• Chapter 6: Fatigue Failure Resulting from Variable loading

Fatigue

Def:- when parts are subjected to fluctuating stress the parts are likely to fail at stress levels that are much less than yielding or ultimate strengths if subjected to large numbers of cycles

- In Ductile material fatigue fracture is the same as the fracture of Brittle material

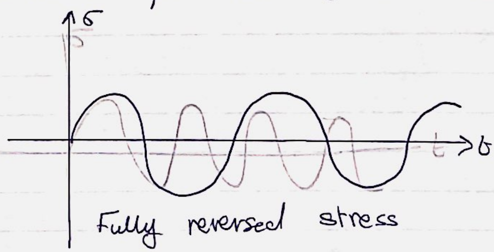
Fatigue Failure Surface

D. Material: Crack In → Crack Prop → Frac
B. Material: ~ → Fracture

- Crack Propagation Zone
→ Smooth surface

- Final fracture zone
→ Rough surface like brittle fracture

Fatigue life Methods:-
- stress life Method
- Testing device:- R.R Moore high speed rotating beam machine
- the machine subjects specimen to pure bending with no transverse shear
- infinite life ??
- The stress is fully Reversed

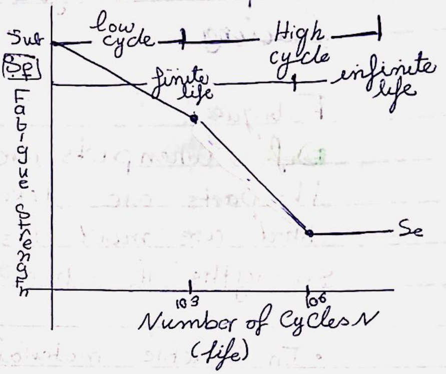


Fully reversed stress

$$S_e' = \begin{cases} 0.5\, S_{ut} & S_{ut} \leq 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

S_e : endurance limit.
highest stress at infinite life without failing

S_{ut} :- ultimate Tensile stress



Fatigue Strength

• This Graph represent how many stress I can apply for a number of cycles

S_f : fatigue Strength
f : fatigue Strength fraction

→ Stress life Method

$$S_e' = \begin{cases} 0.5\, S_{ut}, & S_{ut} \leq 200 \text{ kpsi } (1400) \text{ Mpa} \\ 100 \text{ Kpsi}, & \text{"} > 200 \text{ "} \\ 700 \text{ MPa}, & \text{"} > 1400 \text{ Mpa} \end{cases}$$ *

Fatigue Strength : $S_f = f\, S_{ut}$ at 10^3 Cycle

$$S_f = a N^b$$

$$a = \frac{(f\, S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3} \log \left(\frac{f\, S_{ut}}{S_e} \right)$$

$$S_e = K_a K_b K_c K_d K_e K_f \underline{S_e'}$$ from *

Ka :

$K_a = a (S_{ut})^b$ Table.

Kb :

$$K_b = \begin{cases} \left(\frac{d}{0.3}\right)^{-0.107}, & 0.11 \leq d \leq 2 \text{in} \\ 0.91\, d^{-0.157}, & 2 \leq d \leq 10 \text{in} \\ 1.24\, d^{-0.107}, & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51\, d^{-0.157}, & 51 < d \leq 254 \text{ mm} \end{cases}$$

Axial loading : $K_b = 1$

Kc : → (from tables) = $\begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$

- for Combined torsion and other loading
  $K_c = 1$ (Von Mises stress)

Kd

$Kd = \frac{S_T}{S_{RT}}$ obtain from tables

Ke

$Ke = 1 - 0.08\, Z_a$ obtain from tables

Number of cycles

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b}$$

$$(\sigma_{rev})_{max} = K_f\, (\sigma_{rev})_{nom}$$

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Stress concentration factors

→ K_t, K_{ts} are obtained from graphs

$$K_t = \frac{\sigma_{max}}{\sigma_0} \quad , \quad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

Fatigue stress concentration factors

→ K_f, K_{fs}

Notch Sensivity

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$\sqrt{a}$ ~ Tables

r ~ Notch radius

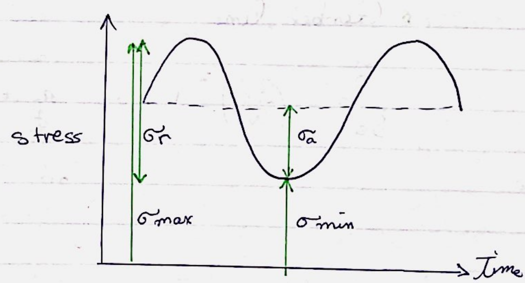
Note: $K_f = 1 + q(K_t - 1)$

$K_{fs} = 1 + q_{shear}(K_{ts} - 1)$

6-11 Characterizing Fluctuating stresses

$$F_m = \frac{F_{max} + F_{min}}{2}$$

$$F_a = \left| \frac{F_{max} - F_{min}}{2} \right|$$



σ_{min} : minimum

σ_{max} : maximum

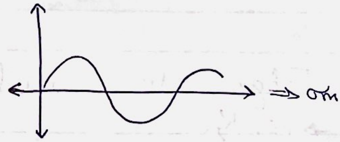
σ_a : amplitude component

$$\sigma_m = \frac{\sigma_{min} + \sigma_{max}}{2}$$

$$n_y = \left|\frac{S_y}{\sigma}\right|$$

$$\sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right|$$

- If the stress is reversed.
  $\sigma_m = 0$



6-12 Fatigue failure criteria for fluctuating stress

- Soderberg line (Ductile)

$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1 \Rightarrow \boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}}$$

amplitude — mean stress — Fatigue Factor of safety

yield strength

- Modified Goodman line :- (Ductile)

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \Rightarrow \boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}}$$

- Gerber line

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \Rightarrow \boxed{n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right]}$$

or $\boxed{\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1}$ $\quad n > 0$

- ASME - Elliptic line (Ductile)

$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$

$$\boxed{\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1}$$

$$\boxed{n_f = \sqrt{\frac{1}{\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2}}}$$

- Langer first Cycle yielding

$$S_a + S_m = S_y \qquad \sigma_a + \sigma_m = \frac{S_y}{n}$$

- Brittle materials has another chriteria which is not included

Combined loadings:-

Remember

$$\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{\frac{1}{2}}$$

$$\sigma_a' = \left\{\left[(K_f)_{bend}(\sigma_a)_{bend} + (K_f)_{axial}\frac{(\sigma_a)_{axial}}{0.85}\right]^2 + 3\left[(K_{fs})_{tors}\tau_{a\,tors}\right]^2\right\}^{\frac{1}{2}}$$

$$\sigma_m' = \left\{\left[(K_f)_{bend}(\sigma_m)_{bend} + (K_f)_{axial}\right]^2 + 3\left[(K_{fs})_{torsion}(\tau_m)_{torsion}\right]^2\right\}^{\frac{1}{2}}$$

General Procedure :-

- ✓ Select material
- Calculate $S_{e'}$
- Calculate Correction factors
- Calculate Corrected S_e
- Draw S-N diagram
- Calculate σ_A (Alternating stress) for desired life
- Set acceptable σ_A with safety factor

$$FoS = \frac{\sigma_{A\,rev} \text{ (at N Desired)}}{\sigma_{A\,Design}}$$