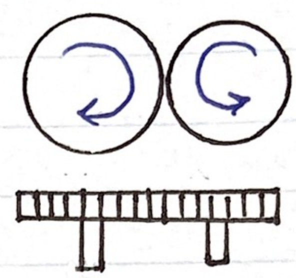
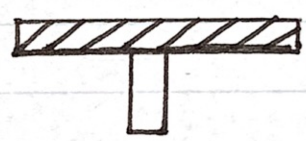
Chapter 13 : Gears

Types of gears

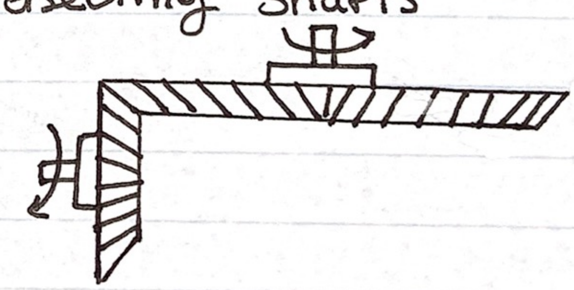
→ Spur Gear : Teeth parallel to axis of rotation
- And shafts are parallel



→ Helical Gear : Teeth inclined to the axis of rotation
- less noisy
- thrust loads and bending couples due to inclined teeth



→ Bevel Gears : teeth formed on conical surfaces
- Transmitts motion between intersecting shafts



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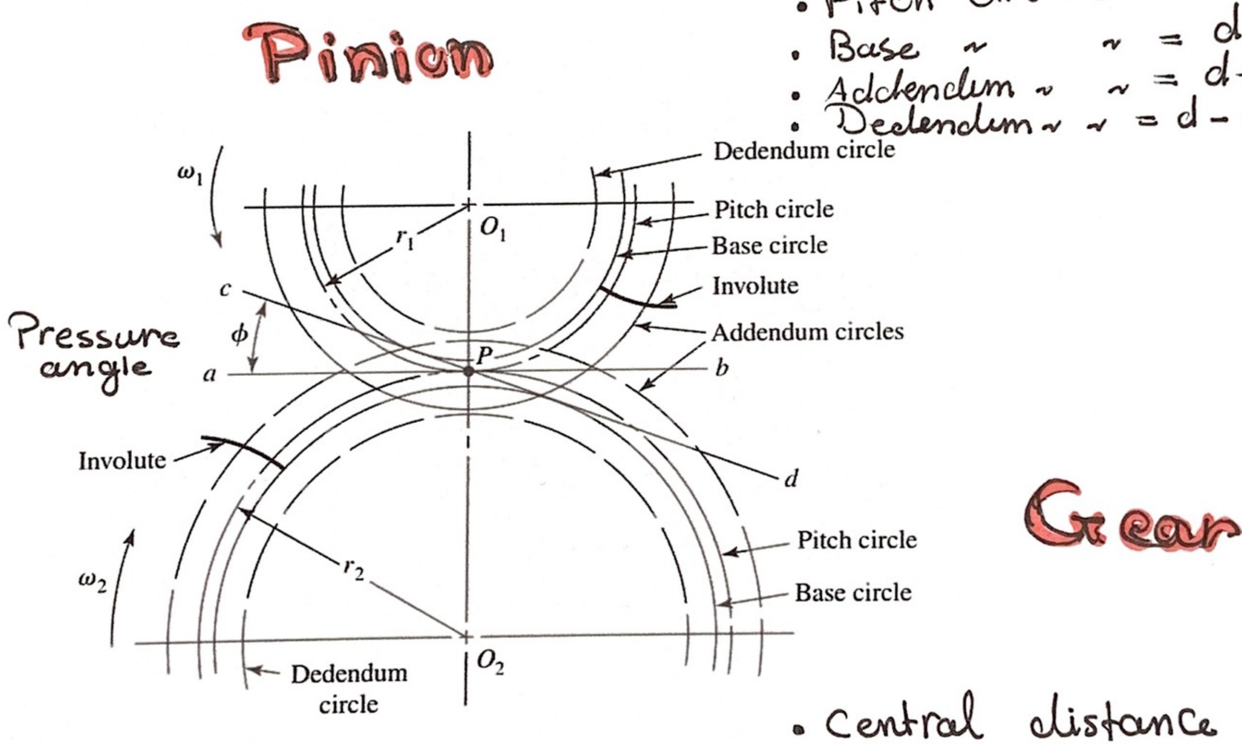
→ Worm and Worm gears:-

