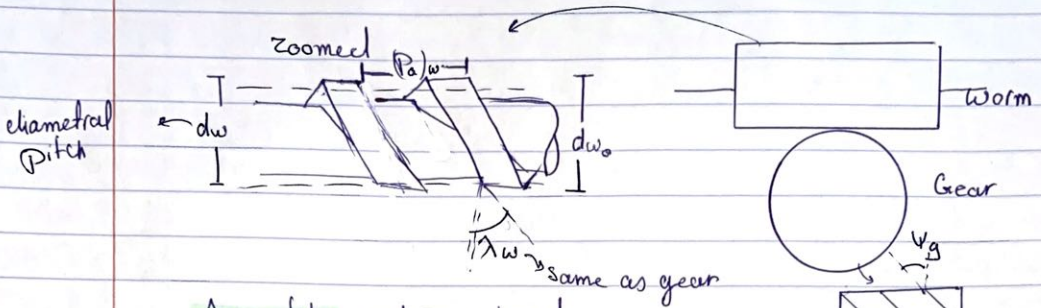


Worm Gear

- Gives high velocity ratio but higher friction (low efficiency)

Worm: power screw



$$\lambda_w = \psi_g \text{ and same hand}$$

$$c = \frac{d_w + d_g}{2}$$

$$(P_a)_w = (P_c)_g$$

$$V_R = \frac{W_w}{W_g} = \frac{N_g}{N_w} \neq \frac{d_g}{d_w}$$

N_w : direction of worm side = Single, double, ...

Number of teeth (N_w) and d_w are not related

But in gear $P_c = \frac{\pi d_g}{N_g}$

$b \leq 0.5 d_w$: Recommended value

$$d_{w0} = d_w + 2a$$

$$(d_g)_0 = d_g + 2a$$

Recommended value: $\frac{C^{0.875}}{3''} < d_w < \frac{C^{0.875}}{1.7''}$ in inch

where: C: center distance

$$\frac{C^{0.875}}{2.002} < d_w < \frac{C^{0.875}}{1.068}$$
 in mm

$$N_g > 24, N_g + N_w > 10$$

(Pa)w has recommended values (P 809)

Lead Angle for the worm: $\tan^{-1} \frac{L}{\pi d_w} = \lambda_w$

Lead = $L = \underbrace{m_w}_{\text{Single, double}} P_a$

$P_a = \frac{\pi}{P_t}$ transverse (Given)

↪ Average not outside

Addendum, dedendum from Table 13-5 P 690

$a_{add} = d_w - 2b$

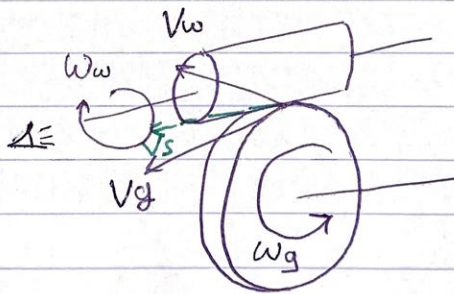
Relation between lead angle and Normal Angle
 From Table 15-9 to prevent interference (The Given is Pressure Angle λ_{max} and $\lambda < \lambda_{max}$)

