• Chapter 5: failures Resulting from static loading

Static load

Def: stationary force or couple moment applied to a member

→ No magnitude change

→ point to point application

• اكثر stress يتعرض له الجسم قبل ان يحصل failure

▸ We consider relation between Strength (max stress) and static load

Static Design criteria

1 Factor of safety (n)

$n = \frac{\text{Strength}}{\text{Stress}}$:- $n > 1$ • this will insure that part won't fail

n can't be negative

2 Reliability approach

$0 < R < 1$: select the material, processing & dimension such that the probability of failure is less than a preselected value

Ex: $R = 0.9 \rightarrow$ 90% chance that the part will function without failure.

• Types of materials:-

1- Ductile material : • $\varepsilon_f > 0.05$ → Design limit

• yield strength for Tension ≃ yield strength for compression

$\boxed{S_{yt} = S_{yc} = S_y}$ (strength)

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2- Brittle material: • $G_f < 0.05$

• No yield strength

Ultimate strength is our design limit

• Ultimate tensile (strength) is less than compression ultimate strength

$\boxed{S_{uc} > S_{uT}}$

• Difference: Brittle material break suddenly with no elongation but Ductile material is subjected to necking and elongation

Ductile material

→ Yield criteria

→ • Max shear stress (MSS) :- Theory 1

• Distortion energy (DE) :- Theory 2

• Ductile coulomb law (DCM) :- Theory 3

Brittle material

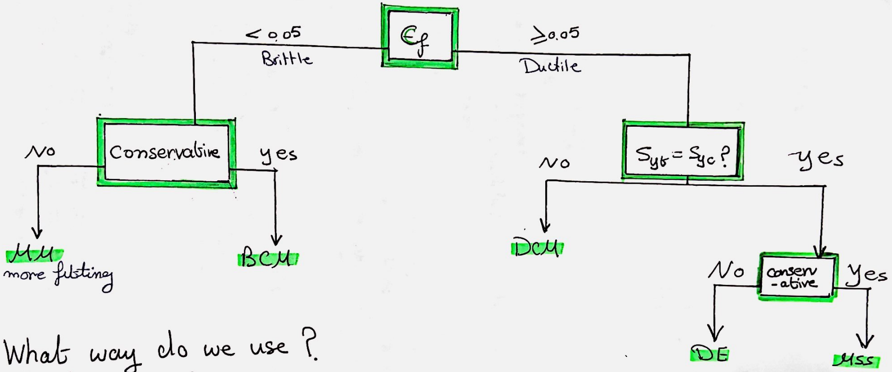
→ Fracture criteria

→ Max Normal stress (MNS)

• Brittle coulomb Mohr (BCM)

• Modified Mohr (MM)

Material



What way do we use?

- Ductile materials

• more accurate → DE (Precise) • if $S_t \neq S_c$ use DCM	• $n_{MSS} < n_{DE}$ • MSS is easy Quick & Conservative

Max Shear stress Theory

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \Rightarrow \sigma_1 - \sigma_3 \geq S_y$ (yield strength

from ↗
Mohr circle

$\boxed{\begin{array}{l} S_y: \text{yield strength (comp or tension)} \\ S_{sy}: \text{yield strength (shear)} \end{array}}$

$$n = \frac{S_y}{\sigma_1 - \sigma_3}$$

$$\tau_{max} = \frac{S_y}{2n}$$

• Cases:-

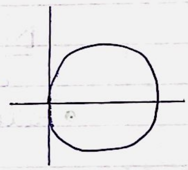
$\left.\begin{array}{ll} \sigma_1 & \text{highest} \\ \sigma_2 & \text{less (middle)} \\ \sigma_3 & \text{lowest} \end{array}\right\}$ Absolute values

Case 1

Define $\sigma_1 = \sigma_A$, $\sigma_2 = \sigma_B$, $\sigma_3 = 0$

$\sigma_A \geq \sigma_B \geq 0$: we take σ_A as the limit

Tension $\sigma_A \geq S_y \Rightarrow n = \frac{S_y}{\sigma_A}$

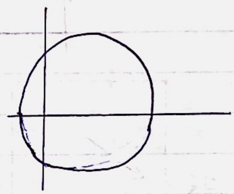


Case 2

Define $\sigma_1 = \sigma_A$, $\sigma_3 = \sigma_B$

$\sigma_A \geq 0 \geq \sigma_B$ ~ negative

$\sigma_A - \sigma_B \geq S_y \Rightarrow n = \frac{S_y}{\sigma_A - \sigma_B}$

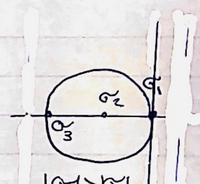


Case 3

$0 \geq \sigma_A \geq \sigma_B$

Define $\sigma_1 = 0$ $\sigma_3 = \sigma_B$

Comp $\sigma_B \leq -S_y \Rightarrow n = \frac{-S_y}{\underbrace{\sigma_B}_{\text{negative}}}$ $n \rightarrow$ positive



