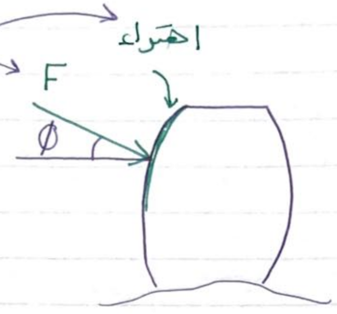
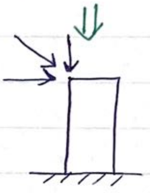
Ch 14: Design of spur gear

Gear design

1. Bending stress
2. Contact stress



Lewis formula

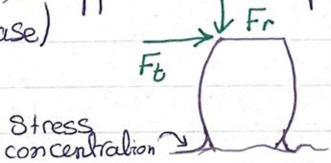


$$\sigma = \frac{Mc}{I}$$

Assumptions

1- All load is applied at tip of single tooth (Worst case)

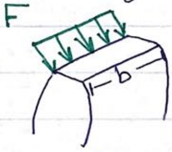
↓ not always ($C.R > 1$ or $C.R > 2$ ....)



2- Fr is negligible (Fr does compression)

3- Stress concentration at fillet is neglicted.

4- load uniformly distributed at full depth of teeth



Design of spur gears

- Bending Stress

lewis equation for spur gear bending

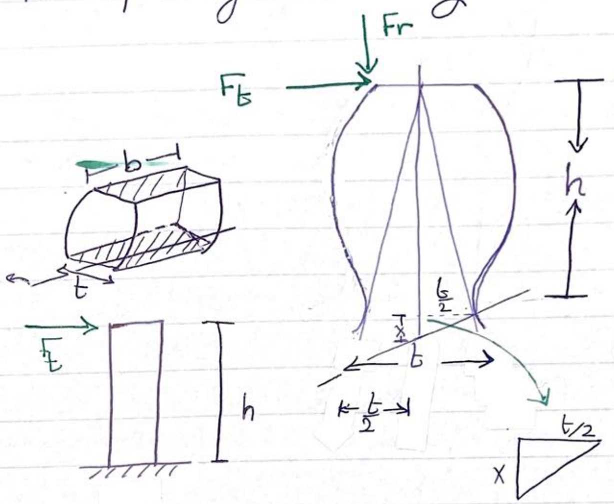
$\sigma = Mc/I$

$M = F_t h$

$I = \frac{bt^3}{12}$

$c = t/2$

moment ال محور



So stress Becomes

$$\sigma = \frac{6 F_t h}{b t^2}$$

$$\frac{x}{t/2} = \frac{t/2}{h} \Rightarrow \frac{t^2}{h} = 4x$$

$$\boxed{\sigma = \frac{6 F_t}{4 x b}}$$

lewis form faction is used: $y = \frac{2x}{3p}$, $P = \frac{\pi}{p}$, $\boxed{Y = \pi y}$

$$\boxed{\sigma = \frac{F_t P}{bY}}$$ if US units are used

↳ not used

$$\boxed{\sigma = \frac{F_t}{b\,m\,Y}}$$ If SI units are used
↳ not used

Factors affecting bending stress of spur gears

1. Velocity Factor (K_v)

- $V\uparrow \rightarrow$ repeated load increases

2. Accuracy of manufacturing the gears accuracy
اذا كانت السرعة عالية وال accuracy قليلة
بصير impact loading بين الأسنان
affects C.R and impact loading

3. C.R
Normally: $1 < C.R < 2$
if $C.R < 1$ load applied at single tooth
if $C.R > 1$ load is shared → يتوزع

برجع بعتمد على ال accuracy : لو كانت قليلة بنختار
نستعمل $C.R<1$ لأنه بدنا ناخذ ال worst case

