

Basic gear rules

External Gears

$$\text{Pitch diameter} = d$$

$$\text{Base circle diameter} = d \cos \phi$$

$$\text{Addendum} \sim \sim = d + 2a = d + 2m = d + \frac{2}{P}$$

$$\text{Dedendum} \sim \sim = d - 2b = d - (2)(1.25)m = \frac{d - 2(1.25)}{P}$$

$$\text{Central distance} = c = \left(\frac{N_p + N_g}{2} \right) m = \frac{d_p + d_g}{2}$$

$$\text{Clearance} = c = b - a$$

$$r_f = 0.35m$$

$$\text{Circular Pitch} = P_c = \frac{\pi d}{P}$$

$$\text{tooth thickness} = \frac{P}{2}$$

$$\left. \begin{aligned} \text{Diametral Pitch} = P &= \frac{N}{d} = \frac{1}{m} \\ \text{Circular Pitch} = P_c = P &= \pi m \end{aligned} \right\} P P = \pi$$

$$\text{Base pitch} = P_b = P_c \cos \phi$$

$$\text{C.r.} = \frac{L_{ab}}{P_b} = \frac{\sqrt{(r_{ap})^2 - (r_{bp})^2} + \sqrt{(r_{ag})^2 - (r_{bg})^2} - C \sin \phi}{\pi m \cos \phi}$$

Internal gears

$$d_a = d - 2m = d - \frac{2}{P}$$

$$d_d = d + (2)(1.25)(m) = d + \frac{(2)(1.25)}{P}$$

$$c = \left(\frac{N_g - N_p}{2} \right) m$$

$$\text{C.r.} = \frac{L_{ab}}{P_b} = \frac{\sqrt{(r_{ag})^2 - (r_{bg})^2} - \sqrt{(r_{ap})^2 - (r_{bp})^2} + C \sin \phi}{\pi m \cos \phi}$$

Interference

External gear

$$N_{p\min} = \frac{2K}{(1+2V_R)(\sin\phi)^2} \left(V_R + \sqrt{(V_R)^2 + (1+2V_R)(\sin\phi)^2} \right)$$

$$N_{g\max} = \frac{(N_g)^2 (\sin\phi)^2 - 4K^2}{4K - 2N_p (\sin\phi)^2}$$

$K = 1 \rightarrow$ full depth
 $= 0,8 \rightarrow$ stub

Internal gear

$$\sqrt{(r_{ag})^2 - (r_{bg})^2} - (r_{bg} - r_{bp}) \tan\phi$$

Backlash

$$B = 2c' (\text{Inv}\phi - \text{Inv}\phi')$$

\uparrow $\tan\phi - \phi$ \uparrow rad

$$\cos\phi' = \frac{c}{c'} \cos\phi$$

$$R_p' = \frac{N_p}{N_p + N_g} c'$$

$$R_g' = c' - R_p'$$

$$t' = 2R' \left(\frac{t}{2R} - (\tan\phi - \phi) - (\tan\phi' - \phi') \right)$$

Thickness \rightarrow

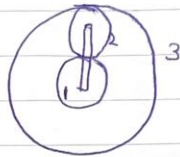
Gear trains

$$V_p = \frac{\omega_{in}}{\omega_{out}} = \frac{N_{out}}{N_{in}} \quad (\text{Ext: neg, Int: pos})$$

Meshed gears have same: m, P, ϕ, t, P_c, a, b

In Planetary gears:-

$$\frac{\omega_3 - \omega_{arm}}{\omega_1 - \omega_{arm}} = -\frac{N_1}{N_2} \times \frac{N_2}{N_3}$$



In general:-

$$\frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \frac{\text{driven teeth}}{\text{driver teeth}}$$

First, last
Same Direction (+)
Negative if opposite direction

Force analysis

$$F_t = \frac{2T}{d}$$

$$F_r = F_t \tan \phi$$

Notes
 $T_g \neq T_p$

Check forces analysis
as w go
Back to

Power :-

$$H_p = \frac{F_t V}{33000}, \quad V = \frac{\pi d n}{12}$$

$$= \frac{T n^2}{63025} \text{ RPM}$$

$$W = F_t V, \quad V = \frac{\pi d n}{60}$$

Re solve
on planetary
force
analysis

Helical gears

$$d = \frac{N}{P} = N m$$

$$a = \frac{1}{P_n} = m_n$$

$$b = \frac{1.25}{P_n} = 1.25 \text{ mm}$$

$$P_a = \frac{P}{\tan \psi} \quad (\text{axial Pitch})$$

$$P = P_n \cos \psi$$

$$m_f = \frac{b}{P_a} > 1.125$$

spur gear
teeth number $N_e = \frac{N}{\cos^3 \psi}$

F_t / Power

$$F_r = F_t \tan \phi$$

$$F_t = F_b \cos \psi$$

$$F_a = F_t \tan \psi$$

$$\tan \phi_n = \tan \phi \cos \psi$$

Note

$$\left\{ \begin{array}{l} F_b \\ F_a \text{ abs} \\ F_t \\ \\ F_n \\ F_b, F_r \text{ abs} \end{array} \right.$$