Force Analysis of Worm gear

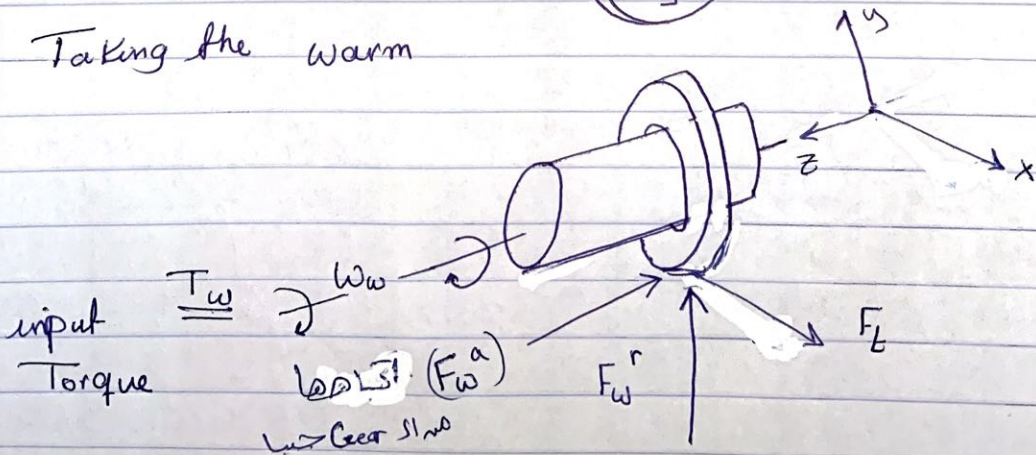
Right hand worm, clock-wise Rotation

Worm is driver

(left hand opposite)



Taking the worm

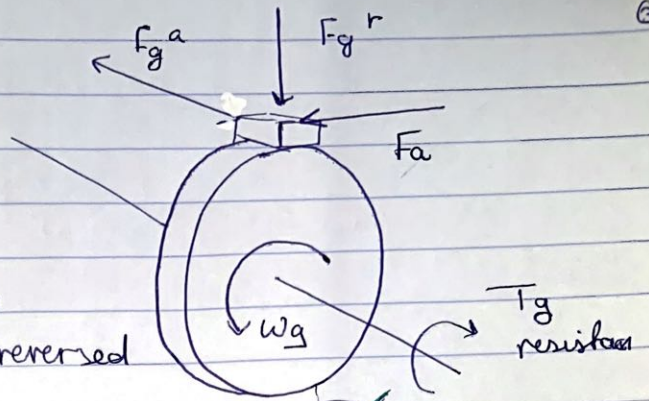


$$F_w^b = F_g^a$$

$$F_w^r = F_g^r$$

$$F_w^a = F_g^b$$

- If left hand, w_g is reversed and F_g^b is reversed



For the worm gear

Worm is Driver

$$F_b = F_n \cos \theta_n$$

$$F_r = F_n \sin \theta_n$$

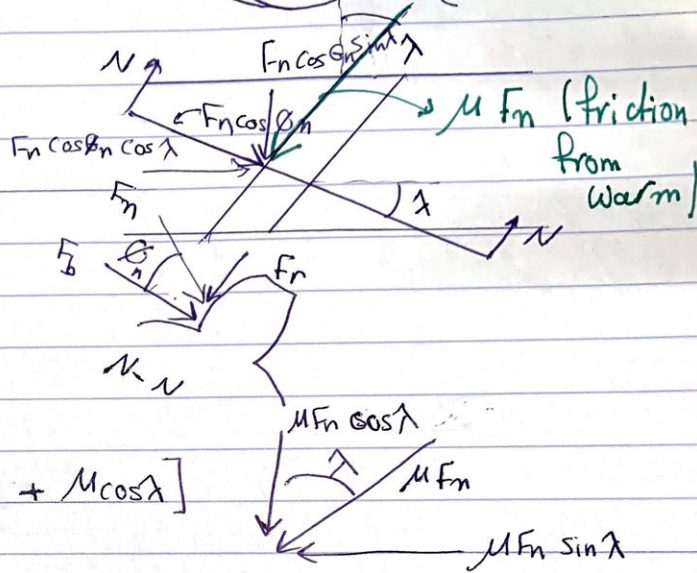
$$F_g^t = F_w^a = F_n [\cos \theta_n \cos \lambda - \mu \sin \lambda]$$

$$F_g^a = F_w^b = F_n [\cos \theta_n \sin \lambda + \mu \cos \lambda]$$

$$F_g^r = F_n \sin \theta_n = F_w^r$$

$$\frac{F_g^t}{F_w^b} = \frac{\cos \theta_n \cos \lambda - \mu \sin \lambda}{\cos \theta_n \sin \lambda + \mu \cos \lambda}$$

$$F_g^r = \frac{F_g^t}{\cos \theta_n \cos \lambda - \mu \sin \lambda} \sin \theta_n = \frac{F_w^t}{\cos \theta_n \sin \lambda + \mu \cos \lambda} \sin \theta_n$$



If the gear is the Driver.