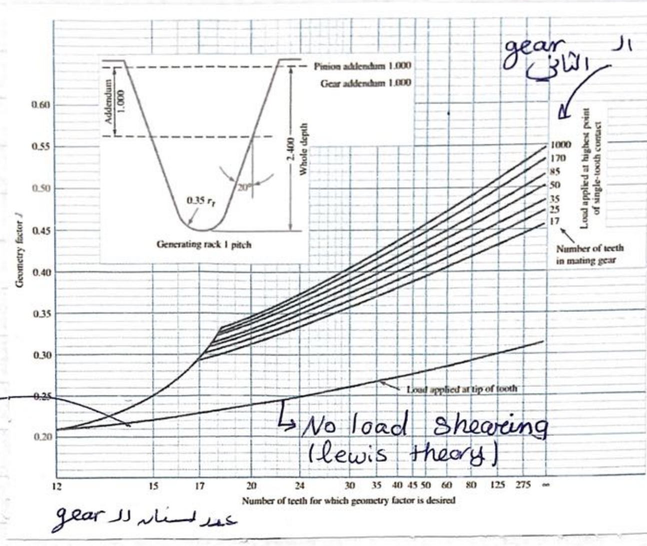
4. Stress concentration at fillet radius
We use K_f, m_N
↳ load sharing factor

$$\bar{J} = \frac{Y}{K_f\, m_N}$$

spur gear geometry factor
obtained from 14-6

→ • Explaining Figure 14-6

If C.R > 1 / acc is high
Don't use lewis theory



We takes it when Q_v is low

5. Load distribution across tooth surface
affected by mounting accuracy and rigidity
اذا كان مسوحله يحني بعض يحس Rigid ، الـ load distrib تقل

6. Impact load ← الشي اللي بتأثر على الـ machine

According to AGMA : $\sigma = \frac{F_t\, P}{b\, J}(K_o\, K_m\, K_v)$ U.S

Factors

$\sigma = \frac{F_t\, K_o\, K_m\, K_v}{b\, J\, m}$ S.I

Back to 1-

* $K_v = \left[\frac{A + \sqrt{200V}}{A}\right]^B$ → m/s $= \frac{\pi d n}{60}$

$B = \frac{(12 - Q_v)^{2/3}}{4}$ → Q_v = gear quality no. → 8–12 high accurate ; → 7 Commercial

Either Q_v is directly given or from Application
ماكنة دقيقة ولا لأ،
عادي ، يعني لستخد
التكلفة أو 8

$A = 50 + 56(1-B)$

If we are using British unit

$$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^{B} \qquad V = \frac{\pi d n}{12} \text{ fpm}$$

K_v can be obtained from Figure 14-9

* K_o = over load factor reflects the impact loading of drive and driven machines

obtained from Table in Figure 14-17 page 758

Driven machine : اللي بيدور → gear اللي بيطلع الحركة

Powersource (Driver) →

Driven →

* K_m mounting factor → Shaft لل (في) Deflection (في) كل
Reflects the rigidity of mounting and shafts

misalignment

Rigidity ↑ → misalignment ↓

Deflection

b in english is F in the Book

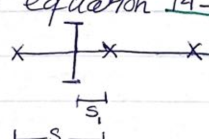
$$K_m = C_{mc}\left[C_{pf}\,C_{pm} + C_{ma}\,C_e\right] + 1 = C_{mf}$$

equation 14-31 → C_{mc} = Crown factor
$\begin{cases} 0.8 & \text{crowned} \\ 1 & \text{uncrowned} \end{cases}$

or Directly from Figure 14-11 ← C_{ma} = housing & gear accuracy
$= A + Bb + CB^2$
A, B, C → Table 14-10

C_{pf} → equation 14-32

C_{pm} : eq 14-33



S_1

$\frac{S}{2}$

C_e :- Alligned 100% → use 0.8
otherwise use 1
eq 14-35

Assumption for K_m to be calculated as previous
1- Straddle mounted (gear mounted between two bearings)

2- $b \leq 40"$

3- $\frac{b}{d} < 10$

Now $\underline{\sigma}$ can be calculated

Bending strength

$$S_e = S_t \frac{K_L K_t}{K_r}$$

S_e: bending endurance limit
S_t: ~ ~ fatigue limit measured for life 10^7 cycles and $R = 99\%$