- The worm is a screw
- The Worm gear → wheel
- Direction of Rotation of wheel Depends on:-
  1- Cut of the warm (right or left hand)
  2- Rotation of worm
- Types, Single & Double enveloping

In a Simple Gear:

- The pinion is the smaller

- Pitch circles should be tangent to each other
- Pitch circle diameter = d
- Base 〃 〃 = $d \cos\phi$
- Addendum 〃 〃 = $d + 2a$
- Dedendum 〃 〃 = $d - 2b$

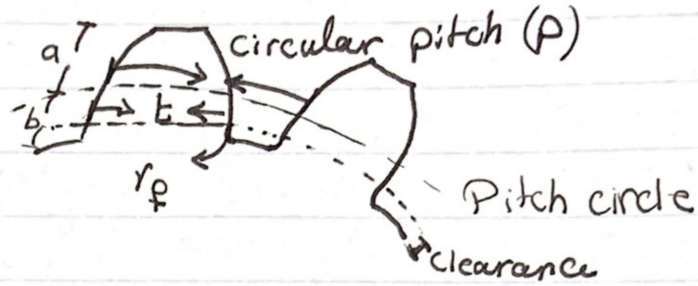


- Central distance

$$C = \left(\frac{N_p + N_g}{2}\right) \cdot m$$

Definitions:-

- Circular Pitch = tooth thickness + width of face
- t: tooth thickness $= \frac{P}{2}$
- Circular Pitch $(P) = \pi m = \frac{\pi d}{N}$
- r_f: fillet radius $= 0.35\,m$



- Diametral Pitch $(P) = \frac{N}{d} = \frac{1}{m}$ (U.S units) teeth / inch

  where

  N: number of teeth

  d: pitch diameter

  m: module (for SI units) mm / teeth
- $P\,P = \pi$

- Addendum $a = m = \frac{1}{P}$
- Dedendum $b = 1.25m = \frac{1.25}{P}$
- Clearance $c = b - a$
- Base pitch $P_b = P_c \cos\phi$ $\qquad P_c$: circular Pitch $= \pi\, d/N = \pi m$

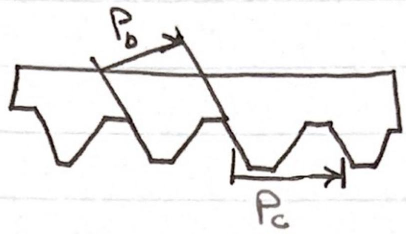
Conjugate Action:-

- To be able to transmitt a uniform rotary motion from one shaft to another → gears produce constant angular velocity ratio
- To do so the point of contact must pass throup pitch point P

- Involute profile provides this

Rack and Pinion

$N = \infty$



Rack

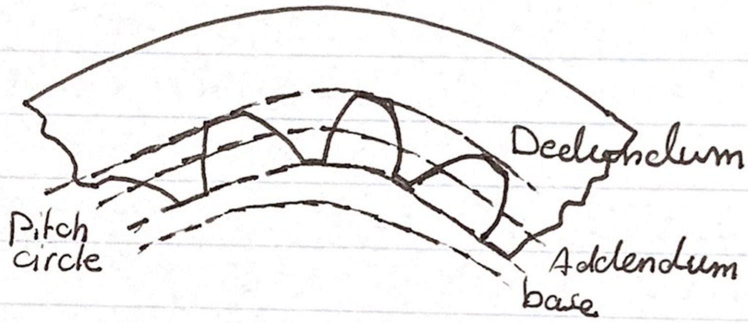
Internal gear

Addendum & dedundum circles are reversed

$d_a = d - 2m$

$d_d = d + (2)(1.25)(m)$

$$C = \left(\frac{N_g - N_p}{2}\right) \cdot m$$



Contact Ratio

Number of teeth in contact at the same moment → must be >1

$$c.r = \frac{L_{ab}}{P_b} = \frac{\sqrt{(r_{ap})^2 - (r_{bp})^2} + \sqrt{(r_{ag})^2 - (r_{bg})^2} - C\sin\phi}{\pi m \cos\phi}$$