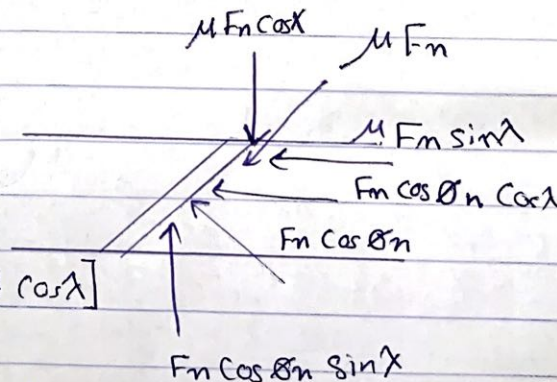
Does the worm move?

$$F_w^b = ?$$

$$F_w^b = F_g^a = F_n [\cos \theta_n \sin \lambda - \mu \cos \lambda]$$

If $F_w^b > 0$: Moves

If $F_w^b < 0$: Self locked $\rightarrow \mu > \cos \theta_n \tan \lambda$



Velocities

$$\tan \lambda = V_g / V_w$$

$$\sin \lambda = V_g / V_s$$

$$\cos \lambda = V_w / V_s$$

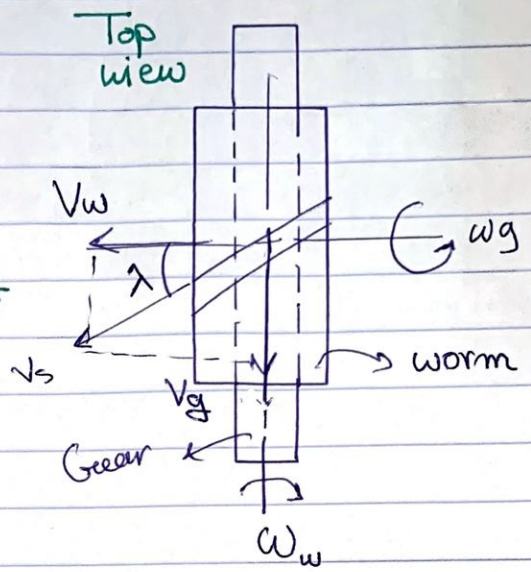
V_s : Friction $\mu \bar{w}$

From Previous \leftarrow

$$e_w (\text{efficiency}) = \frac{F_g b V_g}{F_w b V_w}$$

← output

Input



$$= \left(\frac{\cos \theta_n \cos \lambda}{\cos \theta_n \sin \lambda + \mu \cos \lambda} \right) \left(\frac{V_g}{V_w} \right) \tan \lambda = \frac{\cos \theta_n - \mu \tan \lambda}{\cos \theta_n + \mu \cot \lambda}$$

if the gear is Driver

$$e_g = \frac{\cos \theta_n + \mu \cot \lambda}{\cos \theta_n + \mu \tan \lambda}$$

Figure 13-42: used to find friction coefficient
curve B (friction) < curve A 6

A, B depends on material

Allowable tangential force on worm gear

Contact stress

$$(F_g^t)_{all} = C_s C_m C_v (d_g)^{0.8} b$$

d_g : diameter of gear
 b : face width of gear

at strength = stress ↪ تقوية

AGMA so $F_g^t app < F_g^t all$

C_s : material factor: depends on material (strength)

From Equations: 15-32, 15-33, 15-34
and 15-35

Note D_m = mean diameter of gear = d_g

C_v : velocity factor

From Equation: 15-37

C_m : velocity ratio factor: $m_g = \frac{W_w}{W_g} = \frac{N_g}{N_w}$

Equation 15-38 → f for gear made of bronze (or use curve)

How to find if it is safe?

$$F_g^t = H_g \frac{33000 n K_a}{V_g}$$

n : design factor
 K_a : application factor

$$n = \frac{(F_g^t)_{all}}{(F_g^t)^t K_a}$$

$$H_{gall} = (F_g^t)_{all} V_g$$

$$H_g = (F_g^t V_g)$$

So $H_{gall} = H_{nominal} K_a n$

↑
Given or wanted

Wear stress

Buckingham $(F_g^t)_{all} = \frac{K_w \sqrt{Q} b}{\underbrace{\quad}_{\text{wear factor}}}$ From Table 15-11