كل 100 واحد منهم بصير فيه failure

from Table 14-4 or from Figures 14-3 14-4

H_B given or chosen if σ given (load known)

(Pay attention for Psi and Mpa)

In figure 14-3: heat treatment is hardened steel

K_L: life factor if life $\neq 10^7$

K_R: when $R \neq 99\%$ (Reliability) from Table 14-10
Meaning that at $R = 99\%$ $K_R = 1$

K_t: temperature factor: $K_t = 1$ if $t \leq 120°C$ (250 F°)

$K_t = \frac{460 + T}{620}$ if $t > 250$ F°

T In F°

If K_t is in strength then we flip numerator to deno-

To find Factor of Safety

$$\boxed{n = \frac{S_e}{\sigma}} = 1.25 \rightarrow 1.5$$ هاد الـRange المعقول
ممكن تكون أكبر لا مشكلة

- **Note:-**
Rim factor : Solid مش Gear بتكون ال (special case)

لنتخلص من الوزن

K_B can be larger than 1 or 1

$$m_B = \frac{t_R}{h_t} = \frac{\text{Rim thickness}}{\text{height of tooth}}$$

$m_B < 1.2 \rightarrow K_B = 1.6 \ln\left(\frac{2.242}{m_B}\right)$ Figure 14-16

$m_B \geq 1.2 \rightarrow K_B = 1$

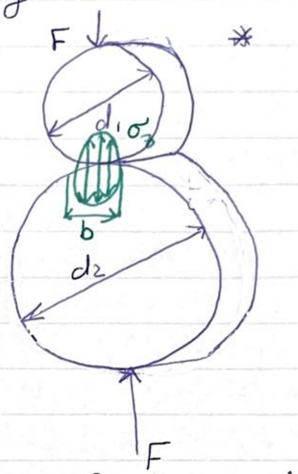
If $K_B > 1 \rightarrow$ put in numerator in σ and denominator in S_e

- **Note**
To find life :-

$L = 60\, L_h\, n_{rpm}$ (rev) and for the meeting gear $\frac{L_P}{L_g} = \frac{\omega_P}{\omega_g}$

Contact stress

- Hertz stress between two cylinders *

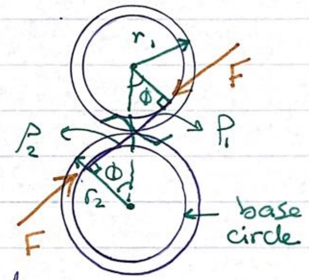


$$\sigma_z = \frac{2F}{\pi b L}$$

b: تقعر نوع التحت

$$b = \sqrt{\frac{\frac{2F}{\pi L}\left(\left(\frac{1-\nu_1^2}{E_1}\right) + \frac{(1-\nu_2^2)}{E_2}\right)}{\left(\frac{1}{d_1}\right) + \left(\frac{1}{d_2}\right)}}$$

- Gear involute is treated as cylinder with radius equals to radius of curvature of involute at contact point



- $\rho_1 = \dfrac{d_1 \sin\phi}{2}$
- $\rho_2 = \dfrac{d_2 \sin\phi}{2}$

- ρ_1 ρ_2 radiuses of curvature for involute

Back to * : ρ_1 represents $\frac{d_1}{2}$ and ρ_2 represents $\frac{d_2}{2}$

Hertz $\longrightarrow$ Contact stress between gears

$d_1 \longrightarrow d_p \sin\phi$

$d_2 \longrightarrow d_g \sin\phi$

$\sigma_z \longrightarrow \sigma_c$

$F \longrightarrow F_t / \cos\phi$

$L \longrightarrow$ b ← face width of teeth not b in *

→ Contact stress becomes

$$\sigma_c = 0.564 \sqrt{\frac{F_t}{\cos\phi\, b}\left[\frac{(2/d_p\sin\phi) + (2/d_g\sin\phi)}{\left(\frac{1-\nu_p^2}{E_p}\right) + \left(\frac{1-\nu_g^2}{E_g}\right)}\right]}$$