Line of contact

For internal gear:

$$c.r = \frac{L_{ab}}{P_b} = \frac{\sqrt{(r_{ag})^2 - (r_{bg})^2} - \sqrt{(r_{ap})^2 - (r_{bp})^2} + C\sin\phi}{\pi m \cos\phi}$$

How did we obtain the equation? Answer in last Page →

Interferance

→ When involute region hits non involute region

→ undercutting is not a good solution

To prevent interference in an external gear :-
Choose N_p , N_g correctly

$$N_{P_{min}} = \frac{2K}{(1+2V_R)(\sin\phi)^2}\left(V_R + \sqrt{(V_R)^2 + (1+2V_R)(\sin\phi)^2}\right)$$

Where : $K = 1$ for full depth
$K = 0.8$ for stub

$$N_{p\,max} = \frac{(N_p)^2(\sin\phi)^2 - 4K^2}{4K - 2N_p(\sin\phi)^2}$$

: N_p is the chosen after previous equation

Effect of increasing center distance $C' = C + \Delta C$

1. Clearance or (back lash) between gears increases
2. pitch radii increase for both gear & pinion
3. V_R remain constant

تتحدد شكل السن → ϕ

4. Pressure angle remains same (cut pressure angle)

تتحدد شكل الحركة → ϕ'

5. operating pressure angle increase

$$\cos\phi' = \frac{C}{C'}\cos\phi$$

$\tan\phi - \phi$

$$B = \text{Backlash} = 2C'(\text{Inv}\,\phi - \text{Inv}\,\phi')$$

$$t' = 2R'\left(\frac{t \rightarrow \frac{P}{2}}{2R} - \underset{\text{rad}}{(\tan\phi - \phi)} - \underset{\text{rad}}{(\tan\phi' - \phi')}\right)$$

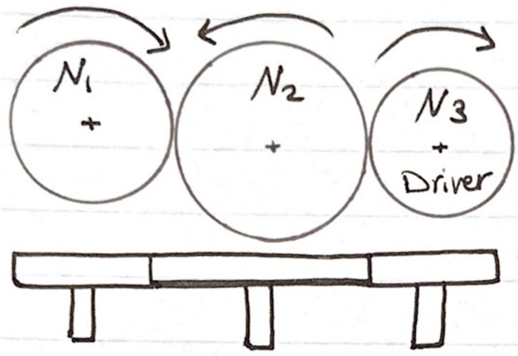
(Video 13)

Note:
$R_p' = \frac{N_p}{N_p + N_g} C'$

$R_g' = C' - R_p'$

Gear trains

1. Simple Gear train : one gear on each shaft



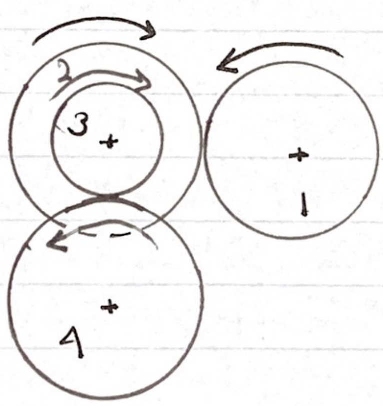
$$V_R = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_1}{\omega_3}$$

$$= -\frac{\omega_1}{\omega_2} \times -\frac{\omega_2}{\omega_3} = -\frac{N_1}{\cancel{N_2}} \times -\frac{\cancel{N_2}}{N_3}$$

$$= \frac{N_1}{N_3}$$ ← last gear / ← first gear

- Intermediate gear/s (idler) has no effect on V_R only direction of Rotation

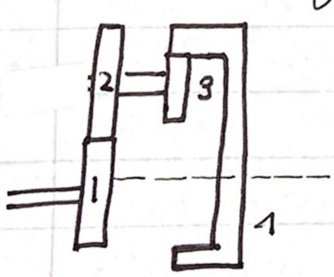
2. Compound Gear train : Multiple gears / shaft