Design and Spur gear Selection ③

Bending stress of spur gear according to AGMA

$$\sigma = \frac{F_t P}{b J} K_o K_m K_v \quad \text{U.S.}$$

$$\sigma = \frac{F_t K_a K_m K_v}{b J m} \quad \text{S.I.}$$

σ ← σ_s
 σ_p

F_t → Power or given

P → given

b → given

J → Figur 14-6

lower line : Q_v is low

higher lines : Q_v is high

$$\text{(S.I.) } K_v = \text{Dynamic Factor} = \left(\frac{A + \sqrt{200V}}{A} \right)^B$$

$$V = \frac{\pi d n}{60} \quad \leftarrow \text{rev/min}$$

$$B = \frac{(12 - Q_v)^3}{4}$$

$$A = 50 + 56(1 - B)$$

Q_v → 8-12 accurate
 For less commercial

$$(U.S) K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B = \frac{\pi d n}{12} \text{ fpm}$$

K_o = over load factor = Table (S)

Power source (motor)

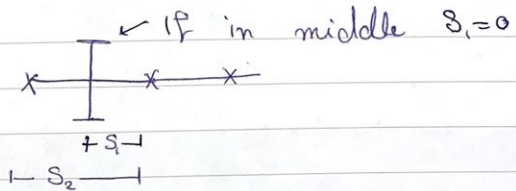
Driven machine (\pm $\frac{S_1}{S_2}$)

$$K_m = C_{mc} \left[C_{pf} C_{pm} + C_{ma} C_e \right] + 1 = C_{mp}$$

$b \leq 40^\circ$ / $\frac{b}{d} < 10$ / straddle

$$C_{mc} = \begin{cases} 0.8 & \text{crowned} \\ 1 & \text{uncrowned} \end{cases}$$

$$C_{pm} = \text{Eq ()}$$



$$C_{pf} = \text{Eq ()}$$

$$C_{ma} = A + Bb + cb^2 \quad (A, B, C) \rightarrow \text{Table 14-10 / Figure 14-11}$$

$$C_e = \text{Eq ()}$$

$$(U.S) K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B = \frac{\pi d n}{12} \text{ fpm}$$

K_o = over load factor = Table (S)

Power source (motor)

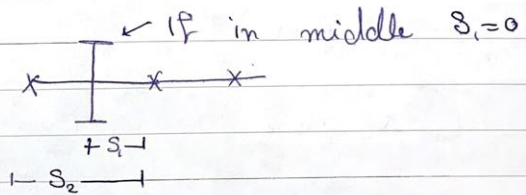
Driven machine (موتور)

$b \leq 40^n / \frac{b}{a} < 10 / \text{straddle}$

$$K_m = C_{mc} [C_{pf} C_{pm} + C_{ma} C_e] + 1 = C_{mp}$$

$$C_{mc} = \begin{cases} 0.8 & \text{crowned} \\ 1 & \text{uncrowned} \end{cases}$$

$$C_{pm} = Eq ()$$



$$C_{pf} = Eq ()$$

$$C_{ma} = A + Bb + Cb^2 \quad (A, B, C) \rightarrow \text{Table 14-10} / \text{Figure 14-11}$$

$$C_e = Eq ()$$

Bending strength of spur gear

$$S_e = S_t \frac{K_L K_b}{K_r}$$

S_e ← endurance limit
 S_t ← fatigue limit
 K_L ← surface finish factor
 K_b ← geometry factor
 K_r ← reliability factor

S_t : Table 14-4 / Figure 14-3, 14-4
 H_B given or assumed if σ known

To find life:-

$$\frac{L_p}{L_g} = \frac{w_p}{w_g}$$

$$L = 60 L_h n \text{ rpm (rev)}$$

Pay ATTENTION for

MPa, PSI

K_L Figure 14.14

K_r : $R \neq 99\%$? Table 14-10

$$K_b: K_b = \frac{160 + T^{\frac{1}{2}}}{620} \text{ if } T > 250F^\circ$$

Factor of safety

$$n = \frac{S_e}{\sigma}$$

flip
 denominator
 or numerals
 (study)

Rim Factor

$$m_B = \frac{bR}{h_t}$$

$$m_B < 1.2 \quad K_B = 1.6 \ln \left(\frac{2.242}{m_B} \right)$$

Figure 14-16

$$m_B \geq 1.2 \quad K_B = 1$$

Contact Stress

←
same
g.f

$$\sigma_c = C_p \sqrt{\frac{F_t \cdot C_a C_m C_v}{b I d p}} = \text{psi or Mpa}$$

$$C_a = K_a$$

$$C_m = K_m$$

$$C_v = K_v$$

$$C_p = \text{Table 14-8}$$

$$I = \frac{\sin \theta \cos \theta}{2} \frac{R}{R+1}, \quad R = \frac{N_g}{N_p} = \frac{+d_g}{d_p}$$