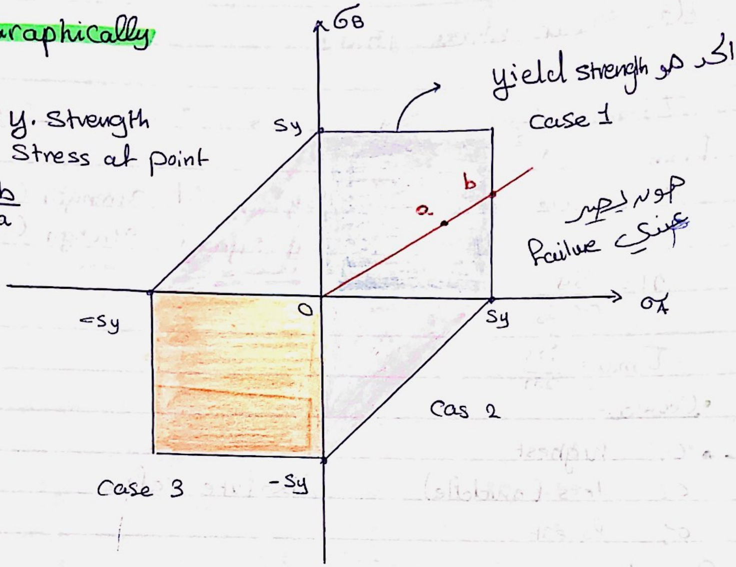
$|\sigma_3| > \sigma_1$

But $\sigma_3 < \sigma_1$

We take absolute values

n Graphically

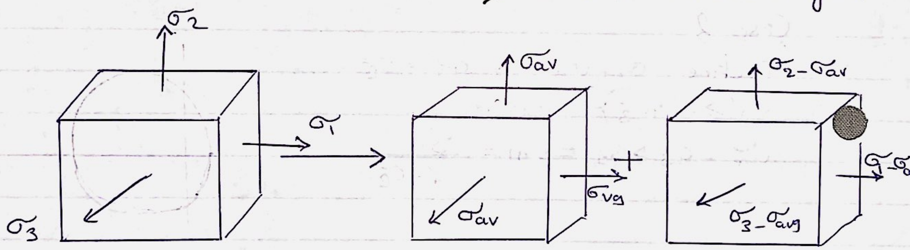
Ob = y. strength
Oa = Stress at point
$$n = \frac{Ob}{Oa}$$



Distoration Energy theory

تشوه

- Failure occurs when → Distoration strain energy / unit volume is $\geq$ D.S.E / unit volume for yield



Hydrostatic
- stress is acting equally at each face

Distortional
- Not equal

$$\sigma_{avg} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$\sigma_1 > \sigma_2 > \sigma_3$

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Distortion energy when $\sigma_1 = \sigma_2 = \sigma_3 = 0$ (which we want)
(No Distoration)

Stress in this Theory is:- (Von Misses Stress)

it's an Ellipse equation

$$\sigma' = \left(\frac{(\sigma_1-\sigma_2)^2}{2} + \frac{(\sigma_2-\sigma_3)^2}{2} + \frac{(\sigma_3-\sigma_1)^2}{2}\right)^{\frac{1}{2}} \quad \text{if 3D}$$

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{\frac{1}{2}} \quad \text{if 2D} \quad \sigma_1 = \sigma_A, \ \sigma_2 = \sigma_B$$

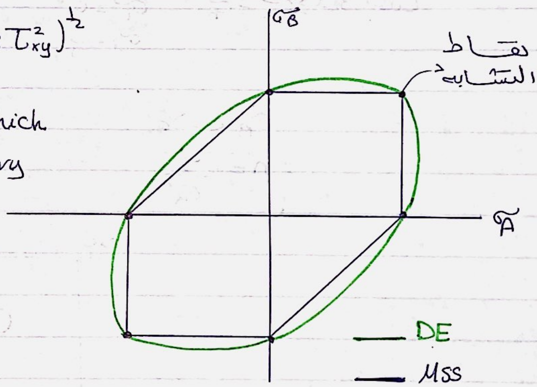
- If I have σ_x, σ_y, τ_{xy} (or + σ_z, τ_{yz}, τ_{zx} if 3D)
Then we can use:-