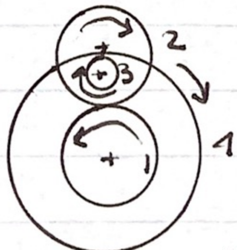


$$V_R = \frac{\omega_1}{\omega_4} \times \frac{\omega_3}{\omega_4} = -\frac{N_2}{N_1} \times -\frac{N_4}{N_3}$$

$$= +\frac{N_2 N_4}{N_1 N_3} = \frac{\text{Product of Driven}}{\text{Product of Driver}}$$

If the gear is internal



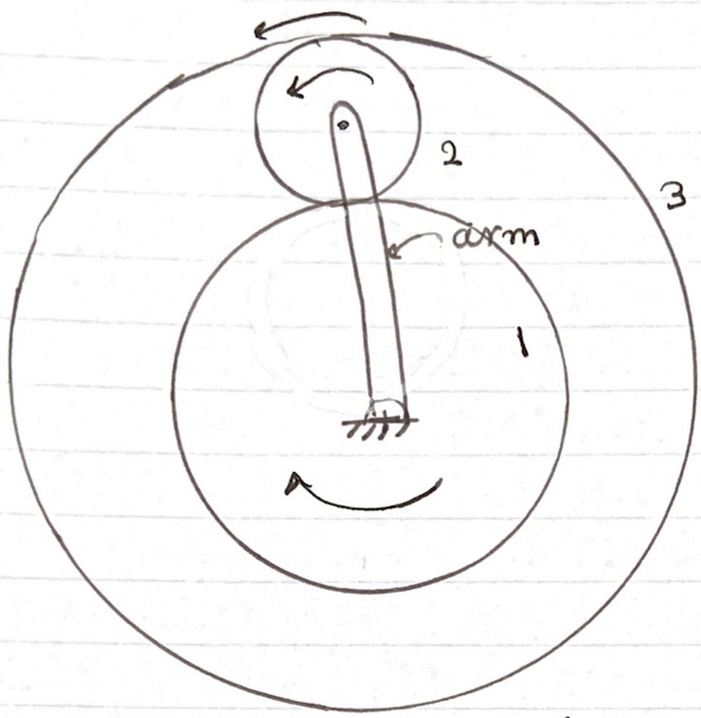


$$V_R = -\frac{N_2}{N_1} \times \frac{+N_4}{N_3}$$ ← Internal

For gears to mesh:

- Same m, P
- Same ϕ
- Same t
- Same P_c
- Same a, b

• Planetary Gears



Assume arm is fixed

$$\frac{\omega_3}{\omega_1} = \frac{-N_1}{N_3}$$

Assume arm is not fixed

$$\frac{\omega_3 - \omega_{arm}}{\omega_1 - \omega_{arm}} = \frac{-N_1}{N_2} \times \frac{N_2}{N_3}$$

In general: $\dfrac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \dfrac{\text{driven teeth}}{\text{driver teeth}}$

