Chapter 14: Energy Methods

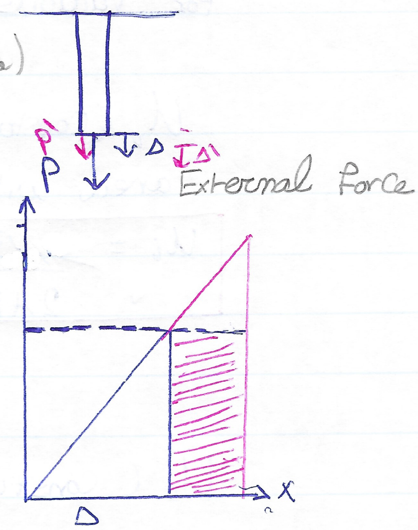
14.1

- Work = Force x displacement (Work of a Force)
  = Moment x twist angle (Work of a moment)

$\boxed{U_e = \frac{1}{2} P \Delta}$

Work / Energy of external forces

and for a moment :- $\boxed{U_e = \frac{1}{2} M \theta}$



- If we have two forces , P, P' *

- Work of P for displacement Δ' :-

$\boxed{U_e' = P \Delta'}$ (rectangular)

and for a moment :

$\boxed{U_e' = M \theta'}$

Note:
P is constant in U_e' because Δ' is caused by P'

- Strain Energy : internal work converted from external work done by the load applied

* Strain energy in terms of Normal stress
Assuming material behaves in a linear elastic manner

$$U_i = \int_V \frac{\sigma^2 dV}{2E}$$ Axial Force or Bending Moment

* U = it's always positive wether σ is Positive or negative
Same for τ

$$U_i = \int_V \frac{\tau^2 dV}{2G}$$ Shear force or Torsion

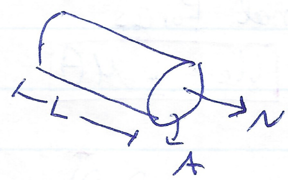
$\boxed{U_e = U_i}$ energy conservation

14.2: Elastic Strain Energy for various loading

U_i (Constant cross-section area with an axial force).

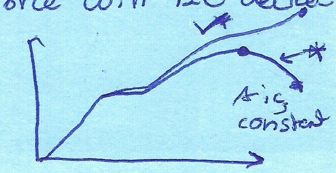
$$\boxed{U_i = \frac{N^2 L}{2AE}}$$

N



Note
- Cross section Area will decrease stress will increas in true fracture stress
- In the conventional* Diagram, we assume A is constant

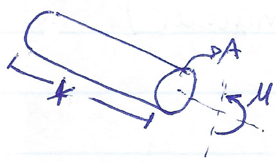
Force will be decrea



U_i (variable cross-section area with a Bending Moment) Using Flexture formula

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

N m

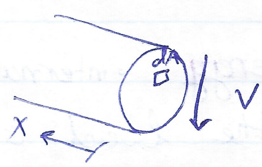


U_i (variable) cross-section area with a transverse shear)

$$U_i = \int_0^L \frac{f_s V^2 dx}{2GA}$$

Given

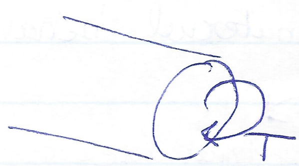
N



U_i (variable cross-section area with a Torsional Moment)

$$= \int_0^L \frac{T^2}{2GJ} dx$$

N.m عزم



How to solve Questions

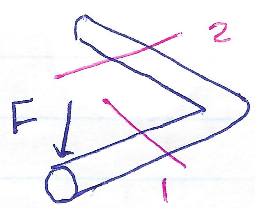
- If a displacement is required, use conservation of energy:-

$$U_e = U_i$$

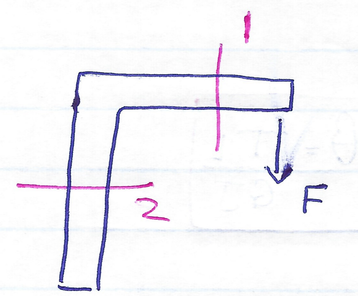
$\frac{1}{2} P \ldots$ = need to be calculated