C_p = Elastic coefficient

$$= 0.564 \sqrt{\frac{1}{\left(\frac{1-\nu_p^2}{E_p}\right) + \left(\frac{1-\nu_g^2}{E_g}\right)}} = \sqrt{Psi}\ ,\ \sqrt{MPa}$$

$$I = \text{geometry factor} = \frac{\sin\phi\cos\phi}{2}\ \frac{R}{R+1}$$

$$R = \frac{\omega_p}{\omega_g} = \frac{d_g}{d_p} = \frac{N_g}{N_p} = +ve\ (ext)$$

$R = -ve$ (int)

So σ_c becomes:-

$$\sigma_c = C_p \sqrt{\frac{F_t}{b\, I\, d_p}} = psi \text{ or } Mpa$$

From Table 14-8

MPa الي بين قوسين

When introducing factors

$$\sigma_c = C_p \sqrt{\frac{F_t\, C_o C_m C_v}{b I\ d_p}}$$

$C_v = K_v$

$C_o = K_o$

$C_m = K_m$

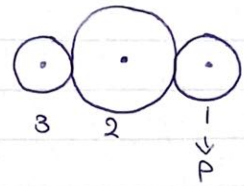
Note:-

$\frac{N_g}{N_p}$ ← if this is taken then we use d_p in σ_c

if $\frac{N_p}{N_g}$ is used then we use d_g in σ_c →

- If we took 1 as Pinion :

$$R = \frac{N_2}{N_1} = \frac{N_g}{N_p} \longrightarrow d_1 \text{ in } \sigma_c$$



- If we took 2 as Pinion

$$R = \frac{N_3}{N_2} = \frac{N_g}{N_p} \longrightarrow d_2 \text{ in } \sigma_c$$

- Contact Strength

$$S_{H_e} = S_c \, \frac{C_L \, C_T \, C_H}{C_R}$$

Figure 14-5
Through hardened steel

S_c → measured for 10^7 life and $R = 99\%$

C_L used if life $\neq 10^7$ — Figure 14-15

C_R 〃 〃 $R \neq 99\%$ — Table 14-10

$C_T = K_T = \frac{620}{460+T}$, $T > 250F$

- C_H = Hardness ratio factor used only for gear
for pinion $C_H = 1$

$1.7 > \frac{H_p}{H_g} > 1.2$ → we take $C_H = 1 + A'(V_R - 1)$

$A' = 8.98\times10^{-3} \frac{H_p}{H_g} - 8.29\times10^{-3}$

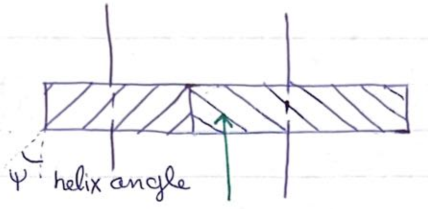
$\frac{H_p}{H_g} < 1.2 \rightarrow C_H = 1$

Exp video 29 + 30

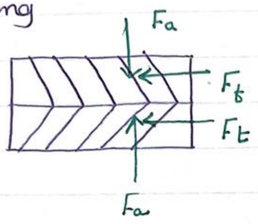
Assignment 31

Helical gear

- No noise : used in cars
- For high power : double helical gear is used

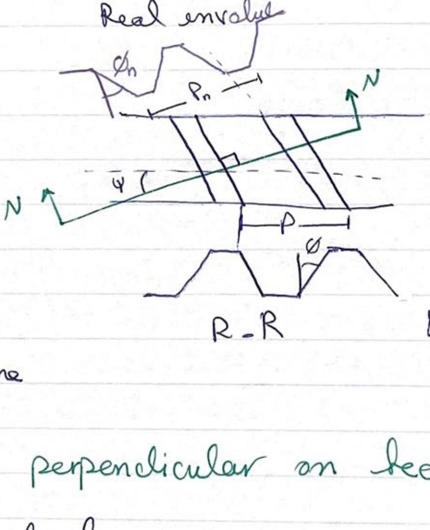


Explaining



Why is it qute:-
Contact spread gradually across fac width

Geometry



N: normal view/plane

R: Rotational plane

- $P_n = P\cos\psi$
- P_n: circular Pitch in normal plane
- P : " " " rotational plane

