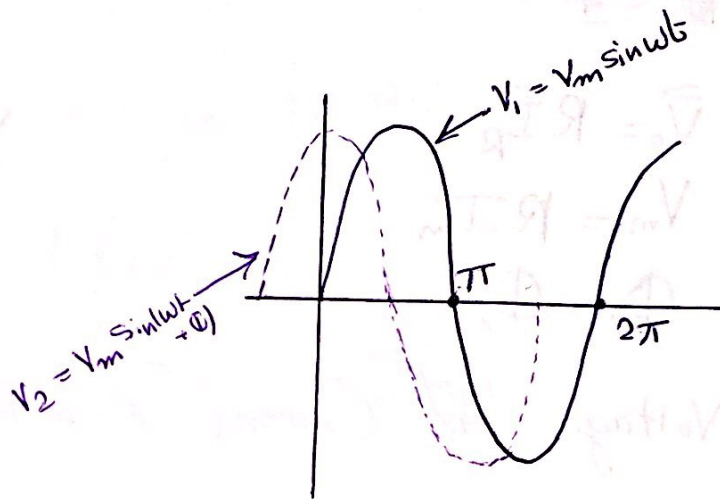
↑
Angular frequency

Amplitude of  
the sinusoid

$$\omega = 2\pi f$$

$$f = \frac{1}{T}$$

- $V_2$  leads  $V_1$  by  $\phi$
- $V_1, V_2$  have the same frequency



## The Sinusoidal Response

$$i(t) = i_n(t) + i_f(t)$$

$$I_0 e^{-t/\tau}$$

Transient  
Component

$$I_1 \cos \omega t + I_2 \sin \omega t = y_p$$

مكونة جيبية لعدد لا نهائي

steady-state  
Component

\* To use Calculator :-

$$Z_1 = 4 + j3 \rightarrow \text{Calculator} \rightarrow 5 \angle 36.9^\circ$$

Pol(4,3) Then [RCI] tan

$$5 \angle 36.9 \rightarrow \text{Calculator}$$

Rec(5, 36.9) Then [RCI] tan

↑ to give Re
↑ to give Im

• Phasors

$$i(t) = I_m \cos(\omega t + \Phi_i)$$

$$v(t) = V_m \cos(\omega t + \Phi_v)$$

$$\vec{I} = I_m \angle \Phi_i$$

$$\vec{V} = V_m \angle \Phi_v$$

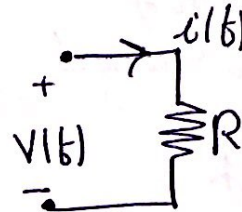
• Phasor Relationships for Circuit Elements

→ Resistor

$$\vec{V}_R = R \vec{I}_R$$

$$V_m = R I_m$$

$$\Phi_v = \Phi_i$$



\* Voltage and Current of a Resistor are in phase

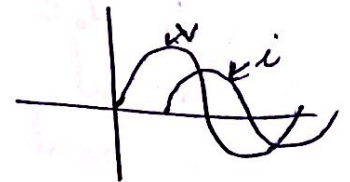
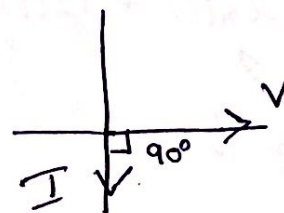
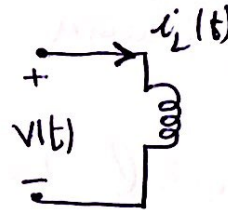
→ Inductor

$$\frac{\vec{V}_L}{\vec{I}_L} = j\omega L$$

↑  
Impedance

$$V_m = \omega L I_m$$

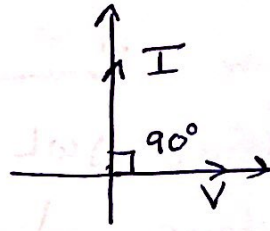
$$\Theta_v = \Theta_i + 90$$



• Voltage leads the Current by 90°

→ Capacitor

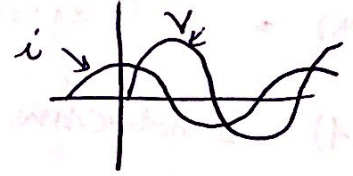
$$\vec{V}_c = \frac{1}{j\omega C} \vec{I}_c$$



$$I_m = \omega C V_m$$

$$\Phi_i = \Phi_v + 90^\circ$$

Current leads the Voltage by  $90^\circ$



### ■ Impedance and Admittance

$$\square Z(j\omega) = \frac{\vec{V}}{\vec{I}} \sim \Omega \quad \text{Imp}$$

$$\square Y(j\omega) = \frac{\vec{I}}{\vec{V}} \sim \Omega^{-1} \quad \text{Adm}$$

$$Z(j\omega) = \frac{1}{Y(j\omega)}$$

•  $\vec{Z}(\text{Imp}) = R + jX$

↑  
Resistive Part

↙  
Reactive Part

$$X = |Z| \sin \theta_Z$$

$$R = |Z| \cos \theta_Z$$

$$\theta_Z = \theta_v - \theta_i$$

→ You can use Current and Voltage Divider Rules  
to find  $Z_1, Z_2, \dots$

• load 5 Types:- (look at  $Z_{eq}$  To know)

- 1) pure Resistive  $R$
- 2) pure Inductance  $j\omega L$
- 3) ~ Capacitance  $\frac{1}{j\omega C}$
- 4) Inductive load  $R + jX$
- 5) Capacitive load  $R - jX$

Remk: If  $Z_L = Z_C$  Then

$$\omega = \frac{1}{\sqrt{LC}}$$

→ where  $Z_{eq} = R$  Resistive

↑ Resonant frequency

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$

$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

where

$$C = \sqrt{A^2 + B^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{B}{A}$$

## Complex Numbers:-

forms:-

1) Rec. form

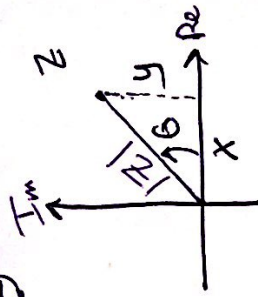
$$Z = x + jy$$

2) Exp. form

$$Z = |Z| e^{j\theta}$$

3) Pol. form

$$Z = |Z| \angle \theta$$



Multiplication:-

$$\rightarrow Z_1 Z_2 = |Z_1| |Z_2| \angle \theta_1 + \theta_2$$

$$\rightarrow \frac{Z_1}{Z_2} = \frac{|Z_1|}{|Z_2|} \angle \theta_1 - \theta_2$$