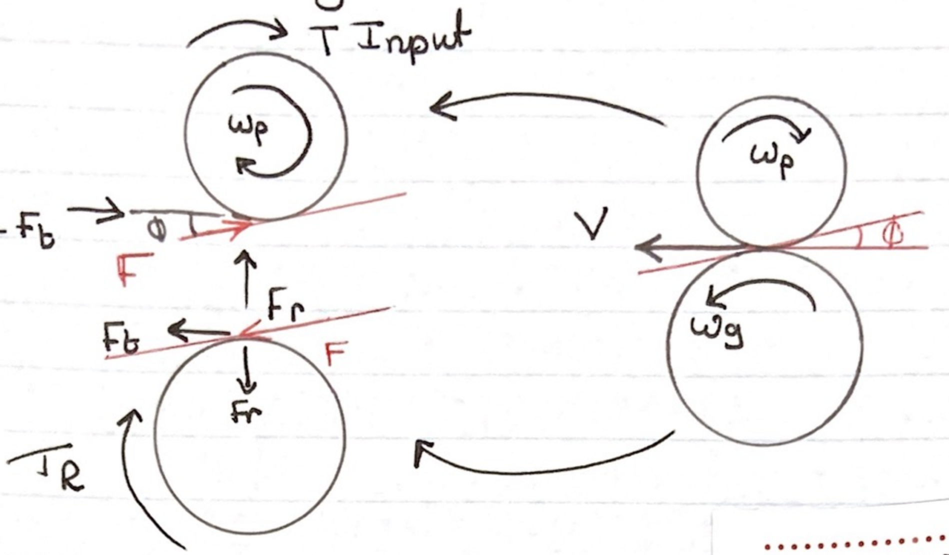
F, L same Direction ⇒ +

F, L opposite " ⇒ −

Gear force analysis

- Simple

T تعاكس ال
عشان $\Sigma T = 0$



$V_p = V_g$
No slipping
$N_p d_p = N_g d_g$

$$F_t = \frac{2T}{d}$$
$$F_r = F_t \tan\theta$$

- $(F_t)_p$: هي الي بتشغل الPower
gear لل

F_r : Does not transmit power

$T = F_t \dfrac{d}{2}$, $T_g \neq T_p$

- Power:

<u>In Hp</u> $\quad Hp = \dfrac{F_t V}{33\,000}$ $\quad F_t \rightarrow$ [lb] , $V = \dfrac{\pi d n}{12}$ in pm

If Torque is Given

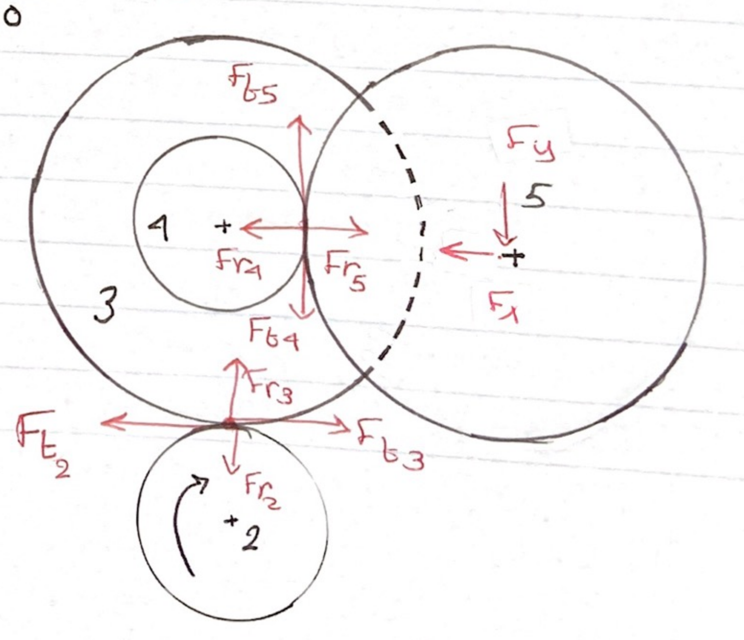
- $Hp = \dfrac{T n}{63\,025}$ $\quad T \rightarrow$ [lb.in] , $n = RPM$

<u>In Watt</u> $\quad W = F_t V$ $\quad F_t \rightarrow$ [N] , $V = \dfrac{\pi d n}{60}$ m/s