N-N plane is perpendicular on teeth (Same as spur)

So Pn is the standard

$15 < \psi < 30°$: $\psi \uparrow \longrightarrow$ Smoother motion $\longrightarrow F_a \uparrow$

$\psi \downarrow \longrightarrow$ Same as Spur $\longrightarrow F_a \downarrow$
useless

So ψ should be between 15, 30°

- Meating gears should have same ψ but opposite in hand (Right & left hand)

- When we calculate diameter (and a, b) we use P :-

$$\frac{\pi}{P_n} = \frac{\pi}{P}\cos\psi \longrightarrow P = P_n\cos\psi$$

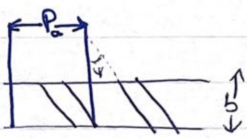
$$\boxed{d = \frac{N}{P}} = Nm$$

only diameter is found using /

$$\rightarrow a = \frac{1}{P_n} = m_n$$

$$\rightarrow b = \frac{1.25}{P_n} = 1.25\, m_n$$

$$\hookrightarrow r_{ap} = r_p + a = \frac{N_p}{2P} + \frac{1}{P_n}$$



Axial Pitch

$$P_a = \frac{P}{\tan\psi}$$

To assure axial overlap or axial contact ratio

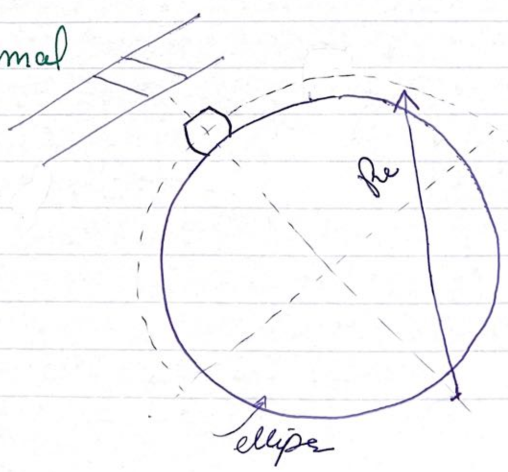
$$m_F = \frac{b}{P_a} > 1.15 \rightarrow 2$$

عشان اتأكد انو ال b طابقة
على اكتر من سن

Involute in normal Plane

Same as ellipse with d = of ellipse = Re

$$R_e = \frac{d}{2}\,\frac{1}{\cos^2\psi}$$



- Virtual number of teeth N_e

No. of teeth of spur gear with $d = 2R_e$

$$P_n = \frac{\pi (2R_e)}{N_e}$$

$$P = \frac{\pi d}{N} = \frac{(2R_e)\pi}{N}$$

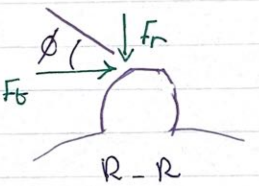
$$N_e = \frac{N}{\cos^3 \psi}$$

عدد أسنان ال spur gear

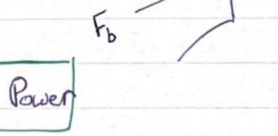
Force analysis of helical gear

ϕ : أنا طالعها

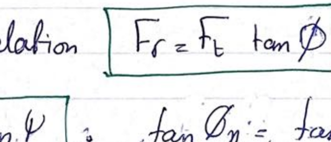
ϕ_n : ثابتة



How to find forces?