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right]^{\frac{1}{2}} \quad (3D)$$

$$\sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{\frac{1}{2}}$$

$$\boxed{n = S_y / \sigma'}$$

- There are 6 points at which DE Theory = MSS Theory



$n_{DE} > n_{MSS}$
So more conservative

- Pure shear stress:-

$$\sigma' = S_y = (3(\tau_{xy})^2)^{\frac{1}{2}}$$

$$\Rightarrow \tau_{xy} = \frac{S_y}{\sqrt{3}} = 0.577\, S_y$$

$S_{sy} = 0.577\, S_y$ in Distoration energy theory this is The Shear yield Strength

(In MSS $S_{sy} = 0.5\, S_y$)

Coulumb Mohr Theory :-

مستثنى من MSS

- Difference here is that $S_t \neq S_c$

Cases

Case 1 :- $\sigma_A \geq \sigma_B \geq 0$, $\sigma_1 = \sigma_A$, $\sigma_3 = 0$

$\Rightarrow \sigma_A \geq S_t \Rightarrow \boxed{n = \frac{S_t}{\sigma_A}}$

Case 2 :- $\sigma_A \geq 0 \geq \sigma_B$, $\sigma_1 = \sigma_A$, $\sigma_3 = \sigma_B$

$$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1$$

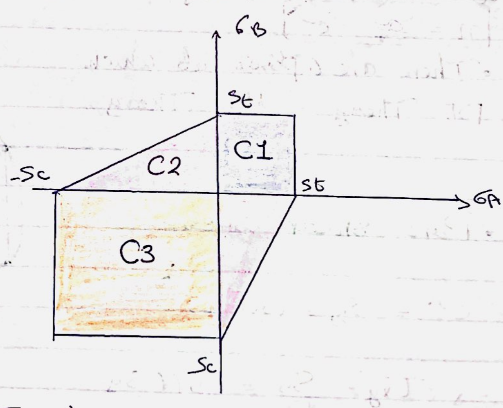
$$\Rightarrow \boxed{\frac{1}{n} = \frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c}}$$

we have to divide because Triangle does not have equal

Case 3: $0 \geq \sigma_A \geq \sigma_B$ $\sigma_1 = 0$, $\sigma_3 = \sigma_B$

$$\sigma_B \leq -S_c$$

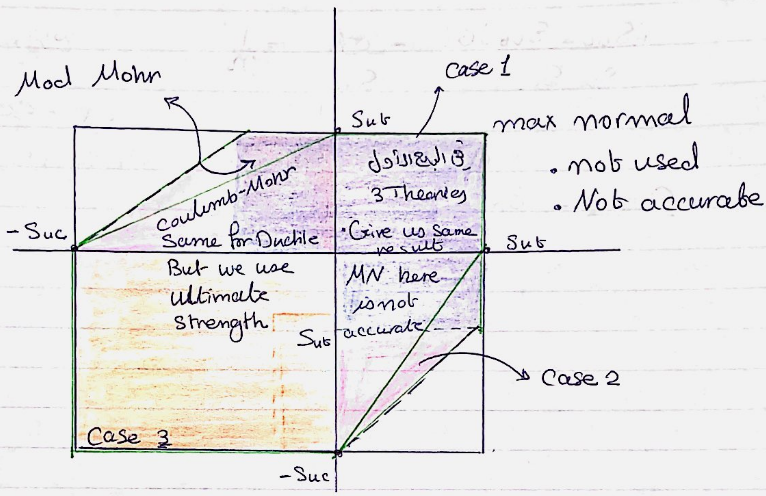
$$\boxed{n = \frac{S_c}{\sigma_B}}$$



To find S_{sy}: (τ_{max})

$$\tau_{max} = S_{sy} = \frac{S_{yt} S_{yc}}{S_{yt} + S_{yc}}$$

Brittle material



- **Coulumb Mohr Theory :-**

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

- **Modefied Mohr**

Case 1:- $\sigma_A = \frac{S_{ut}}{n} \longrightarrow \sigma_A \geq \sigma_B \geq 0$

or $\longrightarrow \sigma_A \geq 0 \geq \sigma_B \;,\; \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$

Case 2:-

$$\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$$

للربع الثاني، الرابع

$\sigma_A \ge 0 \ge \sigma_B$ ، $\left|\frac{\sigma_B}{\sigma_A}\right| > 1$

• لازم انتبه على هذا الشرط

Case 3:- $\sigma_B = -\frac{S_{uc}}{n}$ $\quad 0 \ge \sigma_A \ge \sigma_B$