To calculate U_i
Take Sections in the Body :

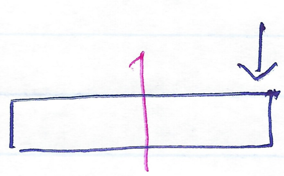
Ex



Here we take two sections
In 1 : V & M
In 2 : T & M & V



Here we take two sections
In 1: V, M
In 2: N, M



Here we take one section
In section: V, M

14.8 :- Castigliano's Theorem

& Elastic System

$$\delta_i = \frac{\partial U}{\partial F_i}$$

internal Energy

displacement

Force

* δ_i is same Direction of F_i

$$\boxed{\delta = \frac{FL}{AE}}$$

twisting

$$\theta_i = \frac{\partial U}{\partial M_i}$$

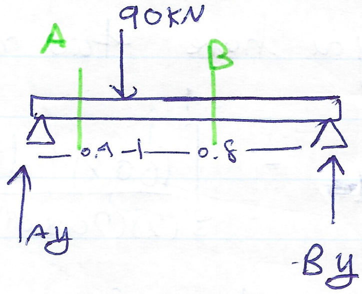
Moment

$$\boxed{\theta = \frac{TL}{GJ}}$$

Example on Elastic strain Energy

1- Find Reactions

$\Sigma M_A = 0$



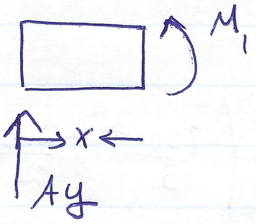
$(B_y)(1.2) - (90)(0.4) = 0$

$\boxed{B_y = 30}$ KN ↑

So $A_y = 60$ KN ↑

We need to find M for two sections (A & B)

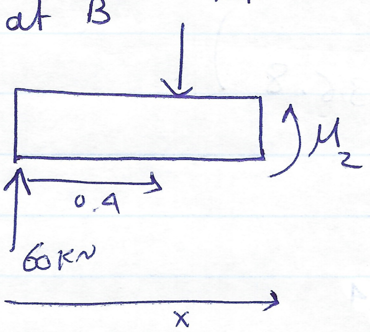
at A :-



$\Sigma M = 0 \qquad M - (A_y)(x) = 0$

$\boxed{M_1 = 60x}$

at B



$\Sigma M = 0 \qquad M_2 - (60)(x) + (90)(x - 0.4) = 0$

$M_2 = 60x - 90(x - 0.4) = 60x - 90x + 36$

$\boxed{M_2 = -30x + 36}$

3- you have two moments so $U_{Tot} = U_1 + U_2$

$$U_1 = \int_0^{0.4} \frac{(60x)^2\,dx\ (10^6)}{(2)(200\times10^9)(3.33\times10^{-6})}$$

Note: Make 60 kN = 60×10^3 N

$$= \int_0^{0.4} \frac{3600x^2\,dx\,(10)^6}{1332000} = \left.\frac{3600\,x^3 (10)^6}{3\times1332000}\right]_0^{0.4} = 5.76\times10^{-5}\times10^6$$

$$= 57.6\ J$$

$$U_2 = \int_{0.4}^{1.2} \frac{(-30x+36)^2 (10)^6\,dx}{1332000}$$

$$= \int_{0.4}^{1.2} \frac{(900x^2 - 2160x + 1296)10^6}{1332000} = \frac{1}{1332000}\left(\frac{900x^3}{3} - \frac{2160x^2}{2} + 1296x\right)_{0.4}^{1.2}(10^6)$$

$$= \frac{(10)^6}{1.332\times10^6}\left(300x^3 - 1080x^2 + 1296x\right)_{0.4}^{1.2}$$

$$= \frac{(10^6)}{1.332\times10^6}\left(300(1.2^3 - 0.4^3) - 1080(1.2^2 - 0.4^2) + 1296(1.2 - 0.4)\right)$$

$$= \frac{(10^6)}{1.332\times10^6}\left(499.2 - 1382.4 + 1036.8\right)$$

$$= 115.3\times10^{-6}\times10^6 = 115.3$$

so $U_{tot} = U_1 + U_2 = 172.9\ J$