Notice that F_n is the resultant of F_r, F_t, F_a (3D force)

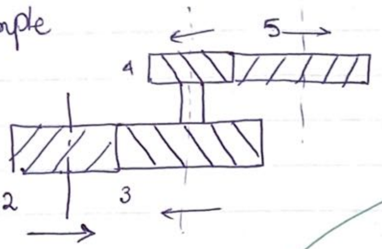


$\boxed{F_t \text{ is obtained from Power}}$

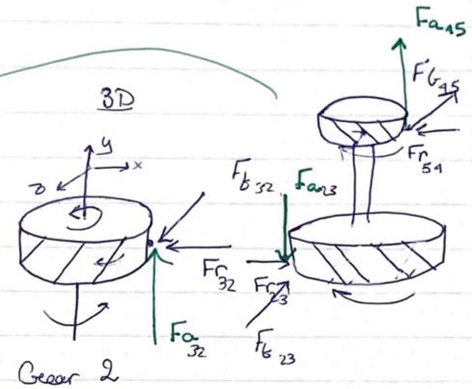
F_r is obtained from relation $\boxed{F_r = F_t \tan \phi}$

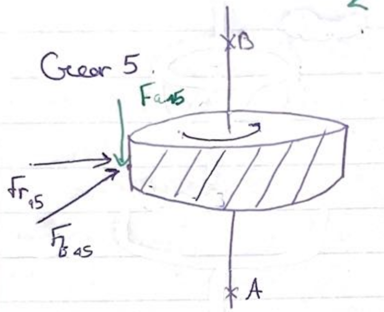
$F_t = F_b \cos \psi$, $\boxed{F_a = F_t \tan \psi}$, $\tan \phi_n = \tan \phi \cos \psi$

Example



Ft

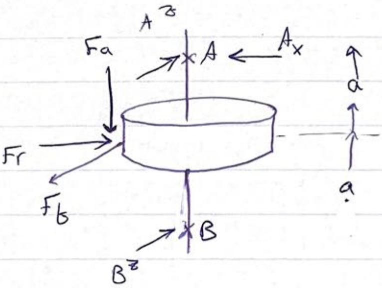




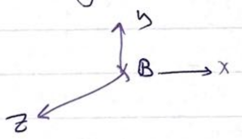
Note Same Shaft
F_t Should do
opposite Torque

If Bearings included : F_t is divided to 2 in A and B , But F_r is not since F_a is introduced

$$A^z = B^z = \frac{F_t}{2}$$



Talking Point B



$$\Sigma M_{B^z} = 0 = F_r(a) - F_a \frac{d}{2} + \overset{\checkmark \text{ found}}{A_x}(2a)$$

Helical gear design

▶ Bending stress

$$\sigma = \frac{F_t \; P \; K_o \; K_m \; K_v}{b \; J}$$

Only difference is J :- Figure 14-7 , J'
14-8

J'_{75} geometry factor = N 75

if $N \neq 75$ use figure 14-8 → K_J

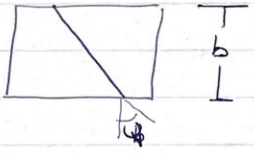
$J = K_J \; J_{75}$

Other analysis

Bending strength , S_e, S_t ---- same as spur gear

▶ Contact stress

$$\sigma_c = C_p \sqrt{\frac{F_t \; C_o C_m C_v}{b \; I \; d_p}}$$



$$\rightarrow b' = \frac{b}{\cos\psi} \cdot (C.R)\;(0.95)$$

• Instead we use new I : $I = \frac{\sin\phi \cos\phi}{2 m_N} \; \frac{R}{R+1}$

$m_N = \frac{1}{0.95\, C.R}$

$m_N = \cancel{\frac{\cos\psi}{0.95\, C.R}}$ and b stays the same

$\frac{1}{0.95\, C.R}$

→ Contact ratio

$C.R = ? = \frac{L_{ab}}{P_{bn}}$ n not P_b

$r_{ap} = r_p + \frac{1}{P_n} = r_p + m_n$

$r_b = r\cos\phi$

$P_b = P_n \cos\phi_n$

→ Sef same as spur gear