- Compound

F_r always in direction of center

<u>Important</u> $F_{t5} \neq F_{r3}$
$F_{r5} \neq F_{t3}$



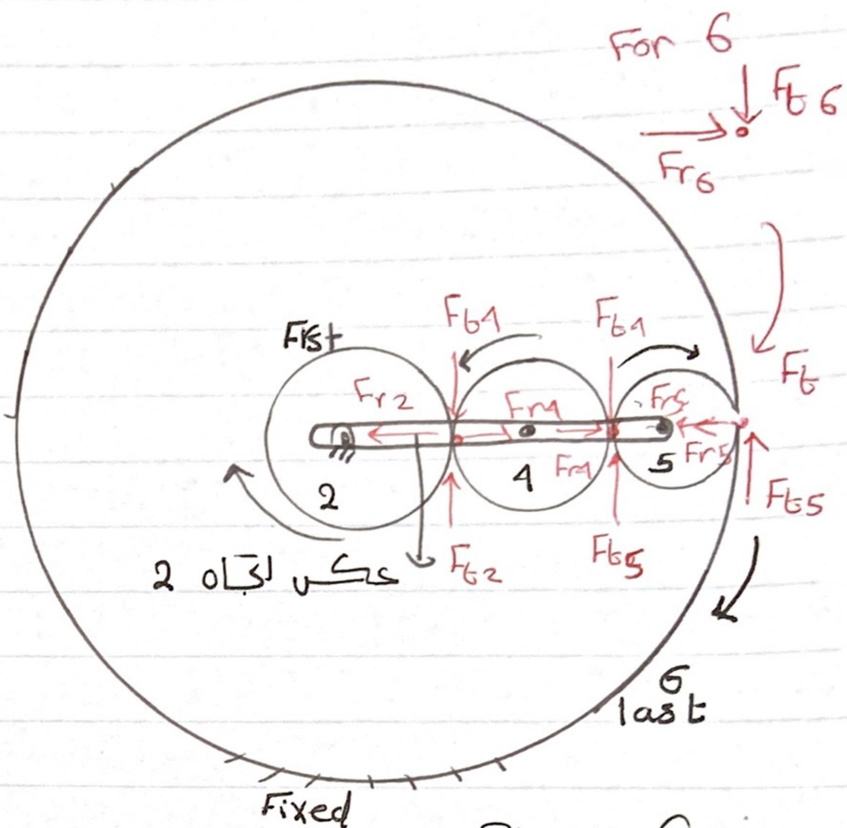
- Planetary

Reaction forces at 4 :-

$F_{x\,4} = 2F_{r4}$

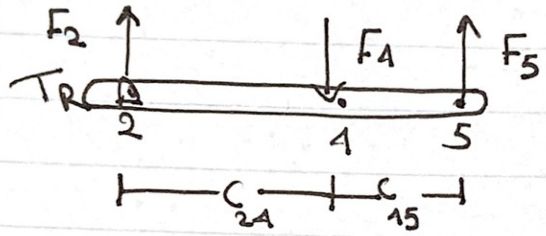
$F_{y\,4} = 2F_{t4}$

Same for 5



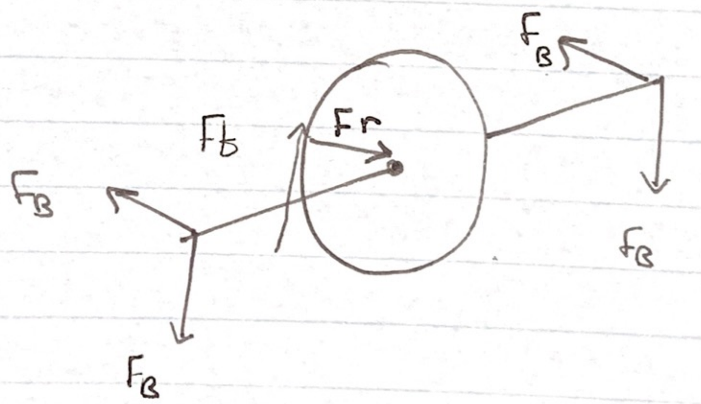
Since 6 is fixed
Then output is at the arm

Taking the arm :

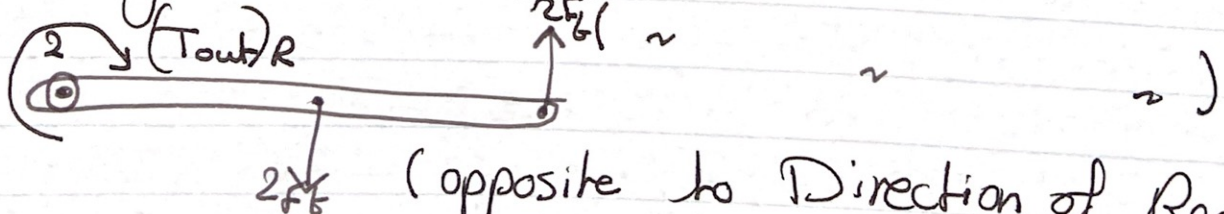


$$T_{about\,2} = (-F_4)C_{24} + (F_5)C_{45}$$
$$= -2F_t C_{24} + 2F_t C_{45}$$

Note : If you have a gear with a shaft



Taking the arm:-



(opposite to Direction of Reaction)

T_{out} is opposite to ΣT from forces