Ext

- If internal

Contact Strength

$$S_{pe} = S_c \frac{C_L C_t C_H}{C_R}$$

S_c : Figure 14-5

C_L : Figure 14-15

C_R : Table 14-10

$$C_t = k_t = \frac{620}{100 + T} \quad t > 250^\circ F$$

$$C_H = \begin{cases} 1 & \text{if } \frac{H_p}{H_g} < 1.2 \\ = 1 + A \cdot (V_R - 1) & , A = 8.98 \cdot 10^{-3} \frac{H_p}{H_g} - 8.2 \alpha \cdot 10^{-3} \end{cases}$$

$$\text{if } \frac{H_p}{H_g} > 1.2$$

Design and Helical gear Selection 6

Bending Stress

$$\sigma = \frac{F_t P K_o K_m K_v}{b J}$$

σ → σ_g σ_p

Same as Spur gear

J , Figure 14-7
 14-8

J_{75} → when $N=75$ of mating gear

$J = K J_{75}$

$K J$, Figure 14-8

Other analysis, S_e, S_t, S_{ef} Same as Spur gear

Contact Strength

$$\sigma_c = C_p \sqrt{\frac{F_t C_o C_m C_v}{b I d_p}}$$

σ_c ← Same as σ_p
 Same

~~$$b = \frac{b}{\cos \phi} (C.R.) (0.95)$$~~

$$I = \frac{\sin \phi \cos \phi}{2 m N} \frac{R}{R+1}$$

$\frac{1}{0.95 C.R.}$

$C.R. = \frac{L a b}{P_{bn}}$ \uparrow changes
 $\frac{1}{P_{bn}} \leftarrow \frac{n}{m}$

$$P_{bn} = P_m \cos \phi_m$$

Bearings

$$C^a L_{10} = F^a L \quad C = F_r \left(\frac{L}{L_{10}} \right)^{\frac{1}{a}}$$

$$a = \begin{cases} 3 & \text{ball bearings} \\ 10/3 & \text{roller } \sim \end{cases}$$

Bearing Selection:-

$$C_{10} = K_a F_e \left(\frac{L}{K_r L_{10}} \right)^{\frac{1}{a}}$$

K_a Table 11-5

$$K_r = X_0 + (G - X_0) (1 - R)^{\frac{1}{b}} \quad \text{Table 11-6}$$

$$F_e = F_r \& F_b \quad \text{No Axial load}$$

$$F_e = \max \begin{cases} V F_r \\ V_x F_r + V F_a \end{cases} \quad \text{with Axial load}$$

and to find C_{10} we need C_0

C_0 : Table 11-2

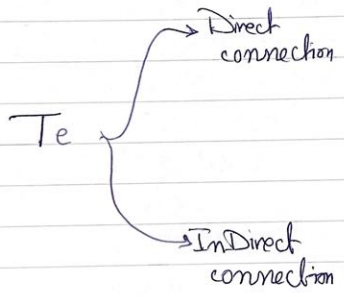
Table 11-1 \rightarrow get $e \rightarrow$ compare $\frac{F_e}{F_r}$ with $e \rightarrow X, Y$

$F_{eq} \checkmark \rightarrow$ get C

Back to Table 11-2 : check if $C_{calc} < C_{table}$ choose another

Tapered Bearings

Timken, $F_a = 0.4 F_r$ $L_{10} = 3000$ hr
 $= 0.75$ \leftarrow Steep angle $K_r = 1.5$ \leftarrow Shallow angle $n = 500$ rpm



Direct connection

$$F_{eA} = 0.4 F_{rA} + K_A \left(\frac{0.4 F_{rB}}{K_B} + T_e \right)$$

$$F_{eB} = 0.4 F_{rB} + K_B \left(\frac{0.4 F_{rA}}{K_A} - T_e \right)$$

Indirect connection

$$F_{eA} = 0.4 F_{rA} + K_A \left(\frac{0.4 F_{rB}}{K_B} - T_e \right)$$

$$F_{eB} = 0.4 F_{rB} + K_B \left(\frac{0.4 F_{rA}}{K_A} + T_e \right)$$

$$F_D = \max \{ F_e, F_r \}$$

\downarrow
 F_{DA} and F_{DB}

$$C = K_a F_D \left(\frac{L_D n_D}{L_0 n_0} \right)^{\frac{3}{T_0}}$$

$$C = k_a F_e \left(\frac{LD \cdot nD}{f_e f_v k_r L R n_R} \right)^{3/10}$$

$$k_r = 4.48 (1-R)^3$$

f_e Figure 11-16

f_v viscosity factor Figure 11-17

Variable loading

$$\frac{l_1}{L_1} + \frac{l_2}{L_2} + \frac{l_3}{L_3} = 1$$

$$F_{eq} = F_1^a \frac{l_1}{L_1} + F_2^a \frac{l_2}{L_2} + F_3^a \frac{l_3}{L_3}$$

To obtain L_i

$$F^a L_i = C^a L_{10} \quad \leftarrow 10^5$$

from table 11-2

$$F_1^a l_1 + F_2^a l_2 + F_3^a l_3 = F_1^a L_1 = F_2^a L_2 = F_3^a L_3 = C^a L_{10}$$