Steel 250-500 (3rd row)

Example: Worm steel $H_B = 308$ } Go to row 3
 Gear chilled Bronze }
 1st, 2nd rows: Hardness > 500

This is also compared to (F_g^t) applied

Bending stress

$$\sigma_b = \frac{W_g^t}{P_n b y} \rightarrow F_g^t$$

$$P_n = P \cos \lambda$$

y: Lewis factor

$$D_n = 1.5 \rightarrow y = 0.1$$

$$D_n = 20 \rightarrow y = 0.125$$

$$D_n = 25 \rightarrow y = 0.15$$

$$D_n = 30 \rightarrow y = 0.175$$

$$\sigma_b = S_e \rightarrow (F_{tg})_{all}$$

- We obtain F_g^t all from three ways wear, contact, bending \rightarrow we take the smallest and Power is calculated

Worm gear thermal capacity
Friction causes heat

$T < 200^\circ\text{F} (93^\circ\text{C}) \rightarrow$ satisfactory operation
We need to dissipate heat

Heat transfer coefficient

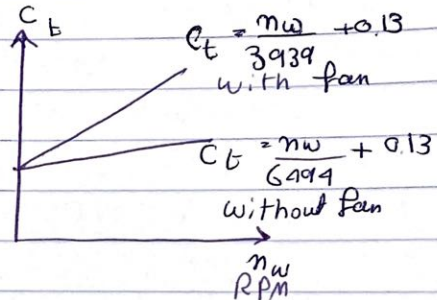
$$H_T = C_t A (t_o - t_a)$$

↙ surface Area
↖ oil temp
↘ ambient temp

$$H_T \begin{cases} \text{ft. lb} \\ \text{min} \\ W \end{cases}$$

$$A \begin{cases} \text{ft}^2 \\ \text{m}^2 \end{cases}$$

To find C_t



t_o between: $T_o = 160 - 200^\circ\text{F}$

Area is Given or estimated

$$A = 0.3 C^{1.7}$$

↙ $F_n \text{ ft}^2$
↘ center distance in in

$$A = 432 C^{1.7}$$

↙ in in²

To calculate friction:

$$H_f = H_w - (H_g) \approx e w H_w$$

$$e = \frac{H_{out}}{H_{in}} = \frac{H_{in} - H_f}{H_{in}}$$

$$H_f = F_f V_s = \mu F_n = \mu F_t \cos \phi \cos \lambda - \mu \sin \lambda$$

$$H_f \approx H_T \quad (\text{Assume})$$

Bevel Gears

Geometry:

Assumed to be two cones intersecting

Pitch cones of bevel gear are analogous to pitch cylinder of spur gear

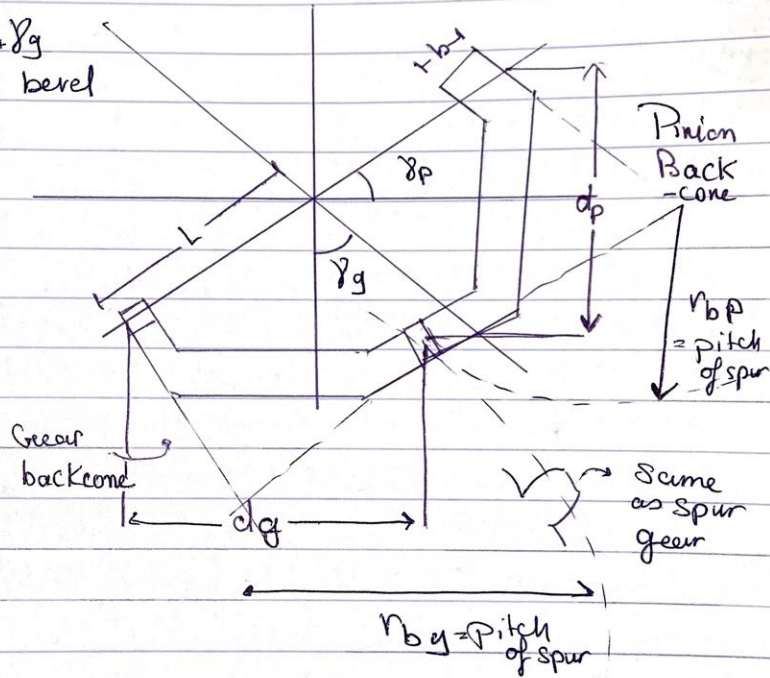


$$d = \frac{N}{P}$$

Σ (shafts) = $90^\circ = \gamma_p + \gamma_g$
in case of straight bevel

$$\begin{aligned} \gamma_p &= \tan^{-1} \left(\frac{d_p}{d_g} \right) \\ &= \tan^{-1} \left(\frac{N_p}{N_g} \right) \end{aligned}$$

$$\begin{aligned} \frac{W_p}{W_g} &= \frac{d_g}{d_p} = \frac{N_g}{N_p} \\ &= \tan \gamma_g = \cot \gamma_p \end{aligned}$$



$$L = \frac{d_g}{2 \sin \gamma_g}$$

Involute in back cone similar to spur gear with radius = r_b

Virtual no of teeth = N'

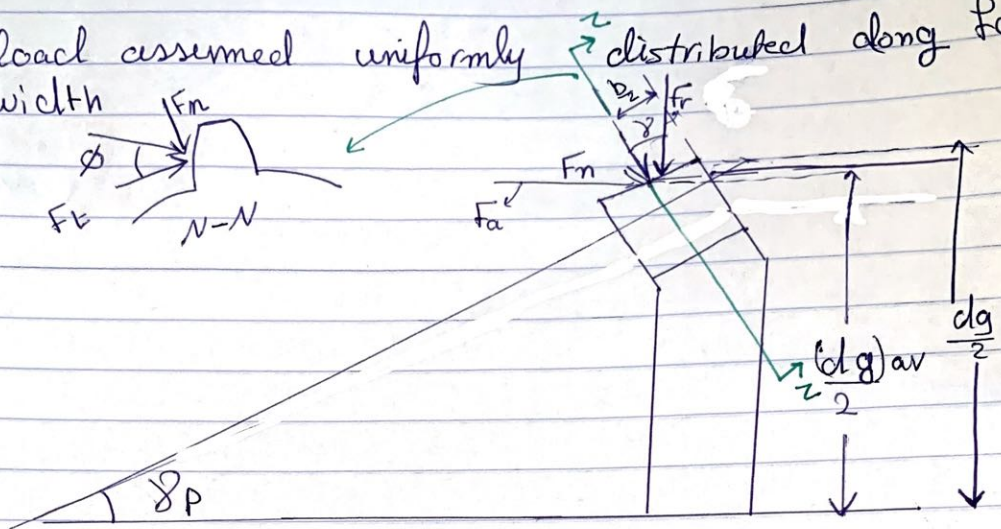
$$P = \frac{2\pi r_b P}{N'}$$

$$P = \frac{N_g'}{2r_b g} = \frac{N_p'}{2r_b p}$$

limitations of b
 $b < \frac{L}{3}$ element
 $b \leq \frac{10}{P}$

Force analysis of Bevel gear

load assumed uniformly distributed along face width



$$d_{av} = d - b \sin \gamma$$

$$V_{av} = \left(\frac{\pi d_{av} n}{12} \right) \text{ for hp} = \left(\frac{\pi d_{av} n}{60} \right) \text{ for W}$$

$$F_t = \begin{cases} = \frac{W \text{ (watts)}}{V_{av}} & \text{lb} \\ \frac{33000 \text{ (hp)}}{V_{av}} & \text{N} \end{cases}$$

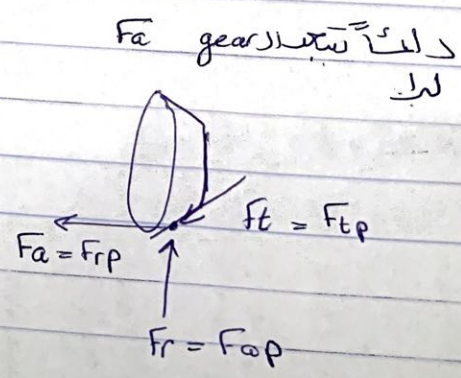
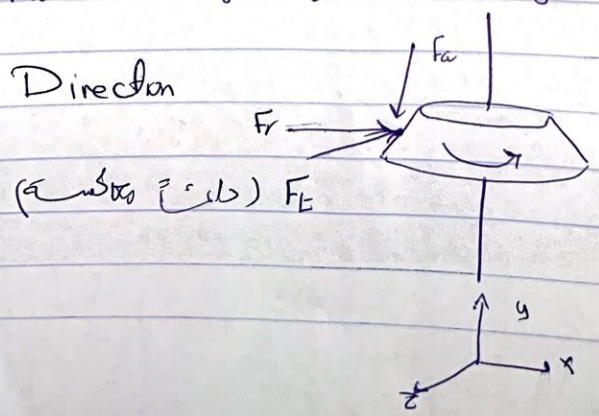
$$F_n = F_t \tan \phi$$

$$F_a = F_n \sin \gamma = F_t \tan \phi \sin \gamma$$

$$F_r = F_n \cos \gamma = F_t \tan \phi \cos \gamma$$

(lec 3 last question)

Direction



Design of bevel gears

Bending

$$\sigma = \frac{F_t P}{b J} K_o K_m K_v K_s$$

K_o : over load factor \rightarrow Table (15-2)

J : Figure [15-7], $\Sigma = 90, \phi = 20$

K_m = mounting factor

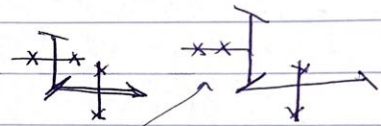
$$K_m = K_{mb} + 0.0036 b^2 \quad b = \text{in}$$

$$K_m = K_{mb} + 5.6 \times 10^{-6} b^2 \quad b = \text{mm}$$

$K_{mb} = 1$ if Both straddle

$K_{mb} = 1.1$ one gear straddle

or the overhang

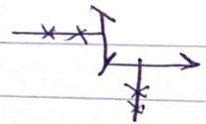


over hung (Both) $K_{mb} = 1.25$
= overboard

$K_v \rightarrow$ Fig [15-5]

K_s = Size factor Equation 15-10

\downarrow same but metric



Strength =

$$S_e = \frac{K_L K_t}{K_r} S_t$$

at $L = 10^7$

$R = 99\%$

S_t : Table 15-6, Fig 15-13

K_L : Fig 15-9 or Eq 15-15

$K_L = \sqrt{L}$ metric

Through hardened

K_r : Table 15-3 or Eq 15-19, 20

$$K_t = \frac{710}{160 + T}$$

$T > 1250^\circ\text{F}$

$$K_t = 1$$

$T < 250^\circ\text{F}$

$$K_t = \frac{893}{273 + T}$$

$T > 120^\circ\text{C}$

Contact

$$\sigma_c = C_p \sqrt{\frac{F_t C_o C_m C_v C_s C_c}{b I d_p}}$$

$$C_o = K_o$$

$$C_m = K_m$$

$$C_v = K_v$$

C_s = size factor From Eq 15-9

C_c = crown factor From Eq 15-12

1.5 crowned 2 uncrowned

I From Fig 15-6 (what I used as pinion here is considered to be d_p)

$$C_p = 2290 \sqrt{\text{Psi}} \leftarrow \text{steel-steel}$$
$$= 190 \sqrt{\text{MPa}}$$

or If not steel-steel $C_p = \sqrt{\frac{1}{\pi \left(1 - \frac{\nu_p^2}{E_p}\right)} + \frac{1 - \nu_g^2}{E_g}}$

Posons

Strength

$$S_{fe} = S_e \frac{C_b C_L C_H}{C_R}$$

C_H used for gear

same spr
as gears
taken
before

C_L = Figure 15-8

S_e Table 15-4, fig 15-12 through hardened

$$C_t = K_t$$

$$C_R = \text{Table [15-3]} \text{ or } \text{eq [15-20], [15-19]}$$
$$= \sqrt{Y_R}$$